

**7**

# **CIRCULAR MOTION**

 $2\pi$ 

 $\delta q$  $\theta_2$ ....  $\Theta_1$  $O_{\kappa}$ 

 $B(t_2)$ 

 $A(t_1)$ 

r⊉

#### **ANGULAR DISPLACEMENT(or)**

The angle described by radius vector is called angular displacement.



Infinitesimal angular displacement is a vector quantity. However, finite angular displacement is a scalar quantity.

$$
\overrightarrow{d\theta_1} + \overrightarrow{d\theta_2} = \overrightarrow{d\theta_2} + \overrightarrow{d\theta_1} \text{ But } \overrightarrow{\theta_1} + \overrightarrow{\theta_2} \neq \overrightarrow{\theta_2} + \overrightarrow{\theta_1}
$$

**S.I. Unit :** Radian ; 1 radian =  $\frac{360}{25}$ 

**Dimension :** M°L°T°

- In 1 revolution  $\Delta\theta = 360^\circ = 2\pi$  radian
- In N revolution  $\Delta\theta = 360^\circ \times N = 2\pi N$  radian

Clockwise rotation is taken as negative  $\overrightarrow{)}$ 

Anticlockwise rotation is taken as positive  $(+)$ 

#### **ANGULAR VELOCITY()**

The rate of change of angular displacement with time is called

angular velocity. It is a vector quantity. The angle traced per unit time by the radius vector is called angular speed. Instantaneous angular velocity

$$
\omega = \lim_{\delta t \to 0} \frac{\delta \theta}{\delta t} \text{ or } \omega = \frac{d\theta}{dt}
$$

**S.I. Unit**: rad/sec ; **Dimension**: 
$$
M^0L^0T^{-1}
$$
 **Direction** : Infinitesimal angular displacement, angular velocity and angular acceleration are vector quantities whose direction is given by right hand rule.



**Right Hand Rule :** Imagine the axis of rotation to be held in the right hand with fingers curled round the axis and the thumb stretched along the axis. If the curled fingers denote the sense of rotation, then the thumb denoted the direction of the angular velocity (or angular acceleration or infinitesimal angular displacement). Anti-Clockwise<br>
he axis of rotation to be held<br>
urled round the axis and the<br>
i. If the curled fingers denote<br>
thumb denoted the direction<br>
r angular acceleration or<br>
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of (i) hour hand of a watch<br>
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ec. =  $2\pi$  rad<br>  $\frac{\pi}{0 \times 60}$ Anti-Clockwise<br>axis of rotation to be held<br>led round the axis and the<br>the curled fingers denote<br>mb denoted the direction<br>ngular acceleration or<br>nt).<br>(i) hour hand of a watch<br>pmpletes one rotation in<br>50 sec. =  $2\pi$  rad<br> $\$ Let dural of the axis and the<br>
urled round the axis and the<br>
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axis of rotation to be held<br>
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the curled fingers denote<br>
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mgular acceleration or<br>
it).<br>
(i) hour hand of a watch<br>
smpletes one rotation in<br>
50 sec. =  $2\$ 

#### **Example 1 :**

Calculate the angular speed of (i) hour hand of a watch and (ii) earth about its own axis.

 $360$   $560$   $121$ Sol. (i) The hour hand of a watch completes one rotation in 12 hours

 $2\pi$  Angle covered in  $12 \times 60 \times 60$  sec. =  $2\pi$  rad Angular speed of hour hand

$$
= \frac{2 \pi}{12 \times 60 \times 60} = \frac{\pi}{21600} \text{ rad/s}
$$

(ii) Earth completes one rotation about its axis in 24 hours angle covered in  $24 \times 60 \times 60$  sec. =  $2\pi$  rad

Angular speed of earth =  $\frac{2 \pi}{12(6.60 \times 10^{19} \text{ rad/s})} = \frac{\pi}{12200} \text{ rad/s}$ 

When we say that the angle is  $2\pi$ , it implies that angle is  $2\pi$  radian. Usually, we do not mention the unit of the angle, when it is expressed in radian.

#### **Relation between angular velocity and linear velocity :**

Suppose the particle moves along a circular path from point A to point B in infinitesimally time  $\delta t$ .

As,  $\delta t \rightarrow 0$ ,  $\delta \theta \rightarrow 0$ .

 $\therefore$  arc AB = chord AB

i.e. displacement of the particle is along a straight line.





RCULAR MOTION	ANGULARACELERATIC
But, $\lim_{\delta t \to 0} \frac{\delta \theta}{\delta t} = \omega = \text{angular velocity}$	ANGULARACELERATIC
∴ $v = r$ . $\omega$ [For circular motion only]	Average angular acceleration. Average angular acc.
i.e. (Linear velocity) = (Radius) × (Angular velocity)	$\overline{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta \omega}{\Delta t}$

 $\therefore$  v = r .  $\omega$  [For circular motion only]

i.e. (Linear velocity) = (Radius)  $\times$  (Angular velocity)

 $\vec{v} = \vec{\omega} \times \vec{r}$  [In general]

**EXECTION**<br>  $\lim_{t\to 0} \frac{\delta \theta}{\delta t} = \omega = \text{angular velocity}$ <br>
The rate of change of ang<br>  $\lim_{t\to 0} \frac{\delta \theta}{\delta t} = \omega = \text{angular velocity}$ <br>
The rate of change of angular acceleration.<br>
Average angular acceleration.<br>
Average angular acceleration.<br>
Average **ICULAR MOTION**<br>
But,  $\lim_{\delta t \to 0} \frac{\delta \theta}{\delta t} = \omega = \text{angular velocity}$ <br>  $\therefore v = r$ .  $\omega$ [For circular motion only]<br>  $\therefore v = r$ .  $\omega$ [For circular motion only]<br>  $\therefore v = r$ .  $\omega$ [For circular motion only]<br>  $\therefore v = r$ .  $\omega$ [For circular motion on Linear velocity of a particle performing circular motion is the vector product of its angular velocity and radius vector. In general,

LAR MOTION	ANGULARACCELLERATION
\n $\lim_{\delta t \to 0} \frac{\delta \theta}{\delta t} = \omega = \text{angular velocity}$ \n	\n        ANGULARACCELLERATION \n        The rate of change of \n        angular acceleration. \n        Average angular momentum of the velocity of a particle performing circular motion is \n        If is a vector quantity, in direction of angular \n        SI. Unit : radian/sec \n        The velocity component perpendicular to the line joining the particle and \n        and observer is a constant. \n        The velocity component perpendicular to the line joining the particle and \n        and observer is a constant. \n        The velocity component perpendicular to the line point of the particle and \n        and observer is a constant. \n        The velocity component perpendicular to the particle and \n        and observer is a constant. \n        The velocity component perpendicular to the particle and \n        and observer is a constant. \n        The velocity component perpendicular to the particle and \n        and observer is a constant. \n        The velocity component perpendicular to the particle and \n        and observer is a constant. \n        The velocity component perpendicular to the particle and \n        and power is a constant. \n        The velocity component perpendicular to the particle and \n        and power is a constant. \n        The velocity of a particle and \n        the velocity of a particle and \n        and observer is a constant. \n        The velocity of the particle is a constant. \n

#### **Example 2 :**

A particle is launched from horizontal plane with speed u and angle of projection  $\theta$ . Find angular velocity as observed from the point of projection of the particle at the time of landing.



w.r.t O, 
$$
\omega = \frac{u \sin \theta}{R}
$$
;  $\omega = \frac{u \sin \theta}{\left(\frac{u^2 \sin 2\theta}{g}\right)} = \frac{g}{2u \cos \theta}$ 

#### **Relative angular velocity :**

Angular velocity of a particle A with respect to other moving particle B is the rate at which position vector of A with respect to B rotates at that instant (or it is simply, angular velocity of A with origin fixed at B). Angular velocity of A w.r.t. B,  $\omega_{AB}$  is mathematically define as **Sol.** The tangential acceleration is given<br>
velocity:<br>
solity of a particle A with respect to other<br>
de B is the rate at which position vector of A<br>
to B rotates at that instant (or it is simply,<br>
w.r.t. B,  $\omega_{\text{A}}$  is becomes or pure in the speed of the state of the state of the state of the rate of A<br>
Fitticle B is the rate at which position vector of A<br>
For a velocity of A with origin fixed at B). Angular<br>  $\frac{a}{r} = \frac{a_1}{r} = \frac{0.5 \text$ gular velocity of a particle A with respect to other<br>
vin particle B is the rate at which position vector of A<sub>t</sub> =  $\frac{a_r}{dt} = \frac{a_r}{12 - t_1} = \frac{a_r}{120}$  m/<br>
h respect to B rotates at that instant (or it is simply,<br>
lular v

Component of relative velocity of A  
\n
$$
\omega_{ab} = \frac{\text{w.r.t. B, perpendicular to line}}{\text{separation between A and B}} = \frac{(V_{AB})_{\perp}}{r_{AB}}
$$
 **KINEMATICS OF CIRCULAR MOTIO**  
\n*Kinematics of circular motion*

#### **Example 3 :**

A particle is moving with constant speed in a circle as shown, find the angular velocity of the particle A with  $\zeta$ respect to fixed point B and C if angular velocity with respect to  $O$  is  $\omega$ .





$$
\omega_{\text{AO}} = \frac{(v_{\text{AO}})_\perp}{r_{\text{AO}}} = \frac{v}{r} = \omega
$$

$$
\therefore \omega_{AB} = \frac{(v_{AB})_{\perp}}{r_{AB}} = \frac{v}{2r} = \frac{\omega}{2}
$$
 and  $\omega_{AC} = \frac{(v_{AC})_{\perp}}{r_{AC}} = \frac{v}{3r} = \frac{\omega}{3}$   $\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2\pi)$ 

#### **ANGULARACCELERATION()**

The rate of change of angular velocity with time is called angular acceleration.

Average angular acc. Instantaneous angular acc.

$$
\alpha = \frac{\mathrm{d}\omega}{\mathrm{dt}} = \frac{\mathrm{d}^2\theta}{\mathrm{dt}^2}
$$

**EDIMADYANGELERATION (a)**<br> **EDIMADYANGED LEARNIFON**<br>
te of change of angular velocity with time is called<br>
r acceleration.<br>
ge angular acc. Instantaneous angular acc.<br>  $\frac{2-\omega_1}{2-t_1} = \frac{\Delta\omega}{\Delta t}$   $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt$ **EXECTE ERATION (a)**<br> **EXECTE ERATION (a)**<br> **EXECTE EXECTED AT AND ANOVANCED LEARNING**<br>
are acceleration.<br>
ge angular acc.<br>
Instantaneous angular acc.<br>
Instantaneous angular acc.<br>  $22 - \omega_1 = \frac{\Delta \omega}{\Delta t}$ <br>  $\alpha = \frac{d\omega}{dt} = \frac{d$ **EXECTED ATION (a)**<br>
THE CONSTANTION (a)<br>
The contract of change of angular velocity with time is called<br>
lar acceleration.<br>
Instantaneous angular acc.<br>
Instantaneous angular acc.<br>  $\frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta \omega}{\Delta t}$ <br>  $\alpha =$ **ILARACCELERATION** (a)<br>
The rate of change of angular velocity with time is called<br>
angular acceleration.<br>
Average angular acc.<br>  $\overline{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta \omega}{\Delta t}$   $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ <br>
It is a vector quant **EXECTED ATTION (a)**<br> **EXECUTERATION (a)**<br>
of change of angular velocity with time is called<br>
acceleration.<br>
angular acc. Instantaneous angular acc.<br>  $-\frac{\omega_1}{-t_1} = \frac{\Delta \omega}{\Delta t}$   $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ <br>
ctor quantity, **SPARID MADWANGED LEARNING**<br>
velocity with time is called<br>
Instantaneous angular acc.<br>  $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ <br>
rection is along the change<br>
ension : M°L°T<sup>-2</sup> It is a vector quantity, whose direction is along the change in direction of angular velocity. **S.I.** Unit : radian/sec<sup>2</sup>; Dimension : M°L°T<sup>-2</sup>

Velocity component perpendicular **Relation between angular acceleration & linear acceleration**

 $v_1$  For perfect circular motion we know,  $v = \omega r$ r Chameron  $=\frac{V_{\perp}}{I}$  On differentiating with respect to time, we get,

$$
\frac{dv}{dt} = r \frac{d\omega}{dt}
$$
; Tangential acceleration,  $a_t = r\alpha$ 

**EXECTELERATION (CO)**<br> **EXECTELERATION (CO)**<br>
ITENT CORRIDGE DIEATRINING<br>
ITENT acceleration.<br>
Instantaneous angular acc.<br>  $\frac{ω_2 - ω_1}{t_2 - t_1} = \frac{Δω}{Δt}$ <br>  $\alpha = \frac{dω}{dt} = \frac{d^2θ}{dt^2}$ <br>
vector quantity, whose direction is **EXACTELERATION (a)**<br> **EXACTELERATION (a)**<br>
ate of change of angular velocity with time is called<br>
at acceleration.<br>
age angular acc.<br>
Instantaneous angular acc.<br>  $\frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta \omega}{\Delta t}$ <br>  $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta$ Remember that  $a_t = \frac{dv}{dt}$  is the rate of change of speed and is not the rate of the change of velocity. It is, therefore not  $\overline{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta \omega}{\Delta t}$   $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ <br>
It is a vector quantity, whose direction is along the change<br>
in direction of angular velocity.<br>
S.I. Unit : radian/sec<sup>2</sup>; Dimension : M°L°T<sup>-2</sup><br>
io

equal to the net acceleration. Infact it is the component of acceleration along the tangent and hence suffix t is used for tangential acceleration.  $a_t$  is known as the tangential acceleration.

 $(tangential acceleration) = (angular acc) \times (radius)$ 

#### **Example 4 :**

**S.1. Unit** : radianxec="; **Dimension** : M<sup>-1</sup><sub>-</sub><sup>-1</sup> = <br>
particle and<br>
particle and<br>
the particle  $\frac{v_{\perp}}{r} = \frac{v_{\perp}}{r}$ <br>
on differentiating with respect to time, we get,<br>
be a with speed u<br>
Find angular velocity as If perpendicular<br>
is particle and<br>  $\frac{v_1}{m} = \frac{v_1}{r}$ <br>  $\frac{dv}{dt} = r \frac{dv}{dt}$ ; Tangential acceleration & linear acceleration<br>
to differentialing with respect to time, we get,<br>
on differentialing with respect to time, we ge  $\theta$  article travels in a circle of radius 20 cm at a speed that S.I. Unit : radian/sec<sup>2</sup> ; Dimension : M<sup>2</sup>L<sup>2</sup>T<sup>-2</sup><br>
aponent perpendicular<br>
time the particle and<br>
the particle and<br>
the particle and<br>
terveen the particle and<br>
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terveen the particle and<br>
tervee ent perpendicular<br>
the particle and<br>
the particle and<br>
the particle and<br>
the particle  $\frac{v_1}{r} = \frac{v_2}{r}$ <br>
on differentiating with respect to time, we get,<br>
on differentiating with respect to time, we get,<br>
during the s Δt dt dt<sup>2</sup><br>
antity, whose direction is along the change<br>
mgular velocity.<br>
an/sec<sup>2</sup>; **Dimension :** M°L°T<sup>-2</sup><br> **gular acceleration & linear acceleration**<br>
alar motion we know, v = ω r<br>
g with respect to time, we get,<br> uniformly increases. If the speed changed from 5.0 m/s to 6.0m/s in 2.0 sec, find the angular acceleration. ember that  $a_t = \frac{d_t}{dt}$  is the rate of change of speed and<br>the rate of the change of velocity. It is, therefore not<br>to the net acceleration.<br>it is is the component of acceleration along the tangent<br>ence suffix t is used the change of velocity. It is, therefore not<br>acceleration.<br>mponent of acceleration along the tangent<br>t is used for tangential acceleration.<br>he tangential acceleration.<br> $\vec{a}_t = \vec{\alpha} \times \vec{r}$ <br>eration) = (angular acc) × (rad oriality what respect to time, we get,<br>  $r \frac{d\omega}{dt}$ ; Tangential acceleration,  $a_t = r\alpha$ <br>
or that  $a_t = \frac{dv}{dt}$  is the rate of change of speed and<br>
rate of the change of velocity. It is, therefore not<br>
the net acceleration. r  $\frac{d\omega}{dt}$  ; Tangential acceleration,  $a_t = r\alpha$ <br>
er that  $a_t = \frac{dv}{dt}$  is the rate of change of speed and<br>
rate of the change of velocity. It is, therefore not<br>
he net acceleration.<br>
the net coerention along the tangent<br> fraction,  $a_t = r\alpha$ <br>  $\left(\frac{r}{r}\right)^2$  is the rate of change of speed and<br>
ange of velocity. It is, therefore not<br>
ration.<br>
Into facceleration along the tangent<br>
ed for tangential acceleration.<br>  $\times \vec{r}$ <br>  $\left(\vec{r}\right) = (\text{angular acc}) \times$ that  $a_t = \frac{d_t}{dt}$  is the rate of change of speed and<br>te of the change of velocity. It is, therefore not<br>net acceleration.<br>ecomponent of acceleration along the tangent<br>uffix t is used for tangential acceleration.<br>as the t

g  $\frac{1}{x}$  **Sol.** The tangential acceleration is given by

$$
a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{6.0 - 5.0}{2.0} \text{ m/s}^2 = 0.5 \text{ m/s}^2.
$$

The angular acceleration is

$$
\alpha = \frac{a_t}{r} = \frac{0.5 \text{ m/s}^2}{20 \text{ cm}} = 2.5 \text{ rad/sec}^2.
$$

#### AB **KINEMATICS OF CIRCULAR MOTION**

AB Kinematics of circular motion resembles with the kinematics W.<br>
We see that it is the component of acceleration along the tangential<br>
and hence sulling tis used for tangential acceleration.<br>
We said for tangential acceleration.<br>
We said the set of reduces the tangential accelerat separation between A and B<br>  $\frac{1}{\text{exp}(\text{det})}$  and B results is used for tangential acceleration.<br>  $\frac{\text{using }\theta_1}{\text{using}}$  :  $\omega = \frac{\text{using }\theta_2}{\left(\frac{m^2 \sin 20}{g}\right)} = \frac{g}{2u\cos\theta}$ <br>  $\frac{1}{\text{exp}(\text{det})}$ <br>  $\frac{1}{\text{exp}(\text{det})}$  =  $\frac{1}{2u\$ Expect to outer<br>
is simply,<br>
visition vector of A<br>
(or it is simply,<br>
and a B). Angular<br>  $\alpha = \frac{a_t}{r} = \frac{0.5 \text{ m/s}^2}{20 \text{ cm}} = 2.5 \text{ rad/sec}^2.$ <br>
or  $\frac{a_t}{r} = \frac{(V_{AB})_+}{r_{AB}}$ <br>
KINEMATICSOF CIRCULARMOTION<br>
of the intermation of The angular acceleration is<br>
(or it is simply,<br>
at B). Angular<br>
lly define as<br>  $\alpha = \frac{a_t}{r} = \frac{0.5 \text{ m/s}^2}{20 \text{ cm}} = 2.5 \text{ rad/sec}^2$ .<br>  $\frac{1}{r_{AB}} = \frac{(V_{AB})_1}{r_{AB}}$ <br> **KINEMATICS OF CIRCULARMOTION**<br>
Kinematics of circular motion A with respect to other<br>
instant (or it is simply,<br>
in the contract of A<sub>t</sub> =  $\frac{a_t}{t} = \frac{a_t}{12 - t_1} = \frac{2.0 \text{ m/s}^2}{2.0 \text{ m/s}^2} = 0.5 \text{ m/s}^2$ ,<br>
in fixed at B). Angular<br>
in fixed at B). Angular<br>
im fixed at B). Angular<br> the change of velocity. It is, therefore not<br>
acceleration.<br>
mponent of acceleration along the tangent<br>
t is used for tangential acceleration.<br>  $\frac{1}{1} = \vec{\alpha} \times \vec{r}$ <br>
eration) = (angular acc) × (radius)]<br>
i in a circle o of linear motion. Displacement in linear motion is equivalent to angular displacement in circular motion. Similarly, linear velocity equivalent to angular velocity and acceleration equivalent to angular acceleration.  $\frac{6.0-3.0}{2.0}$  m/s<sup>2</sup> = 0.5 m/s<sup>2</sup>.<br>
is<br>
= 2.5 rad/sec<sup>2</sup>.<br> **MOTION**<br>
ion resembles with the kinematics<br>
ment in linear motion is equivalent<br>
circular motion. Similarly, linear<br>
gular velocity and acceleration<br>
elerati ged from 5.0 m/s to<br>ged from 5.0 m/s to<br>leration.<br> $s^2 = 0.5$  m/s<sup>2</sup>.<br> $s^2$ .<br>with the kinematics<br>motion is equivalent<br>on. Similarly, linear<br>y and acceleration<br> $t + \frac{1}{2}\vec{\alpha}t^2$ <br> $\left.\right)$ <br>t cle of radius 20 cm at a speed that<br>he speed changed from 5.0 m/s to<br>e angular acceleration.<br>ion is given by<br> $= \frac{6.0 - 5.0}{2.0}$  m/s<sup>2</sup> = 0.5 m/s<sup>2</sup>.<br>a i is<br> $= 2.5$  rad/sec<sup>2</sup>.<br>**RMOTION**<br>toin resembles with the kinematics the speed changed from 5.0 m/s to<br>
e angular acceleration.<br>
ion is given by<br>  $= \frac{6.0 - 5.0}{2.0}$  m/s<sup>2</sup> = 0.5 m/s<sup>2</sup>.<br>
i is<br>  $= 2.5$  rad/sec<sup>2</sup>.<br> **RMOTION**<br>
intimes in the kinematics<br>
ment in linear motion is equivalent<br> e speed changed from 5.0 m/s to<br>angular acceleration.<br>on is given by<br> $\frac{6.0-5.0}{2.0}$  m/s<sup>2</sup> = 0.5 m/s<sup>2</sup>.<br>is<br>= 2.5 rad/sec<sup>2</sup>.<br>**MOTION**<br>ion resembles with the kinematics<br>ment in linear motion is equivalent<br>circular mot

For constant angular acceleration :

$$
Q_1 \omega = \frac{u \sin \theta}{R}; \omega = \frac{u \sin \theta}{\left(\frac{u^2 \sin 2\theta}{g}\right)} = \frac{g}{2u \cos \theta}
$$
\nA particle travels in a circle of radius 20 cm at a speed that  
\nuniformly increases. If the speed changed from 5.0 m/s to  
\n6.0 m/s in 2.0 sec, find the angular acceleration.  
\n**square velocity**;  
\n**angular velocity** of a particle A with respect to other  
\nrespect to B rotates at that instant (or it is simply,  
\n $u \sin \theta = 0.5 \text{ m/s}^2 = 0.5 \text{ m/s}^2$ .  
\n $u \sin \theta = 0.5 \text{ m/s}^2 = 0.5 \text{ m/s}^2$ .  
\n $u \sin \theta = 0.5 \text{ m/s}^2 = 0.5 \text{ m/s}^2$ .  
\n $u \sin \theta = 0.5 \text{ m/s}^2 = 0.5 \text{ m/s}^2$ .  
\n $u \sin \theta = 0.5 \text{ m/s}^2 = 0.5 \text{ m/s}^2 = 0.5 \text{ m/s}^2$ .  
\nThe angular acceleration is given by  
\n $u \sin \theta = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{6.0 - 5.0}{2.0} \text{ m/s}^2 = 0.5 \text{ m/s}^2$ .  
\nThe angular acceleration is  
\n $u \sin \theta = \frac{dv_2}{dt} = \frac{v_2 - v_1}{2.0} = \frac{6.0 - 5.0}{2.0} \text{ m/s}^2 = 0.5 \text{ m/s}^2$ .  
\nThe angular acceleration is  
\n $u \sin \theta = \frac{dv_2}{v_2}$ .  
\n $u \sin \theta = \$ 

ODM ADVANCED LEARNING

Here  $\omega_0$  = initial angular velocity,  $\omega_{(t)}$  = angular velocity after time t,  $\theta_0$  = Initial angular position

- $\theta$ <sub>(t)</sub> = Angular position after time t,
- $\theta_{\text{nth}}^{\sigma}$  = Angular displacement in n<sup>th</sup> second.

These equations are valid if  $\alpha$  is a constant. If angular acceleration is variable then use calculus approach.

#### **Example 5 :**

A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle  $\theta_1$ . In the next 2 sec, it rotates through an additional angle  $\theta_2$ . Find the ratio of  $\Theta_2/\Theta_1$ .

**Sol.** Using relation  $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ .  $2^{\alpha}$ 

In the first case,  $\theta_1 = \frac{1}{2}\alpha$ .  $4 = 2\alpha$ 

In the second case, 
$$
\theta_2 = \omega t + \frac{1}{2} \alpha t^2
$$
 (6)

where  $\omega$  is initial angular speed For next 2 sec. i.e. final angular speed of first 2 sec.

$$
\therefore \omega = 0 + 2\alpha \; ; \; \theta_2 = 2\alpha \cdot 2 + \frac{1}{2}\alpha \cdot 4 = 6\alpha \quad \therefore \; \frac{\theta_2}{\theta_1} = \frac{3}{1}
$$
 The Magnitude of the cen

Alternatively, we can write

$$
\theta_1 + \theta_2 = \frac{1}{2}\alpha (4)^2 = 8\alpha ; \theta_2 = 6\alpha
$$

#### **Example 6 :**

A disc starts rotating with constant angular acceleration

of  $\pi/2$  rad/ $s^2$  about a fixed axis perpendicular to its plane and through its centre. Find

- (a) the angular velocity of the disc after 4s
- (b) the angular displacement of the disc after 4 s and
- (c) number of turns accomplished by the disc in 4 s.

**Sol.** Here 
$$
\alpha = \frac{\pi}{2} \text{rad/s}^2
$$
,  $\omega_0 = 0$ ,  $t = 4$  s

(a) 
$$
\omega_{(4s)} = 0 + \left(\frac{\pi}{2} \text{ rad} / \text{s}^2\right) \times 4 \text{ s} = 2\pi \text{ rad} / \text{s}.
$$

(b) 
$$
\theta_{(4s)} = 0 + \frac{1}{2} \left( \frac{\pi}{2} \text{ rad} / s^2 \right) \times (16s^2) = 4 \pi \text{ radian.}
$$

(c) n.2
$$
\pi
$$
 rad. = 4 $\pi$  radian  $\implies$  n=2.

#### **CENTRIPETAL FORCE**

In circular motion the force acting on the particle along the radius and towards the centre keeps the body moving along the circular path. This force is called centripetal force. **Explanation :**

- (i) Centripetal force is necessary for circular motion.
- (ii) It is along the radius and towards the centre.

(iii) Centripetal force =  $[mass] \times [centripetal acceleration]$ 

$$
= \frac{mv^2}{r} = mr\omega^2
$$

- (iv) Centripetal force is due to known interaction. Therefore it is a real force.
- $\frac{DY\text{MATERIAL: PHYSICS}}{\times$  [centripetal acceleration]<br>= mro<sup>2</sup><br>known interaction.<br>tring is revolved uniformly<br>the centripetal force is due **Ex.** (a) If an object tied to a string is revolved uniformly in a horizontal circle, the centripetal force is due to the tension imparted to the string by the hand.



- (b) When a satellite is revolving in circular orbit round the earth, the centripetal force is due to the gravitational force of attraction between the satellite and the earth.
- d to uniform angular speed of first 2 sec.<br>  $\theta_2 = \alpha t + \frac{1}{2} \alpha t^2$ <br>  $\theta_0 t + \frac{1}{2} \alpha t^2$ <br>  $\theta_1 = \frac{1}{2} \alpha t + \frac{1}{2} \alpha t^2$ <br>  $\theta_2 = \alpha t + \frac{1}{2} \alpha t^2$ <br>
(b) When a satellite is tevrolving in circular orbit<br>  $\theta_1 = \frac{1}{2} \alpha t^2$ ted to uniform angular acceleration about<br>
its angular velocity is zero. In the first<br>
through an angle  $\theta_1$ . In the next 2 sec, it<br>
an additional angle  $\theta_2$ . Find the ratio of<br>  $\theta_1 = \frac{1}{2}\alpha$ ,  $4 = 2\alpha$ <br>  $\theta_1 = \frac{1}{2$ (c) In an atom, an electron revolves in a circular orbit round the nucleus. The centripetal force is due to the electrostatic force of attraction between the positively charged nucleus and negatively charged electron. (b) When a satellite is revolving in circular orbit round<br>the earth, the centripetal force is due to the<br>gravitational force of attraction between the<br>satellite and the earth.<br>(c) In an atom, an electron revolves in a cir (b) When a satellite is revolving in circular orbit round<br>the earth, the centripetal force is due to the<br>gravitational force of attraction between the<br>satellite and the earth.<br>(c) In an atom, an electron revolves in a cir (b) When a satellite is revolving in circular orbit round<br>the earth, the ectripetal force is due to the<br>gravitational force of attraction between the<br>satellite and the earth.<br>(c) In an atom, an electron revolves in a circ

# $\frac{\Theta_2}{\Theta_1} = \frac{3}{1}$  The Magnitude of the centripetal acceleration

1 Suppose that the particle travels from a point P<sup>1</sup> to an adjacent point P<sub>2</sub> in time  $\delta t$  and that the angle P<sub>1</sub>OP<sub>2</sub> is  $\delta \theta$ . Magnitude of change in velocity from  $P_1$  to  $P_2$ 

$$
= 2\tilde{v} \sin(\delta\theta/2)
$$

$$
\begin{aligned} \left[\Delta v = \sqrt{v^2 + v^2 - 2v^2 \cos \delta \theta} \right] &= \sqrt{2v^2(1 - \cos \delta \theta)} \\ &= \sqrt{2v^2 2\sin^2(\delta \theta/2)} = 2v\sin(\delta \theta/2) \end{aligned}
$$

In the direction of  $\overrightarrow{P_1O}$ , the acceleration is given approximately by

In the first case, 
$$
\theta_1 = \frac{1}{2}\alpha.4 = 2\alpha
$$
  
\nIn the second case,  $\theta_2 = \alpha i + \frac{1}{2}\alpha i^2$   
\nIn the second case,  $\theta_2 = \alpha i + \frac{1}{2}\alpha i^2$   
\nIn the second case,  $\theta_2 = \alpha i + \frac{1}{2}\alpha i^2$   
\nFor next 2 sec. i.e. final angular speed of first 2 sec.  
\nFor next 2 sec. i.e. final angular speed of first 2 sec.  
\nFor next 2 sec. i.e. final angular speed of first 2 sec.  
\nFor next 2 sec. i.e. final angular speed of first 2 sec.  
\n $\therefore \omega = 0 + 2\alpha ; \theta_2 = 2\alpha . 2 + \frac{1}{2}\alpha . 4 = 6\alpha . \therefore \frac{\theta_2}{\theta_1} = \frac{3}{1}$   
\nAlternatively, we can write  
\nAlternatively, we can write  
\nAlternatively, we can write  
\nAlternatively, we can write  
\nSuppe6:  
\nA disc starts rotating with constant angular acceleration  
\n $\therefore \theta_1 + \theta_2 = \frac{1}{2}\alpha (4)^2 = 8\alpha ; 0_2 = 6\alpha$   
\n $\theta_1 + \theta_2 = \frac{1}{2}\alpha (4)^2 = 8\alpha ; 0_2 = 6\alpha$   
\n $\Rightarrow (\Delta v^2 + v^2 - 2v^2 \cos \theta) = \sqrt{2v^2(1 - \cos \theta)}$   
\n $= \sqrt{2v^2 2 \sin^2(6\theta/2)} = 2v \sin (6\theta/2)$   
\n $\Rightarrow (\Delta v^2 + v^2 - 2v^2 \cos \theta) = \sqrt{2v^2(1 - \cos \theta)}$   
\nand through its center. Find  
\n(a) for 2 and 3<sup>2</sup> about a fixed axis perpendicular to its plane  
\nand through its center.  
\n(b) the angular velocity of the disc after 4s and  
\n(c) number of turns accomplished by the disc in 4 s.  
\nHere  $\alpha = \frac{\pi}{2}$  rad/s<sup>2</sup>,  $\omega_0 = 0$ ,  $t = 4 s$   
\ni.e.  $\frac{2v \sin (6\theta/2)}{\delta t}$   
\n(c)  $0.4s_0 = 0 + (\frac{\pi}{2} \tan^2/s^2) \times (16s^2) =$ 

So the acceleration at P<sub>1</sub> towards O is v (d $\theta$ /dt) But  $d\theta/dt$  is the angular velocity of the particle, which we will denote by  $\omega$  and we know that  $v = r\omega$ 

Hence, 
$$
v \frac{d\theta}{dt} = v\omega = (r\omega) \omega
$$
 or  $v \left(\frac{v}{r}\right)$ 

i.e. the radial acceleration of a particle travelling with constant speed v in a circle of radius r is towards the centre and is of magnitude.  $v^2/r$  or  $r\omega^2$ 



#### **Example 7 :**

A space shuttle is in a circular orbit at a height of 250 km, where the acceleration of earth's gravity is 93 per cent of its surface value. What is the period of its orbit? ("Period" means the time to complete one orbit). **EXECUTE:**<br>
The is in a circular orbit at a height of 250 km,<br>
celeration of earth's gravity is 93 per cent of<br>
(A) Its velocity and accelerate<br>
alue. What is the period of its orbit?<br>
(B) Its velocity and accelerate<br>
ten **IOTION**<br>
uttle is in a circular orbit at a height of 250 km,<br>
acceleration of earth's gravity is 93 per cent of<br>
value. What is the period of its orbit?<br>
means the time to complete one orbit).<br>
(2) Its velocity and accel **MOTION**<br>
uttle is in a circular orbit at a height of 250 km,<br>
acceleration of earth's gravity is 93 per cent of<br>
value. What is the period of its orbit?<br>
means the time to complete one orbit).<br>
(B) Its velocity and accel

**Sol.** 
$$
a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} \qquad [v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T}]
$$

Here  $r =$  radius of circular orbit of shuttle. So r = R<sub>earth</sub> + 250 km =  $6.6 \times 10^6$  m, where  $R_{earth}^{num} = 6.37 \times 10^6$  m.

$$
T = \left(\frac{4\pi^2 r}{a}\right)^{1/2} = \left(\frac{(4\pi^2) (6.6 \times 10^6 \text{ m})}{(0.93) (9.8 \text{ m/s}^2)}\right) = 5347 \text{s} = 89 \text{ min.}
$$

#### **CENTRIFUGAL FORCE**

The pseudo force experienced by a particle performing uniform circular motion due to accelerated frame of reference which is along the radius and directed away from the centre is called centrifugal force.

#### **Explanation :**

- (i) Centrifugal force is a pseudo force as it is experienced due to accelerated frame of reference. The origin of this force is unknown. (For observer in non-inertial frame)
- (ii) It is along the radius and away from the centre.
- (iii) The centrifugal force is having the same magnitude as that of centripetal force. But, its direction is opposite to that of centripetal force. It is not due to reaction of centripetal force because without action, reaction is not possible, but centrifugal force can exists without centripetal force.

(iv) Magnitude of the centrifugal force is  $mv^2/r = \omega^2$ .  $\sqrt{r}$  .  $\omega^2$  **Q.6 Note :** Pseudo force acts in non inertial frame i.e. accelerated frame of reference in which Newton's Law's of Motion do not hold good.

- **Ex.** (a) When a car moving along a horizontal curve takes a turn, the person in the car experiences a push in the outward direction.
- (b) The coin placed slightly away from the centre of a rotating gramophone disc slips towards the edge of the disc.
- (c) A cyclist moving fast along a curved road has to lean inwards to keep his balance.

#### **Applications of centrifugal force :**

- (i) The centrifugal pump used to lift the water, works on  $\Omega$ . the principle of centrifugal force.
- (ii) A cream-separator used in the dairy work, works on the principle of centrifugal force.
- (iii) Centrifuge used for the separation of suspended particle from the liquid, works on the principle of centrifugal force.
- (iv) The "spin drier" of washing machine works on the principle of centrifugal force.

## **TRY IT YOURSELF-1**

- **THON**<br>
The is in a circular orbit at a height of 250 km,<br>
celeration of earth's gravity is 93 per cent of<br>
the . What is the period of its orbit?<br>
(A) Its velocity and acceleration are bot<br>
the . What is the period of it **EXAMPLE SET ALCONSTRIBUTE AT ALCOLE AT ALCOLE CALCOLE CALCOLE ARENAL AND SURFAMPLE AND SURFAMPLE AND SURFAMPLE AND SURFAMPLE AT A DUST SURFAMPLE AND SURFAMPLE AND SURFAMPLE AND SURFAMPLE AND SURFAMPLE AND SURFAMPLE AND S EXAMPLE SET AND TRIVITYOURSELF-1**<br>
EDENTATIVATIVE SURFAINING<br>
IT ALL AND THE PROPERTION IS SURFAINING (A) Its velocity is one state with a uniform speed:<br>
(A) Its velocity and acceleration are both constant<br>
(B) Its velo **Q.1** When a particle moves in a circle with a uniform speed: (A) Its velocity and acceleration are both constant (B) Its velocity is constant but the acceleration changes (C) Its acceleration is constant but the velocity changes (D) Its velocity and acceleration both change.
	- $v = \frac{\text{distance}}{\text{distance}} = \frac{2\pi r}{T}$  Magnitude of angular acceleration is **Q.2** A body moves with constant angular velocity on a circle.

 $(A)$  r<sub> $\omega$ </sub><sup>2</sup>

- (B) constant
- (C) zero (D) None of the above
- **EXECUTE:**<br> **EXECUTIVE SET AND TRANSFELE-1**<br>
theight of 250 km,<br>
ty is 93 per cent of<br>
(A) Its velocity and acceleration are both constant<br>
(B) Its velocity and acceleration are both constant<br>
(B) Its velocity is constant **MOTION**<br>
Shuttle is in a circular orbit at a height of 250 km,<br>
ERYITYOURSE<br>
e acceleration of earth's gravity is 93 per cent of<br>
e value. What is the period of its orbit?<br>
(A) Its velocity and acceleration<br>
(B) Its velo **CULAR MOTION**<br> **EVERT YOURSELE-1**<br>
We a space shuttle is in a circular orbit at a height of 250 km,<br>
A space shuttle is in a circular orbit at a height of 250 km,<br>
A space shuttle is in a circular orbit at a height of 25 **EXAMPION SERVITYOURSELF:**<br>
shuttle is in a circular orbit at a height of 250 km, **CAL When a particle moves in a circle with a uniform spectation** of earth's gravity is 93 per cent of (A) Its velocity and acceleration ar **AR MOTION**<br>
TREVITYOURSELF-1<br>
constantine is in a circular orbit at a height of 250 km,<br>
ce the acceleration of earth's gravity is 93 per cent of<br>
relation of earth's gravity is 93 per cent of<br>
(A) Its velocity and accel **EXAMIDION**<br>
Fraction of earth of the sight of 250 km,<br>
Fraction and circular orbit at a height of 250 km,<br>
FRAYITYOURSELE-1<br>
ence the acceleration of earth syncing via S9 per cent of<br>
(A) Its velocity and acceleration ar **Q.3** Estimate the acceleration of the moon towards the earth given it orbits it once in 28 days at a radius of about a quarter of a million miles. (I know the units are funny and numbers are approximate. This problem tests your ability to give a quick and decent estimate, say to 10 percent.)
	- **Q.4** The square of the angular velocity  $\omega$  of a certain wheel increases linearly with the angular displacement during 100rev of the wheel's motion as shown. Compute the time



- **Q.5** A helicopter blades has an angular speed of  $\omega$ = 6.50 rev/s and an angular acceleration of  $\alpha = 1.30$  rev/s<sup>2</sup>. For point 1 at a distance 3m and point 2 at a distance 6.70m on the blade, calculate the magnitudes of (a) the tangential speeds and (b) the tangential accelerations.
- The maximum speed of the blades on rotary lawn mowers is limited to reduce the hazard from flying stones and other debris. A currently available model has a rotation rate of 3700 revolutions per minute and a blade 0.25 m in radius. What is the speed at the tip of the blade ?
- **Q.7** A wheel rotates with an angular acceleration given by  $\alpha = 4ar^3 - 3bt^2$ , where t is the time and a and b are constants. If the wheel has initial angular speed  $\omega_0$ , write the equation for the (i) angular speed (ii) angular displacement.
- **Q.8** The lawn mower blade has an angular velocity of 387 rad/ s and a radius of 0.25 m. If it accelerates to this velocity uniformly from rest over a 10s interval, find (a) the angular acceleration; (b) the tangential acceleration at the tip of the blade.
- **Q.9** A spotlight S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/sec. The spot of light P moves along the floor at a distance of 3m. Find the velocity of the spot P when  $\theta = 45^\circ$ .



- **Q.10** A fan is rotating with angular velocity 100 rev/sec. Then it is switched off. It takes 5 minutes to stop.
	- (a) Find the total number of revolution made before it stops. (Assume uniform angular retardation)
	- (b) Find the value of angular retardation.
	- (c) Find the average angular velocity during this interval.

### **ANSWERS**

- **(1)** (D) **(2)**(BC) **(3)** 1/3600 times g **(4)**  $(40\pi/7)$  sec.
- **(5)** (a) Point 1 :  $v_T = 122$  m/s (273 mph); Point 2 :  $v_T = 273$  m/s (611 mph)
- (b) Point 1 :  $a_T = 24.5 \text{ m/s}^2$ ; Point 2 :  $a_T = 54.7 \text{ m/s}^2$ **(6)**  $350 \text{ km h}^{-1}$ .

(7) (i) 
$$
\omega = \omega_0 + at^4 - bt^3
$$
 (ii)  $\theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}$ 

- **(8)** (a) 38.7 rad  $s^{-2}$  (b) 9.68 m  $s^{-2}$
- **(9)** 0.6 m/s

(10) (a) 15000 revol. (b) 
$$
\frac{1}{3}
$$
 rev/sec<sup>2</sup> (c) 50 rev/sec.

#### **UNIFORM CIRCULAR MOTION**

Motion of a particle along the circumference of a circle with a constant speed is called uniform circular motion.  $\qquad \qquad$   $\qquad$ Uniform circular motion is an accelerated motion. In case of uniform circular



$$
motion: \frac{dv}{dt} = 0
$$

(i) Speed remains constant.  $v = constant$  and  $v = \omega r$ Angular velocity  $\omega$  = constant Motion will be periodic with time period

$$
T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}
$$

(ii) **Frequency of Uniform Circular Motion :** The number of revolutions performed per unit time by the particle performing uniform circular motion is called the frequency(n).

$$
\therefore \quad n = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\omega}{2\pi}
$$

S.I. unit of frequency is Hz.

(iii) As  $\omega$  = constant, from  $\omega = \omega_0 + \alpha t$ 

 $\implies$  angular acceleration  $\alpha = 0$ .

As 
$$
a_t = \alpha r
$$
, tangential acc.  $a_t = 0$ 

(iv) As  $a_t = 0$ ,  $a = (a_t^2 + a_t^2)^{1/2}$  yields  $a = a_r$ , i.e., acceleration For is not zero but along radius towards centre and has magnitude  $a = a_r = (v^2/r) = r\omega^2$ . .

- (v) Speed and magnitude of acceleration are constant but their directions are always changing so velocity and acceleration are not constant. Direction of  $\vec{v}$  is always along the tangent while that of  $\vec{a}_r$  along the radius. **STUDY MATERIAL: PHYSICS**<br>Speed and magnitude of acceleration are constant but<br>their directions are always changing so velocity and<br>acceleration are not constant. Direction of  $\bar{v}$  is always<br>along the tangent while tha  $\begin{array}{ccc}\n\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet\n\end{array}$
- **STODYMATE:**<br>
100 rev/sec. Then it<br>
toon made before it<br>
their directions are always change<br>
tretardation)<br>
100 rev/sec. Then it<br>
their directions are always change<br>
acceleration are not constant. Directions<br>
along the ta **STUDY MATERIAL: PHYSIC**<br>
(v) Speed and magnitude of acceleration are constant by<br>
up to stop.<br>
up to stop.<br>
up to stop.<br>
up to stop.<br>
it is always changing so velocity an<br>
up to exceleration are not constant. Direction o **STUDY MATERIAL: PHYSICS**<br>
y 100 rev/sec. Then it<br>
(v) Speed and magnitude of acceleration are constant but<br>
their directions are always changing so velocity and<br>
tractardation)<br>
along the tangent while that of  $\vec{a}_r$  a **STUDY MATERIAL: PHYS**<br>
ular velocity 100 rev/sec. Then it<br>
minutes to stop.<br>
their directions are always changing so velocity in<br>
their directions are always changing so velocity<br>
under retardation)<br>
under retardation.<br> (vi) If the moving body comes to rest, i.e.  $\vec{v} \rightarrow 0$ , the body will move along the radius towards the centre and if radial acceleration  $a_r$  vanishes, the body will fly off along the tangent. So a tangential velocity and a radial acceleration (hence force) is a must for uniform circular motion. **STUDY MATERIAL: PHYSICS**<br>
ed and magnitude of acceleration are constant but<br>
directions are always changing so velocity and<br>
leration are not constant. Direction of  $\vec{v}$  is always<br>
g the tangent while that of  $\vec{a}_r$ **STUDY MATERIAL: PHYSICS**<br>and magnitude of acceleration are constant but<br>irections are always changing so velocity and<br>ation are not constant. Direction of  $\vec{v}$  is always<br>the tangent while that of  $\vec{a}_r$  along the ra **STUDY MATERIAL: PHYSICS**<br>
Speed and magnitude of acceleration are constant but<br>
their directions are always changing so velocity and<br>
acceleration are not constant. Direction of  $\bar{v}$  is always<br>
along the tangent while win move along the tradinal acceleration  $a_k$  vanishes, the body will fly off<br>aradial acceleration  $a_k$  vanishes, the body will fly off<br>along the tangent. So a tangential velocity and a radial<br>acceleration (hence force) i

As 
$$
\overrightarrow{F} = \frac{mv^2}{r} \neq 0
$$
, so the body is not in equilibrium

and linear momentum of the particle moving on the circle is not conserved. However, as the force is central,

 $\vec{\tau} = 0$ , so angular momentum is conserved, i.e.,

 $\vec{p} \neq$  constant but  $\vec{L}$  = constant.  $\mathcal{F}^{\text{max}}_{\text{max}}$ = constant.

(vii) The work done by centripetal force is always zero as it is perpendicular to velocity and hence displacement. By work-energy theorem as : onserved. However, as the force is central,<br>o angular momentum is conserved, i.e.,<br>t but  $\vec{L} = \text{constant}$ .<br>one by centripetal force is always zero as<br>ccular to velocity and hence displacement.<br>regy theorem as :<br>e-hange in ki

Work done = change in kinetic energy  $\Delta K = 0$ So K (Kinetic energy) remains constant.

e.g. Planets revolving around the sun, motion of an

electron around the nucleus in an atom

 $=$  constant,  $\vec{v}$  changing continuously

 $\vec{a}_r \implies$  directed towards the center, continuously changing direction.

#### **Example 8 :**

A stone of mass 1 kg is whirled in a circular path of radius 1m. Find out the tension in the string if the linear velocity is 10 m/s ?

**Sol.** Tension = 
$$
\frac{mv^2}{R} = \frac{1 \times (10)^2}{1} = 100 \text{ N}
$$

#### **Example 9 :**

 $\omega$  with a time period of 30 days at a height of 1600 km. Find = 0, so angular momentum is conserved, i.e.,<br>
nnstant but  $\vec{L}$  = constant.<br>
ork done by centripetal force is always zero as<br>
pendicular to velocity and hence displacement.<br>
k-energy theorem as :<br>
kinetic energy cmanis stant but  $\vec{L}$  = constant.<br>
the done by centripetal force is always zero as<br>
endicular to velocity and hence displacement.<br>
-energy theorem as :<br>
me = change in kinetic energy  $\Delta K = 0$ <br>
inetic energy) remains constant. A satellite of mass  $10<sup>7</sup>$  kg is revolving around the earth out the force of attraction on satellite by earth ?

**Sol.** Force =  $m\omega^2$ R and

T 2 r 2 6 2 2 3.14 6.28 T 30 86400 2.59 10 Force 2 2 6 6.28 m r 2.59 10 × 10<sup>7</sup> × (6400 + 1600) × 10<sup>3</sup> = 2.34 × 10<sup>6</sup> N



#### **Example 10 :**

Two balls of equal masses are attached to a string at distances 1m and 2m from one end as shown in fig.  $\left( \begin{array}{cc} T_1 \\ T_2 \end{array} \right)$  m <sup>T</sup> The string with masses is

then moved in a horizontal circle with constant speed. Find the ratio of the tension  $T_1$  and  $T_2$  ?





#### **NON UNIFORM CIRCULAR MOTION**

A circular motion in which both direction and magnitude of velocity changes.

#### **Examples of non uniform circular motion :**

A merry-go-round spinning up from rest to full speed, or a ball whirling around in a vertical circle.

The net acceleration is neither parallel nor perpendicular to the velocity.

We can resolve the acceleration vector into two components:

> dv dt

a<sup>n</sup>

 $O<sub>1</sub>$ 

 $v^2$ r 2

#### **(a) Radial Acceleration :**

The component of the acceleration towards the centre is  $\omega^2 r = v^2/r$  $a_r$  perpendicular to the  $\bigwedge$ 

velocity

 $\Rightarrow$  changes only the directions of velocity. Acts just like the acceleration in uniform circular motion.

$$
a_c \text{ or } a_r = \frac{v^2}{r} = \omega^2 r
$$

**Centripetal force** 
$$
F_c = \frac{mv^2}{r} = m\omega^2 r
$$
 (ii) I

#### **(b) Tangential Acceleration :**

 $2m \rightarrow$ 

m

 $1<sub>m</sub>$ 

The component along the tangent

(along the direction of motion) is 
$$
\frac{dv}{dt}
$$

 $a_t$  parallel to the velocity (since it is tangent to the path)

 $\Rightarrow$  changes magnitude of the velocity acts just like onedimensional acceleration

(b) Tangential Acceleration :  
\nThe component along the tangent  
\n(along the direction of motion) is 
$$
\frac{dv}{dt}
$$
  
\na<sub>t</sub> parallel to the velocity (since it is tangent to the  
\npath)  
\n⇒ changes magnitude of the velocity acts just like one-  
\ndimensional acceleration  
\n⇒  $a_t = \frac{dv}{dt}$ , where  $v = \frac{ds}{dt}$  and  $s =$  length of arc  
\nTangential force  $F_t = ma_t$   
\nNet acceleration vector is obtained by vector addition of  
\nthese two components.  
\nThe magnitude of the net acceleration  
\n $a = \sqrt{a_t^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$   
\nIn non-uniform circular motion :  
\nspeed |  $\vec{v}$  | ≠ constant angular velocity  $\omega \ne$  constant  
\ni.e. speed ≠ constant i.e. angular velocity ≠ constant

**Net acceleration vector** is obtained by vector addition of these two components.

The magnitude of the net acceleration

$$
a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{d v}{d t}\right)^2}
$$

#### **In non-uniform circular motion :**

tion:<br>
tion:<br>
g the tangent<br>
of motion) is  $\frac{dv}{dt}$ <br>
locity (since it is tangent to the<br>
of the velocity acts just like one-<br>
tion<br>  $\frac{ds}{dt}$  and s = length of arc<br>
= ma<sub>t</sub><br>
is obtained by vector addition of<br>
t accelerat (along the direction of motion) is  $\frac{d\mathbf{r}}{dt}$ <br>  $a_{\mathbf{t}}$  parallel to the velocity (since it is tangent to the<br>
path)<br>  $\Rightarrow$  changes magnitude of the velocity acts just like one-<br>
dimensional acceleration<br>  $\Rightarrow a_{\mathbf{t}}$ speed  $|\vec{v}| \neq$  constant angular velocity  $\omega \neq$  constant i.e. speed  $\neq$  constant i.e. angular velocity  $\neq$  constant If at any instant  $\Rightarrow$  v = magnitude of velocity of particle

- $\Rightarrow$  r = radius of circular path
- $\Rightarrow$   $\omega$  = angular velocity of a particle, then at that instant  $v = r \omega$

**Net force on the particle in non uniform circular motion**

<sup>c</sup> m v <sup>F</sup> F F F c t 2 2 F F F c t F<sup>C</sup> Ft F F C If is the angle made by F with F<sup>c</sup> then tc<sup>F</sup> tan F <sup>1</sup> tc<sup>F</sup> tan F [angle between F<sup>c</sup> and Ft is 90º] Net acceleration, 2 2 net c t a a a 

Angle between F and F<sub>t</sub> is  $(90^{\circ} - \theta)$ 

 $F_{\text{net}}$ m

#### **NOTE**

- (i) In both uniform and non-uniform circular motion  $F_c$  is perpendicular to velocity. So work done by centripetal force will be zero in both the cases.
- $=\frac{m \nu}{r} = m\omega^2 r$  (ii) m any (ii) In uniform circular motion  $F_t = 0$ , as  $a_t = 0$ , so work done will be zero by tangential force. But in non-uniform circular motion  $F_t \neq 0$ , so work done by tangential force is non zero.



 $a<sub>r</sub>$  a

 $a<sub>t</sub>$ t

Rate of work done by net force in non-uniform circular motion = rate of work done by tangential force

$$
\Rightarrow P = \frac{dW}{dt} = \overrightarrow{F_t} \cdot \overrightarrow{v} = \overrightarrow{F_t} \cdot \frac{d \overrightarrow{x}}{dt}
$$

(iii) In a circle as tangent and radius are always normal to each

other, so  $\overrightarrow{a_1}$   $\overrightarrow{a_1}$ .

Net acceleration in case of circular motion

$$
a=\sqrt{a_r^2+a_t^2}
$$

Here it must be noted that  $a_t$  governs the magnitude of  $\vec{v}$ while  $a_r$  its direction of motion so that

if  $a_r = 0$  and  $a_t = 0$ ,  $a \rightarrow 0 \Rightarrow$  motion is uniform translatory if  $a_r = 0$  but  $a_t \neq 0$ ,  $a \rightarrow a_t \Rightarrow$  motion is accelerated translatory if  $a_r \neq 0$  but  $a_t = 0$ ,  $a \rightarrow a_r \Rightarrow$  motion is uniform circular ⇒  $P = \frac{dW}{dt} = \vec{F}_t \cdot \vec{v} = \vec{F}_t \cdot \frac{d\vec{x}}{dt}$ <br>
In a circle as tangent and radius are always normal to each<br>
or a circle as tangent and radius are always normal to each<br>
or a circle as tangent and radius are always nor

circular

#### **NOTE**

- **(i)** In one-dimensional motion, acceleration is always parallel to velocity and changes only the magnitude of the velocity vector.
- **(ii)** In uniform circular motion, acceleration is always perpendicular to velocity and changes only the direction of the velocity vector.
- **(iii)** In the more general case, like projectile motion, acceleration is neither parallel nor perpendicular to velocity. Causing change in both the magnitude and direction of the velocity vector.



#### **Example 11 :**

A road makes a 90° bend with a radius of 190 m. A car enters the bend moving at 20 m/s. Finding this too fast, the driver decelerates at 0.92 m/s<sup>2</sup>. Determine the acceleration of the car when its speed rounding the bend has dropped to 15 m/s.

**Sol.** Since it is rounding a curve, the car has a radial acceleration associated with its changing direction, in addition to the tangential deceleration that changes its speed.

EXERCISE ARISING<br>
EXAMPLE DEFARING<br>
Rate of work done by net force in non-uniform circular<br>
motion = rate of work done by tangential force<br>  $\Rightarrow P = \frac{dW}{dt} = \vec{F}_t \cdot \vec{v} = \vec{F}_t \cdot \frac{d\vec{x}}{dt}$ <br>
In a circle as tangent and radi **STUDYMATERIA**<br>
of work done by net force in non-uniform circular<br>
on = rate of work done by tangential force<br>  $P = \frac{dW}{dt} = F_t \cdot v = F_t \cdot \frac{d\vec{x}}{dt}$ <br>  $F = \frac{dW}{dt} = F_t \cdot v = F_t \cdot \frac{d\vec{x}}{dt}$ <br>  $\therefore$  solid acceleration is directed<br> **STUDYMATERI**<br>
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TOOR done by the force in non-uniform circular<br>
The of work done by tangential force<br>  $\frac{dV}{dt} = \vec{F}_t \cdot \vec{F}_t \cdot \frac{d\vec{X}}{dt}$ <br>
as tangent and radius are always normal to each one by net force in non-uniform circular<br>
f work done by tangential force<br>  $\overrightarrow{r_t} \cdot \overrightarrow{v} = \overrightarrow{F_t} \cdot \frac{d\overrightarrow{x}}{dt}$ <br>  $\overrightarrow{r_t} \cdot \overrightarrow{v} = \overrightarrow{F_t} \cdot \frac{d\overrightarrow{x}}{dt}$ <br>
Solveing down, the tan<br>
acceleration is disposite the velocity.<br> ETAINING<br>
For the by-net force in non-uniform circular<br>  $=$  rate of work done by tangential force<br>  $=$  at  $\frac{dW}{dt} = F_t \cdot v = F_t \cdot \frac{d\vec{x}}{dt}$ <br>
Eleast angent and radius are always normal to each<br>
so  $\vec{a}_t = \frac{v^2}{r} = \frac{(15 \text{$ STU<br>
Rate of work done by net force in non-uniform circular<br>
Rate of work done by the force in non-uniform circular<br>
motion = rate of work done by tangential force<br>  $P = \frac{dW}{dt} = \vec{F}_t \cdot \vec{v} = \vec{F}_t \cdot \frac{d\vec{x}}{dt}$ <br>
In a cir 2 2 **EXECUTE:**<br>
SUP of work done by net force in non-uniform circular<br>
on = rate of work done by tangential force<br>  $P = \frac{dW}{dt} = F_t \cdot \vec{v} = F_t \cdot \frac{d\vec{x}}{dt}$ <br>  $F = \frac{dV}{dt}$  are given that<br>  $P = \frac{dW}{dt} = F_t \cdot \vec{v} = F_t \cdot \frac{d\vec{x}}{dt}$ <br> We are given that  $a<sub>t</sub>=0.92 \text{ m/s}^2$ , since the car is slowing down, the tangential acceleration is directed opposite the velocity. The radial acceleration is **STUDY MATERIAL: PHYSICS**<br>given that<br> $y2 \text{ m/s}^2$ , since the car is<br>g down, the tangential<br>ration is directed<br>te the velocity.<br> $\frac{2}{\pi} = \frac{(15 \text{ m/s})^2}{190 \text{ m}} = 1.2 \text{ m/s}^2$ <br>and the of net acceleration.<br> $= \sqrt{a_r^2 + a_t^2} = [(1$ **STUDY MATERIAL: PHYSICS**<br>We are given that<br> $a_t = -0.92$  m/s<sup>2</sup>, since the car is<br>slowing down, the tangential<br>acceleration is directed<br>ppposite the velocity.<br>The radial acceleration is<br> $a_r = \frac{v^2}{r} = \frac{(15 \text{ m/s})^2}{190 \text{ m}} =$ **STUDY MATERIAL: PHYSICS**<br>
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92 m/s<sup>2</sup>, since the car is<br>
g down, the tangential<br>
tration is directed<br>
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dial acceleration is<br>  $r^2 = \frac{(15 \text{ m/s})^2}{190 \text{ m}} = 1.2 \text{ m/s}^2$ <br>
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-0.92 m/s<sup>2</sup>, since the car is<br>
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osite the velocity.<br>
radial acceleration is<br>  $\frac{v^2}{r} = \frac{(15 \text{ m/s})^2}{190 \text{ m}} = 1.2 \text{ m/s}^2$ <br>
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s<sup>2</sup>, since the car is<br>
wn, the tangential<br>
e velocity.<br>
cceleration is<br>  $\left(\frac{(15 \text{ m/s})^2}{190 \text{ m}}\right) = 1.2 \text{ m/s}^2$ <br>
of net acceleration.<br>  $\frac{2}{r} + a_t^2 = [(1.2 \text{ m/s})^2 + (0.92 \text{ m/s})^2]^{1/2} =$ **STUDY MATERIAL: PHYSICS**<br>
re given that<br>  $0.92 \text{ m/s}^2$ , since the car is<br>
ing down, the tangential<br>
site the velocity.<br>
site the velocity.<br>
radial acceleration is<br>  $\frac{v^2}{r} = \frac{(15 \text{ m/s})^2}{190 \text{ m}} = 1.2 \text{ m/s}^2$ <br>
nitude SIUDY MATERIAL: PHYSICS<br>
that<br>
i, the tangential<br>
i is directed<br>
velocity.<br>
celeration is<br>  $\frac{5 \text{ m/s})^2}{190 \text{ m}} = 1.2 \text{ m/s}^2$ <br>
f net acceleration.<br>  $+a_t^2 = [(1.2 \text{ m/s})^2 + (0.92 \text{ m/s})^2]^{1/2} = 1.5 \text{ m/s}^2$ <br>
an angle<br>  $\frac{1}{a$ **STUDY MATERIAL: PHYSICS**<br>
given that<br>
m/s<sup>2</sup>, since the car is<br>
down, the tangential<br>
thion is directed<br>
the velocity.<br>
al acceleration is<br>  $=\frac{(15 \text{ m/s})^2}{190 \text{ m}} = 1.2 \text{ m/s}^2$ <br>
de of net acceleration.<br>  $\sqrt{a_r^2 + a_t^2} = [($ **STUDY MATERIAL: PHYSICS**<br>
that<br>
since the car is<br>
the tangential<br>
is directed<br>
elocity.<br>
eleration is<br>  $\frac{m/s^2}{90 \text{ m}} = 1.2 \text{ m/s}^2$ <br>
net acceleration.<br>
and  $a_t^2 = [(1.2 \text{ m/s})^2 + (0.92 \text{ m/s})^2]^{1/2} = 1.5 \text{ m/s}^2$ <br>
an angle **STUDY MATERIAL: PHYSICS**<br>
re given that<br>
ing down, the tangential<br>
leleration is directed<br>
site the velocity.<br>
radial acceleration is<br>  $\frac{v^2}{r} = \frac{(15 \text{ m/s})^2}{190 \text{ m}} = 1.2 \text{ m/s}^2$ <br>
nitude of net acceleration.<br>  $a = \sqrt{a$ 

$$
a_r = \frac{v^2}{r} = \frac{(15 \text{ m/s})^2}{190 \text{ m}} = 1.2 \text{ m/s}^2
$$

Magnitude of net acceleration.

$$
a = \sqrt{a_r^2 + a_t^2} = [(1.2 \text{ m/s})^2 + (0.92 \text{ m/s})^2]^{1/2} = 1.5 \text{ m/s}^2
$$
  
and points at an angle

$$
\theta = \tan^{-1} \left( \frac{a_r}{a_t} \right) = \tan^{-1} \left( \frac{1.2 \text{ m/s}^2}{0.92 \text{ m/s}^2} \right) = 53^\circ
$$

relative to the tangent line to the circle.

#### **Example 12 :**

FUDY MATERIA<br>
For the tore in non-uniform circular<br>  $\frac{d}{dt}$  a  $\frac{d}{dt}$  are  $-0.92 \text{ m/s}^2$ , since the car is<br>
slowing down, the tangential<br>
acceleration is directed<br>
(poposite the velocity.<br>
The radia acceleration is **STUDY MATERIAL: PHYSICS**<br>
an that<br>
s<sup>2</sup>, since the car is<br>
wn, the tangential<br>
in is directed<br>
velocity.<br>
cceleration is<br>  $\frac{15 \text{ m/s}}{190 \text{ m}} = 1.2 \text{ m/s}^2$ <br>
of net acceleration.<br>  $\frac{2}{t} + a_t^2 = [(1.2 \text{ m/s})^2 + (0.92 \text{ m/s})^$ A particle is constrained to move in a circular path of radius  $r = 6m$ . Its velocity varies with time according to the relation  $v = 2t$  (m/s). Determine its (i) centripetal acceleration, (ii) tangential acceleration, (iii) instantaneous acceleration at (a)  $t = 0$  sec, and (b)  $t = 3$  sec.  $\theta = \tan^{-1} \left( \frac{a_r}{a_t} \right) = \tan^{-1} \left( \frac{1.2 \text{ m/s}^2}{0.92 \text{ m/s}^2} \right) = 53^\circ$ <br>
sive to the tangent line to the circle.<br>
12:<br>
12:<br>
tricle is constrained to move in a circular path of radius<br>
m. Its velocity varies with time acco 190 m<br>
of orter acceleration.<br>  $\left[\frac{a_1^2}{a_1^2} + a_1^2\right] = [(1.2 \text{ m/s})^2 + (0.92 \text{ m/s})^2]^{1/2} = 1.5 \text{ m/s}^2$ <br>
s at an angle<br>  $\tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{1.2 \text{ m/s}^2}{0.92 \text{ m/s}^2}\right) = 53^\circ$ <br>
the tangent line to the circle.<br>
is tangent line to the circle.<br>
nstrained to move in a circular path of radius<br>
city varies with time according to the relation<br>
determine its (i) centripetal acceleration, (ii)<br>
eleration, (iii) instantaneous acceleration a nts at an angle<br>  $\tan^{-1}\left(\frac{a_r}{a_r}\right) = \tan^{-1}\left(\frac{1.2 \text{ m/s}^2}{0.92 \text{ m/s}^2}\right) = 53^\circ$ <br>
to the tangent line to the circle.<br>
le is constrained to move in a circular path of radius<br>
lts velocity varies with time according to the r at an angle<br>
at an angle<br>  $n^{-1} \left( \frac{a_r}{a_t} \right) = \tan^{-1} \left( \frac{1.2 \text{ m/s}^2}{0.92 \text{ m/s}^2} \right) = 53^\circ$ <br>
the tangent line to the circle.<br>
s constrained to move in a circular path of radius<br>
velocity varies with time according to th <sup>2</sup>/<sub>2</sub>  $\left(\frac{1}{2}\right)$  = 53°<br>
cular path of radius<br>
ding to the relation<br>
al acceleration, (ii)<br>
ous acceleration at<br>  $a_t = \frac{dv}{dt} = 2$  m/s<sup>2</sup><br>
m/s<sup>2</sup><br>
F-2 = 53°<br>ar path of radius<br>g to the relation<br>cceleration, (ii)<br>acceleration at<br> $= \frac{dv}{dt} = 2$  m/s<sup>2</sup>  $\left( \frac{a_1}{a_1} \right)$  - land to the circle.<br>
Therefore, and the set of reduced to move in a circular path of radius<br>
In this velocity varies with time according to the relation<br>  $2t$  (m/s). Determine its (i) centripetal acc

**Sol.** (a) At 
$$
t = 0
$$
,  $v = 0$ , Thus  $a_r = 0$ 

but 
$$
\frac{dv}{dt} = 2
$$
 thus  $a_t = 2$  m/s<sup>2</sup>  
and  $a = \sqrt{a_t^2 + a_r^2} = 2$  m/s<sup>2</sup>

(b) At 
$$
t = 3
$$
 sec,  $v = 6$  m/s,

so 
$$
a_r = \frac{v^2}{r} = \frac{(6)^2}{6} = 6
$$
 m/s<sup>2</sup> and  $a_t = \frac{dv}{dt} = 2$  m/s<sup>2</sup>

## **TRY IT YOURSELF-2**

- **Q.1** A particle moves in a circle of radius 2.0cm at a speed given by  $v = 4t$ , where v is in cm/s and t is in seconds.
	- (a) Find the tangential acceleration at  $t = 1s$ .
	- (b) Find total acceleration at  $t = 1s$ .
- **Q.2** A flywheel of radius 20 cm starts from rest, and has a constant angular acceleration of 60  $\text{rad/s}^2$ . find
	- (a) the magnitude of the linear acceleration of a point on the rim after 0.15s.
	- (b) the number of revolutions completed in 0.25s.
- **Q.3** If a particle moves in a circle describing equal angles in equal times, its velocity vector:
	- (A) remains constant
	- (B) changes in magnitude
	- (C) changes in direction
	- (D) changes both in magnitude and direction

## **CIRCULAR MOTION**



**Q.4** A car is rounding a circular turn of radius 200 m at constant speed. The magnitude of its centripetal acceleration is  $2m/s<sup>2</sup>$ . What is the speed of the car?  $(A)$  400 m/s (B) 20 m/s

 $\mathfrak{D}$ 

 $3\blacksquare$ 

- (C)  $100 \text{ m/s}$  (D)  $10 \text{ m/s}$
- **Q.5** As the object speeds up along the circular path in a counterclockwise direction, shown,its acceleration points:
	- (A) toward the center of the circular path. 1
	- (B) in a direction tangential to the circular path.
	- (C) outward.
	- (D) none of the above.
- **Q.6** An object moves counterclockwise along the circular path shown . As it moves along the path its acceleration vector continuously points toward point S. The object (A) speeds up at P, Q, and R.
	- (B) slows down at P, Q, and R.
	- (C) speeds up at P and slows down at R.
	- (D) slows down at P and speeds up at R.
- **Q.7** A puck of mass m is moving in a circle at constant speed on a frictionless table as shown. The puck is connected by a

string to a suspended bob, also of

mass m, which is at rest below the table. Half of the length of the string is above the tabletop and half below. What is the magnitude of the centripetal acceleration of the moving puck? Let g be the gravitational constant.

- (A) The magnitude of the centripetal acceleration of the moving puck is less than g .
- (B) The magnitude of the centripetal acceleration of the moving puck is equal to g .
- (C) The magnitude of the centripetal acceleration of the moving puck is greater than g .
- (D) The magnitude of the centripetal acceleration of the moving puck is zero.
- **Q.8** You stand on a merry-go-round spinning at f revolutions per second. You are R meters from the center. What is the minimum coefficient of static friction  $\mu_s$  between your shoes and the floor that will keep you from slipping off?
- **Q.9** A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?
- **Q.10** A turn table rotates with constant angular acceleration of 2 rad/s<sup>2</sup> about a fixed vertical axis through its centre and perpendicular to its plane. A coin is placed on it at a distance of 1m from the axis of rotation. The coin is always at rest relative to the turntable. If at  $t = 0$  the turntable was at rest, then find the total acceleration of the coin after one second.

## **ANSWERS**

**(1)** (a)  $4 \text{ cm/s}^2$ , (b)  $4\sqrt{5} \text{ cm/s}^2$ **ANSWERS**<br>
(b)  $4\sqrt{5}$  cm / s<sup>2</sup><br>
(c) 0.3 rev.<br>
(d) (D) (5) (B)<br>
(7) (B) (8)  $\mu_s = \frac{R (2\pi f)^2}{g}$ **(2)** (a)  $20.2 \text{ m/s}^2$ , (b)  $0.3 \text{ rev}$ . **(3)** (C) **(4)** (D) **(5)** (B) **(6)** (C) **(7)** (B) **(8)**  $\mu_s = \frac{R (2\pi f)^2}{g}$ 2 WANCED LEARNING<br>
S =  $\frac{R (2\pi f)^2}{g}$ <br>
2  $\sqrt{5m/s^2}$ DVANGED LEARNING<br>
(B)<br>  $\mu_s = \frac{R (2\pi f)^2}{g}$ <br>
(B)  $2\sqrt{5}m/s^2$ **(9)** 0.86 m/s<sup>2</sup>,  $\tan^{-1}(1.4) = 54^{\circ}28'$ **ANSWERS**<br> **ANSWERS**<br>  $s^2$ , (b)  $4\sqrt{5}$  cm / s<sup>2</sup><br>  $s^2$ , (b) 0.3 rev.<br>
(4) (D) (5) (B)<br>
(7) (B) (8)  $\mu_s = \frac{R (2\pi f)^2}{g}$ <br>
(an<sup>-1</sup> (1.4) = 54°28' (10)  $2\sqrt{5}$ m / s<sup>2</sup><br> **ILARMOTION**<br>
ody suspended by string :<br>
st exampl

## **VERTICAL CIRCULAR MOTION**

#### **Motion of a body suspended by string :**

This is the best example of non-uniform circular motion. Suppose a particle of mass m is attached to an inextensible light string of length r. The particle is moving in a vertical circle of radius r, about a fixed point O. At lowest point A velocity of particle  $=$  u (in horizontal direction) After covering  $\angle \theta$  velocity of particle = v (at point B) Resolve weight (mg) into two components mg cos  $\theta$  (along radial direction);  $mg \sin \theta$  (tangential direction)



Force T– mg cos  $\theta$  provides necessary centripetal force.

$$
T - mg\cos\theta = \frac{mv^2}{r}
$$
...(1)

Force T – mg cos θ provides necessary centripetal force.

\n
$$
T - mg \cos \theta = \frac{mv^{2}}{r}
$$
 ...(1)\n
$$
\Delta OCB, \cos \theta = \frac{r - h}{r}
$$
 ...(2)\n
$$
F = r(1 - \cos \theta)
$$
\nBy conservation of energy at point A and B

\n
$$
\frac{1}{2} m u^{2} = \frac{1}{2} m v^{2} + mgh
$$
\n
$$
V^{2} = u^{2} - 2gh
$$
 ...(3)\nSubstitute value of cos θ and v<sup>2</sup> in eqn. (1)\n
$$
T - mg \left[ \frac{r - h}{r} \right] = \frac{m}{r} (u^{2} - 2gh)
$$
\n
$$
T = \frac{m}{r} [u^{2} - 2gh + gr - gh]
$$
\n
$$
T = \frac{m}{r} [u^{2} + gr - 3gh]
$$
 ...(4)

or  $h = r(1 - \cos \theta)$ 

By conservation of energy at point A and B

$$
\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh
$$

or 
$$
v^2 = u^2 - 2gh
$$
 ...(3)  
Substitute value of cos  $\theta$  and  $v^2$  in eqn. (1)

$$
T - mg \left[ \frac{r - h}{r} \right] = \frac{m}{r} (u^2 - 2gh)
$$

or 
$$
T = \frac{m}{r} [u^2 - 2gh + gr - gh]
$$

or 
$$
T = \frac{m}{r} [u^2 + gr - 3gh]
$$
 .....(4)



R



(i) It velocity becomes zero at height  $h_1$ 

$$
0 = u^{2} - 2gh_{1} \text{ or } 0 = \frac{m}{r} [u^{2} + gr - 3gh_{2}]
$$
  
or  $h_{1} = \frac{u^{2}}{2g}$  ....(5)

(ii) It tension becomes zero at height  $h<sub>2</sub>$ 

$$
u^2 + gr - 3gh_2 = 0
$$
 or  $h_2 = \frac{u^2 + gr}{3g}$  ......(6)

**(A) Case of oscillation**

It  $v = 0$ ,  $T \neq 0$  then  $h_1 < h_2$ 





semi-circular path

v

$$
\frac{u^2}{2g} < \frac{u^2 + gr}{3g} \quad ; \quad 3u^2 < 2u^2 + 2gr
$$
\n
$$
u^2 < 2gr \qquad ; \qquad u < \sqrt{2gr}
$$

#### **(B) Case of leaving the circle**

It  $v \neq 0$ , T = 0 then  $h_1 > h_2$ 

 $u^2 > 2gr$ 



**(C) Case of complete the circle**



#### **NOTE**

2 smooth spherical shell of radius R. The only difference is  $\sum_{n=1}^{\infty}$  1. The same conditions apply if a particle moves inside a **STUDY MATERIAL: PHYSICS**<br>
0 at height h<sub>1</sub><br>
0 =  $\frac{m}{r} [u^2 + gr - 3gh_2]$ <br>
1. The same conditions apply if a particle moves inside a<br>
smooth spherical shell of radius R. The only difference is<br>
that the tension is replaced **STUDYMATERIAL: PHYSICS**<br>
theight h<sub>1</sub><br>  $=\frac{m}{r}[u^2 + gr - 3gh_2]$ <br>
1. The same conditions apply if a particle moves inside a<br>
smooth spherical shell of radius R. The only difference is<br>
that the tension is replaced by the nor o at height h<sub>1</sub><br>
0 =  $\frac{m}{r} [u^2 + gr - 3gh_2]$ <br>
1. The same conditions apply if a particle moves insignal to  $u = \frac{m}{r} [u^2 + gr - 3gh_2]$ <br>
1. The same conditions apply if a particle moves insignated by the normal reaction N.<br>
1 that the tension is replaced by the normal reaction N. This is shown in the figure given below. **STUDY MATERIAL: PHYSICS**<br>same conditions apply if a particle moves inside a<br>oth spherical shell of radius R. The only difference is<br>the tension is replaced by the normal reaction N.<br>is shown in the figure given below.<br> $v$ **STUDY MATERIAL: PHYSICS**<br>
<sup>2</sup><br>
The same conditions apply if a particle moves inside a<br>
smooth spherical shell of radius R. The only difference is<br>
that the tension is replaced by the normal reaction N.<br>
This is shown in

$$
v = \sqrt{gR} \quad ; \qquad N = 0
$$







**2.** If a particle of mass m is connected to a light rod and whirled in a vertical circle of radius R, then to complete the circle, the minimum velocity of the particle at the bottom most points is not  $\sqrt{5gR}$ . Because in this case, velocity of the particle at the topmost point can be zero also. Using conservation of mechanical energy between points A and B as shown in fig.(a) we get  $V = 0$ <br>  $V = 0$ <br>  $N \neq 0$ <br>  $N \neq$ **If the of oscillation is**  $0 < u \le \sqrt{2gR}$ <br>  $V=0$ <br>  $V \ne 0$ <br>  $h \le R$ <br>
cle of mass m is connected to a light rod and<br>
a vertical circle of radius R, then to complete the<br>
minimum velocity o 1 2 mu mg = mg (2R) or u = 2 $\sqrt{gR}$ <br>
2 mu mg = mg (2R) or u = 2 $\sqrt{gR}$ <br>
2 mu mg = minimum velocity of the particle at the bottom<br>
2 mu minimum velocity of the particle at the bottom<br>
that is not  $\sqrt{5gR}$ . Because in th

$$
\frac{1}{2}m(u^2 - v^2) = mgh
$$
  
or 
$$
\frac{1}{2}mu^2 = mg(2R) \text{ or } u = 2\sqrt{gR}
$$





inside a smooth vertical tube as shown in fig. (b).

#### **Example 13 :**

A ball is released from height h as shown in fig. Find the condition for the particle to complete the circular path.



**Sol.** According to law of conservation of energy  $(K.E + P.E)$  at  $A = (K.E + P.E)$  at B

$$
\Rightarrow 0 + \text{mgh} = \frac{1}{2} \text{mv}^2 + 0 \Rightarrow v = \sqrt{2gh}
$$

But velocity at the lowest point of circle,

$$
v \ge \sqrt{5gR} \implies \sqrt{2gh} \ge \sqrt{5gR} \implies h \ge \frac{5R}{2}
$$

#### **Example 14 :**

In a circus a motorcyclist moves in vertical loops inside a 'death well' (a hollow spherical chamber with holes, so lost at any point). Therefore,  $10u^2 - 10g \ge 0 \Rightarrow u^2 \ge g$ that the spectators can watch from outside). Explain clearly<br>why the motorcyclist does not drop down when he is at Hence the least value of u is  $\sqrt{g} = \sqrt{9.8} = 3.1 \text{ ms}^{-1}$ why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required to perform a vertical loop if the radius of the chamber is 25 m. conservation of mechanical conservation of mechanic of mechanic of mechanic of mechanic  $\sqrt{5gR} \Rightarrow \sqrt{2gh} \ge \sqrt{5gR} \Rightarrow h \ge \frac{5R}{2}$ <br>
From (1) and (2)<br>
4:<br>
directions a motorcyclist moves in vertical loops inside a<br>
directions Locity at the lowest point of circle,<br>  $\frac{1}{2}$  radius of the conservation of mechanical<br>  $\frac{1}{2}$  radius of the conservation of mechanical<br>  $\frac{1}{2}$  radius of the conserved and the circles,<br>  $\frac{1}{2}$  radius of the ci

**Sol.** When the motorcyclist is at the highest point of the deathwell, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also acts downwards.These two forces are balanced by the outward centrifugal force acting on him.

$$
\therefore \quad R + mg = \frac{mv^2}{r} \quad \text{....(i)} \quad r = \text{radius of the circle} \, .
$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle). Because of the balancing of the forces, the motorcyclist does not fall down.

The minimum speed required to perform a vertical loop is given by equation (i), when  $R = 0$ .

$$
\therefore \quad mg = \frac{mv^2_{\text{min}}}{r} \quad \text{or} \quad v^2_{\text{min}} = gr
$$

$$
r = v_{\text{min}} = \sqrt{gr} = \sqrt{9.8 \times 25} \text{ ms}^{-1} = 15.65 \text{ ms}^{-1}
$$

Because of the balancing of the forces, the motorcyclist<br>does not fall down.<br>The minimum speed required to perform a vertical loop is<br>given by equation (i), when R = 0.<br> $\therefore$  mg =  $\frac{mv^2_{min}}{r}$  or  $v^2_{min.} = gr$ <br>or  $v_{min} = \sqrt$ **EXECUTE ATAINING**<br>
EXECUTED THE MANUVAL CONDUCTDERTAINING<br>
minimum speed required to perform a vertical loop is<br>
n by equation (i), when R = 0.<br>  $mg = \frac{mv^2_{min}}{r}$  or  $v^2_{min} = gr$ <br>  $v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25} \text{ ms}^{-1} = 15.65 \text{ ms$ So, the minimum speed at the top required to perform a vertical loop is  $15.65 \text{ ms}^{-1}$ .

#### **Example 15 :**

A particle of mass 2 kg is moving on the inside surface of a smooth hollow cylinder of radius 0.2 m whose axis is horizontal. Find the least speed which the particle must have at the lowest point of its path if it travels in complete circles.



**Sol.** Applying Newton's law along BO we have

$$
R + 2g = 2\left(\frac{v^2}{0.2}\right)
$$
 .........(1)

Conservation of mechanical energy from A to B gives

$$
\left(\frac{1}{2}\right)(2u^2) - 2g(0.2) = \left(\frac{1}{2}\right)(2v^2) + 2g(0.2) \dots (2)
$$

2 From  $(1)$  and  $(2)$ 

 $R + 2g = 10 (u^2 - 0.8g)$ or  $R = 10u^2 - 10g$ Hence the least value of u is g 9.8 = 3.1 ms–1

#### **Example 16 :**

A roller coaster of mass M is at the top of the Loop-theloop of radius R at twice the minimum speed possible. What force does the track exert on it? What force does it exert when it is at the bottom of the circle? (Use conservation of energy if needed.)

**Sol.** When a roller coaster is at the top of it's track, it's minimum velocity occurs when the normal force equals 0. If the velocity gets any smaller than this, the roller coaster cannot maintain its circular motion and it falls off the track. The only force acting on the roller coaster is the force due to gravity.

![](_page_10_Picture_0.jpeg)

$$
\frac{Mv_{min}^2}{R} = Mg \Rightarrow v_{min} = \sqrt{Rg}
$$

STUDY MATERIAL:<br>
STUDY MATERIAL:<br>  $\frac{Mv_{min}^2}{R} = Mg \Rightarrow v_{min} = \sqrt{Rg}$   $\therefore T \sin\theta = \frac{mv^2}{r}$  and  $T \cos\theta = mg$ <br>
is case,  $v = 2\sqrt{Rg}$ . When the roller coaster is at the<br>
f the track, the normal force points away from the<br>
downwards **STUDYMATERI**<br>  $= Mg \Rightarrow v_{min} = \sqrt{Rg}$   $\therefore T \sin\theta = \frac{mv^2}{r}$  and  $T \cos\theta = mg$ <br>  $v = 2\sqrt{Rg}$ . When the roller coaster is at the<br>
Rick, the normal force points away from the<br>
rom these equations<br>
From these equations<br>  $T = mg \sqrt{1 + \frac{v^4$ **INVARGED LEARNING**<br>
IN this case,  $v = 2\sqrt{Rg}$ . When the roller coaster is at the<br>
In this case,  $v = 2\sqrt{Rg}$ . When the roller coaster is at the<br>
top of the track, the normal force points away from the<br>
track, downwards top of the track, the normal force points away from the track, downwards towards the center of the circle. Solving for the normal force, **EXERUISING STUDY MATERIAL: PHYS**<br>  $\frac{Mv_{min}^2}{R} = Mg \Rightarrow v_{min} = \sqrt{Rg}$  <br>  $\therefore T \sin\theta = \frac{mv^2}{r}$  and  $T \cos\theta = mg$ <br>  $\therefore$   $T \sin\theta = \frac{mv^2}{r}$  and  $T \cos\theta = mg$ <br>  $\therefore$   $T \sin\theta = \frac{mv^2}{r}$  and  $T \cos\theta = mg$ <br>  $\therefore$   $T \sin\theta = \frac{mv^2}{r}$  and  $T \cos\theta = mg$ <br> **STUDYMA**<br>  $\frac{m}{R} = Mg \Rightarrow v_{min} = \sqrt{Rg}$   $\therefore$   $T \sin\theta = \frac{mv^2}{r}$  and  $T \cos\theta = mg$ <br>  $\sec x = \sqrt{Rg}$ . When the roller coaster is at the<br>
track, the normal force points away from the<br>
track, the normal force points away from the<br>
mular **STUDYMATERIAL: PHY**<br>  $\frac{1}{R} \frac{v_{\text{min}}^2}{R} = Mg \Rightarrow v_{\text{min}} = \sqrt{Rg}$ <br>  $\therefore T \sin \theta = \frac{mv^2}{r}$  and  $T \cos \theta = mg$ <br>  $\csc v = 2\sqrt{Rg}$ . When the roller coaster is at the<br>
the track, the normal force points away from the<br>
normal force,<br>
no  $\frac{Wv_{\text{min}}}{R} = Mg \Rightarrow v_{\text{min}} = \sqrt{Rg}$ <br>  $\therefore T \sin\theta = \frac{W}{r}$  and  $T \cos\theta = mg$ <br>
in this case,  $v = 2\sqrt{Rg}$ . When the roller coaster is at the<br>
root free check, the normal force points wavy from the gequations<br>
rack, downwards towa  $\frac{r_{min}^2}{R} = Mg \Rightarrow v_{min} = \sqrt{Rg}$ <br>  $\therefore T \sin\theta = \frac{mv^2}{r}$  and  $T \cos\theta = mg$ <br>
the track, the normal force points away from these equations<br>
be track, the normal force points away from the term of the circle. Solving<br>
ormal force,<br>

$$
F = \frac{Mv^2}{R} = \frac{M (2\sqrt{Rg})^2}{R} = Mg + N \Rightarrow N = 3 Mg
$$

We can find the velocity of the roller coaster at the bottom of the track using conservation of energy. Taking the potential energy to be zero at the bottom, at the top of the

track, the roller coaster has kinetic energy equal to 
$$
\frac{1}{2}Mv^2
$$

and a potential energy of Mg (2R). This must be equal to its energy at the bottom of the track when it has only kinetic energy. Conserving energy and solving for the final velocity,

top of the track, the normal force points away from the  
track, downwards towards the center of the circle. Solving  
for the normal force,  

$$
F = \frac{Mv^2}{R} = \frac{M (2\sqrt{Rg})^2}{R} = Mg + N \Rightarrow N = 3 Mg
$$
  
We can find the velocity of the roller coaster at the bottom  
of the track using conservation of energy. Taking the  
potential energy to be zero at the bottom, at the top of the  
itis energy at the bottom of the track when it has only  
kinetic energy. Conserving energy and solving for the final  
velocity,  
 $\frac{1}{2}Mv_f^2 = \frac{1}{2}Mv^2 + Mg (2R) = \frac{1}{2}M (4Rg) + Mg (2R)$   
 $v_f = \sqrt{8gR}$   
 $v_f = \sqrt{8gR}$ 

Now, we need to find the normal force when the roller coaster is at the bottom of the track with velocity  $v_f$ . The Exam normal force still points away from the track, but now that is upwards towards the center of the circle.

$$
\frac{Mv_f^2}{R} = 8Mg = N - Mg \Rightarrow N = 9Mg
$$

#### **APPLICATIONS OF CIRCULAR MOTION**

#### **1. The conical pendulum :**

A conical pendulum consists of a body attached to a string, such that it can revolve in a horizontal circle with uniform speed. The string traces out a cone in the space.

![](_page_10_Figure_14.jpeg)

The force acting on the bob are: (a) Tension  $T$  (b) weight mg. The horizontal component  $T \sin \theta$  of the tension  $T$ <br>provides the centrinetal force and the vertical component provides the centripetal force and the vertical component Tcos $\theta$  balances the weight of bob

$$
\therefore T \sin \theta = \frac{mv^2}{r} \text{ and } T \cos \theta = mg
$$

From these equations

**STUDY MATERIAL: PHYSICS**  
\n
$$
\therefore \quad T \sin \theta = \frac{mv^2}{r} \text{ and } T \cos \theta = mg
$$
\nFrom these equations  
\n
$$
T = mg \sqrt{1 + \frac{v^4}{r^2 g^2}} \quad ...(1) \text{ and } \tan \theta = \frac{v^2}{rg} \quad ...(2)
$$
\nIf h = height of conical pendulum  
\n
$$
\tan \theta = \frac{OP}{OS} = \frac{r}{h} \qquad ...(3)
$$
\nFrom (2) and (3),  $\frac{v^2}{rg} = \frac{r}{h} \Rightarrow \omega^2 = \frac{v^2}{r^2} = \frac{g}{h}$   
\nThe time period of revolution,  $T = 2\pi \sqrt{\frac{h}{g}} = 2\pi \sqrt{\frac{\ell \cos \theta}{g}}$   
\n**Hints to solve numerical problem**  
\n(i) First show all force acting on a particle.  
\n(ii) Resultant force along radial direction provides

If  $h =$  height of conical pendulum

$$
\tan \theta = \frac{\text{OP}}{\text{OS}} = \frac{\text{r}}{\text{h}} \tag{3}
$$

From (2) and (3),  $\frac{1}{\sqrt{2}}$  $2\begin{array}{c} 2 \end{array}$   $\begin{array}{c} 2 \end{array}$   $\begin{array}{c} 2 \end{array}$  $r^2$  h

 $\frac{1}{2}Mv^2$  The time period of revolution  $T - 2\pi /h^2 - 2\pi / \ell \cos \theta$  $\theta$ 

#### **Hints to solve numerical problem**

- (i) First show all force acting on a particle.
- (ii) Resolve these forces along radius and tangent.
- (iii) Resultant force along radial direction provides necessary centripetal force.
- (iv) Resultant force along tangent equals to zero.

#### **Example 17 :**

A vertical rod is rotating about its axis with a uniform angular speed  $\omega$ . A simple pendulum of length  $\ell$  is attached to its upper end what is its inclination with the rod ?

2. Example 17<br>
And the strated vising conservation of energy. Taking the<br>
track using conservation of energy. Taking the<br>
track using conservation of energy. Taking the<br>
17 the function of the track when it has only be ze Example 17<br>
Example per section of the relation that the bottom<br>
deline velocity of the roller coaster at the bottom, at the top of the<br>
heler coaster has kinetic energy equal to  $\frac{1}{2}Mv^2$ <br>
and al energy of Mg (2R). T **Sol.** Let the radius of the circle in which the bob is rotating is x, the tension in the string is T, weight of the bob mg, and inclination of the string  $\theta$ . Then T cos $\theta$  balances the weight  $mg$  and T sin $\theta$  provides the centripetal force necessary for circular motion.

![](_page_10_Figure_31.jpeg)

That is T cos  $\theta$  = mg and T sin  $\theta$  = m $\omega^2$  x but  $x = \ell \sin \theta$  :  $T = m\omega^2 \ell$ 

and 
$$
\cos \theta = \frac{mg}{T} = \frac{mg}{m \omega^2 \ell}
$$

or 
$$
\theta = \cos^{-1}\left(\frac{g}{\omega^2 \ell}\right)
$$

**286**

![](_page_11_Picture_1.jpeg)

#### **2. Banked tracks :**

**(A) A Cyclist Making a turn :** Let a cyclist moving on a circular path of radius r bend away from the vertical by an angle  $\theta$ . If R is the reaction of the ground, then R may be resolved into two components horizontal and vertical. The vertical component R cos  $\theta$  balances the weight mg of the cyclist and the horizontal component  $R \sin \theta$  provides the necessary centripetal force for circular motion. **AR MOTION**<br> **Example 10**<br> **Example 10**<br> **Exploit Making a turn**: Let a cyclist moving on a<br> **Cyclist Making a turn**: Let a cyclist moving on a<br>
pircular path of radius r bend away from the vertical<br>
by an angle 0. If R i **10TION**<br> **acks:**<br> **acks:**<br> **acks :**<br> **acks**<br> **cos**<br> **cos** 

R sin θ = 
$$
\frac{mv^2}{r}
$$
 ....(1) and R cos θ = mg ....(2)

Dividing (1) by (2), we get, 
$$
\tan \theta = \frac{v^2}{rg}
$$
 ....(3)  $\frac{mv^2}{g} = \mu_S N$ 

![](_page_11_Figure_6.jpeg)

For less bending of cyclist, his speed v should be smaller and radius r of circular path should be greater. If  $\mu$  is coefficient of friction, then for no skidding of cycle (or overturning of cyclist)

$$
\mu \le \tan \theta \qquad \qquad \dots \dots \dots (4) \quad ; \quad \mu \ge \frac{v^2}{rg}
$$

**(B) An Aeroplane Making a Turn :** In order to make a circular turn, a plane must roll at some angle  $\theta$  in such a manner that the horizontal component of the lift force L provides the necessary centripetal force for circular motion. The vertical component of the lift force (D) balances the weight of the plane.

![](_page_11_Figure_10.jpeg)

![](_page_11_Figure_11.jpeg)

When a car takes a turn on a level road, the portion of the turn can be approximated by an arc of a circle of radius r (see fig). If the car makes the turn at a constant speed v, then there must be some centripetal force acting on the car. This force is generated by the friction between the tyres and the road. (car has a tendency to slip radially out ward, so frictional force acts inwards)  $\mu_S$  is the coefficient of static friction

 $N = mg$  is the normal reaction of the surface.

![](_page_11_Figure_14.jpeg)

The maximum safe velocity v is

$$
\frac{mv^2}{r} = \mu_S \ N = \mu_S mg \text{ or } \mu_S = \frac{v^2}{rg} \quad \text{or } v = \sqrt{\mu_S rg}
$$

It is independent of the mass of the car. The safe velocity is same for all vehicles of larger and smaller mass.

#### **Example 18 :**

A car is travelling at 30 km/h in a circle of radius 60m. What is the minimum value of  $\mu_s$  for the car to make the turn without skidding ?

**Sol.** The minimum  $\mu_s$  should be such that

$$
\cos \theta = mg
$$
 ....(2)  
\n
$$
\cos \theta = mg
$$
 ....(2)  
\nThe maximum safe velocity v is  
\n $mg$  of car  
\n $mg$  of car  
\n $mg$   
\n $mg$  of  $mg$   
\n $g$   
\n $mg$   
\n $g$   
\n $mg$   
\n $g$   
\n

For all values of  $\mu_s$  greater than or equal to the above value, the car can make the turn without skidding.

#### **(D) Bankig of road :**

If a cyclist takes a turn, he can bend from his vertical position. This is not possible in the case of car, truck or train.

![](_page_11_Figure_25.jpeg)

The tilting of the vehicle is achieved by raising the outer edge of the circular track, slightly above the inner edge. This is known as banking of curved track.

The angle of inclination with the horizontal is called the angle of banking.

![](_page_12_Picture_0.jpeg)

(a) If driver moves with slow velocity that friction does not play any role in negotiating the turn.

The various forces acting on the vehicle are :

- (i) Weight of the vehicle (mg) in the downward direction.
- (ii) Normal reaction (N) perpendicular to the inclined surface of the road.

Resolve N in two components.

 $*$  N cos  $\theta$ , vertically upwards which balances weight of the vehicle.

$$
\therefore \qquad \text{N} \cos \theta = \text{mg} \qquad \qquad \text{....(1)}
$$

 $N \sin \theta$ , in horizontal direction which provides necessary centripetal force.

$$
\therefore \quad N \sin \theta = \frac{mv^2}{r}
$$
 ....(2)

On dividing eqn. (ii) by eqn. (i)

![](_page_12_Figure_12.jpeg)

Where m is the mass of the vehicle, r is radius of curvature of the road, v is speed of the vehicle and

$$
\theta
$$
 is the banking angle  $\left(\sin \theta = \frac{h}{b}\right)$ . The

are as follows :

- Velocity of the vehicle
- Radius of the curve
- Acceleration due to gravity Thus, there is no need of mass of the vehicle to

express the value of angle of banking i.e. angle of banking  $\Rightarrow$  does not dependent on the mass of the vehicle.

 $\therefore$   $v^2 = \text{gr} \tan \theta$ 

$$
\therefore \quad v = \sqrt{gr \tan \theta} \quad (maximum \text{ safe speed})
$$

This gives the maximum safe speed of the vehicle. In actual practice, some frictional forces are always present. So, the maximum safe velocity is always much greater than that given by the above equation. While constructing the curved track, the value of  $\theta$ is calculated for fixed values of  $v_{\text{max}}$  and r. This explains why along the curved roads, the speed limit at which the curve is to be negotiated is clearly indicated on sign boards.

The outer side of the road is raised by  $h = b \times \theta$ .

When  $\theta$  is small, then  $\tan \theta \approx \sin \theta = \frac{h}{h}$ ;<br>
With this speed  $\overline{b}$ , of the tyres.

**STUDY MATERIAL: PHYSICS**  
Also 
$$
\tan \theta = \frac{v^2}{rg}
$$
  $\therefore \frac{v^2}{rg} = \frac{h}{b}$  or  $h = \frac{v^2}{rg} \times b$   
This gives us the height through which outer edge  
is raised above the inner edge.  
ne driver moves faster than the safe speed  
tioned above, a friction force must act parallel to

This gives us the height through which outer edge is raised above the inner edge.

**YMATERIAL: PHYSICS**<br>  $\frac{v^2}{rg} = \frac{h}{b}$  or  $h = \frac{v^2}{rg} \times b$ <br>
through which outer edge<br>
er edge.<br>
ter than the safe speed<br>
of force must act parallel to **EXECUTE ALEXAL: PHYSICS**<br>  $\frac{v^2}{rg} = \frac{h}{b}$  or  $h = \frac{v^2}{rg} \times b$ <br>
through which outer edge<br>
er edge.<br>
er than the safe speed<br>
force must act parallel to<br>
centre of the turn. L: PHYSICS<br>  $h = \frac{v^2}{rg} \times b$ <br>
ch outer edge<br>
e safe speed<br>
act parallel to<br>
turn. (b) If the driver moves faster than the safe speed mentioned above, a friction force must act parallel to the road, inwards towards centre of the turn.

![](_page_12_Figure_28.jpeg)

In this case forces acting on the vehicle are :

- Weight of the vehicle (mg) in the downward direction.
- Normal reaction perpendicular to the inclined plane of the road.
- Frictional force f between the tyres and the road. N cos  $\theta$  and N sin $\theta$  are the two rectangular components of N.  $\frac{1}{2}$  fsin  $\theta$ <br>
w = mg<br>
r sase forces acting on the vehicle are :<br>
ight of the vehicle (mg) in the downward<br>
ne of the road.<br>
from all reaction perpendicular to the inclined<br>
ne of the road.<br>
road N sin  $\theta$  are the vehicle are :<br>g) in the downward<br>cular to the inclined<br>ne tyres and the road.<br>the two rectangular<br>he two rectangular<br>vertical motion.<br>ss  $\theta$  together provide<br>rce.<br> $\frac{v^2}{r} = N \sin \theta + \mu N \cos \theta$

f cos  $\theta$  and f sin  $\theta$  are the two rectangular components of f.

The car does not have any vertical motion.

- b) is a set of  $\mathbb{R}^3$  is the set of  $\mathbb{R}^3$  $\therefore$  mg + f sin  $\theta$  = N cos  $\theta$ or  $mg = N \cos \theta - f \sin \theta$ But  $f = \mu N$ , where  $\mu \le \mu_s$ .
	- .  $\therefore$  mg = N cos  $\theta$  –  $\mu$  N sin  $\theta$

The forces N sin  $\theta$  and f cos  $\theta$  together provide the necessary centripetal force.

$$
\therefore \qquad \frac{mv^2}{r} = N\sin\theta + f\cos\theta \text{ or } \frac{mv^2}{r} = N\sin\theta + \mu N\cos\theta
$$

Dividing eqn (ii) by eqn (i) we get

In this case forces acting on the vehicle are :  
\n\* Weight of the vehicle (mg) in the downward  
\ndirection.  
\n\* Normal reaction perpendicular to the inclined  
\nplane of the road.  
\nFritical force F between the types and the road.  
\nN cos θ and N sinθ are the two rectangular  
\ncomponents of N.  
\nfor cos θ and f sin θ are the two rectangular  
\ncomponents of f.  
\nThe car does not have any vertical motion.  
\n
$$
\therefore
$$
 mg + f sin θ = N cos θ  
\nor mg = N cos θ - f sin θ  
\nBut f = μN, where μ ≤ μ,  
\n $\therefore$  mg = N cos θ - μ N sin θ  
\nThe forces N sin θ and f cos θ together provide  
\nthe necessary centripetal force.  
\n $\therefore$   $\frac{mv^2}{r} = N sin θ + f cos θ$  or  $\frac{mv^2}{r} = N sin θ + \mu N cos θ$   
\nDividing eqn (ii) by eqn (i) we get  
\n $\frac{mv^2}{mg} = \frac{N sin θ + \mu N cos θ}{N cos θ - \mu N sin θ}$   
\nor  $\frac{v^2}{rg} = \frac{sin θ + \mu cos θ}{cos θ - \mu sin θ}$  or  $\frac{v^2}{rg} = \frac{cos θ (tan θ + \mu)}{cos θ (1 - \mu tan θ)}$   
\nor  $v^2 = \frac{tan θ + \mu}{1 - \mu tan θ}$  or  $v = \sqrt{\frac{tan θ + \mu}{1 - \mu tan θ}}$ rg  
\nThe best speed to negative a curve is obtained by  
\nputting μ = 0.  $\therefore$   $v = \sqrt{rg tan θ}$   
\nWith this speed, there will be minimum wear and tear  
\nof the types.

or 
$$
v^2 = \frac{\tan \theta + \mu}{1 - \mu \tan \theta}
$$
rg or  $v = \sqrt{\frac{\tan \theta + \mu}{1 - \mu \tan \theta}}$ rg

The best speed to negotiate a curve is obtained by

putting 
$$
\mu = 0
$$
.  $\therefore$   $v = \sqrt{rg \tan \theta}$   
With this speed, there will be minimum wear and tear

**288**

O<sub>i</sub>

![](_page_13_Picture_1.jpeg)

#### **Example 19 :**

At what angle should a highway be banked for cars travelling at a speed of 100 km/h if the radius of the road is 400m and no frictional forces are involved?

**Sol.** The banking should be done at an angle  $\theta$  such that

$$
\tan \theta = \frac{v^2}{rg} = \frac{\frac{250}{9} \times \frac{250}{9}}{400 \times 10}
$$
 or  $\tan \theta = \frac{625}{81 \times 40} = 0.19$  When the particle leaves the sphere N=0,

or  $\theta = \tan^{-1} 0.19 \approx 0.19$  radian  $\approx 0.19 \times 57.3^{\circ} \approx 11^{\circ}$ 

#### **3. Apparent weight of car**

**(A) Convex bridge :** The motion of the motor car over a convex bridge is the motion along the segment of a circle. The centripetal force is provided by the difference of weight mg of the car and the normal reaction N of the bridge.

![](_page_13_Figure_9.jpeg)

Clearly  $N < mg$ , i.e., the apparent weight of the moving car is less than the weight of the stationary car. (or car moving on flat surface)

![](_page_13_Figure_11.jpeg)

Clearly  $N > mg$ , i.e., the apparent weight of the moving car is more than the weight of the stationary car.

#### **Example 20 :**

A particle of mass m slides down from the vertex of semihemisphere, without any initial velocity. At what height from horizontal will the particle leave the sphere.

**Sol.** Let the particle leave them sphere at height h,

$$
\frac{mv^2}{R} = mg \cos \theta - N
$$

![](_page_13_Figure_17.jpeg)

$$
\frac{mv^2}{R} = mg \cos \theta \Rightarrow v^2 = gR \cos \theta \qquad \qquad \dots (1)
$$

According to law of conservation of energy  $(K.E. + P.E.)$  at  $A = (K.E. + P.E.)$  at B

Now we be banked for cars  
\nwith the radius of the road is  
\n
$$
= \frac{625}{81 \times 40} = 0.19
$$

\nWhen the particle leaves the sphere  
\n $N = 0$ ,  
\n $\approx 0.19 \times 57.3^\circ \approx 11^\circ$ 

\nOn the motor car over a  
\n $= \frac{625}{R} = \frac{1}{2}$  m  
\n $= \frac{mv^2}{R} = \frac{1}{2}$  m  
\n $= \frac{1}{2}$  m  
\n $= \frac{mv^2}{r} = \frac{1}{2}$  m  
\n $= \frac{1}{3}R$ , Also  $\cos\theta = \frac{2}{3}$ 

\nFrom (1) and (2),  $h = \frac{2}{3}R$ , Also  $\cos\theta = \frac{2}{3}$ 

\n4. Condition of Overturning : Here, we shall find the condition for the car to overturn. Let the distance between the centers of wheels of the car be 2a and the centre of gravity.

**4. Condition of Overturning :** Here, we shall find the condition for the car to overturn. Let the distance between the centres of wheels of the car be 2a and the centre of gravity be h metres above the ground (road). The different forces acting on the car are shown in the fig.

![](_page_13_Figure_23.jpeg)

- (i) The weight mg of the car acts downwards through centre of gravity G.
- (ii) The normal reactions of the ground  $R_1$  and  $R_2$  on the inner and outer wheels respectively. These act vertically upwards.
- (iii) Let force of friction  $F_1 + F_2$  between wheels and ground towards the centre of the turn. Let the radius of circular path be r and the speed of the

car be v. Since there is no vertical motion, equating the vertical forces, we have

$$
R_1 + R_2 = mg.
$$
 ...(1)

The horizontal force = centripetal force for motion in a

circle. So, 
$$
F = F_1 + F_2 = \frac{mv^2}{r}
$$
 ....(2)

Taking moments about the centre of mass G.

$$
(F1 + F2)h + R1a = R2a
$$
  
∴ F<sub>1</sub> + F<sub>2</sub> = (R<sub>2</sub> - R<sub>1</sub>)  $\frac{a}{h}$  ....(3)

Combining this with equation (2) to eliminate  $F_1 + F_2$  gives

![](_page_14_Picture_0.jpeg)

$$
R_2 - R_1 = \frac{hmv^2}{ar}
$$
 ....(4)

EXAMING<br>  $2-R_1 = \frac{hmv^2}{ar}$  ....(4) Example 21:<br>
e now have two simul-<br>
neous equations, (1)<br>  $d(4)$ , for  $R_1$  and  $R_2$ .<br>
Niving these by adding We now have two simultaneous equations, (1) and (4), for  $R_1$  and  $R_2$ . . <sup>.</sup> Solving these by adding & subtracting, we find that

$$
2R_1 = mg - \frac{hmv^2}{ar} \qquad \qquad \underbrace{\qquad \qquad \qquad \qquad \qquad}_{mg} \qquad \qquad \qquad \qquad \qquad \qquad}_{a} \qquad \qquad \underbrace{F_x \qquad \qquad}_{a}}_{\text{and } 2R_2 = mg + \frac{hmv^2}{ar}}
$$

From these expressions it is clear.

Inner wheels will leave the ground when  $R_1$  will become zero and the car begins to overturn,

i.e., 
$$
mg = \frac{hmv^2}{ar}
$$

So the limiting speed is given by  $v^2 = \frac{gra}{h}$  as required. **6.** Looping the loop the best example

**5. Death well and rotor :** Example of uniform circular motion In 'death well' a person drives a bicycle on a vertical surface of a large wooden well.

In 'death well' walls are at rest while person revolves. In a rotor at a certain angular speed of rotor a person hangs resting against the wall without any floor. In rotor person is at rest and the walls rotate.

![](_page_14_Figure_10.jpeg)

In both these cases friction balances the weight of person while reaction provides the centripetal force necessary for circular motion, i.e.,

Force of friction  $F_S = mg \& \text{Normal reaction } F_N = \frac{mv^2}{r}$  We find that  $\Sigma F$  (in radial d

so 
$$
\frac{F_N}{F_S} = \frac{v^2}{rg}
$$
, i.e.,  $v = \sqrt{\frac{rgF_N}{F_S}}$  Remove

Now for v to be minimum  $F_S$  must be maximum,

i.e., 
$$
v_{\min} = \sqrt{\frac{rg}{\mu}}
$$
 [as  $F_{S \max} = \mu F_N$ ]

#### **Example 21 :**

h

G |

 $R \neq 0$ 

**Example 21:**<br>  $R_2 - R_1 = \frac{hmv^2}{ar}$  ....(4)<br>
We now have two simulations, (1)<br>
and (4), for  $R_1$  and  $R_2$ .<br>  $R_3$  and the subtracting, we find that<br>  $x$  subtracting, we find that<br>  $x$  subtracting, we find that **Example 21 :**<br>  $-R_1 = \frac{hmv^2}{ar}$  ....(4)<br>  $R \ne 0$ <br>  $\therefore$  (4) ....(4)<br>  $\therefore$  A 62 kg woman is a passe<br>
amusement park. A drum of<br>  $\therefore$  A 62 kg woman is a passe<br>
amusement park. A drum of<br>  $\therefore$  A 62 kg woman is a passe<br>
a **EXAMPLE 21:**<br>  $R_2 - R_1 = \frac{hmv^2}{ar}$  .....(4)<br>
We now have two simul-<br>
and (4), for  $R_1$  and  $R_2$ .<br>  $2R_1 = mg - \frac{hmv^2}{ar}$ <br>  $2R_2 = mg + \frac{hmv^2}{ar}$ <br>
From these expressions it is clear.<br>
Then the normal force of the drum rotates **EXAMPLE 21:**<br>  $R_1 = \frac{hmv^2}{ar}$  .....(4)<br>  $R_2 = mg + \frac{hmv^2}{ar}$ <br>  $\therefore R_3 = \frac{hmv^2}{ar}$ <br>  $\therefore R_4 = \frac{hmv^2}{ar}$ <br>  $\therefore R_5 = mg - \frac{hmv^2}{ar}$ <br>  $\therefore R_6 = \frac{hmv^2}{ar}$ <br>  $\therefore R_7 = mg + \frac{hmv^2}{ar}$ <br>  $\therefore R_8 = mg - \frac{hmv^2}{ar}$ <br>  $\therefore R_9 = mg + \frac{hmv^2}{ar}$ <br>  $\therefore$ **EXAMPLE 21:**<br>
SERUING EXAMPLE 21:<br>  $-R_1 = \frac{hmv^2}{\text{ar}}$ <br>
(a) the vall of the rotating drama is a passeng<br>
in the wall of the rotating drum of rangement park. A drum of rangement park. A drum of rangement park. A drum of r A 62 kg woman is a passenger in a "rotor ride" at an amusement park. A drum of radius 5.0 m is spun with an angular velocity of 25 rpm. The woman is pressed against the wall of the rotating drum as shown in fig. (a) Calculate the normal force of the drum on the woman (the centripetal force that prevents her from leaving her circular path). (b) While the drum rotates, the floor is lowered. A vertical static friction force supports the woman's weight. What must the coefficient of friction be to support her weight? **STUDY MATERIAL: PHYSICS**<br> **revert in a** reading the summent park. A drum of radius 5.0 m is spun with an ment park. A drum of radius 5.0 m is spun with an ar velocity of 25 rpm. The woman is pressed against ll of the rot **STUDY MATERIAL: PHYSICS**<br>
ann is a passenger in a "rotor ride" at an<br>
the A drum of radius 5.0 m is spun with an<br>
ty of 25 rpm. The woman is pressed against<br>
rotating drum as shown in fig. (a) Calculate<br>
ce of the drum o **STUDY MATERIAL: PHYSICS**<br>
pman is a passenger in a "rotor ride" at an<br>
park. A drum of radius 5.0 m is spun with an<br>
city of 25 rpm. The woman is pressed against<br>
ne rotating drum as shown in fig. (a) Calculate<br>
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endeding 5.0 m is spun with an<br>
relocity of 25 rpm. The woman is pressed against<br>
of the rotating drum as shown in fig. (a) Calculate<br>
al force of t **STODY MATERIAL: PHYSICS**<br>
n is a passenger in a "rotor ride" at an<br>
c. A drum of radius 5.0 m is spun with an<br>
of 25 rpm. The woman is pressed against<br>
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of the drum on the woman **STUDY MATERIAL: PHYSICS**<br>
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tank A drum of radius 5.0 m is spun with an<br>
ocity of 25 rpm. The woman is pressed against<br>
the rotating drum as shown in fig. (a) Calculate<br>
force **STUDY MATERIAL: PHYSICS**<br>
man is a passenger in a "rotor ride" at an<br>
ark. A drum of radius 5.0 m is spun with an<br>
er otating drum as shown in fig. (a) Calculate<br>
er othe drum on the woman (the centripetal<br>
vents her fro

**Sol.** (a) Normal force exerted by the drum on women towards the centre  $F_N = ma_c = ma^2r$ 

$$
= 62 \text{ kg} \times \left( 25 \frac{\text{rev}}{\text{min}} \times \frac{2 \pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{min}}{60 \text{ s}} \right)^2 \times 5 \text{ m} = 2100 \text{ N}.
$$

(b) 
$$
\mu F_N = F = mg
$$

so 
$$
\mu = \frac{g}{\omega^2 r} = \left(\frac{60}{2\pi \times 25}\right)^2 \times \frac{10}{5} = 0.292
$$

**6. Looping the loop :**This is the best example of non uniform circular motion in vertical plane. For looping the loop pilot of the plane puts off the engine at lowest point and traverses a vertical loop.(with variable velocity)

![](_page_14_Figure_23.jpeg)

#### **Example 22 :**

An aeroplane moves at 64 m/s in a vertical loop of radius 120m, as shown in fig. Calculate the force of the plane's seat on a 72 kg pilot while passing through the bottom part of the loop.

**Sol.** Two forces act on the pilot his downward weight force w and the upward force of the aeroplane's seat  $F_{\text{seat}}$ . Because the pilot moves in a circular path, these forces along the radial direction must, according to Newton's

![](_page_14_Figure_27.jpeg)

 $mv^2$ 

r a bheiltean an t-

.

2 second law  $(\Sigma F = ma)$ , equal the pilot's mass times his centripetal acceleration, where  $a_c = v^2/r$ .

We find that 
$$
\Sigma
$$
 F (in radial direction) = F<sub>seat</sub> – w =  $\frac{mv^2}{r}$ .

**Example 2** An aeroplane moves at 64 m/s in a vertical loop of radius 120m, as shown in fig. Calculate the force of the planes  
seat on a 72 kg  
parallelly  
part of the loop.  
**Sol.** Two forces act on the pilot  
his downward weight force we  
and the upward force of the  
acropales seat F<sub>seat</sub>  
Because the yield of  
equation path, these forces  
calances the weight of  
equation F<sub>N</sub> = 
$$
\frac{mv^2}{r}
$$
  
  
 $= \sqrt{\frac{rgF_N}{F_S}}$   
  
 $= \sqrt{\frac{rgF_N}{F_S}}$   
  
**Remember that forces pointing to Newton's  
elementing the radial direction, where a<sub>c</sub> = v<sup>2</sup>/r.  
  
**Example 2** We find that  $\Sigma F$  (in radial direction) = F<sub>seat</sub> - w =  $\frac{mv^2}{r}$ .  
  
 $= \sqrt{\frac{rgF_N}{F_S}}$   
  
 $= \sqrt{\frac{rgF_N}{F_S}}$   
  
**Remember that forces pointing towards the center of the  
circle (F<sub>seat</sub>) are positive and those pointing away from the  
center (w) are negative. Substituting ω = mg rearranging,  
we find that the force of the aeroplane seat on the pilot is  
we find the force of the aroplane seat on the pilot is  
 $= rL$**** 

$$
F_{\text{seat}} = m \left( \frac{v^2}{r} + g \right) = 72 \,\text{kg} \left[ \frac{\left( 64 \,\text{m/s} \right)^2}{120 \,\text{m}} + 9.8 \,\text{m/s}^2 \right]
$$
\n
$$
= 72 \,\text{kg} \left( 34.1 \,\text{m/s}^2 + 9.8 \,\text{m/s}^2 \right) = 3160.8 \,\text{N}
$$

The pilot in this example feels very heavy. To keep him in the circular path, the seat must push the pilot upwards with a force of 3160 N, 4.5 times his normal weight. He experiences an acceleration of 4.5 g, that is, 4.5 times the acceleration of gravity.

## **TRY IT YOURSELF-3**

- **Q.1** The radius of curvature of a railway line at a place when the train is moving with a speed of 36 km  $h^{-1}$  is 1000 m, the distance between the two rails being 1.5 metre. Calculate the elevation of the outer rail above the inner rail so that there may be no side pressure on the rails.
- **Q.2** An aircraft executes a horizontal loop at a speed of Q.9 720 km/h with its wing banked at 15°. Calculate the radius of the loop.
- **Q.3** A simple pendulum is vibrating with an angular amplitude of 90 $\degree$  as shown in the given figure. For what value of  $\alpha$ , is the acceleration directed?

![](_page_15_Figure_6.jpeg)

(i) vertically upwards (ii) horizontally (iii) vertically downwards

**Q.4** A car moving at a speed of 36 km/hr is taking a turn on a circular road of radius 50 m. A small wooden plate is kept on the seat with its plane perpendicular to the radius of the circular road (figure). A small block of mass 100g is kept on the seat which rests against the plate. The friction coefficient between the block and the plate is For the hoop.<br>
The broad in the solution of the boop.<br>
The broad in order the solution is vibrating with an angular amplitude<br>
of the boop.<br>
In the given figure. For what value of  $\alpha$ , is<br>
the acceleration integrive figu

- (a) Find the normal contact force exerted by the plate on the block.
- (b) The plate is slowly turned so that the angle between the normal to the plate and the radius of the road slowly increases. Find the angle at which the block will just start sliding on the plate.

![](_page_15_Figure_12.jpeg)

**Q.5** A train runs along an unbanked circular track of radius 30 m at a speed of 54 km/h. The mass of the train is  $10^6$  kg.  $\blacksquare$ What provides the centripetal force required for this purpose. The engine or the rails ? What is the angle of banking required to prevent wearing out of the rail ?

![](_page_15_Picture_14.jpeg)

- **Q.6** A car takes a turn around a circular curve if it turns at double the speed, the tendency to overturn is (A) halved (B) doubled (C) quadrupled (D) unchanged
- **Q.7** A particle originally at rest at the highest point of a smooth vertical circle of radius R, is slightly displaced. Find the vertical distance below the highest point where the particle will leave the circle.
- **Q.8** A body of mass 500 g tied to a string of length 1m is revolved in the vertical circle with a constant speed. Find the minimum speed at which there will not be any slack on the string. Take  $g = 10$  m/s<sup>2</sup>
- A hemispherical bowl of radius  $r = 0.1$ m is rotating about its axis (which is vertical) with an angular velocity  $\omega$ . A particle of mass  $10^{-2}$  kg on the frictionless inner surface of the bowl is also rotating with the same  $\omega$ . The particle is at a height h from the bottom of the bowl. (a) Obtain the relation between h and  $\omega$ . What is the minimum value of  $\omega$ needed in order to have a nonzero value of h. (b) It is desired to measure 'g' using this setup by measuring h accurately. Assuming that r and  $\omega$  are known precisely and that the least count in the measurement of h is  $10^{-4}$ m. What is minimum error  $\Delta g$  in the measured value of g.  $[g = 9.8 \text{ m/s}^2]$
- **Q.10** A ring of radius R is placed such that it lies in a vertical plane. The ring is fixed. A bead of mass m is constrained to move along the ring without any friction. One end of the spring is connected with the mass m and other end is rigidly fixed with the topmost point of the ring. Initially the spring is in un-extended position and the bead is at a vertical distance R from the lowermost point of the ring. The bead is now released from rest. simples connected with the mass m and other end is<br>
rigidly fixed with the topmost point of the ring. Initially the<br>
spring is in un-extended position and the bead is at a<br>
vertical distance R from the lowermost point of
	- (a) What should be the value of spring constant K such that the bead is just able

to reach bottom of the ring.

(b) The tangential and centripetal accelerations of the bead at initial and bottom-most position for the same value of spring constant K.

![](_page_15_Figure_23.jpeg)

## **ANSWERS**

- **(1)** 0.0153 m **(2)** 15.24 km.
	-

$$
(5) 37^{\circ} \tag{6}
$$

**(6)** (C) **(7)** R/3<br>**(9)** (a)  $7\sqrt{2}$  rad/s (b)  $-9.8 \times 10^{-3}$  m/s<sup>2</sup>

**(10) (a)** 
$$
K = \frac{mg}{R(3 - 2\sqrt{2})}
$$
; (b) At initial instant  $a_t = g$ ,  $a_c = 0$ 

At bottom position  $a_t = 0$ ,  $a_c = 0$ 

![](_page_16_Picture_1.jpeg)

## **IMPORTANT POINTS**

**1.** The physical quantities which remain constant for a particle moving in circular path are speed, kinetic energy and angular momentum. **EXECUTE ARRIVAT POINTS**<br> **EXECUTE ARRIVAT POINTS**<br>
The physical quantities which remain constant for a<br>
anticle moving in circular path are speed, kinetic energy<br>
a<sub>r</sub> =  $\omega^2 r = \frac{v^2}{r} = \omega v$  (Always applicable)<br>
A is a **IMPORTANT POINTS**<br>
Example 1:<br>
Example 1:<br>
angular momentum.<br>  $= \omega^2 r = \frac{v^2}{r} = \omega v$  (Always applicable)<br>  $= 4\pi^2 n^2 r = \frac{4\pi^2}{T^2} r$  (Applicable in uniform circular<br>  $= \sin \theta$  angular momentum.<br>
Alternative repends on di **EXERIBED**<br> **EXER EXECUTE ANTIFICUTE SET AND STUDY M**<br> **ADDITIONAL EXA**<br>
The physical quantities which remain constant for a<br>
and angular momentum.<br>  $a_r = \omega^2 r = \frac{v^2}{r} = \omega v$  (Always applicable)<br>  $a_r = 4\pi^2 n^2 r = \frac{4\pi^2}{T^2} r$  (Applicable **STI<br>
MPORTANT POINTS**<br> **ADDITIONAL**<br> **EXECUTE IN THE CONSTANT POINTS**<br> **EXECUTE IN THE CONSTANT POINTS**<br> **EXECUTE IT AND CONSTANT**<br> **EXECUTE:**<br> **EXECUTE:**<br> **EXECUTE IN THE VALUATIONAL**<br>
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2. 
$$
a_r = \omega^2 r = \frac{v^2}{r} = \omega v
$$
 (Always applicable)

2  $r = 4\pi$  if  $r = \frac{1}{T^2}$  (Applicable in uniform  $4\pi^2$  and  $\pi$  and  $\pi$  and  $\pi$  and  $\pi$  $T^2$  (represent in uniform viewing  $\pi^2$  , we have a set of  $\pi$ 

 $motion)$ ; n = frequency of rotation,

 $\vec{a}_r = \vec{\omega} \times \vec{v}$ 

- **SULTER EXAMPLES**<br> **IMPORTANT POINTS**<br>
The physical quantities which remain constant for a<br>
and angular momentum path are speed, kinetic energy<br>
and angular momentum.<br>  $a_r = \omega^2 r = \frac{v^2}{r} = \omega v$  (Always applicable)<br>  $a_r = 4\$ **3.** Circular motion with variable speed. For complete circles, **STUDYMATERIAL: PHYSICS**<br>
The physical quantities which remain constant for a<br>
particle moving in circular path are speed, kinetic energy<br>
and angular momentum.<br>
and angular momentum.<br>  $a_x = a^2r = \frac{v^2}{r} = \omega v$  (Always appl becomes slack, i.e. when  $T = 0$ , range of values of u for **STED MANUMERAT POINTS**<br>
1. The physical quantities which remain constant for a<br>
particle moveing in circular path are speed, kinetic energy **Example 1**:<br>
and angular momentum.<br>
2.  $a_r = \omega^2 r = \frac{v^2}{r} = \omega v$  (Always applica
- complete a vertical circle of radius r is  $\sqrt{5rg}$ . The minimum

velocity at highest point then is  $\sqrt{rg}$ .

- **5.** The difference in tension at the highest and the lowest point in a vertical circle is 6mg, i.e. 6 times the weight of the body.
- of suspension from the centre of circular motion.
- **7.** If a body is moving on a curved road with speed greater than the speed limit, the reaction at the inner wheel disappears and it will leave the ground first.
- **8.** On unbanked curved roads the minimum radius of

curvature of the curve for safe driving is  $r = \frac{v^2}{\mu g}$ , where v<br>A circular loop has

is the speed of the vehicle and  $\mu$  is the coefficient of friction.

**9.** To prevent skidding up the inclination (away from centre)

$$
v \leq \sqrt{gr\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)}
$$

**10.** To prevent skidding down the inclination (towards the

centre) 
$$
v \ge \sqrt{gr \left( \frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta} \right)}
$$

- **11.** The skidding of a vehicle will occur if  $\frac{v^2}{r} > \mu g$  i.e., skid-<br>and  $N = m\omega^2 r$ ding will take place if the speed is large, the curve is sharp and  $\mu$  is small.
- **12.** If r is the radius of curvature of the speed breaker, then the maximum speed with which the vehicle can run on its

## **ADDITIONAL EXAMPLES**

#### **Example 1 :**

- The kinetic energy of a particle moving along a circle of radius r depends on distance covered s as  $K = As^2$  where A is a const. Find the force acting on the particle as a function of s. <sup>1</sup> 2 2 mv As **TUDY MATERIAL: PHYSICS**<br> **EXAMPLES**<br>
rticle moving along a circle of<br>
ce covered s as  $K = As^2$  where<br>
ce acting on the particle as a<br>
m<br>  $v = s \sqrt{\frac{2A}{m}}$  ...(1)<br>
...(2)<br>  $= \frac{dv}{ds} \cdot \frac{ds}{dt} = v \frac{dv}{ds}$  ...(3) **STUDY MATERIAL: PHYSICS**<br> **ITIONAL EXAMPLES**<br>
energy of a particle moving along a circle of<br>
energy of a particle moving along a circle of<br>
st. Find the force acting on the particle as a<br>
s.<br>
to given problem<br>  $= As^2$  or **STUDYMATERIAL: PHYSICS**<br> **DDITIONAL EXAMPLES**<br>
inetic energy of a particle moving along a circle of<br>
ir depends on distance covered s as K = As<sup>2</sup> where<br>
it const. Find the force acting on the particle as a<br>
on of s.<br>
id **STUDY MATERIAL: PHYSICS**<br> **DDITIONAL EXAMPLES**<br>
etic energy of a particle moving along a circle of<br>
depends on distance covered s as K = As<sup>2</sup> where<br>
const. Find the force acting on the particle as a<br>
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ing to giv **STUDY MATERIAL: PHYSICS**<br> **IL EXAMPLES**<br>
particle moving along a circle of<br>
fance covered s as K = As<sup>2</sup> where<br>
force acting on the particle as a<br>
blem<br>  $v = s \sqrt{\frac{2A}{m}}$  ...(1)<br>
...(2)<br>  $\frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \frac{dv}{ds}$ **STUDY MATERIAL: PHYSICS**<br> **DNAL EXAMPLES**<br>
v of a particle moving along a circle of<br>
n distance covered s as K = As<sup>2</sup> where<br>
the force acting on the particle as a<br>
n problem<br>
or  $v = s \sqrt{\frac{2A}{m}}$  ...(1)<br>
s<sup>2</sup><br>  $a_t = \frac{dv}{dt}$ **STUDY MATERIAL: PHYSICS**<br> **IAL EXAMPLES**<br>
f a particle moving along a circle of<br>
istance covered s as K = As<sup>2</sup> where<br>
e force acting on the particle as a<br>
roblem<br>
or  $v = s \sqrt{\frac{2A}{m}}$  ...(1)<br>
...(2)<br>
=  $\frac{dv}{dt} = \frac{dv}{ds} \cdot \$ **NDDITIONAL EXAMPLES**<br>
:<br>
it where energy of a particle moving along a circle of<br>
is r depends on distance covered s as K = As<sup>2</sup> where<br>
a const. Find the force acting on the particle as a<br>
origin to given problem<br>  $\frac{1}{$ **STUDY MATERIAL: PHYSICS**<br> **DDITIONAL EXAMPLES**<br>
metic energy of a particle moving along a circle of<br>
r depends on distance covered s as K = As<sup>2</sup> where<br>
const. Find the force acting on the particle as a<br>
m of s.<br>
mv<sup>2</sup> = **ILEXAMPLES**<br>particle moving along a circle of<br>tance covered s as K = As<sup>2</sup> where<br>force acting on the particle as a<br>blem<br> $v = s \sqrt{\frac{2A}{m}}$  ...(1)<br>...(2)<br> $\frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \frac{dv}{ds}$  ...(3)<br> $\frac{dv}{ds} = \sqrt{\frac{2A}{m}}$  ...(4) **IL EXAMPLES**<br>
particle moving along a circle of<br>
fance covered s as K = As<sup>2</sup> where<br>
force acting on the particle as a<br>
blem<br>  $v = s \sqrt{\frac{2A}{m}}$  ...(1)<br>
...(2)<br>  $\frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \frac{dv}{ds}$  ...(3)<br>  $\frac{dv}{ds} = \sqrt{\frac{2A}{m$
- **Sol.** According to given problem

$$
\frac{1}{2}
$$
 mv<sup>2</sup> = As<sup>2</sup> or v = s  $\sqrt{\frac{2A}{m}}$  ...(1)

So, 
$$
a_r = \frac{v^2}{r} = \frac{2As^2}{mr}
$$
 ...(2)

Further more as,  $a_t = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \frac{dv}{ds}$ 

from  $eq^n$ . (1),

$$
v = s \sqrt{\left(\frac{2A}{m}\right)} \implies \frac{dv}{ds} = \sqrt{\frac{2A}{m}}
$$
...(4)

Substitute values from eq<sup>n</sup>. (1) and eq<sup>n</sup>. (4) in eq<sup>n</sup>. (3)

1. Find the first point of the string does not require a vertical circle in the origin. The equation is 
$$
ax = 0
$$
 and  $ax = 0$  and  $ax = 0$  and  $ax = 0$  and  $ax = 0$ . The first point is  $ax = 0$  and  $ax = 0$  and  $ax = 0$ . The first point is  $ax = 0$  and  $ax = 0$ . The first point is  $ax = 0$  and  $ax = 0$ . The first point is  $ax = 0$ 

#### **Example 2 :**

A circular loop has a small bead which can slide on it without friction. The radius of the loop is r. Keeping the loop vertically it is rotated about a vertical diameter at a constant angular speed  $\omega$ . What is the value of angle  $\theta$ , when the bead is in dynamic equilibrium?

![](_page_16_Figure_36.jpeg)

![](_page_17_Picture_1.jpeg)

#### **Example 3 :**

A body weighing 0.4 kg is whirled in a vertical circle making 2 revolutions per second. If the radius of the circle is 1.2 m, find the tension in the string, when the body is (a) at the top of the circle (b) at the bottom of the circle. Given :  $g = 9.8$  ms<sup>-2</sup> and  $\pi = 3.14$ . 4 kg is whirled in a vertical circle making<br>
(a) The centripetal force is provide<br>
econd. If the radius of the circle is 1.2 m,<br>
train. The train causes an equal<br>
the string, when the body is (a) at the<br>
b at the outer r

**Sol.** Mass m = 0.4 kg; time period =  $\frac{1}{2}$  second and radius, (ii) tar  $\frac{1}{2}$  second and radius,

 $r = 1.2 m$ 

Angular velocity,  $\omega = \frac{2\pi}{\sqrt{2}} = 4\pi$  rad s<sup>-1</sup> = 12.56 rad s<sup>-1</sup> Example

(a) At the top of the circle,

$$
T = \frac{mv^2}{r} - mg = mr\omega^2 - mg = m(r\omega^2 - g)
$$
  
= 0.4 (1.2 × 12.56 × 12.56 - 9.8) N = 71.8 N

(b) At the lowest point,  $T = m (r\omega^2 + g) = 79.64$  N

#### **Example 4 :**

A small body of mass  $m = 0.1$  kg swings in a vertical circle at the end of a chord of length 1 m. Its speed is 2 m/s when the chord makes an angle  $\theta = 30^{\circ}$  with the vertical. Find the tension in the chord.

Angular velocity, 
$$
\omega = \frac{2\pi}{1/2} = 4\pi \text{ rad s}^{-1} = 12.56 \text{ rad s}^{-1}
$$

\n(a) At the top of the circle,  $\omega = 2\pi - m\omega^2 - mg = m(r\omega^2 - g)$  of the force at the rod is 0.2, what is the (a) optim distribution between the w trace car and the rod is 0.2, what is the (a) optim distribution between the w trace car and the rod is 0.2, what is the (a) optim distribution between the w trace car and the rod is 0.2, what is the (b) 0.4 (b) At the lowest point,  $T = m(r\omega^2 + g) = 79.64 \text{ N}$ 

\nExample 4:

\nExample 4:

\nExample 5:

\nExample 6:

\nExample 1:  $\omega = 10.1 \times 12.56 \times$ 

#### **Example 5 :**

A train rounds an unbanked circular bend of radius 30 m at a speed of 54 km/h. The mass of the train is  $10^6$  kg. What provides the centripetal force required for this purpose? The engine or the rails? The outer or inner rails? Which Sol. rail will wear out faster, the outer or the inner rail ? What is the angle of banking required to prevent wearing out of the rails?

**Sol.** 
$$
r = 30m
$$
,  $v = 54 \text{ km/h} = \frac{54 \times 5}{18} \text{ m/s} = 15 \text{ m/s}$   
Wl  
m =  $10^6 \text{ kg}$ ,  $\theta = ?$ 

(i) The centripetal force is provided by the lateral thrust by the outer rail on the flanges of the wheel of the train. The train causes an equal and opposite thrust on the outer rail (Newton's third law of motion). Thus, the outer rail wears out faster. **EXERCISE THE CONTROVANCED LEARNING**<br>
The centripetal force is provided by the lateral thrust<br>
by the outer rail on the flanges of the wheel of the<br>
train. The train causes an equal and opposite thrust<br>
on the outer rail **EXECUTE 10**<br> **EXECUTE 10 EXECUTE:**<br> **EXECUTE:**<br> **P**<br> **EXECUTE:**<br> **P**<br> **EXECUT** 

(ii) 
$$
\tan \theta = \frac{v^2}{rg} = \frac{15 \times 15}{30 \times 9.8} = 0.7653
$$
  
or  $\theta = \tan^{-1} (0.7653) = 37.43^{\circ}$ 

#### **Example 6 :**

is whirled in a vertical circle making<br>
10. (i) The centripetal force is provided by<br>
12. If the radius of the circle is 1.2 m,<br>
term, the train causes an equal and<br>
tending, when the body is (a) at the<br>
tending the botto **EMOTION**<br>
Weighing 0.4 kg is whirled in a vertical circle making<br>
weighing 0.4 kg is whirled in a vertical circle making<br>
to the contripetal force is provided by the lateral thrust<br>
tension in the string, when outer rail A circular race track of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race car and the road is 0.2, what is the (a) optimum speed of the race car to avoid wear and tear of tyres, and the (b) maximum permissible speed to avoid slipping?

**AR MOTION**<br>
2 **EXECUTE:**<br>
2 **EXECUTE: Sol.** (a) On a banked road, the horizontal component of the normal reaction and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the component of the normal reaction is enough to provide the required centripetal force. In this case, the frictional force is not required. 53)=37.43°<br>
Fradius 300 m is banked at an angle<br>
to f friction between the wheels of a<br>
0.2, what is the (a) optimum speed<br>
wear and tear of tyres, and the (b)<br>
speed to avoid slipping?<br>
, the horizontal component of the<br> e track of radius 300 m is banked at an angle<br>coefficient of friction between the wheels of a<br>he road is 0.2, what is the (a) optimum speed<br>tr to avoid wear and tear of tyres, and the (b)<br>rmissible speed to avoid slipping f radius 300 m is banked at an angle<br>to of friction between the wheels of a<br>s 0.2, what is the (a) optimum speed<br>d wear and tear of tyres, and the (b)<br>speed to avoid slipping?<br>d, the horizontal component of the<br>md the fri  $30 \times 9.8$ <br>  $-9.7653$ ) = 37.43°<br>
k of radius 300 m is banked at an angle<br>
cient of friction between the wheels of a<br>
dd is 0.2, what is the (a) optimum speed<br>
void wear and tear of tyres, and the (b)<br>
ble speed to avoid s

The optimum speed is given by

$$
v_0 = (rg \tan \theta)^{1/2} = (300 \times 9.8 \tan 15^\circ)^{1/2} \text{ ms}^{-1}
$$
  
= 28.1 m/s

(b) The maximum permissible speed is given by

$$
V_{\text{max}} = \left(\frac{\mu_{\text{s}} + \tan \theta}{1 - \mu_{\text{s}} \tan \theta} \,\text{rg}\right)^{1/2}
$$

 Substituting values and simplifying, we get  $v_{\text{max}} = 38.1 \text{ m/s}.$ 

#### **Example 7 :**

A bob of mass m, suspended by a string of length  $\ell_1$  is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length  $\ell_2$ , which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio  $\ell_1/\ell_2$ 2 is – minimum velocity required to complete a full circle<br>
ertical plane. At the highest point, it collides<br>
y with another bob of mass m suspended by a<br>
length  $\ell_2$ , which is initially at rest. Both the<br>
length  $\ell_2$ , which bstituting values and simplifying,<br>get  $v_{max} = 38.1$  m/s.<br>of mass m, suspended by a string of length  $\ell_1$  is<br>minimum velocity required to complete a full circle<br>vertical plane. At the highest point, it collides<br>ally with of mass m, suspended by a string of length  $\ell_1$  is<br>minimum velocity required to complete a full circle<br>ertical plane. At the highest point, it collides<br>lly with another bob of mass m suspended by a<br>f length  $\ell_2$ , whic

The initial speed of 1<sup>st</sup> bob (suspended by a string of length  $\ell_1$ ) is  $\sqrt{5g\ell_1}$ .

The speed of this bob at highest point will be  $\sqrt{g \ell_1}$ .

When this bob collides with the other bob there speeds will be interchanged.

$$
\sqrt{g\ell_1} = \sqrt{5g\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = 5
$$

![](_page_18_Picture_0.jpeg)

## **QUESTION BANK CHAPTER 7 : CIRCULAR MOTION**

## **EXERCISE - 1 [LEVEL-1]**

## **PART - 1 : KINEMATICS OF CIRCULAR MOTION**

**Q.1** A particle completes 1.5 revolutions in a circular path of radius 2 cm. The angular displacement of the particle will be – (in radian)<br>(A)  $6\pi$  $(B)$  3 $\pi$ 

![](_page_18_Picture_811.jpeg)

**Q.2** A particle revolving in a circular path completes first one third of circumference in 2 sec, while next one third in 1 sec. The average angular velocity of particle will be (in rad/sec)

 $(A) 2\pi/3$  (B)  $\pi/3$ (C)  $4\pi/3$  (D)  $5\pi/3$ 

**Q.3** The ratio of angular speeds of minute hand to hour hand of a watch is -  $(A) 12 : 1$  (B) 6 : 1

![](_page_18_Picture_812.jpeg)

 $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ , where  $\omega_0$  and  $\alpha$  are constant and  $\frac{1}{2}$   $\alpha t^2$ , where  $\omega_0$  and  $\alpha$  are constant and

 $\omega_0 = 1$  rad/sec,  $\alpha = 1.5$  rad/sec<sup>2</sup>. The angular velocity at time,  $t = 2$  sec will be (in rad/sec) - $(A) 1$  (B) 5  $(C)$  3 (D) 4

**Q.5** The magnitude of the linear acceleration, the particle moving in a circle of radius of 10 cm with uniform speed completing the circle in 4 s, will be - (A)  $5\pi$  cm/s<sup>2</sup> (B)  $2.5\pi$  cm/s<sup>2</sup>

(C)  $5\pi^2$  cm/s<sup>2</sup> (D)  $2.5\pi^2$  cm/s<sup>2</sup>

**Q.6** The length of second's hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is -

(A) 0   
 (B) 
$$
\frac{\pi}{30\sqrt{2}} \text{ cm/s}
$$

(C) 
$$
\frac{\pi}{30}
$$
 cm/s (D)  $\frac{\pi\sqrt{2}}{30}$  cm/s (E) 2.25 : 9.8 (D) 2.5 : 9  
Q.16 Choose the correct options –

- **Q.7** A particle moves in a circle of radius 20cm with a linear speed of 10m/s. The angular velocity will be -  $(A)$  50 rad/s  $(B)$  100 rad/s  $(C)$  25 rad/s  $(D)$  75 rad/s
- **Q.8** The angular velocity of a particle is given by  $\omega = 1.5 t - 3t^2 + 2$ , the time when its angular acceleration decreases to be zero will be - (A) 25 sec (B) 0.25 sec (C) 12 sec (D) 1.2 sec
- **Q.9** A particle is moving in a circular path with velocity varying with time as  $v = 1.5t^2 + 2t$ . If 2 cm the radius of circular path, the angular acceleration at  $t = 2$  sec will be  $(A)$  4 rad/sec<sup>2</sup> (B) 40 rad/sec $<sup>2</sup>$ </sup>  $(C)$  400 rad/sec<sup>2</sup> (D)  $0.4$  rad/sec<sup>2</sup>
- **Q.10** A grind stone starts from rest and has a constant-angular acceleration of 4.0 rad/sec<sup>2</sup>. The angular displacement and angular velocity, after 4 sec. will respectively be - (A) 32 rad, 16 rad/sec (B) 16rad, 32 rad/s (C) 64rad, 32 rad/sec (D) 32 rad, 64rad/sec
- **Q.11** The shaft of an electric motor starts from rest and on the application of a torque, it gains an angular acceleration given by  $\alpha = 3t - t^2$  during the first 2 seconds after it starts after which  $\alpha = 0$ . The angular velocity after 6 sec will be - **[LEVEL-1]**<br> **Q.10** A grind stone starts from rest and has a constant-angular<br>
acceleration of 4.0 rad/sec<sup>2</sup>. The angular displacement<br>
and angular velocity, after 4 sec. will respectively be-<br>
(A) 32 rad, 16 rad/sec (B) **IDN**<br> **Example 18**<br> **Example 18**<br> **A** 0 rad/sec<sup>2</sup>. The angular displacement<br>
city, after 4 sec. will respectively be-<br>
d/sec (B) 16rad, 32 rad/s<br>
(D) 32 rad, 64rad/sec<br>
lectric motor starts from rest and on the<br>
torque, **EL-1]**<br>
A grind stone starts from rest and has a constant-angular<br>
acceleration of 4.0 rad/sec<sup>2</sup>. The angular displacement<br>
and angular velocity, after 4 sec. will respectively be-<br>
4 (A) <sup>32</sup> rad, 16 rad/sec (B) 16rad, A grind stone starts from rest and has a constant-angular<br>acceleration of 4.0 rad/see<sup>2</sup>. The angular displacement<br>and angular velocity, after 4 sec. will respectively be-<br>(A) 32 rad, 16 rad/sec (B) 16 rad, 32 rad/sec<br>(C)
	- (A) 10/3 rad/sec (B) 3/10 rad/sec (C) 30/4 rad/sec (D) 4/30 rad/sec
- **Q.12** A motor car is travelling at 30 m/s on a circular road of radius 500 m. It is increasing its speed at the rate of  $2m/s<sup>2</sup>$ . Its net acceleration is  $(in m/s<sup>2</sup>)$  –

(A) 2 (B) 1. 8 (C) 2.7 (D) 0

**Q.4** The angular displacement of a particle is given by **Q.13** What is the value of linear velocity, if  $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$  and  $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$ 

## **PART - 2 : DYNAMICS OF UNIFORM CIRCULAR MOTION**

- of a watch is  $(0, 1)$ <br>
(C) 12:1<br>
(C) 12:1<br>
(C) 11:2<br>
(C) 11:2<br>
(C) 11:2<br>
(C) 11:2<br>
(C) 11:2<br>
(C) 11:6<br>
(C) 11:2<br>
(C) 11:2<br>
(C) 11:2<br>
(C) 11:2<br>
(C) 11:5<br>
(C) 11:5<br>
(A) 11:5<br>
(A) 11:5<br>
(A) 11:5<br>
(A) 11:5<br>
(A) 11:5<br>
(A) 11: **Q.14** An electron is moving in a circular orbit of radius  $5.3 \times 10^{-11}$  metre around the atomic nucleus at a rate of  $6.6 \times 10^{15}$  revolutions per second. The acceleration of the electron and centripetal force acting on it will be - (The mass of the electron is  $9.1 \times 10^{-31}$ kg) (A)  $8.3 \times 10^{-8}$ N (B)  $3.8 \times 10^{-8}$ N  $(C) 4.15 \times 10^{-8}$ N  $(D) 2.07 \times 10^{-8}$ N
	- $\pi$  with a steady speed of 900 km/h. The ratio of centripetal  $\pi\sqrt{2}$  (C) 2.25 : 9.8 (D) 2.5 : 9.8 **Q.15** An air craft executes a horizontal loop of radius 1 km acceleration to that gravitational acceleration will be-  $(A) 1 : 6.38$  (B) 6. 38 : 1
		- (1) Centripetal force is not a real force. It is only the requirement for circular motion.
			- (2) Work done by centripetal force may or may not be zero.
			- (3) Work done by centripetal force is always zero.
			- (4) Centripetal force is a fundamental force.

 $(A) 1, 2$  and 3 are correct (B) 1 and 2 are correct

- (C) 2 and 4 are correct (D) 1 and 3 are correct
- **Q.17** A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration  $a_c$  is varying with time t as  $a_c = k^2 rt^2$ , where k is a constant. The power delivered to the particle by the forces acting on it will be -

(A) 
$$
mk^2t^2r
$$
  
\n(B)  $mk^2r^2t^2$   
\n(C)  $m^2k^2t^2r^2$   
\n(D)  $mk^2r^2t$ 

![](_page_19_Picture_2.jpeg)

- **Q.18** In uniform circular motion
	- (A) Both velocity and acceleration are constant
	- (B) Acceleration and speed are constant but velocity changes
	- (C) Both acceleration and velocity changes
	- (D) Both acceleration and speed are constant
- **Q.19** A particle does uniform circular motion in a horizontal plane. The radius of the circle is 20 cm. The centripetal force acting on the particle is 10 N. It's kinetic energy is  $(A) 0.1 J$  (B) 0.2 J  $(C) 2.0 J$  (D) 1.0 J
- **Q.20** A particle moves with constant angular velocity in circular path of certain radius and is acted upon by a certain centripetal force F. If the angular velocity is doubled, keeping radius the same, the new force will be (A)  $2F$  (B)  $F^2$ niform eircular motion<br>
Both velocity and acceleration are constant<br>  $\text{A} = \text{A} \times \text{B}$ <br>
Acceleration and speed are constant<br>  $\text{A} = \text{A} \times \text{B}$ <br>
acceleration and velocity changes<br>
Both acceleration and velocity chang (B) changes<br>
Both acceleration and velocity changes<br>
Both acceleration and speed are constant<br>
if the cord can sustain me<br>
acting does uniform circular motion in a horizontal<br>
if the cord can sustain me<br>
if the cord can susta
	- $(C)$  4F (D) F/2
- **Q.21** Two bodies of equal masses revolve in circular orbits of radii  $R_1$  and  $R_2$  with the same period. Their centripetal forces are in the ratio

(A) 
$$
\left(\frac{R_2}{R_1}\right)^2
$$
  
\n(B)  $\frac{R_1}{R_2}$   
\n(C)  $\left(\frac{R_1}{R_2}\right)^2$   
\n(D)  $\sqrt{R_1 R_2}$   
\n(D)  $\sqrt{R_1 R_2}$   
\n(D)  $\sqrt{R_1 R_2}$   
\n(D)  $\sqrt{R_1 R_2}$ 

- **Q.22** A stone ties to the end of a string 1m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolution in 44 seconds, what is the magnitude and direction of acceleration of the stone
	- (A)  $(\pi^2/4)$  m/s<sup>2</sup> and direction along the radius towards the centre
	- (B)  $\pi^2$  m/s<sup>2</sup> and direction along the radius away from the centre
	- (C)  $\pi^2$  m/s<sup>2</sup> and direction along the radius towards the centre
	- (D)  $\pi^2$  m/s<sup>2</sup> and direction along the tangent to the circle

## **PART - 3 : NON-UNIFORM CIRCULAR MOTION AND VERTICAL CIRCULAR MOTION**

**Q.23** A cane filled with water is revolved in a vertical circle of radius 4 m and water just does not fall down. The time period of revolution will be –

![](_page_19_Picture_701.jpeg)

- **Q.24** A body of mass m kg is rotating in a vertical circle at the end of a string of length r metre. The difference in the kinetic energy at the top and the bottom of the circle is  $(A)$  mg/r  $(B)$  2mg/r (C) 2mgr (D) mgr
- **Q.25** A car is moving in a circular path of radius 100 m with velocity of 200 m/sec such that in each sec its velocity increases by 100 m/s, the net acceleration of car will be - (in m/sec)
- (A)  $100\sqrt{17}$  (B)  $10\sqrt{7}$
- (C)  $10\sqrt{3}$  (D)  $100\sqrt{3}$
- **Q.26** A 4 kg balls is swing in a vertical circle at the end of a cord 1 m long. The maximum speed at which it can swing if the cord can sustain maximum tension of 163.6 N will be -
	- $(A) 6 \text{ m/s}$  (B)  $36 \text{ m/s}$  $(C) 8 \text{ m/s}$  (D) 64 m/s
- **Q.27** The string of a pendulum is horizontal. The mass of the bob is m. Now the string is released. The tension in the string in the lowest position is -

(A) 1 mg (B) 2 mg (C) 3 mg (D) 4 mg

**Q.28** A weightless thread can support tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2 m in a vertical plane then the maximum angular velocity of the stone will be

(A) 5 rad/s (B) 
$$
\sqrt{30}
$$
 rad/s

$$
R_1
$$
 (C)  $\sqrt{60}$  rad/s (D) 10 rad/s

2 and  $\overline{a}$  $\overline{R_2}$  **Q.29** A body of mass m hangs at one end of a string of length Both acceleration and velocity changes and which it can swing<br>
Both acceleration and speed are constant<br>
if the cord can sustain maximum is proof 163.6 N wi<br>
Hoth acceleration and speed are constant<br>
if the cord can susta **EXECUTE 18**<br>
(A)  $100\sqrt{17}$  (B)  $10\sqrt{7}$ <br>
(C)  $10\sqrt{3}$  (D)  $100\sqrt{3}$ <br>
A 4 kg balls is swing in a vertical circle at the end of a<br>
cord 1 m long. The maximum speed at which it can swing<br>
the cord can sustain maximum t (A) 100  $\sqrt{17}$  (B) 10  $\sqrt{7}$ <br>
(C) 10  $\sqrt{3}$ <br>
A 4 kg balls is swing in a vertical circle at the end of a<br>
cord 1 mlong. The maximum speed at which it can swing<br>
fif the cord can sustain maximum tension of 163.6 N will<br>  $\ell$ , the other end of which is fixed. It is given a horizontal velocity so that the string would just reach where it makes an angle of 60° with the vertical. The tension in the string at mean position is – (C) 8 m/s<br>
The string of a pendulum is horizontal. The mass of the<br>
The string of a pendulum is horizontal. The mass of the<br>
bob is m. Now the string is released. The tension in the<br>
string in the lowest position is -<br>
(A (A) 1 mg<br>
(B) 2 mg<br>
(C) 3 mg<br>
(C) 1 mg<br>
A weightless thread can support tension upto 30 N. A<br>
stone of mass 0.5 kg is tied to it and is revolved in a<br>
sticular path of radius 2 m in a vertical plane then the<br>
maximum angu

![](_page_19_Picture_702.jpeg)

**Q.30** A stone of mass m is tied to a string and is moved in a vertical circle of radius r making n revolutions per minute. The total tension in the string when the stone is at its lowest point is

- 
- (C)  $\sin$ <br>
(A weightless thread can support tension upto 30 N. A<br>
stone of mass 0.5 kg is tied to it and is revolved in a<br>
sticular path of radius 2 m in a vertical plane then the<br>
maximum angular velocity of the stone wil **Q.31** A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical  $(A)$  30<sup>o</sup>  $(D)$   $450$ tension in the string when the stone is at its<br>int is<br>(B) m(g +  $\pi$  n<sup>2</sup>)<br> $+\pi$  nr) (D) m{g + ( $\pi$ <sup>2</sup> n<sup>2</sup> r)/900}<br>pendulum oscillates in a vertical plane. When<br>phrough the mean position, the tension in the<br>times the wei the stone is at its<br>
r<sup>2</sup>)<br>
n<sup>2</sup> r)/900}<br>
cal plane. When<br>
ne tension in the<br>
ulum bob. What<br>
endulum of the<br> **IRCULAR**<br>
lius 100 m with a<br>
which he has to<br>  $\frac{1}{40}$ <br>  $\left(\frac{1}{20}\right)$

$$
(A) 30o \n(C) 60o \n(D) 90o
$$

## **PART - 4 : APPLICATIONS OF CIRCULAR MOTION**

**Q.32** A car driver is negotiating a curve of radius 100 m with a speed of 18 km/hr. The angle through which he has to lean from the vertical will be -

(A) 
$$
\tan^{-1} \frac{1}{4}
$$
 \t\t (B)  $\tan^{-1} \frac{1}{40}$   
(C)  $\tan^{-1} \left(\frac{1}{2}\right)$  \t\t (D)  $\tan^{-1} \left(\frac{1}{20}\right)$ 

![](_page_20_Picture_0.jpeg)

- **Q.33** The vertical section of a road over a canal bridge in the direction of its length is in the form of circle of radius 8.9 metre. Find the greatest speed at which the car can cross this bridge without losing contact with the road at its highest point, the center of gravity of the car being at a height  $h = 1.1$  metre from the ground.  $(g = 10 \text{ m/sec}^2)$ (A) 5 m/s (B) 7 m/s (C) 10 m/s (D) 13 m/s
- **Q.34** The maximum speed at which a car can turn round a curve of 30 metre radius on a level road if the coefficient of friction between the tyres and the road is 0.4, will be - (A) 10.84 m/s (B) 17.84 m/s
- (C)  $11.76 \text{ m/s}$  (D)  $9.02 \text{ m/s}$ **Q.35** The angular speed with which the earth would have to rotate on it axis so that a person on the equator would weight  $(3/5)$ <sup>th</sup> as much as present will be: (Take the equatorial radius as 6400 km)

(A) 
$$
8.7 \times 10^4
$$
 rad/sec (B)  $8.7 \times 10^3$  rad/sec

(C)  $7.8 \times 10^4$  rad/sec rad/sec  $(D) 7.8 \times 10^3$  rad/sec

- **Q.36** The roadway bridge over a canal is the form of an arc of a circle of radius 20 m. What is the minimum speed with which a car can cross the bridge without leaving contact with the ground at the highest point  $(g = 9.8 \text{ m/s}^2)$ (A)  $7 \text{ m/s}$  (B)  $14 \text{ m/s}$  $(C)$  289 m/s  $(D)$  5 m/s as a speed with which the earth would have to<br>
a speed with which the earth would have to  $(0, 0, 0)$  in the wheels leave to<br>
the axis so that a person on the equator would<br>  $10^4$  rad/sec (B)  $8.7 \times 10^3$  rad/sec and is (13/5)<sup>th</sup> as much a person on the equation would be:<br>
(13/5)<sup>th</sup> as much a present will be:<br>  $\times 10^4$  rad/sec (B) 8.7 × 10<sup>3</sup> rad/sec radius (201) and the equatorial radius as 6400 km)<br>  $\times 10^4$  rad/sec (D) 7.8 × 10<sup>3</sup>
- **Q.37** Roads are banked on curves so that
	- (A) The speeding vehicles may not fall outwards (B) The frictional force between the road and vehicle may be decreased
	- (C) The wear and tear of tyres may be avoided
	- (D) The weight of the vehicle may be decreased
- **Q.38** For a body moving in a circular path, a condition for no skidding if  $\mu$  is the coefficient of friction, is

(A) 
$$
\frac{mv^2}{r} \le \mu mg
$$
  
\n(B)  $\frac{mv^2}{r} \ge \mu mg$   
\n(C)  $\frac{v}{r} = \mu g$   
\n(D)  $\frac{mv^2}{r} = \mu mg$   
\nQ.4'

**Q.39** For a heavy vehicle moving on a circular curve of a highway the road bed is banked at an angle  $\theta$ corresponding to a particular speed. The correct angle of banking of the road for vehicles moving at 60 km/hr will be - (If radius of curve  $= 0.1$  km)

(A) 
$$
\tan^{-1}(0.283)
$$
   
\n(B)  $\tan^{-1}(2.83)$    
\n(C)  $\tan^{-1}(0.05)$    
\n(D)  $\tan^{-1}(0.5)$ 

**Q.40** A train has to negotiate a curve of radius 400 m. By how much should the outer rail be raised with respect to inner rail for a speed of 48 km/hr. The distance between the rail is 1 m.

(A) 
$$
12 \text{ m}
$$
 (B)  $12 \text{ cm}$ 

- $(C)$  4.5 cm  $(D)$  4.5 m
- **Q.41** A cyclist turns around a curve at 15 miles/hour. If he turns at double the speed, the tendency to overturn is (A) Doubled (B) Quadrupled (C) Halved (D) Unchanged
- **Q.42** A stone of mass m is tied to a string of length  $\ell$  and rotated in a circle with a constant speed v. If the string is released, the stone flies –
	- (A) Radially outward
	- (B) Radially inward

(C) Tangentially outward

(D) With an acceleration 
$$
\frac{mv^2}{\ell}
$$

- **Q.43** A car sometimes overturns while taking a turn. When it overturns, it is
	- (A) The inner wheel which leaves the ground first
	- (B) The outer wheel which leaves the ground first
	- (C) Both the wheels leave the ground simultaneously
	- (D) Either wheel leaves the ground first
- Sometive that the stream tasks and a ground simultaneously<br>
by would have to (C) Both the wheels leave the ground simultaneously<br>
equator would<br>
(D) Either wheels leave the ground first<br>  $Q.44$  Anotor cyclist moving with 9.44 A motor cyclist moving with a velocity of 72 km/hour on<br>
a flat road takes a turn on the road at a point where the<br>
radius of curvature of the road is 20 meters. The<br>
10<sup>3</sup> rad/sec<br>
acceleration due to gravity is 10 **Q.44** A motor cyclist moving with a velocity of 72 km/hour on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 meters. The acceleration due to gravity is  $10 \text{ m/sec}^2$ . In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than

(A) 
$$
\theta = \tan^{-1} 6
$$
  
\n(B)  $\theta = \tan^{-1} 2$   
\n(C)  $\theta = \tan^{-1} 25.92$   
\n(D)  $\theta = \tan^{-1} 4$ 

**Q.45** A cyclist goes round a circular path of circumference 34.3 m in  $\sqrt{22}$  sec. the angle made by him, with the vertical, will be  $(A)$  $(5)$   $400$ 

![](_page_20_Picture_798.jpeg)

## **PART - 5 : MISCELLANEOUS**

- $\frac{1}{2}$  mv<sup>2</sup><br>in the string is 6N, when the stone is at  $(g = 10 \text{ m/sec}^2)$  $(A)$  Top of the circle **Q.46** A 1 kg stone at the end of 1m long string is whirled in a vertical circle at constant speed of 4 m/sec. The tension  $(B)$  Bottom of the circle (C) Half way down (D) None of the above spect to the vertical<br>
= tan<sup>-1</sup> 2<br>
= tan<sup>-1</sup> 4<br>
the of circumference<br>
de by him, with the<br>
o<sup>o</sup><br> **EOUS**<br>
string is whirled in a<br>  $\frac{1 \text{ m/sec}}{1 \text{ m/sec}}$ . The tension<br>
is at (g = 10 m\sec<sup>2</sup>)<br>
ottom of the circle<br>
one of the = tan<sup>-1</sup> 2<br>
= tan<sup>-1</sup> 4<br>
th of circumference<br>
de by him, with the<br>
o<sup>o</sup><br> **ECUIS**<br>
string is whirled in a<br>  $\pm m/sec$ . The tension<br>
is at (g = 10 m\sec<sup>2</sup>)<br>
ottom of the circle<br>
one of the above<br>
city v on a circular<br>
s speed = tan<sup>-1</sup> 4<br>
th of circumference<br>
de by him, with the<br>
o<br>
o<br>
(o<br> **EOUS**<br>
string is whirled in a<br>  $\frac{1 \text{ m/sec}}{\text{m/sec}}$ . The tension<br>
s at (g = 10 m\sec<sup>2</sup>)<br>
ottom of the circle<br>
one of the above<br>
city v on a circular<br>
speed
- $\frac{mv^2}{2} = \mu mg$  **Q.47** A car is travelling with linear velocity v on a circular road of radius r. If it is increasing its speed at the rate of 'g' meter\sec<sup>2</sup> , then the resultant acceleration will be

acceleration due to gravity is 10 m/sec-. In order to avoid  
skidding, he must not bend with respect to the vertical  
plane by an angle greater than  
(A) 
$$
\theta = \tan^{-1} 6
$$
 (B)  $\theta = \tan^{-1} 2$   
(C)  $\theta = \tan^{-1} 25.92$  (D)  $\theta = \tan^{-1} 4$   
A cyclist goes round a circular path of circumference  
34.3 m in  $\sqrt{22}$  sec. the angle made by him, with the  
vertical, will be  
(A) 45° (B) 40°  
(C) 42° (D) 48°  
**PART - 5 : MISCELLANEOUS**  
A 1 kg stone at the end of 1 m long string is which in a  
vertical circle at constant speed of 4 m/sec. The tension  
in the string is 6N, when the stone is at (g = 10 m/sec<sup>2</sup>)  
(A) Top of the circle (B) Bottom of the circle  
(C) Half way down (D) None of the above  
A car is travelling with linear velocity v on a circular  
road of radius r. If it is increasing its speed at the rate of  
'g' meter/sec<sup>2</sup>, then the resultant acceleration will be  
(A)  $\sqrt{\frac{v^2}{r^2} - g^2}$  (B)  $\sqrt{\frac{v^4}{r^2} + g^2}$   
(C)  $\sqrt{\frac{v^4}{r^2} - g^2}$  (D)  $\sqrt{\frac{v^2}{r^2} + g^2}$   
You may have seen in a circus a motorcyclist driving in  
vertical loops inside a 'deathwell' (a hollow spherical  
chamber with holes, so the spectators can watch from  
outside). What is the minimum speed required at the

**Q.48** You may have seen in a circus a motorcyclist driving in vertical loops inside a 'deathwell' (a hollow spherical chamber with holes, so the spectators can watch from outside).What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m ?

![](_page_20_Picture_799.jpeg)

## **CIRCULAR MOTION QUESTION BANK**

![](_page_21_Picture_2.jpeg)

**Q.49** A child is belted into a Ferris wheel seat that rotates counter clockwise at constant speed in a vertical plane at an amusement park. At the location shown, which direction best represents the total force exerted on the child by the seat and belt –

![](_page_21_Figure_4.jpeg)

- **Q.50** As you ride on a merry-go-round, you feel a strong outward pull that feels just like the force of gravity. This fictitious force occurs because
	- (A) your velocity is toward the center of the merry-goround and you experience a fictitious force in the direction opposite your velocity.
	- (B) your velocity is away from the center of the merrygo-round and you experience a fictitious force in the direction of your velocity.
	- (C) you are accelerating toward the center of the merrygo-round and experience a fictitious force in the direction opposite your acceleration.
	- (D) you are accelerating away from the center of the merry-go-round and experience a fictitious force in the direction of your acceleration.
- **Q.51** A particle P is moving in a circle of radius 'a' with a uniform speed v. C is the centre of the circle and AB is a diameter. When passing through B the angular velocity of P about and A and C are in the ratio.

![](_page_21_Figure_11.jpeg)

**Q.52** A rectangular block is moving along a frictionless path when it encounters the circular loop as shown. The block passes points 1, 2, 3, 4, 1 before returning to the horizontal track. At point 3 :

![](_page_21_Figure_13.jpeg)

- (A) its mechanical energy is a minimum
- (B) it is not accelerating
- (C) its speed is a minimum
- (D) it experiences a net upward force
- **Q.53** Suppose the coefficient of static friction between the road and the tires on a formula one car is 0.6 during a Grand Prix auto race. What speed will put the car on the verge of sliding as it rounds a level curve of 30.5 m radius ? (A) its mechanical energy is a minimum<br>
(C) its speed is a minimum<br>
(C) its speed is a minimum<br>
(D) it experiences a net upward force<br>
Suppose the coefficient of static friction between the<br>
coad and the tires on a formul
	- (A)  $13 \text{ m/s}$  (B)  $26 \text{ m/s}$ (C)  $5 \text{ m/s}$  (D)  $22 \text{ m/s}$
- **Q.54** A small disc in on the top of a hemisphere of radius R. What is the smallest horizontal velocity v that should be given to the disc for it to leave the hemisphere and not slide down it? [There is no friction]

(A) 
$$
v = \sqrt{2gR}
$$
  
\n(B)  $v = \sqrt{gR}$   
\n(C)  $g/R$   
\n(D)  $v = \sqrt{g^2R}$ 

**Q.55** A particle is moving along a circular path as shown in

 $\vec{v} = (4m/s)\hat{i} - (3m/s)\hat{j}$ . The particle is moving through ...... quadrant if it is travelling clockwise and through .............. quadrant if it is travelling anticlockwise, respectively around the circle –

![](_page_21_Figure_24.jpeg)

(C) First, third (D) Third, first

## **EXERCISE - 2 [LEVEL-2]**

#### **ONLY ONE OPTION IS CORRECT**

**Q.1** A particle initially at rest starts moving from point A on the surface of a fixed smooth hemisphere of radius r as shown.

![](_page_21_Figure_29.jpeg)

The particle looses its contact with hemisphere at point B. C is centre of the hemisphere. The equation relating  $\alpha$ and  $\beta$  is –

![](_page_21_Picture_425.jpeg)

**Q.2** Two same masses are tied with equal lengths of strings and are suspended at the same fixed point. One mass is suspended freely whereas another is kept in a way that string is horizontal as shown. This mass is given initial velocity u in vertical downward direction.

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![](_page_22_Picture_0.jpeg)

It strikes the freely suspended of mass elastically that is just able to complete the circular motion after  $\alpha$  $\begin{array}{ccc} 0 & L & \phantom{a} & m \\ \hline & \phantom{aa} & \phantom{aa} & \phantom{aa} \\ \end{array}$  $\overline{O}$ m the collision about point of suspension O. Magnitude of velocity u is – **EXECUTE THE EXECUTE OF THE EXECUTE OF THE EXECUTE OF THE EXECUTE OF THE ENSIGNATION BANK**<br>
The complete the circular motion after of<br>
suspension O. Magnitude of the circular motion after of<br>
suspension O. Magnitude of th **EXERCUBENDING**<br>
It strikes the freely suspended on the collision about an isolated by any and the collision about point of<br>
the collision about point of<br>
the collision about point of<br>
experiment on the collision about po

$$
(C) \sqrt{3 g L} \tag{D}
$$

- **Q.3** A ring of mass  $2\pi$  kg and of radius 0.25m is making 300rpm  $Q.8$ about an axis through its centre perpendicular to its plane. The tension (in newtons) developed in the ring is:  $(A) 50$  (B)100
	- (C) 175 (D) 250
- **Q.4** A self propelled vehicle (assume it as a point mass) runs on a track with constant speed V. It passes through three positions A, B and Con the circular part of the track. Suppose  $N_A$ ,  $N_B$  and  $N_C$  are the normal forces exerted by the track on the vehicle when it is passing through points A, B and C respectively then –

![](_page_22_Figure_8.jpeg)

![](_page_22_Figure_9.jpeg)

 $(D) N_B > N_C > N_A$ **Q.5** A small bead of mass  $m = 1$  kg is carried by a circular hoop having centre at C and radius  $r = 1$  m which rotates about a fixed vertical axis. The coefficient of friction between bead and hoop is  $\mu = 0.5$ . Find the maximum angular speed of the hoop for which the bead does not have relative motion with respect to hoop.

![](_page_22_Figure_11.jpeg)

when it just leaves the track at B.

- $(A) R/2$  (B)  $2R/3$
- $(C) R/3$  (D)  $R/4$ **Q.7** The velocity and acceleration vectors of a particle

**STUDY MATERIAL: PHYSICS**<br>
(A) R/2 (B) 2R/3<br>
(C) R/3 (D) R/4<br>
The velocity and acceleration vectors of a particle<br>
undergoing circular motion are  $\vec{v} = 2\hat{i} \text{ m/s}$  and<br>  $\vec{a} = 2\hat{i} + 4\hat{j} \text{ m/s}^2$  respectively at an undergoing circular motion are  $\vec{v} = 2\hat{i}$  m/s and **STUDY MATERIAL: PHYSICS**<br>
(A) R/2 (B) 2R/3<br>
(C) R/3 (D) R/4<br>
The velocity and acceleration vectors of a particle<br>
undergoing circular motion are  $\vec{v} = 2\hat{i}$  m/s and<br>  $\vec{a} = 2\hat{i} + 4\hat{j}$  m/s<sup>2</sup> respectively at an ins  $\vec{a} = 2\hat{i} + 4\hat{j}$  m / s<sup>2</sup> respectively at an instant of time. The

radius of the circle is –

 $(A) 1m$  (B) 2m  $(C)$  3m  $(D)$  4m

Fig shows a rod of length 20 cm. pivoted near an end which is made to rotate in a horizontal plane with a constant angular speed. A ball of mass m is suspended by a string also of

![](_page_22_Figure_18.jpeg)

length 20 cm. from the other end of the rod. If the angle  $\theta$  made by the string with the vertical is 30°, find the angular speed of rotation. Take  $g = 10 \text{ m/s}^2$ 

(C) 8.4 rad/sec. (D) 12.2 rad/sec.

(B)  $\pi/3$  rad/hr. (D)  $\pi/2$  rad/hr.

**Q.9** Two satellites  $S_1$  and  $S_2$  revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S<sub>1</sub> is  $10^4$  km. When S<sub>1</sub> is closest to  $S_2$ , the angular speed of  $S_2$  as observed by an astronaut in  $S_1$  is  $-$ 

 $(A)$  4.4 rad/sec.  $(B)$  2.4 rad/sec.

(A) 
$$
\pi
$$
 rad/hr.  
(C) 2π rad/hr.

**Q.10** A small coin of mass 40kg is placed on the horizontal surface of rotating disc. The disc starts from rest and is given a constant angular acceleration  $\alpha = 2$  rad/s<sup>2</sup>. The coefficient of static friction between the coil and the disc

![](_page_22_Figure_25.jpeg)

is  $\mu_s = 3/4$  and coefficient of kinetic friction is  $\mu_k = 0.5$ . The coin is placed at a distance  $r = 1m$  from the centre of the disc. Find the magnitude of the resultant force on the coin exerted by the disc just before it starts slipping on the disc.

- $(A) 0.1 N$  (B) 1.0 N  $(C) 0.5 N$  (D) 1.5 N
- **Q.11** A particle of mass moves along the internal smooth surface of a vertical cylinder of radius R. Find the force with which the particle acts on the cylinder wall if at the initial moment of time its velocity equals  $V_0$  and forms an angle  $\alpha$  with the horizontal. c friction is  $\mu_k = 0.5$ .<br>
Im from the centre of<br>
the resultant force on<br>
fore it starts slipping<br>
.0 N<br>
.5 N<br>
the internal smooth<br>
lius R. Find the force<br>
eylinder wall if at the<br>
equals V<sub>0</sub>. and forms<br>  $\left(\frac{mV_0^2}{2R}\$ c friction is  $\mu_k = 0.5$ .<br>
Im from the centre of<br>
ne resultant force on<br>
fore it starts slipping<br>
0N<br>
5N<br>
the internal smooth<br>
lius R. Find the force<br>
cylinder wall if at the<br>
equals V<sub>0</sub>. and forms<br>  $\left(\frac{mV_0^2}{2R}\right) \cos^$

A small coin of mass 40kg is  
\nplaced on the horizontal  
\nsurface of rotating disc.  
\nThe disc starts from rest and is  
\ngiven a constant angular  
\nacceleration 
$$
\alpha = 2
$$
 rad/s<sup>2</sup>. The  
\ncoefficient of static friction  
\nis  $\mu_s = 3/4$  and coefficient of kinetic friction is  $\mu_k = 0.5$ .  
\nThe coin is placed at a distance r = 1 m from the centre of  
\nthe disc. Find the magnitude of the resultant force on  
\nthe coin exerted by the disc just before it starts slipping  
\non the disc.  
\n(A) 0.1 N  
\n(C) 0.5 N  
\n(A) particle of mass moves along the internal smooth  
\nsurface of a vertical cylinder of radius R. Find the force  
\nwith which the particle acts on the cylinder wall if at the  
\ninitial moment of time its velocity equals  $V_0$ , and forms  
\nan angle  $\alpha$  with the horizontal.  
\n(A)  $\left(\frac{mV_0^2}{R}\right) \sin^2 \alpha$  (B)  $\left(\frac{mV_0^2}{2R}\right) \cos^2 \alpha$   
\n(C)  $\left(\frac{2mV_0^2}{R}\right) \cos^2 \alpha$  (D)  $\left(\frac{mV_0^2}{R}\right) \cos^2 \alpha$ 

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B

 $\overline{0}$ 

![](_page_23_Picture_2.jpeg)

**Q.12** A wheel of radius 0.1m (wheel A) is attached by a nonstretching belt to a wheel of radius 0.2m (wheel B). The belt does not slip. By the time wheel B turns through 1 revolution, wheel A will rotate through – **EULAR MOTION**<br>
A wheel of radius 0.1m (wheel A) is attached by a non-<br>
stretching belt to a wheel of radius 0.2m (wheel B). The<br>
belt does not slip. By the time wheel B turns through 1<br>
revolution, wheel A will rotate th

![](_page_23_Figure_4.jpeg)

- (A) 1/2 revolution (B) 1 revolution
- (C) 2 revolution (D) 4 revolution
- **Q.13** A particle is moving in a circular path. The acceleration

 $\vec{a} = 2\hat{i} + 3\hat{j}$  m/s<sup>2</sup> and  $\vec{P} = 6\hat{i} - 4\hat{j}$  kgm/s. Then the

motion of the particle is

- (A) uniform circular motion
- (B) circular motion with tangential acceleration
- (C) circular motion with tangential retardation
- (D) we cannot say anything from a and P only.
- **Q.14** Two particles A and B separated by a distance 2R are moving counter clockwise along the same circular path of radius R each with uniform speed v. At time  $t = 0$ , A is

given a tangential acceleration of magnitude  $a = \frac{72v^2}{25.2}$ 

- (A) the time lapse for the two bodies to collide is  $\frac{6pR}{5V}$
- (B) the angle covered by A is  $11\pi/6$
- (C) angular velocity of A is  $\frac{11V}{5P}$
- (D) radial acceleration of A is  $289v^2/5R$
- **Q.15** A boat is travelling with a speed of 27 kmph due east. An observer is situated at 30m south of the line of travel. The angular velocity of boat relative to the observer in the position shown will be –

![](_page_23_Figure_21.jpeg)

 $(A)$  0.125 rad/sec  $(B)$  zero (C) 0.250 rad/sec (D) 0.67 rad/sec

![](_page_23_Figure_23.jpeg)

**Q.16** The sphere at P is given a downward velocity  $v_0$  and swings in a vertical plane at the end of a rope of  $\ell = 1$ m attached to a support at O. The rope breaks at angle 30° from horizontal, knowing that it can withstand a maximum tension equal to three times the weight of the sphere. Then the value of  $v_0$  will be:  $(g = \pi^2 \text{ m/s}^2)$ 

![](_page_23_Figure_25.jpeg)

(A) 
$$
g/2
$$
 m/s (B)  $2g/3$  m/s

(C) 
$$
\sqrt{3g/2}
$$
 m/s (D) g/3 m/s

**1.** (a) is statehed by a non-<br>
(a)  $g/2$  m/s<br>
wheel of radius 0.2m (wheel B). The<br>
By the time wheel B turns through  $\begin{pmatrix} 0.17 & 0.17 & 0.17 \end{pmatrix}$ <br>
A will rotate through  $\begin{pmatrix} 0.17 & 0.17 & 0.17 \end{pmatrix}$ <br>
A will rotate t (A)  $g/2$  m/s (B)  $2g/3$  m/s<br>
(C)  $\sqrt{3g/2}$  m/s (D)  $g/3$  m/s<br>
A particle moves in a curve  $y = a \log \sec (x/a)$  in such a<br>
way that the tangent to the curve rotates uniformly with<br>
angular speed 2 rad/sec. Find resultant accele **Q.17** A particle moves in a curve  $y = a \log \sec (x/a)$  in such a way that the tangent to the curve rotates uniformly with angular speed 2 rad/sec. Find resultant acceleration of the particle when  $x = (\pi/4)$  a and a is a constant with value 1/2.

(A) 4 m/sec<sup>2</sup> (B) 2 m/sec<sup>2</sup> (A) 3 m/sec<sup>2</sup> (A) 6 m/sec<sup>2</sup>

**Q.18** For a particle moving along circular path, the radial acceleration  $a_r$  is proportional to time t. If  $a_t$  is the tangential acceleration, then which of the following will be independent of time t ?

(A) 
$$
a_t
$$
  
\n(B)  $a_r.a_t$   
\n(C)  $a_r/a_t$   
\n(D)  $a_r (a_t)^2$ 

**Q.19** A small ball of mass m starts from rest from point A (b, c) on a smooth slope which is a parabola. The normal force that the ground exerts at the instant, the ball arrives at lowest point (0, 0) is (take acceleration due to gravity g)

![](_page_23_Figure_33.jpeg)

 $\frac{11 \text{V}}{2 \text{m}}$  **Q.20** In the system shown, the mass m = 2 kg. oscillates in a 5R circular arc of amplitude 60°, the minimum value of  $(C)$  mg  $(D)$  3mg coefficient of friction between mass = 8 kg and surface of table to avoid slipping is -

![](_page_23_Figure_35.jpeg)

**Q.21** A body moving with a constant speed describes a

 $\vec{r}$  = 15 (cos pt  $\hat{i}$  + sin pt  $\hat{j}$ ) m, where p is in rad/s, and t is in second. What is its centripetal acceleration at  $t = 3s$  ?  $(A)$  45 p<sup>2</sup> m/s<sup>2</sup> (B) 5  $p^2$  m/s<sup>2</sup> (C) 15 p m/s<sup>2</sup> (D)  $15 \text{ p}^2 \text{ m/s}^2$ 

**Q.22** A particle of mass m is suspended by a string of length  $\ell$  from a fixed rigid support. A horizontal velocity

> made by the string with the vertical when the acceleration of the particle is inclined to the string by 45° ?  $(A) 45^{\circ}$  (B) 60°  $(C) 0^{\circ}$  (D) 90°

![](_page_24_Picture_0.jpeg)

**Q.23** A particle of mass 5 kg is free to slide on a smooth ring of radius  $r = 20$ cm. fixed in a vertical plane. The particle is attached to one end of a spring whose other end is fixed to the top point O of the ring. Initially the particle is at rest at a

![](_page_24_Figure_4.jpeg)

point A of the ring such that  $\angle$  OCA = 60°, C being the centre of the ring. The natural length of the spring is also equal to  $r = 20$ cm. After the particle is released and slides down the ring the contact force between the particle and the ring becomes zero when it reaches the lowest position B. Determine the force constant of the spring.

![](_page_24_Picture_577.jpeg)

**Q.24** Consider the arrangement shown in which a bob of mass m is suspended by means of a string connected to peg P. If the bob is given a horizontal velocity  $\vec{u}$  having  $\vec{v}$ STUDY MATERIAL: PHYSICS<br>
Consider the arrangement shown in which<br>
a bob of mass m is suspended by means<br>
of a string connected to peg P. If the bob  $\ell$ <br>
is given a horizontal velocity  $\vec{u}$  having<br>
magnitude  $\sqrt{3g\ell}$ **STUDY MATERIAL: PHYSICS**<br>
sider the arrangement shown in which<br>
b of mass m is suspended by means<br>
string connected to peg P. If the bob  $\ell$ <br>
wen a horizontal velocity  $\vec{u}$  having<br>
initude  $\sqrt{3g\ell}$ , find the minimu **STUDY MATERIAL: PHYSICS**<br>
e arrangement shown in which<br>
ass m is suspended by means<br>
connected to peg P. If the bob  $\ell$ <br>
orizontal velocity  $\vec{u}$  having<br>  $\sqrt{3g\ell}$ , find the minimum speed of the bob in<br>
motion.<br>
(B)

![](_page_24_Figure_8.jpeg)

 $2\,\dot{\vartheta}$ 

magnitude  $\sqrt{3gt}$ , find the minimum speed of the bob in subsequent motion.

(A) 
$$
\frac{1}{2}\sqrt{\frac{g\ell}{3}}
$$
 (B)  $\frac{1}{3}\sqrt{\frac{g\ell}{3}}$  (C)  $\frac{1}{3}\sqrt{\frac{g\ell}{2}}$  (D)  $\frac{1}{5}\sqrt{\frac{g\ell}{2}}$ 

**Q.25** A heavy particle hangs from a point O, by a string of length a, it is projected horizontally with a velocity

**STUDY MATERIAL: PHYSICS**<br>
ement shown in which<br>
suspended by means<br>
ed to peg P. If the bob  $\ell$ <br>
all velocity  $\vec{u}$  having<br>
and the minimum speed of the bob in<br>  $\frac{1}{3}\sqrt{\frac{g\ell}{3}}$  (C)  $\frac{1}{3}\sqrt{\frac{g\ell}{2}}$  (D)  $\frac{1}{5}\$ vertical, string makes where string becomes slack is –

**K**  
\n**S TUDY MATERIAL: PHYSICS**  
\nConsider the arrangement shown in which  
\na bob of mass m is suspended by means  
\nof a string connected to peg P. If the bob 
$$
\ell
$$
  
\nis given a horizontal velocity  $\vec{u}$  having  
\nmagnitude  $\sqrt{3gt}$ , find the minimum speed of the bob in  
\nsubsequent motion.  
\n(A)  $\frac{1}{2}\sqrt{\frac{gt}{3}}$  (B)  $\frac{1}{3}\sqrt{\frac{gt}{3}}$  (C)  $\frac{1}{3}\sqrt{\frac{gt}{2}}$  (D)  $\frac{1}{5}\sqrt{\frac{gt}{2}}$   
\nA heavy particle hangs from a point O, by a string of  
\nlength a, it is projected horizontally with a velocity  
\n $v = \sqrt{(2+\sqrt{3}) ag}$ . The angle with the downward  
\nvertical, string makes where string becomes slack is –  
\n(A)  $q = sin^{-1}\frac{a-1}{\xi}\frac{a}{\sqrt{3}}\frac{b}{\delta}$  (B)  $q = cos^{-1}\frac{a-1}{\xi}\frac{b}{\sqrt{3}}\frac{b}{\delta}$   
\n(C)  $q = cos^{-1}\frac{a}{\xi}\frac{1}{\sqrt{2}}\frac{b}{\delta}$  (D)  $q = sin^{-1}\frac{a}{\xi}\frac{1}{\sqrt{2}}\frac{b}{\delta}$   
\n**E BASED QUESTIONS**  
\nA turn table rotates with constant angular acceleration  
\nof 2 rad/s<sup>2</sup> about a fixed vertical axis through its centre  
\nand perpendicular to its plane. A coin is placed on it at a

 $2\dot{\vartheta}$   $\frac{8}{\sqrt{2}}\dot{\vartheta}$ 

**EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)**

#### **NOTE : The answer to each question is a NUMERICAL VALUE.**

**Q.1** A particle moves in a circle of radius  $r = 4/3$ cm. at a speed given by  $v = 2.0 t^2$  where v is in cm/s and t in seconds. Find the magnitude of the acceleration (in  $\text{cm/s}^2$ ) at

 $t = 1s$ .

**Q.2** A particle of mass  $m = 1$  kg moves in a circle of radius

R = 2m with uniform speed  $v = 3\pi$  m/s. The magnitude of impulse given by centripetal force to the particle in one  $Q.5$ 

**Q.3** A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle  $\theta_1$ . In the next 2 sec, it rotates through an additional angle  $\theta_2$ . Find the ratio of  $\theta_2/\theta_1$ .

(C) 300 N/m<br>
(D) 100 N/m<br>
(C)  $q = \cos^{-1} \frac{a}{k} - \frac{1}{3}$  (B)<br>
(C)  $q = \cos^{-1} \frac{a}{k} - \frac{1}{3}$  (B)<br>
(C)  $q = \cos^{-1} \frac{a}{k} - \frac{1}{3}$  (D)<br> **EXERCISE - 3 (NUMERICALVALU Q.4** A turn table rotates with constant angular acceleration of 2 rad/s<sup>2</sup> about a fixed vertical axis through its centre and perpendicular to its plane. A coin is placed on it at a distance of 1m from the axis of rotation. The coin is always at rest relative to the turntable. If at  $t = 0$  the turntable was at rest, then the total acceleration of the Find the second is the second is  $\sqrt{2}$  and  $\sqrt{2}$  A.

**Q.5** A railway line is taken round a circular arc of radius 1000m, and is banked by raising the outer rail h m above the inner rail. If the lateral pressure on the inner rail when a train travels round the curve at 10 m/s is equal to the lateral pressure on the outer rail when the train's speed is 20 m/s, the value of h is  $(A - 1.96)$ m (The distance between the rails is 1.5m). Find the value of A.

![](_page_25_Picture_2.jpeg)

## **EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]**

- **Q.1** The minimum velocity (in ms<sup>-1</sup>) with which a car driver  $Q.6$ must travels on a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is – **[AIEEE-2002]**  $(A) 60$  (B) 30  $(C) 15$  (D) 25
- **Q.2** Which of the following statements is FALSE for a particle moving in a circle with a constant angular speed ? Q.7

 **[AIEEE-2004]**

- (A) The velocity vector is tangent to the circle
- (B) The acceleration vector is tangent to the circle
- (C) The acceleration vector ponits to the centre of the circle
- (D) The velocity and acceleration vectors and perpendicular to each other
- **Q.3** A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that – **[AIEEE-2004]**
	- (A) Its velocity is constant
	- (B) Its acceleration is constant
	- (C) Its kinetic energy is constant
	- (D) It moves in a straight line
- **Q.4** A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length  $s = t^3 + 5$ , where s is in metres and t is in seconds. The radius of the path is<br>20 m. The acceleration of 'P' when  $t = 2$  s is nearly –  $Q.9$ 20 m. The acceleration of 'P' when  $t = 2$  s is nearly –

![](_page_25_Figure_17.jpeg)

**Q.5** For a particle in uniform circular motion the acceleration  $\vec{a}$  at a point P(R,  $\theta$ ) on the circle of radius R is (here  $\theta$  is measured from the x–axis) **[AIEEE 2010]**

(A) 
$$
-\frac{v^2}{R}\cos\theta \hat{i} + \frac{v^2}{R}\sin\theta \hat{j}
$$
 (B)  $-\frac{v^2}{R}\sin\theta \hat{i} + \frac{v^2}{R}\cos\theta \hat{j}$   
(C)  $-\frac{v^2}{R}\cos\theta \hat{i} - \frac{v^2}{R}\sin\theta \hat{j}$  (D)  $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$ 

- **Q.6** Two cars of masses  $m_1$  and  $m_2$  are moving in circles of radii  $r_1$  and  $r_2$ , respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is : **[AIEEE 2012]**  $(A)$  m<sub>1</sub>r<sub>1</sub>: m<sub>2</sub>r<sub>2</sub>  $(B) m_1 : m_2$  $(C) r_1 : r_2$  $(D) 1:1$
- **Q.7** A particle is moving along a circular path with a constant  $n - 1$ . What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

#### **[JEE MAIN 2019]**

(A) zero (B) 10 m/s (C)  $10\sqrt{3}$  m/s (D)  $10\sqrt{2}$  m/s

**Q.8** Two particles A, B are moving on two concentric circles of radii R<sub>1</sub> and R<sub>2</sub> with equal angular speed  $\omega$ . At t = 0, their positions and direction of motion are shown in the

![](_page_25_Figure_26.jpeg)

Example the cone of the moves in a straight line<br>
and P moves in a straight line<br>
are pair outer-clockwise direction on a<br>
are pair is second a length s = i<sup>3</sup> + 5, where s<br>
such that it sweeps out a length s = i<sup>3</sup> + 5, kwise direction on a<br>
re. The movement of<br>
gth s =  $t^3 + 5$ , where s<br>
exaction on a<br>
exaction of  $(A) - \omega (R_1 + R_2) \hat{i}$ <br>
(B)  $\omega (R_1 - R_2) \hat{i}$ <br>
(D)  $\omega (R_2 - R_1) \hat{i}$ <br>
(D)  $\omega (R_2 - R_1) \hat{i}$ <br>
(D)  $\omega (R_2 - R_1) \hat{i}$ <br>
(D)  $\omega (R_$ **Q.9** A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length  $\ell$ . The other end is fixed. The system is given an angular speed  $\omega$  about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is : *[JEE MAIN 2020 (JAN)]* 

 $k + m\omega$ 

(A) 
$$
\frac{m\ell\omega^2}{k + m\omega^2}
$$
 (B)  $\frac{m\ell\omega^2}{k - m\omega^2}$   
(C)  $\frac{m\ell\omega^2}{k - \omega m}$  (D)  $\frac{m\ell\omega^2}{k + m\omega}$ 

$$
301 \boxed{\phantom{0}}
$$

![](_page_26_Picture_2.jpeg)

## **EXERCISE - 5 (PREVIOUS YEARS AIPMT / NEET QUESTIONS)**

- **Q.1** A particle moves in a circle of radius 5cm with constant Q.8 speed and time period  $0.2\pi s$ . The acceleration of the particle is **[AIPMT (PRE) 2011]**  $(A) 5 m/s<sup>2</sup>$ (B)  $15 \text{ m/s}^2$  $(C)$  25 m/s<sup>2</sup> (D)  $36 \text{ m/s}^2$
- **Q.2** A car of mass 1000 kg negotiates a banked curve of radius 90 m on a frictionless road. If the banking angle is 45°, the speed of the car is : **[AIPMT (PRE) 2012]**  $(A) 20 \text{ ms}^{-1}$  (B) 30 ms<sup>-1</sup> (C)  $5 \text{ ms}^{-1}$  (D)  $10 \text{ ms}^{-1}$
- **Q.3** A car of mass m is moving on a level circular track of **O.9** radius R. If  $\mu_s$  represents the static friction between the road and tyres of the car, the maximum speed of the car in circular motion is given by – **[AIPMT (MAINS) 2012]**

(A) 
$$
\sqrt{\mu_s m R g}
$$
 (B)  $\sqrt{R g / \mu_s}$  towards centre  
\n(C)  $\sqrt{m R g / \mu_s}$  (D)  $\sqrt{\mu_s R g}$  string)

- **EXERCISE 5 (PREVIOUS YEARS AIPMIT /NEET QUESTION BANK**<br>
EXERCISE 5 (PREVIOUS YEARS AIPMIT /NEET QUEST<br>
A particle moves in a circle of radius 5cm with constant Q.8 In the given figure,<br>
speed and time period 0.2rs. T **Q.4** Two stones of masses m and 2 m are whirled in horizontal circles, the heavier one in a radius r/2 and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they  $0.10$ experience same centripetal forces. The value of n is : (A) 1 (B) 2 **[RE-AIPMT 2015]** Fig. represents the state include the main speed of the car<br>
rects of the car, the maximum speed of the car<br>
notion is given by - [AIPMT (MAINS) 2012]<br>
(B)  $\sqrt{Rg/k_s}$ <br>
(D)  $\sqrt{\mu_s}Rg$ <br>
(D)  $\sqrt{\mu_s}Rg$ <br>
(B)  $\sqrt{\mu_s}$ <br>
(D)  $\sqrt$  $t_s mRg$  (B)  $\sqrt{R_g R/\mu_s}$  towards centre) will be - (T rep<br>
imag /  $\mu_s$  (D)  $\sqrt{\mu_s R_g}$  string)<br>
ones of masses m and 2 m are whirled in horizontal<br>
us r. The tangential speed of lighter stone is n<br>
us r. The tangential sp tyres of the car, the maximum speed of the car<br>
motion is given by - [AIPMT(MAINS) 2012]<br>
The constrained in a smooth horizontal table. If<br>  $\frac{1}{R_g}$  (B)  $\sqrt{R_g/k_s}$ <br>
(B)  $\sqrt{R_g/k_s}$ <br>
So framsses m and 2 m are whiteled in motion is given by - [AIPMT (MAINS) 2012]<br>  $\frac{1}{\text{Reg}}$  (B)  $\sqrt{\text{Rg}/\text{H}_s}$  (D)  $\sqrt{\text{Rg}/\text{Rg}}$  string)<br>
terele with speed V the net force or<br>
breadvict one in ancidus 1/2 and the lighter one is towards centre) will be
	- $(C)$  3 (D) 4
- **Q.5** A car is negotiating a curved road of radius R. The road is banked at an angle  $\theta$ . The coefficient of friction between the tyres of the car and the road is  $\mu_s$ . The maximum safe velocity on this road is

**[NEET 2016 PHASE 1]**

(A) 
$$
\sqrt{gR^2 \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}}
$$
  
\n(B)  $\sqrt{gR \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}}$   
\n(C)  $\sqrt{\frac{g}{R} \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}}$   
\n(D)  $\sqrt{\frac{g}{R^2} \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}}$ 

**Q.6** A particle of mass 10 g moves along a circle of radius<br>6.4cm with a constant tongential acceleration. What is Q.12 6.4cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to  $8 \times 10^{-4}$  J by the end of the second revolution after the beginning of the motion? **[NEET 2016 PHASE 1]**

(A) 0.1 m/s<sup>2</sup> (B) 0.15 m/s<sup>2</sup> (C) 0.18 m/s<sup>2</sup> (D) 0.2 m/s<sup>2</sup>

- **Q.7** What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop? **[NEET 2016 PHASE 1]**
	- (A)  $\sqrt{gR}$  (B)  $\sqrt{2gR}$  (C) 10 rad/s
	- (C)  $\sqrt{3gR}$  (D)  $\sqrt{5gR}$

In the given figure,  $a = 15$  m/s<sup>2</sup> represents the total acceleration of a particle moving in the clockwise direction in a circle of radius

![](_page_26_Figure_19.jpeg)

 $R = 2.5$  m at a given instant of time. Speed of the particle is **[NEET 2016 PHASE 2]**  $(A)$  4.5 m/s (B) 5.0 m/s (C)  $5.7 \text{ m/s}$  (D)  $6.2 \text{ m/s}$ 

**EXERCISE - 5 (PREVIOUS VEARS AIPMIT /NEET QUESTIONS)**<br>
EXERCISE - 5 (PREVIOUS VEARS AIPMIT /NEET QUESTIONS)<br>
A particle moves in a circle of radius 5 cm with constant Q8 In the given figure,<br>
particle is<br>
(a)  $15 \text{ m/s}^2$ (D) 10 ms<sup>-1</sup><br>
(C) 5.7 m/s<br>
(C) 5.7 m/s<br>
(C) 5.7 m/s<br>
(C) 6.6<br>
in grepresents the static friction between the<br>
or of mass m and the other cal socon<br>
or of mass mann and the other cal soconal<br>
circle with speed v<sup>1</sup> the ne ss m is moving on a level circular track of<br>
Level circular track of<br>
Level come of string of length t is cor<br>
es of the car, the maximum speed of the car<br>
of mass m' and the other end is con<br>
of mass m' and the other end Ethion between the same of mass im' and the other end is connected to a small peg<br>
in speed of the car<br>
(MAINS) 2012] on a smooth horizontal table. If the particle moves in<br>  $\frac{\overline{R}}{R_g}$ <br>  $\frac{R_g}{R_g}$ <br>  $\frac{R_g}{R_g}$ <br>  $\frac{R$ <sup>5-1</sup><br>
circular track of<br>
C(5) 5.7 m/s<br>
circular track of<br> **0.9** One end of string of length *l* is connected to a particle<br>
or an smooth horizontal table. If the particle moves in<br>
circle with speed v'the net force on t circular track of (b) One end of string of length  $\ell$  is connected to a particle<br>since do fthe carrier of mass in and the other end is connected to a small peg<br>speed of the carrier or an smooth horizontal table. If the p One end of string of length  $\ell$  is connected to a particle of mass 'm' and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed 'v' the net force on the particle (directed towards centre) will be – (T represents the tension in the string) **[NEET 2017]**

(A) 
$$
T + \frac{mv^2}{\ell}
$$
 (B)  $T - \frac{mv^2}{\ell}$   
(C) Zero (D) T

**Q.10** A mass m is attached to a thin wire and whirled in a vertical circle. The wire is most likely to break when:

**[NEET 2019]**

(A) the mass is at the highest point.

(B) the wire is horizontal.

(C) the mass is at the lowest point.

(D) inclined at an angle of 60° from vertical.

 $\frac{\tan \theta}{\tan \theta}$  motion in concentric circles of radii  $r_A$  and  $r_B$  with  $gR \frac{F_S - \tan \theta}{1 - \mu_s \tan \theta}$  speed v<sub>A</sub> and v<sub>B</sub> respectively. Their time period of Rg / $\mu_s$  towards centre) will be – (T represents the tension in the<br>
string)<br>
whirded in borizontal<br>
The value of  $\mu_s$  of the lighter one<br>
of lighter stone is n<br>
(C) Zero<br>
(D) T<br>
or stone when they<br>
(C) Zero<br>
(D) T<br>
are speed of the car<br>
on a smooth horizontal table. If the particle moves in<br>
(MAINS) 2012]<br>
in a smooth horizontal table. If the particle moves in<br>
eircle with speed V the net force on the particle (directed<br>
towards centre) **MAINS) 2012**<br>
and announced with speed V the net force on the particle (directed<br>
towards centre) will be – (T represents the tension in the<br>
the lighter one<br>
the lighter one<br>
string)<br>
(A) T +  $\frac{mv^2}{l}$ <br>
(B) T =  $\frac{mv^$ Two particles A and B are moving in uniform circular rotation is the same. The ratio of angular speed of A to that of B will be : **[NEET 2019]**

$$
\frac{g}{c^2} \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}
$$
\n(A)  $r_A : r_B$ \n(B)  $v_A : v_B$ \n(C)  $r_B : r_A$ \n(D) 1 : 1

**Q.12** A block of mass 10 kg is in contact against the inner wall of a hollow cylindrical drum of radius 1m. The coefficient of friction between the block and the inner wall of the cylinder is 0.1. The minimum angular velocity needed for the cylinder to keep the block stationary when the cylinder is vertical and rotating about its axis, will be :  $(g = 10 \text{ m/s}^2)$ ) **[NEET 2019]**

(A) 
$$
\sqrt{10}
$$
 rad/s  
\n(B)  $\frac{10}{2\pi}$  rad/s  
\n(C) 10 rad/s  
\n(D)  $10\pi$  rad/s

![](_page_27_Picture_2.jpeg)

## **ANSWER KEY**

![](_page_27_Picture_510.jpeg)

![](_page_27_Picture_511.jpeg)

## **Q 1 2 3 4 5 A** | 5 | 3 | 3 | 5 | 2 **EXERCISE - 3**

![](_page_27_Picture_512.jpeg)

![](_page_27_Picture_513.jpeg)

![](_page_28_Picture_2.jpeg)

# **CAR MOTION TRY IT YOURSELF-1 EXECULAR MOTION**<br> **EXECULAR MOTION** or  $\theta = \omega_0$ <br> **EXECULAR MOTION** or  $\theta = \omega_1$ <br>
elocity changes as its direction change.<br>  $\omega = \text{constant}$   $\therefore \alpha = 0 = \text{constant}$ <br>  $\omega = 2\pi RT$  and its acceleration towards the earth is<br>  $\omega = 2\pi RT$  an **CIRCULAR MOTION**<br> **CIRCULAR MOTION**<br> **CIRCULAR MOTION**<br>
Velocity changes as its direction change.<br>
Velocity changes as its direction change.<br>
Acceleration changes as its direction change.<br>
Acceleration changes as its dir **CIRCULAR MOTION**<br> **CIRCULAR MOTION**<br> **CIRCULAR MOTION** or  $\theta = \omega_0 t + \frac{at^5}{5} - \frac{b^5}{4}$ <br>
CONCIVIS CONTIGNS<br>
CONCIVIS changes as its direction change.<br>
Acceleration changes as its direction change.<br>
(8) (a) Since the acc

- **(1) (D).** Velocity changes as its direction change. Acceleration changes as its direction change.
- **(2) (BC).**  $\omega$  = constant  $\therefore \alpha$  = 0 = constant
- **(3)** Let the moon orbit the earth in time T at a radius R. Its speed is then  $v = 2\pi R/T$  and its acceleration towards the earth is

$$
a = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}
$$

Put in the given numbers to find an answer which is roughly 1/3600 times g.

#### **(4) (40/7) sec.**

 $\omega_0^2$  = 900 (rad/sec)<sup>2</sup>  $\Rightarrow \omega_0$  = 30 rad/sec  $\omega^2$  = 1600 (rad/sec)<sup>2</sup>  $\Rightarrow \omega$  = 40 rad/sec

$$
\theta = \left(\frac{\omega + \omega_0}{2}\right)t \Rightarrow t = \frac{2 \times 100 \times 2\pi}{40 + 30} = \frac{40\pi}{7} \sec.
$$

**(5)** (a) Converting the angular speed  $\omega$  to from rev/s, we obtain,

$$
\omega = \left(6.50 \frac{\text{rev}}{\text{s}}\right) \left(\frac{2\pi \text{rad}}{1 \text{ rev}}\right) = 40.8 \frac{\text{rad}}{\text{s}} \quad \therefore
$$

The tangential speed of each point is

**Point 1 :**  $v_T = r\omega = (3.00 \text{m}) (40.8 \text{ rad/s}) = 122 \text{ m/s} (273 \text{ mph})$ **Point 2 :**  $v_T = r\omega = (6.70 \text{ m}) 40.8 \text{ rad/s} = 273 \text{ m/s} (611 \text{ mph})$ The radian unit, being dimensionless, does not appear in the final answers.

(b) Converting the angular acceleration  $\alpha$  to rad/s<sup>2</sup> from

rev/s<sup>2</sup>, we find 
$$
\alpha = \left(1.30 \frac{\text{rev}}{\text{s}^2}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 8.17 \frac{\text{rad}}{\text{s}^2}
$$
  $\therefore$  v = 3 sec<sup>2</sup> θ

The tangential acceleration can now be determined : **Point 1 :**  $a_T = r\alpha = (3.00 \text{ m})(8.17 \text{ rad/s}^2) = 24.5 \text{ m/s}^2$ ; **Point 2 :**  $a_T = r\alpha = (6.70 \text{ m}) (8.17 \text{ rad/s}^2) = 54.7 \text{ m/s}^2$ and  $\alpha = (\frac{\pi}{2})^2 + (\frac{1}{2})^2 = 0.1775^2$ <br>
and acceleration can now be determined :<br>  $\frac{\pi}{2} = \pi \alpha = (3.00 \text{ m})(8.17 \text{ rad/s}^2) = 24.5 \text{ m/s}^2$ ;<br>  $\pi \alpha = (6.70 \text{ m})(8.17 \text{ rad/s}^2) = 54.7 \text{ m/s}^2$ <br>  $\therefore \text{ At the ins}$ <br>  $\frac{\pi}{2} = \pi \alpha = (3.00 \text{ m})($ = τα = (3.00 m) (8.17 rad/s<sup>2</sup>) = 24.5 m/s<sup>2</sup>;<br>
ετα = (6.70 m) (8.17 rad/s<sup>2</sup>) = 54.7 m/s<sup>2</sup>;<br>
, we need to find ω. Converting 3700 rev min<sup>-1</sup> (10) (a) θ =  $\left(\frac{\omega + i}{2}\right)$ <br>
, we need to find ω. Converting 3700 rev min<sup></sup>

**(6)** To use  $v = r\omega$ , we need to find  $\omega$ . Converting 3700 rev min<sup>-1</sup> to rad  $s^{-1}$ .

$$
\omega = 3700 \frac{\text{rev}}{\text{min}} \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) \left( \frac{1 \text{ min}}{60 \text{s}} \right) = 387 \text{ rad s}^{-1}
$$
\nthe velocity at the blade tip is

\n
$$
\omega = (0.25 \text{m}) (387 \text{ rad s}^{-1}) = 97 \text{ ms}^{-1},
$$
\nthe is nearly 350 km h<sup>-1</sup>.

\nWe know,  $d\omega = \alpha dt$ 

\nIntegrating both sides, we get

\n
$$
\int_{\omega_0}^{\omega} d\omega = \int_{0}^{t} \alpha dt = \int_{0}^{t} \left( 4at^3 - 3bt^2 \right) dt
$$
\n
$$
\omega = \omega_0 + at^4 - bt^3
$$
\nWe know,  $d\theta = \omega dt$ 

\nOn integrating both the sides, we get

\n
$$
\int_{0}^{t} d\theta = \int_{0}^{t} \omega dt = \int_{0}^{t} \left( \omega_0 + at^4 - bt^3 \right) dt
$$
\n(2) (a) The tang  $a_t = \alpha r = t$ 

\n304

Then the velocity at the blade tip is  $v = r\omega$  = (0.25m) (387 rad s<sup>-1</sup>) = 97 ms<sup>-1</sup>, , which is nearly 350 km  $h^{-1}$ .

**(7)** (i) We know,  $d\omega = \alpha dt$ Integrating both sides, we get

In the velocity at the blade tip is  
\nω= (0.25m) (387 rad s<sup>-1</sup>) = 97 ms<sup>-1</sup>,  
\nthe known, do = αdt  
\nIntegrating both sides, we get  
\n
$$
\int_{\omega_0}^{\omega_0} d\omega = \int_0^t \alpha dt = \int_0^t (4at^3 - 3bt^2) dt
$$
\n
$$
\omega = \omega_0 + at^4 - bt^3
$$
\nWe know, dθ = ω dt  
\nOn integrating both the sides, we get  
\n
$$
\int_0^{\omega_0} d\theta = \int_0^t \omega dt = \int_0^t (\omega_0 + at^4 - bt^3) dt
$$
\n
$$
\omega = \sqrt{a_t^2 + a_c^2}
$$
\n
$$
\int_0^t d\theta = \int_0^t \omega dt = \int_0^t (\omega_0 + at^4 - bt^3) dt
$$
\n(2) (a) The tangential  
\n
$$
a_t = \alpha r = (60 \text{ rad})
$$

or  $\omega = \omega_0 + at^4 - bt^3$ 

(ii) We know,  $d\theta = \omega dt$ On integrating both the sides, we get

$$
\int_{0}^{\theta} d\theta = \int_{0}^{t} \omega dt = \int_{0}^{t} \left( \omega_0 + at^4 - bt^3 \right) dt
$$

or 
$$
\theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}
$$

**(8)** (a) Since the acceleration is uniform,

STUDY MATERIAL: PHYSICS  
\n
$$
\theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}
$$
  
\nSince the acceleration is uniform,  
\n
$$
\alpha = \frac{\Delta \omega}{\Delta t} = \frac{387 \text{ rad s}^{-1}}{10 \text{s}} = 38.7 \text{ rad s}^{-2}
$$
  
\nThe tangential acceleration component is

5 4 <sup>1</sup> 387 rad s t 10s (b) The tangential acceleration component is a<sup>T</sup> = × r = (38.7 rad s–2) (0.25 m) = 9.68 m s–2

(9) 
$$
\omega = \frac{v \cos \theta}{r} \Rightarrow v = \frac{r \omega}{\cos \theta}
$$
, where  $r = \frac{3}{\cos \theta}$ 

![](_page_28_Figure_32.jpeg)

$$
V = 900 \text{ (rad/sec)}^2 \Rightarrow \omega_0 = 30 \text{ rad/sec}
$$
\n
$$
1600 \text{ (rad/sec)}^2 \Rightarrow \omega_0 = 30 \text{ rad/sec}
$$
\n
$$
\frac{(\omega + \omega_0)}{\omega} \text{[to]} = t = \frac{2 \times 100 \times 2}{40 + 30} = \frac{40\pi}{7} \text{ sec.}
$$
\n
$$
V = \frac{3\omega}{\cos^2 \theta}
$$
\n
$$
= \left(6.50 \frac{\text{rev}}{\text{s}}\right) \left(\frac{2\pi \text{rad}}{1 \text{ rev}}\right) = 40.8 \frac{\text{rad}}{\text{s}}
$$
\n
$$
= 1.1 \times r_T = r\omega = (3.00m) (40.8 \text{ rad/s}) = 122 \text{ m/s} (273 \text{ mph})
$$
\n
$$
= 1.1 \times r_T = r\omega = (3.00m) (40.8 \text{ rad/s}) = 122 \text{ m/s} (273 \text{ mph})
$$
\n
$$
= 1.1 \times r_T = r\omega = (3.00m) (40.8 \text{ rad/s}) = 273 \text{ m/s} (611 \text{ mph})
$$
\n
$$
= 1.1 \times r_T = r\omega = (6.70 \text{ m}) (40.8 \text{ rad/s}) = 273 \text{ m/s} (611 \text{ mph})
$$
\n
$$
= 1.1 \times r_T = r\omega = (6.70 \text{ m}) (8.17 \text{ rad/s}^2) = 24 \text{ m/s} (11 \text{ mph})
$$
\n
$$
= 8.17 \frac{\text{at}}{\text{at}} \text{at} \text{as } \text{to } \text{v} = 3 \text{ sec}^2 \theta \cdot \frac{d\theta}{dt} = 3\omega \text{sec}^2 \theta
$$
\n
$$
= 4.1 \times r_T = r\omega = (6.70 \text{ m}) (8.17 \text{ rad/s}^2) = 9.45 \text{ m/s}^2.
$$
\nAt the instant shown,  $v = 3 \times 0.1 \times (\sqrt{2})^2 = 0.6 \text{ m/s}$ \n
$$
= 1.2 \text{ are } r
$$

$$
\therefore \quad v = 3 \sec^2 \theta \cdot \frac{d\theta}{dt} = 3\omega \sec^2 \theta
$$

 $\therefore$  At the instant shown,  $v = 3 \times 0.1 \times (\sqrt{2})^2 = 0.6$  m/s

(10) (a) 
$$
\theta = \left(\frac{\omega + \omega_0}{2}\right) t = \left(\frac{100 + 0}{2}\right) \times 5 \times 60 = 15000 \text{ revol.}
$$

(b) 
$$
\omega = \omega_0 + \alpha t \Rightarrow 0 = 100 - \alpha (5 \times 60) \Rightarrow \alpha = \frac{1}{3} \text{ rev/sec}^2
$$

(c) 
$$
\omega_{\text{av}} = \frac{\text{Total angle of rotation}}{\text{Total time taken}} = \frac{15000}{50 \times 60} = 50 \text{ rev/sec.}
$$

## **TRY IT YOURSELF-2**

**(1)** (a) Tangential acceleration

$$
a_t = \frac{dv}{dt}
$$
 or  $a_t = \frac{d}{dt}(4t) = 4 \text{ cm/s}^2$ 

and answers.  
\nConverting the angular acceleration α to rad/s<sup>2</sup> from  
\n2, we find α = 
$$
\left(1.30 \frac{rev}{s^2}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 8.17 \frac{\text{rad}}{s^2}
$$
  
\n $\frac{1}{2} \text{ cm} = 8.17 \frac{\text{rad}}{s^2}$   
\n $\frac{1}{2} \text{ cm} = 1.30 \text{ cm}$  (8.17 rad/s<sup>2</sup>) = 24.5 m/s<sup>2</sup>;  
\n $1 \text{ cm} = \text{ cm} = (6.70 \text{ m}) (8.17 \text{ rad/s}^2) = 54.7 \text{ m/s}^2$   
\n $\frac{1}{2} \text{ cm} = \text{ cm} = (6.70 \text{ m}) (8.17 \text{ rad/s}^2) = 54.7 \text{ m/s}^2$   
\n $\frac{1}{2} \text{ cm} = \text{ cm} = (6.70 \text{ m}) (8.17 \text{ rad/s}^2) = 54.7 \text{ m/s}^2$   
\n $\frac{1}{2} \text{ cm} = \text{ cm} = (6.70 \text{ m}) (8.17 \text{ rad/s}^2) = 54.7 \text{ m/s}^2$   
\n $\frac{1}{2} \text{ cm} = 3700 \text{ rev}$   
\n $\frac{1}{2} \text{ cm} = 1.300 \text{ rev}$   
\n $\frac$ 

 $(\omega_0 + at' - bt')dt$ <br>  $a_t = \alpha r = (60 \text{ rad/s}^2)(0.2 \text{ m}) = 12 \text{ m/s}^2$ **(2) (a)** The tangential acceleration is constant and given by

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![](_page_29_Picture_2.jpeg)

In order to calculate the radial acceleration we first need to find the angular velocity at the given time

 $\omega = \omega_0 + \alpha t = 0 + (60 \text{ rad/s}^2) (0.15 \text{ s}) = 9 \text{ rad/s}$ 

$$
a_r = \omega^2 r = (81 \text{ rad}^2/\text{s}^2)(0.2 \text{ m}) = 16.2 \text{ m/s}^2
$$

The magnitude of the net linear acceleration is,

$$
a = \sqrt{a_r^2 + a_t^2} = 20.2 \text{ m/s}
$$

**b)** 
$$
\theta = 0 + \frac{1}{2} \alpha t^2 = \frac{1}{2} (60 \text{ rad/s}^2) (0.25 \text{s})^2 = 1.88 \text{ rad}
$$

This corresponds to (1.88 rad) ( 1 rev/ $2\pi$  rad) = 0.3 rev. **(3) (C).**  $|v| = constant$ 

**(4) (B).** The magnitude of the centripetal acceleration for a car undergoing circular motion is given by  $a = v^2/R$ . Therefore the speed of the car is

$$
v = \sqrt{aR} = \sqrt{(2ms^{-2})(200m)} = 20 m/s
$$

- **(5) (D).** The object always has a component of the acceleration pointing inward. When it is speeding up, it has a component of the tangential acceleration in the direction of motion (counterclockwise). The vector sum of these two components points somewhere between the arrow 1 and 2.
- **(6) (C).** At the point P the acceleration has a positive tangential component so it is speeding up. At the point S the acceleration has a zero tangential component so it is moving at a constant speed. At the Point R the acceleration has a negative tangential component so it is slowing down.
- **(7) (B).** Since the suspended object is not accelerating the tension force upwards exactly balances the gravitational force downwards, so  $T = mg$ . The inward force on the puck is due to the tension in the string which is equal in to the mass times the centripetal acceleration,  $T = ma_{centripetal}$ Therefore, mg = ma<sub>centripetal</sub> or g = a<sub>centripetal</sub>  $\cdot$  **(8)** You are spinning on a merry-go-round at a constant distance
- from the center. Your velocity is given by your distance divided by the time it takes you to travel that distance. In a time 1/f seconds you make one full revolution, which is a distance of  $2\pi R$ . Your acceleration is given by ension in es surray when is equal in to the exertified acceleration,  $T = ma_{\text{centripetal}}$  and  $a_{\text{r}} = \omega^2 r = (2 \text{ rad/s})^2 \times 1 \text{ m} = 4 \text{ m/s}^2$ <br>
in the centripetal acceleration,  $T = ma_{\text{centripetal}}$   $\therefore$   $a = \sqrt{a_{\text{T}}^2 + a_{\text{r}}^2} = 2\sqrt{5}$ Is, so T = mg. The inward force on the puck<br>
so T = mg. The inward force on the puck<br>  $a_T = \alpha T = (2 \text{ rad/s}^2 \times 1 \text{ m} = 2 \text{ m/s}^{-1})$ <br>
so in the string which is equal in to the<br>  $a_T = \alpha^2 T = (2 \text{ rad/s})^2 \times 1 \text{ m} = 4 \text{ m/s}^2$ <br>
or a me

 $a = v^2/R = (2\pi Rf)^2/(R)$ . The force of friction is  $F_f \le \mu_s mg$ . To find the minimum  $\mu_s$  to keep you from falling off, you (1) want the force due to friction that will just balance your centripetal acceleration. ou to travel that distance. In a<br>
more full revolution, which is a<br>
core of friction is F<sub>t</sub> ≤ μ<sub>s</sub>mg.<br>
core of friction is F<sub>t</sub> ≤ μ<sub>s</sub>mg.<br>
ceep you from falling off, you (1) Velocity, v = 36 km/h =  $\frac{36 \times 1000}{3600}$ <br>

$$
F = mR (2\pi f)^{2} = \mu_{s}mg \Rightarrow \mu_{s} = \frac{R (2\pi f)^{2}}{g}
$$

(9) Speed, 
$$
v = 27 \text{ km/h} = 27 \times \frac{5}{18} \text{ ms}^{-1} = 7.5 \text{ ms}^{-1}
$$

Centripetal acceleration,

$$
a_c = \frac{v^2}{r}
$$
 or  $a_c = \frac{(7.5)^2}{80}$  ms<sup>-2</sup> = 0.7 ms<sup>-2</sup>

![](_page_29_Figure_21.jpeg)

P is the point at which cyclist applies brakes. At this point,

tangential acceleration  $a_t$ , being negative, will act opposite

to 
$$
\vec{v}
$$
. Total acceleration,  $a = \sqrt{a_c^2 + a_t^2}$ 

1 acceleration, 
$$
a = \sqrt{a_c^2 + a_t^2}
$$
  
\n
$$
a = \sqrt{(0.7)^2 + (0.5)^2} \text{ ms}^{-2} = 0.86 \text{ ms}^{-2}
$$
\n
$$
\tan \theta = \frac{a_c}{a_t} = \frac{0.7}{0.5} = 1.4 \qquad \therefore \quad \theta = 54^{\circ}28'
$$
\nond angular velocity of the turntable and hence\n
$$
\text{coin about the axis of rotation is}
$$
\n
$$
+ 2(\text{rad/s}^2) \times 1 \text{ s} = 2 \text{ rad/s}
$$
\n
$$
\text{ar} = (2 \text{ rad/s}^2) \times 1 \text{ m} = 2 \text{ m/s}^2
$$
\n
$$
\text{r} = (2 \text{ rad/s})^2 \times 1 \text{ m} = 4 \text{ m/s}^2
$$
\n
$$
\frac{a}{a} = \frac{2\sqrt{5} \text{ m/s}^2}{\text{ s}^2}
$$

**(10)** After 1 second angular velocity of the turntable and hence that of the coin about the axis of rotation is

<sup>s</sup> R (2 f ) <sup>5</sup> 1 1 27 ms 7.5ms <sup>T</sup> a r (2 rad / s ) 1m 2m / s a r = <sup>2</sup> r = (2 rad/s)<sup>2</sup> × 1 m = 4 m/s<sup>2</sup> 2 2 2 T r a a a 2 5m / s 36 1000 r = 1000 m; tan <sup>=</sup> <sup>2</sup> v 10 10 1 rg 1000 9.8 98 tan

## **TRY IT YOURSELF-3**

**(1)** Velocity,  $v = 36$  km/h =  $\frac{36 \times 1000}{3600}$  m/s = 10 m/s radius,

$$
r = 1000 \text{ m}; \tan \theta = \frac{v^2}{rg} = \frac{10 \times 10}{1000 \times 9.8} = \frac{1}{98}
$$

Let h be the height through which outer rail is raised. Let  $\ell$  be the distance between the two rails.

Then, 
$$
\tan \theta = \frac{h}{\ell}
$$
  $[\because \theta \text{ is very small}]$ 

or 
$$
h = \ell \tan \theta = 1.5 \times \frac{1}{98} \text{ m} = 0.0153 \text{ m}
$$
 [ $\because \ell = 1.5 \text{ m}$ ]

![](_page_30_Picture_0.jpeg)

(2) Speed, v = 720 km/h = 
$$
\frac{720 \times 1000}{3600}
$$
 m/s = 200 m/s (a) N<sub>1</sub> =  $\frac{mv^2}{R}$  =  $\frac{1 \times 100}{10 \times 50}$  = 0.2 N  
\nand tan θ = tan 15° = 0.2679  
\ntan θ =  $\frac{v^2}{rg}$  or r =  $\frac{v^2}{g \tan \theta}$  =  $\frac{200 \times 200}{9.8 \times 0.2679}$  m  
\n= 15235.7 m = 15.24 km.  
\n(3) (i) 0°, (ii) cos<sup>-1</sup> (1/ $\sqrt{3}$ ), (iii) 90°  
\n(i) At α = 0°  
\n $a_{net} = \frac{v^2}{R}$  T  
\nAcceleration is vertically up  
\n(ii) At α = 90° is at B  
\n $N = \frac{mv^2}{R} cos θ$  ...... (i)

Acceleration is vertically down.

mg

(iii) Horizontally

$$
\tan \alpha = \frac{v^2 / R}{g \sin \alpha} \Rightarrow g \sin \alpha \cdot \tan \alpha = \frac{v^2}{R}
$$
 .........(1)  
(5)  $r = 30m, m = 10^6 \text{ kg}, \theta = ?$ 

![](_page_30_Figure_7.jpeg)

Using energy conservation :

$$
\frac{1}{2}mv^2 = mgR\cos\alpha \qquad \qquad \dots \dots \dots \dots (2)
$$

By eq.  $(1)$  and  $(2)$ ,

$$
\tan \alpha = \frac{1}{\sqrt{2}} \quad \therefore \quad \cos \alpha = \frac{1}{\sqrt{3}} \quad \therefore \quad \alpha = \cos^{-1} \left( \frac{1}{\sqrt{3}} \right) \qquad \qquad \textbf{(7)} \qquad \text{iv.}
$$

**(4) (a) 0.2 N, (b) 30°**

![](_page_30_Figure_13.jpeg)

$$
\tan \theta = \mu = \frac{\sqrt{3}}{3} = \frac{0.58}{0.58} = 1.724 \Rightarrow \theta = 3
$$

$$
v = 54 \text{kmh}^{-1} = \frac{54 \times 1000}{3600} = \text{ms}^{-1} = 15 \text{ms}^{-1}
$$

**(i)** The centripetal force is provided by the lateral thrust by the outer rail on the flanges of the wheel of the train. The train causes an equal and opposite thrust on the outer rail (Newton's third law of motion). Thus, the outer rail wears out faster. slipping,  $\mu N = \frac{N}{R} \sin \theta$  .......(2)<br>
(1) & (2),<br>  $\theta = \mu = \frac{1}{\sqrt{3}} = \frac{1}{0.58} = 1.724 \Rightarrow \theta = 30^{\circ}$ <br>  $\frac{10^6 \text{ kg}, \theta = ?}{3600} = \text{ms}^{-1} = 15 \text{ms}^{-1}$ <br>
ipetal force is provided by the lateral thrust by<br>
1 on the flanges of t

$$
u = \frac{v^2}{g \sin \alpha}
$$
   
\n
$$
u = \frac{v}{g \sin \alpha}
$$
   
\n
$$
u = \frac{v}{g}
$$
   
\n
$$
u = \frac{v}{g}
$$
   
\n
$$
u = 54 \text{ km/h}^{-1} = \frac{54 \times 1000}{3600} = \text{ms}^{-1} = 15 \text{ ms}^{-1}
$$
  
\n
$$
u = 54 \text{ km/h}^{-1} = \frac{54 \times 1000}{3600} = \text{ms}^{-1} = 15 \text{ ms}^{-1}
$$
  
\n
$$
u = 54 \text{ km/h}^{-1} = \frac{54 \times 1000}{3600} = \text{ms}^{-1} = 15 \text{ ms}^{-1}
$$
  
\n
$$
u = \frac{1}{\alpha}
$$
   
\n
$$
u = \frac{v}{\alpha}
$$
   
\n
$$
u = \frac{1}{\alpha}
$$
   
\n
$$
u = \frac{1}{\sqrt{2}}
$$
   
\n
$$
u = \frac{1}{
$$

**(6)** (C)  $(7)$ 

![](_page_31_Picture_2.jpeg)

![](_page_31_Figure_3.jpeg)

$$
N \sin \theta = m\omega^2 R
$$
;  $N \cos \theta = mg$  :  $\tan \theta = \frac{\omega^2 R}{g}$ 

$$
\Rightarrow \frac{R}{r-h} = \frac{\omega^2 R}{g} \Rightarrow \omega^2 = \frac{g}{r-h} \qquad \dots (1)
$$

(a) h > 0  $\Rightarrow$  r  $-\frac{6}{\omega^2}$  > 0  $g \qquad \qquad \overline{g}$  $\omega^2$  (b) 4

$$
\Rightarrow \quad \omega > \sqrt{\frac{g}{r}} = \sqrt{\frac{9.8}{0.1}} = \sqrt{98} = 7\sqrt{2} \frac{\text{rad}}{\text{second}}
$$

(b) 
$$
g = \omega^2 (r - h) \Rightarrow \frac{\Delta g}{g} = -\frac{\Delta h}{h} = -\frac{10^{-4}}{h}
$$

**(10) (a)** 
$$
K = \frac{mg}{R(3 - 2\sqrt{2})}
$$

- **(b)** At initial instant  $a_t = g$ ,  $a_c = 0$ At bottom position  $a_t = 0$ ,  $a_c = 0$
- 2R (a) Applying conservation of energy between initial and  $\frac{2R}{\text{final position is}}$ .  $\tan \theta = \frac{\omega^2 R}{rR}$  final position is : **EXECUTIONS**<br>
(10) (a)  $K = \frac{mg}{R(3-2\sqrt{2})}$ <br>
(b) At initial instant  $a_t = g$ ,  $a_c = 0$ <br>
At bottom position  $a_t = 0$ ,  $a_c = 0$ <br>
(a) Applying conservation of energy between initial and final position is:<br>
Loss in gravitational P  $\frac{mg}{R(3-2\sqrt{2})}$ <br>
initial instant  $a_t = g$ ,  $a_c = 0$ <br>
bottom position  $a_t = 0$ ,  $a_c = 0$ <br>
plying conservation of energy between initial and<br>
al position is :

g Loss in gravitational P.E. of the bead of mass m

 $=$  gain in spring P. E.

r h <sup>1</sup> <sup>2</sup> mgR K (2R 2R) <sup>2</sup> or mg <sup>K</sup> 

(b) At 
$$
t = 0 \Rightarrow a_t = g \Rightarrow a_c = 0
$$

At lowest point  $a_t = 0 \Rightarrow a_c = 0$ 

g 9.8 rad 98 7 2 (10) (a)  $K = \frac{mg}{R(3-2\sqrt{2})}$ <br>
(b) At initial instant  $a_1 = g, a_2 = 0$ <br>  $\Rightarrow 2m = \frac{g}{r} \Rightarrow 0^2 = \frac{g}{r-1}$ <br>
(b) At initial instant  $a_1 = g, a_2 = 0$ <br>  $\Rightarrow 0^2 = \frac{g}{r-1}$ <br>
(c)<br>  $\Rightarrow r - \frac{g}{\sqrt{2}} > 0$ <br>
(b) At  $t = 0 \Rightarrow a_1 = 9 \Rightarrow a_2 = 0$ <br>  $\$ **AR MOTION**<br>
THE TRY SOLUTIONS<br>
IN TRISPERITY (10) (a)  $K = \frac{mg}{R(3-2\sqrt{2})}$ <br>
(b) At initial instant  $a_1 = g, a_2 = 0$ <sup>19</sup> R<br>
<sup>19</sup> R<br> **10** (b) At initial instant  $a_t = 0$ ,  $a_c = 0$ <br>
At bottom position  $a_t = 0$ ,  $a_c = 0$ <br>
At bottom position  $a_t = 0$ ,  $a_c = 0$ <br>  $\therefore \tan \theta = \frac{\omega^2 R}{g}$ <br>
(a) Applying conservation of energy between initial instant  $a_t$ **Example 19 and 19** (10) (a)  $K = \frac{mg}{R(3-2\sqrt{2})}$ <br>
(b) At initial instant  $a_t = g, a_c = 0$ <br>  $x$  b At initial instant  $a_t = g, a_c = 0$ <br>  $x$  b At initial instant  $a_t = g, a_c = 0$ <br>  $x = \frac{g}{r - h}$ <br>
(b) At initial instant  $a_t = g, a_c = 0$ <br>  $x = \frac{g}{r - h}$ <br>  $\therefore mgh = \$ (10) (a)  $K = \frac{mg}{R(3-2\sqrt{2})}$ <br>
(b) At initial instant  $a_t = g$ ,  $a_c = 0$ <br>  $\cos\theta = mg$   $\therefore \tan\theta = \frac{\omega^2 R}{g}$ <br>  $\therefore \tan\theta = \frac{\omega^2 R}{g}$ <br>  $\therefore \tan\theta = \frac{\omega^2 R}{g}$ <br>
(a) Applying conservation of energy between interesting to the sead of mass The centripetal acceleration of bead at the initial and final position is zero because its speed at both position is zero. The tangential acceleration of the bead at initial position is g. The tangential acceleration of the bead at lower most position is zero.

![](_page_32_Picture_0.jpeg)

## **CHAPTER-7 : CIRCULAR MOTION EXERCISE-1**

**(1) (B).** We have angular displacement

$$
= \frac{\text{linear displacement}}{\text{radius of path}} \Rightarrow \Delta\theta = \frac{\Delta S}{r}
$$

Here,  $\Delta S = n (2\pi r) = 1.5 (2\pi \times 2 \times 10^{-2}) = 6\pi \times 10^{-2}$ 

$$
\therefore \Delta \theta = \frac{6\pi \times 10^{-2}}{2 \times 10^{-2}} = 3\pi \text{ radian}
$$

(2) (A). We have 
$$
\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time}}
$$

For first one third part of circle,

angular displacement, 
$$
\theta_1 = \frac{S_1}{r} = \frac{2\pi r/3}{r}
$$

For second one third part of circle,

$$
\theta_2 = \frac{2\pi r/3}{r} = \frac{2\pi}{3} \text{ rad}
$$

Total angular displacement,  $\theta = \theta_1 + \theta_2 = 4\pi/3$  rad Total time  $= 2 + 1 = 3$  sec

$$
\therefore \ \ \frac{\rightarrow}{\omega_{av}} = \frac{4\pi/3}{3} \text{ rad/s} = \frac{4\pi}{6} = \frac{2\pi}{3} \text{ rad/s} \tag{0.00}
$$

**(3) (A).** Angular speed of hour hand,

$$
\omega_1 = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{12 \times 60} \text{ rad/sec}
$$
 (11)

Angular speed of minute hand,

$$
\omega_2 = \frac{2\pi}{60}
$$
 rad/sec  $\Rightarrow \frac{\omega_2}{\omega_1} = \frac{12}{1}$ 

(4) **(D).** We have 
$$
\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \Rightarrow \frac{d\theta}{dt} = \omega_0 + \alpha t
$$

This is angular velocity at time t. Now angular velocity at  $t = 2$  sec will be

3 6 3  
\nA). Angular speed of hour hand,  
\n
$$
\omega_1 = \frac{\Delta \theta}{\Delta t} = \frac{2\pi}{12 \times 60}
$$
 rad/sec  
\n $\omega_2 = \frac{2\pi}{60}$  rad/sec  $\Rightarrow \frac{\omega_2}{\omega_1} = \frac{12}{1}$   
\nA. 1. Given a  
\n $\omega_2 = \frac{2\pi}{60}$  rad/sec  $\Rightarrow \frac{\omega_2}{\omega_1} = \frac{12}{1}$   
\nA. 2. Given a  
\n $\omega_2 = \frac{2\pi}{60}$  rad/sec  $\Rightarrow \frac{\omega_2}{\omega_1} = \frac{12}{1}$   
\nA. 3. Using a  
\n $\omega_2 = \frac{2\pi}{60}$  rad/sec  $\Rightarrow \frac{d\theta}{\omega_1} = \omega_0 + \alpha t$   
\nThis is angular velocity at time t.  
\nNow angular velocity at t = 2 sec will be  
\n $\omega = \left(\frac{d\theta}{dt}\right)_{t=2\sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4$  rad/sec  
\n $\omega = \left(\frac{d\theta}{dt}\right)_{t=2\sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4$  rad/sec  
\n $\omega = \left(\frac{d\theta}{dt}\right)_{t=2\sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4$  rad/sec  
\n $\omega = \left(\frac{d\theta}{dt}\right)_{t=2\sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4$  rad/sec  
\n $\omega = \left(\frac{d\theta}{dt}\right)_{t=2\sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4$  rad/sec  
\n $\omega = \left(\frac{d\theta}{dt}\right)_{t=2\sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4$  rad/sec  
\n $\omega = \left(\frac{d\theta}{dt}\right)_{t=2\sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4$  rad/sec  
\n $\omega = \left(\frac{d\theta}{dt}\right)_{t=2\sec} = \omega_0 + 2\alpha = 1 +$ 

**(5) (D).** The distance covered in completing the circle is  $2\pi r = 2\pi \times 10$  cm

The linear speed is 
$$
v = \frac{2\pi r}{t} = \frac{2\pi \times 10}{4} = 5\pi \text{ cm/s}
$$
 (ii)

The linear acceleration is,  $a = \frac{v^2}{v^2} = \frac{(5\pi)^2}{10} = 2.5 \pi^2$  cm/s<sup>2</sup> Rad  $r = \frac{(5\pi)^2}{10} = 2.5 \pi^2$  cm/s<sup>2</sup><br>Radial and the each other.  $\frac{\pi^2}{4}$  = 2.5  $\pi^2$  cm/s<sup>2</sup> Radial and tang

This acceleration is directed towards the centre of the circle

$$
\omega_2 = \frac{60}{60} \text{ rad/sec} \Rightarrow \frac{60}{61} = \frac{1}{1}
$$
\nAt  $t = 0$ ,  $\omega = 0$   $\therefore c = 0$ ,  $\omega = \frac{3t^2}{2} - \frac{t^3}{3}$   
\n**(4)**\n**(D).** We have  $\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \Rightarrow \frac{d\theta}{dt} = \omega_0 + \alpha t$   
\nThis is angular velocity at time t.  
\nNow angular velocity at  $t = 2$  sec will be  
\n $\omega = \left(\frac{d\theta}{dt}\right)_{t=2\sec} = \omega_0 + 2\alpha = 1 + 2 \times 1.5 = 4 \text{ rad/sec}$   
\n**(5)**\n**(D).** The distance covered in completing the circle is  
\n $2\pi t = 2\pi \times 10 \text{ cm}$   
\nThe linear speed is  $v = \frac{2\pi r}{t} = \frac{2\pi \times 10}{4} = 5\pi \text{ cm/s}$   
\n $2\pi t = 2\pi \times 10 \text{ cm}$   
\nThe linear acceleration is,  $a = \frac{v^2}{r} = \frac{5\pi}{60} = 2.5 \pi^2 \text{ cm/s}^2$   
\nThis acceleration is directed towards the centre of the circle  
\n**(6) (B).** Velocity =  $\frac{2\pi \text{ cm}}{\text{Time of revolution}} = \frac{2\pi \text{ cm}}{60} = \frac{2\pi \text{ cm}}{30} = \frac{$ 

<sup>=</sup> linear displacement radius of path <sup>=</sup> <sup>S</sup> <sup>=</sup> 2 r / 3 Change in velocity v = 2 2 30 30 ; <sup>2</sup> 30 =cm/s 3t 2 2 10 3 2 2

- $\Delta S$  r = 20 cm = 0.2 m :  $\omega$  = 50 rad/s **(7) (A).** The angular velocity is  $\omega = v/r$ . Hence,  $v = 10$  m/s
- **(8) (B).** Given that  $\omega = 1.5t 3t^2 + 2$

$$
\alpha = \frac{d\omega}{dt} = 1.5 - 6t
$$

When,  $\alpha = 0 \implies 1.5 - 6t = 0 \implies t = 1.5/6 = 0.25$  sec

**EXERCISE-1**<br>
EXERCISE-1<br>
TER-7: CIRCULARMOTION<br>
EXERCISE-1<br>
have angular displacement<br>  $\frac{\text{linear displacement}}{\text{radius of path}} \Rightarrow \Delta\theta = \frac{\Delta S}{r}$ <br>  $\alpha = \frac{d\omega}{dt} = 1.5 - 6t$ <br>  $\Delta x \times 10^{-2} = 3\pi \text{ radian}$ <br>  $\Delta \theta = \frac{\Delta S}{r}$ <br>  $\alpha = \frac{d\omega}{dt} = 1.5 - 6t$ <br>  $\Delta x \times$ **EXECULARMOTION**<br>
EXECULARMOTION<br>
EXERCILARMOTION<br>
EXERCISE 1<br>
and any angle of path<br>  $= \frac{\text{linear displacement}}{\text{radius of path}} \Rightarrow \Delta\theta = \frac{\Delta S}{r}$ <br>  $= n(2\pi r) = 1.5 (2\pi \times 2 \times 10^{-2}) = 6\pi \times 10^{-2}$ <br>  $\alpha = \frac{d\omega}{dt} = 1.5 - 6t$ <br>  $= 1.5 - 6t$ <br>
any  $\vec{\omega}_{av} = \frac{\$ **EXERCISE-1**<br>
EXERCISE-1<br>
ave angular displacement<br>  $= \frac{\text{linear displacement}}{\text{radius of path}} \Rightarrow \Delta\theta = \frac{\Delta S}{r}$ <br>  $= \pi (2\pi r) = 1.5 (2\pi \times 2 \times 10^{-2}) = 6\pi \times 10^{-2}$ <br>  $= 3\pi \text{ radian}$ <br>  $= \frac{\pi \times 10^{-2}}{10^{-2}} = 3\pi \text{ radian}$ <br>  $= \frac{\pi \times 10^{-2}}{10^{-2}} = 3\pi \text{ radian}$ <br>  $= \frac$ **(3) CHAPTER-7: CIRCULAR MOTION**<br>
(1) **(B). We have angular displacement**<br>  $= \frac{\text{linear displacement}}{\text{radius of path}} \Rightarrow \Delta\theta = \frac{\Delta S}{r}$ <br>
Here,  $\Delta S = n(2\pi r) = 1.5 (2\pi \times 2 \times 10^{-2}) = 6\pi \times 10^{-2}$ <br>  $\therefore \text{ } \text{A}\theta = \frac{6\pi \times 10^{-2}}{2 \times 10^{-2}} = 3\pi \text{ radian}$ <br>
( **EXERCISE-1**<br> **CAE. SOLUTIONS**<br> **EXERCISE-1**<br>
change in velocity  $\Delta v = \sqrt{\frac{\pi}{30}}^2 + (\frac{\pi}{30})^2$ ,  $\frac{\pi}{30}\sqrt{2}$  = angular displacement<br>
linear displacement<br>  $7\pi r = 1.5 (2\pi \times 2 \times 10^{-2}) = 6\pi \times 10^{-2}$ <br>  $10^{-2} = 3\pi$  radian<br>  $\$ **EXERCISE-1**<br>
We have angular displacement<br>  $\frac{\text{linear displacement}}{2 \times 10^{-2}} = \frac{3\pi \text{ rad/s}}{10^{-2}} = 3\pi \text{ rad/s}$ <br>  $\frac{2\pi r}{3} = \frac{3\pi}{4}$ <br>  $\frac{2\pi r}{3} = 3\pi$ <br>  $\frac{2\pi r}{3} = \frac{3\pi}{3}$ <br>  $\frac{2\pi r}{3} = \frac{3\pi}{3}$ <br>  $\frac{2\pi r}{3} = \frac{3\pi}{3}$ <br>  $\frac{2\pi r}{3$ **EXERCISE-1** Change in velocity  $\Delta v = \sqrt{\frac{\pi}{30}}$ <br>
are angular displacement<br>  $= \frac{\text{linear displacement}}{\text{radius of path}} \Rightarrow \Delta \theta = \frac{\Delta S}{r}$ <br>  $= \frac{\pi}{10}$ <br>  $= \frac{\pi}{10}$ <br>  $= \frac{\pi}{10}$ <br>  $= \frac{1.5 - 3t^2 + 2}{\pi}$ <br>  $= 3\pi$  radian<br>  $= \frac{\pi}{10}$ <br>  $= \frac{3\pi}{10}$ <br>  $= \$ **EXERCISE-1**<br>
EXERCISE-1<br>
Have angular displacement<br>  $\frac{\sin \alpha x}{\alpha}$  and  $\frac{\sin \alpha x}{\alpha} = \frac{1}{2\pi} \tan \alpha$ <br>  $\frac{\sin \alpha x}{\alpha} = \frac{3\pi}{4}$ <br>  $\frac{5\pi}{4} = 3 \sec \alpha$ <br>  $\frac{\pi}{2} = 3\pi$  radian<br>  $\frac{\pi}{2} = \frac{3\pi}{4}$ <br>  $\frac{5\pi}{4} = \frac{2\pi}{3}$ <br>  $\frac{5\pi}{4$ **(9) (C).** Given  $v = 1.5 t^2 + 2t$ Linear acceleration  $a = dv/dt = 3t + 2$ This is the linear acceleration at time t Now angular acceleration at time t

$$
\alpha = \frac{a}{r} \Rightarrow \alpha = \frac{3t + 2}{2 \times 10^{-2}}
$$

Angular acceleration at  $t = 2$  sec

$$
(\alpha)_{\text{at }t=2\text{sec}} = \frac{3 \times 2 + 2}{2 \times 10^{-2}} = \frac{8}{2} \times 10^{2}
$$

$$
= 4 \times 10^2 = 400 \text{ rad/sec}^2
$$

**(10) (A).** Angular displacement after 4 sec is

**EXAMPLE 2.1.2 EXERCISE-1**  
\nWe have angular displacement  
\n
$$
= \frac{\text{linear displacement}}{\text{radius of path}} \Rightarrow \Delta\theta = \frac{\Delta S}{r}
$$
\n
$$
\Rightarrow \Delta\theta = \frac{\Delta S}{r} \Rightarrow \Delta\theta = \Delta\theta \Rightarrow \Delta
$$

(11) (A). Given 
$$
\alpha = 3t - t^2
$$

$$
\frac{1}{2 \times 10^{-2}} = 3\pi \text{ radian}
$$
\nWhen  $\alpha = 0 \Rightarrow 1.5-6 = 0.25 \text{ sec}$   
\nWe have  $\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time}}$   
\nWe have  $\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time}}$   
\n $\alpha = \frac{a}{r} \Rightarrow \alpha = \frac{31 + 2}{2 \times 10^{-2}}$   
\nand one third part of circle,  
\n $\frac{2\pi r}{r} = \frac{3}{3} \text{ rad}$   
\n $\alpha = \frac{a}{r} \Rightarrow \alpha = \frac{3 \times 2 + 2}{2 \times 10^{-2}} = \frac{8}{2} \times 10^2$   
\nand one third part of circle,  
\n $\alpha = \frac{a}{r} \Rightarrow \alpha = \frac{3 \times 2 + 2}{2 \times 10^{-2}} = \frac{8}{2} \times 10^2$   
\nand  $\alpha = 4 \times 10^2 = 400 \text{ rad/sec}^2$   
\n $\alpha = \frac{4\pi}{r} \Rightarrow \frac{3\pi}{r} \Rightarrow \frac{2\pi r}{r} \Rightarrow \frac{4\pi}{3} \text{ rad/s}$   
\n $\alpha = \frac{4\pi}{r} \Rightarrow \frac{3\pi}{r} \Rightarrow \frac{3\pi}{r} \Rightarrow \frac{4\pi}{r} \Rightarrow \frac{2\pi}{r} \Rightarrow \alpha = \frac{31 + 2}{2 \times 10^{-2}} = \frac{8}{2} \times 10^2$   
\n $\alpha = \frac{4\pi}{3} \text{ rad/s} = \frac{4\pi}{6} = \frac{2\pi}{3} \text{ rad/s}$   
\n $\alpha = \frac{4\pi}{3} \text{ rad/s} = \frac{4\pi}{6} = \frac{2\pi}{3} \text{ rad/s}$   
\n $\alpha = \frac{1}{2} \times 60 \text{ rad/sec}^2$   
\n $\alpha = \frac{1}{2} \times 60 \text{ rad/sec} = \frac{9}{\omega_1} = \frac{1}{1}$   
\n $\alpha = \frac{1}{2} \text{ rad} \Rightarrow \frac{d\omega}{r} = \frac{1}{2} \text{ rad} \Rightarrow \frac{d\omega}{dr} =$ 

 $\frac{d\theta}{dt} = \omega_0 + \alpha t$  Angular velocity at  $t = 2 \sec$ ,  $(\omega)_{t=2}$  sec

$$
= \frac{3}{2}(4) - \frac{8}{3} = \frac{10}{3} \text{ rad/sec}
$$

Since there is no angular acceleration after 2 sec  $\therefore$  The angular velocity after 6 sec remains the same.

**(12) (C).** Two types of acceleration are experienced by the car (i) Radial acceleration due to circular path,

$$
a_r = \frac{v^2}{r} = \frac{(30)^2}{500} = 1.8 \text{ m/s}^2
$$

4 (ii) A tangential acceleration due to increase of tangential speed given by  $a_t = \Delta v / \Delta t = 2$  m/s<sup>2</sup>

> Radial and tangential acceleration are perpendicular to each other. Net acceleration of car

$$
a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1.8)^2 + (2)^2} = 2.7 \text{ m/s}^2
$$

At 
$$
t = 0
$$
,  $\omega = 0$   $\therefore c = 0$ ,  $\omega = \frac{3t^2}{2} - \frac{t^3}{3}$   
\n $\omega_0 + \alpha t$  Angular velocity at  $t = 2 \sec$ ,  $(\omega)_{t=2 \sec}$   
\n $= \frac{3}{2}(4) - \frac{8}{3} = \frac{10}{3}$  rad/sec  
\n  
\nSince there is no angular acceleration after 2 sec  
\n $\therefore$  The angular velocity after 6 sec remains the same.  
\ng the circle is  
\n(i) Radial acceleration due to circular path,  
\n $a_r = \frac{v^2}{r} = \frac{(30)^2}{500} = 1.8 \text{ m/s}^2$   
\n $= 5\pi \text{ cm/s}$   
\n(ii) At tangential acceleration due to increase of tangential speed given by  $a_t = \Delta v/\Delta t = 2 \text{ m/s}^2$   
\nRadial and tangential acceleration are perpendicular to each other. Net acceleration of car  
\nthe centre of the  
\n $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1.8)^2 + (2)^2} = 2.7 \text{ m/s}^2$   
\n $\frac{2\pi r}{60}$   
\n(13) (B).  $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$ 

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#### **CIRCULAR MOTION Q.B.- SOLUTIONS**

![](_page_33_Picture_2.jpeg)

**(14) (A).** Let the radius of the orbit be r and the number of revolutions per second be n. Then the velocity of electron is given by  $v = 2\pi n r$ ,

$$
\therefore \text{ Acceleration a} = \frac{v^2}{r} = \frac{4\pi^2 r^2 n^2}{r} = 4\pi^2 r n^2 =
$$

Substituting the given values, we have  $a = 4 \times (3.14)^2 \times (5.3 \times 10^{-11}) (6.6 \times 10^{15})^2$  $= 9.1 \times 10^{22}$  m/s<sup>2</sup> towards the nucleus. The centripetal force is

$$
F_C = ma = (9.1 \times 10^{-31}) (9.1 \times 10^{22})
$$

$$
= 8.3 \times 10^{-8}
$$
 N towards the nucleus.

**(15) (B).** Given that radius of horizontal loop  $r = 1$  km = 1000 m

Speed v = 900 km/h = 
$$
\frac{9000 \times 5}{18}
$$
 = 250 m/s

Centripetal acceleration  $a_c = \frac{v^2}{r} = \frac{250 \times 250}{1000} = 62.5 \text{ m/s}^2$   $\therefore a_{\text{net}} = \sqrt{a_c^2 + a_c^2}$ 

- $\therefore \frac{\text{Cernipean acceleration}}{\text{Gravitational acceleration}} = \frac{c}{g} = \frac{5.38}{9.8} = 6.38:1$ Centripetal acceleration  $\frac{a_c}{g} = \frac{62.5}{9.8} = 6.38 : 1$   $\qquad \qquad = \sqrt{(40.0655 - 1.5 \cdot 1)}$
- **(16) (D).** (a) Centripetal force is not a real force. It is only the requirement for circular motion. It is not a new kind of force. Any of the forces found in nature such as gravitational force, electric friction force, tension in string reaction force may act as centripetal force. Everation  $\frac{1}{2}$   $\frac{1}{2}$  and a relation of the summation  $\frac{1}{2}$  mv<sup>2</sup> = 10 x  $\frac{1}{2}$  forces in a relation of the summation of the summation of the forces found in nature such as<br>
electric friction force, tension eleration  $\frac{R}{8}$  =  $\frac{62.5}{9.8}$  = 6.38 :  $\frac{1}{2}$  =  $\frac{1}{2}$  (400)<sup>2</sup> + (100)<sup>2</sup> = 100  $\sqrt{17}$  m/s<sup>2</sup><br>
eleration is to tat real force. It is only the (26) (A). Maximum tension  $T = \frac{mv^2}{r} + mg$ <br>
coince is not a real (16) **(D).** (a) Centriped force is not a rat force. It is only the **(26)** (A). Maximum tension  $T = \frac{\mu}{r}$  requirement for circular motion. It is not a new kind of<br>
force. Any of the forces found in nature such as<br>
gravit by the force is not areal force. It is only the (26) (A). Maximum tension  $T = \frac{mv}{r} + mg$ <br>
that motion. It is not a new kind of<br>
the scentripetal force, tension in string<br>
the decric friction force, tension in string<br>
the s

(b) Work done by centripetal force is always zero.

(17) **(D).** Centripetal acceleration, 
$$
a_c = \frac{v^2}{r} = k^2 rt^2
$$
  
\n $\therefore$  Variable velocity  $v = \sqrt{k^2 r^2 t^2} = krt$   
\n**EXECUTE:**

The force causing the velocity to varies F = m  $\frac{dv}{dt}$  = m k r then T - mg =  $\frac{mv^2}{dt}$ 

The power delivered by the force is,  $P = Fv = mkr \times krt = mk^2r^2t$ 

**(18) (C).** Both changes in direction although their magnitudes remains constant.

(19) (D). 
$$
\frac{mv^2}{r} = 10 \Rightarrow \frac{1}{2}mv^2 = 10 \times \frac{r}{2} = 1J
$$

If angular velocity is doubled force will becomes four times. Centripetal acceleration,  $a_c = \frac{v^2}{r} = k^2 \pi^2$ <br>
the bob is converted into K.E. hence<br>
triable velocity  $v = \sqrt{k^2 r^2 t^2} = k \tau t$ <br>
orce causing the velocity to varies  $F = m \frac{dv}{dt} = m k r$ <br>
orce causing the velocity to varies  $F =$ Finding the velocity of  $\cos \theta$  and R are constant)<br>  $\cos^2 \theta$  and the proposition,  $a_c = \frac{v^2}{r} = k^2 r t^2$ <br>  $\cos \theta$  welocity  $v = \sqrt{k^2 r^2 t^2} = k r t$ <br>  $\cos \theta$  welocity  $v = \sqrt{k^2 r^2 t^2} = k r t$ <br>  $\cos \theta$  welocity to varies  $F = m \frac{dv}{dt} = m$ Force that we deling the velocity  $v = \sqrt{k^2 r^2 t^2} = k \tau t$ <br>
the bob at the book at he lowest position. In fig.<br>
the book at he lowest position in the structure of the book at the lowest position.<br>
the book at he lowest point power delivered by the force is,<br>
Five mkr × krt = mk<sup>2</sup>r<sup>2</sup>t<br>
Both changes in direction although their magnitudes<br>  $\frac{mv^2}{r} = 10 \Rightarrow \frac{1}{2}mv^2 = 10 \times \frac{r}{2} = 1J$ <br>  $\Rightarrow \frac{30}{0.5} - F = m\omega^2 R$   $\therefore F \propto \omega^2$  (m and R are constant Both changes in direction although their magnitudes<br>
ins constant.<br>  $\frac{mv^2}{r} = 10 \Rightarrow \frac{1}{2}mv^2 = 10 \times \frac{r}{2} = 1J$   $\Rightarrow \frac{30}{0.5} - F = m\omega^2 R$   $\therefore F \propto \omega^2$  (m and R are constant) (29) (A). When<br>
gular velocity is doubled force Variable velocity  $v = \sqrt{k^2 r^2 t^2} = krt$ <br>
F T be the tension in th<br>
force causing the velocity to varies  $F = m$  dv<br>
power delivered by the force is,<br>  $= Fv = mkr \times krt = mk^2r^2t$ <br>
From (1) & (2), T = 3 mg<br>
R. Both changes in directio Example the velocity to varies  $F = m \frac{dv}{dt} = m k r$ <br>
then  $T - mg = \frac{mv^2}{l}$ <br>  $= Fv = mkr \times krt = mk^2t^2t$ <br>
From (1) & (2),  $T = 3$  mg<br>
anis constant.<br>  $\frac{mv^2}{r} = 10 \Rightarrow \frac{1}{2}mv^2 = 10 \times \frac{r}{2} = 1J$ <br>  $\frac{m v^2}{r} = 10 \Rightarrow \frac{1}{2}mv^2 = 10 \times \frac{r}{2$ by the velocity to varies F = m  $\frac{W^2}{dt}$  = m k r<br>
f when T - mg =  $\frac{W^2}{\ell}$ <br>
f F - m k + k t = m k<sup>2</sup>r<sup>2</sup><br>
f V = m k<sup>2</sup>r<sup>2</sup><br>
as onstant.<br>
as constant.<br>
as constant.<br>
T = 10  $\Rightarrow \frac{1}{2}mv^2 = 10 \times \frac{r}{2} = 1J$ <br>
F = m  $\frac$ 

**(21) (B).** 
$$
F = m \left( \frac{4\pi^2}{T^2} \right) R
$$
.

If masses and time periods are same then  $F \propto R$ 

$$
\therefore \frac{F_1}{F_2} = \frac{R_1}{R_2}
$$

(22) 
$$
\text{(C).a} = \frac{v^2}{r} = \omega^2 r = 4\pi^2 n^2 r = 4\pi^2 \left(\frac{22}{44}\right)^2 \times 1 = \pi^2 m s^{-2} = m \left\{ g + \frac{1}{2} \right\}
$$

and its direction is always along the radius and towards the centre.

$$
(23) (D). Time period
$$

**Q.B.- SOLUTIONS**  
\nthe orbit be r and the number of  
\ne n. Then the velocity of electron  
\n(a) (23) (D). Time period  
\n
$$
= \frac{4\pi^2 r^2 n^2}{r} = 4 \pi^2 r n^2
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
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\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 sec
$$
\n
$$
= \frac{2\pi r}{r} = \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times
$$

**(24) (C).** Difference in K.E. = Difference in P.E. = 2mgr

(25) **(A).** We know centripetal acceleration 
$$
a
$$

[11] **AR MOTION**  
\n(A). Let the radius of the orbit be r and the number of  
revolutions per second be n. Then the velocity of electron  
is given by v = 2πn,  
∴ Acceleration = 
$$
\frac{v^2}{r} = \frac{4\pi^2 r^2 n^2}{r} = 4 \pi^2 r n^2
$$
  
\n $\therefore$  Acceleration =  $\frac{v^2}{r} = \frac{4\pi^2 r^2 n^2}{r} = 4 \pi^2 r n^2$   
\n $= 91 \times 10^{12} \text{ m/s}^2 \text{ (S.0 × 1015)} = 64 \times 10^{15} \text{ s}$   
\n $= 91 \times 10^{12} \text{ m/s}^2 \text{ (S.0 × 1015)} = 24 \times 10^{-15} \text{ N} \text{ (S.0 × 102)} = 24 \times 10^{-15} \text{ N} \text{ (S.0 × 102)} = 24 \times 10^{-15} \text{ N} \text{ (S.0 × 102)} = 91 \times 10^{12} \text{ m/s}^2 \text{ (S.0 × 102)} = 83 \times 10^{-8} \text{ N} \text{ towards the nucleus.}$   
\nThe centripetal force is not a model.  
\n $F_C = ma = (9.1 \times 10^{-31})(9.1 \times 10^{22})$   
\n $= 8.3 \times 10^{-8} \text{ N} \text{ towards the nucleus.}$   
\n $F_C = ma = (9.1 \times 10^{-31})(9.1 \times 10^{22})$   
\n $= 8.3 \times 10^{-8} \text{ N} \text{ towards the nucleus.}$   
\n $= 8.3 \times 10^{-8} \text{ N} \text{ towards the nucleus.}$   
\n $\therefore$   $\frac{\text{Central path of the 1}}{100 \times 10^{-2}} = 250 \text{ m/s}$   
\n $\therefore$   $\frac{\text{Central path of the 1}}{100 \times 10^{-2}} = 250 \text{ m/s}$   
\n $\therefore$   $\frac{\text{Central path of the 1$ 

(26) (A). Maximum tension T = 
$$
\frac{mv^2}{r}
$$
 + mg

$$
\therefore \frac{mv^2}{r} = T - mg \text{ or } \frac{mv^2}{r} = 163.6 - 4 \times 9.8 \Rightarrow v = 6 \text{ m/s}
$$

**(27) (C).** The situation is shown in fig. Let v be the velocity of the bob at the lowest position. In this position the P.E. of bob is converted into K.E. hence -  $\Rightarrow$  v = 6 m/s<br>
e velocity of<br>
n the P.E. of<br>
r + g<br>  $\frac{0}{2}$  = 5 rad / s<br>
n p (inclined<br>
an position Maximum tension T =  $\frac{mv^2}{r}$  + mg<br>  $\frac{mv^2}{r}$  = T - mg or  $\frac{mv^2}{r}$  = 163.6 - 4 × 9.8  $\Rightarrow$  v = 6 m/s<br>
The situation is shown in fig. Let v be the velocity of<br>
bob at the lowest position. In this position the P.E. of =  $\sqrt{(400)^2 + (100)^2} = 100 \sqrt{17}$  m/s<br>
aximum tension T =  $\frac{mv^2}{r}$  + mg<br>  $=$  - T - mg or  $\frac{mv^2}{r}$  = 163.6-4 × 9.8  $\Rightarrow$  v = 6 m/s<br>
e situation is shown in fig. Let v be the velocity of<br>
at the lowest position. In this ng<br>  $-4 \times 9.8$  ⇒  $v = 6$  m/s<br>
st v be the velocity of<br>
s position the P.E. of<br>
....(1)<br>
....(2)<br>
....(2)  $\frac{mv^2}{r}$  + mg<br>  $=\frac{mv^2}{r}$  + mg<br>  $\frac{r^2}{r}$  = 163.6 – 4 × 9.8  $\Rightarrow$  v = 6 m/s<br>
xn in fig. Let v be the velocity of<br>
ition. In this position the P.E. of<br>
3. hence -<br>
2gl ....(1)<br>
string,<br>
...(2)<br>
mg  $\Rightarrow \frac{T_{max}}{m} = \omega^2 r + g$ ∴  $\frac{mv^2}{r} = T - mg$  or  $\frac{mv^2}{r} = 163.6 - 4 \times 9.8 \Rightarrow v = 6$  m/s<br>
(C). The situation is shown in fig. Let v be the velocity of<br>
the bob at the lowest position. In this position the P.E. of<br>
pob is converted into K.E. hence -<br>

$$
mg\ell = \frac{1}{2}mv^2 \Rightarrow v^2 = 2g\ell \qquad \qquad \dots (1)
$$

If T be the tension in the string,

then 
$$
T - mg = \frac{mv^2}{\ell}
$$
 ....(2)  
From (1) & (2),  $T = 3$  mg

(28) (A). 
$$
T_{\text{max}} = m\omega_{\text{max}}^2 r + mg \implies \frac{T_{\text{max}}}{m} = \omega^2 r + g
$$

$$
\Rightarrow \frac{30}{0.5} - 10 = \omega^2_{\text{max}} r \Rightarrow \omega_{\text{max}} = \sqrt{\frac{50}{r}} = \sqrt{\frac{50}{2}} = 5 \text{ rad/s}
$$

by the force is,<br>  $\frac{1}{2}$  From (1) & (2),  $T = 3$  mg<br>
direction although their magnitudes<br>
(28) (A). T<sub>max</sub> = mo<sup>2</sup><sub>max</sub> t + mg  $\Rightarrow \frac{T_{\text{max}}}{m} = \omega^2 t + g$ <br>  $\Rightarrow \frac{30}{0.5} - 10 = \omega^2 \text{max} t \Rightarrow \omega \text{max} = \sqrt{\frac{50}{r}} = \sqrt{\frac{50}{2}} = 5$  ra he velocity to varies F = m  $\frac{dv}{dt}$  = m k r<br>  $t = mk^2r^2t$ <br>  $t = mk^2r^2t$ <br>  $t = mk^2r^2t$ <br>  $t = m^2k^2t^2$ <br>  $t = m^2k^2t^2$ <br>  $t = m^2k^2t^2$ <br>  $t = 10 \times \frac{r}{2} = 1$ <br>  $\Rightarrow \frac{30}{0.5} - 10 = \omega^2_{max}r \Rightarrow \omega_{max} = \sqrt{\frac{50}{r}} = \sqrt{\frac{50}{2}} = 5$  rad<br> **(29) (A).** When body is released from the position p (inclined at angle  $\theta$  from vertical) then velocity at mean position mg $\ell = \frac{1}{2}$  mv<sup>2</sup>  $\Rightarrow$  v<sup>2</sup> = 2g $\ell$  ....(1)<br>
ff T be the tension in the string,<br>
then T - mg =  $\frac{mv^2}{\ell}$  ....(2)<br>
From (1) & (2), T = 3 mg<br>
(A). T<sub>max</sub> =  $mo_{max}^2 r + mg$   $\Rightarrow \frac{T_{max}}{m} = ω^2 r + g$ <br>  $\Rightarrow \frac{30}{0.5} - 10 = ω^2_{max}$ If T be the tension in the string,<br>
then T - mg =  $\frac{mv^2}{\ell}$  ....(2)<br>
From (1) & (2), T = 3 mg<br>
(28) (A). T<sub>max</sub> =  $\text{mo}_{\text{max}}^2 r + \text{mg} \Rightarrow \frac{\text{T}_{\text{max}}}{m} = \omega^2 r + g$ <br>  $\Rightarrow \frac{30}{0.5} - 10 = \omega^2_{\text{max}} r \Rightarrow \omega_{\text{max}} = \sqrt{\frac{50}{r}} = \sqrt{\frac{5$ m<br>  $r \Rightarrow \omega_{max} = \sqrt{\frac{50}{r}} = \sqrt{\frac{50}{2}} = 5 \text{ rad/s}$ <br>
leased from the position p (inclined<br>
cal) then velocity at mean position<br>
vest point = mg +  $\frac{mv^2}{\ell}$ <br>  $\approx 60$ )] = mg + mg = 2mg<br>
= m{g + 4 $\pi^2$ n<sup>2</sup>r}<br>  $\begin{pmatrix} 2 \\ r \end{pmatrix}$  $\frac{1}{2}$ <br>  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  are set and  $\frac{1}{2}$  are set and  $\frac{1}{2}$  are set and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  are set and  $\frac{1}{2}$  T - mg =  $\frac{mv^2}{\ell}$  ....(2)<br>
(1) & (2), T = 3 mg<br>
T<sub>max</sub> = mo<sub>2</sub><sub>max</sub> r + mg  $\Rightarrow \frac{T_{max}}{m} = \omega^2 r + g$ <br>  $\frac{0}{0.5} - 10 = \omega^2_{max} r \Rightarrow \omega_{max} = \sqrt{\frac{50}{r}} = \sqrt{\frac{50}{2}} = 5$  rad / s<br>
When body is released from the position p (inclined<br>

$$
\therefore \text{ Tension at the lowest point} = \text{mg} + \frac{\text{mv}^2}{\ell}
$$

$$
= mg + \frac{m}{\ell} [2gl(1 - \cos 60)] = mg + mg = 2mg
$$

**(0) (D).** T = mg + m
$$
\omega^2
$$
r = m{g + 4 $\pi$ 

Then 
$$
T - mg = \frac{mv}{\ell}
$$
 ....(2)

\nFrom (1) & (2),  $T = 3$  mg

\n**(A).**  $T_{\text{max}} = m\omega_{\text{max}}^2 r + mg \Rightarrow \frac{T_{\text{max}}}{m} = \omega^2 r + g$ 

\n $\Rightarrow \frac{30}{0.5} - 10 = \omega^2_{\text{max}} r \Rightarrow \omega_{\text{max}} = \sqrt{\frac{50}{r}} = \sqrt{\frac{50}{2}} = 5 \text{ rad/s}$ 

\n**(A).** When body is released from the position  $p$  (inclined at angle  $\theta$  from vertical) then velocity at mean position  $v = \sqrt{2gt/(1 - \cos \theta)}$ 

\n $\therefore$  Tension at the lowest point  $= mg + \frac{mv^2}{\ell}$ 

\n $= mg + \frac{m}{\ell} [2gt/(1 - \cos 60)] = mg + mg = 2mg$ 

\n**(D).**  $T = mg + m\omega^2 r = m\{g + 4\pi^2 n^2 r\}$ 

\n $= m\left\{g + \left(4\pi^2 \left(\frac{n}{60}\right)^2 r\right)\right\} = m\left\{g + \left(\frac{\pi^2 n^2 r}{900}\right)\right\}$ 

![](_page_34_Picture_0.jpeg)

**(31) (D).** Tension at mean position,  $mg + \frac{mv}{r} = 3mg$ 

$$
v = \sqrt{2g\ell} \qquad ...(i)
$$
  
9 with the vertical

then 
$$
v = \sqrt{2g\ell (1 - \cos \theta)}
$$
 ...(ii)

Comparing (i) and (ii),  $\cos \theta = 0 \Rightarrow \theta = 90^{\circ}$ 

**(32) (B).** We know that,

$$
\tan \theta = \frac{v^2}{rg} = \frac{\left(18 \times \frac{5}{18}\right)^2}{100 \times 10} = \frac{1}{40} \Rightarrow \theta = \tan^{-1} \frac{1}{40}
$$
 Hence

**(33) (C).** Let R be the normal reaction exerted by the road on the car. At the highest point, we have

Q.B. SOLUTIONS		
(D). Tension at mean position, $mg + \frac{mv^2}{r} = 3mg$	∴ $tan \theta = \frac{v^2}{rg} = 0.2$	
$v = \sqrt{2g\ell}$	∴ (i)	(40) (C). We know that the vertical then $v = \sqrt{2g\ell (1 - \cos \theta)}$
(3.10)	Comparing (i) and (ii), $\cos \theta = 0 \Rightarrow \theta = 90^\circ$	∴ (ii)
(3.21)	Linear matrix, $\theta = 0$ and $\theta = \frac{v^2}{rg} = \frac{\left(18 \times \frac{5}{18}\right)^2}{100 \times 10} = \frac{1}{40} \Rightarrow \theta = \tan^{-1} \frac{1}{40}$	Here $v = 48$ km/hr of $\theta = \frac{mv^2}{400 \times 9.8}$
(b) Let R be the normal reaction exerted by the road on the car. At the highest point, we have		
$\frac{mv^2}{(r+h)} = mg - R$ , R should not be negative.	(41) (B). $F = \frac{mv^2}{r} \Rightarrow F$	
$\frac{mv^2}{(r+h)} = mg - R$ , R should not be negative.	(42) (C). Stone flies in the arc of the current.	
(A). Let $W = Mg$ be the weight of the car.	(43) (A). Because the rest of the car.	

 $\therefore$  v<sub>max</sub> = 10 m/sec

- **(34) (A).** Let  $W = Mg$  be the weight of the car. Friction force = 0.4 W
	- Centripetal force =  $\frac{M v^2}{r} = \frac{W v^2}{g r}$ ; 0.4 W =  $\frac{W v^2}{g r}$  $\Rightarrow$   $v^2$  = 0.4  $\times$  g  $\times$  r = 0.4  $\times$  9.8  $\times$  30 = 117.6  $\Rightarrow$  v = 10.84 m/sec
- **(35) (C).** Let v be the speed of earth's rotation. We know that  $W = mg$

Hence 3 5 W = mg – <sup>2</sup> m v r or 3 mg = mg – 5 2 3 mg = <sup>2</sup> m v r or v<sup>2</sup> <sup>=</sup> 2g r 5 Now v<sup>2</sup> <sup>=</sup> <sup>3</sup> 2 9.8 (6400 10 ) <sup>5</sup> Solving, we get v = 5 × 10<sup>9</sup> m/sec, = 7.8 × 10<sup>4</sup>

$$
\omega = \sqrt{\left(\frac{2g}{5r}\right)} = 7.8 \times 10^4 \text{ radian/sec.}
$$

- **(36) (B).** The minimum speed at highest point of a vertical circle is given by  $v_c = \sqrt{rg} = \sqrt{20 \times 9.8} = 14 \text{ m/s}$
- **(37) (A).** By doing so component of weight of vehicle provides centripetal force.
- **(38) (A).** The value of frictional force should be equal or more than required centripetal force. i.e.  $\mu$ mg  $\geq \frac{mv^2}{r}$   $\therefore$  R + mg =  $\frac{mv^2}{r}$

(39) (A). 
$$
v = 60 \text{ km/hr} = \frac{50}{3} \text{ m/s}
$$
;  $r = 0.1 \text{ km} = 100 \text{m}$ 

$$
\therefore \tan \theta = \frac{v^2}{rg} = 0.283 \therefore \theta = \tan^{-1}(0.283)
$$

**O.B.- SOLUTIONS** STUDY MATERIAL: PHYSICS<br>  $+\frac{mv^2}{r} = 3mg$   $\therefore \tan \theta = \frac{v^2}{rg} = 0.283$   $\therefore \theta = \tan^{-1}(0.283)$ <br>  $\therefore$  (i) (40) (C). We know that  $\tan \theta = v^2 / rg$  ......(1)<br>  $\theta$  with the vertical Let h be the relative raising of o **(40) (C).** We know that  $\tan \theta = \frac{v^2}{rg}$  $\dots (1)$ Let h be the relative raising of outer rail with respect to

**EXAMPLE 22**  
\n**IDENTIFY** of the two terms of the one term.  
\n**EXECUTE:** (a) (b) 
$$
x = \sqrt{2g\ell}
$$
 (b)  $x = \sqrt{2g\ell}$   
\n(b) The  $y = \sqrt{2g\ell}$  (c)  $y = \sqrt{2g\ell}$   
\n(c) Let R be the positive number of the one term.  
\n**EXECUTE:** (a)  $\ln(3)$ ,  $\cos\theta = 0 \Rightarrow \theta = 90^\circ$   
\n**EXECUTE:** (a)  $\cos\theta = 0 \Rightarrow \theta = 90^\circ$   
\n(b) We know that,  
\n $\tan\theta = \frac{v^2}{\ell} = \frac{1}{100 \times 10} = \frac{1}{40} \Rightarrow 0 = \tan^{-1}\frac{1}{40}$   
\n**EXECUTE:** (a)  $\tan(6i)$ ,  $\cos\theta = 0 \Rightarrow \theta = 90^\circ$   
\n(b) We know that,  
\n $\tan\theta = \frac{v^2}{\ell} = \frac{1}{100 \times 10} = \frac{1}{40} \Rightarrow 0 = \tan^{-1}\frac{1}{40}$   
\n**EXECUTE:** (a)  $\tan\theta = \frac{V}{\ell} \times \frac{S}{\ell}$   
\n(b) We know that,  
\n $\tan\theta = \frac{V^2}{\ell} = \frac{1}{\sqrt{8}} = \frac{1}{100 \times 10} = \frac{1}{40} \Rightarrow 0 = \tan^{-1}\frac{1}{40}$   
\n**EXECUTE:** (a)  $\tan\theta = \frac{V}{\sqrt{8}} \times \frac{S}{\ell}$   
\n(b)  $\tan\theta = \frac{V}{\sqrt{8}} \times \frac{S}{\ell + \frac{S}{\ell}} = \frac{1}{100 \times 10} = \frac{1}{40} \Rightarrow 0 = \tan^{-1}\frac{1}{40}$   
\n**EXECUTE:** (a)  $\sin\theta = \frac{1}{\sqrt{8}} \Rightarrow 0.045 \text{ m} = 4.5 \text{ cm.}$   
\n(b)  $\cos\theta = 0 \Rightarrow \theta = 0$ 

$$
\therefore h = \frac{(120/9)^2 \times 1}{400 \times 9.8} = 0.045 \text{ m} = 4.5 \text{ cm}.
$$

(41) **(B).** 
$$
F = \frac{mv^2}{r} \Rightarrow F \propto v^2.
$$

If v becomes double then F (tendency to overturn) will become four times.

- **(42) (C).** Stone flies in the direction of instantaneous velocity due to inertia.
- **(43) (A).** Because the reaction on inner wheel decreases and becomes zero. So it leaves the ground first. the state of F (tendency to overturn) will<br>
tion of instantaneous velocity<br>
on inner wheel decreases and<br>
the ground first.<br>  $m/sec$ <br>  $\frac{20 \times 20}{20 \times 10} = \tan^{-1}(2)$ <br>  $\frac{4 \cdot 3}{2 \pi \pi}$  and  $v = \frac{2 \pi r}{T} = \frac{2 \pi r}{\sqrt{22}}$ <br>  $-1 \left(\frac{v^2}{rg}\right) = 4$ (tendency to overturn) will<br>on of instantaneous velocity<br>i inner wheel decreases and<br>ne ground first.<br> $\frac{x}{20}$  = tan<sup>-1</sup>(2)<br> $\frac{x}{x10}$  = tan<sup>-1</sup>(2)<br> $\frac{3}{x}$  and  $v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$ <br> $\left(\frac{v^2}{rg}\right) = 45^\circ$ <br> $+ mg = \frac$

(44) **(B).** 
$$
v = 72 \text{ km/hour} = 20 \text{ m/sec}
$$

1 V becomes double then F (tenency to overturn) win  
\nbecome four times.  
\nC). Stone flies in the direction of instantaneous velocity  
\nthe to inertia.  
\nA). Because the reaction on inner wheel decreases and  
\nbecomes zero. So it leaves the ground first.  
\nB). v = 72 km/hour = 20 m/sec  
\n
$$
θ = tan^{-1}\left(\frac{v^2}{rg}\right) = tan^{-1}\left(\frac{20 \times 20}{20 \times 10}\right) = tan^{-1}(2)
$$
\nA). 2πr = 34.3 ⇒ r =  $\frac{34.3}{2\pi}$  and v =  $\frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$   
\nAngle of binding, θ = tan<sup>-1</sup> $\left(\frac{v^2}{rg}\right)$  = 45°  
\n(A). Tension at top point T + mg =  $\frac{mu^2}{r}$   
\n∴ T =  $\frac{1 \times 4^2}{1}$  -1×10 = 6N  
\nB). a =  $\sqrt{a_c^2 + a_t^2}$  =  $\sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$   
\nD). When the motorcyclist is at the highest point of the

40  
\nthe road on  
\n
$$
\therefore h = \frac{(120/9)^2 \times 1}{400 \times 9.8} = 0.045 \text{ m} = 4.5 \text{ cm.}
$$
\ne.  
\n41) **(B)**  $F = \frac{mv^2}{r} \Rightarrow F \propto v^2$ .  
\nIf v becomes double then F (tendency to overturn) will  
\nbecome four times.  
\n42) **(C)**. Stone flies in the direction of instantaneous velocity  
\ndue to inertia.  
\n43) **(A)**. Because the reaction on inner wheel decreases and  
\nbecomes zero. So it leaves the ground first.  
\n44) **(B)**  $v = 72 \text{ km/hour} = 20 \text{ m/sec}$   
\n $\theta = \tan^{-1} \left(\frac{v^2}{rg}\right) = \tan^{-1} \left(\frac{20 \times 20}{20 \times 10}\right) = \tan^{-1}(2)$   
\n45) **(A)**  $2\pi r = 34.3 \Rightarrow r = \frac{34.3}{2\pi}$  and  $v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$   
\nAngle of binding,  $\theta = \tan^{-1} \left(\frac{v^2}{rg}\right) = 45^\circ$   
\n46) **(A)**. Tension at top point  $T + mg = \frac{mu^2}{r}$   
\n $\therefore T = \frac{1 \times 4^2}{r} = \frac{1 \times 10}{600}$ 

$$
\frac{1}{r}
$$
 Angle of binding,  $\theta = \tan^{-1} \left( \frac{v^2}{rg} \right) = 45^\circ$ 

(46) (A). Tension at top point 
$$
T + mg = \frac{mu^2}{r}
$$

$$
\therefore T = \frac{1 \times 4^2}{1} - 1 \times 10 = 6N
$$

(47) **(B).** 
$$
a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}
$$

 $\frac{1}{2}$ <br>  $\frac{1}{2}$  and  $\sqrt{9}$ ,  $\sqrt{3}$ ,  $\sqrt{3}$ ,  $\sqrt{3}$  and  $\sqrt{2}$  and (45) (A).  $2\pi r = 34.3 \Rightarrow r = \frac{r}{2\pi}$  and  $v = \frac{r}{T} = \frac{r}{\sqrt{22}}$ <br>  $r = mg - \frac{mv^2}{r}$ <br>
Angle of binding,  $\theta = \tan^{-1} \left(\frac{v^2}{rg}\right) = 45^\circ$ <br>
(46) (A). Tension at top point  $T + mg = \frac{mu^2}{r}$ <br>  $\therefore T = \frac{1 \times 4^2}{1} - 1 \times 10 = 6N$ <br>
(47) (B) 34.3  $\Rightarrow$   $r = \frac{34.3}{2\pi}$  and  $v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$ <br>
mding,  $\theta = \tan^{-1} \left(\frac{v^2}{rg}\right) = 45^\circ$ <br>
m at top point  $T + mg = \frac{mu^2}{r}$ <br>  $\frac{4^2}{c^2} - 1 \times 10 = 6N$ <br>  $\frac{2}{c^2} + a_t^2 = \sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$ <br>
the motorcyclist is at 34.3  $\Rightarrow$   $r = \frac{34.3}{2\pi}$  and  $v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$ <br>
mding,  $\theta = \tan^{-1} \left(\frac{v^2}{rg}\right) = 45^\circ$ <br>
m at top point  $T + mg = \frac{mu^2}{r}$ <br>  $\frac{4^2}{c} - 1 \times 10 = 6N$ <br>  $\frac{2}{c} + a_t^2 = \sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$ <br>
the motorcyclist is at the mes zero. So it leaves the ground iffst.<br>  $v = 72 \text{ km/hour} = 20 \text{ m/sec}$ <br>  $\tan^{-1} \left(\frac{v^2}{rg}\right) = \tan^{-1} \left(\frac{20 \times 20}{20 \times 10}\right) = \tan^{-1}(2)$ <br>  $2\pi r = 34.3 \implies r = \frac{34.3}{2\pi} \text{ and } v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$ <br>
e of binding,  $\theta = \tan^{-1} \left(\frac{v^$ tion on inner wheel decreases and<br>
aves the ground first.<br>  $= 20 \text{ m/sec}$ <br>  $\left(\frac{20 \times 20}{20 \times 10}\right) = \tan^{-1}(2)$ <br>  $= \frac{34.3}{2\pi}$  and  $v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$ <br>  $\tan^{-1}\left(\frac{v^2}{rg}\right) = 45^\circ$ <br>
int T + mg =  $\frac{mu^2}{r}$ <br>  $= 6N$ <br> ecause the reaction on inner wheel decreases and<br>  $\text{es zero. So it leaves the ground first.}$ <br>  $= 72 \text{ km/hour} = 20 \text{ m/sec}$ <br>  $n^{-1} \left( \frac{v^2}{rg} \right) = \tan^{-1} \left( \frac{20 \times 20}{20 \times 10} \right) = \tan^{-1} (2)$ <br>  $2\pi \pi = 34.3 \implies r = \frac{34.3}{2\pi} \text{ and } v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$ <br>
o **(48) (D).** When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also acts downwards. These two forces are balanced by the outward centrifugal force acting on him. gle of binding,  $\theta = \tan^{-1} \left(\frac{v^2}{rg}\right) = 45^\circ$ <br>
. Tension at top point  $T + mg = \frac{mu^2}{r}$ <br>  $T = \frac{1 \times 4^2}{1} - 1 \times 10 = 6N$ <br>
.  $a = \sqrt{a_e^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$ <br>
. When the motorcyclist is at the highest point of the thin- $2\pi r = 34.3 \Rightarrow r = \frac{1}{2\pi}$  and  $v = \frac{1}{T} = \frac{1}{\sqrt{22}}$ <br>
e of binding, θ = tan<sup>-1</sup>  $\left(\frac{v^2}{rg}\right) = 45^\circ$ <br>
Fension at top point T + mg =  $\frac{mu^2}{r}$ <br>  $= \frac{1 \times 4^2}{1} - 1 \times 10 = 6N$ <br>  $= \sqrt{a_e^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$ <br>
Wh

$$
\therefore R + mg = \frac{mv^2}{r}
$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle).

#### **CIRCULAR MOTION Q.B.- SOLUTIONS**

![](_page_35_Picture_2.jpeg)

Because of the balancing of the forces, the motorcyclist does not fall down. **CULAR MOTION**<br>
Because of the balancing of the forces, the motorcyclist<br>
does not fall down.<br>
The minimum speed required to perform a vertical loop is<br>  $\therefore mg = \frac{mv_{min}^2}{r}$  or  $v_{min}^2 = gr$ <br>
or  $v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25} \text{ ms}^{-$ 

The minimum speed required to perform a vertical loop is given by equation (1) when  $R = 0$ 

$$
\therefore mg = \frac{mv_{\min}^2}{r} \qquad \text{or } v_{\min}^2 = gr
$$

or 
$$
v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25} \text{ ms}^{-1} = 15.65 \text{ ms}^{-1}
$$
  
So the minimum speed of the top required to no

So, the minimum speed, at the top, required to perform a vertical loop is  $15.65 \text{ ms}^{-1}$ .

CHRCULAR MOTION	Q.B. SOLUTIONS	EXERCISE-2		
does not fall down.	The minimum speed required to perform a vertical loop is	(1)	(C). Let v be the speed of particle at B, just when it about to loose contact.	
given by equation (1) when R = 0	from application of Newton's second law to the particle normal to the spherical surface			
...	mg = $\frac{mv_{min}^2}{r}$ or $v_{min}^2 = gr$ or $v_{min}^2 = \sqrt{9.8 \times 25} \text{ ms}^{-1} = 15.65 \text{ ms}^{-1}$	Applying conservation of energy as the block moves vertical loop is 15.65 ms <sup>-1</sup> .	Applying conservation of energy as the block moves vertical loop is 15.65 ms <sup>-1</sup> .	Applying equation of energy as the block moves from A to B: $\frac{1}{2}mv^2 = mg$ (r cos a - r sinh) ......... (2)
(49)	(D).	Now, P = 0	Subting (1) and (2), we get, 3 sin $\beta = 2 \cos \alpha$ acquiring velocity of colliding mass after the collision. A Iso a required velocity must be equal to $\sqrt{5g}$ to	

**(50) (C).** As you ride, you are moving in a circle at uniform speed. You are experiencing uniform circular motion. As a result, your acceleration is always toward the center of the circle (a centripetal acceleration). Since you always feel a fictitious force in the direction opposite your acceleration, you feel one that is outward away from the center of the merry-go-round.

(51) **(B).** 
$$
\omega_A = \frac{v}{2r}
$$
  $\omega_C = \frac{v}{r}$ ;  $\therefore \frac{\omega_A}{\omega_B} = \frac{1}{2}$  but dn

- **(52) (C).** Its mechanical energy is conserved. It has a centripetal acceleration, downward. Its speed is minimum.
- **(53) (A).** The magnitude of the acceleration of the car as it rounds the curve is given by  $v^2/R$ , where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires provides the force to produce this acceleration.

The horizontal component of Newton's second law is  $f = mv^2/R$ . If N is the normal force of the road on the car (4) and m is the mass of the car, the vertical component of the second law is N – mg = 0. Thus N = mg and  $\mu_e$ N =  $\mu_e$ mg. If the car does not slip,  $f \le \mu_s$  mg.

This means  $v^2/R \le \mu_s g$ , or speed with which the car can round the curve without (5) slipping is

$$
v_{\text{max}} = \sqrt{\mu_s Rg} = \sqrt{(0.60)(30.5 \text{m})(9.8 \text{ m/s}^2)} = 13 \text{ m/s}
$$

**(54) (B).** At top point normal reaction should be zero

![](_page_35_Figure_17.jpeg)

#### **EXERCISE-2**

**(1) (C).** Let v be the speed of particle at B, just when it about to loose contact.

> From application of Newton's second law to the particle normal to the spherical surface

$$
\frac{mv^2}{r} = mg\sin b \qquad \qquad \dots \dots \dots (1)
$$

Applying conservation of energy as the block moves

Q.B. SOLUTIONS  
\nng of the forces, the motorcyclist  
\n(1) (C). Let v be the speed of particle at B, just when it about  
\nthen R = 0  
\n
$$
v_{min}^2 = \text{gr}
$$
  
\n $\frac{1}{2}mv^2 = mg \sin b$  .........(1)  
\n $\frac{1}{2}mv^2 = mg (r \cos a - r \sin b)$  .........(2)  
\n $\frac{1}{2}mv^2 = mg (r \cos a - r \sin b)$  .........(2)  
\nSolving (1) and (2), we get, 3 sin β = 2 cos α  
\n $\frac{1}{2}$  so a  
\n $\frac{1}{2}$  and a circle at uniform  
\n $\frac{1}{2}$  and a circle at uniform  
\n $\frac{1}{2}$  is the point of the velocity of colliding mass after the collision.  
\nAlso acquired velocity must be equal to  $\sqrt{5gL}$  to  
\n $\sqrt{5gL}$  to

Solving (1) and (2), we get,  $3 \sin \beta = 2 \cos \alpha$ 

**(2) (C).** As collision is elastic, freely suspended mass moves acquiring velocity of colliding mass after the collision.

Also acquired velocity must be equal to  $\sqrt{5gL}$  to complete the circular motion. 1 1 <sup>2</sup> mgL mu m(5g ) 2 2 **EXERCISE-2**<br> **EXERCISE-2**<br>
the speed of particle at B, just when it about<br>
act.<br>
act.<br>
act.<br>
act.<br>
act.<br>
servation of Newton's second law to the particle<br>
spherical surface<br>
sinb<br>  $\lim_{x \to 0} (1)$ <br>
is ervation of energy as speed of particle at B, just when it about<br>t.<br>t.<br>t.<br>m of Newton's second law to the particle<br>bherical surface<br> $mv^2 = mg (r \cos a - r \sin b)$  .........(2)<br>(2), we get, 3 sin  $\beta = 2 \cos \alpha$ <br>is elastic, freely suspended mass moves<br>ity of c 2 2 it about<br>particle<br>......... (1)<br>c moves<br>......... (2)<br>s moves<br>ollision.<br> $\overline{5gL}$  to<br> $\frac{q}{2} \otimes \frac{q}{2}$ ) <sup>2</sup> m r <sup>T</sup>

Hence, mgL + 
$$
\frac{1}{2}
$$
mu<sup>2</sup> =  $\frac{1}{2}$ m(5gℓ)  $\Rightarrow$  u =  $\sqrt{3gL}$ 

**(D).** As 
$$
2T \sin \frac{q}{2} = dmw^2r
$$
 (for small angle  $\sin \frac{q}{2} \otimes \frac{q}{2}$ )

d to perform a vertical loop is  
\n
$$
R = 0
$$
  
\n $R = 0$   
\

![](_page_35_Figure_30.jpeg)

Put m =  $2\pi$  kg,  $\omega$  = 10  $\pi$  radian/s and r = 0.25 m  $\therefore$  T = 250 N

Also acquired velocity must be equal to 
$$
\sqrt{5gL}
$$
 to  
complete the circular motion.  
ancing uniform circular motion. As a  
ion is always toward the center of  
all acceleration). Since you always  
we in the direction opposite your  
one that is outward away from the  
 $\frac{V}{}$ ;  $\therefore \frac{\omega_A}{}$  =  $\frac{1}{2}$   
 $\frac{1}{2}$  but  $\frac{dm}{m} = \frac{m}{\ell} dr$ . As  $\ell = 2pr$   $\therefore T = \frac{mv^2r}{2p}$   
 $\frac{V}{}$ ;  $\therefore \frac{\omega_A}{\omega_B} = \frac{1}{2}$   
but  $\frac{dm}{m} = \frac{m}{\ell} dr$ . As  $\ell = 2pr$   $\therefore T = \frac{mv^2r}{2p}$   
energy is conserved.  
coeleration, downward.  
of the acceleration of the car as it  
reational force of the road on the  
ratioal force of the road on the  
the top orduce this acceleration.  
moment of Newton's second law is  
normal force of the road on the car  
the car, the vertical component of the  
= 0. Thus N = mg and  $\mu_s N = \mu_s mg$ .  
 $\mu_s = \frac{m}{\ell} dr$ .  
So  $T = 250 N$   
 $\therefore T = 250 N$   
 $\therefore T = 250 N$   
 $\therefore T = \frac{mv^2}{R}$   
 $\therefore T = 250 N$   
 $\therefore T = \frac{mv^2}{R}$   
 $\$ 

Hence N decreases as  $\theta$  increases.

**(5) (A).** The maximum angular speed of the hoop corresponds to the situation when the bead is just about slide towards. The free body diagram of the bead is

![](_page_35_Figure_35.jpeg)

For the bead not to slide upwards

$$
m\omega^{2} (r\sin 45^{\circ})\cos 45^{\circ} - mg\sin 45^{\circ} < \mu N \dots (1)
$$

**(6) (A).** By energy conservation between A and B

$$
Mg\frac{2R}{5} + 0 = \frac{MgR}{5} + \frac{1}{2}Mv^2 \text{ p } v = \sqrt{\frac{2gR}{5}}
$$

Now, radius of curvature 
$$
r = \frac{v^2}{a_r} = \frac{2gR}{g\cos 37^\circ} = \frac{R}{2}
$$

![](_page_36_Figure_7.jpeg)

**(7) (A).** It can be observed that component of acceleration perpendicular to velocity is  $a = 4$  m/s<sup>2</sup>

$$
\therefore \text{ Radius} = \frac{v^2}{a_c} = \frac{(2)^2}{4} = 1 \text{ metre}
$$

**(8) (A).** Let  $\omega$  be the angular speed of rotation. It is obvious from the figure that the ball moves in a horizontal centre

centre will be equal to  $(\ell + \ell \sin \theta) \omega^2$ . (A). It can be observed that component of acceleration<br>
expendicular to velocity is a = 4 m/s<sup>2</sup><br>  $\therefore$  Radius =  $\frac{v^2}{a_e} = \frac{(2)^2}{4} = 1$  metre<br>
(A). Let to be the angular speed of rotation. It is obvious<br>
from the figur Example that component of acceleration<br>
Let can be observed that component of acceleration<br>
Let to be the angular speed of rotation. It is obvious<br>
Let to be the angular speed of rotation this obvious<br>
diation (*t* + *s* r can be observed una component of acceleration<br>
andicular to velocity is a = 4 m/s<sup>2</sup><br>
adius =  $\frac{v^2}{a_e} = \frac{(2)^2}{4} = 1$  metre<br>
Let  $\omega$  be the angular speed of rotation. It is obvious<br>
the figure that the ball moves in

$$
\therefore \tan \theta = \frac{\omega^2 (\ell + \ell \sin \theta)}{g}
$$

or 
$$
\omega^2 = \frac{g \tan \theta}{\ell(1 + \sin \theta)} = \frac{10 \times (1/\sqrt{3})}{(0.20)[1 + (1/2)]}
$$

Solving, we get,  $\omega$  = 4.4 rad/sec.

(9) **(B).** 
$$
\omega_1 = \frac{2\pi}{1} \text{ rad/hr.}
$$
,  $\omega_2 = \frac{2\pi}{8} \text{ rad/hr.}$  (14) (B).

∴ Radius = 
$$
\frac{v^2}{a_e} = \frac{(2)^2}{4} = 1
$$
 metre  
\n(A). Let  $\omega$  be the angular speed of rotation. It is obvious  
\nfrom the figure that the ball moves in a horizontal centre  
\nof radius ( $\ell + \ell \sin \theta$ ) and its acceleration towards the  
\ncentre will be equal to ( $\ell + \ell \sin \theta$ ) so<sup>2</sup>.  
\nHere, we have T cos θ = mg  
\nand T sin θ = m ∘ $\ell$ ( $\ell + \ell \sin \theta$ ) (12)  
\nand T sin θ = m ∘ $\ell$ ( $\ell + \ell \sin \theta$ ) (20011-1112)  
\nand T sin θ = m ∘ $\ell$ ( $\ell + \ell \sin \theta$ ) (30042-  
\nand T sin θ = m ∩ $\ell$ ( $\ell + \ell \sin \theta$ ) (412)  
\n∴ tan θ =  $\frac{\omega^2(\ell + \ell \sin \theta)}{g}$  (120011-1112)  
\n∴ C. Let R and S be C  
\n $\omega$  and T sin θ = m √ $\omega$  ∴  $\omega$  and T sin θ = m √ $\omega$  ∞<sup>2</sup> ∞<sup>2</sup> ∞<sup>2</sup> ∞<sup>2</sup> √ $\omega$   
\nand T sin θ = m ∞ $\ell$  (1+ sin θ)  
\n $\omega$  = 2π ln θ  
\n∴ tan θ =  $\frac{\omega^2(\ell + \ell \sin \theta)}{g}$  (13) (15) The nature of motion can be determined only if we  
\n $\omega$  is the acceleration as a function of time.  
\nHence acceleration at an instant is given and not known at  
\nthere acceleration at an instant is given and not known at  
\nthere are determined, and the amount of time.  
\nHence acceleration at an instant is given and not known at  
\ntherefore at m is given and not known at  
\ntherefore at m is given and not known at  
\ntherefore at m =  $\frac{\sin \theta}{\tan \theta} = \frac{10 \times (1/\sqrt{3})}{(1/\sqrt{3})}$   
\n∴ A mgle covered by A = p +  $\frac{\sqrt{t}}{t}$  Put t,  
\n $V_1 = \frac{2\pi R_1}{10} = 2\pi \times 10^4$  km/hr  
\n

At closest separation

$$
\omega = \frac{V_{rel} \perp \text{ to line joining}}{\text{length of line joining}} = \frac{\pi \times 10^4 \text{ km/hr}}{3 \times 10^4 \text{ km}} = \frac{\pi}{3} \text{ rad/hr}.
$$

WE DREAD INTERTAIL: PHYSICS<br>
Where N = mg cos 45° + mw<sup>2</sup> (r sin 45°)r sin 45° ....... (2)<br>
From (1) and (2) we get,  $w = \sqrt{30\sqrt{2}}$  rad/s<br>
(A). By energy conservation between A and B<br>  $Mg \frac{2R}{5} + 0 = \frac{MgR}{5} + \frac{1}{2}Mv^2 P$ **CO.B.- SOLUTIONS**<br>
ANDERIAL: PHYSICS<br>
MORE N = mg cos 45° + mw<sup>2</sup> (rsin 45°)rsin 45° ...... (2)<br>
From (1) and (2) we get, w =  $\sqrt{30\sqrt{2}}$  rad/s<br>
(A). By energy conservation between A and B<br>
Mg  $\frac{2R}{5} + 0 = \frac{MgR}{5} + \frac{$ **STUDY MATERIAL: PHYSICS**<br>
losest separation<br>
V<sub>rel</sub>  $\perp$  to line joining  $= \frac{\pi \times 10^4 \text{ km/hr}}{3 \times 10^4 \text{ km}} = \frac{\pi}{3} \text{ rad/hr}$ .<br>
The friction force on coin just before coin is to slip<br>
e: f =  $\mu_s mg$ <br>
mal reaction on the co **STUDY MATERIAL: PHYSICS**<br>
losest separation<br>  $\frac{V_{rel} \perp}{V_{len}}$  to line joining  $= \frac{\pi \times 10^4 \text{ km/hr}}{3 \times 10^4 \text{ km}} = \frac{\pi}{3} \text{ rad/hr}$ .<br>
The friction force on coin just before coin is to slip<br>
e: f =  $\mu_s mg$ <br>
resultant reacti **STUDY MATERIAL: PHYSICS**<br>
st separation<br>  $\perp$  to line joining  $= \frac{\pi \times 10^4 \text{ km/hr}}{3 \times 10^4 \text{ km}} = \frac{\pi}{3} \text{ rad/hr}$ .<br>
friction force on coin just before coin is to slip<br>  $= \mu_s mg$ <br>
eaction on the coin, N = mg S<br>
S<br>
At closest separation<br>  $\omega = \frac{V_{rel} \perp}{I_{ength}}$  to line joining  $= \frac{\pi \times 10^4 \text{ km/hr}}{3 \times 10^4 \text{ km}} = \frac{\pi}{3} \text{ rad/hr}$ .<br>
(C). The friction force on coin just before coin is to slip<br>
will be : f =  $\mu_s mg$ <br>
Normal reaction o **(10) (C).** The friction force on coin just before coin is to slip will be :  $f = \mu_{\rm s}mg$ Normal reaction on the coin,  $N = mg$ 

The resultant reaction y disk to the coin

Q.B. SOLUTIONS	STUDY MATERIAL: PHYSICS
\n        after N = mg cos 45° + mw² (r sin 45°) r sin 45° ...... (2)\n        or (1) and (2) we get, $w = \sqrt{30\sqrt{2}}$ rad/s\n        (10) (C). The friction force on coin just before coin is to slip will be: $f = \mu_s mg$ \n        Now, radius of curvature\n $r = \frac{v^2}{a_r} = \frac{2gR/5}{g\cos 37°} = \frac{R}{2}$ \n	\n        (11) (D). Direction of velocity of the particle may be shown as in fig. This velocity $v_0$ is resolved in two components. Here $V_0 \cos \alpha$ component is tangential in horizontal and line.\n

**EXECUTIONS**<br> **EXECUTIONS (Q.B.- SOLUTIONS**<br>
At closest separation<br>  $^2$ (rsin 45°)rsin 45° .......(2)<br>  $= \sqrt{30\sqrt{2}}$  rad/s<br>  $\sqrt{2}$  rad/s<br>
(10) (C). The friction force on coin just before coin is to slip<br>
between A and B<br>  $\sqrt{2}$  P  $v = \sqrt{\frac{2gR}{5}}$ **STUDY MATERIAL: PHYSICS**<br>
closest separation<br>  $\frac{V_{rel} \perp}{\text{length of line joining}} = \frac{\pi \times 10^4 \text{ km/hr}}{3 \times 10^4 \text{ km}} = \frac{\pi}{3} \text{ rad/hr}$ .<br>
The friction force on coin just before coin is to slip<br>
be : f =  $\mu_s mg$ <br>
resultant reaction y disk to **STUDY MATERIAL: PHYSICS**<br>
eest separation<br>  $\frac{1}{\log \theta} \frac{1}{\theta} + \frac{1}{\theta} \frac{1}{\theta} \frac{1}{\theta} \frac{1}{\theta} \frac{1}{\theta}$  fine joining  $= \frac{\pi \times 10^4 \text{ km/hr}}{3 \times 10^4 \text{ km}} = \frac{\pi}{3} \text{ rad/hr}$ .<br>
If  $\theta = \frac{\pi}{1000} \text{ rad}$  is to slip reaction on th **(11) (D).** Direction of velocity of the particle may be shown as in fig. This velocity  $v_0$  is resolved in two components. Here  $V_0$  cos  $\alpha$  component is tangential horizontal and  $V_0$  sin  $\alpha$  is along the surface but vertically downwards. Here, if the particle exerts a force N on the surface of the cylinder, the cylinder will also apply equal and opposite force on the particle 10<sup>-3</sup> ' 10'  $\sqrt{1 + \frac{9}{16}} = 0.5 \text{ N}$ <br>
ection of velocity of the particle may be shown as<br>
his velocity v<sub>0</sub> is resolved in two components.<br>
cos  $\alpha$  component is tangential horizontal and<br>
the particle exerts a force N 10<sup>-1</sup> 10  $\sqrt{1 + \frac{16}{16}} = 0.5 \text{ N}$ <br>
cction of velocity of the particle may be shown as<br>
his velocity v<sub>0</sub> is resolved in two components.<br>
cos  $\alpha$  component is tangential horizontal and<br>
is along the surfice but vertica

![](_page_36_Figure_23.jpeg)

and this will provide centripetal force to the particle. So, N = m ( $V_0 \cos \alpha$ )<sup>2</sup>/R = mV<sub>0</sub><sup>2</sup> cos<sup>2</sup> $\alpha$ /R

or, N = 
$$
\left(\frac{mV_0^2}{R}\right)\cos^2\alpha
$$

**(12) (C).** Let speed of belt be v Angular speeds of wheels

$$
\omega_{\rm B} = \frac{\rm v}{2\pi R_{\rm B}}
$$
,  $\omega_{\rm A} = \frac{\rm v}{2\pi R_{\rm A}}$ ;  $\frac{\omega_{\rm A}}{\omega_{\rm B}} = \frac{R_{\rm B}}{R_{\rm A}} = 2$ 

- be observed that component of acceleration<br>
lat to velocity is a = 4 m/s<sup>2</sup><br>  $\frac{v^2}{a_c} = \frac{(2)^2}{4} = 1$  metre<br>
gue that the ball moves in a britization. It is obvious<br>
gue that the ball moves in a britization tenter<br>  $f(t + \$ rad / hr. and this will provide centripetal force to the spin of the spin of the spin of the section of the spin of the spi  $\therefore$ <br>  $V_0 \cos \alpha$ <br>  $V_0$ <br>  $R_A$ <br>  $R_A$ <br>  $R_A$ <br>  $V_0$ <br>  $R_B$ <br>  $R_A$ <br>  $V_0$ <br>  $V_0$ <br>  $R_B$ <br>  $V_0$ <br>  $V_0$ <br>  $R_B$ <br>  $V_1$ <br>  $\left\{\n\begin{array}{c}\n\searrow \searrow \\
\searrow \searrow \\
\searrow \searrow\n\end{array}\n\right\}$ <br>  $\frac{A}{B^S} = \frac{R_B}{R_A} = 2$ <br>
letermined only if we<br>
function of time.<br>
ven and not known at **(13) (D).** The nature of motion can be determined only if we know velocity and acceleration as function of time. Here acceleration at an instant is given and not known at other times so D. d this will provide centripetal force to the particle.<br>  $N = m (V_0 \cos \alpha)^2/R = mV_0^2 \cos^2 \alpha/R$ <br>  $N = \left(\frac{mV_0^2}{R}\right) \cos^2 \alpha$ <br>  $N = \left(\frac{mV_0^2}{R}\right) \cos^2 \alpha$ <br>  $N = \left(\frac{mV_0^2}{R}\right) \cos^2 \alpha$ <br>  $N = \frac{V_0}{2\pi R_B}$ ,  $\omega_A = \frac{V_0}{2\pi R_A}$ ;  $\frac{\omega_A}{\omega$ s will provide centripetal force to the particle.<br>
= m (V<sub>0</sub> cos α)<sup>2</sup>/R = mV<sub>0</sub><sup>2</sup> cos<sup>2</sup>α/R<br>
=  $\left(\frac{mV_0^2}{R}\right)$  cos<sup>2</sup>α<br>
=  $\left(\frac{mV_0^2}{R}\right)$  cos<sup>2</sup>α<br>
=  $\frac{v}{2\pi R_B}$ ,  $ω_A = \frac{v}{2\pi R_A}$ ,  $\frac{ω_A}{ω_B} = \frac{R_B}{R_A} = 2$ <br>
en We vising  $\frac{V_{\text{e}}}{V_{\text{e}}}\sqrt{V_{\text{e}}}\cos \alpha$ <br>  $V_{\text{e}}$  will provide centripetal force to the particle.<br>  $\frac{mV_0^2}{R} \cos^2 \alpha/R$ <br>
speed of belt be v<br>
speed of belt be v<br>
speed of wheels<br>  $\frac{v}{2\pi R_B}$ ,  $\omega_A = \frac{v}{2\pi R_A}$ ;  $\$ V<sub>sima</sub><br>
his will provide centripetal force to the particle.<br>
N = m (V<sub>0</sub> cos  $\alpha$ )<sup>2</sup>/R = mV<sub>0</sub><sup>2</sup> cos<sup>2</sup> $\alpha$ /R<br>
Let speed of belt be v<br>
alar speeds of wheels<br>  $a = \frac{v}{2\pi R_B}$ ,  $\omega_A = \frac{v}{2\pi R_A}$ ;  $\frac{\omega_A}{\omega_B} = \frac{R_B}{R_A} = 2$ the particle.<br>
(R<br>  $\frac{R_B}{R_A} = 2$ <br>
mined only if we<br>
ion of time.<br>
and not known at<br>  $\frac{5pR}{6v}$ <br>
Put t, tal force to the particle.<br>  $V_0^2 \cos^2 \alpha / R$ <br>  $\frac{\omega_A}{\omega_B} = \frac{R_B}{R_A} = 2$ <br>
in be determined only if we<br>
on as function of time.<br>
it is given and not known at<br>  $\frac{\pi}{R}$   $\therefore t = \frac{5pR}{6v}$ <br>  $\frac{p}{R}$  Put t,<br>  $\frac{p}{2}$ <br>  $\frac{\pi}{2$ (12) **(C).** Let speed of belt be v<br>
Angular speeds of wheels<br>  $\omega_B = \frac{v}{2\pi R_B}$ ,  $\omega_A = \frac{v}{2\pi R_A}$ ;  $\frac{\omega_A}{\omega_B} = \frac{R_B}{R_A} = 2$ <br> **(13) (D)**. The nature of motion can be determined only if we<br>
know velocity and acceleratio
	- $\pi$  (14) **(B).** As when they collide

$$
vt + \frac{1}{2} \frac{\alpha 72 v^2}{\xi 25pR} \frac{v}{\omega} t^2 - pR = vt \quad \therefore \quad t = \frac{5pR}{6v}
$$

Now, angle covered by  $A = p + \frac{vt}{R}$  Put t,

$$
\therefore \text{ Angle covered by A} = \frac{11p}{6}
$$
  
5) (A). V = 27 kmph = 7.5 m/sec  

$$
\vec{r} = L\hat{i} + 30\hat{j} = 30 \tan \theta \hat{i} + 30\hat{j}
$$

![](_page_37_Picture_2.jpeg)

**EXECUTE** A.10710N  
\n
$$
\dot{V} = \frac{df}{dt} \div 7.5 - 30 \sec^2 \theta \left(\frac{d\theta}{dt}\right) \hat{i}
$$
\n(19) (A).  $v - mg = \frac{mv^2}{R} \div v^2 = 2ge$ ;  $R = \frac{\left[1 + (dy/dx)^2\right]^{2/2}}{d^2y/dx^2}$   
\n
$$
7.5 = 30 \sec^2 \theta \omega \div 7.5 = 30 \sec^2 45^{\circ} \omega
$$
\n
$$
\frac{7.5}{30 \times 2} = \omega = \frac{7.6}{6} = 0.125 \text{ rad/sec}
$$
\n
$$
\frac{V}{V} = \omega = \frac{1}{6} \omega \times 125 \text{ rad/sec}
$$
\n(10) (C). T - mag in  $q = \frac{mv^2}{R}$   
\n
$$
V = \omega = \frac{V}{V}
$$
\n
$$
V = \omega = \frac{V}{V}
$$
\n(11) (A).  $y = \log \sec\left(\frac{x}{a}\right)$ ,  $\frac{d\theta}{dt} = w$  (constant)  
\n
$$
\frac{dy}{dx} = \tan\left(\frac{a}{a}\right)
$$
\n
$$
\frac{dV}{dx} = \frac{1}{2} \sec^2\left(\frac{x}{a}\right)
$$
\n(12) (A).  $y = \log \sec\left(\frac{x}{a}\right)$ ,  $\frac{d\theta}{dt} = w$  (constant)  
\n
$$
\frac{dy}{dx} = \tan\left(\frac{x}{a}\right)
$$
\n
$$
\frac{dV}{dx} = \frac{1}{2} \sec^2\left(\frac{x}{a}\right)
$$
\n(22) (B).  $a_x = a_x = 3 \sin 6x$ ,  $a_y = -2 \sin^{-2} x$   
\n
$$
V = -2 \sec^2 x \sin^2 x
$$
\n(23) (B).  $a_x = a_y = 5 \sin 6x$ ,  $a_y = -\frac{y^2}{c} = \frac{u^2 - 2g'(t - \cos \theta)}{u^2 - 2g(t - \cos \theta)}$   
\nRadius of curvature  $= \left[\frac{1 + (\frac{dy^2}{dy^2})}{d^2x} = \frac$ 

(18) **(D).** 
$$
a_r \propto t
$$
;  $\frac{v^2}{r} = kt$ ;  $v^2 = krt$ ;  $2v \frac{dv}{dt} = kr$   
\n
$$
a_t = \frac{kr}{2v} = \frac{kr}{2\sqrt{rkt}} = \frac{1}{2} \sqrt{\frac{kr}{t}}
$$
;  $a_t^2 a_r = \frac{1}{4} \frac{kr}{t} \times kt$   
\n= independent on t  
\n313

7.5 = 30 sec<sup>2</sup> 
$$
\theta
$$
  $\left(\frac{d\theta}{dt}\right)$   $\hat{i}$   
\n7.5 = 30 sec<sup>2</sup>  $\theta$   $\left(\frac{d\theta}{dt}\right)$   $\hat{i}$   
\n7.6 = 30 sec<sup>2</sup>  $\theta$   $\left(\frac{d\theta}{dt}\right)$   $\hat{i}$   
\n8. (19) (A). N – mg =  $\frac{mv^2}{R}$ ;  $v^2 = 2gc$ ;  $R = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2}$   
\n9. (A). N – mg =  $\frac{mv^2}{R}$ ;  $v^2 = 2gc$ ;  $R = \frac{b^2 dy}{dx}$ ;  $2 = \frac{b^2 dy}{c dx^2}$   
\nA. (0, 0) dy/dx = 0  
\nB.  $\frac{b^2}{2c}$ ; N – mg =  $\frac{m2gc(2c)}{b^2}$ ; N = mg $\left(1 + \frac{4c^2}{b^2}\right)$   
\n(a) (B). T<sub>max</sub> =  $\mu$  8g; T<sub>max</sub> - 2g = mw<sup>2</sup>/r  
\n(b) . a<sub>c</sub> =  $\frac{v^2}{r} = \omega^2 r$ ; T = 15  
\n3. (a)  $\frac{dv}{dx} = \frac{1}{a} \sec^2 \left(\frac{x}{a}\right)$   
\n(b) . a<sub>c</sub> =  $\frac{v^2}{r} = \omega^2 r$ ; T = 15  
\n(b) . A + B, N + kx – mg = mv<sup>2</sup>/r; N = 0  
\n6. A + B, N + kx – mg = mv<sup>2</sup>/r; N = 0  
\n6. A + B, N + kx – mg = mv<sup>2</sup>/r; N = 0  
\n6. A + B, N + kx – mg = mv<sup>2</sup>/r; N = 0  
\n6. A + B, N

$$
R = \frac{b^2}{2c} \quad ; \quad N - mg = \frac{m2gc (2c)}{b^2} \quad ; \quad N = mg \left(1 + \frac{4c^2}{b^2}\right)
$$

$$
x^{2} = 4ay = \frac{b^{2}}{c}y ; 2x = \frac{b^{2}}{c} \frac{dy}{dx} ; 2 = \frac{b^{2}}{c} \frac{d^{2}y}{dx^{2}}
$$
  
At (0, 0) dy/dx = 0  

$$
R = \frac{b^{2}}{2c} ; N - mg = \frac{m2gc (2c)}{b^{2}} ; N = mg \left(1 + \frac{4c^{2}}{b^{2}}\right)
$$
  
(20) **(B).**  $T_{max} = \mu$  8g;  $T_{max} - 2g = mv^{2}/r$   
 $v^{2} = 2gh = 2gr \cos 60^{\circ} = gr$   
 $T_{max} - 2g = 2gr/r ; T_{max} - 4g ; 4g = \mu$  8g  $\Rightarrow \mu = 1/2$   
(21) **(D).**  $a_{c} = \frac{v^{2}}{r} = \omega^{2}r ; r = 15$   
(22) **(D).**  $a_{r} = a_{t} = g \sin \theta ; a_{r} = \frac{v^{2}}{\ell} = \frac{u^{2} - 2g\ell (1 - \cos \theta)}{\ell}$   
 $u^{2} = 3g\ell ; \theta = 90^{\circ}$   
(23) **(B).** At B, N + kx – mg = mv<sup>2</sup>/r ; N = 0  
 $k \frac{20}{100} - 5g = \frac{5v^{2}}{20/100}$  ......... (1)  
To find v use energy conservation,  
 $mgr (1 + \cos 60^{\circ}) = \frac{1}{2} mv^{2} + \frac{1}{2} kx^{2}$  ......... (2)  
Solving eq. (1) and (2), k = 500 N/m  
(24) **(B).** T – mg cos  $\theta = \frac{mv^{2}_{min}}{\ell}$ ; T = 0  
 $v^{2}_{min} = u^{2} - 2g\ell (1 - \cos \theta) = g\ell + 2g\ell \cos \theta$ 

(21) **(D).** 
$$
a_c = \frac{v^2}{r} = \omega^2 r
$$
;  $r = 15$ 

At  $(0, 0)$  dy/dx = 0

(22) **(D).** 
$$
a_r = a_t = g \sin \theta
$$
;  $a_r = \frac{v^2}{\ell} = \frac{u^2 - 2g\ell (1 - \cos \theta)}{\ell}$ 

$$
u^2 = 3g\ell \ ; \ \theta = 90^\circ
$$

**(23) (B).** At B, N + kx – mg = mv<sup>2</sup> /r ; N = 0

$$
k\frac{20}{100} - 5g = \frac{5v^2}{20/100}
$$
 ......(1)

To find v use energy conservation,

$$
mgr (1 + \cos 60^\circ) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2
$$
 ......... (2)  
Solving eq. (1) and (2), k = 500 N/m

(20) (B). T<sub>max</sub> = 
$$
\mu
$$
 8g; T<sub>max</sub> - 2g = m<sup>2</sup>/r  
\n $v^2 = 2gh = 2gr$  (r; T<sub>max</sub> = 4g; 4g =  $\mu$  8g  $\Rightarrow \mu = 1/2$   
\n(21) (D).  $a_c = \frac{v^2}{r} = \omega^2 r$ ; T<sub>max</sub> = 4g; 4g =  $\mu$  8g  $\Rightarrow \mu = 1/2$   
\n(22) (D).  $a_c = \frac{v^2}{r} = \omega^2 r$ ; r = 15  
\n  
\n*(*22) (D).  $a_c = \frac{v^2}{r} = \omega^2 r$ ; r = 15  
\n  
\n*(*23) (B). At B, N + kx = mg = mv<sup>2</sup>/r; N = 0  
\n $\frac{2}{\sqrt{100}} = 5g = \frac{5v^2}{20/100}$  .........(1)  
\nTo find v use energy conservation,  
\n $\frac{dy}{dx} \times \frac{dx}{dt} = \left[\tan\left(\frac{x}{a}\right)\right]aw$  (24) (B). T – mg cos  $\theta = \frac{mv_{min}^2}{m}$ ; T = 0  
\n $v_{min}^2 = u^2 - 2g(f(1 - \cos \theta)) = gf + 2g f \cos \theta$   
\n $v^2 = 3 \cos \theta = g + 2g \cos \theta$ ; T = 0  
\n $v_{min}^2 = u^2 - 2g(f(1 - \cos \theta)) = g(f + 2g f \cos \theta)$   
\n $v^2 = 3 \cos \theta = g + 2g \cos \theta$ ; T = 0  
\n $v_{min}^2 = \frac{u^2}{\sqrt{3}}$ ; v<sub>min</sub> =  $\sqrt{\frac{g^2}{3}}$   
\n $\sqrt{0 + a^2w^4 \sec^4(\frac{x}{a})}$   
\n(25) (B). T =  $\frac{mv^2}{r} - 2mg + 3mg \cos \theta$   
\n $0 = \frac{m(2 + \sqrt{3})ag}{a} - 2mg + 3mg \cos \theta$   
\n $8 \times \frac{1}{4} \times$ 

$$
0 = \frac{m(2+\sqrt{3}) \text{ ag}}{a} - 2\text{mg} + 3\text{mg}\cos\theta
$$

$$
-\sqrt{3}mg = 3mg\cos\theta; \quad \cos\theta = \frac{-1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)
$$

## **EXERCISE-3**

(25) **(B).** T = 
$$
\frac{mv^2}{r}
$$
 - 2mg + 3mg cos θ  
\n
$$
0 = \frac{m(2 + \sqrt{3})ag}{a} - 2mg + 3mg cos \theta
$$
\n
$$
-\sqrt{3}mg = 3mg cos \theta; cos \theta = \frac{-1}{\sqrt{3}} \Rightarrow \theta = cos^{-1}(\frac{-1}{\sqrt{3}})
$$
\n**EXERCISE-3**  
\n(1) 5.  $a_t = \frac{du}{dt} = 4t = 4m/s^2$   
\n
$$
a_n = \frac{v^2}{R} = \frac{4}{4/3} = 3 m/s^2 \Rightarrow a = \sqrt{a_t^2 + a_c^2} = 5
$$

**(2) 3.** The angular displacement of the particle in  $t = 1$  sec. is

![](_page_38_Figure_4.jpeg)

 $\therefore$  The magnitude of impulse by centripetal force in t = 1

**(3) 3.** Using relation  $\theta = \omega_0 t + \frac{1}{2}at^2$ .  $\frac{1}{2}$  at<sup>2.</sup>

In the first case,  $\theta_1 = \frac{1}{2}a$ .  $4 = 2a$ ,  $\omega = 0 + 2a$ ; In the second

case, 
$$
\theta_2 = 2a \cdot 2 + \frac{1}{2}a \cdot 4 = 6a
$$
 {  $\omega_0 = 2a$   $\therefore \frac{\theta_2}{\theta_1} = \frac{3}{1}$  }

**(4) 5.** After 1 second angular velocity of the turntable and hence that of the coin about the axis of rotation is

$$
\omega = 0 + 2(\text{rad/s}) \times 1 \text{ s} = 2 \text{ rad/s} \qquad \text{giv}
$$
\n
$$
a_{\text{T}} = \alpha \text{r} = (2 \text{ rad/s}^2) \times 1 \text{ m} = 2 \text{ m/s}^2
$$
\n
$$
a_{\text{R}} = \omega^2 \text{r} = (2 \text{ rad/s})^2 \times 1 \text{ m} = 4 \text{ m/s}^2 \qquad \text{hen}
$$
\n
$$
\therefore a = \sqrt{a_{\text{T}}^2 + a_{\text{R}}^2} = 2\sqrt{5} \text{ m/s}^2 \qquad \text{But}
$$

**(5) 2.** Let m be the mass of the train and  $\theta$  the angle at which the rail track is banked.

![](_page_38_Figure_12.jpeg)

(a) Horizontally, 
$$
R \sin \theta - P \cos \theta = m \frac{10^2}{10^3}
$$
 (1) (B).

(1)

(1) gives, 
$$
R \sin \theta = \frac{1}{10} m + P \cos \theta
$$
. (2) gives,

$$
R\cos\theta = mg - P\sin\theta
$$

Q.B.- SOLUTIONS STUD<br>
yular displacement of the particle in t = 1 sec. is  $\Rightarrow$  mg sin  $\theta - \frac{1}{10}$  m cos  $\theta = P$ <br>
(b) Horizontally,  $S \cos \theta - P \sin \theta = n$ <br>
vertically,  $S \cos \theta - P \sin \theta = n$ <br>
agnitude of impulse by centripetal force in EARRING<br>
EARRING<br>
angular displacement of the particle in t = 1 sec. is  $\Rightarrow$  mg sin  $\theta - \frac{1}{10}$  m cos  $\theta =$ <br>  $t = \frac{v}{R}t = \frac{3\pi}{2}$ <br>
e magnitude of impulse by centripetal force in t = 1<br>
ds is = change in momentum =  $\sqrt$ WE gular displacement of the particle in t = 1 sec. is  $\Rightarrow$  mg sin  $\theta - \frac{1}{10} \text{ m} \cos \theta = P$  (<br>  $\frac{v}{R}t = \frac{3\pi}{2}$ <br>  $\frac{v}{R} = \frac{3\pi}{2}$ <br>  $\frac{v}{R} = \frac{3\pi}{2}$ <br>  $\frac{2\pi}{2}$ <br>  $\frac{2\pi}{2}$ <br>  $\frac{2\pi}{2}$ <br>  $\frac{2\pi}{2}$ <br>  $\frac{2\pi}{2}$ **13.** The angular displacement of the particle in t = 1 sec. is  $\theta = \omega t = \frac{v}{R} t = \frac{3\pi}{2}$ <br>  $\theta = \omega t = \frac{v}{R} t = \frac{3\pi}{2}$ <br>  $\therefore$  The magnitude of impulse by centripetal force in t = 1 (3) gives,  $S \sin \theta = \frac{4}{10} m - 1$ <br>
secon SECONDENSITY SECOND TIONS<br>
3. The angular displacement of the particle in t = 1 sec. is  $\Rightarrow \text{mg}\sin\theta - \frac{1}{10}\text{m}\cos\theta = P$  ( $\cos^2\theta + \sin^2\theta = 1$ )<br>  $\theta = \omega t = \frac{v}{R}t = \frac{3\pi}{2}$ <br>  $\therefore$  The magnitude of impulse by centripetal force  $\frac{\theta_2}{\theta_3} = \frac{3}{2}$   $\Rightarrow P = \frac{1}{10} \text{ m} \cos \theta - \text{ m} \text{ g} \sin \theta$  $1^{j}$  $\theta_1$   $1^{\frac{3}{2}}$ **Example 1.1.** The magnitarial displacement of the particle in t = 1 sec. is  $\Rightarrow$  mg  $\sin \theta - \frac{1}{10} \text{m} \cos \theta = P \cos^2 \theta + \sin^2 \theta = 1$ <br>  $= \cot - \frac{v}{R}t - \frac{3\pi}{2}$ <br>  $= \cot - \frac{v}{R}t - \frac{3\pi}{2}$ <br>
The magnitude of impulse by eentripetal momentum =  $\sqrt{2}mv = 3\sqrt{2}\pi$  Ns<br>  $y_t + \frac{1}{2}at^2$ <br>  $y_t + \frac{1}{2}at^2$ <br>  $y_t = 2a$ ,  $\omega = 0 + 2a$ ; In the second<br>  $= 6a$  { $\omega_0 = 2a$  .:  $\frac{\theta_2}{\theta_1} = \frac{3}{1}$ }<br>  $= 2rad/s$ <br>  $= 1$ <br>  $y_t = 2d$  and  $y_t = 2d$ <br>  $= 2d$  and  $y_t = 2d$ <br>  $= 2d$ <br> The angular displacement of the particle in t = 1 sec. is  $\Rightarrow$  mg sin  $\theta - \frac{1}{10}$  moos  $\theta = P$  (cos<sup>2</sup>  $\theta + \sin^2 \theta = 1$ )<br>  $-\cot = \frac{v}{R}t - \frac{3\pi}{2}$ <br>
The magnitude of impulse by centripetal force in t = 1<br>
The magnitude of im SEC,  $θ_1 = \frac{1}{2}a$ ,  $4 = 2a$ ,  $ω = 0 + 2a$ ; In the second<br>  $1 \cdot 2 + \frac{1}{2}a$ ,  $4 = 6a$  { $ω_0 = 2a$  ∴  $\frac{θ_2}{θ_1} = \frac{3}{1}$ }<br>
SECOND angular velocity of the turntable and<br>  $2(\text{rad/s}^2) \times 1 \text{ s} = 2 \text{ rad/s}$ <br>  $= (2 \text{ rad/s}^2) \times 1 \text{ m$ (b) Horizontally, Soinet P cose  $\theta = m$  and  $\theta = 2a$ .  $2 + \frac{1}{2}a$ . Because the real of impulse by centripetal force in t = 1<br>
Sing relation  $\theta = \omega_0 t + \frac{1}{2}at^2$ <br>
Sing relation  $\theta = \omega_0 t + \frac{1}{2}at^2$ <br>
Sing relation  $\theta = \omega_$ -oa (a θ<sub>0</sub> - 2a . · θ<sub>1</sub> 1<sup>j</sup><br>
ular velocity of the turntable and<br>
about the axis of rotation is<br>  $\times 1$  s = 2rad/s<br>  $\times 1$  s = 2rad/s<br>  $\times 1$  m = 4 m/s<sup>2</sup><br>  $\times 1$  m = 4 m/s<sup>2</sup><br>
hence tan θ =  $\frac{1}{4g}$  = 0.0255<br>
But t  $\Rightarrow$  mg sin  $\theta - \frac{1}{10}$  m cos  $\theta = P$  (cos<sup>2</sup>  $\theta$  + sin<sup>2</sup>  $\theta$  = 1) **STUDY MATERIAL: PHYSICS**<br>
mg sin  $\theta - \frac{1}{10}$  m cos  $\theta = P$  (cos<sup>2</sup>  $\theta$  + sin<sup>2</sup>  $\theta$  = 1)<br>
Horizontally, Ssin  $\theta$  + P cos  $\theta$  = m  $\frac{20^2}{10^3}$  ......... (3)<br>
tically, Scos  $\theta$  – P sin  $\theta$  = mg ........ (4) STUDY MATERIAL: PHYSICS<br>  $\theta - \frac{1}{10} m \cos \theta = P \quad (\cos^2 \theta + \sin^2 \theta = 1)$ <br>
tally,  $S \sin \theta + P \cos \theta = m \frac{20^2}{10^3} \dots (3)$ <br>  $S \cos \theta - P \sin \theta = mg \qquad \qquad \dots (4)$ (b) Horizontally,  $S \sin \theta + P \cos \theta = m \frac{20^2}{10^3} \dots (3)$ STUDY MATERIAL: PHYSICS<br>  $\cos \theta = P \quad (\cos^2 \theta + \sin^2 \theta = 1)$ <br>  $S \sin \theta + P \cos \theta = m \frac{20^2}{10^3} \dots (3)$ <br>  $P \sin \theta = mg \qquad \dots (4)$ <br>  $\frac{4}{10}m - P \cos \theta \qquad (4) \text{ gives,}$ <br>  $n \theta$ **TUDY MATERIAL: PHYSICS**<br>
= P  $(\cos^2 \theta + \sin^2 \theta = 1)$ <br>  $\theta + P \cos \theta = m \frac{20^2}{10^3}$  ......... (3)<br>  $\theta = mg$  ......... (4)<br>
- P  $\cos \theta$  (4) gives, S<br>
S<br>
S<br>
S<br>
TUDY MATERIAL: PHYSICS<br>  $\Rightarrow$  mg  $\sin \theta - \frac{1}{10} \text{m} \cos \theta = P$  ( $\cos^2 \theta + \sin^2 \theta = 1$ )<br>
(b) Horizontally,  $S \sin \theta + P \cos \theta = m \frac{20^2}{10^3}$  ........ (3)<br>
Vertically,  $S \cos \theta - P \sin \theta = mg$  ........ (4)<br>
(3) gives,  $S \sin \theta = \frac{4}{10$ (3) gives,  $S\sin\theta = \frac{1}{10}m - P\cos\theta$  (4) gives, **STUDY MATERIAL: PHYSICS**<br>  $-\frac{1}{10} \text{m} \cos \theta = P \quad (\cos^2 \theta + \sin^2 \theta = 1)$ <br>
tally, Ssin  $\theta + P \cos \theta = m \frac{20^2}{10^3}$  ........(3)<br>
Scos  $\theta - P \sin \theta = mg$  ........(4)<br>
Ssin  $\theta = \frac{4}{10} \text{m} - P \cos \theta$  (4) gives,<br>  $mg + P \sin \theta$ <br>
ves  $\frac{\sin \theta}{\cos \theta}$ STUDY MATERIAL: PHYSICS<br>  $\cdot$ mcos  $\theta = P$  (cos<sup>2</sup>  $\theta$  + sin<sup>2</sup>  $\theta$  = 1)<br>
Ssin  $\theta$  + Pcos  $\theta$  = m  $\frac{20^2}{10^3}$  ......... (3)<br>  $\theta$  – Psin  $\theta$  = mg ........ (4)<br>  $\theta = \frac{4}{10}$  m – Pcos  $\theta$  (4) gives,<br>
Psin  $\theta$ <br>  $\frac$ STUDY MATERIAL: PHYSICS<br>
mg sin  $\theta - \frac{1}{10}$  m cos  $\theta = P$  (cos<sup>2</sup>  $\theta$  + sin<sup>2</sup>  $\theta$  = 1)<br>
Horizontally, Ssin  $\theta$  + P cos  $\theta$  = m  $\frac{20^2}{10^3}$  .........(3)<br>
pritically, Scos  $\theta$  – P sin  $\theta$  = m  $\theta$  ........(4)<br>
p S<br>
S<br>
S<br>  $\Rightarrow$  mg sin  $\theta - \frac{1}{10}$  m cos  $\theta = P$  (cos<sup>2</sup>  $\theta + \sin^2 \theta = 1$ )<br>
(b) Horizontally, Ssin  $\theta + P \cos \theta = m \frac{20^2}{10^3}$  .........(3)<br>
Vertically, Scos  $\theta - P \sin \theta = mg$  .........(4)<br>
(3) gives, Ssin  $\theta = \frac{4}{10}$  m - P cos STUDY MATERIAL: PHYSICS<br>  $cos \theta = P$   $(cos^2 \theta + sin^2 \theta = 1)$ <br>  $Ssin \theta + Pcos \theta = m \frac{20^2}{10^3}$  .........(3)<br>  $P sin \theta = mg$  .........(4)<br>  $\frac{4}{10}m - Pcos \theta$  (4) gives,<br>  $m \theta$ <br>  $\frac{sin \theta}{cos \theta} = \frac{(4/10)m - Pcos \theta}{mg + Psin \theta}$ <br>  $sin^2 \theta = \frac{4}{10}m cos \theta - Pcos^2 \theta$ <br>  $mg sin$ STUDY MATERIAL: PHYSICS<br>
= P  $(\cos^2 \theta + \sin^2 \theta = 1)$ <br>  $1\theta + P\cos \theta = m \frac{20^2}{10^3}$  .........(3)<br>  $1\theta = mg$  ........(4)<br>  $n - P\cos \theta$  (4) gives,<br>  $\frac{\theta}{\theta} = \frac{(4/10)m - P\cos \theta}{mg + P\sin \theta}$ <br>  $\frac{P}{P} = \frac{4}{10}m\cos \theta - P\cos^2 \theta$ STUDY MATERIAL: PHYSICS<br>
= P  $(\cos^2 \theta + \sin^2 \theta = 1)$ <br>  $n\theta + P \cos \theta = m \frac{20^2}{10^3}$  .........(3)<br>  $n\theta = mg$  ........(4)<br>  $n- P \cos \theta$  (4) gives,<br>  $\theta = \frac{(4/10)m - P \cos \theta}{mg + P \sin \theta}$ <br>  $d^2 \theta = \frac{4}{10} m \cos \theta - P \cos^2 \theta$ <br>  $\sin \theta$ S<br>
S<br>  $\Rightarrow$  mg sin  $\theta - \frac{1}{10} \text{m} \cos \theta = P$  (cos<sup>2</sup>  $\theta + \sin^2 \theta = 1$ )<br>
(b) Horizontally, Ssin  $\theta + P \cos \theta = m \frac{20^2}{10^3}$  .........(3)<br>
Vertically, Scos  $\theta - P \sin \theta = mg$  ........(4)<br>
(3) gives, Ssin  $\theta = \frac{4}{10} \text{m} - P \cos \theta$  (4) gi STUDY MATERIAL: PHYSICS<br>  $\frac{1}{2}$  m cos  $\theta = P$  (cos<sup>2</sup>  $\theta$  + sin<sup>2</sup>  $\theta$  = 1)<br>
(3)<br>  $s\theta - P \sin \theta = mg$  .........(3)<br>  $n\theta = \frac{4}{10}$  m - P cos  $\theta$  (4) gives,<br>  $+ P \sin \theta$ <br>  $\frac{\sin \theta}{\cos \theta} = \frac{(4/10) \text{m} - P \cos \theta}{\text{m}g + P \sin \theta}$ <br>  $\theta + P$ **STUDY MATERIAL: PHYSICS**<br>
mg sin  $\theta - \frac{1}{10}$  m cos  $\theta = P$  (cos<sup>2</sup>  $\theta$  + sin<sup>2</sup>  $\theta$  = 1)<br>
Torizontally, Scos  $\theta$  - P sin  $\theta$  + P cos  $\theta$  = m  $\frac{20^2}{10^3}$  .........(3)<br>
sically, Scos  $\theta$  - P sin  $\theta$  = mg ...... STUDY MATERIAL: PHYSICS<br>
gsin  $\theta - \frac{1}{10}$  m cos  $\theta = P$  (cos<sup>2</sup>  $\theta$  + sin<sup>2</sup>  $\theta$  = 1)<br>
rizontally, Ssin  $\theta$  + P cos  $\theta$  = m  $\frac{20^2}{10^3}$  .........(3)<br>
ally, Scos  $\theta$  – P sin  $\theta$  = mg .........(4)<br>
res, Ssin  $\theta =$ Now from part (a)  $P = n$  $\cos\theta = P$   $(\cos^2\theta + \sin^2\theta = 1)$ <br>  $S\sin\theta + P\cos\theta = m\frac{20^2}{10^3}$ .........(3)<br>  $-P\sin\theta = mg$  ........(4)<br>  $=\frac{4}{10}m-P\cos\theta$  (4) gives,<br>  $\sin\theta$ <br>  $\frac{\sin\theta}{\cos\theta} = \frac{(4/10)m - P\cos\theta}{mg + P\sin\theta}$ <br>  $P\sin^2\theta = \frac{4}{10}m\cos\theta - P\cos^2\theta$ <br>  $-mg\sin\theta$ <br>  $P = mg\sin$ STUDY MATERIAL: PHYSICS<br>  $cos \theta = P$   $(cos^2 \theta + sin^2 \theta = 1)$ <br>  $S sin \theta + P cos \theta = m \frac{20^2}{10^3}$  .........(3)<br>  $P sin \theta = mg$  ........(4)<br>  $\frac{4}{10}m - P cos \theta$  (4) gives,<br>  $m \theta$ <br>  $\frac{sin \theta}{cos \theta} = \frac{(4/10)m - P cos \theta}{mg + P sin \theta}$ <br>  $sin^2 \theta = \frac{4}{10}m cos \theta - P cos^2 \theta$ <br>  $mg sin \$ giving  $\frac{1}{2}$  m cos  $\theta$  = mg sin  $\theta$ 2001 EV mand by: Ssin θ + P cos θ = m  $\frac{20^3}{10^3}$  ......... (3)<br>
3, Scos θ - P sin θ = mg<br>
3, Ssin θ =  $\frac{4}{10}$  m - P cos θ (4) gives,<br>
3 = mg + P sin θ<br>
3 gives  $\frac{\sin \theta}{\cos \theta} = \frac{(4/10)m - P \cos \theta}{mg + P \sin \theta}$ <br>
mg sin θ + = $\text{m}\cos\theta = P$   $(\cos^2 \theta + \sin^2 \theta = 1)$ <br>  $\sin\theta + P\cos\theta = m\frac{20^2}{10^3}$  .........(3)<br>  $\theta - P\sin\theta = mg$  ........(4)<br>  $\theta = \frac{4}{10}m - P\cos\theta$  (4) gives,<br>  $-P\sin\theta$ <br>  $\frac{\sin\theta}{\cos\theta} = \frac{(4/10)m - P\cos\theta}{mg + P\sin\theta}$ <br>  $\theta + P\sin^2\theta = \frac{4}{10}m\cos\theta - P\cos^2\theta$ <br> hence  $\tan \theta = \frac{1}{1} = 0.0255$ ly, Scosθ – Psin θ = mg<br>
s, Ssin θ =  $\frac{4}{10}$  m – Pcosθ (4) gives,<br>
θ = mg + Psin θ<br>
g gives  $\frac{\sin \theta}{\cos \theta} = \frac{(4/10)\text{m} - \text{P}\cos \theta}{\text{m}g + \text{P}\sin \theta}$ <br>
mg sin θ + Psin<sup>2</sup> θ =  $\frac{4}{10}$  mcosθ – Pcos<sup>2</sup> θ<br>  $\frac{4}{10}$  mcosθ  $4g$  and  $4g$ tally,  $\sin \theta + P \cos \theta = m \frac{1}{10^3}$  (3)<br>
Scos  $\theta - P \sin \theta = mg$  (4)<br>
Sin  $\theta = \frac{4}{10}m - P \cos \theta$  (4) gives,<br>  $mg + P \sin \theta$ <br>  $\frac{\sin \theta}{\cos \theta} = \frac{(4/10)m - P \cos \theta}{mg + P \sin \theta}$ <br>  $g \sin \theta + P \sin^2 \theta = \frac{4}{10}m \cos \theta - P \cos^2 \theta$ <br>  $m \cos \theta - mg \sin \theta$ <br>  $\arct(a) P = mg \sin \theta - \frac{1}{$ (3) gives,  $S \sin \theta = \frac{4}{10} m - P \cos \theta$  (4) gives,<br>  $S \cos \theta = mg + P \sin \theta$ <br>
Dividing gives  $\frac{\sin \theta}{\cos \theta} = \frac{(4/10)m - P \cos \theta}{mg + P \sin \theta}$ <br>
Hence,  $mg \sin \theta + P \sin^2 \theta = \frac{4}{10} m \cos \theta - P \cos^2 \theta$ <br>  $\Rightarrow P = \frac{4}{10} m \cos \theta - mg \sin \theta$ <br>
Now from part (a)  $P = mg \sin \theta - \frac$ (s) gives,  $S \sin \theta = \frac{1}{10} \text{m} - \text{r} \cos \theta$  (4) gives,<br>  $S \cos \theta = \text{mg} + \text{P} \sin \theta$ <br>
Dividing gives  $\frac{\sin \theta}{\cos \theta} = \frac{(4/10)\text{m} - \text{P} \cos \theta}{\text{mg} + \text{P} \sin \theta}$ <br>
Hence,  $\text{mg} \sin \theta + \text{P} \sin^2 \theta = \frac{4}{10} \text{m} \cos \theta - \text{P} \cos^2 \theta$ <br>  $\Rightarrow \text{P}$ part (a)  $P = mg \sin \theta - \frac{1}{10} m \cos \theta$ <br>  $m \cos \theta = mg \sin \theta$ <br>  $\theta = \frac{1}{4g} = 0.0255$ <br>
stance between the rails is 1.5m.<br>
= 1.5 sin  $\theta = 1.5 \times 0.0255$ <br>  $n\theta$  because  $\theta$  is small)<br>
= 1.5m<br>
= 1.5m<br>
= 1.5m<br>
= 1.5m<br>
= 1.5m<br>
= 1.5m<br>
= 1.5m dt

But the distance between the rails is 1.5m.

![](_page_38_Figure_22.jpeg)

So the outer rail is raised 0.0383 m above the inner rail.

#### **EXERCISE-4**

**(B).** 
$$
\frac{mv^2}{r} = \mu mg
$$
;  $v = \sqrt{\mu rg} = \sqrt{0.6 \times 150 \times 10} = 30$  m/s

 $10<sup>3</sup>$  **(2) (B).** The acceleration vector is along radius of the circle. **(3) (C).** In uniform circular motion its kinetic energy will

(4) **(D).** S = t<sup>3</sup> + 5 
$$
\therefore
$$
 Speed, v =  $\frac{ds}{dt}$  = 3t<sup>2</sup>

constant.

and rate of change of speed 
$$
\frac{dv}{dt} = 6t
$$

ds <sup>2</sup> v 3t  $\therefore$  Tangential acceleration at t = 2s, at = 6 × 2 = 12 m/s<sup>2</sup> At t = 2s,  $v = 3(2)^2 = 12$  m/s  $\frac{1}{4}$ <br>  $\frac{1}{2}$ <br>  $\frac{1}{3}$  m above the inner rail.<br>  $\frac{1}{2}$ <br>  $= \sqrt{0.6 \times 150 \times 10} = 30 \text{ m/s}$ <br>
along radius of the circle.<br>
its kinetic energy will<br>  $= \frac{ds}{dt} = 3t^2$ <br>
ed  $\frac{dv}{dt} = 6t$ <br>  $t = 2s$ , at  $= 6 \times 2 = 12 \text{ m/s}$ A<br>  $\[\begin{array}{c}\n\frac{1}{h} \\
\frac{1}{h} \\
0.6 \times 150 \times 10 = 30 \text{ m/s}\n\end{array}\]$ <br>
ong radius of the circle.<br>
kinetic energy will<br>  $\[\begin{array}{c}\n\frac{ds}{dt} = 3t^2 \\
\frac{dv}{dt} = 6t \\
2s, at = 6 \times 2 = 12 \text{ m/s}^2\n\end{array}\]$ <br>  $= \frac{v^2}{R} = \frac{144}{20} \text{ m/s}^2$ <br>  $\approx 14 \text$ CISE-4<br>  $\mu$ rg =  $\sqrt{0.6 \times 150 \times 10}$  = 30 m/s<br>
tor is along radius of the circle.<br>
otion its kinetic energy will<br>
d,  $v = \frac{ds}{dt} = 3t^2$ <br>
f speed  $\frac{dv}{dt} = 6t$ <br>
on at t = 2s, at = 6 × 2 = 12 m/s<sup>2</sup><br>
12 m/s<br>
ion,  $a_c = \frac{v^2$ 

**(B).** 
$$
\frac{mv}{r} = \mu mg
$$
;  $v = \sqrt{\mu rg} = \sqrt{0.6 \times 150 \times 10} = 30$  m/s  
\n**(B).** The acceleration vector is along radius of the circle.  
\n**(C).** In uniform circular motion its kinetic energy will  
\nconstant.  
\n**(D).**  $S = t^3 + 5$   $\therefore$  Speed,  $v = \frac{ds}{dt} = 3t^2$   
\nand rate of change of speed  $\frac{dv}{dt} = 6t$   
\n $\therefore$  Tangential acceleration at  $t = 2s$ , at  $= 6 \times 2 = 12$  m/s<sup>2</sup>  
\nAt  $t = 2s$ ,  $v = 3(2)^2 = 12$  m/s  
\n $\therefore$  Centripetal acceleration,  $a_c = \frac{v^2}{R} = \frac{144}{20}$  m/s<sup>2</sup>  
\n $\therefore$  Net acceleration  $= \sqrt{a_t^2 + a_i^2} \approx 14$  m/s<sup>2</sup>

$$
\therefore \quad \text{Net acceleration} = \sqrt{a_t^2 + a_i^2} \approx 14 \text{ m/s}^2
$$

![](_page_39_Picture_2.jpeg)

![](_page_39_Figure_3.jpeg)

**(9) (B).**  $\mathbf{x}^{\mathcal{T}}$  $\frac{m}{\epsilon}$  kx  $\frac{m}{\epsilon}$  $\begin{array}{ccc}\n & x & m \\
 \hline\n\ell & x & m \\
\end{array}$  m  $(\ell+x)\omega^2$  $kx = m\ell\omega^2 + mx\omega^2$ 2 a *z*  $\ell \omega^2$ 

$$
x = \frac{m\ell\omega^2}{k - m\omega^2}
$$

#### **EXERCISE-5**

(1) (A). 
$$
a = \omega^2 R = \left(\frac{2\pi}{0.2\pi}\right)^2 (5 \times 10^{-2}) = 5 \text{ m/s2}
$$

(2) **(B).** For banking 
$$
\tan \theta = \frac{V^2}{Rg}
$$
;  $\tan 45^\circ = \frac{V^2}{90 \times 10} = 1$ ;  $\omega \ge \sqrt{\frac{g}{r\mu}}$ ;  $\omega_{\text{min}}$ 

**(3) (D).** For smooth driving maximum speed of car v then

$$
\frac{mv^2}{R} = \mu_s mg \ ; \ v = \sqrt{\mu_s Rg}
$$

11170NS  
\n**(4) (B)** (F<sub>C</sub>)<sub>heavier</sub> = (F<sub>C</sub>)<sub>lighter</sub>  
\n
$$
\frac{2mV^2}{(r/2)} = \frac{m (nV)^2}{r} \Rightarrow n^2 = 4 \Rightarrow n = 2
$$
\n**(5) (B)** Vertical equilibrium:  
\n
$$
\Rightarrow mg = N\cos \theta - f_1 \sin \theta \qquad f_1 \cos \theta
$$
\n
$$
\Rightarrow mg = N\cos \theta - f_1 \sin \theta \qquad f_1 \cos \theta
$$
\n
$$
\Rightarrow mg = N\cos \theta - f_1 \sin \theta \qquad f_1 \cos \theta
$$
\n
$$
\Rightarrow \text{Im } \theta + f_1 \cos \theta = mv^2/R... (2)
$$
\n
$$
\frac{Fq}{Eqq} (1) \cdot \frac{v^2}{Rg} = \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta}
$$
\n
$$
v = \sqrt{Rg \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta}} = \sqrt{Rg \frac{\tan \theta + \mu_s}{1 - \mu_s \tan \theta}}
$$
\n**(6) (A)** m = 0.01 kg, r = 6.4 cm  
\n
$$
\frac{1}{2}mv^2 = 8 \times 10^{-4} \text{ J} \cdot v^2 = \frac{16 \times 10^{-4}}{0.01} = 16 \times 10^{-2}
$$
\nSpeed v<sup>2</sup> = 2a<sub>1</sub>4\pi r  
\na<sub>1</sub> =  $\frac{v^2}{8\pi r} = \frac{16 \times 10^{-2}}{8 \times 3.14 \times 6.4 \times 10^{-2}} = 0.1 \text{ m/s}^2$   
\n**(7) (D)** v<sub>min</sub> =  $\sqrt{5gR}$   
\n**(8) (C)** a cos 30° =  $\frac{v^2}{r} \Rightarrow 15\frac{\sqrt{3}}{2} = \frac{v^2}{2.5} \Rightarrow v = 5.7 \text{ m/s}$   
\n**(9) (D)** Net force on the particle in uniform circular motion is centripetal force, which is provided by the tension in string.  
\n**(**

$$
\frac{1}{2}mv^2 = 8 \times 10^{-4} \text{ J} \; ; \; v^2 = \frac{16 \times 10^{-4}}{0.01} = 16 \times 10^{-2}
$$
\n
$$
\text{Speed } v^2 = 2a_t s \; ; \; v^2 = 2a_t 4\pi r
$$
\n
$$
a_t = \frac{v^2}{8\pi r} = \frac{16 \times 10^{-2}}{8 \times 3.14 \times 6.4 \times 10^{-2}} = 0.1 \text{ m/s}^2
$$

$$
(7) \qquad (D). \quad v_{\min} = \sqrt{5gR}
$$

(8) (C). 
$$
a \cos 30^\circ = \frac{v^2}{r} \implies 15 \frac{\sqrt{3}}{2} = \frac{v^2}{2.5} \implies v = 5.7 \text{ m/s}
$$

**(9) (D).** Net force on the particle in uniform circular motion is centripetal force, which is provided by the tension in string.

(10) (C). 
$$
T - mg = \frac{mu^2}{\ell}
$$
  

$$
T = mg + \frac{mu^2}{\ell}
$$

$$
\frac{u^2}{\ell} \qquad \qquad \bigg(\bigg\downarrow_{T}\bigg)_{u}
$$

The tension is maximum at the lowest position of mass,so the chance of breaking is maximum.

a<sub>1</sub> = 
$$
\omega^2 r_1
$$
  
\n $\omega^2$   
\n $\omega^3$   
\n $\omega^4$   
\n $\omega^5$   
\n $\sqrt[3]{r_1}$   
\n $\sqrt[3]{r_2}$   
\n(b) At  $m = 0.01 \text{ kg. } r = 6.4 \text{ cm}$   
\n $\sqrt[3]{r_1}$   
\n $\$