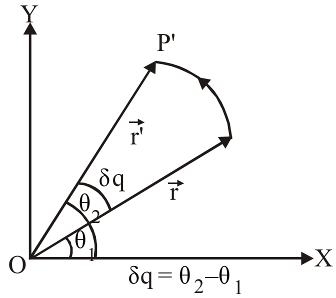
7

CIRCULAR MOTION

ANGULAR DISPLACEMENT ($\delta\theta$ or θ)

The angle described by radius vector is called angular displacement.



Infinitesimal angular displacement is a vector quantity. However, finite angular displacement is a scalar quantity.

$$\vec{d\theta_1} + \vec{d\theta_2} = \vec{d\theta_2} + \vec{d\theta_1} \quad \text{But} \quad \vec{\theta_1} + \vec{\theta_2} \neq \vec{\theta_2} + \vec{\theta_1}$$

S.I. Unit : Radian ; 1 radian $= \frac{360}{2\pi}$

Dimension : M°L°T°

No. of revolution $= \frac{\text{angular displacement}}{2\pi}$

In 1 revolution $\Delta\theta = 360° = 2\pi$ radian

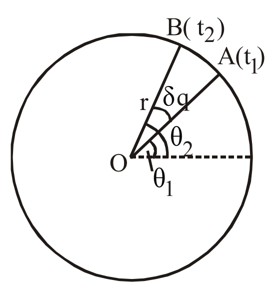
In N revolution $\Delta\theta = 360° \times N = 2\pi N$ radian

Clockwise rotation is taken as negative 

Anticlockwise rotation is taken as positive 

ANGULAR VELOCITY (ω)

The rate of change of angular displacement with time is called angular velocity. It is a vector quantity. The angle traced per unit time by the radius vector is called angular speed.



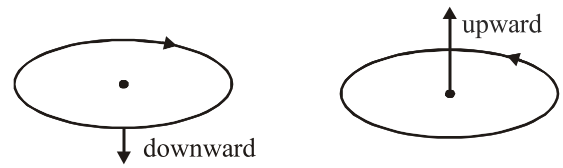
Instantaneous angular velocity

$$\omega = \lim_{\delta t \to 0} \frac{\delta\theta}{\delta t} \quad \text{or} \quad \omega = \frac{d\theta}{dt}$$

Average angular velocity, $\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$

S.I. Unit : rad/sec ; **Dimension :** $M^0L^0T^{-1}$

Direction : Infinitesimal angular displacement, angular velocity and angular acceleration are vector quantities whose direction is given by right hand rule.



Right Hand Rule : Imagine the axis of rotation to be held in the right hand with fingers curled round the axis and the thumb stretched along the axis. If the curled fingers denote the sense of rotation, then the thumb denoted the direction of the angular velocity (or angular acceleration or infinitesimal angular displacement).

Example 1 :

Calculate the angular speed of (i) hour hand of a watch and (ii) earth about its own axis.

Sol. (i) The hour hand of a watch completes one rotation in 12 hours

Angle covered in $12 \times 60 \times 60$ sec. $= 2\pi$ rad

Angular speed of hour hand

$$= \frac{2\pi}{12 \times 60 \times 60} = \frac{\pi}{21600} \text{ rad/s}$$

(ii) Earth completes one rotation about its axis in 24 hours angle covered in $24 \times 60 \times 60$ sec. $= 2\pi$ rad

Angular speed of earth $= \frac{2\pi}{12 \times 60 \times 60} = \frac{\pi}{43200}$ rad/s

When we say that the angle is 2π, it implies that angle is 2π radian. Usually, we do not mention the unit of the angle, when it is expressed in radian.

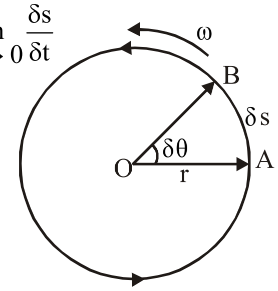
Relation between angular velocity and linear velocity :

Suppose the particle moves along a circular path from point A to point B in infinitesimally time δt.

As, $\delta t \to 0, \delta\theta \to 0$.

$\therefore$ arc AB = chord AB

i.e. displacement of the particle is along a straight line.



$\therefore$ Linear velocity, $v = \lim_{\delta t \to 0} \frac{\delta s}{\delta t}$

But, $\delta s = r.\delta\theta$

$\therefore \quad v = \lim_{\delta t \to 0} \frac{r.\delta\theta}{\delta t}$

$= r . \lim_{\delta t \to 0} \frac{\delta\theta}{\delta t}$

But, $\lim_{\delta t \to 0} \frac{\delta\theta}{\delta t} = \omega$ = angular velocity

$\therefore$ $v = r \cdot \omega$ [For circular motion only]
i.e. (Linear velocity) = (Radius) × (Angular velocity)

In vector notation, $\vec{v} = \vec{\omega} \times \vec{r}$ [In general]

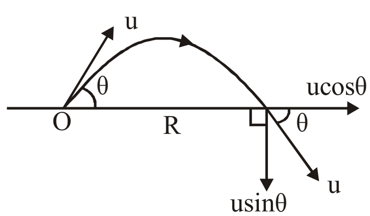
Linear velocity of a particle performing circular motion is the vector product of its angular velocity and radius vector.
In general,

$$\omega = \frac{\text{Velocity component perpendicular to the line joning the particle and the observer}}{\text{Distance between the particle and observer}} = \frac{v_\perp}{r}$$

Example 2 :

A particle is launched from horizontal plane with speed u and angle of projection θ. Find angular velocity as observed from the point of projection of the particle at the time of landing.

Sol.



w.r.t O, $\omega = \frac{u \sin\theta}{R}$; $\omega = \frac{u\sin\theta}{\left(\frac{u^2 \sin 2\theta}{g}\right)} = \frac{g}{2u\cos\theta}$

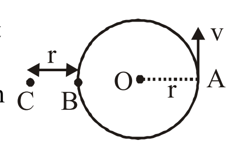
Relative angular velocity :

Angular velocity of a particle A with respect to other moving particle B is the rate at which position vector of A with respect to B rotates at that instant (or it is simply, angular velocity of A with origin fixed at B). Angular velocity of A w.r.t. B, ω_{AB} is mathematically define as

$$\omega_{ab} = \frac{\text{Component of relative velocity of A w.r.t. B, perpendicular to line}}{\text{separation between A and B}} = \frac{(V_{AB})_\perp}{r_{AB}}$$

Example 3 :

A particle is moving with constant speed in a circle as shown, find the angular velocity of the particle A with respect to fixed point B and C if angular velocity with respect to O is ω.



Sol. Angular velocity of A with respect to O is

$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v}{r} = \omega$$

$$\therefore\ \omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{v}{2r} = \frac{\omega}{2} \text{ and } \omega_{AC} = \frac{(v_{AC})_\perp}{r_{AC}} = \frac{v}{3r} = \frac{\omega}{3}$$

ANGULAR ACCELERATION (α)

The rate of change of angular velocity with time is called angular acceleration.

Average angular acc. Instantaneous angular acc.

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

It is a vector quantity, whose direction is along the change in direction of angular velocity.

S.I. Unit : radian/sec^2 ; **Dimension :** $M^\circ L^\circ T^{-2}$

Relation between angular acceleration & linear acceleration

For perfect circular motion we know, $v = \omega r$
On differentiating with respect to time, we get,

$$\frac{dv}{dt} = r\frac{d\omega}{dt} \text{ ; Tangential acceleration, } a_t = r\alpha$$

Remember that $a_t = \frac{dv}{dt}$ is the rate of change of speed and is not the rate of the change of velocity. It is, therefore not equal to the net acceleration.
Infact it is the component of acceleration along the tangent and hence suffix t is used for tangential acceleration.
a_t is known as the tangential acceleration.

In vector form, $\vec{a}_t = \vec{\alpha} \times \vec{r}$

(tangential acceleration) = (angular acc) × (radius)]

Example 4 :

A particle travels in a circle of radius 20 cm at a speed that uniformly increases. If the speed changed from 5.0 m/s to 6.0m/s in 2.0 sec, find the angular acceleration.

Sol. The tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{6.0 - 5.0}{2.0} \text{ m/s}^2 = 0.5 \text{ m/s}^2.$$

The angular acceleration is

$$\alpha = \frac{a_t}{r} = \frac{0.5 \text{ m/s}^2}{20 \text{ cm}} = 2.5 \text{ rad/sec}^2.$$

KINEMATICS OF CIRCULAR MOTION

Kinematics of circular motion resembles with the kinematics of linear motion. Displacement in linear motion is equivalent to angular displacement in circular motion. Similarly, linear velocity equivalent to angular velocity and acceleration equivalent to angular acceleration.
For constant angular acceleration :

$$\vec{\omega}_{(t)} = \vec{\omega}_0 + \vec{\alpha}t \text{ ; } \vec{\theta}_{(t)} = \vec{\theta}_0 + \vec{\omega}_0 t + \frac{1}{2}\vec{\alpha}t^2$$

$$\omega_{(t)}^2 = \omega_0^2 + 2\alpha\theta \text{ ; } \theta = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2n-1)$$

Here ω_0 = initial angular velocity, $\omega_{(t)}$ = angular velocity after time t, θ_0 = Initial angular position

$\theta_{(t)}$ = Angular position after time t,

θ_{nth} = Angular displacement in n^{th} second.

These equations are valid if α is a constant. If angular acceleration is variable then use calculus approach.

Example 5 :

A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1. In the next 2 sec, it rotates through an additional angle θ_2. Find the ratio of θ_2/θ_1.

Sol. Using relation $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$.

In the first case, $\theta_1 = \frac{1}{2}\alpha . 4 = 2\alpha$

In the second case, $\theta_2 = \omega t + \frac{1}{2}\alpha t^2$

where ω is initial angular speed

For next 2 sec. i.e. final angular speed of first 2 sec.

$\therefore \omega = 0 + 2\alpha$; $\theta_2 = 2\alpha . 2 + \frac{1}{2}\alpha . 4 = 6\alpha \quad \therefore \frac{\theta_2}{\theta_1} = \frac{3}{1}$

Alternatively, we can write

$$\theta_1 + \theta_2 = \frac{1}{2}\alpha (4)^2 = 8\alpha ; \theta_2 = 6\alpha$$

Example 6 :

A disc starts rotating with constant angular acceleration of $\pi/2$ rad/s^2 about a fixed axis perpendicular to its plane and through its centre. Find

(a) the angular velocity of the disc after 4s

(b) the angular displacement of the disc after 4 s and

(c) number of turns accomplished by the disc in 4 s.

Sol. Here $\alpha = \frac{\pi}{2} rad/s^2$, $\omega_0 = 0$, $t = 4$ s

(a) $\omega_{(4s)} = 0 + \left(\frac{\pi}{2} rad/s^2\right) \times 4s = 2\pi\ rad/s.$

(b) $\theta_{(4s)} = 0 + \frac{1}{2}\left(\frac{\pi}{2} rad/s^2\right) \times (16 s^2) = 4\pi$ radian.

(c) n.2π rad. = 4π radian $\Rightarrow$ n = 2.

CENTRIPETAL FORCE

In circular motion the force acting on the particle along the radius and towards the centre keeps the body moving along the circular path. This force is called centripetal force.

Explanation :

(i) Centripetal force is necessary for circular motion.

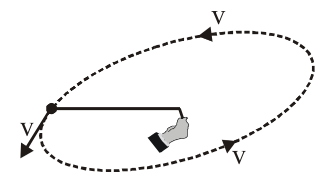
(ii) It is along the radius and towards the centre.

(iii) Centripetal force = [mass] × [centripetal acceleration]

$$= \frac{mv^2}{r} = mr\omega^2$$

(iv) Centripetal force is due to known interaction. Therefore it is a real force.

Ex. (a) If an object tied to a string is revolved uniformly in a horizontal circle, the centripetal force is due to the tension imparted to the string by the hand.



(b) When a satellite is revolving in circular orbit round the earth, the centripetal force is due to the gravitational force of attraction between the satellite and the earth.

(c) In an atom, an electron revolves in a circular orbit round the nucleus. The centripetal force is due to the electrostatic force of attraction between the positively charged nucleus and negatively charged electron.

The Magnitude of the centripetal acceleration

Suppose that the particle travels from a point P_1 to an adjacent point P_2 in time δt and that the angle P_1OP_2 is $\delta\theta$. Magnitude of change in velocity from P_1 to P_2

$= 2v \sin(\delta\theta/2)$

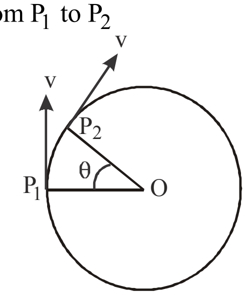
$$[\ \Delta v = \sqrt{v^2 + v^2 - 2v^2 \cos\delta\theta} = \sqrt{2v^2(1-\cos\delta\theta)}$$

$$= \sqrt{2v^2 2\sin^2(\delta\theta/2)} = 2v\sin(\delta\theta/2)\ ]$$

In the direction of $\overrightarrow{P_1O}$, the acceleration is given approximately by

$$\frac{\text{increase in velocity along } \overrightarrow{P_1O} \text{ from } P_1 \text{ to } P_2}{\text{time taken to travel from } P_1 \text{ to } P_2}$$

i.e. $\frac{2v\sin(\delta\theta/2)}{\delta t}$



Now as $\delta\theta \to 0$,

$\sin(\delta\theta/2) \to (\delta\theta/2)$

and $\frac{\delta\theta}{\delta t} \to \frac{d\theta}{dt}$

So the acceleration at P_1 towards O is v $(d\theta/dt)$

But $d\theta/dt$ is the angular velocity of the particle, which we will denote by ω and we know that $v = r\omega$

Hence, $v\frac{d\theta}{dt} = v\omega = (r\omega)\,\omega$ or $v\left(\frac{v}{r}\right)$

i.e. the radial acceleration of a particle travelling with constant speed v in a circle of radius r is towards the centre and is of magnitude. v^2/r or $r\omega^2$

Example 7 :

A space shuttle is in a circular orbit at a height of 250 km, where the acceleration of earth's gravity is 93 per cent of its surface value. What is the period of its orbit? ("Period" means the time to complete one orbit).

Sol. $a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$ $\quad [v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T}]$

Here r = radius of circular orbit of shuttle.

So $r = R_{earth} + 250\text{ km} = 6.6 \times 10^6$ m,

where $R_{earth} = 6.37 \times 10^6$ m.

$$T = \left(\frac{4\pi^2 r}{a}\right)^{1/2} = \left(\frac{(4\pi^2)\,(6.6\times10^6\,\text{m})}{(0.93)\,(9.8\,\text{m}/\text{s}^2)}\right) = 5347\text{s} = 89\text{ min.}$$

CENTRIFUGAL FORCE

The pseudo force experienced by a particle performing uniform circular motion due to accelerated frame of reference which is along the radius and directed away from the centre is called centrifugal force.

Explanation :

(i) Centrifugal force is a pseudo force as it is experienced due to accelerated frame of reference. The origin of this force is unknown. (For observer in non-inertial frame)

(ii) It is along the radius and away from the centre.

(iii) The centrifugal force is having the same magnitude as that of centripetal force. But, its direction is opposite to that of centripetal force. It is not due to reaction of centripetal force because without action, reaction is not possible, but centrifugal force can exists without centripetal force.

(iv) Magnitude of the centrifugal force is mv^2/r or $mr\omega^2$.

Note : Pseudo force acts in non inertial frame i.e. accelerated frame of reference in which Newton's Law's of Motion do not hold good.

Ex. (a) When a car moving along a horizontal curve takes a turn, the person in the car experiences a push in the outward direction.

(b) The coin placed slightly away from the centre of a rotating gramophone disc slips towards the edge of the disc.

(c) A cyclist moving fast along a curved road has to lean inwards to keep his balance.

Applications of centrifugal force :

(i) The centrifugal pump used to lift the water, works on the principle of centrifugal force.

(ii) A cream-separator used in the dairy work, works on the principle of centrifugal force.

(iii) Centrifuge used for the separation of suspended particle from the liquid, works on the principle of centrifugal force.

(iv) The "spin drier" of washing machine works on the principle of centrifugal force.

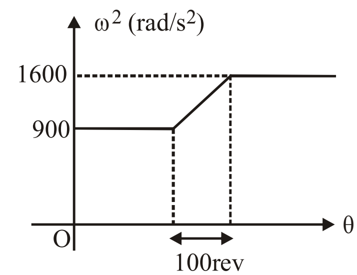
TRY IT YOURSELF-1

Q.1 When a particle moves in a circle with a uniform speed:
(A) Its velocity and acceleration are both constant
(B) Its velocity is constant but the acceleration changes
(C) Its acceleration is constant but the velocity changes
(D) Its velocity and acceleration both change.

Q.2 A body moves with constant angular velocity on a circle. Magnitude of angular acceleration is –
(A) $r\omega^2$ (B) constant
(C) zero (D) None of the above

Q.3 Estimate the acceleration of the moon towards the earth given it orbits it once in 28 days at a radius of about a quarter of a million miles. (I know the units are funny and numbers are approximate. This problem tests your ability to give a quick and decent estimate, say to 10 percent.)

Q.4 The square of the angular velocity ω of a certain wheel increases linearly with the angular displacement during 100rev of the wheel's motion as shown. Compute the time t required for the increase.



Q.5 A helicopter blades has an angular speed of ω= 6.50 rev/s and an angular acceleration of α = 1.30 rev/s². For point 1 at a distance 3m and point 2 at a distance 6.70m on the blade, calculate the magnitudes of (a) the tangential speeds and (b) the tangential accelerations.

Q.6 The maximum speed of the blades on rotary lawn mowers is limited to reduce the hazard from flying stones and other debris. A currently available model has a rotation rate of 3700 revolutions per minute and a blade 0.25 m in radius. What is the speed at the tip of the blade ?

Q.7 A wheel rotates with an angular acceleration given by $\alpha = 4ar^3 - 3bt^2$, where t is the time and a and b are constants. If the wheel has initial angular speed ω_0, write the equation for the (i) angular speed (ii) angular displacement.

Q.8 The lawn mower blade has an angular velocity of 387 rad/s and a radius of 0.25 m. If it accelerates to this velocity uniformly from rest over a 10s interval, find (a) the angular acceleration; (b) the tangential acceleration at the tip of the blade.

Q.9 A spotlight S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/sec. The spot of light P moves along the floor at a distance of 3m. Find the velocity of the spot P when θ = 45°.

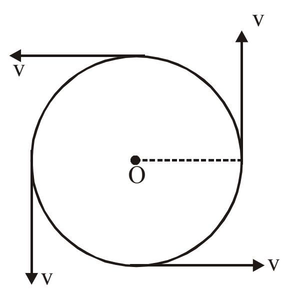
Q.10 A fan is rotating with angular velocity 100 rev/sec. Then it is switched off. It takes 5 minutes to stop.
(a) Find the total number of revolution made before it stops. (Assume uniform angular retardation)
(b) Find the value of angular retardation.
(c) Find the average angular velocity during this interval.

ANSWERS

(1) (D) **(2)** (BC) **(3)** 1/3600 times g **(4)** $(40\pi/7)$ sec.
(5) (a) Point 1 : $v_T = 122$ m/s (273 mph) ;
Point 2 : $v_T = 273$ m/s (611 mph)
(b) Point 1 : $a_T = 24.5$ m/s^2 ; Point 2 : $a_T = 54.7$ m/s^2
(6) 350 km h^{-1}.
(7) (i) $\omega = \omega_0 + at^4 - bt^3$ (ii) $\theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}$
(8) (a) 38.7 rad s^{-2} (b) 9.68 m s^{-2}
(9) 0.6 m/s
(10) (a) 15000 revol. (b) $\frac{1}{3}$ rev/sec^2 (c) 50 rev/sec.

UNIFORM CIRCULAR MOTION

Motion of a particle along the circumference of a circle with a constant speed is called uniform circular motion.
Uniform circular motion is an accelerated motion.
In case of uniform circular motion : $\frac{dv}{dt} = 0$



(i) Speed remains constant. v = constant and $v = \omega r$
Angular velocity ω = constant
Motion will be periodic with time period

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

(ii) **Frequency of Uniform Circular Motion :** The number of revolutions performed per unit time by the particle performing uniform circular motion is called the frequency(n).

$$\therefore \quad n = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\omega}{2\pi}$$

S.I. unit of frequency is Hz.

(iii) As ω = constant, from $\omega = \omega_0 + \alpha t$
$\Rightarrow$ angular acceleration $\alpha = 0$.
As $a_t = \alpha r$, tangential acc. $a_t = 0$

(iv) As $a_t = 0$, $a = (a_r^2 + a_t^2)^{1/2}$ yields $a = a_r$, i.e., acceleration is not zero but along radius towards centre and has magnitude $a = a_r = (v^2/r) = r\omega^2$.

(v) Speed and magnitude of acceleration are constant but their directions are always changing so velocity and acceleration are not constant. Direction of $\vec{v}$ is always along the tangent while that of $\vec{a}_r$ along the radius. $\vec{v} \perp \vec{a}_r$

(vi) If the moving body comes to rest, i.e. $\vec{v} \to 0$, the body will move along the radius towards the centre and if radial acceleration a_r vanishes, the body will fly off along the tangent. So a tangential velocity and a radial acceleration (hence force) is a must for uniform circular motion.

As $\vec{F} = \frac{mv^2}{r} \neq 0$, so the body is not in equilibrium and linear momentum of the particle moving on the circle is not conserved. However, as the force is central, i.e., $\vec{\tau} = 0$, so angular momentum is conserved, i.e.,

$\vec{p} \neq$ constant but $\vec{L}$ = constant.

(vii) The work done by centripetal force is always zero as it is perpendicular to velocity and hence displacement. By work-energy theorem as :
Work done = change in kinetic energy $\Delta K = 0$
So K (Kinetic energy) remains constant.
e.g. Planets revolving around the sun, motion of an electron around the nucleus in an atom
Speed $|\vec{v}|$ = constant, $\vec{v}$ changing continuously
$\vec{a}_r \Rightarrow$ directed towards the center, continuously changing direction.

Example 8 :
A stone of mass 1 kg is whirled in a circular path of radius 1m. Find out the tension in the string if the linear velocity is 10 m/s ?

Sol. Tension $= \frac{mv^2}{R} = \frac{1 \times (10)^2}{1} = 100$ N

Example 9 :
A satellite of mass 10^7 kg is revolving around the earth with a time period of 30 days at a height of 1600 km. Find out the force of attraction on satellite by earth ?

Sol. Force $= m\omega^2 R$ and

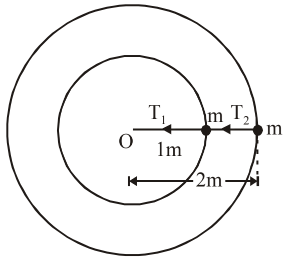
$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{30 \times 86400} = \frac{6.28}{2.59 \times 10^6}$$

$$\text{Force} = m\omega^2 r = \left(\frac{6.28}{2.59 \times 10^6}\right)^2 \times 10^7 \times (6400 + 1600) \times 10^3$$

$$= 2.34 \times 10^6 \text{ N}$$

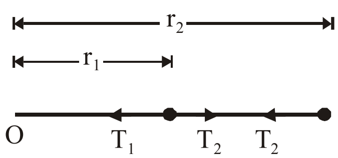
Example 10 :

Two balls of equal masses are attached to a string at distances 1m and 2m from one end as shown in fig.
The string with masses is then moved in a horizontal circle with constant speed. Find the ratio of the tension T_1 and T_2 ?



Sol. Let the radii of the two circles are r_1 and r_2. The linear speed of the two masses are $v_1 = \omega r_1$, $v_2 = \omega r_2$ where ω is the angular speed of the circular motion.
The tension in the strings are such that

$$T_2 = \frac{m v_2^2}{r_2} = m\omega^2 r_2$$



$$T_1 - T_2 = \frac{m v_1^2}{r_1} = m\omega^2 r_1$$

$$\therefore \quad T_1 = m\omega^2 r_1 + T_2 = m\omega^2 (r_1 + r_2)$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{r_1 + r_2}{r_2} = \frac{1+2}{2} = \frac{3}{2}$$

NON UNIFORM CIRCULAR MOTION

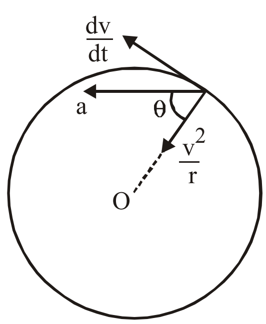
A circular motion in which both direction and magnitude of velocity changes.

Examples of non uniform circular motion :

A merry-go-round spinning up from rest to full speed, or a ball whirling around in a vertical circle.
The net acceleration is neither parallel nor perpendicular to the velocity.
We can resolve the acceleration vector into two components:

(a) Radial Acceleration :

The component of the acceleration towards the centre is $\omega^2 r = v^2/r$
a_r perpendicular to the velocity
$\Rightarrow$ changes only the directions of velocity.
Acts just like the acceleration in uniform circular motion.



$$a_c \text{ or } a_r = \frac{v^2}{r} = \omega^2 r$$

Centripetal force $\quad F_c = \frac{m v^2}{r} = m\omega^2 r$

(b) Tangential Acceleration :

The component along the tangent (along the direction of motion) is $\frac{dv}{dt}$

a_t parallel to the velocity (since it is tangent to the path)

$\Rightarrow$ changes magnitude of the velocity acts just like one-dimensional acceleration

$\Rightarrow \quad a_t = \frac{dv}{dt}$, where $v = \frac{ds}{dt}$ and s = length of arc

Tangential force $F_t = ma_t$

Net acceleration vector is obtained by vector addition of these two components.
The magnitude of the net acceleration

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

In non-uniform circular motion :

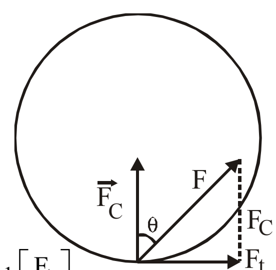
speed $|\vec{v}| \neq$ constant angular velocity $\omega \neq$ constant
i.e. speed $\neq$ constant i.e. angular velocity $\neq$ constant
If at any instant $\Rightarrow$ v = magnitude of velocity of particle

$\Rightarrow$ r = radius of circular path
$\Rightarrow$ ω = angular velocity of a particle,
then at that instant $v = r\,\omega$

Net force on the particle in non uniform circular motion

$$\vec{F} = \vec{F}_c + \vec{F}_t$$

$$\Rightarrow \quad F = \sqrt{F_c^2 + F_t^2}$$



If θ is the angle made by F with F_c

then $\tan\theta = \frac{F_t}{F_c} \quad \Rightarrow \theta = \tan^{-1}\left[\frac{F_t}{F_c}\right]$

[angle between F_c and F_t is 90º]
Angle between F and F_t is $(90º - \theta)$

Net acceleration, $a = \sqrt{a_c^2 + a_t^2} = \frac{F_{net}}{m}$

NOTE

(i) In both uniform and non-uniform circular motion F_c is perpendicular to velocity. So work done by centripetal force will be zero in both the cases.

(ii) In uniform circular motion $F_t = 0$, as $a_t = 0$, so work done will be zero by tangential force.
But in non-uniform circular motion $F_t \neq 0$, so work done by tangential force is non zero.

Rate of work done by net force in non-uniform circular motion = rate of work done by tangential force

$$\Rightarrow \quad P = \frac{dW}{dt} = \vec{F}_t \cdot \vec{v} = \vec{F}_t \cdot \frac{d\vec{x}}{dt}$$

(iii) In a circle as tangent and radius are always normal to each other, so $\vec{a}_t \perp \vec{a}_r$.

Net acceleration in case of circular motion

$$a = \sqrt{a_r^2 + a_t^2}$$

Here it must be noted that a_t governs the magnitude of $\vec{v}$ while a_r its direction of motion so that

if $a_r = 0$ and $a_t = 0$, $a \to 0 \Rightarrow$ motion is uniform translatory

if $a_r = 0$ but $a_t \neq 0$, $a \to a_t \Rightarrow$ motion is accelerated translatory

if $a_r \neq 0$ but $a_t = 0$, $a \to a_r \Rightarrow$ motion is uniform circular

if $a_r \neq 0$ & $a_t \neq 0$, $a \to \sqrt{a_r^2 + a_t^2} \Rightarrow$ motion is nonuniform circular

NOTE

(i) In one-dimensional motion, acceleration is always parallel to velocity and changes only the magnitude of the velocity vector.

(ii) In uniform circular motion, acceleration is always perpendicular to velocity and changes only the direction of the velocity vector.

(iii) In the more general case, like projectile motion, acceleration is neither parallel nor perpendicular to velocity. Causing change in both the magnitude and direction of the velocity vector.

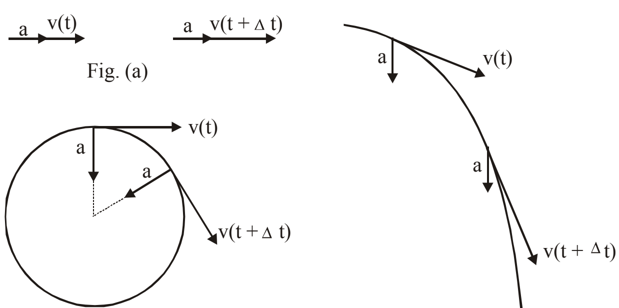


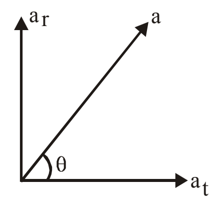
Fig. (a) Fig.(b) Fig. (c)

Example 11 :

A road makes a 90° bend with a radius of 190 m. A car enters the bend moving at 20 m/s. Finding this too fast, the driver decelerates at 0.92 m/s². Determine the acceleration of the car when its speed rounding the bend has dropped to 15 m/s.

Sol. Since it is rounding a curve, the car has a radial acceleration associated with its changing direction, in addition to the tangential deceleration that changes its speed.

We are given that $a_t = -0.92\ m/s^2$, since the car is slowing down, the tangential acceleration is directed opposite the velocity.



The radial acceleration is

$$a_r = \frac{v^2}{r} = \frac{(15\ m/s)^2}{190\ m} = 1.2\ m/s^2$$

Magnitude of net acceleration.

$$a = \sqrt{a_r^2 + a_t^2} = [(1.2\ m/s)^2 + (0.92\ m/s)^2]^{1/2} = 1.5\ m/s^2$$

and points at an angle

$$\theta = \tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{1.2\ m/s^2}{0.92\ m/s^2}\right) = 53°$$

relative to the tangent line to the circle.

Example 12 :

A particle is constrained to move in a circular path of radius r = 6m. Its velocity varies with time according to the relation v = 2t (m/s). Determine its (i) centripetal acceleration, (ii) tangential acceleration, (iii) instantaneous acceleration at (a) t = 0 sec, and (b) t = 3 sec.

Sol. (a) At t = 0, v = 0, Thus $a_r = 0$

but $\frac{dv}{dt} = 2$ thus $a_t = 2\ m/s^2$

and $a = \sqrt{a_t^2 + a_r^2} = 2\ m/s^2$

(b) At t = 3 sec, v = 6 m/s,

so $a_r = \frac{v^2}{r} = \frac{(6)^2}{6} = 6\ m/s^2$ and $a_t = \frac{dv}{dt} = 2\ m/s^2$

Therefore, $a = \sqrt{2^2 + 6^2} = \sqrt{40}\ m/s^2$

TRY IT YOURSELF-2

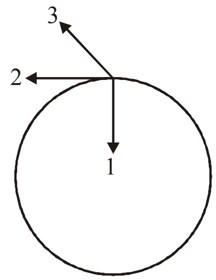
Q.1 A particle moves in a circle of radius 2.0cm at a speed given by v = 4t, where v is in cm/s and t is in seconds.
(a) Find the tangential acceleration at t = 1s.
(b) Find total acceleration at t = 1s.

Q.2 A flywheel of radius 20 cm starts from rest, and has a constant angular acceleration of 60 rad/s^2. find
(a) the magnitude of the linear acceleration of a point on the rim after 0.15s.
(b) the number of revolutions completed in 0.25s.

Q.3 If a particle moves in a circle describing equal angles in equal times, its velocity vector:
(A) remains constant
(B) changes in magnitude
(C) changes in direction
(D) changes both in magnitude and direction

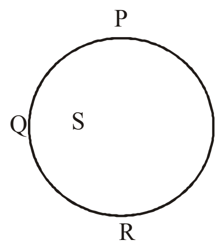
Q.4 A car is rounding a circular turn of radius 200 m at constant speed. The magnitude of its centripetal acceleration is $2 m/s^2$. What is the speed of the car?
(A) 400 m/s (B) 20 m/s
(C) 100 m/s (D) 10 m/s

Q.5 As the object speeds up along the circular path in a counterclockwise direction, shown, its acceleration points:



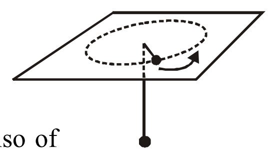
(A) toward the center of the circular path.
(B) in a direction tangential to the circular path.
(C) outward.
(D) none of the above.

Q.6 An object moves counter-clockwise along the circular path shown. As it moves along the path its acceleration vector continuously points toward point S. The object



(A) speeds up at P, Q, and R.
(B) slows down at P, Q, and R.
(C) speeds up at P and slows down at R.
(D) slows down at P and speeds up at R.

Q.7 A puck of mass m is moving in a circle at constant speed on a frictionless table as shown. The puck is connected by a string to a suspended bob, also of mass m, which is at rest below the table. Half of the length of the string is above the tabletop and half below. What is the magnitude of the centripetal acceleration of the moving puck? Let g be the gravitational constant.



(A) The magnitude of the centripetal acceleration of the moving puck is less than g .
(B) The magnitude of the centripetal acceleration of the moving puck is equal to g .
(C) The magnitude of the centripetal acceleration of the moving puck is greater than g .
(D) The magnitude of the centripetal acceleration of the moving puck is zero.

Q.8 You stand on a merry-go-round spinning at f revolutions per second. You are R meters from the center. What is the minimum coefficient of static friction μ_s between your shoes and the floor that will keep you from slipping off?

Q.9 A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?

Q.10 A turn table rotates with constant angular acceleration of $2 rad/s^2$ about a fixed vertical axis through its centre and perpendicular to its plane. A coin is placed on it at a distance of 1m from the axis of rotation. The coin is always at rest relative to the turntable. If at t = 0 the turntable was at rest, then find the total acceleration of the coin after one second.

ANSWERS

(1) (a) $4 cm/s^2$, (b) $4\sqrt{5}\ cm/s^2$

(2) (a) $20.2 m/s^2$, (b) 0.3 rev.

(3) (C) **(4)** (D) **(5)** (B)

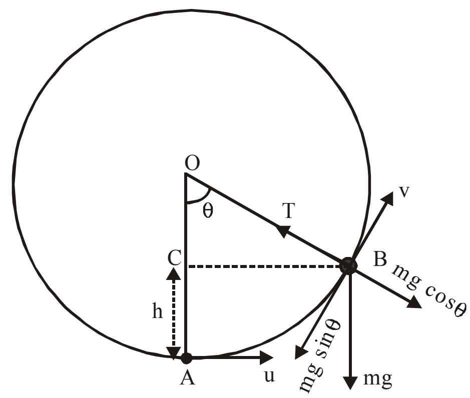
(6) (C) **(7)** (B) **(8)** $\mu_s = \dfrac{R\,(2\pi f)^2}{g}$

(9) $0.86 m/s^2$, $\tan^{-1}(1.4) = 54°28'$ **(10)** $2\sqrt{5} m/s^2$

VERTICAL CIRCULAR MOTION

Motion of a body suspended by string :

This is the best example of non-uniform circular motion.
Suppose a particle of mass m is attached to an inextensible light string of length r. The particle is moving in a vertical circle of radius r, about a fixed point O.
At lowest point A velocity of particle = u
(in horizontal direction)
After covering $\angle\theta$ velocity of particle = v (at point B)
Resolve weight (mg) into two components
mg cos θ (along radial direction) ;
mg sin θ (tangential direction)



Force T – mg cos θ provides necessary centripetal force.

$$T - mg\cos\theta = \frac{mv^2}{r} \qquad ...(1)$$

$$\Delta\, OCB,\ \cos\theta = \frac{r-h}{r} \qquad ...(2)$$

or $h = r(1 - \cos\theta)$

By conservation of energy at point A and B

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh$$

or $v^2 = u^2 - 2gh$ ...(3)

Substitute value of cos θ and v^2 in eqn. (1)

$$T - mg\left[\frac{r-h}{r}\right] = \frac{m}{r}(u^2 - 2gh)$$

or $T = \dfrac{m}{r}[u^2 - 2gh + gr - gh]$

or $T = \dfrac{m}{r}[u^2 + gr - 3gh]$ .....(4)

(i) It velocity becomes zero at height h_1

$$0 = u^2 - 2gh_1 \quad \text{or} \quad 0 = \frac{m}{r}[u^2 + gr - 3gh_2]$$

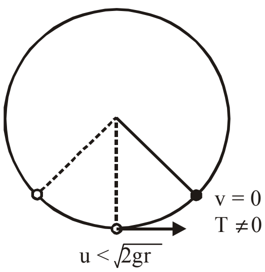
or $\quad h_1 = \dfrac{u^2}{2g}$ .....(5)

(ii) It tension becomes zero at height h_2

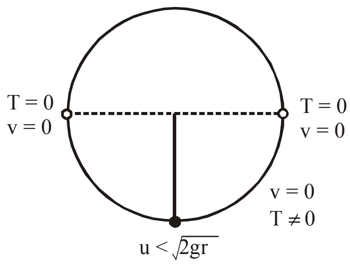
$u^2 + gr - 3gh_2 = 0 \quad \text{or} \quad h_2 = \dfrac{u^2 + gr}{3g}$ ......(6)

(A) Case of oscillation

It $v = 0, T \neq 0$ then $h_1 < h_2$



Case of oscillation



Case of oscillation semi-circular path

$$\frac{u^2}{2g} < \frac{u^2 + gr}{3g} \quad ; \quad 3u^2 < 2u^2 + 2gr$$

$$u^2 < 2gr \quad ; \quad u < \sqrt{2gr}$$

(B) Case of leaving the circle

It $v \neq 0, T = 0$

then $h_1 > h_2$

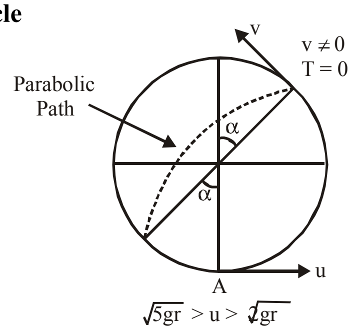
$$\Rightarrow \frac{u^2}{2g} > \frac{u^2 + gr}{3g}$$

$3u^2 > 2u^2 + 2gr$

$u^2 > 2gr$

$u > \sqrt{2gr}$

$\sqrt{5gr} > u > \sqrt{2gr}$



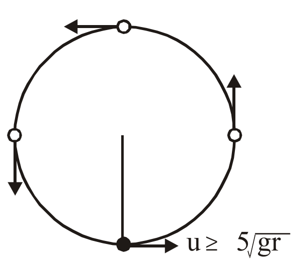
Case of leaving the circle

(C) Case of complete the circle

$u \geq \sqrt{5gr}$

$T \geq 0$

$v \neq 0$

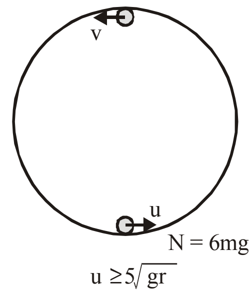


NOTE

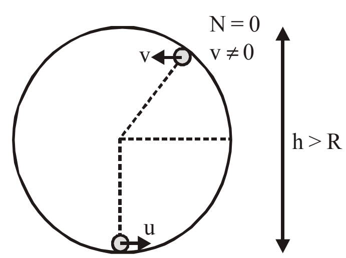
1. The same conditions apply if a particle moves inside a smooth spherical shell of radius R. The only difference is that the tension is replaced by the normal reaction N. This is shown in the figure given below.

$v = \sqrt{gR} \quad ; \quad N = 0$

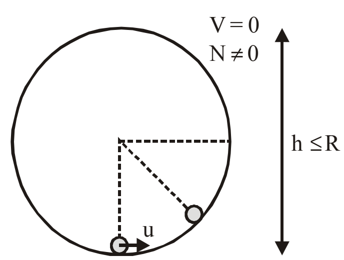
(i) Condition of looping the loop is $u \geq \sqrt{5gR}$



(ii) Condition of leaving the circle $\sqrt{2gR} < u < \sqrt{5gR}$



(iii) Condition of oscillation is $0 < u \leq \sqrt{2gR}$



2. If a particle of mass m is connected to a light rod and whirled in a vertical circle of radius R, then to complete the circle, the minimum velocity of the particle at the bottom most points is not $\sqrt{5gR}$. Because in this case, velocity of the particle at the topmost point can be zero also. Using conservation of mechanical energy between points A and B as shown in fig.(a) we get

$$\frac{1}{2}m(u^2 - v^2) = mgh$$

or $\quad \dfrac{1}{2}mu^2 = mg(2R) \quad \text{or} \quad u = 2\sqrt{gR}$

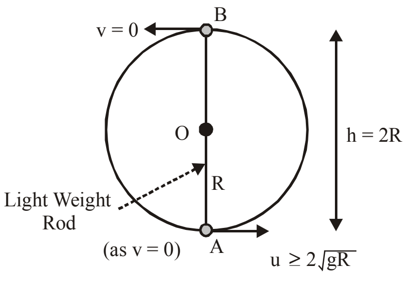


Fig. (a)

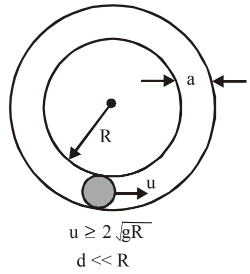


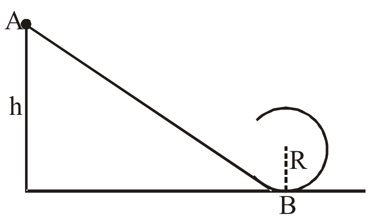
Fig. (b)

Therefore, the minimum values of u in this case is $2\sqrt{gR}$.

Same is the case when a particle is compelled to move inside a smooth vertical tube as shown in fig. (b).

Example 13 :

A ball is released from height h as shown in fig. Find the condition for the particle to complete the circular path.



Sol. According to law of conservation of energy (K.E + P.E) at A = (K.E + P.E) at B

$$\Rightarrow 0 + mgh = \frac{1}{2}mv^2 + 0 \Rightarrow v = \sqrt{2gh}$$

But velocity at the lowest point of circle,

$$v \geq \sqrt{5gR} \Rightarrow \sqrt{2gh} \geq \sqrt{5gR} \Rightarrow h \geq \frac{5R}{2}$$

Example 14 :

In a circus a motorcyclist moves in vertical loops inside a 'death well' (a hollow spherical chamber with holes, so that the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required to perform a vertical loop if the radius of the chamber is 25 m.

Sol. When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also acts downwards.These two forces are balanced by the outward centrifugal force acting on him.

$$\therefore \quad R + mg = \frac{mv^2}{r} \quad .....(i) \qquad r = \text{radius of the circle .}$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle).

Because of the balancing of the forces, the motorcyclist does not fall down.

The minimum speed required to perform a vertical loop is given by equation (i), when R = 0.

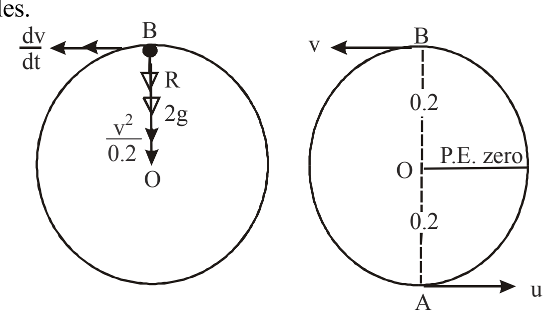
$$\therefore \quad mg = \frac{mv^2_{min}}{r} \quad \text{or} \quad v^2_{min.} = gr$$

$$\text{or} \quad v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25}\ ms^{-1} = 15.65\ ms^{-1}$$

So, the minimum speed at the top required to perform a vertical loop is 15.65 ms^{-1}.

Example 15 :

A particle of mass 2 kg is moving on the inside surface of a smooth hollow cylinder of radius 0.2 m whose axis is horizontal. Find the least speed which the particle must have at the lowest point of its path if it travels in complete circles.



Sol. Applying Newton's law along BO we have

$$R + 2g = 2\left(\frac{v^2}{0.2}\right) \qquad .... (1)$$

Conservation of mechanical energy from A to B gives

$$\left(\frac{1}{2}\right)(2u^2) - 2g\,(0.2) = \left(\frac{1}{2}\right)(2v^2) + 2g\,(0.2) \quad .... (2)$$

From (1) and (2)

$$R + 2g = 10\,(u^2 - 0.8g) \quad \text{or} \quad R = 10u^2 - 10g$$

But, for complete circles, $R \geq 0$ at B (i.e. contact is not lost at any point). Therefore, $10u^2 - 10g \geq 0 \Rightarrow u^2 \geq g$

Hence the least value of u is $\sqrt{g} = \sqrt{9.8} = 3.1\ ms^{-1}$

Example 16 :

A roller coaster of mass M is at the top of the Loop-the-loop of radius R at twice the minimum speed possible. What force does the track exert on it? What force does it exert when it is at the bottom of the circle?

(Use conservation of energy if needed.)

Sol. When a roller coaster is at the top of it's track, it's minimum velocity occurs when the normal force equals 0.

If the velocity gets any smaller than this, the roller coaster cannot maintain its circular motion and it falls off the track. The only force acting on the roller coaster is the force due to gravity.

$$\frac{Mv_{min}^2}{R} = Mg \Rightarrow v_{min} = \sqrt{Rg}$$

In this case, $v = 2\sqrt{Rg}$. When the roller coaster is at the top of the track, the normal force points away from the track, downwards towards the center of the circle. Solving for the normal force,

$$F = \frac{Mv^2}{R} = \frac{M(2\sqrt{Rg})^2}{R} = Mg + N \Rightarrow N = 3\,Mg$$

We can find the velocity of the roller coaster at the bottom of the track using conservation of energy. Taking the potential energy to be zero at the bottom, at the top of the track, the roller coaster has kinetic energy equal to $\frac{1}{2}Mv^2$ and a potential energy of Mg (2R). This must be equal to its energy at the bottom of the track when it has only kinetic energy. Conserving energy and solving for the final velocity,

$$\frac{1}{2}Mv_f^2 = \frac{1}{2}Mv^2 + Mg\,(2R) = \frac{1}{2}M\,(4Rg) + Mg\,(2R)$$

$$= 4MgR$$

$$v_f = \sqrt{8gR}$$

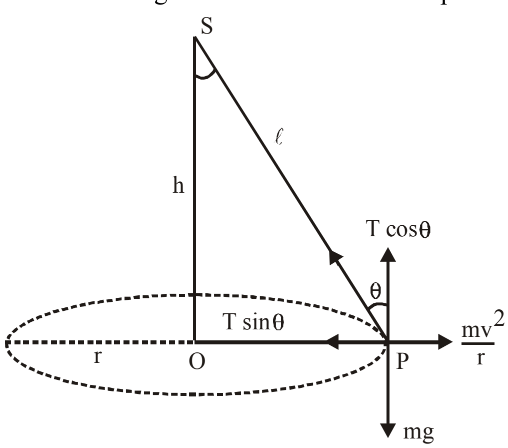
Now, we need to find the normal force when the roller coaster is at the bottom of the track with velocity v_f. The normal force still points away from the track, but now that is upwards towards the center of the circle.

$$\frac{Mv_f^2}{R} = 8Mg = N - Mg \Rightarrow N = 9Mg$$

APPLICATIONS OF CIRCULAR MOTION

1. The conical pendulum :

A conical pendulum consists of a body attached to a string, such that it can revolve in a horizontal circle with uniform speed. The string traces out a cone in the space.



The force acting on the bob are: (a) Tension T (b) weight mg. The horizontal component T sinθ of the tension T provides the centripetal force and the vertical component Tcosθ balances the weight of bob

$$\therefore \quad T\sin\theta = \frac{mv^2}{r} \text{ and } T\cos\theta = mg$$

From these equations

$$T = mg\sqrt{1+\frac{v^4}{r^2g^2}} \quad ...(1) \quad \text{and } \tan\theta = \frac{v^2}{rg} \quad ...(2)$$

If h = height of conical pendulum

$$\tan\theta = \frac{OP}{OS} = \frac{r}{h} \quad ..(3)$$

From (2) and (3), $\frac{v^2}{rg} = \frac{r}{h} \Rightarrow \omega^2 = \frac{v^2}{r^2} = \frac{g}{h}$

The time period of revolution, $T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$

Hints to solve numerical problem

(i) First show all force acting on a particle.

(ii) Resolve these forces along radius and tangent.

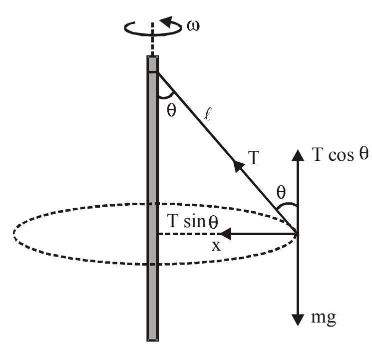
(iii) Resultant force along radial direction provides necessary centripetal force.

(iv) Resultant force along tangent equals to zero.

Example 17 :

A vertical rod is rotating about its axis with a uniform angular speed ω. A simple pendulum of length ℓ is attached to its upper end what is its inclination with the rod ?

Sol. Let the radius of the circle in which the bob is rotating is x, the tension in the string is T, weight of the bob mg, and inclination of the string θ. Then T cosθ balances the weight mg and T sinθ provides the centripetal force necessary for circular motion.



That is $T\cos\theta = mg$ and $T\sin\theta = m\omega^2 x$

but $x = \ell\sin\theta \quad \therefore T = m\omega^2\ell$

and $\cos\theta = \frac{mg}{T} = \frac{mg}{m\omega^2\ell}$

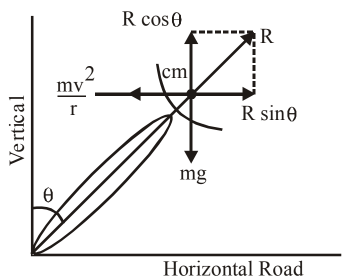
or $\theta = \cos^{-1}\left(\frac{g}{\omega^2\ell}\right)$

2. Banked tracks :

(A) A Cyclist Making a turn : Let a cyclist moving on a circular path of radius r bend away from the vertical by an angle θ. If R is the reaction of the ground, then R may be resolved into two components horizontal and vertical. The vertical component R cos θ balances the weight mg of the cyclist and the horizontal component R sin θ provides the necessary centripetal force for circular motion.

$$R\sin\theta = \frac{mv^2}{r} \quad ....(1) \text{ and } R\cos\theta = mg \quad ....(2)$$

Dividing (1) by (2), we get, $\tan\theta = \frac{v^2}{rg}$ ....(3)



For less bending of cyclist, his speed v should be smaller and radius r of circular path should be greater. If μ is coefficient of friction, then for no skidding of cycle (or overturning of cyclist)

$\mu \le \tan\theta$ ....(4) ; $\mu \ge \frac{v^2}{rg}$

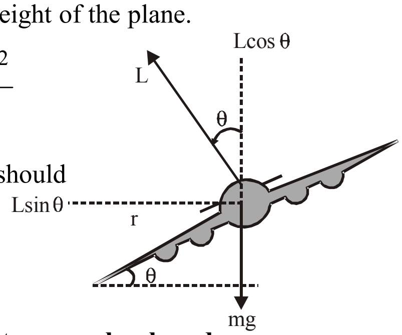
(B) An Aeroplane Making a Turn : In order to make a circular turn, a plane must roll at some angle θ in such a manner that the horizontal component of the lift force L provides the necessary centripetal force for circular motion. The vertical component of the lift force balances the weight of the plane.

$$L\sin\theta = \frac{mv^2}{r}$$

and $L\cos\theta = mg$

or the angle θ should be such that

$$\tan\theta = \frac{v^2}{rg}$$

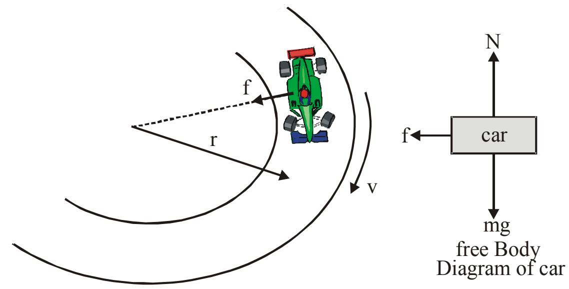


(C) A car taking A turn on a level road :

When a car takes a turn on a level road, the portion of the turn can be approximated by an arc of a circle of radius r (see fig). If the car makes the turn at a constant speed v, then there must be some centripetal force acting on the car. This force is generated by the friction between the tyres and the road. (car has a tendency to slip radially out ward, so frictional force acts inwards)

μ_S is the coefficient of static friction

N = mg is the normal reaction of the surface.



The maximum safe velocity v is

$$\frac{mv^2}{r} = \mu_S N = \mu_S mg \text{ or } \mu_S = \frac{v^2}{rg} \quad \text{or} \quad v = \sqrt{\mu_s rg}$$

It is independent of the mass of the car. The safe velocity is same for all vehicles of larger and smaller mass.

Example 18 :

A car is travelling at 30 km/h in a circle of radius 60m. What is the minimum value of μ_s for the car to make the turn without skidding ?

Sol. The minimum μ_s should be such that

$$\mu_S mg = \frac{mv^2}{r} \quad \text{or} \quad \mu_S = \frac{v^2}{rg}$$

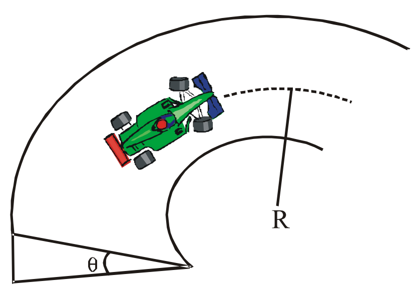
Here, $v = 30\frac{km}{h} = \frac{30\times1000}{3600} = \frac{25}{3} m/s$

$$\mu_S = \frac{25}{3}\times\frac{25}{3}\times\frac{1}{60\times10} = 0.115$$

For all values of μ_s greater than or equal to the above value, the car can make the turn without skidding.

(D) Bankig of road :

If a cyclist takes a turn, he can bend from his vertical position. This is not possible in the case of car, truck or train.



The tilting of the vehicle is achieved by raising the outer edge of the circular track, slightly above the inner edge. This is known as banking of curved track.

The angle of inclination with the horizontal is called the angle of banking.

(a) If driver moves with slow velocity that friction does not play any role in negotiating the turn.
The various forces acting on the vehicle are :
(i) Weight of the vehicle (mg) in the downward direction.
(ii) Normal reaction (N) perpendicular to the inclined surface of the road.
Resolve N in two components.
* N cos θ, vertically upwards which balances weight of the vehicle.
∴ $N\cos\theta = mg$ .....(1)
* N sin θ, in horizontal direction which provides necessary centripetal force.

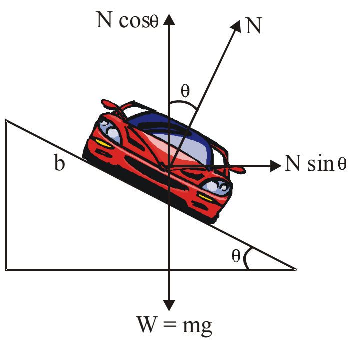
∴ $N\sin\theta = \dfrac{mv^2}{r}$ .....(2)

On dividing eqn. (ii) by eqn. (i)

$$\frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

or $\tan\theta = \dfrac{v^2}{rg}$

∴ $\theta = \tan^{-1}\left(\dfrac{v^2}{rg}\right)$



Where m is the mass of the vehicle, r is radius of curvature of the road, v is speed of the vehicle and θ is the banking angle $\left(\sin\theta = \dfrac{h}{b}\right)$.

Factors that decide the value to angle of banking are as follows :
* Velocity of the vehicle
* Radius of the curve
* Acceleration due to gravity

Thus, there is no need of mass of the vehicle to express the value of angle of banking
i.e. angle of banking ⇒ does not dependent on the mass of the vehicle.

∵ $v^2 = gr\tan\theta$

∴ $v = \sqrt{gr\tan\theta}$ (maximum safe speed)

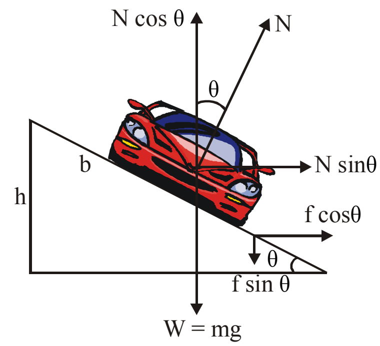
This gives the maximum safe speed of the vehicle. In actual practice, some frictional forces are always present. So, the maximum safe velocity is always much greater than that given by the above equation. While constructing the curved track, the value of θ is calculated for fixed values of v_{max} and r. This explains why along the curved roads, the speed limit at which the curve is to be negotiated is clearly indicated on sign boards.
The outer side of the road is raised by $h = b \times \theta$.

When θ is small, then $\tan\theta \approx \sin\theta = \dfrac{h}{b}$;

Also $\tan\theta = \dfrac{v^2}{rg}$ ∴ $\dfrac{v^2}{rg} = \dfrac{h}{b}$ or $h = \dfrac{v^2}{rg} \times b$

This gives us the height through which outer edge is raised above the inner edge.

(b) If the driver moves faster than the safe speed mentioned above, a friction force must act parallel to the road, inwards towards centre of the turn.



In this case forces acting on the vehicle are :
* Weight of the vehicle (mg) in the downward direction.
* Normal reaction perpendicular to the inclined plane of the road.
* Frictional force f between the tyres and the road.
N cos θ and N sinθ are the two rectangular components of N.
f cos θ and f sin θ are the two rectangular components of f.
The car does not have any vertical motion.

∴ $mg + f\sin\theta = N\cos\theta$
or $mg = N\cos\theta - f\sin\theta$
But $f = \mu N$, where $\mu \le \mu_s$.
∴ $mg = N\cos\theta - \mu N\sin\theta$
The forces N sin θ and f cos θ together provide the necessary centripetal force.

∴ $\dfrac{mv^2}{r} = N\sin\theta + f\cos\theta$ or $\dfrac{mv^2}{r} = N\sin\theta + \mu N\cos\theta$

Dividing eqn (ii) by eqn (i) we get

$$\frac{\frac{mv^2}{r}}{mg} = \frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu N\sin\theta}$$

or $\dfrac{v^2}{rg} = \dfrac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$ or $\dfrac{v^2}{rg} = \dfrac{\cos\theta(\tan\theta + \mu)}{\cos\theta(1 - \mu\tan\theta)}$

or $v^2 = \dfrac{\tan\theta + \mu}{1 - \mu\tan\theta} rg$ or $v = \sqrt{\dfrac{\tan\theta + \mu}{1 - \mu\tan\theta} rg}$

The best speed to negotiate a curve is obtained by putting $\mu = 0$. ∴ $v = \sqrt{rg\tan\theta}$

With this speed, there will be minimum wear and tear of the tyres.

Example 19 :

At what angle should a highway be banked for cars travelling at a speed of 100 km/h if the radius of the road is 400m and no frictional forces are involved?

Sol. The banking should be done at an angle θ such that

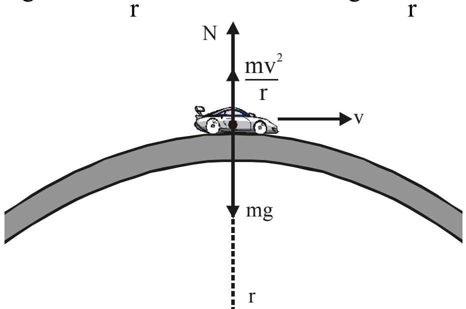
$$\tan\theta = \frac{v^2}{rg} = \frac{\frac{250}{9}\times\frac{250}{9}}{400\times 10} \text{ or } \tan\theta = \frac{625}{81\times 40} = 0.19$$

or $\theta = \tan^{-1} 0.19 \approx 0.19 \text{ radian } \approx 0.19 \times 57.3^\circ \approx 11^\circ$

3. Apparent weight of car

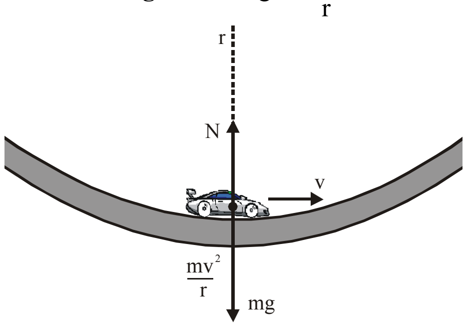
(A) Convex bridge : The motion of the motor car over a convex bridge is the motion along the segment of a circle. The centripetal force is provided by the difference of weight mg of the car and the normal reaction N of the bridge.

$$\therefore \quad mg - N = \frac{mv^2}{r} \quad \text{or} \quad N = mg - \frac{mv^2}{r}$$



Clearly $N < mg$, i.e., the apparent weight of the moving car is less than the weight of the stationary car. (or car moving on flat surface)

(B) Concave bridge : $N - mg = \frac{mv^2}{r}$



Apparent weight $N = mg + \frac{mv^2}{r}$

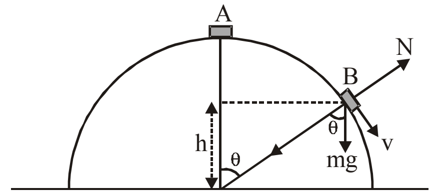
Clearly $N > mg$, i.e., the apparent weight of the moving car is more than the weight of the stationary car.

Example 20 :

A particle of mass m slides down from the vertex of semi-hemisphere, without any initial velocity. At what height from horizontal will the particle leave the sphere.

Sol. Let the particle leave them sphere at height h,

$$\frac{mv^2}{R} = mg\cos\theta - N$$



When the particle leaves the sphere $N = 0$,

$$\frac{mv^2}{R} = mg\cos\theta \Rightarrow v^2 = gR\cos\theta \qquad .....(1)$$

According to law of conservation of energy

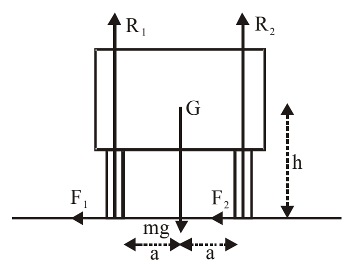
(K.E. + P.E.) at A = (K.E. + P.E.) at B

$$\Rightarrow \quad 0 + mgR = \frac{1}{2}$$

$$mv^2 + mgh \Rightarrow v^2 = 2g(R - h) \qquad .....(2)$$

From (1) and (2), $h = \frac{2}{3}R$, Also $\cos\theta = \frac{2}{3}$

4. Condition of Overturning : Here, we shall find the condition for the car to overturn. Let the distance between the centres of wheels of the car be 2a and the centre of gravity be h metres above the ground (road). The different forces acting on the car are shown in the fig.



(i) The weight mg of the car acts downwards through centre of gravity G.

(ii) The normal reactions of the ground R_1 and R_2 on the inner and outer wheels respectively. These act vertically upwards.

(iii) Let force of friction $F_1 + F_2$ between wheels and ground towards the centre of the turn.

Let the radius of circular path be r and the speed of the car be v. Since there is no vertical motion, equating the vertical forces, we have

$$R_1 + R_2 = mg. \qquad ....(1)$$

The horizontal force = centripetal force for motion in a circle. So,

$$F = F_1 + F_2 = \frac{mv^2}{r} \qquad ....(2)$$

Taking moments about the centre of mass G.

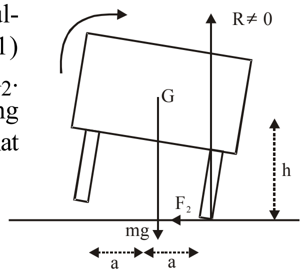
$$(F_1 + F_2)h + R_1 a = R_2 a$$

$$\therefore \quad F_1 + F_2 = (R_2 - R_1)\frac{a}{h} \qquad ....(3)$$

Combining this with equation (2) to eliminate $F_1 + F_2$ gives

$$R_2 - R_1 = \frac{hmv^2}{ar} \qquad ....(4)$$

We now have two simultaneous equations, (1) and (4), for R_1 and R_2. Solving these by adding & subtracting, we find that



$$2R_1 = mg - \frac{hmv^2}{ar}$$

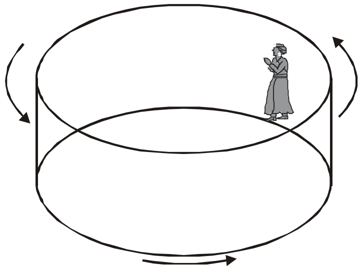
and $$2R_2 = mg + \frac{hmv^2}{ar}$$

From these expressions it is clear.
Inner wheels will leave the ground when R_1 will become zero and the car begins to overturn,

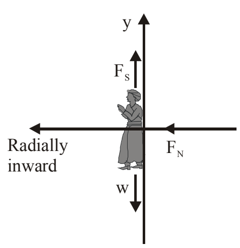
i.e., $$mg = \frac{hmv^2}{ar}$$

So the limiting speed is given by $v^2 = \frac{gra}{h}$ as required.

5. Death well and rotor : Example of uniform circular motion
In 'death well' a person drives a bicycle on a vertical surface of a large wooden well.
In 'death well' walls are at rest while person revolves.
In a rotor at a certain angular speed of rotor a person hangs resting against the wall without any floor.
In rotor person is at rest and the walls rotate.



(a) A passenger on a 'rotor ride'.



(b) A force diagram for the person.

In both these cases friction balances the weight of person while reaction provides the centripetal force necessary for circular motion, i.e.,

Force of friction $F_S = mg$ & Normal reaction $F_N = \frac{mv^2}{r}$

so $\frac{F_N}{F_S} = \frac{v^2}{rg}$, i.e., $v = \sqrt{\frac{rgF_N}{F_S}}$

Now for v to be minimum F_S must be maximum,

i.e., $v_{min} = \sqrt{\frac{rg}{\mu}}$ [as $F_{S\,max} = \mu F_N$]

Example 21 :

A 62 kg woman is a passenger in a "rotor ride" at an amusement park. A drum of radius 5.0 m is spun with an angular velocity of 25 rpm. The woman is pressed against the wall of the rotating drum as shown in fig. (a) Calculate the normal force of the drum on the woman (the centripetal force that prevents her from leaving her circular path). (b) While the drum rotates, the floor is lowered. A vertical static friction force supports the woman's weight. What must the coefficient of friction be to support her weight?

Sol. (a) Normal force exerted by the drum on women towards the centre $F_N = ma_c = m\omega^2 r$

$$= 62\,kg \times \left(25\frac{rev}{min} \times \frac{2\pi\,rad}{1\,rev} \times \frac{1\,min}{60\,s}\right)^2 \times 5\,m. = 2100\,N.$$

(b) $\mu F_N = F = mg$

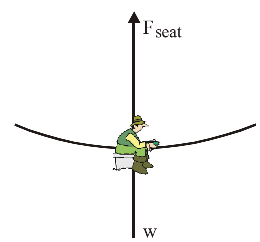
$$\text{so} \quad \mu = \frac{g}{\omega^2 r} = \left(\frac{60}{2\pi \times 25}\right)^2 \times \frac{10}{5} = 0.292$$

6. Looping the loop : This is the best example of non uniform circular motion in vertical plane.
For looping the loop pilot of the plane puts off the engine at lowest point and traverses a vertical loop.(with variable velocity)

Example 22 :

An aeroplane moves at 64 m/s in a vertical loop of radius 120m, as shown in fig. Calculate the force of the plane's seat on a 72 kg pilot while passing through the bottom part of the loop.

Sol. Two forces act on the pilot his downward weight force w and the upward force of the aeroplane's seat F_{seat}. Because the pilot moves in a circular path, these forces along the radial direction must, according to Newton's second law ($\Sigma F = ma$), equal the pilot's mass times his centripetal acceleration, where $a_c = v^2/r$.



We find that ΣF (in radial direction) $= F_{seat} - w = \frac{mv^2}{r}$.

Remember that forces pointing towards the center of the circle (F_{seat}) are positive and those pointing away from the center (w) are negative. Substituting $\omega = mg$ & rearranging, we find that the force of the aeroplane seat on the pilot is

$$F_{seat} = m\left(\frac{v^2}{r} + g\right) = 72\,kg\left[\frac{(64\,m/s)^2}{120\,m} + 9.8\,m/s^2\right]$$

$$= 72\,kg\,(34.1\,m/s^2 + 9.8\,m/s^2) = 3160.8\,N$$

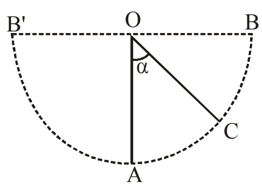
The pilot in this example feels very heavy. To keep him in the circular path, the seat must push the pilot upwards with a force of 3160 N, 4.5 times his normal weight. He experiences an acceleration of 4.5 g, that is, 4.5 times the acceleration of gravity.

TRY IT YOURSELF-3

Q.1 The radius of curvature of a railway line at a place when the train is moving with a speed of 36 km h^{-1} is 1000 m, the distance between the two rails being 1.5 metre. Calculate the elevation of the outer rail above the inner rail so that there may be no side pressure on the rails.

Q.2 An aircraft executes a horizontal loop at a speed of 720 km/h with its wing banked at 15°. Calculate the radius of the loop.

Q.3 A simple pendulum is vibrating with an angular amplitude of 90° as shown in the given figure. For what value of α, is the acceleration directed?

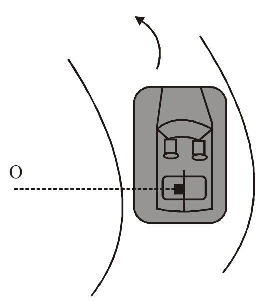


(i) vertically upwards (ii) horizontally
(iii) vertically downwards

Q.4 A car moving at a speed of 36 km/hr is taking a turn on a circular road of radius 50 m. A small wooden plate is kept on the seat with its plane perpendicular to the radius of the circular road (figure). A small block of mass 100g is kept on the seat which rests against the plate. The friction coefficient between the block and the plate is

$\mu = 1/\sqrt{3} = 0.58.$

(a) Find the normal contact force exerted by the plate on the block.
(b) The plate is slowly turned so that the angle between the normal to the plate and the radius of the road slowly increases. Find the angle at which the block will just start sliding on the plate.



Q.5 A train runs along an unbanked circular track of radius 30 m at a speed of 54 km/h. The mass of the train is 10^6 kg. What provides the centripetal force required for this purpose. The engine or the rails ? What is the angle of banking required to prevent wearing out of the rail ?

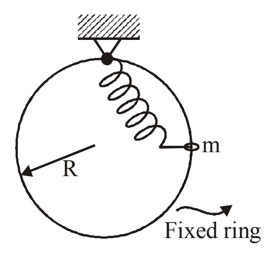
Q.6 A car takes a turn around a circular curve if it turns at double the speed, the tendency to overturn is
(A) halved (B) doubled
(C) quadrupled (D) unchanged

Q.7 A particle originally at rest at the highest point of a smooth vertical circle of radius R, is slightly displaced. Find the vertical distance below the highest point where the particle will leave the circle.

Q.8 A body of mass 500 g tied to a string of length 1m is revolved in the vertical circle with a constant speed. Find the minimum speed at which there will not be any slack on the string. Take g = 10 m/s^2

Q.9 A hemispherical bowl of radius r = 0.1m is rotating about its axis (which is vertical) with an angular velocity ω. A particle of mass 10^{-2} kg on the frictionless inner surface of the bowl is also rotating with the same ω. The particle is at a height h from the bottom of the bowl. (a) Obtain the relation between h and ω. What is the minimum value of ω needed in order to have a nonzero value of h. (b) It is desired to measure 'g' using this setup by measuring h accurately. Assuming that r and ω are known precisely and that the least count in the measurement of h is 10^{-4}m. What is minimum error Δg in the measured value of g.
[g = 9.8 m/s^2]

Q.10 A ring of radius R is placed such that it lies in a vertical plane. The ring is fixed. A bead of mass m is constrained to move along the ring without any friction. One end of the spring is connected with the mass m and other end is rigidly fixed with the topmost point of the ring. Initially the spring is in un-extended position and the bead is at a vertical distance R from the lowermost point of the ring. The bead is now released from rest.
(a) What should be the value of spring constant K such that the bead is just able to reach bottom of the ring.
(b) The tangential and centripetal accelerations of the bead at initial and bottom-most position for the same value of spring constant K.



ANSWERS

(1) 0.0153 m **(2)** 15.24 km.

(3) (i) 0°, (ii) $\cos^{-1}(1/\sqrt{3})$, (iii) 90° **(4)** (a) 0.2 N, (b) 30°

(5) 37° **(6)** (C) **(7)** R/3

(8) $\sqrt{10}$ m / s **(9)** (a) $7\sqrt{2}$ rad / s (b) -9.8×10^{-3} m/s^2

(10) **(a)** $K = \dfrac{mg}{R(3-2\sqrt{2})}$; (b) At initial instant $a_t = g$, $a_c = 0$

At bottom position $a_t = 0$, $a_c = 0$

IMPORTANT POINTS

1. The physical quantities which remain constant for a particle moving in circular path are speed, kinetic energy and angular momentum.

2. $a_r = \omega^2 r = \dfrac{v^2}{r} = \omega v$ (Always applicable)

   $a_r = 4\pi^2 n^2 r = \dfrac{4\pi^2}{T^2} r$ (Applicable in uniform circular motion) ; n = frequency of rotation,

   T = time period of rotation. $\vec{a}_r = \vec{\omega} \times \vec{v}$

3. Circular motion with variable speed. For complete circles, the string must be taut in the highest position, $u^2 \geq 5ga$. Circular motion ceases at the instant when the string becomes slack, i.e. when T = 0, range of values of u for which the string does go slack is $\sqrt{2ga} < u < \sqrt{5ga}$.

4. The minimum velocity for a body at the lowest point to complete a vertical circle of radius r is $\sqrt{5rg}$. The minimum velocity at highest point then is $\sqrt{rg}$.

5. The difference in tension at the highest and the lowest point in a vertical circle is 6mg, i.e. 6 times the weight of the body.

6. Conical pendulum : $w = \sqrt{g/h}$ where h is height of a point of suspension from the centre of circular motion.

7. If a body is moving on a curved road with speed greater than the speed limit, the reaction at the inner wheel disappears and it will leave the ground first.

8. On unbanked curved roads the minimum radius of curvature of the curve for safe driving is $r = \dfrac{v^2}{\mu g}$, where v is the speed of the vehicle and μ is the coefficient of friction.

9. To prevent skidding up the inclination (away from centre)

   $$v \leq \sqrt{gr\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$$

10. To prevent skidding down the inclination (towards the centre) $v \geq \sqrt{gr\left(\dfrac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta}\right)}$

11. The skidding of a vehicle will occur if $\dfrac{v^2}{r} > \mu g$ i.e., skidding will take place if the speed is large, the curve is sharp and μ is small.

12. If r is the radius of curvature of the speed breaker, then the maximum speed with which the vehicle can run on its without leaving contact with the ground is $v = \sqrt{(gr)}$.

ADDITIONAL EXAMPLES

Example 1 :

The kinetic energy of a particle moving along a circle of radius r depends on distance covered s as $K = As^2$ where A is a const. Find the force acting on the particle as a function of s.

Sol. According to given problem

$$\frac{1}{2}mv^2 = As^2 \quad \text{or} \quad v = s\sqrt{\frac{2A}{m}} \qquad ...(1)$$

$$\text{So, } a_r = \frac{v^2}{r} = \frac{2As^2}{mr} \qquad ...(2)$$

$$\text{Further more as, } a_t = \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt} = v\frac{dv}{ds} \qquad ...(3)$$

from eq^n. (1),

$$v = s\sqrt{\left(\frac{2A}{m}\right)} \Rightarrow \frac{dv}{ds} = \sqrt{\frac{2A}{m}} \qquad ...(4)$$

Substitute values from eq^n. (1) and eq^n. (4) in eq^n. (3)

$$a_t = \left[s\sqrt{\frac{2A}{m}}\right]\left[\sqrt{\frac{2A}{m}}\right] = \frac{2As}{m}$$

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left[\frac{2As^2}{mr}\right]^2 + \left[\frac{2As}{m}\right]^2} = \frac{2As}{m}\sqrt{1 + \left[\frac{s}{r}\right]^2}$$

$$\text{so} \quad F = ma = 2As\sqrt{1 + \left[\frac{s}{r}\right]^2}$$

Example 2 :

A circular loop has a small bead which can slide on it without friction. The radius of the loop is r. Keeping the loop vertically it is rotated about a vertical diameter at a constant angular speed ω. What is the value of angle θ, when the bead is in dynamic equilibrium?

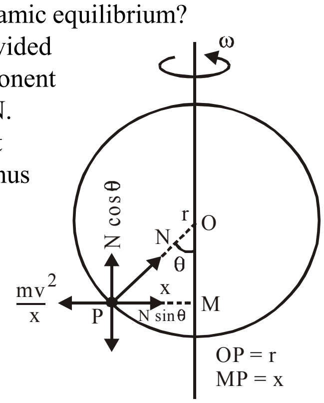
Sol. Centripetal force is provided by the horizontal component of the normal reaction N. The vertical component balances the weight. Thus

$N \sin\theta = m\omega^2 x$

and $N \cos\theta = mg$

Also $x = r\sin\theta$

and $N = m\omega^2 r$



$$\cos\theta = \frac{g}{\omega^2 r}$$

$$\text{or} \quad \theta = \cos^{-1}\left(\frac{g}{r\omega^2}\right)$$

Example 3 :

A body weighing 0.4 kg is whirled in a vertical circle making 2 revolutions per second. If the radius of the circle is 1.2 m, find the tension in the string, when the body is (a) at the top of the circle (b) at the bottom of the circle.
Given : $g = 9.8\ ms^{-2}$ and $\pi = 3.14$.

Sol. Mass m = 0.4 kg; time period = $\frac{1}{2}$ second and radius,

r = 1.2 m

Angular velocity, $\omega = \frac{2\pi}{1/2} = 4\pi\ rad\ s^{-1} = 12.56\ rad\ s^{-1}$

(a) At the top of the circle,

$$T = \frac{mv^2}{r} - mg = mr\omega^2 - mg = m(r\omega^2 - g)$$

$= 0.4\,(1.2 \times 12.56 \times 12.56 - 9.8)\ N = 71.8\ N$

(b) At the lowest point,
$T = m\,(r\omega^2 + g) = 79.64\ N$

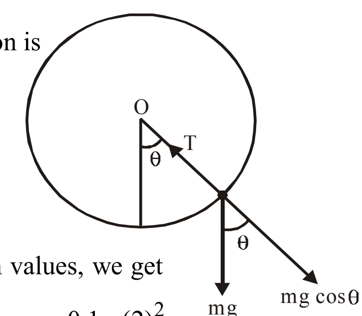
Example 4 :

A small body of mass m = 0.1 kg swings in a vertical circle at the end of a chord of length 1 m. Its speed is 2 m/s when the chord makes an angle θ = 30° with the vertical. Find the tension in the chord.

Sol. The equation of motion is

$$T - mg\cos\theta = \frac{mv^2}{r}$$

or $T = mg\cos\theta + \frac{mv^2}{r}$



Substituting the given values, we get

$$T = 0.1 \times 9.8 \times \cos 30 + \frac{0.1 \times (2)^2}{1}$$

$$= 0.98 \times \left(\frac{\sqrt{3}}{2}\right) + 0.4 = 0.85 + 0.4 = 1.25\ N$$

Example 5 :

A train rounds an unbanked circular bend of radius 30 m at a speed of 54 km/h. The mass of the train is 10^6 kg. What provides the centripetal force required for this purpose? The engine or the rails? The outer or inner rails? Which rail will wear out faster, the outer or the inner rail ? What is the angle of banking required to prevent wearing out of the rails?

Sol. $r = 30m,\ v = 54\ km/h = \frac{54 \times 5}{18}\ m/s = 15\ m/s$

$m = 10^6\ kg,\ \theta = ?$

(i) The centripetal force is provided by the lateral thrust by the outer rail on the flanges of the wheel of the train. The train causes an equal and opposite thrust on the outer rail (Newton's third law of motion). Thus, the outer rail wears out faster.

(ii) $\tan\theta = \frac{v^2}{rg} = \frac{15 \times 15}{30 \times 9.8} = 0.7653$

or $\theta = \tan^{-1}(0.7653) = 37.43°$

Example 6 :

A circular race track of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race car and the road is 0.2, what is the (a) optimum speed of the race car to avoid wear and tear of tyres, and the (b) maximum permissible speed to avoid slipping?

Sol. (a) On a banked road, the horizontal component of the normal reaction and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the component of the normal reaction is enough to provide the required centripetal force. In this case, the frictional force is not required.
The optimum speed is given by
$v_0 = (rg\tan\theta)^{1/2} = (300 \times 9.8 \tan 15°)^{1/2}\ ms^{-1}$
$= 28.1\ m/s$

(b) The maximum permissible speed is given by

$$v_{max} = \left(\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\,rg\right)^{1/2}$$

Substituting values and simplifying,
we get $v_{max} = 38.1\ m/s$.

Example 7 :

A bob of mass m, suspended by a string of length ℓ_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length ℓ_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio ℓ_1/ℓ_2 is –

Sol. The initial speed of 1st bob (suspended by a string of length ℓ_1) is $\sqrt{5g\ell_1}$.

The speed of this bob at highest point will be $\sqrt{g\ell_1}$.

When this bob collides with the other bob there speeds will be interchanged.

$$\sqrt{g\ell_1} = \sqrt{5g\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = 5$$

QUESTION BANK

CHAPTER 7 : CIRCULAR MOTION

EXERCISE - 1 [LEVEL-1]

PART - 1 : KINEMATICS OF CIRCULAR MOTION

Q.1 A particle completes 1.5 revolutions in a circular path of radius 2 cm. The angular displacement of the particle will be – (in radian)

(A) 6π (B) 3π
(C) 2π (D) π

Q.2 A particle revolving in a circular path completes first one third of circumference in 2 sec, while next one third in 1 sec. The average angular velocity of particle will be (in rad/sec)

(A) $2\pi/3$ (B) $\pi/3$
(C) $4\pi/3$ (D) $5\pi/3$

Q.3 The ratio of angular speeds of minute hand to hour hand of a watch is -

(A) 12 : 1 (B) 6 : 1
(C) 1 : 12 (D) 1 : 6

Q.4 The angular displacement of a particle is given by

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, where ω_0 and α are constant and

$\omega_0 = 1$ rad/sec, $\alpha = 1.5$ rad/sec^2. The angular velocity at time, t = 2 sec will be (in rad/sec) -

(A) 1 (B) 5
(C) 3 (D) 4

Q.5 The magnitude of the linear acceleration, the particle moving in a circle of radius of 10 cm with uniform speed completing the circle in 4 s, will be -

(A) 5π cm/s^2 (B) 2.5π cm/s^2
(C) $5\pi^2$ cm/s^2 (D) $2.5\pi^2$ cm/s^2

Q.6 The length of second's hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is -

(A) 0 (B) $\frac{\pi}{30\sqrt{2}}$ cm/s
(C) $\frac{\pi}{30}$ cm/s (D) $\frac{\pi\sqrt{2}}{30}$ cm/s

Q.7 A particle moves in a circle of radius 20cm with a linear speed of 10m/s. The angular velocity will be -

(A) 50 rad/s (B) 100 rad/s
(C) 25 rad/s (D) 75 rad/s

Q.8 The angular velocity of a particle is given by $\omega = 1.5\,t - 3t^2 + 2$, the time when its angular acceleration decreases to be zero will be -

(A) 25 sec (B) 0.25 sec
(C) 12 sec (D) 1.2 sec

Q.9 A particle is moving in a circular path with velocity varying with time as $v = 1.5t^2 + 2t$. If 2 cm the radius of circular path, the angular acceleration at t = 2 sec will be

(A) 4 rad/sec^2 (B) 40 rad/sec^2
(C) 400 rad/sec^2 (D) 0.4 rad/sec^2

Q.10 A grind stone starts from rest and has a constant-angular acceleration of 4.0 rad/sec^2. The angular displacement and angular velocity, after 4 sec. will respectively be -

(A) 32 rad, 16 rad/sec (B) 16rad, 32 rad/s
(C) 64rad, 32 rad/sec (D) 32 rad, 64rad/sec

Q.11 The shaft of an electric motor starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$ during the first 2 seconds after it starts after which $\alpha = 0$. The angular velocity after 6 sec will be -

(A) 10/3 rad/sec (B) 3/10 rad/sec
(C) 30/4 rad/sec (D) 4/30 rad/sec

Q.12 A motor car is travelling at 30 m/s on a circular road of radius 500 m. It is increasing its speed at the rate of 2m/s^2. Its net acceleration is (in m/s^2) –

(A) 2 (B) 1.8
(C) 2.7 (D) 0

Q.13 What is the value of linear velocity, if

$\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$

(A) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (B) $-18\hat{i} - 13\hat{j} + 2\hat{k}$
(C) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (D) $6\hat{i} - 2\hat{j} + 8\hat{k}$

PART - 2 : DYNAMICS OF UNIFORM CIRCULAR MOTION

Q.14 An electron is moving in a circular orbit of radius 5.3×10^{-11} metre around the atomic nucleus at a rate of 6.6×10^{15} revolutions per second. The acceleration of the electron and centripetal force acting on it will be - (The mass of the electron is 9.1×10^{-31}kg)

(A) 8.3×10^{-8}N (B) 3.8×10^{-8}N
(C) 4.15×10^{-8}N (D) 2.07×10^{-8}N

Q.15 An air craft executes a horizontal loop of radius 1 km with a steady speed of 900 km/h. The ratio of centripetal acceleration to that gravitational acceleration will be-

(A) 1 : 6.38 (B) 6.38 : 1
(C) 2.25 : 9.8 (D) 2.5 : 9.8

Q.16 Choose the correct options –

(1) Centripetal force is not a real force. It is only the requirement for circular motion.
(2) Work done by centripetal force may or may not be zero.
(3) Work done by centripetal force is always zero.
(4) Centripetal force is a fundamental force.

(A) 1, 2 and 3 are correct (B) 1 and 2 are correct
(C) 2 and 4 are correct (D) 1 and 3 are correct

Q.17 A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it will be -

(A) mk^2t^2r (B) $mk^2r^2t^2$
(C) $m^2k^2t^2r^2$ (D) mk^2r^2t

Q.18 In uniform circular motion
(A) Both velocity and acceleration are constant
(B) Acceleration and speed are constant but velocity changes
(C) Both acceleration and velocity changes
(D) Both acceleration and speed are constant

Q.19 A particle does uniform circular motion in a horizontal plane. The radius of the circle is 20 cm. The centripetal force acting on the particle is 10 N. It's kinetic energy is
(A) 0.1 J (B) 0.2 J
(C) 2.0 J (D) 1.0 J

Q.20 A particle moves with constant angular velocity in circular path of certain radius and is acted upon by a certain centripetal force F. If the angular velocity is doubled, keeping radius the same, the new force will be
(A) 2F (B) F^2
(C) 4F (D) F/2

Q.21 Two bodies of equal masses revolve in circular orbits of radii R_1 and R_2 with the same period. Their centripetal forces are in the ratio
(A) $\left(\frac{R_2}{R_1}\right)^2$ (B) $\frac{R_1}{R_2}$
(C) $\left(\frac{R_1}{R_2}\right)^2$ (D) $\sqrt{R_1R_2}$

Q.22 A stone ties to the end of a string 1m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolution in 44 seconds, what is the magnitude and direction of acceleration of the stone
(A) (π^2 /4) m/s^2 and direction along the radius towards the centre
(B) π^2 m/s^2 and direction along the radius away from the centre
(C) π^2 m/s^2 and direction along the radius towards the centre
(D) π^2 m/s^2 and direction along the tangent to the circle

PART - 3 : NON-UNIFORM CIRCULAR MOTION AND VERTICAL CIRCULAR MOTION

Q.23 A cane filled with water is revolved in a vertical circle of radius 4 m and water just does not fall down. The time period of revolution will be –
(A) 1 s (B) 10 s
(C) 8 s (D) 4 s

Q.24 A body of mass m kg is rotating in a vertical circle at the end of a string of length r metre. The difference in the kinetic energy at the top and the bottom of the circle is
(A) mg/r (B) 2mg/r
(C) 2mgr (D) mgr

Q.25 A car is moving in a circular path of radius 100 m with velocity of 200 m/sec such that in each sec its velocity increases by 100 m/s, the net acceleration of car will be - (in m/sec)
(A) $100\sqrt{17}$ (B) $10\sqrt{7}$
(C) $10\sqrt{3}$ (D) $100\sqrt{3}$

Q.26 A 4 kg balls is swing in a vertical circle at the end of a cord 1 m long. The maximum speed at which it can swing if the cord can sustain maximum tension of 163.6 N will be -
(A) 6 m/s (B) 36 m/s
(C) 8 m/s (D) 64 m/s

Q.27 The string of a pendulum is horizontal. The mass of the bob is m. Now the string is released. The tension in the string in the lowest position is -
(A) 1 mg (B) 2 mg
(C) 3 mg (D) 4 mg

Q.28 A weightless thread can support tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2 m in a vertical plane then the maximum angular velocity of the stone will be
(A) 5 rad/s (B) $\sqrt{30}$ rad / s
(C) $\sqrt{60}$ rad / s (D) 10 rad/s

Q.29 A body of mass m hangs at one end of a string of length ℓ, the other end of which is fixed. It is given a horizontal velocity so that the string would just reach where it makes an angle of 60° with the vertical. The tension in the string at mean position is –
(A) 2mg (B) mg
(C) 3mg (D) $\sqrt{3}$ mg

Q.30 A stone of mass m is tied to a string and is moved in a vertical circle of radius r making n revolutions per minute. The total tension in the string when the stone is at its lowest point is
(A) mg (B) $m(g+\pi n r^2)$
(C) $m(g+\pi n r)$ (D) $m\{g+(\pi^2 n^2 r)/900\}$

Q.31 A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical
(A) 30^o (B) 45^o
(C) 60^o (D) 90^o

PART - 4 : APPLICATIONS OF CIRCULAR MOTION

Q.32 A car driver is negotiating a curve of radius 100 m with a speed of 18 km/hr. The angle through which he has to lean from the vertical will be -
(A) $\tan^{-1}\frac{1}{4}$ (B) $\tan^{-1}\frac{1}{40}$
(C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) $\tan^{-1}\left(\frac{1}{20}\right)$

Q.33 The vertical section of a road over a canal bridge in the direction of its length is in the form of circle of radius 8.9 metre. Find the greatest speed at which the car can cross this bridge without losing contact with the road at its highest point, the center of gravity of the car being at a height h = 1.1 metre from the ground. (g = 10 m/sec^2)
(A) 5 m/s (B) 7 m/s
(C) 10 m/s (D) 13 m/s

Q.34 The maximum speed at which a car can turn round a curve of 30 metre radius on a level road if the coefficient of friction between the tyres and the road is 0.4, will be -
(A) 10.84 m/s (B) 17.84 m/s
(C) 11.76 m/s (D) 9.02 m/s

Q.35 The angular speed with which the earth would have to rotate on it axis so that a person on the equator would weight (3/5)th as much as present will be:
(Take the equatorial radius as 6400 km)
(A) 8.7×10^4 rad/sec (B) 8.7×10^3 rad/sec
(C) 7.8×10^4 rad/sec (D) 7.8×10^3 rad/sec

Q.36 The roadway bridge over a canal is the form of an arc of a circle of radius 20 m. What is the minimum speed with which a car can cross the bridge without leaving contact with the ground at the highest point (g = 9.8 m/s^2)
(A) 7 m/s (B) 14 m/s
(C) 289 m/s (D) 5 m/s

Q.37 Roads are banked on curves so that
(A) The speeding vehicles may not fall outwards
(B) The frictional force between the road and vehicle may be decreased
(C) The wear and tear of tyres may be avoided
(D) The weight of the vehicle may be decreased

Q.38 For a body moving in a circular path, a condition for no skidding if μ is the coefficient of friction, is
(A) $\frac{mv^2}{r} \le \mu mg$ (B) $\frac{mv^2}{r} \ge \mu mg$
(C) $\frac{v}{r} = \mu g$ (D) $\frac{mv^2}{r} = \mu mg$

Q.39 For a heavy vehicle moving on a circular curve of a highway the road bed is banked at an angle θ corresponding to a particular speed. The correct angle of banking of the road for vehicles moving at 60 km/hr will be - (If radius of curve = 0.1 km)
(A) $\tan^{-1}(0.283)$ (B) $\tan^{-1}(2.83)$
(C) $\tan^{-1}(0.05)$ (D) $\tan^{-1}(0.5)$

Q.40 A train has to negotiate a curve of radius 400 m. By how much should the outer rail be raised with respect to inner rail for a speed of 48 km/hr. The distance between the rail is 1 m.
(A) 12 m (B) 12 cm
(C) 4.5 cm (D) 4.5 m

Q.41 A cyclist turns around a curve at 15 miles/hour. If he turns at double the speed, the tendency to overturn is
(A) Doubled (B) Quadrupled
(C) Halved (D) Unchanged

Q.42 A stone of mass m is tied to a string of length ℓ and rotated in a circle with a constant speed v. If the string is released, the stone flies –
(A) Radially outward
(B) Radially inward
(C) Tangentially outward
(D) With an acceleration $\frac{mv^2}{\ell}$

Q.43 A car sometimes overturns while taking a turn. When it overturns, it is
(A) The inner wheel which leaves the ground first
(B) The outer wheel which leaves the ground first
(C) Both the wheels leave the ground simultaneously
(D) Either wheel leaves the ground first

Q.44 A motor cyclist moving with a velocity of 72 km/hour on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 meters. The acceleration due to gravity is 10 m/sec^2. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than
(A) $\theta = \tan^{-1} 6$ (B) $\theta = \tan^{-1} 2$
(C) $\theta = \tan^{-1} 25.92$ (D) $\theta = \tan^{-1} 4$

Q.45 A cyclist goes round a circular path of circumference 34.3 m in $\sqrt{22}$ sec. the angle made by him, with the vertical, will be
(A) 45^o (B) 40^o
(C) 42^o (D) 48^o

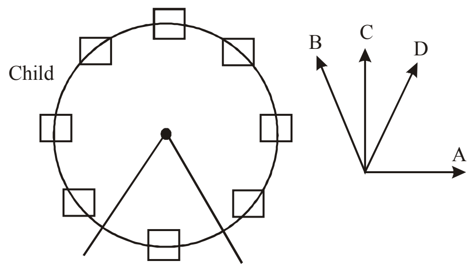
PART - 5 : MISCELLANEOUS

Q.46 A 1 kg stone at the end of 1m long string is whirled in a vertical circle at constant speed of 4 m/sec. The tension in the string is 6N, when the stone is at (g = 10 m\sec^2)
(A) Top of the circle (B) Bottom of the circle
(C) Half way down (D) None of the above

Q.47 A car is travelling with linear velocity v on a circular road of radius r. If it is increasing its speed at the rate of 'g' meter\sec^2, then the resultant acceleration will be
(A) $\sqrt{\left\{\frac{v^2}{r^2} - g^2\right\}}$ (B) $\sqrt{\left\{\frac{v^4}{r^2} + g^2\right\}}$
(C) $\sqrt{\left\{\frac{v^4}{r^2} - g^2\right\}}$ (D) $\sqrt{\left\{\frac{v^2}{r^2} + g^2\right\}}$

Q.48 You may have seen in a circus a motorcyclist driving in vertical loops inside a 'deathwell' (a hollow spherical chamber with holes, so the spectators can watch from outside).What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m ?
(A) 18.65 ms^{-1}. (B) 25.65 ms^{-1}.
(C) 5.65 ms^{-1}. (D) 15.65 ms^{-1}.

Q.49 A child is belted into a Ferris wheel seat that rotates counter clockwise at constant speed in a vertical plane at an amusement park. At the location shown, which direction best represents the total force exerted on the child by the seat and belt –

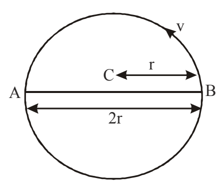


(A) A (B) B
(C) C (D) D

Q.50 As you ride on a merry-go-round, you feel a strong outward pull that feels just like the force of gravity. This fictitious force occurs because

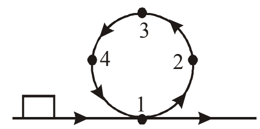
(A) your velocity is toward the center of the merry-go-round and you experience a fictitious force in the direction opposite your velocity.
(B) your velocity is away from the center of the merry-go-round and you experience a fictitious force in the direction of your velocity.
(C) you are accelerating toward the center of the merry-go-round and experience a fictitious force in the direction opposite your acceleration.
(D) you are accelerating away from the center of the merry-go-round and experience a fictitious force in the direction of your acceleration.

Q.51 A particle P is moving in a circle of radius 'a' with a uniform speed v. C is the centre of the circle and AB is a diameter. When passing through B the angular velocity of P about and A and C are in the ratio.



(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 4 : 1

Q.52 A rectangular block is moving along a frictionless path when it encounters the circular loop as shown. The block passes points 1, 2, 3, 4, 1 before returning to the horizontal track. At point 3 :



(A) its mechanical energy is a minimum
(B) it is not accelerating
(C) its speed is a minimum
(D) it experiences a net upward force

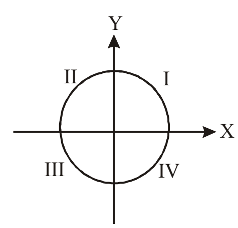
Q.53 Suppose the coefficient of static friction between the road and the tires on a formula one car is 0.6 during a Grand Prix auto race. What speed will put the car on the verge of sliding as it rounds a level curve of 30.5 m radius ?

(A) 13 m/s (B) 26 m/s
(C) 5 m/s (D) 22 m/s

Q.54 A small disc in on the top of a hemisphere of radius R. What is the smallest horizontal velocity v that should be given to the disc for it to leave the hemisphere and not slide down it? [There is no friction]

(A) $v = \sqrt{2gR}$ (B) $v = \sqrt{gR}$
(C) g/R (D) $v = \sqrt{g^2R}$

Q.55 A particle is moving along a circular path as shown in the figure. The instantaneous velocity of the particle is $\vec{v} = (4m/s)\,\hat{i} - (3m/s)\,\hat{j}$. The particle is moving through ...... quadrant if it is travelling clockwise and through .... quadrant if it is travelling anticlockwise, respectively around the circle –

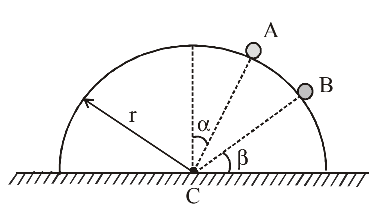


(A) First, first (B) First, second
(C) First, third (D) Third, first

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 A particle initially at rest starts moving from point A on the surface of a fixed smooth hemisphere of radius r as shown.

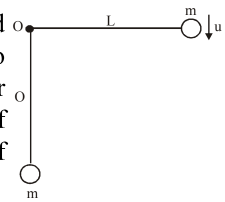


The particle looses its contact with hemisphere at point B. C is centre of the hemisphere. The equation relating α and β is –

(A) 3 sin α = 2 cos β (B) 2 sin α = 3 cos β
(C) 3 sin β = 2 cos α (D) 2 sin β = 3 cos α

Q.2 Two same masses are tied with equal lengths of strings and are suspended at the same fixed point. One mass is suspended freely whereas another is kept in a way that string is horizontal as shown. This mass is given initial velocity u in vertical downward direction.

It strikes the freely suspended mass elastically that is just able to complete the circular motion after the collision about point of suspension O. Magnitude of velocity u is –



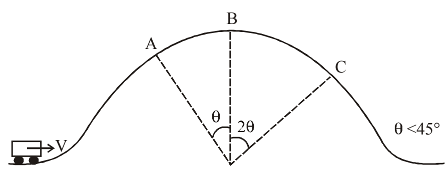
(A) $\sqrt{gL}$ (B) $\sqrt{2.5\,gL}$

(C) $\sqrt{3\,gL}$ (D) $\sqrt{2gL}$

Q.3 A ring of mass 2π kg and of radius 0.25m is making 300rpm about an axis through its centre perpendicular to its plane. The tension (in newtons) developed in the ring is:

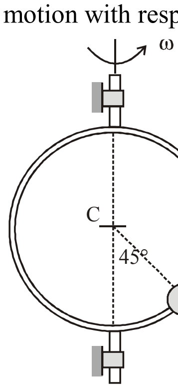
(A) 50 (B) 100
(C) 175 (D) 250

Q.4 A self propelled vehicle (assume it as a point mass) runs on a track with constant speed V. It passes through three positions A, B and Con the circular part of the track. Suppose N_A, N_B and N_C are the normal forces exerted by the track on the vehicle when it is passing through points A, B and C respectively then –



(A) $N_A = N_B = N_C$ (B) $N_B > N_A > N_C$
(C) $N_C > N_A > N_B$ (D) $N_B > N_C > N_A$

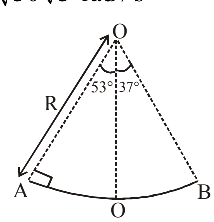
Q.5 A small bead of mass m = 1 kg is carried by a circular hoop having centre at C and radius r = 1 m which rotates about a fixed vertical axis. The coefficient of friction between bead and hoop is μ = 0.5. Find the maximum angular speed of the hoop for which the bead does not have relative motion with respect to hoop.



(A) $\sqrt{30\sqrt{2}}$ rad / s (B) $\sqrt{15\sqrt{2}}$ rad / s

(C) $\sqrt{45\sqrt{2}}$ rad / s (D) $\sqrt{30\sqrt{3}}$ rad / s

Q.6 A section of fixed smooth circular track of radius R in vertical plane is shown in the figure. A block is released from position A and leaves the track at B. Find the radius of curvature of its trajectory when it just leaves the track at B.

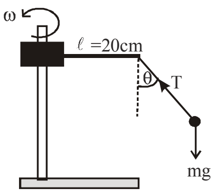


(A) R/2 (B) 2R/3
(C) R/3 (D) R/4

Q.7 The velocity and acceleration vectors of a particle undergoing circular motion are $\vec{v} = 2\hat{i}\ m/s$ and $\vec{a} = 2\hat{i} + 4\hat{j}\ m/s^2$ respectively at an instant of time. The radius of the circle is –

(A) 1m (B) 2m
(C) 3m (D) 4m

Q.8 Fig shows a rod of length 20 cm. pivoted near an end which is made to rotate in a horizontal plane with a constant angular speed. A ball of mass m is suspended by a string also of length 20 cm. from the other end of the rod. If the angle θ made by the string with the vertical is 30°, find the angular speed of rotation. Take g = 10 m/s^2

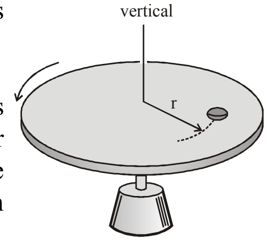


(A) 4.4 rad/sec. (B) 2.4 rad/sec.
(C) 8.4 rad/sec. (D) 12.2 rad/sec.

Q.9 Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km. When S_1 is closest to S_2, the angular speed of S_2 as observed by an astronaut in S_1 is –

(A) π rad/hr. (B) $\pi/3$ rad/hr.
(C) 2π rad/hr. (D) $\pi/2$ rad/hr.

Q.10 A small coin of mass 40kg is placed on the horizontal surface of rotating disc. The disc starts from rest and is given a constant angular acceleration α = 2 rad/s². The coefficient of static friction between the coil and the disc is μ_s = 3/4 and coefficient of kinetic friction is μ_k = 0.5. The coin is placed at a distance r = 1m from the centre of the disc. Find the magnitude of the resultant force on the coin exerted by the disc just before it starts slipping on the disc.



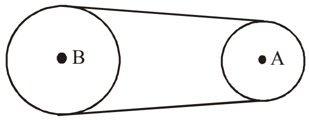
(A) 0.1 N (B) 1.0 N
(C) 0.5 N (D) 1.5 N

Q.11 A particle of mass moves along the internal smooth surface of a vertical cylinder of radius R. Find the force with which the particle acts on the cylinder wall if at the initial moment of time its velocity equals V_0. and forms an angle α with the horizontal.

(A) $\left(\frac{mV_0^2}{R}\right)\sin^2\alpha$ (B) $\left(\frac{mV_0^2}{2R}\right)\cos^2\alpha$

(C) $\left(\frac{2mV_0^2}{R}\right)\cos^2\alpha$ (D) $\left(\frac{mV_0^2}{R}\right)\cos^2\alpha$

Q.12 A wheel of radius 0.1m (wheel A) is attached by a non-stretching belt to a wheel of radius 0.2m (wheel B). The belt does not slip. By the time wheel B turns through 1 revolution, wheel A will rotate through –

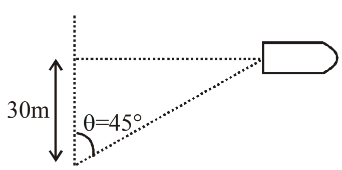


(A) 1/2 revolution (B) 1 revolution
(C) 2 revolution (D) 4 revolution

Q.13 A particle is moving in a circular path. The acceleration and momentum vectors at an instant of time are $\vec{a} = 2\hat{i} + 3\hat{j}$ m/s^2 and $\vec{P} = 6\hat{i} - 4\hat{j}$ kgm/s. Then the motion of the particle is
(A) uniform circular motion
(B) circular motion with tangential acceleration
(C) circular motion with tangential retardation
(D) we cannot say anything from a and P only.

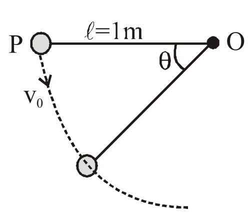
Q.14 Two particles A and B separated by a distance 2R are moving counter clockwise along the same circular path of radius R each with uniform speed v. At time t = 0, A is given a tangential acceleration of magnitude $a = \dfrac{72v^2}{25pR}$
(A) the time lapse for the two bodies to collide is $\dfrac{6pR}{5V}$
(B) the angle covered by A is $11\pi/6$
(C) angular velocity of A is $\dfrac{11V}{5R}$
(D) radial acceleration of A is $289v^2/5R$

Q.15 A boat is travelling with a speed of 27 kmph due east. An observer is situated at 30m south of the line of travel. The angular velocity of boat relative to the observer in the position shown will be –



(A) 0.125 rad/sec (B) zero
(C) 0.250 rad/sec (D) 0.67 rad/sec

Q.16 The sphere at P is given a downward velocity v_0 and swings in a vertical plane at the end of a rope of ℓ = 1m attached to a support at O. The rope breaks at angle 30° from horizontal, knowing that it can withstand a maximum tension equal to three times the weight of the sphere. Then the value of v_0 will be: ($g = \pi^2$ m/s^2)

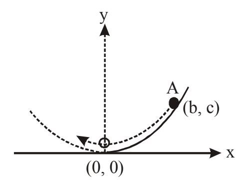


(A) g /2 m/s (B) 2g/3 m/s
(C) $\sqrt{3g/2}$ m/s (D) g/3 m/s

Q.17 A particle moves in a curve y = a log sec (x/a) in such a way that the tangent to the curve rotates uniformly with angular speed 2 rad/sec. Find resultant acceleration of the particle when x = (π/4) a and a is a constant with value 1/2.
(A) 4 m/sec^2 (B) 2 m/sec^2
(A) 3 m/sec^2 (A) 6 m/sec^2

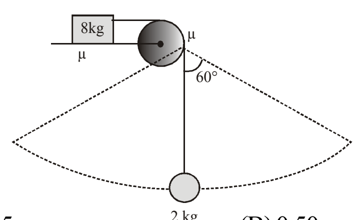
Q.18 For a particle moving along circular path, the radial acceleration a_r is proportional to time t. If a_t is the tangential acceleration, then which of the following will be independent of time t ?
(A) a_t (B) $a_r.a_t$
(C) a_r / a_t (D) $a_r (a_t)^2$

Q.19 A small ball of mass m starts from rest from point A (b, c) on a smooth slope which is a parabola. The normal force that the ground exerts at the instant, the ball arrives at lowest point (0, 0) is (take acceleration due to gravity g)



(A) $mg\left(\dfrac{b^2+4c^2}{b^2}\right)$ (B) $\dfrac{4mgc^2}{b^2}$
(C) mg (D) 3mg

Q.20 In the system shown, the mass m = 2 kg. oscillates in a circular arc of amplitude 60°, the minimum value of coefficient of friction between mass = 8 kg and surface of table to avoid slipping is -

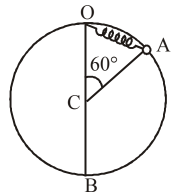


(A) 0.25 (B) 0.50
(C) 0.40 (D) None of these

Q.21 A body moving with a constant speed describes a circular path whose radius vector is given by $\vec{r} = 15(\cos pt\ \hat{i} + \sin pt\ \hat{j})$ m, where p is in rad/s, and t is in second. What is its centripetal acceleration at t = 3s ?
(A) 45 p^2 m/s^2 (B) 5 p^2 m/s^2
(C) 15 p m/s^2 (D) 15 p^2 m/s^2

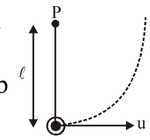
Q.22 A particle of mass m is suspended by a string of length ℓ from a fixed rigid support. A horizontal velocity $v_0 = \sqrt{3g\ell}$ is imparted to it suddenly. What is the angle made by the string with the vertical when the acceleration of the particle is inclined to the string by 45° ?
(A) 45° (B) 60°
(C) 0° (D) 90°

Q.23 A particle of mass 5 kg is free to slide on a smooth ring of radius r = 20cm. fixed in a vertical plane. The particle is attached to one end of a spring whose other end is fixed to the top point O of the ring. Initially the particle is at rest at a point A of the ring such that $\angle OCA = 60°$, C being the centre of the ring. The natural length of the spring is also equal to r = 20cm. After the particle is released and slides down the ring the contact force between the particle and the ring becomes zero when it reaches the lowest position B. Determine the force constant of the spring.



(A) 250 N/m (B) 500 N/m
(C) 300 N/m (D) 100 N/m

Q.24 Consider the arrangement shown in which a bob of mass m is suspended by means of a string connected to peg P. If the bob is given a horizontal velocity $\vec{u}$ having magnitude $\sqrt{3g\ell}$, find the minimum speed of the bob in subsequent motion.



(A) $\frac{1}{2}\sqrt{\frac{g\ell}{3}}$ (B) $\frac{1}{3}\sqrt{\frac{g\ell}{3}}$ (C) $\frac{1}{3}\sqrt{\frac{g\ell}{2}}$ (D) $\frac{1}{5}\sqrt{\frac{g\ell}{2}}$

Q.25 A heavy particle hangs from a point O, by a string of length a, it is projected horizontally with a velocity $v = \sqrt{(2+\sqrt{3})\,ag}$. The angle with the downward vertical, string makes where string becomes slack is –

(A) $\theta = \sin^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ (B) $\theta = \cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

(C) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (D) $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 A particle moves in a circle of radius r = 4/3cm. at a speed given by $v = 2.0\,t^2$ where v is in cm/s and t in seconds. Find the magnitude of the acceleration (in cm/s^2) at t = 1s.

Q.2 A particle of mass m = 1 kg moves in a circle of radius R = 2m with uniform speed $v = 3\pi$ m/s. The magnitude of impulse given by centripetal force to the particle in one second is $A\sqrt{2}\pi$ Ns. Find the value of A.

Q.3 A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1. In the next 2 sec, it rotates through an additional angle θ_2. Find the ratio of θ_2/θ_1.

Q.4 A turn table rotates with constant angular acceleration of 2 rad/s^2 about a fixed vertical axis through its centre and perpendicular to its plane. A coin is placed on it at a distance of 1m from the axis of rotation. The coin is always at rest relative to the turntable. If at t = 0 the turntable was at rest, then the total acceleration of the coin after one second is $2\sqrt{A}\,m/s^2$. Find the value of A.

Q.5 A railway line is taken round a circular arc of radius 1000m, and is banked by raising the outer rail h m above the inner rail. If the lateral pressure on the inner rail when a train travels round the curve at 10 m/s is equal to the lateral pressure on the outer rail when the train's speed is 20 m/s, the value of h is (A – 1.96)m (The distance between the rails is 1.5m). Find the value of A.

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 The minimum velocity (in ms^{-1}) with which a car driver must travels on a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is – **[AIEEE-2002]**

(A) 60 (B) 30

(C) 15 (D) 25

Q.2 Which of the following statements is FALSE for a particle moving in a circle with a constant angular speed ? **[AIEEE-2004]**

(A) The velocity vector is tangent to the circle

(B) The acceleration vector is tangent to the circle

(C) The acceleration vector ponits to the centre of the circle

(D) The velocity and acceleration vectors and perpendicular to each other

Q.3 A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that – **[AIEEE-2004]**

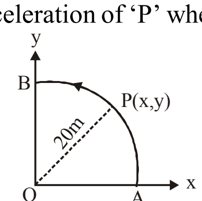
(A) Its velocity is constant

(B) Its acceleration is constant

(C) Its kinetic energy is constant

(D) It moves in a straight line

Q.4 A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2 s is nearly – **[AIEEE 2010]**



(A) 13 m/s^2 (B) 12 m/s^2

(C) 7.2 m/s^2 (D) 14 m/s^2

Q.5 For a particle in uniform circular motion the acceleration $\vec{a}$ at a point P(R, θ) on the circle of radius R is (here θ is measured from the x–axis) **[AIEEE 2010]**

(A) $-\frac{v^2}{R}\cos\theta\,\hat{i}+\frac{v^2}{R}\sin\theta\,\hat{j}$ (B) $-\frac{v^2}{R}\sin\theta\,\hat{i}+\frac{v^2}{R}\cos\theta\,\hat{j}$

(C) $-\frac{v^2}{R}\cos\theta\,\hat{i}-\frac{v^2}{R}\sin\theta\,\hat{j}$ (D) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$

Q.6 Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is : **[AIEEE 2012]**

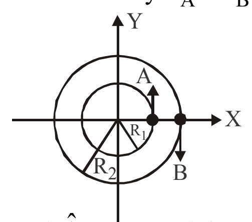
(A) $m_1r_1 : m_2r_2$ (B) $m_1 : m_2$

(C) $r_1 : r_2$ (D) 1 : 1

Q.7 A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **[JEE MAIN 2019]**

(A) zero (B) 10 m/s (C) $10\sqrt{3}$ m/s (D) $10\sqrt{2}$ m/s

Q.8 Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure. The relative velocity $\vec{v}_A - \vec{v}_B$ at $t = \pi/2\omega$ is given by : **[JEE MAIN 2019]**



(A) $-\omega(R_1+R_2)\,\hat{i}$ (B) $\omega(R_1+R_2)\,\hat{i}$

(C) $\omega(R_1-R_2)\,\hat{i}$ (D) $\omega(R_2-R_1)\,\hat{i}$

Q.9 A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length ℓ. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is : **[JEE MAIN 2020 (JAN)]**

(A) $\frac{m\ell\omega^2}{k+m\omega^2}$ (B) $\frac{m\ell\omega^2}{k-m\omega^2}$

(C) $\frac{m\ell\omega^2}{k-\omega m}$ (D) $\frac{m\ell\omega^2}{k+m\omega}$

EXERCISE - 5 (PREVIOUS YEARS AIPMT / NEET QUESTIONS)

Q.1 A particle moves in a circle of radius 5cm with constant speed and time period 0.2πs. The acceleration of the particle is **[AIPMT (PRE) 2011]**
(A) $5\ m/s^2$ (B) $15\ m/s^2$
(C) $25\ m/s^2$ (D) $36\ m/s^2$

Q.2 A car of mass 1000 kg negotiates a banked curve of radius 90 m on a frictionless road. If the banking angle is 45°, the speed of the car is : **[AIPMT (PRE) 2012]**
(A) $20\ ms^{-1}$ (B) $30\ ms^{-1}$
(C) $5\ ms^{-1}$ (D) $10\ ms^{-1}$

Q.3 A car of mass m is moving on a level circular track of radius R. If μ_s represents the static friction between the road and tyres of the car, the maximum speed of the car in circular motion is given by– **[AIPMT (MAINS) 2012]**
(A) $\sqrt{\mu_s mRg}$ (B) $\sqrt{Rg/\mu_s}$
(C) $\sqrt{mRg/\mu_s}$ (D) $\sqrt{\mu_s Rg}$

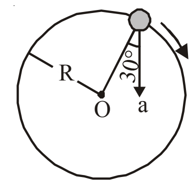
Q.4 Two stones of masses m and 2 m are whirled in horizontal circles, the heavier one in a radius r/2 and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they experience same centripetal forces. The value of n is :
(A) 1 (B) 2 **[RE-AIPMT 2015]**
(C) 3 (D) 4

Q.5 A car is negotiating a curved road of radius R. The road is banked at an angle θ. The coefficient of friction between the tyres of the car and the road is μ_s. The maximum safe velocity on this road is **[NEET 2016 PHASE 1]**
(A) $\sqrt{gR^2\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$ (B) $\sqrt{gR\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$
(C) $\sqrt{\frac{g}{R}\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$ (D) $\sqrt{\frac{g}{R^2}\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$

Q.6 A particle of mass 10 g moves along a circle of radius 6.4cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion? **[NEET 2016 PHASE 1]**
(A) $0.1\ m/s^2$ (B) $0.15\ m/s^2$
(C) $0.18\ m/s^2$ (D) $0.2\ m/s^2$

Q.7 What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop? **[NEET 2016 PHASE 1]**
(A) $\sqrt{gR}$ (B) $\sqrt{2gR}$
(C) $\sqrt{3gR}$ (D) $\sqrt{5gR}$

Q.8 In the given figure, $a = 15\ m/s^2$ represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. Speed of the particle is **[NEET 2016 PHASE 2]**



(A) 4.5 m/s (B) 5.0 m/s
(C) 5.7 m/s (D) 6.2 m/s

Q.9 One end of string of length ℓ is connected to a particle of mass 'm' and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed 'v' the net force on the particle (directed towards centre) will be – (T represents the tension in the string) **[NEET 2017]**
(A) $T+\frac{mv^2}{\ell}$ (B) $T-\frac{mv^2}{\ell}$
(C) Zero (D) T

Q.10 A mass m is attached to a thin wire and whirled in a vertical circle. The wire is most likely to break when: **[NEET 2019]**
(A) the mass is at the highest point.
(B) the wire is horizontal.
(C) the mass is at the lowest point.
(D) inclined at an angle of 60° from vertical.

Q.11 Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. The ratio of angular speed of A to that of B will be : **[NEET 2019]**
(A) $r_A : r_B$ (B) $v_A : v_B$
(C) $r_B : r_A$ (D) 1 : 1

Q.12 A block of mass 10 kg is in contact against the inner wall of a hollow cylindrical drum of radius 1m. The coefficient of friction between the block and the inner wall of the cylinder is 0.1. The minimum angular velocity needed for the cylinder to keep the block stationary when the cylinder is vertical and rotating about its axis, will be : ($g = 10\ m/s^2$) **[NEET 2019]**
(A) $\sqrt{10}$ rad/s (B) $\frac{10}{2\pi}$ rad/s
(C) 10 rad/s (D) 10π rad/s

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	B	A	A	D	D	B	A	B	C	A	A	C	B	A	B	D	D	C	D	C
Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	B	C	D	C	A	A	C	A	A	D	D	B	C	A	C	B	A	A	A	C
Q	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55					
A	B	C	A	B	A	A	B	D	D	C	B	C	A	B	C					

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	C	D	B	A	A	A	A	B	C	D	C	D	B	A	C	A	D	A	B
Q	21	22	23	24	25															
A	D	D	B	B	B															

EXERCISE - 3

Q	1	2	3	4	5
A	5	3	3	5	2

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9
A	B	B	C	D	C	C	B	D	B

EXERCISE - 5

Q	1	2	3	4	5	6	7	8	9	10	11	12
A	A	B	D	B	B	A	D	C	D	C	D	C

CIRCULAR MOTION

TRY IT YOURSELF-1

(1) **(D).** Velocity changes as its direction change. Acceleration changes as its direction change.

(2) **(BC).** ω = constant $\quad \therefore\ \alpha = 0$ = constant

(3) Let the moon orbit the earth in time T at a radius R. Its speed is then $v = 2\pi R/T$ and its acceleration towards the earth is

$$a = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

Put in the given numbers to find an answer which is roughly 1/3600 times g.

(4) **($40\pi/7$) sec.**

$\omega_0^2 = 900\ (\text{rad/sec})^2 \Rightarrow \omega_0 = 30$ rad/sec

$\omega^2 = 1600\ (\text{rad/sec})^2 \Rightarrow \omega = 40$ rad/sec

$$\theta = \left(\frac{\omega + \omega_0}{2}\right) t \Rightarrow t = \frac{2 \times 100 \times 2\pi}{40 + 30} = \frac{40\pi}{7}\ \text{sec}.$$

(5) (a) Converting the angular speed ω to from rev/s, we obtain,

$$\omega = \left(6.50\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{rad}}{1\ \text{rev}}\right) = 40.8\frac{\text{rad}}{\text{s}}$$

The tangential speed of each point is

Point 1 : $v_T = r\omega = (3.00\text{m})(40.8\ \text{rad/s}) = 122$ m/s (273 mph)

Point 2 : $v_T = r\omega = (6.70\ \text{m})\ 40.8\ \text{rad/s}) = 273$ m/s (611 mph)

The radian unit, being dimensionless, does not appear in the final answers.

(b) Converting the angular acceleration α to rad/s^2 from rev/s^2, we find $\alpha = \left(1.30\ \frac{\text{rev}}{\text{s}^2}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 8.17\ \frac{\text{rad}}{\text{s}^2}$

The tangential acceleration can now be determined :

Point 1 : $a_T = r\alpha = (3.00\ \text{m})\ (8.17\ \text{rad/s}^2) = 24.5\ \text{m/s}^2$;

Point 2 : $a_T = r\alpha = (6.70\ \text{m})\ (8.17\ \text{rad/s}^2) = 54.7\ \text{m/s}^2$

(6) To use $v = r\omega$, we need to find ω. Converting 3700 rev min^{-1} to rad s^{-1}.

$$\omega = 3700\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\text{s}}\right) = 387\ \text{rad s}^{-1}$$

Then the velocity at the blade tip is

$v = r\omega = (0.25\text{m})\ (387\ \text{rad s}^{-1}) = 97\ \text{ms}^{-1}$,

which is nearly 350 km h^{-1}.

(7) (i) We know, $d\omega = \alpha dt$

Integrating both sides, we get

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha\ dt = \int_0^t \left(4at^3 - 3bt^2\right) dt$$

or $\omega = \omega_0 + at^4 - bt^3$

(ii) We know, $d\theta = \omega\, dt$

On integrating both the sides, we get

$$\int_0^{\theta} d\theta = \int_0^t \omega\ dt = \int_0^t \left(\omega_0 + at^4 - bt^3\right) dt$$

or $\theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}$

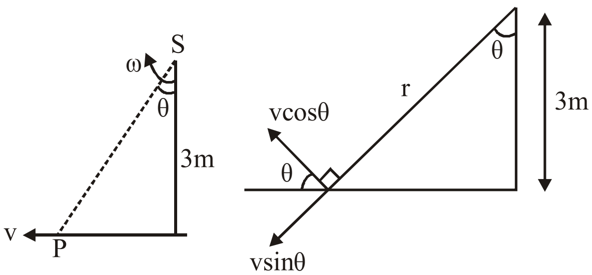
(8) (a) Since the acceleration is uniform,

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{387\ \text{rad s}^{-1}}{10\text{s}} = 38.7\ \text{rad s}^{-2}$$

(b) The tangential acceleration component is

$a_T = \alpha \times r = (38.7\ \text{rad s}^{-2})\ (0.25\ \text{m}) = 9.68\ \text{m s}^{-2}$

(9) $\omega = \frac{v\cos\theta}{r} \Rightarrow v = \frac{r\omega}{\cos\theta}$, where $r = \frac{3}{\cos\theta}$



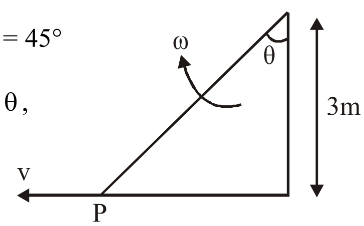
$\therefore\quad v = \frac{3\omega}{\cos^2\theta}$

At the instant shown, $\theta = 45°$

$\therefore\ v = 0.6$ m/s

Alternatively, $x = 3\tan\theta$,

also, $v_p = \frac{dx}{dt}$



$\therefore\quad v = 3\sec^2\theta \cdot \frac{d\theta}{dt} = 3\omega\sec^2\theta$

$\therefore\quad$ At the instant shown, $v = 3 \times 0.1 \times (\sqrt{2})^2 = 0.6\ \text{m/s}$

(10) (a) $\theta = \left(\frac{\omega + \omega_0}{2}\right) t = \left(\frac{100 + 0}{2}\right) \times 5 \times 60 = 15000$ revol.

(b) $\omega = \omega_0 + \alpha t \Rightarrow 0 = 100 - \alpha(5 \times 60) \Rightarrow \alpha = \frac{1}{3}$ rev/sec^2

(c) $\omega_{av} = \frac{\text{Total angle of rotation}}{\text{Total time taken}} = \frac{15000}{50 \times 60} = 50$ rev/sec.

TRY IT YOURSELF-2

(1) (a) Tangential acceleration

$$a_t = \frac{dv}{dt} \quad \text{or} \quad a_t = \frac{d}{dt}(4t) = 4\ \text{cm/s}^2$$

(b) $a_c = \frac{v^2}{R} = \frac{(4)^2}{2} = 8\ \text{cm/s}^2$

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(4)^2 + (8)^2} = 4\sqrt{5}\ \text{cm/s}^2$$

(2) **(a)** The tangential acceleration is constant and given by $a_t = \alpha r = (60\ \text{rad/s}^2)\ (0.2\text{m}) = 12\ \text{m/s}^2$

In order to calculate the radial acceleration we first need to find the angular velocity at the given time

$\omega = \omega_0 + \alpha t = 0 + (60\ rad/s^2)\ (0.15\ s) = 9\ rad/s$

$a_r = \omega^2 r = (81\ rad^2/s^2)(0.2\ m) = 16.2\ m/s^2$

The magnitude of the net linear acceleration is,

$$a = \sqrt{a_r^2 + a_t^2} = 20.2\ m/s^2$$

(b) $\theta = 0 + \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(60\ rad/s^2\right)(0.25s)^2 = 1.88\ rad$

This corresponds to (1.88 rad) (1 rev/2π rad) = 0.3 rev.

(3) **(C).** | v | = constant

(4) **(B).** The magnitude of the centripetal acceleration for a car undergoing circular motion is given by $a = v^2/R$. Therefore the speed of the car is

$$v = \sqrt{aR} = \sqrt{(2ms^{-2})\,(200m)} = 20\ m/s$$

(5) **(D).** The object always has a component of the acceleration pointing inward. When it is speeding up, it has a component of the tangential acceleration in the direction of motion (counterclockwise). The vector sum of these two components points somewhere between the arrow 1 and 2.

(6) **(C).** At the point P the acceleration has a positive tangential component so it is speeding up. At the point S the acceleration has a zero tangential component so it is moving at a constant speed. At the Point R the acceleration has a negative tangential component so it is slowing down.

(7) **(B).** Since the suspended object is not accelerating the tension force upwards exactly balances the gravitational force downwards, so T = mg . The inward force on the puck is due to the tension in the string which is equal in to the mass times the centripetal acceleration, $T = ma_{centripetal}$

Therefore, $mg = ma_{centripetal}$ or $g = a_{centripetal}$.

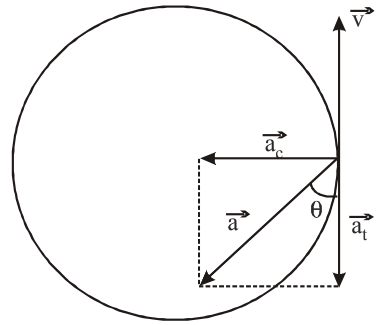
(8) You are spinning on a merry-go-round at a constant distance from the center. Your velocity is given by your distance divided by the time it takes you to travel that distance. In a time 1/f seconds you make one full revolution, which is a distance of 2πR. Your acceleration is given by $a = v^2/R = (2\pi Rf)^2/(R)$. The force of friction is $F_f \le \mu_s mg$. To find the minimum μ_s to keep you from falling off, you want the force due to friction that will just balance your centripetal acceleration.

$$F = mR\,(2\pi f)^2 = \mu_s mg \Rightarrow \mu_s = \frac{R\,(2\pi f)^2}{g}$$

(9) Speed, $v = 27\ km/h = 27 \times \frac{5}{18} ms^{-1} = 7.5 ms^{-1}$

Centripetal acceleration,

$$a_c = \frac{v^2}{r} \text{ or } a_c = \frac{(7.5)^2}{80} ms^{-2} = 0.7 ms^{-2}$$



P is the point at which cyclist applies brakes. At this point, tangential acceleration a_t, being negative, will act opposite to $\vec{v}$. Total acceleration, $a = \sqrt{a_c^2 + a_t^2}$

$$a = \sqrt{(0.7)^2 + (0.5)^2}\ ms^{-2} = 0.86 ms^{-2}$$

$$\tan\theta = \frac{a_c}{a_t} = \frac{0.7}{0.5} = 1.4 \qquad \therefore\ \theta = 54°28'$$

(10) After 1 second angular velocity of the turntable and hence that of the coin about the axis of rotation is

$$\omega = 0 + 2(rad/s^2) \times 1\ s = 2\ rad/s$$

$$a_T = \alpha r = (2\ rad/s^2) \times 1m = 2\ m/s^2$$

$$a_r = \omega^2 r = (2\ rad/s)^2 \times 1\ m = 4\ m/s^2$$

$$\because\ a = \sqrt{a_T^2 + a_r^2} = 2\sqrt{5}\ m/s^2$$

TRY IT YOURSELF-3

(1) Velocity, $v = 36\ km/h = \frac{36 \times 1000}{3600}\ m/s = 10\ m/s$ radius,

$r = 1000\ m$; $\tan\theta = \frac{v^2}{rg} = \frac{10 \times 10}{1000 \times 9.8} = \frac{1}{98}$

Let h be the height through which outer rail is raised.

Let ℓ be the distance between the two rails.

Then, $\tan\theta = \frac{h}{\ell}$ [∵ θ is very small]

or $h = \ell \tan\theta = 1.5 \times \frac{1}{98}\ m = 0.0153\ m$ [∵ ℓ = 1.5 m]

(2) Speed, $v = 720 \text{ km/h} = \frac{720 \times 1000}{3600} \text{ m/s} = 200 \text{ m/s}$

and $\tan\theta = \tan 15° = 0.2679$

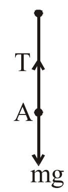
$$\tan\theta = \frac{v^2}{rg} \quad \text{or} \quad r = \frac{v^2}{g\tan\theta} = \frac{200 \times 200}{9.8 \times 0.2679} \text{ m}$$

$$= 15235.7 \text{ m} = 15.24 \text{ km.}$$

(3) **(i) 0°, (ii) cos^{-1} $(1/\sqrt{3})$, (iii) 90°**

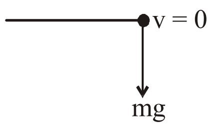
(i) At $\alpha = 0°$

$$a_{net} = \frac{v^2}{R} \uparrow$$



Acceleration is vertically up

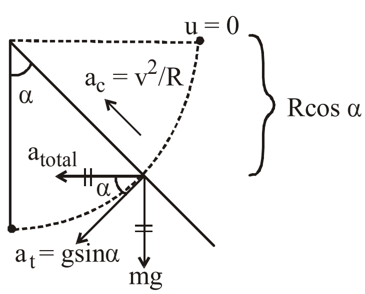
(ii) At $\alpha = 90°$ is at B



Acceleration is vertically down.

(iii) Horizontally

$$\tan\alpha = \frac{v^2/R}{g\sin\alpha} \Rightarrow g\sin\alpha \cdot \tan\alpha = \frac{v^2}{R} \quad \text{........... (1)}$$



Using energy conservation :

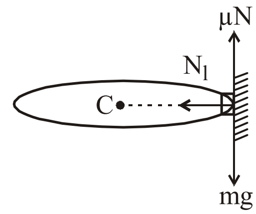
$$\frac{1}{2}mv^2 = mgR\cos\alpha \quad \text{........... (2)}$$

By eq. (1) and (2),

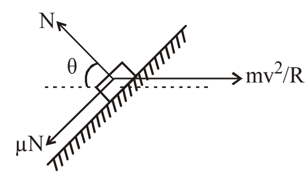
$$\tan\alpha = \frac{1}{\sqrt{2}} \quad \therefore \cos\alpha = \frac{1}{\sqrt{3}} \quad \therefore \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

(4) **(a) 0.2 N, (b) 30°**

(a) $N_1 = \frac{mv^2}{R} = \frac{1 \times 100}{10 \times 50} = 0.2 \text{ N}$



(b)



$$N = \frac{mv^2}{R}\cos\theta \quad \text{...... (1)}$$

For just slipping, $\mu N = \frac{mv^2}{R}\sin\theta$ ...... (2)

From eq. (1) & (2),

$$\tan\theta = \mu = \frac{1}{\sqrt{3}} = \frac{1}{0.58} = 1.724 \Rightarrow \theta = 30°$$

(5) $r = 30\text{m}$, $m = 10^6 \text{ kg}$, $\theta = ?$

$$v = 54\text{kmh}^{-1} = \frac{54 \times 1000}{3600} = \text{ms}^{-1} = 15\text{ms}^{-1}$$

(i) The centripetal force is provided by the lateral thrust by the outer rail on the flanges of the wheel of the train. The train causes an equal and opposite thrust on the outer rail (Newton's third law of motion). Thus, the outer rail wears out faster.

(ii) $\tan\theta = \frac{v^2}{rg} = \frac{15 \times 15}{30 \times 9.8} = 0.7653$

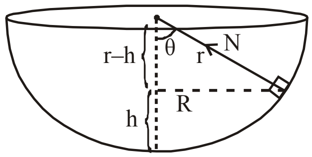
$\therefore \theta = \tan^{-1}(0.7653) = 37.43°$

(6) (C)

(7) R/3

(8) $\sqrt{10} \text{ m/s}$

(9) (a) $7\sqrt{2} \text{ rad/s}$ **(b) –9.8 × 10^{-3} m/s^2**



$$N\sin\theta = m\omega^2 R\ ;\ N\cos\theta = mg \qquad \therefore\ \tan\theta = \frac{\omega^2 R}{g}$$

$$\Rightarrow \frac{R}{r-h} = \frac{\omega^2 R}{g} \Rightarrow \omega^2 = \frac{g}{r-h} \qquad ......(1)$$

(a) $h > 0 \Rightarrow r - \frac{g}{\omega^2} > 0$

$$\Rightarrow \omega > \sqrt{\frac{g}{r}} = \sqrt{\frac{9.8}{0.1}} = \sqrt{98} = 7\sqrt{2}\ \frac{\text{rad}}{\text{second}}$$

(b) $g = \omega^2 (r-h) \Rightarrow \frac{\Delta g}{g} = -\frac{\Delta h}{h} = -\frac{10^{-4}}{h}$

(10) (a) $K = \frac{mg}{R(3-2\sqrt{2})}$

(b) At initial instant $a_t = g, a_c = 0$

At bottom position $a_t = 0, a_c = 0$

(a) Applying conservation of energy between initial and final position is :

Loss in gravitational P.E. of the bead of mass m

= gain in spring P. E.

$$\therefore\ mgR = \frac{1}{2}K(2R - \sqrt{2}R)^2 \quad \text{or} \quad K = \frac{mg}{R(3-2\sqrt{2})}$$

(b) At $t = 0 \Rightarrow a_t = g \Rightarrow a_c = 0$

At lowest point $a_t = 0 \Rightarrow a_c = 0$

The centripetal acceleration of bead at the initial and final position is zero because its speed at both position is zero. The tangential acceleration of the bead at initial position is g. The tangential acceleration of the bead at lower most position is zero.

CHAPTER-7 : CIRCULAR MOTION
EXERCISE-1

(1) **(B).** We have angular displacement

$$= \frac{\text{linear displacement}}{\text{radius of path}} \Rightarrow \Delta\theta = \frac{\Delta S}{r}$$

Here, $\Delta S = n(2\pi r) = 1.5(2\pi \times 2 \times 10^{-2}) = 6\pi \times 10^{-2}$

$$\therefore \Delta\theta = \frac{6\pi \times 10^{-2}}{2 \times 10^{-2}} = 3\pi \text{ radian}$$

(2) **(A).** We have $\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time}}$

For first one third part of circle,

angular displacement, $\theta_1 = \frac{S_1}{r} = \frac{2\pi r/3}{r}$

For second one third part of circle,

$$\theta_2 = \frac{2\pi r/3}{r} = \frac{2\pi}{3} \text{ rad}$$

Total angular displacement, $\theta = \theta_1 + \theta_2 = 4\pi/3$ rad
Total time $= 2 + 1 = 3$ sec

$$\therefore \vec{\omega}_{av} = \frac{4\pi/3}{3} \text{ rad/s} = \frac{4\pi}{6} = \frac{2\pi}{3} \text{ rad/s}$$

(3) **(A).** Angular speed of hour hand,

$$\omega_1 = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{12 \times 60} \text{ rad/sec}$$

Angular speed of minute hand,

$$\omega_2 = \frac{2\pi}{60} \text{ rad/sec} \Rightarrow \frac{\omega_2}{\omega_1} = \frac{12}{1}$$

(4) **(D).** We have $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow \frac{d\theta}{dt} = \omega_0 + \alpha t$

This is angular velocity at time t.
Now angular velocity at t = 2 sec will be

$$\omega = \left(\frac{d\theta}{dt}\right)_{t=2\,sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4 \text{ rad/sec}$$

(5) **(D).** The distance covered in completing the circle is $2\pi r = 2\pi \times 10$ cm

The linear speed is $v = \frac{2\pi r}{t} = \frac{2\pi \times 10}{4} = 5\pi$ cm/s

The linear acceleration is, $a = \frac{v^2}{r} = \frac{(5\pi)^2}{10} = 2.5\,\pi^2$ cm/s^2

This acceleration is directed towards the centre of the circle

(6) **(B).** Velocity $= \frac{\text{Circumference}}{\text{Time of revolution}} = \frac{2\pi r}{60}$

$$= \frac{2\pi \times 1}{60} = \frac{\pi}{30} \text{ cm/s}$$

Change in velocity $\Delta v = \sqrt{\left(\frac{\pi}{30}\right)^2 + \left(\frac{\pi}{30}\right)^2}$; $\frac{\pi}{30}\sqrt{2}$ =cm/s

(7) **(A).** The angular velocity is $\omega = v/r$. Hence, v = 10 m/s
r = 20 cm = 0.2 m $\therefore \omega = 50$ rad/s

(8) **(B).** Given that $\omega = 1.5t - 3t^2 + 2$

$$\alpha = \frac{d\omega}{dt} = 1.5 - 6t$$

When, $\alpha = 0 \Rightarrow 1.5 - 6t = 0 \Rightarrow t = 1.5/6 = 0.25$ sec

(9) **(C).** Given $v = 1.5\,t^2 + 2t$
Linear acceleration a = dv/dt = 3t + 2
This is the linear acceleration at time t
Now angular acceleration at time t

$$\alpha = \frac{a}{r} \Rightarrow \alpha = \frac{3t + 2}{2 \times 10^{-2}}$$

Angular acceleration at t = 2 sec

$$(\alpha)_{at\ t=2sec} = \frac{3 \times 2 + 2}{2 \times 10^{-2}} = \frac{8}{2} \times 10^2$$

$$= 4 \times 10^2 = 400 \text{ rad/sec}^2$$

(10) **(A).** Angular displacement after 4 sec is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 4 \times 4^2 = 32 \text{ rad}$$

Angular velocity after 4 sec
$\omega = \omega_0 + \alpha t = 0 + 4 \times 4 = 16$ rad/sec

(11) **(A).** Given $\alpha = 3t - t^2$

$$\Rightarrow \frac{d\omega}{dt} = 3t - t^2 \Rightarrow d\omega = (3t - t^2)dt \Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} + c$$

At $t = 0, \omega = 0$ $\therefore c = 0$, $\omega = \frac{3t^2}{2} - \frac{t^3}{3}$

Angular velocity at $t = 2$ sec, $(\omega)_{t=2\,sec}$

$$= \frac{3}{2}(4) - \frac{8}{3} = \frac{10}{3} \text{ rad/sec}$$

Since there is no angular acceleration after 2 sec
$\therefore$ The angular velocity after 6 sec remains the same.

(12) **(C).** Two types of acceleration are experienced by the car
(i) Radial acceleration due to circular path,

$$a_r = \frac{v^2}{r} = \frac{(30)^2}{500} = 1.8 \text{ m/s}^2$$

(ii) A tangential acceleration due to increase of tangential speed given by $a_t = \Delta v/\Delta t = 2$ m/s^2
Radial and tangential acceleration are perpendicular to each other. Net acceleration of car

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1.8)^2 + (2)^2} = 2.7 \text{ m/s}^2$$

(13) **(B).** $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$

(14) **(A).** Let the radius of the orbit be r and the number of revolutions per second be n. Then the velocity of electron is given by $v = 2\pi nr$,

$\therefore$ Acceleration $a = \dfrac{v^2}{r} = \dfrac{4\pi^2 r^2 n^2}{r} = 4\pi^2 r n^2$

Substituting the given values, we have

$a = 4 \times (3.14)^2 \times (5.3 \times 10^{-11})(6.6 \times 10^{15})^2$

$= 9.1 \times 10^{22}$ m/s^2 towards the nucleus.

The centripetal force is

$F_C = ma = (9.1 \times 10^{-31})(9.1 \times 10^{22})$

$= 8.3 \times 10^{-8}$ N towards the nucleus.

(15) **(B).** Given that radius of horizontal loop r = 1 km = 1000 m

Speed $v = 900$ km/h $= \dfrac{9000 \times 5}{18} = 250$ m/s

Centripetal acceleration $a_c = \dfrac{v^2}{r} = \dfrac{250 \times 250}{1000} = 62.5$ m/s^2

$\therefore \dfrac{\text{Centripetal acceleration}}{\text{Gravitational acceleration}} = \dfrac{a_c}{g} = \dfrac{62.5}{9.8} = 6.38 : 1$

(16) **(D).** (a) Centripetal force is not a real force. It is only the requirement for circular motion. It is not a new kind of force. Any of the forces found in nature such as gravitational force, electric friction force, tension in string reaction force may act as centripetal force.

(b) Work done by centripetal force is always zero.

(17) **(D).** Centripetal acceleration, $a_c = \dfrac{v^2}{r} = k^2 rt^2$

$\therefore$ Variable velocity $v = \sqrt{k^2 r^2 t^2} = k\,r\,t$

The force causing the velocity to varies $F = m\dfrac{dv}{dt} = m\,k\,r$

The power delivered by the force is,

$P = Fv = mkr \times krt = mk^2r^2t$

(18) **(C).** Both changes in direction although their magnitudes remains constant.

(19) **(D).** $\dfrac{mv^2}{r} = 10 \Rightarrow \dfrac{1}{2}mv^2 = 10 \times \dfrac{r}{2} = 1\,J$

(20) **(C).** $F = m\omega^2 R \;\; \therefore \; F \propto \omega^2$ (m and R are constant)

If angular velocity is doubled force will becomes four times.

(21) **(B).** $F = m\left(\dfrac{4\pi^2}{T^2}\right)R$.

If masses and time periods are same then $F \propto R$

$\therefore \dfrac{F_1}{F_2} = \dfrac{R_1}{R_2}$

(22) **(C).** $a = \dfrac{v^2}{r} = \omega^2 r = 4\pi^2 n^2 r = 4\pi^2\left(\dfrac{22}{44}\right)^2 \times 1 = \pi^2 ms^{-2}$

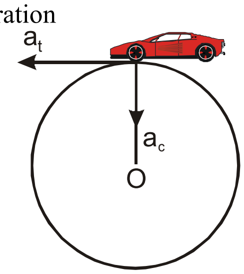
and its direction is always along the radius and towards the centre.

(23) **(D).** Time period

$= \dfrac{\text{Circumference}}{\text{Critical speed}} = \dfrac{2\pi r}{\sqrt{gr}} = \dfrac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4$ sec

(24) **(C).** Difference in K.E. = Difference in P.E. = 2mgr

(25) **(A).** We know centripetal acceleration



$a_c = \dfrac{(\text{tangential velocity})^2}{\text{radius}}$

$= \dfrac{(200)^2}{100} = 400$ m/sec^2

Tangential acceleration

$a_t = 100$ m/sec^2 (given)

$\therefore a_{net} = \sqrt{a_c^2 + a_t^2 + 2a_c a_t \cos 90^o} = \sqrt{a_c^2 + a_t^2}$

$= \sqrt{(400)^2 + (100)^2} = 100\sqrt{17}$ m/s^2

(26) **(A).** Maximum tension $T = \dfrac{mv^2}{r} + mg$

$\therefore \dfrac{mv^2}{r} = T - mg$ or $\dfrac{mv^2}{r} = 163.6 - 4 \times 9.8 \Rightarrow v = 6$ m/s

(27) **(C).** The situation is shown in fig. Let v be the velocity of the bob at the lowest position. In this position the P.E. of bob is converted into K.E. hence -

$mg\ell = \dfrac{1}{2}mv^2 \Rightarrow v^2 = 2g\ell$ ....(1)

If T be the tension in the string,

then $T - mg = \dfrac{mv^2}{\ell}$ ....(2)

From (1) & (2), T = 3 mg

(28) **(A).** $T_{max} = m\omega^2_{max} r + mg \Rightarrow \dfrac{T_{max}}{m} = \omega^2 r + g$

$\Rightarrow \dfrac{30}{0.5} - 10 = \omega^2_{max} r \Rightarrow \omega_{max} = \sqrt{\dfrac{50}{r}} = \sqrt{\dfrac{50}{2}} = 5$ rad / s

(29) **(A).** When body is released from the position p (inclined at angle θ from vertical) then velocity at mean position

$v = \sqrt{2g\ell\,(1 - \cos\theta)}$

$\therefore$ Tension at the lowest point $= mg + \dfrac{mv^2}{\ell}$

$= mg + \dfrac{m}{\ell}[2gl(1 - \cos 60)] = mg + mg = 2mg$

(30) **(D).** $T = mg + m\omega^2 r = m\{g + 4\pi^2 n^2 r\}$

$= m\left\{g + \left(4\pi^2\left(\dfrac{n}{60}\right)^2 r\right)\right\} = m\left\{g + \left(\dfrac{\pi^2 n^2 r}{900}\right)\right\}$

(31) **(D).** Tension at mean position, $mg + \frac{mv^2}{r} = 3mg$

$v = \sqrt{2g\ell}$ ...(i)

and if the body displaces by angle θ with the vertical

then $v = \sqrt{2g\ell(1-\cos\theta)}$ ...(ii)

Comparing (i) and (ii), $\cos\theta = 0 \Rightarrow \theta = 90°$

(32) **(B).** We know that,

$$\tan\theta = \frac{v^2}{rg} = \frac{\left(18 \times \frac{5}{18}\right)^2}{100 \times 10} = \frac{1}{40} \Rightarrow \theta = \tan^{-1}\frac{1}{40}$$

(33) **(C).** Let R be the normal reaction exerted by the road on the car. At the highest point, we have

$\frac{mv^2}{(r+h)} = mg - R$, R should not be negative.

Therefore $v^2 \le (r+a)g = (8.9+1.1) \times 10$

or $v^2 \le 10 \times 10 v \le 10$ m/sec

$\therefore v_{max} = 10$ m/sec

(34) **(A).** Let W = Mg be the weight of the car.

Friction force = 0.4 W

Centripetal force $= \frac{Mv^2}{r} = \frac{Wv^2}{gr}$; $0.4\,W = \frac{Wv^2}{gr}$

$\Rightarrow v^2 = 0.4 \times g \times r = 0.4 \times 9.8 \times 30 = 117.6$

$\Rightarrow v = 10.84$ m/sec

(35) **(C).** Let v be the speed of earth's rotation.

We know that W = mg

Hence $\frac{3}{5}W = mg - \frac{mv^2}{r}$ or $\frac{3}{5}mg = mg - \frac{mv^2}{r}$

$\therefore \frac{2}{3}mg = \frac{mv^2}{r}$ or $v^2 = \frac{2gr}{5}$

Now $v^2 = \frac{2 \times 9.8 \times (6400 \times 10^3)}{5}$

Solving, we get $v = 5 \times 10^9$ m/sec,

$\omega = \sqrt{\left(\frac{2g}{5r}\right)} = 7.8 \times 10^4$ radian/sec.

(36) **(B).** The minimum speed at highest point of a vertical circle is given by $v_c = \sqrt{rg} = \sqrt{20 \times 9.8} = 14$ m/s

(37) **(A).** By doing so component of weight of vehicle provides centripetal force.

(38) **(A).** The value of frictional force should be equal or more than required centripetal force. i.e. $\mu mg \ge \frac{mv^2}{r}$

(39) **(A).** $v = 60$ km/hr $= \frac{50}{3}$ m/s ; $r = 0.1$ km $= 100$m

$\therefore \tan\theta = \frac{v^2}{rg} = 0.283 \therefore \theta = \tan^{-1}(0.283)$

(40) **(C).** We know that $\tan\theta = v^2/rg$ ..... (1)

Let h be the relative raising of outer rail with respect to inner rail. Then $\tan\theta = \frac{h}{\ell}$ ...... (2)

(ℓ = separation between rails)

From (1) & (2), $h = \frac{v^2}{rg} \times \ell$

Hence $v = 48$ km/hr $= \frac{120}{9}$ m/s, ($r = 400$ m, $\ell = 1$m),

$\therefore h = \frac{(120/9)^2 \times 1}{400 \times 9.8} = 0.045$ m $= 4.5$ cm.

(41) **(B).** $F = \frac{mv^2}{r} \Rightarrow F \propto v^2$.

If v becomes double then F (tendency to overturn) will become four times.

(42) **(C).** Stone flies in the direction of instantaneous velocity due to inertia.

(43) **(A).** Because the reaction on inner wheel decreases and becomes zero. So it leaves the ground first.

(44) **(B).** $v = 72$ km / hour $= 20$ m / sec

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20 \times 20}{20 \times 10}\right) = \tan^{-1}(2)$$

(45) **(A).** $2\pi r = 34.3 \Rightarrow r = \frac{34.3}{2\pi}$ and $v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$

Angle of binding, $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = 45°$

(46) **(A).** Tension at top point $T + mg = \frac{mu^2}{r}$

$\therefore T = \frac{1 \times 4^2}{1} - 1 \times 10 = 6N$

(47) **(B).** $a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$

(48) **(D).** When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also acts downwards. These two forces are balanced by the outward centrifugal force acting on him.

$\therefore R + mg = \frac{mv^2}{r}$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle).

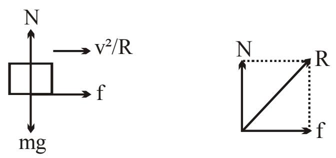
Because of the balancing of the forces, the motorcyclist does not fall down.
The minimum speed required to perform a vertical loop is given by equation (1) when R = 0

$\therefore\ mg = \frac{mv_{min}^2}{r}$ or $v_{min}^2 = gr$

or $v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25}\ ms^{-1} = 15.65\ ms^{-1}$

So, the minimum speed, at the top, required to perform a vertical loop is 15.65 ms^{-1}.

(49) (D).



(50) (C). As you ride, you are moving in a circle at uniform speed. You are experiencing uniform circular motion. As a result, your acceleration is always toward the center of the circle (a centripetal acceleration). Since you always feel a fictitious force in the direction opposite your acceleration, you feel one that is outward away from the center of the merry-go-round.

(51) (B). $\omega_A = \frac{v}{2r}$ $\omega_C = \frac{v}{r}$; $\therefore\ \frac{\omega_A}{\omega_B} = \frac{1}{2}$

(52) (C). Its mechanical energy is conserved.
It has a centripetal acceleration, downward.
Its speed is minimum.

(53) (A). The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R, where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires provides the force to produce this acceleration.

The horizontal component of Newton's second law is $f = mv^2/R$. If N is the normal force of the road on the car and m is the mass of the car, the vertical component of the second law is $N - mg = 0$. Thus $N = mg$ and $\mu_s N = \mu_s mg$. If the car does not slip, $f \leq \mu_s mg$.

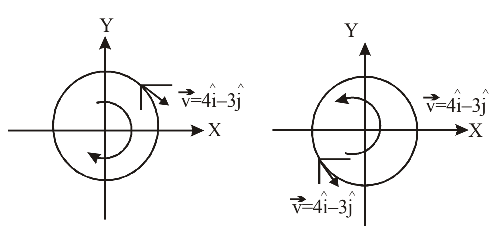
This means $v^2/R \leq \mu_s g$, or $v \leq \sqrt{\mu_s Rg}$. The maximum speed with which the car can round the curve without slipping is

$$v_{max} = \sqrt{\mu_s Rg} = \sqrt{(0.60)\ (30.5 m)\ (9.8\ m/s^2)} = 13\ m/s$$

(54) (B). At top point normal reaction should be zero

$mg - N = \frac{mv^2}{R}$ $\therefore\ v = \sqrt{gR}$

(55) (C).



EXERCISE-2

(1) (C). Let v be the speed of particle at B, just when it about to loose contact.
From application of Newton's second law to the particle normal to the spherical surface

$\frac{mv^2}{r} = mg \sin b$ .... (1)

Applying conservation of energy as the block moves from A to B: $\frac{1}{2}mv^2 = mg\ (r \cos a\ -\ r \sin b)$ .... (2)

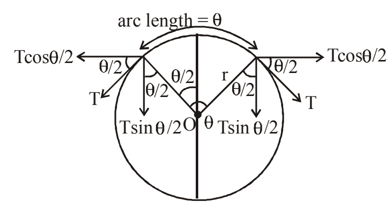
Solving (1) and (2), we get, $3 \sin \beta = 2 \cos \alpha$

(2) (C). As collision is elastic, freely suspended mass moves acquiring velocity of colliding mass after the collision.
Also acquired velocity must be equal to $\sqrt{5gL}$ to complete the circular motion.

Hence, $mgL + \frac{1}{2}mu^2 = \frac{1}{2}m(5g\ell) \Rightarrow\ u = \sqrt{3gL}$

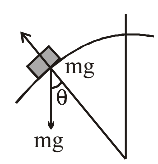
(3) (D). As $2T \sin\frac{q}{2} = dmw^2 r$ (for small angle $\sin\frac{q}{2}$ ® $\frac{q}{2}$)

but $dm = \frac{m}{\ell}\theta r$. As $\ell = 2pr$ $\therefore\ T = \frac{mw^2 r}{2p}$



Put m = 2π kg, ω = 10 π radian/s and r = 0.25 m
∴ T = 250 N

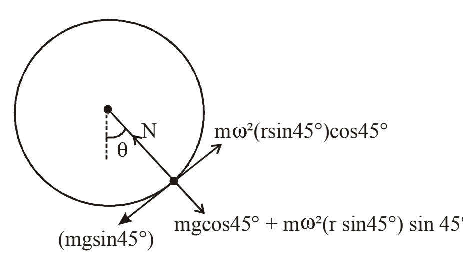
(4) (B). $mg \cos\theta - N = \frac{mv^2}{R}$



or $N = mg \cos\theta - \frac{mv^2}{R}$

Hence N decreases as θ increases.

(5) (A). The maximum angular speed of the hoop corresponds to the situation when the bead is just about slide towards. The free body diagram of the bead is



For the bead not to slide upwards

$m\omega^2 (r \sin 45°) \cos 45° - mg \sin 45° < \mu N$ .... (1)

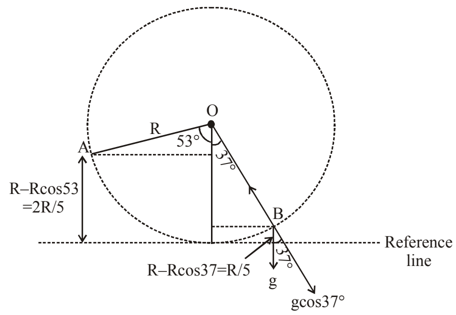
where $N = mg\cos 45° + mw^2(r\sin 45°)r\sin 45°$ .... (2)

From (1) and (2) we get, $w = \sqrt{30\sqrt{2}}\ \text{rad/s}$

(6) **(A).** By energy conservation between A and B

$$Mg\frac{2R}{5} + 0 = \frac{MgR}{5} + \frac{1}{2}Mv^2 \Rightarrow v = \sqrt{\frac{2gR}{5}}$$

Now, radius of curvature $r = \frac{v_\perp^2}{a_r} = \frac{2gR/5}{g\cos 37°} = \frac{R}{2}$



(7) **(A).** It can be observed that component of acceleration perpendicular to velocity is $a = 4\ m/s^2$

$$\therefore \text{Radius} = \frac{v^2}{a_c} = \frac{(2)^2}{4} = 1\ \text{metre}$$

(8) **(A).** Let ω be the angular speed of rotation. It is obvious from the figure that the ball moves in a horizontal centre of radius $(\ell + \ell\sin\theta)$ and its acceleration towards the centre will be equal to $(\ell + \ell\sin\theta)\,\omega^2$.

Here, we have $T\cos\theta = mg$ .... (1)

and $T\sin\theta = m\omega^2(\ell + \ell\sin\theta)$ .... (2)

$$\therefore \tan\theta = \frac{\omega^2(\ell + \ell\sin\theta)}{g}$$

$$\text{or } \omega^2 = \frac{g\tan\theta}{\ell(1+\sin\theta)} = \frac{10\times(1/\sqrt{3})}{(0.20)[1+(1/2)]}$$

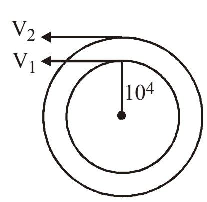
Solving, we get, ω = 4.4 rad/sec.

(9) **(B).** $\omega_1 = \frac{2\pi}{1}\ \text{rad/hr.},\ \omega_2 = \frac{2\pi}{8}\ \text{rad/hr.}$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \frac{R_2}{R_1} = 4 \Rightarrow R_2 = 4\times 10^4\ \text{km}$$

$$V_1 = \frac{2\pi R_1}{1h} = 2\pi\times 10^4\ \text{km/hr},$$

$$V_2 = \frac{2\pi R_2}{8h} = \pi\times 10^4\ \text{km/hr}$$



At closest separation

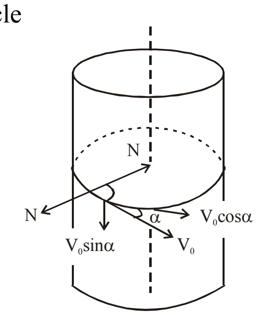
$$\omega = \frac{V_{rel} \perp \text{ to line joining}}{\text{length of line joining}} = \frac{\pi\times 10^4\ \text{km/hr}}{3\times 10^4\ \text{km}} = \frac{\pi}{3}\ \text{rad/hr.}$$

(10) **(C).** The friction force on coin just before coin is to slip will be : $f = \mu_s mg$
Normal reaction on the coin, N = mg
The resultant reaction y disk to the coin

$$= \sqrt{N^2 + f^2} = \sqrt{(mg)^2 + \mu_s(mg)^2} = mg\sqrt{1+\mu^2}$$

$$= 40\times 10^{-3}\times 10\times\sqrt{1+\frac{9}{16}} = 0.5\ N$$

(11) **(D).** Direction of velocity of the particle may be shown as in fig. This velocity v_0 is resolved in two components. Here $V_0\cos\alpha$ component is tangential horizontal and $V_0\sin\alpha$ is along the surface but vertically downwards. Here, if the particle exerts a force N on the surface of the cylinder, the cylinder will also apply equal and opposite force on the particle



and this will provide centripetal force to the particle.
So, $N = m(V_0\cos\alpha)^2/R = mV_0^2\cos^2\alpha/R$

$$\text{or, } N = \left(\frac{mV_0^2}{R}\right)\cos^2\alpha$$

(12) **(C).** Let speed of belt be v
Angular speeds of wheels

$$\omega_B = \frac{v}{2\pi R_B},\ \omega_A = \frac{v}{2\pi R_A};\ \frac{\omega_A}{\omega_B} = \frac{R_B}{R_A} = 2$$

(13) **(D).** The nature of motion can be determined only if we know velocity and acceleration as function of time.
Here acceleration at an instant is given and not known at other times so D.

(14) **(B).** As when they collide

$$vt + \frac{1}{2}\left(\frac{72v^2}{25\pi R}\right)t^2 - \pi R = vt \quad \therefore t = \frac{5\pi R}{6v}$$

Now, angle covered by $A = \pi + \frac{vt}{R}$ Put t,

$$\therefore \text{Angle covered by } A = \frac{11\pi}{6}$$

(15) **(A).** V = 27 kmph = 7.5 m/sec

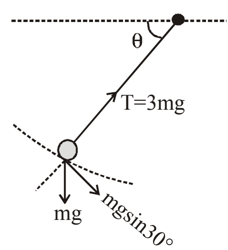
$$\vec{r} = L\hat{i} + 30\hat{j} = 30\tan\theta\hat{i} + 30\hat{j}$$

$\vec{V} = \frac{d\vec{r}}{dt}$; $7.5 = 30\sec^2\theta\left(\frac{d\theta}{dt}\right)\hat{i}$

$7.5 = 30\sec^2\theta\,\omega$; $7.5 = 30\sec^2 45°\,\omega$

$\frac{7.5}{30\times 2} = \omega = \frac{7.5}{60} = 0.125 \text{ rad/sec}$

(16) **(C).** $T - mg\sin q = \frac{mv^2}{R}$



$\text{Þ}\;\; 3mg - mg\sin 30°$

$= \frac{m(v_0^2 + 2g\ell\sin 30°)}{\ell}$

$\therefore\; v_0 = \sqrt{3g/2}$

(17) **(A).** $y = a\log\sec\left(\frac{x}{a}\right)$, $\frac{d\theta}{dt} = w$ (constant)

$\frac{dy}{dx} = \tan\left(\frac{x}{a}\right), \frac{d^2y}{dx^2} = \frac{1}{a}\sec^2\left(\frac{x}{a}\right)$

Radius of curvature $= \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = a\sec\frac{x}{a}$

$\frac{dy}{dx} = \tan\theta = \tan\left(\frac{x}{a}\right) \Rightarrow \theta = \frac{x}{a}; x = a\theta$

$\frac{dx}{dt} = a\frac{d\theta}{dt} = aw,\; \frac{d^2x}{dt^2} = 0,\; \frac{dy}{dt} = \frac{dy}{dx}\times\frac{dx}{dt} = \left[\tan\left(\frac{x}{a}\right)\right]aw$

$\frac{d^2y}{dt^2} = aw\sec^2\left(\frac{x}{a}\right).\frac{1}{a}dx = aw^2\sec^2\frac{x}{a}$

Now resultant acceleration,

$a = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2} = \sqrt{0 + a^2w^4\sec^4\left(\frac{x}{a}\right)}$

$= aw^2\sec^2\frac{x}{a} = \left(\frac{w^2}{a}\right)R^2$

$= \frac{2\times 2}{1/2}\times\left(\frac{1}{2}\right)^2\times\sec^2\left(\frac{\pi}{4}\frac{a}{a}\right) = 8\times\frac{1}{4}\times 2 = 4 \text{ m/sec}^2$

(18) **(D).** $a_r \propto t$; $\frac{v^2}{r} = kt$; $v^2 = krt$; $2v\frac{dv}{dt} = kr$

$a_t = \frac{kr}{2v} = \frac{kr}{2\sqrt{rkt}} = \frac{1}{2}\sqrt{\frac{kr}{t}}$; $a_t^2 a_r = \frac{1}{4}\frac{kr}{t}\times kt$

= independent on t

(19) **(A).** $N - mg = \frac{mv^2}{R}$; $v^2 = 2gc$; $R = \frac{\left[1+(dy/dx)^2\right]^{3/2}}{d^2y/dx^2}$

$x^2 = 4ay = \frac{b^2}{c}y$; $2x = \frac{b^2}{c}\frac{dy}{dx}$; $2 = \frac{b^2}{c}\frac{d^2y}{dx^2}$

At (0, 0) dy/dx = 0

$R = \frac{b^2}{2c}$; $N - mg = \frac{m2gc\,(2c)}{b^2}$; $N = mg\left(1 + \frac{4c^2}{b^2}\right)$

(20) **(B).** $T_{max} = \mu\, 8g$; $T_{max} - 2g = mv^2/r$

$v^2 = 2gh = 2gr\cos 60° = gr$

$T_{max} - 2g = 2gr/r$; $T_{max} = 4g$; $4g = \mu\, 8g \Rightarrow \mu = 1/2$

(21) **(D).** $a_c = \frac{v^2}{r} = \omega^2 r$; $r = 15$

(22) **(D).** $a_r = a_t = g\sin\theta$; $a_r = \frac{v^2}{\ell} = \frac{u^2 - 2g\ell\,(1-\cos\theta)}{\ell}$

$u^2 = 3g\ell$; $\theta = 90°$

(23) **(B).** At B, $N + kx - mg = mv^2/r$; $N = 0$

$k\frac{20}{100} - 5g = \frac{5v^2}{20/100}$ .... (1)

To find v use energy conservation,

$mgr\,(1 + \cos 60°) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ .... (2)

Solving eq. (1) and (2), k = 500 N/m

(24) **(B).** $T - mg\cos\theta = \frac{mv_{min}^2}{\ell}$; $T = 0$

$v_{min}^2 = u^2 - 2g\ell\,(1-\cos\theta) = g\ell + 2g\ell\cos\theta$

$0 - g\cos\theta = g + 2g\cos\theta$; □ $\cos\theta = -1/3$

$\frac{g}{3} = \frac{v_{min}^2}{\ell}$; $v_{min} = \sqrt{\frac{g\ell}{3}}$

(25) **(B).** $T = \frac{mv^2}{r} - 2mg + 3mg\cos\theta$

$0 = \frac{m(2+\sqrt{3})\,ag}{a} - 2mg + 3mg\cos\theta$

$-\sqrt{3}mg = 3mg\cos\theta$; $\cos\theta = \frac{-1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

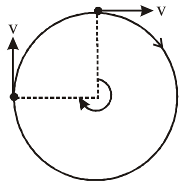
EXERCISE-3

(1) **5.** $a_t = \frac{du}{dt} = 4t = 4 \text{ m/s}^2$

$a_n = \frac{v^2}{R} = \frac{4}{4/3} = 3 \text{ m/s}^2 \Rightarrow a = \sqrt{a_t^2 + a_c^2} = 5$

(2) **3.** The angular displacement of the particle in t = 1 sec. is

$\theta = \omega t = \frac{v}{R}t = \frac{3\pi}{2}$



$\therefore$ The magnitude of impulse by centripetal force in t = 1 seconds is = change in momentum = $\sqrt{2}mv = 3\sqrt{2}\pi$ Ns

(3) **3.** Using relation $\theta = \omega_0 t + \frac{1}{2}at^2$.

In the first case, $\theta_1 = \frac{1}{2}a.4 = 2a, \omega = 0 + 2a$; In the second case, $\theta_2 = 2a.2 + \frac{1}{2}a.4 = 6a$ $\{\omega_0 = 2a \therefore \frac{\theta_2}{\theta_1} = \frac{3}{1}\}$

(4) **5.** After 1 second angular velocity of the turntable and hence that of the coin about the axis of rotation is

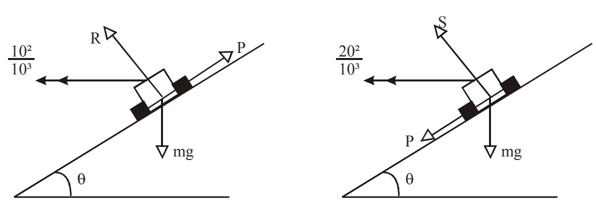
$\omega = 0 + 2(rad/s^2) \times 1\,s = 2\,rad/s$

$a_T = \alpha r = (2\ rad/s^2) \times 1m = 2\,m/s^2$

$a_R = \omega^2 r = (2\ rad/s)^2 \times 1\ m = 4\ m/s^2$

$\because a = \sqrt{a_T^2 + a_R^2} = 2\sqrt{5}m/s^2$

(5) **2.** Let m be the mass of the train and θ the angle at which the rail track is banked.



(a) (b)

(a) Horizontally, $R\sin\theta - P\cos\theta = m\frac{10^2}{10^3}$ .... (1)

Vertically, $R\cos\theta + P\sin\theta = mg$ .... (2)

(1) gives, $R\sin\theta = \frac{1}{10}m + P\cos\theta$. (2) gives,

$R\cos\theta = mg - P\sin\theta$

Dividing gives, $\frac{\sin\theta}{\cos\theta} = \frac{(1/10)m + P\cos\theta}{mg - P\cos\theta}$

Hence, $mg\sin\theta - P\sin^2\theta = \frac{1}{10}m\cos\theta + P\cos^2\theta$

$\Rightarrow mg\sin\theta - \frac{1}{10}m\cos\theta = P \quad (\cos^2\theta + \sin^2\theta = 1)$

(b) Horizontally, $S\sin\theta + P\cos\theta = m\frac{20^2}{10^3}$ .... (3)

Vertically, $S\cos\theta - P\sin\theta = mg$ .... (4)

(3) gives, $S\sin\theta = \frac{4}{10}m - P\cos\theta$ (4) gives,

$S\cos\theta = mg + P\sin\theta$

Dividing gives $\frac{\sin\theta}{\cos\theta} = \frac{(4/10)m - P\cos\theta}{mg + P\sin\theta}$

Hence, $mg\sin\theta + P\sin^2\theta = \frac{4}{10}m\cos\theta - P\cos^2\theta$

$\Rightarrow P = \frac{4}{10}m\cos\theta - mg\sin\theta$

Now from part (a) $P = mg\sin\theta - \frac{1}{10}m\cos\theta$

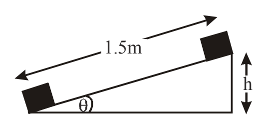
giving $\frac{1}{2}m\cos\theta = mg\sin\theta$

hence $\tan\theta = \frac{1}{4g} = 0.0255$

But the distance between the rails is 1.5m.

Hence, $h = 1.5\sin\theta = 1.5 \times 0.0255$

($\sin\theta \approx \tan\theta$ because θ is small)



So the outer rail is raised 0.0383 m above the inner rail.

EXERCISE-4

(1) **(B).** $\frac{mv^2}{r} = \mu mg$; $v = \sqrt{\mu rg} = \sqrt{0.6 \times 150 \times 10} = 30\,m/s$

(2) **(B).** The acceleration vector is along radius of the circle.

(3) **(C).** In uniform circular motion its kinetic energy will constant.

(4) **(D).** $S = t^3 + 5$ $\therefore$ Speed, $v = \frac{ds}{dt} = 3t^2$

and rate of change of speed $\frac{dv}{dt} = 6t$

$\therefore$ Tangential acceleration at t = 2s, $a_t = 6 \times 2 = 12\ m/s^2$

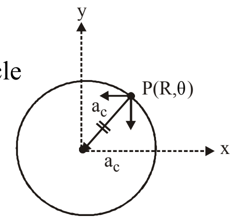
At t = 2s, $v = 3(2)^2 = 12$ m/s

$\therefore$ Centripetal acceleration, $a_c = \frac{v^2}{R} = \frac{144}{20}m/s^2$

$\therefore$ Net acceleration = $\sqrt{a_t^2 + a_i^2} \approx 14\ m/s^2$

(5) **(C).** For a particle in uniform circular motion,

$$\vec{a} = \frac{v^2}{R} \text{ towards centre of circle}$$



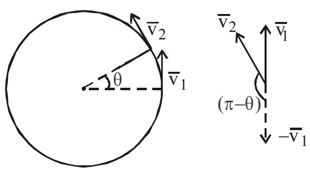
$$\therefore \quad \vec{a} = \frac{v^2}{R}(-\cos\theta\,\hat{i} - \sin\theta\hat{j})$$

or $\quad \vec{a} = -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

(6) **(C).** They have same ω.
Centripetal acceleration $= \omega^2 r$

$$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$$

(7) **(B).**

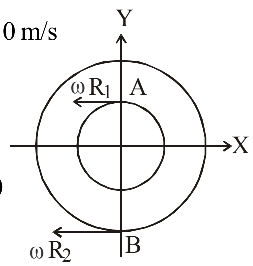


$$|\Delta\vec{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\pi-\theta)}$$

$= 2v\sin\frac{\theta}{2}$ Since, $[\,|\vec{v}_1| = |\vec{v}_2|\,]$

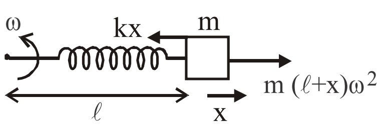
$= (2\times 10)\times\sin(30°) = 10$ m/s

(8) **(D).** $\theta = \omega t = \omega\frac{\pi}{2\omega} = \frac{\pi}{2}$



$$\vec{V}_A - \vec{V}_S = \omega R_1(-\hat{i}) - \omega R_2(-\hat{i})$$

(9) **(B).**



$kx = m\ell\omega^2 + mx\omega^2$

$$x = \frac{m\ell\omega^2}{k - m\omega^2}$$

EXERCISE-5

(1) **(A).** $a = \omega^2 R = \left(\frac{2\pi}{0.2\pi}\right)^2 (5\times 10^{-2}) = 5$ m/s2

(2) **(B).** For banking $\tan\theta = \frac{V^2}{Rg}$; $\tan 45° = \frac{V^2}{90\times 10} = 1$;

$V = 30$ m/s

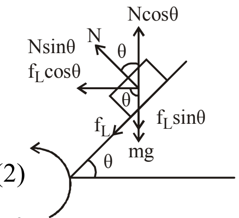
(3) **(D).** For smooth driving maximum speed of car v then

$$\frac{mv^2}{R} = \mu_s mg \;;\; v = \sqrt{\mu_s Rg}$$

(4) **(B).** $(F_C)_{heavier} = (F_C)_{lighter}$

$$\frac{2mV^2}{(r/2)} = \frac{m(nV)^2}{r} \Rightarrow n^2 = 4 \Rightarrow n = 2$$

(5) **(B).** Vertical equilibrium :
$N\cos\theta = mg + f_L\sin\theta$
$\Rightarrow mg = N\cos\theta - f_L\sin\theta \ldots(1)$
Horizontal equilibrium
$N\sin\theta + f_L\cos\theta = mv^2/R \ldots(2)$



$$\frac{\text{Eqn (2)}}{\text{Eqn (1)}} : \frac{v^2}{Rg} = \frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}$$

$$v = \sqrt{Rg\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}} = \sqrt{Rg\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}}$$

(6) **(A).** $m = 0.01$ kg, $r = 6.4$ cm

$\frac{1}{2}mv^2 = 8\times 10^{-4}$ J ; $v^2 = \frac{16\times 10^{-4}}{0.01} = 16\times 10^{-2}$

Speed $v^2 = 2a_t s$; $v^2 = 2a_t 4\pi r$

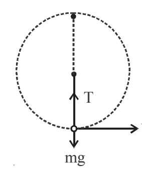
$$a_t = \frac{v^2}{8\pi r} = \frac{16\times 10^{-2}}{8\times 3.14\times 6.4\times 10^{-2}} = 0.1 \text{ m/s}^2$$

(7) **(D).** $v_{min} = \sqrt{5gR}$

(8) **(C).** $a\cos 30° = \frac{v^2}{r} \Rightarrow 15\frac{\sqrt{3}}{2} = \frac{v^2}{2.5} \Rightarrow v = 5.7$ m/s

(9) **(D).** Net force on the particle in uniform circular motion is centripetal force, which is provided by the tension in string.

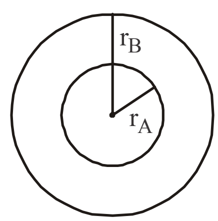
(10) **(C).** $T - mg = \frac{mu^2}{\ell}$



$T = mg + \frac{mu^2}{\ell}$

The tension is maximum at the lowest position of mass,so the chance of breaking is maximum.

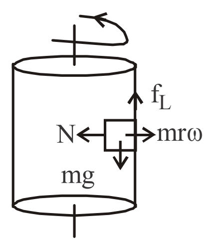
(11) **(D).** $T_A = T_B = T$



$\omega_A = \frac{2\pi}{T_A}$; $\omega_B = \frac{2\pi}{T_B}$

$$\frac{\omega_A}{\omega_B} = \frac{T_B}{T_A} = \frac{T}{T} = 1$$

(12) **(C).** For equilibrium of the block limiting friction
$f_L \geq mg$
$\mu N \geq mg$
$\mu m r\omega^2 \geq mg$



$\omega \geq \sqrt{\frac{g}{r\mu}}$; $\omega_{min} = \sqrt{\frac{g}{r\mu}} = \sqrt{\frac{10}{0.1\times 1}}$

$= 10$ rad/s