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BINOMIAL THEOREM

INTRODUCTION

Given two number a and b, if we are required to find $(a+b)^2$, we can just add a and b then multiply the sum by itself. Another way of doing it is to find $a^2 + 2ab + b^2$. We also know that $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$, Now we may need to use the expression for $(a+b)^5$, $(a+b)^7$, etc. But we cannot remember all the expressions. Binomial theorem helps to find these expressions.

Binomial Expression : An algebraic expression containing two term is called a binomial expression for example,

$(2x+3)$, $(x^2 - \frac{1}{x})$, $(x+a)$, etc are binomial expression.

Binomial theorem : The formula by which any power of a binomial can be expanded in the form of a series is called binomial theorem.

TERMINOLOGY USED IN BINOMIAL THEOREM

Factorial notation : n! is pronounced as factorial n and is

defined as $n! = \begin{cases} n(n-1)(n-2).......3.2.1 \,;\, \text{if } n \in N \\ 1 \quad ; \text{ if } n = 0 \end{cases}$

Note : $n! = n.(n-1)!$, $n \in N$

Mathematical meaning of ^nC_r (Other symbol $\binom{n}{r}$ & C(n, r))

The term ^nC_r denotes number of combinations of r things chosen from n distinct things mathematically,

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

Properties related to ^nC_r :

(i) $^nC_r = {^nC_{n-r}}$

(ii) $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

(iii) If $^nC_x = {^nC_y}$ then either $x = y$ or $x + y = n$

(iv) $^nC_r = \frac{n}{r}\,.\,{^{n-1}C_{r-1}}$

(v) $\frac{1}{r+1}\,{^nC_r} = \frac{1}{n+1}\,{^{n+1}C_{r+1}}$

(vi) $\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$

(vii) If n and r are relatively prime, then ^nC_r is divisible by n, but converse is not necessarily true.

BINOMIAL THEOREM FOR POSITIVE INTEGRAL INDEX

Theorem : If x and a are real number, then for all $n \in N$,

$(x+a)^n = {^nC_0}\,x^n a^0 + {^nC_1}\,x^{n-1} a^1 + {^nC_2}\,x^{n-2} a^2 + ...+ {^nC_r}\,x^{n-r} a^r + ...+ {^nC_{n-1}}\,x^1 a^{n-1} + {^nC_n}\,x^0 a^n$

i.e. $(x+a)^n = \sum_{r=0}^{n} {^nC_r}\, x^{n-r}\, a^r$

Properties :

(i) There are $(n+1)$ terms in the expansion of $(x+a)^n$

(ii) The sum of powers of a and x in each term of expansion is n.

(iii) The first and last term being x^n and a^n respectively.

(iv) The binomial coefficients in the expansion of $(x+a)^n$ equidistant from the beginning and the end are equal.

$^nC_0 = {^nC_n}$, $^nC_1 = {^nC_{n-1}}$ .....

General Term : In the expansion of $(x+a)^n$, $(r+1)^{th}$ term is called the general term which can be represented by T_{r+1}.

$T_{r+1} = {^nC_r}\,x^{n-r} a^r$

$= {^nC_r}\,(\text{first term})^{n-r}\,(\text{second term})^r$

To find a term from the end in the expansion of $(x+a)^n$:

It can be easily seen that in the expansion of $(x+a)^n$,

$(r+1)^{th}$ term from end $= (n-r+1)^{th}$ term from beginning.

i.e. $T_{r+1}(E) = T_{n-r+1}(B)$

$T_r(E) = T_{n-r+2}(B)$

Some deduction of binomial expansion :

(i) Expansion of $(x-a)^n$

$(x-a)^n = {^nC_0}x^n a^0 - {^nC_1}x^{n-1}a^1 + {^nC_2}x^{n-2}a^2 - {^nC_3}x^{n-3}a^3 + ...+(-1)^r\,{^nC_r}x^{n-r}a^r + ...+(-1)^n\,{^nC_n}\,x^0 a^n$

Put $(-a)$ in place of a in the expansion of $(x+a)^n$.

General term $= (r+1)^{th}$ term

$T_{r+1} = {^nC_r}(-1)^r.x^{n-r}\,a^r$

(ii) By putting $x = 1$ and $a = x$ in the expansion of $(x+a)^n$

$(1+x)^n = {^nC_0} + {^nC_1}x + {^nC_2}x^2 + ...+ {^nC_r}x^r + ...+ {^nC_n}\,x^n$

which is the standard form of binomial expansion

General term $= (r+1)^{th}$ term

$$T_{r+1} = {^nC_r}x^r = \frac{n(n-1)(n-2).....(n-r+1)}{r!}\,x^r$$

(iii) By putting $(-x)$ in place of x in the expansion of $(1+x)^n$

$(1-x)^n = {^nC_0} - {^nC_1}x + {^nC_2}x^2 - {^nC_3}x^3 + ... + (-1)^r\,{^nC_r}x^r + ...+ {^nC_n}x^n.$

General term $= (r+1)^{th}$ term

$$T_{r+1} = (-1)^r.{^nC_r}x^r = (-1)^r\,\frac{n(n-1)(n-2).....(n-r+1)}{r!}\,x^r$$

Number of terms in the expansion of $(x+y+z)^n$:

$(x+y+z)^n$ can be expanded as –

$(x+y+z)^n = \{(x+y)+z\}^n$

$= (x+y)^n + {}^nC_1(x+y)^{n-1}.z + {}^nC_2(x+y)^{n-2}z^2 + ... + {}^nC_nZ^n.$

$=$ (n + 1) terms + n terms + (n–1) terms +..... + 1 term

$\therefore$ Total number of terms = $(n+1)+n+(n-1)+.....+1$

$$= \frac{(n+1)(n+2)}{2}$$

The number of distinct terms in the expansion of $(x_1+x_2+x_3+.....x_r)^n$ is given by ${}^{n+r-1}C_{r-1}$

BINOMIAL COEFFICIENTS & THEIR PROPERTIES

In the expansion of $(1+x)^n$

i.e. $(1+x)^n = {}^nC_0 + {}^nC_1x + ... + {}^nC_rx^r + ... + {}^nC_nx^n$

The coefficients ${}^nC_0, {}^nC_1, {}^nC_n$ of various powers of x, are called binomial coefficients and they are written as $C_0, C_1, C_2, ...C_n$.

Hence, $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_rx^r + ... + C_nx^n$ ....(1)

where $C_0 = 1,\ C_1 = n,\ C_2 = \frac{n(n-1)}{2!}$

$C_r = \frac{n(n-1).......(n-r+1)}{r!},\ C_n = 1$

Now, we shall obtain some important expressions involving binomial coefficients-

(a) Sum of coefficient : Putting x = 1 in (1), we get

$C_0 + C_1 + C_2 + ...+ C^n = 2^n$ ....(2)

i.e. $\sum_{r=0}^{n} {}^nC_r = 2^n$

(b) Sum of coefficients with alternate signs:

Putting x = –1 in (1) We get

$C_0 - C_1 + C_2 - C_3 + ... = 0$ ....(3)

i.e. $\sum_{r=0}^{n} (-1)^r\ {}^nC_r = 0$

(c) Sum of coefficient of even and odd terms

From (3), we have

$C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ...$ ....(4)

i.e. Sum of coefficient of even and odd terms are equals

From (2) and (4) we have

$\Rightarrow\ C_0 + C_2 + ... = C_1 + C_3 + ... = 2^{n-1}$

(d) Sum of products of coefficients

Replacing x by 1/x in (1) We get

$\left(1+\frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + ... + \frac{C_n}{x^n} + ...$ ....(5)

Multiplying (1) by (5), we get

$\frac{(1+x)^{2n}}{x^n} = (C_0 + C_1x + C_2x^2 + ...)\left(C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + ...\right)$

Now, comparing coefficients of x^r on both the sides, we get

$C_0C_r + C_1C_{r+1} + ... + C_{n-r}C_n = {}^{2n}C_{n-r}$

$= \frac{2n!}{(n+r)!(n-r)!}$ ....(6)

(e) Sum of squares of coefficients

Putting r = 0 in (6), we get

$C_0^2 + C_1^2 + C_2^2 + ... + C_n^2 = \frac{2n!}{n!n!}$

(f) Putting r = 1 in (6) we get

$C_0C_1 + C_1C_2 + C_2C_3 + ... + C_{n-1}C_n$

$= {}^{2n}C_{n-1} = \frac{2n!}{(n+1)!(n-1)!}$ ....(7)

(g) Putting r = 2 in (6), we get

$C_0C_2 + C_1C_3 + C_2C_4 + ... + C_{n-2}C_n$

$= {}^{2n}C_{n-2} = \frac{2n!}{(n+2)!(n-2)!}$ ....(8)

Example 1 :

Find the tenth term in the expansion of $(2x^2 + 1/x)^{12}$

Sol. Comparing $(2x^2 + 1/x)^{12}$ with $(X + a)^n$

$n = 12, X = 2x^2, a = 1/x.$

$\therefore\ 10^{th}$ term $= T_{10} = {}^{12}C_9\,(2x^2)^{12-9}\,(1/x)^9 = {}^{12}C_9.\,8.1/x^3$

or $T_{10} = 1760/x^3$

Example 2 :

Find the 7th term from the end in the expansion of $\left(x-\frac{2}{x^2}\right)^{10}$.

Sol. The 7th term from the end = 5th term from beginning

$T_5 = {}^{10}C_4x^6\left(-\frac{2}{x^2}\right)^4 = {}^{10}C_4\cdot 2^4\left(\frac{1}{x^2}\right)$

Example 3 :

If the second, third and fourth terms in the expansion of $(a+b)^n$ are 135, 30 and 10/3 respectively, then find the value of n.

Sol. $T_2 = {}^nC_1\,ab^{n-1} = 135$ ....(i)

$T_3 = {}^nC_2\,a^2b^{n-2} = 30$ ....(ii)

$T_4 = {}^nC_3\,a^3b^{n-3} = \frac{10}{3}$ ....(iii)

Dividing (i) by (ii)

$\frac{{}^nC_1ab^{n-1}}{{}^nC_2a^2b^{n-2}} = \frac{135}{30}\ ;\ \frac{n}{\frac{n}{2}(n-1)}\,\frac{b}{a} = \frac{9}{2}$ ....(iv)

$\frac{b}{a} = \frac{9}{4}(n-1)$ ....(v)

Dividing (ii) & (iii)

$\dfrac{\frac{n(n-1)}{2}}{\frac{n(n-1)(n-2)}{3.2}} \cdot \frac{b}{a} = \frac{10}{3} = 9$ ....(vi)

Eliminating a and b from (v) and (vi) $\Rightarrow n = 5$

Example 4 :

Find the number of terms in the expansion of $(x + y + 2z)^8$.

Sol. $\because$ n = 8 and from the above given formula we have

Number of terms $= \frac{(8+1)(8+2)}{2} = 45$

Example 5 :

Find the value of $aC_0 + (a + b) C_1 + (a + 2b)C_2 + ... + (a + nb) C_n$

Sol. $a(C_0 + C_1 + C_2 + ... + C_n) + b(C_1 + 2C_2 + ... + {}^nC_n)$

$= a.2^n + b.n2^{n-1} = 2^n \left(\frac{2a+nb}{2}\right) = (2a + nb)\, 2^{n-1}$

MIDDLE TERM IN THE EXPANSION OF $(x + a)^n$

There are (n +1) terms in the expansion of $(x + a)^n$. The middle term depends upon the value of n.

(a) When n is even, then only one middle term exists and it is $\left(\frac{n}{2}+1\right)^{th}$ term ; Middle term $= {}^nC_{\frac{n}{2}} x^{\frac{n}{2}} . a^{\frac{n}{2}}$

(b) When n is odd, there will be two middle terms and they are $\frac{(n+1)^{th}}{2}$ and $\frac{(n+3)^{th}}{2}$ terms.

The first middle term $= {}^nC_{\frac{n-1}{2}} x^{\frac{n+1}{2}} . a^{\frac{n-1}{2}}$

The second middle term $= {}^nC_{\frac{n+1}{2}} x^{\frac{n-1}{2}} . a^{\frac{n+1}{2}}$

Example 6 :

Find the middle term in the expansion of $(x + 4)^4$.

Sol. Here n = 4 is even so there is only one middle term which is $(4/2) + 1 = 3^{rd}$ term.

Therefore, middle term $T_3 = {}^4C_2(x)^2(4)^2 = 96x^2$

Example 7 :

If the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924\, x^6$, then find the value of n.

Sol. Since n is even therefore $\left(\frac{n}{2}+1\right)^{th}$ term is middle term,

hence ${}^nC_{n/2}\,(x^2)^{n/2}\left(\frac{1}{x}\right)^{n/2} = 924\, x^6 \Rightarrow x^{n/2} = x^6 \Rightarrow n = 12$

GREATEST BINOMIAL COEFFICIENTS

In a binomial expansion binomial coefficients of the middle terms are called as greatest binomial coefficients.

(i) If n is even : When $r = \frac{n}{2}$ i.e. ${}^nC_{n/2}$ takes maximum value.

(ii) if n is odd : $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$ i.e. ${}^nC_{\frac{n-1}{2}} = {}^nC_{\frac{n+1}{2}}$ and take maximum value.

Example 8 :

Find the greatest coefficient in the expansion of $(1 + x)^{2n}$.

Sol. The greatest coefficient = the coefficient of the middle term

$= {}^{2n}C_n = \frac{1.3.5.........(2n-1)}{n!} . 2^n$

Example 9 :

Find the term which has the greatest binomial coefficient in the expansion of $(x^2 + 2/x)^6$

Sol. We know that Binomial Coefficient of middle term is the greatest Binomial coefficient.

Since n = 6 is even, So the middle term is $T_{n/2+1}$

$\therefore$ middle term $= n/2 + 1 = 3 + 1 \Rightarrow 4^{th}$ term

TO DETERMINE A PARTICULAR TERM IN THE EXPANSION

In the expansion of $\left(x^\alpha \pm \frac{1}{x^\beta}\right)^n$, if x^m occurs in T_{r+1}, then r is given by $n\alpha - r(\alpha + \beta) = m \Rightarrow r = \frac{n\alpha - m}{\alpha + \beta}$

Thus in above expansion if constant term i.e. the term which is independent of x, occurs in T_{r+1} then r is determined by

$n\alpha - r(\alpha + \beta) = 0 \Rightarrow r = \frac{n\alpha}{\alpha + \beta}$

Example 10 :

Find the coefficient of x^{39} in the expansion of $(x^4 - 1/x^3)^{15}$

Sol. From above formula, $r = \frac{15(4) - 39}{4+3} = 3$

$\therefore$ The required term $= T_4 = {}^{15}C_3\,(x^4)^{12}\,(-1/x^3)^3 = -455\, x^{39}$

$\therefore$ coefficient of $x^{39} = -455$

Example 11 :

Find the term independent of x in the expansion of

$$\left(\sqrt[6]{x}-\frac{1}{\sqrt[3]{x}}\right)^9$$

Sol. $T_{r+1} = {}^9C_r\left(\sqrt[6]{x}\right)^{9-r}\left(-\frac{1}{\sqrt[3]{x}}\right)^r$

$$= {}^9C_r(-1)^r \cdot x^{\frac{9-r}{6}-\frac{r}{3}} = {}^9C_r \cdot x^{\left(\frac{9-3r}{6}\right)}$$

Now $\frac{9-3r}{6} = 0 \Rightarrow r = 3$; $T = -{}^9C_3$

Example 12 :

Find the term independent of x in the expansion of

$$\left(\frac{4}{3}x^2-\frac{3}{2x}\right)^9$$

Sol. Here comparing $\left(\frac{4}{3}x^2-\frac{3}{2x}\right)^9$ with $\left(x^{\alpha}\pm\frac{1}{x^{\beta}}\right)^n$

We get $\alpha = 2$, $\beta = 1$, $n = 9$ and $r = \frac{9(2)}{2+1} = 6$

$\therefore$ $(6+1) = 7^{th}$ term is independent of x.

NUMERICALLY GREATEST TERM IN THE EXPANSION OF $(x+a)^n$

Let T_{r+1} be the greatest term in $(x+a)^n$

Then $\frac{T_{r+1}}{T_r} \ge 1$ and $\frac{T_{r+2}}{T_{r+1}} \le 1$

$$\left|\frac{T_{r+1}}{T_r}\right| = \left|\frac{{}^nC_r x^{n-r}a^r}{{}^nC_{r-1}x^{n-r+1}a^{r+1}}\right| = \left(\frac{n-r+1}{r}\right)\left|\frac{a}{x}\right|$$

Now $\left(\frac{n-r+1}{r}\right)\left|\frac{a}{x}\right| \ge 1 \Rightarrow \frac{n+1}{r} \ge \left|\frac{x}{a}\right|+1 \Rightarrow r \le \frac{n+1}{\left|\frac{x}{a}\right|+1}$

Algorithm :

Step I : Calculate $\frac{n+1}{\left|\frac{x}{a}\right|+1} = k$ (say)

Step II : (a) If k is an integer than T_k and T_{k+1} are the numerically greatest term.

(b) If k is not an integer. Let m is its integral part than T_{m+1} is the numerically greatest term.

Example 13 :

If the sum of the coefficients is expansion of $(1+2x)^n$ is 6561, find the greatest term in the expansion for $x = 1/2$

Sol. Sum of the coefficient in the expansion of $(1+2x)^n = 6561$

$\Rightarrow (1+2x)^n = 6561$, when $x = 1$

$\Rightarrow 3^n = 6561 \Rightarrow 3^n = 3^8 \Rightarrow n = 8$

Now, $\frac{T_{r+1}}{T_r} = \frac{{}^8C_r(2x)^r}{{}^8C_{r-1}(2x)^{r-1}} = \frac{9-r}{r}\times 2x$

$\Rightarrow \frac{T_{r+1}}{T_r} = \frac{9-r}{r}$ $\left[x = \frac{1}{2}\right]$

$\therefore \frac{T_{r+1}}{T_r} > 1 \Rightarrow \frac{9-r}{r} > 1 \Rightarrow 9 - r > r \Rightarrow 2r < 9 \Rightarrow r < 4\frac{1}{2}$

Hence 5^{th} term is the greatest term.

TRY IT YOURSELF-1

Q.1 Find the 6^{th} term in the expansion of $(2x^2 - 1/3x^2)^{10}$.

Q.2 Find the coefficient of x^k in $1+(1+x)+(1-x)^2+...+(1+x)^n$ $(0 \le k \le n)$.

Q.3 Find the sum ${}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9$.

Q.4 If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is α and and the coefficients of middle terms in the expansion of $(1+x)^{2n+1}$ are β and γ, then relate α, β and γ.

Q.5 Find the greatest coefficient in the expansion of $(1+2x/3)^{15}$.

Q.6 Find the sum $3\,{}^nC_0 - 8\,{}^nC_1 + 13\,{}^nC_2 - 18\,{}^nC_3 + ...$

Q.7 The number of dissimilar terms in the expansion of $(1-3x+3x^2-x^3)^{20}$ is –

(A) 21 (B) 31

(C) 41 (D) 61

Q.8 Find 7^{th} term of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^9$

Q.9 Find the middle term(s) in the expansion of $\left(1-\frac{x^2}{2}\right)^{14}$.

Q.10 Find the coefficient of x^{32} and x^{-17} in $\left(x^4-\frac{1}{x^3}\right)^{15}$.

ANSWERS

(1) $-\frac{896}{27}$ **(2)** ${}^{n+1}C_{k+1}$ **(3)** 2^9

(4) $\beta+\gamma=\alpha$ **(5)** ${}^{15}C_6(2/3)^6$ **(6)** 0

(7) (D) **(8)** $\frac{10500}{x^3}$ **(9)** $-\frac{429}{16}x^{14}$

(10) Coefficient of x^{32} is 1365, x^{-17} is –1365.

MULTINOMIAL THEOREM

If $(n \in N)$, then general terms in expansion of

$\left(x_1+x_2+x_3+.........+x_k\right)^n$ is

$$\frac{n!}{a_1!a_2!a_3!.....a_k!}.x_1^{a_1}.x_2^{a_2}.x_3^{a_3}........x_k^{a_k} \text{ where}$$

$a_1 + a_2 + a_3 + ... + a_k = n, 0 \le a_i \le n, i = 1, 2, 3, ...k$ and the number of terms in the expansion are ${}^{n+k-1}C_{k-1}$.

Number of terms in $(x + y)^n = {}^{n+1}C_1$;

Number of terms in $(x + y + z)^n = {}^{n+2}C_2$

Example 14 :

Find the coefficient of $a^2b^3c^4d$ in the expansion of $(a - b - c + d)^{10}$.

Sol. $(a - b - c + d)^{10}$

$$= \sum_{r_1+r_2+r_3+r_4=10} \frac{(10)!}{r_1!\,r_2!\,r_3!\,r_4!}(a)^{r_1}(-b)^{r_2}(-c)^{r_3}(d)^{r_4}$$

We want to get $a^2b^3c^4d$ this implies that

$r_1 = 2, r_2 = 3, r_3 = 4, r_4 = 1$

$\therefore$ Coefficient of $a^2b^3c^4d$ is

$$\frac{(10)!}{2!3!4!1!}(-1)^3(-1)^4 = -12600$$

BINOMIAL THEOREM FOR NEGATIVE AND FRACTIONAL INDICES

When n is a negative integer or a fraction then the expansion of a binomial is possible only when

(i) Its first term is 1, and

(ii) Its second term is numerically less than 1.

when $n \notin N$ and $|x| < 1$, then it states

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...+ \frac{n(n-1)(n-r+1)}{r!}x^r + ...\infty$$

General term : $T_{r+1} = \dfrac{n(n-1)(n-2)........(n-r+1)}{r!}x^r$

Note :

(i) In this expansion the coefficient of different terms can not be expressed as ${}^nC_0$, ${}^nC_1$, ${}^nC_2$,...... because n is not a positive integer.

(ii) In this case there are infinite terms in the expansion.

Some important Expansions :

If $|x| < 1$ and $n \in Q$ but $n \notin N$, then

(a) $(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 +......+ \dfrac{n(n-1).....(n-r+1)}{r!}x^r + ...$

(b) $(1-x)^n = 1 - nx + \dfrac{n(n-1)}{2!}x^2 - \dfrac{n(n-1)(n-2)}{3!}x^3 +......+ \dfrac{n(n-1).....(n-r+1)}{r!}(-x)^r + ...$

(c) $(1-x)^{-n} = 1 + nx + \dfrac{n(n+1)}{2!}x^2 + \dfrac{n(n+1)(n+2)}{3!}x^3 +......+ \dfrac{n(n+1).....(n+r-1)}{r!}x^r + ...$

(d) $(1+x)^{-n} = 1 - nx + \dfrac{n(n+1)}{2!}x^2 - \dfrac{n(n+1)(n+2)}{3!}x^3 +......+ \dfrac{n(n+1)(n+r-1)}{r!}(-x)^r + ...$

By putting n = 1, 2, 3 in the above results (c) and (d), we get the following results

(e) $(1-x)^{-1} = 1 + x + x^2 + x^3 + ...+ x^r + ...$

General term $T_{r+1} = x^r$

(f) $(1+x)^{-1} = 1 - x + x^2 - x^3 + ...(-x)^r + ...$

General term $T_{r+1} = (-x)^r$

(g) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + ...+ (r+1)x^r + ...$

General term $T_{r+1} = (r+1)x^r$

(h) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + ...+ (r+1)(-x)^r + ...$

General term $T_{r+1} = (r+1)(-x)^r$

(i) $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + ...+ \dfrac{(r+1)(r+2)}{2!}x^r + ...$

General term $= \dfrac{(r+1)(r+2)}{2!}x^r$

(j) $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + ...+ \dfrac{(r+1)(r+2)}{2!}(-x)^r + ...$

General term $= \dfrac{(r+1)(r+2)}{2!}(-x)^r$

Example 15 :

Find the term independent of x in the expansion of $\left(\dfrac{1-x}{1+x}\right)^2$.

Sol. $(1-x)^2(1+x)^{-2} = (1 - 2x + x^2)(1 - 2x + 3x^2 + ...)$

The term independent of x is 1.

Example 16 :

If $|x| < 2/3$ then find the fourth term in the expansion of $\left(1+\dfrac{3}{2}x\right)^{1/2}$.

Sol. $T_4 = \dfrac{1/2(1/2-1)(1/2-2)}{3!}.\left(\dfrac{3x}{2}\right)^3 = \dfrac{27}{128}x^3$

Example 17 :

If $\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{\sqrt{4-x}}$ is approximately equal to a + bx for small values of x, then find (a,b)

Sol. $$\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{2\left[1-\frac{x}{4}\right]^{1/2}}=\frac{\left[1+\frac{1}{2}(-3x)+\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}(-3x)^2+........\right]+\left[1+\frac{5}{3}(-x)+\frac{5}{3}\frac{2}{3}\frac{1}{2}(-x)^2+....\right]}{2\left[1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}\left(-\frac{x}{4}\right)^2+....\right]}$$

$$=\frac{\left[1-\frac{19}{12}x+\frac{53}{144}x^2-......\right]}{\left[1-\frac{x}{2}-\frac{1}{8}x^2-.....\right]}=1-\frac{35}{24}x+........$$

Neglecting higher powers of x, then

$$a+bx=1-\frac{35}{24}x \Rightarrow a=1,\ b=-\frac{35}{24}$$

SOME APPLICATIONS OF BINOMIAL THEOREM

1. **Integral and fractional part :**

   If $(\sqrt{A}+B)^n=I+f$, where I and n are positive integers, n being odd and $0<f<1$, then

   $(I+f).f=K^n$ where $A-B^2=K>0$ and $\sqrt{A}-B<1$

   If n is an even integer, then $(I+f)(1-f)=K^n$.

2. To find the remainder when a^n is divided b, we adjust the power of a to a^m which is very close to b say with difference 1. Also, the remainder is always positive. When number of the type $3k-1$ is divided by 3, we have

   $$\frac{3k-1}{3}=\frac{3k-3+2}{3}=k-1+\frac{2}{3}$$

   Hence, the remainder is 2.

3. $2\le\left(1+\frac{1}{n}\right)^n<3,\ n\ge1,\ n\in N$

4. **To find the sum of Infinite series :**

   We can compare the given infinite series with the expansion of $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+.....$ and by finding the value of x and n and putting in $(1+x)^n$ the sum of series is determined.

Example 18 :

Find the value of cube root of 1001 up to five decimal places.

Sol. $(1001)^{1/3}=(1000+1)^{1/3}=10\left(1+\frac{1}{1000}\right)^{1/3}$

$$=10\left\{1+\frac{1}{3}\cdot\frac{1}{1000}+\frac{1/3(1/3-1)}{2!}\frac{1}{1000^2}+.....\right\}$$

$=10\{1+0.0003333-0.00000011+....\}=10.00333$

Example 19 :

Find the remainder when 5^{99} is divided by 13.

Sol. Here $5^2=25$ which is close to $26=13\times2$.

Hence, $E=5^{99}=5\times5^{98}=5\times(5^2)^{49}=5\,(26-1)^{49}$

$E=5\,[{}^{49}C_0\,26^{49}-{}^{49}C_1\,26^{48}+{}^{49}C_2\,26^{48}-....+{}^{49}C_{48}\,26-{}^{49}C_{49}]$

$=5\times26k-5$

Now, $\frac{E}{13}=10k-\frac{5}{13}=10k-1+\frac{8}{13}$.

Hence, the remainder is 8.

Example 20 :

Find the positive integer just greater than $(1+0.0001)^{10000}$.

Sol. $(1+0.0001)^{10000}=\left(1+\frac{1}{10000}\right)^{10000}$

Now, we know that $2\le\left(1+\frac{1}{n}\right)^n<3,\ n\ge1,\ n\in N$

Hence, positive integer just greater than $(1+0.0001)^{10000}$ is 3.

Example 21 :

The sum of $1+\frac{1}{4}+\frac{1\cdot3}{4\cdot8}+\frac{1\cdot3\cdot5}{4\cdot8\cdot12}+....\infty$ is–

(A) $\sqrt{2}$ (B) $1/\sqrt{2}$

(C) $\sqrt{3}$ (D) $2^{3/2}$

Sol. Comparing with $1+nx+\frac{n(n-1)}{2!}x^2+......$

$nx=1/4$ ...(1)

and $\frac{n(n-1)x^2}{2!}=\frac{1.3}{4.8}$

or $\frac{nx(nx-x)}{2!}=\frac{3}{32}\Rightarrow\frac{1}{4}\left(\frac{1}{4}-x\right)=\frac{3}{16}$ (by (1))

$\Rightarrow\left(\frac{1}{4}-x\right)=\frac{3}{4}\Rightarrow x=\frac{1}{4}-\frac{3}{4}=-\frac{1}{2}$ ... (2)

putting the value of x in (1)

$n(-1/2)=1/4\Rightarrow n=-1/2$

Sum of series $=(1+x)^n=(1-1/2)^{-1/2}=(1/2)^{-1/2}=\sqrt{2}$

Example 22 :

If x is so small so that its square and higher power can be neglected. Find the value of $\dfrac{\left(1+\frac{2x}{3}\right)^{-5}+(4+2x)^{1/2}}{(4+x)^{3/2}}$.

Sol. $\dfrac{\left(1+\frac{2x}{3}\right)^{-5}+(4+2x)^{1/2}}{(4+x)^{3/2}}=\dfrac{\left(1-\frac{10x}{3}\right)+2\left(1+\frac{x}{4}\right)}{8\left(1+\frac{3x}{8}\right)}$

$=\frac{1}{8}\left(3-\frac{10x}{3}+\frac{x}{2}\right)\left(1+\frac{3x}{8}\right)^{-1}=\frac{3}{8}\left(1-\frac{17x}{18}\right)\left(1-\frac{3x}{8}\right)$

$=\frac{3}{8}\left(1-\frac{17x}{18}-\frac{3x}{8}\right)=\frac{72-95x}{24\times 8}$

Example 23 :

If $(6\sqrt{6}+14)^{2n+1}$ = [N] + F and F = N – [N]; where [.] denotes greatest integer function, then NF is equal to

(A) 20^{2n+1} (B) an even integer
(C) odd integer (D) 40^{2n+1}

Sol. **(AB).** Since $(6\sqrt{6}+14)^{2n+1}=[N]+F$

Let us assume that $f=(6\sqrt{6}-14)^{2n+1}$; where $0\le f<1$.

$[N]+F-f=(6\sqrt{6}+14)^{2n+1}-(6\sqrt{6}-14)^{2n+1}$

$=2\left[{}^{2n+1}C_1(6\sqrt{6})^{2n}(14)+{}^{2n+1}C_3(6\sqrt{6})^{2n-2}(14)^3+....\right]$

$\Rightarrow$ $[N]+F-f=$ even integer.

Now, $0<F<1$ and $0<f<1$

So, $-1<F-f<1$ and $F-f$ is an integer so it can only be zero.

Thus, NF = $(6\sqrt{6}+14)^{2n+1}(6\sqrt{6}-14)^{2n+1}=20^{2n+1}$.

Example 24 :

Find the last three digits in 11^{50}.

Sol. Expansion of $(10+1)^{50}={}^{50}C_0 10^{50}+{}^{50}C_1 10^{49}$
$+.....+{}^{50}C_{48}10^2+{}^{50}C_{49}10+{}^{50}C_{50}$

$=\underbrace{{}^{50}C_0 10^{50}+{}^{50}C_1 10^{49}+......+{}^{50}C_{47}10^3}_{1000K}$
$+49\times 25\times 100+500+1$

$\Rightarrow$ $1000K+123001$

$\Rightarrow$ Last 3 digits are 001.

Example 25 :

Prove that $2222^{5555}+5555^{2222}$ is divisible by 7.

Sol. When 2222 is divided by 7 it leaves a remainder 3. So adding & subtracting 3^{5555}, we get :

$E=\underbrace{2222^{5555}-3^{5555}}_{E_1}+\underbrace{3^{5555}+5555^{2222}}_{E_2}$

For E_1 : Now since 2222–3 = 2219 is divisible by 7, therefore E_1 is divisible by 7 ($\because$ x^n-a^n is divisible by x –a)

For E_2 : 5555 when divided by 7 leaves remainder 4.

So adding and subtracting 4^{2222}, we get :

$E_2=3^{5555}+4^{2222}+5555^{2222}-4^{2222}$

$=(243)^{1111}+(16)^{1111}+(5555)^{2222}-4^{2222}$

Again $(243)^{1111}+16^{1111}$ and $(5555)^{2222}-4^{2222}$ are divisible by 7 ($\because$ x^n+a^n is divisible by x + a when n is odd)

Hence $2222^{5555}+5555^{2222}$ is divisible by 7.

TRY IT YOURSELF-2

Q.1 Find (i) the last digit, (ii) the last two digits and (iii) the last three digits of 17^{256}.

Q.2 Find the remainder when $x=5^{5^{5^{\cdot^{\cdot^{5}}}}}$ (24 times 5) is divided by 24.

Q.3 Find the values of x, for which $1/(\sqrt{5+4x})$ can be expanded by binomial theorem.

Q.4 Find the sum $1-\frac{1}{8}+\frac{1}{8}\times\frac{3}{16}-\frac{1\times3\times5}{8\times16\times24}+....$

Q.5 Using binomial theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Q.6 Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

Q.7 What is the remainder when 5^{99} is divided by 13.

Q.8 Find the last two digits of the number $(17)^{10}$.

Q.9 Coefficient of x^{11} in the expansion of $(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$ is –

(A) 1051 (B) 1106
(C) 1113 (D) 1120

ANSWERS

(1) 681, 81, 1 **(2)** 5 **(3)** $|x|<5/4$

(4) $2/\sqrt{5}$ **(5)** $(1.1)^{10000}>1000$ **(6)** 0.951

(7) 8 **(8)** 49 **(9)** (C)

EXPONENTIAL & LOGARITHMIC SERIES

'e' Series : The sum of infinite Series $1+\frac{1}{1!}+\frac{1}{2!}+.....\infty$ is denoted by the number e.

If n tends to infinity then value of $\left(1+\frac{1}{n}\right)^n$ is right value of given series therefore.

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \quad \text{or } e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+.......\infty$$

$$e^x = 1 + x + \frac{x^2}{2!} + ...\infty$$

Logarithmic series : If $|x| < 1$ then

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\infty$$

SOME IMPORTANT RESULTS

Related to exponential series :

(i) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + ... + \frac{x^r}{r!} + ...$ or $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$

(ii) Replacing x by –x the above result becomes.

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + ... + (-1)^r\frac{x^r}{r!} + ...$$

or $e^{-x} = \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$

(iii) The value of e lies between 2 and 3

(iv) Generally, value of e taken as $e \cong 2.7183$ up to four places of decimal.

(v) e is an irrational number lying between 2.71 and 2.73.

(vi) To find the sum of exponential series.

(a) Write n^{th} term of given series

(b) Numerator of T_n must be independent from n

(c) By putting n = 1, 2, 3, ... we find T_1, T_2, ...

(d) The sum of T_1, T_2, ... is value of series

(vii) General term :

(a) In the expansion of e^{ax}.

General Term $= T_{n+1} = \frac{(ax)^n}{n!}$; coefficient of $x^n = \frac{a^n}{n!}$

(b) In the expansion of e^{-ax}

General term $= T_{n+1} = \frac{(-a)^n x^n}{n!}$; coefficient of $x^n = \frac{(-a)^n}{n!}$

(c) In the expansion of e^{ax+b}

General term $= T_{r+1} = e^b\frac{a^n x^n}{n!}$

coefficient of $x^n = e^b\frac{a^n}{n!}$

Related to Logarithmic series:

(i) $a^x = 1 + x(\log_e a) + \frac{x^2}{2!}(\log_e a)^2 + \frac{x^3}{3!}(\log_e a)^3 + ...\infty$

(ii) $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... \infty$ when $|x| < 1$

(iii) $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}$ .... ∞ when $|x| < 1$

(iv) $\log\frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + ...\infty\right)$

(v) $\log\frac{x+1}{x-1} = 2\left(\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + ...\infty\right)$

(vi) $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$

Example 26 :

Find the value of $e^{-1/5}$ correct to four places of decimal.

Sol. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ to ∞ ...... (1)

Putting x = (–1/5) in eq. (1), we get

$$e^{-1/5} = 1 - \frac{1}{5} + \frac{1}{2!}\left(-\frac{1}{5}\right)^2 + \frac{1}{3!}\left(-\frac{1}{5}\right)^3 + \frac{1}{4!}\left(-\frac{1}{5}\right)^4 + ... \text{ to } \infty$$

$$e^{-1/5} = 1 - \frac{2}{10} + \frac{2^2}{2!}\cdot\frac{1}{10^2} - \frac{2^3}{3!}\cdot\frac{1}{10^3} + \frac{2^4}{4!}\cdot\frac{1}{10^4} + ...$$

$e^{-1/5} = 1 - 0.200000 + 0.0200000 - 0.001333 + 0.000066$

$e^{-1/5} = 0.8187$ (correct to 4 decimal places)

Example 27 :

If α, β are the roots of the eq. $x^2 - px + q = 0$, prove that

$$\log_e(1 + px + qx^2) = (\alpha+\beta)x - \frac{\alpha^2+\beta^2}{2}x^2 + \frac{\alpha^3+\beta^3}{3}x^3 - ....$$

Sol. $\text{RHS} = \left[\alpha x - \frac{\alpha^2 x^2}{2} + \frac{\alpha^3 x^3}{3} - ...\right] + \left[\beta x - \frac{\beta^2 x^2}{2} + \frac{\beta^3 x^3}{3} - ...\right]$

$= \log_e(1 + \alpha x) + \log_e(1 + \beta x)$

$= \log_e(1 + (\alpha+\beta)x + \alpha\beta x^2)$

$= \log_e(1 + px + qx^2) = \text{LHS}$

Here, we have used the facts $\alpha + \beta = p$ and $\alpha\beta = q$. We know this from the given roots of the quadratic equation. We have also assumed that both $|\alpha . x| < 1$ and $|\beta x| < 1$.

NOTE

1. $\sum_{n=0}^{\infty}\frac{1}{n!} = \sum_{n=0}^{\infty}\frac{1}{(n-1)!} = \sum_{n=0}^{\infty}\frac{1}{(n-k)!} = e$

2. $\sum_{n=1}^{\infty}\frac{1}{n!} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \infty = e - 1$

3. $e^{ax} = 1 + \frac{(ax)}{1!} + \frac{(ax)^2}{2!} + \frac{(ax)^3}{3!} + ... + \frac{(ax)^n}{n!} + ... \infty$

4. $\sum_{n=1}^{\infty} \frac{n^2}{n!} = \sum_{n=1}^{\infty} \frac{n}{(n-1)!} = 2e$

5. $\sum_{n=1}^{\infty} \frac{n^3}{n!} = \sum_{n=1}^{\infty} \frac{n^2}{(n-1)!} = 5e$

6. $\sum_{n=1}^{\infty} \frac{n^4}{n!} = \sum_{n=1}^{\infty} \frac{n^3}{(n-1)!} = 15e$

7. The coefficient of x^n in the expansion of

$e^{a+bx} = 1 + \frac{(a+bx)}{1!} + \frac{(a+bx)^2}{2!} + ...\infty$ is $\frac{e^a . b^n}{n!}$

ADDITIONAL EXAMPLES

Example 1 :

Find the coefficient of x^{20} in the expansion of

$(1+x^2)^{40} . \left(x^2 + 2 + \frac{1}{x^2}\right)^{-5}$

Sol. Expression $= (1+x^2)^{40} . \left(x + \frac{1}{x}\right)^{-10} = (1+x^2)^{30} . x^{10}$

The coefficient of x^{20} in $x^{10}(1+x^2)^{30}$

= the coefficient of x^{10} in $(1+x^2)^{30} = {}^{30}C_5 = {}^{30}C_{30-5} = {}^{30}C_{25}$

Example 2 :

If in the expansion of $(1+y)^n$, find the coefficient of 5^{th}, 6^{th} and 7^{th} terms are in A.P., then find the value of n.

Sol. As given ${}^nC_4$, ${}^nC_5$, ${}^nC_6$ are in A.P.

$\Rightarrow {}^nC_4 + {}^nC_6 = 2.\,{}^nC_5$

$\Rightarrow \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!} = 2\frac{n}{(n-5)!5!}$

$\Rightarrow 30 + (n-5)(n-4) = 2.6\,(n-4)$

$\Rightarrow n^2 - 21n + 98 = 0 \Rightarrow (n-7)(n-14) = 0 \;\therefore\; n = 7, 14$

Example 3 :

Find the order of polynomial $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

Sol. $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

$= 2\,[x^5 + {}^5C_2\, x^3(x^3-1) + {}^5C_4\, x\,(x^3-1)^2]$

$= 2\,[x^5 + 10x^3\,(x^3-1) + 5x\,(x^6 - 2x^3 + 1]$

$= 10x^7 + 20x^6 + 2x^5 - 20x^4 - 20x^3 + 10x$

$\therefore$ Polynomial has order of 7

Example 4 :

Find the sum of the last ten coefficients in the expansion of $(1+x)^{19}$ when expanded in ascending powers of x

Sol. The required sum $= {}^{19}C_{10} + {}^{19}C_{11} + ... + {}^{19}C_{19}$

$= {}^{19}C_0 + {}^{19}C_1 + {}^{19}C_2 + ... + {}^{19}C_9$ (since ${}^nC_r = {}^nC_{n-r}$)

Adding, $2 \times$ (required sum) $= {}^{19}C_0 + {}^{19}C_1 + ... + {}^{19}C_{19} = 2^{19}$

Example 5 :

If $(1+x-2x^2)^8 = a_0 + a_1x + a_2x^2 + ... + a_{16}x^{16}$ then find the sum $a_1 + a_3 + a_5 + ... + a_{15}$.

Sol. Sum $= \frac{1}{2}\{(a_0 + a_1 + a_2 + ... + a_{16}) - (a_0 - a_1 + a_2 - .... + a_{16})\}$

$= \frac{1}{2}\left\{(1+1-2)^8 - (1-1-2)^8\right\} = \frac{1}{2}\left(-2^8\right) = -2^7$

Example 6 :

Find the sum of the coefficients of even powers of x in the expansion of $(1+x+x^2+x^3)^5$.

Sol. $(1+x+x^2+x^3)^5$

$= (1+x)^5\,(1+x^2)^5 = (1+5x+10x^2+10x^3+5x^4+x^5)$
$(1+5x^2+10x^4+10x^6+5x^8+x^{10})$

$\Rightarrow$ Coefficient of even powers of x

$= (1+10+5) \times 2^5 = 16 \times 32 = 512$

Example 7 :

Find the sum of the coefficient of all the integral powers of x in the expansion of $(1+2\sqrt{x})^{40}$

Sol. The coefficient of all the integral powers of x are ${}^{40}C_0$, ${}^{40}C_2.2^2$, ${}^{40}C_4.2^4$,......, ${}^{40}C_{40}.2^{40}$

$(1+2)^{40} = {}^{40}C_0 + {}^{40}C_1.2 + {}^{40}C_2.2^2 + ... + {}^{40}C_{40}.2^{40}$

$(1-2)^{40} = {}^{40}C_0 - {}^{40}C_1.2 + {}^{40}C_2.2^2 - .... + {}^{40}C_{40}.2^{40}$

Adding $3^{40} + 1 = 2 \times$ (required sum) $= \frac{1}{2}(3^{40}+1)$

Example 8 :

If x is numerically very small as compared with 1, then find the value of $(1-7x)^{1/3}\,(1+2x)^{-3/4}$.

Sol. $(1-7x)^{1/3} = 1 + \frac{1}{3}(-7x) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{1.2}(-7x)^2 + ...$ ...... (i)

$(1+2x)^{-3/4} = 1 + (-3/4)(2x) + \frac{(-3/4)(-7/4)}{1.2}(4x^2) + ...$ .. (ii)

Multiply (i) and (ii), we get

$\Rightarrow (1-7x)^{1/3}\,(1+2x)^{-3/4} = 1 - \frac{7}{3}x - \frac{3}{2}x + ... = 1 - \frac{23x}{6}$

(Neglected rest term as x small)

Example 9 :

Find the number of terms with integral coefficients in the expansion of $(7^{1/3} + 5^{1/2}.\,x)^{600}$

Sol. $t_{r+1} = {}^{600}C_r .\, 7^{\frac{600-r}{3}} .\, 5^{r/2} .x^r$

Here $0 \le r \le 600$ and $\frac{r}{2}$, $200 - \frac{r}{3}$ are integers

$\therefore$ r should be a multiple of 6

$\therefore$ r = 0, 6, 12, ..., 600

Example 10 :

If $(1 + ax)^n = 1 + 8x + 24x^2 + ..$ then find the values of a and n

Sol. $na = 8 \Rightarrow n^2a^2 = 64, \frac{n(n-1)}{2}a^2 = 24$. since $\frac{2n}{n-1} = \frac{8}{3}$

$\Rightarrow \quad 6n = 8n - 8 \Rightarrow \quad n = 4, a = 2$

Example 11 :

If $(1 - x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$ then find the value of $a_0 + a_2 + a_4 + ... + a_{2n}$

Sol. Put $x = 1 \Rightarrow 1 = a_0 + a_1 + a_2 + ... + a_{2n}$

Put $x = -1 \Rightarrow 3^n = a_0 - a_1 + a_2 - a_3 + ... + a_{2n}$

Adding, $3^n + 1 = 2(a_0 + a_2 + a_4 + ... + a_{2n})$

Example 12 :

Find the largest coefficient in the expansion of $(4 + 3x)^{25}$

Sol. $(4 + 3x)^{25} = 4^{25}\left(1 + \frac{3}{4}x\right)^{25}$

Let $(r + 1)^{th}$ term will have largest coefficient

$$\Rightarrow \frac{\text{Coefficient of } T_{r+1}}{\text{Coefficient of } T_r} \geq 1 \Rightarrow \frac{{}^{25}C_r\left(\frac{3}{4}\right)^r}{{}^{25}C_{r-1}\left(\frac{3}{4}\right)^{r-1}} \geq 1$$

$$\Rightarrow \left(\frac{25 - r + 1}{r}\right)\frac{3}{4} \geq 1 \Rightarrow r \leq \frac{78}{7}$$

Largest possible value of r is 11

$\therefore$ Coefficient of $T_{12} = 4^{25} \times {}^{25}C_{11} \times (3/4)^{11}$

Example 13 :

Find the coefficient of x^{13} in the expansion of $(1 - x)^5 (1 + x + x^2 + x^3)^4$ is

Sol. Expression $= (1 - x)^5 .(1 + x)^4 (1 + x^2)^4$

$= (1 - x)(1 - x^2)^4 (1 + x^2)^4 = (1 - x)(1 - x^4)^4$

$\therefore$ so the coefficient of $x^{13} = -{}^4C_3(-1)^3 = 4$

Example 14 :

Find the value of

$$\frac{(18^3 + 7^3 + 3.18.7.25)}{3^6 + 6.243.2 + 15.81.4 + 20.27.8 + 15.9.16 + 6.3.32 + 64}$$

Sol. The numerator is of the form $a^3 + b^3 + 3ab(a + b) = (a + b)^3$

Where a = 18, and b = 7 $\therefore N^r = (18 + 7)^3 = (25)^3$

Denominator can be written as

$3^6 + {}^6C_1.3^5.2^1 + {}^6C_2.3^4.2^2 + {}^6C_3.3^3.2^3 + {}^6C_4.3^2.2^4 + {}^6C_5$

$3.2^5 + {}^6C_6 2^6 = (3 + 2)^6 = 5^6 = (25)^3 \Rightarrow \frac{Nr}{Dr} = \frac{(25)^3}{(25)^3} = 1$

Example 15 :

If the coefficients of r^{th} and $(r + 1)^{th}$ terms in the expansion of $(3 + 7x)^{29}$ are equal, then find the value of r

Sol. We have, $T_{r+1} = {}^{29}C_r.3^{29-r}.(7x)^r = ({}^{29}C_r . 3^{29-r} . 7^r)x^r$

$\therefore a_r$ = coefficient of $(r+1)^{th}$ term $= {}^{29}C_r . 3^{29-r} . 7^r$

Now, $a_r = a_{r-1} \Rightarrow {}^{29}C_r . 3^{29-r} . 7^r = {}^{29}C_{r-1} . 3^{30-r} . 7^{r-1}$

$$\Rightarrow \frac{{}^{29}C_r}{{}^{29}C_{r-1}} = \frac{3}{7} \Rightarrow \frac{30 - r}{r} = \frac{3}{7} \Rightarrow r = 21$$

Example 16 :

If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then find the value of $3C_0 - 5C_1 + 7C_2 + ... + (-1)^n(2n+3)C_n$

Sol. We have $3C_0 - 5C_1 + 7C_2 + ... + (-1)^n(2n+3)C_n$

$= 3C_0 - 3C_1 + 3C_2 + ... + (-1)^n 3C_n - 2C_1 + 4C_2 + ... + (-1)^n 2n\,C_n$

$= 3(C_0 - C_1 + C_2 + ... + (-1)^nC_n) - 2(C_1 - 2C_2 + ... (-1)^n nC_n)$

$= 3 \times 0 - 2 \times 0 = 0$

Example 17 :

Find the sum of ${}^{10}C_3 + {}^{11}C_3 + {}^{12}C_3 + ... + {}^{20}C_3$

Sol. Expression = coefficient of x^3 in

$\{(1 + x)^{10} + (1 + x)^{11} + (1 + x)^{12} + ... + (1 + x)^{20}\}$

$= \text{Coefficient of } x^3 \text{ in } \frac{(1 + x)^{10}\{1 - (1 + x)^{11}\}}{1 - (1 + x)}$

Coefficient of x^4 in $\{(1 + x)^{21} - (1 + x)^{10}\}$

$= {}^{21}C_4 - {}^{10}C_4 = {}^{21}C_{17} - {}^{10}C_6$

Example 18 :

Find the sum of series $1 + \frac{1}{3}\left(\frac{1}{2}\right)^2 + \frac{1}{5}\left(\frac{1}{2}\right)^4 + ... \infty$

Sol. Given : $1 + \frac{1}{3}\left(\frac{1}{2}\right)^2 + \frac{1}{5}\left(\frac{1}{2}\right)^4 + ... \infty$

$$= 2\left[\frac{1}{2} + \frac{1}{3}\left(\frac{1}{2}\right)^3 + \frac{1}{5}\left(\frac{1}{2}\right)^5 + ...\right]$$

$$= 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + ...\right], \text{ where } x = \frac{1}{2}$$

$$= 2 . \frac{1}{2}\log\frac{1 + x}{1 - x} = \log\frac{1 + x}{1 - x}$$

$$= \log\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = \log\frac{3/2}{1/2} = \log_e 3$$

QUESTION BANK

CHAPTER 7 : BINOMIAL THEOREM

EXERCISE - 1 [LEVEL-1]

PART 1 : GENERAL TERM, COEFFICIENT, INDEPENDENT TERM, MIDDLE TERM AND GREATEST TERM

Q.1 The middle term of the expansion $\left(x-\frac{2}{x}\right)^8$ is-
(A) 560 (B) –560
(C) 1120 (D) –1120

Q.2 Independent from x in the expansion of $\left(\sqrt{x}-\frac{3}{x^2}\right)^{10}$ is -
(A) 3240 (B) – 3240
(C) 405 (D) – 405

Q.3 The middle terms in the expansion $(x + 6)^7$ is-
(A) $7560\,x^4, -45360\,x^3$ (B) $-7560\,x^4, -45360\,x^3$
(C) $7560\,x^4, 45360\,x^3$ (D) None of these

Q.4 Expansion of $(1 + 3x + 2x^2)^6$, find the coefficient of x^{11}.
(A) 576 (B) 460
(C) 148 (D) 450

Q.5 The 4th term from the end in the expansion of $(2x - 1/x^2)^{10}$
(A) $960\,x^{-11}$ (B) $960\,x^{-12}$
(C) $-960\,x^{-12}$ (D) $-960\,x^{-11}$

Q.6 The term which has the greatest coefficient in the expansion of $(x + a)^8$ is–
(A) 3rd (B) 4th
(C) 5th (D) 6th

Q.7 The greatest term in the expansion of $(2x + 7)^{10}$, when $x = 3$ is-
(A) T_5 (B) T_6
(C) T_7 (D) None of these

Q.8 The first four terms of the expansion of $\left(ax-\frac{1}{bx^2}\right)^5$ are

(A) $a^5x^5 - 5\frac{a^4}{b}x^2 + 10\frac{a^3}{b^2x} - 10\frac{a^2}{b^3x^4}$

(B) $a^5x^5 + 5\frac{a^4}{b}x^2 - 10\frac{a^3}{b^2x} + 10\frac{a^2}{b^3x^4}$

(C) $a^5x^5 - 5\frac{a^4}{b}x^2 - 10\frac{a^3}{b^2x} - 10\frac{a^2}{b^3x^4}$

(D) $a^5x^5 + 5\frac{a^4}{b}x^2 + 10\frac{a^3}{b^2x} + 10\frac{a^2}{b^3x^4}$

Q.9 The sixth term in the expansion of $\left(3x^2-\frac{1}{2x}\right)^8$ is-
(A) $\frac{189}{4}x$ (B) $-\frac{189}{4}x$ (C) $\frac{189}{4}x^2$ (D) $\frac{189}{4}x^3$

Q.10 The first four terms in the expansion of $(3x + 1/x)^4$ is -
(A) $81\,x^4 - 108\,x^2 + 54 - 12\,x^{-2}$
(B) $81\,x^4 + 108\,x^2 + 54 + 12\,x^{-2}$
(C) $-81\,x^4 - 108\,x^2 - 54 - 12\,x^{-2}$
(D) None of these

Q.11 The tenth term in the expansion of $(2x^2 + 1/x)^{12}$ is –
(A) $1760/x^3$ (B) $-1760/x^3$
(C) $1760/x^2$ (D) None of these

Q.12 If in the expansion of $\left(x^3-\frac{3}{x^2}\right)^{15}$ the r^{th} term is independent of x, then r equals-
(A) 8 (B) 9
(C) 10 (D) None of these

Q.13 Find the coefficients of x^n in $\left(1+\frac{x}{1!}+\frac{x^2}{2!}+.....+\frac{x^n}{n!}\right)^2$.
(A) $6^n/n!$ (B) $4^n/n!$
(C) $2^n/n!$ (D) $7^n/n!$

Q.14 In the expansion of $(4–3x)^7$, the numerically greatest term at $x = 2/3$ is -
(A) T_4 (B) T_5 (C) T_3 (D) T_2

Q.15 Find the coefficient of x^5 in the expansion of $(1 + x)^{21} + (1 + x)^{22} + ... (1 + x)^{30}$.
(A) ${}^{31}C_6 - {}^{21}C_6$ (B) ${}^{31}C_5 - {}^{21}C_3$
(C) ${}^{21}C_6 - {}^{31}C_6$ (D) ${}^{11}C_6 - {}^{12}C_6$

Q.16 The coefficient of x^4 in the expansion of $\frac{1+2x+3x^2}{(1-x)^2}$
(A) 13 (B) 14
(C) 20 (D) 22

Q.17 If the fourth term in the expansion of $(px + 1/x)^n$ is 5/2 then the value of n and p are respectively-
(A) 6, 1/2 (B) 1/2, 6
(C) 3,1 (D) 3, 1/2

Q.18 The coefficient of x^4 in the expansion of $(1+ x + x^2 + x^3)^n$ is -
(A) ${}^nC_4$ (B) ${}^nC_4 + {}^nC_2$
(C) ${}^nC_1 + {}^nC_2 + {}^nC_4 . {}^nC_2$ (D) ${}^nC_4 + {}^nC_2 + {}^nC_1 . {}^nC_2$

Q.19 If $(2 – x – x^2)^{2n} = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$, then the value of $a_0 + a_2 + a_4 + ...$ is-
(A) $2^{n–1}$ (B) 2^{2n}
(C) $2^{2n–1}$ (D) None of these

Q.20 If x^m occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{2n}$, the coefficient of x^m is -
(A) $\frac{(2n)!}{m!(2n-m)!}$ (B) $\frac{(2n)!3!3!}{(2n-m)!}$
(C) $\frac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$ (D) None of these

Q.21 If the third term in the expansion of $\left[x + x^{\log_{10} x}\right]^5$ is equal to 10,00,000 then x equals-
(A) 10 (B) 10^2
(C) 10^3 (D) No such x exists

Q.22 The greatest integer in the expansion of $(1+x)^{2n+2}$ is -
(A) $\frac{(2n)!}{(n!)^2}$ (B) $\frac{(2n+2)!}{[(n+1)!]^2}$
(C) $\frac{(2n+2)!}{n!(n+1)!}$ (D) $\frac{(2n)!}{n!(n+1)!}$

Q.23 If $\frac{T_2}{T_3}$ in the expansion of $(a+b)^n$ and $\frac{T_3}{T_4}$ in the expansion of $(a+b)^{n+3}$ are equal, then n =
(A) 3 (B) 4
(C) 5 (D) 6

Q.24 Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$ for every value of θ, then
(A) $b_0 = 1, b_1 = 3$ (B) $b_0 = 0, b_1 = n$
(C) $b_0 = -1, b_1 = n$ (D) $b_0 = 0, b_1 = n^2 - 3n + 3$

Q.25 If $(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then $a_0 + a_2 + a_4 + ... + a_{2n} =$
(A) $\frac{3^n+1}{2}$ (B) $\frac{3^n-1}{2}$ (C) $\frac{1-3^n}{2}$ (D) $3^n + \frac{1}{2}$

Q.26 If the sum of the coefficients in the expansion of $(x+y)^n$ is 1024, then the value of the greatest coefficient in the expansion is –
(A) 356 (B) 252
(C) 210 (D) 120

Q.27 The interval in which x must lie so that the numerically greatest term in the expansion of $(1-x)^{21}$ has the numerically greatest coefficient is
(A) $\left[\frac{5}{6}, \frac{6}{5}\right]$ (B) $\left(\frac{5}{6}, \frac{6}{5}\right)$ (C) $\left(\frac{5}{6}, \frac{6}{5}\right)$ (D) $\left[\frac{4}{5}, \frac{5}{4}\right]$

Q.28 If the 6^{th} term in the expansion of the binomial
$$\left[\sqrt{2^{\log(10-3^x)}} + \sqrt[5]{2^{(x-2)\log 3}}\right]^m$$
is equal to 21 and it is known that the binomial coefficients of the 2nd, 3rd and 4th terms in the expansion represent respectively the first, third and fifth terms of an A.P. (the symbol log stands for logarithm to the base 10), then x =
(A) 0 (B) 1
(C) 2 (D) Both (A) and (C)

Q.29 If for positive integers r > 1, n > 2 the coefficient of the $(3r)^{th}$ and $(r+2)^{th}$ powers of x in the expansion of $(1+x)^{2n}$ are equal, then
(A) n = 2r (B) n = 3r
(C) n = 2r + 1 (D) None of these

Q.30 If the coefficient of the second, third and fourth terms in the expansion of $(1+x)^n$ are in A.P., then n is equal to
(A) 7 (B) 2
(C) 6 (D) None of these

Q.31 In the binomial expansion of $(a-b)^n, n \geq 5$, the sum of the 5^{th} and 6^{th} terms is zero. Then a/b is equal to
(A) $\frac{1}{6}(n-5)$ (B) $\frac{1}{5}(n-4)$ (C) $\frac{5}{(n-4)}$ (D) $\frac{6}{(n-5)}$

Q.32 Given that 4th term in the expansion of $\left(2+\frac{3}{8}x\right)^{10}$ has the maximum numerical value, the range of value of x for which this will be true is given by
(A) $-\frac{64}{21} < x < -2$ (B) $-\frac{64}{21} < x < 2$
(C) $\frac{64}{21} < x < 4$ (D) None of these

Q.33 6^{th} term in expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$ is
(A) $\frac{4580}{17}$ (B) $-\frac{896}{27}$ (C) $\frac{5580}{17}$ (D) None

Q.34 If the $(r+1)^{th}$ term in the expansion of $\left(\sqrt[3]{\frac{a}{\sqrt{b}}} + \sqrt{\frac{b}{\sqrt[3]{a}}}\right)^{21}$ has the same power of a and b, then the value of r is
(A) 9 (B) 10
(C) 8 (D) 6

Q.35 In the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$, the coefficient of x^4 is
(A) $\frac{405}{256}$ (B) $\frac{504}{259}$ (C) $\frac{450}{263}$ (D) None of these

Q.36 If coefficients of $(2r+1)^{th}$ term and $(r+2)^{th}$ term are equal in the expansion of $(1+x)^{43}$, then the value of r
(A) 14 (B) 15
(C) 13 (D) 16

Q.37 If the second, third and fourth term in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, then n =
(A) 15 (B) 20
(C) 10 (D) 5

Q.38 The term independent of x in $\left(2x - \frac{1}{2x^2}\right)^{12}$ is
(A) – 7930 (B) – 495
(C) 495 (D) 7920

Q.39 Independent of x in the expansion of $\left(x^2 - \frac{3\sqrt{3}}{x^3}\right)^{10}$ is

(A) 153090 (B) 150000
(C) 150090 (D) 153180

Q.40 If the sum of the coefficients in the expansion of $(1+2x)^n$ is 6561, the greatest term in the expansion for x = 1/2 is -

(A) 4^{th} (B) 5^{th}
(C) 6^{th} (D) None of these

Q.41 If $(r+1)^{th}$ term is $\frac{3.5....(2r-1)}{r!}\left(\frac{1}{5}\right)^r$, then this is the term of binomial expansion-

(A) $\left(1-\frac{2}{5}\right)^{1/2}$ (B) $\left(1-\frac{2}{5}\right)^{-1/2}$
(C) $\left(1+\frac{2}{5}\right)^{-1/2}$ (D) $\left(1+\frac{2}{5}\right)^{1/2}$

Q.42 The coefficient of the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is-

(A) 1/3 (B) 19/54
(C) 17/54 (D) 1/4

Q.43 If the coefficient of 4^{th} term in the expansion of $\left(x+\frac{\alpha}{2x}\right)^n$ is 20, then the respective values of α & n are

(A) 2, 7 (B) 5, 8
(C) 3, 6 (D) 2, 6

Q.44 The coefficient of x^m in $(1+x)^m + (1+x)^{m+1} + ... + (1+x)^n, m \le n$ is –

(A) ${}^{n+1}C_{m+1}$ (B) ${}^{n-1}C_{m-1}$
(C) ${}^{n}C_{m}$ (D) ${}^{n}C_{m+1}$

Q.45 Coefficient of x^{25} in expansion of expression $\sum_{r=0}^{50} {}^{50}C_r\,(2x-3)^r(2-x)^{50-r}$ is –

(A) ${}^{50}C_{25}$ (B) $-\,{}^{50}C_{30}$
(C) ${}^{50}C_{30}$ (D) $-\,{}^{50}C_{25}$

Q.46 The term independent of x in $\left[\sqrt{\frac{x}{3}} + \sqrt{\frac{3}{2x^2}}\right]^{10}$ is –

(A) 1 (B) ${}^{10}C_1$
(C) 5/12 (D) None of these

Q.47 The $(p+2)^{th}$ term from end in $\left(x - \frac{1}{x}\right)^{2n+1}$ is –

(A) $(-1)^p \frac{(2n+1)!}{(2n-p)!\,(p+1)!} x^{2p-2n+1}$

(B) $(-1)^p \frac{(2n+1)!}{(2n-p)!\,(p+1)!} x^{2n-2p+1}$

(C) $(-1)^p \frac{(2n+1)!}{(2n-p)!\,(p+1)!} x^{2p-2n-1}$

(D) None of these

Q.48 If sum of the coefficients in expansion $(1+x)^n$ is 4096 then value of largest binomial coefficients is –

(A) 792 (B) 924
(C) 462 (D) None of these

Q.49 Numerically greatest term in expansion of $(3-5x)^{11}$ at x = 1/5 is –

(A) $(55).3^9$ (B) $(55).3^8$
(C) $(55).3^{10}$ (D) None of these

Q.50 In the binomial expansion of $(1+x)^{15}$, the coefficients of x^r and x^{r-3} are equal. Then r is –

(A) 8 (B) 7
(C) 4 (D) 6

Q.51 If 21^{st} and 22^{nd} terms in the expansion of $(1+x)^{44}$ are equal, then x is equal to

(A) 8/7 (B) 21/22
(C) 7/8 (D) 23/24

Q.52 The middle term of expansion $\left(\frac{10}{x} + \frac{x}{10}\right)^{10}$

(A) ${}^{7}C_5$ (B) ${}^{8}C_5$
(C) ${}^{9}C_5$ (D) ${}^{10}C_5$

Q.53 The 11^{th} term in the expansion of $\left(x + \frac{1}{\sqrt{x}}\right)^{14}$ is –

(A) $\frac{1001}{x}$ (B) $\frac{x}{1001}$
(C) $\frac{999}{x}$ (D) i

PART 2 : PROPERTIES OF BINOMIAL COEFFICIENTS

Q.54 Maximum value of ${}^{20}C_r$, is equal to

(A) ${}^{20}C_{11}$ (B) ${}^{20}C_{12}$
(C) ${}^{20}C_{10}$ (D) none of these

Q.55 If $\alpha_1, \alpha_2, \alpha_3, ...\alpha_n$ are the n^{th} roots of unity, then ${}^{n}C_1\,\alpha_1 + {}^{n}C_2\,\alpha_2 + {}^{n}C_3\,\alpha_3 + ... + {}^{n}C_n\,\alpha_n$ equals –

(A) $\frac{\alpha_1}{\alpha_2}$ (B) $\frac{\alpha_1}{\alpha_2}((\alpha_1+\alpha_2)^{2n}-1)$
(C) $\frac{\alpha_1}{\alpha_2}((1+\alpha_2)^n-1)$ (D) $\frac{\alpha_1}{\alpha_2}((\alpha_1+\alpha_2)^n+1)$

Q.56 Find the sum of the series : $1^2.\,C_1 + 2^2.\,C_2 + 3^2.\,C_3 + ... + n^2.\,C_n$

(A) $(n+1)2^{n-2}$ (B) $2n\,(n+1)2^{n-2}$
(C) $n\,(n-1)2^{n-2}$ (D) $n\,(n+1)2^{n-2}$

Q.57 The sum of the coefficient of the terms of the expansion of polynomial $(1+x-3x^2)^{2143}$ is-

(A) 2^{2143} (B) 1
(C) –1 (D) 0

Q.58 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ then $C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ is equal to-
(A) $2^{n-1}(n+2)$ (B) $2^n(n+1)$
(C) $2^{n-1}(n+1)$ (D) $2^n(n+2)$

Q.59 Find the sum $\sum_{r=1}^{n} \frac{r\,{}^nC_r}{{}^nC_{r-1}}$
(A) $\frac{n(n-1)}{2}$ (B) $\frac{n(n+1)}{2}$
(C) $\frac{(n+1)}{2}$ (D) None

Q.60 The value of $\sum_{r=1}^{10} r.\frac{{}^nC_r}{{}^nC_{r-1}}$ is equal to
(A) $5(2n-9)$ (B) $10n$
(C) $9(n-4)$ (D) None of these

Q.61 Value of $2C_0 - \frac{2^2}{2}C_1 + \frac{2^3}{3}C_2 + ... + (-1)^{n+1}\frac{2^{n+1}}{(n+1)}$.
(A) 0 (B) 1
(C) 2 (D) 3

Q.62 If $\sum_{r=0}^{2n} a_r (x-1)^r = \sum_{r=0}^{2n} b_r (x-2)^r$ and $b_r = (-1)^{r-n}$ for all $r \geq n$, find a_n
(A) ${}^{n+1}C_n$ (B) ${}^{2n-1}C_n$
(C) ${}^{n-1}C_n$ (D) ${}^{2n+1}C_n$

Q.63 Find the value of $\sum_{r=0}^{n} {}^nC_r \sin rx \cos(n-r)x$.
(A) $2^{n-1}\sin nx$ (B) $3^{n-1}\sin nx$
(C) $7^{n-1}\cos nx$ (D) $5^{n-1}\cos nx$

Q.64 If $C_0, C_1, C_2, ... C_n$ are binomial coefficients then $\frac{1}{n!0!} + \frac{1}{(n-1)!} + \frac{1}{(n-2)!2!} + ... + \frac{1}{0!\,n!}$ is equal to
(A) 2^n (B) $\frac{2^{n-1}}{n!}$
(C) $\frac{2^n}{n!}$ (D) none of these

Q.65 Set of value of r for which, ${}^{18}C_{r-2} + 2\,.\,{}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$ contains :
(A) 4 elements (B) 5 elements
(C) 7 elements (D) 10 elements

Q.66 Let $(1+x)^n = \sum_{r=0}^{n} a_r x^r$ then $\left(1+\frac{a_1}{a_0}\right).\left(1+\frac{a_2}{a_0}\right)......\left(1+\frac{a_n}{a_{n-1}}\right)$ is equal to –
(A) $\frac{(n+1)^{n+1}}{n!}$ (B) $\frac{(n+1)^n}{n!}$
(C) $\frac{n^{n-1}}{(n-1)!}$ (D) $\frac{(n+1)^n}{(n+1)!}$

Q.67 $\frac{1}{1!(n-1)} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ...$ is equal to
(A) $\frac{2^{n-1}}{n!}$ for even values of n only
(B) $\frac{2^{n-1}+1}{n!} - 1$ for odd values of n only
(C) $\frac{2^{n-1}}{n!}$ for all $n \in N$
(D) None of these

Q.68 The value of ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + ... + {}^{10}C_9$ is –
(A) $2^{10} - 1$ (B) 2^{10}
(C) 2^{11} (D) $2^{10} - 2$

Q.69 If the value of $C_0 + 2.C_1 + 3.C_2 + ... + (n+1).C_n = 576$, then n is –
(A) 7 (B) 5
(C) 6 (D) 9

PART 3 : BINOMIAL THEOREM FOR ANY INDEX

Q.70 If $|x| < 1/2$, then expansion of $(1-2x)^{1/2}$ is-
(A) $1 - x - \frac{1}{2}x^2$ .... (B) $1 - x + \frac{1}{2}x^2$ ....
(C) $1 + x - \frac{1}{2}x^2$ .... (D) None of these

Q.71 The tenth term in the expansion of $(1+x)^{-3}$ is -
(A) $-55x^9$ (B) $55x^9$
(C) $-66x^{10}$ (D) $66x^{10}$

Q.72 The coefficient of x^5 in the expansion of $(1-x)^{-6}$ is –
(A) 1260 (B) – 1260
(C) – 252 (D) 252

Q.73 To expand $(1+2x)^{-1/2}$ as an infinite series, the range of x should be –
(A) $\left[\frac{-1}{2}, \frac{1}{2}\right]$ (B) $\left(\frac{-1}{2}, \frac{1}{2}\right)$
(C) $[-2, 2]$ (D) $(-2, 2)$

Q.74 If r^{th} and $(r+1)^{th}$ terms in the expansion of $(p+q)^n$ are equal, then $\frac{(n+1)q}{r(p+q)}$ is –
(A) 1/2 (B) 1/4
(C) 1 (D) 0

Q.75 The 13th term in the expansion of $\left(x^2+\frac{2}{x}\right)^n$ is independent of x then the sum of the divisors of n is –

(A) 39 (B) 36

(C) 37 (D) 38

PART 4 : APPLICATIONS OF BINOMIAL THEOREM

Q.76 The value of $\sqrt{99}$ upto three decimals is -

(A) 9.949 (B) 9.958

(C) 9.948 (D) None of these

Q.77 The sum of the series $1+\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+....=$

(A) $1/\sqrt{5}$ (B) $1/\sqrt{2}$

(C) $\sqrt{5/3}$ (D) $\sqrt{5}$

Q.78 The sum of the series

$$\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+.....\text{m terms}\right)$$ is

(A) $\frac{2^{mn}-1}{2^{mn}(2^n-1)}$ (B) $\frac{2^{mn}-1}{2^n-1}$

(C) $\frac{2^{mn}+1}{2^n+1}$ (D) None

Q.79 $49^n + 16n - 1$ is divisible by

(A) 3 (B) 19

(C) 64 (D) 29

Q.80 The difference between an integer and its cube is divisible

(A) 4 (B) 6

(C) 9 (D) None of these

Q.81 The greatest integer which divides $101^{100} - 1$ is -

(A) 100 (B) 1000

(C) 10,000 (D) 100,000

Q.82 The sum of the rational terms in the expansion of $(\sqrt{2}+3^{1/5})^{10}$ is equal to

(A) 40 (B) 41

(C) 42 (D) 0

Q.83 The integer just greater than $(3+\sqrt{5})^{2n}$ is divisible by $(n \in N)$

(A) 2^{n-1} (B) 2^{n+1}

(C) 2^{n+2} (D) not divisible by 2

Q.84 The remainder when 27^{40} is divided by 12 is –

(A) 3 (B) 7

(C) 9 (D) 11

Q.85 The last two digits of the number $(23)^{14}$ are –

(A) 01 (B) 03

(C) 09 (D) None of these

Q.86 If $(1 - x - 2x^2)^6 = 1 + a_1x + a_2x^2 + ... + a_{12}x^{12}$. Then $\frac{a_2}{2^2}+\frac{a_4}{2^4}+\frac{a_6}{2^6}+......+\frac{a_{12}}{2^{12}}$ is equal to –

(A) –1 (B) –1/2

(C) 0 (D) 1/2

Q.87 The remainder when 7^{103} is divided by 25 is –

(A) 0 (B) 18

(C) 9 (D) None of these

Q.88 The number $(49^2 - 4)(49^3 - 49)$ is divisible by –

(A) 7! (B) 9!

(C) 6! (D) 5!

Q.89 The digit in the unit's place of $7^{171} + (177)!$

(A) 0 (B) 1

(C) 1 (D) 3

Q.90 The remainder when, $10^{10} . (10^{10} + 1)(10^{10} + 2)$ is divided by 6 is –

(A) 2 (B) 4

(C) 0 (D) 6

Q.91 The remainder obtained when $1! + 2! + 3! + …. + 11!$ is divided by 12 is –

(A) 9 (B) 8

(C) 7 (D) 6

PART 5 : MISCELLANEOUS

Q.92 Given that the term of the expansion $(x^{1/3} - x^{-1/2})^{15}$ which does not contain x is 5 m where $m \in N$, then m =

(A) 1100 (B) 1010

(C) 1001 (D) none

Q.93 If the coefficients of x^7 & x^8 in the expansion of $\left[2+\frac{x}{3}\right]^n$ are equal, then the value of n is –

(A) 15 (B) 45

(C) 55 (D) 56

Q.94 The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is

(A) 4 (B) 15
(C) 0 (D) 18

Q.95 If $n \in N$ & n is even, then

$$\frac{1}{1.(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ... + \frac{1}{(n-1)!1!} =$$

(A) 2^n (B) $\frac{2^{n-1}}{n!}$

(C) $2^n n!$ (D) none of these

Q.96 If in the expansion of $\left(2^x + \frac{1}{4^x}\right)^n$, $T_3 = 7T_2$ and sum of the binomial coefficients of second and third terms is 36, then the value of x is –

(A) –1/3 (B) – 1/2
(C) 1/3 (D) 1/2

Q.97 $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ...a_{2n} x^{2n}$, then $a_0 + a_2 + a_4 + ... a_{2n} = 0$

(A) $\frac{3^n + 1}{2}$ (B) $\frac{3^n - 1}{2}$

(C) $\frac{3^{n-1} + 1}{2}$ (D) $\frac{3^{n-1} - 1}{2}$

Q.98 If $(11)^{27} + (21)^{27}$ when divided by 16 leaves the remainder

(A) 0 (B) 1
(C) 2 (D) 14

Q.99 If $(1 + x)^{2n} = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$ then

(A) $a_0 + a_2 + a_4 + ... = (1/2)(a_0 + a_1 + a_2 + a_3 + ...)$
(B) $a_{n+1} < a_n$
(C) $a_{n-3} = a_{n+3}$
(D) All of these

Q.100 The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$, is

(A) 440 (B) 770
(C) 990 (D) 1001

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 The coefficient of x^{49} in the expansion of

$$(x-1)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2^2}\right).....\left(x-\frac{1}{2^{49}}\right) \text{ is equal to}$$

(A) $-2\left(1-\frac{1}{2^{50}}\right)$ (B) + ve coefficient of x

(C) – ve coefficient of x (D) $-2\left(1-\frac{1}{2^{49}}\right)$

Q.2 If $6^{83} + 8^{83}$ is divided by 49, then the remainder is

(A) 35 (B) 5

(C) 1 (D) 0

Q.3 When $11^{27} + 21^{27}$ is divided by 16, the remainder is –

(A) 1 (B) 14

(C) 0 (D) 2

Q.4 If the 3rd term in the expansion of $(x + x^t)^5$ is 10^6 where $t = \log_{10} x$ then the number of possible values of x is –

(A) 2 (B) 0

(C) 1 (D) infinite

Q.5 $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$ is a polynomial of the order of –

(A) 5 (B) 6

(C) 7 (D) 8

Q.6 Last three digits of the number $N = 7^{100} - 3^{100}$ are

(A) 100 (B) 300

(C) 500 (D) 000

Q.7 If $(1 + x - 2x^2)^8 = a_0 + a_1x + a_2x^2 + ... + a_{16}x^{16}$ then the sum $a_1 + a_3 + a_5 + ... + a_{15}$ is equal to

(A) -2^7 (B) 2^7

(C) 2^8 (D) None of these

Q.8 The coefficient of x^{83} in $(1 + x + x^2 + x^3 + x^4)^n (x - 1)^{n+3}$, is equal to

(A) ${}^nC_7(-1)^n$ (B) $-{}^nC_{16}$

(C) ${}^nC_{13}$ (D) ${}^nC_9$

Q.9 The coefficient of x^4 of in the expansion $(1 + 5x + 9x^2 + ...\infty)(1 + x^2)^{11}$ is –

(A) ${}^{11}C_2 + 4\,{}^{11}C_1 + 3$ (B) ${}^{11}C_2 + 3\,{}^{11}C_1 + 4$

(C) $3\,{}^{11}C_2 + 4\,{}^{11}C_1 + 3$ (D) 171

Q.10 The number formed by last two digits of the number $(17)^{256}$ is

(A) 81 (B) 80

(C) 91 (D) 93

Q.11 The last two digits of the number 3^{400} are :

(A) 81 (B) 43

(C) 29 (D) 01

Q.12 In the binomial $(2^{1/3} + 3^{-1/3})^n$, if the ratio of the seventh term from the beginning of the expansion to the seventh term from its end is 1/6, then n equal to

(A) 6 (B) 9

(C) 12 (D) 15

Q.13 If $(1 + x + x^2)^{25} = a_0 + a_1x + a_2x^2 + ... + a_{50} . x^{50}$ then $a_0 + a_2 + a_4 + ... + a_{50}$ is :

(A) even

(B) odd & of the form 3n

(C) odd & of the form (3n – 1)

(D) odd & of the form (3n + 1)

Q.14 $(2n + 1)(2n + 3)(2n + 5) .... (4n - 1)$ is equal to :

(A) $\frac{(4n)!}{2^n.(2n)!(2n)!}$ (B) $\frac{(4n)!\ n!}{2^n.(2n)!(2n)!}$

(C) $\frac{(4n)!\ n!}{(2n)!(2n)!}$ (D) $\frac{(4n)!\ n!}{2^n\ !(2n)!}$

Q.15 The coefficient of $\lambda^n\mu^n$ in the expansion of

$$\left[(1+\lambda)(1+\mu)(\lambda+\mu)\right]^n \text{ is}$$

(A) $\sum_{r=0}^{n} C_r^2$ (B) $\sum_{r=0}^{n} C_{r+2}^2$

(C) $\sum_{r=0}^{n} C_{r+3}^2$ (D) $\sum_{r=0}^{n} C_r^3$

Q.16 The coefficient of the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is

(A) 5/4 (B) 7/4

(C) 9/4 (D) none of these

Q.17 The expansion of $(1 + x)^n$ has 3 consecutive terms with coefficients in the ratio 1 : 2 : 3 and can be written in the form ${}^nC_k : {}^nC_{k+1} : {}^nC_{k+2}$. The sum of all possible values of $(n + k)$ is –

(A) 18 (B) 21

(C) 28 (D) 32

Q.18 The sum of the coefficient of all the terms in the expansion of $(2x - y + z)^{20}$ in which y do nto appear at all while x appears in even powers and z appears in odd powers is –

(A) 0 (B) $\frac{2^{20}-1}{2}$ (C) 2^{19} (D) $\frac{3^{20}-1}{2}$

Q.19 If the second term of the expansion $\left[a^{1/13}+\frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$ then the value of $\frac{{}^nC_3}{{}^nC_2}$ is :

(A) 4 (B) 3

(C) 12 (D) 6

Q.20 If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$ then

(A) $a_0 - a_2 + a_4 - a_6 + ... = 0$, if n is odd
(B) $a_1 - a_3 + a_5 - a_7 + ... = 0$, if n is even
(C) $a_0 - a_2 + a_4 - a_6 + ... = 0$, if $n = 4p$, $p \in I^+$
(D) $a_1 - a_3 + a_5 - a_7 + ... = 0$, if $n = 4p + 1$, $p \in I^+$

Q.21 The last term in the binomial expansion of $\left(\sqrt[3]{2}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3.\sqrt[3]{9}}\right)^{\log_3 8}$. Then the 5^{th} term from the beginning is

(A) ${}^{10}C_6$
(B) $2.\ {}^{10}C_4$
(C) $^{1/2,}\ {}^{10}C_4$
(D) None of these

Q.22 In the expansion of $(1 + x)^{15}$, the value of $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ... + \frac{15C_{15}}{C_{14}}$ is

(A) 24
(B) 2^{15}
(C) 0
(D) 120

Q.23 The sum of the co–efficients of all the even powers of x in the expansion of $(2x^2 - 3x + 1)^{11}$ is :

(A) $2 . 6^{10}$
(B) $3 . 6^{10}$
(C) 6^{11}
(D) none

Q.24 Co-efficient of α^t in the expansion of,
$(\alpha + p)^{m-1} + (\alpha + p)^{m-2}(\alpha + q) + (\alpha + p)^{m-3}(\alpha + q)^2 + ... (\alpha + q)^{m-1}$
where $\alpha \ne -q$ and $p \ne q$ is :

(A) $\frac{{}^mC_t\ (p^t - q^t)}{p-q}$
(B) $\frac{{}^mC_t\ (p^{m-t} - q^{m-t})}{p-q}$
(C) $\frac{{}^mC_t\ (p^t + q^t)}{p-q}$
(D) $\frac{{}^mC_t\ (p^{m-t} + q^{m-t})}{p-q}$

Q.25 If $(1 + x)^n = a_0 + a_1x + a_2x^2 + . . . + a_nx^n$, then $\left(1+\frac{a_1}{a_0}\right)\left(1+\frac{a_2}{a_1}\right)\left(1+\frac{a_3}{a_2}\right)...\left(1+\frac{a_n}{a_{n-1}}\right)$ is equal to

(A) $\frac{n^n}{n!}$
(B) $\frac{(n+1)^n}{n!}$
(C) $\frac{n^{n+1}}{(n+1)!}$
(D) none

Q.26 The value of $\left\{\frac{3^{2003}}{28}\right\}$, where $\{\cdot\}$ denotes the fractional part, is equal to

(A) 15/28
(B) 5/28
(C) 19/28
(D) 9/28

Q.27 $(1 + x)(1 + x + x^2)(1 + x + x^2 + x^3)\ ......\ (1 + x + x^2 + ... + x^{100})$ when written in the ascending power of x then the highest exponent of x is –

(A) 4950
(B) 5050
(C) 5150
(D) none

Q.28 The term independent of x in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x > 0$, is a times the corresponding binomial coefficient then a is

(A) 3
(B) 1/3
(C) –1/3
(D) None of these

Q.29 $(1 + \sqrt{2}x^2)^9 - 1 - 9\sqrt{2}x^2 - 70x^4$ is divisible by

(A) x^6
(B) x^8
(C) x^{10}
(D) none of these

Q.30 $aC_0 + (a + b)C_1 + (a + 2b)C_2 + ... + (a + nb)C_n$ is equal to

(A) $(2a + nb)2^n$
(B) $(2a + nb)2^{n-1}$
(C) $(na + 2b)2^n$
(D) $(na + 2b)2^{n-1}$

Q.31 x^r occurs in the expansion of $\left(x^3 + \frac{1}{x^4}\right)^n$ provided –

(A) 2n – r is divisible by 5
(B) 3n – r is divisible by 5
(C) 2n – r is divisible by 7
(D) 3n – r is divisible by 7

Q.32 In the expansion of $\left(\sqrt[3]{\frac{a}{b}} + \sqrt[3]{\frac{b}{\sqrt{a}}}\right)^{21}$ the term containing same powers of a and b is –

(A) 11^{th}
(B) 13^{th}
(C) 12^{th}
(D) 6^{th}

Q.33 The middle term in the expansion of $\left(1+\frac{1}{x^2}\right)(1+x^2)^n$ is

(A) ${}^{2n}C_n\ x^{2n}$
(B) ${}^{2n}C_n\ x^{-2n}$
(C) ${}^{2n}C_n$
(D) ${}^{2n}C_{n-1}$

Q.34 If n is even positive integer, then the condition that the greatest term in the expansion of $(1 + x)^n$ may have the greatest coefficient also is

(A) $\frac{n}{n+2} < x < \frac{n+2}{n}$
(B) $\frac{n+1}{n} < x < \frac{n}{n+1}$
(C) $\frac{n}{n+4} < x < \frac{n+4}{n}$
(D) none of these

Q.35 The coefficient of x^{n-2} in the polynomial $(x - 1)(x - 2)(x - 3)\ ....\ (x - n)$ is –

(A) $\frac{n(n^2+2)(3n+1)}{24}$
(B) $\frac{n(n^2-1)(3n+2)}{24}$
(C) $\frac{n(n^2+1)(3n+4)}{24}$
(D) None of these

ASSERTION AND REASON QUESTIONS

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.
(E) Statement -1 is False, Statement-2 is False.

Q.36 **Statement–1:** 2^{60} when divided by 7 leaves the reminder 1.
Statement–2 : $(1+x)^n = 1 + n_1 x$, where $n, n_1 \in N$.

Q.37 **Statement–1 :** $C_0^2 + C_1^2 + C_2^2 + C_3^2 + ... + C_n^2 = \frac{2n!}{(n!)^2}$
Statement–2 : ${}^nC_0 - {}^nC_1 + {}^nC_2 .... + (-1)^n\, {}^nC_n = 0$

Q.38 Let n be a positive integers and k be a whole number, k< 2n.
Statement–1 : The maximum value of ${}^{2n}C_k$ is ${}^{2n}C_n$.
Statement–2 : $\frac{{}^{2n}C_{k+1}}{{}^{2n}C_k} > 1$, for $k = 0, 1, 2, \dots, n-1$.

Q.39 **Statement-1:** Any positive integral power of $(\sqrt{2}-1)$ can be expresses as $\sqrt{N}-\sqrt{N-1}$ for some natural number $N > 1$.
Statement-2: Any positive integral power of $\sqrt{2}-1$ can be expressed as $A + B\sqrt{2}$ where A and B are integers.

Q.40 **Statement 1 :** $\sum_{k=1}^{n} K.({}^nC_K)^2 = n.\,{}^{2n-1}C_{n-1}$
Statement 2 : If 2^{2003} is divided by 15 the remainder is 1.

Q.41 **Statement–1 :** The coefficient of x^{203} in the expression $(x-1)(x^2-2)(x^2-3)\dots(x^{20}-20)$ must be 13.
Statement–2 : The coefficient of x^8 in the expression $(2+x)^2(3+x)^3(4+x)^4$ is equal to 30.

MATCH THE COLUMN TYPE QUESTIONS

Q.42 Match the column –

Column I	Column II
(a) If (r + 1)th term is the first negative term in the expansion of $(1+x)^{7/2}$, then the value of r (where $\|x\| < 1$) is	(p) divisible by 2
(b) The coefficient of y in the expansion of $(y^2 + 1/y)^5$ is	(q) divisible by 5
(c) If the second term in the expansion $\left(a^{1/13} + \frac{a}{\sqrt{a^{-1}}}\right)^n$ is $14a^{5/2}$, then the value of n is	(r) divisible by 10
(d) The coefficient of x^4 in the expression $(1+2x+3x^2+4x^3+...\text{upto }\infty)^{1/2}$ is c, ($c \in N$), then c + 1 (where $\|x\| < 1$) is	(s) a prime number

Code :
(A) a-qs, b-pqr, c-p, d-ps (B) a-ps, b-q, c-s, d-qr
(C) a-r, b-pqr, c-ps, d-pq (D) a-qr, b-ps, c-pr, d-rs

Q.43 Match the column –

Column I
(a) ${}^mC_1\, {}^nC_m - {}^mC_2\, {}^{2n}C_m + {}^mC_3\, {}^{3n}C_m + ...$ is
(b) ${}^nC_m + {}^{n-1}C_m + {}^{n-2}C_m + ... + {}^mC_m$ is
(c) $C_0C_1 + C_1C_{n-1} + ... + C_nC_0$ is
(d) $2^m\, {}^nC_0 - 2^{m-1}\, {}^nC_1\, {}^{n-1}C_{m-1} + ... + (-1)^m\, {}^nC_m\, {}^{n-m}C_0$ is
(t) the coefficient of x^n in $(1+x)^{2n}$

Column II
(p) the coefficient of x^m in the expansion of $(1-(1+x)^n)^m$.
(q) the coefficient of x^m in $\frac{(1+x)^{n+1}}{x}$
(r) the coefficient of x^{n+1} in $(1+x)^{2n}$
(s) the coefficient of x^m in the expansion of $(1+x)^n$
(t) the coefficient of x^n in $(1+x)^{2n}$

Code :
(A) a-p, b-t, c-s, d-p (B) a-p, b-q, c-t, d-s
(C) a-r, b-q, c-s, d-t (D) a-r, b-s, c-p, d-q

Q.44 Match the column –

Column I	Column II
(a) Let $a = 3^{\frac{1}{223}} + 1$ and for all $n \geq 3$, let $f(n) = {}^nC_0.a^{n-1} - {}^nC_1a^{n-2} + {}^nC_2a^{n-3} - .... + (-1)^{n-1}.{}^nC_{n-1}a^0$. If the value of $f(2007) + f(2008) = 2187k$, wehre $k \in N$, then values lesser than k are.	(p) 3
(b) The power of x which has the greatest coefficient in the expansion of $\left(1+\frac{x}{2}\right)^{10}$ is r then values greater than r are	(q) 4
(c) If the coefficient of 4th term in the expansion of $\left(x+\frac{\alpha}{2x}\right)^n$ is 20, then the values greater than α are	(r) 5
	(s) 6
	(t) 7

Code :
(A) a-pqrst, b-qrst, c-pqrst (B) a-pqrs, b-qr, c-st
(C) a-pqr, b-pqrs, c-pqs (D) a-qr, b-pqs, c-prst

PASSAGE BASED QUESTIONS

Passage 1-(Q.45-Q.47)

Consider the mutinomial expansion $(a+b+c)^{10}$, then answer the following questions.

Q.45 Total number of terms in the expansion of $(a+b+c)^{10}$ are
(A) 65 (B) 66
(C) 67 (D) 68

Q.46 Coefficient of a^8bc in the expansion of $(a + b + c)^{10}$ is–
(A) 95 (B) 85
(C) 91 (D) 90

Q.47 Coefficient of $a^4b^5c^3$ in the expansion of $(a + b + c)^{10}$ is–
(A) 1 (B) 2
(C) 3 (D) None of these

Passage 2-(Q.48-Q.50)

Consider the binomial expansion $R = (1 + 2x)^n = I + f$ where I is the integral part of R and f is the fractional part of R $n \in N$. Also the sum of the coefficients of R is 6561.

Q.48 The value of $(n + R – Rf)$ for $x = 1/\sqrt{2}$ equals –
(A) 7 (B) 8
(C) 9 (D) 10

Q.49 If i^{th} terms is the greatest term for x = 1/2, then 'i' equals
(A) 4 (B) 5
(C) 6 (D) 7

Q.50 If k^{th} terms is having greatest coefficient then sum of all possible value(s) of k is –
(A) 6 (B) 7
(C) 11 (D) 13

Passage 3-(Q.51-Q.53)

If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ .... (1)
then sum of the series $C_0 + C_k + C_{2k} + ...$ can be obtained by putting all the roots of the equation $x^k – 1 = 0$ in (1) and then adding vertically.

For example : Sum of these $C_0 + C_2 + C_4 + ...$ can be obtained by putting roots of the equation x
$x^2 – 1 = 0 \Rightarrow x = \pm 1$ in (1) and then adding vertically.

$x = 1$ $C_0 + C_1 + C_2 + ... = 2^n$
$x = – 1$ $C_0 – C_1 + C_2 + ... = 0$

$2(C_0 + C_2 + C_4 + ...) = 2^n$
$C_0 + C_2 + C_4 + ... = 2^{n–1}$

Q.51 Values of x, we should substitute in (1) to get the sum of the series $C_0 + C_3 + C_6 + C_9$ ......, are –
(A) $1, –1, \omega$ (B) $\omega, \omega^2, \omega^3$
(C) $\omega, \omega^2, –1$ (D) None of these

Q.52 If n is a multiple of 3, then $C_0 + C_3 + C_6 + ...$ is equal to
(A) $\dfrac{2^n + 2}{3}$ (B) $\dfrac{2^n - 2}{3}$
(C) $\dfrac{2^n + 2(-1)^n}{3}$ (D) $\dfrac{2^n - 2(-1)^n}{3}$

Q.53 Sum of values of x, which we should substitute in (1) to give the sum of the series : $C_0 + C_4 + C_8 + C_{12} + ...$, is
(A) 2 (B) $2(1 + i)$
(C) $2(1 – i)$ (D) 0

Passage 4-(Q.54-Q.56)

Consider the identity $(1+x)^{6m} = \sum_{r=0}^{6m} {}^{6m}C_r . x^r$.

By putting different values of x on both sides, we can get summation of several series involving binomial coefficients. For example, by putting x = 1/2 we get $\sum_{r=0}^{6m} {}^{6m}C_r \dfrac{1}{2^r} = \left(\dfrac{3}{2}\right)^{6m}$.

Q.54 The value of $\sum_{r=0}^{6m} {}^{6m}C_r 2^{r/2}$ is equal to –
(A) $\dfrac{3^{6m}}{2}$ (B) $(1+\sqrt{2})^{3m}$
(C) $(3+2\sqrt{2})^{3m}$ (D) None of these

Q.55 The value of $\sum_{r=0}^{3m} (-1)^r \, {}^{6m}C_{2r}$ is –
(A) 2^{3m} (B) 0 if m is odd
(C) $–2^{3m}$ if m is even (D) None of these

Q.56 The value of $\sum_{r=0}^{3m} (-3)^{r-1} \, {}^{6m}C_{2r-1}$ is –
(A) 0 (B) 1
(C) depend on m (D) None of these

Passage 5-(Q.57-Q.59)

Coefficient of x^r in expansion of $(1 + x)^n$ is ${}^nC_r$.
To determine the numerically greatest term (absolute value) in the expansion of $(a + x)^n$, when n is a positive integer. Consider

$$\left|\frac{T_{r+1}}{T_r}\right| = \left|\frac{{}^nC_r a^{n-r} x^r}{{}^nC_{r-1} a^{n-r+1} x^{r-1}}\right| = \left|\frac{{}^nC_r}{{}^nC_{r-1}}\right| \left|\frac{x}{a}\right|$$

$$= \left|\frac{n-r+1}{r}\right| . \left|\frac{x}{a}\right| = \left|\frac{n+1}{r} - 1\right| \left|\frac{x}{a}\right|$$

Thus, $|T_{r+1}| > |T_r|$ if $\left\{\dfrac{n+1}{r} - 1\right\}\left|\dfrac{x}{a}\right| > 1$

i.e., $\dfrac{n+1}{r} > 1 + \dfrac{a}{x} \Rightarrow \dfrac{n+1}{1+\left|\dfrac{a}{x}\right|} > r$

Q.57 If the sum of the coefficient in the expansion of $(1 + 2x)^n$ is 6561, the greatest term in the expansion for x = 1/2 is –
(A) 4^{th} (B) 5^{th}
(C) 6^{th} (D) None of these

Q.58 If the coefficient of x^7 and x^8 in $\left(2 + \dfrac{x}{3}\right)^n$ are equal, then n is –
(A) 56 (B) 55
(C) 45 (D) 15

Q.59 Given the integers r > 1, n > 2, and coefficient of $(3r)^{th}$ and $(r + 2)^{th}$ term in the binomial expansion of $(1 + x)^{2n}$ are equal, then –
(A) n = 2r (B) n = 3r
(C) n = 2r + 1 (D) None of these

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 If $\sum_{p=1}^{n}\sum_{m=p}^{n} {}^nC_m \cdot {}^mC_p = 19$, then find value of n.

Q.2 Given $(1-2x+5x^2-10x^3)(1+x)^n = 1+a_1x+a_2x^2+....$ and that $a_1^2 = 2a_2$ then the value of n is

Q.3 If $(1+x-3x^2)^{2145} = a_0+a_1x+a_2x^2+.........$ then $a_0-a_1+a_2-a_3+.....$ ends with

Q.4 The remainder, if $1+2+2^2+2^3+......+2^{1999}$ is divided by 5 is

Q.5 Sum of all the rational terms is the expansion of $(3^{1/4}+4^{1/3})^{12}$, is

Q.6 If the coefficient of x^n is the expansion of $\frac{(1+x)^2}{(1-x)^2}$ is 32 then the value of n equals

Q.7 The number of values of 'r' satisfying the equation , ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is :

Q.8 Number of rational terms in the expansion of $\left(\sqrt{2}+\sqrt[4]{3}\right)^{100}$ is :

Q.9 Sum of last two digits of 21 to the $(100)^{th}$ power, is

Q.10 The coefficient of x^3 in the expansion of $\left(\frac{x^2}{4}+\frac{2}{x}\right)^{12}$, is

Q.11 The coefficient of x^3 in the expansion of $(1+x+x^2)^{12}$, is

Q.12 In the expansion $1+(1+x)+(1+x)^2+.......+(1+x)^{19}$, if the coefficient of x^p is the greatest, then the value of p is

Q.13 If in the expansion of $(1+x)^m(1-x)^n$, coefficients of x and x^2 are 3 and – 6 respectively, then m is –

Q.14 The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, where $\left[\binom{p}{q}=0, \text{ if } p<q\right]$ is maximum, when m is

Q.15 The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n = _______

Q.16 The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1 : 3 : 5. Find n +r.

Q.17 Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6.

Q.18 Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$.

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 If the coefficient of $(r+2)^{th}$ and $(3r)^{th}$ term in the exp. of $(1+x)^{2n}$ are equal then – **[AIEEE 2002]**
(A) $n = 2r + 1$ (B) $n = 2r - 1$
(C) $n = 2r$ (D) None of these

Q.2 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then
$\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + ... + \frac{nC_n}{C_{n-1}} =$ **[AIEEE-2002]**
(A) $\frac{n}{2}$ (B) $n(n+1)$ (C) $\frac{n(n+1)}{12}$ (D) $\frac{n(n+1)}{2}$

Q.3 The coefficient of x^{39} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is- **[AIEEE-2002]**
(A) –455 (B) –105
(C) +455 (D) +105

Q.4 If x is nearly equal to 1 then the value of $\frac{ax^n - bx^a}{x^b - x^a}$ is –
(A) $\frac{a+b}{1-x}$ (B) $\frac{1}{1-x}$ (C) $\frac{1}{1+x}$ (D) $\frac{a+b}{1+x}$

Q.5 The number of integral terms in the expansion of $\left(\sqrt{3} + \sqrt[8]{5}\right)^{256}$ is - **[AIEEE-2003]**
(A) 35 (B) 32
(C) 33 (D) 34

Q.6 The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals- **[AIEEE 2004]**
(A) – 5/3 (B) 10/3
(C) – 3/10 (D) 3/5

Q.7 The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is- **[AIEEE 2004]**
(A) $(n-1)$ (B) $(-1)^n(1-n)$
(C) $(-1)^{n-1}(n-1)^2$ (D) $(-1)^{n-1}n$

Q.8 If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to-
(A) $\frac{1}{2}n$ (B) $\frac{1}{2}n - 1$ **[AIEEE 2004]**
(C) $n-1$ (D) $\frac{2n-1}{2}$

Q.9 If the coefficients of rth, (r + 1)th and (r + 2)th terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation - **[AIEEE-2005]**
(A) $m^2 - m(4r-1) + 4r^2 - 2 = 0$
(B) $m^2 - m(4r+1) + 4r^2 + 2 = 0$
(C) $m^2 - m(4r+1) + 4r^2 - 2 = 0$
(D) $m^2 - m(4r-1) + 4r^2 + 2 = 0$

Q.10 If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation - **[AIEEE-2005]**
(A) $a - b = 1$ (B) $a + b = 1$
(C) $a/b = 1$ (D) $ab = 1$

Q.11 If x is so small that x^2 and higher power of x may be neglected, then $\frac{(1+x)^{3/2} - (1+x/2)^3}{(1-x)^{1/2}}$ may be approximated as – **[AIEEE-2005]**
(A) $1 - \frac{3}{8}x^2$ (B) $3x + \frac{3}{8}x^2$ (C) $-\frac{3}{8}x^2$ (D) $\frac{x}{2} - \frac{3}{8}x^2$

Q.12 If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + ...$ then a_n is
(A) $\frac{a^n - b^n}{b-a}$ (B) $\frac{a^{n+1} - b^{n+1}}{b-a}$
(C) $\frac{b^{n+1} - a^{n+1}}{b-a}$ (D) $\frac{b^n - a^n}{b-a}$

Q.13 For natural numbers m, n if $(1-y)^m (1+y)^n = 1 + a_1y + a_2y^2 + ...$, and $a_1 = a_2 = 10$, then (m, n) is– **[AIEEE 2006]**
(A) (35, 20) (B) (45, 35)
(C) (35, 45) (D) (20, 45)

Q.14 In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5^{th} and 6^{th} terms is zero, then a/b equals- **[AIEEE 2007]**
(A) $\frac{5}{n-4}$ (B) $\frac{6}{n-5}$ (C) $\frac{n-5}{6}$ (D) $\frac{n-4}{5}$

Q.15 The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ...$ $..... + {}^{20}C_{10}$ is- **[AIEEE 2007]**
(A) $-{}^{20}C_{10}$ (B) $\frac{1}{2}\,{}^{20}C_{10}$
(C) 0 (D) ${}^{20}C_{10}$

Q.16 **Statement- 1:** $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n+2)\,2^{n-1}$
Statement -2: $\sum_{r=0}^{n}(r+1)\,{}^nC_r\,x^r = (1+x)^n + nx(1+x)^{n-1}$ **[AIEEE-2008]**
(A) Statement-1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is true, Statement -2 is true; Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is true, Statement -2 is false
(D) Statement-1 is false, Statement-2 is true

Q.17 The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is - **[AIEEE-2009]**

(A) 0 (B) 2
(C) 7 (D) 8

Q.18 Let $S_1 = \sum_{j=1}^{10} j(j-1)\, {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\, {}^{10}C_j$, $S_3 = \sum_{j=1}^{10} j^2\, {}^{10}C_j$

Statement-1: $S_3 = 55 \times 2^9$ **[AIEEE 2010]**

Statement-2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is false
(C) Statement-1 is false, Statement-2 is true
(D) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1

Q.19 Coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(A) 144 (B) – 132 **[AIEEE 2011]**
(C) – 144 (D) 132

Q.20 If n is a positive integer, then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is

(A) an irrational number **[AIEEE 2012]**
(B) an odd positive integer
(C) an even positive integer
(D) a rational number other than positive integers

Q.21 The term independent of x in expansion of

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$$ is – **[JEE MAIN 2013]**

(A) 4 (B) 120
(C) 210 (D) 310

Q.22 If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to **[JEE MAIN 2014]**

(A) (16, 251 / 3) (B) (14, 251 / 3)
(C) (14, 272 / 3) (D) (16, 272 / 3)

Q.23 The sum of coefficients of integral powers of x in the binomial expansion of $(1 - 2\sqrt{x})^{50}$ is **[JEE MAIN 2015]**

(A) $\frac{1}{2}(3^{50})$ (B) $\frac{1}{2}(3^{50} - 1)$
(C) $\frac{1}{2}(2^{50} + 1)$ (D) $\frac{1}{2}(3^{50} + 1)$

Q.24 If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$ is 28, then the sum of the coefficients of the terms in this expansion, is : **[JEE MAIN 2016]**

(A) 2187 (B) 243
(C) 729 (D) 64

Q.25 The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + ... + ({}^{21}C_{10} - {}^{10}C_{10})$, is **[JEE MAIN 2017]**

(A) $2^{20} - 2^9$ (B) $2^{20} - 2^{10}$
(C) $2^{21} - 2^{11}$ (D) $2^{21} - 2^{10}$

Q.26 The sum of the co-efficients of all odd degree terms in the expansion of $(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$, $(x > 1)$

(A) 1 (B) 2 **[JEE MAIN 2018]**
(C) –1 (D) 0

Q.27 If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k = **[JEE MAIN 2019 (JAN)]**

(A) 14 (B) 6
(C) 4 (D) 8

Q.28 The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is **[JEE MAIN 2019 (JAN)]**

(A) 12 (B) 15
(C) 10 (D) 14

Q.29 The sum of the series $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... + 62.{}^{20}C_{20}$ is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) 2^{24} (B) 2^{25}
(C) 2^{26} (D) 2^{23}

Q.30 The sum of the co-efficients of all even degree terms in x in the expansion of $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$, $(x > 1)$ is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) 32 (B) 26
(C) 29 (D) 24

Q.31 If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{1/12}\right)^6$ is equal to 200, and x>1, then the value of x is : **[JEE MAIN 2019 (APRIL)]**

(A) 10^3 (B) 100
(C) 10^4 (D) 10

Q.32 If some three consecutive in the binomial expansion of $(x + 1)^n$ is powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is :

[JEE MAIN 2019 (APRIL)]

(A) 964 (B) 625

(C) 227 (D) 232

Q.33 If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

[JEE MAIN 2019 (APRIL)]

(A) (28, 315) (B) (–54, 315)

(C) (–21, 714) (D) (24, 861)

Q.34 The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

[JEE MAIN 2019 (APRIL)]

(A) 35 (B) 38

(C) 23 (D) 58

Q.35 The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1+x+x^2)^9$ is : **[JEE MAIN 2019 (APRIL)]**

(A) –84 (B) 84

(C) 126 (D) –126

Q.36 If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + ... + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to:

[JEE MAIN 2019 (APRIL)]

(A) (420, 18) (B) (380, 19)

(C) (380, 18) (D) (420, 19)

Q.37 The term independent of x in the expansion of

$$\left(\frac{1}{60} - \frac{x^8}{81}\right)\cdot\left(2x^2 - \frac{3}{x^2}\right)^6$$ is equal to :

[JEE MAIN 2019 (APRIL)]

(A) 36 (B) – 108

(C) – 72 (D) – 36

Q.38 If sum of all the coefficient of even powers in $(1-x+x^2-x^3 \ldots\ldots x^{2n})(1+x+x^2+x^3 \ldots\ldots + x^{2n})$ is 61 then n is equal to **[JEE MAIN 2020 (JAN)]**

Q.39 Let coefficient of x^4 and x^2 in the expansion of

$$(x+\sqrt{x^2-1})^6 + (x-\sqrt{x^2-1})^6$$ is α and β then

[JEE MAIN 2020 (JAN)]

(A) $\alpha + \beta = 48$ (B) $\alpha + \beta = 60$

(C) $\alpha - \beta = -132$ (D) $\alpha - \beta = -60$

Q.40 The coefficient of x^4 is the expansion of $(1 + x + x^2)^{10}$ is **[JEE MAIN 2020 (JAN)]**

Q.41 In the expansion of $\left(\frac{x}{\cos\theta} + \frac{1}{x\sin\theta}\right)^{16}$, if ℓ_1 is the least value of the term independent of x when $\frac{\pi}{8} \le \theta \le \frac{\pi}{4}$ and ℓ_2 is the least value of the term independent of x when $\frac{\pi}{16} \le \theta \le \frac{\pi}{8}$, then the ratio $\ell_2 : \ell_1$ is equal to :

[JEE MAIN 2020 (JAN)]

(A) 1 : 8 (B) 1 : 16

(C) 8 : 1 (D) 16 : 1

Q.42 If $C_r \equiv {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$, then k is equal to _____. **[JEE MAIN 2020 (JAN)]**

Q.43 Coefficient of x^7 in $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \ldots\ldots + x^{10}$ is-

[JEE MAIN 2020 (JAN)]

(A) 330 (B) 210

(C) 420 (D) 260

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	C	C	A	D	C	B	A	B	B	A	C	C	C	A	D	A	D	C	C	A	B	C	B	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	B	B	D	C	A	B	A	B	A	A	A	D	D	A	B	B	C	D	A	D	D	A	B	A	D
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	C	D	A	C	C	D	C	A	B	A	A	D	A	C	C	B	C	D	A	A	A	D	B	C	A
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
A	A	C	A	C	B	C	B	B	C	C	B	B	D	D	C	A	C	C	C	B	A	A	A	D	C

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	A	C	A	C	D	A	B	D	A	D	B	A	B	D	A	A	A	A	A	A	D	B	B	B
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	C	B	D	D	B	D	B	C	A	B	A	B	A	A	C	C	A	B	A	B	D	D	C	B	D
Q	51	52	53	54	55	56	57	58	59																
A	B	C	D	C	B	A	B	B	A																

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	3	6	3	0	283	8	2	26	1	99	352	9	12	15	6	10	4	10

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
A	C	D	A	B	C	C	B	A	C	D	C	C	C	D	B	A	B	B	C	A	C	D	D	C	B	B	D	B
Q	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43													
A	B	D	D	D	A	B	B	A	D	30	C	615	D	51	A													

CHAPTER- 7 : BINOMIAL THEOREM
SOLUTIONS TO TRY IT YOURSELF
TRY IT YOURSELF-1

(1) $T_{r+1} = {}^{n}C_r x^{n-r} y^r$ for $(x+y)^n$.

$$T_6 = {}^{10}C_5 (2x^2)^5 \left(-\frac{1}{3x^2}\right)^5 = -\frac{10!}{5!5!} 32 \times \frac{1}{243} = -\frac{896}{27}$$

(2) The expression being in G.P., we have

$E = 1 + (1+x) + (1+x)^2 + ... + (1+x)^n$

$$= \frac{(1+x)^{n+1} - 1}{(1+x) - 1} = x^{-1}[(1+x)^{n+1} - 1]$$

Therefore, the coefficient of x^k in E is equal to the coefficient of x^{k+1} in $[(1+x)^{n+1} - 1]$, which is given by ${}^{n+1}C_{k+1}$.

(3) We know that

$2^{n-1} = {}^{n}C_0 + {}^{n}C_2 + {}^{n}C_4 + ... = {}^{n}C_1 + {}^{n}C_3 + {}^{n}C_5 + ...$

So, ${}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + ...+ {}^{10}C_9 = 2^{10-1} = 2^9$.

(4) Since, $(n+2)^{th}$ term is the middle term in the expansion of $(1+x)^{2n+2}$, therefore $\alpha = {}^{2n+2}C_{n+1}$.

Since $(n+1)^{th}$ and $(n+2)^{th}$ terms are middle terms in the expansion of $(1+x)^{2n+1}$, therefore,

$\beta = {}^{2n+1}C_n$ and $\gamma = {}^{2n+1}C_{n+1}$

But ${}^{2n+1}C_n + {}^{2n+1}C_{n+1} = {}^{2n+2}C_{n+1} \Rightarrow \beta + \gamma = \alpha$

(5) The greatest coefficient is equal to the greatest term when $x = 1$.

For $x = 1$, $\dfrac{T_{r+1}}{T_r} = \dfrac{15-r+1}{r}\dfrac{2}{3}$

Let $\dfrac{T_{r+1}}{T_r} \geq 1 \Rightarrow \dfrac{15-r+1}{r}\dfrac{2}{3} \geq 1 \Rightarrow 32 - 2r \geq 3r$

$r \leq 32/5 \Rightarrow r = 6$

Hence, 7^{th} term has the greatest coefficient and its value is $T_{6+1} = {}^{15}C_6 (2/3)^6$.

(6) The general term of the series is $T_r = (-1)^r (3+5r)\, {}^{n}C_r$ where, r = 0, 1, 2, ... n. Therefore, sum of the series is

given by $S = \sum_{r=0}^{n} (-1)^r (3+5r)\ {}^{n}C_r$

$$= 3\left(\sum_{r=0}^{n} (-1)^r\ {}^{n}C_r\right) + 5\left(\sum_{r=1}^{n} (-1)^r\ n\ {}^{n-1}C_{r-1}\right)$$

$$= 3\left(\sum_{r=0}^{n} (-1)^r\ {}^{n}C_r\right) - 5n\left(\sum_{r=1}^{n} (-1)^{r-1}\ {}^{n-1}C_{r-1}\right)$$

$= 3(1-1)^n - 5n(1-1)^{n-1} = 0$

(7) **(D).** $(1 - 3x + 3x^2 - x^3)^{20} = [(1-x)^3]^{20} = (1-x)^{60}$

Therefore, number of dissimilar terms in the expansion of $(1 - 3x + 3x^2 - x^3)^{20}$ is 61.

(8) 7^{th} term of $\left(\dfrac{4x}{5} - \dfrac{5}{2x}\right)^9$

$$T_{6+1} = {}^{9}C_6 \left(\frac{4x}{5}\right)^{9-6}\left(-\frac{5}{2x}\right)^6$$

$$= \frac{9!}{3!6!}\left(\frac{4x}{5}\right)^3\left(\frac{5}{2x}\right)^6 = \frac{10500}{x^3}$$

(9) Here, n is even, therefore middle term is $\left(\dfrac{14+2}{2}\right)^{th}$ term.

It means T_8 is middle term

$$T_8 = {}^{14}C_7\left(-\frac{x^2}{2}\right)^7 = -\frac{429}{16}x^{14}.$$

(10) Let $(r+1)^{th}$ term contains x^m

$$T_{r+1} = {}^{15}C_r (x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r = {}^{15}C_r\, x^{60-7r}(-1)^r$$

(i) For x^{32}, $60 - 70r = 32 \Rightarrow 7r = 28 \Rightarrow r = 4$, so 5^{th} term

$T_5 = {}^{15}C_4\, x^{32}(-1)^4$

Hence, coefficient of x^{32} is 1365.

(ii) For x^{-17}, $60 - 7r = -17 \Rightarrow r = 11$, so 12^{th} term.

$T_{12} = {}^{15}C_{11}\, x^{-17}(-1)^{11}$

Hence, coefficient of x^{-17} is -1365.

TRY IT YOURSELF-2

(1) We have, $17^{256} = (17^2)^{128} = (289)^{128} = (290-1)^{128}$

$\therefore\ 17^{256} = {}^{128}C_0 (290)^{128} - {}^{128}C_1 (290)^{127}$

$+ {}^{128}C_2 (290)^{126} - ...... - {}^{128}C_{125} (290)^3$

$+ {}^{128}C_{126} (290)^2 - {}^{128}C_{127} (290) + 1$

$= [{}^{128}C_0 (290)^{128} - {}^{128}C_1 (290)^{127} + {}^{128}C_2 (290)^{126} - .....$

$- {}^{128}C_{125} (290)^3] + {}^{128}C_{126} (290)^2 - {}^{128}C_{127} (290) + 1$

$= 1000m + {}^{128}C_2 (290)^2 - {}^{128}C_1 (290) + 1 \qquad (m \in IN)$

$$= 1000m + \frac{(128)(127)}{2}(290)^2 - 128 \times 290 + 1$$

$= 1000m + (128)(127)(290)(145) - 128 \times 290 + 1$

$= 1000m + (128)(290)(127 \times 145 - 1) + 1$

$= 1000m + (128)(290)(18414) + 1$

$= 1000m + 683527680 + 1 = 1000m + 683527000 + 680 + 1$

$= 1000(m + 683527) + 681$

Hence, the last three digits of 17^{256} must be 681. As a result, the last two digits of 17^{256} are 81 and the last digit of 17^{256} is 1.

(2) Here $5^{5^{5^{\cdot^{\cdot^{5}}}}}$ (23 times 5) is an odd natural number.

Therefore, $x = 5^{2m+1} = 5 \times (25^m)$, where m is a natural number. Thus, $x = 5 \times (24+1)^m = 5 +$ a multiple of 24.

Hence, the remainder is 5.

(3) The given expression can be written as

5 – 1/2 {1 + (4/5x) – 1/2] and is valid only when

$$\left|\frac{4}{5}x\right| < 1 \Rightarrow |x| < \frac{5}{4}$$

(4) Comparing the given expression to

$$1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots = (1+x)^n$$

We get, $nx = -\frac{1}{8}$ and $\frac{n(n-1)}{2!}x^2 = \frac{3}{128}$

$$\Rightarrow \quad x = \frac{1}{4},\ n = -\frac{1}{2}$$

Hence, $1 - \frac{1}{8} + \frac{1}{8}\frac{3}{16} - \ldots = \left(1 + \frac{1}{4}\right)^{-1/2} = \frac{2}{\sqrt{5}}$

(5) We have, $(1.1)^{10000} = [1 + (0.1)]^{10000} = 1$

$+ 10000\,C_1\,(0.1) + {}^{10000}C_2\,(0.1)^2 + \ldots\ldots$

$= 1 + 10000 \times (0.1) +$ other terms

$= 1001 +$ other positive terms of the expansion.

Hence, $(1.1)^{10000} > 1000$

(6) We have, $(0.99)^5 = (1 - 0.01)^5$

$= {}^5C_0 - {}^5C_1\,(0.01) - {}^5C_2\,(0.01)^2 \ldots$

$= 1 - (5 \times 0.01) + (10 \times 0.0001) \ldots\ldots$

$= 1 - 0.05 + 0.001 \ldots\ldots = 0.951$

(7) $5^{99} = 5 \cdot 5^{98} = 5 \cdot (25)^{49} = 5(26-1)^{49}$

$= 5\,[{}^{49}C_0\,(26)^{49} - {}^{49}C_1\,(26)^{48} + \ldots\ldots + {}^{49}C_{48}\,(26)^1 - {}^{49}C_{49}\,(26)^0]$

$= 5\,[{}^{49}C_0\,(26)^{49} - {}^{49}C_1\,(26)^{48} + \ldots\ldots + {}^{49}C_{48}\,(26)^1 - 1]$

$= 5\,[{}^{49}C_0\,(26)^{49} - {}^{49}C_1\,(26)^{48} + \ldots\ldots + {}^{49}C_{48}\,(26)^1 - 13] + 60$

$= 13(k) + 52 + 8$ (where k is a positive integer)

$= 13(k+4) + 8$.

Hence, remainder is 8.

(8) $(17)^{10} = (289)^5 = (290-1)^5$

$= {}^5C_0\,(290)^5 - {}^5C_1\,(290)^4 + \ldots + {}^5C_4\,(290)^1 - {}^5C_5\,(290)^0$

$= {}^5C_0\,(290)^5 - {}^5C_1\,(290)^4 + \ldots + {}^5C_3\,(290)^2 + 5 \times 290 - 1$

= A multiple of 1000 + 1449

Hence, last two digits are 49.

(9) **(C).**

Power of x^2	x^3	x^4	Coefficient of x^{11}
0	1	2	${}^4C_0 \times {}^7C_1 \times {}^{12}C_2$
2	1	1	${}^4C_2 \times {}^7C_1 \times {}^{12}C_1$
4	1	0	${}^4C_4 \times {}^7C_1 \times {}^{12}C_0$
1	3	0	${}^4C_1 \times {}^7C_3 \times {}^{12}C_0$
			1113

CHAPTER- 7 : BINOMIAL THEOREM

EXERCISE-1

(1) **(C).** Since (n = 8) is even then there is only one middle term i.e. $T_{\frac{8+2}{2}} = T_5$

$\therefore\ T_5 = {}^8C_4 (x)^4 (-2/x)^4$
$= {}^8C_4 . (-2)^4 = 16.{}^8C_4 = 1120$

(2) **(C).** Since we require term independent from x

$\therefore\ n\alpha - r(\alpha+\beta) = 0$

$\Rightarrow 10 \times \frac{1}{2} - r\left(\frac{1}{2}+2\right) = 0$

$\Rightarrow\ r = 2$ i.e. 3^{rd} term.

$\therefore\ T_3 = {}^{10}C_2 (\sqrt{x})^8 (-3/x^2)^2$

$= {}^{10}C_2 .(-3)^2 . x^{\circ} = \frac{10.9}{2.1} . 9 = 405$

(3) **(C).** Here n = 7 is odd so there are two middle terms which are $\left(\frac{7+1}{2}\right) = 4^{th}$ term and $\frac{7+3}{2} = 5^{th}$ term.

Hence middle terms $T_4 = {}^7C_3 x^4 . 6^3 = 7560\, x^4$
$T_5 = {}^7C_4 x^3 . 6^4 = 45360\, x^3$

(4) **(A).** $(1+3x+2x^2)^6 = [1+x(3+2x)]^6$
$= 1 + {}^6C_1\, x(3+2x) + {}^6C_2\, x^2(3+2x)^2 + {}^6C_3\, x^3 (3+2x)^3 + {}^6C_4\, x^4 (3+2x)^4 + {}^6C_5\, x^5 (3+2x)^5 + {}^6C_6\, x^6 (3+2x)^6$

We get x^{11} only from ${}^6C_6\, x^6 (3+2x)^6$.

Hence, coefficient of x^{11} is ${}^6C_5 \times 3 \times 2^5 = 576$

(5) **(D).** Required term $= T_{10-4+2} = T_8 = {}^{10}C_7 (2x)^3 (-1/x^2)^7$
$= -960\, x^{-11}$

(6) **(C).** Since n = 8 is even, therefore the term with greatest coefficient $= \left(\frac{8+2}{2}\right)^{th}$ term $= 5^{th}$ term.

(7) **(B).** Here $\frac{(n+1)a}{x+a} = \frac{(10+1).7}{6+7} = \frac{77}{13} = 5\frac{12}{13}$

$\therefore$ Greatest term $= T_{5+1} = T_6$

(8) **(A).** $\left(ax - \frac{1}{bx^2}\right)^5$

$= {}^5C_0 (ax)^5 + {}^5C_1 (ax)^4 \left(-\frac{1}{bx^2}\right) + {}^5C_2 (ax)^3 \left(-\frac{1}{bx^2}\right)^2$

$+ {}^5C_3 (ax)^2 \left(-\frac{1}{bx^2}\right)^3 + \ldots$

$= a^5x^5 - 5\frac{a^4}{b}x^2 + 10\frac{a^3}{b^2x} - 10\frac{a^2}{b^3x^4}$

(9) **(B).** $T_6 = {}^8C_5 (3x^2)^3 \left(-\frac{1}{2x}\right)^5$

$= 56 \times (27x^6) \times \left(-\frac{1}{32x^5}\right) = -\frac{189}{4}x$

(10) **(B).** $= {}^4C_0 (3x)^4 + {}^4C_1 (3x)^3 (1/x) + {}^4C_2 (3x)^2 (1/x)^2 + {}^4C_3 (3x)(1/x)^3$
$= 81\,x^4 + 108\,x^2 + 54 + 12\,x^{-2}$

(11) **(A).** Comparing $(2x^2 + 1/x)^{12}$ with $(X+a)^n$.
$n = 12, X = 2x^2, a = 1/x.$

$\therefore\ 10^{th}$ term $= T_{10} = {}^{12}C_9 (2x^2)^{12-9} (1/x)^9$
$= {}^{12}C_9 . 8.1/x^3$

or $T_{10} = 1760/x^3$

(12) **(C).** If r^{th} term is independent of x, then by the formula
$15 \times 3 - (r-1)(3+2) = 0$
$\Rightarrow r - 1 = 9 \Rightarrow r = 10$

(13) **(C).** We have, $\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!}\right)^2$

$\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!}\right)\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}\right)$

Now, x^n term is generated if terms of the two brackets are multiplied as shown in loops above.

Hence, the coefficient of x^n is

$1 \times \frac{1}{n!} + \frac{1}{1!} \times \frac{1}{(n-1)!} + \frac{1}{2!} \times \frac{1}{(n-2)!} + \ldots + \frac{1}{n!}$

$= \frac{1}{n!}\left(\frac{n!}{n!} \times \frac{n!}{(n-1)!\,1!} + \frac{n!}{(n-2)!\,2!} + \ldots + \frac{n!}{n!}\right)$

$= \frac{1}{n!}({}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n) = \frac{2^n}{n!}$

(14) **(C).** $(4-3x)^7 = 4^7\left(1 - \frac{3x}{4}\right)^7$

$\therefore\ \frac{T_{r+1}}{T_r} = \left|\frac{7-r+1}{r} . \frac{-3x}{4}\right| = \frac{8-r}{2r} \qquad \left(\because x = \frac{2}{3}\right)$

Now $T_{r+1} \geq T_{r+1}$ if $8 - r \geq 2r$

$\Rightarrow 3r \leq 8 \Rightarrow r \leq 2\frac{2}{3}$

$\therefore\ T_1 \leq T_2 \leq T_3 \geq T_4 \geq T_5 \ldots\ldots$

$\therefore$ Numerical value of T_3 is greatest.

(15) **(A).** $(1+x)^{21}+(1+x)^{22}+.....(1+x)^{30}$

$= (1+x)^{21}\left[\frac{(1+x)^{10}-1}{(1+x)-1}\right]=\frac{1}{x}\left[(1+x)^{31}-(1+x)^{21}\right]$

$\Rightarrow$ Coefficient of x^5 in the given expression

$=$ Coefficient of x^5 in $\left\{\frac{1}{x}\left[(1+x)^{31}-(1+x)^{21}\right]\right\}$

$=$ Coefficient of x^6 in $(1+x)^{31}-(1+x)^{21}] = {}^{31}C_6 - {}^{21}C_6$

(16) **(D).** Exp. $=(1+2x+3x^2)(1-x)^{-2}$

$=(1+2x+3x^2)(1+2x+3x^2+4x^3+5x^4+...)$

$\therefore$ Coefficient of $x^4 = 5+8+9 = 22$

(17) **(A).** The fourth term in expansion of $(px+1/x)^n$

$T_4 = {}^nC_3 . (px)^{n-3}(1/x)^3 = 5/2.$

$\Rightarrow ({}^nC_3.p^{n-3}) . x^{n-6} = 5/2. x^0$

Compairing the coefficient of x and constant term

$n-6=0 \Rightarrow n=6$ and ${}^nC_3 (p)^{n-3} = 5/2$

putting n = 6 in it

$6C_3\, p^3 = 5/2 \Rightarrow p^3 = 1/8$

$\Rightarrow p^3 = (1/2)^3 \Rightarrow p = 1/2$

(18) **(D).** Exp. $=(1+x)^n(1+x^2)^n$

$=(1+{}^nC_1x+{}^nC_2x^2+{}^nC_3x^3+{}^nC_4x^4+.....+x^n)$

$(1+{}^nC_1x^2+{}^nC_2x^4+...+x^{2n})$

$\therefore$ Coefficient of $x^4 = {}^nC_4+{}^nC_2.{}^nC_1+{}^nC_2$

(19) **(C).** Putting x = 1 and x = –1 in the given expansion, we get

$a_0+a_1+a_2+a_3+a_4+.... = 0$

$a_0-a_1+a_2-a_3+a_4-.... = 2^{2n}$

Adding $2(a_0+a_2+a_4+.....) = 2^{2n}$

$\Rightarrow a_0+a_2+a_4+... = 2^{2n-1}$

(20) **(C).** The general term in the expansion of the given expression is $T_{r+1} = {}^{2n}C_r\, x^{2n-r}\left(\frac{1}{x^2}\right)^r = {}^{2n}C_r\, x^{2n-3r}$

For the coefficient of x^m, we must have

$2n-3r = m \Rightarrow r = \frac{2n-m}{3}$

So, coefficient of

$x^m = {}^{2n}C_{\frac{2n-m}{3}} = \frac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$

(21) **(A).** Here $T_3 = {}^5C_2x^3(x\log_{10}x)^2 = 10^6$

or $x^3\, x^{2\log_{10}x} = 10^5$

Taking log of both sides, we get

$3\log_{10}x + 2(\log_{10}x)^2 = 5$

or $2(\log_{10}x)^2+5\log_{10}x-2\log_{10}x-5=0$

or $(\log_{10}x-1)(2\log_{10}x+5)=0$

or $x = 10$ or $2\log_{10}x+5=0$

(22) **(B).** The coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^{n+2}$ will be maximum.

If $r \le \frac{(2n+2)+1}{2}$

$r \le (n+1)+1/2$

$r = n+1 =$ Maximum coefficient $= {}^{2n+2}C_{n+1}$

$= \frac{(2n+2)!}{(n+1)!(n+1)!} = \frac{(2n+2)!}{[(n+1)!]^2}$

(23) **(C).** Accordingly, $\frac{T_2}{T_3} = \frac{{}^nC_1a^{n-1}b}{{}^nC_2a^{n-2}b^2}$ ......(i)

$\frac{T_3}{T_4} = \frac{{}^{n+3}C_2a^{n+1}b^2}{{}^{n+3}C_3a^nb^3}$ ......(ii)

$(i)=(ii) \Rightarrow \frac{2n}{n(n-1)} = \frac{6(n+3)(n+2)}{2(n+3)(n+2)(n+1)}$

$\Rightarrow 2(n+1) = 3(n-1) \Rightarrow n = 5$.

(24) **(B).** Given $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r\theta$

$\Rightarrow \sin n\theta = b_0\sin^0\theta + b_1\sin^1\theta$

$+ b_2\sin^2\theta + b_3\sin^3\theta + ... + b_n\sin^n\theta$

$\Rightarrow \sin n\theta = b_0 + b_1\sin\theta + b_2\sin^2\theta + ... + b_n\sin^n\theta$

(n is an odd integer)

$\because \sin n\theta = {}^nC_1\sin\theta\cos^{n-1}\theta - {}^nC_3\sin^3\theta\cos^{n-3}\theta + ...$

$= {}^nC_1\sin\theta.(1-\sin^2\theta)^{(n-1)/2} - {}^nC_3\sin^3\theta(1-\sin^2\theta)^{(n-3)/2} + ...$

$\therefore b_0 = 0, b_1 =$ coefficient of $\sin\theta = {}^nC_1 = n$

($\because$ n – 1 = n – 3 are all even integers)

(25) **(A).** $(1-x+x^2)^n = a_0+a_1x+a_2x^2+....+a_{2n}x^{2n}$

Putting $x=1$, we get

$(1-1+1)^n = a_0+a_1+a_2+.....+a_{2n}$

$\Rightarrow 1 = a_0+a_1+a_2+....+a_{2n}$ .....(i)

Putting x = –1, we get

$\Rightarrow 3^n = a_0-a_1+a_2-....+a_{2n}$ ......(ii)

Adding (i) and (ii), we get

$\frac{3^n+1}{2} = a_0+a_2+a_4+....+a_{2n}$.

(26) **(B).** Given $2^n = 1024$, $\therefore n = 10$

$\therefore$ The greatest coefficient is ${}^{10}C_5 = 252$.

(27) **(B).** If n is odd, then numerically the greatest coefficient in the expansion of $(1-x)^n$ is ${}^nC_{(n-1)/2}$ or ${}^nC_{(n+1)/2}$,

Therefore in case of $(1-x)^{21}$, the numerically greatest coefficient is ${}^{21}C_{10}$ or ${}^{21}C_{11}$.

Therefore the numerically greatest term

$= {}^{21}C_{11}x^{11}$ or ${}^{21}C_{10}x^{10}$

$\therefore\ {}^{21}C_{11}x^{11} > {}^{21}C_{12}x^{12}$ and ${}^{21}C_{10}x^{10} > {}^{21}C_9x^9$

$\Rightarrow \frac{21!}{10!\,11!} > \frac{21!}{9!\,12!}x$ and $\frac{21!}{11!\,10!}x > \frac{21!}{9!\,12!}$

$\Rightarrow \frac{6}{5} > x$ and $x < \frac{5}{6} \Rightarrow x \in \left(\frac{5}{6}, \frac{6}{5}\right)$

(28) **(D).** Since coefficients ${}^mC_1$, ${}^mC_2$ and ${}^mC_3$ of T_2, T_3, T_4 i.e. are the first, third and fifth terms of an A. P., which will also be in A. P. of common difference 2d.

Hence $2\,{}^mC_2 = {}^mC_1 + {}^mC_3 \Rightarrow (m-2)(m-7)=0$.

Since 6th term is 21, m = 2 is ruled out and we have m = 7

and $T_6 = 21 = {}^7C_5\left[\sqrt{2^{\log(10-3^x)}}\right]^{7-5} \times \left[\sqrt[5]{2^{(x-2)}\log 3}\right]^5$

$\Rightarrow 21 = 21\,.\,2^{\log(10-3^x)+\log 3^{x-2}}$

$\Rightarrow 2^{\log[(10-3^x)\,3^{x-2}]} = 1 = 2^0$

Which on simplification gives x = 0, 2.

(29) **(C).** In the expansion of $(1+x)^{2n}$, the general term

$= {}^{2n}C_k, 0 \le k \le 2n$

As given for $r > 1, n > 2, {}^{2n}C_{3r} = {}^{2n}C_{r+2}$

$\Rightarrow$ Either $3r = r+2$

or $3r = 2n-(r+2)$, $\qquad (\because {}^nC_r = {}^nC_{n-r})$

$\Rightarrow r = 1$ or $n = 2r+1 \Rightarrow n = 2r+1$, $\quad (\because r > 1)$.

(30) **(A).** In the expansion of $(1+x)^n$, it is given that ${}^nC_1, {}^nC_2, {}^nC_3$ are in A.P.

$\Rightarrow 2.{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow 2.\frac{n(n-1)}{1.2} = \frac{n}{1} + \frac{n(n-1)(n-2)}{1.2.3}$

$\Rightarrow 6(n-1) = 6 + (n-2)(n-1)$

$\Rightarrow n^2 - 9n + 14 = 0 \Rightarrow n = 2$ or $n = 7$.

But n = 2 is not acceptable because, when n=2, there are only three terms in the expansion of $(1+x)^2$, $\therefore\ n = 7$.

(31) **(B).** $T_{r+1} = {}^nC_r(a)^{n-r}(-b)^r$.

$T_5 = T_{4+1} = {}^nC_4a^{n-4}(-b)^4 = {}^nC_4a^{n-4}b^4$ and 6th term

$= (T_6) = T_{5+1} = {}^nC_5\,a^{n-5}(-b)^5 = -{}^nC_5\,a^{n-5}\,b^5$

Since $T_5 + T_6 = 0$, therefore

${}^nC_4\,a^{n-4}\,b^4 - {}^nC_5\,a^{n-5}\,b^5 = 0 \Rightarrow \frac{a^{n-4}\,b^4}{a^{n-5}\,b^5} = \frac{{}^nC_5}{{}^nC_4}$

$\Rightarrow \frac{a}{b} = \frac{n!}{(n-5)!\,5!}.\frac{4!\,(n-4)!}{n!} \Rightarrow \frac{a}{b} = \frac{n-4}{5}$.

(32) **(A).** T_3, T_4, T_5 in the given expansion are respectively

${}^{10}C_2 2^8\left(\frac{3x}{8}\right)^2, {}^{10}C_3 2^7\left(\frac{3x}{8}\right)^3, {}^{10}C_4 2^6\left(\frac{3x}{8}\right)^4$

or $1620x^2, 810x^3, \frac{8505}{32}x^4$

We are given that T_4 is numerically the greatest term so that $|T_4| \nless |T_3|$ and $|T_4| \nless |T_5|$

$\therefore\ |x| > 2$ and $\frac{64}{21} \nless |x|$; $\quad 2 \nless |x| < \frac{64}{21}$ .....(i)

The above inequality (i) is equivalent to two inequalities

$2 < x < \frac{64}{21}$ and $-\frac{64}{21} < x < -2$

(33) **(B).** Applying $T_{r+1} = {}^nC_rx^{n-r}a^r$ for $(x+a)^n$

Hence $T_6 = {}^{10}C_5(2x^2)^5\left(-\frac{1}{3x^2}\right)^5$

$= -\frac{10!}{5!5!}32 \times \frac{1}{243} = -\frac{896}{27}$

(34) **(A).** We have $T_{r+1} = {}^{21}C_r\left(\sqrt[3]{\frac{a}{\sqrt{b}}}\right)^{21-r}\left(\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^r$

$= {}^{21}C_ra^{7-(r/2)}b^{(2/3)r-(7/2)}$

Since the powers of a and b are the same, therefore

$7 - \frac{r}{2} = \frac{2}{3}r - \frac{7}{2} \Rightarrow r = 9$

(35) **(A).** In the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$, the general term is

$T_{r+1} = {}^{10}C_r\left(\frac{x}{2}\right)^{10-r}.\left(-\frac{3}{x^2}\right)^r$

$= {}^{10}C_r(-1)^r.\frac{3^r}{2^{10-r}}x^{10-r-2r}$

Here, the exponent of x is $10-3r = 4 \Rightarrow r = 2$

$\therefore\ T_{2+1} = {}^{10}C_2\left(\frac{x}{2}\right)^8\left(-\frac{3}{x^2}\right)^2 = \frac{10.9}{1.2}.\frac{1}{2^8}.3^2.x^4$

$= \frac{405}{256}x^4$

$\therefore$ The required coefficient $= \frac{405}{256}$.

(36) **(A).** Coefficient of $(2r+1)^{th}$ term in expansion of $(1+x)^{43} = {}^{43}C_{2r}$ and coefficient of $(r+2)^{th}$ term $=$ coefficient of $\{(r+1)+1\}^{th}$ term $= {}^{43}C_{r+1}$

According to question ${}^{43}C_{2r} = {}^{43}C_{r+1} = {}^{43}C_{43-(r+1)}$

then $2r = 43-(r+1)$ or $3r = 42$ or $r = 14$.

(37) **(D).** $T_2 = n(x)^{n-1}(a)^1 = 240$ .....(i)

$T_3 = \frac{n(n-1)}{1.2}x^{n-2}a^2 = 720$ .....(ii)

$T_4 = \frac{n(n-1)(n-2)}{1.2.3}x^{n-3}a^3 = 1080$ .....(iii)

To eliminate x,

$\frac{T_2 \, . \, T_4}{T_3^{\,2}} = \frac{240\,.\,1080}{720\,.\,720} = \frac{1}{2} \Rightarrow \frac{T_2}{T_3}\cdot\frac{T_4}{T_3} = \frac{1}{2}$

Now, $\frac{T_{r+1}}{T_r} = \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

Putting r = 3 and 2 in above expression, we get

$\Rightarrow \frac{n-2}{3}\cdot\frac{2}{n-1} = \frac{1}{2} \Rightarrow n = 5$.

(38) **(D).** We have $(x)^{12-r}\left(\frac{1}{x^2}\right)^r = x^0 \Rightarrow x^{12-3r} = x^0 \Rightarrow r = 4$

Hence the required term is ${}^{12}C_4\,2^8\left(-\frac{1}{2}\right)^4 = 7920$.

(39) **(A).** $T_{r+1} = {}^{10}C_r(x^2)^{10-r}\left(\frac{-3\sqrt{3}}{x^3}\right)^r$

For term independent of x, $20-2r-3r=0 \Rightarrow r=4$

$\therefore\ T_{4+1} = {}^{10}C_4(-3)^4(\sqrt{3})^4 = 153090$.

(40) **(B).** Sum of the coefficients in the expansion of $(1+2x)^n = 6561$

$\Rightarrow (1+2x)^n = 6561$ when $x = 1$

$\Rightarrow 3^n = 6561 \Rightarrow 3^n = 3^8 \Rightarrow n = 8$

Now, $\frac{T_{r+1}}{T_r} = \frac{{}^8C_r(2x)^r}{{}^8C_{r-1}(2x)^{r-1}} = \frac{9-r}{r}.2x$

$\Rightarrow \frac{T_{r+1}}{T_r} = \frac{9-r}{r}$ $[\because x = 1/2]$

$\therefore \frac{T_{r+1}}{T_r} > 1 \Rightarrow \frac{9-r}{r} > 1 \Rightarrow 9-r > r \Rightarrow 2r < 9 \Rightarrow r < 4\frac{1}{2}$

Hence, 5^{th} term is the greatest term.

(41) **(B).** $T_{r+1} = \frac{3.5....(2r-1)}{r!}\left(\frac{1}{5}\right)^r$

$= \frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)...\left(\frac{2r-1}{2}\right)}{r!}\left(\frac{2}{5}\right)^r$

$= \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)...\left(-\frac{1}{2}-r+1\right)}{r!}\left(-\frac{2}{5}\right)^r$

which is the $(r+1)^{th}$ term of $\left(1-\frac{2}{5}\right)^{-1/2}$

(42) **(C).** $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$

$=(1+x+2x^3)\left[\sum_{r=0}^{9}{}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r\right]$

$=(1+x+2x^3)\left[\sum_{r=0}^{9}{}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{19-3r}\right]$

$+\ 2\left[\sum_{r=0}^{9}{}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{21-3r}\right]$

Clearly, first and third expansions contain term independent of x and are obtained by equation $18-3r=0$ and $21-3r=0$ respectively. So, coefficient of the term independent of

$x = {}^9C_6\left(\frac{3}{2}\right)^{9-6}\left(-\frac{1}{3}\right)^6 + 2\left({}^9C_7\left(\frac{3}{2}\right)^{9-7} - \left(\frac{1}{3}\right)^7\right)$

$= \frac{7}{18} - \frac{7}{27} = \frac{17}{54}$

(43) **(D).** $T_4 = {}^nC_3\,x^{n-3}\left(\frac{\alpha}{2x}\right)^3 \Rightarrow {}^nC_3\,x^{n-3}\left(\frac{\alpha}{2}\right)^3 = 20$

$n = 6,\ {}^6C_3\left(\frac{\alpha}{2}\right)^3 = 20 \Rightarrow \alpha = 2$

(44) **(A).** ${}^mC_m + {}^{m+1}C_m + ... + {}^nC_m$

$= {}^{m+1}C_{m+1} + {}^{m+1}C_m + ... + {}^nC_m$

$= {}^{m+2}C_{m+1} + {}^{m+2}C_m + ... + {}^nC_m$

$= ....$

$= {}^nC_{m+1} + {}^nC_m = {}^{n+1}C_{m+1}$

(45) $\sum_{r=0}^{50} {}^{50}C_r\,(2x-3)^r(2-x)^{50-r}$

$= ((2-x)+(2x-3))^{50}$
$= (x-1)^{50}$
$= (1-x)^{50}$
$= {}^{50}C_0 - {}^{50}C_1\,x .... - {}^{50}C_{25}\,x^{25} + ...$
coefficient of x^{25} is $-{}^{50}C_{25}$

(46) **(D).** The general term, $T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\sqrt{\frac{3}{2x^2}}\right)^r$

$= {}^{10}C_r\left(\frac{1}{3}\right)^{5-\frac{r}{2}}\left(\frac{3}{2}\right)^{\frac{r}{2}}x^{5-\left(\frac{3r}{2}\right)}$

For independent term of x

$5-\frac{3r}{2}=0 \Rightarrow r=\frac{10}{3}$

which is not a positive integer. So there is no term independent of x.

(47) **(A).** The $(p+2)^{th}$ term from the end
= The $(2n-p+1)^{th}$ term from beginning

$= {}^{2n+1}C_{2n-p}\,x^{(2n+1)-(2n-p)}\left(-\frac{1}{x}\right)^{2n-p}$

$= (-1)^p\frac{(2n+1)!}{(2n-p)!\,(p+1)!}x^{2p-2n+1}$

(48) **(B).** Sum of coeff. $= (1+1)^n = 4096, n = 12$
${}^{12}C_6$ = greatest binomial coefficient = 924

(49) **(A).** $n = \frac{(11+1).\left|5.\frac{1}{5}\right|}{|3|+\left|5.\frac{1}{5}\right|} = \frac{12\times 1}{4} = 3$

T_3, T_4 are numerically greatest $= {}^{11}C_2.(3)^9.(1)^2$

$= \frac{11\times 10}{2}.(3)^9 = (55).(3)^9$

(50) **(D).** $T_{r+1} = {}^{15}C_r\,x^r$. Co-efficient of $x^r = {}^{15}C_{r+3}$,
$\therefore$ Coefficient of $x^{r+3} = {}^{15}C_{r+3}$.
Given, ${}^{15}C_r = {}^{15}C_{r+3} \Rightarrow r + r + 3 = 15 \Rightarrow r = 6$

(51) **(C).** ${}^{44}C_{20}x^{20} = {}^{44}C_{21}x^{21} \Rightarrow x = \frac{{}^{44}C_{20}}{{}^{44}C_{21}}$

$\frac{(44-21)!\,21!}{(44-20)!\,20!} = \frac{23!}{24!}\times\frac{21!}{20!} = \frac{21}{24} = \frac{7}{8}$

(52) **(D).** It has 11 terms

$\therefore$ Middle term $= T_6\,(r=5) = {}^{10}C_5\left(\frac{10}{x}\right)^5\times\left(\frac{x}{10}\right)^5 = {}^{10}C_5$

(53) **(A).** $T_{11} = {}^{14}C_{10}$

$x^4\cdot\frac{1}{x^5} = \frac{{}^{14}C_4}{x} = \frac{1001}{x}$

(54) **(C).** We know that ${}^nC_r|_{max\,i} = \begin{cases} {}^nC_{n/2}, & n = \text{even} \\ {}^nC_{\frac{n-1}{2}}, & n = \text{odd}\end{cases}$

$\Rightarrow {}^{20}C_r\Big|_{max\,i} = {}^{20}C_{10}$

(55) **(C).** $(1+x)^n = {}^nC_0 + {}^nC_1\,x + {}^nC_2\,x^2 + ... + {}^nC_n\,x^n$
$(1+x)^n = 1 + {}^nC_1\,x + {}^nC_2\,x^2 + ... + {}^nC_n\,x^n$
Put $x = \alpha_2$.
$(1+\alpha_2)^n = 1 + \alpha_2\,({}^nC_1 + {}^nC_2\,\alpha_2 + {}^nC_3\,\alpha_2{}^2 + ... + {}^nC_n\,\alpha_2{}^{n-1})$
$(1+\alpha_2)^n = 1 + \alpha_2\,({}^nC_1\,\alpha_1 + {}^nC_2\,\alpha_2 + ... + {}^nC_n\,\alpha_n)$

$\alpha_1\left[\frac{(1+\alpha_2)^n-1}{\alpha_2}\right] = {}^nC_1\,\alpha_1 + {}^nC_2\,\alpha_2 + ... + {}^nC_n\,\alpha_{n7}$

(56) **(D).** $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ ...(i)
Differentiating equation (i) w.r.t. x, we have
$n\,(1+x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + ... + nC_nx^{n-1}$ ...(ii)
Multiplying equation (ii) throughout by x, we have
$nx\,(1+x)^{n-1} = C_1x + 2C_2x^2 + 3C_3x^3 + ... + nC_nx^n$ ...(iii)
Differentiating equation (iii) w.r.t. x, we have
$n\,(1+x)^{n-1} + n\,(n-1)(1+x)^{n-2}$
$= C_1 + 2^2.\,C_2x + 3^2.\,C_3x^2 + ... + n^2.\,C_n\,x^{n-1}$ ...(iv)
Putting x = 1 in equation (iii), we have
$1^2.\,C_1 + 2^2.\,C_2 + 3^2.\,C_3 + ... + n^2.\,C_n$
$= n.\,2^{n-1} + n(n-1).\,2^{n-2} = (n^2+n)\,2^{n-2}$
$= n\,(n+1)2^{n-2}$.

(57) **(C).** We get the sum of the coefficients of terms by putting x = 1 in the polynomial $(1+x-3x^2)^{2143}$
$\therefore\ (1+1-3)^{2143} = (-1)^{2143} = (-1)^{2142}.\,(-1)$
$= [(-1)^2]^{1021}.\,(-1) = 1\times -1 = -1$

(58) **(A).** Putting x = 1 in the given expansion, we get
$C_0 + C_1 + C_2 + C_3 + ...C_n = 2^n$ ....(1)
Now, differentiating the given expansion with respect to x and then putting x = 1, we get
$C_1 + 2C_2 + 3C_3 + ... + nC_n = n.2^{n-1}$ ....(2)
Given Exp. $= C_0 + 2C_1 + 3C_2 + ... + (n+1)\,C_n$
$= (C_0 + C_1 + C_2 + ... + C_n) + (C_1 + 2C_2 + 3C_3 + ... + nC_n)$
$= 2^n + n.\,2^{n-1}$ [from (1) and (2)]
$= 2^{n-1}\,(n+2)$

(59) **(B).** $\sum_{r=1}^{n}\frac{r\,{}^nC_r}{{}^nC_{r-1}}=\sum_{r=1}^{n}\frac{n-r+1}{r}=(n+1)\sum_{r=1}^{n}-\sum_{r=1}^{n}r$

$=n(n+1)-\frac{n(n+1)}{2}=\frac{n(n+1)}{2}$

(60) **(A).** $\frac{r.{}^nC_r}{{}^nC_{r-1}}=\frac{n.{}^{n-1}C_{r-1}}{{}^nC_{r-1}}$

$=n.\frac{(n-1)!}{(r-1)!(n-r)!}\times\frac{(r-1)!(n-r+1)!}{n!}=n-r+1$

Sum $=n+(n-1)+....+(n-9)$

(61) $(1+x)^n=C_0+C_1x+C_2x^2+....+C_nx^n$

$\frac{(1+x)^{n+1}-1}{n+1}=C_0x+C_1\frac{x^2}{2}+C_2\frac{x^3}{3}+....+C_n\frac{x^{n+1}}{n+1}$

Put $x=-2$:

$\frac{(-1)^{n+1}-1}{n+1}=-2C_0+C_1\frac{2^2}{2}-C_2\frac{2^3}{3}+....+C_n\frac{(-2)^{n+1}}{(n+1)}$

If n is odd, then L.H.S. = 0.

(62) **(D).** Let, $x-1=t$ then $\sum_{r=0}^{2n}a_rt^r=\sum_{r=0}^{2n}b_r(t-1)^r$

$\therefore a_n=$ coefficient of t^n in $\sum_{r=0}^{2n}b_r(t-1)^r$

= coefficient of t^n in

$(b_0+b_1)(t-1)+......+b_n(t-1)^n+b_{n+1}(t-1)^{n+1}$
$+.........+b_{2n}(t-1)^{2n})$

$=b_n\,{}^nC_0+b_{n+1}\,{}^{n+1}C_1(-1)^1+b_{n+2}\,{}^{n+2}C_2(-1)^2$
$+.........b_{2n}\,{}^{2n}C_n(-1)^n$

$=(-1)^n\,{}^nC_0+(-1)^{n+1-n+1}\,{}^{n+1}C_1+.........+(-1)^{2n-n+n}\,{}^{2n}C_n$

$={}^nC_0+{}^{n+1}C_1+{}^{n+2}C_2+.........{}^{2n}C_n={}^{2n+1}C_{n+1}$

$={}^{2n+1}C_n$

(63) **(A).** Here sum is given by

$S=\sum_{r=0}^{n}{}^nC_r\sin rx\cos(n-r)x$

$\Rightarrow S=\sum_{r=0}^{n}{}^nC_{n-r}\sin(n-r)x\cos rx$ (replacing r by n– r)

$\Rightarrow 2S=\sum_{r=0}^{n}{}^nC_n\sin nx=\sin nx\times 2^n$

$\Rightarrow S=2^{n-1}\sin nx$

(64) **(C).** $C_0+C_1+C_2+...+C_n=2^n$

$\frac{n!}{(n-1)!}+\frac{n!}{(n-1)!\,1!}+\frac{n!}{(n-1)!\,2!}+....+\frac{n!}{0!\,n!}=\frac{2^n}{n!}$

(divide it by n !)

(65) **(C).** LHS : ${}^{20}C_r\ge{}^{20}C_{13}\Rightarrow{}^{20}C_r\ge{}^{20}C_7$

$\Rightarrow r=7,8,9,10,11,12,13$

(66) **(B).** Expression

$=\left(\frac{{}^nC_0+{}^nC_1}{{}^nC_0}\right).\left(\frac{{}^nC_1+{}^nC_2}{{}^nC_1}\right)....\left(\frac{{}^nC_{n-1}+{}^nC_n}{{}^nC_{n-1}}\right)$

$=\left(\frac{{}^{n+1}C_1}{{}^nC_0}\right).\left(\frac{{}^{n+1}C_2}{{}^nC_1}\right)....\left(\frac{{}^{n+1}C_n}{{}^nC_{n-1}}\right)$

$=\left(\frac{1.{}^{n+1}C_1}{{}^nC_0}\right).\left(\frac{2.{}^{n+1}C_2}{{}^nC_1}\right)....\left(\frac{n.{}^{n+1}C_n}{{}^nC_{n-1}}\right).\frac{1}{n!}$

$=\left(\frac{(n+1).{}^nC_0}{{}^nC_0}\right).\left(\frac{(n+1).{}^nC_1}{{}^nC_1}\right)....\left(\frac{(n+1).{}^nC_{n-1}}{{}^nC_{n-1}}\right).\frac{1}{n!}$

$=\frac{(n+1)^n}{n!}$

(67) **(C).** Expression $=\frac{1}{n!}\{{}^nC_1+{}^nC_3+{}^nC_5+.....\}=\frac{1}{n!}.2^{n-1}$

for all $n\in N$

(68) **(D).** ${}^{10}C_1+{}^{10}C_2+.........{}^{10}C_9=2^{10}-{}^{10}C_0-{}^{10}C_1$
$=2^{10}-1-1=2^{10}-2$

(69) **(A).** $aC_0+(a+d)C_1+(a+2d)C_2+.......+(a+nd)C_n$
$=(2a+nd)2^{n-1}$

$a=1,d=1,\ (2+n)2^{n-1}=576\Rightarrow n=7$

(70) **(A).** $(1-2x)^{1/2}=1+\frac{1}{2}(-2x)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-2x^2)+......$

$=1-x-\frac{1}{2}x^2....$

(71) **(A).** The tenth term of the expansion is

$T_{10}=\frac{(-3)(-4)(-5)....(-3-8)}{9!}(x)^9$

$=\frac{-3(-4)(-5)...(-11)}{9!}x^9=-55x^9$

(72) **(D).** x^5 occurs in T_6 of the expansion, so

$$T_6 = T_{5+1} = \frac{6.7.8.9.10}{5!}x^5 = 252\,x^5$$

$\therefore$ Coefficient of $x^5 = 252$

(73) **(B).** $(1+2x)^{-1/2}$ can be expanded if $|2x|<1$ i.e. if $|x| < \frac{1}{2}$, i.e. if $-\frac{1}{2}<x<\frac{1}{2}$ i.e. if $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$.

(74) **(C).** $T_r = {}^nC_{r-1}\,p^{n-r+1}.\,q^{r-1} = (T_{r+1}) = {}^nC_r\,p^{n-r}.q^r$ (given) $\Rightarrow$

$$\frac{n!}{(n-r+1)!\,(r-1)!}p^{n-r+1}.q^{r-1} = \frac{n!}{(n-1)!\,r!}p^{n-r}.q^r$$

$$\frac{p}{(n-r+1)} = \frac{q}{r} \Rightarrow pr = nq - rq + q$$

$$(p+q)\,r = q\,(n+1)\ ;\ \frac{(n+1)\,q}{(p+q)\,r} = 1$$

(75) **(A).** We have, $t_{r+1} = {}^xC_r\,(x^2)^{n-r}\left(\frac{2}{x}\right)^r$; Put r = 12,

$$t_{13} = {}^nC_1(x^2)^{x-12}\left(\frac{2}{x}\right)^{12} = {}^xC_{12}.x^{2x-24}.2^{12}.x^{-12}$$

$$t_{13} = {}^nC_{12}\,2^{12}.x^{2n-36}\ ;\ 2n-36=0 \Rightarrow n=18$$

$$18 = 2\times 3^2\ S(18) = \frac{2^{1+1}-1}{2-1}.\frac{3^{2+1}-1}{3-1} = \frac{3}{1}.\frac{26}{2} = 39$$

(76) **(A).** $\because\ \sqrt{99} = (100-1)^{1/2} = 10\left(1-\frac{1}{10^2}\right)^{1/2}$

$$= 10\left[1 - \frac{1}{2}.\frac{1}{10^2} + \frac{1/2.\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{1}{10^2}\right)^2 + ...\right]$$

$= 10\,[1 - 0.005 - 0.0000125]$

$= 10\,[0.9949] = 9.949$

(77) **(C).** $S = 1 + \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + ...$

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ..$$

$$\Rightarrow nx = \frac{1}{5} \text{ and } \frac{n(n-1)x^2}{2!} = \frac{1.3}{5.10}$$

$$\Rightarrow n = -\frac{1}{2} \text{ and } x = \frac{-2}{5}$$

$$\therefore\ S = \left(1-\frac{2}{5}\right)^{-1/2} = \left(\frac{3}{5}\right)^{-1/2} = \sqrt{\frac{5}{3}}.$$

(78) **(A).** $\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + ...\text{upto } m \text{ terms}\right)$

$$= \sum_{r=0}^{n}(-1)^r\ {}^nC_r.\frac{1}{2^r} + \sum_{r=0}^{n}(-1)^r.{}^nC_r\frac{3^r}{2^{2r}} + \sum_{r=0}^{n}(-1)^r\ {}^nC_r\frac{7^r}{2^{3r}} + ...$$

$$= \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + ... \text{up to m terms.}$$

$$= \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{8^n} + ... \text{upto m terms}$$

$$= \frac{\frac{1}{2^n}\left(1-\frac{1}{2^{nm}}\right)}{\left(1-\frac{1}{2^n}\right)} = \frac{2^{mn}-1}{2^{mn}(2^n-1)}$$

(79) **(C).** $49^n + 16n - 1 = (1+48)^n + 16n - 1$

$1 + {}^nC_1(48) + {}^nC_2(48)^2 + ... + {}^nC_n(48)^n + 16n - 1$

$= (48n + 16n) + {}^nC_2(48)^2 + {}^nC_3(48)^3 + ... + {}^nC_n(48)^n$

$= 64n + 8^2[{}^nC_2.6^2 + {}^nC_3.6^3.8 + {}^nC_4.6^4.8^2 + .. + {}^nC_n.6^n.8^{n-2}]$

Hence, $49^n + 16n - 1$ is divisible by 64.

(80) **(B).** Let the number is x.

Then $x - x^3 = x(1-x^2) = (1-x)(x)(1+x)$

According to Langrange's theorem it is divisible by 3! i.e., 6.

(81) **(C).** $101^{100} - 1 = (100+1)^{100} - 1$

$= 100^{100} + {}^{100}C_1\ 100^{99} + {}^{100}C_2\ 100^{98} + ... + 1 - 1$

$= 100^{100} + {}^{100}C_1 100^{99} + {}^{100}C_2\ 100^{98} + ... + {}^{100}C_{99}\ 100^1$

$= 100(100^{99} + {}^{100}C_1\ 100^{98} + ... + {}^{100}C_{99})$

$= 100\ (100^{99} + {}^{100}C_1\ 100^{98} + ... + {}^{100}C_{98}\ 100 + {}^{100}C_{99})$

$= 100\ (100^{99} + {}^{100}C_1\ 100^{98} + ... + {}^{100}C_{98}\ 100 + 100)$

$= 100^2\ (100^{98} + {}^{100}C_1\ 100^{97} + ... + {}^{100}C_2 + 1)$

$\therefore$ the greatest integer which divides given number $= 100^2 = 10{,}000$

(82) **(B).** Here $T_{r+1} = {}^{10}C_r\,(\sqrt{2})^{10-r}\,(3^{1/5})^r$,

where r = 0, 1, 2, ...,10.

We observe that in general term T_{r+1} powers of 2 and 3 are $\frac{1}{2}(10-r)$ and $\frac{1}{5}$ r respectively and $0 \le r \le 10$.

So both these powers will be integers together only when r = 0 or 10

$\therefore$ Sum of required terms $= T_1 + T_{11}$

$= {}^{10}C_0\,(\sqrt{2})^{10} + {}^{10}C_{10}\,(3^{1/5})^{10} = 32 + 9 = 41$

(83) **(B).** $R = (3+\sqrt{5})^{2n}$, $G = (3-\sqrt{5})^{2n}$

Let $[R]+1 = I$ ($\because$ [.] greatest integer function)

$\Rightarrow R+G = I$ ($\because 0 < G < 1$)

$(3+\sqrt{5})^{2n} + (3-\sqrt{5})^{2n} = I$

seeing the option put n = 1

I = 28 is divisible by 4 i.e. 2^{n+1}

(84) **(C).** $27^{40} = 3^{120}$

$3^{119} = (4-1)^{119} = {}^{119}C_0 4^{119} - {}^{119}C_1 4^{118}$

$+ {}^{119}C_2 4^{117} - {}^{119}C_3 4^{116} + ... + (-1)$

$\therefore 3^{119} = 4k - 1 \quad \therefore 3^{120} = 12k - 3 = 12(k-1) + 9$

$\therefore$ The required remainder is 9

(85) **(C).** $(23)^{14} = (529)^7 = (530-1)^7$

$= {}^7C_0(530)^7 - {}^7C_1(530)^6 + ... - {}^7C_5(530)^2 + {}^7C_6 530 - 1$

$= {}^7C_0(530)^7 - {}^7C_1(530)^6 + ... + 3710 - 1 = 100m + 3709$

$\therefore$ last two digits are 09.

(86) **(B).** $(1-x-2x^2)^6 = (1+x)^6(1-2x)^6$

$= 1 + a_1x + a_2x^2 + ... + a_{12}x^{12}$

Putting x = 1/2 we have

$0 = 1 + \frac{a_1}{2} + \frac{a_2}{2^2} + ... + \frac{a_{12}}{2^{12}}$ .... (1)

Putting x = – 1/2 we have

$1 = 1 - \frac{a_1}{2} + \frac{a_2}{2^2} - \frac{a_3}{2^3} + ... + \frac{a_{12}}{2^{12}}$ .... (2)

Adding eq. (1) and eq. (2)

$\frac{a_2}{2^2} + \frac{a_4}{2^4} + ... + \frac{a_{12}}{2^{12}} = -\frac{1}{2}$

(87) **(B).** $\frac{7^{103}}{25} = \frac{7[49]^{51}}{25} = \frac{7[50-1]^{51}}{25}$

$= \frac{7[25k-1]}{25} = \frac{25(k^1) - 7 + 25 - 25}{25}$

$= \frac{25(k^1-1)+18}{25} \Rightarrow$ Remainder = 18

(88) **(D).** Now, x = (364, 420) = 28

$(49^2-4)(49^3-49) = (49^2-2^2)(49^2-1)\,49$

= 51 . 47 . 50 . 48 . 49,

which is the product of five consecutive integers, and hence divisible by 5!.

(89) **(D).** $7^2 \equiv -1 \pmod{10}$

$(7^2)^{85} \equiv -1 \pmod{10}$

$7^{170} . 7 \equiv -7 \pmod{10} \equiv 3 \pmod{10}$

unit digit = 3, but unit digit in (177)! = 0

(90) **(C).** $10^{10} . (10^{10}+1)(10^{10}+2)$ is a product of 3 consecutive integers and hence is divisible by 3! = 6.

$\therefore$ Remainder = 0

(91) **(A).** 1! + 2! + 3! = 1 + 2 + 6 = 9 ; 4! = 24 ; 5! = 120

$\therefore$ n! is divisible by 12 for n > 3.

$\therefore$ Required remainder is 9

(92) **(C).** $T_{r+1} = {}^{15}C_r (x^{1/3})^{15-r}(-x^{-1/2})^r$

$\Rightarrow \frac{15-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 6$

Hence T_7 is independent of x

and $T_7 = {}^{15}C_6 = 5005 = 5m \Rightarrow m = 1001$

(93) **(C).** $2^n\left(1+\frac{x}{6}\right)^n \Rightarrow T_{r+1} = 2^n . {}^nC_r \left(\frac{x}{6}\right)^r$

$\Rightarrow 2^n . {}^nC_7 . \frac{1}{6^7} = 2^n . {}^nC_8 . \frac{1}{6^8}$

$\Rightarrow 6 . {}^nC_7 = {}^nC_8 \Rightarrow n - 7 = 48 \Rightarrow n = 55$

(94) **(C).** $E = (19-4)^{23} + (19+4)^{23}$

$= 2\,[19^{23} + {}^{23}C_2 \cdot 19^{21} \cdot 4^2 + ... + {}^{23}C_{22} \cdot 19 \cdot 4^{22}]$

$= 2 \cdot 19\,[19^{22} + {}^{23}C_2 \cdot 19^{20} \cdot 4^2 + ...+ {}^{23}C_{22} \cdot 4^{22}]$

$\Rightarrow$ E is divisible by 19 $\Rightarrow$ Remainder = 0

(95) **(B).** ${}^nC_r = \frac{n!}{r!(n-r)!}$; $\frac{{}^nC_r}{n!} = \frac{1}{r!(n-r)!}$;

put r = 1, 3, 5, ... and add

(96) **(A).** ${}^nC_1 + {}^nC_2 = 36 \Rightarrow n = 8$

$T_3 = 7\,T_2 \Rightarrow (2^x)^3 = 1/2$

$3x = -1 \Rightarrow x = -1/3$

(97) **(A).** Put x = 1 and x = – 1 and then on addition we get

$a_0 + a_2 + a_4 + ... a_{2n} = \frac{3^n+1}{2}$

(98) **(A).** $a^n + b^n = (a+b)\,(Q\,(a, b))$ if n is odd i.e. $a^n + b^n$ is divisible by a + b if n is odd

alternatively : interpret from $(16-5)^{27} + (16+5)^{27}$

(99) **(D).** $a_0 + a_1 + a_2 + ... = 2^{2n}$ and $a_0 + a_2 + a_4 + ... = 2^{2n-1}$

$a_n = {}^{2n}C_n$ = the greatest coefficient, being the middle coefficient

$a_{n-3} = {}^{2n}C_{n-3} = {}^{2n}C_{2n-(n-3)} = {}^{2n}C_{n+3} = a_{n+3}$

(100) **(C).** We have Coefficient of x^4 in $(1+x+x^2+x^3)^{11}$

= coefficient of x^4 in $(1+x^2)^{11}(1+x)^{11}$

= coefficient of x^4 in $(1+x)^{11}$ + coefficient of x^2 in $11.(1+x)^{11}$ + constant term is

${}^{11}C_2.(1+x)^{11} = {}^{11}C_4 + 11.\,{}^{11}C_2 + {}^{11}C_2 = 990.$

EXERCISE-2

(1) **(A).** Coeff.. of x^{49} in this series is

$$-\left[1+\frac{1}{2}+\frac{1}{2^2}+......+\frac{1}{2^{49}}\right] = -\left[\frac{1-\frac{1}{2^{50}}}{1-\frac{1}{2}}\right]$$

$$= -2.\left[1-\frac{1}{2^{50}}\right]$$

(2) **(A).** $(1+7)^{83}+(7-1)^{83}=(1+7)^{83}-(1-7)^{83}$
$=2[^{83}C_1\cdot 7+{}^{83}C_3\cdot 7^3+..........+{}^{83}C_{83}\cdot 7^{83}]$
$=(2\cdot 7\cdot 83)+49I$ where I an integer
$\therefore 14\times 83=1162$

$$\frac{1162}{49}=23\frac{35}{49}$$

$\therefore$ Remainder is 35

(3) **(C).** $(16-5)^{27}+(16+5)^{27}=16^{27}-{}^{27}C_1\,16^{26}.5$
$+{}^{27}C_2\,16^{25}.5^2+..............+{}^{27}C_{26}\,16\times 5^{26}-{}^{27}C_{27}.5^{27}$
$+16^{27}+{}^{27}C_1\,16^{26}.5+{}^{27}C_2\,16^{25}.5^2+..............+{}^{27}C_{26}$
$16\times 5^{26}+{}^{27}C_{27}.5^{27}$
$=2[16^{27}+{}^{27}C_2\,16^{25}.5^2+............+{}^{27}C_{26}.16.5^{26}]$
$\therefore$ remainder $=0$

(4) **(A).** $T_3={}^5C_2\,x^3.x^{2t}=10^6$
$x^{3+2t}=10^5$
$(3+2t)\log_{10}x=5$
$\therefore (3+2t)\,t=5$ i.e., $2t^2+3t-5=0$ i.e., $t=1,-5/2$
$x=10^t=10,\ 10^{-5/2}$

(5) **(C).** $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$=2[x^5+{}^5C_2\,x^3(x^3-1)+{}^5C_4\,x(x^3-1)^2]$
$=2[x^5+10x^3(x^3-1)+5x(x^6-2x^3+1]$
$=10x^7+20x^6+2x^5-20x^4-20x^3+10x$
$\therefore$ polynomial has order of 7.

(6) **(D).** Consider $(5+2)^{100}-(5-2)^{100}$
$=2[^{100}C_1\,5^{99}\cdot 2+{}^{100}C_3\,5^{97}\cdot 2^3+.......+{}^{100}C_{99}5\cdot 2^{99}]$
$=2[1000\cdot 5^{98}+{}^{100}C_3\,5^{94}+.......+1000\cdot 2^{98}]$
$\Rightarrow$ minimum 000 as last three digits.

(7) **(A).** Sum

$$=\frac{1}{2}\{(a_0+a_1+a_2+....+a_{16})-(a_0-a_1+a_2-....+a_{16})\}$$

$$=\frac{1}{2}\left\{(1+1-2)^8-(1-1-2)^8\right\}=\frac{1}{2}\left(-2^8\right)=-2^7$$

(8) **(B).** We have, $(1+x+x^2+x^3+x^4)^n\,(x-1)^{n+3}$

$$=\left(\frac{1-x^5}{1-x}\right)^n(1-x)^{n+3}=\left(1-x^5\right)^n(1-x)^3$$

$$=(-x^3+3x^2-3x+1)\sum_{r=0}^{n}{}^nC_r(-1)^r\,x^{5r}$$

$$=-\sum_{r=0}^{n}{}^nC_r(-1)^r\,x^{5r+3}+3\sum_{r=0}^{n}{}^nC_r(-1)^r\,x^{5r+2}$$

$$-3\sum_{r=0}^{n}{}^nC_r(-1)^r\,x^{5r+1}+3\sum_{r=0}^{n}{}^nC_r(-1)^r\,x^{5r}$$

For term containing x^{83}, we have $5r+3=83\Rightarrow r=16$ whereas $5r+2=83$, $5r+1=83$ and $5r=83$ give no integral value of r. Hence, their is only one term containing x^{83}

whose coefficient $=-{}^nC_{16}$.

(9) **(D).** Coefficient of x^4 is
$(1+5x+9x^2+..........)(1+x^2)^{11}$
$=(1+5x+9x^2+..........)(1+x^2+{}^{11}C_2\,(x^2)^2+........)^{11}$
$=(1+5x+9x^2+13x^3+17x^4+.....)(1+11x^2+{}^{11}C_2\,x^4$ ....)

Coefficient of $x^4={}^{11}C_2+9\times 11+17=55+99+17=171$

(10) **(A).** $(17)^{256}=(289)^{128}=(300-11)^{128}$

$$={}^{128}C_0\left(-11\right)^{128}+100\,m\text{ , for some integer m}$$

$$=11^{128}+100\,m=\left(10+1\right)^{128}+100\,m$$

$={}^{128}C_0\,1^{128}+{}^{128}C_1\,10+100\,m_1+100\,m$ for some integer $m_1=1+1280+100\,k$, $m+m_1=k=1281+100k$
Hence the required number is 81.

(11) **(D).** $3^{400}=81^{100}=(1+80)^{100}={}^{100}C_0+{}^{100}C_1\,80+.......+{}^{100}C_{100}\,80^{100}$

$\Rightarrow$ Last two digits are 01

(12) **(B)** $T_{r+1}={}^nC_r\,a^{n-r}.b^r$ where $a=2^{1/3}$ and $b=3^{-1/3}$
T_7 from beginning $={}^nC_6\,a^{n-6}\,b^6$ and T_7 from end $={}^nC_6\ b^{n-6}\ a^6$

$$\Rightarrow\ \frac{a^{n-12}}{b^{n-12}}=\frac{1}{6}\ \Rightarrow\ 2^{\frac{n-12}{3}}.3^{\frac{n-12}{3}}=6^{-1}$$

$\Rightarrow n-12=-3\Rightarrow n=9$

(13) **(A).** Putting $x=1$ and -1 and adding

$$a_0+a_2+......+a_{50}=\frac{3^{25}+1}{2}=\frac{(1+2)^{25}+1}{2}$$

$$\left(\frac{odd+1}{2}=\frac{even}{2}\right)$$

$$=\frac{25C_0+{}^{25}C_1.2+{}^{25}C_2.2^2+{}^{25}C_{25}.2^{25}+1}{2}$$

$$=\frac{2\left[1+{}^{25}C_1+{}^{25}C_2.2+......+{}^{25}C_{25}.2^{24}\right]}{2}$$

$=2[13+{}^{25}C_2+......+{}^{25}C_{25}.2^{23}]\Rightarrow$ even

(14) **(B).** $E = (2n+1)(2n+3)(2n+5)......(4n-1)$
Multiply numerator and denominator by $(2n+2)(2n+4)......(4n)$ & also by $(2n)!$ and $n!$.

$$E = \frac{(2n)!\,(2n+1)(2n+2)(2n+3)...(4n-1).4n}{(2n)!\,(2n+2)(2n+4)...(2n+2n)}$$

$$= \frac{(4n)! \times (n)!}{(2n)!\,2^n\,[(n+1)(n+2)...(2n)]\,n!}$$

$$= \frac{(n!).(4n)!}{2^n.((2n)!)^2}$$

(15) **(D).** Coefficient of $\lambda^n\mu^n \Rightarrow (1+\lambda)^n(1+\mu)^n(\lambda+\mu)^n$

$$\lambda^n\mu^n = (\lambda^r.\mu^{n-r}.\lambda^{n-r}.\mu^r)$$

Coefficient of λ^r in $(1+\lambda)^n$ is ${}^nC_r$

Coefficient of μ^{n-r} in $(1+\mu)^n$ is ${}^nC_{n-r}$

Coefficient of $\lambda^{n-r}.\mu^r$ in $(\lambda+\mu)^n$ is ${}^nC_{n-r}$

So net coefficient is $\sum_{r=0}^{n}\left({}^nC_r\right)^3$

(16) **(A).** The $(r+1)$th term in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is given by

$$T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r = {}^{10}C_r\,\frac{x^{5-(r/2)}}{3^{5-(r/2)}}.\frac{3^r}{2^r x^{2r}}$$

$$= {}^{10}C_r\,\frac{3^{(3r/2)-5}}{2^r}x^{5-(5r/2)}.$$

For T_{r+1} to be independent of x, we must have
$5-(5r/2)=0$ or $r=2$.
Thus, the 3rd term is independent of x and is equal to

$${}^{10}C_2\,\frac{3^{3-5}}{2^2} = \frac{10\times 9}{2}\times\frac{3^{-2}}{4} = \frac{5}{4}.$$

(17) **(A).** $\frac{{}^nC_k}{{}^nC_{k+1}} = \frac{1}{2} \Rightarrow \frac{n!}{k!(n-k)!}\frac{(k+1)!(n-k-1)!}{n!} = \frac{1}{2}$

or $\frac{k+1}{n-k} = \frac{1}{2}$ or $2k+2 = n-k$ or $n-3k=2$ .... (1)

Similarly $\frac{{}^nC_{k+1}}{{}^nC_{k+2}} = \frac{2}{3}$

$$\frac{n!}{(k+1)!(n-k-1)!}.\frac{(k+2)!(n-k-2)!}{n!} = \frac{2}{3}$$

$$\frac{k+2}{n-k-1} = \frac{2}{3}$$

$3k+6 = 2n-2k-2$
$2n-5k = 8$ .... (2)
From (1) and (2)
$n = 14$ and $k = 4$
$\therefore n+k = 18$

(18) **(A).** $\frac{20!}{p!q!r!}(2x)^p(-y)^q(z)^r = \frac{20!}{p!q!r!}2^p(-1)^q x^p y^q z^r$

$p+q+r = 20,\ q=0$
$p+r = 20$ p is even and r is odd.
even + odd = even (never possible)
Coefficient of such power never occur
$\therefore$ coefficient is zero

(19) **(A).** $T_2 = {}^nC_1\left(a^{1/13}\right)^{n-1}\cdot a\sqrt{a} = 14a^{5/2}$

or $n\cdot a^{\frac{n-1}{13}} = 14a$

$n\cdot a^{\frac{n-14}{13}} = 14$ hence $\frac{n-14}{13} = 0 \Rightarrow n = 14$

Now, $\frac{{}^{14}C_3}{{}^{14}C_2} = \frac{14!}{3!\cdot 11!}\ \frac{2!\cdot 2!}{14!} = \frac{12}{3} = 4$

(20) **(A).** $(1+x+x^2)n = a_0 + a_1x + a_2x^2 + ... + a_nx^{2n}$
Put $x = i$
$i^n = a_0 + a_1 i - a_2 - a_3 i + a_4 ....a_{2n}(+i)2n$
$n \in$ odd real part $= 0$
$a_0 - a_2 + a_4 + ... = 0$

(21) **(A).** The last term $= {}^nC_n.\left(-\frac{1}{\sqrt{2}}\right)^n = \left(\frac{1}{3.\sqrt[3]{9}}\right)^{\log_3 8}$

(from the question)

$$(-1)^n.\left(\frac{1}{2}\right)^{n/2} = \left(\frac{1}{3^{5/3}}\right)^{\log_3 8}$$

$$= 3^{-5/3.3\log_3 2} = 3^{\log_3 2^{-5}} = 2^{-5} = \left(\frac{1}{2}\right)^5$$

$n = 10$; So, $t_5 = {}^{10}C_4.(2^{1/3})^6.\left(-\frac{1}{\sqrt{2}}\right)^4$

$$= {}^{10}C_4 = {}^{10}C_{10-4} = {}^{10}C_6$$

(22) **(D).** $\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r} = r\,.\,\frac{C_r}{C_{r-1}} = n\,.\,r+1$

Here in this case $n = 15$.

So, $\frac{C_1}{C_0} + \frac{{}^2C_2}{C_1} + \frac{{}^3C_3}{C_2} + ... + \frac{{}^{15}C_{15}}{C_{14}}$

$= \sum_{r=1}^{15} (n-r+1) = n + (n-1) + (n-2) + ... + 1 = \frac{n\,(n+1)}{2}$

(23) **(B).** Let $(2x^2 - 3x + 1)^{11} = a_0 + a_1 x + a_2 x^2 + ... + a_{22} x^{22}$

$S_E + S_0 = P(1) = 0$ .... (1)

when $P(x) = (2x^2 - 3x + 1)^{11}$

$S_E - S_0 = P(-1) = 6^{11}$ .... (2)

$\Rightarrow 2S_E = 6^{11} \Rightarrow S_E = 3\,.\,6^{10}$

(24) **(B).** $E = (\alpha + p)^{m-1}$

$\left[1 + \frac{\alpha+q}{\alpha+p} + \left(\frac{\alpha+q}{\alpha+p}\right)^2 + ... + \left(\frac{\alpha+q}{\alpha+p}\right)^{m-1}\right]$

$\Rightarrow$ Coefficient of $\alpha^t = \frac{(\alpha+p)^m - (\alpha+q)^m}{p-q}$

$= \frac{(p+\alpha)^m - (q+\alpha)^m}{p-q} = \frac{{}^mC_t\,(p^{m-t} - q^{m-t})}{p-q}$

(25) **(B).** Clearly $a_r = {}^nC_r$

$\Rightarrow \frac{a_r}{a_{r-1}} = \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{(n-r+1)}{r} \Rightarrow 1 + \frac{a_r}{a_{r-1}} = \frac{n+1}{r}$

$\Rightarrow \prod_{r=1}^{n}\left(1 + \frac{a_r}{a_{r-1}}\right) = \prod_{r=1}^{n}\frac{(n+1)}{r} = \frac{(n+1)^n}{n!}$

(26) **(C).** $3^{2003} = 3^{2001}\,.\,3^2 = 9(27)^{667} = 9(28-1)^{667}$

$= 9({}^{667}C_0\,28^{667} - {}^{667}C_1\,(28)^{666} + \ldots + {}^{667}C_{667}(-1)^{667})$

that means if we divide 3^{2003} by 28, remainder is 19.

Thus, $\left\{\frac{3^{2003}}{28}\right\} = \frac{19}{28}$

(27) **(B).** Highest exponent in the product of first two is

$3 = 1 + 2$

Highest exponent in the product of first three is

$6 = 1 + 2 + 3$

Similarly, Highest exponent in the product of first hundred

$= 1 + 2 + ... + 100 = 5050$

(28) **(D).** $T_{r+1} = {}^{18}C_r(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r = a\,{}^{18}C_r$

is independent of x provided r = 12 and then a = 1.

(29) **(D).** $\left(1 + \sqrt{2}\,x^2\right)^9 = 1 + 9\sqrt{2}\,x^2 + 36\,.\,2x^4$

$+ {}^9C_3\,2\sqrt{2}\,x^6 + ...$

$\left(1 + \sqrt{2}\,x^2\right)^{-9} = 1 - 9\sqrt{2}\,x^2 - 70\,x^4 - 2x^4$

$+ {}^9C_3\,2\sqrt{2}\,x^6 + ...$

The expression is divisible by x, x^2, x^3, x^4 only.

(30) **(B).** $a(C_0 + C_1 + C_2 + ... + C_n) + b(C_1 + 2C_2 + ... + {}^nC_n)$

$= a.2^n + b.n2^{n-1} = 2^n\left(\frac{2a+nb}{2}\right) = (2a+nb)\,2^{n-1}$

(31) **(D).** $T_{p+1} = {}^nC_p\,(x^3)^{n-p}\,(x^{-4})^p = {}^nC_p\,x^{3n-7p}$ and x^r

occurs provided $p = \frac{3n-r}{7}$ is an integer.

(32) **(B).** Given : $\left(\sqrt[3]{\frac{a}{b}} + \sqrt[3]{\frac{b}{\sqrt{a}}}\right)^{21}$

$t_{r+1} = {}^{21}C_r\left(\frac{a}{b}\right)^{\frac{21-r}{3}}\frac{b^{r/3}}{a^{r/6}} = {}^{21}C_r\,a^{\frac{42-3r}{6}}\,b^{\frac{2r-21}{3}}$

$\therefore 42 - 3r = 4r - 42$ i.e. $r = 12$

$\therefore$ 13^{th} term contains same power of a and b.

(33) **(C).** $\left(1 + \frac{1}{x^2}\right)^n\,(1 + x^2)^n = \frac{\left(1+z^2\right)^{2n}}{x^{2n}}$,

numerator has $(2n+1)$ terms.

The middle terms is $\frac{1}{x^{2n}}\,x\,{}^{(2n)}C_n\,(x^2)^n = {}^{(2n)}C_n$.

(34) **(A).** If n is even, the greatest coefficient is ${}^nC_{n/2}$

$\therefore$ the greatest term $= {}^nC_{n/2}\,x^{n/2}$

$\therefore {}^nC_{n/2} > {}^nC_{n/2-1}\,x^{n/2-1}$

and ${}^nC_{n/2}\,x^{n/2} > {}^nC_{n/2+1}\,x^{n/2+1}$

$\Rightarrow \frac{n - \frac{n}{2} + 1}{\frac{n}{2}}\,x > 1$ and $\frac{n - \left(\frac{n}{2}+1\right) + 1}{\frac{n}{2}+1}\,x < 1$

$\Rightarrow x > \frac{\frac{n}{2}}{\frac{n}{2}+1}$ & $x < \frac{\frac{n}{2}+1}{\frac{n}{2}} \Rightarrow x > \frac{n}{n+2}$ and $x < \frac{n+2}{n}$

Hence $\frac{n}{n+2} < x < \frac{n+2}{n}$

(35) **(B).** $E = (x-\alpha_1)(x-\alpha_1)(x-\alpha_3)+.....(x-\alpha_n)$

where $\alpha_1 = 1, \alpha_2 = 2$ etc.

$= x^n - (\Sigma\alpha_1)x^{n-1} + (\Sigma\,\alpha_1\alpha_2)x^{n-2} + ...$

Hence coefficient of x^{n-2} = sum of the product of the first 'n' natural numbers taken two at a time

$= 1\times 2 + 2\times 3 + 3\times 4$ ....n terms

$= \dfrac{(1+2+3+.......+n)^2 - (1^2+2^2+.........+n^2)}{2}$

(36) **(A).** $2^{60} = 8^{20} = (1+7)^{20} = 1 + 7n_1$

$\therefore$ remainder is 1.

(37) **(B).** $(1+n)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ ... (1)

Put $x = -1$

$C_0 - C_1 + C_2 - C_3 + ... (-1)^n C_n = 0$

also $(x+1)^n = C_0x^n + C_1x^{n-1} + ... + C_nx^6$ ... (2)

multiplying (1) and (2) and comp. coeff. of x^n

$C_0{}^2 + C_1{}^2 + ... + C_n{}^2 = {}^{2n}C_n = \dfrac{n2!}{(n!)^2}$

(38) **(A).** Statement – II is true (can be checked easily) and that's why

${}^{2n}C_0 < {}^{2n}C_1 < {}^{2n}C_2 < ... < {}^{2n}C_{n-1} < {}^{2n}C_n \; ... \; {}^{2n}C_{2n}$.

Hence statement – I is true

(39) **(A).** Obviously statement – II is true and the correct reasoning of statement – I.

(40) **(C).** Statement-1 is true but statement-2 s false.

$2^{2000} = (2^4)^{500} = (16)^{500} = (15+1)^{500} = 15\,.\,m + 1, m \in I^+$.

$\therefore 2^{2003} = 2^3.(15m+1) = 15.8m + 8$

$\therefore$ Remainder = 8

(41) **(C).** Given expression

$= x.\,x^2.\,x^3 ... x^{20}\left(1-\dfrac{1}{x}\right)\left(1-\dfrac{2}{x^2}\right)...\left(1-\dfrac{20}{x^{20}}\right) = x^{210}.\,P$

where, $P = \left(1-\dfrac{1}{x}\right)\left(1-\dfrac{2}{x^2}\right)\left(1-\dfrac{3}{x^3}\right)...\left(1-\dfrac{20}{x^{20}}\right)$

Now, coefficient of x^{203} in original expression = coefficient of x^{-7} in P. But,

$P = 1 - \left(\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{x^3}+...\right)+\left(\dfrac{1}{x}.\dfrac{6}{x^6}+\dfrac{2}{x^2}.\dfrac{5}{x^3}+\dfrac{3}{x^3}.\dfrac{4}{x^4}\right)$

$-\left(\dfrac{1}{x}.\dfrac{2}{x^2}.\dfrac{4}{x^4}+...\right);$

Coefficient of $x^{-7} = -7+6+10+12-8 = 13$.

The expression $(2+x)^2(3+x)^3(4+x)^4$

$= x^9 + (2+2+3+3+3+3+4+4+4+4)x^8 + ..$

$\Rightarrow$ coefficient of $x^8 = 29$.

(42) **(A).**

(a) $T_{r+1} = \dfrac{\frac{7}{2}\left(\frac{7}{2}-1\right)\left(\frac{7}{2}-2\right)....\left(\frac{7}{2}-r+1\right)x^r}{r!}$

First negative term if $\dfrac{7}{2} - r + 1 + < 0$ i.e. $r > \dfrac{9}{2}$.

Hence r = 5

(b) $T_{r+1} = {}^5C_r(y^2)^{5-r}\left(\dfrac{1}{y}\right)^r = {}^5C_ry^{10-3r}$

$10 - 3r = 1 \Rightarrow r = 3$

So, coefficient of x y $= {}^5C_3 = 10$

(c) $T_2 = 14a^{5/2} \Rightarrow {}^nC_1(a^{1/13})^{n-1}(a^{3/2})^1 = 14a^{5/2}$

$\Rightarrow na^{\frac{n-1}{13}+\frac{3}{2}} = 14a^{5/2} \Rightarrow n = 14$

(d) $(1+2x+3x^2+4x^3+.......)^{1/2}$

$= [(1-x)^{-2}]^{1/2} = (1-x)^{-1} = 1 + x + x^2 + ... + x^n + ...$

Hence coefficient of $x^4 = 1$

$\therefore$ c = 1, so c + 1 = 2

(43) **(B).**

(a) $({}^mC_1\,{}^nC_m - {}^mC_2\,{}^{2n}C_m + {}^mC_3\,{}^{3n}C_m - ...$
$(-1)^{m-1}\,{}^mC_m\,{}^{mn}C_m)$

= Coefficient of x^m in the expansion of

$({}^mC_1(1+x)^n - {}^mC_2(1+x)^{2n} + {}^mC_3(1+x)^{3n}$ .....
$+ (-1)^{m-1}\,{}^mC_m(1+x)^{mn})$

= Coefficient of x^m in the expansion of

$({}^mC_0 - [{}^mC_0 - {}^mC_1(1+x)^n + {}^mC_2(1+x)^{2n} - {}^mC_3(1+x)^{3m}$
$+ ..... + (-1)^m\,{}^mC_m(1+x)^{mn}])$

= Coefficient of x^m in the expansion of $(1-(1-(1+x)^n)^m)$

= Coefficient of x^m in the expansion of $(1-(1+x)^n)^m$.

(b) ${}^nC_m + {}^{n-1}C_m + {}^{n-2}C_m + ... + {}^mC_m$ is the coefficient of x^m in the expnasion of

$(1+x)^n + (1+x)^{n-1} + (1+x)^{n-2} + ... + (1+x)^m$

$= (1+x)^m\,[1 + (1+x) + (1+x)^2 + ... + (1+x)^{n-m}]$

$= (1+x)^m\left(\dfrac{1-(1+x)^{n-m+1}}{1-(1+x)}\right) = \dfrac{(1+x)^{n-1}-(1+x)^m}{x}$

The the given expression is equal to the coefficient of x^m in the expansion of $\dfrac{(1+x)^{n+1}}{x}$

(c) $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3$
$+ .... + {}^nC_nx^n - A$

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + ... + {}^nC_nx^n$

multiplying eq. A and B and equating coefficients of x^n

on both the sides.

Coefficient of x^n in the expansion of $(1+x)^n\,(x+x)^n = {}^nC_0\,{}^nC_1 + {}^nC_1\,{}^nC_{n-1} + {}^nC_2\,{}^nC_{n-2} + ... + {}^nC_n\,{}^nC_0$.

Coefficient of of xn in the expansion of $(1+x)^n = {}^{2n}C_n$

(d) $2^m\ {}^nC_m$ = Coefficient of x^m in the expansion of $(1+x)^{2n}$

$2^{m-1}\ {}^{n-1}C_{m-1}$ = Coefficient of x^{m-1} in the expansion of $(1+x)^{2n-1}$

= Coefficient of x^m in the expansion of $x\,(1+2x)^{n-1}$

Given expression = Coefficient of x^m in the expansion of ${}^nC_0\,(1+2x)^n - {}^nC_1\,x\,(1+2x)^{n-1} + {}^nC_2\,x^2\,(1+2x)^{n-2} - ....$

= Coefficient of xm in the expansion of $(1+2x-x)^n = {}^nC_m$

(44) (A).

(a) We know that,

$$(a-1)^n = {}^nC_0.a^n - {}^nC_1.a^{n-1} + {}^nC_2a^{n-2} - .... + (-1)^{n-1}.{}^nC_{n-1}a + (-1)\,{}^nC_n$$

$$\therefore\ \frac{(a-1)^n}{a} = {}^nC_0a^{n-1} - {}^nC_1a^{n-2} + {}^nC_2a^{n-3} - .... + (-1)^{n-1}.{}^nC_{n-1} + \frac{(-1)^n}{a}\ {}^nC_n$$

Hence, $f(n) = \dfrac{(a-1)^n - (-1)^n}{a}$

Now, $f(2007) + f(2008) = \dfrac{(a-1)^{2007}+1}{a} + \dfrac{(a-1)^{2008}-1}{a}$

$$= \frac{(a-1)^{2007}(1+a-1)}{a} = (a-1)^{2007}$$

$$= \left(3^{\frac{1}{223}}\right)^{2007} = 3^9 = 3^2\cdot 3^7 = 9\,(2187)$$

Hence, $k = 9$

Alternatively, $f(n) = \dfrac{1}{a}\ [{}^nC_0.a^n - {}^nC_1.a^{n-1} + {}^nC_2a^{n-2} - .... + (-1)^{n-1}.{}^nC_{n-1}a + (-1)\,{}^nC_n] - \dfrac{(-1)^n}{a}$

$$\therefore\quad f(n) = \frac{1}{a}[a-1]^n - \frac{(-1)^n}{a} \qquad ....(1)$$

Now given, $a = 3^{\frac{1}{223}} + 1$ or $a - 1 = 3^{\frac{1}{223}}$

$$\text{Hence,}\quad f(n) = \frac{1}{a}\left(3^{\frac{1}{223}}\right)^n - \frac{(-1)^n}{a} \qquad ....(2)$$

$$\therefore\quad f(2007) = \frac{1}{a}\left(3^{\frac{1}{223}}\right)^{2007} - \frac{(-1)^{2007}}{a}$$

$$\text{or}\quad f(2007) = \frac{1}{a}(3^9) + \frac{1}{a}\ \left(\frac{2007}{223} = 9\right)$$

$$f(2007) = \frac{1}{a}(3^9+1) \qquad ....(3)$$

$$f(2008) = \frac{1}{a}\left(3^{\frac{2008}{223}}\right) - \frac{1}{a} = \frac{1}{a}\left(3^9.3^{\frac{1}{223}}\right) - \frac{1}{a}$$

$$\text{or } f(2008) = \frac{1}{a}\left(3^9.3^{\frac{1}{223}} - 1\right) \qquad ....(4)$$

$$\text{Hence,}\ f(2007) + f(2008) = \frac{1}{a}(3^9+1) + \frac{1}{a}\left(3^9.3^{\frac{1}{223}} - 1\right)$$

$$= \frac{1}{a}\left(3^9 + 3^9.3^{\frac{1}{223}}\right)$$

$$= \frac{3^9}{a}\left(1 + 3^{\frac{1}{223}}\right) = \frac{3^9}{a}.a = 3^9 = 9.3^7 = 9\,(2187) \Rightarrow k = 9$$

(b) $T_{r+1} = {}^{10}C_r\,\dfrac{x^r}{2^r}$

For $r = 2$; ${}^{10}C_2\,\dfrac{x^2}{2^2} \Rightarrow$ coefficient of $x^2 = \dfrac{45}{4} = 11\dfrac{1}{4}$

For $r = 3$; ${}^{10}C_3\,\dfrac{x^3}{2^3} \Rightarrow$ coefficient of $x^3 = 15$

For $r = 4$; ${}^{10}C_4\,\dfrac{x^x}{2^4}$

$\Rightarrow$ coefficient of $x^4 = \dfrac{210}{16} = \dfrac{105}{8} = 13\dfrac{1}{8} \Rightarrow r = 3$

(c) $T_4 = {}^nC_3\ x^{n-3}\left(\dfrac{\alpha}{2x}\right)^3 \Rightarrow {}^nC_3\ x^{n-3}\left(\dfrac{\alpha}{2}\right)^3 = 20$

$n = 6,\ {}^6C_3\left(\frac{\alpha}{2}\right)^3 = 20 \Rightarrow \alpha = 2$

(45) **(B).** Total number of terms is ${}^{10+3-1}C_{10} = {}^{12}C_{10} = 66$

(46) **(D).** Coefficient of $a^8bc = \frac{10!}{8!1!1!} = 90$

(47) **(D).** Coefficeint of $a^4b^5c^3$ is 0 $\because\ 4+5+3 = 12 > 10$

(48) (C), (49) (B), (50) (D).

$R = (1+2x)^n$

Put x = 1 to get sum of all coefficients

$\therefore\ 3^n = 6561 = 3^8 \Rightarrow n = 8$

(i) For $x = \frac{1}{\sqrt{2}},\ R = (\sqrt{2}+1)^8$

Consider

$\underbrace{(\sqrt{2}+1)^8 + (\sqrt{2}-1)^8}_{I+f+f'} = 2\left[{}^8C_0(\sqrt{2})^8 + ...\right] = \text{even integer}$

Since I is integer $\Rightarrow f + f'$ must be an integer

but $0 < f + f' < 2 \Rightarrow f + f' = 1 \Rightarrow f' = 1 - f$

Now, $n + R - Rf$

$n + R(1-f) = 8 + (\sqrt{2}+1)^n(\sqrt{2}-1)^n = 8 + 1 = 9$

(ii) T_{r+1} in $(1+2x)^8 = {}^8C_r(2x)^r = {}^8C_r$ when $x = 1/2$

Now $T_{r+1} \geq T_r$

$\frac{T_{r+1}}{T_r} \gtrless 1 \Rightarrow \frac{8C_r}{{}^8C_{r-1}} \gtrless 1$

$T_{r+1} \gtrless T_r\ ;\ \frac{8!}{r!(8-r)!}.\frac{(r-1)!(9-r)!}{8!} \geq 1$

$(9-r) \geq r \Rightarrow 9 \geq 2r$

For r = 1, 2, 3, 4 this is true

i.e. $T_5 > T_4$

but for r = 5, $T_6 < T_5 \Rightarrow T_5$ is the greatest term $\Rightarrow$ (B)

(iii) Again, $T_{k+1} = {}^8C_k.\ 2^k.x^k\ ;\ T_k = {}^8C_{k-1}.2^{k-1}.x^{k-1}$

$T_{k-1} = {}^8C_{k-2}.2^{k-2}.x^{k-2}$

We want to find the term having the greatest coefficient

$\therefore\ 2^{k-1}.\ {}^8C_{k-1} > 2^k.{}^8C_k$ .... (1)

and $2^{k-1}.\ {}^8C_{k-1} > 2^{k-2}.{}^8C_{k-2}$ .... (2)

From (1), $\frac{8!2^{k-1}}{(k-1)!(9-k)!} > \frac{2^k.8!}{k!(8-k)!} \Rightarrow \frac{1}{(9-k)} > \frac{2}{k}$

$\Rightarrow k > 18 - 3k \Rightarrow k > 6$

Again $2^{k-1}.\ {}^8C_{k-1} > 2^{k-2}.{}^8C_{k-2}$

$\frac{8!.2^{k-1}}{(k-1)!(9-k)!} > \frac{2^{k-2}.8!}{(k-2)!(10-k)!} \Rightarrow \frac{2}{k-1} > \frac{1}{10-k}$

$\Rightarrow 20 - 2k > k - 1 \Rightarrow 21 > 3k \Rightarrow k < 7$

$\Rightarrow 6 < k < 7 \Rightarrow T_6$ and T_7 term has the greatest coefficient

$\Rightarrow k = 6$ or $7 \Rightarrow$ Sum $= 6 + 7 = 13$.

(51) (B), (52) (C), (53) (D).

$x^3 - 1$

$x = 1, \omega, \omega^2$ or $x = \omega, \omega^2, \omega^3$

$x = 1 : C_0 + C_1 + C_2 + C_3 + C_4 + ... = 2^n$

$x = \omega : C_0 + C_1\omega + C_2\omega^2 + C_3\omega^3 + C_4\omega^4 + ... = (1+\omega)^n$

$x = \omega^2 : C_0 + C_1\omega^2 + C_2\omega^4 + C_3\omega^6 + ... = (1+\omega^2)^n$

$3(C_0 + 0 + 0 + C_3 + 0 + 0 + C_6 + ...) = 2^n + (-\omega^2)^n + (-\omega)^n$

$= 2^n + (-1)^n + (-1)^n$

$\therefore\ C_0 + C_3 + C_6 + ... = \frac{2^n + 2(-1)^n}{3}$

$x^4 - 1 = 0 \Rightarrow x = \pm 1, \pm i$

$\therefore$ Sum of values $x = 1 + (-1) + i + (-i) = 0$

(54) **(C).** $\sum_{r=0}^{6m} {}^{6m}C_r 2^{r/2}$ put $x = \sqrt{2}$

$= (1+\sqrt{2})^{6m} = (3+2\sqrt{2})^{3m}$

(55) **(B).** $\sum_{r=0}^{3m} (-1)^r\ {}^{6m}C_{2r}$

$= (\sqrt{2})^{6m}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^{6m} + (\sqrt{2})^{6m}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)^{6m}$

$= 2^{3m}.2\cos\frac{3m\pi}{2} = \begin{cases} 0 & \text{if m is odd} \\ (-1)^{m/2}2^{3m+1} & \text{if m is even} \end{cases}$

(56) **(A).** $\sum_{r=0}^{3m} (-3)^{r-1}\ {}^{6m}C_{2r-1}$

$= \frac{1}{\sqrt{3}i}\left\{\begin{matrix}\sqrt{3}i\ {}^{6m}C_1 + (\sqrt{3}i)^3\ {}^{6m}C_3 + (\sqrt{3}i)^5\ {}^{6m}C_5 \\ + ... + (\sqrt{3}i)^{6m-1}\ {}^{6m}C_{6m-1}\end{matrix}\right\}$

$(1+\sqrt{3}i)^{6m} = {}^{6m}C_0 + \sqrt{3}i\ {}^{6m}C_1 + (\sqrt{3}i)^2\ {}^{6m}C_2 + (\sqrt{3}i)^3\ {}^{6m}C_3 + ...$

$(1-\sqrt{3}i)^{6m} = {}^{6m}C_0 - \sqrt{3}i\ {}^{6m}C_1 + (\sqrt{3}i)^2\ {}^{6m}C_2 - (\sqrt{3}i)^3\ {}^{6m}C_3 + ...$

$(1+\sqrt{3}i)^{6m} - (1-\sqrt{3}i)^{6m}$

$= 2[\sqrt{3}i\ {}^{6m}C_1 + (\sqrt{3}i)^3\ {}^{6m}C_3 + ...]$

Given expression

$= \frac{2^{6m}}{2\sqrt{3}i}(\cos 2m\pi + i\sin 2m\pi - \cos 2m\pi + i\sin 2m\pi) = 0$

(57) **(B).** Sum of the coefficient in the expansion of $(1+2x)^n = 6561$

$\Rightarrow (1+2x)^n = 6561$, when $x = 1$

$\Rightarrow 3^n = 6561 \Rightarrow 3^n = 3^8 \Rightarrow n = 8$

Now, $\dfrac{T_{r+1}}{T_r} = \dfrac{^8C_r(2x)^r}{^8C_{r-1}(2x)^{r-1}} = \dfrac{9-r}{r} \times 2x$

$\Rightarrow \dfrac{T_{r+1}}{T_r} = \dfrac{9-r}{r}$ $[\because x = 1/2]$

$\therefore \dfrac{T_{r+1}}{T_r} > 1 \Rightarrow \dfrac{9-r}{r} > 1 \Rightarrow 9-r > r \Rightarrow 2r < 9 \Rightarrow r < 4\dfrac{1}{2}$

Hence, 5^{th} term is the greatest term.

(58) **(B).** $^nC_7 \dfrac{2^{n-7}}{3^7} = {^nC_8} \dfrac{2^{n-8}}{3^8} \Rightarrow n = 55$

(59) **(A).** Coefficient of $(3r)^{th}$ and $(r+2)^{th}$ terms will be $^{2n}C_{3r-1}$ and $^{2n}C_{r+1}$.

These are equal $\Rightarrow (3r-1)+(r+1) = 2n \Rightarrow n = 2r$

EXERCISE-3

(1) **3.** $^nC_m \cdot {^mC_p} = \dfrac{n!}{(n-m)!m!} \times \dfrac{m!}{(m-p)!p!}$

$= \dfrac{n!}{(n-p)!p!} \times \dfrac{(n-p)!}{(n-m)!(m-p)!} = {^nC_p} \cdot {^{n-p}C_{m-p}}$

Now, $\sum\limits_{p=1}^{n} \sum\limits_{m=p}^{n} {^nC_p} \cdot {^{n-p}C_{m-p}} =$

$\sum\limits_{m=p}^{n} {^nC_p}({^{n-p}C_0} + {^{n-p}C_1} + {^{n-p}C_2} + ... + {^{n-p}C_{n-p}})$

$= \sum\limits_{p=1}^{n} {^nC_p} . 2^{n-p} = \sum\limits_{p=0}^{n} {^nC_p} . 2^{n-p} - 2^n = 3^n - 2^n$

further $3^n - 2^n = 19 \Rightarrow n = 3$

(2) **6.** $(1 - 2x + 5x^2 + 10x^3)\ [C_0 + C_1x + C_2x^2 + ...]$
$= 1 + a_1x + a_2x^2 + ...$

$a_1 = n - 2$ and $a_2 = \dfrac{n(n-1)}{2} - 2n + 5$

put $a_1{}^2 = 2a_2$

$(n-2)^2 = n(n-1) - 4n + 10$

$n^2 - 4n + 4 = n^2 - 5n + 10$

$n = 6$

(3) **3.** Put $x = -1$; $(-3)^{2145} = a_0 - a_1 + a_2 - a_3 + ...$

$-(3)^{2145} = -(3^4)^{536} \cdot 3 \Rightarrow$ ends in 3

(4) **0.** $\dfrac{1(2^{2000}-1)}{1} = 2^{2000} - 1$

$(5-1)^{1000} - 1 = (1-5)^{1000} - 1$

$1 - {^{1000}C_1} \cdot 5 + {^{1000}C_2} \cdot 5^2 + ... + {^{1000}C_{1000}} \cdot 5^{1000} - 1$

which is divisible by 5

(5) **283.** $T_{r+1} = {^{12}C_r}\,(3^{1/4})^{12-r} \cdot (4)^{r/3}$

$= {^{12}C_r} \cdot 3^{(3-r/4)} \cdot (4)^{r/3}$

For rational term $r = 0$ or 12

$\therefore$ Sum $= T_1 + T_{13} = {^{12}C_0} \cdot 3^3 + {^{12}C_{12}} \cdot 4^4 = 3^3 + 4^4$
$= 27 + 256 = 283$

(6) **8.** Coefficient of x^2 in $(1-x)^{-n}$, $n \in N$ is $^{n+r-1}C_r$

now coefficient of x^n in $(1+x)^2\,(1-x)^{-2}$

or coefficient of x^n in $(1+2x+x^2)(1-x)^{-2}$

or coeff. of x^n in $(1-x)^{-2} + 2 \cdot$ coeff. of x^{n-1} in $(1-x)^{-2}$ + coeff. of x^{n-2} in $(1-x)^{-2}$

$= {^{n+1}C_n} + 2 \cdot {^nC_{n-1}} + {^{n-1}C_{n-2}}$

$= (n+1) + 2n + n - 1 = 4n$

hence $4n = 32 \Rightarrow n = 8$

(7) **2.** $r = 3$ or 5 ; $r = 0$ is not possible

(8) **26.** $T_{r+1} = {^{100}C_r}\ 2^{\frac{100-r}{2}} \cdot 3^{r/4}$

$\Rightarrow r$ must be even and divisible by 4

$\Rightarrow r = 0, 4, 8, ..., 100$

(9) **1.** $(21)^{100} = (1+20)^{100} = 1 + {^{100}C_1} \cdot 20 + {^{100}C_2} \cdot 20^2 + ...$
$+ {^{100}C_{100}} \cdot 20^{100}$

hence last two digit are 01

(10) **99.** $\left(\dfrac{x^2}{4} + \dfrac{2}{x}\right)^{12}$ general term

$T_{r+1} = {^{12}C_r} \dfrac{x^{2(12-r)}}{4^{12-r}} \times \dfrac{2^r}{x^r} \Rightarrow 24 - 3r = 3 \Rightarrow r = 7$

Coefficient $= {^{12}C_7} \times \dfrac{2^7}{4^5} = \dfrac{12 \times 11 \times 10 \times 9 \times 8}{120} \times \dfrac{1}{2^3} = 99$

(11) **352.** $(1+x+x^2)^{12} = [1+(x+x^2)]^{12}$

$T_{r+1} = {^{12}C_r} \cdot x^r(1+x)^r$

coefficient of $x^3 \Rightarrow r = 2, 3$

${^{12}C_2} \cdot 2 + {^{12}C_3} = 132 + 220 = 352$

(12) **9.** Given expression $= 1 + (1+x) + (1+x)^2 + ... + (1+x)^{19}$

$= \dfrac{(1+x)^{20} - 1}{(1+x) - 1} = \dfrac{(1+x)^{20} - 1}{x}$

(It is a G.P. with first term $= 1 + x$, common ratio $= 1 + x$)

Now coefficient of x^p in $\dfrac{(1+x)^{20} - 1}{x}$

$=$ coefficient of x^{p+1} in $\{(1+x)^{20} - 1\}$

$T_{r+1} = {^{20}C_r}\, x^{20-r}$ $\quad \because n = 20$ even

$\Rightarrow T_{(n/2)+1}$ is the term with greatest coefficient

$\therefore T_{(20/2)+1} = T_{11} = {}^{20}C_r\, x^{20-10}$
$\therefore p + 1 = 10 \Rightarrow p = 9$

(13) **12.** $(1+x)^m (1-x)^n = \left[1 + mx + \frac{m(m-1)}{2}x^2 + ...\right]$

$$\left[1 - nx + \frac{n(n-1)}{2}x^2 - ......\right]$$

$$= 1 + (m-n)x + \left[\frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn\right]x^2 + ...$$

Term containing power of $x \ge 3$.
Now, $m - n = 3$ .... (1)
[$\because$ coefficient of $x = 3$ given]

and $\frac{1}{2}m(m-1) + \frac{1}{2}n(n-1) - mn = -6$

$\Rightarrow m(m-1) + n(n-1) - 2mn = -12$
$\Rightarrow m^2 - m + n^2 - n - 2mn = -12$
$\Rightarrow (m-n)^2 - (m+n) = -12$
$\Rightarrow m + n = 9 + 12 = 21$ .... (2)
On solving eqs. (1) and (2), we get $m = 12$.

(14) **15.** $\sum_{i=0}^{m} {}^{10}C_i\ {}^{20}C_{m-i} = {}^{10}C_0\ {}^{20}C_m + {}^{10}C_1\ {}^{20}C_{m-1}$

$$+ {}^{10}C_2\ {}^{20}C_{m-2} + ... + {}^{10}C_m\ {}^{20}C_0$$

= Coefficient of x^m in the expansion of product $(1+x)^{10}(x+1)^{20}$
= Coefficient of x^m in the expansion of $(1+x)^{30} = {}^{30}C_m$
Hence, the maximum value ${}^{30}C_m$ is ${}^{30}C_{15}$.

(15) **6.** Let T_{r-1}, T_r, T_{r+1} are three consecutive terms of $(1+x)^{n+5}$

$T_{r-1} = {}^{n+5}C_{r-2}(x)^{r-2}$, $T_r = {}^{n+5}C_{r-1}x^{r-1}$,

$T_{r+1} = {}^{n+5}C_r x^r$

Where, ${}^{n+5}C_{r-2} : {}^{n+5}C_{r-1} : {}^{n+5}C_r = 5 : 10 : 14$.

So, $\frac{{}^{n+5}C_{r-2}}{5} = \frac{{}^{n+5}C_{r-1}}{10} = \frac{{}^{n+5}C_r}{14}$

So, $\frac{{}^{n+5}C_{r-2}}{5} = \frac{{}^{n+5}C_{r-1}}{10} \Rightarrow n - 3r = -3$ .... (1)

$\frac{{}^{n+5}C_{r-1}}{10} = \frac{{}^{n+5}C_r}{14} \Rightarrow 5n - 12r = -30$ .... (2)

From eq. (1) and (2), $n = 6$

(16) **10.** According to the question,

${}^nC_{r-2} : {}^nC_{r-1} : {}^nC_r = 1 : 3 : 5$

$$\frac{{}^nC_{r-2}}{1} = \frac{{}^nC_{r-1}}{3} = \frac{{}^nC_r}{5}$$

$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{5}{3}$ and $\frac{{}^nC_{r-1}}{{}^nC_{r-2}} = \frac{3}{1}$

$\Rightarrow \frac{n-r+1}{r} = \frac{5}{3}$ and $\frac{n-r+2}{r-1} = \frac{3}{1}$

$$\left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$\Rightarrow 3n - 8r + 3 = 0$ and $n - 4r + 5 = 0$
Solving these for n and r, we get $n = 7$ and $r = 3$.

(17) **4.** T_{r+1} of $(1+x)^m = {}^mC_r x^r$
Here, $r = 2$

Coefficient of $x^2 = {}^mC_2 = \frac{m(m-1)}{2}$

$\Rightarrow 6 = \frac{m(m-1)}{2} \Rightarrow m^2 - m = 12$

$\Rightarrow m^2 - m - 12 = 0 \Rightarrow (m-4)(m+3) = 0$
$\Rightarrow m = 4$ or $m = -3$
Positive value of $m = 4$.

(18) **10.** We have, $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$

5^{th} term from the beginning $= {}^nC_4 (\sqrt[4]{2})^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^4$

5^{th} term from the end $= (n+1-5+1)^{th}$ term from beginning
$= (n-3)^{th}$ term from beginning

$$= {}^nC_{n-4} (\sqrt[4]{2})^4 \left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$$

Now, $\frac{{}^nC_4 (\sqrt[4]{2})^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^4}{{}^nC_{n-4}(\sqrt[4]{2})^4\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}} = \frac{\sqrt{6}}{1}$

$\Rightarrow (2)^{\frac{n-8}{4}} . (3)^{\frac{n-8}{4}} = 2^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

$\Rightarrow \frac{n-8}{4} = \frac{1}{2} \Rightarrow n - 8 = 2 \Rightarrow n = 10$

EXERCISE-4

(1) **(C).** $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + {}^{2n}C_3 x^3 + \ldots\ldots + {}^{2n}C_{2n} x^{2n}$

Coefficient of $(r+2)^{th}$ term is ${}^{2n}C_{r+1}$

Coefficient of $(3r)^{th}$ term is ${}^{2n}C_{3r-1}$

According to question

${}^{2n}C_{r+1} + {}^{2n}C_{3r-1}$

$\Rightarrow r+1+3r-1 = 2n$

$\Rightarrow 4r = 2n \Rightarrow 2r = n$

(2) **(D).** $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots.. + C_nx^n$

$$\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \ldots.. + \frac{nC_n}{C_{n-1}} = ?$$

$$T_r = \frac{r.\,{}^nC_r}{{}^nC_{r-1}} = r.\frac{n-r+1}{r}$$

$T_r = n-r+1$

$\Rightarrow T_1 = n$

$T_2 = (n-1)$

$T_3 = (n-2)$

....

$T_n = 1$

On adding, $T_1 + T_2 + T_3 + \ldots\ldots\ldots. + T_n$

$$= n + (n-1) + (n-2) + \ldots\ldots. + 1 = \frac{n\,(n+1)}{2}$$

(3) **(A).** Let x^{39} comes in $(r+1)^{th}$ no. of term in the expansion

of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ $\therefore\ r = \frac{15\times4-39}{4+3} = \frac{21}{7} = 3 \Rightarrow r = 3$

$\therefore x^{39}$ comes in 4^{th} term

$$\therefore T_{3+1} = {}^{15}C_3\,(x^4)^{15-3}\left(-\frac{1}{x^3}\right)^3 = -\,{}^{15}C_3\,x^{39}$$

$$\therefore \text{Coefficient of } x^{39} \text{ is } -\,{}^{15}C_3 = -\frac{15}{3}\times\frac{14}{2}\times\frac{13}{1} = -455$$

$\because$ If x^m comes in $(r+1)^{th}$ no. of term in expansion of

$\left(ax^\alpha \pm \frac{b}{x^\beta}\right)^n$ then $r = \frac{n\alpha - m}{\alpha+\beta}$

(4) **(B).** $\because$ x is nearly equal to 1

$\therefore x = 1+h$, where h is very small

$\Rightarrow h = x-1$

$$\text{Now, } \frac{ax^b - bx^a}{x^b - x^a} = \frac{a\,(1+h)^b - b\,(1+h)^a}{(1+h)^b - (1+h)^a}$$

$$= \frac{a\,(1+bh) - b\,(1+ah)}{(1+bh)-(1+ah)}$$

$$= \frac{a-b}{h\,(b-a)} = \frac{-1}{h} = \frac{-1}{x-1} = \frac{1}{1-x}$$

[If x is very very small then $(1+x)^n = 1+nx$]

(5) **(C).** $(\sqrt{3} + \sqrt[8]{5})^{256}$

Let T_{r+1} term is integeral term

$$T_{r+1} = {}^{256}C_r\,(\sqrt{3})^{256-r}\,(\sqrt[8]{5})^r$$

$$= {}^{25}C_r\ 3^{\frac{256-r}{2}}\ 5^{r/8} \qquad \ldots\ldots (1)$$

In (1), ${}^{25}C_r$ is integral and for $3^{\frac{256-r}{2}}$ to be integral

$256-r$ must be multiple of 2.

$\therefore\ r = 0, 2, 4, 6 \ldots\ldots. 256 \qquad \ldots\ldots.. (2)$

and for $5^{r/8}$ to be integral r must be multiple of 8

$\therefore\ r = 0, 8, 16, 24 \ldots\ldots. 256 \qquad \ldots\ldots.. (3)$

From (2) and (3) common values of r are

$r = 0, 8, 16, 24 \ldots\ldots.$ 33 values

$\therefore$ 33 integral terms in expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$

(6) **(C).** $(1+\alpha x)^4$ total no. of term are 5

$\therefore$ middle term is 3^{rd}

$\therefore\ T_{2+1} = {}^4C_2\,(1)^{4-2}\,(\alpha x)^2$

$\therefore$ Coeff. is ${}^4C_2\,\alpha^2$

and in $(1-\alpha x)^6$ total no. of term are 7

$\therefore$ middle term is 4^{th}

$\therefore\ T_{3+1} = {}^6C_3\,(-\alpha x)^3$

$\therefore$ Coefficient of $(-\alpha)^3\,{}^6C_3$

according to question, ${}^4C_2\,\alpha^2 = -\,\alpha^3\,{}^6C_3$

$\Rightarrow 6\alpha^2 = -\,\alpha^3\,(20) \Rightarrow -\alpha = 3/10 \Rightarrow \alpha = -3/10$

(7) **(B).** $(1+x)\,(1-x)^n = (1+x)\,({}^nC_0 - {}^nC_1\,x - {}^nC_2\,x^2 - {}^nC_3\,x^3 + \ldots\ldots. + {}^nC_{n-1} + (-x)^{n-1} + {}^nC_n\,(-x)^n)$

$= (1+x)\,({}^nC_0 - {}^nC_1\,x + {}^nC_2\,x^2 - {}^nC_3\,x^3 + \ldots\ldots. + (-1)^{n-1}\,{}^nC_{n-1}\,x^{n-1} + (-1)^n\ {}^nC_n\,x^n)$

$\therefore$ Coefficient of x^n is $(-1)^n\ {}^nC_n + (-1)^{n-1}\,{}^nC_{n-1}$

$(-1)^n + (-1)^{n-1}\,n = (-1)^n\,[1-n]$ $\quad \{\because {}^nC_n = 1, {}^nC_{n-1} = n\}$

(8) **(A).** $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$ (given)

$$S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r} = \frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{1}{{}^nC_2} + \frac{1}{{}^nC_3} + \ldots.. + \frac{1}{{}^nC_n} \qquad \ldots\ldots (1)$$

$$\text{and } t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$$

$$= \frac{0}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{2}{{}^nC_2} + \frac{3}{{}^nC_3} + \ldots.. + \frac{n}{{}^nC_n}$$

$$= \frac{n}{^nC_n} + \frac{n-1}{^nC_{n-1}} + \frac{n-2}{^nC_{n-2}} + \ldots\ldots \frac{2}{^nC_2} + \frac{2}{^nC_1} + \frac{0}{^nC_0}$$

in reverse order}

$$= \frac{n-0}{^nC_n} + \frac{n-1}{^nC_{n-1}} + \frac{n-2}{^nC_{n-2}} + \ldots\ldots$$

$$+ \frac{n-(n-2)}{^nC_2} + \frac{n-(n-1)}{^nC_1} + \frac{n-n}{^nC_0}$$

$$= \left(\frac{n}{^nC_n} - \frac{0}{^nC_n}\right) + \left(\frac{n}{^nC_{n-1}} - \frac{1}{^nC_{n-1}}\right) + \ldots\ldots$$

$$+ \left(\frac{n}{^nC_1} - \frac{n-1}{^nC_1}\right) + \left(\frac{n}{^nC_0} - \frac{n}{^nC_0}\right)$$

$$= n\left[\frac{1}{^nC_n} + \frac{1}{^nC_{n-1}} + \frac{1}{^nC_{n-2}} + \ldots + \frac{1}{^nC_0}\right]$$

$$- \left[\frac{0}{^nC_n} + \frac{1}{^nC_{n-1}} + \frac{2}{^nC_{n-2}} + \ldots + \frac{n}{^nC_0}\right]$$

$$= nS_n - \left[\frac{0}{^nC_0} + \frac{1}{^nC_1} + \frac{2}{^nC_2} + \ldots + \frac{n}{^nC_n}\right]$$

$$\left\{\because {}^nC_0 = {}^nC_n \text{ or } {}^nC_r = {}^nC_{n-r} \quad {}^nC_1 = {}^nC_{n-1}\right\}$$

$$t_n = nS_n - t_n \Rightarrow 2t_n = nS_n \Rightarrow \frac{t_n}{S_n} = \frac{n}{2}$$

(9) **(C).** $(1+y)^m = C_0 + C_1y + C_2y^2 + \ldots\ldots + C_my^m$

Coefficient of r^{th} term is $^mC_{r-1}$

and Coefficient of $(r+1)^{th}$ term is ^mC_r

and coefficient of $(r+2)^{th}$ term is $^mC_{r+1}$

According to question they are in A.P.

$\therefore\ 2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$

$$\Rightarrow \frac{2.m!}{(m-r)!\,r!} = \frac{m!}{(r-1)!\,(m-r+1)!} + \frac{m!}{(m-r-1)!\,(r+1)!}$$

$$= \frac{2.1}{(m-r)\,(m-r-1)!\,r\,(r-1)!}$$

$$= \frac{1}{(r-1)!\,(m-r+1)\,(m-r)\,(m-r-1)!}$$

$$+ \frac{1}{(m-r-1)!\,(r+1)\,r\,(r-1)!}$$

$$= \frac{2}{(m-r)\,r} = \frac{1}{(m-r+1)\,(m-r)} + \frac{1}{(r+1)\,r}$$

$$= \frac{2}{(m-r)\,r} = \frac{(r+1)\,r + (m-r+1)\,(m-r)}{(m-r+1)\,(m-r)\,(r+1)\,(r)}$$

$\Rightarrow 2\,(m-r+1)\,(r+1) = r^2 + r + m^2 + r^2 - 2mr + m - r$

$\Rightarrow 2\,[mr - r^2 + r + m - r + 1] = m^2 + 2r^2 - 2mr + m$

$\Rightarrow m^2 - 4mr - m + 4r^2 - 1 = 0$

$\Rightarrow m^2 - m\,(4r+1) + 4r^2 - 2 = 0$

(10) **(D).** If x^m comes in $(r+1)^{th}$ term in expansion of

$$\left(ax^\alpha \pm \frac{b}{x^\beta}\right)^n \text{ then } r = \frac{n\alpha - m}{\alpha+\beta}$$

$\therefore$ Let x^7 comes in $(r+1)^{th}$ term in expansion of

$$\left[ax^2 + \frac{1}{bx}\right]^{11}$$

$$\therefore\ r = \frac{11\times 2 - 7}{2+1} = \frac{15}{3} = 5 \ \therefore\ r = 5$$

Now, $T_{5+1} = {}^{11}C_5\,(ax^2)^{11-5}\left(\frac{1}{bx}\right)^5$

$\therefore$ Coefficient is ${}^{11}C_5\left(\frac{a^6}{b^5}\right)$ .... (1)

and let x^{-7} comes in $(r+1)^{th}$ no. term in expansion of

$$\left(ax - \frac{1}{bx^2}\right)^{11} \quad \therefore\ r = \frac{11\times 1 - (-7)}{1+2} = \frac{18}{3} = 6$$

Now, $T_{6+1} = {}^{11}C_6\,(ax)^{11-6}\left(\frac{-1}{bx}\right)^6$

$\therefore$ Coefficient is ${}^{11}C_6\left(\frac{a^5}{b^6}\right)$ .... (2)

According to question,

$${}^{11}C_5\frac{a^6}{b^5} = {}^{11}C_6\frac{a^5}{b^6} \Rightarrow a = 1/b \Rightarrow ab = 1$$

(11) **(C).** x is so small that x^3 and higher power of x may be neglected $\therefore\ (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2$

Now, $\dfrac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$

$$= \frac{\left[1+\frac{3}{2}x+\frac{3/2\,(3/2-1)}{2!}x^2\right]-\left[1+\frac{x^3}{8}+3.1^2\,\frac{x}{2}+3.1\left(\frac{x}{2}\right)^2\right]}{(1-x)^{1/2}}$$

$$= \left[\frac{\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)-\left(1+\frac{3}{2}x+\frac{3}{4}x^2\right)}{(1-x)^{1/2}}\right]$$

$\{\because\ x^{3/8} \to \text{neglected}\}$

$= (-3/8\,x^2)\,(1-x)^{-1/2}$

$$= \left(\frac{-3}{8}x^2\right)\left[1+\frac{x}{2}+\frac{-\frac{1}{2}\left(\frac{-1}{2}-1\right)}{2!}(-x)^2\right]$$

$$= \left(\frac{-3}{8}x^2\right)\left[1+\frac{x}{2}+\frac{3}{8}x^2\right] = -\frac{3}{8}x^2-\frac{3}{16}x^3-\frac{9}{64}x^4$$

$= -\frac{3}{8}x^2$ {term containing x^3 and higher power are neglected}

(12) $\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+a_3x^3+.....+a_nx^n$ $+......\infty$ ...... (1)

Now, $\frac{1}{(1-ax)(1-bx)} = (1-ax)^{-1}(1-bx)^{-1}$

$= [1+ax+(ax)^2+(ax)^3+.....]\,[1+(bx)+(bx)^2+(bx)^3+......]$

Now, coefficient of x^n is

$b^n + a^1b^{n-1} + a^2b^{n-2} + a^3b^{n-3} + ... + a^n$ .... (2)

$$= \frac{b^n\left[1-\left(\frac{a}{b}\right)^{n+1}\right]}{1-\frac{a}{b}} = \frac{b^n\left[\frac{b^{n+1}-a^{n+1}}{b^{n+1}}\right]}{\left(\frac{b-a}{b}\right)} = \frac{b^{n+1}-a^{n+1}}{b-a}$$

$\{\because$ (1) is G.P. and sum of (n + 1) term of G.P. is $\frac{a(1-r^{n+1})}{1-r}\}$

(13) **(C).** $(1-y)^m(1+y)^n$

$[({}^mC_0-{}^mC_1\,y+{}^mC_2\,y^2-{}^mC_3\,y^3+.......+{}^mC_m\,(-y)^m)$
$({}^nC_0+{}^nC_1\,y+{}^nC_2\,y^2+{}^nC_3\,y^3+.......+{}^nC_n\,(y)^n)]$ ...... (1)

but $(1-y)^m(1+y)^n = 1+a_1y+a_2y^2+$ .... (given)

and $a_1 = a_2 = 10$ .... (2)

$\therefore$ RHS of (1) and (2) are same

On comparing

${}^mC_0\times{}^nC_1-{}^mC_1\times{}^nC_0 = 10$

$\Rightarrow n-m = 10$ .... (3)

and ${}^mC_0\times{}^nC_2-{}^mC_1\times{}^nC_1+{}^mC_2\times{}^nC_0 = 10$

$$\frac{n(n-1)}{2}-mn+\frac{m(m-1)}{2} = 10$$

$\Rightarrow n^2-n-2mn+m^2-m = 20$ .... (4)

Put value of n from (3) in (4)

$(m+10)^2-(m+10)-2m(m+10)+m^2-m = 20$

$\Rightarrow m^2+100+20m-m-10-2m^2-20m+m^2-m = 20$

$\Rightarrow -2m+70 = 0 \Rightarrow m = 35$

$\therefore$ $n = 45 \Rightarrow (m, n) = (35, 45)$

(14) **(D).** $(a-b)^n = {}^nC_0\,a^n+{}^nC_1\,a^{n-1}(-b)+{}^nC_2\,a^{n-2}(-b)^2$ $+ ....+{}^nC_n(-b)^n$

Now, $T_5 = {}^nC_4a^{n-4}(-b)^4 = {}^nC_4\,a^{n-4}b^4$

$T_6 = {}^nC_5a^{n-5}(-b)^5 = -\,{}^nC_5\,a^{n-5}b^5$

According to question, $T_5+T_6 = 0$

$\Rightarrow {}^nC_4a^{n-4}\,b^4 - {}^nC_5a^{n-5}\,b^5 = 0$

$\Rightarrow {}^nC_4a^{n-4}\,b^4 = {}^nC_5a^{n-5}\,b^5$

$$\Rightarrow \frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{\frac{n!}{(n-5)!\,5!}}{\frac{n!}{(n-4)!\,4!}}\ ;\quad \frac{a}{b} = \frac{n-4}{5}$$

(15) **(B).** ${}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.........+{}^{20}C_{10}$

$\because$ We know that

${}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+{}^{20}C_4-{}^{20}C_5+.........+{}^{20}C_{17}$ $+{}^{20}C_{18}-{}^{20}C_{19}+{}^{20}C_{20} = 0$

$\Rightarrow 2({}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+........-{}^{20}C_9)+{}^{20}C_{10}=0$

$\Rightarrow 2({}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+......+{}^{20}C_{10})-{}^{20}C_{10}=0$

$$\Rightarrow ({}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.......+{}^{20}C_{10}) = \frac{1}{2}\,{}^{20}C_{10}$$

$\therefore$ ${}^nC_r = {}^nC_{n-r}$ $\therefore$ ${}^{20}C_0 = {}^{20}C_{20}$

${}^{20}C_1 = {}^{20}C_{19}$; ${}^{20}C_2 = {}^{20}C_{18}$

(16) **(A).** Statement 1 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n+2)\,2^{n-1}$

$$\text{L.H.S.} = \sum_{r=0}^{n}(r+1)\,{}^nC_r = \sum_{r=0}^{n}(r.{}^nC_r+{}^nC_r)$$

$$\sum_{r=0}^{n} r.\frac{n}{r}\,{}^{n-1}C_{r-1}+\sum_{r=0}^{n}{}^nC_r$$

$$n\sum_{r=0}^{n}{}^{n-1}C_{r-1}+2^n = n.2^{n-1}+2^n = 2^{n-1}(n+2) = \text{R.H.S.}$$

$\therefore$ Statement (1) is correct.

Statement 2 :

$$\sum_{r=0}^{n}(r+1)\ {}^nC_r x^r = (1+x)^n + nx\,(1+x)^{n-1}$$

$$\text{L.H.S.} = \sum_{r=0}^{n}(r+1)\ {}^nC_r x^r = \sum_{r=0}^{n}(r.\ {}^nC_r x^r + {}^nC_r x^r)$$

$$= \sum_{r=0}^{n} r.\frac{n}{r}\ {}^{n-1}C_{r-1}x^r + \sum_{r=0}^{n}{}^nC_r x^r$$

$$= n\sum_{r=0}^{n}{}^{n-1}C_{r-1}x^r + (1+x)^n = nx\sum_{r=0}^{n}{}^{n-1}C_{r-1}x^{r-1} + (1+x)^n$$

$= nx\,(1+x)^{n-1} + (1+x)^n = \text{RHS}$. $\therefore$ Statement (2) is true

$\therefore$ If we put $x = 1$ in statement (2) we get statement (1)

$\therefore$ statement (2) is correct explanation for statement (1).

(17) **(B).** $8^{2n} - (62)^{2n+1}$

$\Rightarrow (9-1)^{2n} - (63-1)^{2n+1}$

$\Rightarrow ({}^{2n}C_0 9^{2n} - {}^{2n}C_1 9^{2n-1} + ... + 1)$

$\quad - ({}^{2n+1}C_0 63^{2n+1} - {}^{2n+1}C_1 63^{2n} + ... - 1)$

$\Rightarrow 9K + 2$. So remainder is 2.

(18) **(B).** $S_1 = \sum_{j=1}^{10} j\,(j-1)\frac{10!}{j\,(j-1)\,(j-2)!\,(10-j)!}$

$$= 90\sum_{j=2}^{10}\frac{8!}{(j-2)!\,(8-(j-2))!} = 90.2^8$$

$$S_2 = \sum_{j=1}^{10} j\frac{10!}{j\,(j-1)!\,(9-(j-1))!}$$

$$= 10\sum_{j=1}^{10}\frac{9!}{(j-1)!\,(9-(j-1))!} = 10.2^9$$

$$S_3 = \sum_{j=1}^{10}[j\,(j-1) + j]\frac{10!}{j!\,(10-j)!} = \sum_{j=1}^{10} j\,(j-1)\ {}^{10}C_j$$

$$= \sum_{j=1}^{10} j\ {}^{10}C_j = 90\,.\,2^8 + 10\,.\,2^9$$

$= 90\,.\,2^8 + 20\,.\,2^9 = 110\,.\,2^8 = 55\,.\,2^9$

(19) **(C).**

$(1 - x - x^2 + x^3)^6$

$(1-x)^6\,(1-x^2)^6$

$({}^6C_0 - {}^6C_1 x^1 + {}^6C_2 x^2 - {}^6C_3 x^3 + {}^6C_4 x^4 - {}^6C_5 x^5 + {}^6C_6 x^6)$

$({}^6C_0 - {}^6C_1 x^2 + {}^6C_2 x^4 - {}^6C_3 x^6 + {}^6C_4 x^8 + ...+ {}^6C_6 x^{12})$

Now coefficient of $x^7 = {}^6C_1\ {}^6C_3 - {}^6C_3\ {}^6C_2 + {}^6C_5\ {}^6C_1$

$= 6\times20 - 20\times15 + 36 = 120 - 300 + 36 = 156 - 300 = -144$

(20) **(A).** $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$

$= 2\,[{}^{2n}C_1\,(\sqrt{3})^{2n-1} + {}^{2n}C_3\,(\sqrt{3})^{2n-3} + {}^{2n}C_5\,(\sqrt{3})^{2n-5} + ...]$

which is an irrational number.

(21) **(C).** $\left((x^{1/3}+1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$; $(x^{1/3} - x^{-1/2})^{10}$

$$T_{r+1} = {}^{10}C_r\,(x^{1/3})^{10-r}(-x^{-1/2})^r$$

$$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0 \Rightarrow r = 4$$

$$T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$$

(22) **(D).** $1\,(1-2x)^{18} + ax\,(1-2x)^{18} + bx^2(1-2x)^{18}$

Coefficient of x^3 :

$(-2)^3\ {}^{18}C_3 + a\,(-2)^2\ {}^{18}C_2 + b\,(-2)\ {}^{18}C_1 = 0$

$$\frac{4\times(17\times16)}{(3\times2)} - 2a\cdot\frac{17}{2} + b = 0 \qquad ....(1)$$

Coefficient of x^4 :

$(-2)^4\ {}^{18}C_4 + a\,(-2)^3\ {}^{18}C_3 + b\,(-2)^2\ {}^{18}C_2 = 0$

$$(4\times20) - 2a\cdot\frac{16}{3} + b = 0 \qquad ....(2)$$

From eq. (1) and (2), we get

$$4\left(\frac{17\times8}{3} - 20\right) + 2a\left(\frac{16}{3} - \frac{17}{2}\right) = 0$$

$$4\left(\frac{17\times8-60}{3}\right) + \frac{2a\,(-19)}{6} = 0$$

$$a = \frac{4\times76\times6}{3\times2\times19} = 16\ ;\quad b = \frac{2\times16\times16}{3} - 80 = \frac{272}{3}$$

(23) **(D).** $(1-2\sqrt{x})^{50} = {}^{50}C_0 - {}^{50}C_1(2\sqrt{x})^1 + {}^{50}C_2(2\sqrt{x})^2$

$+ .... + {}^{50}C_{50}(-2\sqrt{x})^{50}$

Sum of coefficient of integral power of x

$= {}^{50}C_0\,2^0 + {}^{50}C_2\,2^2 + {}^{50}C_4\,2^4 + ... + {}^{50}C_{50}\,2^{50}$

We know that

$(1+2)^{50} = {}^{50}C_0 + {}^{50}C_1\,2 + ... + {}^{50}C_{50}\,.\,2^{50}$

Then, ${}^{50}C_0 + {}^{50}C_2\,.\,2^2 + ... + {}^{50}C_{50}\,.\,2^{50} = \dfrac{3^{50}+1}{2}$

(24) **(C).** $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$

Assuming all dissimilar terms ${}^{n+2}C_2 = 28$; $n = 6$

Sum of all coefficients $= 3^6 = 729$

(25) **(B).** $({}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \ldots\ldots {}^{21}C_{10})$
$- ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \ldots\ldots {}^{10}C_{10}) = S_1 - S_2$

$S_1 = {}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \ldots\ldots {}^{21}C_{10}$

$S_1 = \frac{1}{2}({}^{21}C_1 + {}^{21}C_2 + \ldots\ldots + {}^{21}C_{20})$

$= \frac{1}{2}({}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{20} + {}^{21}C_{21} - 2)$

$S_1 = 2^{20} - 1$

$S_2 = ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \ldots\ldots {}^{10}C_{10}) = 2^{10} - 1$

Therefore, $S_1 - S_2 = 2^{20} - 2^{10}$

(26) **(B).** Let $\sqrt{x^3 - 1} = y$

$(x+y)^5 + (x-y)^5$
$= ({}^5C_0 x^5 + {}^5C_1 x^4 y + \ldots\ldots + {}^5C_5 y^5)$
$+ ({}^5C_0 x^5 - {}^5C_1 x^4 y + \ldots\ldots - {}^5C_5 y^5)$
$= 2[{}^5C_0 x^5 + {}^5C_2 x^3 y^2 + {}^5C_4 x y^4]$
$= 2[C_0 x^5 + {}^5C_2 x^3 y^2 + {}^5C_4 x y^4]$
$= 2[x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2]$
$= 2[x^5 + 10x^6 - 10x^3 + 5x(x^6 + 1 - 2x^3)]$
$= 2[x^5 + 10x^6 - 10x^3 + 5x^7 + 5x - 10x^4]$
$= 2[1 - 10 + 5 + 5] = 2$

(27) **(D).** $\frac{2^{403}}{15} = \frac{2^3 \cdot (2^4)^{100}}{15} = \frac{8}{15}(15+1)^{100}$

$= \frac{8}{15}(15\lambda + 1) = 8\lambda + \frac{8}{15}$ $\because 8\lambda$ is integer

$\Rightarrow$ Fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15} \Rightarrow k = 8$

(28) **(B).** $(1 - t^6)^3 (1 - t)^{-3}$
$(1 - t^{18} - 3t^6 + 3t^{12})(1 - t)^{-3}$
Cofficient of t^4 in $(1-t)^{-3}$ is ${}^{3+4-1}C_4 = {}^6C_2 = 15$

(29) **(B).** $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + \ldots + 62.{}^{20}C_{20}$

$= \sum_{r=0}^{20}(3r+2)\,{}^{20}C_r = 3\sum_{r=0}^{20} r \cdot {}^{20}C_r + 2\sum_{r=0}^{20} {}^{20}C_r$

$= 3\sum_{r=0}^{20} r\left(\frac{20}{r}\right){}^{19}C_{r-1} + 2 \cdot 2^{20} = 60.2^{19} + 2.2^{20} = 2^{25}$

(30) **(D).** $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$
$= 2[{}^6C_0 x^6 + {}^6C_2 x^4 (x^3 - 1)$
$+ {}^6C_4 x^2 (x^3 - 1)^2 + {}^6C_6 (x^3 - 1)^3]$
$= 2[{}^6C_0 x^6 + {}^6C_2 x^7 - {}^6C_2 x^4 + {}^6C_4 x^8 + {}^6C_4 x^2$
$- 2{}^6C_4 x^5 + (x^9 - 1 - 3x^6 + 3x^3)]$

$\Rightarrow$ Sum of coefficient of even powers of x
$= 2[1 - 15 + 15 + 15 - 1 - 3] = 24$

(31) **(D).** $200 = {}^6C_3\left(\frac{1}{x^{1+\log_{10} x}}\right)^{3/2} \times x^{1/4}$

$\Rightarrow\ 10 = x^{\frac{3}{2(1+\log_{10} x)} + \frac{1}{4}} \Rightarrow\ 1 = \left(\frac{3}{2(1+t)} + \frac{1}{4}\right) t$

where $t = \log_{10} x$

$\Rightarrow\ t^2 + 3t - 4 = 0 \Rightarrow t = 1, -4 \Rightarrow x = 10, 10^{-4}$

$\Rightarrow\ x = 10$ (As $x > 1$)

(32) **(D).** $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15}$; $\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$; $\frac{r}{n-r+1} = \frac{2}{15}$

$15r = 2n - 2r + 2$; $17r = 2n + 2$

$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$; $\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$; $\frac{r+1}{n-r} = \frac{3}{14}$

$14r + 14 = 3n - 3r$
$3n - 17r = 14$
$2n - 17r = -2$

$n = 16$
$17r = 34,\ r = 2$
${}^{16}C_1,\ {}^{16}C_2,\ {}^{16}C_3$

$\frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16 + 120 + 560}{3}$

$\frac{680 + 16}{3} = \frac{696}{3} = 232$

(33) **(A).** Coefiicient of $x^2 = {}^{15}C_2 \times 9 - 3a({}^{15}C_1) + b = 0$
$-45a + b + {}^{15}C_2 \times 9 = 0$ ....(i)
$-27 \times {}^{15}C_3 + 9a \times {}^{15}C_2 - 3b \times {}^{15}C_1 = 0$
$\Rightarrow\ 9 \times {}^{15}C_2\, a - 45\, b - 27 \times {}^{15}C_3 = 0$
$\Rightarrow\ 21a - b - 273 = 0$ ...(ii)
(i) + (ii) : $-24a + 672 = 0 \Rightarrow a = 28$
So, $b = 315$

(34) **(B).** $T_r = \sum_{r=0}^{n} {}^nC_r\, x^{2n-2r} \cdot x^{-3r}$

$2n - 5r = 1 \Rightarrow 2n = 5r + 1$
for $r = 15$. $n = 38$
Smallest value of n is 38.

(35) **(B).** $(1+x)(1-x)^{10}(1+x+x^2)^9$
$(1-x^2)(1-x^3)^9$
${}^9C_6 = 84$

(36) **(A).** $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$
Diff. w.r.t. x
$n(1+x)^{n-1} = {}^nC_1 + {}^nC_2(2x) + \ldots\ldots + {}^nC_n\, n(x)^{n-1}$
Multiply by x both side

$nx(1+x)^{n-1} = {}^nC_1 x + {}^nC_2(2x^2) + ... + {}^nC_n(nx^n)$

Diff w.r.t. x

$n[(1+x)^{n-1} + (n-1)x(1+x)^{n-2}]$

$= {}^nC_1 + {}^nC_2 2^2 x + ... {}^nC_n(n^2)x^{n-1}$

Put x = 1 and n = 20

${}^{20}C_1 + 2^2\,{}^{20}C_2 + 3^2\,{}^{20}C_3 + ... + (20)^2\,{}^{20}C_{20}$

$= 20 \times 2^{18}[2+19] = 420(2^{18}) = A(2^\beta)$

(37) **(D).** $\frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right) - \frac{1}{81}x^8\left(2x^2 - \frac{3}{x^2}\right)^6$

Its general term

$\frac{1}{60}\,{}^6C_r 2^{6-r}(-3)^r x^{12-r} - \frac{1}{81}\,{}^6C_r 2^{6-r}(-3)^r 12^{20-4r}$

For term independent of x, r for I[st] expression is 3 and r for second expression is 5

$\therefore$ Term independent of x = – 36

(38) **30.** Let $(1 - x + x^2 .....)(1 + x + x^2 ...)$

$= a_0 + a_1 x + a_2 x^2 + ...$

Put x = 1

$1(2n+1) = a_0 + a_1 + a_2 + ...a_{2n}$ ..(i)

put x = –1

$(2n+1) \times 1 = a_0 - a_1 + a_2 + ...a_{2n}$ ..(ii)

Form eq. (i) + (ii)

$4n + 2 = 2(a_0 + a_2 + ...) = 2 \times 61$

$\Rightarrow 2n + 1 = 61 \Rightarrow n = 30$

(39) **(C).** $2[{}^6C_0 x^6 + {}^6C_2 x^4(x^2-1) + {}^6C_4 x^2(x^2-1)^2 + {}^6C_6(x^2-1)^3]$

$= 2[x^6 + 15(x^6 - x^4) + 15x^2(x^4 - 2x^2 + 1) + (-1 + 3x^2 - 3x^4 + x^6)]$

$= 2(32x^6 - 48x^4 + 18x^2 - 1)$

$\alpha = -96$ and $\beta = 36$

$\therefore \alpha - \beta = -132$

(40) **(615.00)**

$(1 + x + x^2)^{10} = {}^{10}C_0 + {}^{10}C_1 x(1+x) + {}^{10}C_2 x^2(1+x)^2 + {}^{10}C_3 x^3(1+x)^3 + {}^{10}C_4 x^4(1+x)^4 + ...$

Coeff. of $x^4 = {}^{10}C_2 + {}^{10}C_3 \times {}^3C_1 + {}^{10}C_4 = 615.$

(41) **(D).** $T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^r$

$= {}^{16}C_r(x)^{16-2r} \times \frac{1}{(\cos\theta)^{16-r}(\sin\theta)^r}$

For independent of x; $16 - 2r = 0 \Rightarrow r = 8$

$\Rightarrow T_9 = {}^{16}C_8 \frac{1}{\cos^8\theta\sin^8\theta} = {}^{16}C_8\frac{2^8}{(\sin 2\theta)^8}$

For $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ ℓ_1 is least for $\theta_1 = \frac{\pi}{4}$

For $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$ ℓ_2 is least for $\theta_2 = \frac{\pi}{8}$

$\frac{\ell_2}{\ell_1} = \frac{(\sin 2\theta_1)^8}{(\sin 2\theta_2)^8} = (\sqrt{2})^8 = \frac{16}{1}$

(42) **(51)**

$S = 1.{}^{25}C_0 + 5.{}^{25}C_1 + 9.{}^{25}C_2 + ... + (101)^{25}C_{25}$

$S = 101\,{}^{25}C_{25} + 97\,{}^{25}C_1 + ... + 1\,{}^{25}C_{25}$

$2S = (102)(2^{25})$

$S = 51(2^{25})$

(43) **(A).** $\frac{(1+x)^{10}\left[1 - \left(\frac{x}{1+x}\right)^{11}\right]}{\left(1 - \frac{x}{1+x}\right)}$

$= \frac{(1+x)^{10}[(1+x)^{11} - x^{11}]}{(1+x)^{11} \times \frac{1}{(1+x)}} = (1+x)^{11} - x^{11}$

Coefficient of x^7 is ${}^{11}C_7 = {}^{11}C_4 = 330$