8

PERMUTATIONS AND COMBINATIONS

INTRODUCTION

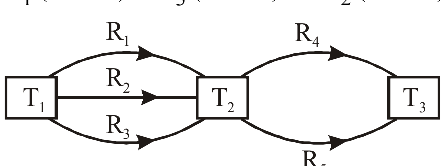
* The most fundamental application of mathematics is counting. There are many natural methods used for counting.
* This chapter is deals with various known techniques those are much faster than the usual counting methods.
* We mainly focus, our methods, on counting the number of arrangements, (permutations) and the number of selections (combinations), even although we may use these techniques for counting in some other situations also.

FUNDAMENTAL PRINCIPLE OF COUNTING

FPC (Fundamental Principle of Counting) is used to count some event without actually counting them.

* **Multiplication Principle :** If an operation can be performed in 'm' different ways; following which a second operation can be performed in 'n' different ways, then the two operations in succession can be performed in m × n ways. This can be extended to any finite number of operations.
* **Addition principle :** If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in 'n' different ways. Then either of the two operations can be performed in (m + n) ways. This can be extended to any finite number of mutually exclusive operations.

Model- I : Find number of ways of in which one can travel from T_1 (town 1) to T_3 (town 3) via T_2 (town 2).



Total ways :

$T_1 — R_1 — T_2 — R_4 — T_3$

$T_1 — R_1 — T_2 — R_5 — T_3$

$T_1 — R_2 — T_2 — R_4 — T_3$

$T_1 — R_2 — T_2 — R_5 — T_3$

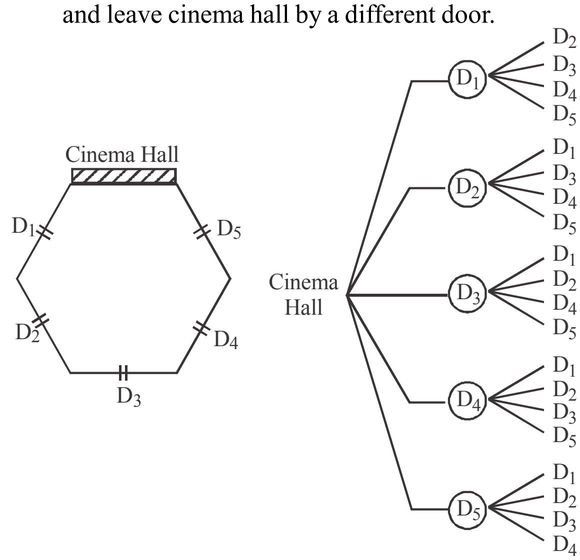
$T_1 — R_3 — T_2 — R_4 — T_3$

$T_1 — R_3 — T_2 — R_5 — T_3$ = 6 ways

It is easy to proceed by FPC T_1 to $T_2 \longrightarrow 3$
T_2 to $T_3 \longrightarrow 2$
Total ways = 3 × 2 = 6

Model- II :

(i) To find the number of ways by which a person can enter and leave cinema hall by a different door.



4 + 4 + 4 + 4 + 4 = 20

By F.P.C. : A person can enter in cinema hall by 5 ways & leave by 4 ways = 5 × 4 = 20.

(ii) If he can enter and leave by any door then number of ways = 5 × 5 = 25.

Basic Steps to Remember :

Step-I : Identify the independent events involved in a given problem.

Step-II : Find the number of ways performing/occuring each event

Step-III : Multiply these numbers to get the total number of ways of performing/occuring all the events

Example 1 :

A person wants to go from station P to station R via station Q. There are 4 routes from P to Q and 5 routes from Q to R.In how many ways can he travel from P to R -

Sol. He can go from P to Q in 4 ways and Q to R in 5 ways
So number of ways of travel from P to R is 4 × 5 = 20

Example 2 :

A person wants to leave station Q there are 4 routes from station Q to P and 5 routes from Q to R. In how many ways can he travel the station Q.

Sol. He can go from Q to P in 4 ways and Q to R in 5 ways
he can leave station Q in 4 + 5 = 9 ways.

Example 3 :

How many 3 digit numbers can be formed by the digit 1, 2, 3, 4, 5 without repetition.

Sol. Hundred's place digit can be selected in 5 ways.
Ten's place digit can be selected in 4 ways.
Unit's place digit can be selected in 3 ways.
So, $5 \times 4 \times 3 = 60$

Example 4 :

In an examination of 10 T/F question, How many sequence of answers are possible.

Sol. Any question can be answered in two ways , i.e. true or false. So total task of anwering tan question can be done in

$2 \times 2 \times 2 \times \ldots\ldots$ 10 times $= 2^{10}$ ways

Example 5 :

10 students complete in a swiming race. In how many ways can they occupy the first 3 positions.

Sol. 1^{st} place can be occupied in 10 ways.
2^{nd} place can be occupied in 9 ways.
3^{rd} place can be occupied in 8 ways.
So total number of ways $= 10 \times 9 \times 8 = 720$.

Example 6 :

There are 7 flags of diffrent colour. Find the number of diffrent signals that can be transmitted by the use of 2 flags one above the other.

Sol. 1^{st} place can be occupied in 7 ways.
2^{nd} place can be occupied in 6 ways.
So total number of ways $= 7 \cdot 6 = 42$.

Example 7 :

A letter lock consists of 3 rings each marked with 10 different letters. In how many ways, its is possible to make an unsuccessful attempt to open the lock?

Sol. 2 ring may have same letters at a time. One ring can have any one of 10 different letters in 10 ways.
Similarly 2^{nd} and 3^{rd} ring can have any one of 10 different letters in 10 ways respectively.
Total number of attempts $= 10 \times 10 \times 10 = 10^3 = 1000$
But out of these 1000 attempt, only one attempt is successful.
Required number of unsuccessful attempt $= 1000 - 1 = 999$

Example 8 :

Number of ways in which m diffrent toys can be distributed in "n" children if every child may receive any number of toys

Sol. Object of distribution toys
One toy $\rightarrow$ n children
Total number of ways $= n \times n \ldots. n = n^m$

Example 9 :

There are m men and n monkey. Number of ways in which every monkey has a master, if a man can have any number of monkey.

Sol. Monkey is distributed among in masters, like 1 monkey can go to $\rightarrow$ m masters
Total number of ways $= m \times m \times \ldots\ldots\ldots. m = m^n$

Example 10 :

How many integers greater than 5000 can be formed with the digit 7, 6, 5, 4 & 3 using each digit at most once.

Sol. For 4 digits number $\Rightarrow$ First position can be filled by 7, 6 or 5 (that is in three ways). Hence
4 digit number $= 3 \times 4 \times 3 \times 2 = 72$
5 digit number $= 5 \times 4 \times 3 \times 2 \times 1 = 120$
Total number = 192

Example 11 :

How many natural number less than 30000 can be made from the digits 0, 1, 2, 3, 4, 5, 6.

Sol. Let a five digit number be denoted by

a	b	c	d	e

Each of the places can be filled by either of 0, 1, 2, 3, 4, 5, 6 in 7 ways.
The place marked by "a" can be filled by the digits 0, 1 or 2 (since number is to be less than 30000)
Hence number of intergers $= 3 \times 7 \times 7 \times 7 \times 7 = 3 \times 7^4$
In these numbers one case includes "00000" which is not a natural numbers.
Hence number of natural numbers $= 3 \times 7^4 - 1$

Example 12 :

Find the number of ways in which we can post 5 letters in 10 letter boxes.

Sol. 1^{st} letter $\Rightarrow$ 10 boxes
2^{nd} letter $\Rightarrow$ 10 boxes
3^{rd} letter $\Rightarrow$ 10 boxes
4^{th} letter $\Rightarrow$ 10 boxes
5^{th} letter $\Rightarrow$ 10 boxes
$= 10^5$.

* **Lexicography illustration (Lexicography is called science of making words) :**

(a) Find total number of 5 letter word that can be formed from letters of word "TOUGH".

↑	↑	↑	↑	↑

$5 \times 4 \times 3 \times 2 \times 1 = 120$

(b) Find the rank of "TOUGH" if all the letters of the word are arranged in all possible orders & written out as in a dictionary.
The number of letters in the word "TOUGH" is 5 & all the five letters are different.
Alphabatical order of all the letters is G, H, O, T, U
Number of words begining with G $= 4 \times 3 \times 2 \times 1$
Number of words begining with H $= 4 \times 3 \times 2 \times 1$
Number of words begining with O $= 4 \times 3 \times 2 \times 1$
Number of words begining with TG $= 3 \times 2 \times 1$
Number of words begining with TH $= 3 \times 2 \times 1$

Number of words begining with TOG = 2 × 1
Number of words begining with TOH = 2 × 1
Next words begining with "TOU" and it is "TOUGH" = 1.
Rank = 24 + 24 + 24 + 6 + 6 + 2 + 2 + 1 = 89

FACTORIALS

* If n is a natural number then the product of all natural number upto n is called factorial n and it is denoted by -
  n ! or $\underline{|n}$. Thus, n ! = n (n – 1) (n – 2) ..... 3.2.1.
  $n! = n(n-1)! = n(n-1)(n-2)!$
  $= n(n-1)(n-2)(n-3)!$ etc.
  Note : 0 ! = 1, 1 ! = 1, 2 ! = 2, 3 ! = 6, 4 ! = 24, 5 ! = 120, 6 ! = 720
* Factorial of negative number is undefined
* $(2n!) = 2^n \cdot n! [1 \cdot 3 \cdot 5 \ldots\ldots (2n-1)]$
  $(2n)! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \ldots\ldots (2n-1)(2n-2)$
  Take 2 common each n even terms
  $= 2^n (1 \cdot 3 \cdot 5 \ldots\ldots (2n+1))(1 \cdot 2 \cdot 3 \ldots\ldots n)$
  $= 2^n (n!)(1 \cdot 3 \cdot 5 \ldots\ldots (2n+1))$

Example 13 :
(i) Find n if $(n+1)! = 12 \times (n-1)!$
(ii) $(n+2)! = 2550\, n!$

Sol. (i) $(n+1)\, n\, (n-1)! = 12 \times (n-1)!$
$n^2 + n - 12 = 0;$
$(n+4)(n-3) = 0 \therefore n = 3$
(ii) $(n+2)(n+1) = 2550;$
$(n+52)(n-49) = 0 \therefore n = 49$

* **Finding exponent of prime in integer:**
Let $E_p(n)$ denote the exponent of the prime p in the positive integer n. Then,

$$E_p(n!) = E_p(1 \cdot 2 \cdot 3 \ldots\ldots (n-1)n) = E_p\left(p \cdot 2p \cdot 3p \ldots\ldots \left[\frac{n}{p}\right] p\right)$$

$\left[\because \begin{array}{l}\text{Remaining integers between}\\ \text{1 and n are not divisible by p}\end{array}\right]$

$$= \left[\frac{n}{p}\right] + E_p\left(1 \cdot 2 \cdot 3 \ldots \left[\frac{n}{p}\right]\right)$$

Now, the last integer amongst 1, 2, 3, ……, $\left[\frac{n}{p}\right]$ which is divisible by p is $\left[\frac{n/p}{p}\right] p = \left[\frac{n}{p^2}\right] p$

$$\therefore \quad E_p(n!) = \left[\frac{n}{p}\right] + E_p\left(p \cdot 2p \cdot 3p \ldots\ldots \left[\frac{n}{p^2}\right] p\right)$$

$\left[\because \begin{array}{l}\text{Remaining integers between}\\ \text{1 and } \left[\frac{n}{p}\right] \text{ are not divisible by p}\end{array}\right]$

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots\ldots + \left[\frac{n}{p^s}\right]$$

where s is the largest positive integer such that $p^s \le n < p^{s+1}$

Example 14 :
Find exponent of prime 2 in (100 !)

Sol. $(2 \cdot 50)! = 2^{(50)} \cdot 50!\,(1 \cdot 3 \cdot 5 \ldots\ldots 99) \to 50$ 2's
$(50!) = (2 \cdot 25)! = 2^{(25)} \cdot 25!\,(1 \cdot 3 \cdot 5 \ldots\ldots 49) \to 25$ 2's
$(24!) = (2 \cdot 12)! = 2^{(12)} \times 12!\,(1 \cdot 3 \cdot 5 \ldots\ldots 23) \to 12$ 2's
$(12!) = (2 \cdot 6)! = 2^{(6)} \cdot 6!\,(1 \cdot 3 \cdot 5 \ldots\ldots 11) \to 6$ 2's
$(6!) = (2 \cdot 3)! = 2^{(3)} \cdot 3!\,(1 \cdot 3 \cdot 5 \ldots\ldots 5)$
$= 2^{(3)} \cdot 3!\,(1 \cdot 3 \cdot 5) \to 3$ 2's
$(3!) = 3 \cdot 2 \cdot 1 = 2^{(1)} \cdot (1 \cdot 3) \to 1$ 2's
$50 + 25 + 12 + 6 + 3 + 1 = 97$

Alternative Method :

$$E_2(100!) = \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right]$$
$$= 50 + 25 + 12 + 6 + 3 + 1$$

Example 15 :
Find number of zeros of the end of (1000)!

Sol. In any usual factorial of a natural number of 2s are more than number of 5s. Hence number of 10s are same as number of 5s.
Now we decide number of 5s in (1000)! (1000!)
5, 10, 15 ……… 1000 : 200
⇒ 200 number containing at least one 5
25, 50 ……… 1000 : 40
⇒ 40 number containing at least two 5
125, 250 ……… 1000 : 8
⇒ 8 number containing at least three 5
625 1
⇒ 1 number containing at least four 5
Total = 200 + 40 + 8 + 1 = 249

Alternative Method :

$$E_5(1000!) = \left[\frac{1000}{5}\right] + \left[\frac{1000}{5^2}\right] + \left[\frac{1000}{5^3}\right] + \left[\frac{1000}{5^4}\right]$$
$$= 200 + 40 + 8 + 1 = 249$$

Example 16 :
Find number of zeros at the end of 2007!

Sol. $$E_5(2007!) = \left[\frac{2007}{5}\right] + \left[\frac{2007}{5^2}\right] + \left[\frac{2007}{5^3}\right] + \left[\frac{2007}{5^4}\right]$$
$$= 401 + 80 + 16 + 3 = 500$$

PERMUTATION & COMBINATION

* **Permutation :** Permutation means arrangement in a definte order of things which may be alike or different taken some or all at a time. Hence permutation refers to the situation where order of occurrence of the events is important.

* **Combination :** Combination/selection/collection/committee refers to the situation where order of occurrence of the event is not important. Combination is selection of one or more things out of n things which may be alike or different taken some or all at a time.

Note :

* Things which are alike and which are different. All god made things in general are treated to be different and all man made things are to be spelled weather like or different. Hence we say that permutation is arrangement of things in definite order.

Example :

(i) Out of A, B, C, D take 3 letters & form number plate of car. [Pemutation]

(ii) Out of four letters A, B, C, D take any 3 letters & form triangle (possible). [Combination]

In 1st case arrangement of letters are there, in 2nd case only selection will form the triangle, arrangment is not required.

COUNTING FORMULAS FOR PERMUTATIONS

Without Repetition :

(i) The number of permutations of n different things, taking r at a time is denoted by ${}^nP_r$ or P(n, r)

then ${}^nP_r = \dfrac{n!}{(n-r)!}$ $(0 \le r \le n)$

$= n(n-1)(n-2) \ldots\ldots (n-r+1),\ n \in N$ and $r \in W$

(ii) The number of arrangements of n different objects taken all at a time is ${}^nP_n = n!$

Note : ${}^nP_1 = n,\quad {}^nP_r = n.\ {}^{n-1}P_{r-1},\quad {}^nP_r = (n-r+1).\ {}^nP_{r-1},\quad {}^nP_n = {}^nP_{n-1}$

Example 17 :

Find the number of ways in which four persons can sit on six chairs.

Sol. ${}^6P_4 = 6.5.4.3 = 360$

With Repetition :

(i) The number of permutations of n things taken all at a time, p are alike of one kind, q are alike of second kind and r are alike of a third kind and the rest n – (p + q + r) are all different is $\dfrac{n!}{p!\,q!\,r!}$

(ii) The number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^r.

Example 18 :

In how many ways can 5 prizes be distributed among 4 boys when every boy can take one or more prizes ?

Sol. First prize may be given to any one of the 4 boys, hence first prize can be distributed in 4 ways.

similarly every one of second, third, fourth and fifth prizes can also be given in 4 ways.

$\therefore$ The number of ways of their distribution

$= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$

Example 19 :

Find the number of words that can be formed out of the letters of the word COMMITTEE.

Sol. There are 9 letters in the given word in which two T's, two M's and two E's are identical. Hence the required number of

words $= \dfrac{9!}{2!2!2!} = \dfrac{9!}{(2!)^3}$

Example 20 :

Find total number of word's formed by using all letters of the word "IITJEE".

Sol. Ways are $= \dfrac{6!}{2!\,(2!)}$

Example 21 :

Four faces of a tetrahedral dice are marked with 2, 3, 4, 5. The lowest face being considered as the outcome. In how many way a total of 30 can occur in 7 throws.

Sol. 7 throws outcome whose sum is equal to 30 can be obtained in following way.

Category	Numebr of ways
5, 5, 5, 5, 5, 2, 3	$\dfrac{7!}{5!} = 42$
5, 5, 5, 5, 4, 4, 2	$\dfrac{7!}{(4!)(2!)} = 105$
5, 5, 5, 5, 4, 3, 3	$\dfrac{7!}{(4!)(2!)} = 105$
5, 5, 5, 4, 4, 4, 3	$\dfrac{7!}{(3!)(3!)} = 140$
5, 5, 4, 4, 4, 4, 4	$\dfrac{7!}{(2!)(5!)} = 21$

Total ways = 42 + 105 + 105 + 140 + 21 = 413

Number of Permutations under certain conditions:

(a) The number of permutation of n different things taken all together when r particular things are to be place at some r given places $= {}^{n-r}P_{n-r} = (n-r)!$

(b) The number of permutations of n different things taken r at a time when m particular things are to be placed at m given places $= {}^{n-m}P_{r-m}$.

(c) Number of permutations of n different things, taken r at a time, when a particular things is to be always included in each arrangement, is r. ${}^{n-1}P_{r-1}$

(d) Number of permutation of n different things, taken r at a time, when particular thing is never taken in each arrangement is $^{n-1}P_r$.

(e) Number of permutations of n different things, taken all at a time, when m specified things always come together is $m! \times (n-m+1)!$

(f) Number of permutations of n different things, taken all at a time, when m specified things never come together is $n! - m! \times (n-m+1)!$

Example 22 :

How many different 4-letter words can be formed with the letters of the word 'JAIPUR' when A and I are always to be included ?

Sol. Since A and I are always to be included, so first we select 2 letters from the remaining 4, which can be done in $^4C_2 = 6$ ways. Now these 4 letters can arranged $4! = 24$ ways, so the required number $= 6 \times 24 = 144$.

Example 23 :

How many different words can be formed with the letter of the word 'JAIPUR' which start with 'A' and end with 'I'.

Sol. After putting A and I at their respective places (only in one way) we shall arrange the remaining 4 different letters at 4 places in 4! ways. Hence the required number $= 1 \times 4! = 24$

Example 24 :

How many different 3-letter words can be formed with the letter of word 'JAIPUR' when A & I are always to be excluded?

Sol. After leaving A and I, we are remained with 4 different letters which are to be used for forming 3-letter words. Hence the required number $= {}^4P_3 = 4.3.2 = 24$

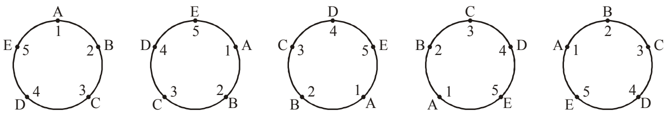
CIRCULAR PERMUTATION

Permutation of objects in a row is called as linear permutation. If we arrange the objects along a closed curve it is called as circular permutation.

Thus in, circular permutation, we consider one object fixed and the remaining objects are arranged.

(i) The number of circular permutation of **n** distinct objects is **(n – 1) !**

Consider 5 objects A, B, C, D, E to be arranged around a closed curve is called circular permutation.



All are Same

Let the total number of circular permutation be x. Above circular permutation is equivalent to 5 linear permutations given by ABCDEF, EABCD, DEABC, CDEAB, BCDEA that is one circular permutation is equivalent to 5x linear permutation given by $x . 5 = 5 !$

$$x = \frac{5!}{5} = \frac{5.(5-1)!}{5} = (5-1)!$$

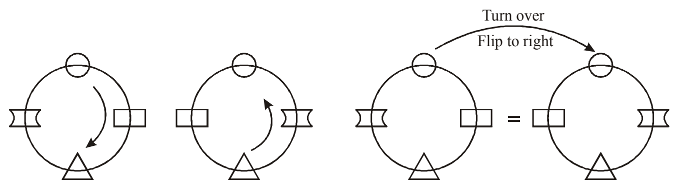
Similarly for n objects $nx = n !$

$$\boxed{x = \frac{n!}{n} = (n-1)!}$$

Note : In the above theorem anti–clockwise and clockwise order of arrangements are considered as distinct permutations.

(ii) If aniclockwise and clockwise are considered to be same total number of circular permutation given by $\frac{(n-1)!}{2}$.

If we arrange flowers or garland beads in a neckless then there is no distinction between clockwise & anticlockwise direction.



Note: The distinction between clockwise and anticlockwise is ignored when a number of people have to be seated around a table so as not to have the same neighours.

(iii) Number of circular permutations of n different things taken r at a time :

Case - I : If clockwise and anticlockwise orders are taken as different, then the required number of circular permutations $= \frac{^nP_r}{r}$.

Case- II : If clockwise and anticlockwise orders are taken as not different, then the required number of circular permutations $= \frac{^nP_r}{2r}$

(iv) Restricted Circular Permutation

When there is a restriction in a circular permutation then first of all we shall perform the restricted part of the operation and then perform the remaining part treating it similar to a linear permutation.

Example 25 :

In how many ways can 5 boys and 5 girls be seated at a round table so that no two girls may be together ?

Sol. Leaving one seat vacant between two boys, 5 boys may be seated in 4! ways. Then at remaining 5 seats, 5 girls any sit in 5! ways. Hence the required number $= 4! \times 5!$

Example 26 :

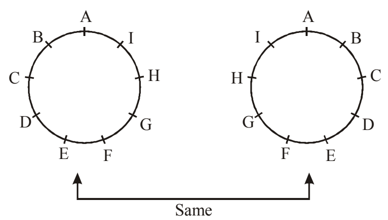
In how many ways can 4 beads out of 6 different beads be strung into a ring ?

Sol. In this case a clockwise and corresponding anticlockwise ordered will give the same circular permutation. So the required number $= \frac{^6P_4}{4.2} = \frac{6.5.4.3}{4.2} = 45$

Example 27 :

Find the number of ways in which 9 people can be seated on a round table so that all shall not have the same neighbours in any 2 arrangements.

Sol. For same neighbour, clockwise and anticlockwise arrangements are same. So total number of ways will be arrangement of 9 people taken clockwise and anticlockwise same and equal to $\frac{(9-1)!}{2} = \frac{8!}{2}$



Example 28 :

Find the number of ways in which 10 children can sit in a mary go round relative to one another.

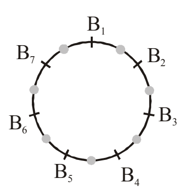
Sol. Here clockwise and anticlockwise arrangements are diffrent. Thus required ways = (10 – 1)! = 9!

Example 29 :

Find number of ways in which 7 American and 7 British people can be seated on a round table so that no two Americans are consecutive.

Sol. Circular arrangement of 7 British = (7 – 1) !

There are 7 gap among 7 British.



Out of 7 gap's 7 American can be filled by (7!) ways.

Total ways = (7!) (7 – 1)!

Example 30 :

Find the number of ways in which 10 persons can sit round a circular table so that none of them has the same neighbours in any two arrangements.

Sol. 10 persons can sit round a circular table in 9! ways. But here clockwise and anticlockwise orders will give the same neighbours. Hence the required number of ways = 9!/2.

Example 31 :

Find number of circular permutation of n persons if two specific people are never together.

Sol. Required ways = Total – when A & B are always together

= (n – 1) – (n – 2)! × 2 = (n – 2)! [n – 1 – 2]

= (n – 2) ! (n – 3).

Example 32 :

In how many ways 7 diffrent flowers can be formed into a garland.

Sol. Here clockwise and anticlockwise permutations are same

Hence total ways = 6! / 2.

TRY IT YOURSELF-1

Q.1 A college offers 6 courses in the morning and 4 in the evening. Find the possible number of choices with the student if he wants to study one course in the morning and one in the evening.

Q.2 If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$, then find the value of n.

Q.3 A college offers 6 courses in the morning and 4 in the evening. Find the number of ways a student can select exactly one course, either in the morning or in the evening.

Q.4 How many words can be formed with the letters of the word 'MATHEMATICS' by rearranging them.

Q.5 If the best and the worst papers never appear together, find in how many ways six examination papers can be arranged.

Q.6 In how many ways can 5 boys and 3 girls sit in a row so that no two girls are together?

Q.7 Find the number of words that can be made out of the letters of the word 'MOBILE' when consonants always occupy odd places.

Q.8 A round-table conference is to be held among 20 delegates belonging from 20 different countries. In how many ways can they be seated if two particular delegates are (i) always to sit together; (ii) never to sit together.

Q.9 Find the number of ways in which 10 different diamonds can be arranged to make a necklace.

Q.10 Find the number of ways in which six persons can be seated at a round table, so that all shall not have the same neighbours in any two arrangements.

Q.11 Find the number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together.

Q.12 How many 4-letter words, with or without meaning, can be formed out of the letters in the word 'LOGARITHMS', if repetition of letters is not allowed?

ANSWERS

(1) 24	**(2)** 4	**(3)** 10
(4) 11!/(2!2!2!)	**(5)** 480	**(6)** ${}^6P_3 \times 5!$
(7) 36	**(8)** (i) 2 × (18)!, (ii) 17 × 18!	
(9) 181440	**(10)** 60	**(11)** 6! × 5!
(12) 5040		

COMBINATION

Each of the different groups or selections which can be made by some or all of a number of given things without reference to the order of the things in each group is called a Combination.

Difference between permutation and combination :

	Problems of Permutations	**Problems of Combinations**
1	Arrangements	Selections, choose
2	Standing in a line seated in a row	Distributed group is formed
3	Problems on digits	Committee
4	Problems on letters from a word	Geometrical problems

COUNTING FORMULAS FOR COMBINATION

(1) Selection of objects without Repetition

The number of combinations of n different things taken r at a time is denoted by ^nC_r or C (n, r) or $\binom{n}{r}$

Then $^nC_r = \frac{n!}{r!(n-r)!}$; $(0 \le r \le n)$

$= \frac{^nP_r}{r!} = \frac{n(n-1)(n-2)....(n-r+1)}{r(r-1)(r-2).........2.1}$; $n \in N$ and $r \in W$

If $r > n$, then $^nC_r = 0$

Some Important Results :

(i) $^nC_n = 1$, $^nC_0 = 1$, (ii) $^nC_r = \frac{^nP_r}{r!}$

(iii) $^nC_r = {}^nC_{n-r}$, (iv) $^nC_x = {}^nC_y \Rightarrow x + y = n$

(v) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ (vi) $^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$

(vii) $^nC_r = \frac{1}{r}(n-r+1)\,{}^nC_{r-1}$ (viii) $^nC_1 = {}^nC_{n-1} = n$

MAXIMISE ^nC_r :

^nC_r is maximum at $\begin{cases} r = \frac{n}{2} & \text{if n is even} \\ r = \frac{n-1}{2} \text{ or } \frac{n+1}{2} & \text{if n is odd} \end{cases}$

e.g. $^{15}C_r$ is maximum when r = 7 or 8, $^{12}C_r$ is maximum when r = 6

Example 33 :

If $^{20}C_r = {}^{20}C_{r-10}$, then find the value of $^{18}C_r$

Sol. $^{20}C_r = {}^{20}C_{r-10} \Rightarrow r + (r - 10) = 20 \Rightarrow r = 15$

$\therefore {}^{18}C_r = {}^{18}C_{15} = {}^{18}C_3 = \frac{18.17.16}{1.2.3} = 816$

Example 34 :

How many combination of 4 letters can be made of the letters of the word 'JAIPUR' ?

Sol. The number of combinations $= {}^6C_4 = \frac{6.5.4.3}{4.3.2.1} = 15$

(2) Selection of Objects with Repetition :

The total number of selections of r things from n different things when each thing may be repeated any number of times is $^{n+r-1}C_r$

Example 35 :

8 pens are to be selected from pens of 3 colours (pens of each colour being available any number of times), then find the total number of selections.

Sol. $^{3+8-1}C_8 = {}^{10}C_8 = 45$

(3) Restricted Selection/Arrangement

(i) The number of combinations of n different things taken r at a time when k particular objects occurs is $^{n-k}C_{r-k}$. If k particular objects never occur is $^{n-k}C_r$.

(ii) The number of arrangements of n distinct objects taken r at a time so that k particular object are always included $= {}^{n-k}C_{r-k} \cdot r!$ and never included $= {}^{n-k}C_r \cdot r!$

(iii) The number of combinations of n objects, of which p are non-identical, taken r at a time is

$^{n-p}C_r + {}^{n-p}C_{r-1} + {}^{n-p}C_{r-2} + ...+ {}^{n-p}C_0$ if $r \le p$.

$^{n-p}C_r + {}^{n-p}C_{r-1} + {}^{n-p}C_{r-2} + ...+ {}^{n-p}C_{r-p}$ if $r > p$.

Example 36 :

From 4 gentlemen and 6 ladies a committee of five is to be selected. Find the number of ways in which the committee can be formed so that gentlemen are in majority

Sol. The committee will consist of 4 gentlemen and 1 lady or 3 gentlemen and 2 ladies

$\therefore$ the number of committees $= {}^4C_4 \times {}^6C_1 + {}^4C_3 \times {}^6C_2 = 66$

(4) Selection from distinct objects :

The number of ways (or combinations) of n different things selecting at least one of them is

$^nC_1 + {}^nC_2 + {}^nC_3 + ...+ {}^nC_n = 2^n - 1.$

This can also be stated as the total number of combination of n different things.

Example 37 :

Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner ?

Sol. He can invite one, two three, four, five or six friends at the dinner. So total number of ways of his invitation

$= {}^6C_1 + {}^6C_2 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 2^6 - 1 = 63$

(5) Selection from identical objects

The number of combination of n identical things taking r $(r \le n)$ at a time is 1.

The number of ways of selecting r things out of n alike things is n + 1 (where r = 0, 1, 2 .....n).

The number of ways to select some or all out of (p + q + r) things where p are alike of first kind, q are alike of second kind and r are alike of third kind is = (p + 1) (q + 1) (r+ 1) – 1

Example 38 :

There are n different books and p copies of each in a library. Find the number of ways in which one or more than one book can be selected.

Sol. Total cases = p + 1 (if selected or not)

Required number of ways

$= (p + 1)(p + 1)......\text{n terms} - 1 = (p + 1)^n - 1$

Example 39 :

A bag contains 3 one rupee coins, 4 fifty paise coins and 5 ten paise coins. How many selection of money can be formed by taking atleast one coin from the bag.

Sol. Here are 3 things of first kind, 4 things of second kind and 4 things of third kind so the total number of selections

$= (3 + 1)(4 + 1)(5 + 1) - 1 = 119$

(6) Selection when both identical and distinct objects are present: If out of (p + q + r + t) things, p are alike one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of combinations is

$$(p+1)(q+1)(r+1)\,2^t - 1$$

DIVISION AND DISTRIBUTION OF OBJECTS

1. The number of ways in which (m + n) different things can be divided into two groups which contain m and n things respectively is $^{m+n}C_m\ ^{n}C_n = \frac{(m+n)!}{m!n!}$, $m \neq n$

Particular case :

When m = n, then total number of combination is

$\frac{(2m)!}{(m!)^2}$ when order of groups is considered.

$\frac{(2m)!}{2!(m!)^2}$ when order of groups is not considered.

2. The number of ways in which (m + n + p) different things can be divided into three groups which contain m, n and p things respectively is

$$^{m+n+p}C_m \cdot {}^{n+p}C_p \cdot {}^{p}C_p = \frac{(m+n+p)!}{m!n!p!},\ m \neq n \neq p$$

Particular case :

When m = n = p, then total number of combination is

$\frac{(3m)!}{(m!)^3}$ when order of groups is considered.

$\frac{(3m)!}{3!(m!)^3}$ when order of groups is not considered.

3. Total number of ways to divide n identical things among r person is $^{n+r-1}C_{r-1}$

Also total number of ways to divide n identical things among r persons so that each gets atleast one is $^{n-1}C_{r-1}$

4. For distribution of n distinct objects into r different boxes, if in any box any number of objects can be placed for each object, there are r possibilities. Then total number of possibilities for n objects is $r \times r \times r$ .... n times $= r^n$.

5. Distribution of n identical objects in r different boxes if empty boxes are not allowed $= {}^{n-1}C_{r-1}$.

6. Distribution of n identical objects in r different boxes if empty boxes are allowed $= {}^{n+r-1}C_{r-1}$.

7. Grouping and then arrangement.

If (m + n + p) different thing's can be divided in 3 group's & can be distributed to three person's.

$$\text{Required ways} = \frac{(m+n+p)!}{m!\,n!\,p!} \times 3!$$

8. The number of ways in which 'r' group's of n different object's can be formed in such a way that 'p' groups of n_1 object, q group of n_2 object each is ……

$$\text{Required ways} = \frac{n!}{(n_1!)^p (n_2!)^q (p!)(q!)}$$

$n = (n_1 + n_1$ .... p times $) + (n_2 + n_2$ .... q times)

Divide by factorial of number of equal size group.

Note :

(i) Number of non-negative integral solutions of the equation $x_1 + x_2 + ... + x_r = n$

This is equivalent to the number of ways of distributing n identical objects into r different boxes if empty boxes are allowed which is $^{n+r-1}C_{r-1} = {}^{n+r-1}C_n$.

(ii) Number of positive integral solutions of the equation $x_1 + x_2 + ... + x_r = n$

This is equivalent to the number of ways of distributing n identical objects into r different boxes if empty boxes are not allowed which is $^{n-1}C_{r-1}$.

Example 40 :

In how many ways 20 identical mangoes may be divided among 4 persons and if each person is to be given atleast one mango, then find the number of ways .

Sol. 20 identical mangoes may be divided among 4 persons in $^{20+4-1}C_{4-1} = {}^{19}C_3 = 1771$ ways.

If each person is to be given atleast one mango, then number of ways will be $^{20-1}C_{4-1} = {}^{19}C_3 = 969$.

Example 41 :

In how many ways can a pack of 52 cards be divided in 4 sets, three of them having 17 cards each and fourth just one card?

Sol. Since the cards are to be divided into 4 sets, 3 of them having 17 cards each and 4th just one card, so number of

$$\text{ways} = \frac{52!}{(17!)^3 3!1!} = \frac{52!}{(17!)^3 3!}$$

Example 42 :

In how many ways 3 team's of 11 player's each, 4 team's 6 player's each, 2 team's of 15 player's each can be formed out of 87 player's

Sol. $\text{Required ways} = \frac{87!}{(11!)^3 (6!)^4 (15!)^2} \times \frac{1}{(3!)(4!)(2!)}$

Example 43 :

In how many ways 6 bundles of 12 different toys be made such that 2 bundles are of 3 toys each, 2 bundles are 2 toys each & 2 bundle of 1 toy each

Sol. $\text{Required ways} = \frac{(12!)}{(3!)^2 (2!)^2 (1!)^2} \times \frac{1}{(2!)\,(2!)\,(2!)}$

Example 44 :

Total number of ways in which 200 person's can be divided into 100 equal group's.

Sol. $\text{Required ways} = \frac{200!}{(2!)^{100} (100)!}$

Example 45 :

Find number of ways by which 30 Jawan's can be divided into three group's of 12, 10 & 8 and send to three different boarder's.

Sol. Total ways = $\frac{(30!)\times 3!}{(8!)(10!)(12!)}$

In above case if group's are equal size(i.e. group of 10 each)

Send to three boarder's
↑
$= \frac{(30!)\times(3!)}{(10!)^3(3)!}$
↓
Three equal size groups

Example 46 :

Find number of ways by which five different objects given to three students.

Sol. Two cases possible {1, 1, 3} {1, 2, 2}

$$\left[\frac{5!}{(1!)^2\ 3!\times 2!}+\frac{5!}{1!(2!)^2\times 2!}\right]3!$$

Example 47 :

During election's 3 districts are to be canvassed by 20, 15, 10 people respectively. If 45 volunter's there then number of ways in which they can be sent.

Sol. Required ways = $\frac{45!}{20!(5!)(10!)}$

Example 48 :

Find the number of non-negative integral solutions of the equation $x+y+z=10$.

Sol. The no. of solutions is equivalent to the number of ways. Ten identical objects are distributed in 3 distinct boxes if empty boxes are allowed, which is ${}^{10+3-1}C_3 = {}^{12}C_3$.

Example 49 :

Find number of different terms in expansion of $(x+y+z)^{10}$

Sol. ${}^{10+3-1}C_{3-1} = {}^{12}C_2$.

TRY IT YOURSELF-2

Q.1 Twenty-eight games were played in a football tournament with each team playing once against each other. How many teams were there?

Q.2 Out of 10 consonants and 4 vowels, how many words can be formed each containing 3 consonants and 2 vowels?

Q.3 Find the total number of ways of selecting five letters from the word 'INDEPENDENT'.

Q.4 If there are 12 persons in a party, and if each two of them shake hands with each other, how many handshakes happen in the party?

Q.5 In an election, number of candidates exceeds the number to be elected by 2. A man can vote in 56 ways. Find the number of candidates.

Q.6 Find the number of ways of dividing 52 cards among four players equally.

Q.7 Find the number of ways in which n different prizes can be distributed among m (< n) persons if each is entitled to receive at most n – 1 prizes.

Q.8 Find the number of positive integral solutions of the equation $x+y+z=12$.

Q.9 Let C be the set of 6 consonants (b, c, d, f, g, h) and V be a set of 5 vowels (a, e, i, 0, u) and W be the set of seven letter words that can be formed with these 11 letters using both the following rules.

(a) The vowels and consonant in the word must alternate.

(b) No letter can be used more than once in a single word.

If the number of words in the set W are 10 K. Find K.

(A) 1200 (B) 2400

(C) 3600 (D) 3200

Q.10 Sanjeev has 7 friend's. In how many ways can be invite one or more of them to dinner.

(A) 2^5-1 (B) 2^3-1

(C) 2^7-1 (D) 2^9-1

ANSWERS

(1) 8	**(2)** 86400	**(3)** 72
(4) 66	**(5)** 6	**(6)** $\frac{52!}{(13!)^4}$
(7) m^n-m	**(8)** 55	**(9)** (C)
(10) (C)		

NEGATIVE BINOMIAL EXPANSION

$(1-x)^{-n} = 1 + {}^nC_1\,x + {}^{n+1}C_2\,x^2 + {}^{n+2}C_3\,x^3 + \ldots$ to ∞, if $-1<x<1$

Coefficient of x^r in this expansion ${}^{n+r-1}C_1$ $(n \in N)$

Result : Number of ways in which it is possible to make a selection form $m+n+p=N$ things, where p are alike of one kind, m alike of second kind & n alike of third kind taken r at a time is given by coefficient of x^r in the expansion of $(1+x+x^2+....+x^p)(1+x+x^2+...+x^m)(1+x+x^2+....+x^n)$.

For example the number of ways in which a selection of four letters can be made from the letters of the word PROPORTION is given by coefficient of x^4 in $(1+x+x^2+x^3)(1+x+x^2)(1+x+x^2)(1+x)(1+x)(1+x)$.

METHOD OF FICTION PARTITION

Number of ways in which n identical things may be distributed among p persons if each person may receive none, one or more things is, ${}^{n+p-1}C_n$.

Example 50 :

Find the number of solutions of the equation x + y + z = 6, where x, y, z ∈ W.

Sol. Number of solutions

= Coefficient of x^6 in $(1+x+x^2+\ldots\ldots\ldots.\ x^6)^3$

= Coefficient of x^6 in $(1-x^7)^3(1-x)^{-3}$

= Coefficient of x^6 in $(1-x)^{-3} = {}^{3+6-1}C_6 = {}^8C_2 = 28$

DIVISORS

Let $N = p^a q^b r^c$....... where p, q, r....... are distinct primes and a, b, c.......... are natural numbers then :

(a) The total numbers divisors of N including 1 & N is $(a+1)(b+1)(c+1)$ ....

(b) The sum of these divisors
$(p^0 + p^1 + p^2 + ... + p^a)(q^0 + q^1 + q^2 + ... + q^b)(r^0 + r^1 + r^2 + ... + r^c)$ ....

(c) Number of ways in which N can be resolved as a product of two factors is

$$= \begin{cases} \frac{1}{2}(a+1)(b+1)(c+1)........... & \text{If N is not a perfect square} \\ \frac{1}{2}[(a+1)(b+1)(c+1)......+1] & \text{If N is perfect square} \end{cases}$$

(d) Numbers of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N.

Example 51 :

Find the number of divisors of 1350. Also find the sum of all divisors.

Sol. $1350 = 2 \times 3^3 \times 5^2$

Number of divisors $= (1+1)(3+1)(2+1) = 24$

Sum of divisors $= (1+2)(1+3+3^2+3^3)(1+5+5^2) = 3720$

Example 52 :

Consider the number $N = 2^5 \times 3^4 \times 5^7 \times 7^2$ then find the number of divisors divisible by 5.

Sol. Atleast one 5 must be there in $2^5 \times 3^4 \times 5^7 \times 7^2$

$(5+1)(4+1)(7)(2+1) = 630$

SUM OF NUMBERS

(i) For given n different digits a_1, a_2, a_3 ......a_n the sum of the digits in the unit place of all numbers formed (if numbers are not repeated) is $(a_1 + a_2 + a_3 + ... + a_n)(n-1)!$
i.e. (sum of the digits) $(n-1)!$

(ii) Sum of the total numbers which can be formed with given n different digits a_1, a_2, ...a_n is
$(a_1 + a_2 + a_3 + ... + a_n)(n-1)! \,.\, (111$ ....n times)

Example 53 :

Find the sum of all the numbers greater then 10000 formed by the digits 1,3,5,7,9 if no digit being repeated.

Sol. Method 1 : All possible numbers = 5! = 120

If one occupies the units place then total numbers = 24.

Hence 1 enjoys units place 24 times

|||ly 1 enjoys each place 24 times

				1

Sum due to 1 = $1 \times 24(1 + 10 + 10^2 + 10^3 + 10^4)$

|||ly Sum due to the digit 3 = $3 \times 24(1 + 10 + 10^2 + 10^3 + 10^4)$

⋮ ⋮ ⋮ ⋮ ⋮ ⋮ ⋮

Required total sum

$= 24(1 + 10 + 10^2 + 10^3 + 10^4)(1 + 3 + 5 + 7 + 9)$

Method –2 : In 1^{st} column there are twenty four 1's , Twenty four 3's & so on and their sum is = 24 × 25 = 600

Hence add. in vertical column normally we get = 6666600

	5^{th}			2^{nd}	1^{st}
120 Number	X	X	X	X	X
	X	X	X	X	X
	⋮	⋮	⋮	⋮	⋮
	⋮	⋮	⋮	⋮	⋮
	X	X	X	X	X

666 6 6 0 0 = 6666600

Method–3 : Applicable only if the digits used are such that they have the same common difference. (valid even if the digits are repeating)

Writing all the numbers in ascending order of magnitude

$S = (13579 + 13597 + ... + 97513 + 97531)$

$S = (13579 + 99531) + (13597 + 97513) + ...$

$= (111110)$ 60 time $= 6666600$ Ans

$S = \frac{n}{2}(\ell + L)$ where n = number of numbers, ℓ = smallest,

L = Largest

Example 54 :

Find the sum of all 4 digit numbers formed with the digits 1,2,4 and 6.

Sol. Sum $= (a_1 + a_2 + a_3 + ... + a_n)(n-1)!\,(111$ ....n times)

Sum $= (1+2+4+6).3!.(1111) = 13 \times 6\,1111 = 86658$

Example 55 :

Find the sum of all the four digit numbers that can be formed with the digits 3, 2, 3, 4.

Sol. The number of numbers having 2 in units place $= \frac{3!}{2!} = 3$

[∵ the other three places are to be filled by 3, 3 and 4]

The number of numbers having 4 in units places $= \frac{3!}{2!} = 3$

[∵ the other three places are to be filled by 3, 3 and 2]

and the number of numbers having 3 in units places $= 3! = 6$

[∵ the other three places are to be filled by 2, 3 and 4]

Thus, sum of the digits occurring in the units place
$= 2 \times 3 + 3 \times 6 + 4 \times 3 = 36$

We can see that the given digits (3, 2, 3, 4) occur at the tens, hundreds and thousands place, the same number of times as they occur at the units place.

Hence, the required sum of the numbers formed
$= 36(1 + 10 + 100 + 1000) = 39996$

DERANGEMENT THEOREM

Any change in the given order of the thing is called a Derangement.

(i) If n items are arranged in a row, then the number of ways in which they can be rearranged so that no one of them occupies the place assigned to it is

$$n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+....+(-1)^n\frac{1}{n!}\right]$$

(ii) If n things are arranged at n places then the number of ways to rearrange exactly r things at right places is

$$\frac{n!}{r!}\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+....+(-1)^{n-r}\frac{1}{(n-r)!}\right]$$

Example 56 :

There are 3 letters and 3 envelopes. Find the number of ways in which all letters are put in the wrong envelopes.

Sol. The required no. of ways $= 3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right] = 3-1 = 2$

Example 57 :

There are 4 balls of different colour and 4 boxes of colours same as those of the balls. Then find the number of ways to place two balls in boxes with respect to their colour.

Sol. Number of ways $= \frac{4!}{2!}\left[1-\frac{1}{1!}+\frac{1}{2!}\right] = 4.3\left[1-1+\frac{1}{2}\right] = 6$

IMPORTANT RESULTS ABOUT POINTS

If there are n points in a plane of which m (< n) are collinear, then

(a) Total number of different straight lines obtained by joining these n points is $^nC_2 - {}^mC_2 + 1$

(b) Total number of different triangles formed by joining these n points is $^nC_3 - {}^mC_3$

(c) Number of diagonals in polygon of n sides is

$$^nC_2 - n \text{ i.e. } \frac{n(n-3)}{2}$$

(d) If m parallel lines in a plane are intersected by a family of other n parallel lines. Then total number of parallelograms so formed is

$$^mC_2 \times {}^nC_2 \quad \text{i.e.} \quad \frac{mn(m-1)(n-1)}{4}$$

(e) Number of triangles formed by joining vertices of convex polygon of n sides is $^nC_3$ of which

(i) Number of triangles having exactly two sides common to the polygon = n

(ii) Number of triangles having exactly one side common to the polygon = n(n – 4)

(iii) Number of triangles having no side common to the polygon

$$= \frac{n(n-4)(n-5)}{6}$$

Example 58 :

There are 10 points in a plane and 4 of them are collinear. Find the number of straight lines joining any two of them.

Sol. A straight line can be drawn joining two points, so there will be $^{10}C_2$ straight lines joining 10 points. But 4 of them are collinear, so we shall get only one line joining any two of these 4 points.

Hence the total number of lines $= {}^{10}C_2 - {}^4C_2 + 1 = 40$

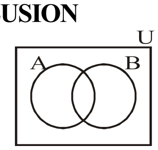
PRINCIPLE OF INCLUSION AND EXCLUSION

In the Venn's diagram (i), we get

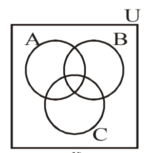
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A' \cap B') = n(U) - n(A \cup B)$

In the Venn's diagram (ii), we get



(i)



(ii)

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$n(A' \cap B' \cap C') = n(U) - n(A \cup B \cup C)$

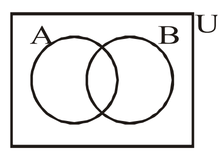
In general, we have $n(A_1 \cup A_2 \cup .... \cup A_n)$

$$= \sum n(A_i) - \sum_{i \neq j} n(A_i \cap A_j) + \sum_{i \neq j \neq k} n(A_i \cap A_j \cap A_k)$$

$$+ ..... + (-1)^n \sum n(A_1 \cap A_2 \cap ... \cap A_n)$$

Example 59 :

Find the number of permutations of letters a,b,c,d,e,f,g taken all at a time if neither 'beg' nor 'cad' pattern appear.



Sol. The total number of permutations without any restrictions; n (U) = 7!

Let A be the set of all possible permutations in which 'beg' pattern always appears : n (A) = 5!

(b e g) a c d f

Let B be the set of all possible permutations in which 'cad' pattern always appears : n (B) = 5!

(c a d) b e f g

$n(A \cap B)$: Number of all possible permutations when both 'beg' and 'cad' patterns appear.

(c a d) (b e g) f

$n(A \cap B) = 3!$.

Therefore, the total number of permutations in which 'beg' and 'cad' patterns do not appear

$$n(A' \cap B') = n(U) - n(A \cup B) = n(U) - n(A) - n(B) + n(A \cap B) = 7! - 5! - 5! + 3!.$$

PROMBLEMS BASED ON NUMBER THEORY

Note that every natural number except 1 has atleast 2 divisors. If it has exactly two divisors then it is called a prime. System of prime numbers begins with 2. All primes except 2 are odd. A number having more than 2 divisors is called a composite. **2 is the only even number which is not composite.** A pair of natural numbers are said to be relatively prime or coprime if their HCF is one. For two natural numbers to be relatively it is not necessay that one or both should be prime. It is possible that they both are composite but still coprime. eg. 4 and 25. Note that 1 is neither prime nor composite however it is coprime with every other natural number.A pair of primes are said to be twin if their non-negative difference is 2 e.g. 3 & 5 ; 5 & 7 e.t.c.

Number of divisors and their sum :

(a) Every natural number N can always be put in the form

$N = p_1^{\alpha 1}, p_2^{\alpha 2} \ldots p_k^{\alpha k}$ where $p_1, p_2, \ldots p_k$ are distinct primes and $\alpha_1, \alpha_2, \ldots, \alpha_k$ are non-negative integers.

(b) If $N = p_1^{\alpha 1}, p_2^{\alpha 2} \ldots p_k^{\alpha k}$ then number of divisor of N is equivalent of number of ways of selecting zero or more objects from the groups of identical objects, $(p_1, p_1, \ldots, \alpha_1$ times) $(p_2, p_2, \ldots \alpha_2$ times), $(p_k, p_k, \ldots, \alpha_k$ times) $= (\alpha_1 + 1)(\alpha_2 + 1) \ldots (\alpha_k + 1)$ which includes 1 and N also.

(c) All the divisors excluding 1 and N are called proper divisors. Also number of divisions of N can be seen as number of different terms in the expansion of

$$(p_1^0 + p_1^1 + p_1^2 + \ldots + p_1^{\alpha 1}) \times (p_2^0 + p_2^1 + p_2^2 + \ldots + p_2^{\alpha 2}) \times \ldots\ldots \times (1 + p_k + p_k^2 + \ldots + p_k^{\alpha k})$$

Hence, sum of the divisors of N is

$$(1 + p_1 + p_1^2 + \ldots + p_1^{\alpha 1}) \times (1 + p_2 + p_2^2 + \ldots + p_2^{\alpha 2}) \ldots\ldots (1 + p_k + p_k^2 + \ldots + p_k^{\alpha k})$$

$$= \frac{p_1^{\alpha 1+1} - 1}{p_1 - 1} \, \frac{p_2^{\alpha 2+1} - 1}{p_2 - 1} \ldots \frac{p_k^{\alpha k+1} - 1}{p_k - 1}$$

(d) Number of ways of putting N as a product of two natural numbers is $\frac{1}{2}(a_1 + 1)(a_2 + 1) \ldots\ldots (a_k + 1)$ if N is not a perfect square.

If N is a perfect square, then this is

$$\frac{1}{2}[(a_1 + 1)(a_2 + 1) \ldots\ldots (a_k + 1) + 1].$$

(e) If $N = p^a \times q^b \times r^c \ldots\ldots$ p, q are prime number & a, b are natural number.

$$I = \frac{(\text{Total number of divisors})}{2}$$

where I is the number of required ways if N is not a perfect square.

$$I = \frac{(\text{Total number of divisors} + 1)}{2}$$

where I is the number of required ways if N is a perfect square.

Example 60 :

Consider the number $N = 2^5 \times 3^4 \times 5^7 \times 7^2$, find

(i) Total number of divisor,

(ii) Number of proper divisor

(iii) Number of odd divisor

(iv) Number of even divisor.

(v) Number of divisors divisible by 5.

(vi) Number of divisors divisible by 10.

(vii) Number of divisors divisible by 2 but not by 4.

(viii) Sum of all divisors

(ix) Sum of even divisors

(x) Sum of odd divisors

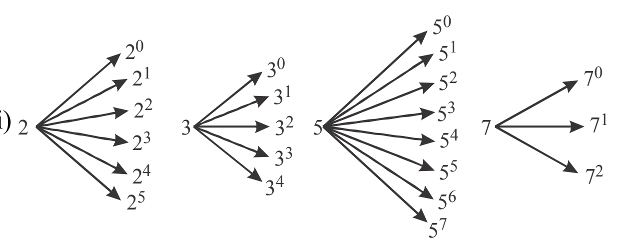
(xi) Sum of divisor of the form $(4n + 2),\ n \in N\ (4n + 2)$

$\Rightarrow$ Even number but not divisible by 4, so exactly one 2.

(xii) Number of ways in which N can be resolved as product of two divisor.

Sol.

(i)



Number of divisor $= (5 + 1)(4 + 1)(7 + 1)(2 + 1)$
$= 6 \times 5 \times 8 \times 3 = 720$

(ii) Proper divisor will be other than 1 and number itself.
So proper divisor $= 720 - 2 = 718$.

(iii) Number of odd divisor can be obtained by choosing $3^4 \times 5^7 \times 7^2$
Which can be formed by $(4 + 1)(7 + 1) \times (2 + 1) = 120$ ways

(iv) It can be obtain by selecting atleast one '2' in $2^5 \cdot 3^4 \cdot 5^7 \cdot 7^2$ which can be done in $5 \times (4 + 1) \times (7 + 1) \times (2 + 1) = 600$

(v) Atleast one 5 must be there in $2^5 \times 3^4 \times 5^7 \times 7^2$
$(5 + 1)(4 + 1)(7)(2 + 1) = 630$

(vi) For divisibility by 10 number most be divisible by 2 & 5.
So total divisor must contain atleast one 2 and atleast one 5 in $2^5 \times 3^4 \times 5^7 \times 7^2$
So that ways $= 5 \times (4 + 1) \times (7) \times (2 + 1) = 525$

(vii) Exact one 2 should be there
Total ways $= 1 \times 5 \times 8 \times 3 = 120$

(viii) It can be obtained from
$(2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)$
$(5^0 + 5^1 + 5^2 + 5^3 + 5^4 + \ldots\ldots + 5^7)(7^0 + 7^1 + 7^2)$

$$= (2^6 - 1) \times \frac{(3^5 - 1)}{2} \times \frac{(5^8 - 1)}{4} \times \frac{(7^3 - 1)}{6}$$

(ix) At least one 2 must be there
$\text{Sum} = (2^1 + 2^2 + \ldots + 2^5)(3^0 + 3^1 + \ldots + 3^4)(5^0 + 5^1 + \ldots + 5^7)(7^0 + 7^1 + 7^2)$

$$\text{Sum} = 2 \cdot (2^5 - 1) \times \frac{(3^5 - 1)}{2} \times \frac{(5^8 - 1)}{4} \times \frac{(7^3 - 1)}{6}$$

(x) No 2 can be selected
$\text{Sum} = (3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + \ldots + 5^7)(7^0 + 7^1 + 7^2)$

(xi) $\text{Sum} = (2)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + \ldots\ldots + 5^7)(7^0 + 7^1 + 7^2)$

$$= 2 \times \frac{(3^5 - 1)}{2} \times \frac{(5^8 - 1)}{4} \times \frac{(7^3 - 1)}{6} = \frac{(3^5 - 1)(5^8 - 1)(7^3 - 1)}{24}$$

(xii) When N is perfect square $N = 2^5 \times 3^4 \times 5^7 \times 7^2$ is not a perfect square

$$I = \frac{(\text{Total number of divisors})}{2} = \frac{720}{2} = 360$$

TRY IT YOURSELF-3

Q.1 Find the maximum number of points of intersection of 6 circles.

Q.2 There are 10 points on a plane of which 5 points are collinear. Also, no three of the remaining 5 points are collinear. Then find (i) the number of straight lines joining these points; (ii) the number of triangles formed joining these points.

Q.3 There are four balls of different colours and four boxes of colours same as those of the balls. Find the no. of ways in which the balls, one in each box, could be placed in such a way that a ball does not go to box of its own colour.

Q.4 Consider the 10 digits 0, 1, 2, 3, ... 9
S-1 : Number of four digit even numbers that can be formed if each digit is to used only once in the number is 2268.
S-2 : Total 4 -digits numbers that can be formed if each digit is used only once is 4536.
(A) S-1 is true, S-2 is true & S-2 is correct explanation for S-1.
(B) S-1 is true, S-2 is true & S-2 is not the correct explanation for S-1.
(C) S-1 is true, S-2 is false.
(D) S-1 is false, S-2 is true.

Q.5 Find the number of non-negative integral solutions of $x + y + z + w \le 20$.

Q.6 In how many ways 8100 can be resolved into product of two factors?

Q.7 Consider the number $N = 2^5 \times 3^4 \times 5^7 \times 7^2$ then find the number of divisors divisible by 2 but not by 4.

Q.8 Let $n \ge 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then find the value of n.

Q.9 If 5 parallel straight lines are intersected by 4 parallel straight lines, then find the number of parallelograms thus formed.

ANSWERS

(1) 30	**(2)** (i) 36, (ii) ${}^{10}C_3 - {}^5C_3$	
(3) 9	**(4)** (D)	**(5)** ${}^{20+5-1}C_{5-1}$.
(6) 23	**(7)** 120	**(8)** 5
(9) 60		

ADDITIONAL EXAMPLES

Example 1 :
There are 13 players of cricket, out of which 4 are bowlers. In how many ways a team of eleven be selected from them so as to include atleast two bowlers ?

Sol. A team of eleven may be selected in the following three ways
(i) 2 bowlers + 9 others
(ii) 3 bowlers + 8 others
(iii) 4 bowlers + 7 others
There are 4 bowlers and 9 other, so the total number of selection $= ({}^4C_2 \times {}^9C_9) + ({}^4C_3 \times {}^9C_8) + ({}^4C_4 \times {}^9C_7)$
$= 6 + 36 + 36 = 78$

Example 2 :
There are 10 points in a plane of which no three points are collinear and 4 points are concyclic. Find the number of different circles that can be drawn through at least 3 points of these points.

Sol. The number of circles $= ({}^{10}C_3 - {}^4C_3) + 1 = 117$

Example 3 :
A bag contains 3 black, 4 white and 2 red balls, all the balls being different. Find the number of selections of atmost 6 balls containing balls of all the colours.

Sol. The required number of selections
$= {}^3C_1 \times {}^4C_1 \times {}^2C_1 ({}^6C_3 + {}^6C_2 + {}^6C_0) = 42 \times 4!$

Example 4 :
To fill up 12 vacancies, there are 25 candidates of which 5 are from SC. If 3 of these vacancies are reserved for SC candidates while the remaining are open to all then find the number of ways in which the selection can be made.

Sol. 3 vacancies for SC candidates can be filled up from 5 candidates in ${}^5C_3$ ways.
After this for remaining $12 - 3 = 9$ vacancies, there will be $25 - 3 = 32$ candidates. These vacancies can be filled up in ${}^{22}C_9$ ways. Hence required number of ways $= {}^5C_3 \times {}^{22}C_9$

Example 5 :
Find the number of 6 digit numbers that can be made with the digits 1, 2, 3 and 4 and having exactly two pairs of digits.

Sol. The number will have 2 pairs and 2 different digits
The number of selections $= {}^4C_2 \times {}^2C_2$, and for each selection, number of arrangements $= \frac{6!}{2!2!}$.

The required number of nos = $^4C_2 \times {^2C_2} \times \frac{6!}{2!2!} = 1080$

Example 6 :

Find the number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being. 10 pearls of each colour.

Sol. Ten pearls of one colour can be arranged in $\frac{1}{2}.(10-1)!$ ways. The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour $= 10!$

$\therefore$ the required number of ways $= \frac{1}{2} \times 9! \times 10! = 5\,(9!)^2$

Example 7 :

Find the number of ways of arranging six persons (having A, B, C and D among them) in a row so that A, B, C and D are always in order ABCD (not necessarily together).

Sol. The number of ways of arranging ABCD is 4!. For each arrangement of ABCD, the number of ways of arranging six persons is same. Hence required number is $\frac{6!}{4!} = 30$.

Example 8 :

Find the number of words each containing 3 consonants and 2 vowels that can be formed out of 5 consonants and 4 vowels.

Sol. 3 consonants and 2 vowels from 5 consonants and 4 vowels can be selected in $^5C_3 \times {^4C_2} = 60$ ways.
But total number of words with $3 + 2 = 5$ letters $= 5! = 120$
$\therefore$ The required number of words $= 60 \times 120 = 7200$

Example 9 :

Find the number of numbers less than 1000 that can be formed out of the digits 0,1,2,4 & 5 , no digit being repeated.

Sol. $^4C_1 + {^4C_1} \times {^4C_1} + {^4C_1} \times {^4C_1} \times {^3C_1} = 4 + 16 + 48 = 68$

Example 10 :

Find the total number of ways of selecting five letters from letters of the word INDEPENDENT.

Sol. There are 11 letters in the given word which are as follows (NNN) (EEE) (DD)IPT
Five letters can be selected in the following manners :
(i) All letters different : $^6C_5 = 6$
(ii) Two similar and there different : $^3C_1 \cdot {^5C_3} = 30$
(iii) Three similar and two different : $^2C_1 . {^5C_2} = 20$
(iv) Three similar and two similar : $^2C_1 . {^2C_1} = 4$
(v) Two similar, two similar and one different : $^3C_2 . {^4C_1} = 12$
$\therefore$ Total selections $= 6 + 30 + 20 + 4 + 12 = 72$

Example 11 :

How many numbers greater than 10 lac be formed from 2, 3, 0, 3, 4, 2, 3 ?

Sol. A number greater than 10 lac contains atleast 7 digits. So every number which is formed using all the given 7 digits will be greater than 10 lac. But in given digits 2 occurs twice and 3 occurs thrice and 0 is also there. Hence the required number of numbers $= \frac{7!}{2!3!} - \frac{6!}{2!3!} = 420 - 60 = 360$

Example 12 :

There are three coplanar parallel lines. If any p points are taken on each of the lines, find the maximum number of triangles with vertices at these points.

Sol. The number of triangles with vertices on different lines $= {^pC_1} \times {^pC_1} \times {^pC_1} = p^3$
The number of triangles with 2 vertices on one line and the third vertex on any one of the other two lines

$= {^3C_1}\,\{^pC_2 \times {^{2p}C_1}\} = 6p.\frac{p(p-1)}{2}$

$\therefore$ the required number of triangles
$= p^3 + 3p^2(p-1) = p^2\,(4p-3)$

Example 13 :

Find the total number of 6 digit numbers in which all the odd digits and only digits appear.

Sol. Clearly, one of the odd digits 1, 3, 5, 7, 9 will be repeated
The number of selections of the sixth digit $= {^5C_1} = 5$
$\therefore$ The required number of numbers $= 5 \times (6!/2)$

Example 14 :

Find the number of words of four letters containing equal number of vowels and consonants, repetition being allowed.-

Sol. The number of selections of 1 pair of vowels and 1 pair of consonants $= {^5C_1} \times {^{21}C_1}$
The number of selections of 2 different vowels and 2 different consonants $= {^5C_2} \times {^{21}C_2}$

Number of wards $= {^5C_1} \times {^{21}C_1} \times \frac{4!}{2!\,2!} + {^5C_2} \times {^{21}C_2} \times 4!$

Example 15 :

Find the number of ways in which a couple can sit around a table with 6 guests if the couple take consecutive seat.

Sol. A couple and 6 guests can be arranged in $(7-1)!$ ways. But in two people forming the couple can be arranged among themselves in 2! ways.
$\therefore$ The required number of ways $= 6! \times 2! = 1440$

Example 16 :

Find the number of 5 digit numbers that can be made using the digits 1 and 2 and in which at least one digit is different.

Sol. Total number of numbers without restriction $= 2^5$
Two numbers have all the digits equal. So, the required numbers of numbers $= 2^5 - 2 = 30$

Example 17 :

In how many ways can a committee of 5 be made out of 6 men and 4 women containing atleast one woman ?

Sol. A committee of 5 out of 6 + 4 = 10 can be made in $^{10}C_5 = 252$ ways.

If no woman is to be included, then number of ways $= {}^{6}C_5 = 6$

$\therefore$ The required number = 252 – 6 = 246

Example 18 :

The sum of all the numbers of four different digits that can be made by using the digits 0, 1, 2 and 3.

Sol. The number of numbers with 0 in the units place = 3! = 6

The number of numbers with 1 or 2 or 3 in the units place $= 3! - 2! = 4$

$\therefore$ The sum of the digits in the unit place

$= 6 \times 0 + 4 \times 1 + 4 \times 2 + 4 \times 3 = 24$

Similarly for the tens and the hundreds places.

The number of numbers with 1 or 2 or 3 in the thousands place = 3!

$\therefore$ The sum of the digits in the thousands place

$= 6 \times 1 + 6 \times 2 + 6 \times 3 = 36$

Required sum $= 36 \times 1000 + 24 \times 100 + 24 \times 10 + 24 = 38664$

Example 19 :

If 3n different things can be equally distributed among 3 person in k ways then find the number of ways to divide the 3n things in 3 equal groups.

Sol. (The number of ways of dividing in 3 equal groups) × (3!) = the number of ways to distribute equally among 3 persons.

Example 20 :

Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of (m/n) is

Sol. $n = 5! \times 6!$; $m = 5! \times {}^{6}C_2 \times {}^{5}C_4 . 2! . 4!$

$$\frac{m}{n} = \frac{5! \times 15 \times 2 \times 5!}{6!} = 5$$

Example 21 :

In a car plate number containing only 3 or 4 digits not containing the digit 0. What is the maximum numbers of cars that can be numbered?

Sol. Here repetition of digits is allowed.

Also, numbers are formed with the digit1, 2, 3, …… 9

Case-I : When car plate numbers contain 3 digit number of places to be filled up r = 3.

Out of the 9 digit first place can be filled by 9 ways.

Similarly, 2nd and 3rd place can be filled in 9 ways respectively.

So, when car plate number contais 3 digit, maximum number of cars = 9^3.

Case-II : When car plate number contains 4 digit, in this case number of cars to be filled up r = 4.

1st place can be filled in 9 ways.

2nd place can be filled by 9 ways and so on.

Maximum number of cars that can be numbered.

$= 9^3 + 9^4 = 7290$

Example 22 :

How many 6 digits odd number greater than 6,00,000 can be formed from the digits 5,6,7, 8, 9, 0 if repetition of digit is allowed ?

Sol. Numbers greater than 6,00,000 and formed with the digit 5, 6, 7, 8, 9, 0 are of 6 digit but begin with 6, 7, 8 or 9.

Also, the numbers which end with 5, 7, 9 are odd.

Hence, first place can be filled by 4 ways (out of 6, 7, 8 or 9).

Last place can be filled by 3 ways. Hence, first and last place can be filled by 4 × 3 ways.

Also2nd place can be filled by 6 ways.

3rd place can be filled by 6 ways

4th place can be filled by 6 ways.

5th place can be filled by 6 ways

Hence, all the 6 places can be filled by

$4 \times 3 \times 6 \times 6 \times 6 \times 6 = 15552$ ways.

Example 23 :

Consider the word DAUGHTER. How many 4 letter word can be formed from the letter of above word so that each word contain letter G.

Sol. 4 possible positions for G.

Remaining three by $\Rightarrow$ $4 \times 7 \times 6 \times 5 = 28 \times 30 = 840$.

Alternative method :

Total number of ways by which 4 letter word can be formd

$= 8 \times 7 \times 6 \times 5$

Number of four letter word without G $= 7 \times 6 \times 5 \times 4$

$= 8 \times 7 \times 6 \times 5 - 7 \times 6 \times 5 \times 4.$

Example 24 :

How many diffrent words can be formed using all the letters in the word "MIRACLE".

(a) If vowels may accupy the even position.

(b) If vowels may occupy odd position.

Sol. (a) Even position $\Rightarrow$

1	2	3	4	5	6	7
	×		×		×	

Vowels $\rightarrow$ I, E, A

Consonants $\rightarrow$ M, R, C, L

Three vowels at three position $\Rightarrow$ $3 \times 2 \times 1 = 6$

Four consonants at four position $\Rightarrow 4 \times 3 \times 2 \times 1 = 24$

Total number of ways = 6 × 24 = 144.

(b)

1	2	3	4	5	6	7
×		×		×		×

1st position can be filled by any one of the four vowel.

2nd position can be filled by any one of the three vowel.

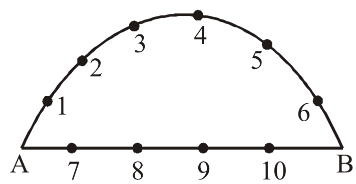
3rd position can be filled by any one of the two vowel.

Thus total ways $= (4 \times 3 \times 2) \times (4 \times 3 \times 2 \times 1) = 576$

Example 25 :

There are 10 points in a plane of which 4 are collinear and rest are non-collinear. Find (i) Number of lines, (ii) Number of triangles

Sol. (i) Number of lines



Method-I : Given 4 points are collinear so total number of ways of selecting any two points = $^{10}C_2$.
If 4 collinear points give only one line (AB). So over counted number of lines formed by collinear points = $^4C_2 - 1$.
Thus total lines = $^{10}C_2 - {}^4C_2 + 1$

Method-II : Take any two points from upper arc = $^6C_2$ ways.
Take one point from upper arc and one point from line AB = $^6C_1 \times {}^4C_1$ ways.
Take both the points from line AB then number of lines = 1 (Line AB).
Total number of lines = $^6C_1 \times {}^4C_1 + {}^6C_2 + 1$

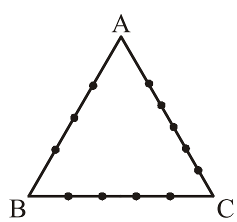
(ii) Method-I : Number of ways of taking any three points = $^{10}C_3$
Number of ways of taking any three points from four collinear points = $^4C_3$
Number of triangles formed = $^{10}C_3 - {}^4C_3$

Method-II : Two points from upper arc and one point from line AB = $^6C_2 \times {}^4C_1$
Two points from line AB and one point from upper arc = $^6C_1 \times {}^4C_2$
All the three points from upper arc = $^6C_3$
Total number of triangle = $^6C_2 \times {}^4C_1 + {}^6C_1 \times {}^4C_2 + {}^6C_3$

Example 26 :

The sides AB, BC, CA of a triangle ABC have 3, 4, 5 points respectively on them. Find the number of triangle that can be constructed using these points as vertices.

Sol. Total ways = $^{12}C_3 - {}^3C_3 - {}^4C_3 - {}^5C_3 = 205$



Example 27 :

How many diffrent words can be formed from the letters of the word GANESHPURI, when the letters E, H, P are never together.

Sol. | | EHP | | | | | | | | EHP → String

(Consider EHP as a letter or string)
So, we have to arrange one string & 7 letters
Hence total number of ways = 10! – 8! × 3!
(EHP can be arranged amongst themselves in 3! ways.)

Example 28 :

How many ways can the seven diffrent colour of a rainbow be arranged so that the blue and green never come together

Sol. Total number of arrangement without constraint = 7!

| | BG | | | | | BG → String

(Consider BG as a letter or string)
If BG always come together then number of ways

= (1 + 5)! 2!
(1 → string; 5 → letters; 2! → Arrangement of B & G)

Required number of ways
= Total possible (without restriction)
– (ways when BG together)
= 7! – 6! × 2!

Example 29 :

4 Boys & 4 Girls are to be seated in a line find number of ways

(i) They can be seated so that "No two girls are together"
(ii) If not all the girls are together
or
If at least one girl is separated from rest of girls.
(iii) Boys and girls are alternate
(iv) If there are 4 married couples then number of ways in which they can be seated so theat each couple is together.

Sol. (i) Out of four standing boys five gaps are there

| B_1 | B_2 | B_3 | B_4 |

Out of five select any four gaps by $^5C_4$ ways in which girls are arranged by = $^5C_4 \times 4!$ ways.
Also standing boys are arranged by 4! ways
Required ways = (4!) × ($^5C_4 \cdot 4!$)
Note : Arrangement by this method is called as gap method.
(ii) Consider $G_1\,G_2\,G_3\,G_4 \rightarrow$ as one string.
If all girls are together then total ways

= (1 + 4)! (4!) = (5! × 4!)
(1 → String; 4 → Four boys; (4!) → Arrangement of four girls in string)

Total number of arrangment without any restriction = 8!
Total number of ways by which not all girls are together
= 8 ! – (5 ! × 4 !)

(iii) If boys and girls are alternate then two ways of arranging respective position of boys and girls are :

OR

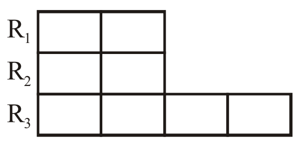
Number of ways of arranging boys $\Rightarrow$ 4!
Number of ways of arranging girls $\Rightarrow$ 4!
Required ways = $2 \times 4! \times 4!$
(iv) Let the couples are as below

B_1G_1	B_2G_2	B_3G_3	B_4G_4

There are four string and each string has husband and wife which can be arranged in 2! ways.
Ways of arranging strings = 4!
Required ways = $(2!)^4 \times 4!$

Example 30 :

In how many ways the letters of the word 'PERSON' can be placed in the squares of the given figure shown so that no row remain empty?



Sol. There are 6 different letters in the word 'PERSON'
Total required way
= Total possible way – (1^{st} row empty + 2^{nd} row empty)
Total possible ways
= ${}^8P_6$ (arrangement of 6 different letters in 8 boxes)
Ways when 1^{st} row empty = 6! (arrangement of 6 different letters in remaining 6 boxes)
Way when 2^{nd} row empty = 6! (arrangement of 6 different letters in remaining 6 boxes)
Total required ways = ${}^8P_6 - (6! + 6!) = 18720$

Example 31 :

In how many ways six different books can be distributed between four persons, so that each person gets atleast one book.

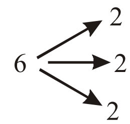
Sol. Two cases possible {1, 1, 1, 3}, {1, 1, 2, 2}

$$\text{Groups}\left[\frac{6!}{(1!)^3\ 3!\,3!} + \frac{6!}{(1!)^2\ (2!)^2\ 2!\,2!}\right]4!$$

Example 32 :

In a jeep there are 3 seat in front and three in the back, number of different ways in which six persons of different height can be seated so that every one in front is shorter than the person directly behind him,

Sol.



$$\frac{6! \times 3!}{(2!)^3\ 3!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{8 \times 6} \times 6 = 15 \times 6 = 90$$

×	×	×
×	×	×

Example 33 :

Consider word ASSASSINATION, find number of ways of arranging the letters.
(i) Number of words using all.
(ii) If no two vowels are together.
(iii) If all S are seperated.
(iv) Atleast one S is seperated from rest of the S's
(v) vowels are in the same order.
(vi) Relative position of vowels and consonant remain same.

Sol. (i) ASSASSINATION contains four S, three A, two N and two I.

$$\text{Total ways} = \frac{13!}{(4!)\,(3!)\,(2!)\,(2!)}$$

(ii) We have six vowels as A, A, A, I, I, O and seven consonants as S, S, S, S, N, T, N
| S | S | S | S | N | T | N |
Six vowels in 8 gap's

$$\text{Total ways} = {}^8C_6 \times \frac{6!}{(3!)\,(2!)} \times \frac{7!}{(4!)\,(2!)}$$

(iii) | A | A | I | N | A | T | I | O | N |
Out of 10 gaps select 4

$$\text{Total ways} = {}^{10}C_4 \times \frac{9!}{(3!)\,(2!)\,(2!)}$$

(iv) Total – all four S together

$$\frac{13!}{(4!)\,(3!)\,(2!)\,(2!)} - \frac{10!}{(3!)(2!)\,(2!)}$$

$\Rightarrow$ Consider [S S S S] as one string.

(v) $$\text{Total ways} = \underbrace{{}^{13}C_6 \times 1}_{\text{arrangement of vowels}} \times \underbrace{\frac{7!}{4!\,2!\,1!}}_{\text{arrangement of consonants}}$$

(vi)

v			v		v		v		v	v	

$$\text{Total ways} = \underbrace{\frac{6!}{3!\ 2!}}_{\text{arrangement of vowels}} \times \underbrace{\frac{7!}{4!\,2!}}_{\text{arrangement of consonants}}$$

Example 34 :

How many words can be formed using all the letters of the word HONOLULU if no two alike letters are together.

Sol. Let A represent's ways when OO together, B when LL together, C when UU together.

Required ways = Total ways – [When all three alike letters together + when 2 alike letters together + when one alike letter together]

$= \text{Total ways} - [n(E_3) + n(E_2) + n(E_1)]$ ...(i)

$$\text{Total ways} = \frac{8!}{(2!)\,(2!)\,(2!)} = 5040$$

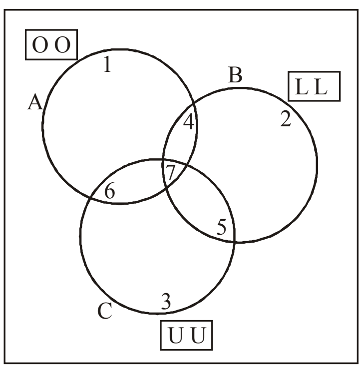
(a) $n(E_3) = A \cap B \cap C$ (Region 7)

i.e., | H | N | OO | LL | UU |

$n(E_3) = 5! = 120$

(b) $n(E_2) = 3\,[(A \cap B) - (A \cap B \cap C)]$ or [Region 4 + 5 + 6]

$$= 3\left(\frac{6!}{2!} - 5!\right) = 720$$



i.e., | H | OO | N | LL | UU |

(c) $n(E_1) = 3\,[A - \{(A \cap B) + (A \cap C)\} + (A \cap B \cap C)]$

$$= \left[\frac{7!}{(2!)(2!)} - \left(\frac{6!}{2!} \times 2\right) + 5!\right] = 1980$$

Put in 1st

Required ways = 5040 – [120 + 720 + 1980] = 2220

Example 35 :

How many 5 lettered words can be formed using the letters of the words "INDEPENDENCE".

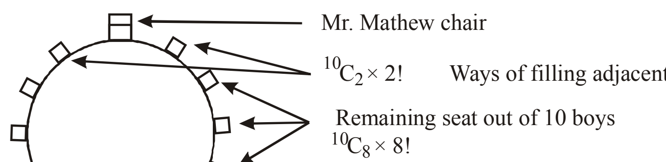
Sol. There are $E \to 4, N \to 3, D \to 2, P \to 1, I \to 1, C \to 1$

Category	Selection	Arrangement
4 alike & diff.	$1 \times {}^5C_1 = 5$ eg : EEEED	$\frac{5!}{4!} \times 5 = 25$
3 alike & 2 diff.	${}^2C_1 \times {}^5C_2 = 20$ EEEDD	$\frac{5!}{3!} \times 20 = 400$
3 alike & 2 alike of diff. kind	${}^2C_1 \times {}^2C_1 = 4$ EEEDD	$\frac{5!}{3! \times 2!} \times 4 = 40$
2 alike & 3 diff.	${}^3C_1 \times {}^5C_3 = 30$ EENDI	$\frac{5!}{2!} \times 30 = 1800$
2 alike + 2 other alike and 1 diff.	${}^3C_2 \times 4 = 12$ EENND	$\frac{5!}{2! \times 2!} \times 12 = 360$
all five diff.	${}^6C_5 = 6$ EDIPC	$6 \times 5! = 720$

Example 36 :

The 10 students of batch B feel they have some conceptual doubt on "Circular permutation". Mr. Mathew called them in disussion room and asked them to sit down around a circular table which is surrounded by 13 chairs. Mr mathew told that his adjacent seat should not remain empty. The number of ways, in which the students can sit around a round table if Mr. Mathew also sit around a chair.

Sol.



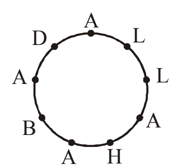
∴ Required number of arrangements

$= {}^{10}C_2 \times 2! \times {}^{10}C_8 \; 8!$

Example 37 :

In how many ways letter's word ALLAHABAD can be arranged in a circle.

Sol. There are four A's & two L's



$$\text{Required ways} = \frac{(9-1)!}{4!\,2!} = \frac{8!}{4!\,2!}$$

Example 38 :

Find the sum of the five digit numbers that can be formed using the digits 3, 4, 5, 6, 7 not using any digit more than once in any number.

Sol. If 3 is placed at units place, the remaining 4 places can be filled in 4! = 24 ways.

Thus, 3 occurs at unit place 24 times.

The other digits similarly, each occurs at the unit places 24 times.

Similarly, each of the digit occurs at the other places tens, hundreds and so on, 24 times.

Hence, the required sum, is

$= 24\,(3 + 4 + 5 + 6 + 7)\,(10^0 + 10^1 + 10^2 + 10^3 + 10^4)$

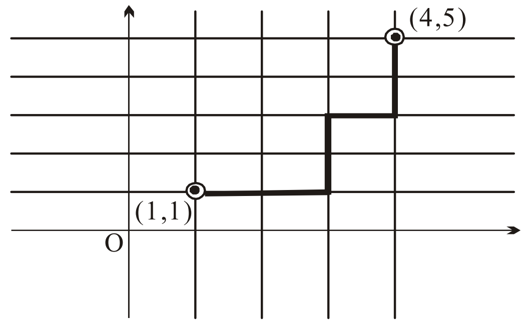
$= 24 \times 25 \times 11111 = 6666600$

Example 39 :

Number of ways in which an ant can reach from (1, 1) to (4, 5) via shortest path.

Sol. Whatever may be the mode of travel of the ant; it has to traverse 3H (Horizontal) and 4V (Vertical) paths.

Hence required number of ways = $\frac{7!}{4!\ 3!} = {}^7C_3$



Note : If there are n vertical and m horizontal lines then there will be (n – 1) horizontal and (m – 1) vertical paths.

QUESTION BANK

CHAPTER 8 : PERMUTATIONS AND COMBINATIONS

EXERCISE - 1 [LEVEL-1]

PART 1 : FUNDAMENTAL PRINCIPLE OF COUNTING, PERMUTATION

Q.1 The number of different words (meaningful or meaningless) can be formed by taking four different letters from English alphabets is-
(A) $(26)^4$ (B) 358800
(C) $(25)^4$ (D) 15600

Q.2 The value of $^8P_3$ is -
(A) 336 (B) 56
(C) 386 (D) None

Q.3 The number of numbers which can be formed with the digits 2, 3, 4, 5, 6 by taking 4 digits at a time are-
(A) 135 (B) 120
(C) 150 (D) None

Q.4 In how many ways can three persons sit on 6 chairs?
(A) 150 (B) 140
(C) 120 (D) 110

Q.5 How many different signals can be made by 5 flags from 8 flags of different colours?
(A) 6720 (B) 5720
(C) 4720 (D) None

Q.6 How many numbers lying between 100 and 1000 can be formed with the digits 1,2,3,4,5,6 if the repetition of digits is not allowed?
(A) 30 (B) 120
(C) 50 (D) None

Q.7 How many four digit numbers are there with distinct digits?
(A) 4536 (B) 4526
(C) 4516 (D) None

Q.8 How many different words can be formed with the letters of the word "ALLAHABAD" ?
(A) 10080 (B) 8640
(C) 15120 (D) 7560

Q.9 How many numbers can be formed with the digits 2,3,3,4,2,3 taken all at a time.
(A) 460 (B) 60
(C) 260 (D) None

Q.10 There are 6 pockets in the coat of a person. In how many ways can he put 4 pens in these pockets ?
(A) 360 (B) 1296
(C) 4096 (D) None

Q.11 The number of three digit numbers can be formed without using the digits 0,2,3,4, 5 and 6 is (if repetition of digit is allowed)–
(A) 54 (B) 64
(C) 44 (D) None

Q.12 The number of numbers are there between 100 and 1000 in which all the digits are distinct is –
(A) 648 (B) 548
(C) 448 (D) None

Q.13 The number of three digit numbers greater than 600 can be formed by using the digits 2,3,4, 6,7 if repetition of digits is allowed-
(A) 50 (B) 20
(C) 30 (D) None

Q.14 In how many ways 3 prizes can be distributed among 5 students, when-
(a) no student receives more than a prize.
(b) a student can receive any number of prizes.
(c) a student does not get all prizes.
(A) 60,125,120 (B) 125,60,120
(C) 125,120,60 (D) None of these

Q.15 How many numbers lying between 1000 and 2000 can be formed with the digits 1, 2, 3, 4, 5 which are divisible by 5.
(A) 3 (B) 6
(C) 12 (D) 18

Q.16 How many different words beginning with S and ending with K can be made by using the letters of the word 'SIKAR'?
(A) 6 (B) 12
(C) 48 (D) 60

Q.17 How many six digit numbers can be formed by using the digits 0,1,2,3,4,5 and 6?
(A) 5040 (B) 4320
(C) 720 (4) 5760

Q.18 If $^{56}P_{r+6}$: $^{54}P_{r+3}$ = 30800 : 1 then the value of r is -
(A) 14 (B) 41
(C) 51 (D) 10

Q.19 The number of ways in which 2 vacancies can be filled up by 13 candidates is-
(A) 25 (B) 78
(C) 156 (D) 169

Q.20 How many different words beginning with A and ending with L can be formed by using the letters of the word "ANILMANGAL'?
(A) 10080 (B) 40320
(C) 20160 (D) None

Q.21 How many numbers can be formed between 20000 and 30000 by using digits 2, 3, 5, 6, 9 when digits may be repeated?
(A) 125 (B) 24
(C) 625 (D) 1250

Q.22 The number of three letters words can be formed from the letters of word 'SACHIN' when I do not come in any word
(A) 120 (B) 60
(C) 24 (D) 48

Q.23 The number of numbers lying between 100 and 1000 which can be formed with the digits 0, 1, 2, 3, 4, 5, 6 is-
(A) 180 (B) 216
(C) 200 (D) None

Q.24 How many numbers between 1000 and 4000 (including 4000) can be formed with the digits 0,1,2,3,4 if each digit can be repeated any number of times?
(A) 125 (B) 275
(C) 375 (D) 500

Q.25 P, Q, R and S have to give lectures to an audience. The organiser can arrange the order of their presentation in
(A) 4 ways (B) 12 ways
(C) 256 ways (D) 24 ways

Q.26 How many numbers can be made with the help of the digits 0, 1, 2, 3, 4, 5 which are greater than 3000 (repetition is not allowed)
(A) 180 (B) 360
(C) 1380 (D) 1500

Q.27 If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is
(A) 324 (B) 341
(C) 359 (D) None of these

Q.28 We are to form different words with the letters of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R, then m_1/m_2 is equal to
(A) 30 (B) 60
(C) 90 (D) 180

Q.29 The numbers of arrangements of the letters of the word SALOON, if the two O's do not come together, is
(A) 360 (B) 720
(C) 240 (D) 120

Q.30 The number of 5 digit telephone numbers having at least one of their digits repeated is
(A) 90,000 (B) 100,000
(C) 30,240 (D) 69,760

Q.31 The number of arrangements of the letters of the word CALCUTTA
(A) 2520 (B) 5040
(C) 10,080 (D) 40,320

Q.32 The number of words which can be made out of the letters of the word MOBILE when consonants always occupy odd places is
(A) 20 (B) 36
(C) 30 (D) 720

Q.33 In how many ways can 5 keys be put in a ring
(A) (1/2) 4! (B) (1/2) 5!
(C) 4! (D) 5!

Q.34 The number of ways in which 5 male and 2 female members of a committee can be seated around a round table so that the two female are not seated together is
(A) 480 (B) 600
(C) 720 (D) 840

Q.35 The number of ways in which 7 persons be seated at 5 places round a table are-
(A) 252 (B) 504
(C) 2520 (D) None of these

Q.36 In how many ways can 5 beads out 7 different beads be strung into a ring ?
(A) 504 (B) 2520
(C) 252 (D) None of these

Q.37 In how many ways can 6 persons be seated round a circular table when two particular persons sit together ?
(A) 120 (B) 240
(C) 48 (D) 24

Q.38 The number of ways in which 7 girls can be stand in a circle so that they do not have the same neighbour in any two arrangements?
(A) 720 (B) 380
(C) 360 (D) None of these

Q.39 The number of ways in which 7 men and 7 women can sit on a circular table so that no two women sit together is
(A) 7! . 7! (B) 7! . 6!
(C) $(6!)^2$ (D) 7!

Q.40 There are three copies each of 4 different books. The number of ways in which they can be arranged on a shelf is
(A) $\frac{12!}{(3!)^4}$ (B) $\frac{11!}{(3!)^2}$
(C) $9!/(3!)^2$ (D) None of these

Q.41 The number of ways in which the letters of the word 'VOWEL' can be arranged so that the letters O, E occupy only even places is –
(A) 12 (B) 24
(C) 18 (D) 36

PART 2 : COMBINATIONS, DIVISION AND DISTRIBUTION OF OBJECTS

Q.42 In how many ways can a committee of 6 persons be made out of 10 persons ?
(A) 210 (B) 300
(C) 151200 (D) None

Q.43 In how many ways a committee of 5 members can be selected from 6 men and 5 women, consisting of 3 men and 2 women?
(A) 200 (B) 100
(C) 300 (D) None

Q.44 Out of 5 men and 2 women, a committee of 3 is to be formed. In how many ways can it be formed if atleast one woman is to be included?
(A) 20 (B) 30
(C) 25 (D) None

Q.45 In how many ways 11 players can be selected out of 15 players when (a) one particular player is always to be selected.(b) one particular player is never to be selected.
(A) 364,1365 (B) 1001,364
(C) 3003, 364 (D) 3003,1001

Q.46 In how many ways can I purchase one or more shirts if 6 different shirts are available ?
(A) 64 (B) 62
(C) 63 (D) 126

Q.47 A bag contains 3 one rupee coins, 4 fifty paise coins and 5 ten paise coins. How many selections of money can be formed by taking atleast one coin from the bag ?
(A) 120 (B) 60
(C) 119 (D) 59

Q.48 The number of straight lines that can be formed by joining 20 points no three of which are in the same straight line except 4 of them which are in the same line –
(A) 183 (B) 186
(C) 197 (D) 185

Q.49 Find the number of different ways in which 8 persons can stand in a row so that between two particular person A and B there are always two person.
(A) 16 (B) 4
(C) 2 (D) None

Q.50 Four couples (husband and wife) decide to form a committee of four members. Find the number of different committees that can be formed in which no couple finds a place.
(A) 14 (B) 41
(C) 51 (D) 16

Q.51 3 copies each of 4 different books are available. The number of ways in which these can be arranged on the shelf is-
(A) 12! (B) $\frac{12!}{3!\,4!}$
(C) 369,600 (D) 369,000

Q.52 The number of ways of dividing 20 persons into 10 couples
(A) $\frac{20!}{2^{10}}$ (B) ${}^{20}C_{10}$ (C) $\frac{20!}{(2!)^9}$ (D) None

Q.53 How many words can be formed containing 4 consonants and 3 vowels out of 6 consonants and 5 vowels ?
(A) ${}^6C_4 \times {}^5C_3$ (B) ${}^6C_4 \times {}^5C_3 \times 7!$
(C) ${}^6P_4 \times {}^5P_3$ (D) ${}^6P_4 \times {}^5P_3 \times 7!$

Q.54 In how many ways can 7 persons be seated round two circular tables when 4 persons can sit on the first table and 3 can sit on the other ?
(A) 420 (B) 35
(C) 210 (D) 2520

Q.55 The number of words by taking 4 letters out of the letters of the word 'COURTESY', when T and S are always included are-
(A) 120 (B) 720
(C) 360 (D) None

Q.56 The number of ways in which five identical balls can be distributed among ten identical boxes such that no box contains more than one ball, is
(A) 10 ! (B) $\frac{10!}{5!}$ (C) $\frac{10!}{(5!)^2}$ (D) None

Q.57 A committee of 12 is to be formed from 9 women and 8 men in which at least 5 women have to be included in a committee. Then the number of committees in which the women are in majority and men are in majority are respectively
(A) 4784, 1008 (B) 2702, 3360
(C) 6062, 2702 (D) 2702, 1008

Q.58 A lady gives a dinner party for six guests. The number of ways in which they may be selected from among ten friends, if two of the friends will not attend the party together is
(A) 112 (B) 140
(C) 164 (D) None

Q.59 In how many ways a team of 10 players out of 22 players can be made if 6 particular players are always to be included and 4 particular players are always excluded
(A) ${}^{22}C_{10}$ (B) ${}^{18}C_3$
(C) ${}^{12}C_4$ (D) ${}^{18}C_4$

Q.60 There are 12 volleyball players in all in a college, out of which a team of 9 players is to be formed. If the captain always remains the same, then in how many ways can the team be formed
(A) 36 (B) 108
(C) 99 (D) 165

Q.61 In a conference of 8 persons, if each person shake hand with the other one only, then the total number of shake hands shall be –
(A) 64 (B) 56
(C) 49 (D) 28

Q.62 On the occasion of Deepawali festival each student of a class sends greeting cards to the others. If there are 20 students in the class, then the total number of greeting cards exchanged by the students is
(A) ${}^{20}C_2$ (B) $2\,.\,{}^{20}C_2$
(C) $2\,.\,{}^{20}P_2$ (D) None

Q.63 In a touring cricket team there are 16 players in all including 5 bowlers and 2 wicket-keepers. How many teams of 11 players from these, can be chosen, so as to include three bowlers and one wicket-keeper
(A) 650 (B) 720
(C) 750 (D) 800

Q.64 In how many ways can 15 students
(i) be divided into 3 groups of 5 each
(ii) be sent to three different colleges in groups of 5 each.
(A) $\frac{15!}{3!\,(5!)^3}, \frac{15!}{(5!)^3}$ (B) $\frac{15!}{(5!)^3}, \frac{15!}{(5!)^3}$
(C) $\frac{15!}{3!\,(5!)^3}, \frac{15!}{3!\,(5!)^3}$ (D) $\frac{15!}{(5!)^3}, \frac{15!}{3!\,(5!)^3}$

Q.65 Find the number of ways in which 16 identical toys are to be distributed among 3 children such that each child does not receive less than 3 toys.
(A) 36 (B) 18
(C) 72 (D) 54

Q.66 Find the number of non-negative integral solutions of $x_1 + x_2 + x_3 + 4x_4 = 20$.
(A) 436 (B) 418
(C) 536 (D) 318

Q.67 In how many ways can 10 identical toys be distributed among 3 children such that the first receives a maximum of 6 toys, the second receives a maximum of 7 toys and the third receives a maximum of 8 toys.
(A) 51 (B) 37
(C) 27 (D) 47

Q.68 In how many ways 5 identical balls can be distributed into 3 different boxes so that no box remains empty?
(A) 36 (B) 18
(C) 6 (D) 12

Q.69 The number of ways of dividing 15 men and 15 women into 15 couples, each consisting of a man and a woman is
(A) 1240 (B) 1840
(C) 1820 (D) 2005

Q.70 A boat is to be manned by eight men of whom 2 can only row on bow side and 3 can only row on stroke side, the number of ways in which the crew can be arranged is –
(A) 4360 (B) 5760
(C) 5930 (D) None of these

Q.71 There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is –
(A) 10^2 (B) 1023
(C) 10! (D) 2^{10}

Q.72 A cricket team of 11 players is to be selected from 13 players of which 4 are bowlers and 2 wicket keepers. The number of ways to select the team, consisting 1 wicket keeper and atleast 3 bowlers is –
(A) 8 (B) 22
(C) 112 (D) None of these

PART 3 : GEOMETRICAL PROBLEMS

Q.73 The number of diagonals in a polygon of m sides is
(A) $\frac{1}{2!}m(m-5)$ (B) $\frac{1}{2!}m(m-1)$
(C) $\frac{1}{2!}m(m-3)$ (D) $\frac{1}{2!}m(m-2)$

Q.74 Six points in a plane be joined in all possible ways by indefinite straight lines, and if no two of them be coincident or parallel, and no three pass through the same point (with the exception of the original 6 points). The number of distinct points of intersection is equal to
(A) 105 (B) 45
(C) 51 (D) None

Q.75 There are 16 points in a plane, no three of which are in a straight line except 8 which are all in a straight line. The number of triangles that can be formed by joining them equals
(A) 504 (B) 552
(C) 560 (D) 1120

Q.76 If 7 points out of 12 are in the same straight line, then number of triangles formed is –
(A) 19 (B) 185
(C) 201 (D) None of these

Q.77 Find the total number of ways in which six '+' & four '–' signs can be arranged in a line such that no two '–' signs occur together, is
(A) 12 (B) 24
(C) 18 (D) 35

PART 4 : DERANGEMENT & MISCELLANEOUS PROBLEMS

Q.78 There are 3 letters and 3 envelopes. Find the number of ways in which all letters are put in the wrong envelopes.
(A) 6 (B) 4
(C) 2 (D) None

Q.79 The total number of seven digit numbers the sum of whose digits is even is
(A) 9000000 (B) 4500000
(C) 8100000 (D) None

Q.80 The sum of all 4 digit numbers that can be formed by using the digits 2, 4, 6, 8 (repetition of digits not allowed)
(A) 133320 (B) 533280
(C) 53328 (D) None

Q.81 Find the number of permutation of 4 letters taken from the word EXAMINATION.
(A) 1504 (B) 2520
(C) 2552 (D) 2454

Q.82 The sum of all numbers which can be formed with digits 1, 2 and 3 is-
(A) 716 (B) 1432
(C) 2148 (D) None of these

Q.83 There are four balls of different colours and four boxes of colours same as those of the balls. The number of ways in which the balls, one each box, could be placed such that a ball does not go to box of its own colour is-
(A) 8 (B) 7
(C) 9 (D) None of these

Q.84 The number of positive integral solution of the equation $x_1 x_2 x_3 x_4 x_5 = 1050$ is –
(A) 1800 (B) 1600
(C) 1400 (D) None of these

Q.85 A seven digit number divisible by 9 is to be formed by using 7 out of number $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. The number of ways in which this can be done is –
(A) 7! (B) 2 (7)!
(C) 3 (7)! (D) 4 (7)!

Q.86 A student is allowed to select at most n books from a collection of (2n + 1) books. If the total number of ways in which he can select a book is 63, then the value of n is
(A) 6 (B) 3
(C) 4 (D) None of these

Q.87 A forecast is to be made of the results of five cricket matches , each of which can be a win or a draw or a loss for Indian team.
Let p = number of forecasts with exactly 1 error
q = number of forecasts with exactly 3 error and
r = number of forecasts with all five errors
then the incorrect statement is –
(A) 8p = 5r (B) 2q = 5r
(C) 8p = q (D) 2 (p + r) > q

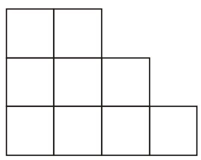
Q.88 Golden Temple Express going from Amritsar to Mumbai stops at 5 intermediate stations. 10 passangers enter the train during the journey with 10 different tickets of k classes. If the number of different sets of tickets they have is ${}^{45}C_{35}$ then k equals –

(A) 1 (B) 2
(C) 3 (D) 4

Q.89 In a bag there is a minimum of six old Indian coins of every denominations (i.e. Athanni, Chavanni, Duanni, Ekanni). Number of ways in which one can take 6 coins from the bag is –

(A) 120 (B) 90
(C) 84 (D) 60

Q.90 The number of ways in which 5 X's can be placed in the squares of the figure so that no horizontal row remains empty is –



(A) 97 (B) 98
(C) 100 (D) 126

Q.91 The number of ways of selecting exactly 4 fruits out of 4 apples, 5 mangoes, 6 organges is –

(A) 10 (B) 15
(C) 20 (D) 25

Q.92 Three dice are rolled. The number of possible outcomes in which at least one die shows 5 is

(A) 215 (B) 36
(C) 125 (D) 91

Q.93 Number of ways of selecting three squares on a chess-board so that all the three be on a diagonal line of the board or parallel to it is

(A) 196 (B) 126
(C) 252 (D) 392

Q.94 The digits, from 0 to 9 are written on 10 slips of paper (one digit on each slip) and placed in a box. If three of the slips are drawn and arranged, then the number of possible different arrangements is –

(A) 1000 (B) 720
(C) 810 (D) None of these

Q.95 If ${}^{20}C_r = {}^{20}C_{r-10}$ then ${}^{18}C_r$ is equal to

(A) 4896 (B) 816
(C) 1632 (D) none of these

Q.96 The number of ways in which 7 different books can be given to 5 students if each can receive none, one or more books is

(A) 5^7 (B) 7^5
(C) $11C_5$ (D) 12!

Q.97 The number of all three elements subsets of the set $\{a_1, a_2, a_3 \ldots a_n\}$ which contain a_3 is

(A) ${}^nC_3$ (B) ${}^{n-1}C_3$
(C) ${}^{n-1}C_2$ (D) none of these

Q.98 Out of 10 consonants and four vowels, the number of words that can be formed using six consonants and three vowels is

(A) ${}^{10}P_6 \times {}^6P_3$ (B) ${}^{10}C_6 \times {}^6C_3$
(C) ${}^{10}C_6 \times {}^4C_3 \times 9!$ (D) ${}^{10}P_6 \times {}^4P_3$

Q.99 The total number of ways in which 8 men and 6 women can be arranged in a line so that no 2 women are together is

(A) 48 (B) ${}^8P_8 . {}^9P_6$
(C) 8! (84) (D) ${}^8C_8 . {}^9C_8$

Q.100 Let there be 9 fixed points on the circumference of a circle. Each of these points is joined to every one of the remaining 8 points by a straight line and the points are so positioned on the circumference that atmost 2 straight lines meet in any interior point of the circle . The number of such interior intersection points is

(A) 126 (B) 351
(C) 756 (D) none of these

Q.101 Number of ways in which the letters of word GARDEN can be arranged with vowels in alphabetical order, is –

(A) 360 (B) 240
(C) 120 (D) 480

Q.102 The sides AB, BC and CA of a triangle ABC have 3,4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices, is

(A) 205 (B) 208
(C) 220 (D) 380

Q.103 Number of integers greater than 7000 and divisible by 5 that can be formed using only the digits 3, 6, 7, 8 and 9, no digit being repeated, is –

(A) 46 (B) 48
(C) 72 (D) 42

Q.104 The number of ways in which the number 94864 can be resolved as a product of two factors is

(A) 30 (B) 23
(C) 45 (D) 46

Q.105 The letters of the word TOUGH are written in all possible orders and these words are written out as in a dictionary, then the rank of the word TOUGH is

(A) 120 (B) 88
(C) 89 (D) 90

Q.106 The streets of a city are arranged like the lines of a chess board . There are m streets running North to South and 'n' streets running East to West . The number of ways in which a man can travel from NW to SE corner going the shortest possible distance is

(A) $\sqrt{m^2 + n^2}$ (B) $\sqrt{(m-1)^2 \, . \, (n-1)^2}$

(C) $\dfrac{(m+n)\,!}{m!\,.\,n!}$ (D) $\dfrac{(m+n-2)\,!}{(m-1)\,!\,.\,(n-1)\,!}$

Q.107 The number of ways in which a mixed double tennis game can be arranged from amongst 9 married couple if no husband and wife plays in the same game is

(A) 756 (B) 3024
(C) 1512 (D) 6048

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 Number of ways in which 15 indistinguishable oranges can be distributed in 3 different boxes so that every box has atmost 8 oranges, is –
(A) 52 (B) 108
(C) 76 (D) 28

Q.2 The number of ten digit numbers that contain only 2 and 3 as its digits, but no any pairwise 3's joins together, is
(A) 145 (B) 143
(C) 129 (D) None of these

Q.3 If the sum of all even positive divisors of 100000 can be expressed in the form $k(5^2+5+1)(5^3+1)$ then the value of k is –
(A) 31 (B) 62
(C) 64 (D) 93

Q.4 There are n white different and n black different balls marked 1, 2, 3, . . ., n. The number of ways in which we can arrange these balls in a row so that neighboring balls are of different colours is
(A) n! (B) (2n)!
(C) $2(n!)^2$ (D) $\frac{(2n)!}{(n!)^2}$

Q.5 3 Indian and 3 American men and their wives are to be seated round a circular table. Let m denotes the number of ways when the Indian couples are together and n denotes the number of ways when all the six couples are together. If m = kn then k equals –
(A) 36 (B) 42
(C) 45 (D) 48

Q.6 If the letters of the word 'PARKAR' are written down in all possible manner as they are in a dictionary, then the rank of the word 'PARKAR' is –
(A) 98 (B) 99
(C) 100 (D) 101

Q.7 The number of positive integers satisfying the inequality $^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$ is
(A) 9 (B) 8
(C) 5 (D) none of these

Q.8 The word 'PATALIPUTRA' without changing the relative order of the vowels and consonants then how many words can be formed is
(A) 3600 (B) 3300
(C) 6300 (D) none of these

Q.9 The number of positive integral solution of abc = 30 is
(A) 30 (B) 27
(C) 8 (D) none of these

Q.10 There are three piles of identical red, blue and green balls and each pile contains at least 10 balls. The number of ways of selecting 10 balls if twice as many red balls as green balls are to be selected, is
(A) 3 (B) 4
(C) 6 (D) 8

Q.11 The number of three digit numbers having only two consecutive digits identical is
(A) 153 (B) 162
(C) 180 (D) 161

Q.12 If all the letters of the word "QUEUE" are arranged in all possible manner as they are in a dictionary, then the rank of the word QUEUE is –
(A) 15^{th} (B) 16^{th}
(C) 17^{th} (D) 18^{th}

Q.13 Let $S = \{(x, y) \mid (x-1)^2 + (y+2)^2 \le 25 \text{ and } x, y \in Z\}$. The number of elements in S is-
(A) 81 (B) 80
(C) 85 (D) 102

Q.14 In a jet there are 3 seats in front and 3 in the back. Number of different ways can six persons of different heights be seated in the jeep, so that every one in front is shorter than the person directly behind is –
(A) 90 (B) 60
(C) 54 (D) 15

Q.15 Numbers of ways in which atleast three fruits be selected out of 20 fruits in which 10-mangoes, 5-apples, 2-oranges and rest are different are –
(A) 1583 (B) 1577
(C) 1559 (D) none of these

Q.16 The number of ways in which 20 different pearly of two colours can be set alternately on a necklace (10 pearls of each colour), is -
(A) $9! \times 10!$ (B) $5(9!)^2$
(C) $(9!)^2$ (D) none of these

Q.17 The total number of integral solutions for (x, y, z) such that xyz = 24 is
(A) 36 (B) 90
(C) 120 (D) none of these

Q.18 A shopkeeper has 10 copies of each of nine different books, then number of ways in which atleast one book can be selected is–
(A) $9^{11} - 1$ (B) $10^{10} - 1$
(C) $11^9 - 1$ (D) 10^9

Q.19 Let $A = \{x/x \text{ is a prime number and } x < 30\}$. The number of different rational numbers whose numerator and denominator belong to A is -
(A) 90 (B) 180
(C) 91 (D) none of these

Q.20 Rajdhani express going from Bombay to Delhi stops at 5 intermediate stations . 10 passengers enter the train during the journey with ten different ticket of two classes .The number of different sets of tickets they may have is
(A) $^{15}C_{10}$ (B) $^{20}C_{10}$
(C) $^{30}C_{10}$ (D) none

Q.21 Let there be $n \ge 3$ circles in a plane. The value of n for which the number of radical centres, is equal to the number of radical axes is (Assume that all radical axes and radical centre exist and are different)
(A) 7 (B) 6
(C) 5 (D) none of these

Q.22 Every body in a room shakes hands with every else. If total number of hand-shaken is 66, then number of persons in the room is
(A) 11 (B) 12
(C) 13 (D) 14

Q.23 The number of three digit numbers of the form xyz such that $x < y$ and $z \le y$ is
(A) 276 (B) 285
(C) 240 (D) 244

Q.24 The total number of six digit numbers $x_1\ x_2\ x_3\ x_4\ x_5\ x_6$ having the property that $x_1 < x_2 \le x_3 < x_4 < x_5 \le x_6$ is equal to –
(A) ${}^{10}C_6$ (B) ${}^{12}C_6$
(C) ${}^{11}C_6$ (D) none of these

Q.25 The number of 6-digit numbers between 1 and 300000 which are divisible by 4 and are obtained by rearranging the digits of 112233, is –
(A) 12 (B) 15
(C) 18 (D) 90

Q.26 Two tour guides are leading 6 tourist. The guides decide to split up. Each tourist must choose one of the guides, but with the stipulation that each guide must take at least one tourist. Number of possible different groupings of guides and tourist is –
(A) 56 (B) 58
(C) 60 (D) 62

Q.27 Six persons A, B, C, D, E and F are to be seated at a circular table . The number of ways this can be done if A must have either B or C on his right and B must have either C or D on his right is
(A) 36 (B) 12
(C) 24 (D) 18

Q.28 The total number of eight digit numbers in which all digits are different is -
(A) 9!9 (B) 9!9/2
(C) 9! (D) none of these

Q.29 If the letters of the word RACHIT are arranged in all possible ways and these words are written out as in a dictionary, then the rank of this word is -
(A) 365 (B) 702
(C) 481 (D) none of these

Q.30 The number of non negative integral solution of the equation, $x + y + 3z = 33$ is
(A) 120 (B) 135
(C) 210 (D) 520

Q.31 ABCD is a convex quadrilateral. 3, 4, 5 and 6 points are marked on the sides AB, BC, CD and DA respectively. The number of triangles with vertices on different sides is -
(A) 270 (B) 220
(C) 282 (D) 342

Q.32 Let m denote the number of four digit numbers such that the left most digit is odd, the second digit is even and all four digits are different and n denotes the number of four digit numbers such that the left most digit is even, an odd second digit and all four different digits. If m = nk then the value of k equals –
(A) 6/5 (B) 5/4
(C) 4/3 (D) 3/2

Q.33 A committee of 10 is chosen from 8 men and 7 women. The number of committees on which men are in the majority is equal to
(A) $\frac{5}{11}$ of the total (B) $\frac{6}{13}$ of the total
(C) $\frac{61}{143}$ of the total (D) $\frac{117}{143}$ of the total

Q.34 With 17 consonants and 5 vowels the number of words of four letters that can be formed having two different vowels in the middle and one consonant, repeated or different at each end is-
(A) 5780 (B) 2890
(C) 5440 (D) 2720

Q.35 Number of permuations 1, 2, 3, 4, 5, 6, 7, 8 and 9 taken all at a time are such that the digit.
1 appearing somewhere to the left of 2, 3 appearing to the left of 4 and 5 somewhere to the left of 6, is
(e.g. 815723946 would be one such permutation)
(A) 9 . 7! (B) 8!
(C) 5! . 4! (D) 8! . 4!

ASSERTION AND REASON QUESTIONS

Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 5 choices (A), (B), (C), (D) & (E) out of which ONLY ONE is correct.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.
(E) Statement -1 is False, Statement-2 is False.

Q.36 Consider the word 'SMALL'
Statement–1 : Total number of 3 letter words from the letters of the given word is 13.
Statement–2 : Number of words having all the letters distinct = 4 and number of words having two are alike and third different = 9

Q.37 **Statement–1:** Number of non integral solution of the equation $x_1 + x_2 + x_3 = 10$ is equal to 34.
Statement–2 : Number of non integral solution of the equation $x_1 + x_2 + x_3 + \ldots x_n = r$ is equal to ${}^{n+r-1}C_r$

Q.38 **Statement–1 :** The number of ways of selecting 5 students from 12 students (of which six are boys and six are girls), such that in the selection there are at least three girls is ${}^6C_3\ {}^9C_2$.
Statement–2 : If a work has two independent parts, of which first part can be done in m way and for each choice of first part, the second part can be done in n ways, then the work can be completed in $m \times n$ ways.

Q.39 **Statement–1:** The number of ways of writing 1400 as a product of two positive integers is 12.
Statement–2 : 1400 is divisible by exactly three prime numbers.

Q.40 **Statement–1 :** A polygon has 44 diagonals and number of sides are 11.
Statement–2 : From n distinct object r object can be selected in ^nC_r ways.

Q.41 **Statement-1:** The number of positive integral solutions of the equation $x_1x_2x_3x_4x_5 = 1050$ is 1875.
Statement-2: The total number of divisor of 1050 is 25.

Q.42 **Statement-1:** $\left(\sum_{r=0}^{100} {}^{500-r}C_3\right) + {}^{400}C_4 = {}^{501}C_4$

Statement-2 : $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

MATCH THE COLUMN TYPE QUESTIONS

Q.43 **Match the column**

Column I	**Column II**
(a) If the number of ways in which n different toys can be distributed in n children if exactly one child does not get any toy is 1200 then n equals	(p) 3
(b) There are 2n white and 2n red counters. Counters are all alike except for the colour. If the number of ways in which they can be arranged in a line so that they are symmetric w.r.t a central mark is 70 then n equals to	(q) 4
(c) Total number of divisors of the number N = 360 which are of the form of 4n + 2, n ≥ 0 is	(r) 5
	(s) 6

Code :
(A) a-r, b-q, c-s (B) a-r, b-p, c-s
(C) a-p, b-q, c-r (D) a-r, b-q, c-p

Q.44 **Match the column**

Column I	**Column II**
(a) Number of five digit numbers of the form $d_1d_2d_3d_4d_5$ where $d_{j,s}$ j =1,2,3,4,5 are digits and satisfying $d_1 < d_2 \le d_3 < d_4 \le d_5$ is	(p) $^{10}C_5$
(b) Number of five digit numbers of the form $dd_1d_2d_3d_4d_5$ where $d_{j,s}$ j =1,2,3,4,5 are digits satisfying $d_1 > d_2 \ge d_3 > d_4 > d_5$ is	(q) $^{11}C_4$
(c) Boby Fischer and Spassky play a unique game series in a chess tournament. They decide to play on till one of them wins 5 matches. If each match end only in win or Loss. Number of ways in which series can be won by either of them, is	(r) $^{11}C_6$
(d) A badminton team has to be selected . $^9C_5$ comprising of 5 students out of 10 students for inter school tournament. Number ofways this can be done if a particular players is to be always included or always excluded from the team, is	(s) 2

Code :
(A) a-qs, b-pr, c-p, d-ps
(B) a-r, b-r, c-ps, d-ps
(C) a-r, b-qr, c-ps, d-pq
(D) a-qr, b-ps, c-pr, d-rs

Q.45 Match the column –

Column I	**Column II**
(a) Number of increasing permutations of m symbols are there from the n set numbers $\{a_1, a_2, \ldots, a_n\}$ where the order among the numbers is given by $a_1 < a_2 < a_3 < \ldots a_{n-1} < a_n$ is	(p) n^m
(b) There are m men and n monkeys. Number of ways in which every monkey has a master, if a man can have any number of monkeys.	(q) ^mC_n
(c) Number of ways in which n red balls and (m – 1) green balls can be arranged in a line, so that no two red balls are together, is (balls of the same colour are alike)	(r) ^nC_m
(d) Number of ways in which 'm' different toys can be distributed in 'n' children if every child may receive any number of toys, is	(s) m^n

Code :
(A) a-p, b-q, c-r, d-s (B) a-p, b-s, c-q, d-r
(C) a-r, b-q, c-s, d-p (D) a-r, b-s, c-q, d-p

Q.46 Match the column

Column I	**Column II**
(a) The total number of selections of fruits which can be made from, 3 bananas, 4 apples & 2 oranges is	(p) Greater than 50
(b) If 7 points out of 12 are in the same straight line, then the number of traingles formed is.	(q) Greater than 100
(c) The number of ways of selecting 10 balls from unlimited number of red, black, white and green balls is	(r) Greater than 150
(d) The total number of proper divisors of 38808 is	(s) Greater than 200

Code :
(A) a-p, b-pqr, c-pqrs, d-p
(B) a-pqr, b-r, c-pqs, d-ps
(C) a-pr, b-pqr, c-ps, d-pq
(D) a-qr, b-ps, c-pr, d-prs

PASSAGE BASED QUESTIONS

Passage 1- (Q.47-Q.50)

If there are n A's, nB's making nC's making 3n letters. Then the number of ways of selecting r letters out of them, The total number of ways of selecting r letters from n A' s, n B's and n C' s is equal to, coefficient of x^r in

$$(1+x+x^2+...+x^n)^3 = \text{coefficient of } x^r \text{ in } \left(\frac{1-x^{n+1}}{1-x}\right)^3$$

$$= \text{coefficient of } x^r \text{ in } (1-3x^{n+1}+3x^{2n+2}-x^{3n+3}).(1-x)^{-3}$$

$$= \text{coefficient of } x^r \text{ in } (1-x)^{-3} \{\text{coeff. of } x^{r-n+1} \text{ in } (1-x)^{-3}\}$$
$$+ 3\{\text{coeff. of } x^{r-2n-2} \text{ in } (1-x)^{-3}\}$$

Q.47 When the value of $r \in [1, n]$, is

(A) ${}^rC_2$ (B) ${}^{r+1}C_2$
(C) ${}^{r+2}C_2$ (D) none of these

Q.48 When the value of $r \in [n+1, 2n+1]$, is

(A) ${}^{r+2}C_2 - 3.{}^{r-n+1}C_2$ (B) ${}^rC_2 - 3.{}^{r+n-1}C_2$
(C) ${}^{r+1}C_2 - 3.{}^{r+n-1}C_2$ (D) ${}^rC_2 - 3.{}^{r-n+1}C_2$

Q.49 When the value of $r \in [2n+2, 3n]$, is

(A) ${}^{r+2}C_2 - 3.{}^{r-n+1}C_2 + {}^{r-2n}C_2$
(B) ${}^{r+2}C_2 - 3.{}^{r-n+1}C_2 + 3.{}^rC_2$
(C) ${}^{r+2}C_2 - 3.{}^{r-n+1}C_2 + 3.{}^{r-2n}C_2$
(D) none of these

Q.50 Is maximum when n is even where $(n < r < 2n+1)$, is

(A) $\{3(n+1)^2+1\}$ (B) $(n+1)^2$
(C) $\frac{1}{4}\{3(n+1)^2+1\}$ (D) none of these

Passage 2- (Q.51-Q.53)

From 5 novels – A, B, C, D and E and four biographies – F, G, H and I – Reena has to choose five books that she will assign over the summer to the participants in the book club she leads. Her selection will be made in accordance with the following requirements.

(i) Exactly two of the books selected must be biographies.

(ii) If B is selected, both D and G must be selected.

(ii) A cannot be selected unless F is selected.

(iv) If either C or E is selected, then the other is must.

(v) If I is selected, neither F nor H can be selected.

Q.51 Which of the following could be a list of the five books that Reena selects –

(A) A, B, D, F, H (B) B, C, D, F, G
(C) A, C, E, F, I (D) A, B, D, F, G

Q.52 If Reena's first two selections are F and H, which of the following must she also select –

(A) B (B) C
(C) D (D) G

Q.53 If E is not selected, then all of the following must be selected except –

(A) A (B) D
(C) F (D) H

Passage 3- (Q.54-Q.56)

Consider the word "W" = COMMISSIONER containing 12 letters of which five vowels and 7 consonants.

Q.54 Number of 5 lettered word each comprising of 2 vowels and 3 consonants is –

(A) 5120 (B) 6720
(C) 4960 (D) None

Q.55 Number of ways in which the letter of word 'W' can be arranged if alike letters are together but separated from the other alike letters is

(A) 2880 (B) 1120
(C) $\frac{12!-8!}{16}$ (D) None

Q.56 Number of ways in which the letters of the word 'W' can be arranged without changing the order of alike letters is

(A) $\frac{12!}{(2!)^4}$ (B) ${}^{12}C_8$
(C) ${}^{12}P_8$ (D) ${}^{12}P_4$

Passage 4- (Q.57-Q.59)

Consider a polygon of sides n which satisfies the equation, $3.{}^nP_4 = {}^{n-1}P_5$.

Q.57 Rajdhani express travelling from Delhi to Mumbai has n stations enroute. Number of ways in which a train can be stopped at 3 stations if no two of the stopping stations are consecutive, is –

(A) 20 (B) 35
(C) 56 (D) 85

Q.58 Number of quadrilaterals that can be made using the vertices of the polygon of sides n if exactly two adjacent sides of the quadrilateral are common to the sides of the n-gon, is –

(A) 50 (B) 60
(C) 70 (D) None of these

Q.59 Number of quadrilaterals that can be formed using the vertices of a polygon of sides n if exactly 1 side of the quadrilateral is common with the side of the n-gon, is –

(A) 150 (B) 100
(C) 96 (D) None of these

Passage 5- (Q.60-Q.61)

Consider the digits 1, 2, 2, 3, 3, 3 and answer the following

Q.60 If all the 6 digit numbers using these digits only are formed and arranged in ascending order of their magnitude then 29^{th} number will be

(A) 213332 (B) 233321
(C) 233312 (D) none

Q.61 Let M denotes the number of six digit numbers using only the given digits if not all the 2's are together and N denotes the corresponding figure if no 3's are together then M – N equals

(A) 16 (B) 28
(C) 54 (D) 36

Passage 6- (Q.62-Q.64)

Let Set S = {1, 2, 3, ...n} be a set of first n natural numbers and $A \subseteq S$. Suppose n(A) represents cardinal number and min (A) represents least number among the elements of set A.

Q.62 The greatest value of min (A), where $A \subseteq S$ and $n(A) = r, 1 \le r \le n$ is –

(A) r (B) n – r
(C) n – r + 1 (D) r + 1

Q.63 The number of subsets A of S for which n (A) = r and min (A) = k, is –

(A) ${}^{n-k}C_{r-1}$ (B) ${}^{n}C_{r-1}$
(C) ${}^{n-k+1}C_{r-1}$ (D) ${}^{n-k-1}C_{r-1}$

Q.64 The value of $\sum_{n(A)=r} (\min(A)) = k$ is –

(A) $n.{}^{n-k}C_{r-1}$
(B) $(n+1)\,{}^{n-k}C_{r-1} - r\,{}^{n-k+1}C_r$
(C) $k.\,{}^{n-k}C_{r-1} + n.\,{}^{n-k+1}C_r$
(D) ${}^{n}C_r$

Passage 7- (Q.65-Q.67)

Consider the letters of the word MATHEMATICS. There are eleven letters some of them are identical. Letters are classified as repeating and non-repeating letters. Set of repeating letters = {M, A, T}. Set of non-repeating letters = {H, E, I, C, S}

Q.65 Possible number of words taking all letters at a time such that atleast one repeating letter is at odd position in each word, is –

(A) $\frac{9!}{2!2!2!}$ (B) $\frac{11!}{2!2!2!}$

(C) $\frac{11!}{2!2!2!} - \frac{9!}{2!2!}$ (D) $\frac{9!}{2!2!}$

Q.66 Possible number of words taking all letters at a time such that in each word both M's are together and both T's are together but both A's are not together, is –

(A) $7!\,{}^{8}C_2$ (B) $\frac{11!}{2!2!2!} - \frac{9!}{2!2!}$

(C) $\frac{6!4!}{2!2!}$ (D) $\frac{9!}{2!2!2!}$

Q.67 Possible number of words in which no two vowels are together, is –

(A) $\frac{7!}{2!2!}.{}^{8}C_4\frac{4!}{2!}$ (B) $\frac{7!}{2!}.{}^{8}C_4\frac{4!}{2!}$

(C) $7!.{}^{8}C_4\frac{4!}{2!}$ (D) $\frac{7!}{2!2!2!}.{}^{8}C_4\frac{4!}{2!}$

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 Consider the set of all triangles OPQ where O is the origin and P and Q are distinct points in the plane with non-negative integral coordinates (x, y) such that $5x + y = 99$. Number of such distinct triangles whose area is a positive integer, is

Q.2 There are 3 men and 7 women taking a dance class. Number of different ways in which each man be paired with a woman partner, and the four remaining women be paired into two pairs each of two, is –

Q.3 Number of ways in which 5 A's and 6 B's can be arranged in a row which reads the same backwards and forwards, is

Q.4 Let A = {a, b, c, d, e, f} and B = {1, 2, 3} are two sets.
Let m denotes the number of mappings which are into from A to B.
Let n denotes the number of mappings which are injective from B to A. Find (m + n).

Q.5 Find the number of 4 digit numbers starting with 1 and having exactly two identical digits.

Q.6 A child has a set of 96 distinct blocks. Each block is one of two material (plastic, wood), 3 sizes (small, medium, large), 4 colours (blue, green, red, yellow), and 4 shapes (circle, hexagon, square, triangle). How many blocks in the set are different from "Plastic medium red circle" in exactly two ways? ("The wood medium red square" is such a block)

Q.7 Number of selections that can be made of 6 letters from the word "COMMITTEE" is

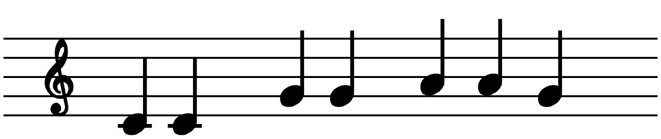
Q.8 Suppose that there are 5 red points and 4 blue points on a circle. Find the number of convex polygons whose vertices are among the 9 points and having at least one blue vertex.

Q.9 Number of regular polygons that have integral interior angle measure, is

Q.10 The number of permutation of the letters AAAABBBC in which the A's appear together in a block of four letters or the B's appear in a block of 3 letters, is

Q.11 Two balls are drawn from a bag containing 3 white, 4 black and 5 red balls then the number of ways in which the two balls of different colours are drawn is

Q.12 The tune 'Twinkle Twinkle Little Star' has 7 notes in its first line, CCGGAAG. All notes are held for the same length of time. If the notes are rearranged at random, number of different melodies that can be composed, is



Q.13 Number of divisors of the form $4n + 2$ $(n \geq 0)$ of the integer 240 is

Q.14 How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that odd digits occupy even positions ?

Q.15 Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of 'n' sides. If $T_{n+1} - T_n = 21$, then 'n' equals

Q.16 The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is

Q.17 If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$, where $n > 1$, and the runs scored in the k^{th} match are given by $k \cdot 2^{n+1-k}$, where $1 \leq k \leq n$. Find n.

Q.18 If r, s, t are prime numbers and p, q are the positive integers such that LCM of p, q is $r^2t^4s^2$, then the number of ordered pair (p, q) is

Q.19 The letters of the COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

Q.20 Consider the set of eight vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k} \,;\, a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is ____

Q.21 Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is –

Q.22 Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

Q.23 Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is –

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 Find the no. of numbers which can be formed with digits 0,1,2,3,4 greater than 1000 and less than 4000 if repetition is allowed- **[AIEEE 2002]**

(A) 125 (B) 400
(C) 375 (D) 374

Q.2 If repetition of the digits is allowed, then the number of even natural numbers having three digits is-**[AIEEE-2002]**

(A) 250 (B) 350
(C) 450 (D) 550

Q.3 If ${}^nC_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals- **[AIEEE 2003]**

(A) ${}^{n+1}C_{r+1}$ (B) ${}^{n+2}C_r$
(C) ${}^{n+2}C_{r+1}$ (D) ${}^{n+1}C_r$

Q.4 A student is to answer 10 out of 13 questions, an examination such that he must choose least 4 from the first five questions. The number of choices available to him, is- **[AIEEE 2003]**

(A) 346 (B) 140
(C) 196 (D) 280

Q.5 The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by **[AIEEE 2003]**

(A) $(7!) \times (5!)$ (B) $(6!) \times (5!)$
(C) 30 (D) $(5!) \times (4!)$

Q.6 How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order ?

(A) 120 (B) 240 **[AIEEE 2004]**
(C) 360 (D) 480

Q.7 The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is-

(A) 5 (B) 21 **[AIEEE 2004]**
(C) 3^8 (D) ${}^8C_3$

Q.8 If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number - **[AIEEE-2005]**

(A) 601 (B) 600
(C) 603 (D) 602

Q.9 The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is - **[AIEEE-2005]**

(A) ${}^{55}C_4$ (B) ${}^{55}C_3$
(C) ${}^{56}C_3$ (D) ${}^{56}C_4$

Q.10 At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is **[AIEEE 2006]**

(A) 6210 (B) 385
(C) 1110 (D) 5040

Q.11 The set S : {1, 2, 3, …, 12} is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is- **[AIEEE 2007]**

(A) $12!/3!(4!)^3$ (B) $12!/3!(3!)^4$
(C) $12!/(4!)^3$ (D) $12!/(3!)^4$

Q.12 How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which not two S are adjacent ? **[AIEEE 2008]**

(A) $6 . 7 . {}^8C_4$ (B) $6 . 8 . {}^7C_4$
(C) $7 . {}^6C_4 . {}^8C_4$ (D) $8 . {}^6C_4 . {}^7C_4$

Q.13 In a shop there are five types of ice-creams available . A child buys six ice-creams.

Statement-1: The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$

Statement -2: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.**[AIEEE 2008]**

(A) Statement-1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is true, Statement -2 is true; Statement-2 is not a correct explanation for Statement-1.
(C) Statement-1 is true, Statement -2 is false.
(D) Statement-1 is false, Statement-2 is true.

Q.14 From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is -

(A) Less than 500 **[AIEEE 2009]**
(B) At least 500 but less than 750
(C) At least 750 but less than 1000
(D) At least 1000

Q.15 There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is –

(A) 36 (B) 66 **[AIEEE 2010]**
(C) 108 (D) 3

Q.16 **Statement-1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$. **[AIEEE 2011]**

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

Q.17 Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is **[AIEEE 2012]**

(A) 880 (B) 629
(C) 630 (D) 879

Q.18 Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is – **[JEE MAIN 2013]**
(A) 256 (B) 220
(C) 219 (D) 211

Q.19 Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is – **[JEE MAIN 2013]**
(A) 7 (B) 5
(C) 10 (D) 8

Q.20 The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is **[JEE MAIN 2015]**
(A) 861 (B) 820
(C) 780 (D) 901

Q.21 The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, **[JEE MAIN 2015]**
(A) 192 (B) 120
(C) 72 (D) 216

Q.22 Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is **[JEE MAIN 2015]**
(A) 256 (B) 275
(C) 510 (D) 219

Q.23 If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary ; then the position of the word SMALL is **[JEE MAIN 2016]**
(A) 59^{th} (B) 52^{nd}
(C) 58^{th} (D) 46^{th}

Q.24 A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in the party, is : **[JEE MAIN 2017]**
(A) 469 (B) 484 (C) 485 (D) 468

Q.25 From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is: **[JEE MAIN 2018]**
(A) at least 500 but less than 750
(B) at least 750 but less than 1000
(C) at least 1000
(D) less than 500

Q.26 Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is **[JEE MAIN 2019 (JAN)]**
(A) 200 (B) 300
(C) 500 (D) 350

Q.27 All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is : **[JEE MAIN 2019 (APRIL)]**
(A) 175 (B) 162
(C) 160 (D) 180

Q.28 The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is : **[JEE MAIN 2019 (APRIL)]**
(A) 288 (B) 306
(C) 360 (D) 310

Q.29 A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then : **[JEE MAIN 2019 (APRIL)]**
(A) $m = n = 78$ (B) $n = m - 8$
(C) $m + n = 68$ (D) $m = n = 68$

Q.30 The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is : **[JEE MAIN 2019 (APRIL)]**
(A) 36 (B) 60
(C) 48 (D) 72

Q.31 Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number beams is : **[JEE MAIN 2019 (APRIL)]**
(A) 210 (B) 190
(C) 170 (D) 180

Q.32 The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is : **[JEE MAIN 2019 (APRIL)]**
(A) 2^{20} (B) $2^{20} - 1$
(C) $2^{20} + 1$ (D) 2^{21}

Q.33 A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to : **[JEE MAIN 2019 (APRIL)]**
(A) 25 (B) 28
(C) 27 (D) 24

Q.34 Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 & 9 appear, is **[JEE MAIN 2020 (JAN)]**
(A) $\frac{5}{2}(6!)$ (B) 5^6
(C) $\frac{1}{2}(6!)$ (D) 6!

Q.35 If $({}^{36}C_{r+1}) \times (k^2 - 3) = {}^{35}C_r$ 6, then number of ordered pairs (r, k) are (where $k \in I$). **[JEE MAIN 2020 (JAN)]**
(A) 6 (B) 2
(C) 3 (D) 4

Q.36 If maximum value of ${}^{19}C_p$ is a, ${}^{20}C_q$ is b, ${}^{21}C_r$ is c, then relation between a, b, c is **[JEE MAIN 2020 (JAN)]**
(A) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$ (B) $\frac{a}{22} = \frac{b}{11} = \frac{c}{42}$
(C) $\frac{a}{22} = \frac{b}{42} = \frac{c}{11}$ (D) $\frac{a}{21} = \frac{b}{11} = \frac{c}{22}$

Q.37 The number of four letter words that can be made from the letters of word "EXAMINATION" is

[JEE MAIN 2020 (JAN)]

Q.38 If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is 336 k, then k is equal to

[JEE MAIN 2020 (JAN)]

(A) 8 (B) 6

(C) 4 (D) 7

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	A	B	C	A	B	A	D	B	B	B	A	A	A	B	A	B	B	C	A	C	B	A	C	D
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	C	A	A	C	D	B	B	A	A	B	C	C	C	B	A	A	A	A	C	B	C	C	D	A	D
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	C	D	B	A	C	C	D	B	C	D	D	B	B	A	A	C	D	C	A	B	B	B	C	C	A
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
A	B	D	C	B	A	D	B	C	D	D	B	A	C	C	B	B	D	D	B	B	A	C	C	B	A
Q	101	102	103	104	105	106	107																		
A	A	A	D	B	C	D	C																		

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	B	B	C	B	B	B	A	B	B	B	C	A	A	C	B	C	C	C	C	C	B	A	C	B
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	D	D	B	C	C	D	B	C	A	A	A	D	D	B	A	C	A	A	B	D	A	C	A	C	C
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67								
A	D	B	D	B	A	D	C	A	B	C	B	C	C	B	B	A	A								

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
A	90	630	10	309	432	29	35	450	22	44	47	210	4	60	7	40	7	225	96	5	7	5	53

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	C	C	C	C	B	C	B	A	D	B	A	C	D	D	C	B	D	C	B	C	A	D	C	C	C	B
Q	27	28	29	30	31	32	33	34	35	36	37	38														
A	D	D	A	B	C	A	A	A	D	A	2454	A														

CHAPTER- 8 : PERMUTATIONS AND COMBINATIONS

SOLUTIONS TO TRY IT YOURSELF

TRY IT YOURSELF-1

(1) The student has 6 choices from the morning courses out of which he can select one course in 6 ways.
For the evening course, he has 4 choices out of which he can select one in 4 ways.
Hence the total number of ways = 6 × 4 = 24

(2) ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$

$$\Rightarrow \frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{3}{5} \Rightarrow \frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{3}{5}$$

$\Rightarrow 10(2n+1) = 3(n+2)(n+1)$
$\Rightarrow 3n^2 - 11n - 4 = 0 \Rightarrow (n-4)(3n+1) = 0 \Rightarrow n = 4$

(3) The student has 6 choices from the morning courses out of which he can select one course in 6 ways.
For the evening course, he has 4 choices out of which he can select one in 4 ways. Hence the total number of ways = 6 + 4 = 10.

(4) Since there are 2M's, 2A's and 2 T's, the required number of ways is 11!/(2!2!2!).

(5) If the best and worst papers appear always together, the number of ways is 5! × 2. Therefore, required number of ways is as follows.
Total number of ways without any restrictions – number of ways when best and worst paper are together
= 6! – 5! × 2 = 480

(6) Five boys can sit in 5! ways; in this case, there are 6 vacant places where the girls can sit in ${}^6P_3$ ways. Therefore, the required number of ways is ${}^6P_3 \times 5!$

(7) The word 'MOBILE' has three even places and three odd places. It has 3 consonants and 3 vowels. In three odd places, we have to fix up 3 consonants, which can be done in ${}^3P_3$ ways.
Now, in the remaining three places, we have to fix up the remaining three, which can be done in ${}^3P_3$ ways.
Hence, the total number of ways is ${}^3P_3 \times {}^3P_3 = 36$.

(8) (i) Let the two particular delegates who wish to sit together be treated as one delegate. So we have 19 delegates who can be arranged on a round table in (19 – 1)!, i.e., 18! ways.
After this, the two particular delegates can be permuted between themselves in 2! = 2 ways. Hence, by product rule, number of required arrangements is 2 × (18)!.

(ii) The total number of arrangements of 20 delegates on a round table is 19!.
Hence, the number of arrangements in which the two particular delegates never sit together is
19! – 2 × 18! = 18! (19 – 2) = 17 × 18!

(9) Since diamonds do not have natural order of left and right so clockwise and anticlockwise arrangements are taken as identical. Therefore, the number of arrangements of 10 different diamonds to make a necklace is (1/2) × 9 = 181440.

(10) In this case, anticlockwise and clockwise arrangements are the same.
Hence, the number of ways of arrangements is 5!/2 = 60.

(11) 6 men can dine at a round table in 5! ways.
Now, there are 6 vacant spaces between any two men.
These can be occupied by 5 women in ${}^6P_5$ i.e. 6! ways.
Therefore, the total number of ways is 6! × 5!.

(12) There are 10 letters in the word 'LOGARITHMS'. So, the number of 4-letter words is equal to number of arrangements of 10 letters, taken 4 at a time, i.e.,
${}^{10}P_4 = 5040$.

TRY IT YOURSELF-2

(1) Let the number of teams be n. Then number of matches to be played is ${}^nC_2 = 28$.

$$\therefore \frac{n(n-1)}{2} = 28$$

$\Rightarrow n^2 - n - 56 = 0$
$\Rightarrow (n-8)(n+7) = 0$
$\Rightarrow n = 8$ as $n \neq -7$

(2) The number of ways of selection of three consonants from 10 is ${}^{10}C_3$. The number of ways of selection of two vowels from 4 is ${}^4C_2$. Permutation of these 5 letters (all distinct) is 5!. Therefore, number of words that can be formed is
${}^{10}C_3 \times {}^4C_3 \times 5! = 86400$

(3) Given letters are I, (N, N, N), (D, D), (E, E, E), P, T
The choices are as follows :

Choice	Ways
All the letters are distinct [different letters are I,N,D,E,P,T]	${}^6C_5 = 6$
3 distinct, 2 alike	${}^3C_1 \times {}^5C_3 = 30$
2 distinct, 3 alike	${}^2C_1 \times {}^5C_2 = 20$
2 alike, 2 alike, 1 distinct	${}^3C_2 \times {}^4C_1 = 12$
3 alike, 2 alike	${}^2C_1 \times {}^2C_1 = 4$
	Total = 72

(4) It is to be noted here that, when two persons shake hands, it is counted as on handshake, not two. So, this is a problem on combinations.
The total number of handshake is same as the number of ways of selecting 2 persons among 12 persons is
${}^{12}C_2 = 12!/(10! \times 2!) = 66$

(5) Let the number of candidates be n. Therefore, n – 2 are to be elected and no one can vote up to n – 2. Hence, the number of ways in which one can vote is

$^nC_1 + {}^nC_2 + ... + {}^nC_{n-2} = 56$ (given)

$\Rightarrow 2^n - ({}^nC_0 + {}^nC_{n-1} + {}^nC_n) = 56$

$\Rightarrow 2^n - n = 58$

$\Rightarrow 2^n = 58 + n$

which is satisfied by n = 6 only.

(6) We can divide 52 cards equally into four groups in $52!/(13!)^4\,(4!)$ ways.

Now, these four groups can be distributed among four players as 4! ways.

Therefore, the total number of ways of dividing the cards among four players equally is

$$\frac{52!}{(13!)^4\,(4!)} \times 4! = \frac{52!}{(13!)^4}$$

(7) The total number of ways is $m \times m \times ....$ n times $= m^n$.

The number of ways in which one gets all the prizes is m.

Therefore, the required number of ways is $m^n - m$.

(8) Here, the number of solutions is equivalent to the number of ways. Twelve identical objects are distributed in 3 distinct boxes if empty boxes are not allowed, which is $^{12-1}C_{3-1} = {}^{11}C_2 = 55$.

(9) **(C).** Consonant $\rightarrow$ b, c, d, f, g, h (6)

Vowels $\rightarrow$ a, e, i, o, u (5)

W :

×		×		×		×

Case I : If word begins with consonants then

$({}^6C_4 \times 4!) \times ({}^5C_3 \times 3!) = 360 \times 60 = 21600$

Case II : If word begins with vowels

$({}^5C_4 \times 4!) \times ({}^6C_3 \times 3!) = 120 \times 120 = 14400$

Total = 36000 $\Rightarrow$ 10K = 36000 $\Rightarrow$ K = 3600

(10) **(C).** $^7C_1 + {}^7C_2 + {}^7C_3 .... {}^7C_7 = 2^7 - 1$

TRY IT YOURSELF-3

(1) Two circles intersect maximum at two distinct points. Now, two circles can be selected in $^6C_2$ ways. Again, each selection of two circles gives two points of intersection. Therefore, the total number of points of intersection is

$^6C_2 \times 2 = 30$.

(2) (i) Line is formed joining two points. Hence, number of lines is $^{10}C_2$. But joining any points from 5 collinear points gives the same line. Again, 2 points are selected from 5 in $^5C_2$ ways or lines joining collinear points is taken $^5C_2$ (= 10) times. Then the number of straight lines = $^{10}C_2 - 10 + 1 = 36$.

(ii) For a triangle, three non-collinear points are required. Three points can be selected in $^{10}C_3$ ways. Now, the selection of three points from 5 collinear points does not form triangle. Hence, number of triangles is $^{10}C_3 - {}^5C_3$.

(3) Number of derangements in such problems is given by

$$n!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - .... + (-1)^n \frac{1}{n!}\right\}$$

Hence, the required number of derangements is'

$$4!\left\{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\} = 12 - 4 + 1 = 9$$

(4) **(D).** Four digit even number

Case I : If zero as least digit

$9 \times 8 \times 7$ [0] $= 9 \times 8 \times 7$

Case II : Zero is not last digit

$8 \times 8 \times 7$ [ ] $= 8 \times 8 \times 7$

4 ways (2,4,6,8)

Four digit even number = $17 \times 56 = 952$

Total four digit numbers

[ ] [ ] [ ] [ ]

$9 \times 9 \times 8 \times 7 = 4536$

(5) Let, $x + y + z + w + t = 20$ .... (1)

where $t \geq 0$.

Now, we find the non-negative integral solution of eq. (1).

The total number of such solutions is $^{20+5-1}C_{5-1}$.

(6) $8100 = 2^2 \times 3^4 \times 5^2$

Number of ways $= \frac{1}{2}[(2+1)(4+1)(2+1)+1] = 23$

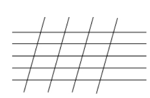
(7) Exact one 2 should be there

Total ways = $1 \times 5 \times 8 \times 3 = 120$

(8) Number of line joining adjacent points = n

$n = {}^nC_2 - n$; $2n = {}^nC_2$

$2n = \frac{n(n-1)}{2}$; n = 0 or n = 5

(9) 

No. of parallelograms = $^5C_2 \times {}^4C_2 = 60$.

CHAPTER- 8 : PERMUTATIONS AND COMBINATIONS

EXERCISE-1

(1) **(B).** The first letter of four letter word can be chosen by 26 ways, second by 25 ways, third by 24 ways and fourth by 23 ways. So number of four letter words

$= 26 \times 25 \times 24 \times 23 = 358800$

(2) **(A).** ${}^8P_3 = 8.7.6 = 336$

With the help of formula

$${}^8P_3 = \frac{8!}{(8-3)!} = \frac{8.7.6.5.4.3.2.1}{5.4.3.2.1} = 8.7.6 = 336$$

(3) **(B).** The required numbers $= {}^5P_4 = \frac{5!}{(5-4)!} = 5.4.3.2.1 = 120$

(4) **(C).** The Required number of ways are

${}^6P_3 = 6.5.4 = 120$

(5) **(A).** The total number of signals is the number of arrangements of 8 flags by taking 5 flags at a time.

Hence required number of signals

$$= {}^8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!} = 6720$$

(6) **(B).** Every number lying between 100 and 1000 is a three digits number. Therefore we have to find the number of permutations of six digits 1,2,3,4,5,6 taken three at a time.

Hence, the required number of numbers

$$= {}^6P_3 = \frac{6 \times 5 \times 4 \times 3!}{3!} = 120$$

(7) **(A).** The total number of arrangements of ten digits 0,1,2,3,4,5,6,7,8,9 taking 4 at a time is ${}^{10}P_4$. But these arrangements also include those number which have 0 at thousand's place. Such numbers are not four digit numbers. When 0 is fixed at thousand's place, we have to arrange remaining 9 digits by taking 3 at a time. The number of such arrangements is ${}^9P_3$.

So, the total number of numbers having 0 at thousand's place $= {}^8P_3$.

Hence, the total numbers four digit numbers

$= {}^{10}P_4 - {}^9P_3 = 5040 - 504 = 4536$

(8) **(D).** There are in all 9 letters in the given word. Out of them there are 4 'A's, 2 'L's and the remaining 3 are different

So the total number of permutations $\frac{9!}{4!\,2!} = 7560$

(9) **(B).** We have to arrange these six digits, out of which 2 occurs twice, 3 occurs thrice and rest are distinct.

The number of such arrangement $= \frac{6!}{2! \times 3!} = 60$

(10) **(B).** First pen can be put in 6 ways.

Similarly each of second, third and fourth pen can be put in 6 ways.

Hence total number of ways $= 6 \times 6 \times 6 \times 6 = 1296$

(11) **(B).** We have to determine the total number of three digit numbers formed by using the digits 1,7,8,9. clearly the repetition of digits is allowed.

A three digit number has three places viz. unit's, ten's and hundred's. Unit's place can be filled by any of digit 1,7,8,9, so units place can be filled in 4 ways. Similarly each one of the ten's and hundred's places can be filled in 4 ways.

$\therefore$ Total number of required numbers $= 4 \times 4 \times 4 = 64$

(12) **(A).** 4 number between 100 and 1000 has three digits, so we have to form all possible 3- digit numbers with distinct digits. We cannot have at the hundred's place so the hundred's place can be filled with any of the digits 1,2,3, ...9. So there are 9 ways of filling the hundred's place.

Now 9 digits are left including 0. so ten's place can be filled with any of the remaining 9 digits in 9 ways. Now, the unit's place can be filled within any of the numbering 8 digits, so there are 8 ways of filling the unit's place.

Hence the total number of required numbers

$= 9 \times 9 \times 8 = 648$

(13) **(A).** Since three digit number greaterthan 600 will have 6 or 7 at hundred's place. So hundred's place can be filled in 2 ways. Each of ten's and one's place can be filled in 5 ways. Hence total no. of required numbers $= 2 \times 5 \times 5 = 50$

(14) **(A).** (a) Since the first prize can be given to any one of the student so it can be given in 5 ways. The second prize can be given in 4 ways and the third prize can be given in 3 ways as no student can get more than one prize. Therefore, number of ways of distributing 3 prizes among 5 boys $= 5 \times 4 \times 3 = 60$

(b) Since a student can receive any number of prizes, therefore each prize can be distributed in 5 ways. Hence the number of ways of distribution of 3 prizes among 5 students is $= 5 \times 5 \times 5 = 5^3 = 125$.

(c) Here the number of ways so that a student can receive all prizes will be 5, therefore if a student doesn't receive all prizes, then number of ways of distributing 3 prizes among 5 students $= 5^3 - 5 = 120$

(15) **(B).** The required numbers will contain 4 digits in which first and last digits will be 1 and 5 respectively. The remaining two places are to be filled up from the remaining three digits and it can be done in ${}^3P_2 = 6$ ways.

Hence the required number $= 1 \times 6 = 6$

(16) **(A).** After putting S and K at their respective places the remaining 3 letters can be arranged in 3! ways. Therefore required number of words $= 1 \times 3! = 6$

(17) **(B).** First Method : The total numbers by taking any six digits from the given seven digits $= {}^7P_6 = 5040$

The six digits numbers containing zero at first place

$= {}^6P_5 = 720$

$\therefore$ Required Numbers $= 5040 - 720 = 4320$

Second Method : Here first we make six squares in a row in the following way

6 6 5 4 3 2

Obviously first square on extreme left can be filled in 6 ways, since any non zero six digits 1,2,3,4,5,6 can occupy this square .

The remaining squares can be filled in
$6 \times 5 \times 4 \times 3 \times 2 = 720$ no. of ways
$\therefore$ Required Numbers $= 6 \times 720 = 4320$

(18) (B). $\frac{^{56}P_{r+6}}{^{54}P_{r+3}} = \frac{30800}{1} \Rightarrow \frac{56!}{(56-r-6)!} = \frac{(30800) \times 54!}{(54-r-3)!}$

$\Rightarrow 56 \times 55 \times (51-r) = 30800$

$\Rightarrow (51-r) = \frac{30800}{56 \times 55} = 10 \Rightarrow r = 51 - 10 = 41$

(19) (C). The no. of ways to fill up 2 vacancies by 13 candidates is $^{13}P_2 = 13 \times 12 = 156$

(20) (A). After fixing the letters A and L in the first and last places, the total number of available places are 8 and the letters are also 8. Out of these 8 letters there are 2 groups of alike letters. Therefore no. of words $= \frac{8!}{2!2!} = 10080$

(21) (C). First digit between 20000 and 30000 will be 2 which can be chosen by one way. Every number will be of five digits and all the digits can be anything from the given five digit except first digit. So each digit of the remaining four digits can be chosen in 5 ways
$\therefore$ required numbers $= 1 \times 5 \times 5 \times 5 \times 5 = 625$

(22) (B). There are 6 letters in the given word. Then the number of three letters words from the remaining 5 letters after removing I is $= {}^5P_3 = 5 \times 4 \times 3 = 60$

(23) (A). Required numbers will have 3 digits so their total number $= {}^7P_3 - {}^6P_2 = 180$

(24) (C). Required number will have 4 digits and their thousand digit will be 1 or 2 or 3 or 4. The number of such numbers will be 124, 125, 125 and 1 respectively.
$\therefore$ Total numbers $= 124 + 125 + 125 + 1 = 375$

(25) (D). The arrangement can be done in ${}^4P_4 = 24$ ways.

(26) (C). All the 5 digit numbers and 6 digit numbers are greater than 3000. Therefore number of 5 digit numbers $= {}^6P_5 - {}^5P_5 = 600$.
{Since the case that 0 will be at ten thousand place should be omit}.
Similarly number of 6 digit numbers $6! - 5! = 600$.
Now the numbers of 4 digit numbers which are greater than 3000, having 3, 4 or 5 at first place, this can be done in 3 ways and remaining 3 digit may be filled from remaining 5 digits *i.e.* required number of 4 digit numbers are ${}^5P_3 \times 3 = 180$. Hence total required number of numbers $= 600 + 600 + 180 = 1380$.

(27) (A). Words starting from A are $5! = 120$
Words starting from I are $5! = 120$
Words starting from KA are $4! = 24$
Words starting from KI are $4! = 24$
Words starting from KN are $4! = 24$
Words starting from KRA are $3! = 6$
Words starting from KRIA are $2! = 2$
Words starting from KRIN are $2! = 2$
Words starting from KRISA are $1! = 1$
Words starting from KRISNA are $1! = 1$
Hence rank of the word KRISNA is 324.

(28) (A). We have 5 letters other than 'I' and 'N' of which two are identical (E's). We can arrange these letters in a line in $\frac{5!}{2!}$ ways. In any such arrangement, 'I' and 'N' can be placed in 6 available gaps in ${}^6P_2$ ways, so required number $= \frac{5!}{2!} \cdot {}^6P_2 = m_1$. Now, if word start with 'I' and end with 'R' then the remaining letters are 5. So total no. of ways $= \frac{5!}{2!} = m_2$

$\therefore \frac{m_1}{m_2} = \frac{5!}{2!} \cdot \frac{6!}{4!} \cdot \frac{2!}{5!} = 30$.

(29) (C). Total number of arrangements are $\frac{6!}{2!} = 360$.
The number of ways in which O's come together $= 5! = 120$.
Hence required number of ways $= 360 - 120 = 240$.

(30) (D). Using the digits 0, 1, 2, ..., 9 the number of five digit telephone numbers which can be formed is 10^5 (since repetition is allowed)
The number of five digit telephone, numbers which have none of the digits repeated $= {}^{10}P_5 = 30240$.
$\therefore$ The required number of telephone numbers $= 10^5 - 30240 = 69760$.

(31) (B). Required number of ways $= \frac{8!}{2!2!2!} = 5040$.

(32) (B). The word MOBILE has three even places and three odd places. It has 3 consonants and 3 vowels. In three odd places we have to fix up 3 consonants which can be done in ${}^3P_3$ ways.
Now, remaining three places we have to fix up remaining three places we have to fix up remaining three which can be done in ${}^3P_3$ ways.
The total number of ways $= {}^3P_3 \times {}^3P_3 = 36$.

(33) (A). Required number of ways $= \frac{1}{2}(5-1)! = \frac{4!}{2}$.
{Since clockwise and anticlockwise are same in case of ring}.

(34) (A). Fix up a male and the remaining 4 male can be seated in 4! ways. Now no two female are to sit together and as such the 2 female are to be arranged in five empty seats between two consecutive male and number of arrangement will be ${}^5P_2$. Hence by fundamental theorem the total number of ways is $= 4! \times {}^5P_2 = 24 \times 20 = 480$ ways.

(35) **(B).** The required number

$$= \frac{^7P_5}{5} = \frac{7.6.5.4.3}{5} = 504$$

(36) **(C).** In this case a clockwise and corresponding anti clockwise order will give the same circular permutation.

So the required number $= \frac{^7P_5}{2.5} = \frac{7.6.5.4.3}{2.5} = 252$

(37) **(C).** First of all let two particular persons be seated together. They can sit together in 2! = 2 ways.
Then the remaining four persons may sit on remaining four places in 4 ! = 24 ways, so the total number of ways $= 2 \times 24 = 48$

(38) **(C).** Seven girls can keep stand in a circle by $\frac{(7-1)!}{2!}$ number of ways, because there is no difference in anticlockwise and clockwise order of their standing in a circle. $\therefore \frac{(7-1)!}{2!} = 360$

(39) **(B).** Here one women will sit between two men. Now fixing the place of one man the remaining 6 men on the circular table can sit in 6! ways. Since there are seven places between 7 men. Therefore seven women can sit on these places in 7! ways. Thus 7 men and 7 women under the given condition can sit in 7! **.** 6! ways.

(40) **(A).** Total number of books $= 3 \times 4 = 12$ in which each of 4 different books is repeated 3 times.
Hence the required number of arrangements

$$= \frac{12!}{3! \times 3! \times 3! \times 3!} = \frac{12!}{(3!)^4}$$

(41) **(A).**

V	O	W	E	L
	⊗		⊗	
1	2	3	4	5

O and E are only even places [⊗ marked places] can be arranged in $^2P_2$ ways = 2
Remaining letters can be arranged in (3!) ways = 6
Required number of words $= {^2P_2} \times 3! = 12$

(42) **(A).** We have to select 6 persons from 10 given persons. This can be done in $^{10}C_6$ ways therefore number of committees $= {^{10}C_6} = {^{10}C_4} = \frac{10!}{4!\,6!} = \frac{10.9.8.7}{4.3.2.1} = 210$

(43) **(A).** Three men out of 6 men can be selected in $^6C_3$ ways. Two women out of 5 women can be selected in $^5C_2$ ways. Therefore total number of ways $= {^6C_3} \times {^5C_2} = 200$ ways.

(44) **(C).** The committee can be formed in the following ways:
(i) By selecting 2 men and 1 woman
(ii) By selecting 1 man and 2 women
2 men out of 5 men and 1 woman out of 2 women can be chosen in $^5C_2 \times {^2C_1}$ ways.
And 1 man out of 5 men and 2 women out of 2 women can be chosen in $^5C_1 \times {^2C_2}$ ways.
Total number of ways of forming the committee
$= {^5C_2} \times {^2C_1} + {^5C_1} \times {^2C_2} = 20 + 5 = 25$

(45) **(B).** (a) In this case 10 players are to be selected out of 14 players and it can be done in

$$^{14}C_{10} = {^{14}C_4} = \frac{14.13.12.11}{1.2.3.4} = 1001 \text{ ways}$$

(b) In this case 11 players are to be selected out of 14 players and it can be done in

$$^{14}C_{11} = {^{14}C_3} = \frac{14.13.12}{1.2.3} = 364 \text{ ways}$$

(46) **(C).** Total number of ways
$= {^6C_1} + {^6C_2} + {^6C_3} + {^6C_4} + {^6C_5} + {^6C_6} = 2^6 - 1 = 63$

(47) **(C).** Here are 3 things of first kind, 4 things of second kind and 5 things of third kind so the total number of selections $= (3+1)(4+1)(5+1) - 1 = 119$

(48) **(D).** Required number $= {^{20}C_2} - {^4C_2} + 1$

$$= \frac{20 \times 19}{2} - \frac{4 \times 3}{2} + 1 = 190 - 6 + 1 = 185.$$

(49) **(A).** The number of 4 persons including A, B $= {^6C_2}$
Considering these four as a group, number of arrangements with the other four = 5!
But in each group the number of arrangements = 2! × 2!
$\therefore$ The required number of ways $= {^6C_2} \times 5! \times 2! \times 2! = 16$

(50) **(D).** The number of committees of 4 gentlemen $= {^4C_4} = 1$
The number of committees of 3 gentlemen,
1 wife $= {^4C_3} \times {^1C_1}$
($\because$ after selecting 3 gentlemen only 1 wife is left who can be included)
The number of committees of 2 gentlemen, 2 wives
$= {^4C_2} \times {^2C_2}$
The number of committees of 1 gentleman, 3 wives
$= {^4C_1} \times {^3C_3}$
The number of committees of 4 wives = 1
$\therefore$ The required number of committees
$= 1 + 4 + 6 + 4 + 1 = 16$

(51) **(C).** The total no. of ways to arrange 3 copies each of 4 different books $= \frac{12!}{(3!)^4} = 369{,}600$

(52) **(D).** Here the order of the couples is not important.

So, required number of ways is $\frac{20!}{2^{10}10!}$

(53) **(B).** First we select 4 consonant out of 6 and 3 vowels out of 5. This can be done in $^6C_4 \times {^5C_3}$ ways. After such a selection of 7 letters then can be arranged in 7! ways.
So the total number of words $= {^6C_4} \times {^5C_3} \times 7!$

(54) **(A).** First we divide 7 persons into two groups of 4 and 3 persons. The total number of such division

$$= \frac{7!}{4!\,3!} = 35$$

Now for such a division of 4 and 3 persons there are 3! × 2! ways of sitting round the given two tables.
Hence total number of required arrangements
$= 35 \times 12 = 420$

(55) **(C).** Since T and S are to be included in every word, therefore first we choose 2 letters out of the remaining letters which can be done in $^6C_2$ ways. Now each group of letters will give 4! words. Therefore number of words $= {}^6C_2 \times 4! = 15 \times 24 = 360$

(56) **(C).** Out of 10 boxes we have to choose only 5 boxes because the balls are identical and the boxes are also identical (but they can occupy different places), the required number of ways $= {}^{10}C_5 = \frac{10!}{(5!)^2}$.

(57) **(D).** The number of ways in which at least 5 women can be included in a committee is

$${}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$$

$= 1008 + 2352 + 2016 + 630 + 56 = 6062$ ways

(i) The women are in majority in $(2016 + 630 + 56) = 2702$ cases.

(ii) Men are in majority in 1008 cases.

(58) **(B).** Either 6 selected out of 8 or one from 2 and 5 from 8
$= {}^8C_6 + {}^2C_1 + {}^8C_5 = 140$.

(59) **(C).** 6 particular players are always to be included and 4 are always excluded so total no. of selection, now,4 players out of 12, hence number of ways = $^{12}C_4$.

(60) **(D).** Required number of ways = $^{11}C_8 = 165$.
{Since, captain already be chosen, so now from 11 players 8 are to be chosen}.

(61) **(D).** Total number of shake hands when each person shake hands with the other once only = $^8C_2 = 28$ ways.

(62) **(B).** $2 \cdot {}^{20}C_2$ {Since two students can exchange cards each other in two ways}.

(63) **(B).** Required number of ways
$= {}^5C_3 \times {}^2C_1 \times {}^9C_7 = 10 \times 2 \times 36 = 720$.

(64) **(A).** (i) Total number of combinations of 15 students into 3 groups of 5 each $= \frac{15!}{3!(5!)^3}$

(ii) In this case the groups are associated with different colleges, so the required number $= \frac{15!}{(5!)^3}$

(65) **(A).** Let x_1, x_2, x_3 be the number of toys received by the three children
Then, $x_1, x_2, x_3, \geq 3$ and $x_1, x_2, x_3 = 16$
Let $u_1 = x_1 - 3$, $u_2 = x_2 - 3$ and $u_3 = x_3 - 3$
Then, $u_1, u_2, u_3 \geq 0$ and $u_1 + u_2 + u_3 = 7$
Here, $n = 7$ and $r = 3$
∴ Number of ways $= {}^{n+r-1}C_{r-1} = {}^9C_2 = 36$

(66) **(C).** Here, clearly $0 \leq x_4 \leq 5$, $x_1, x_2, x_3, \geq 0$ and $x_1 + x_2 + x_3 = 20 - 4x_4$
⇒ $r = 3$ and $n = 20 - 4x_1$
If $x_4 = 0$ number of ways $^{20+3-1}C_{3-1} = {}^{22}C_2$
If $x_4 = 1$ number of ways $^{16+3-1}C_{3-1} = {}^{18}C_2$
Similarly, if $x_4 = 2, 3, 4, 5$, number of ways
$= {}^{14}C_2, {}^{10}C_2, {}^6C_2, {}^2C_2$ respectively
∴ Total number of ways
$= {}^{22}C_2 + {}^{18}C_2 + {}^{14}C_2 + {}^{10}C_2 + {}^6C_2 + {}^2C_2 = 536$

(67) **(D).** Let x_1, x_2 and x_3 be the number of toys received by the three children. Then, $x_1+x_2+x_3=10$, $0 \leq x_1 \leq 6$, $0 \leq x_2 \leq 7$, $0 \leq x_3 \leq 8$
⇒ Required number of ways = coeff of x^n in $(x^0 + x^1 + x^2 + \ldots\ldots x^6)(x^0 + x^1 + x^2 + \ldots\ldots x^7)$ $(x^0 + x^1 + x^2 + \ldots\ldots x^8)$ = coeff of x^{10} in

$$\left[\frac{1(1-x^7)}{1-x}\right]\left[\frac{1(1-x^8)}{1-x}\right]\left[\frac{1(1-x^9)}{1-x}\right]$$

= coeff of x^{10} in $[(1-x^7)(1-x^8)(1-x^9)][1-x]^{-3}$
= coeff of x^{10} in $(1 - x^7 - x^8 - x^9 \ldots\ldots)(1 + {}^3C_1x + {}^4C_2x^2 + {}^5C_3x^3 + \ldots\ldots {}^{12}C_{10}x^{10} + \ldots\ldots)$
[Note that powers >10 are unimportant and hence ignored]
$= {}^{12}C_{10} - {}^5C_3 - {}^4C_2 - {}^3C_1 = 66 - 10 - 6 - 3 - = 47$

(68) **(C).** The required number of ways $= {}^{5-1}C_{3-1}$
$= {}^4C_2 = \frac{4.3}{1.2} = 6$

(69) **(A).** The number of ways of choosing first couple is $({}^{15}C_1)({}^{15}C_1) = 15^2$
The number of ways of choosing second couple is $({}^{14}C_1)({}^{14}C_1) = 14^2$ and so on.
Thus the number of ways of choosing the couples is $15^2 + 14^2 + 13^2 + \ldots + 2^2 + 1^2$

$$= \frac{15 \times (15+1)(2(15)+1)}{6} = 1240$$

(70) **(B).** First we have to select 2 men for bow side and 3 for stroke side.
No. of selections of the crew for two sides $= {}^5C_2 \times {}^3C_3$.
For each selection there are 4 persons each on both sides who can be arranged in 4! × 4! ways.
Required number of arrangement $= {}^5C_2 \times {}^3C_3 \times 4! \times 4!$
$= 5760$

(71) **(B).** At least one lamp is to be kept switched on, the total number of ways are
$^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10} = 2^{10} - 1 = 1023$

(72) **(B).** Bowler (4) (wk) (2) rest (7)

Bowler (4)	(wk) (2)	rest (7)	
3	1	7	$= {}^4C_3 \times {}^2C_1 \times {}^7C_7$
4	1	6	

$^4C_4 \times {}^2C_1 \times {}^4C_6 = (4 \times 2 \times 1) + (1 \times 2 \times 7)$
$8 + 14 = 22$

(73) **(C).** Required number of diagonals $= {}^mC_2 - m$

$$= \frac{m(m-1)}{2!} - m = \frac{m}{2!}(m-3).$$

(74) **(C).** Number of lines from 6 points $= {}^6C_2 = 15$. Points of intersection obtained from these lines $= {}^{15}C_2 = 105$.

Now we find the number of times, the original 6 points come.

Consider one point say A_1. Joining A_1 to remaining 5 points, we get 5 lines, and any two lines from these 5 lines give A_1 as the point of intersection.

$\therefore$ A_1 come ${}^5C_2 = 10$ times in 105 points of intersections.

Similar is the case with other five points.

$\therefore$ 6 original points come $6 \times 10 = 60$ times in points of intersection.

Hence the number of distinct points of intersection $= 105 - 60 + 6 = 51$.

(75) **(A).** ${}^{16}C_3 - {}^8C_3 = 504$.

(76) **(B).**

Number of triangles formed $= {}^{12}C_3 - {}^7C_3 = 220 - 35 = 185$

(77) **(D).** First we write six '+' signs at alternate places i.e by leaving one place vacant between two successive '+' signs. Now there are 5 places vacant between these signs and there are two places vacant at the ends. If we write 4 '–' signs at these 7 places then no two '–' will come together. Hence total number of ways $= {}^7C_4 = 35$

(78) **(C).** The required number of ways

$$= 3!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right] = 3! - 3! + \frac{3!}{2!} - 1 = 3 - 1 = 2$$

(79) **(B).** Suppose $x_1x_2x_3x_4x_5x_6x_7$ represents a seven digit number. Then x_1 takes the value $1, 2, 3, \ldots\ldots, 9$ and $x_2, x_3, \ldots x_7$ all take values 0, 1, 2, 3, ..., 9.

If we keep $x_1, x_2, \ldots\ldots, x_6$ fixed, then the sum $x_1 + x_2 + \ldots.. + x_6$ is either even or odd. Since x_7 takes 10 values 0, 1, 2, ..., 9, five of the numbers so formed will be even and 5 odd.

Hence the required number of numbers

$= 9 \, . \, 10 \, . \, 10 . \, 10 \, . \, 10 . \, 10 \, . \, 5 = 4500000$.

(80) **(A).** Sum of the digits in the unit place is $6(2 + 4 + 6 + 8) = 120$ units. Similarly, sum of digits in ten place is 120 tens and in hundredth place is 120 hundreds etc. Sum of all the 24 numbers is

$120(1 + 10 + 10^2 + 10^3) = 120 \times 1111 = 133320$.

(81) **(D).** Number of permutations are :

(AA), (I I), (NN), E, X, M, T, O

$$= \text{Coefficient of } x^4 \text{ in } 4!\left(1 + \frac{x}{1!} + \frac{x^2}{2!}\right)^3\left(1 + \frac{x}{1!}\right)^5$$

$$= \text{Coefficient of } x^4 \text{ in } 4!\left(1 + x + \frac{x^2}{2!}\right)^3 (1 + x)^5$$

$= $ Coefficient of x^4 in

$$4!\left\{(1+x)^3 + \frac{x^6}{8} + \frac{3}{2}(1+x)^2x^2 + \frac{3}{4}x^4(1+x)\right\}.(1+x)^5$$

$= $ Coefficient of x^4 in

$$4!\left\{(1+x)^8 + \frac{x^6}{8}(1+x)^5 + \frac{3}{2}x^2(1+x^7) + \frac{3}{4}x^4(1+x)^6\right\}$$

$$= 4!\left\{{}^8C_4 + 0 + \frac{3}{2}{}^7C_2 + \frac{3}{4}\right\} = 24\left\{\frac{8.7.6.5}{1.2.3.4} + \frac{3}{2} + \frac{7.6}{1.2} + \frac{34}{3}\right\}$$

$= 8.7.6.5 + 6(3.7.6) + 6.3 = 1680 + 756 + 18 = 2454$

(82) **(B).** The sum of the digits $= 1 + 2 + 3 = 6$ and $n = 3$, so the sum of all numbers formed

$= 6 \, . \, 2! \, (111) = 12 \times 111 = 1432$

(83) **(C).** Number of derangements are

$$= 4!\left\{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\} = 12 - 4 + 1 = 9$$

Since number of derangements in such a problems is given

$$\text{by } n! \left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots(-1)^n\frac{1}{n!}\right\}$$

(84) **(D).** $x_1x_2x_3x_4x_5 = 2 \times 3 \times 5^2 \times 7$ we can assign 2, 3 or 7 to any of variables. We can assign entire 5^2 to just one variable in 5 ways or can assign

$5^2 = 5 \times 5$ to two variable in ${}^5C_2$ ways

${}^5C_1 + {}^5C_2 = 5 + 10 = 15$ ways

Required number of solutions $= 5 \times 5 \times 5 \times 15 = 1875$

(85) **(D).** Sum of 7 digits = a multiple of 9

We know sum of numbers 1, 2, 3, ... 8, 9 is 45.

So, two left number should also have sum as 9.

The pair to be left are (1, 8), (2, 7), (3, 6), (4, 5) with each pair left number of 7 digit numbers in 7!.

So, with all 4 pairs $= 4 \times 7!$

(86) **(B).** ${}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots {}^{2n+1}C_n = 63$

$2\left({}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots {}^{2n+1}C_n\right) = 126$

$\left({}^{2n+1}C_1 + {}^{2n+1}C_{2n}\right) + \left({}^{2n+1}C_2 + {}^{2n+1}C_{2n-1}\right) + \ldots\ldots\ldots + \left({}^{2n+1}C_n + {}^{2n+1}C_{n+1}\right) = 126$

${}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots + {}^{2n+1}C_{2n} = 126$

${}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots\ldots + {}^{2n+1}C_{2n+1}$

$= 126 + {}^{2n+1}C_0 + {}^{2n+1}C_{2n+1}$

$2^{2n+1} = 126 + 1 + 1$

$2^{2n+1} = 2^7 \qquad \therefore n = 3$

(87) **(A).** $p = 10$; $q = 80$; $r = 32$

(88) **(C).** Number of different tickets of the same class ${}^6C_2 = 15$

$S_1 \; S_2 \; S_3 \; S_4 \; S_5 \; M$

Number of different tickets of k class = 15k

Hence ${}^{15k}C_{10} = {}^{45}C_{35} = {}^{45}C_{10} \Rightarrow k = 3$

(89) **(C).**

A	B	C	D	
Athanni	Chavanni	Duanni	Ekanni	(beggar)

$\underbrace{0\;0\;0\;0\;0\;0}\;\underbrace{\varnothing\;\;\varnothing\;\;\varnothing}_{\text{False}}$

Using beggar method number of ways $= {}^9C_3 = 84$

(90) **(B).** ${}^9C_5 -$ [only top remains empty + middle empty + bottom empty]

${}^9C_5$ - $[{}^7C_5 + {}^6C_5 + {}^5C_5] = 126 - (21 + 6 + 1) = 98$

(91) **(B).** 4 apples, 5 mangoes and 6 oranges
coeff. of x^4 in $(1+x+x^2+x^3+x^4)3$ = coeff. of x^4 in $(1-x)^{-3} = {}^6C_2 = 15$

(92) **(D).** Required number of possible outcomes
= Total number of possible outcomes – Number of possible outcomes in which 5 does not appear on any dice
$= 6^3 - 5^3 = 91$.

(93) **(D).** Number of ways

$$= \left[\left({}^3C_3 + {}^4C_3 + {}^5C_3 + {}^6C_3 + {}^7C_3\right)\times 2 + {}^8C_3\right]\times 2 = 392$$

(94) **(B).** ${}^{10}P_3 = 720$

(95) **(B).** ${}^{20}C_r = {}^{20}C_{r-10} \Rightarrow r + (r-10) = 20 \Rightarrow r = 15$

$$\therefore\ {}^{18}C_r = {}^{18}C_{15} = {}^{18}C_3 = \frac{18.17.16}{1.2.3} = 816$$

(96). **(A).** Ist book can be given to any of the five students. Similarly other six books also have 5 choices. Hence the total number of ways is 5^7.

(97) **(C).** The number of three elements subsets containing a_3 is equal to the number of ways of selecting 2 elements out of n – 1 elements. So, the required number of subsets is ${}^{n-1}C_2$.

(98) **(C).** Six consonants and three vowels can be selected from 10 consonants and 4 vowels in ${}^{10}C_6 \times {}^4C_3$ ways. Now, these 9 letters can be arranged in 9! ways. So, required number of words $= {}^{10}C_6 \times {}^4C_3 \times 9!$

(99) **(B).** 8 men can sit in a row in ${}^8P_8$ ways. Then for the 6 women, there are 9 seats to sit
$\therefore$ the women can sit in ${}^9P_6$ ways
$\therefore$ total numbe of ways $= {}^8P_8 \,.\, {}^9P_6$

(100) **(A).** Any interior intersection point corresponds to 4 of the fixed points , namely the 4 end points of the intersecting segments . Conversely, any 4 labled points determine a quadrilateral, the diagonals of which intersect once within the circle .
Number of interior intersection points $= {}^9C_4 = 126$.

(101) **(A).** Order of vowels of fixed

$$\therefore \text{ required number of ways are } \frac{6!}{2!}$$

(102) **(A).** ${}^{13}C_3 - {}^3C_3 - {}^4C_3 - {}^5C_3 = 205$

(103) **(D).** Four digit numbers = 3 . 3. 2. 1 = 18
Five digit numbers = 4. 3. 2. 1. 1 = 24
$\therefore$ Total number of numbers = 42

(104) **(B).** As $94864 = 2^4 . 7^2 . 11^2$
$\Rightarrow$ No. of ways = ½ [(4 + 1) (2 + 1) (2 + 1) + 1] = 23

(105) **(C).** Rank $= (4!\times 3) + (3!\times 2) + (2!\times 2) + 1$
$= 72 + 12 + 4 + 1 = 89$

(106) **(D).** Number of ways = Arrangement of (m – 1) things of one kind and (n – 1) things of the other kind

$$= \frac{(m+n-2)!}{(m-1)!(n-1)!}$$

(107) **(C).** Selection of two husbands $= {}^9C_2$
Selection of two wifes whose husbannds are not chosen yet $= {}^7C_2$.
Total number of ways to form two teams
$= {}^9C_2 \,.\, {}^7C_2 \,.\, 2! = 1512$

EXERCISE-2

(1) **(A).** $x + y + z = 15;\quad x \le 8;\ y \le 8, z \le 8$
Total solution $= {}^{17}C_2 = 136$
Suppose x or y or z is given 9 oranges
$x + y + z = 6$
${}^3C_1 \,.\, {}^8C_2 = 84$ invalid
effective ways = 136 – 84 = 52

(2) **(B).**
When exactly one 3 and nine 2's $\rightarrow$ ${}^{10}C_1$
When exactly two 3's and eight 2's $\rightarrow$ ${}^9C_2$
When three 3's and seven 2's $\rightarrow$ ${}^8C_3$
When four 3's and six 2's $\rightarrow$ ${}^7C_4$
When five 3's and five 2's $\rightarrow$ ${}^6C_5$
Total = 143

(3) **(B).** $10^5 = 2^5 \cdot 5^5$
Sum $= (2 + 2^2 + 2^3 + 2^4 + 2^5)(5^0 + 5^1 + 5^2 + 5^3 + 5^4 + 5^5)$

$$= 2\,\frac{(2^5-1)}{2-1}.\frac{(5^6-1)}{5-1} = \frac{2\cdot 31}{4}(5^3-1)(5^3+1)$$

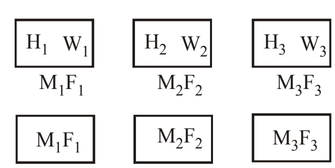
$$= \frac{31}{2}(5-1)(5^2+5+1)(5^3+1) = 62\,(5^2+5+1)(5^3+1)$$

$\Rightarrow k = 62$

(4) **(C).** We can arrange n white and n black balls alternately in the following ways
(i) W B W B . . . (ii) B W B W . . .
So required number of ways $= n! \times n! + n! \times n! = 2(n!)^2$

(5) **(B).** $m = 8!\, 2^3$
$n = 5!\, 2^6$

$$\frac{m}{n} = \frac{8\cdot 7\cdot 6\cdot 5!\cdot 2^3}{5!\cdot 2^6}$$



k = 42

(6) **(B).** Letters of the word PARKAR written in alphabatical order are A A K P R R
Number of words starting with A is = 60
Number of words starting with K is = 30
Number of words starting with PAA is = 3
Number of words starting with PAK is = 3
Number of words starting with PARA is = 20
Number of words starting with PARKAR is = 1

$\therefore$ Rank of word PARKAR is 99

(7) **(B).** ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$

$\Rightarrow {}^{n+1}C_3 - {}^{n+1}C_2 \le 100$

$\Rightarrow \frac{(n+1)n(n-1)}{6} - \frac{(n+1)n}{2} \le 100$

$\Rightarrow (n + 1)\, n\, (n - 1) - 3n\, (n + 1) \le 600$

$\Rightarrow (n + 1)\, (n)\, (n - 4) \le 600$

The value of n satisfying this inequality are 2, 3, 4, 5, 6, 7, 8 and 9 only. Hence required number is 8.

(8) **(A).** There are eleven letters in the word 'PATALIPUTRA' and there are two P's, two T's, three A's and four other different letters.

Number of consonants = 6, number of vowels = 5

Since relative order of the vowels and consonants remains unchanged, therefore, vowels will occupy only vowels's place and consonants will occupy only consonant's place.

Now 6 consonants can be arranged among themselves in

$\therefore \frac{6!}{2!2!}$ ways [since there are two P's and two T's]

and five vowels can be arranged among themselves in

$\frac{5!}{3!}$ ways, since A occurs thrice

$\therefore$ Required number $= \frac{6!}{2!2!} \cdot \frac{5!}{3!} = 3600$

(9) **(B).** We have : $30 = 2 \times 3 \times 5$. So, 2 can be assigned to either a or b or c i.e. 2 can be assigned in 3 ways.

Similarly, each of 3 and 5 can be assigned in 3 ways. Thus, the number of solutions is $3 \times 3 \times 3 = 27$.

(10) **(B).** Let the number of green balls be x. Then the number of red balls is 2x. Let the number of blue balls be y.

Then, $x + 2x + y = 10 \Rightarrow 3x + y = 10 \Rightarrow y = 10 - 3x$

Clearly, x can take values 0, 1, 2, 3. The corresponding values of y are 10, 7, 4 and 1. Thus, the possibilities are (0, 10, 0), (2, 7, 1), (4, 4, 2) and (6, 1, 3), where (r, b, g) denotes the number of red, blue and green balls.

(11) **(B).** When first two consecutive digits are 11, 22, etc

$= 9 \times 9 = 81$

When last two consecutive digits are $0, 0 = 9$

When last two consecutive digits are

$11, 22, 33, \ldots = 9 \,.\, 8 = 72 \Rightarrow$ Total $81 + 9 + 72 = 162$

(12) **(C).** E □□□□ $\frac{4!}{2}$ Q E □□□□ $\frac{3!}{2!}$

Q U E E U Q U E U E

Total $= 12 + 3 + 1 + 1 = 17$

(13) **(A)** The number of elements in

$S = \{(x, y) \mid (x-1)^2 + (y+2)^2 \le 25 \text{ and } x, y > \in z\}$ is same as the number of lattice points, the points having both its co-ordinates integers satisfying $x^2 + y^2 \le 25$.

Let $T = \{(x, y) \mid x^2 + y^2 \le 25, x, y \in z\}$.

Then we have |T| = (S) we must do some systematic counting.

In general if (x, y) is a solution of $x^2 + y^2 \le 25$, then so are (x, –y), (–x, y), (–x, –y), (y, x), (y, –x), (–y, x) and (–y, –x).

So any solution, in this way can generate a maximum of 8 solutions. Considering the change of sign and an interchange of x and y. Let's count our solutions.

With a, b > 0 (a < b)

any solution (a, b) indeed realizes all possible eight solution, viz, (a, –b), (–a, b), (–a, –b), (b, a), (b, –a), (–b, a), (–b, –a).

$\left.\begin{matrix}(1,2)\\(1,3)\\(1,4)\\(2,3)\\(2,4)\\(3,4)\end{matrix}\right\} \to 6 \times 8 = 48 \text{ solutions}$

With a, b > 0, a = b; any solution (a, a) gives rise to (a, a), (a, –a), (–a, a), (–a, –a) 4 solutions.

$\left.\begin{matrix}(1,1)\\(2,2)\\(3,3)\end{matrix}\right\} \to 3 \times 4 = 12 \text{ solutions}$

With b > 0 any solution (0, b) gives rise to (0, –b), (b, 0) and (–b, 0), i.e. in all 4 solutions.

$\left.\begin{matrix}(0,1)\\(0,2)\\(0,3)\\(0,4)\\(0,5)\end{matrix}\right\} \to 5 \times 4 = 20 \text{ solutions}$

Also (0, 0) doesn't produce any other solution

$(0, 0)\} \to 1$ solution

The number of solutions $= 48 + 12 + 20 + 1 = 81$

(14) **(A).** Group 6 persons can be divided into 3 equal groups

in $\frac{6!}{2!2!2!3!}$ ways

$P_1 \quad P_2 \quad P_3$

$P_4 \quad P_6 \quad P_6$

say P_1P_4 ; P_2P_5 ; P_3P_6

Now each elements of a group can be arranged in 3! ways.

Total ways $= \frac{6!3!}{2!2!2!3!} = \frac{720}{8} = 90$

(15) **(C).** Total number of ways of selecting any number of fruits $= 11 \times 6 \times 3 \times 2 \times 2 \times 2 = 1584$

Number of ways in which no fruit is selected = 1

Number of ways in which only one fruit is selected = 6

Number of ways in which two fruit are selected

$= {}^6C_2 + 3 = 18$

$\therefore$ Number of ways in which at least three fruits are selected $= 1584 - (1 + 6 + 18) = 1559$

(16) **(B).** Ten pearls of one colour can be arranged in $\frac{1}{2} \cdot (10 - 1)!$ ways. The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour = 10!

$\therefore$ the required number of ways = $\frac{1}{2} \times 9! \times 10! = 5\,(9!)^2$

(17) **(C).** 24 = 2.3.4, 2.2.6.4, 1.3.8, 1.2.12, 1.1.24 (as product of three positive integers)
$\therefore$ the total number of positive integral solution of

xyz = 24 is equal to $3! + \frac{3!}{2!} + 3! + 3! + \frac{3!}{2!}$, i.e., 30

Any two of the factors in each factorization may be negative.
$\therefore$ the total number of integral solution = 30 + 3 × 30 = 120

(18) **(C).** $\underbrace{B_1}_{10}\ \underbrace{B_2}_{10}\ \underbrace{B_3}_{10}\ ....\underbrace{B_9}_{10}$. Selection of atleast one book

$\underbrace{(10+1)\,(10+1).......(10+1)}_{9\text{ times}} - 1 = 11^9 - 1$

(19) **(C).** A = {2, 3, 5, 7, 11, 13, 19, 23, 29}. A rational number is made by taking any two in any order.
So, the required number of rational numbers
$= {}^{10}P_2 + 1$(including 1) = 91

(20) **(C).** For a particular class total number of different tickets from first intermediate station = 5
Similarly number of different tickets from second intermediate station = 4
So total number of different tickets = 5 + 4 + 3 + 2 + 1 = 15
And same number of tickets for another class
$\Rightarrow$ total number of different tickets = 30 and number of selection = ${}^{30}C_{10}$

(21) **(C).** For a radical centre 3 circles are required
$\Rightarrow$total number of radical centres = ${}^nC_3$
$\Rightarrow$ total number of radical axis = ${}^nC_2$
${}^nC_2 = {}^nC_3 \Rightarrow n = 5$

(22) **(B).** If number of persons be n, then total number of hand-shaken = ${}^nC_2 = 66$
$\Rightarrow n\,(n-1) = 132 \Rightarrow (n+11)\,(n=12) = 0$
$\therefore$ n =12 ($\therefore n \neq -11$)

(23) **(A).** If zero is included it will be at z $\Rightarrow {}^9C_2$ no's

If zero is excluded

$$\begin{cases} x,\ y,\ z \ \text{ all diff.} & \Rightarrow\ {}^9C_3 \times 2! \\ x = z < y & \Rightarrow\ {}^9C_2 \\ x < y = z_1 & \Rightarrow\ {}^9C_2 \ \text{No's} \end{cases}$$

Total number of ways = 276

(24) **(C).**
Case - I : $x_1 < x_2 < x_3 < x_4 < x_5 < x_6 \ \Rightarrow {}^9C_6$ ways
Case - II : $x_1 < x_2 = x_3 < x_4 < x_5 < x_6 \ \Rightarrow {}^9C_5$ ways
Case - III : $x_1 < x_2 < x_3 < x_4 < x_5 = x_6 \ \Rightarrow {}^9C_5$ ways
Case - IV : $x_1 < x_2 = x_3 < x_4 < x_5 = x_6 \ \Rightarrow {}^9C_4$ ways
Thus total number of such numbers is equal to
${}^9C_8 + {}^9C_5 + {}^9C_5 + {}^9C_4 = {}^{10}C_6 + {}^{10}C_5 = {}^{11}C_6$

(25) **(B).**

2	•	•	•	1	2

6 ways

1		•	•		2

(2 cannot be ten's place) 3 × 3 = 9 ways
Total number of ways = 15

(26) **(D).** $\frac{6!}{1!\cdot 5!}\cdot 2! + \frac{6!}{2!\cdot 4!}\cdot 2! + \frac{6!}{3!\cdot 3!\cdot 2!}\cdot 2!$

(concept of grouping)

G_1	G_2
1	5
2	4
3	3

12 + 30 + 20 = 62

(27) **(D).** when A has B or C to his right we have AB or AC
when B has C or D to his right we have BC or BD
Thus we must have ABC or ABD or AC and BD for ABC D, E, F on a circle number of ways = 3 ! = 6
for ABD C, E, F on a circle number of ways = 3 ! = 6
for AC, BD, E, F the number of ways = 3 ! = 6
$\Rightarrow$ Total = 18

(28) **(B).** There are ten digits 0, 1, 2, ..., 9. Permutations of these digits taken eight at a time = ${}^{10}P_8$ which include permutations having 0 at the first.
When 0 is fixed at the first place, then number of such

permutations $= {}^9P_7 - {}^9P_7 = \frac{10!}{2} - \frac{9!}{2} = \frac{9!9}{2}$

(29) **(C).** The number of words beginning with A (i.e., in which A comes in first place) is ${}^5P_5 = 5!$. Similarly number of words beginning with is 5!, beginning with H is 5! and beginning with I is also 5!.
Now letters of RACHIT in alphabetic order are as ACHIRT
So before , four letters A, C, H, I can occur in 4(5!) = 480 ways. Now word RACHIT happens to be the first word beginning with R.
Therefore the rank of this word = 480 + 1 = 481.

(30) **(C).** Consider cases when z = 0, 1, 2, ... , 11
$\Rightarrow$ x + y = 33, 33, 27,...
Total number of solution of x + y = 33, 30, 27,...

$= 34 + 31 + 28 + ... + 1$ (12 times) $= \frac{12}{2}(1 + 34) = 210$

(31) **(D).** The number of triangles with vertices on sides AB, BC, CD = ${}^3C_1 \times {}^4C_1 \times {}^5C_1$
Similarly for other cases
$\therefore$ the total number of triangles
$= {}^3C_1 \times {}^4C_1 \times {}^5C_1 + {}^3C_1 \times {}^4C_1 \times {}^6C_1 + {}^3C_1 \times {}^5C_1 \times {}^6C_1$
= 342

(32) **(B).** m = 5 . 5. 8. 7 = 1400

5	5	8	7

n = 1400 – (5 . 8. 7) = 1400 – 280 = 1120

0			

$\Rightarrow k = \frac{1400}{1120} = \frac{5}{4}$

(33) **(C)** Men can be in a majority, if the distribution is as follows.

M	6	7	8
W	4	3	2

and the number of ways of choosing them
$= {}^8C_6 \times {}^7C_4 + {}^8C_7 \times {}^7C_3 + {}^8C_8 \times {}^7C_2 = 1281$
Total number of ways of forming the committee
$= {}^{15}C_{10} = 3003$

Hence number of committees $= \frac{1281}{3003}$ of the total $= \frac{61}{134}$

of the total

(34) **(A).** The two letters, the first and the last of the four lettered word can be chosen in $(17)^2$ ways, as repetition is allowed for consonants. The two vowels in the middle are distinct so that the number of ways of filling up the two places is ${}^5P_2 = 20$. The number of different words $= (17)^2 . 20 = 5780$.

(35) **(A).** Number of digits are 9
Select 2 places for the digit 1 and 2 in ${}^9C_2$ ways
from the remaining 7 places select any two places for 3 and 4 in ${}^7C_2$ ways
and from the remaining 5 places select any two for 5 and 6 in ${}^5C_2$ ways
Now, the remaining 3 digits can be filled in 3! ways

$\therefore$ Total ways $= {}^9C_2 . {}^7C_2 . {}^5C_2 . 3! = \frac{9!}{2!.7!} . \frac{7!}{2!.5!} . \frac{5!}{2!.3!} . 3!$

$= \frac{9!}{8!} = \frac{9.8.7!}{8} = 9.7!$

(36) **(A).** Number of words having all the letters distinct $= {}^4P_1 = 4$
Number of words having two are alike and third different

$= {}^1C_1 . {}^3C_1 . \frac{3!}{2!} = 9$

(37) **(D).** Number of solution $= {}^{12}C_{10} = 66$.

(38) **(D).** Statement – II is true, known as the rule of product. Statement – I is not true, as the two parts of the work are not independent . Three girls can be chosen out of six girls in ${}^6C_3$ ways, but after this choosing 3 students out of remaining nine students depends on the first part.

(39) **(B).** Since, $1400 = 2^3.5^2.7^1$
$\Rightarrow$ Total no. of factors = (3 + 1) (2 + 1) (1 + 1) = 24
$\Rightarrow$ No. of ways of expressing 1400 as a product of two

numbers $= \frac{1}{2} \times 24 = 12$.

But this does not follow from statement – II which is obviously true.

(40) **(A).** Let no of sides are n.
${}^nC_2 - n = 44 \Rightarrow n = -8$ or $11 \Rightarrow n = 11$.

(41) **(C).** $x_1x_2x_3x_4 = 1050 = 2 \times 3 \times 5^2 \times 7$
Thus 5^2 can as sign in ${}^5C_1 + {}^5C_2 = 15$ ways
We can assign 2, 3, or 7 to any. of 5 variables.
Hence req. number of solutions $= 5 \times 5 \times 5 \times 15 = 1875$

(42) **(A).** $({}^{400}C_4 + {}^{400}C_3) + {}^{401}C_3 + ... + {}^{500}C_3$
$= ({}^{401}C_4 + {}^{401}C_3) + {}^{402}C_3 + ... + {}^{500}C_3$ ....
$= ({}^{500}C_4 + {}^{500}C_3) = {}^{501}C_4$

(43) **(A).**
One child can be rejecte in ${}^nC_1$ ways remaining children $= (n - 1)$

Number of toys = n
divide n toys in (n – 1) groups

$\underbrace{T_1 \quad T_2}_{2} \quad \underbrace{\overset{T_3}{1} \quad \overset{T_4}{1} .... \overset{T_n}{1}}_{(n-2)\text{ toys}} = \frac{n!}{2!(n-2)!}$

can be distributed in $= \frac{n!(n-1)!}{2!(n-2)!}$

Total ways

$= \frac{{}^nC_1 n!(n-1)(n-2)!}{2!(n-2)!} = (n!).{}^nC_2 = 1200 \Rightarrow n = 5$

(B) 2n red counters; 2n white counters arrange is red and

n white counters in $\frac{2n!}{n!n!}$ ways

remaining nR and nW will be the mirror image of each arrangement and
$\therefore$ can be arranged only in one way
$\therefore {}^{2n}C_n = 70 \therefore n = 4$
(C) $360 = 2^3 . 3^2 . 5$
$4n + 2 \Rightarrow$ even that not divisible by 4 hence exactly one 2 must be taken number of divisor (1)(3)(2) = 6

(44) **(B).**
If $d_1 < d_2 < d_3 < d_4 < d_5 \Rightarrow$ number of numbers ${}^9C_5$ (0 can not be included)
if $d_1 < d_2 = d_3 < d_4 < d_5 \Rightarrow$ number of numbers ${}^9C_4$
if $d_1 < d_2 < d_3 < d_4 = d_5 \Rightarrow$ number of numbers ${}^9C_4$
if $d_1 < d_2 = d_3 < d_4 = d_5 \Rightarrow$ number of numbers ${}^9C_3$
Total $= {}^9C_5 + {}^9C_4 + {}^9C_4 + {}^9C_3$
$= {}^{10}C_5 + {}^{10}C_4 = {}^{11}C_5 = {}^{11}C_6$
(b) $d_1 > d_2 \geq d_3 > d_4 > d_5 = {}^{10}C_5 + {}^{10}C_4 = {}^{11}C_5 = {}^{11}C_6$
(c) Total number of ways in which the tournament can be won by either players $= {}^{10}C_5$
(d) (Think !) ${}^{10}C_5 = 2 . {}^9C_4 = 2 . {}^9C_5$

(45) **(D).**

(46) **(A).**
(a) Required number of ways
$= (2 + 1) (3 + 1) (4 + 1) – 1 = 59$
(b) The number of ways of selecting 3 points out of 12 points is ${}^{12}C_3$. Three points out of 7 collinear points can be selected in ${}^7C_3$ ways. Hence,
the number of triangles formed is ${}^{12}C_3 - {}^7C_3 = 185$

(c) Required number of ways = Coefficient of x^{10} in $(1+x+x^2+......)^4$
$= $ Coefficient of x^{10} in $(1-x)^{-4} = {}^{10+4-1}C_{4-1} = {}^{13}C_3$
$=286$

(d) Factorizing the given number, we have
$38808 = 2^3 . 3^2 . 7^2 . 11$
The total number of divisors of this number is same as the number of ways of selecting some or all of two 2's, two 3's, two 7's and one 11. Therefore, the total number of divisors $= (3+1)(2+1)(2+1)(1+1) = 72$.
Hence, the required number of divisors $= 72 - 2 = 70$

(47) **(C).** ${}^{r+2}C_2, 1 \le r \le n$

(48) **(A)**. ${}^{r+2}C_2 - 3.{}^{r-n+1}C_2, \quad n+1 \le r \le 2n+1$

(49) **(C)**. ${}^{r+2}C_2 - 3.{}^{r-n+1}C_2 + 3.{}^{r-2n}C_2, \quad 2n+2 \le r \le 3n$

(50) **(C).** When n is even. N is maximum for r = 3n/2

So $N_{max} = \frac{1}{4}[3(n+1)^2 + 1]$

(51) (D), (52) (B), (53) (D).

(i) Only choice (D) doesn't violate any of the conditions.
(ii) Since Reena cannot select another biography, we eliminate choice (D), also since if B is elected, G must be too, eliminate choice (A). Therefore, the other selections must be three of the four novels A, C, D, and E. If she did not select C, she could not select E either ($C \leftrightarrow E$), and then she would have only two novels. So C must be selected.
$\therefore$ (B) holds.
If E is not selected, then neither is C ($C \leftrightarrow E$), so the three novels selected are A, B and D. Since $A \to F$ and $B \to G$, F and G must be the selected biographies, H can't be selected.
$\therefore$ (D) holds.

(54) **(B), (55) (A), (56) (D).**

(i) Vowels O O I I E
Consonants M M S S R C N

MM = 2	E = 1
S S = 2	R = 1
OO = 2	C = 1
I I = 2	N = 1
——	——
8	4 12 letters

2 Vowels
(a) both alike; selected in 2 ways and can be arranged in 2 ways
(b) both different; selected in ${}^3C_2 = 3$ and can be arranged in 6 ways
Total = 8 ways
3 Consonants

(a) 2 alike + 1 different ${}^2C_1 . {}^4C_1 = 8 \to 8 \times \frac{3!}{2!} = 24$ ways

(b) all 3 different ${}^5C_3 = 10 \to 10 . 3! = 60$ ways
Total = 84 ways

Hence number of words ${}^5C_3 . 8 . 84 = 6720$

(ii) | M M | | I I | | O O | | I I |

Other letter | E | R | C | N | can be arranged in 4! ways and 4 gaps out of 5 can be selected in ${}^5C_4$ ways and can be arranged in 4! ways
$\therefore$ Total $4! . {}^5C_4 . 4! = 576 \times 5 = 2880$

(iii) Let all alike letter be denoted by V

Hence number of ways $= \frac{12!}{8!} = {}^{12}P_4$

(57) (C), (58) (A), (59) (B).

$3.{}^nP_4 = {}^{n-1}P_5 \Rightarrow 3.\frac{n!}{(n-4)!} = \frac{(n-1)!}{(n-6)!}$

$\Rightarrow 3n = (n-4)(n-5) \Rightarrow 3n = n^2 - 9n + 20$
$\Rightarrow n^2 - 12n + 20 = 0$
$\Rightarrow (n-10)(n-2) = 0 \Rightarrow n = 10$ as $n \ne 2$

(i) From 10 stations train to be stopped at 3
$\therefore$ 7 remains. In between these 7 stations there are 8 gaps
$\therefore$ out of 8 gaps select any 3 $\Rightarrow {}^8C_3 = 56$.

(ii) Any 3 consecutive vertices can be selected in 10 ways
(1, 2, 3) or (2, 3, 4) or .... or (10, 1, 2)
say e.g. (1, 2, 3)
now 4 and 10 cannot be selected
from the remaining five vertices from 5 to 9 any one vertex can be selected in ${}^5C_1$ ways.
$\therefore$ number of such quadraliterals $= 10 . {}^5C_1 = 50$

(iii) Any 2 consecutive vertices can be selected in 10 ways
(1, 2) or (2, 3) or .... or (10, 1)
say e.g. (1, 2)
now 3 and 10 cannot be selected and from the remaining 6 vertices any two non-consecutive vertices can be slected in ${}^5C_2$ ways.
$\therefore$ number of such quadraliterals $= 10 . {}^5C_2 = 100$

(60) **(C).** Starting with 1 | 1 | | | | | |

$\frac{5!}{3!2!} = 10$

Starting with 2 | 2 | | | | | |

$\frac{5!}{3!} = 20$

Hence 30th number is 233321
$\therefore$ 29th number is 233312

(61) **(B).** M = Total – When all 2's together

$= \frac{6!}{2!3!} - \frac{5!}{3!} = 60 - 20 = 40$

N = When no 3's are together

$= \frac{3!}{2!} \times {}^4C_3 = 12 \quad |1|2|2|$

$M - N = 40 - 12 = 28$

(62) **(C).** Consider $A = \{n-r+1, n-r+2, ..., n\} \subset S$.
Clearly $n(A) = r$ and $\min(A) = n-r+1$
For all other cases $\min(A) < n-r+1$
$\therefore$ Greatest value of $\min(A) = n-r+1$.

(63) **(C).** k is the least member of A and rest of the r – 1 members of A are among the numbers.
$k+1, k+2, k+3, ... n$
so number of ways $= {}^{n-k}C_{r-1}$.

(64) **(B).** $\sum_{\substack{A \subseteq S \\ |A|=r}} (\min(A)) = \sum_{k=1}^{n-r+1} (k.{}^{n-k}C_{r-1})$

$k\,{}^{n-k}C_{r-1} = [(n+1)-(n-k+1)]\,.\,{}^{n-k}C_{r-1}$
$= (n+1)\,{}^{n-k}C_{r-1} - (n-k+1)\,.\,{}^{n-k}C_{r-1}$

$(n-k+1)\,.\,{}^{n-k}C_{r-1} = \dfrac{(n-k+1).(n-k)!}{(r-1)![n-k-r+1)!}$

$= \dfrac{r.(n-k+1)!}{r!(n-k+1-r)!} = r.\,{}^{n-k+1}C_r$

$k\,.\,{}^{n-k}C_{r-1} = (n+1)\,.\,{}^{n-k}C_{r-1} - r\,.\,{}^{n-k+1}C_r$

(65) (B), (66) (A), (67) (A).
Since thre are 5 even places and 3 pairs of repeated letters therefore at least one of these must be at an odd place.

$\therefore$ the number of ways $= \dfrac{11!}{2!\,2!\,2!}$

Make a bundle of both M's and another bundle of T's. Then except A's we have 5 letters remaining so M's, T's and the letters except A's can be arranged in 7! ways.
$\therefore$ Total number of arrangements $= 7! \times {}^8C_2$

Consonants can be placed in $\dfrac{7!}{2!\,2!}$ ways

Then there are 8 places and 4 vowels

$\therefore$ Number of ways $= \dfrac{7!}{2!\,2!}.\,{}^8C_4\,\dfrac{4!}{2!}$

EXERCISE-3

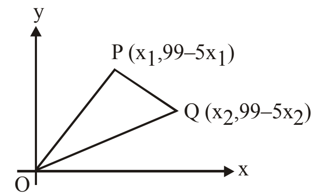
(1) **90.**

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x_1 & 99-x_1 & 1 \\ x_2 & 99-x_2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[x_1(99-x_2) - x_2(99-x_1)]$$

$$\text{Area} = \frac{1}{2}|[(x_1-x_2)\,99]|$$

Area is an integer then both x_1 and x_2 are simultaneously either even or both odd.



Now x_1 varies from 0 to 19 < 10 even, 10 odd

hence ${}^{10}C_2 + {}^{10}C_2 = 2 \cdot {}^{10}C_2 = 90$

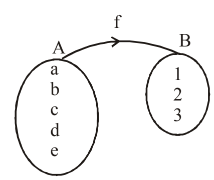
(2) **630.** $10\begin{cases} 3m \\ 7w \end{cases}$; 3 women can be selected in ${}^7C_3$ ways and can be paired with 3 men in 3! ways.
Remaining 4 women can be grouped into two couples in

$\dfrac{4!}{2!\,2!\,2!} = 3$ $\therefore$ Total $= {}^7C_3\,.\,3!\,.\,3 = 630$

(3) **10.** A A A A A | B B B B B B
M
Middle digit must be A (think !)
× × × × × ↓ × × × × ×
so that even number of A's and B's are available
Take AABBB on one side of M (6th place) and then their image about M in a unique way

$\therefore$ Number of ways $= \dfrac{5!}{2!.3!} = 10$

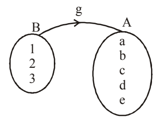
(4) **309.** m = into mappings from A to B
Case-I: If exactly one element of set B is not the image of any of the elements of set A then total number of into functions are ${}^3C_1 \times (2^6 - 2) = 3 \times 62 = 186$



Case-II: If exactly two elements of set B is not the image of any of the elements of set A then total number of into functions are ${}^3C_2 \times 1 = 3$ and hence $m = 186 + 3 = 189$
Alternatively: Total mappings $= 3^6 = 729$
Surjective mappings

2	2	2	$\rightarrow \dfrac{6! \times 3!}{(2!)^3 3!} = \dfrac{720}{8} = 90$
2	3	1	$\rightarrow \dfrac{6! \times 3!}{2!\,3!} = \dfrac{720}{2} = 360$
1	1	4	$\rightarrow \dfrac{6! \times 3!}{2!\,4!} = \dfrac{720}{2 \times 4} = 90$

Total surjective mapping = 540



$\therefore$ not surjective mappings = 729 – 540 = 189 = m

n = number of injective mapping from B to A.

Injective mapping = ${}^6C_3 \times 3! = 20 \times 6 = 120 = n$

$\therefore$ (m + n) = 189 + 120 = 309

(5) **432. Case-I :** When the two identical digits are both unity as shown.

| 1 | x | y | 1 | any one place out of 3 block for unity can be taken in 3 ways and the remaining two blocks can be filled in 9 · 8 ways. Total ways in this case = 3 · 9 · 8 = 216

Case-II : When the two identical digit are other than unity.

| 1 | x | x | y | ; | 1 | x | y | x | ; | 1 | y | x | x |

two x's can be taken in 9 ways and filled in three ways and y can be taken in 8 ways.

Total ways in this case = 9 · 3 · 8 = 216

Total of both case = 432

(6) **29.** There are ${}^4C_2 = 6$ ways a block can differ from the given block in exactly two ways

(1) material and size, (2) material and colour, (3) material and shape, (4) size and colour, (5) size and shape, and (6) colour and shape. Since there is only 1 choice for different material, 2 choices for different size, 3 choices for a different colour, and 3 choices for a different shape, it follows that the number of blocks in each of the above categories is

(1×2), (1×3), (1×3), (2×3), (2×3) and (3×3), respectively.

The answer is the sum of these six numbers = 29

(7) **35.** 2 alike + 2 other alike + 2 other alike = 1

2 alike + 2 other alike + 2 different = ${}^3C_2 \cdot {}^4C_2 = 18$

$$\begin{cases} M's = 2; & T's = 2; & E's = 2 \\ C's = 1; & O's = 1; & I's = 1 \end{cases}$$

2 alike + 4 different = ${}^3C_1 \cdot {}^5C_1$ = 15

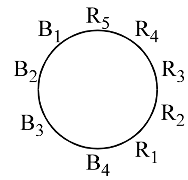
All 6 different = 1

———

= 35

(8) **450.** 9 < 5 R, 4 B

Sr.No.		n(A)
1	Triangle	${}^9C_3 - {}^5C_3$
2	Quadrilateral	${}^9C_4 - {}^5C_4$
3	Pentagon	${}^9C_5 - {}^5C_5$
4	Hexagon	${}^9C_6$
5	Heptagon	${}^9C_7$
6	Octagon	${}^9C_8$
7	Nonagon	${}^9C_9$



When 6 or more vertices are taken then at least one blue has to be taken.

n (A) = (84 – 10) + (126 – 5) + (126 – 1) + 84 + 36 + 3 + 1

= 74 + 121 + 125 + 130 = 450

(9) **22.** Exterior angle = $2\pi/n$

$\therefore$ Interior angle = $\pi - \dfrac{2\pi}{n} = 180° - \dfrac{360°}{n}$

where n is the number of sides

now $\dfrac{360°}{n}$ must be an integer < 180°

hence n ≠ 1, 2 (think !). We have to find the number of divisors of 360 other than 1 and 2.

now $360 = 2^3 \cdot 3^2 \cdot 5^1$

number of divisors = 4 · 3 · 2 = 24

$\therefore$ required number of divisors = 24 – 2 = 22

(10) **44.** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

where A = A's together and B = B's together

now $n(A) = \dfrac{5!}{3!} = 20$ [A A A A] B B B C

$n(B) = \dfrac{6!}{4!} = 30$ [B B B] A A A A C

$n(A \cap B) = 3! = 6$ [A A A A] [B B B] C

$n(A \cup B) = 50 - 6 = 44$

(11) **47.** Bag < 3W, 4B, 5R —2 balls are drawn→

${}^{12}C_2 - [{}^3C_2 + {}^4C_2 + {}^5C_2]$

66 – [3 + 6 + 10]

66 – 19 = 47

(12) **210.** $\dfrac{7!}{3! \cdot 2! \cdot 2!} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{24} = 210$

(13) **4.** Since, $240 = 2^4 . 3 . 5$

$\therefore$ Total number of divisors = (4 + 1) (2) (2) = 20

Out of these 2, 6, 10, and 30 are of the form 4n + 2.

(14) **60.** X – X – X – X – X

The four digits 3, 3, 5, 5 can be arranged at (–) places in $\dfrac{4!}{2!2!} = 6$ ways. The five digits 2, 2, 8, 8, 8 can be arranged at (X) place in $\dfrac{5!}{2!3!} = 10$ ways

Total number of arrangements is 6 × 10 = 60

(15) **7.** A regular polygon of n sides has n vertices, no two . . . . $^nC_3$ triangles can be formed. $\therefore\ T_n = {}^nC_3$; $T_{n+1} = {}^{n+1}C_3$
Given, $T_{n+1} - T_n = 21$; ${}^{n+1}C_3 - {}^nC_3 = 21$

$$\frac{(n+1)\,n\,(n-1)}{3\times2\times1} - \frac{n\,(n-1)\,(n-2)}{3\times2\times1} = 21$$

$n\,(n-1)\,(n+1-n+2) = 126 \Rightarrow n\,(n-1) = 42$
$\Rightarrow\ n\,(n-1) = 7\times6 \Rightarrow n = 7$

(16) **40.** Total number of ways of arranging the letters of the word BANANA is $\frac{6!}{2!3!} = 60$

Number of words in which 2N's come together is $\frac{5!}{3!} = 20$

Hence, the required number is $60 - 20 = 40$.

(17) **7.** Given that runs scored in kth match
$= k\,.\,2^{n+1-k},\ 1 \le k \le n$

and runs scored in n matches $= \frac{n+1}{4}(2^{n+1} - n - 2)$

$$\therefore\ \sum_{k=1}^{n} k.2^{n+1-k} = \frac{n+1}{4}(2^{n+1} - n - 2)$$

$$\Rightarrow\ 2^{n+1}\left[\sum_{k=1}^{n}\frac{k}{2^k}\right] = \frac{n+1}{4}(2^{n+1} - n - 2)$$

$$\Rightarrow\ 2^{n+1}\left[\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots.. + \frac{n}{2^n}\right]$$

$$= \frac{n+1}{4}(2^{n+1} - n - 2) \qquad \ldots\ldots..(1)$$

Let $S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots. + \frac{n}{2^n}$

$$\frac{1}{2}S = \frac{1}{2^2} + \frac{2}{2^3} + \ldots. + \frac{n-1}{2^n} + \frac{n}{2^{n+1}}$$

Subtracting the above two, we get

$$\frac{1}{2}S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots. + \frac{1}{2^n} - \frac{n}{2^{n+1}}$$

$$\Rightarrow\ \frac{1}{2}S = \frac{1}{2}\left(\frac{1 - \frac{1}{2^n}}{1 - \frac{1}{2}}\right) - \frac{n}{2^{n+1}} \Rightarrow S = 2\left[1 - \frac{1}{2^n} - \frac{n}{2^{n+1}}\right]$$

$\therefore$ Eqation (1) becomes

$$2\cdot2^{n+1} = \left[1 - \frac{1}{2^n} - \frac{n}{2^{n+1}}\right] = \frac{n+1}{4}[2^{n+1} - n - 2]$$

$$\Rightarrow\ 2\cdot[2^{n+1} - 2 - n] = \frac{n+1}{4}[2^{n+1} - n - 2]$$

$$\Rightarrow\ \frac{n+1}{4} = 2 \Rightarrow n = 7$$

(18) **225.** If L.C.M. of p and q is $r^2t^4s^2$, then distribution of factors r is as follows :

p	q
r^0	r^2
r^1	r^2
r^2	r^2
r^2	r^0
r^2	r^1

Thus, factor r can be distributed in $2\times3-1$ ways. Similarly, factors t and s can be distributed in $2\times5-1$ and $2\times3-1$ ways, respectively.
Hence, number of ordered pairs are
$(2\times3-1)\times(2\times5-1)\times(2\times3-1) = 225.$

(19) **96.** The letters of COCHIN in alphabetic order are C, C, H, I, N, O. Fixing first letter C and keeping C at second place, rest 4 can be arranged in 4! ways.
Similarly, the total number of words starting with CH, CI, CN is 4! in each case.
Then fixing first two letters as CO, next four places when filled in alphabetic order gives the work COCHIN.
Therefore, number of words coming before COCHIN is $4\times4! = 4\times24 = 96$

(20) **5.** Let (1, 1, 1), (–1, 1, 1), (1, –1, 1), (–1, –1, 1) be vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ rest of the vectors are $-\vec{a}, -\vec{b}, -\vec{c}, -\vec{d}$ and let us find the number of ways of selecting co-planar vectors.
Observe that out of any 3 coplanar vectors two will be collinear (anti parallel)
Number of ways of selecting the anti parallel pair = 4
Number of ways of selecting the third vector = 6
Total = 24
Number of non co-planar selections
$= {}^8C_3 - 24 = 32 = 2^5, p = 5$

Alternate Solution: Required value $= \frac{8\times6\times4}{3!}$ $\therefore\ p = 5$

(21) **7.** $n_1 + n_2 + n_3 + n_4 + n_5 = 20$
Maximum of $n_5 = 10,\ n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 4$
$n_5 = 09,\ n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 5$
$n_5 = 08,\ n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 6$
or $n_3 = 4, n_4 = 5$
$n_5 = 07,\ n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6$
or $n_2 = 3,\ n_3 = 4, n_4 = 5$
$n_5 = 06,\ n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 5$
Total number of ways = 7.

(22) **5.** Number of line joining adjacent points = n
$n = {}^nC_2 - n$
$2n = {}^nC_2$

$2n = \frac{n(n-1)}{2}$

$n = 0$ or $n = 5$

(23) 53. $\frac{6!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right)}{5} = \frac{265}{5} = 53$

EXERCISE-4

(1) **(C).** No. of number greater than 1000 and less than 4000, formed with digits 0, 1, 2, 3, 4 and repetition is allowed.
$\therefore$ No. will be of four digit
No. start with 1 : $(5 \times 5 \times 5) = 125$
No. start with 2 : $(5 \times 5 \times 5) = 125$
No. start with 3 : $(5 \times 5 \times 5) = 125$
Total no. are $125 + 125 + 125 = 375$

(2) **(C).** Even natural numbers having 3 digits are ....
Unit place can be filled by 0, 2, 4, 6, 8 = 5 ways
10^{th} place can be filled by 10 ways
100^{th} place can be filled by 9 ways
$\therefore$ total no. of numbers $= 9 \times 10 \times 5 = 450$

(3) **(C).** ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r = {}^nC_{r+1} + {}^nC_{r-1} + {}^nC_r + {}^nC_r$
$= ({}^nC_r + {}^nC_{r+1}) + {}^nC_{r-1} + {}^nC_r$
$= {}^{n+1}C_{r+1} + {}^{n+1}C_r$ $\quad \{\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\}$
$= {}^{n+2}C_{r+1}$

(4) **(C).** A student has to answer 10 out of 13 question
He must choose least 4 from the first five question.
$\therefore$ No. of way he can choose the question.
(i) 4 from first five and 6 from next 8

$${}^5C_4 \times {}^8C_6 = 5 \times {}^8C_2 = 5 \times \frac{8}{2} \times \frac{7}{1} = 140$$

(ii) 5 from first five and 5 from next 8

$${}^5C_5 \times {}^8C_5 = 1 \times {}^8C_3 = 1 \times \frac{8}{3} \times \frac{7}{2} \times \frac{6}{7} = 56$$

$\therefore$ Total way $= 140 + 56 = 196$

(5) **(B).** There are 6 men and 5 women.
First we place 6 men around a round table that can be done in 5! ways.
Now amongs the 6 men there are 6 places between them.
Now at these six place 5 women can be sit in ${}^6P_5$ way
$\therefore$ Total no. of ways $5! \times {}^6P_5 = 5! \times 6!$

(6) **(C).** Total no. of arrangement of letter of word GARDEN so that vowels in alphabetical order.
There are two vowel A and E
According to question A must be before E

$\therefore$ Total no. of ways $= \frac{6!}{2!} = 360$

$\because$ Six letter of words GARDEN can be arranged in 6! way
But in 50% cases A will be ahead of E and in 50% cases E will be ahead of A

$\therefore$ Required arrangement $= \frac{6!}{2!} = 360$

(7) **(B).** We have to distribute 8 identical balls in 3 distinct boxes so that none of the boxes is empty.

$\therefore$ Total no. of ways $= {}^{8-1}C_{3-1} = {}^7C_2 = \frac{7}{2} \times \frac{6}{1} = 21$ way

Reason : No. of way in which n identical things can be distributed amongs r person so that each get at least 1 $= {}^{n-1}C_{r-1}$

(8) **(A).** In SACHIN order of alphabets is A, C, H, I, N, S
$\therefore$ Number of words starting with A = 5!
Number of words starting with C = 5!
Number of words starting with H = 5!
Number of words starting with I = 5!
Number of words starting with N = 5!
Now words start with 5 and after that ACHIN are in ascending order of position.
$\therefore$ $5 \times 5! = 600$ words are in dictionary before words with 5 start and position of this word is 601.

(9) **(D).** ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$
$= {}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3$
$= {}^{50}C_4 + {}^{50}C_3 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$ $\quad \{\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\}$

(10) **(B).** A person can vote at most 4 candidate
$\therefore$ Required no. of way $= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$
$= 10 + 45 + 120 + 210 = 385$

(11) **(A).** Using the formula of dividing the number of items into group of equal size where order is not imp, $(mn)!/[(n!)m(m!)]$, where $n = 4$ and $m = 3$.

(12) **(C).** We have to arranged letter of word MISSISSIPPI so that no two are adjacent except S there are total no. of letter = {M, I, I, I, I, P, P} = 7 letter

These can be arranged in $\frac{7!}{4!2!}$ ways

Now in each arrrangement of 7 letters there are 8 place where we can put 4S to satisfy required condition. This can be done in ${}^8C_4$ ways.

$\therefore$ Total no. of way $\frac{7!}{4!2!} \times {}^8C_4 = \frac{7!6!}{4!2!} \times {}^8C_4$
$= 7.{}^6C_4.{}^8C_4$

(13) **(D).** In a shop there are five types of ice-cream available. A child buys six ice cream
this can be done in ${}^{n+r-1}C_r$ way

$= {}^{5+6-1}C_6 = {}^{10}C_6 = {}^{10}C_4$ way $= \frac{10!}{4!6!}$ ways

$\therefore$ Statement 1 is wrong

and according to statement (2), 6A's and 4B's can arrange in a row $= \frac{10!}{4!6!}$ way.

$\therefore$ Statement (2) is correct.

Reason : Total no. of combination of r things taking 0, 1, 2, ... r at a time out of n differnet things when each thing is available in any no. of time $= {}^{n+r-1}C_r$.

(14) **(D).** Required number of arrangement
$= {}^6C_4 \times {}^3C_1 \times {}^4C_2\,(2!)^2 = 15 \times 3 \times 6 \times 4 = 1080$

(15) **(C).** Total number of ways $= {}^3C_2 \times {}^9C_2$
$= 3 \times \frac{9 \times 8}{2} = 3 \times 36 = 108$

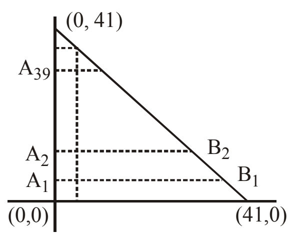
(16) **(B).** Statement - 1 : $B_1 + B_2 + B_3 + B_4 = 10$
= coefficient of x^{10} in $(x^1 + x^2 + ...+ x^7)^4$
= coefficient of x^6 in $(1 - x^7)^4\,(1 - x)^{-4}$
$= {}^{4+6-1}C_6 = {}^9C_3$
Statement - 2 : Obviously ${}^9C_3$

(17) **(D).** $(10 + 1)(9 + 1)(7 + 1) - 1 = 11.10.8 - 1 = 879$

(18) **(C).** $n(A) = 2, n(B) = 4 ; n(A \times B) = 8$
${}^8C_3 + {}^8C_4 + \ldots\ldots + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$
$= 256 - 1 - 8 - 28 = 219$

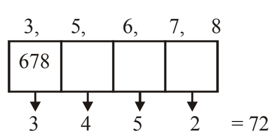
(19) **(B).** $T_n = {}^nC_3 ; T_{n+1} = {}^{n+1}C_3$
$T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3$
$\Rightarrow {}^nC_2 = 10 \Rightarrow n = 5.$

(20) **(C).**

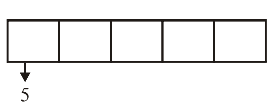


Total number of integral coordinates as required
$= 39 + 38 + 37 + ... + 1 = \frac{39 \times 40}{2} = 780$

(21) **(A).** 4 digit numbers



5 digit numbers



$5 \times 4 \times 3 \times 2 \times 1 = 120$
Total numbers of integers = 72 + 120 = 192

(22) **(D).** $n(A) = 4, n(B) = 2 ;\ n(A \times B) = 8$
Required numbers $= {}^8C_3 + {}^8C_4 + ... + {}^8C_8$
$= 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2) = 256 - 37 = 219$

(23) **(C).** $A : \frac{4!}{2!} = 12,\ L : 4! = 24,\ M : \frac{4!}{2!} = 12,\ SA = \frac{3!}{2!} = 3$
SL : 3! = 6 ; Total 57
Next word is SMALL.

(24) **(C).** Let M_m denotes male relative of man X = 3
M_w denotes female relative of man X = 4
W_m denotes male relative of woman Y = 4
W_w denotes female relative of woman X = 3

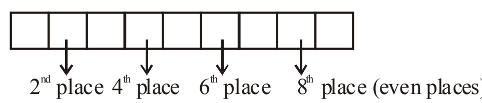
Case I :	$3M_m + 3W_w$	1
Case II :	$3M_w + 3W_m$	${}^4C_3 \times {}^4C_3 = 16$
Case III :	$2M_m + 1M_w + 2W_w + 1W_m$	3 4 3 4 ${}^3C_3 \times {}^4C_1 \times {}^3C_2 \times {}^4C_1 = 144$
Case IV :	$1M_m + 2M_w + 2W_m + 1W_w$	3 4 3 4 ${}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 = 324$

Total = 324 + 144 + 16 + 1 = 485

(25) **(C).** ${}^6C_4 \,.\, {}^3C_1 \times 1 \times 4!$
$\frac{6 \times 5}{2} \times 3 \times 24 \equiv 45 \times 24 = 1080$

(26) **(B).** Required number of ways = Total number of ways – When A and B are always included.
$= {}^5C_2 \,.\, {}^7C_3 - {}^5C_1 \,.\, {}^5C_2 = 300$

(27) **(D).**



Number of such numbers $= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{2!\,4!} = 180$

(28) **(D).**
(A) The number of four-digit numbers Starting with 5 is equal to $6^3 = 216$
(B) Starting with 44 and 55 is equal to $36 \times 2 = 72$
(C) Starting with 433,434 and 435 is equal to $6 \times 3 = 18$
(D) Remaining numbers are 4322,4323,4324,4325is equal to 4. So total numbers are 216 + 72 + 18 + 4 = 310

(29) **(A).** Since there are 8 males and 5 females. Out of these 13, if we select 11 persons, then there will be at least 6 males and atleast 3 females in the selection.
$m = n = \binom{13}{11} = \binom{13}{2} = \frac{13 \times 12}{2} = 78$

(30) **(B).** Sum of given digits 0, 1, 2, 5, 7, 9 is 24.
Let the six digit number be abcdef and to be divisible by 11. So $|(a + c + e) - (b + d + f)|$ is multiple of 11.
Hence only possibility is $a + c + e = 12 = b + d + f$
Case-I : {a, c, e} = {9, 2, 1} & {b, d, f} = {7, 5, 0}
So, Number of numbers = $3! \times 3! = 36$
Case-II : {a,c,e} = {7,5,0} and {b,d,f} = {9,2,1}
So, Number of numbers $2 \times 2! \times 3! = 24$
Total = 60

(31) **(C).** Total cases = number of diagonals = $^{20}C_2 - 20 = 170$

(32) **(A).**

10 Identical Object	21 Distinct 1	0
0	10	$^{21}C_{10} \times 1$
1	9	$^{21}C_9 \times 1$
⋮	⋮	⋮
10	0	$^{21}C_0 \times 1$

$^{21}C_0 + ... + ^{21}C_{10} + ^{21}C_1 + ... + ^{21}C_0 = 2^{21}$

$(^{21}C_0 + ... + ^{21}C_{10}) = 2^{20}$

(33) **(A).** $^5C_1 \,.\, ^nC_2 + ^5C_2 \,.\, ^nC_1 = 1750$

$n^2 + 3n = 700 \qquad \therefore n = 25$

(34) **(A).** 1, 3, 5, 7, 9

For digit to repeat we have $^5C_1$ choice

And six digits can be arrange in $\frac{\lfloor 6}{\lfloor 2}$ ways.

Hence total such numbers = $\frac{5\lfloor 6}{\lfloor 2}$

(35) **(D).** $\frac{36}{r+1} \times \frac{35}{C_r}(k^2 - 3) = \frac{35}{C_r}$

$k^2 - 3 = \frac{r+1}{6} \Rightarrow k^2 = 3 + \frac{r+1}{6}$

r can be 5, 35

for $r = 5, k = \pm 2$

$r = 35, k = \pm 3$. Hence number of order pair = 4

(36) **(A).** We know ^nC_r is max at middle term

$a = ^{19}C_p = ^{19}C_{10} = ^{19}C_9$

$b = ^{20}C_q = ^{20}C_{10}$

$c = ^{21}C_6 = ^{21}C_{10} = ^{21}C_{11}$

$$\frac{a}{^{19}C_9} = \frac{b}{\frac{20}{10} \cdot ^{19}C_9} = \frac{c}{\frac{21}{11} \cdot \frac{20}{10} \cdot ^{19}C_9}$$

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{42/11} \; ; \; \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

(37) **2454**

EXAMINATION

2N, 2A, 2I, E, X, M, T, O

Case I : All are different so

$$^8P_4 = \frac{8!}{4!} = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$$

Case II : 2 same and 2 different so

$$^3C_1 \cdot ^7C_2 \cdot \frac{4!}{2!} = 3 \cdot 21 \cdot 12 = 756$$

Case III : 2 same and 2 same so

$$^3C_2 \cdot \frac{4!}{2!2!} = 3 \cdot 6 = 18$$

$\therefore$ Total = 1680 + 756 + 18 = 2454

(38) **(A).** _ _ _ 2 _

No. of five digits numbers

= No. of ways of filling remaining 4 places

$= 8 \times 8 \times 7 \times 6$

$$\therefore \; k = \frac{8 \times 8 \times 7 \times 6}{336} = 8$$