6

WORK, ENERGY, POWER AND CONSERVATION LAWS

WORK DONE BY A CONSTANT FORCE

Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

Work depends upon two factors :

(i) Force must be applied.

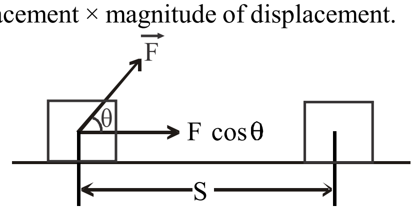
(ii) Distance travelled by the body in the direction of the force.

Work done by the force is measured by the product of magnitude of force and the displacement of the point of application in the direction of force.

i.e. W = FS.

Let a force $\vec{F}$ moves through a displacement $\vec{S}$ and say the directions of these two vectors are not the same. If the angle between the displacement vector and the force vector is θ. The component of the force in the direction of the displacement is F cos θ.

Work done = component of force in the direction of the displacement × magnitude of displacement.



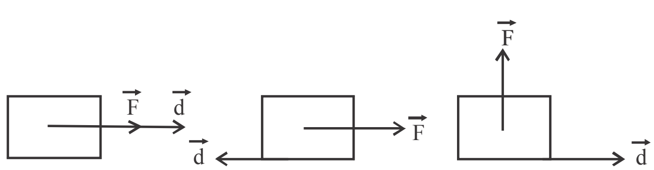
$W = (F\cos\theta)\,S = F\,S\cos\theta$ or $W = \vec{F}.\vec{S}$

Although work is the dot product of two vector quantities force and displacement, it is itself a scalar quantity.

(a) By the definition of work the aspect which must be emphasized is that only the component of the force parallel to the displacement contributes to the work performed.

(b) Any arbitrary force can always be resolved into two components, one parallel and one perpendicular to the displacement of the particle, and only force in the direction of the displacement contributes to the work.

Work done can be positive, negative or zero.

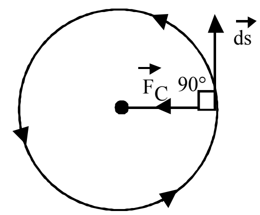


positive work $(0 \le \theta < \pi/2)$ | negative work $(\pi/2 < \theta \le \pi)$ | zero work $(\theta = \pi/2)$

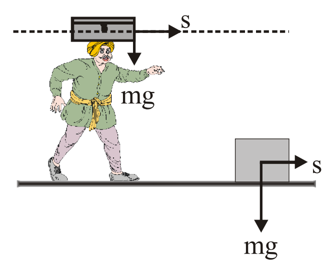
In terms of rectangular components work done

$$W = \vec{F}\cdot\vec{d} = (F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k}) . (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$
$$= F_x dx + F_y dy + F_z dz$$

* When θ = 90°, then
$dW = F\,ds\cos 90° = 0$
Even though the centripetal force mv^2/r acts during a displacement, but no work is performed. The reason for this is that the force is perpendicular to the displacement.



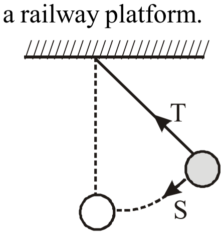
* If a body sliding over a smooth horizontal surface. The work done by the force of gravity and the reaction of the surface will be zero.



This is because both the force of gravity and the reaction act normally to the displacement.

The same argument can be applied to a man carrying a load on his head and walking on a railway platform.

* The tension in the string of a simple pendulum is always perpendicular to displacement. So, work done by the tension is zero.



Units of work

SI Unit : joule (J).

joule : One joule of work is said to be done when a force of one newton displaces a body by one meter in the direction of force.

1 joule = 1 newton × 1 meter = 1 kg $m^2\,s^{-2}$

erg : One erg of work is said to be done when a force of one dyne displaces a body by one cm. in the direction of force.

1 erg = 1 dyne × 1 cm. = 1 gm. $cm^2\,s^{-2}$

Other Units :

(a) 1 joule = 10^7 erg

(b) 1 erg = 10^{-7} joule

(c) 1 eV = 1.6×10^{-19} joule

(d) 1 joule = 6.25×10^{18} eV

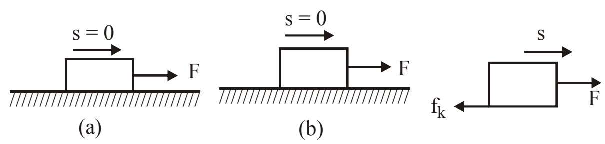
(e) 1 MeV = 1.6×10^{-13} J

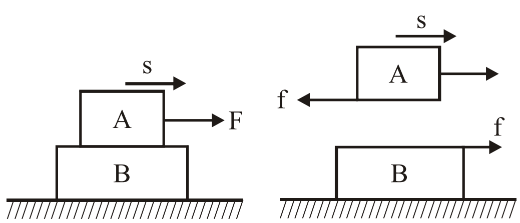
(f) 1 J = 6.25×10^{12} MeV

(g) 1 kilo watt hour (KWH) = 3.6×10^6 joule

Work Done by friction :

There is a misconception that the force of friction always does negative work. In reality, the work done by fricition may be zero, postive or negative depending upon the situation as shown in the figure.





(c)

(a) When a block is pulled by a force F and the block does not move, the work done by friction is zero.

(b) When a block is pulled by a force F on a stationary surface, the work done by the kinetic friction is negative.

(c) Block A is placed on the block B. When the block A is pulled with force F, the friction force does negative work on block A and positive work on block B, which is being accelerated by a force f, the displacement of A relative to the table is in the forward direction. The work done by kinetic friction on block B is positive.

Minimum work done in pulling an object

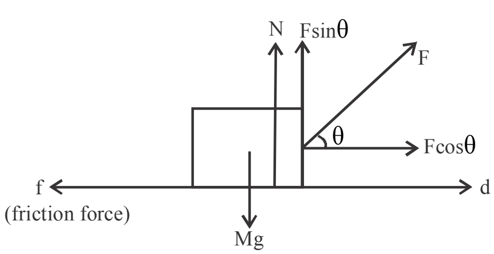
From the figure, $F \sin\theta + N = Mg$

$\therefore$ $N = Mg - F \sin\theta$

$F \cos\theta = f = \mu N = \mu[Mg - F \sin\theta]$

$F(\cos\theta + \mu \sin\theta) = \mu Mg$

$\therefore$ $F = \dfrac{\mu Mg}{\cos\theta + \mu \sin\theta}$ = Minimum force required to pull



Work done in pulling an object

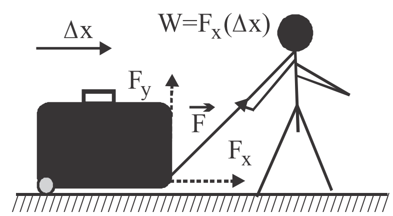
$$W = F\,d = \frac{\mu Mg\,d}{\cos\theta + \mu \sin\theta}$$

Similarly work done in pushing an object

$$W = \frac{\mu Mg\,d}{\cos\theta - \mu \sin\theta}$$

Example 1 :

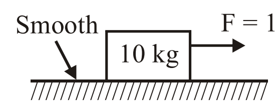
The airline passenger in the figure exerts a 80 N force on his suitcase pulling at an angle 60° with the horizontal. What work he does on suitcase while pulling it 50 m on the floor ?



Sol. W = (80 N) (50 m) cos 60° = 2000J = 2 KJ

Example 2 :

Force shown acts for 2 seconds. Find out work done by force F on 10 kg in seconds.



Sol. $W = \vec{F}\cdot\Delta\vec{S} \Rightarrow W = \vec{F}\cdot\Delta\vec{S}\cos 0° = 10\,\Delta\vec{S}$

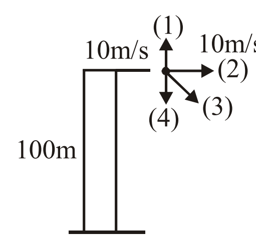
Now, $10 = 10a$

$\therefore\ a = 1\ m/s^2 \Rightarrow S = \frac{1}{2}at^2 = \frac{1}{2}\times 1\times 2^2 = 2m$

$W = 10 \times 2 = 20$ J

Example 3:

The mass of the particle is 2 kg. It is projected as shown in four different ways with same speed 10 m/s. Find out the work done by gravity by the time the stone falls on ground.



Sol. $W = |\vec{F}||\vec{S}|\cos\theta = 2g \times 100 = 2000$ J in each case.

Example 4 :

A body is displaced $10\,\hat{j}$ m under the force $-2\hat{i} + 15\hat{j} + 6\hat{k}$ N. Calculate the work done.

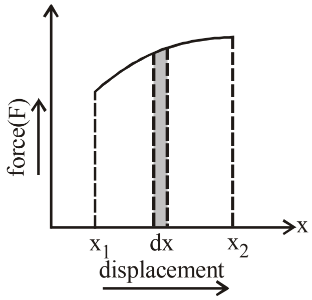
Sol. $W = \vec{F}.\vec{d} = (-2\hat{i} + 15\hat{j} + 6\hat{k}).\,10\hat{j}$

$= 0 + 15 \times 10 + 0 = 150$ joule

WORK DONE BY A VARIABLE FORCE

Work done by a variable force is $W = \int_{x_1}^{x_2} F dx$

=area under F-x curve with proper algebraic sign



Example 5 :

A force $\vec{F} = x\hat{i} + y^2\hat{j}$ N acts on a particle and the particle moves from (1, 2) m to (–3, 4) m. Find work done by the force $\vec{F}$.

Sol. $dW = \vec{F} \cdot d\vec{S}$, where $d\vec{S} = dx\,\hat{i} + dy\,\hat{j}$

$\therefore \quad dW = x\,dx + y^2\,dy$ and

$$W = \int dW = \int_1^{-3} x\,dx + \int_2^4 y^2\,dy = \left.\frac{x^2}{2}\right|_1^{-3} + \left.\frac{y^3}{3}\right|_2^4 = \frac{68}{3} \text{ J}$$

Example 6 :

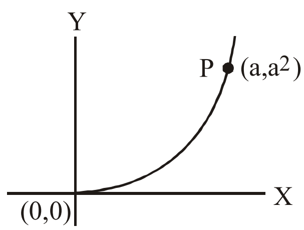
$\vec{F} = xy^2\hat{i} + yx^2\hat{j}$ a particle is moving along parabolic path given by $y = x^2$. Find the work done by this force in moving the particle from (0, 0) to (a, a^2).

Sol. Given : $\vec{F} = xy^2\hat{i} + yx^2\hat{j}$ But $d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$

$$\therefore \quad W = \int_{Path} \vec{F}.d\vec{r}$$

$$= \int_{Path} xy^2dx + yx^2dy$$

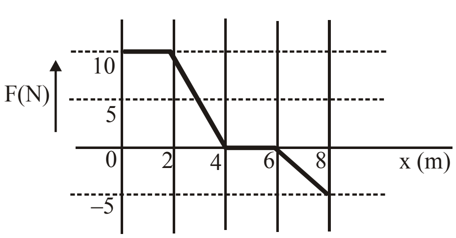
$$= \int_{x=0}^{a} xy^2dx + \int_{y=0}^{a^2} yx^2\,dy$$



Put $y = x^2$ in 1st part of integration & $x^2 = y$ in 2nd part of integration $\int_0^a x^5dx + \int_0^{a^2} y^2dy = \frac{a^6}{6} + \frac{a^6}{3} = \frac{a^6}{2}$

Example 7 :

A 5 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in the figure. Find the work done by this force as the block moves from the origin to x = 8m.



Sol. The work from x = 0 to x = 8 m is the area under the curve.

$$W = 10 \times 2 + \frac{1}{2}(10)(4-2) + 0 + \frac{1}{2}(-5)(8-6) = 25 \text{ J}$$

TRY IT YOURSELF - 1

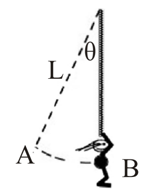
Q.1 A particle is displaced from point A (1, 2) to B (3, 4) by applying force $\vec{F} = 2\hat{i} + 3\hat{j}$. Find the work done by $\vec{F}$ to move the particle from point A to B.

Q.2 A block of mass 5 kg is being raised vertically upwards by the help of a string attached to it. It rises with an acceleration of 2 m/s^2. Find the work done by the tension in the string if the block rises by 2.5 m. Also find the work done by the gravity and the net work done.

Q.3 When a person walks, the force of friction between the floor and the person's feet accelerates the person forward. The floor does –

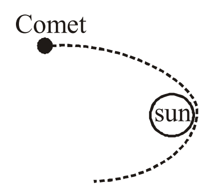
(A) Positive work on the person.
(B) Negative work on the person.
(C) No work on the person.

Q.4 A person swings down on an inextensible rope that is attached to a fixed point. The rope exerts a tension T on the person. The work done by tension on the person as she moves from A to B is –



(A) T (B) TL
(C) TLθ (D) zero.

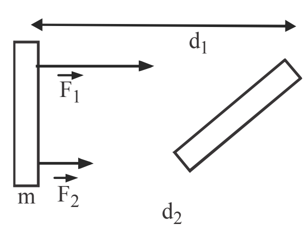
Q.5 A comet is speeding along a hyperbolic orbit toward the Sun. While the comet is moving away from the Sun, the work done by the Sun on the comet is:



(A) positive (B) zero (C) negative

Q.6 If F (x) = $Ax^{3/2}$ find the work done by it in moving a body from x = 0 to x = A.

Q.7 A rectangular object of mass m is pulled by two forces, $\vec{F}_1$ & $\vec{F}_2$ as shown in the figure. The force $\vec{F}_1$ acts through a distance d_1 and the force $\vec{F}_2$ acts through a distance d_2 as shown in the Figure. What is the total work done on the mass?



Q.8 A force F = (6**i** – 2**j**) N acts on a particle that undergoes a displacement d = (3**i** + **j**) m. Find (a) the work done by the force on the particle and (b) the angle between **F** and **d**.

Q.9 A particle is moving along a straight line from point A to point B. The position vectors for points A and B are $(2\hat{i}+7\hat{j}-3\hat{k})$ m and $(5\hat{i}-3\hat{j}-6\hat{k})$ m respectively. One of the force acting on the particle is $\vec{F}=20\hat{i}-30\hat{j}+15\hat{k}$ N. Find the work done by this force.

Q.10 An object is displaced from position vector $\vec{r}_1=(2\hat{i}+3\hat{j})$ m to $\vec{r}_2=(4\hat{i}+6\hat{j})$ m under the action of a force $\vec{F}=(3x^2\hat{i}+2y\hat{j})$ N. Find the work done by this force.

ANSWERS

(1) 10 units.
(2) Work done by the tension = 147.5 J
Work done by gravity = –122.5 J, Net work done = 25 J
(3) (C) **(4)** (D) **(5)** (C)
(6) $(2/5)A^{7/2}$ **(7)** $F_1d_1+F_2d_2$
(8) (a) 16.0 J, (b) $\cos^{-1}(0.8012)=36.8°$ **(9)** 315 J
(10) 83 J

CONSERVATIVE FORCE

A force is said to be conservative if the work done by or against the conservative force :

(a) Is independent of path and depends only on initial and final positions

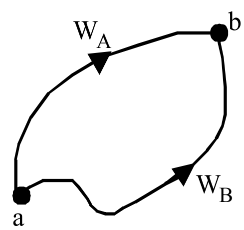
(b) Does not depend on the nature of path followed between the initial and final positions.

Examples of Conservative Force : All central forces are conservative like gravitational, electrostatic, elastic force, restoring force due to spring etc.

In presence of conservative forces mechanical energy remains constant.

To summarise with :

1. When force or more than one forces are acting on a particle in such a way that if particle come back to its initial position and kinetic energy of particle is equal to its previous value which was initially, than forces are known as conservative forces.



2. A force is conservative if the work done by it on a particle that moves between two points a and b along two different paths A and B. Such that $W_A = W_B$.

3. If work done by a force acting on a particle over a close path is zero then force is conservative $W_{ab} + W_{ba} = 0$ or $W_{ab} = -W_{ba}$. These three definition are statements of the same thing in different way.

Special Point :

(a) Work done along a closed path or in a cyclic process is zero. i.e. $\oint F.dx = 0$

(b) If $\vec{F}$ conservative force then $\vec{\nabla}\times\vec{F}=0$.

NON CONSERVATIVE FORCE

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

Work done in a closed path is not zero in a non-conservative force field.

The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

Examples of non-conservative force : Air resistance, viscous force etc. are non-conservative forces.

S.N.	Conservative Forces	Non-conservative Forces
1.	Work done does not depend upon path.	Work done depends upon path.
2.	Work done in a round trip is zero.	Work done in a round trip is not zero.
3.	Central forces, spring forces etc.	Forces are velocity - dependent & retarding in nature.
4.	When only a conservative force acts within a system, the kinetic energy and potential energy can change. However, their sum, the mechanical energy of the system, does not change.	Work done against a non-conservative force may be dissipated as heat energy.
5.	Work done is completely recoverable.	Work done is not completely recoverable.

Example 8 :

A particle moves in x-y plane from (0, 0) to (a, a) and is acted upon by a force $\vec{F}=k(y^2\hat{i}+x^2\hat{j})$ N, where k is a constant and x and y are coordinates in meter. Find work done by this force, if the particle moves along

(i) Two straight lines first from (0, 0) to (a, 0) and then from (a, 0) to (a,a).

(ii) A single straight line .

Sol. $dW=\vec{F}\cdot d\vec{s}=k(y^2\hat{i}+x^2\hat{j})\cdot(dx\,\hat{i}+dy\,\hat{j})$

$=ky^2\,dx+k\,x^2\,dy.$

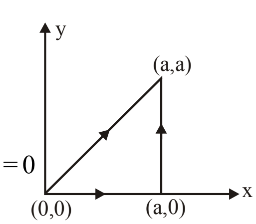
(i) When it moves from (0, 0) to (a, 0)

$y = \text{constant} = 0$

$\Rightarrow dy = a$

$\Rightarrow dW_A = k(0)\,dx + kx^2(0) = 0$

$\Rightarrow W_A = \int dW_A = 0$



When it moves from (a, 0) to (a, a)
x = constant = a $\Rightarrow$ dx = 0 and y changes from 0 to a

$\therefore\ dW_B = ky^2\,(0) + ka^2\,dy = ka^2\,dy$

$$\therefore\ W_B = ka^2\int_0^a dy = ka^3$$

$\therefore\ W = W_A + W_B = ka^3$

(ii) When moves from (0, 0) to (a, a) as shown in above figure, along path C which is a straight line for which $y = x \Rightarrow dy = dx$

$\therefore\ dW = kx^2\,dx + k\,x^2\,dx = 2k\,x^2\,dx$

$$\therefore\ W = \int dW = 2k\int_0^a x^2 dx = \frac{2ka^3}{3}$$

In above example, the work done by the force is different for different paths for different paths taken, so it provides an example of non-conservative force.

WORK ENERGY THEOREM

Consider a constant force $\vec{F}$ acting on a particle of mass m. Let a acceleration 'a' be produced in the direction of force $\vec{F}$, say along X-axis.

Let the resultant force vary in magnitude only, not in direction. The work done by the resultant force in displacing the particle from x_0 to x is

$$W = \int_i^f \vec{F}\cdot d\vec{x} = \int_i^f F\cdot dx$$

Since F = ma, and $a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx}$

Hence, $W = \int_i^f ma\ dx = \int_i^f mv\frac{dv}{dx}dx$

$$W = m\int_i^f v\ .\ dv = m\left(\frac{v^2}{2}\right)_i^f \text{ or } W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

"The work done by the resultant force acting on a particle is equal to the change in the kinetic energy of the particle".

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \text{ [Work Energy Theorem]}$$

Let a particle of mass m is moving with velocity v. This particle is stopped by some resistive force i.e. stopping force and due to action of this force particle comes in rest.

Then $W = 0 - \frac{1}{2}mv^2 = -\frac{1}{2}mv^2$

Negative sign shows that particle is doing work against the stopping force. It means capacity of doing work of a moving particle is $\frac{1}{2}mv^2$.

So that it is known as kinetic energy of particle.

$$K = \frac{1}{2}mv^2$$

The energy possessed by a body by virtue of its motion is known as kinetic energy.

The work done on a particle by the resultant force is equal to the change in Kinetic energy of the particle.

So, the work done by the resultant force

$W = K_f - K_i = \Delta K$

Salient features of the Work energy theorem

* When the speed of the particle is constant, there is no change in Kinetic energy and the work done by the resultant force is zero. For example, in case of uniform circular motion, the speed of the particle is constant and so the centripetal force does no work on the particle.
* It can also be concluded that if no external work is done, the Kinetic energy before a process must be equal to its Kinetic energy at the conclusion of the process.
* Kinetic energy of a particle decreases by an amount just equal to the amount of work which the particle performs. A body is said to have energy associated with it because of its motion; as it does work it slows down and loses some of this energy.
* Work and energy are interchangeable quantities. When work is done, it appears as energy. The energy can be decreased by permitting the particle to do work on other particles.
* Kinetic energy of a body frame dependent as velocity is a frame dependent quantity. Therefore, pseudo force work has to be considered.

Example 9 :

A 60gm tennis ball thrown vertically up at 24 m/s rises to a maximum height of 26m. What was the work done by resistive forces?

Sol. By Work-energy theorem, $W_{net} = \Delta$ K.E.

$$W_g + W_{res} = 0 - \frac{1}{2}mu^2$$

$$-mgh + W_{res} = -\frac{1}{2}mu^2$$

$$W_{res} = 0.06\times 10\times 26 - \frac{1}{2}\times 0.06\times (24)^2 = -1.68J$$

Example 10 :

A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2m?

Sol. $m = 0.5$ kg, $v = ax^{3/2}$, $a = 5\ m^{-1/2}\ s^{-1}$, W = ?

Initial velocity at x = 0, $v_0 = a\times 0 = 0$;

Final velocity at x = 2, $v_2 = a\times 2^{3/2} = 5\times 2^{3/2}$

Work done = increase in kinetic energy

$$= \frac{1}{2}m\,(v_2^2 - v_0^2) = \frac{1}{2}\times 0.5\,[(5\times 2^{3/2})^2 - 0] = 50\ J$$

Example 11 :

A force of $(3\hat{i}-1.5\hat{j})$ N acts on a 5 kg body. The body is at a position of $(2\hat{i}-3\hat{j})$ m and is travelling at 4 m/s. The force acts on the body until it is at the position $(\hat{i}+5\hat{j})$ m. Assuming no other force does work on the body, find the final speed of the body.

Sol. Given; Mass of the body = 5 kg, Force $\vec{F}=3\hat{i}-1.5\hat{j}$

Displacement $\Delta\vec{s}=\{(\hat{i}+5\hat{j})-(2\hat{i}-3\hat{j})\}$ m $=(-\hat{i}+8\hat{j})$ m

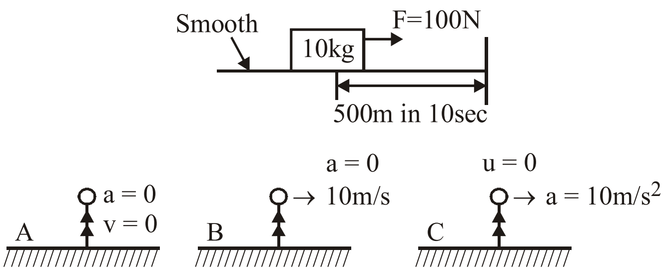
From Work Energy theorem

$$W=\vec{F}\cdot\Delta\vec{s}=\frac{1}{2}m(v^2-u^2)$$

$$-3-12=\frac{1}{2}\times 5[v^2-(4)^2] \Rightarrow v=\sqrt{10}\ \text{m/s}$$

Example 12 :

A block of mass 10 kg is pulled by force F = 100 N. It covers a distance 500m in 10 sec. From initial point this motion is observed by three observers A, B and C as shown in figure.



Find out work done by the force F in 10 seconds as observed by A, B and C.

Sol. $(W_F)_{\text{on block w.r.t. A}}=100\times 500\ \text{J}=50{,}000\ \text{J}$

$(W_F)_{\text{on block w.r.t. B}}=100\,[500-10\times 10]=40{,}000\ \text{J}$

$(W_F)_{\text{on block w.r.t. C}}=100\,[500-500]=0$

TRY IT YOURSELF - 2

Q.1 In some demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 m s^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Q.2 A body of mass 5 kg initially at rest is subjected to a force of 20 N. What is the kinetic energy acquired by the body at the end of 10 s ?

Q.3 Compared to the amount of energy required to accelerate a car from rest to 10 miles per hour, the amount of energy required to accelerate the same car from 10 mph to 20 mph is

(A) the same (B) twice as much

(C) three times as much (D) four times as much

Q.4 The same horizontal force, of magnitude F, is applied to two different blocks, of mass m and 3m respectively. The blocks move on a frictionless surface and both blocks begin from rest. If the force is applied for the same time to each block, which one of the following sentences is true?

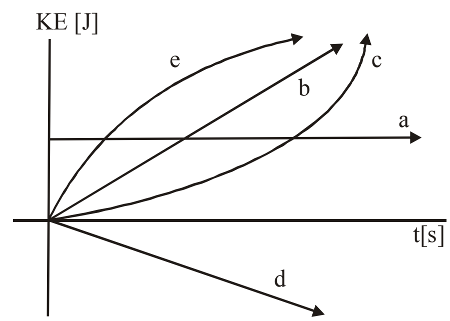
(A) The heavier block acquires 9 times as much kinetic energy as the lighter block.

(B) The heavier block acquires 3 times as much kinetic energy as the lighter block.

(C) The two blocks acquire the same kinetic energy.

(D) The lighter block acquires 3 times as much kinetic energy as the heavier block.

Q.5 An object is dropped to the earth from a height of 10m. Which of the following graphs of kinetic energy vs. time best represent the kinetic energy of the object as it approaches the earth (neglect friction).



(A) a (B) b

(C) c (D) d

Q.6 A car moving at 70 km/h collides rams into an immobile steel wall. Its front is compressed by 0.94m. What average force must a seat belt exert in order to restrain a 75 kg passenger?

Q.7 A net force F stops a car in distance d. In terms of F, how much force must be applied to stop the car in the same distance if its velocity is tripled?

Q.8 A car slows from 40m/s to 20m/s, then from 20 m/s to 0m/s. In which instance (if any) was more energy pulled out of the system? Reversing the question, going from zero to 20 m/s requires more, the same, or less energy than is required to go from 20 m/s to 40 m/s? Explain.

Q.9 A force acts on a body of mass 3 kg causing its speed to increase from 4 metre per second to 5 m/s. How much work has the force done ?

Q.10 A bullet leaving the muzzle of a rifle barrel with a velocity v penetrates a plank and loses one fifth of its velocity. It then strikes second plank, which it just penetrates through. Find the ratio of the thickness of the planks supposing average resistance to the penetration is same in both the cases.

ANSWERS

(1) 63.2 ms^{-1}	**(2)** 4000 joule	**(3)** (C)
(4) (D)	**(5)** (C)	**(6)** –15 kN
(7) nine	**(8)** less, more	**(9)** 13.5 J
(10) 9 : 16		

POTENTIAL ENERGY

The energy stored in a body or system by virtue of its configuration or its position in a field is called potential energy. The potential energy is equal to negative of work done in shifting an object from some reference position to a given position for conservative forces.

Therefore, $\Delta U = -\int_i^f \vec{F} \cdot \vec{dr}$ or $U_f - U_i = -\int_i^f \vec{F} \cdot \vec{dr}$

1. Regarding potential energy U it is worth noting that :

(a) Potential energy can be defined only for conservative forces. It does not exist for non-conservative forces.

(b) Potential energy can be positive, negative or zero.

(c) Potential energy depends on frame of reference.

(d) A moving body may or may not have potential energy.

(e) Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.

2. Potential energy depends on position of reference level.

At reference level F = 0 and potential energy U = 0. e.g.

(a) For gravitational force $F = -\frac{GMm}{r^2}$

(b) For electrostatic force $F = \frac{kq_1q_2}{r^2}$

$F = 0$ at $r = \infty$ so we select reference level at ∞ and hence potential energy at point P is

$$U = -\int_{\infty}^{P} \vec{F} \cdot d\vec{r} = -W$$

(c) For springs $F = -kx$

F = 0 at x = 0 so reference level is natural length of spring. $U = -\int_0^x \vec{F} \cdot d\vec{x} = -\int_0^x -k\vec{x} \cdot d\vec{x}$ or $U = \frac{1}{2}kx^2$

(d) for intermolecular forces reference level is at infinity.

3. Potential energy depends on nature of force

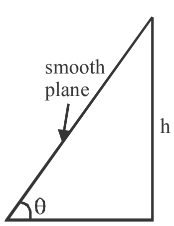
(a) for attractive forces U is negative

(b) for repulsive forces U is positive

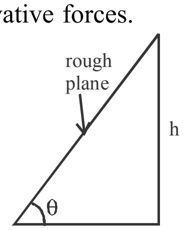
4. When work is done by the force i.e. body moves in direction of force potential energy will decrease.

5. When work is done against the force i.e. body is displaced opposite to direction of force potential energy will increase.

6. For potential energy determination we do not include the work done against non-conservative forces.



potential energy = work done = mgh



work done = mgh + work done against friction but potential energy = mgh

7. The relation between force and potential energy is

$$\vec{F} = -\text{gradient } U = -\vec{\nabla} U$$

$$\vec{F} = -\left(\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right)$$

8. Gravitational potential energy

(a) Gravitational potential energy between two masses

$$U = -\frac{Gm_1m_2}{r}$$

(b) Potential energy of mass m at surface of earth

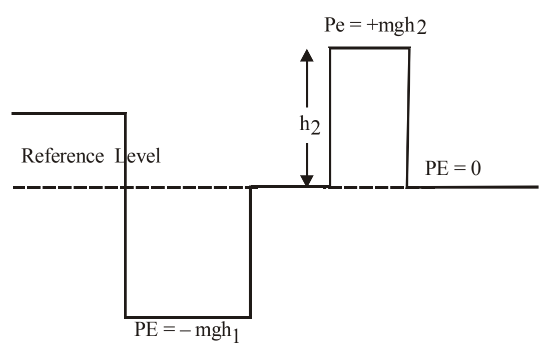
$$U = -\frac{GmM}{R} \quad (r = R)$$

(c) Potential energy at height h from surface of earth

$$U = -\frac{GmM}{R + h}$$

Furthermore if V is the gravitational potential at a point, the potential energy of a particle of mass m at that point will be $U = mV$

Note : Positive gravitational potential energy means that the body will do work in returning to its reference position while negative potential energy means work will be done on the body to bring it back to the reference position. Gravitational potential energy also depends on frame of reference, i.e., reference position.



(d) Change in potential energy

$$\Delta U = U(R + h) - U(R) = \frac{GmMh}{R^2\left(1 + \frac{h}{R}\right)} = \frac{mgh}{1 + \frac{h}{R}}$$

where R is the radius of earth. If $h \ll R$, $(h/R) \ll 1$.

So, $U = mgh$

9. Electrostatic potential energy between two charges

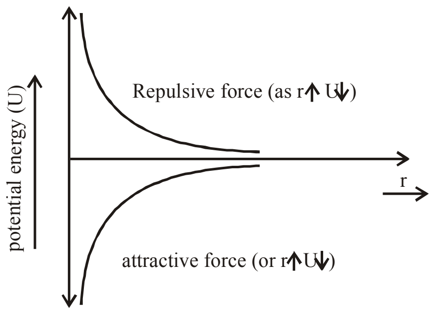
$$U = \frac{kq_1q_2}{r}$$

U can be positive or negative depending on nature of charge

10. Potential energy of a compressed or stretched spring

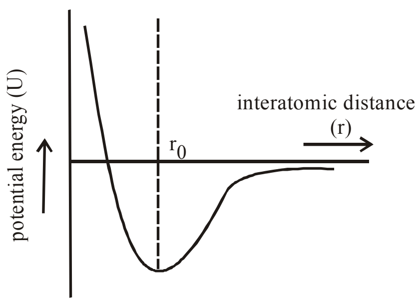
$$U = \frac{1}{2}kx^2$$

11. The potential energy diagram for attractive and repulsive forces between two atoms and inter molecular distance is



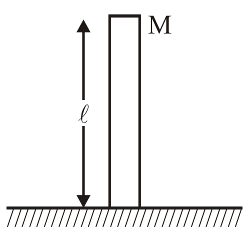
12. The potential energy for a system of two atoms is

$U(r) = \frac{a}{r^{12}} - \frac{b}{r^6}$ where r is interatomic separation.



Example 13 :

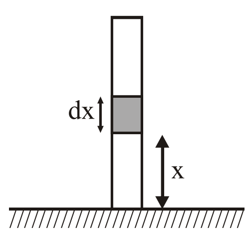
Calculate potential energy of a uniform vertical rod of mass M and length ℓ.



Sol. $dU = (dm)\,gx$

$$\int_0^U dU = \int_0^{\ell}\left(\frac{M}{\ell}dx\right)gx$$

$$U = \frac{Mg\ell}{2}$$



Example 14:

The potential energy of a particle in a space is given by $U = x^2 + y^2$. Find the force associated with this potential energy.

Sol. $F_x = \frac{-\partial u}{\partial x} = -[2x+0] = -2x$; $F_y = \frac{-\partial u}{\partial y} = -[2y+0] = -2y$

$\vec{F} = -2x\hat{i} - 2y\hat{j}$

LAW OF CONSERVATION OF MECHANICAL ENERGY

The sum of the potential energy and the kinetic energy is called the total mechanical energy.

Total mechanical energy of a system remains constant if only conservative forces are acting on a system of particles and the work done by all other forces is zero.

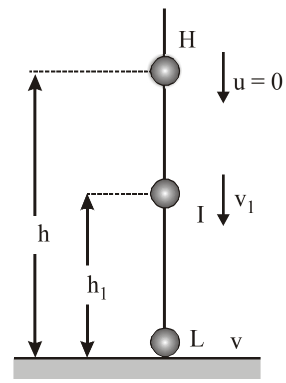
$\Delta K + \Delta U = 0$

or $K_f - K_i + U_f - U_i = 0$

or $K_f + U_f = K_i + U_i = \text{constant}$

Examples based on Law of conservation of mechanical energy:

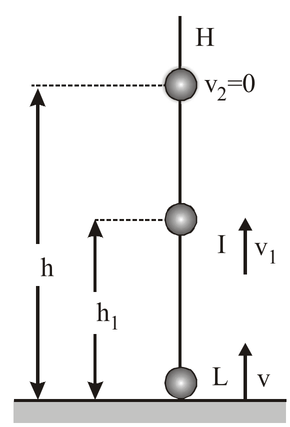
(a) Freely falling body : At the highest point, total energy is in the form of potential energy. At an intermediate point, energy is in the form of both PE and KE. At the lowest point, total energy is in the form of only KE.



$\therefore$ $TE = (PE)_h = (KE)_{h1} + (PE)_{h1} = (KE)_s$

$TE = mgh = mgh_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv^2$

(b) Body projected vertically upwards : At the starting point total energy is in the form of KE only, at an intermediate point energy is in the form of both PE and KE and at the highest point total energy is in the form of only PE.



$TE = (KE)_s = (KE)_{h1} + (PE)_{h1} = (PE)_h$

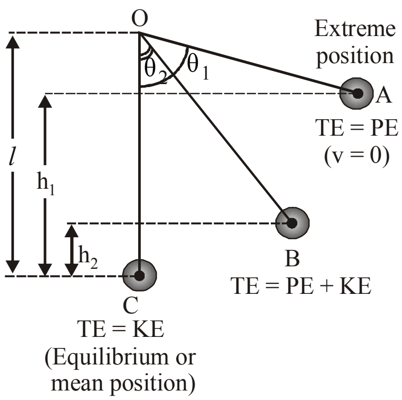
$TE = \frac{1}{2}mv^2 = mgh_1 + \frac{1}{2}mv_1^2 = mgh$

(c) Oscillating pendulum :

(i) $(KE)_C = (PE)_A$

$$\frac{1}{2}mv_c^2 = mgh_1 = mg\ell\,(1 - \cos\theta_1)$$

$$\therefore \quad v_c = v_{max.} = \sqrt{2g\ell(1-\cos\theta_1)}$$



(ii) $(PE)_B + (KE)_B = (PE)_A$

$(KE)_B = (PE)_A - (PE)_B = mg\,(h_1 - h_2)$

$$\frac{1}{2}mv_B^2 = mg\ell\,(\cos\theta_2 - \cos\theta_1)$$

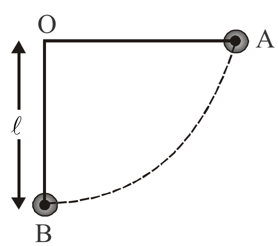
$$\therefore \quad v_B = v_I = \sqrt{2g\ell(\cos\theta_2 - \cos\theta_1)}$$

(iii) If a pendulum of length ℓ is held horizontal as shown in figure and released.

$(PE)_A = (KE)_B$ or $mg\ell$

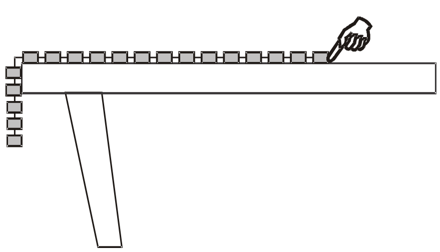
$$= \frac{1}{2}mv_B^2$$

$$\therefore \quad v_B = v_{equal} = \sqrt{2g\ell}$$



Example 15 :

In figure, a chain is held on a frictionless table with one-fourth of its length hanging over the edge. If the chain has length L and mass m, how much work is required to pull the hanging part back onto the table?



Sol. The work required is the change in the gravitational potential energy of the chain-Earth system as the chain is pulled onto the table. Take the potential energy to be zero when the whole chain is on the table. Divide the hanging chain into a large number of infinitesimal segments, each of length dy.

The mass of segment is (m/L) dy and the potential energy of the segment a distance y below the table top is

$dU = -(m/L)\,gy\,dy.$

The total potential energy is

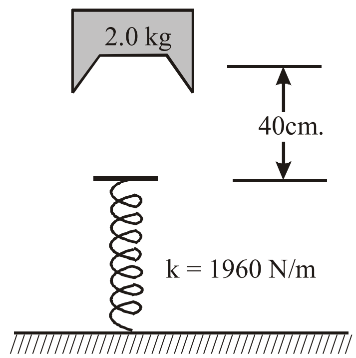
$$U = -\frac{m}{L}g\int_0^{L/4} y\,dy = -\frac{1}{2}\frac{m}{L}g\left(\frac{L}{4}\right)^2 = -\frac{mgL}{32}$$

The work required to pull the chain onto the table is

$$-U = \frac{mgL}{32}$$

Example 16 :

A 2.0 kg block is dropped from a height of 40 cm onto a spring of constant k = 1960 N/m. (figure). Find the maximum distance the spring is compressed.



Sol. Let m be the mass of the block, h the height from which it dropped, and x the compression of the spring.

Take the potential energy of the block-Earth system to be zero when the block is at its initial position. The block drops a distance h + x and the final gravitational potential energy is – mg (h + x). Here x is taken to be positive for a compression of the spring. The spring potential energy is initially zero and finally $\frac{1}{2}kx^2$.

The kinetic energy is zero at both the beginning and end.

Since energy is conserved $0 = -mg\,(h + x) + \frac{1}{2}kx^2$.

This is a quadratic equation for x. It solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k}$$

Now $mg = (2.0\text{ kg})\,(9.8\text{ m/s}^2) = 19.6\text{ N}$ and

$hk = (0.40\text{m})\,(1960\text{ N/m}) = 784\text{ N}$, so

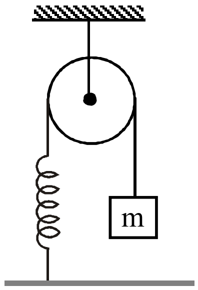
$$x = \frac{19.6\text{N} \pm \sqrt{(19.6\text{N})^2 + 2(19.6\text{N})\,(784\text{N})}}{1960\text{ N/m}} = 0.10\text{m}$$

or -0.080m.

Since x must be positive (a compression) we accept the positive solution and reject the negative solution.

Example 17 :

In the figure shown spring of stiffness k and mass of the block is m. The pulley is fixed. Initially the block m is held such that, the elongation in the spring is zero and then released from rest. Find the maximum elongation in the spring. Neglect the mass of the spring, pulley and that of the string.



Sol. Let the maximum elongation in the spring be x, when the block is at position 2 (velocity = 0).

The displacement of the block m is also x. If E_1 and E_2 are the energies of the system when the block is at position 1 and 2 respectively. Then $E_1 = U_{1g} + U_{1s} + KE_1$, where

U_{1g} = gravitational P.E. with respect to surface S.

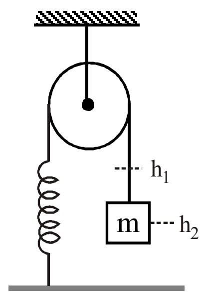
U_{1s} = P.E. stored in the spring.

KE_1 = initial K.E. of the block.

$\Rightarrow E_1 = mgh_1 + 0 + 0 = mgh_1$ .... (i)

and $E_2 = U_{2g} + U_{2s} + KE_2$

$= mgh_2 + \frac{1}{2}kx^2 + 0$ .... (ii)



From conservation of energy $E_1 = E_2$

$$\Rightarrow mgh_1 = mgh_2 + \frac{1}{2}kx^2$$

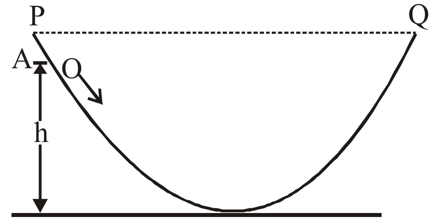
$$\Rightarrow \frac{1}{2}kx^2 = mg(h_1 - h_2) = mgx$$

$$\Rightarrow x = 2mg/k$$

Example 18 :

A ball of mass m is pushed down the wall of hemispherical bowl from point A. It just rises up to edge Q of the bowl. Find the speed at which ball is pushed down.

Sol. From law of conservation of energy



Total energy at A = total energy at Q

$$\frac{1}{2}mv^2 + mgh = 0 + mgR$$

$v^2 = 2g(R - h)$ or $v = \sqrt{2g(R-h)}$

EQUILIBRIUM

A body is said to be in translatory equilibrium if net force acting on the body is zero, $\vec{F}_{net} = 0$

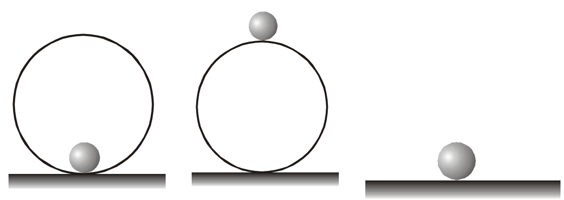
If the force are conservative then $F = -\frac{dU}{dr}$

For equilibrium F = 0. So, $-\frac{dU}{dr} = 0$, or $\frac{dU}{dr} = 0$

At equilibrium position slope of U-r graph is zero or the potential energy is optimum (maximum or minimum or constant).

Equilibrium are of three types.

The situation where F = 0 and $\frac{dU}{dr} = 0$ can be obtained under three conditions.



(a) Stable equilibrium (b) Unstable equilibrium
(c) Neutral equilibrium.

Three identical balls are placed in equilibrium in positions as shown in fig. (a), (b) and (c) respectively.

(a) Ball is placed inside a smooth spherical shell. This ball is in stable equilibrium position.

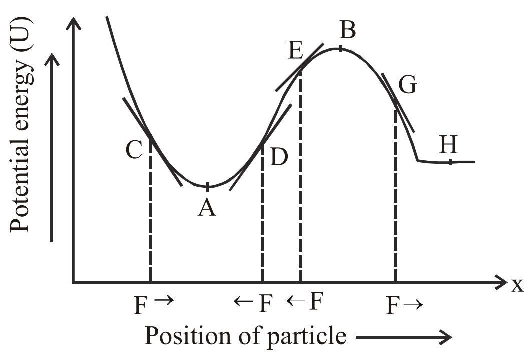
(b) The ball is placed over a smooth sphere. This is in unstable equilibrium position.

(c) The ball is placed on a smooth horizontal ground. This ball is in neutral equilibrium position.

S.No.	Stable Equilibrium	Unstable Equilibrium	Neutral Equilibrium
1.	Net force is zero	Net force is zero	Net force is zero
2.	$\frac{dU}{dr} = 0$ or slope of U–r graph is zero	$\frac{dU}{dr} = 0$ or slope of U–r graph is zero.	$\frac{dU}{dr} = 0$ or slope of U–r graph is zero
3.	When displaced slightly, from its equilibrium position a net restoring force starts acting on the body which has a tendency to bring the body back to its equilibrium position.	When displaced slightly from its equilibrium position, a net force starts acting on the body which moves the body in the direction of displacement or away from the equilibrium position.	When displaced slightly from its equilibrium position the body has neither the tendency to come back to original position nor to move away from the original position.
4.	Potential energy in equilibrium position is minimum as compared to its neighbouring points or $\frac{d^2U}{dr^2} = \text{positive}$	Potential energy in equilibrium position is maximum as compared to its neighbouring points or $\frac{d^2U}{dr^2} = \text{negative}$	Potential energy remains constant even if the body is displaced from its equilibrium position. $\frac{d^2U}{dr^2} = 0$
5.	When displaced from equilibrium position the centre of gravity of the body goes up.	When displaced from equilibrium position the centre of gravity of the body comes down.	When displaced from equilibrium position the centre of gravity of the body remains at the same level.

POTENTIAL ENERGY CURVE AND EQUILIBRIUM

It is a curve which shows change in potential energy with position of a particle.



Stable Equilibrium : When a particle is slightly displaced from equilibrium and it tends to come back towards equilibrium then it is said to be in stable equilibrium

At point **C :** slope $\frac{dU}{dx}$ is negative (as it is in 2nd quadrant) so F is positive

At point **D :** slope $\frac{dU}{dx}$ is positive so F is negative

At point **A :** it is the point of stable equilibrium.

At **A** $U = U_{min}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} = \text{positive}$

Unstable equilibrium : When a particle is displaced from equilibrium and it tends to move away from equilibrium position then it is said to be in unstable equilibrium

At point **E :** $\frac{dU}{dx}$ is positive so F is negative

At point **G :** $\frac{dU}{dx}$ is negative so F is positive

At point **B :** it is the point of unstable equilibrium.

At **B** $U = U_{max}$ $\frac{dU}{dx} = 0;$ $\frac{d^2U}{dx^2} = \text{negative}$

Neutral equilibrium : When a particle is displaced from equilibrium and no force acts on it then equilibrium is said to be neutral equilibrium

at point **H :** is at neutral equilibrium U = const.

$$\frac{dU}{dx} = 0; \quad \frac{d^2U}{dx^2} = 0$$

Binding energy : The difference in energies of top and bottom of a potential well is called binding energy.

Example 19 :

The potential energy of a conservative system is given by $U = ax^2 - bx$, where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable unstable or neutral.

Sol. In a conservative field,

$$F = -\frac{dU}{dx} = -\frac{d}{dx}(ax^2 - bx) = -(2ax - b)$$

$\therefore \quad F = b - 2ax$

For equilibrium, $F = 0$ or $b - 2ax = 0 \ \therefore \ x = b/2a$

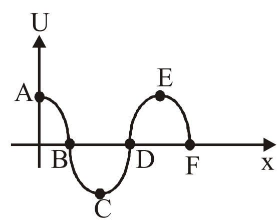
From the given equation we can see that $\frac{d^2U}{dx^2} = 2a$ (+ve),

i.e., U minimum.

Therefore, $x = b/2a$ is the stable equilibrium position.

Example 20:

Identify the points of equilibrium and discuss their nature.



Sol. C, E, F are points of equilibrium because $F = \frac{-\partial U}{\partial x}$

$\therefore$ When slope of U-x curve is zero then F is zero.

Check stability through slopes at near by points.

If we move right then slope should be positive for stable equilibrium and vice versa. In short, it is like a hill and plateau.

Example 21 :

The potential energy of a particle in a certain field has the form $U = a/r^2 - b/r$, where a and b are positive constant, r is the distance from the centre of the field. Find the value of r_0 corresponding to the equilibrium position of the particles ; examine whether this position is stable.

Sol. $U(r) = a/r^2 - b/r$

$$\text{Force} = F = -\frac{dU}{dr} = -\left(\frac{-2a}{r^3} + \frac{b}{r^2}\right) = -\frac{(br - 2a)}{r^3}$$

At equilibrium $F = \frac{dU}{dr} = 0$. Hence $br - 2a = 0$ at equilibrium

$r = r_0 = 2a/b$ corresponds to equilibrium

At stable equilibrium, the potential energy is minimum and at unstable equilibrium, it is maximum. From calculus, we know that for minimum value around a point $r = r_0$, the first derivative should be zero and the second derivative should be positive. For minimum potential energy,

$$\frac{dU}{dr} = 0 \ \text{ and } \ \frac{d^2U}{dr^2} > 0 \quad \text{at } r = r_0$$

We already know $dU/dr = 0$ to get $r = r_0 = 2a/b$.

Let us investigate the second derivative.

$$\frac{d^2U}{dr^2} = \frac{d}{dr}\left(\frac{dU}{dr}\right) = \frac{d}{dr}\left(-\frac{2a}{r^3} + \frac{b}{r^2}\right) = \frac{6a}{r^4} - \frac{2b}{r^3}$$

At $r = r_0 = 2a/b$, $\frac{d^2U}{dr^2} = \frac{6a - 2br_0}{r_0^4} = \frac{2a}{r_0^4} > 0.$

Hence the potential energy function U(r) has a minimum value at $r_0 = 2a/b$.

The system has a stable equilibrium at minimum potential energy state.

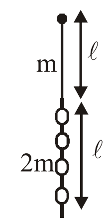
TRY IT YOURSELF-3

Q.1 Calculate the forces F(y) associated with the following one-dimensional potential energies :

(a) $U = -\omega t$, (b) $U = ay^3 - by^2$, (c) $U = U_0 \sin \beta y$.

Q.2 Potential energy of a particle is given by (2x + 3y + 4z) joule. Calculate force on the particle.

Q.3 A rope of length ℓ and mass m is connected to a chain of length ℓ and mass 2m and hung vertically as shown in figure. What is the change in gravitational potential energy if the system is inverted and hung from same point.



(A) $mg\ell$ (B) $4mg\ell$

(C) $3mg\ell$ (D) $2mg\ell$

Q.4 Find out the potential energy of the force $\vec{F} = y\hat{i} + x\hat{j}$.

Q.5 You lift a ball at constant velocity from a height h_i to a greater height h_f. Considering the ball and the earth together as the system, which of the following statements is true?

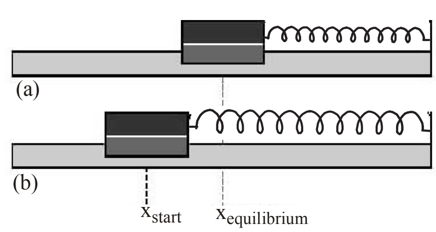
(A) The potential energy of the system increases.

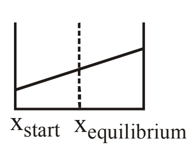
(B) The kinetic energy of the system decreases.

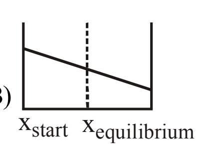
(C) The earth does negative work on the system.

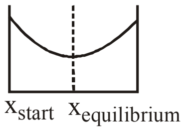
(D) You do negative work on the system.

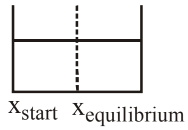
Q.6 In part (a) of the figure, an air track cart attached to a spring rests on the track at the position $x_{equilibrium}$ and the spring is relaxed. In (b), the cart is pulled to the position x_{start} and released. It then oscillates about $x_{equilibrium}$. Which graph correctly represents the potential energy of the spring as a function of the position of the cart?



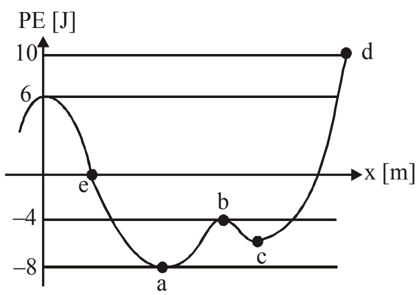
(A)


(B)


(C) 

(D) 

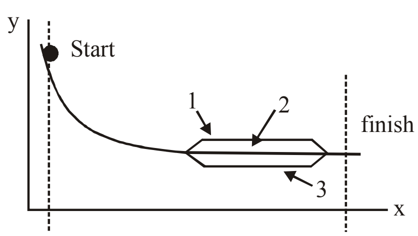
Q.7 Consider the following sketch of potential energy for a particle as a function of position. There are no dissipative forces or internal sources of energy. What is the minimum total mechanical energy that the particle can have if you know that it has traveled over the entire region of X shown?



(A) –8 (B) 6 (C) 10
(D) It depends on direction of travel

Q.8 A spring-loaded toy dart gun is used to shoot a dart straight up in the air, and the dart reaches a maximum height h_{max}. The same dart is shot straight up a second time from the same gun, but this time the spring is compressed only half as far before firing. How far up does the dart go this time, neglecting friction and assuming an ideal spring?
(A) h_{max} (B) $h_{max}/2$
(C) $h_{max}/4$ (D) $4h_{max}$

Q.9 A marble starts from rest and slides down hill. Which path leads to the highest speed at the finish?



(A) 1 (B) 2 (C) 3
(D) all result in the same final speed

Q.10 In above question, which path results in the shortest time to the finish?
(A) 1 (B) 2 (C) 3
(D) all result in the same time.

ANSWERS

(1) (a) ω, (b) $-3ay^2 + 2by$ (c) $-\beta U_0 \cos\beta y$

(2) $\sqrt{29}$ N **(3)** (A) **(4)** $-xy + c$
(5) (A) **(6)** (C) **(7)** (C)
(8) (C) **(9)** (D) **(10)** (C)

POWER

When we purchase a car or jeep we are interested in the horsepower of its engine. We know that usually an engine with large horsepower is most effective in accelerating the automobile.
In many cases it is useful to know not just the total amount of work being done, but how fast the work is done.
We define power as the rate at which work is being done.

$$\text{Average Power} = \frac{\text{Work done}}{\text{Time taken to do work}}$$

$$= \frac{\text{Total change in kinetic energy}}{\text{Total time in change}}$$

If ΔW is the amount of work done in the time interval Δt.

then $$P = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}$$

When work is measured in joules and t is in seconds, the unit for power is the joule per second, which is called watt. For motors and engines, power is usually measured in horsepower, where horsepower is 1 hp = 746 W
The definition of power is applicable to all types of work like mechanical, electrical, thermal.

Instantaneous power, $$P = \frac{dW}{dt}$$

$$W = \vec{F}.\vec{x} \qquad \therefore P = \frac{d}{dt}(\vec{F}.\vec{x}) = \vec{F}.\frac{d\vec{x}}{dt} = \vec{F}.\vec{v}$$

Where v is the instantaneous velocity of the particle and dot product is used as only that component of force will contribute to power which is acting in the direction of instantaneous velocity.
If velocity vector makes an angle θ with the force vector,

then $P = \vec{F}\cdot\vec{v} = Fv\cos\theta$

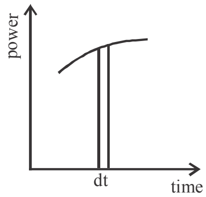
$$P = \frac{dK}{dt} \text{ (time rate of change of kinetic energy)}$$

Note :

1. Power is a scalar with dimension $M^1L^2T^{-3}$
2. SI unit of power is J/s or watt
   1 Horse power = 746 watt = 550 ft-lb/sec.
3. Area under power-time graph gives the work done.
   $$W = \int P dt \text{ (Fig. a)}$$
4. The slope of tangent at a point on work time graph gives instantaneous power (Fig. b)
5. The slope of a straight line joining two points on work time graph gives average power between two points (Fig. c)
6. For a system of varying mass
   $$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

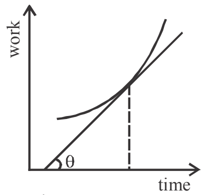
If v = constant then $F = v\dfrac{dm}{dt}$

then $P = \vec{F}.\vec{v} = v^2\dfrac{dm}{dt}$



$W = \int P\,dt$

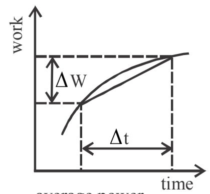
fig(a)



instantaneous power

$P = \dfrac{dW}{dt} = \tan\theta$

fig(b)



average power

$\bar{P} = P_{av} = \dfrac{W_2 - W_1}{t_2 - t_1} = \dfrac{\Delta W}{\Delta t}$

fig(c)

7. In rotatory motion : $P = \tau\dfrac{d\theta}{dt} = \tau\omega$

8. Units : (a) CGS system ; erg/sec
   (b) FPS system : horse power (= 746 watt)
   (c) SI : J/sec or watt

9. (a) If a block of mass m is pulled along the smooth inclined plane of angle θ, with constant velocity v, then the power spent is : $P = (mg\sin\theta)v$.
   (b) If the same block is pulled along rough inclined plane, then the power is $P = (mg\sin\theta + \mu mg\cos\theta)v$

10. If a machine gun fires n bullets per second, each of mass m and velocity v, then the power of gun is :

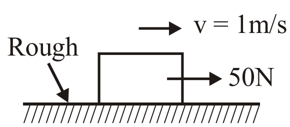
$$P = \frac{1}{2}mnv^2$$

11. (a) When water is coming out from a hose pipe of area of cross section A with a velocity v and hits a wall normally and slides along the wall, then force exerted by the water on the wall is $Av^2\rho$. Then power of water is
$P = Fv = Av^2\rho v = Av^3\rho$
(b) If in the above case, water reflects with the same velocity v after striking the wall, then $F = 2Av^2\rho$ and
$P = Fv = 2Av^3\rho$.

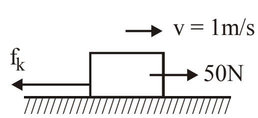
12. If a body moves along a rough horizontal surface, with a constant velocity v, then the power required is $P = \mu mgv$.

Example 22 :

A block moves with constant velocity 1 m/s under the action of horizontal force 50 N on a horizontal surface. What is the power of external force and friction?



Sol. Since $a = 0$ i.e., $f_k = 50$ N



$P_{ext} = 50 \times 1 = 50W$

$P_f = -50 \times 1 = -50W$

Example 23:

A boy is able to chew 20g of ice in 5 minute. Calculate the power of the boy in horse power.

Sol. $W = JH = JML$ $\quad\quad L = 80$ cal/gm

$$P = \frac{W}{t} = \frac{JML}{t}$$

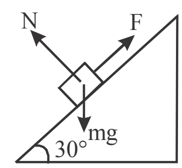
$$= \frac{4.2\,J/cal \times 20gm \times 80cal/gm}{5 \times 60\,sec} = 22.4 \text{ watt}$$

$$P = \frac{22.4}{746} = 0.03 \text{ hp}$$

Example 24 :

A block of mass 2 kg is pulled up on a smooth incline of angle 30° with horizontal. If the block moves with an acceleration of 1 m/s^2, find the power delivered by the pulling force at a time 4 seconds after motion starts. What is the average power delivered during these four seconds after the motion starts?

Sol. The forces acting on the block are shown in figure.



Resolving forces parallel to incline,

$F - mg\sin\theta = ma$

$\Rightarrow F = mg\sin\theta + ma$

$= 2 \times 9.8 \times \sin30° + 2 \times 1 = 11.8$ N

The velocity after 4 seconds = u + at = 0 + 1 × 4 = 4 m/s

Power delivered by force at t = 4 seconds

= Force × Velocity = 11.8 × 4 = 47.2 W

The displacement during 4 seconds is given by the formula

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 1 \times S \quad \therefore \quad S = 8$ m

Work done in four seconds = Force × distance

= 11.8 × 8 = 94.4 J

$\therefore$ Average power delivered $= \dfrac{\text{workdone}}{\text{time}} = \dfrac{94.4}{4} = 23.6$ W

TRY IT YOURSELF-4

Q.1 A car of mass 1500 kg has a maximum speed of 150 kmh^{-1} on the level when working at its maximum power against resistances of 60N. Find the acceleration of the car when it is travelling at 60 kmh^{-1} on the level with the engine working at maximum power assuming that the resistance to motion remains constant.

Q.2 Force acting on a particle moving in a straight line varies with the velocity v of the particle as F = K/v, where K is a constant. The work done by this force in time t is –

(A) $\dfrac{K}{v^2}t$ $\quad\quad$ (B) 2Kt

(C) Kt $\quad\quad$ (D) $\dfrac{2Kt}{v^2}$

Q.3 The units of power could be which of the following –
(i) Joules/sec (ii) Watts/sec.
(iii) $Kg.m^2/s^3$. (iv) Nt.m/s.
(A) i and ii (B) ii and iii
(C) i and iv (D) i and iii

Q.4 An engine pumps up 1000 kg of coal from a mine 100m deep in 50sec. The pump is running with diesel and efficiency of diesel engine is 25%. Then its power consumption will be ($g = 10m/sec^2$):
(A) 10 kW (B) 80 kW
(C) 20 kW (D) 24 kW

Q.5 A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is his power output?

Q.6 A truck can move up a road having a grade of 1.0m rise every 50m with a speed of 24 km/ hour. The resisting force is equal to one twenty - fifty the weight of the truck. With what speed same truck can move down the hill with the same power ?

Q.7 The mass of a cyclist and his machine together is 80 kg. The cyclist finds that on a hill of slope 1 in 30, he can just free - wheel down at constant speed. Calculate the power at which he has to work to maintain a speed of 9 km/ h up this hill, assuming that the resistance to motion is unchanged.

Q.8 A force of 5.0 N acts on a 15 kg body initially at rest. Compute the work done by the force in (a) the first (b) the second, and (c) the third seconds and (d) the instantaneous power due to the force at the end of the third second.

Q.9 An engine develops 10 kW of power. How much time will it take to lift a mass of 200 kg through a height of 40m ? Given $g = 10\ m/s^2$.

Q.10 How much minimum power of a water pump needed to lift water from a level 20m below the ground at a rate of 20 kg/min. (746 W = 1 hp) ($g = 10\ m/s^2$)

ANSWERS

(1) $0.06\ ms^{-2}$	**(2)** (C)	**(3)** (C)
(4) (B)	**(5)** 875 W	**(6)** 72 km / hour.
(7) 130. 65 watt		
(8) (a) 0.83 J (b) 2.5 J (c) 4.2 J (d) 5.0 W		
(9) 8 sec.	**(10)** $\frac{100}{1119}$ HP	

MOMENTUM

The linear momentum of a body is defined as the product of the mass of the body and its velocity, i.e., $\vec{p} = m\vec{v}$. It is a vector and points in the same direction as the velocity vector. In terms of momentum. Newton's second law can be expressed as : $\vec{F} = (d\vec{p}/dt)$. When a resultant applied force $\vec{F}$ acts on a body, it causes the linear momentum of that body to change with time. Linear momentum depends on frame of reference, e.g., linear momentum of a body at rest in a moving train, is zero relative to a man sitting in the train, while is not zero for a man standing on the ground.

Relation between momentum and kinetic energy :
Consider a body of mass m moving with velocity v.
Linear momentum of the body, p = mv.
KE of a particle can be expressed as

$$E = \frac{1}{2}mv^2 = \frac{mv^2}{2m} = \frac{p^2}{2m} \quad \text{or} \quad E = \frac{p^2}{2m} \quad \text{and} \quad p = \sqrt{2mE}$$

This implies that a body cannot possess KE without having momentum and vice-versa.
A body cannot have momentum without energy (as whenever a body possesses momentum, it also possesses KE) but the body may have energy (i.e., potential energy) without momentum.

Note :

(i) KE of a body cannot be negative ($\because K \propto p^2$) but momentum of a body may be negative ($\because \vec{p} = m\vec{v}$).

(ii) KE of a body is independent of direction of motion of the body, but momentum depends upon the direction of motion.

(iii) A single external force acting on a particle need not necessarily change its KE but it changes the momentum.

(iv) (a) The slope of p versus t curve gives the force.
(b) The area under F versus t curve gives the change in momentum.

Example 25 :
Can a body have energy without having momentum?

Sol. Yes; If momentum p = 0, KE = $(p^2/2m) = 0$.
So for a body E = (KE + PE) will reduce to E = PE which may not be zero. So a body or may not have energy without having momentum.

Example 26 :
Can a body have momentum without having energy? Explain.

Sol. Yes ; If for a body E = 0, K + U = 0 ; so, either both are zero or PE is equal to KE but negative, i.e., KE may or may not be zero, Now as $p = \sqrt{2m(KE)}$. So, a body may or not have momentum without having energy.

LAW OF CONSERVATION OF LINEAR MOMENTUM

Through experimental observations it has been found that if no external force acts on a system (called isolated) of constant mass (called closed) the total momentum of the system remains constant (with time).
If the total external force acting on a system is equal to zero, then the final value of the total momentum of the system is equal to the initial value of the total momentum of the system.

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt} \quad \text{If} \quad \vec{F}_{ext} = 0 \ , \ \text{i.e.,} \ \frac{d\vec{p}}{dt} = 0 \ \text{then}$$

$$\vec{p} = \text{constant} \quad \text{or} \quad \vec{p}_f = \vec{p}_i$$

The law may be extended to a system of particle or to the centre of mass of a system of particles.

(a) For a system of particles it takes the form :

If net force (or the vector sum of all the forces) on a system of particles is zero, the vector sum of linear momentum of all the particles remain conserved, or

If $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n = 0$

Then, $\vec{p}_1 + \vec{p}_2 + ... + \vec{p}_n = \text{constant}$

(Sum of initial momentums = Sum of final momentums)

This equation shows that in absence of external force for a closed system the linear momentum of individual particles may change but their sum remains unchanged with time.

(b) For centre of mass of a system of particles.

If $\vec{F}_{cm} = 0$, then $\vec{P}_{cm} = \text{constant}$.

Thus, the law of conservation of linear momentum can be applied to a single particle, to a system of particles or even to the centre of mass of the particles.

A gas contained in a jar at rest the linear momentum of the system (jar) always remains zero, though according to kinetic theory of gas, each gas molecule has momentum that changes with time due to collisions with other molecules.

Conservation of linear momentum is equivalent to Newton's III law of motion.

Law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.

As linear momentum depends on frame of reference, observers in different frames would find different values of linear momentum of a given system but each would agree that his own values of linear momentum does not change with time provided the system is isolated and closed.

This law is universal, i.e., it applies to both macroscopic as well as microscopic systems.

It holds good even in atomic and nuclear physics where classical mechanics falls. Further it is more generally applicable than the law of 'conservation of mechanical energy' because 'internal forces' are often non-conservative and so mechanical energy is not conserved but momentum remains conserved (if $F_{ext} = 0$)

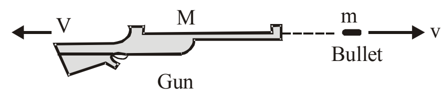
APPLICATIONS

(A) Firing a Bullet from a Gun :

(i) If the bullet is the system, the force exerted by trigger will be external and so the linear momentum of the bullet will change from 0 to mv. This is not the violation of law of conservation of linear momentum as linear momentum is conserved only in absence of external force .

(ii) If the bullet and gun is the system, the force exerted by trigger will be internal so, total momentum of the system $\vec{p}_S = \vec{p}_B + \vec{p}_G = \text{constt.}$

Now as initially both bullet and gun are at rest



$\vec{p}_B + \vec{p}_G = 0$

From this it is evident that :

(1) $\vec{p}_G = -\vec{p}_B$, i.e., if bullet acquires forward momentum, the gun will acquire equal and opposite (backward) momentum.

(2) As $\vec{p} = m\vec{v}$, $m\vec{v} + M\vec{V} = 0$, i.e. $\vec{V} = -\frac{m}{M}\vec{v}$

i.e, if the bullet moves forward, gun 'recoils' or 'kicks' backward.

Heavier the gun lesser will be the recoil velocity V.

(3) Now as $K = \frac{p^2}{2m}$ and $|\vec{p}_B| = |\vec{p}_G| = p$

Kinetic energy of gun, $K_G = \frac{p^2}{2M}$

Kinetic energy of bullet, $K_b = \frac{p^2}{2m}$

$\therefore \frac{K_G}{K_b} = \frac{m}{M} < 1 \qquad (\because M \gg m)$

Thus kinetic energy of gun is smaller than bullet. i.e., kinetic energy of bullet and gun will not be equal.

(4) Initial KE of the system is zero as both are at rest initially.

Final KE of the system $[(1/2)(mv^2 + MV^2)] > 0$.

So, here KE of the system is not constant but increases. If PE is assumed to be constt., ME = (KE + PE) will also increase.

However, energy is always conserved. Here chemical energy of gun powder is converted into KE.

Example 27 :

A bullet of mass 100gm is fired by a gun of 10kg with a speed 2000 m/sec. Find recoil velocity of gun.

Sol. According to conservation of momentum mv + MV = 0

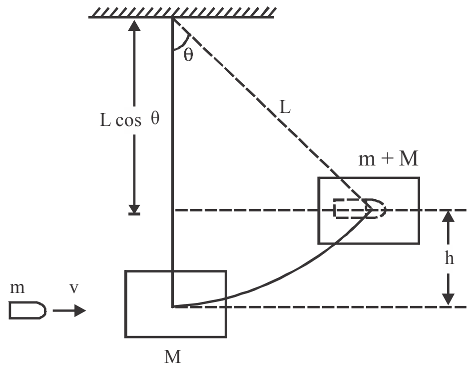
Velocity of gun, $V = -\frac{mv}{M} = -\frac{0.1 \times 2000}{10} = -20 \text{ m/s}$

(B) Block Bullet System

(a) When bullet remains in the block :

Conserving momentum of bullet and block

$$mv + 0 = (M+m)V$$



Velocity of block $V = \dfrac{mv}{M+m}$ ....(1)

conserving energy $\dfrac{1}{2}(M+m)V^2 = (M+m)gh$

$V = \sqrt{2gh}$ ....(2)

from eq^n. (1) and eq^n. (2), $\dfrac{mv}{M+m} = \sqrt{2gh}$

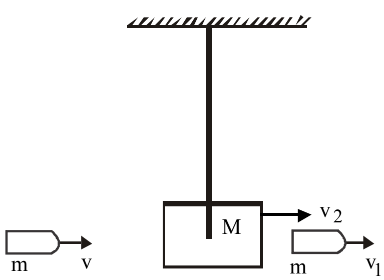
Speed of bullet $v = \dfrac{(M+m)\sqrt{2gh}}{m}$

Height gained by block

$$h = \frac{V^2}{2g} = \frac{m^2v^2}{2g(M+m)^2} \;;\; h = L - L\cos\theta$$

$$\therefore \cos\theta = 1 - \frac{h}{L} \Rightarrow \theta = \cos^{-1}\left(1 - \frac{h}{L}\right)$$

(b) If bullet moves out of the block



Conserving momentum, $mv + 0 = mv_1 + Mv_2$

$m(v - v_1) = Mv_2$ ....(1)

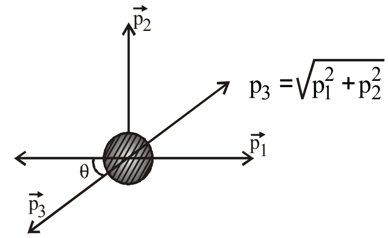
Conserving energy, $\dfrac{1}{2}Mv_2^2 = Mgh$

$v_2 = \sqrt{2gh}$ ....(2)

From eq^n. (1) & eq^n. (2)

$$m(v - v_1) = M\sqrt{2gh} \quad \text{or} \quad h = \frac{m^2(v - v_1)^2}{2gM^2}$$

(C) Explosion of a Bomb :



Conserving momentum, $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

or $\vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$ or $p_3 = \sqrt{p_1^2 + p_2^2}\left(\theta = \dfrac{\pi}{2}\right)$

Angle made by $\vec{p}_3$ from $\vec{p}_1 = \pi + \theta$

Angle made by $\vec{p}_3$ from $\vec{p}_2 = \dfrac{\pi}{2} + \theta$

Energy released in explosion $= K_f - K_i$

$$K_f = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} \;;\; K_i = 0$$

Example 28 :

A shell is fired from a cannon with a speed of 100 m/s at an angle 30° with the vertical (y-direction). At the highest point of its trajectory, the shell explodes into two fragments of masses in the ratio 1 : 2. The lighter fragment moves vertically upwards with an initial speed of 200 m/s. What is the speed of the heavier fragment at the time of explosion.

Sol. Velocity at highest point = u cos θ = 100 cos 60 = 50m/s.

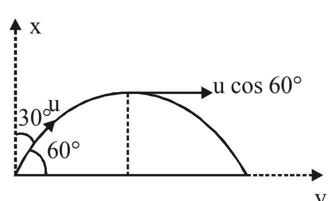
Taking, x and y axes as shown in figure,

Velocity of m = $50\hat{i}$ m/sec

m explodes into m/3 and 2m/3

Velocity of

$\dfrac{m}{3} = 200\hat{j}$ m/sec



Let, velocity of $\dfrac{2m}{3} = \vec{V}$

Applying law of conservation of momentum

$$m \times 50\hat{i} = \frac{m}{3} \times 200\hat{j} + \frac{2m}{3}\vec{V} \Rightarrow 50\hat{i} - \frac{200}{3}\hat{j} = \frac{2}{3}\vec{V}$$

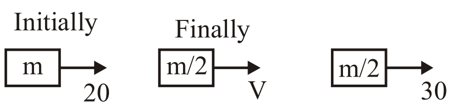
So, $\vec{V} = 75\hat{i} - 100\hat{j}$

Speed $= |\vec{V}| = \sqrt{(75)^2 + (100)^2} = 25\sqrt{9+16} = 125$ m/sec

Example 29 :

A block moving horizontally on a smooth surface with a speed of 20 m/s bursts into two equal parts continuing in the same direction. If one of the parts moves at 30 m/s, with what speed does the second part move and what is the fractional change in the kinetic energy of the system.

Sol.



Applying momentum conservation;

$$m \times 20 = \frac{m}{2}V + \frac{m}{2} \times 30 \Rightarrow 20 = \frac{V}{2} + 15$$

So, V = 10m/s

$$\text{Initial kinetic energy} = \frac{1}{2} m \times (20)^2 = 200m$$

$$\text{Final kinetic energy} = \frac{1}{2} \times \frac{m}{2} \times (10)^2 + \frac{1}{2} \times \frac{m}{2} \times (30)^2$$

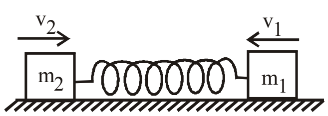
$$= 25\,m + 225\,m = 250\,m$$

Fractional change in kinetic energy

$$= \frac{(\text{final K. E.}) - (\text{initial K. E.})}{\text{initial K. E.}} = \frac{250m - 200m}{200m} = \frac{1}{4}$$

(D) Motion of Two Masses Connected to a Spring

Consider two blocks, resting on a frictionless surface and connected by a massless spring as shown in figure.



If the spring is stretched (or compressed) and then released from rest, then as $F_{ext} = 0$

$$\vec{p}_s = \vec{p}_1 + \vec{p}_2 = \text{constt.}$$

However, initially both the blocks were at rest so,

$$\vec{p}_1 + \vec{p}_2 = 0$$

It is clear that :

(i) $\vec{p}_2 = -\vec{p}_1$, i.e., at any instant the two blocks will have momentum equal in magnitude but opposite in direction (Though they have different values of momentum at different positions).

(ii) As $\vec{p} = m\vec{v}$, $m_1\vec{v}_1 + m_2\vec{v}_2 = 0$,

or $\vec{v}_2 = -(m_1/m_2)\,\vec{v}_1$

i.e., the two blocks always move in opposite directions with lighter block moving faster.

(ii) Now as $K = \frac{p^2}{2m}$ and $|\vec{p}_1| = |\vec{p}_2|$, $\frac{K_1}{K_2} = \frac{m_2}{m_1}$ or the KE of two blocks will not be equal but in the inverse ratio of their masses and so lighter block will have greater KE.

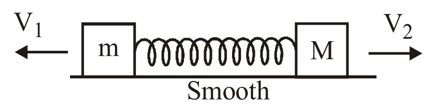
(iv) Initially KE of the blocks is zero (as both are at rest) and after some time KE of the blocks is not zero (as both are in motion). So, KE is not constant but changes. Here during motion of blocks KE is converted into elastic potential energy of the spring and vice-versa such that KE + PE = ME = constant

Example 30 :

A light spring of constant k is kept compressed between two blocks of masses m & M on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions. The spring looses contact with the blocks when it acquires natural length. If the spring was initially compressed through distance x, find the speed of two blocks.

Sol. As net external force acting on the system is zero (net $F_{ext} = 0$)

Applying conservation of momentum in horizontal direction



$$P_f = P_i$$

$$MV_2 - mV_1 = 0 \qquad \text{.... (1)}$$

Applying conservation of mechanical energy (as no work is done)

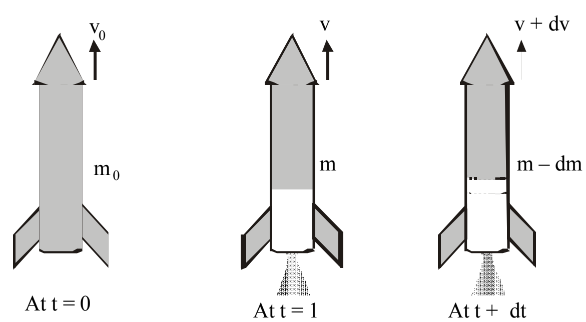
$$\frac{1}{2}kx^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}MV_2^2 \qquad \text{..... (2)}$$

$$V_1 = \sqrt{\frac{MK}{m\,(m+M)}}\ ; \quad V_2 = \sqrt{\frac{mK}{M\,(m+M)}}$$

(E) Rocket Propulsion :

u = velocity of ejected gases w.r.t rocket

$\frac{dm}{dt}$ = mass of gases ejected per unit time



m_0 = initial mass of (rocket + fuel) system

Resultant force on the rocket = upward thrust on the rocket - weight of rocket

$$ma = u\frac{dm}{dt} - mg \qquad \text{.........(1)}$$

$$\text{or} \quad m\frac{dv}{dt} = u\frac{dm}{dt} - mg \qquad \left(\because a = \frac{dv}{dt}\right)$$

$$\text{or} \quad dv = u\frac{dm}{m} - g\,dt$$

Integrate on both the sides

$$\int_{v_0}^{v} dv = -u\int_{m_0}^{m}\frac{dm}{m} - g\int_{0}^{t} dt$$

(negative sign shows reduction in mass)

$$v - v_0 = -u\log_e\frac{m}{m_0} - gt \quad \text{or} \quad v = v_0 + u\log_e\frac{m_0}{m} - gt$$

at $t = 0 \quad v_0 = 0$

so $\quad v = u\log_e\frac{m_0}{m} - gt$ ....(2)

Acceleration of the rocket

$$a = \frac{u\frac{dm}{dt} - mg}{m} \quad \text{or} \quad a = \frac{u}{m}\frac{dm}{dt} - g \qquad \text{......... (3)}$$

In free space $g = 0$

So from eq^n. (2), $v = u\log_e\frac{m_0}{m} = 2.3026\,u\log\frac{m_0}{m}$

Thrust on rocket $F = m\frac{dv}{dt}$ (Since $F = ma$)

or $F = u\frac{dm}{dt}$ [From eq^n. (1)]

Acceleration of rocket $a = \frac{u}{m}\frac{dm}{dt}$ in free space.

Example 31 :

A rocket burns 50 g of fuel per second ejecting it as a gas with velocity of 5×10^5 cm/s. What force is exerted by gas on rocket.

Sol. Upward thrust $= u\frac{dm}{dt}$, (where $u = 5\times10^5$cm/s $= 5\times10^3$m/s

& $\frac{dm}{dt} = 50$ gm/s $= 0.05$ kg/sec)

$= 5 \times 10^3 \times 0.05 = 250$ N

IMPULSE AND MOMENTUM

We know that the force is related to momentum as

$$\vec{F} = \frac{d\vec{P}}{dt} \Rightarrow \vec{F}dt = d\vec{P}$$

We can find the change in momentum of the body during a collision (from $\vec{P}_i$ to $\vec{P}_f$) by integrating over the time of collision and assuming that the force during collision has a constant direction, $\vec{P}_f - \vec{P}_i = \int_{P_i}^{P_f} d\vec{P} = \int_{t_1}^{t_f}\vec{F}dt$

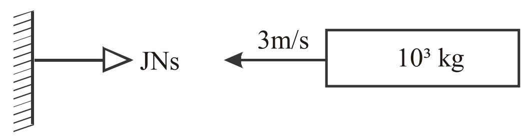
in which the subscripts i (= initial) and f (= final) refer to the times before and after the collision. The integral of a force over the time interval during which the forces acts is called the impulse of the force.

The impulse of this force, $\int_{t_i}^{t_f}\vec{F}dt$, is represented in magnitude by area under the force - time curve.

Impulsive tension : When a string jerks, equal and opposite tensions act suddenly at each end. Consequently equal and opposite impulses act on the objects to which the two ends of the string are attached.

Example 32 :

A truck of mass 10^3 kg travelling at 3 m/s is brought to rest in 2 seconds when it strikes a buffer. What force (assumed constant) is exerted by the buffer ?



Sol. If the impulse exerted by the buffer is +J newton second then the initial velocity of the truck is -3 ms^{-1} and its final velocity is zero.

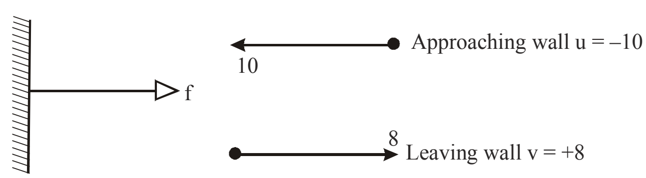
Then $\quad J = mv - mu \Rightarrow J = 0 - 10^3(-3) = 3 \times 10^3$

But $J = Ft = 2F$. Hence, $F = \frac{J}{2} = 1500$.

The force exerted by the buffer is 1500 N.

Example 33 :

A ball of mass 0.5 kg is thrown towards a wall so that it strikes the wall normally with a speed of 10 ms^{-1}. If the ball bounces at right angles away from the wall with a speed of 8ms^{-1}, what impulse does the wall exert on the ball ?



Sol. Taking the direction of the impulse J as positive and using

$J = mv - mu$

we have $J = \frac{1}{2} \times 8 - \frac{1}{2}(-10) = 9$

Therefore the wall exerts an impulse of 9 N-s on the ball.

TRY IT YOURSELF-5

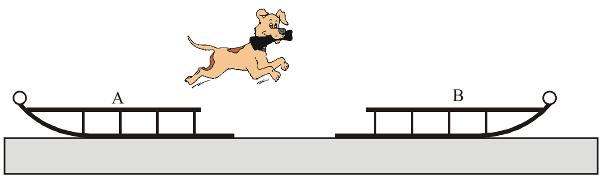
Q.1 Can kinetic energy of a system be changed without changing its momentum.

Q.2 Can momentum of a system be changed without changing its kinetic energy.

Q.3 A meteorite burns in the atmosphere before it reaches earth's surface. What happens to its momentum?

Q.4 Can a sail boat be propelled by air-blown at the sails from a fan attached to the boat.

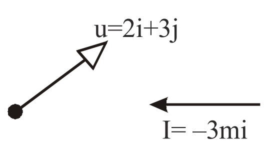
Q.5 Two 22.7 kg ice sleds A and B are placed a short distance apart, one directly behind the other, as shown in fig. A 3.63kg dog, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of 3.05 ms^{-1} relative to the ice. Find the final speeds of the two sleds.



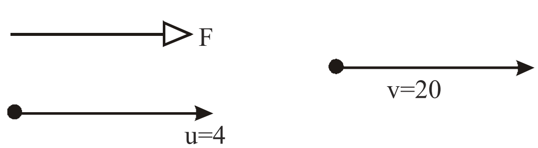
Q.6 Drop a stone from the top of a high cliff. Consider the earth and the stone as a system. As the stone falls, the momentum of the system
(A) increases in the downward direction.
(B) decreases in the downward direction.
(C) stays the same.
(D) not enough information to decide.

Q.7 An explosion splits an object initially at rest into two pieces of unequal mass. Which piece has the greater kinetic energy?
(A) The more massive piece.
(B) The less massive piece.
(C) They both have the same kinetic energy.
(D) There is not enough information to tell.

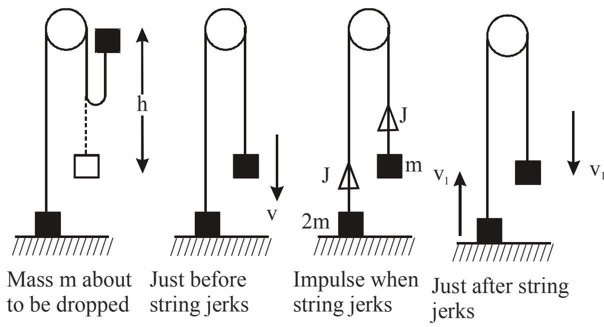
Q.8 A ball of mass m, travelling with velocity $2\hat{i}+3\hat{j}$, receives an impulse $-3m\hat{i}$. What is the velocity of the ball immediately afterwards ?



Q.9 What constant force acting in the direction of motion of a particle of mass 2 kg will increase its speed from 4 m/s to 20 m/s in 4 seconds –



Q.10 A mass 2m rests on a horizontal table. It is attached to a light inextensible string which passes over a smooth pulley and carries a mass m at the other end. If the mass m is raised vertically through a distance h and is then dropped, find the speed with which the mass 2m begins to rise.



ANSWERS

(1) Yes **(2)** Yes **(3)** Constant **(4)** No

(5) Velocity of B = 0.975 ms^{-1}. Velocity of A = 0.841 ms^{-1}.

(6) (C) **(7)** (B) **(8)** $-\hat{i}+3\hat{j}$ **(9)** 8N

(10) $\frac{1}{3}\sqrt{2gh}$

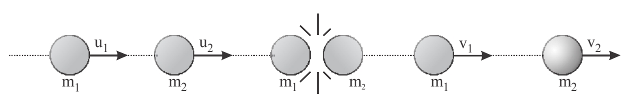
COLLISIONS

When exchange of momentum takes place between two physical bodies only due to their mutual interaction force, is defined as collision between two bodies.

Momentum can be transferred from one body to another by any mutual interaction force (only internal forces), doesn't maters whether there is physical contact or not.

The cases of collision in which physical contact takes place are known as "Impact".

A process in which there is a change in velocity, momentum, mechanical energy and direction of motion due to mutual interaction is called collision



1. For collision it is not necessary for particles to come in contact.
2. Before collision particle come closer and after collision they recede or stick together.
3. In all collisions linear momentum is conserved i.e.,

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

NEWTON'S LAW OF COLLISION

According to Newton's experimental law of impact, the ratio of relative speed of separation after collision and the relative speed of approach before collision is a constant. This constant is called coefficient of restitution or coefficient of resilience. For a given pair of materials. It is denoted by e.

Relative velocity of separation after collision $\propto$ relative velocity of approach before collision

$$\vec{v}_2 - \vec{v}_1 \propto \vec{u}_1 - \vec{u}_2$$

$$\vec{v}_2 - \vec{v}_1 = e\,(\vec{u}_1 - \vec{u}_2)$$

Coefficient of restitution $e = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{u}_1 - \vec{u}_2}$

$$e = \frac{\text{velocity of separation after collision}}{\text{velocity of approach before collision}}$$

The value of 'e' lies between zero and 1. It is zero when the materials stick together and approaches unity for highly elastic materials.

The coefficient of restitution of certain common bodies are given below :

Materials	**Coefficient of Restitution (e)**
Lead-lead	0.20
Lead-soft brass	0.16
Glass-glass	0.94
Cork-cork	0.65
Ivory-Ivory	0.81
Lead-glass	0.25
Lead-ivory	0.44
Cast iron-glass	0.91

Newton distinguished these three classes of collisions in terms of coefficient of restitution.

(i) $e = 1$, the collision is perfectly elastic.

(ii) $e = 0$, the collisions is perfectly inelastic.

(iii) $0 < e < 1$, the collisions is inelastic.

CLASSIFICATION OF COLLISIONS

(a) Perfectly Elastic Collision (Ideally elastic)

Collision between two bodies is said to be elastic if both the bodies come to their original shape and size after the collision, i.e., no fraction of mechanical energy remains stored as deformation potential energy in the bodies. Thus in addition to the linear momentum, kinetic energy also remains conserved before and after collision.

The total kinetic energy of the bodies involved before collision is equal to the total Kinetic energy after collision.

1. Collision in which total kinetic energy is conserved
2. Linear momentum is conserved.
3. Total mechanical energy is conserved.
4. It takes place under influence of conservative forces.
5. $e = 1$ and $\vec{v}_2 - \vec{v}_1 = \vec{u}_1 - \vec{u}_2$

Collisions between atomic, nuclear and fundamental particles only may be truly elastic.

Collisions between gross bodies for example, ivory or glass balls are approximately elastic.

(b) Perfectly Inelastic Collision

It is also known as Absolutely non-elastic collision.

A inelastic impact means an encounter between two bodies whereby these two bodies stick together after collision.

1. Maximum loss in kinetic energy takes place
2. Linear momentum is conserved
3. $e = 0$; $\vec{v}_2 = \vec{v}_1$ i.e. particle stick after collision .

Examples of perfectly inelastic impacts are :

(i) Collision between a bullet and its target when the bullet remains embedded in the target.

(ii) Collisions between clay spheres.

(iii) Collisions between oppositely charged ions resulting in the formation of a molecule.

(iv) Capture of an electron by a positive ion.

(v) Jumping of a man on to a moving trolley, etc.

The term completely inelastic collision does not mean that all the initial kinetic energy is lost. The kinetic energy loss in a completely inelastic collision is the maximum possible consistent with momentum conservation. An indeterminate amount of kinetic energy may be lost to heat, sound or other forms of energy.

(c) Inelastic collision : In an inelastic collision, the colliding bodies do not return to their original shape and size completely after collision and some part of the mechanical energy of the system goes to the deformation potential energy, And some part of energy may be lost to heat, sound or other forms of energy. Thus, only linear momentum remains conserved in case of an inelastic collision.

1. Kinetic energy is not conserved . Loss in K.E takes place
2. Linear momentum is conserved
3. Total energy is conserved
4. It takes place under influence of non conservative force.
5. $0 < e < 1$ $\quad \vec{v}_2 - \vec{v}_1 = e\,(\vec{u}_1 - \vec{u}_2)$

Majority of collisions fall in category in between the above two ideal or extreme cases and are called simply inelastic collisions. e.g. collisions of daily life.

PERFECTLY ELASTIC HEAD ON COLLISION

Collision of two elastic bodies moving in a straight line, known as head-on collision.

Before and after collision particles move along a straight line.

Before collision After collision

Let two particles of mass m_1 and m_2 are moving along a straight line with constant velocities u_1 and u_2 respectively. No external force or forces acting on them so the collide due to difference of their velocities only.

Let after collision their velocities are v_1 and v_2 respectively.

According to law of conservation of momentum

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \text{.........(1)}$$

or $m_1(v_1 - u_1) = -m_2(v_2 - u_2)$ ....(1a)

According to law of conservation of kinetic energy

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \qquad \text{........(2)}$$

or $m_1(v_1^2 - u_1^2) = -m_2(v_2^2 - u_2^2)$ ....(2a)

Divide eq^n. (2a) by eq^n. (1a)

$v_1 + u_1 = v_2 + u_2$ or $v_1 - v_2 = -(u_1 - u_2)$ ....(3)

| Velocity of separation after collision |

= | Velocity of approach before collision |

Solving eq^n. (3) & eq^n. (1) gives us

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + 2\left(\frac{m_2}{m_1 + m_2}\right) u_2 \qquad ....(4)$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right) u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 \qquad ....(5)$$

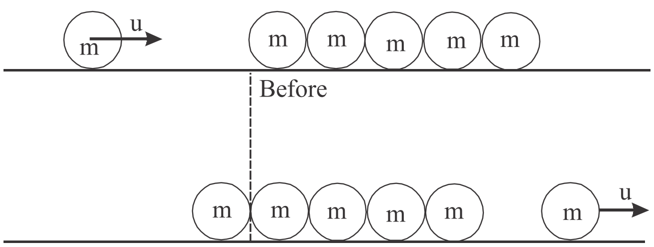
Special Cases :

Case-1 : Equal masses $m_1 = m_2$ when both the colliding bodies are of the same mass.

From eqns. (4) and (5) we get

$v_1 = u_2$ and $v_2 = u_1$.

Thus, if masses of bodies are equal, velocities after collision are interchanged. If the second particle is at rest, after collision first comes to rest and second moves with the velocity of the first.



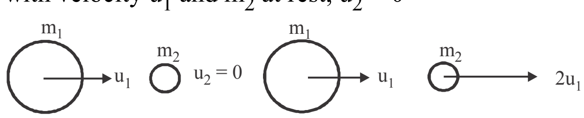
If target body m_2 is at rest i.e. $u_2 = 0$

then $v_1 = 0$ and $v_2 = u_1$

so, maximum transfer of energy and momentum takes place. There is hundred percent transfer of energy.

When a body suffers an elastic collision with another body of the same mass at rest, it is stopped dead whereas the second body starts moving with the same velocity as that of the first.

Case-2 : Massive projectile if $m_1 >> m_2$, and m_1 is moving with velocity u_1 and m_2 at rest, $u_2 = 0$



neglecting m_2 in compression with m_1

$m_1 + m_2 \approx m_1$ and $m_1 - m_2 \approx m_1$.

From eq^n. (4) $v_1 \approx \frac{m_1}{m_1} u_1$ or $v_1 = u_1$

So, the velocity of the massive body after collision is nearly equal to its velocity before collision. In other words, massive body loses only a very small fraction of its kinetic energy.

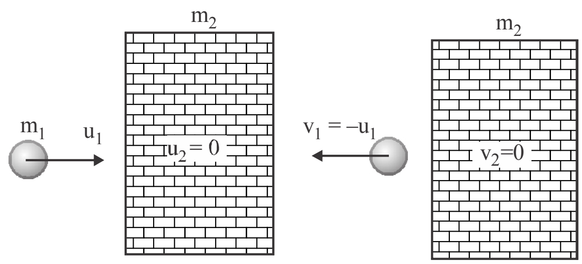
From eq^n. (5), $v_2 \approx \frac{2m_1}{m_1} u_1$ or $v_2 = 2u_1$

So, the velocity of the light body after collision is nearly double the velocity of the massive body before collision. When a massive body suffers an elastic collision with a stationary light body, there is practically no change in the velocity of the massive body but the light body acquires a velocity which is nearly double the initial velocity of massive body.

Case-3 : Massive target if $m_2 >> m_1$, and m_1 is moving with velocity u_1 and m_2 at rest, $u_2 = 0$

neglecting m_1 in compression with m_2

$m_1 + m_2 \approx m_2$ and $m_1 - m_2 \approx -m_2$.



From eq^n. (4), $v_1 \approx \frac{-m_2}{m_2} u_1$ or $v_1 = -u_1$

From eq^n. (5), $v_2 \simeq \frac{2m_1}{m_1 + m_2} u_1 = \frac{2m_1}{m_2} u_1$

So, the velocity of the massive body after collision is very small. or $v_2 \approx 0$ $\left(\because \frac{2m_1}{m_2} \approx 0\right)$

So, the velocity of the light body after collision is nearly equal (in magnitude) and opposite to its velocity before collision. In other words, the light body transfers only a small fraction of its kinetic energy to the massive body. When a light body suffers an elastic collision with a stationary massive body, the velocity of the light body is reversed and the massive body remains practically at rest. Example, A rubber ball thrown against a wall.

Example 34 :

Explain why heavy water which contains $_1H^2$ is more effective in slowing down neutrons in a reactor than heavy element like Pb^{206}.

Sol. Let u_1 and v_1 be the velocity of neutron with mass m before and after collision.

Initial KE of neutron $K_i = \frac{1}{2} m u_1^2$

Final KE of neutron $K_f = \frac{1}{2} m v_1^2$

Fractional loss of KE of neutron

$$f = \frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = 1 - \left(\frac{v_1}{u_1}\right)^2$$

$$f = 1 - \left(\frac{m-M}{m+M}\right)^2 = \frac{4mM}{(m+M)^2}$$

For lead, $f = \frac{4 \times 1 \times 206}{(1+206)^2} = 0.02 = 2\%$

For ${}_1H^2$, $f = \frac{4 \times 2}{(1+2)^2} = 0.89 = 89\%$

Neutrons loose only 2% of energy on collision with lead while it loses 89% of energy on collision with ${}_1H^2$.
So heavy water is a better moderator.

Example 35 :
A ball is moving with velocity 3 m/s towards a heavy wall moving towards the ball with speed 1 m/s. Assuming collision to be elastic, find the velocity of ball immediately after the collision.

Sol. The speed of wall will not change after the collision. So, let v be the velocity of the ball after collision. Since collision is elastic (e = 1),
Relative speed of separation = relative speed of approach

$v_2 - v_1 = (u_1 - u_2)$

$u_1 = 3$ m/s, $u_2 = -1$ m/s, $v_1 = ?$, $v_2 = -1$ m/s

or $-1 - v_1 = 3 - (-1)$ or $v_1 = -5$ m/s

Example 36 :
A ball of 0.1kg collides elastically with a ball of unknown mass at rest. If 0.1kg ball rebound at 1/3 of original speed find mass of other.

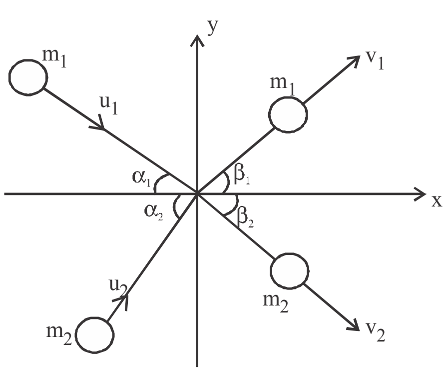
Sol. $v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2}$ (Given $m_1 = 0.1$ kg, $u_1 = u$ & $v_1 = \frac{-u}{3}$)

$$\frac{-u}{3} = \left(\frac{0.1 - m_2}{0.1 + m_2}\right)u$$

Solving $m_2 = 0.2$ kg

ELASTIC COLLISION IN TWO DIMENSIONS (OBLIQUE COLLISION)

If the colliding bodies do not move along the same straight line path, then the collision is said to be an oblique collision.



Particles move before and after collision in a plane.
Conserving momentum along X-axis,

$m_1u_1 \cos\alpha_1 + m_2u_2 \cos\alpha_2$
$= m_1v_1 \cos\beta_1 + m_2v_2 \cos\beta_2$ ....(1)

Y-axis,

$-m_1u_1 \sin\alpha_1 + m_2u_2 \sin\alpha_2 = m_1v_1 \sin\beta_1 - m_2v_2 \sin\beta_2$ ....(2)

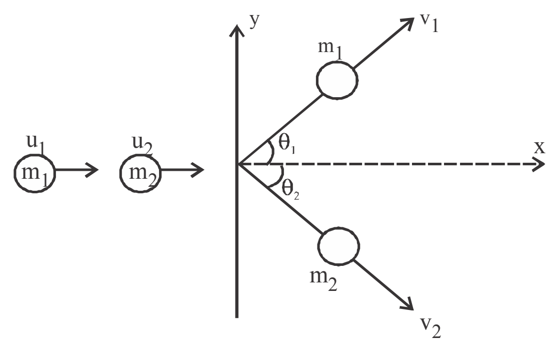
For elastic collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad ....(3)$$

With the help of these three eq^{n}s. we can determine unknown quantity.

Example 37 :
If a particle elastically collides obliquely with a particle of same mass at rest then show that they move perpendicular to each other after collision.

Sol.



Law of conservation of momentum

along x-axis $mu_1 + 0 = m v_1 \cos\theta_1 + mv_2 \cos\theta_2$

$u_1 = v_1 \cos\theta_1 + v_2 \cos\theta_2$. .....(1)

along y-axis $0 = v_1 \sin\theta_1 - v_2 \sin\theta_2$ ......(2)

From energy conservation

$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$u_1^2 = v_1^2 + v_2^2 \quad ......(3)$$

Using eq^n. (1) and eq^n. (2) in eq^n. (3) gives

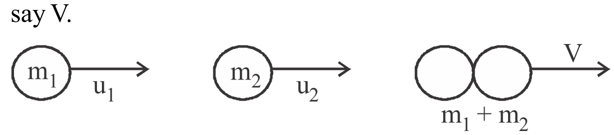
$2v_1v_2 \cos(\theta_1 + \theta_2) = 0$

$$\cos(\theta_1 + \theta_2) = \cos\frac{\pi}{2} \quad \therefore \quad \theta_1 + \theta_2 = \frac{\pi}{2}$$

i.e., The particles move perpendicular to each other after collision.

PERFECTLY INELASTIC COLLISION

Collision is said to be perfectly inelastic if both the particle stick together after collision and move with same velocity, say V.



Let two bodies of masses m_1 and m_2 are moving with velocities u_1 and u_2 before collision.

Total momentum before collision = $m_1 u_1 + m_2 u_2$.
After collision the two bodies stick together and have a common mass $(m_1 + m_2)$.
Total momentum after collision = $(m_1 + m_2) . V$
Applying conservation of momentum

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) . V$$

or $$V = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

Special Cases :

(a) If the motion of the bodies colliding head on is along a straight line, then after impact the bodies will follow the direction of the body having the originally larger momentum.

(b) If the momentum of the bodies are equal in magnitude, that is, $m_1u_1 = - m_2u_2$.
$V = 0$, that is, the colliding bodies come to a standstill.

(c) An inelastic collision is accompanied by a transformation of energy.
Kinetic energy of the system before collision is

$$K_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

Kinetic energy of the system after collision is

$$K_f = \frac{1}{2}(m_1 + m_2)\ V^2$$

Loss of Kinetic energy during collision is

$$\Delta K = K_i - K_f = \frac{1}{2}\left[m_1u_1^2 + m_2u_2^2 - \frac{(m_1u_1 + m_2u_2)^2}{(m_1 + m_2)}\right]$$

or $$\Delta K = \frac{1}{2}\left[\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2\right]$$

Since $(u_1 - u_2)^2$ is +ive, so ΔK is also positive.
As m_1 and m_2 are positive and square of a real quantity (number) is necessarily positive. Thus in an inelastic collision there is always a loss of kinetic energy.

(i) In case of mechanical collisions the loss in energy is converted into heat, sound etc., and

(ii) In case of atomic collisions it is converted into potential energy of the orbital electrons resulting into excitation of the atoms.

Special Case :

If the target is initially at rest, that is, $u_2 = 0$, then

Loss of kinetic energy, $\Delta K = \frac{1}{2}\left[\frac{m_1m_2}{m_1 + m_2}.u_1^2\right]$

But as $K_i = \frac{1}{2}m_1u_1^2$, so $\frac{\Delta K}{K_i} = \frac{m_2}{(m_1 + m_2)}$

Now if the target is massive, that is, $m_2 > m_1$.

$$\frac{\Delta K}{K_i} = 1 \qquad \text{or} \qquad \Delta K = K_i$$

If a light moving body strikes a heavy target at rest and sticks to it, practically all its Kinetic energy is lost.

INELASTIC COLLISION (HEAD ON)

Particles do not stick after collision in an inelastic collision.
Let two particles of masses m_1 and m_2 are moving with velocities u_1 and u_2 respectively
The final velocities of m_1 and m_2 after collision are v_1 and v_2 and coefficient of restitution is e.
On application of law of conservation of momentum

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \qquad ......(1)$$

According to Newton's Law of impact

$$v_2 - v_1 = e\,(u_1 - u_2) \qquad ......(2)$$

From eq^n. (1) – m_2 × eq^n. (2)

$$v_1 = \frac{(m_1 - em_2)}{m_1 + m_2}u_1 + \frac{m_2(1+e)}{m_1 + m_2}u_2$$

From eq^n. (1) + m_1 × eq^n. (2)

$$v_2 = \frac{m_1(1+e)}{m_1 + m_2}u_1 + \frac{(m_2 - em_1)}{m_1 + m_2}u_2$$

Loss in kinetic energy

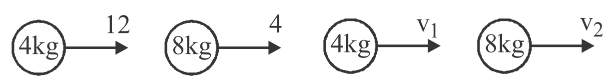
$$\Delta K = K_i - K_f = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad \Delta K = \frac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2(1 - e^2)$$

Example 38 :

A ball of mass 4 kg moving with a velocity of 12 m/s impinges directly on another ball of mass 8 kg moving with a velocity of 4 m/s in the same direction. Find their velocities after impact if e = 0.5.

Sol.



Just before collision Just after collision

$u_1 = 12$ m/s, $m_1 = 4$ kg, $u_2 = 4$m/s, $m_2 = 8$ kg
Let v_1 and v_2 be the velocity after impact.
By conservation of momentum:

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$
$$\Rightarrow 4v_1 + 8v_2 = 80 \qquad .... (1)$$

By Newton's experimental Law:

$$v_2 - v_1 = e\,(u_1 - u_2)$$
$$v_2 - v_1 = 0.5\,(12 - 4) = 4$$

Solving (1) and (2), we get:

$v_1 = 4$ m/s and $v_2 = 8$ m/s

Example 39 :

A ball is dropped from a height h on the floor. In each collision its speed becomes e times of its striking value.

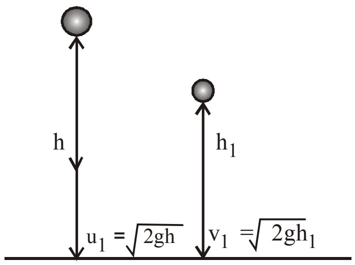
(a) find total distance covered by ball

(b) find time taken to stop rebounding

Sol. (a) $v_2 - v_1 = e\,(u_1 - u_2)$ from newton's law of impact

$0 - (-v_1) = e\,(u_1 - 0)$

$v_1 = eu_1$



$\sqrt{2gh_1} = e\sqrt{2gh}$

$h_1 = e^2h,\ h_2 = e^2h_1 = e^4h \ ......\ h_n = e^{2n}\, h$

Total height $H = h + 2h_1 + 2h_2 + ...$

$= h + 2e^2h + 2e^4h + 2e^6h + ...$

$= h\,[1 + 2e^2\,(1 + e^2 + e^4 + ...)]$

$$H = h\left[1 + 2e^2\left(\frac{1}{1-e^2}\right)\right] = h\left[\frac{1+e^2}{1-e^2}\right]$$

(b) total time $T = t_0 + 2t_1 + 2t_2 + ...$

$$t_0 = \sqrt{\frac{2h}{g}},\ t_1 = \sqrt{\frac{2h_1}{g}} = e\sqrt{\frac{2h}{g}}\ ....\ t_n = e^n\sqrt{\frac{2h}{g}}$$

$$T = \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + ...$$

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}(1 + e + e^2 + ...)$$

$$\text{total time } T = \sqrt{\frac{2h}{g}}\left[1 + 2e\left(\frac{1}{1-e}\right)\right] = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

TRY IT YOURSELF-6

Q.1 Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of v/3 in the same direction. What type of collision has occured?

(A) inelastic
(B) elastic
(C) completely inelastic
(D) Cannot be determined from the information given

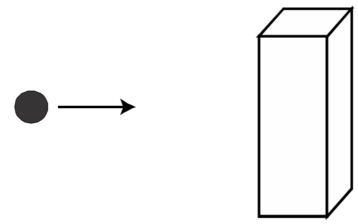
Q.2 A ball strikes a smooth horizontal ground at an angle of 45° with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision.
(Assume $e \le 1$).

(A) 45° (B) 30°
(C) 53° (D) 60°

Q.3 A bullet of mass 0.04 kg travelling horizontally at 100 ms^{-1} hits a stationary block of wood of mass 8 kg, passes through it and emerges horizontally with a speed of 40 ms^{-1}. If the block is free to move on a smooth horizontal plane find the speed with which it is moving after the bullet has passes through it.

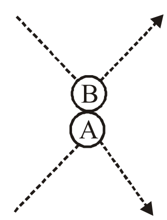
Q.4 Two particles, each of mass m, collide head on when their speeds are 2u and u. If they stick together on impact, find their combined speed in terms of u.

Q.5 Two small, uniform balls of identical density and size are fired from a toy gun toward a wooden block. Ball A is highly elastic and bounces backward after striking the block. Ball B is made of clay and sticks to the wooden block upon impact. Which of the following statements best describes the effects of the collision with the block?

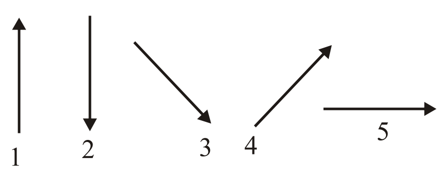


(A) Ball A transfers more momentum and more energy to the block than Ball B.
(B) Ball A transfers more momentum and less energy to the block than Ball B.
(C) Ball A transfers less momentum and more energy to the block than Ball B.
(D) Ball A transfers less momentum and less energy to the block than Ball B.

Q.6 The figure below depicts the paths of two colliding steel balls, A and B.



Which of the arrows 1-5 best represents the impulse applied to ball B by ball A during the collision? Explain your answer.



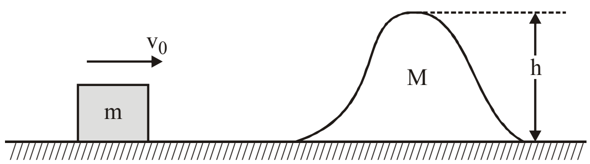
Q.7 Cart A is at rest. An identical cart B, moving to the right, collides inelastically with cart A. They stick together. After the collision, which of the following is true?

(A) Carts A and B are both at rest.
(B) Carts A and B moves to the right with speed greater than Cart B's original speed.
(C) Carts A and B move to the right with a speed less than cart B's original speed.
(D) Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.

Q.8 Two balls that are dropped from a height h_i above the ground, one on top of the other. Ball 1 is on top and has mass m_1, and ball 2 is underneath and has mass m_2 with $m_2 >> m_1$. Ball 2 first collides with the ground and rebounds with speed v_0 then, as ball 2 starts to move upward, it collides elastically with the ball 1 which is still moving downwards also with speed v_0. The final relative speeds after ball 1 and ball 2 collide is –

(A) Zero (B) v_0
(C) $2v_0$ (D) $3v_0$

Q.9 For the system shown in the figure, a small block of mass m and a smooth irregular shaped block of mass M, both free to move, are placed on a smooth horizontal plane. Find the minimum velocity v_0 imparted to block so that it will overcome the highest point of M.



Q.10 A block of mass 1.2 kg moving at a speed of 20 cm/sec collides head on with a similar block kept at rest. The coefficient of restitution is 0.6. Find the loss of kinetic energy during collision.

ANSWERS

(1)	(B)	**(2)**	(B)	**(3)**	0.3 m/s
(4)	u/2	**(5)**	(B)	**(6)**	(1)
(7)	(C)	**(8)**	(C)	**(9)**	$\sqrt{2gh\left(1+\frac{m}{M}\right)}$
(10)	7.7×10^{-3}J				

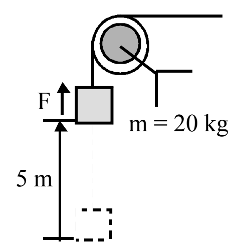
IMPORTANT POINTS

1. If a light body and a heavy body have equal kinetic energy, then heavy body has greater momentum.
2. If a body moves with constant power, distance travelled by it (s) in a time (t) is related by the equation $s \propto t^{3/2}$ and velocity (v) is related to distance travelled (x) by the formula $v \propto x^{3/2}$.
3. Potential energy depends on frame of reference but change in potential energy is independent of reference frame.
4. $\Sigma\, K = \Sigma\, \Delta K$, $\Sigma\, W \Rightarrow$ total work due to all kinds of forces, $\Sigma\, \Delta K \Rightarrow$ total change in kinetic energy.
5. $\Sigma W_{conservative} = -\Sigma\, \Delta U$; $\Sigma W_{conservative} \Rightarrow$ Total work due to all kinds of conservative forces.
 $\Sigma\, \Delta U \Rightarrow$ Total change in all kinds of potential energy.
6. $W_{nonconservative} + W_{pseudo} + W_{other} = \Delta K - W_{cons.} = \Delta K + \Delta U$
 $W_{nonconservative} + W_{pseudo} + W_{other} = \Delta E$
 [ΔE = change in mechanical energy)
7. $W_{conservative} + W_{nonconservative} + W_{pseudo} + W_{other} = \Delta K$
 If $W_{nonconservative} + W_{pseudo} + W_{other} = 0$
 then $W_{conservative} = \Delta K$
 $-\Delta U = \Delta K$; $\Delta U + \Delta K = 0$; $U + K =$ constant
 E = constant (Conservation of mechanical energy)
8. For a conservative force in one dimension, we may define a potential energy function U (x) such that
 $$F(x) = -\frac{dU(x)}{dx} \quad \text{or} \quad U_i - U_f = \int_{x_i}^{x_f} F(x)\,dx$$
9. Impulse : $\Delta\vec{p} = \vec{F}\Delta t,\ p_2 - p_1 = \int_1^2 F\,dt$
10. During a collision : (a) the total linear momentum is conserved at each instant of the collision ; (b) the kinetic energy conservation (even if the collision is elastic) applies after the collision is over and does not hold at every instant of the collision. In fact the two colliding objects are deformed and may be momentarily at rest with respect to each other.
11. $e = -\dfrac{\text{velocity of separation}}{\text{velocity of approach}}$

ADDITIONAL EXAMPLES

Example 1 :

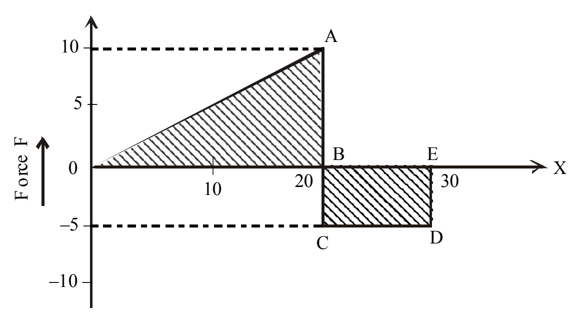
Calculate the work done when a mass of 20 kg is lifted vertically upward through a distance of 5m.



Sol. $F = mg = 20 \times 9.8$ N ; $d = 5$ m
$\theta = 0^o$ $\cos 0^o = 1$
$W = F\, d \cos\theta$
$= 20 \times 9.8 \times 5 \times 1 = 980$ J

Example 2 :

Calculate work done from the graph.



Sol. Work done=area of triangle (OAB)–area of rectangle (BCDE)

$$W = \frac{1}{2} \times 20 \times 10 - 10 \times 5 = 50 \text{ J}$$

Example 3 :

Determine the average force necessary to stop a bullet of mass 20 g and speed 250 ms^{-1} as it penetrates wood to a distance of 12 cm.

Sol. If F newton be the retarding force, then the work done by force is given by $W = F \times S = F \times 0.12$ joule

$$\text{Loss of KE} = \frac{1}{2} \times \frac{20}{1000} \times 250 \times 250 \text{ joule} = 625 \text{ joule}$$

(This kinetic energy is consumed in stopping the bullet and is converted into heat energy)

Applying work-energy theorem, $F \times 0.12 = 625$

$$\text{or} \quad F = \frac{625}{0.12} N = 5.2 \times 10^3 N$$

It is interesting to note that the retarding force is nearly 30,000 times the weight of the bullet.

Example 4 :

What is the stopping distance for a vehicle of mass m moving with speed v along a level road, if the coefficient of friction between the tyres and the road is μ?

Sol. When the vehicle of mass m is moving with velocity v, the kinetic energy of the vehicle $K = (1/2)mv^2$ and if s is the stopping distance, work done by friction

$W = f s \cos\theta = \mu m g s \cos 180 = -\mu m g s.$

So by Work-Energy Theorem, $W = \Delta K = K_F - K_I$

$$\text{i.e.,} \quad -\mu m g s = 0 - \frac{1}{2}mv^2 \quad \text{or} \quad s = \frac{v^2}{2\mu g}$$

Example 5 :

A pump is used to deliver water at a certain rate from a given pipe. To obtain n times water from same pipe in the same time by what amount should power of motor be increased.

Sol. $\frac{dm}{dt} = Av\rho$; v = velocity of flow, A is area of cross-section

ρ = density of liquid.

$$\text{to get n times water in same time} \left(\frac{dm}{dt}\right)' = n\frac{dm}{dt}$$

$$A v'P = n A v \rho \Rightarrow v' = nv. \quad \text{Since,} \quad P = v^2 \frac{dm}{dt}$$

$$\text{So} \quad \frac{P'}{P} = \frac{v'^2 (dm/dt)}{v^2 (dm/dt)} = \frac{n^2 v^2 n \, dm/dt}{v^2 \, dm/dt} = n^3$$

or $P' = n^3 P$

Thus to get n times water the power must be increased n^3 times.

Example 6 :

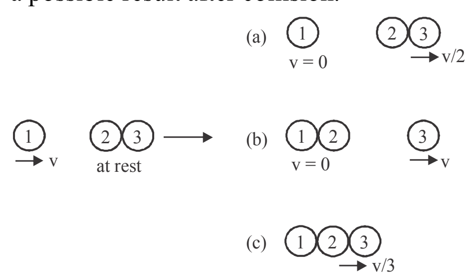
A bullet of mass 0.2 kg is moving with velocity of 1000m/s and passes through a suspended block of mass 10 kg. It comes out with half its initial velocity. Find height gained by the block.

Sol. $$h = \frac{m^2 (v - v_1)^2}{2gM^2} \quad v_1 = \frac{v}{2} = 500 \text{ m/s}$$

$$h = \frac{(0.02)^2 (1000 - 500)^2}{2 \times 10 \times 10 \times 10} = 0.05\text{m}$$

Example 7 :

Two identical balls kept in contact with each other are at rest. They are hit head on by another identical ball of same mass moving with a speed v. If collision in elastic which is a possible result after collision.



Sol. **(b).** Kinetic energy of system before collision

$$K_i = \frac{1}{2}mv^2 + 0 + 0 = \frac{1}{2}mv^2$$

$$\text{for (a) KE after collision} = 0 + \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$$

$$\text{for (b) KE after collision} = 0 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$$

$$\text{for (c) KE after collision} = \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{1}{6}mv^2$$

In elastic collision KE is conserved so (b) is only possible result of collision.

Example 8 :

After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the particles becomes half the initial speed. Find the angle between the two before collision.

Sol. Let θ be the desired angle. Linear momentum of the system will remain conserved. Resultant initial momentum

$$P^2 = P_1^2 + P_2^2 + 2P_2 P_2 \cos\theta$$

As per conservation of momentum

$$\left\{2m\left(\frac{v}{2}\right)\right\}^2 = (mv)^2 + (mv)^2 + 2(mv)(mv)\cos\theta$$

$$\text{or} \quad 1 = 1 + 1 + 2\cos\theta \quad \text{or} \quad \cos\theta = -\frac{1}{2} \quad \therefore \theta = 120°$$

Example 9 :

A spring is fixed at the bottom end of an incline of inclination 37°. A small block is released from rest on an incline from a point 4.8 m away from the spring. The block compresses the spring by 20cm, stops momentarily and then rebounds through a distance of 1m up the incline. Find (a) the friction coefficient between the plane and the block and (b) the spring constant of the spring. [Take g = 10 m/s^2]

Sol.

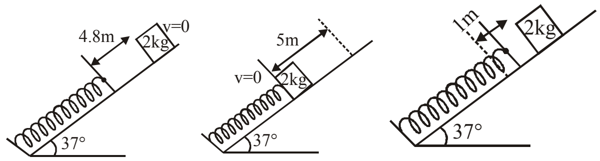


Fig (a) Just after release   Fig (b) When stopped for the first time   Fig (c) When stopped for the second time

Applying work energy theorem for motion from (a) to (b)

$$W_{gravity} + W_{friction} + W_{spring} = \Delta\, KE = \frac{1}{2} m\,(0-0) = 0$$

$$\therefore \quad 20 \times 5 \sin 37° - \mu\,(20 \cos 37°)\, 5 - \frac{1}{2}\, k\,[(0.2)^2 - 0] = 0 \qquad \text{..........(1)}$$

Applying work energy for motion form (b) to (c)

$$-20 \times 1 \times \sin 37° - \mu\,(20 \cos 37°) \times 1 - \frac{1}{2} k\,[0 - (0.2)^2] = 0 \qquad \text{..........(2)}$$

Adding equation (1) and (2)

$$-20\,(5-1) \times \frac{3}{5} - \mu\,(20 \times \frac{4}{5})\,(5+1) = 0$$

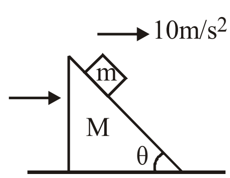
$\Rightarrow \quad \mu = 0.5$

Putting this value in equation (1), we get $k = 1000$ N/m

Here velocity is maximum at equilibrium since before this, spring force was less than the weight of the block and the block was accelerating and after this, the spring force is greater than the weight thus retarding the block to zero velocity upto the lowest position.

Example 10 :

In the figure shown all the surfaces are frictionless, and mass of the block, m = 1kg. The block and wedge are held initially at rest.



Now wedge is given a horizontal acceleration of 10 m/s^2 by applying a force on the wedge, so that the block does not slip on the wedge. Find work done by the normal force in ground frame on the block in $\sqrt{3}$ seconds

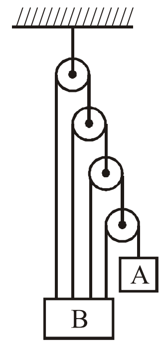
Sol. $W_N = N \cos 45° \times \left(\frac{1}{2} at^2\right)$

$$N = \frac{20}{\sqrt{2}} N \; ; a = 10 \text{ m/s}^2, \; t = \sqrt{3} \text{ sec}.$$

$W_N = 150$ Joule

Example 11 :

The velocity of block A of the system shown in figure is V_A at any instant. Calculate velocity of block B at that instant.

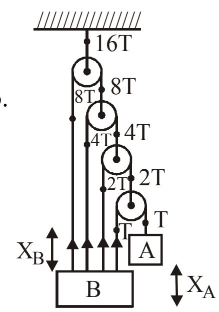


Sol. Work done by internal tension is zero.

$\therefore \quad 15T \times X_B - T \times X_A = 0$

$X_A = 15X_B$

$\therefore \quad V_A = 15V_B$



Example 12 :

A truck moving on horizontal road towards east with velocity 20 m/s collides elastically with a light ball moving with velocity 25 m/s along west. The velocity of the ball just after collision –

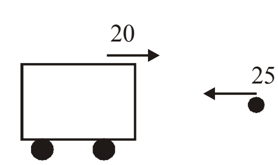
(A) 65 m/s towards east (B) 25 m/s towards west

(C) 65 m/s towards west (D) 20 m/s towards east

Sol. **(A).** Let ball-1 truck-2

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2$$

$$= \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} u_1 + \frac{2}{\frac{m_1}{m_2} + 1} u_2$$



Since, $m_1 << m_2 \Rightarrow \frac{m_1}{m_2} << 1$

$\Rightarrow \quad v_1 = -u_1 + 2u_2 = -(-25) + 2\,(20) = 65$

QUESTION BANK

CHAPTER 6 : WORK, ENERGY, POWER AND CONSERVATION LAWS

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : WORK

Q.1 A man pushes a wall and fails to displace it. He does –
(A) Negative work
(B) Positive but not maximum work
(C) No work at all
(D) Maximum work

Q.2 A force of $(3\hat{i}+4\hat{j})$ Newton acts on a body and displaces it by $(3\hat{i}+4\hat{j})$ m. The work done by the force is
(A) 10 J (B) 12 J
(C) 16 J (D) 25 J

Q.3 A body is acted upon by a force $\vec{F}=-\hat{i}+2\hat{j}+3\hat{k}$. The work done by the force in displacing it from (0,0,0) to (0,0,4m) will be -
(A) 12 J (B) 10 J
(C) 8 J (D) 6 J

Q.4 The work done in pulling a body of mass 5 kg along an inclined plane (angle 60º) with coefficient of friction 0.2 through 2 m, will be -
(A) 98.08 J (B) 94.67 J
(C) 90.08 J (D) 91.08 J

Q.5 A force $\vec{F}=5\hat{i}+6\hat{j}-4\hat{k}$ acting on a body, produces a displacement $\vec{s}=6\vec{i}+5\vec{k}$. Work done by the force is
(A) 18 units (B) 15 units
(C) 12 units (D) 10 units

Q.6 A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is –
(A) + 2000 J (B) – 200 J
(C) zero (D) – 20,000 J

Q.7 A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}s^{-1}$. The work done by the net force during its displacement from x = 0 to x = 2 m is –
(A) 1.5 J (B) 50 J
(C) 10 J (D) 100 J

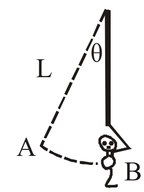
Q.8 A particle is displaced from point A (1, 2) to B (3, 4) by applying force $\vec{F}=2\hat{i}+3\hat{j}$. Find the work done by $\vec{F}$ to move the particle from point A to B.
(A) 10 units (B) 12 units
(C) 8 units (D) 6 units

Q.9 A block of mass 5 kg is being raised vertically upwards by the help of a string attached to it. It rises with an acceleration of 2 m/s^2. Find the work done by the tension in the string if the block rises by 2.5 m.
(A) 147.5 J (B) 125.5 J
(C) 127.5 J (D) 107.5 J

Q.10 In the above question find the net work done on the block–
(A) 15 J (B) 25 J
(C) 122.5 J (D) 147.5 J

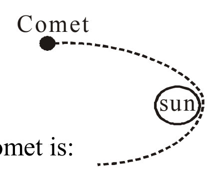
Q.11 When a person walks, the force of friction between the floor and the person's feet accelerates the person forward. The floor does
(A) +ve work on the person. (B) –ve work on the person.
(C) No work on the person. (D) None of these

Q.12 A person swings down on an inextensible rope that is attached to a fixed point. The rope exerts a tension T on the person. The work done by tension on the person as she moves from A to B is –



(A) T (B) TL
(C) TLθ (D) zero.

Q.13 A comet is speeding along a hyperbolic orbit toward the Sun. While the comet is moving away from the Sun, the work done by the Sun on the comet is:



(A) positive (B) zero
(C) negative (D) Can't say

PART - 2 : ENERGY

Q.14 A body of mass 10kg at rest is acted upon simultaneously by two forces 4 N and 3N at right angles to each other. The kinetic energy of the body at the end of 10 sec is –
(A) 100 J (B) 300 J
(C) 50 J (D) 125 J

Q.15 Two springs of spring constants 1500 N/m and 3000 N/m respectively are stretched with the same force. They will have potential energy in the ratio –
(A) 4 : 1 (B) 1 : 4
(C) 2 : 1 (D) 1 : 2

Q.16 A spring with spring constant k when stretched through 1cm, the potential energy is U. If it is stretched by 4 cm. The potential energy will be
(A) 4U (B) 8U
(C) 16 U (D) 2U

Q.17 A body at rest may have –
(A) Energy (B) Momentum
(C) Speed (D) Velocity

Q.18 The kinetic energy possessed by a body of mass m moving with a velocity v is equal to (1/2) mv^2, provided
(A) The body moves with velocities comparable to that of light.
(B) The body moves with velocities negligible compared to the speed of light.
(C) The body moves with velocities greater than that of light.
(D) None of the above statement is correct.

Q.19 If the momentum of a body is increased n times, its kinetic energy increases
(A) n times (B) 2n times
(C) $\sqrt{n}$ times (D) n^2 times

Q.20 If the K.E. of a particle is doubled, then its momentum will
(A) Remain unchanged (B) Be doubled
(C) Be quadrupled (D)Increase $\sqrt{2}$ times

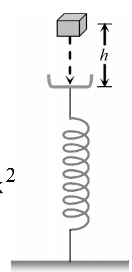
Q.21 If the stone is thrown up vertically and return to ground, its potential energy is maximum –
(A) During the upward journey
(B) At the maximum height
(C) During the return journey
(D) At the bottom

Q.22 A body of mass 2 kg is projected vertically upwards with a velocity of 2 m/sec. The K.E. of the body just before striking the ground is
(A) 2 J (B) 1 J
(C) 4 J (D) 8 J

Q.23 The decrease in the potential energy of a ball of mass 20 kg which falls from a height of 50 cm is
(A) 968 J (B) 98 J
(C) 1980 J (D) None of these

Q.24 An object of 1 kg mass has a momentum of 10kg m/sec then the KE of the object will be –
(A) 100 J (B) 50 J
(C) 1000 J (D) 200 J

Q.25 A block of mass m initially at rest is dropped from a height h on to a spring of force constant k. the maximum compression in the spring is x then



(A) $mgh = \frac{1}{2}kx^2$ (B) $mg(h+x) = \frac{1}{2}kx^2$
(C) $mgh = \frac{1}{2}k(x+h)^2$ (D) None of these

Q.26 Two springs have their force constant as k_1 and k_2 $(k_1 > k_2)$. When they are stretched by the same force
(A) No work is done in case of both the springs.
(B) Equal work is done in case of both the springs.
(C) More work is done in case of second spring.
(D) More work is done in case of first spring.

Q.27 Two bodies of masses m_1 and m_2 have equal kinetic energies. If p_1 and p_2 are their respective momentum, then ratio $p_1 : p_2$ is equal to
(A) $m_1 : m_2$ (B) $m_2 : m_1$
(C) $\sqrt{m_1} : \sqrt{m_2}$ (D) $m_1^2 : m_2^2$

Q.28 In which case does the potential energy decrease
(A) On compressing a spring
(B) On stretching a spring
(C) On moving a body against gravitational force
(D) On the rising of an air bubble in water

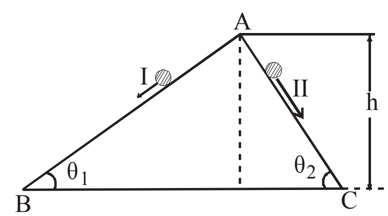
Q.29 Two bodies with kinetic energies in the ratio of 4 : 1 are moving with equal linear momentum. The ratio of their masses is –
(A) 1 : 2 (B) 1 : 1 (C) 4 : 1 (D) 1 : 4

Q.30 A ball of mass 2kg and another of mass 4kg are dropped together from a 60 feet tall building. After a fall of 30 feet each towards earth, their respective kinetic energies will be in the ratio of
(A) $\sqrt{2}:1$ (B) 1 : 4
(C) 1 : 2 (D) $1:\sqrt{2}$

Q.31 A tennis ball dropped from a height of 2 m rebounds only 1.5 metre after hitting the ground. What fraction of energy is lost in the impact -
(A) 1/2 (B) 1/4
(C) 1/8 (D) 1/16

Q.32 If the force acting on a body is inversely proportional to its speed, the kinetic energy of the body is –
(A) constant
(B) directly proportional to time
(C) inversely proportional to time
(D) directly proportional to the square of time

Q.33 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in Fig. Which of the following statement is correct?



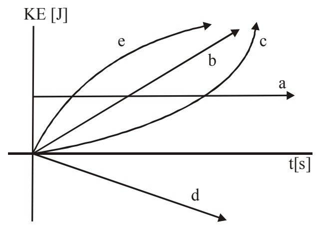
(A) Both the stones reach the bottom at the same time but not with the same speed.
(B) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.
(C) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.
(D) Both the stones reach the bottom at different times and with different speeds.

Q.34 Compared to the amount of energy required to accelerate a car from rest to 10 miles per hour, the amount of energy required to accelerate the same car from 10 mph to 20 mph is
(A) the same (B) twice as much
(C) three times as much (D) four times as much

Q.35 The same horizontal force, of magnitude F, is applied to two different blocks, of mass m and 3m respectively. The blocks move on a frictionless surface and both blocks begin from rest. If the force is applied for the same time to each block, which one of the following sentences is true?
(A) The heavier block acquires 9 times as much kinetic energy as the lighter block.
(B) The heavier block acquires 3 times as much kinetic energy as the lighter block.

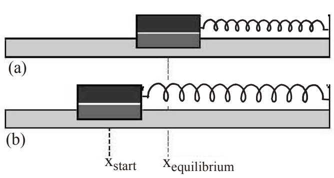
(C) The two blocks acquire the same kinetic energy.
(D) The lighter block acquires 3 times as much kinetic energy as the heavier block.

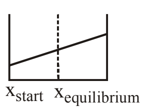
Q.36 An object is dropped to the earth from a height of 10m. Which of the following graphs of kinetic energy vs. time best represent the kinetic energy of the object as it approaches the earth (neglect friction).

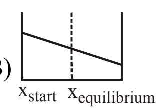


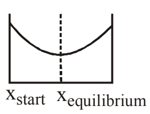
(A) a (B) b
(C) c (D) d

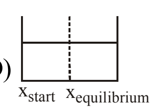
Q.37 In part (a) of the figure, an air track cart attached to a spring rests on the track at the position $x_{equilibrium}$ and the spring is relaxed. In (b), the cart is pulled to the position x_{start} and released. It then oscillates about $x_{equilibrium}$. Which graph correctly represents the potential energy of the spring as a function of the position of the cart?



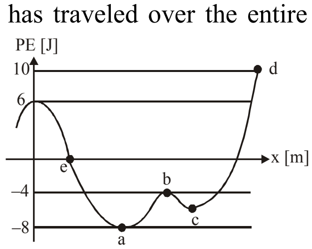
(A)


(B)


(C)


(D)


Q.38 Consider the following sketch of potential energy for a particle as a function of position. There are no dissipative forces or internal sources of energy. What is the minimum total mechanical energy that the particle can have if you know that it has traveled over the entire region of X shown?



(A) –8
(B) 6
(C) 10
(D) It depends on direction of travel

PART - 3 : POWER

Q.39 A sports car accelerates from zero to a certain speed in t seconds. How long does it take for it to accelerate to twice that speed starting from rest, assuming the power of the engine to be constant (independent of velocity) and neglecting any resistance to motion ?
(A) $\sqrt{2}$ t sec. (B) 2t sec.
(C) 3t sec. (D) 4t sec.

Q.40 A force of $2\hat{i}+3\hat{j}+4\hat{k}$ N acts on a body for 4second, produces a displacement of $(3\hat{i}+4\hat{j}+5\hat{k})$m.. The power used is
(A) 9.5 W (B) 7.5 W
(C) 6.5 W (D) 4.5 W

Q.41 Power supplied to a particle of mass 2 kg varies with time as $P=\dfrac{3t^2}{2}$ W . Here t is in second. Velocity of particle at t = 0 is v = 0. The velocity of particle at time t = 2s will be –
(A) 1 m/s (B) 4 m/s
(C) 2 m/s (D) $2\sqrt{2}$ m/s

Q.42 From an automatic gun a man fires 360 bullet per minute with a speed of 360 km/hour. If each weighs 20 g, the power of the gun is
(A) 600W (B) 300W
(C) 150 W (D) 75W

Q.43 A weight lifter lifts 300 kg from the ground to a height of 2m in 3 second. The average power generated by him is
(A) 5880 watt (B) 4410 watt
(C) 2205 watt (D) 1960 watt

Q.44 Force acting on a particle moving in a straight line varies with the velocity v of the particle as F = K/v, where K is a constant. The work done by this force in time t is –
(A) Kt/v^2 (B) 2Kt
(C) Kt (D) $2Kt/v^2$

Q.45 The units of power could be which of the following –
6(i) Joules/sec (ii) Watts/sec.
(iii) $Kg.m^2/s^3$. (iv) Nt.m/s.
(A) i and ii (B) ii and iii
(C) i and iv (D) i and iii

Q.46 An engine pumps up 1000 kg of coal from a mine 100m deep in 50sec. The pump is running with diesel and efficiency of diesel engine is 25%. Then its power consumption will be (g = 10m/sec^2):
(A) 10 kW (B) 80 kW
(C) 20 kW (D) 24 kW

Q.47 A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to –
(A) $t^{1/2}$ (B) t
(C) $t^{3/2}$ (D) t^2

PART - 4 : COLLISIONS

Q.48 A heavy steel ball of mass greater than 1 kg moving with a speed of 2m/sec collides head on with a stationary ping-pong ball of mass less than 0.1 gm. The collision is elastic. After the collision the ping-pong ball moves approximately with speed –

(A) 2 m/sec (B) 4 m/sec
(C) 2×10^4 m/sec (D) 2×10^3 m/sec

Q.49 A billiard ball moving with a speed of 5 m/s collides with an identical ball originally at rest. If the first ball stops after collision, then the second ball will move forward with a speed of –

(A) 10 m/s (B) 5 m/s
(C) 2.5 m/s (D) 1.0 m/s

Q.50 Two equal masses m_1 and m_2 moving along the same straight line with velocities + 3 m/s and – 5 m/s respectively collide elastically. Their velocities after the collision will be respectively

(A) + 4 m/s for both (B) –3 m/s & +5 m/s
(C) – 4 m/s & + 4 m/s (D) –5 m/s & +3 m/s

Q.51 A ball of mass 10 kg is moving with a velocity of 10 m/s. It strikes another ball of mass 5 kg which is moving in the same direction with a velocity of 4 m/s. If the collision is elastic, their velocities after the collision will be, respectively

(A) 6 m/s, 12 m/s (B) 12 m/s, 6 m/s
(C) 12 m/s, 10 m/s (D) 12 m/s, 25 m/s

Q.52 A neutron makes a head-on elastic collision with a stationary deuteron. The fractional energy loss of the neutron in the collision is

(A) 16/81 (B) 8/9
(C) 8/27 (D) 2/3

Q.53 A mass of 10 gm moving with a velocity of 100cm/s strikes a pendulum bob of mass 10gm. The two masses stick together. The maximum height reached by the system now is

(A) Zero (B) 5 cm
(C) 2.5 cm (D) 1.25 cm

Q.54 In an inelastic collision-

(A) momentum is conserved but kinetic energy is not.
(B) momentum is not conserved but kinetic energy is conserved.
(C) neither momentum nor kinetic energy is conserved.
(D) both the momentum and kinetic energy are conserved.

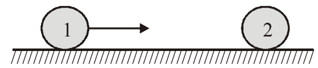
Q.55 Which of the following hold when two particles of masses m_1 and m_2 undergo elastic collision?

(A) when $m_1 = m_2$ and m_2 is stationary, there is maximum transfer of kinetic energy in head an collision.
(B) when $m_1 = m_2$ and m_2 is stationary, there is maximum transfer of momentum in head on collision .
(C) when $m_1 >> m_2$ and m_2 is stationary, after head on collision m_2 moves with twice the velocity of m_1.
(D) All of these

Q.56 Two balls at the same temperature collide inelastically. Which of the following is not conserved-

(A) kinetic energy (B) velocity
(C) temperature (D) momentum

Q.57 Ball 1 collides with an another identical ball 2 at rest as shown in figure. For what value of coefficient of restitution e, the velocity of second ball becomes two times that of 1 after collision.



(A) 1/3 (B) 1/2
(C) 1/4 (D) 1/6

Q.58 A body of mass 2 kg moving with a velocity of 3 m/sec collides head on with a body of mass 1kg moving in opposite direction with a velocity of 4 m/sec. After collision, two bodies stick together and move with a common velocity which in m/sec is equal to –

(A) 1/4 (B) 1/3
(C) 2/3 (D) 3/4

Q.59 A ball of mass m moving with a certain velocity collides against a stationary ball of mass m. The two balls stick together during collision. If E be the initial kinetic energy, then the loss of kinetic energy in the collision is

(A) E (B) E/2
(C) E/3 (D) E/4

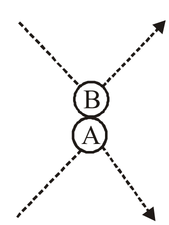
Q.60 Two billiard balls undergo a head-on collision. Ball l is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of v/3 in the same direction. What type of collision has occurred?

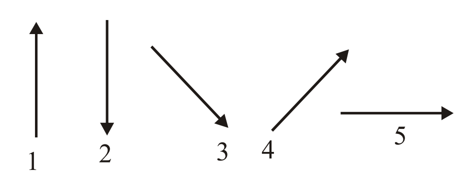
(A) inelastic
(B) elastic
(C) completely inelastic
(D) Cannot be determined from the information given

Q.61 A ball strikes a smooth horizontal ground at an angle of 45° with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision. (Assume $e \le 1$).

(A) 45° (B) 30°
(C) 53° (D) 60°

Q.62 The figure below depicts the paths of two colliding steel balls, A and B. Which of the arrows 1-5 best represents the impulse applied to ball B by ball A during the collision?





(A) 1 (B) 2
(C) 3 (D) 4

Q.63 Cart A is at rest. An identical cart B, moving to the right, collides inelastically with cart A. They stick together. After the collision, which of the following is true?

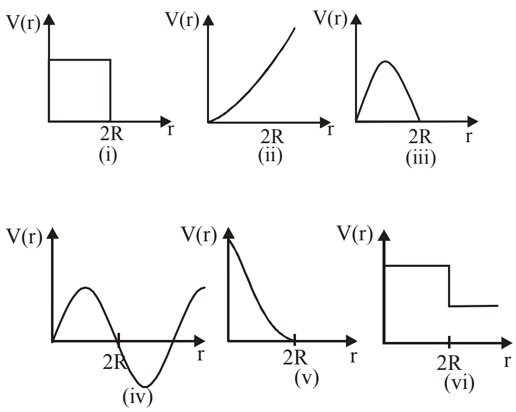
(A) Carts A and B are both at rest.

(B) Carts A and B moves to the right with speed greater than Cart B's original speed.

(C) Carts A and B move to the right with a speed less than cart B's original speed.

(D) Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.

Q.64 Which of the following potential energy curves in figure cannot possibly describe the elastic collision of two billiard balls ? Here r is the distance between centres of the balls.



(A) (i), (ii), (iii), (iv) and (vi)

(B) (i), (ii), (iii), (v), and (vi)

(C) (iii), (iv), and (vi)

(D) (i), (ii), and (vi)

PART - 5 : MISCELLANEOUS

Q.65 Due to retarding force train stops after 80 m. If the speed is doubled, then the distance in which train stops by applying same retarding force is –

(A) The same (B) Doubled

(C) Halved (D) Four times

Q.66 A cylinder of mass 10kg is sliding on a plane with an initial velocity of 10m/s. If coefficient of friction between surface and cylinder is 0.5, then before stopping it will describe –

(A) 12.5 m (B) 5 m

(C) 7.5 m (D) 10 m

Q.67 A 50kg man with 20kg load on his head climbs up 20 steps of 0.25m height each. The work done in climbing is –

(A) 5 J (B) 350 J

(C) 100 J (D) 3430 J

Q.68 A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is –

(A) 1 : 2 : 3 (B) 1 : 4 : 9

(C) 1 : 3 : 5 (D) 1 : 5 : 3

Q.69 The potential energy of a certain spring when stretched through a distance 'S' is 10 joule. The amount of work (in joule) that must be done on this spring to stretch it through an additional distance 'S' will be –

(A) 30 (B) 40

(C) 10 (D) 20

Q.70 A sphere of mass m moving with constant velocity hits another stationary sphere of the same mass. If e is the coefficient of restitution, then the ratio of velocities of the two spheres after the collision will be

(A) $\frac{1}{e}$ (B) $\frac{1-e}{1+e}$

(C) $\frac{e}{1+e}$ (D) $\frac{e+1}{e}$

Q.71 A ball is released from certain height. It loses 50% of its kinetic energy on striking the ground. It will attain a height again equal to

(A) One fourth the initial height

(B) Half the initial height

(C) Three fourth initial height

(D) None of these

Q.72 If the momentum of a body increases by 0.01%, its kinetic energy will increase by

(A) 0.01% (B) 0.02%

(C) 0.04% (D) 0.08%

Q.73 A car of mass 'm' is driven with acceleration 'a' along a straight level road against a constant external resistive force 'R'. When the velocity of the car is 'V', the rate at which the engine of the car is doing work will be –

(A) RV (B) maV

(C) (R + ma) V (D) (ma – R) V

Q.74 The average power required to lift a 100 kg mass through a height of 50 metres in approximately 50 seconds would be

(A) 50 J/s (B) 5000 J/s

(C) 100 J/s (D) 980 J/s

Q.75 A chain of linear density 3 kg /m and length 8 m is lying on the table with 4 m of chain hanging from the edge. The work done in lifting the chain on the table will be -

(A) 117.6 J (B) 235.2 J

(C) 98 J (D) 196 J

Q.76 A force acts on a 30 gm particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 seconds is –

(A) 5.28 J (B) 4.5 J

(C) 7.28 J (D) 6.3 J

Q.77 Due to a force of $(6\hat{i}+2\hat{j})N$ the displacement of a body is $(3\hat{i}-\hat{j})m$, then the work done is

(A) 16 J (B) 12 J
(C) 8 J (D) Zero

Q.78 A spring 40 mm long is stretched by the application of a force. If 10 N force required to stretch the spring through 1mm, then work done in stretching the spring through 40mm

(A) 84 J (B) 68 J
(C) 23 J (D) 8 J

Q.79 When a 1.0kg mass hangs attached to a spring of length 50cm, the spring stretches by 2 cm. The mass is pulled down until the length of the spring becomes 60 cm. What is the amount of elastic energy stored in the spring in this condition, if g = 10 m/s^2

(A) 1.5 Joule (B) 2.0 Joule
(C) 2.5 Joule (D) 3.0 Joule

Q.80 The potential energy between two atoms in a molecule is given by $U(x)=\frac{a}{x^{12}}-\frac{b}{x^6}$; where a and b are positive constants and x is the distance between the atoms. The atom is in stable equilibrium when

(A) $x=\sqrt[6]{\frac{11a}{5b}}$ (B) $x=\sqrt[6]{\frac{a}{2b}}$
(C) x = 0 (D) $x=\sqrt[6]{\frac{2a}{b}}$

Q.81 A 12 kg bomb at rest explodes into two pieces of 4 kg and 8 kg. If the momentum of 4 kg piece is 20 Ns, the kinetic energy of the 8 kg piece is

(A) 40 J (B) 50 J
(C) 20 J (D) 25 J

Q.82 A 0.5 kg ball is thrown up with an initial speed 14 m/s and reaches a maximum height of 8.0m. How much energy is dissipated by air drag acting on the ball during the ascent

(A) 19.6 Joule (B) 4.9 Joule
(C) 10 Joule (D) 9.8 Joule

Q.83 A body of mass 5 kg is moving with a momentum of 10 kg-m/s. A force of 0.2 N acts on it in the direction of motion of the body for 10 seconds. The increase in its kinetic energy is

(A) 2.8 Joule (B) 3.2 Joule
(C) 3.8 Joule (D) 4.4 Joule

Q.84 A car of mass 1250 kg is moving at 30m/s. Its engine delivers 30 kW while resistive force due to surface is 750N. What maximum acceleration can be given in the car

(A) $\frac{1}{3}m/s^2$ (B) $\frac{1}{4}m/s^2$
(C) $\frac{1}{5}m/s^2$ (D) $\frac{1}{6}m/s^2$

Q.85 A 60 kg man runs up a staircase in 12 seconds while a 50 kg man runs up the same staircase in 11, seconds, the ratio of the rate of doing their work is

(A) 6 : 5 (B) 12 : 11
(C) 11 : 10 (D) 10 : 11

Q.86 If the heart pushes 1 cc of blood in one second under pressure 20000 N/m^2 the power of heart is –

(A) 0.02 W (B) 400 W
(C) 5×10^{-10} W (D) 0.2 W

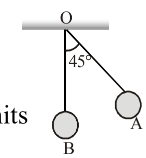
Q.87 A lead ball strikes a wall and falls down, a tennis ball having the same mass and velocity strikes the wall and bounces back. Check the correct statement

(A) The momentum of the lead ball is greater than that of the tennis ball.
(B) The lead ball suffers a greater change in momentum compared with the tennis ball.
(C) The tennis ball suffers a greater change in momentum as compared with the lead ball.
(D) Both suffer an equal change in momentum.

Q.88 A particle of mass m moving with a velocity $\vec{V}$ makes a head on elastic collision with another particle of same mass initially at rest. The velocity of the first particle after the collision will be

(A) $\vec{V}$ (B) $-\vec{V}$
(C) $-2\vec{V}$ (D) Zero

Q.89 The bob A of a simple pendulum is released when the string makes an angle of 45° with the vertical. It hits another bob B of the same material and same mass kept at rest on the table. If the collision is elastic –



(A) Both A and B rise to the same height.
(B) Both A and B come to rest at B.
(C) Both A and B move with the same velocity of A.
(D) A comes to rest and B moves with the velocity of A.

Q.90 A 50 g bullet moving with velocity 10 m/s strikes a block of mass 950 g at rest and gets embedded in it. The loss in kinetic energy will be

(A) 100% (B) 95%
(C) 5% (D) 50%

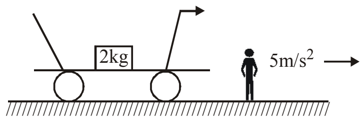
Q.91 A neutron having mass of 1.67×10^{-27} kg and moving at 10^8m/s collides with a deutron at rest and sticks to it. If the mass of the deutron is 3.34×10^{-27} kg then the speed of the combination is

(A) 2.56×10^3 m/s (B) 2.98×10^5 m/s
(C) 3.33×10^7 m/s (D) 5.01×10^9 m/s

EXERCISE - 2 [LEVEL-2]

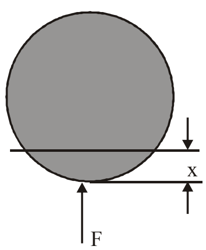
Choose one correct response for each question.

Q.1 An observer and a vehicle, both start moving together from rest (towards right) with acceleration 5 m/s^2 and 2 m/s^2, respectively. There is a 2 kg block on the floor of the vehicle and coefficient of friction is $\mu = 0.3$ between their surface. Then the work done by frictional force on the 2 kg block observed by the running observer, during first 2 seconds of the motion –



(A) 24 J (B) – 24 J
(C) 16 J (D) 36 J

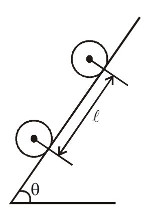
Q.2 Experimentally it has been found that the force F needed to compress elastically a ball through a distance x (as shown in the figure) follows the formula
$F(x) = ax + bx^2 + cx^3$ where a, b and c are constants. The small ball of mass m at rest is dropped from a great height h. It bounces elastically off the floor and is compressed a maximum distance 'd' during the bounce. The height h is –



(A) $\frac{1}{mg}\left(\frac{1}{2}ad^2+\frac{1}{3}bd^3+\frac{1}{4}cd^4\right)$ (B) $\frac{1}{mg}(ad + bd^2 + cd^3)$

(C) $\frac{1}{mg}(ad^2 + bd^3 + cd^4)$ (D) $\frac{1}{mg}(ad^2 + 2bd^3 + 3cd^4)$

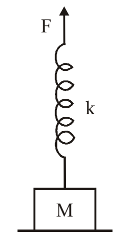
Q.3 A solid sphere of mass m is placed on rough inclined plane as shown in figure. The coefficient of friction μ is insufficient to start pure rolling. The sphere slides a length ℓ on incline from rest and its kinetic energy becomes K . Then work done by friction will be –



(A) $\mu mg\cos\theta$ (B) $\mu mg\sin\theta$

(C) $\frac{2}{5}\mu mg\sin\theta - K$ (D) $K - mg\ell\sin\theta$

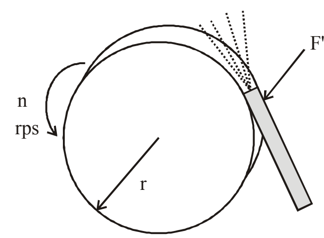
Q.4 A block of mass m kept on rough horizontal surface and is attached with a massless spring of force constant K. The minimum impulsive force applied on the other end of the spring to lift the block is –



(A) Mg (B) $\frac{Mg}{2}$

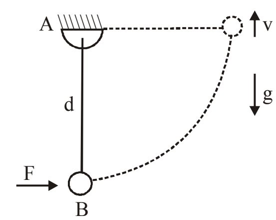
(C) 2Mg (D) $Mg(1+k)$

Q.5 Coefficient of friction between a tool and grinding wheel is μ. Power developed in watt by the wheel of radius r running at n revolutions per second when tool is pressed to the wheel with F'kgf is –



(A) $\mu F'r(2\pi n)$ (B) $\mu F'rg(2\pi n)$
(C) $\mu F'r$ (D) $\mu F'g$

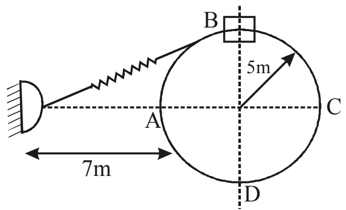
Q.6 A small sphere B of mass M is connected at one end of a light rigid rod whose other end is hinged so that the sphere hangs vertically. At some instant of time a strong wind begins to apply a constant horizontal force to sphere B. As a result, the sphere rotates about A in a vertical plane. The speed of sphere B at the instant when the rod becomes horizontal –



(A) $\sqrt{\left(\frac{F}{M}-g\right)\pi d}$ (B) $\sqrt{\frac{Fd-2Mgd}{M}}$

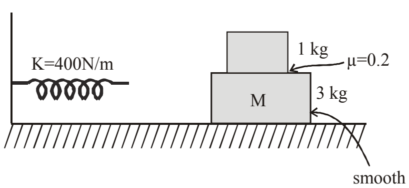
(C) $\sqrt{\frac{2Fd-2Mgd}{M}}$ (D) $\sqrt{\frac{Fd\pi-Mgd}{M}}$

Q.7 A collar 'B' of mass 2 kg is constrained to move along a horizontal smooth and fixed circular track of radius 5 m. The spring lying in the plane of the circular track and having spring constant 200 N/m is undeformed when the collar is at 'A'. If the collar starts from rest at 'B', the normal reaction exerted by the track on the collar when it passes through 'A' is –



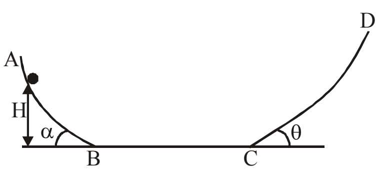
(A) 360 N (B) 720 N
(C) 1440 N (D) 2880 N

Q.8 As shown in the figure, there is no friction between the horizontal surface and the lower block (M = 3 kg) but friction coefficient between both the blocks is 0.2. Both the blocks move together with initial speed V towards the spring, compresses it and due to the force exerted by the spring, moves in the reverse direction of the initial motion. What can be the maximum value of V (in cm/s) so that during the motion, there is no slipping between the blocks.(use $g = 10 m/s^2$)



(A) 20 cm/s (B) 10 cm/s
(C) 30 cm/s (D) 40 cm/s

Q.9 A body is released from rest at height from the bottom of the crate shown. The portion AB of crate has fixed inclination α. The very long portion CD can be set into inclinations $\theta_1 = 30°$, $\theta_2 = 45°$ and $\theta_3 = 60°$. The body always remain in contact with the crate and rises upto heights h_1, h_2, h_3 respectively from the bottom for the given angles. All the surfaces are frictionless. Which of the following is correct –

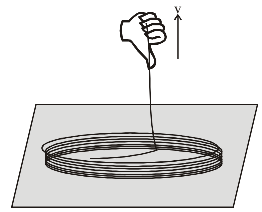


(A) $h_1 > h_2 > h_3$ (B) $h_1 < h_2 < h_3$
(C) $h_1 = h_3 < h_2$ (D) $h_1 = h_2 = h_3$

Q.10 One of the strength training machines in the exercise facility has an interesting behavior: as you lift its handles upward, you must push up with a force of 450 newtons (100 pounds). But as you lower its handles back downward, you must push up with a force of 600 newtons (133 pounds). During your exercise routine, you move these handles slowly up and down a dozen times, and leave them exactly where you found them. As the result of this routine, your total energy

(A) stays the same but you convert food energy into thermal energy.
(B) increases (C) decreases
(D) stays the same but you convert thermal energy into food energy.

Q.11 A uniform rope of linear mass density λ and length ℓ is coiled on a smooth horizontal surface. One end is pulled up with constant velocity v. Then the average power applied by the external agent in pulling the entire rope just off the horizontal surface is –



(A) $\frac{1}{2}\lambda\ell v^2 + \frac{\lambda\ell^2 g}{2}$ (B) $\lambda\ell gv$

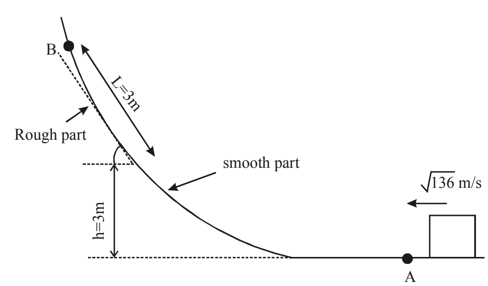
(C) $\frac{1}{2}\lambda v^3 + \frac{\lambda\ell vg}{2}$ (D) $\lambda\ell vg + \frac{1}{2}\lambda v^3$

Q.12 In the above question the maximum power delivered by the agent in pulling up the rope is –

(A) $1\,\ell gv$ (B) $1\,\ell gv + \frac{v^3 1}{2}$

(C) $1\,\ell gv + v^3 1$ (D) $\frac{1\,\ell gv}{2} + \frac{1\,v^3}{2}$

Q.13 A small block slides along a path that is without friction until the block reaches the section L = 3m, which begins at height h = 3m on a flat incline of angle 37°, as shown. In that section, the coefficient of kinetic friction is 0.50. The block passes through point A with a speed of $\sqrt{136}$ m/s. The speed of the block as it passes through point B where the friction ends, in m/s (Take $g = 10 m/s^2$)



(A) 4 m/s (B) 6 m/s
(C) 8 m/s (D) 10 m/s

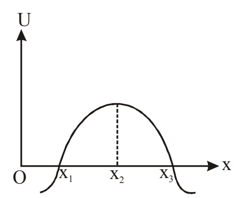
Q.14 You are riding on a swing at the local playground. As you swing back and forth, you begin to think about your speed and kinetic energy (this is obviously a fictional story). These two quantities clearly change between the top of each swing (when you are reversing directions) and the bottom of each swing (when you are passing directly beneath the supporting beam). You wonder when each of these two quantities is at its maximum value. Actually, your speed is at its maximum
(A) at the bottom of a swing and your kinetic energy is at its maximum at the bottom of a swing.

(B) at the bottom of a swing and your kinetic energy is at its maximum at the top of a swing.
(C) at the top of a swing and your kinetic energy is at its maximum at the bottom of a swing.
(D) at the top of a swing and your kinetic energy is at its maximum at the top of a swing.

Q.15 A particle is projected along a horizontal field whose coefficient of friction varies as $\mu = A/r^2$ where r is the distance from the origin in meters and A is a positive constant. The initial distance of the particle is 1m from the origin and its velocity is radially outwards. The minimum initial velocity at this point so that particle never stops is –

(A) ∞ (B) $2\sqrt{gA}$
(C) $\sqrt{2gA}$ (D) $4\sqrt{gA}$

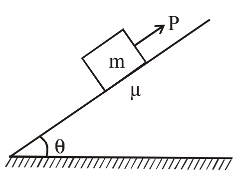
Q.16 In the figure shown the potential energy U of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct. A particle at –



(A) x_1 is in stable equilibrium
(B) x_2 is in stable equilibrium
(C) x_3 is in stable equilibrium
(D) none of these

Q.17 A fire hose has a diameter of 2.5 cm and is required to direct a jet of water to a height of at least 40 m. The minimum power of the pump needed for this hose is :
(A) 21.5 kW (B) 40 kW
(C) 36.5 kW (D) 48 kW

Q.18 A block of mass m is being pulled up the rough incline by an agent delivering constant power P. The coefficient of friction between the block and the incline is μ. The maximum speed of the block during the course of ascent is –



(A) $v = \dfrac{P}{mg\sin q + mmg\cos q}$

(B) $v = \dfrac{P}{mg\sin q - mmg\cos q}$

(C) $v = \dfrac{2P}{mg\sin q - mmg\cos q}$

(D) $v = \dfrac{3P}{mg\sin q - mmg\cos q}$

Q.19 The potential energy function associated with the force $\vec{F} = 4xy\hat{i} + 2x^2\hat{j}$ is :
(A) $U = -2x^2y$ (B) $U = -2x^2y + \text{constant}$
(C) $U = 2x^2y + \text{constant}$ (D) not defined

Q.20 A wooden wedge is pushed horizontally to raise a heavy object. All surfaces are frictionless. Apply the law of conservation of energy to determine the mechanical advantage in terms of the wedge angle θ.
(A) sin θ (B) tan θ
(C) cos θ (D) cot θ

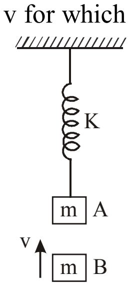
Q.21 A particle with total energy E moves in one dimension in a region where the potential energy is U(x). The acceleration of the particle is zero where
(A) U(x) = E (B) U(x) = 0
(C) $\dfrac{dU(x)}{dx} = 0$ (D) $\dfrac{d^2U(x)}{dx^2} = 0$

Q.22 Suppose potential energy between electron and proton at separation r is given by U = K ln (r), where K is a constant. For such a hypothetical hydrogen atom, the ratio of energy difference between energy levels (n = 1 and n = 2) and (n = 2 and n = 4) is –
(A) 1 (B) 2
(C) 3 (D) 4

Q.23 One end of an unstretched vertical spring is attached to the ceiling and an object attached to the other end is slowly lowered to its equilibrium position. If S be gain in spring energy & G be loss in gravitational potential energy in the process, then –
(A) S = G (B) S = 2G
(C) G = 2S (D) None of these

Q.24 A certain system has potential energy given by the function $U(x) = -ax^2 + bx^4$ with constants, a, b > 0. Which of the following value of x is an unstable equilibrium point?
(A) 0 (B) $\sqrt{a/2b}$
(C) $-\sqrt{a/2b}$ (D) $\sqrt{a/b}$

Q.25 Block A is hanging from a vertical spring and is at rest. Block B strikes the block A with velocity v and sticks to it. Then the value of v for which the spring just attains natural length is –

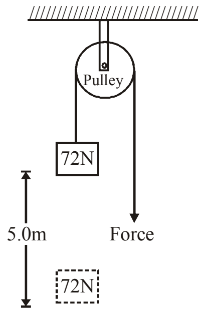


(A) $\sqrt{\dfrac{60mg^2}{k}}$ (B) $\sqrt{\dfrac{6mg^2}{k}}$
(C) $\sqrt{\dfrac{10mg^2}{k}}$ (D) none of these

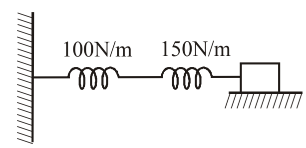
EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

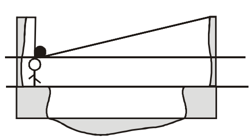
Q.1 In the diagram below, 400 joules of work is done raising a 72-newton weight a vertical distance of 5.0 meters with uniform velocity. How much work (in Joule) is done to overcome friction as the weight is raised ?



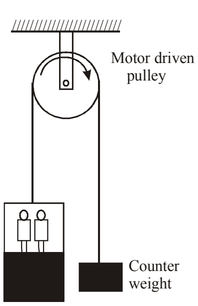
Q.2 Two springs of force constant 100 N/m and 150 N/m are in series as shown. The block is pulled by a distance of 2.5cm. to the right from equilibrium position. The ratio of work done by the spring at left to the work done by the spring at
right is 3/A. Find the value of A.



Q.3 A soldier has to cross a 80m wide river on a ropeway grabbing on a pulley. The length of the rope is 100m, its ends are fixed at the same height on both banks of the river and the rope is tight at shown. The solider starts from a platform under one of the fixed points with zero initial velocity. (Friction is absent everywhere). If the highest speed of soldier during the crossing is v (in m/s). What is v^2 ?



Q.4 An elevator cab, whose total mass with its passengers is 600 kg, ascends uniformly. The mass of the counter-weight is 300 kg. The mechanical power of the motor is 10kW and the efficiency is 60%. Find the speed of the lift.

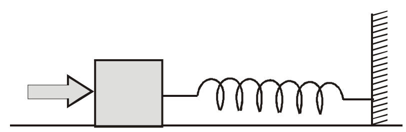


Q.5 A block of mass 2 kg was moving along a straight line on a smooth surface with a speed of 5 m/s. At t = 0, a force given by F = (3 + 2t) N directed in the direction of motion of the body starts acting on the block. Find the kinetic energy of the block after 2 sec (in J)

Q.6 The energy content of gasoline is 3.6×10^7 J/L. A motor with an efficiency of 20% is needed at full output power of 45kW for 50.0 minutes. How many litres of gasoline are required to operate the motor for this amount of time?

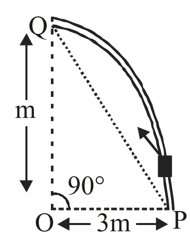
Q.7 Denis is riding his bicycle at 12.0 km/h when he applies the brakes, locking the wheels and sliding to a stop. Denis has a mass of 15.0 kg, while the bicycle has a mass of 10.0 kg. Assuming any energy conversion into heat, calculate the amount of heat (in Joules) generated by the friction between the tires and the road while coming to a stop.

Q.8 A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06m and comes to rest for the first time. The initial velocity of the block in m/s is V= N/10. Then N is



Q.9 A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in m/s) of the particle is zero, the speed (in m/s) after 5s is –

Q.10 Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})



EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is **[AIEEE-2002]**
(A) 16 J (B) 8 J
(C) 32 J (D) 24 J

Q.2 A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is – **[AIEEE-2003]**
(A) 18.75 N-m (B) 25.00 N-m
(C) 6.25 N-m (D) 12.50 N-m

Q.3 Consider the following two statements : **[AIEEE-2003]**
(i) Linear momentum of a system of particle is zero
(ii) Kinetic energy of a system of particle is zero. Then :
(A) (i) implies (ii) but (ii) does not imply (i)
(B) (i) does not imply (ii) but (ii) implies (i)
(C) (i) implies (ii) and (ii) implies (i)
(D) (i) does not imply (ii) and (ii) does not imply (i)

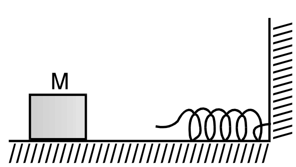
Q.4 A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total of the chain is 4 kg. What is the work done in pulling the entire chain on the table– **[AIEEE-2004]**
(A) 7.2 J (B) 3.6 J
(C) 120 J (D) 1200 J

Q.5 A force $\vec{F} = (5\hat{i} + 3j + 2\hat{k})$ N is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - j)$ m. The work done on the particle in joules is **[AIEEE-2004]**
(A) – 7 (B) + 7
(C) + 10 (D) + 13

Q.6 A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by **[AIEEE-2005]**
(A) $\frac{mv^2}{T^2}.t$ (B) $\frac{mv^2}{T^2}.t^2$ (C) $\frac{1}{2}\frac{mv^2}{T^2}.t$ (D) $\frac{1}{2}\frac{mv^2}{T^2}.t^2$

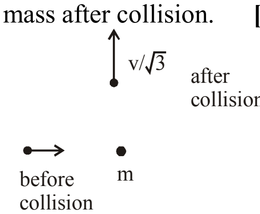
Q.7 A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is **[AIEEE-2005]**
(A) 40 m/s (B) 20 m/s
(C) 10 m/s (D) 10√30 m/s

Q.8 The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant K and compresses it by length L. The maximum momentum of the block after collision is **[AIEEE-2005]**



(A) $\sqrt{MK}$ L (B) $\frac{KL^2}{2M}$
(C) zero (D) $\frac{ML^2}{K}$

Q.9 A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass. After collision the Ist mass moves with velocity $v/\sqrt{3}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision. **[AIEEE-2005]**



(A) v
(B) √3 v
(C) $\frac{2}{\sqrt{3}}$v
(D) $v/\sqrt{3}$

Q.10 A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is – **[AIEEE 2006]**
(A) $\frac{Mg}{\sqrt{2}}$ (B) Mg ($\sqrt{2} - 1$)
(C) Mg ($\sqrt{2} + 1$) (D) Mg $\sqrt{2}$

Q.11 A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is – **[AIEEE 2006]**
(A) 1.25 J (B) 0.5 J
(C) – 0.5 J (D) – 1.25 J

Q.12 A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force.
Consider g = 10 m/s^2. **[AIEEE 2006]**
(A) 20 N (B) 22 N
(C) 4 N (D) 16 N

Q.13 The potential energy of a 1 kg particle free to move along the x-axis is given by V(x) = $\left(\frac{x^4}{4} - \frac{x^2}{2}\right)$ J. The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is – **[AIEEE 2006]**
(A) $1/\sqrt{2}$ (B) 2
(C) $3/\sqrt{2}$ (D) $\sqrt{2}$

Q.14 A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is –
(A) 192 J (B) 96 J **[AIEEE 2006]**
(C) 144 J (D) 288 J

Q.15 Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position – **[AIEEE 2006]**

(A) $\frac{m_1}{m_2}d$ (B) d

(C) $\frac{m_2}{m_1}d$ (D) $\frac{m_1}{m_1+m_2}d$

Q.16 A 2kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by **[AIEEE 2007]**

(A) 5.5 cm (B) 2.5 cm

(C) 11.0 cm (D) 8.5 cm

Q.17 An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range **[AIEEE 2008]**

(A) 2×10^5 J – 3×10^5 J (B) 20,000 J – 50,000 J

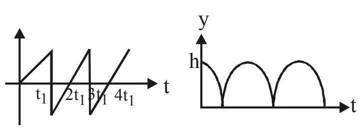
(C) 2,000 J – 5,000 J (D) 200 J – 500 J

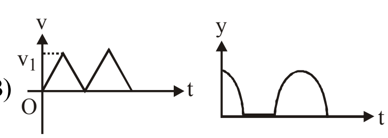
Q.18 A block of mass 0.50 kg is moving with a speed of 2 m/s on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **[AIEEE 2008]**

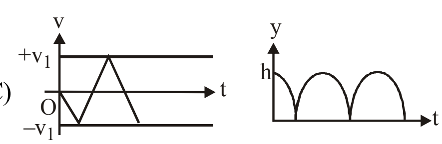
(A) 1.00 J (B) 0.67 J

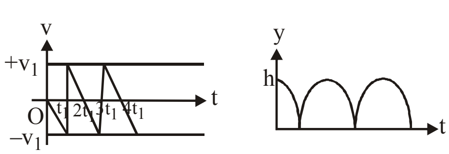
(C) 0.34 J (D) 0.16 J

Q.19 Consider a rubber ball freely falling from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be **[AIEEE 2009]**

(A)


(B)


(C)


(D)


Q.20 The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x=\infty) - U_{\text{at equilibrium}}]$, D is – **[AIEEE 2010]**

(A) $b^2/2a$ (B) $b^2/12a$

(C) $b^2/4a$ (D) $b^2/6a$

Q.21 If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2. **[AIEEE 2012]**

Statement 2 : $k_1 < k_2$

(A) Statement 1 is false, Statement 2 is true.

(B) Statement 1 is true, Statement 2 is false

(C) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for statement 1

(D) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1

Q.22 **Statement-I :** A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$. **[JEE MAIN 2013]**

Statement-II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(A) S-I is true, S -II is true, S-II is a explain S-I.

(B) S-I is true, S-II is true, S-II does not explain S-I.

(C) Statement-I is true,Statement-II is false.

(D) Statement-I is false, Statement-II is true.

Q.23 When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber band by L is : **[JEE MAIN 2014]**

(A) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (B) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

(C) $aL^2 + bL^3$ (D) $\frac{1}{2}(aL^2 + bL^3)$

Q.24 A particle of mass m moving in the x direction with speed 2v is hit by another particle of mass 2m moving in the y direction wth speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to **[JEE MAIN 2015]**

(A) 50% (B) 56%

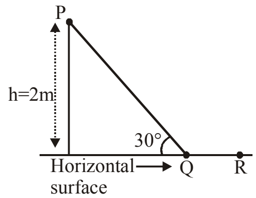
(C) 62% (D) 44%

Q.25 A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$. **[JEE MAIN 2016]**

(A) 6.45×10^{-3} kg (B) 9.89×10^{-3} kg

(C) 12.89×10^{-3} kg (D) 2.45×10^{-3} kg

Q.26 A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ.



The particle is released, from rest, from the point P and it comes to rest at a point R.
The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.
The values of the coefficient of friction μ and the distance $x (= QR)$, are, respectively close to : **[JEE MAIN 2016]**

(A) 0.2 and 3.5 m (B) 0.29 and 3.5 m
(C) 0.29 and 6.5 m (D) 0.2 and 6.5 m

Q.27 A time dependent force F = 6t acts on a particle of mass 1kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be: **[JEE MAIN 2017]**

(A) 22 J (B) 9 J
(C) 18 J (D) 4.5 J

Q.28 It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively. **[JEE MAIN 2018]**

(A) (0, 0) (B) (0, 1)
(C) (–89, –28) (C) (–28, –89)

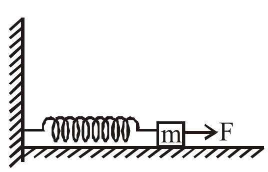
Q.29 A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is : **[JEE MAIN 2018]**

(A) Zero (B) $-\frac{3}{2}\frac{k}{a^2}$ (C) $-\frac{k}{4a^2}$ (D) $\frac{k}{2a^2}$

Q.30 In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is: **[JEE MAIN 2018]**

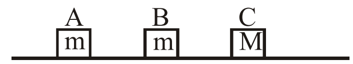
(A) $\frac{v_0}{2}$ (B) $\frac{v_0}{\sqrt{2}}$ (C) $\frac{v_0}{4}$ (D) $\sqrt{2}v_0$

Q.31 A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is : **[JEE MAIN 2019 (JAN)]**



(A) $\frac{\pi F}{\sqrt{mk}}$ (B) $\frac{2F}{\sqrt{mk}}$
(C) $\frac{F}{\sqrt{mk}}$ (D) $\frac{F}{\pi\sqrt{mk}}$

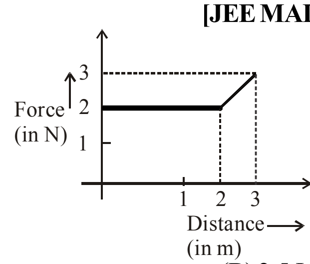
Q.32 Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $(5/6)^{th}$ of the initial kinetic energy is lost in whole process. What is value of M/m ?



[JEE MAIN 2019 (JAN)]

(A) 4 (B) 5
(C) 3 (D) 2

Q.33 A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3m is : **[JEE MAIN 2019 (APRIL)]**



(A) 6.5 J (B) 2.5 J
(C) 4 J (D) 5 J

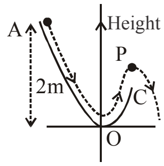
Q.34 A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If
$m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is :- **[JEE MAIN 2019 (APRIL)]**

(A) $v_4 - \frac{v_2}{4}$ (B) $v_4 - \frac{v_2}{2}$
(C) $v_4 - v_2$ (D) $v_4 + v_2$

Q.35 A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms^{-2}) **[JEE MAIN 2020 (JAN)]**

(A) 1.9 m/s (B) 1.7 m/s
(C) 2 m/s (D) 1.5 m/s

Q.36 A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale; take g=10 ms^{-2})_____. **[JEE MAIN 2020 (JAN)]**



Q.37 An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920kg and it moves with a constant speed 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10\ m/s^2$) must be at least :

[JEE MAIN 2020 (JAN)]

(A) 66000 W (B) 63248 W
(C) 48000 W (D) 56320 W

Q.38 A body A, of mass m = 0.1 kg has an initial velocity of $3\hat{i}\ m/s$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}\ m/s$. After collision, A moves with a velocity $\vec{v} = 4(\hat{i}+\hat{j})$. The energy of B after collision is written as $\frac{x}{10}J$. The value of x is ______.

[JEE MAIN 2020 (JAN)]

Q.39 A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is :

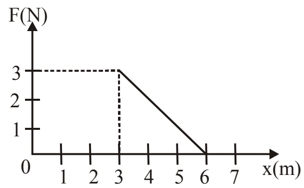
[JEE MAIN 2020 (JAN)]

(A) 1/2 (B) $\sqrt{1/2}$
(C) $\sqrt{3/4}$ (D) $\sqrt{3/2}$

EXERCISE - 5 (PREVIOUS YEARS AIPMT/NEET EXAM QUESTIONS)

Choose one correct response for each question.

Q.1 A force F acting on an object varies with distance x as shown here. The force is in N and x in m. The work done by the force in moving the object from x = 0 to x = 6m is **[AIPMT 2005]**



(A) 18.0 J (B) 13.5 J
(C) 9.0 J (D) 4.5 J

Q.2 A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 m/s. The kinetic energy of the other mass is –
(A) 324 J (B) 486 J **[AIPMT 2005]**
(C) 256 J (D) 524 J

Q.3 A body of mass 3 kg is under a constant force which causes a displacement s in metres in it, given by the relation $S = (1/3)\,t^2$, where t is in seconds. Work done by the force in 2 seconds is – **[AIPMT 2006]**
(A) (3/8) J (B) (8/3) J
(C) (19/5) J (D) (5/19) J

Q.4 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10m. Taking $g = 10\ m/s^2$, work done against friction is – **[AIPMT 2006]**
(A) 200 J (B) 100 J
(C) Zero (D) 1000 J

Q.5 A vertical spring with force constant k is fixed on a table. A ball of mass m at a height h above the free upper end of the spring falls vertically on the spring so that the spring is compressed by a distance d. The net work done in the process is – **[AIPMT 2007]**

(A) $mg\,(h + d) - \frac{1}{2}kd^2$ (B) $mg\,(h - d) - \frac{1}{2}kd^2$

(C) $mg\,(h - d) + \frac{1}{2}kd^2$ (D) $mg\,(h + d) + \frac{1}{2}kd^2$

Q.6 A roller coaster is designed such that riders experience "weightlessness" as they go round the top of a hill whose radius of curvature is 20m. The speed of the car at the top of the hill is between – **[AIPMT 2008]**
(A) 13 m/s and 14 m/s (B) 14 m/s and 15 m/s
(C) 15 m/s and 16 m/s (D) 16 m/s and 17m/s

Q.7 Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional forces are 10 % of energy. How much power is generated by the turbine? ($g = 10\ m/s^2$) **[AIPMT 2008]**
(A) 7.0 kW (B) 8.1 kW
(C) 10.2 kW (D) 12.3 kW

Q.8 A shell of mass 200 gm is ejected from a gun of mass 4 kg by an explosion that generates 1.05kJ of energy. The initial velocity of the shell is **[AIPMT 2008]**
(A) $120\ ms^{-1}$ (B) $100\ ms^{-1}$
(C) $80\ ms^{-1}$ (D) $40\ ms^{-1}$

Q.9 An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water jet. What is the rate at which kinetic energy is imparted to water? **[AIPMT 2009]**

(A) mv^2 (B) $\frac{1}{2}mv^2$ (C) $\frac{1}{2}m^2v^2$ (D) $\frac{1}{2}mv^3$

Q.10 An explosion blows a rock into three parts. Two parts go off at right angles to each other. Out of these two first part of 1 kg moves with a velocity of $12\ ms^{-1}$ and second part of 2 kg moves with a velocity of $8\ ms^{-1}$. If the third part flies off with a velocity of $4\ ms^{-1}$, its mass would be:
(A) 7 kg (B) 17 kg **[AIPMT 2009]**
(C) 3 kg (D) 5 kg

Q.11 A body of mass 1 kg is thrown upwards with a velocity 20 m/s. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction? **[AIPMT 2009]**
(A) 30 J (B) 40 J
(C) 10 J (D) 20 J

Q.12 A ball moving with velocity 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5, then their velocities (in m/s) after collision will be **[AIPMT (PRE) 2010]**
(A) 0, 1 (B) 1, 1
(C) 1, 0.5 (D) 0, 2

Q.13 An engine pumps water through a hose pipe. Water passes through the pipe and leaves it with a velocity of 2 m/s. The mass per unit length of water in the pipe is 100 kg/m. What is the power of the engine **[AIPMT (PRE) 2010]**
(A) 400W (B) 200W
(C) 100W (D) 800W

Q.14 A particle of mass M, starting from rest, undergoes uniform acceleration. If the speed acquired in time T is V, the power delivered to the particle is – **[AIPMT (MAINS) 2010]**

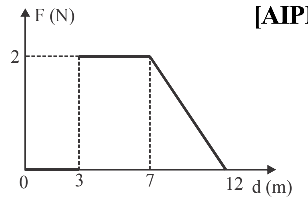
(A) $\frac{MV^2}{T}$ (B) $\frac{1}{2}\frac{MV^2}{T^2}$ (C) $\frac{MV^2}{T^2}$ (D) $\frac{1}{2}\frac{MV^2}{T}$

Q.15 A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest **[AIPMT (PRE) 2011]**
(A) At the instant just after the body is projected
(B) At the highest position of the body.
(C) At the instant just before the body hits the earth.
(D) It remains constant all through.

Q.16 A body of mass M hits normally a rigid wall with velocity V and bounces back with the same velocity. The impulse experienced by the body is – **[AIPMT (PRE) 2011]**
(A) Zero (B) MV
(C) 1.5 MV (D) 2 MV

Q.17 The potential energy of a system increases if work is done **[AIPMT (PRE) 2011]**
(A) Upon the system by a conservative force.
(B) Upon the system by a non-conservative force.
(C) By the system against a conservative force.
(D) By the system against a non-conservative force.

Q.18 Force F on a particle moving in a straight line varies with distance d as shown in the figure. The work done on the particle during its displacement of 12 m is **[AIPMT (PRE) 2011]**



(A) 13 J (B) 18 J
(C) 21 J (D) 26 J

Q.19 A mass m moving horizontally (along the x-axis) with velocity v collides and sticks to mass of 3m moving vertically upward (along the y-axis) with velocity 2v. The final velocity of the combination is : **[AIPMT (MAINS) 2011]**
(A) $\frac{1}{4}v\hat{i}+\frac{3}{2}v\hat{j}$ (B) $\frac{1}{3}v\hat{i}+\frac{2}{3}v\hat{j}$
(C) $\frac{2}{3}v\hat{i}+\frac{1}{3}v\hat{j}$ (D) $\frac{3}{2}v\hat{i}+\frac{1}{4}v\hat{j}$

Q.20 The potential energy of particle in a force field is $U=\frac{A}{r^2}-\frac{B}{r}$, where A and B are positive constant and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is : **[AIPMT (PRE) 2012]**
(A) B/2A (B) 2A/B
(C) A/B (D) B/A

Q.21 Two spheres A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along x-axis. After collision B has a velocity v/2 in a direction perpendicular to the original direction. The mass A moves after collision in the direction.
(A) same as that of B **[AIPMT (PRE) 2012]**
(B) Opposite to that of B
(C) $\theta = \tan^{-1}(1/2)$ to the x-axis
(D) $\theta = \tan^{-1}(-1/2)$ to the x-axis

Q.22 A stone is dropped from a height h. It hits the ground with a certain momentum P. If the same stone is dropped from a height 100% more than the previous height, the momentum when it hits the ground will change by : **[AIPMT (MAINS) 2012]**
(A) 68% (B) 41% (C) 200% (D) 100%

Q.23 A car of mass m starts from rest and accelerates so that the instantaneous power delivered to the car has a constant magnitude P_0. The instantaneous velocity of this car is proportional to – **[AIPMT (MAINS) 2012]**
(A) t^2P_0 (B) $t^{1/2}$
(C) $t^{-1/2}$ (D) $t/\sqrt{m}$

Q.24 A uniform force of $(3\hat{i}+\hat{j})$ newton acts on a particle of mass 2kg. Hence the particle is displaced from position $(2\hat{i}+\hat{k})$ meter to position $(4\hat{i}+3\hat{j}-\hat{k})$ meter. The work done by the force on the particle is – **[NEET 2013]**
(A) 15 J (B) 9 J
(C) 6 J (D) 13 J

Q.25 A body of mass (4m) is lying in x-y plane at rest. It suddenly explodes into three pieces. Two pieces each of mass (m) move perpendicular to each other with equal speeds (v). The total kinetic energy generated due to explosion is – **[AIPMT 2014]**
(A) mv^2 (B) $(3/2)\,mv^2$
(C) $2\,mv^2$ (D) $4\,mv^2$

Q.26 Two particles of masses m_1, m_2 move with initial velocities u_1 and u_2 . On collision, one of the particles get excited to higher level, after absorbing energy ε. If final velocities of particles be v_1 and v_2 then we must have: **[AIPMT 2015]**
(A) $\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2-\varepsilon$
(B) $\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2-\varepsilon=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$
(C) $\frac{1}{2}m_1^2u_1^2+\frac{1}{2}m_2^2u_2^2-\varepsilon=\frac{1}{2}m_1^2v_1^2+\frac{1}{2}m_2^2v_2^2$
(D) $m_1^2u_1+m_1^2u_2-\varepsilon=m_1^2v_1+m_1^2v_2$

Q.27 A block of mass 10 kg, moving in x-direction with a constant speed of 10m/s, is subject to a retarding force F = 0.1 x J/m during its travel from x = 20 m to 30 m. Its final KE will be: **[AIPMT 2015]**
(A) 450 J (B) 275 J
(C) 250 J (D) 475 J

Q.28 Two similar springs P and Q have spring constants K_P and K_Q, such that $K_P > K_Q$. They are stretched, first by the same amount (case a), then by the same force (case b). The work done by the springs W_P and W_Q are related as, in case (a) and case (b), respectively : **[AIPMT 2015]**
(A) $W_P = W_Q$; $W_P = W_Q$ (B) $W_P > W_Q$; $W_Q > W_P$
(C) $W_P < W_Q$; $W_Q < W_P$ (D) $W_P = W_Q$; $W_P > W_Q$

Q.29 A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest the force on the particle at time t is : **[AIPMT 2015]**
(A) $\sqrt{mk}\,t^{-1/2}$ (B) $\sqrt{2mk}\,t^{-1/2}$
(C) $\frac{1}{2}\sqrt{mk}\,t^{-1/2}$ (D) $\sqrt{\frac{mk}{2}}\,t^{-1/2}$

Q.30 A nucleus of uranium decays at rest into nuclei of thorium and helium. Then– **[RE-AIPMT 2015]**
(A) The helium nucleus has less kinetic energy than the thorium nucleus.
(B) The helium has more kinetic energy than the thorium nucleus.

(C) The helium nucleus has less momentum than the thorium nucleus.
(D) The helium nucleus has more momentum than the thorium nucleus.

Q.31 A ball is thrown vertically downwards from a height of 20m with an initial velocity u_0. It collides with the ground, loses 50 percent of its energy in collision and rebounds to the same height. The initial velocity u_0 is : **[RE-AIPMT 2015]**
(A) 10 m/s (B) 14 m/s
(C) 20 m/s (D) 28 m/s

Q.32 On a frictionless surface, a block of mass. M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed (v/3). The second block's speed after the collision is **[RE-AIPMT 2015]**
(A) $\frac{\sqrt{3}}{2}v$ (B) $\frac{2\sqrt{2}}{3}v$
(C) $\frac{3}{4}v$ (D) $\frac{3}{\sqrt{2}}v$

Q.33 A particle of mass 10 g moves along a circle of radius 6.4cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion? **[NEET 2016 PHASE 1]**
(A) 0.1 m/s^2 (B) 0.15 m/s^2
(C) 0.18 m/s^2 (D) 0.2 m/s^2

Q.34 What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop? **[NEET 2016 PHASE 1]**
(A) $\sqrt{gR}$ (B) $\sqrt{2gR}$
(C) $\sqrt{3gR}$ (D) $\sqrt{5gR}$

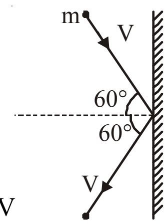
Q.35 A body of mass 1 kg begins to move under the action of a time dependent force $F = (2t\,\hat{i} + 3t^2\hat{j})$ N, where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axis. What power will be developed by the force at the time t? **[NEET 2016 PHASE 1]**
(A) $(2t^2 + 3t^2)$ W (B) $(2t^2 + 4t^4)$ W
(C) $(2t^3 + 3t^4)$ W (D) $(2t^3 + 3t^5)$ W

Q.36 A bullet of mass 10 g moving horizontally with a velocity of 400 ms^{-1} strikes a wood block of mass 2 kg which is suspended by light inextensible string of length 5 m. As a result, the centre of gravity of the block found to rise a vertical distance of 10 cm. The speed of the bullet after it emerges out horizontally from the block will be – **[NEET 2016 PHASE 2]**
(A) 100 ms^{-1} (B) 80 ms^{-1}
(C) 120 ms^{-1} (D) 160 ms^{-1}

Q.37 Two identical balls A and B having velocities of 0.5 m/s and –0.3 m/s respectively collide elastically in one dimension. The velocities of B and A after the collision respectively will be **[NEET 2016 PHASE 2]**
(A) –0.5 m/s and 0.3 m/s (B) 0.5 m/s and –0.3 m/s
(C) –0.3 m/s and 0.5 m/s (D) 0.3 m/s and 0.5 m/s

Q.38 A particle moves from a point $(-2\hat{i} + 5\hat{j})$ to $(4\hat{j} + 3\hat{k})$ when a force of $(4\hat{i} + 3\hat{j})$ N is applied. How much work has been done by the force? **[NEET 2016 PHASE 2]**
(A) 8 J (B) 11 J
(C) 5 J (D) 2 J

Q.39 A rigid ball of mass m strikes a rigid wall at 60° and gets reflected without loss of speed as shown in the figure. The value of impulse imparted by the wall on the ball will be **[NEET 2016 PHASE 2]**



(A) mV (B) 2mV
(C) mV/2 (D) mV/3

Q.40 Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take 'g' constant with a value 10 m/s^2. The work done (in J) by the (i) gravitational force and the (ii) resistive force of air is **[NEET 2017]**
(A) (i)1.25 (ii) – 8.25 (B) (i) 100 (ii) 8.75
(C) (i) 10 (ii) – 8.75 (D) (i) –10 (ii) – 8.25

Q.41 A moving block having mass m, collides with another stationary block having mass 4m. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution (e) will be **[NEET 2018]**
(A) 0.8 (B) 0.25
(C) 0.5 (D) 0.4

Q.42 When a block of mass M is suspended by a long wire of length L, the length of the wire becomes $(L + \ell)$. The elastic potential energy stored in the extended wire is : **[NEET 2019]**
(A) $Mg\ell$ (B) MgL
(C) 1/2 $Mg\ell$ (D) 1/2MgL

Q.43 Body A of mass 4m moving with speed u collides with another body B of mass 2m, at rest. The collision is head on and elastic in nature. After the collision the fraction of energy lost by the colliding body A is : **[NEET 2019]**
(A) 1/9 (B) 8/9 (C) 4/9 (D) 5/9

Q.44 A force F = 20 + 10 y acts on a particle in ydirection where F is in newton and y in meter. Work done by this force to move the particle from y = 0 to y = 1 m is **[NEET 2019]**
(A) 30 J (B) 5 J
(C) 25 J (D) 20 J

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	D	A	B	D	C	B	A	A	B	C	D	C	D	C	C	A	B	D	D	B	C	B	B	B
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	C	C	D	D	C	B	B	C	C	D	C	C	C	D	A	C	A	D	C	C	B	B	B	B	D
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	A	B	D	A	D	D	A	C	B	B	B	A	C	A	D	D	D	C	A	B	B	B	C	D	B
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91									
A	A	A	D	C	D	D	D	D	C	C	A	C	D	D	B	C									

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	A	D	B	B	C	C	A	D	B	C	B	A	A	C	D	A	A	B	D	C	A	C	A	B

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10
A	40	2	240	2	100	19	139	4	5	5

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	A	B	B	B	A	A	A	C	B	D	B	C	D	A	A	C	B	D	C	A	D	A	B	C
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39											
A	B	D	C	A	D	C	A	A	C	A	10	A	1	D											

EXERCISE - 5

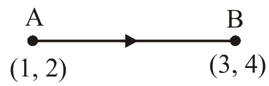
Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	B	B	B	A	B	B	B	D	D	D	A	D	D	C	D	C	A	A	B	D	B	B	B	B
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44						
A	B	D	B	D	B	C	B	A	D	D	C	B	C	A	C	B	C	B	C						

SOLUTIONS

WORK, ENERGY, POWER AND CONSERVATION LAWS

TRY IT YOURSELF-1

(1) $dW = \vec{F}\cdot\Delta\vec{S}$



$\Delta S = (3-1)\,\hat{i} + (4-2)\,\hat{j}$

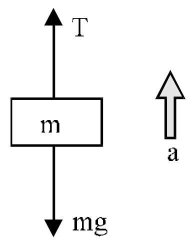
$= (2\hat{i}+3\hat{j})\cdot(2\hat{i}+2\hat{j})$

$= 2\times 2 + 3\times 2 = 10$ units.

(2) Let us first calculate the tension.
From force diagram :

$T - mg = 5a$

$T = 5\,(9.8+2) = 59\,N.$



As the T and displacement $\bar{s}$ are in same direction (upwards), work done by the tension T is W.

$W = Ts = 59\,(2.5) = 147.5\,J$

Work done by the gravity $= -\,mgs = -\,5\,(9.8)\,(2.5) = -\,122.5\,J$

Net work done on block = work done by T + work done by mg $= 147.5 + (-\,122.5) = 25\,J$

(3) **(C).** The friction is static and there is no displacement of the foot on the floor, $d\vec{r} = 0$, when the force is applied $\vec{F}$. So the contribution to the work $dW \equiv \vec{F}\cdot d\vec{r} = 0$. Keep in mind that a human being is not a rigid body. The correct energy transformation is chemical energy is transformed into the motion of the muscles which is transformed into kinetic energy of the center of mass.

(4) **(D).** The tension is perpendicular to the direction of motion so the work done by the tension force is zero.

(5) **(C).** The displacement of the comet has a component in the opposite direction as the force on the comet so the work done is negative. (The comet's acceleration is always toward the Sun; when the comet moves away from the Sun, the work is negative.)

(6) Work done, $W = \int_0^A Ax^{3/2}dx = \left.\frac{Ax^{5/2}}{5/2}\right|_0^A = \frac{2}{5}A^{7/2}$

(7) $W = \Sigma\,\vec{F}_i\,\Delta\vec{r}_i$.

Since the forces act through distances that are parallel to the forces, we simply get

$W = F_1d_1 + F_2d_2$

(8) Assume all numbers in this problem have 3 significant figures:

(a) $W = \mathbf{F}\,.\,\mathbf{d} = ((6\mathbf{i} - 2\mathbf{j})\,N).\,((3\mathbf{i} - \mathbf{j})\,m)$

$= (6.00\,N)\,(3.00m) + (-2.00\,N)\,(1.00m)$

$W = 18.0\,J - 2.00\,J = 16.0\,J$

(b) $|F| = \sqrt{F_x^2 + F_y^2} = \sqrt{6.00^2 + (-2.00)^2}\,N = 6.32\,N$

$|d| = \sqrt{d_x^2 + d_y^2} = \sqrt{3.00^2 + 1.00^2}\,m = 3.16m$

$W = Fd\cos\theta,$

$\cos\theta = \frac{W}{Fd} = \frac{16.0J}{(6.32N)\,(3.16m)} = 0.8012$

Thus, $\theta = \cos^{-1}(0.8012) = 36.8°$

(9) $\vec{S} = (5\hat{i} - 3\hat{j} - 6\hat{k}) - (2\hat{i} + 7\hat{j} - 3\hat{k}) = (3\hat{i} - 10\hat{j} + 3\hat{k})\,m$

$\vec{F} = 20\hat{i} - 30\hat{j} + 15\hat{k}$

$W = \vec{F}\cdot\vec{S} = 60 + 300 - 45 = 315\,J$

(10) $W = \int_{\vec{r}_i}^{\vec{r}_f}\vec{F}\cdot d\vec{r} = \int_{\vec{r}_1}^{\vec{r}_2}(3x^2\hat{i} + 2y\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

$= \int_{\vec{r}_1}^{\vec{r}_2}(3x^2dx + 2y\,dy) = [x^3 + y^2]_{(2,3)}^{(4,6)} = 83\,J$

TRY IT YOURSELF-2

(1) Initial kinetic energy, $K_i = \frac{1}{2}\times\frac{50}{1000}\times 200\times 200J = 1000J$

Final kinetic energy, $K_f = \frac{10}{100}\times 1000J = 100J$

If v_f is emergent speed of the bullet, then

$\frac{1}{2}\times\frac{50}{1000}\times v_f^2 = 100$

or $v_f^2 = 4000$ or $v_f = 63.2\,ms^{-1}$.

Note that the speed is reduced by approximately 68% and not 90%.

(2) $m = 5$ kg; $u = 0$, $F = 20N$; $t = 10$ s, $\frac{1}{2}mv^2 = ?$

$a = \frac{F}{m} = \frac{20}{5} = 4\,ms^{-2}$

$v = u + at = 0 + 4\times 10\,ms^{-1} = 40\,ms^{-1}$

$K.E. = \frac{1}{2}\times 5\,kg\times(40\,ms^{-1})^2 = 4000$ joule

(3) **(C).** The energy increase in going from zero speed to speed v is $\frac{1}{2}mv^2$. To go from v to 2v is

$\frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{3}{2}mv^2$

so the amount of energy required is three times as much.

(4) **(D).** With the same force, the heavier block will accelerate more slowly by a factor of 3. Since $v = at$ and t is the same for both blocks, the final speed v_h of the heavier block will be related to the speed v_ℓ of the lighter block by $v_h = v_\ell/3$.

So, the final kinetic energy of the heavier block is

$$K_h = \frac{1}{2}(3m)\,v_h^2 = \frac{1}{2}(3m)\,(v_\ell/3)^2$$

$$= \frac{1}{3}\left(\frac{1}{2}mv_\ell^2\right) = \frac{1}{3}K_\ell$$

(5) **(C).** The velocity increases linearly with time (for constant acceleration from rest, $v_y = -gt$), and kinetic energy is proportional to v_y^2 so K is proportional to t^2.

(6) The average force is the change in energy divided by the change in distance. Assuming the seat belt keeps the passenger safely in the seat,

$$\vec{F} = \frac{\Delta(E)}{\Delta(x)} = \frac{1}{x_f - x_0}\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\right)$$

$$= \frac{1}{0.94m}\left(\frac{-1}{2}\right)(75kg)\,(70km/h)^2$$

$$= \frac{1}{0.94m}\left(\frac{-1}{2}\right)(75kg)\left(70\frac{km}{h}\frac{1000m}{1\,km}\frac{1h}{3600s}\right)^2$$

$$= -15\ kN$$

(7) Using the definition of work (i.e., **F.d**) and **F.d** = $0.5m(\Delta v)^2$. In this case, the final velocity is zero. Clearly, the left hand side of this equation is proportional to the velocity squared. With d constant and F proportional to v^2, increasing v by a factor of three increases F by a factor of three squared, or nine.

(8) The work, hence energy, required to accelerate a car from zero to 20 m/s will be less than the work required to accelerate the car from 20 m/s to 40 m/s. Why? Because according to the work/energy theorem, work is not linearly related to velocity, it is related to the square of the velocity. Specifically, for the zero to 20 m/s situation, the work/energy expression yields $W_{net} = 0.5m\,(20)^2 - 0.5m\,(0)^2 = 200m$ joules.
For the 20 to 40 m/s situation, the work/energy expression yields $W_{net} = 0.5m\,(40)^2 - 0.5m\,(20)^2 = 600m$ joules. Clearly it takes more energy to accelerate a vehicle that is already moving than it does to accelerate one that starts from rest. Likewise, one must take more energy out of a fast moving car to slow it down by 20 m/s than it does to slow a slow moving car down by 20 m/s.

(9) Initial KE. $= \frac{1}{2}mv_1^2 = \frac{1}{2}(3)\,(4)^2\,J = 24\ J$

Final K.E. $= \frac{1}{2}mv_2^2 = \frac{1}{2}(3)\,(5)^2\,J = 37.5\ J$

Work done = Change in energy
Hence, work done by force = (37.5 – 24) J = 13.5 J

(10) Let R = resistance force offered by the planks,
t_1 = thickness of first plank, t_2 = thickness of second plank.
For first plank :
Loss in KE = work against resistance

$$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{4}{5}v\right)^2 = Rt_1$$

$$\Rightarrow \frac{1}{2}mv^2\left(\frac{9}{25}\right) = Rt_1 \qquad ....(1)$$

For second plank

$$\frac{1}{2}m\left(\frac{4}{5}v\right)^2 - 0 = Rt_2$$

$$\Rightarrow \frac{1}{2}mv^2\left(\frac{16}{25}\right) = Rt_2 \qquad ....(2)$$

Dividing (1) & (2) $\Rightarrow \frac{t_1}{t_2} = \frac{9}{16}$

TRY IT YOURSELF-3

(1) (a) $F = -\frac{dU}{dy} = \omega$ (b) $F = -\frac{dU}{dy} = -3ay^2 + 2by$

(c) $F = -\frac{dU}{dy} = -\beta U_0 \cos\beta y$

(2)

$$\vec{F} = -\vec{\nabla}U = -\left[\hat{i}\frac{\partial}{\partial x}(2x+3y+4z) + \hat{j}\frac{\partial}{\partial y}(2x+3y+4x) + \hat{k}\frac{\partial}{\partial z}(2x+3y+4y)\right]$$

$$\vec{F} = -[2\hat{i} + 3\hat{j} + 4\hat{k}]$$

Magnitude of force, $|\vec{F}| = \sqrt{4+9+16} = \sqrt{29}$ N

(3) **(A).** Let top must point as reference for u_g.

$$\Delta U = U_f - U_i$$

$$= \left(-2mg\frac{\ell}{2} - mg\frac{3\ell}{2}\right) - \left(-mg\frac{\ell}{2} - 2mg\frac{3\ell}{2}\right)$$

$$= -\frac{5mg\ell}{2} + \frac{mg\ell}{2} = \frac{2mg\ell}{2} = mg\ell$$

(4) $dU = -dW$

$$dU = -(y\hat{i} + x\hat{j})\cdot(dx\,\hat{i} + dy\hat{j})$$

$$\int dU = \int -y\,dx + \int -x\,dy$$

$$\int dU = -\int dxy \Rightarrow U = -xy + c$$

(5) **(A).** Here the positive work that you do produces an increase in the system's potential energy. Now the earth is in the system so it cannot do work on the system.

(6) **(C).** The cart starts at x_{start} with no kinetic energy, and so the spring's potential energy is a maximum. Once released, the cart accelerates to the right and its kinetic energy increases as the potential energy of the spring is converted into kinetic energy of the cart.
As the cart passes the equilibrium position, its kinetic energy is a maximum and so the spring's potential energy is a minimum. Once to the right of $x_{equilibrium}$, the cart starts to

compress the spring and it slows down as its kinetic energy is converted back to potential energy of the recompressed spring. At the rightmost point it reaches, the cart reverses its direction of travel. At that instant, it has no kinetic energy and the spring again has maximum potential energy.

(7) **(C).** The mechanical energy is the sum of kinetic and potential energy. If the mechanical energy were less that 10 J, then when the particle was at the position corresponding to the point d on the figure, the PE is equal to 10 J so the kinetic energy would be less than zero, but this is forbidden for a particle using just the principles of classical mechanics. (In quantum mechanics, one would have a small but finite probability of finding the particle in a classical forbidden region.)

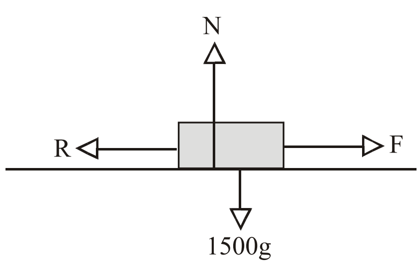
(8) **(C).** This problem is most simply solved using energy considerations. Specifically, if the spring is compressed only half as much, the potential energy stored in the spring, which is the energy delivered to the dart, is one-fourth as much. In the absence of friction, all of this energy becomes potential energy when the dart is at its maximum height. Note that the intermediate calculation of the dart's speed when leaving the gun did not need to be calculated.

(9) **(D).** We are assuming that there is no friction on the paths so the mechanical energy is constant. The change in potential energy is the same for all three paths therefore the change in kinetic energy is also the same for all three paths.

(10) **(C).** We are assuming that there is no friction on the paths so the mechanical energy is constant. Because the speed is greatest on path 3, the horizontal component of the velocity is greater on path 3 than at any point on path 2 or path 1, (included the starting and ending segments). Therefore it takes less time on path 3 to traverse the horizontal distance between the starting and ending points of each path.

TRY IT YOURSELF-4

(1) At maximum speed there is no acceleration.



So, $F = R = 60$

Also $P = Fv = 60 \times 150 \times (5/18) = 2500$

When the speed is 60 kmh^{-1}, i.e., $60 \times \frac{5}{18} ms^{-1}$,

we have $F = \frac{P}{v} = \frac{2500 \times 18}{60 \times 5} = 150$

As the car is accelerating, there is a resultant force in the direction of this acceleration of magnitude $F - R$ where $F - R = 90$

If the acceleration is a ms^{-2}, Newton's Law gives, $F - R = ma$

Therefore, $a = \frac{90}{1500} = 0.06$

The acceleration is 0.06 ms^{-2}.

(2) **(C).** It can be observed that power delivered to particle by force F is : $P = Fv = K$.

The power is constant. Hence work done by force in time t is

$\Delta W = Pt = Kt$

(3) **(C).**

(i) Power is defined as work per unit time, so at the very least you would expect joules/second to do the job.

(ii) Watts is the name for the units of power, so watts/second isn't a power quantity.

(iii) A joule is a nt.m which, in turn, is a (kg.m/second^2)(m), or (kg.m^2/second^2).

Dividing this by seconds yields a perfectly good representation of power units.

(iv) As was pointed out in Part c, a joule/second is really a nt.m/s.

(4) **(B).** $P = \frac{d}{dt}(mgh)$

$$P_{act} = \frac{1000 \times 10 \times 100}{50} = 20000 \text{ W}$$

$$P_{consumption} = \frac{2000}{0.25} W = 80kW$$

(5) The marine must exert a 700 N upward force opposite the force of gravity to lift his body at constant speed. Then his muscles do work:

$W = \mathbf{F} \cdot \mathbf{d} = (700\mathbf{j}\text{ N})(10.0\,\mathbf{j}\text{ m}) = 7000\text{ J}$

The power he puts out is

$$\bar{P} = \frac{W}{\Delta t} = \frac{7000J}{8.00s} = 875 \text{ W}$$

(6) If W is the weight of truck. Then the total force required for constant speed up the incline.

$F = W \sin\theta + \frac{W}{25}$ (Since component of weight $W \sin\theta$ and resisting force. (W/25) are downwards parallel to incline)

$\therefore\ F = \frac{W}{50} + \frac{W}{25} = \frac{3W}{50}$

If v is the speed of truck up the incline, then

$\therefore$ Power of engine , $P = Fv = \left(\frac{3W}{50}\right) x\ v$ .... (1)

If truck moves down the incline. The resisting force is up the incline parallel to plane, while component of weight W sin θ is still down the incline. Therefore the force required for constant speed down the incline

$$F' = \frac{W}{25} - W \sin\theta = \frac{W}{25} - \frac{W}{50} = \frac{W}{50}$$

$\therefore$ If v' is the constant speed of truck down the incline, then

Power of engine, $P' = F'v' = \frac{W}{50}v'$ .... (2)

But given P'= P. Therefore from (1) and (2) ,

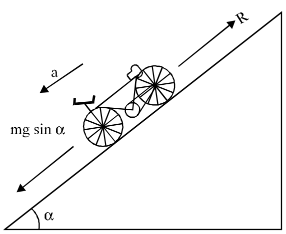
we have $\frac{W}{50}v' = \frac{3W}{50}$

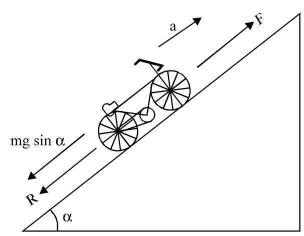
$\therefore \quad v' = 3v = 3\ (24\ km/hour) = 72\ km/hour.$

(7) Let the mass of cyclist and his machine be m.

When the cyclist free - wheels down the incline at a constant speed there is no unbalanced force and hence

mg sin α = frictional resistance to motion , R





or $\quad R = (80\ kg)\ (9.8\ m/s^2)\left(\frac{1}{30}\right) = 26.13\ N$

When he cycles up the hill, the total resistance to his motion

$F = mg\sin\alpha + R = 26.13 + 26.13 = 52.26\ N$

The speed of the cyclist = 9 km / h = 5/2 m/s

$\therefore$ power developed by the cyclist = F v = 52.26 × (5/2)

= 130. 65 watt

(8) (a) The power is given by P = Fv and the worked done by $\vec{F}$ from time t_1 to time t_2 is given by

$$W = \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} Fv\,dt$$

Since $\vec{F}$ is the net force the magnitude of the acceleration is a = F/m and, since the initial velocity is $v_0 = 0$, the velocity as a function of time is given by $v = v_0 + at = (F/m)\ t$.

Thus, $W = \int_{t_1}^{t_2} (F^2/m)t\,dt = \frac{1}{2}(F^2/m)\,(t_2^2 - t_1^2)$

For $t_1 = 0$ and $t_2 = 1.0s$, $\quad W = \frac{1}{2}\left[\frac{(5.0N)^2}{15kg}\right](1.0s)^2 = 0.83\ J$

(b) For $t_1 = 1.0s$ and $t_2 = 2.0s$,

$$W = \frac{1}{2}\left[\frac{(5.0N)^2}{15kg}\right][(2.0s)^2 - (1.0s)^2] = 2.5\ J$$

(c) For $t_1 = 2.0s$ and $t_2 = 3.0s$,

$$W = \frac{1}{2}\left[\frac{(5.0N)^2}{15kg}\right][(3.0s)^2 - (2.0s)^2] = 4.2\ J$$

(d) Substitute v = (F/m) t into P = Fv to obtain $P = F^2t/m$ for the power at any time t.

At the end of the third second, $\quad P = \frac{(5.0N)^2(3.0s)}{15kg} = 5.0W$

(9) $10 \times 10^3 = \frac{mgh}{t}$

$t = \frac{200 \times 10 \times 40}{10 \times 10^3} = 8\ sec.$

(10) $P = \frac{dm}{dt}gh = \frac{20}{60} \times 10 \times 20 = \frac{200}{3}W = \frac{200}{3 \times 746}HP$

$= \frac{100}{1119}HP$

TRY IT YOURSELF-5

(1) Yes; In explosion of a bomb or inelastic collision between two bodies as force is internal, momentum is conserved while kinetic energy changes.

(2) Yes ; If a force acts perpendicular to motion, work done will be zero and so kinetic energy will remain constant. However the force will change the direction of motion and so, the momentum, e.g., in case of uniform circular motion of a body KE is constant while momentum changes.

(3) The momentum of meteorite is transferred to air molecules by air drag and so, the momentum of meteorite plus air molecules (system) always remains constant.

(4) As sails and fan both are attached to the boat, force due to the air-blown on the sails by the fan is an internal force. Now as by an internal force momentum of the system cannot be changed, so the system (boat-fan+sails) cannot be propelled by blowing air at the sails from a fan attached to the boat.

(5) Total momentum imparted to B = 2 × 3.63 × 3.05 kg ms^{-1}.

Velocity of $B = \frac{2 \times 3.63 \times 3.05}{22.7}\ ms^{-1} = 0.975\ ms^{-1}$.

Velocity of A when the dog jumps away from

$A = \frac{3.63 \times 3.05}{22.7}\ ms^{-1} = 0.4877\ ms^{-1}$.

When the dog comes back to A,

Velocity of $A = \frac{22.7 \times 0.4877 + 3.63 \times 3.05}{22.7 + 3.63} = 0.841\ ms^{-1}$.

(6) **(C).** The system is approximately isolated with no external forces acting on the system so the momentum stays the same. (We are ignoring the effects of the sun and moon), The forces between the earth and the stone are internal forces and hence cancel in pairs.

(7) **(B).** The pieces have the same magnitude of momentum. The kinetic energy of an object of mass m and momentum magnitude p is $p^2/2m$, so the piece with the smaller mass has the larger kinetic energy. (This assumes that the pieces are not rotating, but that's not a consideration in this problem.)

(8) Using $I = m(v - u)$

gives $-3m\hat{i} = m(v - \{2\hat{i} + 3\hat{j}\})$

$$v = -3\hat{i} + \{2\hat{i} + 3\hat{j}\} = -\hat{i} + 3\hat{j}$$

Note that the velocity component in the direction of $\hat{j}$ is unchanged. This is because there is no impulse component in this direction.

(9) The constant force F, u and v are all in the same sense i.e. all are positive.

Since $Ft = mv - mu$

$$F \times 4 = 2 \times 20 - 2 \times 4 = 8$$

The required force is 8N.

(10) When mass m falls vertically under gravity we have

$u = 0, a = g, s = h$

Using, $v^2 - u^2 = 2as$ gives $v = \sqrt{2gh}$

Using impulse = change in momentum for each mass gives, for mass 2m, $J = 2mv_1 - 0$;

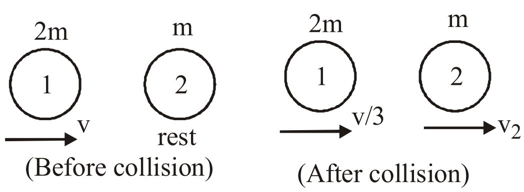
for mass m, $J = mv - mv_1$

From these equations, $v_1 = (1/3)\,v$.

Hence the particle of mass 2m begins to rise with speed $\frac{1}{3}\sqrt{2gh}$.

TRY IT YOURSELF-6

(1) **(B).** Using conservation of linear momentum



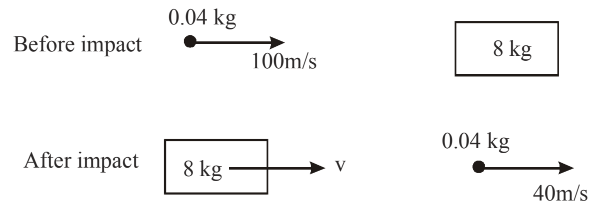
$$mv_2 + 2m\frac{v}{3} = 2mv + m(0) \Rightarrow v_2 = \frac{4v}{3}$$

Now, $e = \dfrac{\frac{4v}{3} - \frac{v}{3}}{v - 0} = 1 \Rightarrow$ Collision is elastic.

(2) **(B).** $\tan\theta' = \dfrac{\tan\theta}{e}$ and $e \le 1$

$\Rightarrow \quad \tan\theta' \ge \tan\theta \Rightarrow \theta' \ge \theta$

(3)



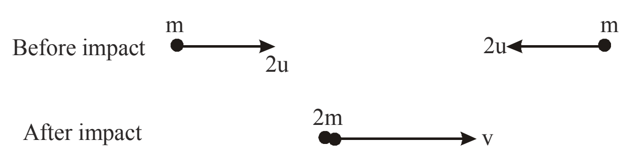
If the speed of the block is v then, using conservation of linear momentum (in the direction of motion) we have

$$0.04 \times 100 = (v) + (0.04 \times 40) \Rightarrow \quad v = \frac{2.4}{8} = 0.3$$

Therefore the block has a speed of 0.3 ms^{-1}.

(4) Using conservation of linear momentum (in the direction of the velocity 2u) we have

$$(m)(2u) - mu = 2m \times v \Rightarrow \quad v = \frac{1}{2}u$$



The combined mass will travel at speed u/2.

(Note that the momentum of the second particle before impact is negative because its sense is opposite to that specified as positive.)

(5) **(B).** Both balls have the same initial momentum prior to striking the block. Following the collision, however, the elastic ball, Ball A, bounces backward, transferring up to twice its initial momentum to the block through the larger impulse.

Ball B, however, sticks to the block, transferring its initial momentum to the block through an impulse equal to its initial momentum. Therefore, Ball A transfers more momentum to the block.

With respect to energy transfer, however, the story changes. Ball A maintains some kinetic energy as it rebounds off the block, therefore it cannot transfer as much kinetic energy to the block as Ball B, which transfers all of its kinetic energy to the block as it comes to rest. Therefore, Ball B transfers more kinetic energy to the block.

(6) **(1).** Ball B has changed its momentum in the upward direction in the figure, and as far as the figure can show, there is no change in its horizontal (rightward) velocity.

(7) **(C).** From conservation of momentum, $mv_{B,0} = 2mv_f$.

So $v_f = v_{B,0}/2$. Thus they move away with speed less one half the original speed of cart B.

(8) **(C).** The relative speed between two objects in an elastic collision does not change before the collision the relative speed was $2v_0$. Therefore after the collision the relative speed is also $2v_0$.

(9) As no external force is acting on the system in horizontal direction, its momentum remains conserved in this direction.

For minimum value of v_0, relative velocity of m wrt M has to be zero at the highest point of M.

So, $mv_0 = (M + m)\, v$

and $\dfrac{mv_0^2}{2} = \dfrac{(M+m)\, v^2}{2} + mgh \Rightarrow v_0 = \sqrt{2gh\left(1+\dfrac{m}{M}\right)}$

(10) Suppose the fist block moves at a speed v_1 and the second at v_2 after collision. Since the collision is head on, the two blocks move along the original direction of motion of first block. Using the principle of conservation of momentum,

$$(1.2 \times 0.2) = 1.2v_1 + 1.2v_2 \;;\; v_1 + v_2 = 0.2 \qquad \text{............ (i)}$$

By Newton's law of restitution,

$$v_2 - v_1 = -\, e\, (u_2 - u_1)$$

$$v_2 - v_1 = -\,0.6\,(0 - 0.2) \quad ; \; v_2 - v_1 = 0.12 \qquad \text{........... (ii)}$$

Adding equations (i) and (ii), $2v_2 = 0.32$

$v_2 = 0.16$ m/s or 16 cm/s ; $v_1 = 0.2 - 0.16 = 0.04$ m/s = 4cm/s

Loss of K. E. =

$$\frac{1}{2}\times 1.2\times(0.2)^2 - \frac{1}{2}\times 1.2\times(0.16)^2 - \frac{1}{2}\times 1.2\times(0.04)^2$$

$$= 0.6\,[0.04 - 0.0256 - 0.0016] = 0.6\times 0.0128 = 7.7\times 10^{-3}\text{J}$$

CHAPTER-6 : WORK, ENERGY, POWER AND CONSERVATION LAWS

EXERCISE-1

(1) **(C).** No displacement is there.

(2) **(D).** $W = \vec{F}.\vec{s} = (3\hat{i}+4\hat{j}).(3\hat{i}+4\hat{j}) = 9+16 = 25\text{ J}$

(3) **(A).** Here $\vec{F} = -\hat{i}+2\hat{j}+3\hat{k}$,

$\vec{d} = (0-0)\ \hat{i} + (0-0)\ \hat{j} + (4-0)\ \hat{k} = 4\ \hat{k}$

$\therefore$ W (Work done) $= \vec{F}.\vec{d} = (-\hat{i}+2\hat{j}+3\hat{k}).4\hat{k} = 12\text{ J}$

(4) **(B).** The minimum force with a body is to be pulled up along the inclined plane is mg (sin θ + μ cos θ)

Work done, $W = \vec{F}.\vec{d}$

$= \ Fd\cos\theta = mg\,(\sin\theta + \mu\cos\theta)\ \times d$

$= 5\times 9.8\,(\sin 60^o + 0.2\cos 60^o)\times 2 = 94.67\text{ J}$

(5) **(D).** $W = \vec{F}.\vec{s} = (5\hat{i}+6\hat{j}-4\hat{k}).(6\hat{i}+5\hat{k}) = 30-20 = 10$ units

(6) **(C).** As the road does not move at all, therefore, work done by the cycle on the road must be zero.

(7) **(B).** Acceleration, $a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ As $v^2 = k^2x^3$

Differentiating both sides with respect to x, we get

$2v\frac{dv}{dx} = 3k^2x^2$; $a = \frac{3}{2}k^2x^2$; $F = ma = \frac{3}{2}m\,k^2x^2$

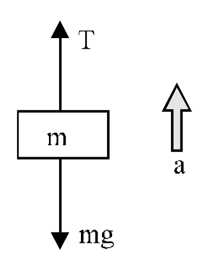
Work done, $W = \int F\,dx = \int_0^2 \frac{3}{2}mk^2x^2dx$

$$W = \frac{3}{2}m\,k^2\left[\frac{x^3}{3}\right]_0^2 = \frac{3}{6}\times 0.5\times 5^2\times[2^3-0] = 50\text{ J}$$

(8) **(A).** $dW = \vec{F}\cdot\Delta\vec{S}$; $\Delta S = (3-1)\,\hat{i} + (4-2)\,\hat{j}$

$W = (2\hat{i}+3\hat{j})\cdot(2\hat{i}+2\hat{j}) = 2\times 2 + 3\times 2 = 10$ units.

(9) **(A).** Let us first calculate the tension.



From force diagram :

$T - mg = 5a$

$T = 5\,(9.8+2) = 59$ N.

As the T and displacement $\bar{S}$ are in same direction (upwards), work done by the tension T is W.

$W = Ts = 59\,(2.5) = 147.5$ J

(10) **(B).** Work done by the gravity = – mgs

$= -5\,(9.8)\,(2.5) = -122.5$ J

Net work done on block = work done by T + work done by mg = 147.5 + (– 122.5) = 25 J

(11) **(C).** The friction is static and there is no displacement of the foot on the floor, $d\vec{r} = 0$, when the force is applied $\vec{F}$. So the contribution to the work

$dW \equiv \vec{F}\cdot d\vec{r} = 0.$

(12) **(D).** The tension is perpendicular to the direction of motion so the work done by the tension force is zero.

(13) **(C).** The displacement of the comet has a component in the opposite direction as the force on the comet so the work done is negative. (The comet's acceleration is always toward the Sun; when the comet moves away from the Sun, the work is negative.)

(14) **(D).** Net force on body $= \sqrt{4^2+3^2} = 5\text{N}$

$\therefore\quad a = F/m = 5/10 = 1/2\ m/s^2$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2 = 125$ Joule

(15) **(C).** $U = \frac{F^2}{2k} \Rightarrow \frac{U_1}{U_2} = \frac{k_2}{k_1}$ (if force are same)

$\therefore\quad \frac{U_1}{U_2} = \frac{3000}{1500} = \frac{2}{1}$

(16) **(C).** Potential energy $U = \frac{1}{2}kx^2$

$\therefore\quad U \propto x^2$ [if k = constant]

If elongation made 4 times then potential energy will become 16 times.

(17) **(A).** Body at rest may possess potential energy.

(18) **(B).** Due to theory of relativity.

(19) **(D).** $E = \frac{P^2}{2m}$ $\therefore$ $E \propto P^2$ i.e. if P is increased n times then E will increase n^2 times.

(20) **(D).** $P = \sqrt{2mE}$ $\therefore$ $P \propto \sqrt{E}$

i.e. if kinetic energy of a particle is doubled the its momentum will becomes $\sqrt{2}$ times.

(21) **(B).** Potential energy = mgh

Potential energy is maximum when h is maximum

(22) **(C).** If particle is projected vertically upward with velocity of 2m/s then it returns with the same speed.

$KE = \frac{1}{2}mv^2 = \frac{1}{2}\times 2\times(2)^2 = 4\text{ J}$

(23) **(B).** $\Delta U = mgh = 20\times 9.8\times 0.5 = 98$ J

(24) **(B).** $E = \frac{P^2}{2m} = \frac{(10)^2}{2\times 1} = 50$ J

(25) **(B).** Change in gravitational potential energy = Elastic potential energy stored in compressed spring $\Rightarrow mg\,(h+x) = \frac{1}{2}kx^2$

(26) **(C).** $W = \frac{F^2}{2k}$. If both springs are stretched by same force then $W \propto 1/k$

As $k_1 > k_2$ therefore $W_1 < W_2$.

i.e. more work is done in case of second spring.

(27) **(C).** $P=\sqrt{2mE}\quad \therefore P\propto\sqrt{m}\quad (\text{if } E=\text{const.})\ \therefore \dfrac{P_1}{P_2}=\sqrt{\dfrac{m_1}{m_2}}$

(28) **(D).** In compression or extension of a spring work is done against restoring force. In moving a body against gravity work is done against gravitational force of attraction. It means in all three cases potential energy of the system increases.
But when the bubble rises in the direction of upthrust force then system works so the potential energy of the system decreases.

(29) **(D).** $E=\dfrac{p^2}{2m}\quad \therefore\ m\propto\dfrac{1}{E}$ (If momentum are same)

$$\frac{m_1}{m_2}=\frac{E_2}{E_1}=\frac{1}{4}$$

(30) **(C).** Kinetic energy $=\dfrac{1}{2}mv^2$

As both balls are falling through same height therefore they possess same velocity.

but $KE\propto m$ (If v = constant)

$$\therefore\ \frac{(KE)_1}{(KE)_2}=\frac{m_1}{m_2}=\frac{2}{4}=\frac{1}{2}$$

(31) **(B).** The fraction of energy lost is given by,

$$\frac{\Delta E}{E}=\frac{mg(h-h')}{mgh}=\frac{h-h'}{h}$$

Given that, h = 2 meter and h' = 1.5 meter

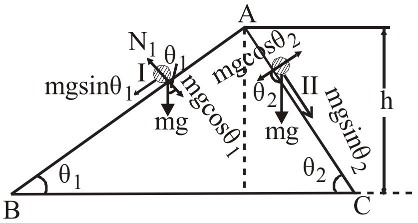
$$\therefore\ \frac{\Delta E}{E}=\frac{2-1.5}{2}=\frac{1}{4}$$

(32) **(B).** $F\propto\dfrac{1}{v}$ or $\dfrac{mdv}{dt}=\dfrac{k}{v}$; $\quad mv\,dv=k\,dt$

$$\frac{mv^2}{2}=kt\quad ;\ KE\propto t$$

(33) **(C).** According to law of conservation of mechanical energy, PE at the top = KE at the bottom

$$mgh=\frac{1}{2}mv_1^2\ \ldots\ (1)\ \text{and}\ \ mgh=\frac{1}{2}mv_2^2\ \ldots\ (2)$$



From (1) and (2), we get $v_1=v_2$
As is clear from figure, acceleration of the two stones are $a_1=g\sin\theta_1$ and $a_2=g\sin\theta_2$ respectively.

As $\theta_2>\theta_1\qquad \therefore\ a_2>a_1$
From $v=u+at=0+at$ or $t=v/a$
As $t\propto 1/a$, and $a_2>a_1\ \ \therefore\ t_2<t_1$
Hence, stone II will take lesser time and reach the bottom earlier than stone 1.

(34) **(C).** The energy increase in going from zero speed to speed v is $\dfrac{1}{2}mv^2$.

To go from v to 2v is $\dfrac{1}{2}m(2v)^2-\dfrac{1}{2}mv^2=\dfrac{3}{2}mv^2$

So the amount of energy required is three times as much.

(35) **(D).** With the same force, the heavier block will accelerate more slowly by a factor of 3. Since v = at and t is the same for both blocks, the final speed v_h of the heavier block will be related to the speed v_ℓ of the lighter block by $v_h=v_\ell/3$. So, the final KE of the heavier block is

$$K_h=\frac{1}{2}(3m)\,v_h^2=\frac{1}{2}(3m)\,(v_\ell/3)^2=\frac{1}{3}\left(\frac{1}{2}mv_\ell^2\right)=\frac{1}{3}K_\ell$$

(36) **(C).** The velocity increases linearly with time (for constant acceleration from rest, $v_y=-gt$), and kinetic energy is proportional to $v_y{}^2$ so K is proportional to t^2.

(37) **(C).** The cart starts at x_{start} with no kinetic energy, and so the spring's potential energy is a maximum. Once released, the cart accelerates to the right and its kinetic energy increases as the potential energy of the spring is converted into kinetic energy of the cart.
As the cart passes the equilibrium position, its kinetic energy is a maximum and so the spring's potential energy is a minimum.

(38) **(C).** The mechanical energy is the sum of kinetic and potential energy. If the mechanical energy were less that 10 J, then when the particle was at the position corresponding to the point d on the figure, the PE is equal to 10 J so the kinetic energy would be less than zero, but this is forbidden for a particle using just the principles of classical mechanics.

(39) **(D).** $Pt=\dfrac{1}{2}mv^2$; v is doubled $\Rightarrow$ t is 4 times

(40) **(A).** $P=\dfrac{\vec{F}.\vec{s}}{t}=\dfrac{(2\hat{i}+3\hat{j}+4\hat{k}).(3\hat{i}+4\hat{j}+5\hat{k})}{4}=\dfrac{38}{4}=9.5\ W$

(41) **(C).** $P=\dfrac{dK}{dt}\Rightarrow\int dK=\int P\,dt$,

$$\frac{1}{2}mv^2=\int_0^2\left(\frac{3}{2}t^2\right)dt\ ,\ v=\ 2m/s$$

(42) **(A).** Power of gun $=\dfrac{\text{Total K.E. of fired bullet}}{\text{time}}$

$$=\frac{n\times\frac{1}{2}mv^2}{t}=\frac{360}{60}\times\frac{1}{2}\times2\times10^{-2}\times(100)^2=600\ W$$

(43) **(D).** $P=\dfrac{\text{Workdone}}{\text{Time}}=\dfrac{mgh}{t}=\dfrac{300\times 9.8\times 2}{3}=1960\,W$

(44) **(C).** It can be observed that power delivered to particle by force F is : $P = Fv = K$.
The power is constant. Hence work done by force in time t is $\Delta W = Pt = Kt$

(45) **(C).** Power is defined as work per unit time

(46) **(B).** $P=\dfrac{d}{dt}(mgh)$; $P_{act}=\dfrac{1000\times 10\times 100}{50}=20000\ W$

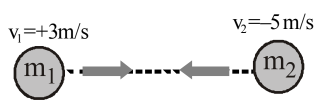
$$P_{consumption}=\frac{2000}{0.25}W=80kW$$

(47) **(B).** From $v = u + at = 0 + at$
As power $P = Fv = ma \times at = ma^2t$.
As m and a are constant $P \propto t$.

(48) **(B).** We know that when heavier body strikes elastically with a lighter body then after collision lighter body will move with double velocity that of heavier body.
i.e. the ping pong ball move with speed of 2×2=4m/s

(49) **(B).** In elastic head on collision velocities gets interchanged.

(50) **(D).**



As $m_1 = m_2$, therefore after elastic collision velocities of masses get interchanged .
i.e. velocity of mass $m_1 = -5$m/s
and velocity of mass $m_2 = +3$ m/s

(51) **(A).** $v_1=\left(\dfrac{m_1-m_2}{m_1+m_2}\right)u_1+\left(\dfrac{2m_2}{m_1+m_2}\right)u_2$

$$v_2=\left(\frac{2m_1}{m_1+m_2}\right)u_1+\left(\frac{m_1-m_2}{m_1+m_2}\right)u_2$$

On putting the values v_1 = 6m/s and v_2 = 12m/s

(52) **(B).** Fractional decrease in KE of neutron

$$=1-\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 \qquad [\text{As } m_1=1 \text{ and } m_2=2]$$

$$=1-\left(\frac{1-2}{1+2}\right)^2=1-\left(\frac{1}{3}\right)^2=1-\frac{1}{9}=\frac{8}{9}$$

(53) **(D).** By applying the conservation of momentum

$10000 = (10+10)\times v_{sys.} \Rightarrow v_{sys.} = 50$ cm/s

If system rises upto height h then

$$h=\frac{v_{sys.}^2}{2g}=\frac{50\times 50}{2\times 1000}=\frac{2.5}{2}=1.25\text{ cm}$$

(54) **(A).** In an inelastic collision momentum is conserved but kinetic energy is not.

(55) **(D).**
(i) When $m_1 = m_2$ and m_2 is stationary, there is maximum transfer of kinetic energy in head an collision
(ii) When $m_1 = m_2$ and m_2 is stationary, there is maximum transfer of momentum in head on collision
(iii) When $m_1 >> m_2$ and m_2 is stationary, after head on collision m_2 moves with twice the velocity of m_1.

(56) **(D).** Momentum remains conserved

(57) **(A).** After collision, $v_2=\left(\dfrac{1+e}{2}\right)u$ and $v_1=\left(\dfrac{1-e}{2}\right)u$

Here, u = initial speed of ball 1
$v_2 = 2v_1$, when e = 1/3

(58) **(C).** $m_1v_1 - m_2v_2 = (m_1 + m_2)v$
$\Rightarrow 2\times 3 - 1\times 4 = (2+1)\,v \Rightarrow v = (2/3)$ m/s

(59) **(B).** From conservation of linear momentum
mu = (m+m) v ; v = u/2

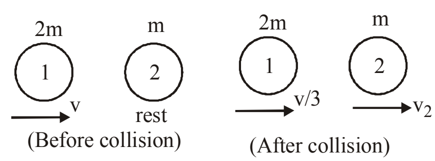
$$\text{Final } E=\frac{1}{2}2mv^2=\frac{1}{2}2m\frac{u^2}{4}=\frac{mu^2}{4}$$

$$\text{Initial K.E.}=\frac{1}{2}mu^2=E$$

Loss = Initial KE – Final KE

$$=\frac{mu^2}{2}-\frac{mu^2}{4}=\frac{mu^2}{4}=\frac{E}{2}$$

(60) **(B).** Using conservation of linear momentum



$$mv_2+2m\frac{v}{3}=2mv+m\,(0)\Rightarrow v_2=\frac{4v}{3}$$

$$e=\frac{\frac{4v}{3}-\frac{v}{3}}{v-0}=1\Rightarrow \text{Collision is elastic.}$$

(61) **(B).** $\tan\theta'=\dfrac{\tan\theta}{e}$ and $e\le 1\Rightarrow \tan\theta'\ge\tan\theta\Rightarrow\theta'\ge\theta$

(62) **(A).** Ball B has changed its momentum in the upward direction in the figure, and as far as the figure can show, there is no change in its horizontal (rightward) velocity.

(63) **(C).** From conservation of momentum, $mv_{B,0} = 2mv_f$.
So $v_f = v_{B,0}/2$. Thus they move away with speed less one half the original speed of cart B.

(64) **(A).** The potential energy of a system of two masses is inversely proportional to the separation between them. In the given case, the potential energy of the system of the two balls will decrease as they come closer to each other. It will become zero (i.e., V(r) = 0) when the two balls touch each other, i.e., at r = 2R, where R is the radius of each billiard ball. The potential energy curves given in figures (i), (ii), (iii), (iv), and (vi) do not satisfy these two conditions. Hence, they do not describe the elastic collisions between them.

(65) **(D).** Stopping distance $S \propto u^2$. If the speed is doubled then the stopping distance will be four times.

(66) **(D).** $s = \frac{u^2}{2\mu g} = \frac{10 \times 10}{2 \times 0.5 \times 10} = 10\,m$

(67) **(D).** $W = mgh = 70 \times 9.8 \times 20 \times 0.25 = 3430J$

(68) **(C).** When the ball is released from the top of tower then ratio of distances covered by the ball in first, second and third second $h_I : h_{II} : h_{III} = 1:3:5$

[because $h_n \propto (2n-1)$]

$\therefore$ Ratio of work done $mgh_I : mgh_{II} : mgh_{III} = 1:3:5$

(69) **(A).** $\frac{1}{2}kS^2 = 10\,J$ (given in the problem)

$$\frac{1}{2}k\left[(2S)^2 - (S)^2\right] = 3 \times \frac{1}{2}kS^2 = 3 \times 10 = 30\,J$$

(70) **(B).** The velocity of sphere of mass $m = u_1$.
The velocity of another sphere of mass, $m = u_2 = 0$
The velocity after collision of first sphere

$$v_1 = \left(\frac{m - em}{m+m}\right)u_1 + 0$$

The velocity after collision of second sphere

$$v_2 = \left[\frac{(1+e)\,m}{m+m}\right]u_1 + 0 \quad \therefore \frac{v_1}{v_2} = \frac{1-e}{1+e}$$

(71) **(B).** Because 50% loss in kinetic energy will affect its potential energy and due to this ball will attain only half of the initial height.

(72) **(B).** $E \propto P^2$ (if m = constant)
% increase in E = 2 (% increase in P)
$= 2 \times 0.01\% = 0.02\%$

(73) **(C).** Force required to move with constant velocity
$\therefore$ Power = FV
Force is required to oppose the resistive force R and also to accelerate the body of mass with acceleration a. $\therefore$ Power = (R + ma) V

(74) **(D).** $P = \frac{mgh}{t} = \frac{100 \times 9.8 \times 50}{50} = 980\,J/s$

(75) **(B).** Mass of the chain hanging = $4 \times 3 = 12$ kg
Shift in center of gravity = 4/2 = 2m
$W = mgh = 12 \times 9.8 \times 2 = 235.2\,J$

(76) **(A).** $v = \frac{dx}{dt} = 3 - 8t + 3t^2$

$\therefore$ $v_0 = 3$ m/s and $v_4 = 19$m/s

$W = \frac{1}{2}m(v_4^2 - v_0^2)$ (According to work energy theorem)

$$= \frac{1}{2} \times 0.03 \times (19^2 - 3^2) = 5.28\,J$$

(77) **(A).** Work done $= \vec{F}.\vec{s}$

$$= (6\hat{i} + 2\hat{j}) \cdot (3\hat{i} - \hat{j}) = 6 \times 3 - 2 \times 1 = 16J$$

(78) **(D).** Here $k = \frac{F}{x} = \frac{10}{1 \times 10^{-3}} = 10^4\,N/m$

$$W = \frac{1}{2}kx^2 = \frac{1}{2} \times 10^4 \times (40 \times 10^{-3})^2 = 8\,J$$

(79) **(C).** Force constant of a spring

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{1 \times 10}{2 \times 10^{-2}} = 500\text{ N/m}$$

Increment in the length = 60 – 50 = 10 cm

$$U = \frac{1}{2}kx^2 = \frac{1}{2}500(10 \times 10^{-2})^2 = 2.5\,J$$

(80) **(D).** Condition for stable equilibrium

$$F = -\frac{dU}{dx} = 0 \Rightarrow -\frac{d}{dx}\left[\frac{a}{x^{12}} - \frac{b}{x^6}\right] = 0$$

$$\Rightarrow -12ax^{-13} + 6bx^{-7} = 0$$

$$\Rightarrow \frac{12a}{x^{13}} = \frac{6b}{x^7} \Rightarrow \frac{2a}{b} = x^6 \Rightarrow x = \sqrt[6]{\frac{2a}{b}}$$

(81) **(D).** For 8 kg piece, momentum p = 20 N

$$KE = \frac{p^2}{2m} = \frac{(20)^2}{2 \times 8} = \frac{400}{16} = 25\,J$$

(82) **(D).** If there is no air drag then maximum height

$$H = \frac{u^2}{2g} = \frac{14 \times 14}{2 \times 9.8} = 10\,m$$

But due to air drag ball reaches up to height 8m only.
Loss in energy $= mg(10-8) = 0.5 \times 9.8 \times 2 = 9.8\,J$

(83) **(D).** Change in momentum = Force × time

$P_2 - P_1 = F \times t = 0.2 \times 10 = 2$

$\Rightarrow P_2 = 2 + P_1 = 2 + 10 = 12$kg-m/s

Increase in K.E.

$$= \frac{1}{2m}(P_2^2 - P_1^2) = \frac{1}{2 \times 5}[(12)^2 - (10)^2] = \frac{44}{10} = 4.4J$$

(84) **(C).** Force produced by the engine

$$F = \frac{P}{v} = \frac{30 \times 10^3}{30} = 10^3 N$$

$$\text{Acceleration} = \frac{\text{Forward force by engine-resistive force}}{\text{mass of car}}$$

$$= \frac{1000 - 750}{1250} = \frac{250}{1250} = \frac{1}{5}\,m/s^2$$

(85) **(C).** $P = \frac{mgh}{t} \Rightarrow \frac{P_1}{P_2} = \frac{m_1}{m_2} \times \frac{t_2}{t_1}$ (As h = constant)

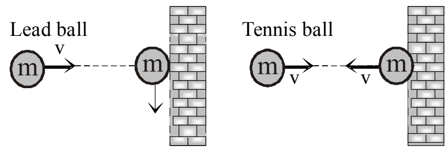
$$\therefore \frac{P_1}{P_2} = \frac{60}{50} \times \frac{11}{12} = \frac{11}{10}$$

(86) **(A).** Power $= \dfrac{\text{workdone}}{\text{time}}$

$= \dfrac{\text{pressure} \times \text{change in volume}}{\text{time}} = \dfrac{20000 \times 1 \times 10^{-6}}{1}$

$= 2 \times 10^{-2} = 0.02\ \text{W}$

(87) **(C).** Change in the momentum
= Final momentum – initial momentum



For lead ball $\Delta\vec{P}_{lead} = 0 - m\vec{v} = -m\vec{v}$

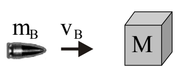
For tennis ball , $\Delta\vec{P}_{tennis} = -m\vec{v} - m\vec{v} = -2m\vec{v}$

i.e. tennis ball suffers a greater change in momentum.

(88) **(D).** In perfectly elastic head on collision of equal masses velocities gets interchanged.

(89) **(D).** Due to the same mass of A and B as well as due to elastic collision velocities of spheres get interchanged after the collision.

(90) **(B).**



Initial K.E. of system = K.E. of the bullet $= \frac{1}{2} m_B v_B^2$

By the law of conservation of linear momentum

$m_B v_B + 0 = m_{sys.} \times v_{sys.}$

$\Rightarrow\ v_{sys.} = \dfrac{m_B v_B}{m_{sys.}} = \dfrac{50 \times 10}{50 + 950} = 0.5\ \text{m/s}$

Fractional loss in K.E. $= \dfrac{\frac{1}{2} m_B v_B^2 - \frac{1}{2} m_{sys.} v_{sys.}^2}{\frac{1}{2} m_B v_B^2}$

By substituting , $m_B = 50 \times 10^{-3}\text{kg},\ v_B = 10\ \text{m/s}$

$m_{sys.} = 1\text{kg},\ v_s = 0.5\ \text{m/s}$ we get

Fractional loss $= \dfrac{95}{100}$ $\therefore$ Percentage loss = 95%

(91) **(C).** According to law of conservation of momentum.
Momentum of neutron = Momentum of combination

$\Rightarrow 1.67 \times 10^{-27} \times 10^{8} = (1.67 \times 10^{-27} + 3.34 \times 10^{-27})\ v$

$\therefore\ v = 3.33 \times 10^{7}\ \text{m/s}$

EXERCISE-2

(1) **(B).** $f_{limiting} = \mu\, mg = 0.3 \times 2.10 = 6N$
Force needed for accn. of 2 m/s^2
$F = ma = 4N$
So friction force static, $f = 4N$

Displacement w.r.t. observer in t = 2 sec. is

$S = u_{rel} t + \frac{1}{2} a_{rel} t^2 = 0 + \frac{1}{2}(-3) \times 4 = -6\text{m}$

$W = \vec{f}.\vec{S} = 4\,(-6) = -24\ \text{J}$

(2) **(A).** By conservation of energy mgh $= \int_0^d F\,dx$

(3) **(D).** Work done by all force = change in kinetic energy
$w_f + mg\ell \sin\theta = K$

(4) **(B).** As $W_{ext} = \Delta E = \frac{1}{2} Kx^2$

$F.x = \frac{1}{2} Kx^2$ or $F = \dfrac{Kx}{2}$

For lifting $Kx = Mg$ or $F = \dfrac{Mg}{2}$

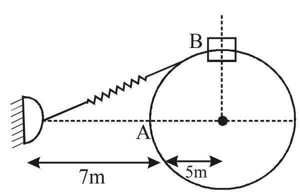
(5) **(B).** Using $P = Fv$, we get $P = Fr\omega$ $(\therefore\ v = r\omega)$
Here $F = \mu N$ (where N is normal reaction)
$= \mu F'g$ $(\because$ 1kgf = g newton)
$\therefore\ P = \mu F' g r\omega = 2\pi\mu n F' g r$ $(\because\ \omega = 2\pi r)$

(6) **(C).** $W_g + W_f = \Delta KE$

$-mgd + Fd = \frac{1}{2} mv^2$; $v = \sqrt{\dfrac{2Fd - 2Mgd}{M}}$

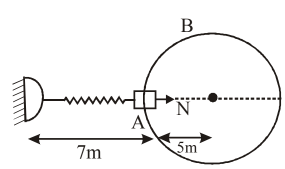
(7) **(C).** Initial extension will be equal to 6m.

$\therefore$ Initial energy $= \frac{1}{2}(200)(6)^2 = 3600\text{J}$



Reaching A : $\frac{1}{2} mv^2 = 3600\text{J} \Rightarrow mv^2 = 7200\ \text{J}$

From F.B.D. at A : $N = \dfrac{mv^2}{R} = \dfrac{7200}{5} = 1440\text{N}$



(8) **(A).** Maximum chance of slipping occurs when spring is maximum compressed. At this moment, as force exerted by the spring is maximum, acceleration of the system is maximum. Hence maximum friction force is required at this moment. By W/E theorem

$\frac{1}{2}(M + m)V^2 = \frac{1}{2} Kx_m^2 \Rightarrow x_m = \sqrt{\dfrac{(M + m)V^2}{K}}$.

Now for upper block $a_m = \frac{Kx_m}{M+m}$

Force on upper block is provided by the friction force.

Therefore, $\mu mg \ge \frac{Kx_m.m}{M+m}$.

For limiting values $V = \mu g\sqrt{\frac{M+m}{K}}$

Using values $V_{max} = 20\text{ cm/s}$

(9) **(D).** From conservation of energy,
$K_1+U_1=K_2+U_2$; $0+mgH=0+mgh_1$
$\Rightarrow h_1=H$. Thus, $h_1=h_2=h_3=H$

(10) **(B).** You do work on the machine on the way up but it does even more work on you on the way down. That's because the two of you push harder on one another on the way down than on the way up. The work you do on the machine is the product of how far you lift it times the upward force you exert on it. The work it does on you is the product of how far it lowers you times the downward force it exerts on you. The latter work is larger than the former, so it does work on you overall.

(11) **(C).** Initial KE $=0$, Initial P.E. $=0$
When the rope is just pulled off the table,

Final K.E. $=\frac{1}{2}(\lambda\ell)v^2$, final P.E. $=(\lambda\ell)g\ell/2$,

Time taken $=t=\ell/v$

Average power $=\frac{\text{net change in energy}}{\text{time}}$

$$=\frac{\frac{1}{2}\lambda\ell v^2+\lambda\ell g\ell/2}{\ell/v}=\frac{1}{2}\lambda v^3+\frac{\lambda\ell vg}{2}$$

(12) **(B).** Instantaneous force $= mg+\frac{vdm}{dt}$ where mg is the force needed for supporting the weight of already hanging section of the rope and $\frac{vdm}{dt}$ is the force needed to supply momentum to the portion of the rope which is to be pulled up

$\frac{vdm}{dt}=v.\frac{d}{dt}(\lambda\ell)=\lambda v^2$, a constant

The force mg is maximum when the whole rope has been pulled up, $(mg)_{max}=(\lambda\ell)g$

Now power due to the force (mg) = mg.v

While power due to force

$$\left(v\frac{dm}{dt}\right)=v.\frac{dm}{dt}\bar{v}=\frac{v^2}{2}\frac{dm}{dt}$$

Where $\bar{v}$ = average velocity of mass dm which is pulled off the ground in the time interval at dt = v/2.

Hence $F_{max}=\lambda\ell g+\lambda v^2$ and $P_{max}=\lambda\ell gv+\frac{1}{2}\lambda v^3$

(13) **(A).** Applying work-energy theorem between A and B.

$$\frac{1}{2}mV_B^2-\frac{1}{2}mV_A^2=W_{gravity}+W_{friction}$$

$$\Rightarrow \frac{1}{2}mV_B^2-\frac{1}{2}m(136)=-mg\,(3+3\sin 37^\circ)-\mu mg\cos 37\times 3$$

$$\Rightarrow \frac{V_B^2}{2}-\frac{136}{2}=-48-12\Rightarrow V_B=4\text{ m/s}$$

(14) **(A).** As you turn around at the end of each swing, you come to a complete stop. Your speed and kinetic energy are both zero. All of your energy is gravitational potential energy. It's as you pass through the bottom of the swing that you are moving fastest and have the most kinetic energy.

(15) **(C).** Work done against friction must equal the initial kinetic

energy $\frac{1}{2}mv^2=\int_1^\infty \mu mgdx$; $\frac{v^2}{2}=Ag\int_1^\infty\frac{1}{x^2}dx$

$\frac{v^2}{2}=Ag\left[-\frac{1}{x}\right]_1^\infty$; $v^2=2gA \Rightarrow v=\sqrt{2gA}$

(16) **(D).** $x=x_1$ and $x=x_3$ are not equilibrium positions because

$\frac{dU}{dx}\ne 0$ at these points.

$x=x_2$ is unstable, as U is maximum at this point.

(17) **(A).** The speed of the water leaving the hose must be $\sqrt{2gh}$ if it is to reach a height h when directed vertically upward. If the diameter is d, the volume of water ejected at this speed is

$$(A.v.)=\frac{1}{4}\pi d^2\times\sqrt{2gh}\,\frac{m^3}{s}$$

$\Rightarrow$ Mass ejected is $\frac{1}{4}\pi d^2\times\sqrt{2gh}\times\rho\,\frac{kg}{s}$

The kinetic energy of this water leaving the hose

$$=\frac{1}{2}mv^2=\frac{1}{8}\pi d^2\times(2gh)^{3/2}\times\rho=21.5kW$$

(18) **(A).** Let at any time the speed of the block along the incline upwards be v.
Then from Newtons's second law

$$\frac{P}{v}-mg\sin\theta-\mu mg\cos\theta=\frac{mdv}{dt}$$

The speed is maximum when $\frac{dv}{dt}=0$

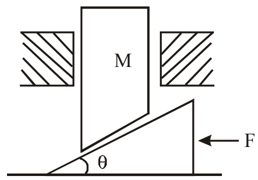
$$\therefore\ v_{max}=\frac{P}{mg\sin\theta+\mu mg\cos\theta}$$

(19) **(B).** $dU = -\vec{F}.d\vec{S} = -\vec{F}.(dx\hat{i} + dy\hat{j})$

Also by reverse method using

$F_x = -\frac{\partial U}{\partial X}$ and $F_y = -\frac{\partial U}{\partial Y}$, only (B) satisfies the criteria.

(20) **(D).** As indicated in figure, we assume that the object to be raised is held between fixed vertical guiding surfaces.



If the wedge moves leftward a distance Δs under the application of the force $\vec{F}$, the object is elevated by a distance $\Delta h = (\Delta s) \tan\theta$. We have $F\,\Delta s = Mg\,\Delta h$ from energy conservation. Therefore the mechanical advantage $\frac{Mg}{F} = \frac{\Delta s}{\Delta h} = \cot\theta$

(21) **(C).** $F = 0$ when $\frac{dU(x)}{dx} = 0 \Rightarrow a = 0$

(22) **(A).** $-\frac{dU}{dr} = F$ (conservative force field)

$\Rightarrow$ $F = \frac{-K}{r}$ provides the centrifugal force for circular motion of electron.

$$\frac{mv^2}{r} = \frac{K}{r} \Rightarrow r = \frac{nh}{2\pi\sqrt{mK}}$$

K.E. of electron $= \frac{1}{2}mv^2 = \frac{1}{2}K$

P.E. of electron $= K \ln r$

$E(n) =$ Total energy $=$ K.E. $+$ P.E.

$$= \frac{1}{2}K + K\ln r = \frac{K}{2}\left[1 + \log\frac{n^2h^2}{4\pi^3 mk}\right]$$

Required ratio $= \frac{E(2) - E(1)}{E(4) - E(2)} = 1$

(23) **(C).** At equilibrium position $x = \frac{mg}{k}$;

$$U_{spring} = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)x = \frac{mgx}{2}$$

$$= \frac{1}{2}(\text{loss in G.P.E.}) \Rightarrow G = 2S$$

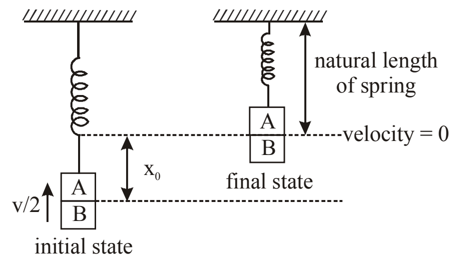
(24) **(A).** $F = 0$ at $x = 0$ and $\pm\sqrt{a/2b}$

$U(x) = 0$ at $x = 0$ and $U(x) < 0$ at $\pm\sqrt{a/2b}$

(25) **(B).** The initial extension in spring is $x_0 = \frac{mg}{k}$

Just after collision of B with A the speed of combined mass is v/2.

For the spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$ (fig.) and comes to rest.



Applying conservation of energy initial and final states, $\frac{1}{2}2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$

Solving we get, $v = \sqrt{\frac{6mg^2}{k}}$

EXERCISE-3

(1) **40.** According to work energy theorem

$W_g + W_F + W_f = \Delta KE = 0$

$W_F = 400$ J

$W_g = -72 \times 5 = -360$ $\quad\therefore W_f = 40$

(2) **2.** $k_{eq} = \frac{100 \times 150}{250} = 60$ N/m

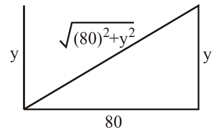
$F = k_{eq}x = 60 \times \frac{2.5}{100} = \frac{3}{2}$ N

For left spring, $x_1 = \frac{3}{2(100)}$

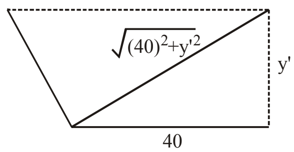
For right spring, $x_2 = \frac{3}{2(150)}$

So, $\frac{\frac{1}{2}\times(100)\left(\frac{3}{2}\right)^2\left(\frac{1}{100}\right)^2}{\frac{1}{2}\times(150)\left(\frac{3}{2}\right)^2\left(\frac{1}{150}\right)^2} = \frac{150}{100} = \frac{3}{2}$

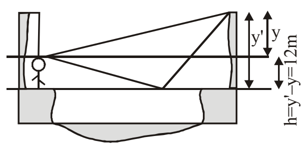
(3) **240.**



$\sqrt{(80)^2 + y^2} = 100$; $y = 18m$



$\sqrt{(40)^2 + y'^2} = 100$



$y' = 30m$. So, $h = 12m$

$mgh = \frac{1}{2}mv^2 \Rightarrow v^2 = 2gh = 2 \times 10 \times 12 = 240$

(4) **2.** $(600g - 300g)\, v = 10 \times 10^3\,(0.6)$

$3000v = 6 \times 10^3$; $v = 2$ m/s

(5) **100.**

$\int_{P_i}^{P_f} \vec{P} = \int \vec{F}.dt \Rightarrow \vec{P}_f = \vec{P}_i + \int \vec{F}.dt = 10\text{ kg-m/s} + \int_0^2 (3+2t)\,dt$

$P_f = 20$ kg-m/sec ; $KE_f = \frac{P_f^2}{2m} = \frac{400}{4} = 100J$

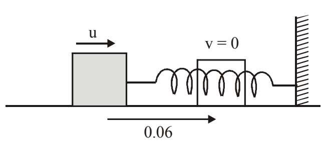
(6) **19.** $E = Pt = 1.35 \times 10^8$; $V = \frac{1.35 \times 10^8}{3.6 \times 10^7} = 3.75\,L$

Since the motor is 20% efficient, five times as much gasoline is needed.

(7) **139.** 12.0 km/h $= 3.33$ m/s

$E_{heat} = E_k = \frac{1}{2}(25)\,(3.33)^2 = 139\,J$

(8) **4.**



From work energy theorem :

$\frac{1}{2}mu^2 = \mu mg \times 0.06 + \frac{1}{2}kx^2$

$\frac{1}{2} \times 0.18\,u^2 = 0.1 \times 0.18 \times 10 \times 0.06 ; 0.4 = \frac{N}{10}$

$N = 4$

(9) **5.** Power $= \frac{dW}{dt} \Rightarrow W = 0.5 \times 5 = 2.5 = KE_f - KE_i$

$2.5 = \frac{M}{2}\,(v_f^2 - v_i^2) \Rightarrow v_f = 5$

(10) **5.** $W_g = -mgh = -1 \times 10 \times 4 = -40\,J$

$W_f = F \times d = 18 \times 5 = +90$

$KE = 90 - 40 = 50 = 10n \Rightarrow n = 5$

EXERCISE-4

(1) **(B).** $W = \frac{1}{2}k\,(x_2^2 - x_1^2) = \frac{1}{2} \times 800\,(225 - 25) \times 10^{-4}$

$= 400\,(200) \times 10^{-4} = 8\,J$

(2) **(A).** $W = \frac{1}{2}k\,(x_2^2 - x_1^2)$

$W = \frac{1}{2} \times 5 \times 10^3\,(10^2 - 5^2) \times 10^{-4} = 18.75\,N-m$

(3) **(B).** If momentum is zero then kinetic energy may or may not be zero. But if kinetic energy is zero, momentum must be zero.

(4) **(B).** Work done = Change in potential energy

$= \frac{1}{2}\,(1.2)\,(10)\,(60 \times 10^{-2}) = 3.6\,J$

(5) **(B).** $W = \vec{F}.\vec{r} = (5\hat{i} + 3\hat{j} + 2\hat{k}).(2\hat{i} - \hat{j}) = 10 - 3 = 7J$

(6) **(A).** $P = Fv$

$P = ma\,v'd$; $P = m\left(\frac{v}{T}\right)\left(\frac{v}{T}\right)t$ $\quad a = \frac{v}{T}$

$v' = 0 + at$; $v' = \frac{v}{T}t$

(7) **(A).** By energy conservation law, $TE_i = TE_f$

$mgh = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right) + mgh'$

$(10)\,(100) = \frac{1}{2}v^2\left(1 + \frac{2}{5}\right) + 10 \times 20$

$100 = \frac{7}{10}v^2 + 200$; $\frac{7}{10}v^2 = 800$; $v = 10\sqrt{\frac{80}{7}}$

(8) **(A).** By energy conservation law

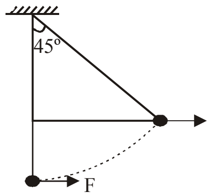
$\frac{P^2}{2M} = \frac{1}{2}KL^2$; $P = \sqrt{MK}\,L$

(9) **(C).** By conservation of momentum, $p_i = p_f$

$mv\,\hat{i} = m\frac{v}{\sqrt{3}}\hat{j} + mv'$; $v' = v\left(\hat{i} - \frac{\hat{j}}{3}\right)$

$|\vec{v}'| = v\sqrt{1 + \frac{1}{3}} = \frac{2v}{\sqrt{3}}$

(10) **(B).**



According to work energy theorem

$W_r + W_g = \Delta K = 0$

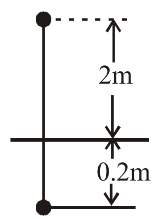
$F(\ell \sin 45^o) - Mg(\ell - \ell \cos 45^o) = 0$

$$\frac{F}{\sqrt{2}} = Mg\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) ;\ F = Mg(\sqrt{2}-1)$$

(11) **(D).** According to work energy theorem, $W = \Delta K$

$$W_g = -\frac{1}{2}mv^2 = -\frac{1}{2}100\times10^{-3}\times5^2$$

(12) **(B).** By Work – Energy theorem



$W_r + W_g = \Delta K = 0$

$F(0.2) - mg(2.2) = 0$

$F(0.2) = 0.2\times10(2.2)$

$F = 22N$

(13) **(C).** $V = \dfrac{x^4}{4} - \dfrac{x^2}{2}$

P.E. will be minimum when $\dfrac{dV}{dx} = 0$

$\dfrac{dV}{dx} = x^3 - x = 0\ ;\ x(x^2-1) = 0\ ;\ x = 0, \pm 1$

So min. value of P.E. $(x = 1)$

$$V_{min} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

Total energy $= K_{max} + V_{min}$

$$2 = K_{max} - \frac{1}{4}$$

Max. kinetic energy $K_{max} = 9/4$

$$\frac{1}{2}mv^2 = \frac{9}{4}\ ;\ \frac{1}{2}(1)v^2 = \frac{9}{4};\ v^2 = \frac{9}{2},\ v = \frac{3}{\sqrt{2}}\,m/s$$

(14) **(D).** By conservation of momentum

$0 = 12(4) + 4(v)$; $v = -12m/s$

So K.E. of the particle $= \dfrac{1}{2}\times4(12)^2 = 288$ J

(15) **(A).** $m_1r_1 = m_2r_2\ ;\ m_1d = m_2r_2\ ;\ r_1 = \dfrac{m_1}{m_2}d$

(16) **(A).** By work energy theorem, $W_g + W_{spring} = \Delta K$

$$(15)\times f + \frac{1}{2}Kx^2 = \frac{1}{2}mv^2$$

(17) **(C).** Mass of a average person m = 60 kg

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\times60\times10^2 = 3000\text{ J}$$

(18) **(B).** Loss of kinetic energy

$$= \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(u_1-u_2)^2 = \frac{1}{2}\times\frac{(0.5)(1)}{0.5+1}(2-0)^2$$

$$= \frac{0.5}{3}\times4 = \frac{2}{3} = 0.67\text{ J}$$

(19) **(D).** Slope of velocity-time graph will remain constant except during collision where it will not be defined. Also variation of height with time will be parabolic.

(20) **(C).** $U(x) = \dfrac{a}{x_{12}} - \dfrac{b}{x^6}$; $U(x=\infty) = 0$

as, $F = -\dfrac{dU}{dx} = -\left[\dfrac{12a}{x^{13}} + \dfrac{6b}{x^7}\right]$

at equilibrium, $F = 0$ $\therefore\ x^6 = \dfrac{2a}{b}$

$$\therefore\ U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4A}$$

$$\therefore\ d = [U(x=\infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

(21) **(A).** $k_1x_1 = k_2x_2 = F$

$$W_1 = \frac{1}{2}k_1x_1^2 = \frac{(k_1x_1)^2}{2k_1} = \frac{F^2}{2k_1}$$

Similarly, $W_2 = \dfrac{F^2}{2k_2} \Rightarrow W \propto \dfrac{1}{k}$

$W_1 > W_2$

$\Rightarrow k_1 < k_2$ statement-2 is true.

Statement 1 : $W_1 = \dfrac{1}{2}k_1x^2$, $W_2 = \dfrac{1}{2}k_2x^2$

So, $W_2 > W_1$. Statement-1 is false.

(22) **(D).** Maximum energy loss $= \dfrac{P^2}{2m} - \dfrac{P^2}{2(m+M)}$

$$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$$

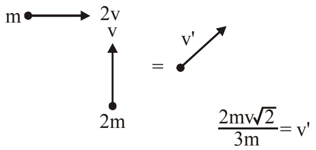
$$\left(f = \frac{M}{m+M}\right)$$

Hence Statement -I is wrong and statement II is correct.

(23) **(A).** $F = ax + bx^2$; $dW = F\,dx$

$$W = \int_0^1 (ax+bx^2)\,dx\ ;\ W = \frac{aL^2}{2} + \frac{bL^3}{3}$$

(24) (B).



$$\text{KE loss} = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 - \frac{1}{2}\times(3m)\left(\frac{2mv\sqrt{2}}{3m}\right)^2 = \frac{5}{3}mv^2$$

$$\text{Reqd. \%} = \frac{\frac{5}{3}mv^2}{2mv^2 + mv^2}\times 100 = 56\%$$

(25) (C). $PE = mgh \times 1000 = 10\times 9.8\times 1\times 1000$
$PE = 10^4 \times 9.8$ J. If mass lost is m, then

$$\text{Energy} = m\times\frac{2}{10}\times 3.8\times 10^7 \text{ J}$$

$$\text{So, } 9.8\times 10^4 = m\times\frac{1}{5}\times 3.8\times 10^7$$

$$m = \frac{5}{3.8}\times 10^{-3}\times 9.8 = 1.289\times 10^{-2}\,\text{kg}$$

(26) (B). $W_{PQ} = \mu mg\cos\theta\times\frac{2}{\sin\theta} = \frac{\mu 2mg}{\tan\theta} = 2mg\sqrt{3}\mu$

$W_{QR} = \mu mg\cdot x \quad \therefore \quad x = 2\sqrt{3} = 3.5\text{m}$

P.E. lost $= mgh = 2mg$

$2mg = 4mg\sqrt{3}\,\mu\ ;\ \frac{1}{2\sqrt{3}} = \mu = 0.29$

(27) (D). $F = 6t\ ;\ \int F\,dt = \Delta P \Rightarrow P = \int_0^1 6t\,dt = \frac{6}{2} = 3$

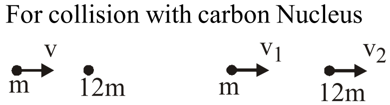
$$\Delta K = \frac{P^2}{2m} = \frac{3^2}{2\times 1} = 4.5\ ;\ W = \Delta K = 4.5\text{ J}$$

(28) (C). For collision with deuterium:

$mv + 0 = mv_1 + 2mv_2$ .... (1)
(Conservation of momentum)
$v_2 - v_1 = v \quad (e = 1)$ .... (2)
By (1) and (2) $v_1 = -v/3$

$$P_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{8}{9} = 0.89$$

For collision with carbon Nucleus



$mv + 0 = mv_1 + 12mv_2$ .... (1)
(Conservation of momentum)
$v = v_2 - v_1 \quad (e = 1)$ .... (2)

By (1) and (2) $v_1 = -\frac{11}{13}v$

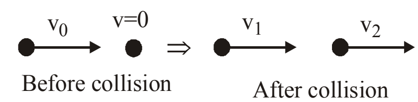
$$P_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2} = \frac{48}{169} \approx 0.28$$

(29) (A). $U = -\frac{k}{2r^2}\ ;\ F = -\frac{dU}{dr}\hat{r} = -\frac{K}{r^3}\hat{r}\ ;\ \frac{K}{r^3} = \frac{mv^2}{r}$

(As Force towards center $= \frac{mv^2}{r}$); $KE = \frac{1}{2}mV^2 = \frac{K}{2r^2}$

$$\text{Total energy} = KE + PE = \frac{K}{2r^2} - \frac{K}{2r^2} = \text{Zero}$$

(30) (D). It is given that final total kinetic energy has increased, so some internal energy of the system must have been converted into kinetic energy.



$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 1.5\left(\frac{1}{2}mv_0^2\right)$$

$\Rightarrow \quad v_1^2 + v_2^2 = 1.5v_0^2$ ... (i)

Since, there is no external force, momentum can be conserved $mv_1 + mv_2 = mv_0 \Rightarrow v_1 + v_2 = v_0$ ...... (ii)
From (i) & (ii), $v_1 + (v_0 - v_1)^2 = 1.5\,v_0^2$
Relative velocity $= |v_2 - v_1|$

$$= \text{Difference of roots} = \frac{\sqrt{D}}{a} = \sqrt{2}v_0$$

(31) (C). Maximum speed is at mean position (equilibrium). $F = kx \Rightarrow x = F/k$

$$W_F + W_{sp} = \Delta KE\ ;\ F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2 \Rightarrow v_{max} = \frac{F}{\sqrt{mk}}$$

(32) (A). $\frac{P^2}{2m} - \frac{P^2}{2(2m+M)} = \frac{5}{6}\frac{P^2}{2m}$

$$\frac{1}{m} - \frac{1}{2m+M} = \frac{5}{6m} \Rightarrow \frac{M}{m} = 4$$

(33) (A). According to work energy theorem
Work done by force on the particle = Change in KE
Work done = Area under F-x graph

$$= \int F\cdot dx = 2\times 2 + \frac{(2+3)\times 1}{2}$$

$W = KE_{final} - KE_{initial} = 6.5$
$KE_{initial} = 0\ ;\ KE_{final} = 6.5$ J

(34) **(C).** Applying linear momentum conservation

$$m_1v_1\hat{i}+m_2v_2\hat{i}=m_1v_3\hat{i}+m_2v_4\hat{i}$$
$$m_1v_1+0.5\,m_1v_2=m_1(0.5\,v_1)+0.5\,m_1v_4$$
$$0.5\,m_1v_1=0.5\,m_1\,(v_4-v_2)$$
$$v_1=v_4-v_2$$

(35) **(A).** $4000\times V+mg\times V=P$

$$\frac{60\times 746}{4000+20000}=V$$

$V=1.86$ m/s. ≈ 1.9 m/s.

(36) **10.** $KE=PE_1-PE_2=mgh_1-mgh_2$
$=1\times 10\times 2-1\times 10\times 1=10$ J

(37) **(A).** Net force on motor will be
$F_m=[920+68\,(10)]\,g+6000=22000$ N
So, required power for motor

$$P_m=\vec{F}_m\cdot\vec{v}=22000\times 3=66000\text{ watt}$$

(38) **1.** By conservation of linear momentum :

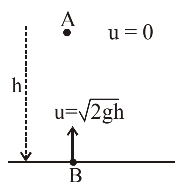
$$(0.1)\,(3\hat{i})+(0.1)\,(5\hat{j})=(0.1)\,(4)\,(\hat{i}+\hat{j})+(0.1)\,v$$

$\Rightarrow\quad \vec{v}=-\hat{i}+\hat{j}$

$\therefore\quad$ Speed of B after collision $|\vec{v}|=\sqrt{2}$

Kinetic energy $=\frac{1}{2}mV^2=\frac{1}{2}(0.1)\,(2)=\frac{1}{10}\quad\therefore\quad x=1$

(39) **(D).**

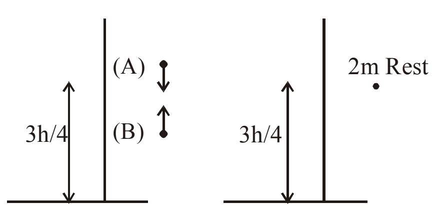


Particles will collide after time $t_0=\frac{h}{\sqrt{2gh}}$

At collision, $v_A=gt_0$; $v_B=u_B-gt_0$

$\Rightarrow\quad v_A=-v_B$

Before collision After collision

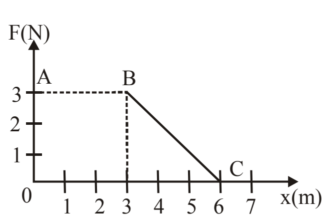


Time taken by combined mass to reach the ground

$$\text{Time}=\sqrt{\frac{2\times(3h/4)}{g}}=\sqrt{\frac{3h}{2g}}$$

EXERCISE-5

(1) **(B).**



Work done = area under F-x graph

$$=3\times 3+\frac{1}{2}\times 3\times 3=13.5\text{ J}$$

(2) **(B).** From conservation of linear momentum
$m_1v_1+m_2v_2=0$

$$v_2=\left(\frac{-m_1}{m_2}\right)v_1=\left(\frac{-18}{12}\right)6=-9\text{m/s}$$

$$\text{K.E.}=\frac{1}{2}m_2v_2^2=\frac{1}{2}\times 12\times 9^2=486\text{ J}$$

(3) **(B).** Acceleration $=\frac{d^2S}{dt^2}=\frac{2}{3}$

Force $=3\times\frac{2}{3}=2$ newton.

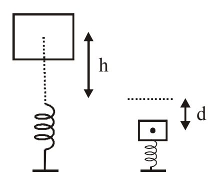
Displacement $=\frac{1}{3}\times 2\times 2=\frac{4}{3}$m

(in 2 secs.)

Work done $=2\times\frac{4}{3}=\frac{8}{3}$J

(4) **(B).** By using work energy theorem $W=\Delta KE$
(here $\Delta KE=0$)
$\Rightarrow\quad 300-W_{gravity}-W_{friction}=0$
$\Rightarrow\quad W_{friction}=300-mgh=300-(2)\,(10)\,(10)=100$J

(5) **(A).** Gravitational potential energy of ball gets converted into elastic potential energy of the spring.



$$mg\,(h+d)=\frac{1}{2}kd^2$$

New work done $=mg\,(h+d)-\frac{1}{2}kd^2=0$

(6) **(B).** $\frac{MV^2}{R}=Mg$

$V=\sqrt{Rg}$; $V=\sqrt{20\times 10}=14.14$ m/s

(7) **(B).** $P_{generated}=P_{input}\times\frac{90}{100}=\frac{Mgh}{t}\times\frac{90}{100}$

$$=\frac{15\times 10\times 60}{1}\times\frac{90}{100}=8.1\text{ KW}$$

(8) **(B).** From momentum conservation
$4V+0.2\,v=0$ ..... (1)

$\frac{1}{2}\times 4\times V^2+\frac{1}{2}\times 0.2\times v^2=1050$ ..... (2)

Solving (1) and (2), $v=100$ m/s

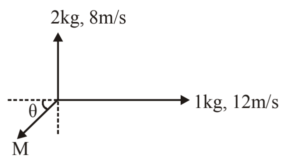
(9) **(D).** m : mass per unit length

$\therefore$ Rate of mass per sec = mv

(Think on dimension basis)

$$\text{Rate of K.E.} = \frac{1}{2}(mv)\,v^2 = \frac{1}{2}mv^3$$

(10) **(D).** $Mv\cos\theta = 12,\ \ Mv\sin\theta = 16$



$$\tan\theta = \frac{16}{12} = \frac{4}{3}\ ;\quad M = \frac{12\times 5}{4\times 3} = \frac{60}{12} = 5\text{ kg}$$

(11) **(D).** $\frac{1}{2}mv^2 - mgh = \frac{1}{2}\times 1\times 400 - 1\times 18\times 10$

$= 200 - 180 = 20$ J

(12) **(A).** Here, $m_1 = m$, $m_2 = 2m$, $u_1 = 2$ m/s, $u_2 = 0$

Coefficient of restitution, e = 0.5

Let v_1 and v_2 be their respective velocities after collision. Applying the law of conservation of linear momentum, we get $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\therefore\ m\times 2 + 2m\times 0 = m\times v_1 + 2m\times v_2$

or $2m = mv_1 + 2mv_2$ or $2 = (v_1 + 2v_2)$ ...... (i)

By definition of coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2} \quad\text{or}\quad e\,(u_1 - u_2) = v_2 - v_1$$

$0.5\,(2-0) = v_2 - v_1\ ;\quad 1 = v_2 - v_1$ ...... (ii)

Solving equations (i) and (ii), we get

$v_1 = 0$ m/s, $v_2 = 1$ m/s

(13) **(D).** Here, Mass per unit length of water, $\mu = 100$ kg/m

Velocity of water, v = 2m/s

Power of engine,

$P = \mu v^3 = (100\text{ kg/m})\,(2\text{m/s})^3 = 800\text{W}$

(14) **(D).** Power delivered in time T is

$$P = F.V. = MaV \ \text{ or } P = MV\frac{dV}{dT}$$

$$\Rightarrow\ P\,dT = MV\,dV \Rightarrow PT = \frac{MV^2}{2}\ \text{ or }\ P = \frac{1}{2}\frac{MV^2}{T}$$

(15) **(C).** $P = \vec{F}.\vec{v} = Fv\cos\theta$

Just before hitting θ is zero and both F, v are maximum.

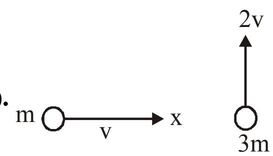
(16) **(D).** Impulse = MV – (–MV) = 2 MV

(17) **(C).** The potential energy of a system increases if work is done by the system against a conservative force.

(18) **(A).** Work = Area under (F – d) graph

= 8 + 5 = 13 J

(19) **(A).**



From momentum conservation

$$mv\hat{i} + 3m\,(2v)\hat{j} = (4m)\,\vec{v}$$

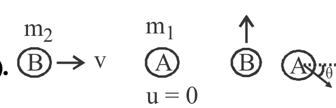
$$\vec{v} = \frac{v}{4}\hat{i} + \frac{6}{4}v\hat{j} = \frac{v}{4}\hat{i} + \frac{3}{2}v\hat{j}$$

(20) **(B).** For equilibrium, $\frac{dU}{dr} = 0 \Rightarrow \frac{-2A}{r^3} + \frac{B}{r^2} = 0\ ;\ r = 2A/B$

For stable equilibrium $\frac{d^2U}{dr^2}$ should be positive

Here $\frac{d^2U}{dr^2} = \frac{6A}{r^4} - \frac{2B}{r^3}$ is +ve value for $r = \frac{2A}{B}$

(21) **(D).**



Conservation of linear momentum along x-direction

$$m_2v = m_1v_x\ ;\quad \frac{m_2v}{m_1} = v_x$$

Along y-direction

$$0 = m_2\times\frac{v}{2} + m_1v_y\ ;\ \tan\theta = \frac{v_y}{v_x} = \frac{1}{2}$$

(22) **(B).** When stone hits the ground momentum

$P = m\sqrt{2gh}$

When same stone dropped from 2h (100% of initial) then momentum

$P' = m\sqrt{2g\,(2h)} = \sqrt{2}P$

which is changed by 41% of initial.

(23) **(B).** Constant power of car $P_0 = F.V = ma.v$

$$P_0 = m\frac{dv}{dt}.v\ \ ;\ \ P_0dt = mv\,dv$$

$P_0 t = \frac{mv^2}{2}$; $v = \sqrt{\frac{2P_0 t}{m}}$; $v \propto \sqrt{t}$

(24) **(B).** $W = \vec{F}.\vec{S} = (3\hat{i} + \hat{j}).[(4-2)\,\hat{i} + (3-0)\,\hat{j} + (-1-1)\,\hat{k}]$

$= (3\hat{i} + \hat{j}).(2\hat{i} + 3\hat{j} - 2\hat{k}) = 3\,(2) + 1\,(3) + 0\,(-2) = 9$ J

(25) **(B).** Initial momentum $= P_i = 0$

Final momentum $P_f = 0 = mv\hat{i} + mv\hat{j} + \vec{P}_3$

$\Rightarrow$ $P_3 = mv\sqrt{2}$

Total KE $= \frac{P_3^2}{2 \times 2m} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

$= \frac{2m^2v^2}{4m} + mv^2 = \frac{3mv^2}{2}$

(26) **(B).** By energy conservation,

$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon$

(27) **(D).** $a = \frac{0.1x}{10} = 0.01x = V\frac{dV}{dx}$

So, $\int_{v_1}^{v_2} V\,dV = \int_{20}^{30} \frac{x}{100}dx$

$\left.\frac{V^2}{2}\right|_{v_1}^{v_2} = \left.\frac{x^2}{200}\right|_{20}^{30} = \frac{30 \times 30}{200} - \frac{20 \times 20}{200} = 4.5 - 2 = 2.5$

$\frac{1}{2}m\,(v_2^2 - v_1^2) = 10 \times 2.5\text{ J} = -25\text{ J}$

$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - 25 = \frac{1}{2} \times 10 \times 10 \times 10 - 25$

$= 500 - 25\text{ J} = 475\text{ J}$

(28) **(B).** If extension is same $W = \frac{1}{2}Kx^2$, so, $W_P > W_Q$

If spring force is same $W = \frac{F^2}{2K}$, so $W_Q > W_P$

(29) **(D).** $\frac{dw}{dt} = P$; $w = Pt = \frac{1}{2}mV^2$; $\sqrt{\frac{2Pt}{m}} = V$

Hence, $a = \frac{dV}{dt} = \sqrt{\frac{2P}{m}} \cdot \frac{1}{2\sqrt{t}}$

$ma = \sqrt{\frac{2Pm^2}{m}} \cdot \frac{1}{2\sqrt{t}} = \sqrt{\frac{Pm}{2t}}$

(30) **(B).** By conservation of linear momentum:

$p_f = p_i = 0 \Rightarrow p_{He} - p_{Th} = 0 \Rightarrow p_{He} = p_{Th}$

but $K \propto 1/m$ and $m_{He} < m_{Th}$

So, $K_{He} > K_{Th}$

(31) **(C).** Let ball rebounds with speed v

So, $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20$ m/s

Energy just after rebound

$E = \frac{1}{2}mv^2 = 200$ m

50% energy loses in collision means just before collision energy is 400 m

By using energy conservation

$\frac{1}{2}mv_0^2 + mgh = 400m$

$\Rightarrow \frac{1}{2}mv_0^2 + m \times 10 \times 20 = 400m$; $v_0 = 20$ m/s

(32) **(B).** In elastic collision energy of system remains same so, $(K.E)_{\text{before collision}} = (K.E)_{\text{After collision}}$

Let speed of second body after collision is V'

$\frac{1}{2}mv^2 + 0 = \frac{1}{2}m\left(\frac{v}{3}\right)^2 + \frac{1}{2}m\,(v')^2$

$\Rightarrow$ $v' = \frac{2\sqrt{2}}{3}v$

(33) **(A).** m = 0.01 kg, r = 6.4 cm

$\frac{1}{2}mv^2 = 8 \times 10^{-4}$ J

$v^2 = \frac{16 \times 10^{-4}}{0.01} = 16 \times 10^{-2}$

Speed $v^2 = 2a_t s$; $v^2 = 2a_t 4\pi r$

$\Rightarrow$ $a_t = \frac{v^2}{8\pi r} = \frac{16 \times 10^{-2}}{8 \times 3.14 \times 6.4 \times 10^{-2}} = 0.1\text{ m/s}^2$

(34) **(D).** $v_{min} = \sqrt{5gR}$

(35) **(D).** $F = (2t\,\hat{i} + 3t^2\hat{j}),\ \vec{a} = 2t\hat{i} + 3t^2\hat{j}$

$v = \int_0^t a\,dt = t^2\hat{i} + t^3\hat{j}$

$P = \vec{F}.\vec{v} = 2t \cdot t^2 + 3t^2 \cdot t^3 = 2t^3 + 3t^5$

(36) **(C).** Apply conservation of linear momentum.

CM rises through height h, so its velocity after collision $= \sqrt{2gh}$

$$0.01 \times 400 = 2 \times \sqrt{2gh} + 0.01 \times v$$

$v = 120$ m/s

(37) **(B).** They will exchange their velocity, so

$v_B = 0.5$ m/s and $v_A = -0.3$ m/s

(38) **(C).** $\vec{s} = \vec{r}_2 - \vec{r}_1 = (4\hat{j} + 3\hat{k}) - (-2\hat{i} + 5\hat{j}) = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{F} = 4\hat{i} + 3\hat{j}$; $W = \vec{F} \cdot \vec{s} = 8 - 3 = 5J$

(39) **(A).** $\vec{J} = |\vec{P}_2 - \vec{P}_1| = 2mV\cos\theta = mV$

(40) **(C).** Work done by the gravity (Wg) = mgh

$$= 10^{-3} \times 10 \times 10^3 = 10 \text{ J}$$

By work-energy theorem,

$W_g + W_{res} = \Delta KE$

$$10 + W_{res} = \frac{1}{2} \times 10^{-3} \times (50)^2 \;;\; W_{res} = -8.75 \text{ J}$$

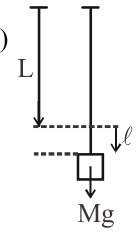
(41) **(B).** According to law of conservation of linear momentum, $mv + 4m \times 0 = 4mv' + 0$; $v' = v/4$

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$$= \frac{v/4}{v} = \frac{1}{4} = 0.25$$

(42) **(C).** $U = \frac{1}{2}$ (work done by gravity)

$U = \frac{1}{2} mg\ell$



(43) **(B).** Fractional loss of KE of ccolliding body

$$\frac{\Delta KE}{KE} = \frac{4(m_1 m_2)}{(m_1 + m_2)^2} = \frac{4(4m)\,2m}{(4m + 2m)^2} = \frac{32m^2}{36m^2} = \frac{8}{9}$$

(44) **(C).** Work done by variable force is $W = \int_{y_i}^{y_f} F\,dy$

Here, $y_i = 0$, $y_f = 1$ m

$$W = \int_0^1 (20 + 10y)\,dy = \left[20 + \frac{10y^2}{2}\right]_0^1 = 25 \text{ J}$$