# 8

# ROTATIONAL MOTION

## CENTRE OF MASS

**Definition:**

**1.** For a system of particles centre of mass is that point at which its total mass is supposed to be concentrated.

**2.** It is a point about which vector sum of moment of masses of all the particles in the system is always zero.

**Moment of mass:** It is the product of mass of the particle and its position vector w.r.t. the reference point.

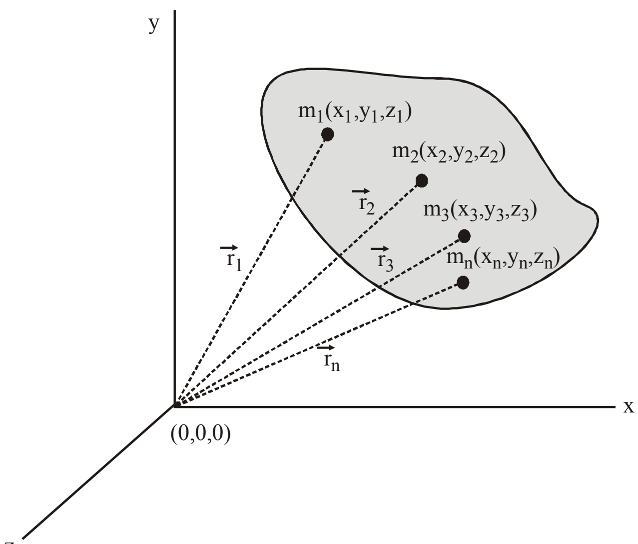
i.e. moment of mass = $m \times \vec{r}$ ; Unit : kg × m

**Calculation of centre of mass :** For a discrete system of particles centre of mass is defined as

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3} = \frac{1}{M}\Sigma m_i\vec{r}_i \quad \text{where}$$

$M = m_1 + m_2 + ...... + m_n$ (total mass of the system)

If co-ordinates of positions of the particles of masses $m_1, m_2,$ ......... are given as $(x_1, y_1, z_1), (x_2, y_2, z_2)$ .......




then position vector of their center of mass is

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$\vec{r}_{cm} = \frac{m_1(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + m_2(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) + m_3(x_3\hat{i} + y_3\hat{j} + z_3\hat{k})}{m_1 + m_2 + m_3 + ......}$$

$$x_{cm} = \left(\frac{m_1x_1 + m_2x_2 + ....}{m_1 + m_2 + m_3 + ......}\right) = \frac{1}{M}\Sigma m_ix_i$$

$$y_{cm} = \left(\frac{m_1y_1 + m_2y_2 + ....}{m_1 + m_2 + m_3 + ......}\right) = \frac{1}{M}\Sigma m_iy_i$$

$$z_{cm} = \left(\frac{m_1z_1 + m_2z_2 + ....}{m_1 + m_2 + m_3 + ......}\right) = \frac{1}{M}\Sigma m_iz_i$$

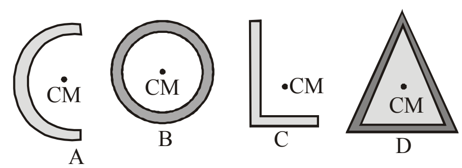
If the system has continuous distribution of mass, treating the mass element dm at position $\vec{r}$ as a point mass and replacing summation by integration.

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\, dm$$

So that $x_{cm} = \frac{1}{M}\int x\, dm$ , $y_{cm} = \frac{1}{M}\int y\, dm$

and $z_{cm} = \frac{1}{M}\int z\, dm$

**NOTE**




* There may or may not be any mass present physically at centre of mass (See Fig. A, B, C, D)

* It may be inside or outside of the body (See Fig. A,B,C, D)

* Its position depends on the shape of the body. (See Fig. A, B, C, D)

* For a given shape it depends on the distribution of mass of within the body and is closer to massive part. (See Fig)

* For symmetrical bodies having homogeneous distribution of mass it coincides with centre of symmetry of geometrical centre. (See Figure A, C, D).

* If we know the centre of mass of parts of the system and their masses, we can get the combined centre of mass by treating the parts as point particles placed at their respective centre of masses.

* It is independent of the co-ordinate system, e.g., the centre of mass of a ring is at its centre whatever be the co-ordinate system.

* If the origin of co-ordinate system is at centre of mass, i.e., $\vec{r}_{cm} = 0$ , then by definition.

$$\frac{1}{M}\Sigma m_i\vec{r}_i = 0 \Rightarrow \Sigma m_i\vec{r}_i = 0$$

the sum of the moments of the masses of a system about its centre of mass is always zero.

* **Centre of mass of some commonly used systems:**

| Body | |
|---|---|
| a. Uniform rod of length L. | L / 2 |
| b. rod having linear mass density $\lambda = \alpha x$ | 2L / 3 |
| c. Quadrant of a uniform circular ring, radius R. | $2R / \pi$ |
| d. Uniform semi circular ring of radius R. | $2R / \pi$ |
| e. Uniform semi circular disc of radius R. | $4R / 3\pi$ |
| f. Uniform hemispherical shell of radius R. | R / 2 |
| g. Uniform solid hemisphere of radius R. | 3R / 8 |
| h. Hollow cone of base radius R & height h. | h/3 from base of the cone. |
| i. Solid cone of base radius R and height h. | h/4 from base of the cone. |

**MOTION OF CENTRE OF MASS**

As for a system of particles, position of centre of mass is

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ....}{m_1 + m_2 + m_3 + ....}$$

$$\text{so } \frac{d}{dt}(\vec{r}_{cm}) = \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + ....}{m_1 + m_2 + m_3 + ....}$$

Velocity of CM

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ....}{m_1 + m_2 + ......} = \frac{1}{M}\Sigma\, m_i\vec{v}_i \quad \left[\because \frac{d\vec{r}}{dt} = \vec{v}\right]$$

$$\text{Similarly, } \vec{a}_{cm} = \frac{d}{dt}\vec{v}_{cm}$$

acceleration of CM

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ....}{m_1 + m_2 + ......} = \frac{1}{M}\Sigma\, m_i\vec{a}_1 \quad \left[\because \vec{a} = \frac{d\vec{v}}{dt}\right]$$

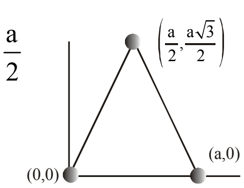
**Example 1 :**

Three bodies of equal masses are placed at (0, 0), (a, 0) and at $\left(\frac{a}{2}, \frac{a\sqrt{3}}{2}\right)$. Find out the co-ordinates of centre of mass.

**Sol.** $$x_{cm} = \frac{0 \times m + a \times m + \frac{a}{2} \times m}{m + m + m} = \frac{a}{2}$$

$$y_{cm} = \frac{0 \times m + 0 \times m + \frac{a\sqrt{3}}{2} \times m}{m + m + m}$$

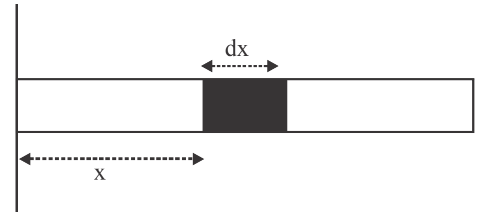
$$= \frac{a\sqrt{3}}{6}$$




**Example 2 :**

If the linear density of a rod of length L varies as $\lambda = A + Bx$, compute its centre of mass.

**Sol.** Let the x-axis be along the length of the rod and origin at one of its end as shown in Fig. As rod is along x-axis, for all points on it y and z will be zero so,




$y_{CM} = 0$ and $z_{CM} = 0$

i.e., centre of mass will be on the rod.

Now consider an element of rod of length dx at a distance x from the origin

. mass of this element $dm = \lambda dx = (A + Bx)\, dx$

$$\text{So, } x_{CM} = \frac{\int_0^L x\, dm}{\int_0^L dm} = \frac{\int_0^L x\,(A + Bx)\, dx}{\int_0^L (A + Bx)\, dx}$$

$$= \frac{\frac{AL^2}{2} + \frac{BL^3}{3}}{AL + \frac{BL^2}{2}} = \frac{L\,(3A + 2BL)}{3\,(2A + BL)}$$

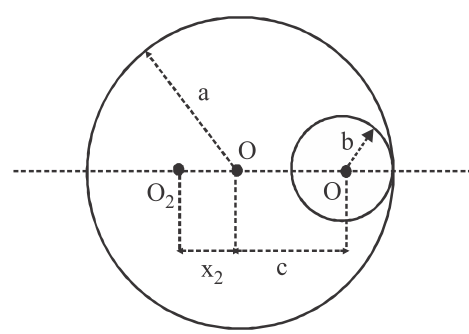
**Note:** (i) If the rod is uniform density then $\lambda$ = constant and $B = 0$ ; $X_{CM} = L/2$

(ii) If the density of rod varies linearly with x, then $\lambda = Bx$ and $A = 0$ . ; $X_{CM} = 2L/3$

**Example 3 :**

Find the centre of mass of a uniform disc of radius 'a' from which a circular section of radius b has been removed. The centre of the hole is at a distance c from the centre of the disc.

**Sol.** Let the circular disc of radius 'a' is made up of the circular section of radius b and remainder. Further let the line of symmetry joining the centres O and $O_1$ be the x-axis with O as origin.



The centre of mass of the disc of radius 'a' will be given by $X_{CM} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$ ........ (i)

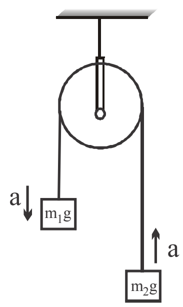
while $Y_{CM}$ and $Z_{CM}$ will be zero (as for all points on x-axis y and z = 0). If $\sigma$ is the density of the material of disc.
$m_1 = \pi b^2\sigma$ and $x_1 = c$
$m_2 = \pi (a^2 - b^2)\ \sigma$ and $x_2 = ?$
$M = m_1 + m_2 = \pi a^2\sigma$ and $X_{CM} = 0$
So substituting these values in equation (i)

$$0 = \frac{\pi b^2\sigma c + \pi (a^2 - b^2)\ \sigma x_2}{\pi a^2\sigma} \quad ....... (ii)$$

i.e., centre of mass of the remainder (say $O_2$) is at a distance $cb^2/(a^2 - b^2)$ to the left of O on the line joining the centres O and $O_1$.

**Example 4 :**
Two blocks of masses $m_1$ and $m_2$ are connected by a light inextensible string passing over a smooth fixed pulley of negligible mass. Find the acceleration of the centre of mass of the system when blocks move under gravity.

**Sol.** $\therefore$ $m_1 > m_2$ so $m_1$ will move downwards and $m_2$ upwards.
Magnitude of acc of each block




$$a = \frac{\text{net pulling force}}{\text{mass to be pulled}} = \frac{(m_1 - m_2)\ g}{m_1 + m_2}$$

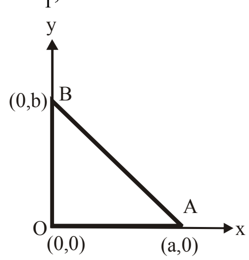
$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} = \frac{m_1a + m_2a}{m_1 + m_2}$$

$$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) a \text{ (+ve downwards and -ve upwards)}$$

$$\vec{a}_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \times \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

In the direction of acceleration of $m_1$, downwards

**Example 5 :**
Three rods of the same mass are placed as shown in the figure. Calculate the coordinates of the centre of mass of the system.




**Sol.** CM of rod OA is at (a/2, 0), CM of rod OB is at (0, a/2), CM of rod AB is at (a/2, a/2)

For system : $x_{CM} = \dfrac{m \times \frac{a}{2} + m \times 0 + m \times \frac{a}{2}}{m + m + m} = \dfrac{m}{3}$

$$y_{CM} = \frac{m \times 0 + m \times \frac{a}{2} + m \times \frac{a}{2}}{m + m + m} = \frac{m}{3}$$

**CENTRE OF MASS AND MOMENTUM OF CONSERVATION**

For a system of particles $m_1$, $m_2$ ...........

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + ...$$

or $M\vec{v}_{cm} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...$ $\quad [\because \vec{p} = m\vec{v}]$

or $M\vec{v}_{cm} = \vec{p}_{cm}$ $\quad [\because \Sigma\ \vec{p}_i = \vec{p}_{cm}]$

Linear momentum of a system of particles is equal to the product of mass of the system with velocity of its centre of mass.

From Newtons second law, $\vec{F}_{ext} = \dfrac{d\ (M\vec{v}_{cm})}{dt}$

If $\vec{F}_{ext} = 0$ then $\vec{v}_{cm}$ = constant

If no external force acts on a system the velocity of its centre of mass remains constant, i.e., velocity of centre of mass is unaffected by internal forces.

**Displacement of objects using centre of mass concept :**

Consider a system of two masses $m_1$ and $m_2$ if first mass is displaced by distance d towards centre of mass to find a displacement of second mass. Assume centre of mass at distance $x_1$ and $x_2$ from $m_1$ and $m_2$

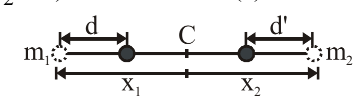
From centre of mass definition

$m_1x_2 = m_2x_2$ ........(i)

and $m_1(x_1 - d) = m_2\ (x_2 - d')$. ........(ii)

Solve (i) and (ii)

$m_1d = m_2d'$




or $d' = \dfrac{m_1}{m_2} d$ (d' is displacement of $m_2$ wrt ground)

If displacement of $m_1$ wrt $m_2$ = $\vec{d}_{12} = \vec{d}_1 - \vec{d}_2$

$$d_{12} = d - (-d') = d + d'$$

$$d' = \frac{m_1}{m_2} d \Rightarrow d' = \frac{m_1}{m_2}(d_{12} - d') \ ; \quad d' = \frac{m_1}{m_1 + m_2} d_{12}$$

Note : From above relation we can write velocity expression

also $v' = \dfrac{m_1}{m_2} v$ and $v' = \dfrac{m_1}{m_1 + m_2} v_{12}$

**Example 6 :**
A man of mass M stands at one end of a plank oflength L which lies at rest on a frictionless surface. The man walks to other end of the plank. If the mass of the plank is M/3 then the distance that the man moves relative to ground is
(A) 3L/4 (B) L/4
(C) 4L/5 (D) L/3

**Sol.** **(B).** Let the distance moved by plank is x.

$$m_1\Delta x_1 + m_2\Delta x_2 = 0$$

$$M\ (L - x) + \frac{M}{3}(-x) = 0 \Rightarrow x = \frac{3L}{4}$$

$\therefore$ Distance moved by person w.r.t. to ground = L – x = L/4

**Example 7 :**

A particle of mass 3m is projected from the ground at some angle with horizontal. The horizontal range is R. At the highest point of its path it breaks into two pieces m and 2m. The smaller mass comes to rest and larger mass finally falls at a distance x from the point of projection where x =

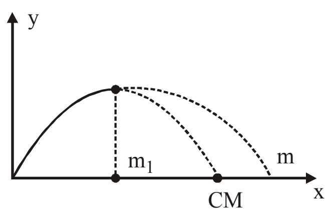
(A) 3R/4 (B) 3R/2

(C) 5R/4 (D) 3R

**Sol. (C).** $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$$R = \frac{m\left(\frac{R}{2}\right) + 2m\,(x_2)}{3m}$$

$$\Rightarrow x_2 = \frac{5R}{4}$$



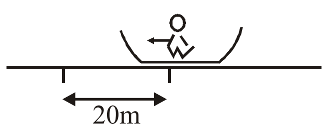

**Example 8 :**

A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time?

(A) 11.2 m (B) 13.8 m

(C) 14.3 m (D) 15.4 m

**Sol. (C).** Let the distance moved by boat be x.




$$80\,\{-(8-x)\} + 200\,(+x) = 0 \Rightarrow x = 16/7 = 2.3$$

Distance of person from shore = 20 – (8 – 2.3) = 14.3m

**Example 9 :**

A man of mass m climbs a rope of length L suspended below a balloon of mass M. The balloon is stationary with respect to ground. If the man begins to climb up the rope at a speed v (relative to rope) in what direction and with what speed (relative to ground) will the balloon move?

**Sol.** Balloon is stationary

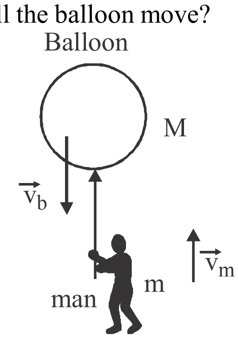
$\Rightarrow$ No net external force acts on it.

$\Rightarrow$ The conservation of linear momentum of the system (balloon + man) is valid

$\Rightarrow$ $M\vec{v}_b + m\vec{v}_{mb} = 0$,

where $\vec{v}_m = \vec{v}_{mb} + \vec{v}_b$

$\Rightarrow$ $M\vec{v}_b + m\,[\vec{v}_{mb} + \vec{v}_b] = 0$




where $v_{mb}$ = velocity of man relative to the balloon (rope)

$\Rightarrow$ $\vec{v}_b = -\dfrac{m\vec{v}_{mb}}{M+m}$

Where $v_{mb} = v \Rightarrow v_b = \dfrac{mv}{M+m}$ and directed opposite to that of the man.

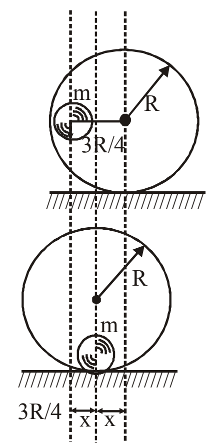
# TRY IT YOURSELF-1

**Q.1** Calculate the position of the centre of mass of a system consisting of two particles of masses $m_1$ and $m_2$ separated by a distance L apart.

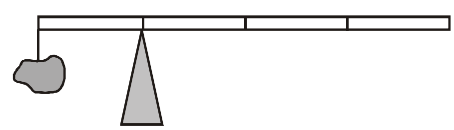
**Q.2** A circle of radius R is cut from a uniform thin sheet of metal. A circular hole of radius R/2 is now cut out of the circle, with the hole tangent to the rim. Find the distance of centre of mass from the center of the original uncut circle to the CM.

**Q.3** A rigid body consists of a 3 kg mass connected to a 2 kg mass by a massless rod. The 3 kg mass is located at $\vec{r}_1 = (2\hat{i} + 5\hat{j})$ m and 2 kg mass at $\vec{r}_2 = (4\hat{i} + 2\hat{j})$ m . Find the length of rod & the coordinates of the centre of mass.

**Q.4** Inside a smooth spherical shell of the radius R a ball of the same mass is released from the shown position (Fig.) Find the distance travelled by the shell on the horizontal floor when the ball comes to the lowest point of the shell.




**Q.5** A 1-kg rock is suspended by a massless string from one end of a 1-m measuring stick. What is the weight of the measuring stick if it is balanced by a support force at the 0.25-m mark?



(A) 0.25 kg (B) 0.5 kg

(C) 1.0 kg (D) 2.0 kg

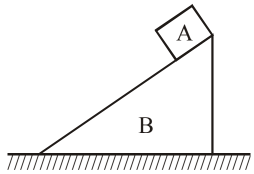
**Q.6** A fisherman in a small fishing boat at rest in a lake hooks a giant log floating in the lake 30 meters away. The fisherman reels the log in. During this process, the boat moves 12meters in the direction of the log. If the mass of the boat and fisherman is 400 kg, what is the mass of the log? Assume frictionless.

**Q.7** Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speed of 2m/sec and 6 m/sec respectively on a smooth horizontal surface. Find the speed of centre of mass of the system.

**Q.8** Two particles of mass 2 kg & 4 kg are approaching towards each other with accelerations 1m/sec² and 2m/sec² respectively on a smooth horizontal surface. Find the acceleration of centre of mass of the system.

**Q.9** A man of mass m is standing on a platform of mass M kept on smooth ice. If the man starts moving on the platform with a speed v relative to the platform, with what velocity relative to the ice does the platform recoil ?

**Q.10** A block A (mass = 4M) is placed on the top of a wedge B of base length $\ell$ (mass = 20M) as shown in figure. When the system is released from rest. Find the distance moved by the wedge B till the block A reaches at lowest end of wedge. Assume all surfaces are frictionless.




## ANSWERS

**(1)** $y_{CM} = 0,\ z_{CM} = 0\ ;\ x_{CM} = \dfrac{m_2 L}{m_1 + m_2}$

**(2)** (0, –R/6) **(3)** $\left(\dfrac{14}{5}\hat{i} + \dfrac{19}{5}\hat{j}\right)$ m **(4)** –3R/8

**(5)** (C) **(6)** $m_{log} = 267$ kg **(7)** 3.33 m/sec

**(8)** 1 m/sec$^2$ **(9)** $V = \dfrac{mv}{M+m}$ **(10)** $\ell/6$

### RIGID BODY

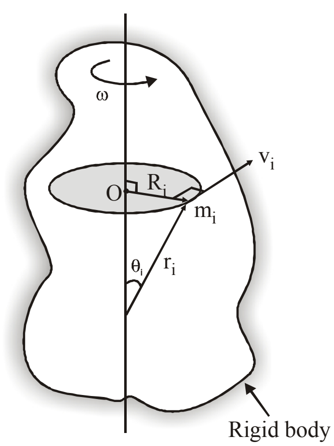
System of particle which does not change its shape under the influence of external force or torque and distance between the particle remains unchanged is called rigid body.

(i) No body in universe is perfectly rigid. However the bodies in which strain effect is quite negligible under the influence of external force may be said to be rigid bodies e.g. earth, billiard ball, stone, piece of steel, etc.

(ii) The internal structure of a rigid body and its shape and size do not change in state of motion.

### ROTATORY MOTION

A body rotating about a fixed axis then every particle of the body moves in a circle and the centres of all these circles lie at axis of rotation.




$\vec{v}_1$ = velocity of i$^{th}$ particle of mass $m_i$.

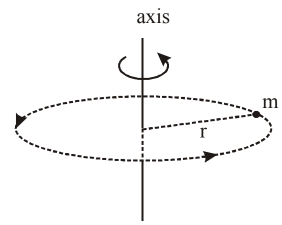
$\vec{r}_i$ = position vector of i$^{th}$ particle

$R_i$ = perpendicular distance of i$^{th}$ particle from axis of rotation.

All particles of rigid body moves with same angular velocity $\vec{\omega}$ .

### MOMENT OF INERTIA

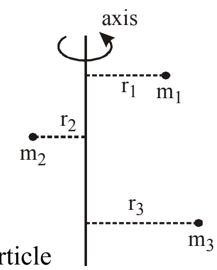
The virtue by which a body revolving about an axis opposes the change in rotational motion is known as moment of inertia.




The moment of inertia of a particle with respect to an axis of rotation is equal to the product of mass of the particle and square of distance from rotational axis.

$I = mr^2$

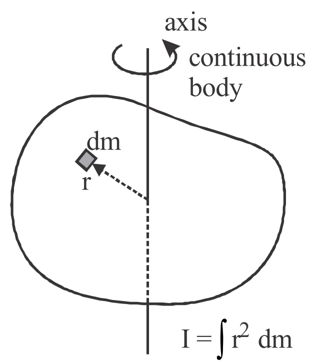
r = perpendicular distance from axis of rotation.




Moment of inertia of system of particle

$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots\ldots\ldots$

$I = \Sigma\, mr^2$




**Moment of inertia depends on**

(a) Mass of the body
(b) Mass distribution of body
(c) Position of axis of rotation
$\Rightarrow$ its shape, size and density

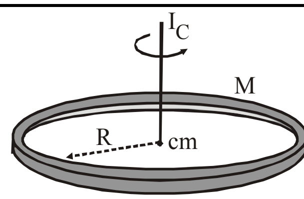
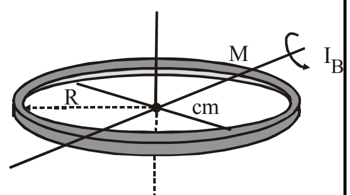
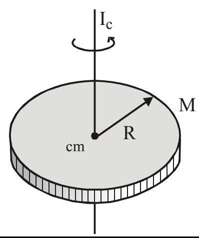
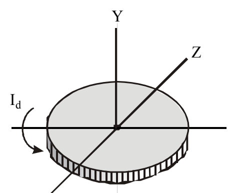
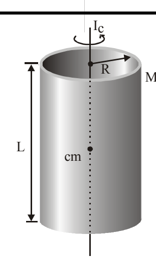
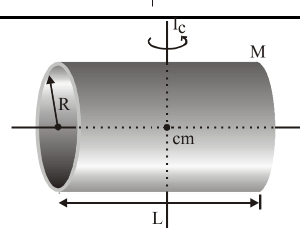
**Moment of inertia does not depend on**

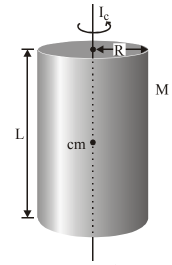
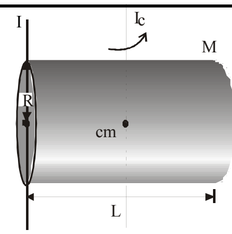
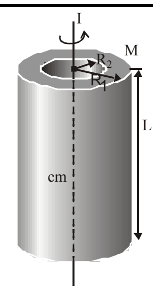
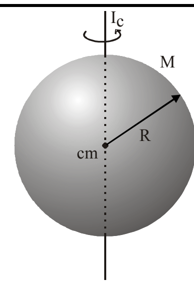
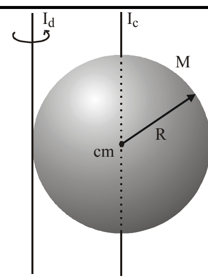
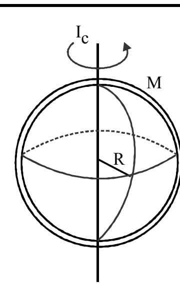
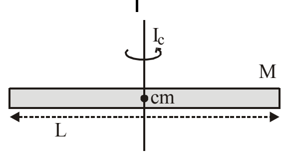
(a) Angular velocity
(b) Angular acceleration
(c) Torque
(d) Angular momentum

Unit : SI : kg-m$^2$, CGS : g-cm$^2$, Dimensions : $M^1L^2T^0$

As the distance of mass increases from the rotational axis, the moment of inertia (M.I.) increases.

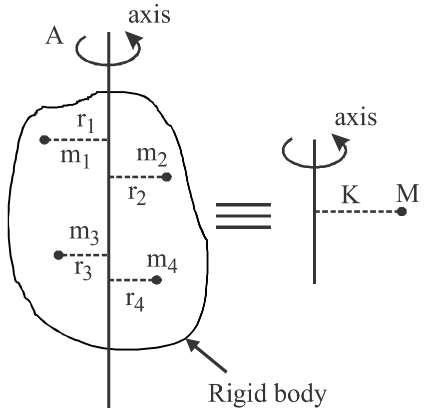
**Moment of inertia of different objects**

| Shape of Body | Rotational axis | Figure | Moment of inertia | Radius of Gyration |
|---|---|---|---|---|
| **(1) Ring**<br>M : mass<br>R : radius | (a) Perpendicular to plane passing through centre of mass |  | $MR^2$ | R |
| | (b) Diameter in the plane |  | $\frac{1}{2}MR^2$ | $\frac{R}{\sqrt{2}}$ |
| **(2) Disc** | (a) Perpendicular to plane passing through centre of mass |  | $\frac{1}{2}MR^2$ | $\frac{R}{\sqrt{2}}$ |
| | (b) Diameter in the plane |  | $\frac{MR^2}{4}$ | $\frac{R}{2}$ |
| **(3) Thin walled cylinder** | (a) Geometrical axis |  | $MR^2$ | R |
| | (b) Perpendicular to length passing through centre of mass |  | $M\left(\frac{R^2}{2}+\frac{L^2}{12}\right)$ | $\sqrt{\frac{R^2}{2}+\frac{L^2}{12}}$ |

| | | | | |
|---|---|---|---|---|
| **(4) Solid cylinder** | (a) Geometrical axis |  | $\frac{MR^2}{2}$ | $\frac{R}{\sqrt{2}}$ |
| | (b) Perpendicular to length passing through centre of mass |  | $M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$ | $\sqrt{\frac{R^2}{4}+\frac{L^2}{12}}$ |
| **(5) Hollow cylinder** | Geometrical axis |  | $M\left[\frac{R_1^2+R_2^2}{2}\right]$ | $\sqrt{\frac{R_1^2+R_2^2}{2}}$ |
| **(6) Solid sphere** | (a) Diameter |  | $\frac{2}{5}MR^2$ | $\sqrt{\frac{2}{5}}.R$ |
| | (b) Tangent |  | $\frac{7}{5}MR^2$ | $\sqrt{\frac{7}{5}}.R$ |
| **(7) Thin spherical shell** | Diameter |  | $\frac{2}{3}MR^2$ | $\sqrt{\frac{2}{3}}R$ |
| **(8) Thin rod** | Perpendicular to length passing through centre of mass |  | $\frac{ML^2}{12}$ | $\frac{L}{2\sqrt{3}}$ |

## RADIUS OF GYRATION (K)

The radius of gyration of a body is the distance from axis of rotation, the square of this distance when multiplied by the mass of body then it gives the moment of the body ($I = MK^2$) about same axis of rotation




$I = MK^2$ but $I = \Sigma mr^2$ so $MK^2 = \Sigma mr^2$

$$\Rightarrow K^2 = \frac{m_1r_1^2 + m_2r_2^2 + .......m_nr_n^2}{M}$$

$$\Rightarrow K = \sqrt{\frac{m_1r_1^2 + m_2r_2^2 + .......m_nr_n^2}{m_1 + m_2 + ..... + m_n}}$$

If $m_1 = m_2 = m_3 = m$ then, $M = mn$

$$K = \sqrt{\frac{r_1^2 + r_2^2 + .......r_n^2}{n}} \quad [n = \text{total number of particles}]$$

Radius of gyration $K = \sqrt{\frac{I}{M}}$

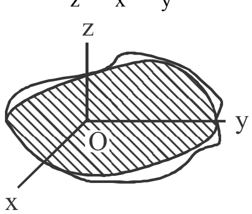
K has no meaning without axis of rotation, K is scalar quantity.

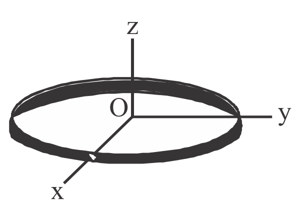
## THEOREMS OF MOMENT OF INERTIA

**Theorem of perpendicular axes (applicable only for two dimensional bodies or plane laminas**

The moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its own plane intersecting each other at the point through which the perpendicular axis passes.

$I_z = I_x + I_y$




where $I_x$ = MI of the body about X-axis

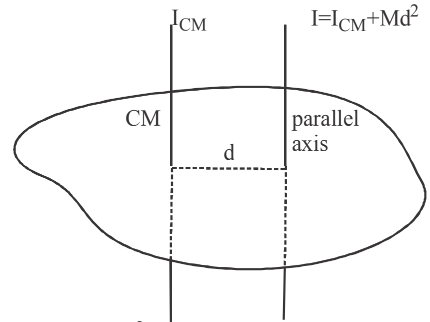
$I_y$ = MI of the body about Y-axis

$I_z$ = MI of the body about Z-axis

applicable only for two dimensional bodies and cannot be used for three dimensional bodies.

**Theorem of parallel axes (for all type of bodies)**

Moment of inertia of a body about any axis is equal to the moment of inertia about a parallel axis passing through the centre of mass plus product of mass of the body and the square of distance between these two parallel axis.




$I = I_{CM} + Md^2$

$I_{CM}$ = Moment of inertia about the axis passing through centre of mass.

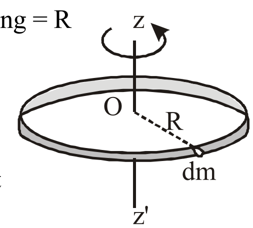
## MOMENT OF INERTIA OF SOME REGULAR BODIES

**Moment of inertia of ring :**

**(i)** About an axis passing through the centre of ring and perpendicular to its plane

Mass of ring = M and radius of ring = R

The ring is assumed to be made up of small elements. Consider one such element of mass dm.




Moment of inertia of this element about the axis ZOZ' = dI

$dI = dm \, . \, R^2$.

Moment of inertia of the whole ring abotu axis ZOZ' is

$$I = \int dI = R^2 \int dm = MR^2$$

$I = MR^2$

**(ii) About the diameter of the ring**

Let moment of inertia of the ring about each diameter $= I_d$ (i.e. XX' and YY')

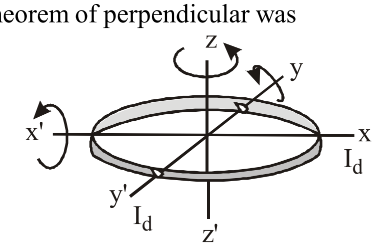
Both the diameters are perpendicular to the axis ZZ' which is passing through the centre of the ring and perpendicular to its plane, by theorem of perpendicular was

$I_{xx'} + I_{yy'} = I_{zz'}$

or $I_d + I_d + I_z$

$\Rightarrow 2I_d = MR^2$

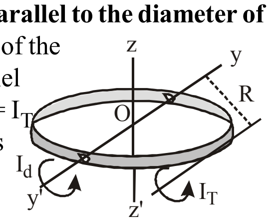
$$\Rightarrow I_d = \frac{1}{2}MR^2$$




**(iii) About an axis tangential and parallel to the diameter of the ring :** Let moment of inertia of the ring about the tangent AB parallel to the diameter YY' of the ring $= I_T$

Applying theorem of parallel axes

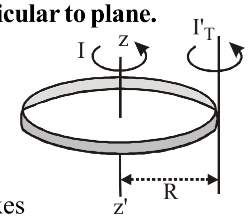
$I_T$ = moment of inertia of ring about diameter YY' + $MR^2$




$$I_T = \frac{1}{2}MR^2 + MR^2 \Rightarrow I_T = \frac{3}{2}MR^2$$

**(iv) About the tangent parallel to the axis passing through the centre of ring and perpendicular to plane.**

Let M.I. of the ring about the tangent parallel to an axis passing through the centre of the ring and perpendicular to its plane = $I'_T$



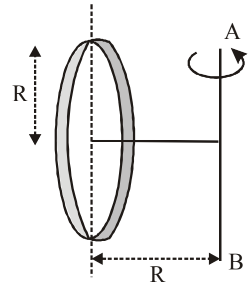

Applying theorem of parallel axes

$I'_T$ = moment of inertia about ZZ' + $MR^2 = MR^2 + MR^2$

$I'_T = 2MR^2$

**Example 10 :**

Find the moment of inertia about axis AB.

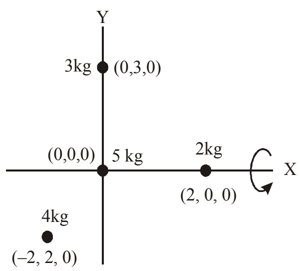



**Sol.** $I_{AB} = I_{dia} + MR^2$

$$= \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

**Example 11 :**

Four bodies of masses 5 kg, 2 kg, 3 kg and 4 kg are respectively placed at position (0, 0, 0), (2, 0, 0), (0, 3, 0) and (–2, –2, 0). Calculate the moment of inertia about x-axis, y-axis and z-axis.




**Sol.** $I_x = 3 \times (3)^2 + 4 \times (2)^2 = 43$ unit

$I_y = 2 \times (2)^2 + 4 \times (2)^2 = 24$ unit

$I_z = 2 \times (2)^2 + 3 \times (3)^2 + 4 \times (2\sqrt{2})^2 = 8 + 27 + 32 = 67$ unit

$(I_z = I_x + I_y = 43 + 24 = 67)$

**Example 12 :**

The moment of inertia of sphere is 40 kg-m$^2$ about the diameter. Determine the moment of inertia about any tangent.

**Sol.** Given that $\frac{2}{5}MR^2 = 40$

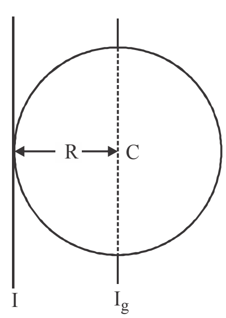
or $MR^2 = 100$

By theorem of parallel axes

$I = I_{CM} + MR^2$

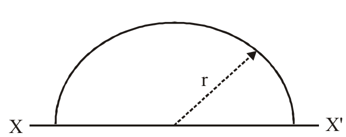
$$= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

$$= \frac{7}{5} \times 100 = 140 \text{ kg-m}^2$$




**Example 13 :**

A uniform wire of length $\ell$ and mass M bented in the shape of semicircle of radius r as shown in figure. Calculate moment of inertia about XX'.



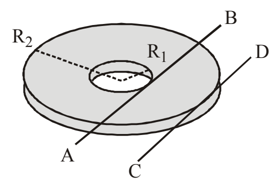

**Sol.** Length of the wire $\ell = \pi r \Rightarrow r = \frac{\ell}{x} \Rightarrow I_{xx'} = \frac{Mr^2}{2} = \frac{M\ell^2}{2\pi^2}$

**Example 14 :**

Calculate the moment of inertia of an annular disc about an axis which lying in the plane of the disc and tangent to the (i) inner circle and (ii) outer circle.

**Sol.** (i) M.I. about tangent to the inner circle is

$$I_{AB} = \frac{M}{4}(R_1^2 + R_2^2) + MR_1^2$$



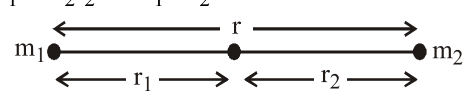

(ii) M.I. about tangent to the outer circle is

$$I_{AB} = \frac{M}{4}(R_1^2 + R_2^2) + MR_2^2$$

**Example 15 :**

Two masses $m_1$ and $m_2$ are placed at a distance r from each other. Find out the moment of inertia of system about an axis passing through their centre of mass.

**Sol.** $m_1r_1 = m_2r_2$ and $r_1 + r_2 = r$




$$\Rightarrow r_1 = \frac{m_2 r}{m_1 + m_2}, \quad r_2 = \frac{m_1 r}{m_1 + m_2}$$

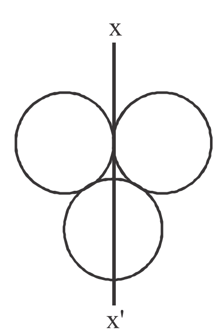
Moment of inertia $I = m_1r_1^2 + m_2r_2^2$

$$= m_1\left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1 + m_2}\right)^2 = \left(\frac{m_1 m_2}{m_1 + m_2}\right)r^2$$

Note : Here $I = \mu r^2$ where $\mu$ = reduced mass = $\frac{m_1 m_2}{m_1 + m_2}$

**Example 16 :**

Adjoining diagram has three disc, in which each has mass M and radius R. Find the moment of inertia of this system about axis xx'.

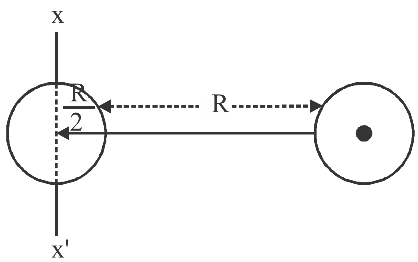



**Sol.** $I_{system} = 2 \times I_{upper} + I_{inner}$

$$= 2 \times \frac{5}{4}MR^2 + \frac{MR^2}{4} = \frac{11}{4}MR^2$$

**Example 17 :**

Diameter of each spherical shell is R and mass M, they are joined by a light and massless road. Calculate the moment of inertia about xx' axis.

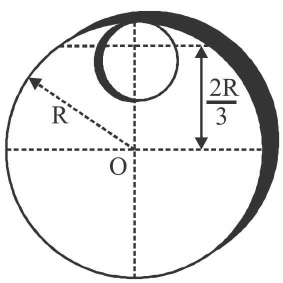



**Sol.** $I_{system} = \frac{2}{3}MR^2 + \left[\frac{2}{3}MR^2 + M\,(2R)^2\right]$

$= \frac{4}{3}MR^2 + 4MR^2 = \frac{16}{3}MR^2$

**Example 18 :**

A thin uniform disc of mass 9M and of radius R. A disc of radius R/3 is cut as shown in figure. Find the moment of inertia of the remaining disc about an axis passing through O and perpendicular to the plane of disc.




**Sol.** As the mass is uniformly distributed on the disc,

so mass density (mass per unit area) $= \frac{9M}{\pi R^2}$

Mass of removed portion $= \frac{9M\pi}{\pi R^2} \times \left[\frac{R}{3}\right]^2 = M$

So the moment of inertia of the removed portion about the stated axis by theorem of parallel axis

$$I_1 = \frac{M}{2}\left[\frac{R}{3}\right]^2 + M\left[\frac{2R}{3}\right]^2 \qquad \text{........ (1)}$$

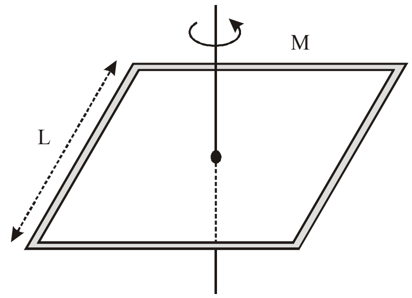
If the disc would not have been removed, then the moment of inertia of complete disc about the stated axis is $I_2$

then $\quad I_2 = 9M\frac{R^2}{2} \qquad$ ........ (2)

So the moment of inetia of the disc shown figure is $I_2 - I_1$.
i.e., $I_2 - I_1 = 4\,MR^2$

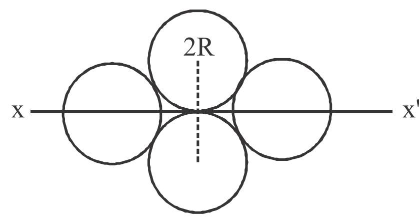
## TRY IT YOURSELF-2

**Q.1** Four rods are placed in the form of a square. Calculate the moment of inertia about an axis passes through the centre and perpendicular to the plane.
(Assume mass M and length L of each rod)



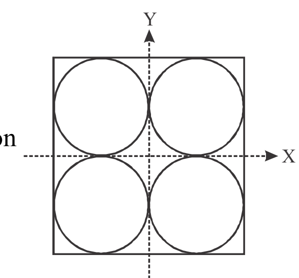

**Q.2** Three rods are placed in the form of equilateral triangle. Calculate the M.I. about axis passing through the centre and perpendicular to the plane. (Assume mass and length of each rod is M and L respectively)

**Q.3** The moment of inertia of a sphere about its diameter is I. Four such spheres are arranged as shown in figure. Find the moment of inertia of the system about the axis XX'.

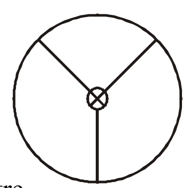



**Q.4** Two rings have their moments of inertia in the ratio 4 : 1 and their diameters are in the ratio 4 : 1. Find the ratio of their masses.

**Q.5** Four holes of radius R are cut from a thin square plate of side 4R & mass M. Determine inertia of the remaining portion about z-axis.

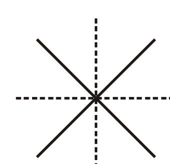



**Q.6** Each wheels has an outer ring having radius R and mass m. Other than outer ring the wheels comprise of some uniform rods (each of mass m and length R).



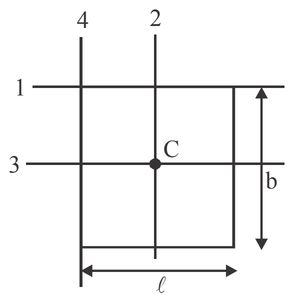
Calculate radius of gyration about centre and perpendicular to plane.

**Q.7** Two uniform identical rods each of mass M and length $\ell$ are joined to form a cross as shown in figure.



Find the moment of inertia of the cross about a bisector as shown dotted in the figure.

**Q.8** In the figure shown find moment of inertia of a plate having mass M, length $\ell$ and width b about axis 1, 2, 3 and 4. Assume that mass is uniformly distributed.




**Q.9** Calcualte the moment of inertia of a hollow cylinder of mass M and radius R about a line parallel to the axis of the cylinder and on the surface of the cylinder.

**Q.10** The diameter of flywheel increases by 1%. Find the percentage increase in moment of inertia about axis of symmetry.

## ANSWERS

**(1)** $\frac{4}{3}ML^2$ **(2)** $\frac{ML^2}{2}$ **(3)** 9I **(4)** 1 : 4

**(5)** $\left[\frac{8}{3}-\frac{10\pi}{16}\right]MR^2$ **(6)** $\frac{R}{\sqrt{2}}$ **(7)** $M\ell^2/12$

**(8)** $I_1 = Mb^2/3,\ I_2 = M\ell^2/12, I_3 = Mb^2/12, I_4 = M\ell^2/3$

**(9)** $\frac{5}{3}MR^2$ **(10)** 2%

## TORQUE (OR MOMENT OF FORCE)

If a pivoted, hinged or suspended body tends to change in state of rotation under the action of a force, it is said to be acted on by a torque $\vec{\tau} = \vec{r} \times \vec{F}$

Unit of torque (N-m) cannot be written as joule, because joule is used specifically for work or energy.

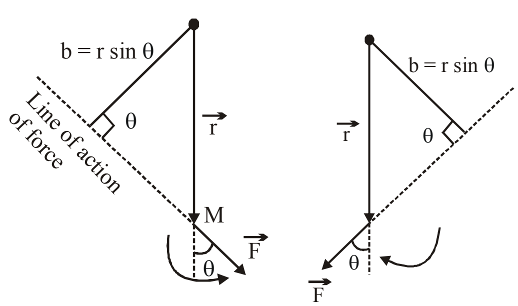
Unit : N-m (same as that of work or energy)

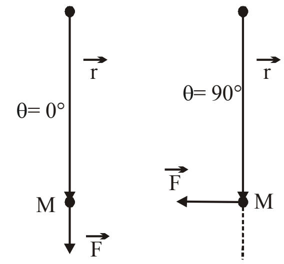
Dimensions : $[ML^2T^{-2}]$

* It is an axial vector, i.e. its direction is always perpendicular to the plane containing vector $\vec{r}$ and $\vec{F}$ .

* It's direction is determined by right hand screw rule.

* Positive sign to all torques acting to turn a body anti-clockwise and a minus to all torques tending to turn it clockwise.




**The magnitude of torque :**

Torque = Force × perpendicular distance of line of action of force from the axis of rotation.

$\tau = r\,F \sin\theta = F\,(r \sin\theta) = Fb$

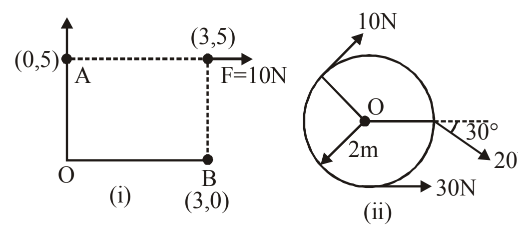
**Relation between torque and angular acceleration :**

$\tau = I\,\alpha$

For complete equilibrium, $\Sigma F = 0, \Sigma\tau = 0$

**Example 19 :**

Find out the torque about point A, O and B for fig. (i) & about O for fig. (ii).



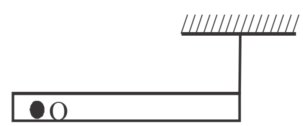

**Sol.** $\tau_A = 0$ ; $\tau_B = 10 \times 5 = 50$ N-m ; $\tau_0 = 10 \times 5 = 50$ N-m

Torque about O

$\tau = -10 \times 2 - 20 \sin 30° \times 2 + 30 \times 2 = 20$ N-m

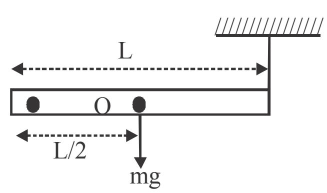
**Example 20 :**

A rod is pivoted at its one edge about point O. Other edge of rod is suspended from the ceiling through rope as shown. If the rope is suddenly cut then find the angular acceleration of rod.




**Sol.** When the rope is cut, weight of rod due to force of gravity will produce torque about point O.

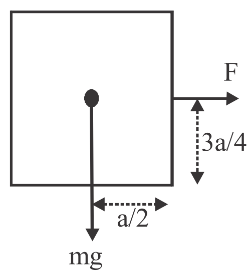
$\tau = I\alpha$, consider force amy acting as shown in figure at CM of rod (i.e. middle point of the rod)




$$mg\left[\frac{L}{2}\right] = \frac{mL^2}{3}\alpha \Rightarrow \frac{g}{2} = \frac{L\alpha}{3} \Rightarrow \alpha = \frac{3}{2}\frac{g}{L}$$

**Example 21 :**

What should be the value of force F for a block of mass m as shown, if it is toppled about O.

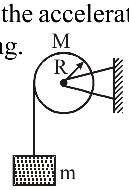



**Sol.** F tends to move the block clockwise while mg tends to move the block anticlockwise. If block topples then

$$F \times \frac{3a}{4} > mg \times \frac{a}{2} \Rightarrow F > \frac{2}{3}mg$$

**Example 22 :**

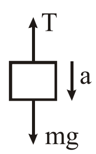
A uniform disc of radius R and mass M is free to rotate about a fixed axis perpendicular to its plane and passing through its centre. A string is wrapped over its rim and a block of mass m is attached to the free end of the string as shown in the figure. The block is released from rest. If string does not slip on the rim then find the acceleration of the block. Neglect the mass of the string.

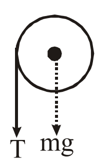



**Sol.** Since string does not slip on the disc hence tangential acceleration of the point on the rim which is in contact with the string is equal to the acceleration of the block.

Let angular acceleration of the disc about axis be $\alpha$, hence acceleration of the block $a = \alpha R$

**F.B.D. of the Block** **F.B.D. of the Disc.**




$\Rightarrow mg - T = m\alpha R$, as $a = \alpha R$ ...(i)

Torque on the disc is $\tau = \tau_{Tension} + \tau_{mg}$

$\Rightarrow I\alpha = TR$ as $\tau_{mg} = 0$

(where, I = M.I. of the disc about the axis)

$\Rightarrow T = I\alpha/R$ ...(ii)

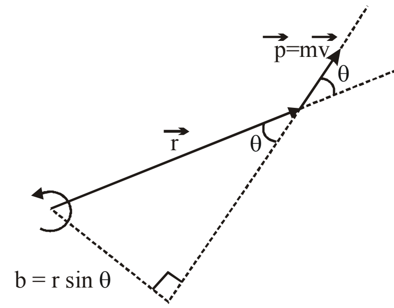
Eliminating T for (i) and (ii),

$$\alpha = \frac{mg}{\left(mR + \frac{I}{R}\right)} = \frac{2mg}{(2m+M)R}, \text{ as } I = \frac{MR^2}{2}$$

Hence $a = \dfrac{2mg}{2m+M}$.

**ANGULAR MOMENTUM**

Angular momentum of a body about a given axis is the product of its linear momentum and perpendicular distance of line of action of linear momentum vector from the axis of rotation.



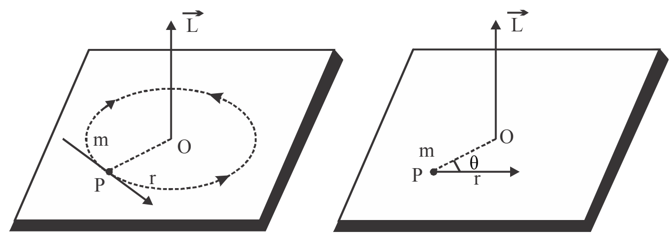

Angular momentum = Linear momentum × Perpendicular distance of line of action of momentum from the axis of rotation. $L = mv \times r\sin\theta \Rightarrow \vec{L} = \vec{r} \times \vec{p}$

Here $\vec{L}$ is the angular momentum of a moving particle abotu point O, $\vec{p}$ is the linear momentum of the particle & $\vec{r}$ is the position vector of the particle regarding the point.

Unit : S.I. J-sec or kg-$m^2$/sec

Dimensions : $[ML^2T^{-1}]$

Angular momentum is an axial vector.




As torque $(\vec{r} \times \vec{F})$ is defined as the 'normal of force', angular momentum is also define as moment of linear momentum. In Cartesian coordinates angular momentum :

$\vec{L} = (\vec{r} \times \vec{p}) = m(\vec{r} \times \vec{v})\ [\because \vec{p} = m\vec{v}]$

$$\left[ L = m \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix} \right]$$

$$\vec{L} = m[(x\hat{i} + y\hat{j} + z\hat{k}) \times (v_x\hat{i} + v_y\hat{j} + v_z\hat{k})]$$

i.e., $\vec{L} = m[\hat{i}(yv_z - zv_y) - \hat{j}(xv_z - zv_x) + \hat{k}(xv_y - yv_x)]$

Magnitude of the angular momentum, $L = mvr\sin\theta$

**(a)** For $\theta = 0°$ or $180°$ i.e. $\vec{r}$ and $\vec{v}$ are parallel or anti-parallel $\sin\theta = 0$ : L will be minimum.

If the axis of rotation is on the line of motion of moving particle then angular momentum is minimum and zero.

**(b)** For $\theta = 90°$, i.e. angular momentum is maximum when $\vec{r}$ and $\vec{v}$ are orthogonal $\sin\theta = 1$ : L will be maximum. i.e., in case of circular motion of a particle, angular momentum is maximum about centre of circle and is mvr.

As $|\vec{L}| = mvr\sin\theta$ so if the point is not on the line of motion i.e. $\theta \neq 0$ or $180°$ i.e., a particle in translatory motion always have an angular momentum unless the point is on the line of motion.

In case of circular motion of a particle

$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$ $\quad [\because \vec{p} = m\vec{v}]$

$\Rightarrow \vec{L} = mvr \sin 90\, \hat{n}\ [\because \theta = 90°]$

[Where $\hat{n}$ is a unit vector perpendicular to the plane of motion in accordance with right hand Screw rule]

or $\vec{L} = mr^2\omega\hat{n}\ [\because v = r\omega]$

or $\vec{L} = I\vec{\omega}\ [\because mr^2 = I\ \&\ \omega\hat{n} = \vec{\omega}]$

In case of circular motion, angular momentum is equal to the product of moment of inertia with angular velocity. This result is rotational analogue of $\vec{p} = m\vec{v}$

In case of rotational motion

$$L = I\omega\ \&\ E_R = \frac{1}{2}I\omega^2 \Rightarrow E_R = \frac{1}{2}I\left[\frac{L}{I}\right]^2 = \frac{L^2}{2I}$$

This result is rotational analogue of $E_T = \frac{p^2}{2m}$

In case of circular motion of a particle

$$\vec{L} = I\vec{\omega} \Rightarrow \frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} \qquad [\because \text{ I is const.}]$$

$$\frac{d\vec{L}}{dt} = I\vec{\alpha} = \vec{\tau} \qquad [\because \frac{d\omega}{dt} = \vec{\alpha} \text{ and } \vec{\tau} = I\vec{\alpha}\ ]$$

The rate of change of angular momentum is equal to the net torque acting on the particle. This expression is rotational analogue of $\frac{d\vec{p}}{dt} = \vec{F}$ and so also referred as newtons II law for rotational motion.

## ANGULAR IMPULSE

If a large torque acts on a body for a small time then

Angular impulse = $\vec{\tau}\, dt$

Angular impulse = $\vec{\tau}_{av.}\Delta t = \Delta\vec{L}$ = change in angular momentum. $\quad [\because \vec{\tau} = \frac{\Delta\vec{L}}{\Delta t}]$

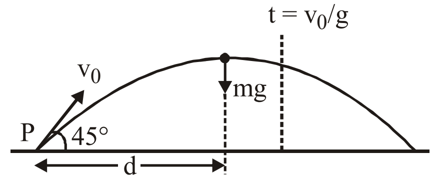
Action of angular impulse is to change the angular momentum. It has same unit, dimensions and direction as angular momentum.

**Example 23 :**

A particle of mass m is projected at t = 0 from the point P on the ground with speed $v_0$ at an angle of 45° to the horizontal. Find the magnitude and direction of the angular momentum of the particle at time $t = v_0/g$.

**Sol.** Only force acting on projectile (particle) is its weight mg so at time t torque of mg about P is

$$\tau = mgd = mg\frac{v_0}{\sqrt{2}} \times t$$




$\therefore$ change in angular momentum = angular impulse

$$\Delta L = \int_0^t \tau\, dt = \frac{mgv_0}{\sqrt{2}}\int_0^t \tau\, dt = \frac{mgv_0}{\sqrt{2}}\frac{t^2}{2} = \frac{1}{2\sqrt{2}}\frac{mv_0^3}{g}$$

Initial angular momentum about P is zero (line of action of linear momentum is passing through P) so angular momentum at time $t = \frac{v_0}{g}$ is $\frac{1}{2\sqrt{2}}\frac{mv_0^3}{g}$ and its direction is perpendicular to the plane of page inwards.

**Example 24 :**

The diameter of a solid disc is 0.5m and its mass is 16 kg. What torque will increase its angular velocity from zero to 120 rotations/minute in 8 seconds ?

**Sol.** Moment of inertia of solid disc

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 16 \times \left[\frac{0.5}{2}\right]^2 = \frac{1}{2}$$

Angular velocity, $\omega = \frac{120 \times 2\pi}{60} = 4\pi$

and change in angular momentum, $\Delta L = I\omega - 0 = I\omega$

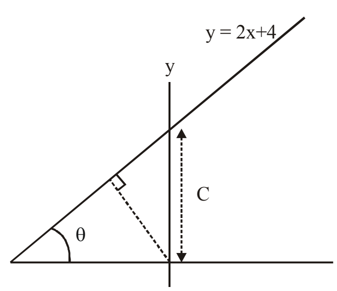
Angular impulse $\tau \times t = \Delta L$

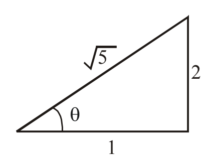
$$\Rightarrow \tau = \frac{\Delta L}{t} = \frac{1}{8} \times \frac{1}{2} \times 4\pi = \frac{\pi}{4}\text{ N-m}$$

**Example 25 :**

A particle having mass 5 kg is moving on a straight line $y = 2x + 4$ with velocity $3\sqrt{5}$ m/s. Find its angular momentum about origin.

**Sol.** $m = \tan\theta = 2$ and $c = 4$ $\qquad (y = mx + c)$



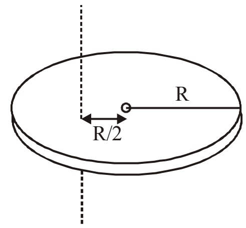




$$\frac{d}{4} = \cos\theta = \frac{1}{\sqrt{5}} \Rightarrow d = \frac{4}{\sqrt{5}}$$

$$\Rightarrow L = mvd = 5 \times 3\sqrt{5} \times \frac{4}{\sqrt{5}} = 60\text{ kg-m}^2\text{/sec.}$$

**Example 26 :**

A disc of mass M and radius R rotating at an angular velocity ω about an axis perpendicular to its plane at a distance R/2 from the centre, as shown in the figure. What is its angular momentum? The moment of inertia of a disc about the central axis is $\frac{1}{2}MR^2$.




The axis of rotation at a distance R/2 from the center of the disc.

**Sol.** The moment of inertia of the disc about the given axis may be found from the parallel axes theorem, equation

$I = I_{cm} + Mh^2$, where h is the distance between the given axis and a parallel axis throug the center of mass. Here, h = R/2 therefore,

$$I = \frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2 = \frac{3}{4}MR^2$$

The angular momentum is,

$$L = I\omega = \frac{3}{4}MR^2\omega .$$

## CONSERVATION OF ANGULAR MOMENTUM

$$\vec{\tau} = \frac{\Delta\vec{L}}{\Delta t} \Rightarrow \tau = \frac{L_f - L_i}{\Delta t} = \frac{I\omega_f - I\omega_i}{\Delta t} = \frac{I(\omega_f - \omega_i)}{\Delta t}$$

If the resultant external torque acting on a system is zero then the total angular momentum of the system remains constant.

If $\tau = 0$ then $\frac{\Delta L}{\Delta t} = 0 \Rightarrow L = \text{constant}$

$\Rightarrow L_f = L_i$ or $I_1\omega_1 = I_2\omega_2$

If a system is isolated from its surrounding i.e. any internal interaction between part of the system cannot alter its total angular momentum.

For example,

**(i)** If a person skating on ice folds his arms then his M.I. decreases and ω increases.

**(ii)** A diver jumping from a height folds his arms and legs (I decrease) in order to increase number of rotation in air by increasing ω.

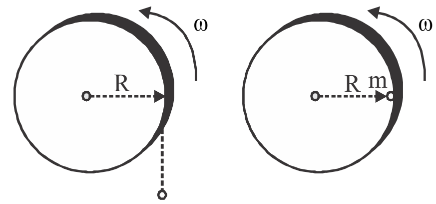
**(iii)** If a person moves towards the centre of rotating platform then I decrease and ω increase.

**Example 27 :**

A solid cylinder of mass M and radius R is rotating along its axis with angular velocity ω without friction. A particle of mass m moving with velocity v collide against the cylinder and sticks to its rim. After the impact calculate angular velocity of cylinder, and that will be initial and final kinetic energy?

**Sol.** Initial angular momentum of cylinder = Iω

Initial angular momentum of particle = mvR




Before striking total angular momentum,

$L_1 = I\omega + mvR$

After striking total angular momentum,

$L_2 = (I + mR^2)\,\omega'$

$L_1 = L_2 \Rightarrow (I + mR^2)\,\omega' = I\omega + mvR$

New angular velocity, $\omega' = \dfrac{I\omega + mvR}{I + mR^2}$

Initial kinetic energy of system $= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

Final kinetic energy of system $= \frac{1}{2}(I + mR^2)\,\omega'^2$

**Example 28 :**

Keeping the mass of earth constant. If its radius is halved then what will be the duration of the day will be.

**Sol.** $I_1\omega_1 = I_2\omega_2$

$$\Rightarrow \frac{2}{5}MR^2 \times \frac{2\pi}{T_1} = \frac{2}{5}M\left(\frac{R}{2}\right)^2 \times \frac{2\pi}{T_2}$$

$\Rightarrow T_2 = \frac{T_1}{4}$ $\quad\because T_1 = 24$ hr.

$\therefore T_2 = 6$ hr.

**Example 29 :**

Explain with reason why if ice melts at pole then moment of inertia of earth increases, angular velocity ω decrease and day will be longer.

**Sol.** If ice of the pole is melts then it will come towards the equator and moment of inertia of earth will increase because mass particles at equator are at more distant from rotational axis as compare to pole.

Time period (T) = 2π/ω

so if I increases then ω decreases $\Rightarrow$ T increases

Due to increment of time period the duration of day and night will increase.

**Example 30 :**

A rotating table has angular velocity ω and moment of inertia $I_1$. A person of mass stands on centre of rotating table. If the person moves a distance r along its radius then what will be the final angular velocity of rotating table.

**Sol.** Initial angular momentum = Final angular momentum

$$I_1\omega = (I_1 + mr^2)\omega_2 \Rightarrow \quad \omega_2 = \frac{I_1\omega}{I_1 + mr^2}$$

**ROTATIONAL KINETIC ENERGY**

* The energy due to rotational motion of a body is known as rotational kinetic energy.
* A rigid body is rotating about an axis with uniform angular velocity ω. The body is assumed to be composed of particles of masses $m_1$, $m_2$, .....

The linear velocity of the particles is

$v_1 = \omega r_1$, $v_2 = \omega r_2$, .........

therefore the kinetic energy of the rotating body is

$$KE_r = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + ........$$

$$KE_r = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + ........)\,\omega^2 = \frac{1}{2}I\omega^2$$

* It is a scalar quantity.
* Rotational kinetic energy

$$KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}I \times \frac{4\pi^2}{T^2} = \frac{1}{2}MK^2\omega^2$$

$$KE_r = \frac{1}{2}I \times \frac{v^2}{R^2} = \frac{L.\omega}{2} = \frac{L^2}{2I}$$

* If external torque acting on a body is equal to zero (τ = 0),

L = constant. $\quad E \propto \frac{1}{I} \propto \frac{1}{MK^2}$

**WORK ENERGY THEOREM IN ROTATIONAL MOTION**

The work done by torque

= Change in kinetic energy of rotation.

$$W = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$$

The change in the rotational kinetic energy of a rigid body equals the work done by torques exerted from outside the body. This equation is analogous to equation to work-energy theorem for a particle.

**Rotational power :** The power associated with the work done by a torque acting on a rotational body.

Divide both sides of equation dW = τ dθ by the time interval dt during which the angular displacement occurs.

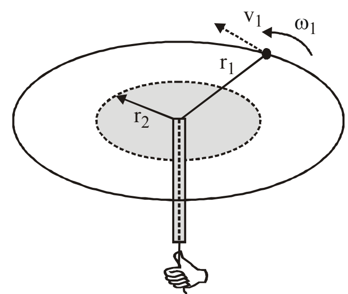
$$\frac{dW}{dt} = \tau\frac{d\theta}{dt} \quad \text{instantaneous rotational power } P_r = \tau\,\omega$$

**Example 31 :**

A point mass is tie to one end of a cord whose other end passes through a vertical hollow tube, caught in one hand. The point mass is being rotated in a horizontal circle of radius 2m with speed of 4 m/s. The cord is then pulled down so that the radius of the circle reduces to 1m. Compute the new linear and angular velocities of the point mass and compute the ratio of kinetic energies under the initial and final states.

**Sol.** The force on the point mass due to cord is radial and hence the torque about the centre of rotation is zero. Therefore, the angular momentum must remain constant as the chord is shortened.

Let mass of the particle is m and in the circle of radius $r_1$ linear velocity $v_1$ and angular velocity $\omega_1$. Further let in a circle of radius $r_2$ the linear velocity $v_2$ and angular velocity $\omega_2$.




∵ Initial angular momentum = Final angular momentum

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \quad mr_1^2\frac{v_1}{r_1} = mr_2^2\frac{v_2}{r_2} \Rightarrow r_1 v_1 = r_2 v_2$$

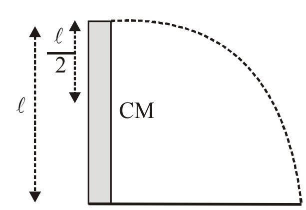
$$\therefore \; v_2 = \frac{r_1}{r_2}v_1 = \frac{2}{1}\times 4 = 8\text{ m/s} \;\&\; \omega_2 = \frac{v_2}{r_2} = \frac{8}{1} = 8\text{ rad/s}$$

$$\frac{\text{Final K.E.}}{\text{Initial K.E.}} = \frac{\frac{1}{2}I_2\omega_2^2}{\frac{1}{2}I_1\omega_1^2} = \frac{mr_2^2 \times \left[\frac{v_2}{r_2}\right]^2}{mr_1^2 \times \left[\frac{v_1}{r_1}\right]^2} = \frac{v_2^2}{v_1^2} = \frac{(8)^2}{(4)^2} = 4$$

**Example 32 :**

A thin meter scale is kept vertical by placing its one end on floor, keeping the end in contact stationary, it is allowed to fall. Calculate the velocity of its upper end when it hits the floor.

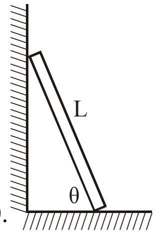
**Sol.** Loss in PE = gain in rotational KE




$$\frac{mg\ell}{2} = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{m\ell^2}{3}\times\frac{v^2}{\ell^2} \Rightarrow v = \sqrt{3g\ell}$$

**Example 33 :**

A uniform ladder of length L rests against a smooth frictionless wall. The floor is rough and the coefficient of static friction between the floor & ladder is $\mu$. When the ladder is positioned at angle $\theta$, it is just about to slip. What is $\theta$.




**Sol.** At equilibrium $\Sigma F = 0$,

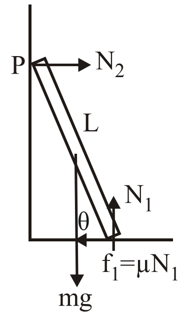
$\Rightarrow N_1 = mg$

$\Sigma\tau_p = 0$

$$\Rightarrow mg\frac{L}{2}\cos\theta + f_1 L\sin\theta = N_1 L\cos\theta$$

$$\frac{mg\cos\theta}{2}L + \mu mgL\sin\theta = mgL\cos\theta$$

$\tan\theta = 1/2\mu$



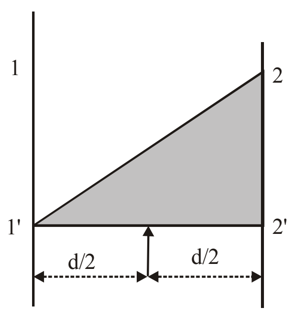

## TRY IT YOURSELF-3

**Q.1** If the kinetic energy of a body is increased by 300% then determine percentage increase in its angular momentum.

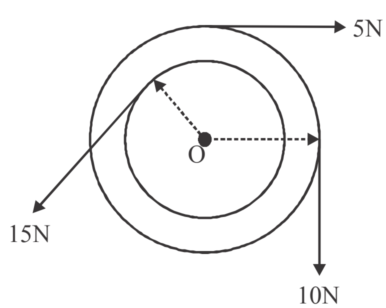
**Q.2** If the angular momentum of a body is increased by 200% then find increase in its rotational kinetic energy.

**Q.3** The power output of an automobile engine is advertised to be 200 hp to 600 rpm. What is the corresponding torque.

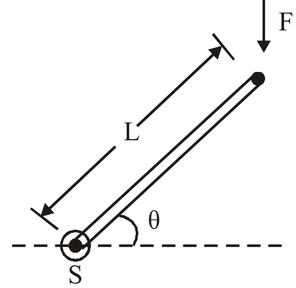
**Q.4** About which axis angular acceleration will be more in diagram. Force is applied at the middle point of triangular lamina.




**Q.5** In the following figure $r_1$ and $r_2$ are 10 cm. and 20 cm. respectively. If the moment of inertia of the wheel is 1500 kg-m$^2$, then determine its angular acceleration.



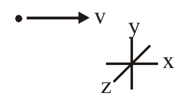

**Q.6** In the figure, a force of magnitude F is applied to one end of a lever of ength L. What is the magnitude of the torque about the point S?




(A) FL $\sin\theta$ (B) FL $\cos\theta$

(C) FL $\tan\theta$ (D) None of the above

**Q.7** In the situation where a particle is moving in the x-y plane with a constant velocity, the magnitude of the angular momentum $|\vec{L}_0|$ about the origin



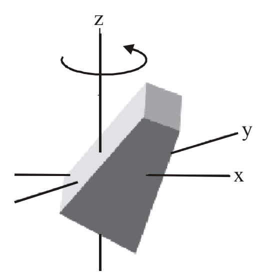

(A) decreases then increases

(B) increases then decrease

(C) is constant

(D) is zero because this is not circular motion.

**Q.8** A non-symmetric body rotates with an angular speed $\omega$ about the z axis. Relative to the origin




(A) $\vec{L}_0$ is constant.

(B) $|\vec{L}_0|$ is constant but $\vec{L}_0 / |\vec{L}_0|$ is not.

(C) $\vec{L}_0 / |\vec{L}_0|$ is constant but $|\vec{L}_0|$ is not.

(D) $\vec{L}_0$ has no z-component. .

**Q.9** You are trying to open a door that is stuck by pulling on the doorknob in a direction perpendicular to the door. Assume the doorknob is at the same height as the center of mass of the door. If instead you tie a rope to the door and then pull perpendicularly with the same force, the torque about the center of mass of the door you exert is

(A) increased (B) decreased

(C) the same (D) unsure

**Q.10** A man on a unicycle pedaling northward begins to slow down. The direction of the cycle's angular acceleration vector will be –

(A) North (B) South

(C) East (D) West

## ANSWERS

| | | | |
|---|---|---|---|
| **(1)** 100% | **(2)** 800% | **(3)** 237.5 N-m | |
| **(4)** $\alpha_{22'} > \alpha_{11'}$ | **(5)** $10^{-3}$ rad$^{-2}$ | **(6)** (B) | |
| **(7)** (C) | **(8)** (B) | **(9)** (C) | **(10)** (C) |

**COMPARISON BETWEEN FORMULA OF TRANSLATORY MOTION AND ROTATORY MOTION**

| Translatory motion | Rotatory motion |
|---|---|
| 1. $\vec{F}=\frac{d\vec{p}}{dt}, \vec{F}=m\vec{a}$ | 1. $\vec{\tau}=\frac{d\vec{L}}{dt}, \vec{\tau}=I\vec{\alpha}$ |
| 2. Linear momentum $(\vec{P}), \vec{p}=m\vec{v}$ | 2. Angular momentum $(\vec{L}), \vec{L}=I\vec{\omega}$ |
| 3. Linear kinetic energy, KE $=\frac{1}{2}mv^2$ | 3. Rotational kinetic energy, E $=\frac{1}{2}I\omega^2$ |
| 4. Work done $W=\vec{F}.\vec{S}$ (constant force) | 4. Work done $W=\vec{\tau}.\vec{\theta}$ (constant torque) |
| 5. Variable force $W=\int \vec{F}\cdot d\vec{s}$ | 5. Variable torque = $W=\int \vec{\tau}.\,d\vec{\theta}$ |
| 6. Power in linear motion, $P=\frac{dW}{dt}=\frac{\vec{F}.d\vec{s}}{dt}=\vec{F}.\vec{v}$ | 6. Power in rotational motion, $P=\frac{dW}{dt}=\frac{\vec{\tau}.\,d\vec{\theta}}{dt}=\vec{\tau}.\vec{\omega}$ |
| 7. Work energy theorem $W=\frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2$ | 7. Work energy theorem $W=\frac{1}{2}I\omega_2^2-\frac{1}{2}I\omega_1^2$ |
| 8. Linear impulse | 8. Angular impulse |
| It is produced a large force for small time $\vec{F}=\frac{d\vec{p}}{\Delta t}$ | It is product of large torque for small time $\vec{\tau}=\frac{\Delta\vec{L}}{\Delta t}$ |
| Impulse momentum theorem | Angular impulse momentum theorem |
| $\Delta\vec{p}=\vec{F}\Delta t=$ Impulse | $\Delta\vec{L}=\vec{\tau}\,\Delta t=$ Angular impulse |

**ROLLING MOTION**

It is a combination of translatory and rotational motion. In this type of motion axis of rotation is not stationary. If a body rotates about an axis with angular velocity $\omega$ then with respect to the axis of rotation linear velocity of any particle in the body at a position $\vec{r}$ from the axis of rotation is equal to $\vec{v}=\vec{\omega}\times\vec{r}$

If the axis of rotation also moves with velocity $\vec{v}_0$ then net velocity of the particle relative to stationary frame will be

$$\vec{V}=\vec{\omega}\times\vec{r}+\vec{v}_0.$$

If a regular rigid body (like sphere, disc or ring) is spined to a certain angular velocity and placed on a surface such that plane of rotation must be perpendicular to the surface and a velocity $\vec{v}_0$ is given to a centre of mass of the body then net velocity of the particle of the body at a distance r from the centre is equal to $\vec{\omega}\times\vec{r}+\vec{v}_0$.

"A body is said to be in pure rolling motion if relative velocity between the point of contacts is zero".

Pure rolling motion is a combination of pure translatory and pure rotational motion. It is equivalent to pure rotation about point of contact. If velocity of the surface on which the body has to roll is $\vec{v}_s$ and R be the radius of the body then, for rolling $\vec{\omega}\times\vec{R}+\vec{v}_0=\vec{v}_s$

If $\vec{v}_s=0$ then the condition for rolling is $v_0=\omega R$.

The steps for analyzing combined rotation and translation is as follows:

(i) List the external forces acting on the body.

(ii) The vector sum of external forces divided by the mass of the body gives the acceleration of the centre of mass.

(iii) Then find the torque of external forces and the moment of inertia of the body about a line through the centre of mass and perpendicular to the plane of motion of the particles. $a_{cm}=\frac{\sum F_{ext}}{M}$ ; $\alpha=\frac{\sum \tau_{ext}}{I}$.

If motion of the body is studied from non-inertial frame of reference having an acceleration a in a fixed direction with respect to an inertial frame, we have to apply a pseudo force (-ma) to each particle. These pseudo forces produce a pseudo torque about the axis. In such a case we do not hope $\tau_{ext}=I\alpha$ to hold.

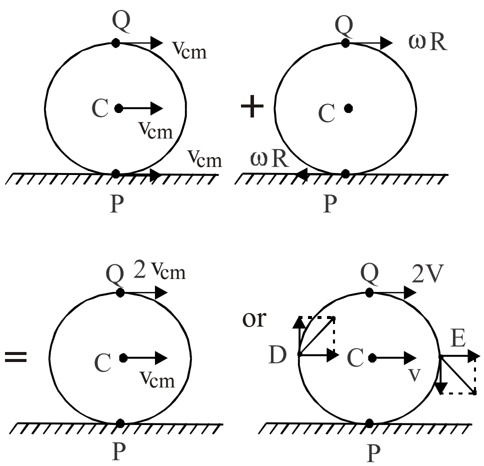
But there exists a very special and very useful case when $\tau_{ext}=I\alpha$ does hold even if the angular acceleration $\alpha$ is measured from a non-inertial frame A. That special case is when axis of rotation passes through the centre of mass. Take the origin at the centre of mass. The total torque of the pseudo forces is, $\sum \vec{r}_i\times(-m_i\vec{a})=-\left(\sum m_i\vec{r}_i\right)\times\vec{a}$

where $r_i$ is the position vector of the $i^{th}$ particle as measured from the centre of mass.

But, $\sum m_i \vec{r}_i = 0 \Rightarrow$ Pseudo torque is zero and we get,

$$\vec{\tau}_{ext} = I_{cm}\vec{\alpha}$$

How the translational motion and rotation motion about the centre of mass are superimposed to get the motion of a rigid body (say a disc of radius **R**) are illustrated in the following figure.




To get the instantaneous velocity of any point on the rigid body we calculate the instantaneous velocity of that point in the pure translation and in pure rotation and add then vectorially.

**Velocity of instantaneous point of contact P,**

= Velocity of point of contact in translation + velocity of instantaneous point of contact in pure rotation

= $v_{cm}$, in the forward direction + Rω in the forward direction

= $v_{cm}$ – Rω, in the forward direction.

**Velocity of the instantaneous top most point Q**

= Velocity of top most point in pure translation + velocity of instantaneous top most point in pure rotation

= $v_{cm}$ in the forward direction + Rω, in the forward direction

= $v_{cm}$ + Rω, in the forward direction.

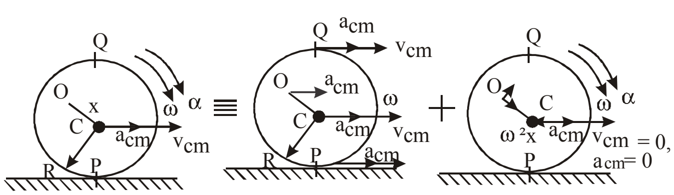
**Velocity of point O**

= Velocity of point in pure translation + Instantaneous velocity of point O in pure rotation

= $v_{cm}$ in the forward direction + xω, perpendicular to CO as shown in the figure.

= $\xrightarrow{v_{cm}} + x\omega$

The figure below illustrates how the instantaneous acceleration are superimposed:




**Acceleration of the instantaneous point of contact w.r.t. supporting surface**

= Acceleration of point of contact in pure translation + acceleration of instantaneous point of contact in pure rotation

= ($a_{cm}$ in the forwards direction) + (Rα, in the backward direction + $\omega^2$R, vertically upward towards centre of mass)

= $\xrightarrow{a_{cm}} + \xleftarrow{R\alpha} + \uparrow \omega^2 R$

**Acceleration of the instantaneous top most point**

= Acceleration of the top most point in pure translation + Acceleration of the instantaneous top most point in pure rotation

= ($a_{cm}$, in the forward direction)+(Rα, in the forward direction + $\omega^2$R, vertically downwards towards the centre of mass)

= $\xrightarrow{a_{cm}} + \xleftarrow{R\alpha} + \downarrow \omega^2 R$

**Acceleration of point O**

= Acceleration of point O in pure translation + Acceleration of point O in pure rotation

= ($a_{cm}$, in the forward direction) + (xα, perpendicular to CO as shown)+($\omega^2$x,along OC directed towards C)

= $\xrightarrow[a_{cm}]{} + x\alpha + \omega^2 R$

**Note :**

1. When the body on the surface, the frictional force on the body (if may) will be static in nature less than its limiting value, $f_s < \mu_s N$. The value and the direction of the friction can be obtained using $\Sigma F = m\,a_{cm}$, $\tau_{cm} = I_{cm}\alpha$, and given constants (basically equation relating the acceleration under rolling condition).
2. When the point of contact moves w.r.t. the supporting surface, $V_{pc} \neq 0$, the frictional force (if the surface is rough) is equal to $\mu_k N$, opposite to the direction of motion of point of contact w.r.t. the surface.
3. Rolling motion of a rigid body is also equivalent to pure rotation about the point of contact.

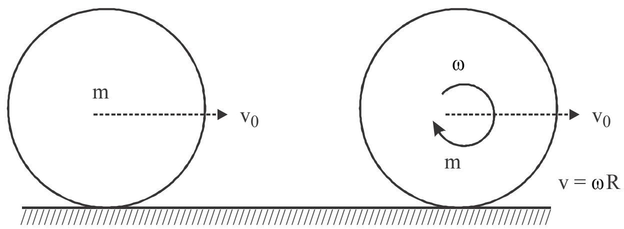
Total KE (in pure rolling) = $\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

$= \frac{1}{2}m\omega^2R^2 + \frac{1}{2}I_{cm}\omega^2$

$= \frac{1}{2}(mR^2 + I_{cm})\,\omega^2 = \frac{1}{2}I_{\text{Point of contact}}\,\omega^2$

**Example 34 :**

A body of radius R and mass m is placed on horizontal rough surface with linear velocity $v_0$, after some time it comes in the condition of pure rolling then determine

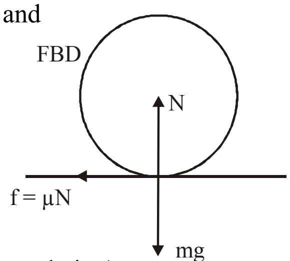



(i) Time t at which body starts pure rolling.
(ii) Linear velocity of body at time t.
(iii) Work done by frictional force in this time t.

**Sol.** For translatory motion : v = u + at

Initial velocity $u = v_0$

Let after time t pure rolling starts and at this t final velocity = v and acceleration = a.




From FBD :

Normal reaction N = mg

Friction force f = μN = μ mg

$\Rightarrow$ ma = μ mg [ ∵ f = ma]

Retardation

a = μg

$v = v_0 - gt$ (–ve sign for retardation)

$v = v_0 - \mu gt$ ........ (i)

For rotatory motion

$\omega = \omega_0 + \alpha t$ (Initial angular velocity $\omega_0 = 0$)

$\omega = \alpha t$ ........ (ii) $\quad \because \tau = I\alpha$

$$\Rightarrow \alpha = \frac{\tau}{I} = \frac{fR}{mK^2} = \frac{\mu mgR}{mK^2} \Rightarrow \alpha = \frac{\mu gR}{K^2} \quad ........ (iii)$$

From eq. (ii) and eq. (iii), $\omega = \dfrac{\mu gR}{K^2}t$ ........ (iv)

∵ For pure rolling, $v = \omega R \Rightarrow \omega = \dfrac{v}{R}$ ........ (v)

From eq. (iv) and (v), $\dfrac{v}{R} = \dfrac{\mu gR}{K^2}t$ or $v = \dfrac{\mu gR^2 t}{K^2}$ ........ (vi)

Substitute v from eq. (vi) into eq. (i)

$$\frac{\mu gR^2 t}{K^2} = v_0 - \mu gt \Rightarrow t = \frac{v_0}{\mu g\left[1 + \frac{R^2}{K^2}\right]}$$

Putting the value of n t in eq. (i)

$$v = v_0 - \mu g\frac{v_0}{\mu g\left[1 + \frac{R^2}{K^2}\right]} = v_0 - \frac{v_0}{1 + \frac{R^2}{K^2}} = \frac{v_0}{1 + \frac{K^2}{R^2}}$$

Velocity of body when pure rolling starts $v = \dfrac{v_0}{1 + \frac{K^2}{R^2}}$

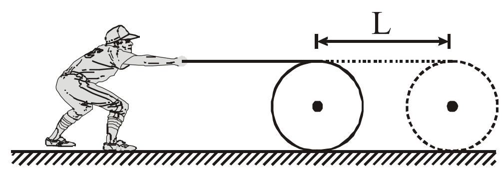
Work done in sliding by frictional force
= Initial kinetic energy – Final kinetic energy

Work done by friction, $W_f = \dfrac{1}{2}Mv_0^2 - \dfrac{1}{2}Mv^2\left(1 + \dfrac{K^2}{R^2}\right)$

$$= \frac{1}{2}Mv_0^2 - \frac{1}{2}\frac{Mv_0^2}{\left(1 + \frac{K^2}{R^2}\right)} = \frac{Mv_0^2}{2\left(1 + \frac{K^2}{R^2}\right)}$$

**Example 35 :**

A cylindrical drum, pushed along by a board rolls forward on the ground. There is no slipping at any contact. Find the distance moved by the man who is pushing the board, when axis of the cylinder covers a distance L.




**Sol.** Let $v_0$ be the linear speed of the axis of the cylinder and ω be its angular speed about the axis. As it does not slip on the ground, $\omega = \dfrac{v_0}{R}$, where R is the radius of the cylinder.

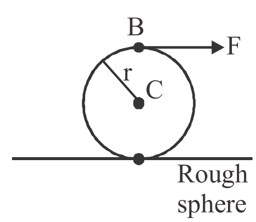
Speed of the topmost point is $v = v_0 + \omega R = 2v_0$

Since time taken by the axis to move a distance L is equal to $t = L/v_0$. In the same interval of time distance moved by the topmost point is $S = 2v_0 \times \dfrac{L}{v_0} = 2L$.

As there is no slipping between any point of contact hence distance moved by the man is 2L.

**Example 36 :**

A constant force F acts tangentially at the highest point of a uniform disc of mass m kept on a rough horizontal surface as shown in figure. If the disc rolls without slipping, calculate the acceleration of the centre (C)and point A and B of the disc.




**Sol.** The situation is shown in figure. As the force F rotates the disc, the point of contact has a tendency to slip towards left so that the static friction on the disc will act towards right. Let r be the radius of the disc and a be the linear acceleration of the centre of the disc. The angular acceleration about the centre of the disc is α = a/r, as there is no slipping.

For the linear motion of the centre,

$$F + f = ma \qquad \text{........ (1)}$$

and for the rotational motion about the centre,

$$Fr - fr = I\alpha = \left(\frac{1}{2}mr^2\right)\left(\frac{a}{r}\right) \text{ or } F - f = \frac{1}{2}ma \qquad \text{........ (2)}$$

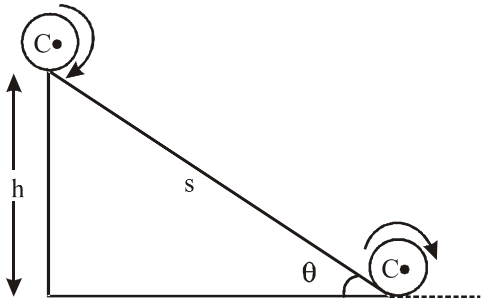
From (1) and (2), $2F = \frac{3}{2}ma$ or $a = \frac{4F}{3m}$

Acceleration of point A is zero.

Acceleration of point B is $2a = 2\left(\frac{4F}{3m}\right) = \frac{8F}{3m}$

**ROLLING MOTION ON AN INCLINED PLANE**

For pure rolling motion (pure translational + pure Rotational) on in inclined plane friction is must to provide the necessary torque.




It is interesting to observe work done by friction in pure rolling is zero (no sliding $\rightarrow$ no work against friction) hence PE at the top converts into KE ( $KE_r + KE_t$ ) at the bottom.

$$\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = mgh$$

Velocity, $v = \dfrac{(2gh)^{1/2}}{\left(1 + \dfrac{K^2}{R^2}\right)^{1/2}} = \dfrac{(2g\sin\theta)^{1/2}}{\left(1 + \dfrac{K^2}{R^2}\right)^{1/2}}$

$v^2 = 2as$ ; Acceleration $a = \dfrac{g\sin\theta}{\left(1 + \dfrac{K^2}{R^2}\right)}$ ; $s = \frac{1}{2}at^2$

Time $t = \sqrt{\dfrac{2s}{a}} = t = \sqrt{\dfrac{2s}{a}} = \dfrac{(2s)^{1/2}\left(1 + \dfrac{K^2}{R^2}\right)^{1/2}}{(g\sin\theta)^{1/2}}$

**Different forces acting on body :**

(i) Weight of the body vertically downward

(ii) Normal reaction perpendicular to the surface

(iii) Friction force (f) opposite to the direction of motion of body.

$$Mg\sin\theta - f = Ma$$

$$Mg\sin\theta - f = M\left[\frac{g\sin\theta}{1 + k^2/R^2}\right]$$

$$f = Mg\sin\theta\left[1 - \frac{1}{1 + k^2/R^2}\right]$$

$$\mu\,[Mg\cos\theta] = Mg\sin\theta\left[\frac{\frac{k^2}{R^2}}{1 + \frac{k^2}{R^2}}\right] \; ; \; \mu = \frac{\tan\theta}{1 + \frac{R^2}{k^2}}$$

For cylinder and disc $R^2/k^2 = 2$

Body slides without slipping on inclined plane, if

$$\mu_s \geq \frac{1}{3}\tan\theta$$

**Example 37 :**

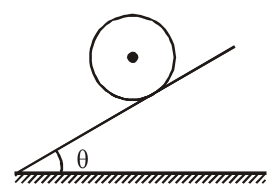
When a sphere of moment of inertia I rolls down an inclined plane. Find the percentage of rotational kinetic energy to the total kinetic energy.

**Sol.** $\dfrac{E_r}{E_T} \times 100 = \dfrac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2} \times 100 = \dfrac{\frac{2}{5}mr^2 \times \frac{v^2}{r^2}}{mv^2\left(1 + \frac{K^2}{r^2}\right)} \times 100$

$= 28\%$

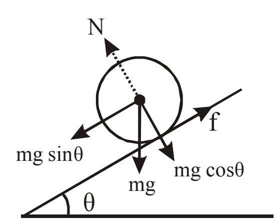
**Example 38 :**

A uniform disc of mass m and radius R is rolling without slipping up a rough inclined plane which makes an angle 30º with the horizontal. If the coefficient of static & kinetic friction are each equal to μ and the only force acting on the disc are gravitational and frictional, then find direction and magnitude of the frictional force acting on it.




**Sol.** Since disc does not slip hence frictional force is static and static friction can have any value between 0 and μN.

Component of mg parallel to the plane is mg sin θ which is opposite to the direction of motion of the centre of the disc, and hence speed of the centre of mass decreases.




For pure rolling the relation $v_{cm} = \omega R$ must be obeyed. Therefore ω must decrease. Only frictional force can provide a torque about the centre.

Torque due to friction must be opposite to the $\vec{\omega}$. Therefore frictional force will act up the plane

Now, for translational motion

$$mg\sin\theta - f = ma_{c.m.} \qquad \ldots(i)$$

For rotational motion

$fr = I\alpha$, where I = M.I. of the disc about centre.

$$= I\frac{a_{cm}}{R}, \text{ as } a = \alpha R \Rightarrow a_{c.m.} = \frac{f R^2}{I} \qquad \ldots\text{(ii)}$$

From (i) and (ii) we get,

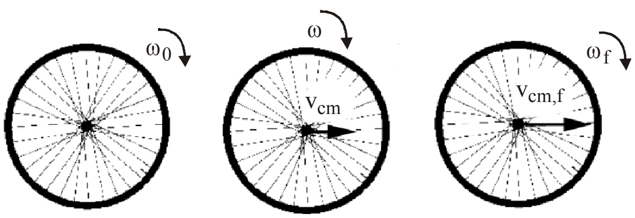
$$f = \frac{mg\sin\theta}{1 + \frac{mR^2}{I}}$$

Using eq. (i) and (ii), we get $f = mg/6$.

## TRY IT YOURSELF-4

**For Q.1-6**

Consider a bicycle wheel of radius R and mass m with moment of inertia $I_{cm}$ about an axis passing perpendicular to the plane of the wheel and through the center-of-mass. The bicycle wheel is initially spinning with angular velocity $\omega_0$ about the center-of-mass. The wheel is lowered to the ground without bouncing. As soon as the wheel touches the level ground, the wheel starts to accelerate forward until it begins to roll without slipping with a final angular velocity $\omega_f$ and center-of-mass velocity $v_{cm,f}$ .




**Q.1 Bicycle Wheel: Forces**

How many forces (in a frame fixed to the ground) are acting on the bicycle when it is translating and rotating but is not yet rolling without slipping?

(A) One (B) Two

(C) Three (D) Four

**Q.2 Bicycle Wheel: Direction of Kinetic Friction**

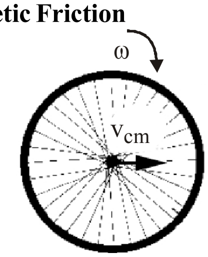
What is the direction of the kinetic friction force on the bicycle wheel in the figure?

(A) Points to the right

(B) Points to the left

(C) Points up

(D) Points down.



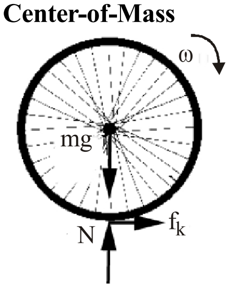

**Q.3 Bicycle Wheel: Torque about the Center-of-Mass**

Is the torque about the center-of-mass zero?

(A) Yes

(B) No

(C) Not sure



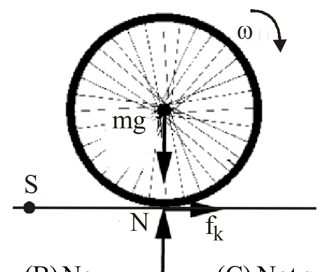

**Q.4 Bicycle Wheel: Angular Momentum about the Center-of-Mass**

Is the angular momentum constant about the center-of-mass?

(A) Yes (B) No (C) Not sure

**Q.5 Bicycle Wheel: Torque about S due to Friction Force**

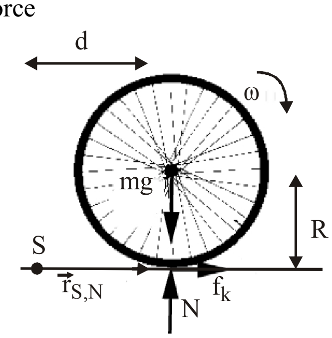
Consider the point S shown in the figure. S lies on the line of contact between the wheel and the ground. Is the torque about S due to the kinetic friction force zero?




(A) Yes (B) No (C) Not sure

**Q.6 Bicycle Wheel: Torque about S due to Normal Force**

Consider the point S shown in the figure. S lies on the line of contact between the wheel and the ground. Is the magnitude and direction of the torque about S due to the normal force



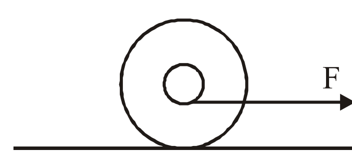

(A) $|\vec{\tau}_{S,N}| = Nd$ , out of the page.

(B) $|\vec{\tau}_{S,N}| = Nd$ , into the page.

(C) $|\vec{\tau}_{S,N}| = NR$ , out of the page.

(D) $|\vec{\tau}_{S,N}| = NR$ , into the page.

**Q.7** Two disks are separated by a spindle of smaller diameter. A string is wound around the spindle and pulled gently. In which direction does it roll?



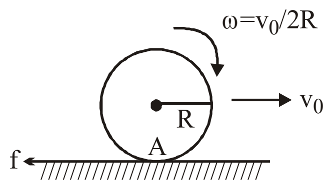

(A) To the right, in the direction of F, winding up the string

(B) To the left, opposite to F, unwinding the string.

(C) It does not roll, it slides to the right

(D) It does not roll, it slides to the left

**Q.8** A hollow sphere of mass M and radius R as shown in figure slips on a rough horizontal plane. At some instant it has linear velocity $v_0$ and angular velocity about the centre $v_0/2R$ as shown in figure. Calculate the linear velocity after the sphere starts pure rolling.




**Q.9** A sphere of mass m rolls without slipping on an inclined plane of inclination θ. What should be the minimum coefficient of static friction to support pure rolling ?

**Q.10** Sphere, disc, ring and solid cylinder of same radius on mass are placed on frictionless inclined plane. Comment on the order of time when they will reach the bottom of inclined plane.

## ANSWERS

| | | | |
|---|---|---|---|
| **(1)** (B) | **(2)** (A) | **(3)** (B) | **(4)** (B) |
| **(5)** (A) | **(6)** (A) | **(7)** (A) | **(8)** $4v_0/5$ |

**(9)** $\mu > \frac{2}{7}\tan\theta$ **(10)** All will reach simultaneoulsy.

## IMPORTANT POINTS

1. If C.M. of system is initially at rest and system is set free to move under the influence of resultant force of fixed line of action then system as whole may move in translatory as well as in rotatory motion but C.M. of system will move in translatory motion only in the direction of resultant force.

2. The angular momentum of a system of n particles about the origin is $L = \sum_{i=1}^{n} r_i \times p_i$

   The torque or moment of force on a system of n particles about the origin is $\tau = \sum_i r_i \times F_i$

   The force $F_i$ acting on the $i^{th}$ particle includes the external as well as internal forces. Assuming Newton's third law and that forces between any two particles act along the line joining the particles we can show $\tau_{int} = 0$ and $\frac{dL}{dt} = \tau_{ext}$

   The angular acceleration of a rigid body rotating about a fixed axis is given by $I\alpha = \tau$. If the external torque τ is zero, the component of angular momentum about the fixed axis Iα of such a rotating body is constant. The angular momentum L and the angular velocity ω are not necessarily parallel vectors. However, for the simpler situations discussed in this chapter when rotation is about a fixed axis which is an axis of symmetry of the rigid body, the relation L = Iω holds good, where I is the moment of the inertia of the body about the rotation axis.

3. The moment of inertia of a rigid body about an axis is defined by the formula $I = \sum m_i r_i^2$ where $r_i$ is the perpendicular distance of the $i^{th}$ point of the body from the axis. The kinetic energy of rotation is $K = \frac{1}{2}I\omega^2$

4. The theorem of parallel axes : $I_z' = I_z + Ma^2$, allows us to determine the moment of inertia of a rigid body about an axis as the sum of the moment of inertia of the body about a parallel axis through its centre of mass and the product of mass and square of the perpendicular distance between these two axes.

   Theorem of perpendicular Axes - This theorem states that the moment of inertia of a plane about an axis perpendicular to its plane is equal to the sum of its moments of inertia about any two mutually perpendicular axes in its plane and intersecting each other at the point where the perpendicular axis pass through it.

   Thus, $I_z = I_x + I_y$

5. For rolling motion without slipping $v_{cm} = R\omega$, where $v_{cm}$ is the velocity of translation (i.e. of the centre of mass), R is the radius and m is the mass of the body. The kinetic energy of such a rolling body is the sum of kinetic energies of translation and rotation: $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$

6. A rigid body is in mechanical equilibrium if
   (a) It is translational equilibrium i.e., the total external force on it is zero : $\Sigma F_i = 0$.
   (b) It is rotational equilibrium i.e., the total external torque on it is zero : $\Sigma \tau_i = \Sigma r_i \times F_i = 0$.

7. $\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$ ; $\vec{L} = I\vec{\omega}$ (for rotatory motion only)

   $\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} + \vec{\omega}\frac{dI}{dt}$. If I is constant $\frac{d\vec{L}}{dt} = I\vec{\alpha}$ ; $\vec{\tau} = I\vec{\alpha}$

   (applicable for rotatory motion with constant M.I.)

   If $\vec{\tau} = 0$ ; $\frac{d\vec{L}}{dt} = 0$ ; $d\vec{L} = 0$

   $\vec{L}$ =const.(conservation of A.M.)

8. If a body is released from rest on rough inclined plane, then for pure rolling $\mu_r \geq \frac{n}{n+1}\tan\theta$ $(I_c = nmr^2)$

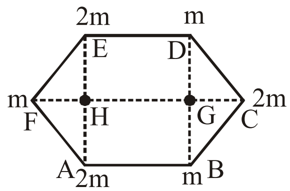
   Rolling with sliding

   $$0 < \mu_s < \left(\frac{n}{n+1}\right)\tan\theta \ ; \ \frac{g\sin\theta}{n+1} < a < g\sin\theta .$$

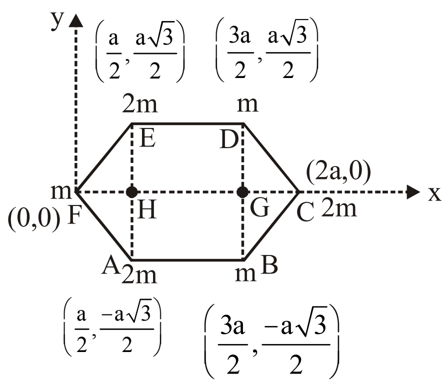
## ADDITIONAL EXAMPLES

**Example 1 :**

Find the position of centre of mass for a system of particles places at the vertices of a regular hexagon as shown in fig.




**Sol.**




$$x_{cm} = \frac{\Sigma\, m_i x_i}{\Sigma m_i}$$

$$= \frac{m \times 0 + 2m \times \frac{a}{2} + m \times \frac{3a}{2} + 2m \times 2a + m \times \frac{3a}{2} + 2m \times \frac{a}{2}}{m + 2m + m + 2m + m + 2m}$$

$$x_{cm} = a$$

$$y_{cm} = \frac{\Sigma\, m_i y_i}{\Sigma m_i}$$

$$= \frac{m \times 0 + 2m \times \frac{a\sqrt{3}}{2} + m \times \frac{a\sqrt{3}}{2} + 2m \times 0 + m \times \left(\frac{-a\sqrt{3}}{2}\right) + 2m \times \left(\frac{-a\sqrt{3}}{2}\right)}{9m}$$

$$y_{cm} = 0\ ; \quad \text{c.m.} : (a, 0)$$

**Example 2:**

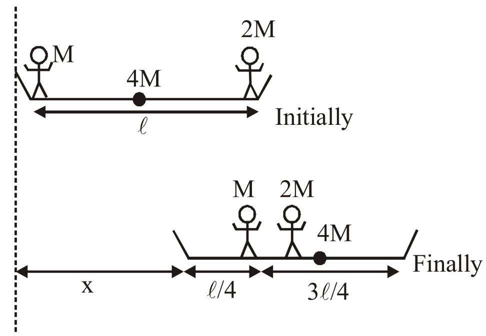
In a boat of mass 4 M and length $\ell$ on a frictionless water surface. Two men A (mass = M) and B (mass 2M) are standing on the two opposite ends. Now A travels a distance $\ell/4$ relative to boat towards its center and B moves a distance $3\ell/4$ relative to boat and meet A. Find the distance travelled by the boat on water till A and B meet.

**Sol.** Let x is distance travelled by boat.

Initial position of center of mass

$$= \frac{M_{Boat}X_{Boat} + M_A X_A + M_B X_B}{M_{Boat} + M_A + M_B}$$

$$= \frac{4M\,(\ell/2) + M.0 + 2M.\ell}{4M + M + 2M} = \frac{4M\ell}{7M} = \frac{4}{7}\ell$$




Final position of center of mass

$$= \frac{4M\left\{\frac{\ell}{2} + x\right\} + M\left\{x + \frac{\ell}{4}\right\} + 2M\left\{x + \frac{\ell}{4}\right\}}{7M}$$

$$= \frac{2M\ell + \frac{M\ell}{4} + \frac{M\ell}{2} + 7Mx}{7M} = \frac{\frac{11M\ell}{4} + 7x}{7}$$

Since there is no horizontal force, position of center of mass remains unchanged.

Center of mass initially = Center of mass finally

$$\Rightarrow \frac{4}{7}\ell = \frac{\frac{11\ell}{4} + 7x}{7} \Rightarrow 4\ell = \frac{11\ell}{4} + 7x \Rightarrow x = \frac{5\ell}{28}$$

**Example 3 :**

A circular disc of radius 2m is revolving at 240 rev/min. A torque is applied which makes it slow at constant rate $\pi$ radius/s$^2$ in what time disc completely stop ? Calculate how many revolutions the disc makes before it come to rest.

**Sol.** $\because \quad \omega_2 = \omega_1 - \alpha t \qquad \therefore\ 0 = 2\pi n - \alpha\, t$

$$\Rightarrow \ t = \frac{2\pi n}{\alpha} = \frac{2\pi \times 240}{\pi \times 60} = 8s$$

Now, $\because\ \omega_2^2 = \omega_1^2 - 2\alpha\theta \ \therefore \quad 0 = \omega_1^2 - 2\alpha\theta$

$$\Rightarrow \ \theta = \frac{\omega_1^2}{2\alpha} = \frac{4\pi^2 n^2}{2\alpha} = \frac{4\pi^2}{2\pi} \times \left(\frac{240}{60}\right)^2 = 32\pi$$

$$\Rightarrow \ \text{Number of revolutions} = \frac{\theta}{2\pi} = \frac{32\pi}{2\pi} = 16$$

**Example 4 :**

A cylinder of mass 5 kg and radius 30 cm. and free to rotate about its axis, receives an angular impulse of 3 kg $m^2s^{-1}$ initially followed by a similar impulse every 4s. What is the angular speed of the cylinder after 30s of the initial impulse? The cylinder is at rest initially.

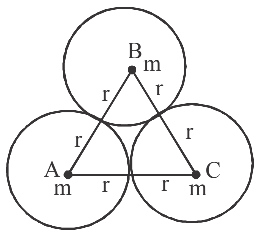
**Sol.** $\because$ Change in angular momentum = total angular impulse

$\therefore$ $I\omega = 8 \times 3 = 24$

$$\Rightarrow \quad \omega = \frac{24}{(MR^2/2)} = \frac{24}{\frac{5}{2}\times(0.3)^2} = 106.7 \text{ rad/s}$$

**Example 5 :**

Find the magnitude of force on sphere B. Also find moment of inertia of system about an axis passing through centre of any one of the sphere.




**Sol.** Force on B,

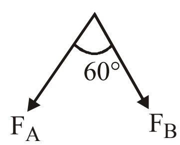
$$F_B = 2F\cos\left(\frac{60°}{2}\right) = 2\frac{GM^2}{(2r)^2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}Gm^2}{4r^2}$$

$F_A = F_C = F$

Moment of inetia (about an axis passing through A)

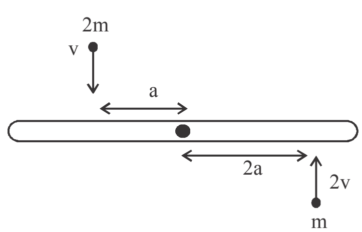
$$I = \frac{2}{5}mr^2 + 2\times\left[\frac{2}{5}mr^2 + m(2r)^2\right]$$

$$= \left(\frac{6}{5}+8\right)mr^2 = \frac{46}{5}mr^2$$




**Example 6 :**

A uniform rod of mass 8m and length 6a is lying on a horizontal table, two point masses m and 2m moving with speed 2v and v respectively strike the rod and stick to it as shown in figure then –




(a) Calculate the speed of centre of mass of rod after the collision.

(b) Calculate angular velocity of the rod abotu an axis passing through its centre of mass.

**Sol.** (a) Let velocity of centre of mass be $v_c$ then by conservation of linear momentum

$2mv - m(2v) = (8m + 2m + m)\,v_c \Rightarrow v_c = 0$

(b) By conservation of angular momentum

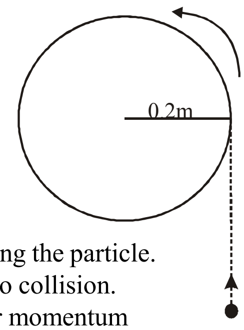
$2mav + m(2a)(2v) = I\omega$

$$= \left[8m\frac{(6a)^2}{12} + 2ma^2 + m(2a)^2\right]\omega$$

$\Rightarrow$ $\omega = v/5a$ anticlockwise.

**Example 7 :**

A solid cylinder of mass 2 kg and radius 0.2m is rotating with angular velocity 3 rad/s. A particle of mass 0.5 kg moving with velocity 5 m/s strike at the circumference and stick on it then calculate.




(a) Angular velocity after sticking the particle.

(b) Loss in kinetic energy due to collision.

**Sol.** (a) By conservation of angular momentum

$$I\omega + mvR = (I + mR^2)\,\omega'$$

$$\Rightarrow \omega' = \frac{I\omega + mvR}{I + mR^2} \Rightarrow \omega' = \frac{\frac{1}{2}MR^2\omega + mvR}{\frac{1}{2}MR^2 + mR^2} = 10.3 \text{ rad/s}$$

(b) Now, $(K.E.)_i = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$$= \frac{1}{2}\times\frac{1}{2}\times 2\times(0.2)^2\times(3)^3 + \frac{1}{2}\times(0.5)\times(5)^2 = 6.43 \text{ J}$$

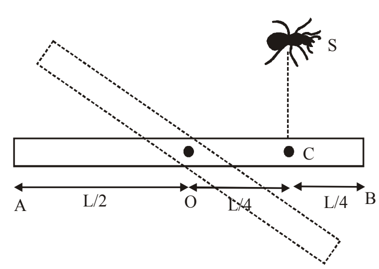
$$(K.E.)_f = \frac{1}{2}(I + \frac{1}{2}mR^2)\omega'^2$$

$$= \frac{1}{2}\times\left(\frac{1}{2}\times 2\times(0.2)^2 + (0.5)\times(0.2)^2\right)(10.3)^2 = 3.18 \text{ J}$$

$\Rightarrow$ $E_{loss} = KE_i - KE_f = 6.43 - 3.18 = 3.25$ J

**Example 8 :**

A homogeneous rod AB of length L and mass M is pivoted at centre O is such a way that it can rotate freely in the vertical plane as shown in figure. The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed v on the point C, midway between the points O and B. Immediately after falling determine angular velocity of rod.



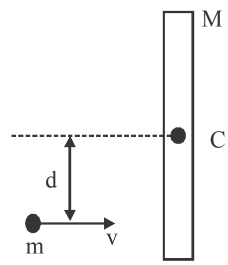
**Sol.** By applying conservation of angular momentum

$$mvr = I\omega \Rightarrow Mv\left(\frac{L}{4}\right) = \left[\frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2\right]\omega$$

$$\Rightarrow \; Mv\frac{L}{4} = \frac{7}{48}ML^2\omega \Rightarrow \omega = \frac{12v}{7L}$$

**Example 9 :**

A stick of length L and mass M lies on a frictionless horizontal surface on which it is free to move in anyway. A ball of mass m moving with speed v as shown in figure. What must be the mass of the ball so that it remains at rest immediately after collision.




**Sol.** Let velocity of CM of stick be V then by conservation of linear momentum $mv = m \times 0 + MV \Rightarrow V = \frac{mv}{M}$

By conservation of angular momentum

$$mvd = I\omega \Rightarrow \omega = \frac{mvd}{I}$$

By conservation of mechanical energy

$$\frac{1}{2}mv^2 = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

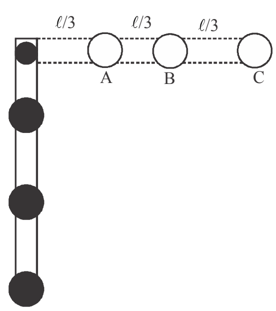
Substituting the value of V and ω

$$\frac{1}{2}mv^2 = \frac{1}{2}M\left(\frac{mv}{M}\right)^2 + \frac{1}{2}I\left(\frac{mvd}{I}\right)^2 \quad \text{Here, } I = \frac{ML^2}{12}$$

$$\text{So, } 1 = \frac{m}{M} + \frac{md^2}{ML^2/12} \Rightarrow m = \frac{ML^2}{(L^2 + 12d^2)}$$

**Example 10 :**

A light rod carries three equal masses A, B and C as shown in figure. What will be velocity of B in vertical position of rod, if it is released from horizontal position as shown in figure.




**Sol.** Loss in P.E. = Gain in K.E.

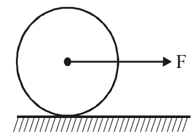
$$mg\frac{\ell}{3} + mg\left(\frac{2\ell}{3}\right) + mg\ell$$

$$= \frac{1}{2}\left(m\left(\frac{\ell}{3}\right)^2 + m\left(\frac{2\ell}{3}\right)^2 + m\ell^2\right)\omega^2$$

$$\Rightarrow \; \omega = \sqrt{\frac{36g}{14\ell}} \Rightarrow v_B = \omega\ell_B = \frac{2\ell}{3}\sqrt{\frac{36g}{14\ell}} = \sqrt{\frac{8g\ell}{7}}$$

**Example 11 :**

A horizontal force F acts on the sphere at its centre as shown. Coefficient of friction between ground and sphere μ. What is maximum value of F, for which there is no slipping ?



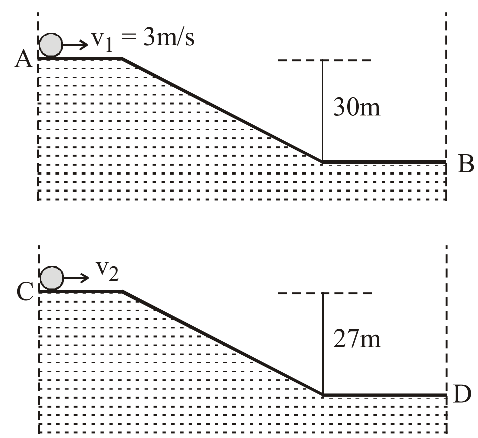

**Sol.** For linear motion, $F - f = Ma$
and for rotational motion, $\tau = I\alpha$

$$\Rightarrow \; f.R = \frac{2}{5}MR^2.\frac{a}{R} \Rightarrow f = \frac{2}{5}Ma \Rightarrow F - f = \frac{5}{2}f$$

$$\text{or } \; F = \frac{7}{2}f \qquad \because \; f \le \mu Mg \; \text{ so } \; F \le \frac{7}{2}\mu Mg$$

**Example 12 :**

Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A ... $_1$ and $v_2$, respectively, and always remain in contact with ths surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s, then $v_2$ in m/s is ($g = 10$ m/s$^2$)




**Sol.** 7. Final kinetic energy of both discs is same

$$\left[\frac{3}{2}\right]\frac{1}{2}m\,(3)^2 + mg\,(30) = \frac{3}{2}\frac{1}{2}mv_2^2 + mg\,(27)$$

$$\frac{3}{4}\cdot 9 + 300 = \frac{3}{4}v_2^2 + 270$$

$$\frac{27}{4} + 30 = \frac{3}{4}v_2^2 \Rightarrow v_2^2 = 9 + 40 \Rightarrow v_2 = 7$$

**Example 13 :**

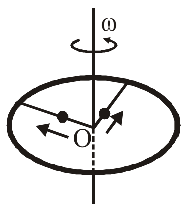
A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass M/8 at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is (8/9) ω and one of the masses is at a distance of (3/5) R from O. At this instant the distance of the other mass from O is :

(A) (2/3) R

(B) (1/3) R

(C) (3/5) R

(D) (4/5) R



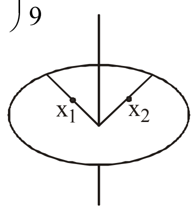

**Sol.** **(D).** By conservation of angular momentum

$$MR^2\omega = \left(MR^2 + \frac{M}{8}\frac{9R^2}{25} + \frac{Md^2}{8}\right)\frac{8\omega}{9}$$

$$R^2 = \left(\frac{200R^2 + 9R^2 + 25d^2}{8 \times 25}\right)\frac{8}{9}$$

$$225\,R^2 - 209\,R^2 = 25\,d^2$$

$$d = \frac{16R^2}{25} = \frac{4R}{5}$$



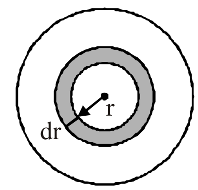

**Example 14 :**

The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k(r/R)$ and $\rho_B(r) = k(r/R)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are $I_A$ and $I_B$, respectively, If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is –

**Sol.** **6.** Consider a shell of radius r and thickness dr

$$dI = \frac{2}{3}(\rho \cdot 4\pi r^2 dr)\,r^2$$

$$I = \int dI$$




$$\frac{I_B}{I_A} = \frac{\int_0^R \frac{2}{3}k\frac{r^5}{R^5}.4\pi r^2 dr\, r^2}{\int_0^R \frac{2}{3}k\frac{r}{R}.4\pi r^2 dr\, r^2} = \frac{6}{10}$$

# QUESTION BANK

# CHAPTER 8 : ROTATIONAL MOTION

## EXERCISE - 1 [LEVEL-1]

## PART - 1 : POSITION OF CENTRE OF MASS

**Q.1** A system consists of mass M and m(<M). The centre of mass of the system is –
(A) At the middle
(B) Nearer to M
(C) Nearer to m
(D) At the position of large mass

**Q.2** For which of the following does the centre of mass lie outside the body ?
(A) A pencil (B) A shotput
(C) A dice (D) A bangle

**Q.3** A circle of radius R is cut from a uniform thin sheet of metal. A circular hole of radius R/2 is now cut out of the circle, with the hole tangent to the rim. Find the centre of mass from the center of the original uncut circle to the CM.
(A) (0, R/6) (B) (0, –R/3)
(C) (0, –R/6) (D) (0, –R/2)

**Q.4** If the linear density of the rod of length L varies as $\lambda = A + Bx$, then its centre of mass is given by –
(A) $\frac{L\,(2A+BL)}{3\,(3A+2BL)}$ (B) $\frac{L\,(3A+2BL)}{3\,(2A+BL)}$
(C) $\frac{L\,(3A+2BL)}{3}$ (D) $\frac{L\,(2A+3BL)}{3}$

## PART - 2 : MOMENTUM CONSERVATION AND CENTRE OF MASS MOTION

**Q.5** If no external force acts on a system. Choose the INCORRECT statement.
(A) Velocity of centre of mass remains constant.
(B) Velocity of centre of mass is not constant.
(C) Velocity of centre of mass may be zero.
(D) Acceleration of centre of mass is zero.

**Q.6** A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to other end of the plank. If the mass of the plank is M/3 then the distance that the man moves relative to ground is
(A) 3L/4 (B) L/4
(C) 4L/5 (D) L/3

**Q.7** A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time?
(A) 11.2 m (B) 13.8 m
(C) 14.3 m (D) 15.4 m

**Q.8** Consider a system of two particles having masses $m_1$ and $m_2$. If the particle of mass $m_1$ is pushed towards the mass centre of particles through a distance d, by what distance would the particle of mass $m_2$ move so as to keep the mass centre of particles at the original position
(A) $\frac{m_2}{m_1}d$ (B) $\frac{m_1}{m_1+m_2}d$
(C) $\frac{m_1}{m_2}d$ (D) d

**Q.9** Two blocks of masses $m_1$ and $m_2$ are connected by a light inextensible string passing over a smooth fixed pulley of negligible mass. Find the acceleration of the centre of mass of the system when blocks move under gravity.
(A) $\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 g$ (B) $\left(\frac{m_1-m_2}{m_1+m_2}\right) g$
(C) $\left(\frac{m_1+m_2}{m_1-m_2}\right)^2 g$ (D) $\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 2g$

**Q.10** Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speed of 2m/sec and 6 m/sec respectively on a smooth horizontal surface. Find the speed of centre of mass of the system.
(A) 3.33 m/sec (B) 2.33 m/sec
(C) 1.33 m/sec (D) 9.33 m/sec

**Q.11** Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration $\vec{a}$. The centre of mass has an acceleration –
(A) zero (B) $\vec{a}/2$
(C) $\vec{a}$ (D) $2\vec{a}$

**Q.12** An isolated particle of mass m is moving in horizontal plane (x-y), along the x-axis, at a certain height above the ground. It suddenly explodes into two fragment of masses m/4 and 3m/4. An instant later, the smaller fragment is at y = +15 cm. The larger fragment at this instant is at –
(A) y = –5cm (B) y = + 20 cm
(C) y = + 5 cm (D) y = – 20 cm

## PART - 3 : MOMENT OF INERTIA

**Q.13** Moment of inertia of HCl molecule about self axis having bond length r and masses of H and Cl atoms $m_H$ and $m_{Cl}$ will be –
(A) $\frac{m_H.m_{Cl}}{m_H+m_{Cl}}r^2$ (B) $\frac{m_H+m_{Cl}}{m_H.m_{Cl}}r^2$
(C) $\frac{2m_H.m_{Cl}}{m_H+m_{Cl}}r^2$ (D) $\frac{m_H+m_{Cl}}{2m_H+m_{Cl}}r^2$

**Q.14** The diameter of flywheel increases by 1%. The percentage increase in moment of inertia about axis of symmetry will be-

(A) 1 % (B) 2%
(C) 3% (D) 4%

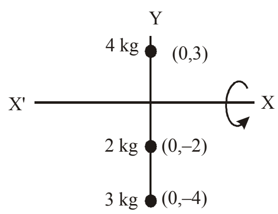
**Q.15** The moment of inertia of sphere is 20 kg-m$^2$ about the diameter. The moment of inertia about any tangent will be-

(A) 70 kg-m$^2$ (B) 35 kg-m$^2$
(C) 50 kg-m$^2$ (D) 20 kg-m$^2$

**Q.16** If the moment of inertia of a disc about an axis tangentially and parallel to its surface be I, what will be the moment of inertia about the axis tangential but perpendicular to the surface-

(A) 6I/5 (B) 3I/4
(C) 3I/2 (D) 5I/4

**Q.17** Calculate moment of inertia w.r.t. rotational axis XX' in figure.




(A) 1 kg-m$^2$ (B) 90 kg-m$^2$
(C) 92 kg-m$^2$ (D) 3 kg-m$^2$

**Q.18** If the radius of solid sphere is 35 cm. The ratio of radius of gyration, when the axis is along a diameter to that when the axis is along a tangent will be-

(A) $\sqrt{\frac{10}{35}}$ (B) $\sqrt{\frac{35}{10}}$
(C) 7/1 (D) 1/7

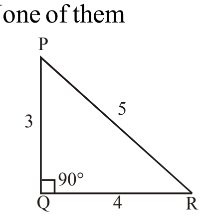
**Q.19** Two circular discs A and B of equal masses and thickness but made of metals with densities $d_A$ and $d_B$ ($d_A > d_B$). If their moments of inertia about an axis passing through the centre and normal to the circular faces be $I_A$ and $I_B$, then-

(A) $I_A = I_B$ (B) $I_A > I_B$
(C) $I_A < I_B$ (D) $I_A \geq I_B$

**Q.20** Three point masses of 1 kg, 2kg and 4 kg are placed at position of (2, 0), (3, 0) and (4, 0) respectively in X-Y plane. Calculate moment of inertia about X-axis :-

(A) 32 kg m$^2$ (B) 95 kg m$^2$
(C) 70 kg m$^2$ (D) None of them

**Q.21** PQR is a right angled triangular plate of uniform thickness as shown in the figure. If $I_1$, $I_2$ and $I_3$ are moments of inertia about PQ, QR and PR axes respectively, then

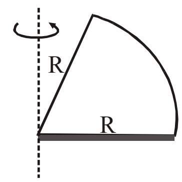



(A) $I_3 < I_2 < I_1$ (B) $I_1 = I_2 = I_3$
(C) $I_2 > I_1 > I_3$ (D) $I_3 > I_1 > I_2$

**Q.22** The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is

(A) $1 : \sqrt{2}$ (B) 1 : 3
(C) 2 : 1 (D) $\sqrt{5} : \sqrt{6}$

**Q.23** One quarter section is cut from a uniform circular disc of radius R. This section has a mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is :

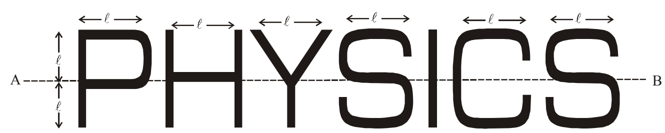



(A) $\frac{1}{2}MR^2$ (B) $\frac{1}{4}MR^2$ (C) $\frac{1}{8}MR^2$ (D) $\sqrt{2}MR^2$

**Q.24** The moment of inertia of a disc, of mass M and radius R, about an axis which is a tangent and parallel to its diameter is –

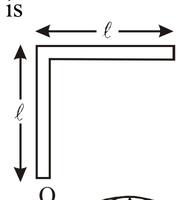
(A) (1/2) MR$^2$ (B) (3/4) MR$^2$
(C) (1/4) MR$^2$ (D) (5/4) MR$^2$

**Q.25** Find out the moment of inertia of the following structure (written as (PHYSICS) about axis AB made of thin uniform rods of mass per unit length λ.



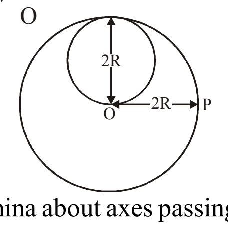

(A) 131 $\ell^3$ (B) 101 $\ell^3$
(C) 71 $\ell^3$ (D) 111 $\ell^3$

**Q.26** Two thin rods of mass m and length $\ell$ each are joined to form L shape as shown. The moment of inertia of rods about an axis passing through free end (O) of a rod and perpendicular to both the rods is




(A) $\frac{2}{7} m\ell^2$ (B) $\frac{m\ell^2}{6}$
(C) $m\ell^2$ (D) $\frac{5m\ell^2}{3}$

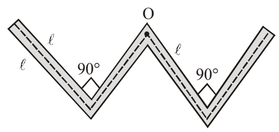
**Q.27** A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure.




The moment of inertia of this lamina about axes passing through O and P is $I_O$ and $I_P$ respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $I_P/I_O$ to the nearest integer is

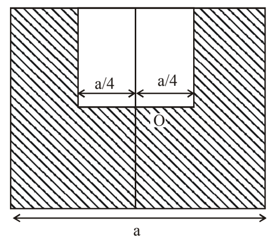
(A) 1 (B) 2
(C) 3 (D) 4

**Q.28** A thin rod of length $4\ell$, mass 4m is bent at the points as shown in the figure. What is the moment of inertia of the rod about the axis passing point O and perpendicular to the plane of the paper.



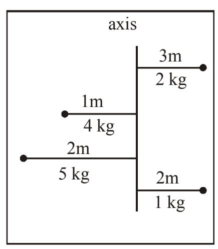

(A) $\frac{m\ell^2}{3}$ (B) $\frac{10m\ell^2}{3}$ (C) $\frac{m\ell^2}{12}$ (D) $\frac{m\ell^2}{24}$

**Q.29** A square plate of edge a/2 is cut out from a uniform square plate of edge 'a' as shown in figure. The mass of the remaining portion is M. Find the moment of inertia of the shaded portion about an axis passing through O (centre of the square of side a) and perpendicular to plane of the plate.




(A) $\frac{9}{16}Ma^2$ (B) $\frac{3}{16}Ma^2$ (C) $\frac{5}{12}Ma^2$ (D) $\frac{Ma^2}{6}$

**Q.30** The radius of gyration in the following figure will be




(A) $\sqrt{\frac{45}{12}}$ (B) $\sqrt{\frac{23}{6}}$

(C) $\sqrt{\frac{11}{3}}$ (D) 2

## PART - 4 : TORQUE AND EQUILIBRIUM OF RIGID BODIES

**Q.31** Given that, $\vec{r} = 2\hat{i} + 3\hat{j}$ and $\vec{F} = 2\hat{i} + 6\hat{k}$. The magnitude of torque (N-m) will be-

(A) $\sqrt{405}$ (B) $\sqrt{410}$

(C) $\sqrt{504}$ (D) $\sqrt{510}$

**Q.32** A constant torque acting on a uniform circular wheel changes its angular momentum from $A_0$ to $4A_0$ in 4 sec. The value of torque will be-

(A) $4A_0$ (B) $12A_0$

(C) $A_0$ (D) $3A_0/4$

**Q.33** A wheel having moment of inertia 2 kg-m$^2$ about its vertical axis, rotates at the ratio of 60rpm about this axis. The torque which can stop the wheel's rotation in one minute would be –

(A) $\frac{\pi}{18}$ N-m (B) $\frac{2\pi}{15}$ N-m

(C) $\frac{\pi}{12}$ N-m (D) $\frac{\pi}{15}$ N-m

**Q.34** Moment of inertia of a body about an axis is 4kg-m$^2$. The body is initially at rest and a torque of 8 N-m starts acting on it along the same axis. Work done by the torque in 20 s, in joules, is

(A) 40 (B) 640

(C) 2560 (D) 3200

**Q.35** The rotational analogue of force in linear motion is :

(A) torque (B) weight

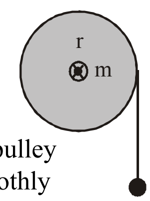
(C) moment of inertia (D) angular momentum

**Q.36** A force $\vec{F} = -10\hat{k}$ N acts on the origin of Cartesian coordinate system. The torque about the point (1, – 1) is

(A) $10\hat{i} - 10\hat{j}$ (B) $-10\hat{i} + 10\hat{j}$

(C) $10\hat{i} + 10\hat{j}$ (D) $-10\hat{i} - 10\hat{j}$

**Q.37** A uniform disc of mass m and radius r and a point mass m are arranged as shown in the figure.




The acceleration of point mass is:

(Assume there is no slipping between pulley and thread and the disc can rotate smoothly about a fixed horizontal axis passing through its centre and perpendicular to its plane)

(A) g/2 (B) g/3

(C) 2g/3 (D) none of these

**Q.38** You are trying to open a door that is stuck by pulling on the doorknob in a direction perpendicular to the door. Assume the doorknob is at the same height as the center of mass of the door. If instead you tie a rope to the door and then pull perpendicularly with the same force, the torque about the center of mass of the door you exert is

(A) increased (B) decreased

(C) the same (D) unsure

**Q.39** A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?

(A) 66gm (B) 56gm

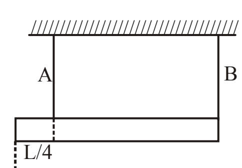
(C) 76gm (D) 46gm

**Q.40** Following objects each having same mass and same radius are rotated about their respective self axes. Which will have greatest angular acceleration if same tangential force is applied on each :-

(A) Disc (B) Ring

(C) Solid sphere (D) Hollow sphere

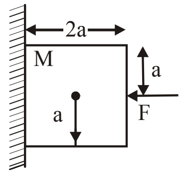
**Q.41** A uniform rod of mass M and length L is horizontally suspended from the ceiling by two vertical light cables as shown. Cable A is connected 1/4 th distance from the left end. Cable B is attached at right end. What is the tension is cable A –




(A) $\frac{Mg}{4}$ (B) $\frac{Mg}{3}$

(C) $\frac{2Mg}{3}$ (D) $\frac{3Mg}{4}$

**Q.42** In the figure shown, a cubical block is held stationary against a rough wall by applying a force 'F' then incorrect statement among the following is –
(A) Frictional force, f = Mg
(B) F = N, N is normal reaction
(C) F does not apply any torque
(D) N does not apply any torque



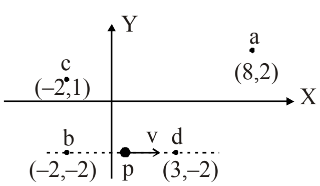

**Q.43** A uniform thin rod of mass 'm' and length L is held horizontally by two vertical strings attached to the two ends. One of the string is cut. Find the angular acceleration soon after it is cut –
(A) g/2L (B) g/L
(C) 3g/2L (D) 2g/L

## PART - 5 : ANGULAR MOMENTUM

**Q.44** The z-component of angular momentum in terms of linear momentum will be-
(A) $J_z = xp_y - yp_x$ (B) $J_z = yp_y - xp_x$
(C) $J_z = zp_y - yp_z$ (D) $J_z = zp_x - xp_z$

**Q.45** A body of mass 1.0 kg is rotating on a circular path of diameter 2.0 m at the rate of 10 rotations in 31.4 sec. The angular momentum of the body is- (in kg. $m^2$/s)
(A) 3 (B) 4
(C) 2 (D) 1

**Q.46** Figure shows a particle P moving with constant velocity V along the positive x-axis and four points a, b, c and d with their x and y coordinates. If $L_1, L_2, L_3$ and $L_4$ are the magnitudes of angular momentum of the particle about the points a, b, c and d respectively then which of the following is INCORRECT :




(A) $L_1 > L_3$ (B) $L_2 = L_4$
(C) $L_1 = L_3$ (D) $L_1 > L_2$

**Q.47** A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically) below the particle on the ground. Then pickup the incorrect alternative –
(A) Angular momentum of the particle about O is increasing
(B) Torque of the gravitational force on the particle about O is decreasing.
(C) The moment of inertia of the particle about O is decreasing.
(D) The angular velocity of the particle about O is increasing.

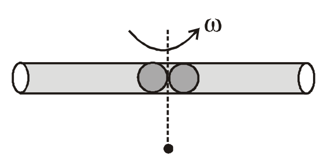
**Q.48** The angular momentum of the earth rotating about its own axis will be- (Mass of earth = $5.98 \times 10^{27}$ gm and mean radius R of earth = $9.37 \times 10^6$ m)
(A) $1.53 \times 10^{34}$ kg $m^2$/sec (B) $1.23 \times 10^{34}$ kg $m^2$/sec
(C) $2.53 \times 10^{34}$ kg $m^2$/sec (D) $1.51 \times 10^{34}$ kg $m^2$/sec

**Q.49** A particle is moving in x-y plane and the components of its velocity along x and y axis are $V_x$ and $V_y$. The angular momentum about the origin will be-
(A) $m\,\hat{k}\left(xV_y - yV_X\right)$ (B) $\frac{\hat{k}}{2}\left(xV_y - yV_X\right)$
(C) $m\,\hat{k}\,\sqrt{xV_y - yV_X}$ (D) $\frac{\hat{k}}{2}\,\sqrt{xV_y - yV_X}$

## PART - 6 : CONSERVATION OF ANGULAR MOMENTUM

**Q.50** A cockroach of mass m is moving on rim of a disc of radius r with velocity v in anticlockwise direction. The moment of inertia of the disc about its own axis is I and it is rotating in the clockwise direction with angular speed ω. If the cockroach stops moving then the angular speed of the disc will be-
(A) $\frac{I\omega}{I + mR^2}$ (B) $\frac{I\omega + mvr}{I + mr^2}$
(C) $\frac{I\omega - mvr}{I + mr^2}$ (D) $\frac{I\omega - mvr}{I}$

**Q.51** A smooth tube of certain mass is rotated in gravity free space and released. The two balls shown in the figure move towards ends of the tube. For the whole system which of the following quantity is not conserved –




(A) Angular momentum (B) Linear momentum
(C) Kinetic energy (D) Angular speed

**Q.52** A disc is rotating with angular velocity ω. If a child sits on it, what is conserved ?
(A) linear momentum (B) angular momentum
(C) kinetic energy (D) moment of inertia

**Q.53** A rotating star has a period of 30 days about an axis passing through its centre. The star undergoes an internal explosion and converts to a neutron star. Initial radius of the core was $1.0 \times 10^4$ km, whereas final radius is 3.0km. Determine the period of rotation of the neutron star.
(A) $2.7 \times 10^{-6}$ days (B) $1.7 \times 10^{-6}$ days
(C) $4.1 \times 10^{-6}$ days (D) $5.2 \times 10^{-6}$ days

## PART - 7 : ROTATIONAL KINETIC ENERGY AND WORK ENERGY THEOREM

**Q.54** A uniform thin rod of length $\ell$ is suspended from one of its ends and is rotated at f rotations per second. The rotational kinetic energy of the rod will be-
(A) $(2/3)\,\pi^2 f^2 m\ell^2$ (B) $(4/3)\,f^2 m\ell^2$
(C) $4\pi^2 f^2 m\ell^2$ (D) 0

**Q.55** The moments of inertia of two rotating bodies A and B are $I_A$ and $I_B$ $(I_A > I_B)$ and their angular momentum are equal. If their kinetic energies be $K_A$ and $K_B$, respectively, then-

(A) $\frac{K_A}{K_B} > 1$ (B) $\frac{K_B}{K_A} > 1$

(C) $\frac{K_A}{K_B} = 1$ (D) $\frac{K_A}{K_B} = \frac{1}{2}$

**Q.56** The moment of inertia of a body about a given axis 1.2 kg-m$^2$. Initially, the body is at rest. In order to produce a rotational K.E. of 1500 joule, an angular acceleration of 25 rad/s$^2$ must be applied about that axis for a duration of-

(A) 4 s (B) 2 s
(C) 8 s (D) 10 s

**Q.57** If a flywheel of mass 20 kg and diameter 1m is rotating 300 revolutions per minute, its kinetic energy will be -

(A) 2465 J (B) 2.465 J
(C) 24.65 J (D) 246.5 J

**Q.58** A cord is wound round the circumference of a wheel of radius r. The axis of the wheel is horizontal and moment of inertia about it is I. A weight mg is attached to end of the cord and falls from rest. After falling through a distance h, the angular velocity of the wheel will be-

(A) $\sqrt{\frac{2gh}{I+mr}}$ (B) $\sqrt{\frac{2mgh}{I+mr^2}}$

(C) $\sqrt{\frac{2mgh}{I+2m}}$ (D) $\sqrt{2gh}$

**Q.59** A rod of length $\ell$ is held vertically stationary with its lower end located at a position P on the horizontal plane. When the rod is released to topple about P, the velocity of the upper end of the rod with which it hits the ground is

(A) $\sqrt{g/\ell}$ (B) $\sqrt{3g\ell}$

(C) $3\sqrt{g/\ell}$ (D) $\sqrt{3g/\ell}$

**Q.60** Circular disc of mass 2 kg and radius 1 m is rotating about an axis perpendicular to its plane and passing through its centre of mass with a rotational kinetic energy of 8 J. The angular momentum in (J-s) is

(A) 8 (B) 4
(C) 2 (D) 1

## PART - 8 : ROLLING MOTION

**Q.61** A solid cylinder of mass M and radius R rolls without slipping down an inclined plane of length L and height h. What is the speed of its centre of mass when the cylinder reaches its bottom ?

(A) $\sqrt{4gh}$ (B) $\sqrt{2gh}$

(C) $\sqrt{\frac{3}{4}gh}$ (D) $\sqrt{\frac{4}{3}gh}$

**Q.62** A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is-

(A) 2/5 (B) 3/5
(C) 2/7 (D) 3/7

**Q.63** A solid sphere and a solid cylinder having the same mass and radius, roll down the same incline. The ratio of their acceleration will be -

(A) 15 : 14 (B) 14 : 15
(C) 5 : 3 (D) 3 : 5

**Q.64** When a ring of moment of inertia I rolls down on an inclined plane, then the percentage of its rotational kinetic energy in total energy is –

(A) 100 % (B) 50 %
(C) 28 % (D) 72 %

**Q.65** A sphere is rolling down without slipping in the incline plane from a vertical height h. The linear velocity as it reaches the ground, if its mass is m and radius is r, will be-(k is radius of gyration of sphere)

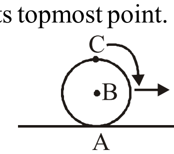
(A) $\sqrt{\frac{2gh}{1+2k^2/r^2}}$ (B) $\sqrt{\frac{2gh}{1+k^2/2r^2}}$

(C) $\sqrt{\frac{2gh}{1+k^2/r^2}}$ (D) $\sqrt{\frac{gh}{1+k^2/r^2}}$

**Q.66** Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement (s) is (are) correct?

(A) Both cylinders P and Q reach the ground at the same time.
(B) Cylinder P has larger acceleration than cylinder Q.
(C) Both cylinders reach the ground with same translational kinetic energy.
(D) Cylinder Q reaches the ground with larger angular speed.

**Q.67** A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,



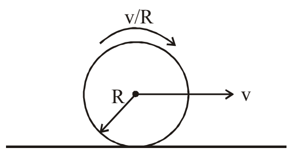

(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$
(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$
(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$
(D) Both (B) and (C)

**Q.68** A sphere is released on a smooth inclined plane from the top. When it moves down its angular momentum is –

(A) conserved about every point
(B) conserved about the point of contact only
(C) conserved about the centre of the sphere only
(D) conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

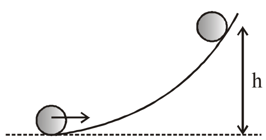
**Q.69** A uniform ring of radius R is given a back spin of angular velocity $V_0/2R$ and thrown on a horizontal rough surface with velocity of center to be $V_0$. The velocity of the centre of the ring when it starts pure rolling will be –
(A) $V_0/2$ (B) $V_0/4$
(C) $3V_0/4$ (D) 0

**Q.70** A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lower most point of the disc




(A) Velocity is v, acceleration is zero
(B) Velocity is zero, acceleration is zero
(C) velocity is v, acceleration is $v^2/R$
(D) velocity is zero, acceleration is $v^2/R$

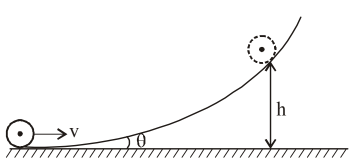
**Q.71** In the figure shown a ball rolls without sliding. On a horizontal surface. It ascends a curved track upto height h and returns.




Value of h is $h_1$ for sufficiently rough curved track to avoid sliding and $h_2$ for smooth curved track, then –
(A) $h_1 = h_2$ (B) $h_1 < h_2$
(C) $h_1 > h_2$ (D) $h_2 = 2\,h_1$

**Q.72** A body of mass m slides down an incline and reaches the bottom with a velocity v. If the same mass were in the form of a ring, which rolls down this incline, the velocity of the ring at bottom would have been-
(A) v (B) $\sqrt{2}v$
(C) $\frac{1}{\sqrt{2}}v$ (D) $\sqrt{2/5}v$

**Q.73** A disc of mass M and radius R rolls on a horizontal surface and then rolls up an inclined plane as shown in the figure. If the velocity of the disc is v, find the height h to which the disc will rise on the inclined will be :-

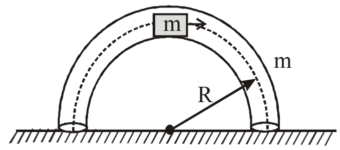



(A) $4v^2/3g$ (B) $4v^2/g$
(C) $3v^2/2g$ (D) $3v^2/4g$

## PART - 9 : MISCELLANEOUS

**Q.74** A train of mass M is moving on a circular track of radius R with constant speed V. The length of the train is half of the perimeter of the track. The linear momentum of the train will be –
(A) 0 (B) $2MV/\pi$
(C) MVR (D) MV

**Q.75** In a vertical plane inside a smooth hollow thin tube a block of same mass as that of tube is released as shown in figure. When it is slightly disturbed it moves towards right. By the time the block reaches the right end of the tube then the displacement of the tube will be (where 'R' is mean radius of tube). Assume that the tube remains in vertical plane.




(A) $2R/\pi$ (B) $4R//\pi$
(C) R/2 (D) R

**Q.76** When two particles of masses $m_1$ and $m_2$ are moving under the action of their internal forces $\vec{f}_1$ and $\vec{f}_2$ –
(A) The motion of one particle relative to the other is the same as the motion of a particle of mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$ under the force $\vec{f}_1$.
(B) Their relative motion is the same as that of one of the particles with its mass replaced by the reduced mass of the system, with the other particle remaining at rest.
(C) Their relative motion can be obtained by assuming one of the particles to have an infinite mass and by replacing the mass of the second particle by the reduced mass of the system, the force between them of the same as before.
(D) Both the particles move with uniform velocity.

**Q.77** If the external force acting on a system has zero resultant the centre of mass
(A) Must not move (B) Must not accelerate
(C) May accelerate (D) None of these

**Q.78** A loaded spring gun of mass M fires a shot of mass m with a velocity V at an angle of elevation θ. The gun is initially at rest on a horizontal frictionless surface. Just after firing, the centre of mass of the gun-shot system :
(A) moves with a velocity Vm/M
(B) moves with a velocity $\frac{Vm}{M}\cos\theta$ in the horizontal direction
(C) remains at rest
(D) moves with a velocity $\frac{V(M-m)}{(M+m)}$ in the horizontal direction.

**Q.79** Consider the following two statements
(a) Linear momentum of the system remains constant
(b) Centre of mass of the system remains at rest.
(A) a implies b and b implies a
(B) a does not imply b and b does not imply a
(C) a implies b but b does not imply a
(D) b implies a but a does not imply b.

**Q.80** A particle at rest suddenly disintegrates into two particles of equal masses which start moving The two fragments will-
(A) Move in the same direction with equal speeds
(B) Move in any directions with any speed
(C) Move in opposite directions with equal speeds
(D) Move in opposite directions with unequal speeds

**Q.81** In a free space, a rifle of mass ' M' shoots a bullet of mass 'm' at a stationary block of mass M distance 'D' away from it. When the bullet has moved through a distance 'd' towards the block, the centre of mass of the bullet block system is at a distance of

(a) $\frac{(D-d)m}{M+m}$ from the block (b) $\frac{md+MD}{M+m}$ from the rifle

(c) $\frac{2dm+DM}{M+m}$ from the rifle (d) $(D-d)\frac{M}{M+m}$ from the bullet

(A) b, c (B) a, d
(C) a, b, c (D) a, b

**Q.82** A body has its centre of mass at the origin. The x-coordinates of the particles
(A) May be all positive
(B) May be all negative
(C) May be all non-negative
(D) May be positive for some cases and negative in other cases

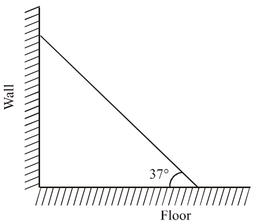
**Q.83** Two particles A and B initially at rest move towards each other under a mutual force of attraction. The speed of centre of mass at the instant when the speed of A is v and the speed of B is 2v is :
(A) v (B) Zero
(C) 2v (D) 3v/2

**Q.84** A bomb travelling in a parabolic path under the effect of gravity. explodes in mid air. The centre of mass of fragments will:
(A) Move vertically upwards and then downwards
(B) Move vertically downwards
(C) Move in irregular path
(D) Move in the parabolic path which the unexploded bomb would have travelled.

**Q.85** Two balls are thrown in air. The acceleration of the centre of mass of the two balls while in air (neglect air resistance)
(A) depends on the direction of the motion of the balls
(B) depends on the masses of the two balls
(C) depends on the speeds of the two balls
(D) is equal to g

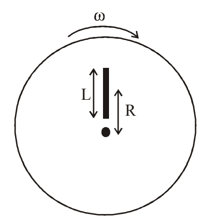
**Q.86** Two particles are moving towards each other along a line joining them so that their centre of mass does not move. After elastic collision between them –
(A) their centre of mass will move
(B) their velocities will not change
(C) their speeds will not change
(D) their velocities will not change if they have equal mass

**Q.87** A uniform ladder of length 5m and mass 100 kg is in equilibrium between vertical smooth wall and rough horizontal surface. Find minimum friction coefficient between floor and ladder for this equilibrium –




(A) 2/3 (B) 1/2
(C) 3/4 (D) 1/3

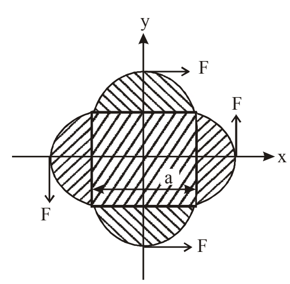
**Q.88** A uniform rod of mass M and length L lies radially on a disc rotating with angular speed ω in a horizontal plane about its axis. The rod does not slip on the disc and the centre of the rod is at a distance R from the centre of the disc. Then the kinetic energy of the rod is –




(A) $\frac{1}{2}mw^2\left(R^2+\frac{L^2}{12}\right)$ (B) $\frac{1}{2}mw^2R^2$

(C) $\frac{1}{24}mw^2L^2$ (D) None of these

**Q.89** A planar object made up of a uniform square plate and four semicircular discs of the same thickness and material is being acted upon by four forces of equal magnitude as shown in figure. The coordinates of point of application of forces is given by –




(A) (0, a) (B) (0, –a)

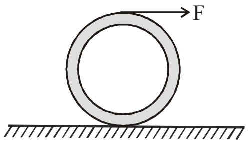
(C) (a, 0) (D) (–a, 0)

**Q.90** The moment of inertia of a door of mass m, length $2\ell$ and width $\ell$ about its longer side is –

(A) $\frac{11m\ell^2}{24}$ (B) $\frac{5m\ell^2}{24}$

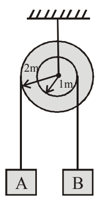
(C) $\frac{m\ell^2}{3}$ (D) None of these

**Q.91** A ring of mass m and radius R rolls on a horizontal rough surface without slipping due to an applied force 'F'. The friction force acting on ring is :



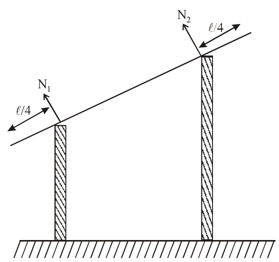

(A) F/3 (B) 2F/3
(C) F/4 (D) zero

**Q.92** In the pulley system shown, if radii of the bigger and smaller pulley are 2m and 1m respectively and the acceleration of block A is 5 $m/s^2$ in the downward direction, then the acceleration of block B will be –



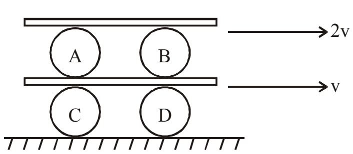

(A) 0 $m/s^2$ (B) 5 $m/s^2$
(C) 10 $m/s^2$ (D) 5/2 $m/s^2$

**Q.93** A uniform rod of length $\ell$ is placed symmetrically on two walls as shown in figure. The rod is equilibrium. If $N_1$ and $N_2$ are the normal forces exerted by the walls on the rod then –



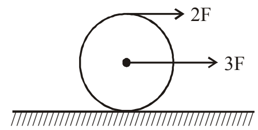

(A) $N_1 > N_2$
(B) $N_1 = N_2$
(C) $N_1 < N_2$
(D) $N_1$ and $N_2$ would be in the vertical directions

**Q.94** Four identical spheres and two long planks are arranged as shown in figure. The planks are being pulled by constant velocity as shown in figure. Assuming no slipping at any surface, the speeds of centres of mass of A and C are respectively



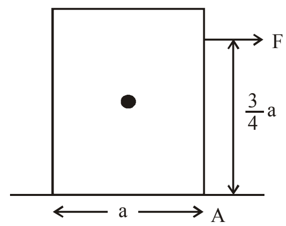

(A) 2v, v (B) v, v/2
(C) 3v/2, v/2 (D) None of these

**Q.95** Two forces of magnitude 2F and 3F are acting on a uniform solid sphere initially kept at rest on a horizontal surface as shown in the figure. Friction force by the horizontal surface on the sphere will be –



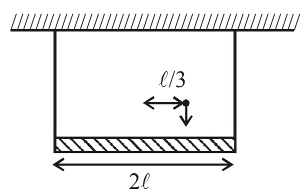

(A) in forward direction
(B) in backward direction
(C) zero
(D) depend on the value of F

**Q.96** A cube of side a, mass m is to be tilted about point A by applying a force F as shown in figure. The minimum force required is



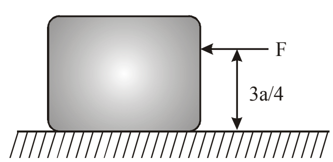

(A) mg (B) (2/3) mg
(C) (3/2) mg (D) (3/4) mg

**Q.97** A rod of mass m and length $2\ell$ hangs by two identical light threads tied to its ends. An insects of mass (3/8)m hits the rod with a speed v at a distance $\ell/3$ from the centre of the rod as shown and sticks to it. As a result one of the thread breaks. The acceleration of the insect just after the thread breaks given that the insect remains at rest with respect to the rod –




(A) g/4 (B) 3g/4
(C) 2g/3 (D) g

**Q.98** A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face, at a height 3a/4 above the base. The minimum value of F for which the cube begins to topple an edge is (Assume that cube does not slide)




(A) mg/3 (B) mg/2
(C) 2mg/3 (D) 3mg/4

# EXERCISE - 2 [LEVEL-2]

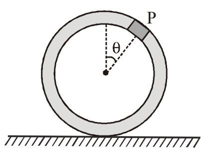
**ONLY ONE OPTION IS CORRECT**

**Q.1** A string is wrapped around a cylinder of mass m and radius R. The string is pulled vertically upward to prevent the centre of mass from falling as the cylinder unwinds the string. The length of the string unwound when the cylinder has reached a speed ω will be :

(A) $\frac{R\omega}{4g}$  (B) $\frac{R^2\omega^2}{4g}$

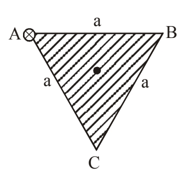
(C) $\frac{R\omega}{8g}$  (D) $\frac{R^2\omega^2}{8g}$

**Q.2** A small block of mass 'm' is rigidly attached at 'P' to a ring of mass '3m' and radius 'r'. The system is released from rest at $\theta = 90°$ and rolls without sliding. The angular acceleration of hoop just after release is –




(A) g/4r  (B) g/8r
(C) g/3r  (D) g/2r

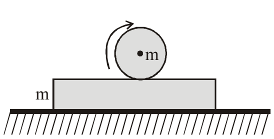
**Q.3** A uniform triangular plate ABC of moment of inertia I (about an axis passing through A and perpendicular to plane of the plate) can rotate freely in the vertical plane about point 'A' as shown in figure. The plate is released from the position shown in the figure. Line AB is horizontal. The acceleration of centre of mass just after the release of plate is –




(A) $\frac{mga^2}{\sqrt{3}I}$  (B) $\frac{mga^2}{4I}$

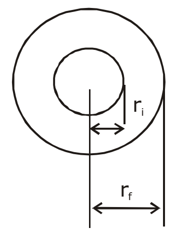
(C) $\frac{mga^2}{2\sqrt{3}I}$  (D) $\frac{mga^2}{3I}$

**Q.4** A sphere of mass 'm' is given some angular velocity about a horizontal axis through its centre and gently placed on a plank of mass 'm'. The coefficient of friction between the two is μ. The plank rests on a smooth horizontal surface. The initial acceleration of the plank is –




(A) zero  (B) (7/5) μg
(C) μg  (D) 2 μg

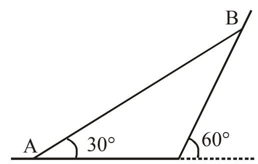
**Q.5** A magnetic tape is found on an empty spool rotating at a constant angular velocity. The final radius $r_f$ of the winding was found to be three times as large as the initial radius $r_1$ (Fig.). The winding time of the tape is $t_1$. What is the time $t_2$ required for winding a tape whose thickness is half that of the initial tap?




(A) $(\sqrt{3} - 1)\, t_1$  (B) $(\sqrt{5} + 1)\, t_1$

(C) $(\sqrt{5} - 1)\, t_1$  (D) $(\sqrt{2} - 1)\, t_1$

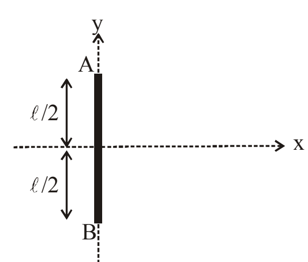
**Q.6** In the figure shown, the instantaneous speed of end A of the rod is v to the left. The angular velocity of the rod of length L, must be –




(A) v/2L  (B) v/L

(C) $v\sqrt{3}/2L$  (D) None of these

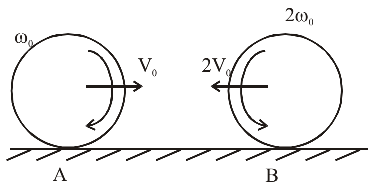
**Q.7** A uniform rod of mass m, length $\ell$ is placed over a smooth horizontal surface along y-axis and is at rest as shown in figure. An impulsive force F is applied for a small time Δt along x-direction at point A. The x-coordinate of end A of the rod when the rod becomes parallel to x-axis for the first time is (initially the coordinate of centre of mass of the rod is (0, 0))




(A) $\frac{\pi\ell}{12}$  (B) $\frac{\ell}{2}\left(1+\frac{\pi}{12}\right)$

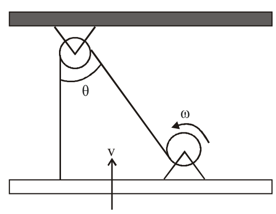
(C) $\frac{\ell}{2}\left(1-\frac{\pi}{6}\right)$  (D) $\frac{\ell}{2}\left(1+\frac{\pi}{6}\right)$

**Q.8** Two identical balls are rolling without slipping on a horizontal plane as shown in figure. They undergo a perfectly elastic collision. Just after the collision the speeds of bottom points of balls A & B will be respectively. (Assume that there is no friction between the balls)




(A) Zero, Zero (B) $2V_0, 4V_0$
(C) $3V_0, 3V_0$ (D) None of these

**Q.9** In the instant shown in the diagram, the board is moving up (vertically with velocity v. The drum winds up at a constant rate ω. If the radius of the drum is R and the board always remains horizontal, find the value of velocity v.




(A) $\frac{wR}{1+\cos\theta}$ (B) $\frac{2wR}{1+\cos\theta}$

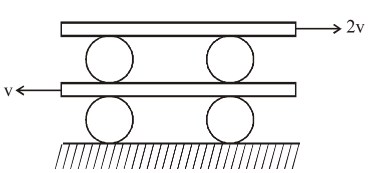
(C) $\frac{wR}{1-\cos\theta}$ (D) $\frac{wR}{1+\sin\theta}$

**Q.10** A uniform disc of radius R lies in the x-y plane, with its centre at origin, its moment of inertia about z-axis is equal to its moment of inertia about line y = x + c. The value of c will be –

(A) $-\frac{R}{2}$ (B) $\pm\frac{R}{\sqrt{2}}$

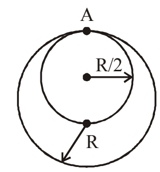
(C) +R/4 (D) – R

**Q.11** A system of uniform cylinders and plates is shown. All the cylinders are identical and there is no slipping at any contact. Velocity of lower and upper plates is V and 2V respectively as shown. Then the ratio of angular speeds of the upper cylinders to lower cylinders is –




(A) 1/3 (B) 3
(C) 1 (D) None of these

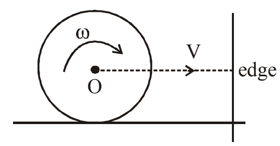
**Q.12** Two rings made of same material and thickness, one of radius R and other of radius R/2 are welded together at point A. Now, it is hanged on a nail at wall, the nail touching both the rings at A. Now it is slightly displaced in the plane of rings and released. The period of small oscillations is –




(A) $2\pi\sqrt{\frac{2R}{5g}}$ (B) $2\pi\sqrt{\frac{5R}{6g}}$

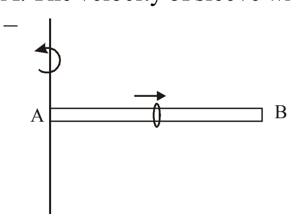
(C) $2\pi\sqrt{\frac{9R}{5g}}$ (D) $2\pi\sqrt{\frac{5R}{2g}}$

**Q.13** A uniform solid sphere of radius r is rolling on a smooth horizontal surface with velocity V and angular velocity $\omega\ (V = \omega r)$. The sphere collides with a sharp edge on the wall as shown. The coefficient of friction between the sphere and the edge $\mu = 1/5$. Just after the collision the angular velocity of the sphere becomes equal to zero. The linear velocity of the sphere just after the collision is equal to –




(A) V (B) V/5
(C) 3V/5 (D) 2V/5

**Q.14** Smooth uniform rod AB of mass M and length $\ell$ rotates freely with an angular velocity $\omega_0$ in a horizontal plane about a stationary vertical axis passing through its end A. A small sleeve of mass m sliding along the rod from the point A. The velocity of sleeve when it reaches other end B, is –




(A) $\frac{\omega_0 \ell}{\sqrt{1+\frac{3m}{M}}}$ relative ground

(B) $\frac{\omega_0 \ell}{\sqrt{1+\frac{3m}{M}}}$ relative to rod

(C) zero relative to rod

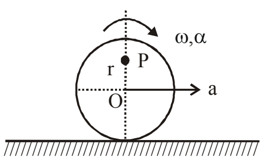
(D) $\frac{12\omega_0 \ell}{\sqrt{1+\frac{3m}{M}}}$ relative to ground

**Q.15** A solid homogeneous cylinder of height h and base radius r is kept vertically on a conveyer belt moving horizontally with an increasing velocity v = a + bt$^2$. If the cylinder is not allowed to slip find the time when the cylinder is about to topple.

(A) $\frac{2rg}{bh}$ (B) $\frac{rg}{2bh}$

(C) $\frac{2}{3}\frac{rg}{bh}$ (D) $\frac{rg}{bh}$

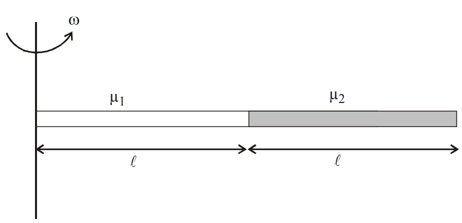
**Q.16** A disc of radius R rolls on a horizontal ground with linear acceleration a and angular acceleration α as shown in figure. The magnitude of acceleration of point P shown in fig. at an instant when its angular velocity is ω, will be




(A) $\sqrt{(a+r\alpha)^2+(r\omega^2)^2}$ (B) $\frac{ar}{R}$

(C) $\sqrt{r^2\alpha^2+r^2\omega^4}$ (D) r α

**Q.17** Two thin rods of same length but of uniform mass per unit length $_1$ and $\mu_2$ respectively are joined together. The system is rotated in horizontal plane as shown in figure. The tension at the joint will be –




(A) $\frac{3}{2}\mu_2\ell^2\omega^2$ (B) $\frac{3}{2}(\mu_1+\mu_2)\,\ell^2\omega^2$

(C) $\frac{3}{2}\mu_1\ell^2\omega^2$ (D) $\frac{1}{2}\mu_1\ell^2\omega^2$

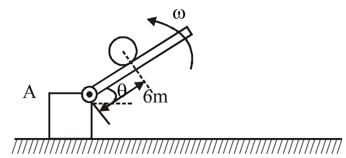
**Q.18** A uniform smooth rod is placed on a smooth horizontal floor is hit by a particle moving on the floor, at a distance $\ell/4$ from one end. Then the distance travelled by the centre of the rod after the collision when it has completed three revolution will be :

[e ≠ 0, $\ell$ is the length of the rod]

(A) $2\pi\ell$ (B) can't be determined

(C) $\pi\ell$ (D) None of these

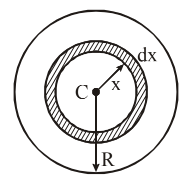
**Q.19** A cylinder weighing 450 N with a radius of 30cm is held fixed on an incline that is rotating at 0.5 rad/s. The cylinder is released when the incline is at position θ equal to 30°. If the cylinder is 6 meter from the bottom A at the instant of release, what is the initial acceleration of the centre of the cylinder relative to the incline, if there is no slipping (g = 10 m/s$^2$)




(A) 2.33 m/s$^2$ (B) 4.66 m/s$^2$

(C) 1.33 m/s$^2$ (D) None of these

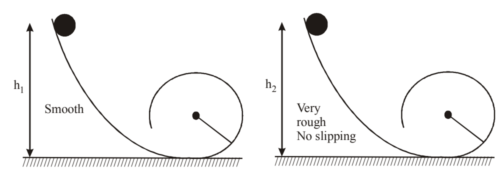
**Q.20** A uniform disc of radius R and mass M is spinned to an angular speed $\omega_0$ in its own plane about its centre and then placed on a rough horizontal surface such that plane of the disc is parallel to the horizontal plane. If co-efficient of friction between the disc and the surface is μ then how long will it take for the disc to come to stop.




(A) $\frac{2\omega_0 R}{3\mu g}$ (B) $\frac{3\omega_0 R}{4\mu g}$

(C) $\frac{3\omega_0 R}{2\mu g}$ (D) $\frac{\omega_0 R}{4\mu g}$

**Q.21** The following figure shows two situations in which a uniform round rigid body is released from rest from the positions shown, such that it is just able to loop the loop without leaving contact with the track. Assuming that radius of the track is large in comparison to the radius of round body, the ratio $h_1/h_2$.



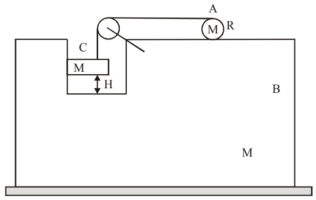

(A) must be greater than 1

(B) must be less than 1

(C) must be equal to 1

(D) can be greater than or less than 1, depending on the moment of inertia of the round body

**Q.22** A wedge B of mass M is placed on a smooth horizontal surface. An ideal string is wrapped over a cylinder A of mass M and radius R which is kept over the wedge and other end of the string is connected to block C of mass M passing over an ideal pulley as shown in the figure. If system is released from rest then after how much time the block C will hit the wedge. Friction between cylinder and wedge is sufficient to prevent slipping. All other surfaces are frictionless.




(A) $\sqrt{\dfrac{H}{g}}$ (B) $\sqrt{\dfrac{2H}{g}}$

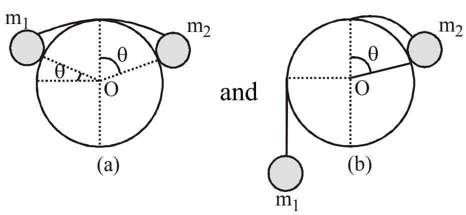
(C) $\sqrt{\dfrac{3H}{g}}$ (D) $\sqrt{\dfrac{4H}{g}}$

**Q.23** The line of action of the resultant of two like parallel forces shifts by one-fourth of the distance between the forces when the two forces are interchanged. The ratio of the two forces is

(A) 1/3 (B) 5/3

(C) 2/3 (D) 5/4

**Q.24** Two masses $m_1$ and $m_2$ connected through a massless and inextensible strings are placed on a smooth cylindrical surface as shown in figure. Angle of equilibrium in each case is –



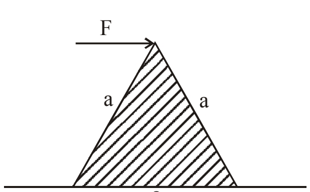

(A) $\theta = \tan^{-1}\left(\dfrac{m_1}{m_2}\right)$ and $\theta = \sin^{-1}\left(\dfrac{m_1}{m_2}\right)$

(B) $\theta = \tan^{-1}\left(\dfrac{m_2}{m_1}\right)$ and $\theta = \sin^{-1}\left(\dfrac{m_2}{m_1}\right)$

(C) $\theta = \tan^{-1}\left(\dfrac{m_1}{m_2}\right)$ and $\theta = \sin^{-1}\left(\dfrac{m_2}{m_1}\right)$

(D) $\theta = \tan^{-1}\left(\dfrac{m_2}{m_1}\right)$ and $\theta = \sin^{-1}\left(\dfrac{m_1}{m_2}\right)$

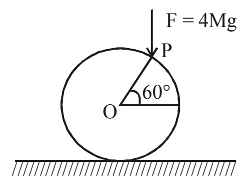
**Q.25** An equilateral prism of mass m rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the prism as shown in the figure. If the coefficient of friction is sufficiently high so that the prism does not slide before toppling, then the minimum force required to topple the prism is –




(A) $\dfrac{mg}{\sqrt{3}}$ (B) $\dfrac{mg}{4}$

(C) $\dfrac{\mu mg}{\sqrt{3}}$ (D) $\dfrac{\mu mg}{4}$

**Q.26** A solid sphere of mass M and radius R is lying on a rough horizontal plane. A constant force F = 4 Mg acts vertically at point P such that OP makes 60° with horizontal. Find the minimum value of coefficient of friction μ so that sphere starts pure rolling.




(A) 3/7 (B) 4/7

(C) 2/7 (D) 2/5

**Q.27** A sphere of mass m is rolling without sliding along positive x-axis on a rough horizontal surface of coefficient of friction μ. It elastically collides with a wall and then returns back. The correct statement of friction force (f) acting on the sphere is –

(A) $f = \mu mg\,\hat{i}$ before collision and $f = -\mu mg\,\hat{i}$ after collision.

(B) f = 0 before collision and $f = +\mu mg\,\hat{i}$ after collision

(C) $f < \mu mg\,\hat{i}$ before collision and $f = \mu mg\,\hat{i}$ just after collision.

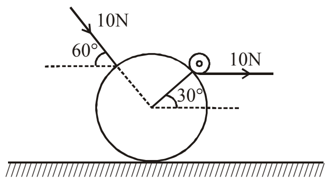
(D) f = μ mg before collision and just after collision.

**Q.28** A billiard ball of at rest is struck horizontally one tenth of the diameter below the top. If P be the impulse of the blow find the initial kinetic energy of the ball, the mass of the ball is being m.

(A) $\dfrac{13P^2}{10m}$ (B) $\dfrac{11P^2}{10m}$

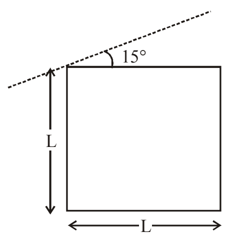
(C) $\dfrac{13P^2}{9m}$ (D) $\dfrac{9P^2}{10m}$

**Q.29** Thread is wrapped over a cylinder of mass 1 kg. It is pulled with 10N force horizontally. The thread passes below a light smooth pulley fixed to centre of the cylinder by a light rigid rod making an angle 30° with horizontal as shown. The pulley just touches the surface of the cylinder. Another force of 10N is applied on the cylinder making 60° with horizontal and passing through the centre of mass of the cylinder. Find the velocity (in m/s) of centre of mass of the cylinder when it purely rolls a distance 60m on the horizontal surface.




(A) 40 m/s (B) 20 m/s
(C) 30 m/s (D) 15 m/s

**Q.30** A square plate of mass M and edge L is shown in figure. The moment of inertia of the plate about the axis in the plane of plate and passing through one of its vertex making an angle 15° from horizontal is –




(A) $\frac{ML^2}{12}$ (B) $\frac{11ML^2}{24}$

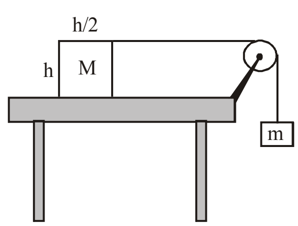
(C) $\frac{7ML^2}{12}$ (D) None of these

**Q.31** Determine the minimum coefficient of friction between a thin rod and a floor at which a person can slowly lift the rod from the floor without slipping, to the vertical position applying to its end a force always perpendicular to its length.

(A) $1/2\sqrt{2}$ (B) 1/3

(C) $1/\sqrt{2}$ (D) $1/\sqrt{3}$

**Q.32** A cylinder of height h, diameter h/2 and mass M and with a homogeneous mass distribution is placed on a horizontal table. One end of a string running over a pulley is fastened to the top of the cylinder, a body of mass m is hung from the other end and the system is released. Friction is negligible everywhere. At what minimum ratio m/M will the cylinder tilt ?



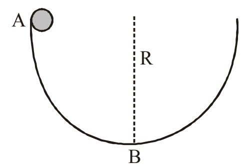

(A) 1 (B) 2
(C) 3 (D) 4

**Q.33** A spherical ball of mass M and radius R is projected along a rough horizontal surface so that initially (t = 0) it slides with a linear speed $v_0$ but does not rotate. As it slides, it begins to spin and eventually rolls without slipping. How long does it take to begin rolling without slipping ?

(A) $\frac{2v_0}{7\mu_k g}$ (B) $\frac{2v_0}{5\mu_k g}$

(C) $\frac{2v_0}{3\mu_k g}$ (D) $\frac{3v_0}{7\mu_k g}$

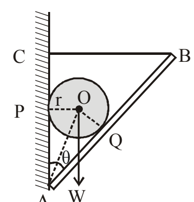
**Q.34** A small sphere A of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius R (>> r) as shown in figure. If the sphere starts from rest at the top point of the hemisphere find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.




(A) $\frac{10}{7}mg$ (B) $\frac{17}{7}mg$

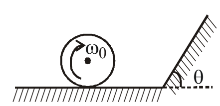
(C) $\frac{5}{7}mg$ (D) $\frac{7}{5}mg$

**Q.35** A cylinder of weight W and radius r is supported in a horizontal position against a wall by a rod AB of negligible weight. The rod is hinged to the wall at A and supported at the other end B by a horizontal rope BC. Neglecting friction find the angle θ that the rod should make with the wall so that the tension in the rope is minimum.




(A) 30° (B) 45°
(C) 60° (D) 75°

**Q.36** The figure shows a frictionless horizontal track which smoothly turns into a frictionless inclined surface of inclination θ. A ring of radius R rolling without slipping on the horizontal surface with angular speed $\omega_0$ moves towards the inclined surface. The maximum distance moved up the inclined surface is –




(A) $\frac{\omega_0^2 R^2}{2g}$ (B) $\frac{\omega_0^2 R^2}{2g\sin\theta}$

(C) $\frac{\omega_0^2 R^2}{2g}\sin\theta$ (D) zero

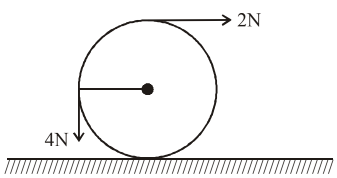
## EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

**NOTE : The answer to each question is a NUMERICAL VALUE.**

**Q.1** A uniform stick of mass m and length $\ell$ spins around on a frictionless horizontal plane, with its Centre of Mass stationary. A mass M is placed on the plane, and the stick collides elastically with it, as shown (with the contact point being the end of the stick). What should M (in kg) be so that after the collision the stick has translational motion, but no rotational motion?
Take m = 24 kg.

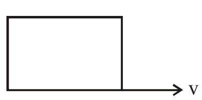
**Q.2** The radius of curvature of the path traced by a point situated at the edge of a rolling disc of radius R when the point attains its highest position is XR. Find the value of X.

**Q.3** A uniform solid disc of mass 1 kg and radius 1m is kept on a perfectly rough horizontal surface. Two forces of magnitude 2N and 4N have been applied on the disc, one is acting along horizontal direction and other is along vertical direction as shown in the figure. Linear acceleration of the centre of mass of the disc is (in m/s$^2$).



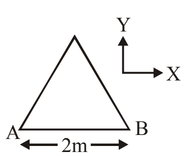

**Q.4** A wheel of radius R rolls on the ground with a uniform velocity v. The relative acceleration of topmost point of the wheel with respect to the bottommost point is Av$^2$/R. Find the value of A.

**Q.5** A cubical block of mass M and side a is kept on a smooth horizontal surface. One corner is given an impulse so that the corner attains velocity V as shown. The instantaneous angular velocity is XV/4a. Find the value of X.

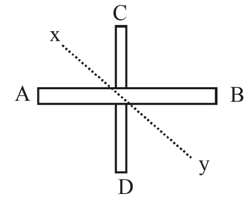



Top view

**Q.6** A rigid equilateral triangular plate ABC of side 2m, is in motion in the x-y plane. At the instant shown in the figure, the point B has velocity $\vec{v}_B = (3\hat{i} + 8\hat{j})\ m/s$ and the plate has angular velocity $\vec{\omega} = 2\hat{k}\ r/s$. Find the speed of point A (in m/s)



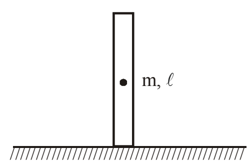

**Q.7** AB and CD are two identical rods each of length $\ell$ and mass m joined to form a cross. The moment of inertia of these two rods about a bisector of the angle between the rods (xy) is $\frac{m\ell^2}{A}$. Find the value of A.



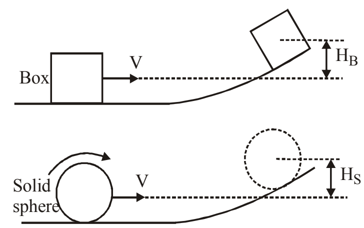

**Q.8** Two equal rods of weight 10N are freely joined. Their free ends are attached by strings to a fixed point. A circular disc of weight 60N and radius r rest in the angle between the rods and the whole hangs in a vertical plane if 4m is the length of each rod and 90° is the angle between the rods then find r (in m).

**Q.9** A rod of mass m and length $\ell$ is released from rest on a smooth surface shown in figure. Find the angular speed (in rad/sec.) of the rod when the centre of rod has descended by $\ell$/4 from its initial position.

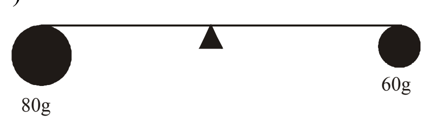
[where $\ell = \frac{240}{13}$ cm ]



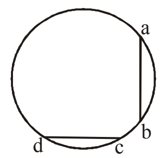

**Q.10** A box and a solid sphere of equal mass are moving with the same velocity across a horizontal floor. The sphere rolls without slipping and the box slides without friction. They encounter an upward slope in the floor and each move up the slope some distance before momentarily stopping and then moving down again. The maximum vertical heights reached by the box and the sphere are $H_B$ and $H_S$ respectively. What is the ratio $\frac{10H_S}{H_B}$ ?



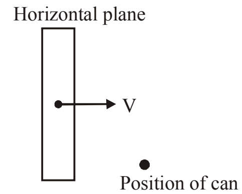

**Q.11** A 60g and 80g ball are fixed to the ends of a 28cm rod of negligible mass. The rod is balanced by a pivot such that it is horizontal and stays at rest. With what angular acceleration (in rad/s$^2$) does it start to move, if the pivot is shifted 2cm. closer to the heavier ball ? (Take g = 9.8 m/s$^2$)



**Q.12** The two ends of a uniform thin rod of length $\sqrt{2}R$ and of mass $2\sqrt{2}$kg can move without friction along a vertical circular path of radius R. The rod is released from the vertical position (ab). Find the force (in N) exerted by an end of the rod on the path when the rod passes the horizontal position (cd).




**Q.13** A 200 kg. beam 2.0m in length slides broadside down the ice with a speed of 16m/s (figure). A 50 kg small favicol can (guaranteed sticking) at rest sticks at one end as it goes past, both go spinning down the ice. Assume frictionless motion. What is the angular velocity of combined mass in rad/s.




## EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

**Q.1** A ring of mass M and radius R is moving in horizontal plane at angular speed ω about self axis. If two equal point masses are placed at the ends of any diameter. Find final angular speed of system - **[AIEEE-2002]**

(A) $\frac{M}{2m}\omega$ (B) $\frac{M}{M+2m}\omega$

(C) $\frac{m}{M+2m}\omega$ (D) none of above

**Q.2** A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling) **[AIEEE-2002]**

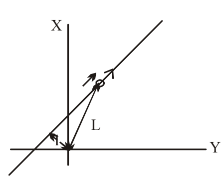
(A) Solid sphere (B) Hollow-sphere
(C) Ring (D) All same

**Q.3** Moment of inertia of a circular wire of mass M and raidus R about its diameter is – **[AIEEE-2002]**

(A) $MR^2$ (B) $\frac{MR^2}{2}$

(C) $2\,MR^2$ (D) $\frac{MR^2}{4}$

**Q.4** In the following figure angular momentum of particle of mass m and speed v about origin is – **[AIEEE-2002]**




(A) mvL (B) mvℓ
(C) mv/L (D) mv/ℓ

**Q.5** A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius 4R is made from an iron plate of thickness t/4. Then the relation between the moment of inertia $I_X$ and $I_Y$ is – **[AIEEE-2003]**

(A) $I_Y = 16\,I_X$ (B) $I_Y = I_X$
(C) $I_Y = 64\,I_X$ (D) $I_Y = 32\,I_X$

**Q.6** A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is **[AIEEE-2003]**

(A) 2 L (B) 4 L
(C) L / 2 (D) L / 4

**Q.7** A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected ? **[AIEEE-2004]**

(A) Moment of inertia
(B) Angular momentum
(C) Angular velocity
(D) Rotational kinetic energy

**Q.8** One solid sphere A and another hollow sphere B are of same mass and same outer radi. Their moment of inertia about their diameters are respectively $I_A$ and $I_B$ such that **[AIEEE-2004]**

(A) $I_A = I_B$ (B) $I_A > I_B$

(C) $I_A < I_B$ (D) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

where $d_A$ and $d_B$ are their densities.

**Q.9** An annular ring with inner and outer radii $R_1$ and $R_2$ is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, $F_1/F_2$ is **[AIEEE-2005]**

(A) $\frac{R_2}{R_1}$ (B) $\left(\frac{R_1}{R_2}\right)^2$

(C) 1 (D) $\frac{R_1}{R_2}$

**Q.10** The moment of inertia of uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is **[AIEEE-2005]**

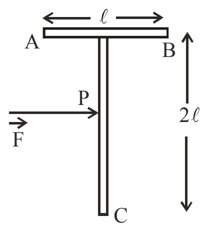
(A) $\frac{1}{4}Mr^2$ (B) $\frac{2}{5}Mr^2$

(C) $Mr^2$ (D) $\frac{1}{2}Mr^2$

**Q.11** A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C. **[AIEEE-2005]**

(A) $\frac{2}{3}\ell$ (B) $\frac{3}{2}\ell$

(C) $\frac{4}{3}\ell$ (D) $\ell$

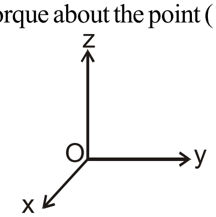



**Q.12** A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attachecd gently to the opposite ends of a diameter of the ring. The ring now ........................ ω′ = **[AIEEE 2006]**

(A) $\frac{\omega m}{(m+M)}$ (B) $\frac{\omega m}{(m+2M)}$

(C) $\frac{\omega(m+2M)}{m}$ (D) $\frac{\omega(m-2M)}{(m+2M)}$

**Q.13** A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1,– 1) is **[AIEEE 2006]**




(A) $F(\hat{i}+\hat{j})$ (B) $-F(\hat{i}-\hat{j})$

(C) $F(\hat{i}-\hat{j})$ (D) $-F(\hat{i}+\hat{j})$

**Q.14** Four point masses, each of value m, are placed at the corners of a square ABCD of side $\ell$. The moment of inertia of this system about an axis passing through A and parallel to BD is – **[AIEEE 2006]**

(A) $3\,m\ell^2$ (B) $m\ell^2$

(C) $2\,m\ell^2$ (D) $\sqrt{3}\,m\ell^2$

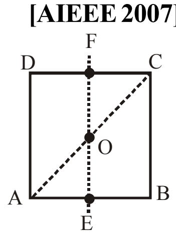
**Q.15** For the given uniform square lamina ABCD, whose centre is O, **[AIEEE 2007]**

(A) $\sqrt{2}\,I_{AC} = I_{EF}$

(B) $I_{AD} = 3I_{EF}$

(C) $I_{AC} = I_{EF}$

(D) $I_{AC} = \sqrt{2}\,I_{EF}$




**Q.16** A circular disc of radius R is removed from a bigger circular disc of radius 2R such that the circumferences of the discs coincide. The centre of mass of the new disc is $\alpha R$ from the centre of the bigger disc. The value of α is **[AIEEE 2007]**

(A) 1/3 (B) 1/2

(C) 1/6 (D) 1/4

**Q.17** A round uniform body of radius R, mass M and moment of inertia 'I', rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is **[AIEEE 2007]**

(A) $\frac{g\sin\theta}{1+I/MR^2}$ (B) $\frac{g\sin\theta}{1+MR^2/I}$

(C) $\frac{g\sin\theta}{1-I/MR^2}$ (D) $\frac{g\sin\theta}{1-MR^2/I}$

**Q.18** Angular momentum of the particle rotating with a central force is constant due to **[AIEEE 2007]**

(A) Constant Force

(B) Constant linear momentum

(C) Zero Torque

(D) Constant Torque

**Q.19** Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is **[AIEEE 2008]**

(A) $\frac{1}{12}ma^2$ (B) $\frac{7}{12}ma^2$

(C) $\frac{2}{3}ma^2$ (D) $\frac{5}{6}ma^{2]}$

**Q.20** A thin uniform rod of length $\ell$ and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of - **[AIEEE 2009]**

(A) $\frac{1}{6}\frac{\ell\omega}{g}$ (B) $\frac{1}{2}\frac{\ell^2\omega^2}{g}$

(C) $\frac{1}{6}\frac{\ell^2\omega^2}{g}$ (D) $\frac{1}{3}\frac{\ell^2\omega^2}{g}$

**Q.21** A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc –

(A) remains unchanged **[AIEEE 2011]**

(B) continuously decreases

(C) continuously increases

(D) first increases and then decreases

**Q.22** A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is – **[AIEEE 2011]**

(A) (3/2) g (B) g

(C) (2/3) g (D) g/3

**Q.23** A pulley of radius 2m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg $m^2$, the number of rotations made by the pulley before its direction of motion is reversed, is : **[AIEEE 2011]**

(A) less than 3
(B) more than 3 but less than 6
(C) more than 6 but less than 9
(D) more than 9

**Q.24** Adiatomic molecule is made of two masses $m_1$ and $m_2$ which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by – (n is an integer) **[AIEEE 2012]**

(A) $\dfrac{(m_1+m_2)^2 n^2h^2}{2m_1^2m_2^2r^2}$ (B) $\dfrac{n^2h^2}{2(m_1+m_2)r^2}$

(C) $\dfrac{2n^2h^2}{(m_1+m_2)r^2}$ (D) $\dfrac{(m_1+m_2)^2n^2h^2}{2m_1m_2r^2}$

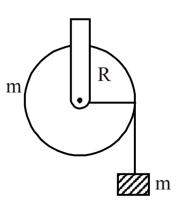
**Q.25** A hoop of radius r and mass m rotating with an angular velocity $\omega_0$ is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero.What will be the velocity of the centre of the hoop when it ceases to slip ? **[JEE MAIN 2013]**

(A) $r\omega_0/2$ (B) $r\omega_0/3$
(C) $r\omega_0/2$ (D) $r\omega_0$

**Q.26** A bob of mass m attached to an inextensible string of length $\ell$ is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension **[JEE MAIN 2014]**

(A) Angular momentum changes in direction but not in magnitude.
(B) Angular momentum changes both in direction and magnitude.
(C) Angular momentum is conserved.
(D) Angular momentum changes in magnitude but not in direction.

**Q.27** A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **[JEE MAIN 2014]**




(A) 5g/6 (B) g
(C) 2g/3 (D) g/2

**Q.28** Distance of the centre of mass of a solid uniform cone from its vertex is $z_0$. If the radius of its base is R and its height h then $z_0$ is equal to **[JEE MAIN 2015]**

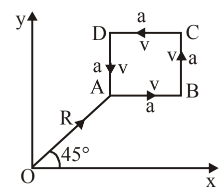
(A) 3h/4 (B) 5h/8
(C) $3h^2/8R$ (D) $h^2/4R$

**Q.29** From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is **[JEE MAIN 2015]**

(A) $\dfrac{MR^2}{16\sqrt{2}\pi}$ (B) $\dfrac{4MR^2}{9\sqrt{3}\pi}$

(C) $\dfrac{4MR^2}{3\sqrt{3}\pi}$ (D) $\dfrac{MR^2}{32\sqrt{2}\pi}$

**Q.30** A particle of mass m is moving along the side of square of side 'a', with a uniform speed v in the x-y plane as shown in the figure: Which of the following statements is false for the angular momentum $\vec{L}$ about the origin? **[JEE MAIN 2016]**



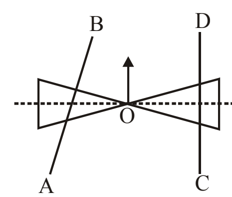

(A) $\vec{L} = mv\left[\dfrac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

(B) $\vec{L} = mv\left[\dfrac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(C) $\vec{L} = \dfrac{mv}{\sqrt{2}} R\,\hat{k}$ when the particle is moving from D to A.

(D) $\vec{L} = \dfrac{mv}{\sqrt{2}} R\,\hat{k}$ when the particle is moving from A to B.

**Q.31** A roller is made by joining, together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to: **[JEE MAIN 2016]**



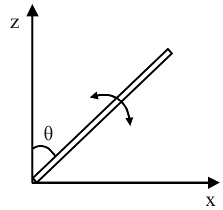

(A) turn right. (B) go straight.
(C) turn left and right alternately. (D) turn left.

**Q.32** The moment of inertia of a uniform cylinder of length $\ell$ and radius R about its perpendicular bisector is I. What is the ratio $\ell/R$ such that the moment of inertia is minimum? **[JEE MAIN 2017]**

(A) $\sqrt{3}/2$ (B) 1

(C) $3/\sqrt{2}$ (D) $\sqrt{3/2}$

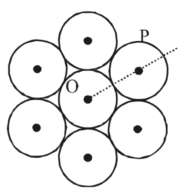
**Q.33** A slender uniform rod of mass M and length $\ell$ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is: **[JEE MAIN 2017]**




(A) $\frac{2g}{3\ell}\sin\theta$ (B) $\frac{3g}{2\ell}\cos\theta$

(C) $\frac{2g}{3\ell}\cos\theta$ (D) $\frac{3g}{2\ell}\sin\theta$

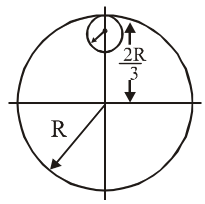
**Q.34** Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is **[JEE MAIN 2018]**




(A) $\frac{73}{2}MR^2$ (B) $\frac{181}{2}MR^2$

(C) $\frac{19}{2}MR^2$ (D) $\frac{55}{2}MR^2$

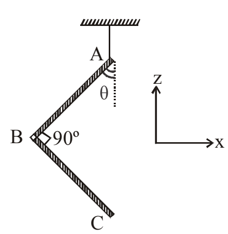
**Q.35** From a uniform circular disc of radius R and mass 9 M, a small disc of radius R/3 is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is : **[JEE MAIN 2018]**




(A) $10\,MR^2$ (B) $\frac{37}{9}MR^2$

(C) $4\,MR^2$ (D) $\frac{40}{9}MR^2$

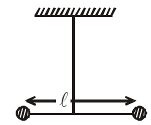
**Q.36** An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then : **[JEE MAIN 2019(JAN)]**




(A) $\tan\theta = 2/\sqrt{3}$ (B) $\tan\theta = 1/3$

(C) $\tan\theta = 1/2$ (D) $\tan\theta = 1/(2/\sqrt{3})$

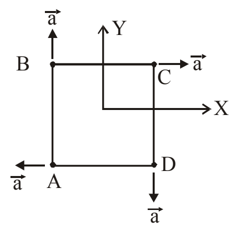
**Q.37** Two masses m and m/2 are connected at the two ends of a massless rigid rod of length $\ell$. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by $\theta_0$ and released, the tension in it when it passes through its mean position will be: **[JEE MAIN 2019(JAN)]**




(A) $3k\theta_0^2/\ell$ (B) $k\theta_0^2/2\ell$

(C) $2k\theta_0^2/\ell$ (D) $k\theta_0^2/\ell$

**Q.38** Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is : **[JEE MAIN 2019 (APRIL)]**




(A) $\frac{a}{5}(\hat{i}-\hat{j})$ (B) $\frac{a}{5}(\hat{i}+\hat{j})$

(C) Zero (D) $a(\hat{i}+\hat{j})$

**Q.39** A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with $\rho_0$ as constant and r is the distance from its centre. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is: **[JEE MAIN 2019 (APRIL)]**

(A) 3/2 (B) 1/2

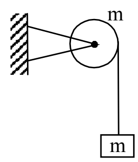
(C) 3/5 (D) 8/5

**Q.40** A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights $h_{sph}$ and $h_{cyl}$ on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by : **[JEE MAIN 2019 (APRIL)]**

(A) 14 / 15 (B) 4 / 5

(C) 1 (D) $2/\sqrt{5}$

**Q.41** A block of mass m is suspended from a pulley in form of a circular disc of mass m & radius R. The system is released from rest, find the angular velocity of disc when block has dropped by height h. (there is no slipping between string & pulley)

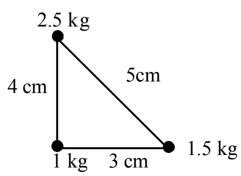



**[JEE MAIN 2020 (JAN)]**

(A) $\frac{1}{R}\sqrt{\frac{4gh}{3}}$ (B) $\frac{1}{R}\sqrt{\frac{2gh}{3}}$

(C) $R\sqrt{\frac{2gh}{3}}$ (D) $R\sqrt{\frac{4gh}{3}}$

**Q.42** Three point masses 1kg, 1.5 kg, 2.5 kg are placed at the vertices of a triangle with sides 3cm, 4cm and 5cm as shown in the figure. The location of centre of mass with respect to 1kg mass is :




**[JEE MAIN 2020 (JAN)]**

(A) 0.6 cm to the right and 2 cm above 1kg mass.
(B) 0.9 cm to the right and 2 cm above 1 kg mass.
(C) 0.9 cm to the left and 2 cm above 1 kg mass.
(D) 0.9cm to the right and 1.5cm above 1 kg mass.

**Q.43** The radius of gyration of a uniform rod of length L, about an axis passing through a point L/4 away from the centre of the rod, and perpendicular to it, is :

**[JEE MAIN 2020 (JAN)]**

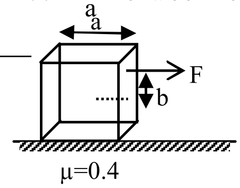
(A) $\sqrt{\frac{7}{48}}L$ (B) $\sqrt{\frac{5}{48}}L$ (C) $\sqrt{\frac{7}{24}}L$ (D) $\sqrt{\frac{19}{24}}L$

**Q.44** The surface mass density of a disc of radius a varies $\sigma = A + Br$ where A & B are positive constants then moment of inertia of the disc about an axis passing through its centre and perpendicular to the plane **[JEE MAIN 2020 (JAN)]**

(A) $2\pi a^4\left(\frac{A}{4}+\frac{Ba}{5}\right)$ (B) $2\pi a^4\left(\frac{Aa}{4}+\frac{B}{5}\right)$

(C) $\pi a^4\left(\frac{A}{4}+\frac{Ba}{5}\right)$ (D) $2\pi a^4\left(\frac{A}{5}+\frac{Ba}{4}\right)$

**Q.45** Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for a box not to topple before moving is ______ **[JEE MAIN 2020 (JAN)]**




**Q.46** Consider a uniform rod of mass M = 4m and length $\ell$ pivoted about its centre. A mass m moving with velocity v making angle $\theta = \pi/4$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is :

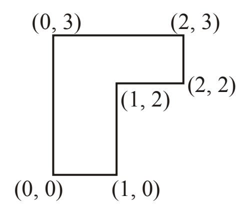
**[JEE MAIN 2020 (JAN)]**

(A) $\frac{3}{7\sqrt{2}}\frac{v}{\ell}$ (B) $\frac{3\sqrt{2}}{7}\frac{v}{\ell}$

(C) $\frac{4}{7}\frac{v}{\ell}$ (D) $\frac{3}{7}\frac{v}{\ell}$

**Q.47** The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are :

**[JEE MAIN 2020 (JAN)]**




(A) (1.25m, 1.50m) (B) (1m, 1.75m)
(C) (0.75m, 0.75m) (D) (0.75m, 1.75m)

# EXERCISE - 5 (PREVIOUS YEARS AIPMT/NEET EXAM QUESTIONS)

**Choose one correct response for each question.**

**Q.1** Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio – **[AIPMT 2005]**

(A) 2 : 1 (B) 1 : 2

(C) $\sqrt{2}:1$ (D) $1:\sqrt{2}$

**Q.2** A drum of radius R and mass M, rolls down without slipping along an inclined plane of angle θ. The frictional force – **[AIPMT 2005]**

(A) Dissipates energy as heat

(B) Decreases the rotational motion

(C) Decreases the rotational and translational motion

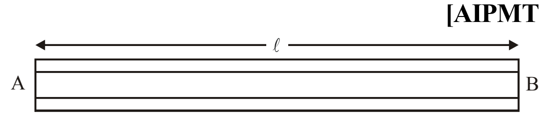
(D) Converts translational energy to rotational energy

**Q.3** The moment of inertia of a uniform circular disc of radius R and mass M about an axis touching the disc at its diameter and normal to the disc is – **[AIPMT 2006]**

(A) $\frac{2}{5}MR^2$ (B) $\frac{3}{2}MR^2$

(C) $\frac{1}{2}MR^2$ (D) $MR^2$

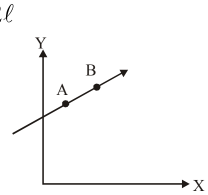
**Q.4** A uniform rod of length $\ell$ and mass m is free to rotate in a vertical plane about A. The rod initially in horizontal position is released. The initial angular acceleration of the rod is (Moment of inertia of rod about A is $\frac{m\ell^2}{3}$) **[AIPMT 2006]**




(A) $\frac{2\ell}{3g}$ (B) $\frac{3g}{2\ell^2}$

(C) $\frac{mg\ell}{2}$ (D) $\frac{3g}{2\ell}$

**Q.5** A particle of mass m moves in the XY plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is $L_A$ when it is at A and $L_B$ when it is at B, then **[AIPMT 2007]**




(A) $L_A = L_B$

(B) the relationship between $L_A$ and $L_B$ depends upon the slope of the line AB.

(C) $L_A < L_B$

(D) $L_A > L_B$

**Q.6** A wheel has angular acceleration of 3.0 rad/sec$^2$ and an initial angular speed of 2.00 rad/sec. In a time of 2 sec it has rotated through an angle (in radian) of **[AIPMT 2007]**

(A) 10 (B) 12

(C) 4 (D) 6

**Q.7** The ratio of the radii of gyration of a circular disc to that of a circular ring, each of same mass and radius, around their respective axes is **[AIPMT 2008]**

(A) $\sqrt{2}:\sqrt{3}$ (B) $\sqrt{3}:\sqrt{2}$

(C) $1:\sqrt{2}$ (D) $\sqrt{2}:1$

**Q.8** Thin rod of length L and mass M is bent at its midpoint into two halves so that the angle between them is 90°. The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is **[AIPMT 2008]**

(A) $\frac{\sqrt{2}ML^2}{24}$ (B) $\frac{ML^2}{24}$

(C) $\frac{ML^2}{12}$ (D) $\frac{ML^2}{6}$

**Q.9** A thin circular ring of mass M and radius R is rotating in a horizontal plane about an axis vertical to its plane with a constant angular velocity ω. If two objects each of mass m be attached gently to the opposite ends of a diameter of the ring, the ring will then rotate with an angular velocity: **[AIPMT 2009]**

(A) $\frac{\omega M}{M+2m}$ (B) $\frac{\omega\,(M+2m)}{M}$

(C) $\frac{\omega M}{M+m}$ (D) $\frac{\omega\,(M-2m)}{M}$

**Q.10** If $\vec{F}$ is the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin, then **[AIPMT 2009]**

(A) $\vec{r}.\vec{\tau} > 0$ and $\vec{F}.\vec{\tau} < 0$ (B) $\vec{r}.\vec{\tau} = 0$ and $\vec{F}.\vec{\tau} = 0$

(C) $\vec{r}.\vec{\tau} = 0$ and $\vec{F}.\vec{\tau} \neq 0$ (D) $\vec{r}.\vec{\tau} \neq 0$ and $\vec{F}.\vec{\tau} = 0$

**Q.11** Four identical thin rods each of mass M and length $\ell$, form a square frame. Moment of inertia of this frame about an axis through the centre of the square and perpendicular to its plane is : **[AIPMT 2009]**

(A) $\frac{2}{3}M\ell^2$ (B) $\frac{13}{3}M\ell^2$

(C) $\frac{1}{3}M\ell^2$ (D) $\frac{4}{3}M\ell^2$

**Q.12** Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$ , respectively. The centre of mass of this system has a position vector: **[AIPMT 2009]**

(A) $-2\hat{i}-\hat{j}+\hat{k}$ (B) $2\hat{i}-\hat{j}-2\hat{k}$

(C) $-\hat{i}+\hat{j}+\hat{k}$ (D) $-2\hat{i}+2\hat{k}$

**Q.13** A circular disk of moment of inertia $I_t$ is rotating in a horizontal plane, about its symmetry axis, with a constant angular speed $\omega_r$. Another disk of moment of inertia $I_b$ is dropped coaxially onto the rotating disk. Initially the second disk has zero angular speed. Eventually both the disks rotate with a constant angular speed $\omega_f$. The energy lost by the initially rotating disc to friction is – **[AIPMT (PRE) 2010]**

(A) $\frac{1}{2}\frac{I_b^2}{(I_t+I_b)}\omega_i^2$ (B) $\frac{1}{2}\frac{I_t^2}{(I_t+I_b)}\omega_i^2$

(C) $\frac{I_b-I_t}{(I_t+I_b)}\omega_i^2$ (D) $\frac{1}{2}\frac{I_bI_t}{(I_t+I_b)}\omega_i^2$

**Q.14** Two particles which are initially at rest,move towards each other under the action of their internal attraction. If their speeds are v and 2v at any instant, then the speed of centre of mass of the system will be – **[AIPMT (PRE) 2010]**

(A) 2v (B) zero

(C) 1.5 v (D) v

**Q.15** From a circular disc of radius R and mass 9M, a small disc of mass M and radius R/3 is removed concentrically. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through its centre is – **[AIPMT (MAINS) 2010]**

(A) $(40/9)MR^2$ (B) $MR^2$

(C) $4\,MR^2$ (D) $MR^2$

**Q.16** Asolid cylinder and a hollow cylinder, both of the samemass and same external diameter are released from the same height at the same time on an inclined plane. Both roll down without slipping.Which one will reach the bottom first ? **[AIPMT (MAINS) 2010]**

(A) Both together only when angle of inclination is 45°

(B) Both together

(C) Hollow cylinder

(D) Solid cylinder

**Q.17** (i) Centre of gravity (C.G.) of a body is the point at which the weight of the body acts.

(ii) Centre of mass coincides with the centre of gravity if the earth is assumed to have infinitely large radius.

(iii) To evaluate the gravitational field intensity due to any body at an external point, the entire mass of the body can be considered to be concentrated at its C.G.

(iv) The radius of gyration of any body rotating about an axis is the length of the perpendicular dropped from the C.G. of the body to the axis.

Which one of the following pairs of statements is correct? **[AIPMT (MAINS) 2010]**

(A) (iv) and (i) (B) (i) and (ii)

(C) (ii) and (iii) (D) (iii) and (iv)

**Q.18** A thin circular ring of mass M and radius r is rotating about its axis with constant angular velocity ω. Two objects each of mass m are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with angular velocity given by – **[AIPMT (MAINS) 2010]**

(A) $\frac{(M+2m)\,\omega}{2m}$ (B) $\frac{2M\omega}{M+2m}$

(C) $\frac{(M+2m)\,\omega}{M}$ (D) $\frac{M\omega}{M+2m}$

**Q.19** The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its midpoint and perpendicular to its length is $I_0$. Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is – **[AIPMT (PRE) 2011]**

(A) $I_0+ML^2$ (B) $I_0+\frac{ML^2}{2}$

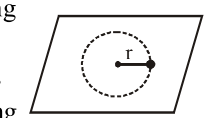
(C) $I_0+\frac{ML^2}{4}$ (D) $I_0+2ML^2$

**Q.20** The instantaneous angular position of a point on a rotating wheel is given by the equation $\theta(t) = 2t^3 - 6t^2$. The torque on the wheel becomes zero at **[AIPMT (PRE) 2011]**

(A) t = 2 s (B) t = 1 s

(C) t = 0.2 s (D) t = 0.25 s

**Q.21** A small mass attached to a string rotates on frictionless table top as shown. If the tension in the string is increased by pulling the string causing the radius of the circular motion to decrease by a factor of 2, the KE of the mass will: **[AIPMT (MAINS) 2011]**




(A) remain constant

(B) increase by a factor of 2

(C) increase by a factor of 4

(D) decrease by a factor of 2

**Q.22** When a mass is rotating in a plane about a fixed point, its angular momentum is directed along : **[AIPMT (PRE) 2012]**

(A) a line perpendicular to the plane of rotation

(B)the line making an angle of 45° to the plane of rotation.

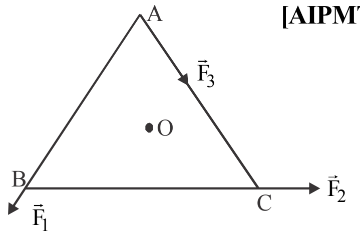
(C) the radius

(D) the tangent to the orbit.

**Q.23** A solid cylinder of mass 3 kg is rolling on a horizontal surface with velocity $4ms^{-1}$. It collides with a horizontal spring of force constant $200\ Nm^{-1}$. The maximum compression produced in the spring will be – **[AIPMT (PRE) 2012]**

(A) 0.5 m (B) 0.6 m
(C) 0.7 m (D) 0.2 m

**Q.24** ABC is an equilateral triangle with O as its centre $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ represent three forces acting along the sides AB, BC and AC respectively. If the total torque about O is zero the magnitude of $\vec{F}_3$ is **[AIPMT (PRE) 2012]**




(A) $F_1 + F_2$ (B) $F_1 - F_2$
(C) $\frac{F_1 + F_2}{2}$ (D) $2(F_1 + F_2)$

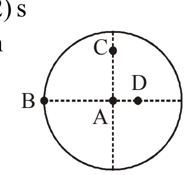
**Q.25** Two persons of masses 55 kg and 65 kg respectively, are at the opposite ends of a boat. The length of the boat is 3.0 m and weighs 100 kg. The 55 kg man walks up to the 65kg man and sits with him. If the boat is in still water the centre of mass of the system shifts by : **[AIPMT (PRE) 2012]**

(A) 3.0 m (B) 2.3 m
(C) zero (D) 0.75 m

**Q.26** A circular platform is mounted on a frictionless vertical axle. Its radius R = 2m and its moment of inertia about the axle is $200\ kgm^2$. It is initially at rest. A50 kg man stands on the edge of the platform and begins to walk along the edge at the speed of $1ms^{-1}$ relative to the ground. Time taken by the man to complete one revolution is– **[AIPMT (MAINS) 2012]**

(A) $\pi$ s (B) $(3\pi/2)$ s
(C) $2\pi$s (D) $(\pi/2)$ s

**Q.27** The moment of inertia of a uniform circular disc is maximum about an axis perpendicular to the disc and passing through **[AIPMT (MAINS) 2012]**

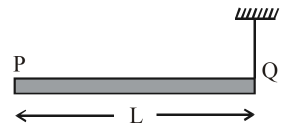



(A) B (B) C
(C) D (D) A

**Q.28** Three masses are placed on the x-axis : 300 g at origin, 500g at x = 40 cm and 400g at x = 70 cm. The distance of the centre of mass from the origin is – **[AIPMT (MAINS) 2012]**

(A) 40 cm (B) 45 cm
(C) 50 cm (D) 30 cm

**Q.29** A rod PQ of mass M and length L is hinged at end P. The rod is kept horizontal by a massless string tied to point Q as shown in figure. When string is cut, the initial angular acceleration of the rod is – **[NEET 2013]**




(A) 2g/3L (B) 3g/2L
(C) g/L (D) 2g/L

**Q.30** A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches upto a maximum height of $3v^2/4g$ with respect to the initial position. The object is **[NEET 2013]**

(A) Disc (B) Ring
(C) Solid sphere (D) Hollow sphere

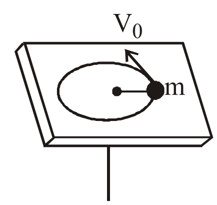
**Q.31** A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 revolutions $s^{-2}$ is – **[AIPMT 2014]**

(A) 25 N (B) 50 N
(C) 78.5 N (D) 157 N

**Q.32** The ratio of the accelerations for a solid sphere (mass m and radius R) rolling down an incline of angle θ without slipping and slipping down the incline without rolling is **[AIPMT 2014]**

(A) 5 : 7 (B) 2 : 3
(C) 2 : 5 (D) 7 : 5

**Q.33** A mass m moves in a circle on a smooth horizontal plane with velocity $v_0$ at a radius $R_0$. The mass is attached to string wihich passes through a smooth hole in the plane as shown.




The tension in the string is increased gradually and finally m moves in a circle of radius $R_0/2$. The final value of the kinetic energy is – **[AIPMT 2015]**

(A) $\frac{1}{4}mv_0^2$ (B) $2mv_0^2$

(C) $\frac{1}{2}mv_0^2$ (D) $mv_0^2$

**Q.34** A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is : **[AIPMT 2015]**

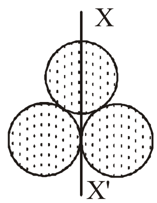
(A) $\frac{Wd}{x}$ (B) $\frac{W(d-x)}{x}$

(C) $\frac{W(d-x)}{d}$ (D) $\frac{Wx}{d}$

**Q.35** Two spherical bodies of mass M and 5 M and radii R and 2R released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is: **[AIPMT 2015]**

(A) 4.5 R (B) 7.5 R
(C) 1.5 R (D) 2.5 R

**Q.36** Three identical spherical shells, each of mass M and radius R are placed as shown in figure. Consider an axis XX' which is touching to two shells and passing through diameter of third shell. Moment of inertia of the system consisting of these three sphereical shell about XX' axis is : **[AIPMT 2015]**




(A) 3 $MR^2$ (B) (16/5) $MR^2$
(C) 4 $MR^2$ (D) (11/5) $MR^2$

**Q.37** An automobile moves on a road with a speed of 54km/h. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is 3 $kgm^2$. If the vehicle is brought to rest in 15s, the magnitude of average torque transmitted by its brakes to wheel is – **[RE-AIPMT 2015]**

(A) 2.86 $kg\,m^2s^{-2}$ (B) 6.66 $kg\,m^2s^{-2}$
(C) 8.58 $kg\,m^2s^{-2}$ (D) 10.86 $kg\,m^2s^{-2}$

**Q.38** Point masses $m_1$ and $m_2$ are placed at the opposite ends of a rigid rod of length L, and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point P on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity $\omega_0$ is minimum, is given by **[RE-AIPMT 2015]**

(A) $x = \frac{m_2 L}{m_1 + m_2}$ (B) $x = \frac{m_1 L}{m_1 + m_2}$

(C) $x = \frac{m_1}{m_2} L$ (D) $x = \frac{m_2}{m_1} L$

**Q.39** A force $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$. The value of $\alpha$ for which angular momentum about origin is conserved is:

(A) 1 (B) –1 **[RE-AIPMT 2015]**
(C) 2 (D) zero

**Q.40** From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre? **[NEET 2016 PHASE 1]**

(A) $15MR^2/32$ (B) $13MR^2/32$
(C) $11MR^2/32$ (D) $9MR^2/32$

**Q.41** A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad $s^{-2}$. Its net acceleration in $ms^{-2}$ at the end of 2.0s is approximately **[NEET 2016 PHASE 1]**

(A) 8.0 (B) 7.0
(C) 6.0 (D) 3.0

**Q.42** A disk and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first? **[NEET 2016 PHASE 1]**

(A) Disk
(B) Sphere
(C) Both reach at the same time
(D) Depends on their masses

**Q.43** Two rotating bodies A and B of masses m and 2m with moments of inertia $I_A$ and $I_B$ ($I_B > I_A$) have equal kinetic energy of rotation. If $L_A$ and $L_B$ be their angular momenta respectively, then **[NEET 2016 PHASE 2]**

(A) $L_A = L_B/2$ (2) $L_A = 2L_B$
(C) $L_B > L_A$ (D) $L_A > L_B$

**Q.44** A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation ($E_{sphere} / E_{cylinder}$) will be **[NEET 2016 PHASE 2]**

(A) 2 : 3 (B) 1 : 5
(C) 1 : 4 (D) 3 : 1

**Q.45** A light rod of length $\ell$ has two masses $m_1$ and $m_2$ attached to its two ends. The moment of inertia of the system about an axis perpendicular to the rod and passing through the centre of mass is **[NEET 2016 PHASE 2]**

(A) $\frac{m_1 m_2}{m_1 + m_2}\ell^2$ (B) $\frac{m_1 + m_2}{m_1 m_2}\ell^2$

(C) $(m_1 + m_2)\ell^2$ (D) $\sqrt{m_1 m_2}\,\ell^2$

**Q.46** A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N ?

(A) 0.25 $rad/s^2$ (B) 25 $rad/s^2$ **[NEET 2017]**
(C) 5 $m/s^2$ (D) 25 $m/s^2$

**Q.47** Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities $\omega_1$ and $\omega_2$. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is– **[NEET 2017]**

(A) $\frac{1}{4}I(\omega_1-\omega_2)^2$ (B) $I(\omega_1-\omega_2)^2$

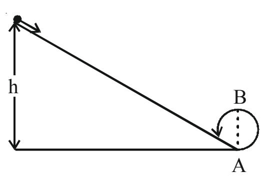
(C) $\frac{1}{8}(\omega_1-\omega_2)^2$ (D) $\frac{1}{2}I(\omega_1+\omega_2)^2$

**Q.48** Which of the following statements are correct ? **[NEET 2017]**

(a) Centre of mass of a body always coincides with the centre of gravity of the body.
(b) Central of mass of a body is the point at which the total gravitational torque on the body is zero.
(c) A couple on a body produce both translational and rotation motion in a body.
(d) Mechanical advantage greater than one means that small effort can be used to lift a large load

(A) (a) and (b) (B) (b) and (c)
(C) (c) and (d) (D) (b) and (d)

**Q.49** A body initially at rest and sliding along a frictionless track from a height h (as shown in the figure) just completes a vertical circle of diameter AB = D. The height h is equal to **[NEET 2018]**




(A) (7/5) D (B) D
(C) (3/2) D (D) (5/4) D

**Q.50** Three objects, A : (a solid sphere), B : (a thin circular disk) and C : (a circular ring), each have the same mass M and radius R. They all spin with the same angular speed $\omega$ about their own symmetry axes. The amounts of work (W) required to bring them to rest, would satisfy the relation. **[NEET 2018]**

(A) $W_B > W_A > W_C$ (B) $W_A > W_B > W_C$
(C) $W_C > W_B > W_A$ (D) $W_A > W_C > W_B$

**Q.51** The moment of the force, $\vec{F}=4\hat{i}+5\hat{j}-6\hat{k}$ at (2, 0, –3), about the point (2, –2, –2), is given by **[NEET 2018]**

(A) $-7\hat{i}-8\hat{j}-4\hat{k}$ (B) $-4\hat{i}-\hat{j}-8\hat{k}$

(C) $-8\hat{i}-4\hat{j}-7\hat{k}$ (D) $-7\hat{i}-4\hat{j}-8\hat{k}$

**Q.52** A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere? **[NEET 2018]**

(A) Rotational kinetic energy
(B) Moment of inertia
(C) Angular velocity
(D) Angular momentum

**Q.53** A solid sphere is in rolling motion. In rolling motion a body possesses translational kinetic energy ($K_t$) as well as rotational kinetic energy ($K_r$) simultaneously. The ratio $K_t : (K_t + K_r)$ for the sphere is **[NEET 2018]**

(A) 10 : 7 (B) 5 : 7
(C) 7 : 10 (D) 2 : 5

**Q.54** A disc of radius 2 m and mass 100 kg rolls on a horizontal floor. Its centre of mass has speed of 20cm/s. How much work is needed to stop it? **[NEET 2019]**

(A) 3 J (B) 30 kJ
(C) 2 J (D) 1 J

**Q.55** A solid cylinder of mass 2 kg and radius 4 cm rotating about its axis at the rate of 3 rpm. The torque required to stop after $2\pi$ revolutions is **[NEET 2019]**

(A) $2\times10^{-6}$ N m (B) $2\times10^{-3}$ N m
(C) $12\times10^{-4}$ N m (D) $2\times10^{6}$ N m

# ANSWER KEY

**EXERCISE - 1**

| Q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | D | C | B | B | B | C | C | A | A | B | A | A | B | A | A | C | A | C | D | A | D | A | D | A | D | C | B | B | B |
| Q | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| A | C | D | D | D | A | D | C | C | A | C | C | D | C | A | C | C | B | A | A | C | D | B | A | A | B | B | A | B | B | B |
| Q | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| A | D | C | A | B | C | D | D | D | B | D | C | C | D | B | C | C | B | C | D | C | B | B | B | D | D | C | A | A | B | C |
| Q | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | | | | | | | | | | | | | | | | | | | | | | |
| A | D | D | B | C | C | B | D | C | | | | | | | | | | | | | | | | | | | | | | |

**EXERCISE - 2**

| Q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | B | C | C | C | B | D | C | A | B | B | C | A | B | D | A | A | B | A | B |
| Q | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | | | | |
| A | B | C | B | A | A | C | B | A | A | B | A | A | A | B | C | B | | | | |

**EXERCISE - 3**

| Q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 12 | 4 | 0 | 2 | 3 | 5 | 12 | 3 | 10 | 14 | 10 | 50 | 6 |

**EXERCISE - 4**

| Q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | D | B | B | C | D | B | C | D | D | C | B | A | A | C | A | A | C | C | C |
| Q | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| A | D | C | B | D | C | A | D | A | B | AC | D | D | D | B | C | B | D | A | D | A |
| Q | 41 | 42 | 43 | 44 | 45 | 46 | 47 | | | | | | | | | | | | | |
| A | A | B | A | A | 50 | B | D | | | | | | | | | | | | | |

**EXERCISE - 5**

| Q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | D | D | B | D | A | A | C | C | A | B | D | A | D | B | A | D | A | D | C | B | C | A | B | A | C | C | A | A | B | A |
| Q | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | | | | | |
| A | D | A | B | C | B | C | B | A | B | B | A | B | C | B | A | B | A | D | D | C | D | D | B | A | A | | | | | |

# ROTATIONAL MOTION

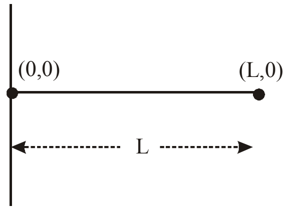
## TRY IT YOURSELF-1

**(1)** Treating the line joining the two particles as x axis

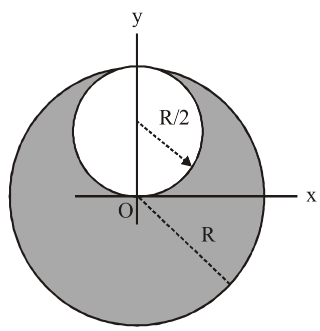
$$x = \frac{m_1 \times 0 + m_2 \times L}{m_1 + m_2}$$

$y_{CM} = 0,\ z_{CM} = 0$

$$x_{CM} = \frac{m_2 L}{m_1 + m_2}$$




**(2)** We treat the hole as a negative mass (imaginary) object that is combined with the original uncut circle. (When the two are added together, the hole region then has zero mass) By symmetry, the CM lies along the +y-axis in figure, so $x_{CM} = 0$. With the origin at the center of the original circle whose mass is assumed to be m.




| | Mass | Location of CM |
|---|---|---|
| Original uncut circle | $m_1 = m$ | $(0, 0)$ |
| Hole of negative axis | $m_2 = \frac{m}{4}$ | $\left(0, \frac{R}{2}\right)$ |

Thus: $$y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m(0) + \left(-\frac{m}{4}\right)\left(\frac{R}{2}\right)}{m + \left(-\frac{m}{4}\right)} = \frac{R}{6}$$

The CM is at the point $(0, -R/6)$

**(3)** Length of rod $= |\vec{r}_2 - \vec{r}_1| = \sqrt{(4-2)^2 + (2-5)^2} = \sqrt{13}\,m$

Centre of mass

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{3}{5}(2\hat{i} + 5\hat{j}) + \frac{2}{5}(4\hat{i} + 2\hat{j}) = \left(\frac{14}{5}\hat{i} + \frac{19}{5}\hat{j}\right) m$$

**(4)** As initial momentum of the system in x-direction is zero, and there is no net external force in x-direction the momentum of system remains zero in x-direction and thus the center of mass of the system undergoes zero displacement in x-direction $m_1\vec{s}_1 + m_2\vec{s}_2 = 0$

When the ball comes to the lowest position; shell moves backwards say by a distance x.

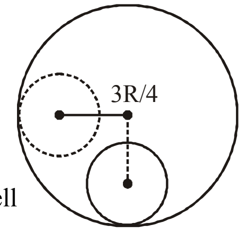
Displacement of ball in x-direction

= Displacement of ball w.r.t. shell + displacement of shell.

Displacement of shell = (–x)

Displacement of ball in x-direction is $\left(\frac{3R}{4} + (-x)\right)$

$$m\left(\frac{3R}{4} - x\right) - mx = 0$$




$\therefore\ x = \frac{3R}{8}$

If we do not consider that the shell moves back ward, we can take its forward displacement to be x,

$\therefore$ Displacement of ball in x-direction $= \frac{3R}{4} + x$

$$m\left(\frac{3R}{4} + x\right) + mx = 0\ ;\ x = -\frac{3R}{8}$$

(–ve sign indicates its backwards motion)

**(5)** **(C).** Because the stick is a uniform, symmetric body, we can consider all its weight as being concentrated at the center of mass at the 0.5-m mark. Therefore the point of support lies midway between the two weights, and the system is balanced only if the total weight on the right is also 1 kg.

**(6)** $\mathbf{m_{log} = 267\ kg}$

Because there are no external forces on the system, and the center of mass is initially at rest, the center of mass must remain at rest. Set x = 0 as the center of mass of the fisherman / log system:

$$x_{cm} = \frac{x_{fisherman}\, m_{fisherman} + x_{log}\, m_{log}}{m_{fisherman} + m_{log}} = 0$$

$$\Rightarrow\ x_{fisherman}\, m_{fisherman} + x_{log}\, m_{log} = 0$$

$$\Rightarrow\ m_{log} = \frac{-x_{fisherman}\, m_{fisherman}}{x_{log}}$$

$$= \frac{(12m)\ (400\ kg)}{(18m)} = 267\ kg$$

**(7)** Velocity of c.m. of the system is given as

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

Since the particles $m_1$ and $m_2$ are moving unidirectionally, $m_1\vec{v}_1$ and $m_2\vec{v}_2$ are parallel $\Rightarrow$

$|m_1\vec{v}_1 + m_2\vec{v}_2| = m_1v_1 + m_2v_2$

Therefore, $$v_{CM} = \frac{|m_1\vec{v}_1 + m_2\vec{v}_2|}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$= \frac{(1/2) + (1/2)(6)}{[1 + (1/2)]}\,m/s = 3.33\ m/sec$$

**(8)** The acceleration of c.m. of the system is given by

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} \Rightarrow a_{cm} = \frac{|m_1\vec{a}_1 + m_2\vec{a}_2|}{m_1 + m_2}$$

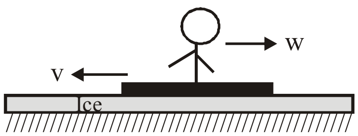
Since $\vec{a}_1$ and $\vec{a}_2$ are anti-parallel, $a_{cm} = \frac{|m_1a_1 - m_2a_2|}{m_1 + m_2}$

$\Rightarrow a_{cm} = \frac{|(2)(1) - (4)(2)|}{2+4} m/sec^2 = 1 m/sec^2$

Since, $m_2a_2 > m_1a_1$, the direction of acceleration of c.m. will be directed in the direction of $a_2$.

**(9)** Consider the situation shown in figure. Suppose the man moves at a speed w towards right and the platform recoils at a speed V towards left, both relative to the ice. Hence, the speed of the man relative to the platform is V + w.

By the question, $V + w = v$, or $w = v - V$ (i)




Taking the platform and the man to be the system, there is no external horizontal force on the system. The linear momentum of the system remains constant.

Initially both the man and the platform were at rest. Thus,

$0 = MV - mw$ or $MV = m(v - V)$ [Using eq. (i)]

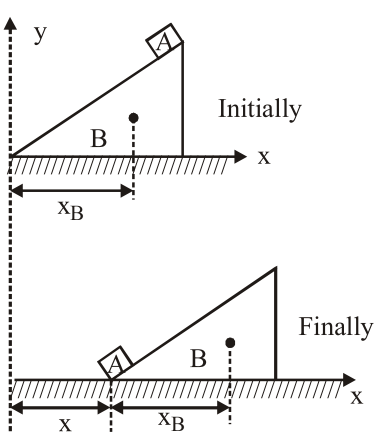
or $V = \frac{mv}{M+m}$

**(10)** Initial position of center of mass

$= \frac{X_BM_B + X_AM_A}{M_B + M_B} = \frac{X_B \cdot 20M + \ell \cdot 4M}{24M} = \frac{5X_B + \ell}{6}$

Final position of center of mass

$= \frac{(X_B + x)\,20M + 4Mx}{24M} = \frac{5(X_B + x) + x}{6}$




Since there is no horizontal force on system

Center of mass initially = center of mass finally.

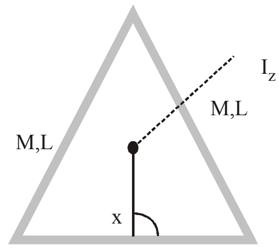
$5X_B + \ell = 5X_B + 5x + x$

$\ell = 6x \Rightarrow x = \ell/6$

## TRY IT YOURSELF-2

**(1)** $I = 4I_{CM} + 4M\left(\frac{L}{2}\right)^2 = 4\left(\frac{ML}{12}\right)^2 + 4\frac{ML^2}{4} = \frac{4}{3}ML^2$

**(2)**




$I = 3I_{CM} + 3Mx^2 = \frac{3ML^2}{12} + 3M\left(\frac{L}{2\sqrt{3}}\right)^2$

$= \frac{ML^2}{4} + \frac{ML^2}{4} = \frac{ML^2}{2}$

**(3)** $I = \frac{2}{5}M(2R)^2 \Rightarrow M(2R)^2 = \frac{5}{2}I$

$\Rightarrow I_{system} = I + 2[I + M(2R)^2] + I$

$= I + 2I + 2 \times \frac{5}{2}I + I = 9I$

**(4)** $\frac{I_1}{I_2} = \frac{M_1}{M_2}\left[\frac{R_1}{R_2}\right]^2$

$\Rightarrow \frac{M_1}{M_2} = \frac{I_1}{I_2} \times \left[\frac{R_2}{R_1}\right]^2 = \frac{4}{1} \times \left[\frac{1}{4}\right]^2 = \frac{1}{4}$

**(5)** M = Mass of square plate before cutting the holes

Mass of one hole $m = \left[\frac{M}{16R^2}\right]\pi R^2 = \frac{\pi M}{16}$

$\therefore$ Moment of inertia of the remaining portion,

$I = I_{square} - 4\,I_{hole}$

$= \frac{M}{12}(16R^2 + 16R^2) - 4\left[\frac{mR^2}{2} + m(\sqrt{2}R)^2\right]$

$= \frac{8}{3}MR^2 - 10MR^2 = \left[\frac{8}{3} - \frac{10\pi}{16}\right]MR^2$

**(6)** $I_{total} = m(R^2) + 3 \times \left(\frac{mR^2}{3}\right) = 2mR^2$

Also, $M_{total} = 4m$ $\therefore$ $K = \sqrt{\frac{2mR^2}{4m}} = \frac{R}{\sqrt{2}}$

**(7)** Consider the line perpendicular to the plane of the figure through the centre of the cross. The moment of inertia of each rod about this line is $M\ell^2/12$ and hence the moment of inertia of the cross is $M\ell^2/6$. The moment of inertia of the cross about the two bisector are equal by symmetry and according to the theorem of perpendicular axes, the moment of inertia of the cross about the bisector is $M\ell^2/12$.

**(8)** Moment of inertia of the plate about axis 1
(by taking rods perpendicular to axis $I_1 = Mb^2/3$
Moment of inertia of the plate about axis 2
(by taking rods perpendicular to axis 2) $I_2 = M\ell^2/12$
Moment of inertia of the plate about axis 3
(by taking rods perpendicular to axis 3) $I_3 = Mb^2/12$
Moment of inertia of the plate about axis 4
(by taking rods perpendicular to axis 4) $I_4 = M\ell^2/3$.

**(9)** The moment of inertia of the cylinder about its axis = $MR^2$.
Using parallel axes theorem,
$I = I_0 + MR^2 = MR^2 + MR^2 = 2MR^2$
Similarly, the moment of inertia of a hollow sphere about a ........

$$\frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$$

**(10)** The moment of inertia of flywheel is given by, $I = MR^2$.
Taking log, $\log I = \log M + 2\log R$

Differentiating, $\frac{dI}{I} = 0 + 2\frac{dR}{R}$

$\therefore$ % change in moment of inertia $= \frac{dI}{I} \times 100 = 2 \times 1\% = 2\%$

## TRY IT YOURSELF-3

**(1)** Percentage increase in momentum $= \frac{L_2 - L_1}{L_1} \times 100$

$L \propto \sqrt{E} \Rightarrow E_1 = E$

$E_2 = E + \frac{300}{100}E \Rightarrow E_2 = 4E$

Increase in momentum

$$= \frac{\sqrt{E_2} - \sqrt{E_1}}{\sqrt{E_1}} \times 100 = \frac{\sqrt{4E} - \sqrt{E}}{\sqrt{E}} \times 100 = 100\%$$

**(2)** $L_1 = L$

$L_2 = L + \frac{200}{100} = 3L$

Percentage increase in rotational energy

$$= \frac{E_2 - E_1}{E_1} \times 100 = \left[\frac{E_2}{E_1} - 1\right] \times 100$$

$$= \left[\frac{L_2^2}{L_1^2} - 1\right] \times 100 = \left[\frac{9L^2}{L^2} - 1\right] \times 100 = 800\%$$

**(3)** $P = 200\text{ hp} = 200\text{ hp} \times \frac{746\text{W}}{1\text{ hp}} = 1.49 \times 10^5\text{ W}$

$\omega = 6000\text{ rev/min} = 6000 \times \frac{2\pi}{60} = 628\text{ rad/sec.}$

$$\tau = \frac{P}{\omega} = \frac{1.49 \times 10^5}{628} = 237.5\text{ N-m}$$

**(4)** $I = m_1r_1^2 + m_2r_2^2 + ..... + m_nr_n^2$ ; $I = \Sigma\, mr^2$
(mass distribution is at more distance from axis 11' as compare to axis 22')

$I_{11'} > I_{22'}$ ; $\tau = F\frac{d}{2} \Rightarrow \tau_{11'} = \tau_{22'}$

$\Rightarrow I_{11'}\,\alpha_{11'} = I_{22'}\,\alpha_{22'}$

$\frac{\alpha_{22'}}{\alpha_{11'}} = \frac{I_{11'}}{I_{22'}} \Rightarrow \alpha_{22'} > \alpha_{11'}$

**(5)** $\alpha = \frac{\tau}{I} = \frac{(5+10)\,r_2 - 15r_1}{I} = \frac{15\,(20-10) \times 10^{-2}}{1500} = 10^{-3}\text{rad}^{-2}$

**(6)** **(B).** Choose units vectors such that $\hat{i} \times \hat{j} = \hat{k}$, with $\hat{i}$ pointing to the right and $\hat{j}$ pointing up. The torque about the point S is given by $\vec{\tau}_S = \vec{r}_{SF} \times \vec{F}$, where
$\vec{r}_{SF} = L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}$ and $\vec{F} = -F\,\hat{j}$ then
$\vec{\tau}_S = (L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}) \times -F\,\hat{j} = -FL\cos\theta\,\hat{k}$
So the magnitude of the torque about S is $|\vec{\tau}_S| = FL\cos\theta$.
Note that the perpendicular moment arm is $r_\perp = L\cos\theta$, so the magnitude of the torque about S is $|\vec{\tau}_S| = r_\perp F$.

**(7)** **(C).** As the particle moves in the positive x-direction, the perpendicular distance from the origin to the line of motion does not change and so the magnitude of the angular momentum about the origin is constant. Recall that for the motion of the particle x (t) varies with time but (y, z) are fixed. The angular momentum about the origin is
$\vec{L}_0 = (x\,(t)\,\hat{i} + y\,\hat{j} + z\hat{k}) \times mv\,\hat{i} = -ymv\,\hat{k} + zmv\,\hat{j}$
which is constant. In the above, the relations
$\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{i} = -\hat{k},\ \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \vec{0}$
were used. Note that the magnitude of $|\vec{L}_0|$ is given by
$|\vec{L}_0| = ((ymv)^2 + (zmv)^2)^{1/2} = mv\,(y^2 + z^2)^{1/2} = mvr_\perp$
where $r_\perp = (y^2 + z^2)^{1/2}$ is constant.

**(8)** **(B).** For this non-symmetric rigid body, the angular momentum about the origin has time varying components in the x-y plane, $\vec{L}_0 = L_z\hat{k} + L_r\hat{r}\,(t)$, (where $\hat{r}\,(t)$ is a radial unit vector pointing outward from the origin) The magnitude of $|\vec{L}_0|$ is constant because both $L_z$ and $L_r$ are constant. The direction of $\hat{r}\,(t)$ in the x-y plane depends on the instantaneous orientation of the body and so the direction of $\vec{L}_0 \,/\, |\vec{L}_0|$ is changing.

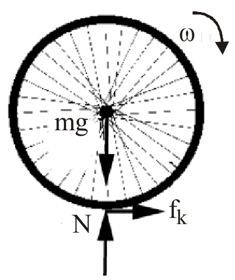
**(9)** **(C).** Because you have not changed the moment arm about the center of mass and you are pulling with the same force, the torque is the same.

**(10)** **(C).** By definition, the direction of any angular acceleration vector will always coincide with the direction of the axis about which the rotation occurs. The axis of rotation for a bike moving northward will be east/west. But which is it, east or west? Think about the direction of the angular velocity. If you make the fingers of your right hand curl to mimic the wheel's motion (i.e., curl the fingers to follow the rotation of the wheel), your right-hand thumb will point west. That is the direction of the angular velocity of the wheel. If the angular acceleration's direction were also to the west, the sign of the angular velocity and angular acceleration would be the same and the bike would be speeding up. But the bike is slowing down, so the direction of the angular acceleration must be opposite west, or east.

## TRY IT YOURSELF-4

**(1)** **(B).** There are two forces acting on the wheel, the contact force between the wheel and the ground and the gravitational force between the wheel and the earth. If you said three, that's ok because the contact force has two components the normal force and kinetic friction.

**(2)** **(A). Points to the right.**

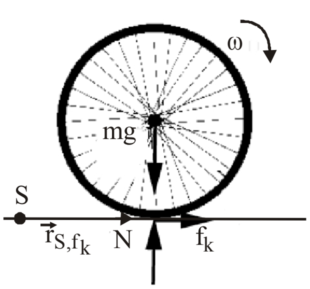



The kinetic friction force on the bicycle wheel points to the right. This is the force that is responsible for the acceleration of the wheel. (All the forces are shown in the figure.)

**(3)** **(B).** The kinetic friction produces a torque about the center-of mass that points out of the page.

**(4)** **(B).** Since the torque is non-zero, the angular momentum about the center-of-mass is changing. The torque due to the kinetic friction decreases the angular velocity.

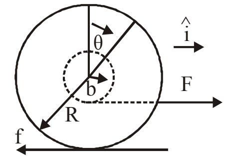
**(5)** **(A).**




Since the vector from S to the where the kinetic friction is acting is parallel to the kinetic friction the torque about S is zero.

**(6)** **(A).** The torque about S due to the normal force is out of the page and has magnitude $\left|\vec{\tau}_{S,N}\right| = Nr_{s,\perp} = Nd$

**(7)** **(A).** For forces below a fixed maximum value, the torque due to the force of friction is larger in magnitude than the torque due to the pulling force. Therefore the cylinder has an angular acceleration pointing into the page (in the clockwise direction) hence the cylinder rolls to the right, in the direction of F, winding up the string.




**(8)** **$4v_0/5$.**

Velocity of the centre = $v_0$ and the angular velocity about the centre = $v_0/2R$ . Thus $v_0 > \omega_0 R$. The sphere slips forward and thus the friction by the plane on the sphere will act backward. As the friction is kinetic, its value is $\mu N = \mu Mg$ and the sphere will be decelerated by $a_{cm} = f/M$.

Hence, $v(t) = v_0 - \dfrac{f}{M}t$ ....... (1)

This friction will also have a torque $\tau = fr$ about the centre. This torque is clockwise and in the direction of $\omega_0$. Hence the angular acceleration about the centre will be

$$\alpha = f\frac{R}{(2/3)MR^2} = \frac{3f}{2MR}$$

and the clockwise angular velocity at time t will be

$$\omega(t) = \omega_0 + \frac{3f}{2MR}t = \frac{v_0}{2R} + \frac{3f}{2MR}t$$

Pure rolling starts when $v(t) = R\omega(t)\cdot$,i.e.,

$$v(t) = \frac{v_0}{2} + \frac{3f}{2M}t \quad \text{....... (2)}$$

Eliminating t from (1) and (2),

$$\frac{3}{2}v(t) + v(t) = \frac{3}{2}v_0 + \frac{v_0}{2} \quad \text{or} \quad v(t) = \frac{2}{5} \times 2v_0 = \frac{4}{5}v_0$$

Thus, the sphere rolls with linear velocity $4v_0/5$ in the forward direction.

**(9)** Suppose the radius of the sphere is r. The forces acting on the sphere. They are (a) weight mg. (b) normal force N and (c) friction f.

Let the linear acceleration of the sphere down the plane be a. The equation for the linear motion of the centre of mass is

$$mg\sin\theta - f = ma \quad \text{...(i)}$$

As the sphere rolls without slipping, its angular acceleration about the centre is a/r. The equation of rotational motion about the centre of mass is,

$$fr = \left(\frac{2}{5}mr^2\right)\left(\frac{a}{r}\right) \quad \text{or} \quad f = \frac{2}{5}ma \quad \text{...(ii)}$$

From (i) and (ii), $a = \dfrac{5}{7}g\sin\theta$ and $f = \dfrac{2}{7}mg\sin\theta$

The normal force is equal to mg cosθ as there is no acceleration perpendicular to the incline. The maximum friction that can act is, therefore, $\mu\, mg\cos\theta$, where $\mu$ is the coefficient of static friction. Thus, for pure rolling

$$\mu\, mg\cos\theta > \frac{2}{7}mg\sin\theta \quad \text{or} \quad \mu > \frac{2}{7}\tan\theta$$

# CHAPTER-8 : ROTATIONAL MOTION
## EXERCISE-1

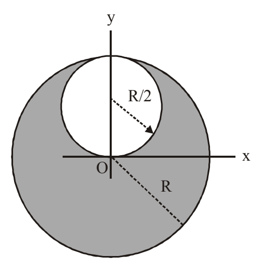
**(1)** **(B).** Centre of mass is nearer to heavier mass.

**(2)** **(D).** Out of the four given bodies, the centre of mass of a bangle lies outside it whereas in all other three bodies it lies within the body.

**(3)** **(C).** We treat the hole as a negative mass (imaginary) object that is combined with the original uncut circle. (When the two are added together, the hole region then has zero mass) By symmetry, the CM lies along the +y-axis in figure, so $x_{CM} = 0$. With the origin at the center of the original circle whose mass is assumed to be m.

| | Mass | Location of CM |
|---|---|---|
| Original uncut circle | $m_1 = m$ | (0, 0) |
| Hole of negative axis | $m_2 = \frac{m}{4}$ | (0, R/2) |

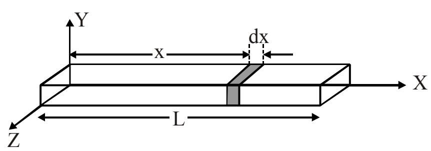
$$y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$




$$= \frac{m(0) + \left(-\frac{m}{4}\right)\left(\frac{R}{2}\right)}{m + \left(-\frac{m}{4}\right)} = \frac{R}{6}$$

The CM is at the point (0, –R/6)

**(4)** **(B).** As the rod is along x-axis, for all points on it y and z will be zero, so $Y_{CM} = 0$ and $Z_{CM} = 0$




i.e., the centre of mass will lie on the rod. Now, consider an element of rod of length dx at a distance x from the origin, then $dm = \lambda\, dx = (A + Bx)\, dx$

$$\text{So, } X_{CM} = \frac{\int_0^L x\, dm}{\int_0^L dm} = \frac{\int_0^L x\,(A + Bx)\, dx}{\int_0^L (A + Bx)\, dx}$$

$$= \frac{\frac{AL^2}{2} + \frac{BL^3}{3}}{AL + \frac{BL^2}{2}} = \frac{L\,(3A + 2BL)}{3\,(2A + BL)}$$

(i) If the rod is of uniform density, then $\lambda = \text{constant} = A$ and $B = 0$. Hence, $X_{CM} = L/2$

(ii) If the density of rod varies linearly with x, i.e., $\lambda = Bx$ and $A = 0$ then $X_{CM} = 2L/3$

**(5)** **(B).** If no external force acts on a system then velocity of centre of mass remains constant.

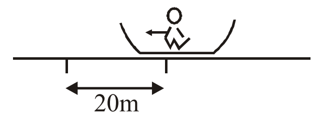
**(6)** **(B).** Let the distance moved by plank is x.

$$m_1\Delta x_1 + m_2\Delta x_2 = 0$$

$$M\,(L - x) + \frac{M}{3}(-x) = 0 \Rightarrow x = \frac{3L}{4}$$

$\therefore$ Distance moved by person w.r.t. to ground $= L - x = L/4$

**(7)** **(C).** Let the distance moved by boat be x.



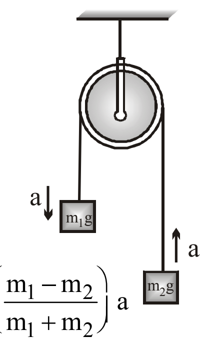

$80\,\{-(8 - x)\} + 200\,(+x) = 0 \Rightarrow x = 16/7 = 2.3$

Distance of person from shore = 20 – (8 – 2.3) = 14.3m

**(8)** **(C).** $m_1d_1 = m_2d_2$

$$\Rightarrow d_2 = \frac{m_1d_1}{m_2} = \frac{m_1}{m_2}d$$

**(9)** **(A).** $m_1 > m_2$ so $m_1$ will move downwards and $m_2$ upwards. Magnitude of acc of each block




$$a = \frac{\text{net pulling force}}{\text{mass to be pulled}} = \frac{(m_1 - m_2)\,g}{m_1 + m_2}$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} = \frac{m_1a + m_2a}{m_1 + m_2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)a$$

(+ve downwards and -ve upwards)

$$\vec{a}_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \times \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

In the direction of acceleration of $m_1$, downwards

**(10)** **(A).** Velocity of c.m. of the system is given as

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}.$$ Since the particles $m_1$ & $m_2$ are moving unidirectionally, $m_1\vec{v}_1$ and $m_2\vec{v}_2$ are parallel

$$\Rightarrow |m_1\vec{v}_1 + m_2\vec{v}_2| = m_1v_1 + m_2v_2$$

$$\text{Therefore, } v_{CM} = \frac{|m_1\vec{v}_1 + m_2\vec{v}_2|}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$= \frac{(1/2) + (1/2)(6)}{[1 + (1/2)]}\,m/s = 3.33\ m/sec$$

**(11)** **(B).** Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration $\vec{a}$. The centre of mass has an acceleration is $\vec{a}/2$.

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

**(12)** **(A).** Before explosion, particle was moving along x-axis i.e., it has no y-component of velocity. Therefore, the centre of mass will not move in y-direction or we can say $y_{CM} = 0$. Now, $y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

Therefore, $0 = \frac{(m/4)(+15) + (3m/4)(y)}{\frac{m}{4} + \frac{3m}{4}}$ or $y = -5$cm.

**(13)** **(A).** For any two particle system of masses $m_1$ and $m_2$ and distance r moment of inertia is $\frac{m_1 m_2}{m_1 + m_2} r^2$

**(14)** **(B).** Moment of inertia of flywheel is given by, $I = MR^2$
Taking log, $\log I = \log M + 2 \log R$

Differentiating, $\frac{dI}{I} = 0 + 2\frac{dR}{R}$

$\therefore$ % change in moment of inertia $= \frac{dI}{I} \times 100 = 2 \times 1\% = 2\%$

**(15)** **(A).** According to the theorem of parallel axes, the have

$I = I_G + Ma^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2 \; (\because a = R)$

Given that $\frac{2}{5}MR^2 = 20$ or $MR^2 = \frac{20 \times 5}{2} = 50,$

$\therefore \; I = \frac{7}{5} \times 50 = 70$ kg-m$^2$

**(16)** **(A).** According to the theorem of parallel axes, the moment of inertia of the disc about an axis tangentially and parallel to the surface is given by

$I = I_{parallel} = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$

The moment of inertia of the disc about an axis tangential but perpendicular to the surface is given by $I' = I_{perpendicular} = \frac{MR^2}{2} + MR^2$

$= \frac{3}{2}MR^2 = \frac{6}{5}\left[\frac{5}{4}MR^2\right] = \frac{6}{5}I$

**(17)** **(C).** $I_{XX'} = 4 \times (3)^2 + 2 \times (2)^2 + 3 \times (4)^2 = 92$ kg-m$^2$

**(18)** **(A).** Along the diameter, $I_g = (2/5)\,mR^2$ or $K_g^2 = (2/5)R^2$

or $K_g = R\sqrt{\frac{2}{5}} = 35\sqrt{\frac{2}{5}} = 7\sqrt{10}$ cm

Along tangent, $I = Ig + mR^2$

$\therefore \; mK^2 = mK_g^2 + mR^2$ or $K^2 = K_g^2 + R^2$

$= (2/5)R^2 + R^2 = (7/5)R^2$

$K = R\sqrt{\frac{7}{5}} = 35\sqrt{\frac{7}{5}}$, Now $\frac{K_g}{K} = \sqrt{\frac{10}{35}}$

**(19)** **(C).** $I_A = \frac{m_A r_A^2}{2}$ and $I_B = \frac{m_B r_B^2}{2}$,

$\therefore \; \frac{I_A}{I_B} = \frac{r_A^2}{r_B^2} \qquad (\because m_A = m_B)$ .........(1)

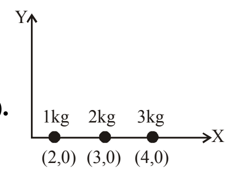
Now, $m_A = \pi r_A^2 \, t d_A$, $m_B = \pi r_B^2 \, t d_B$

So, $\pi r_A^2 \, t d_A = \pi r_B^2 \, t d_B$ or $\frac{r_A^2}{r_B^2} = \frac{d_B}{d_A}$ ......(2)

From equations (1) and (2)

$\frac{I_A}{I_B} = \frac{d_B}{d_A}$. As $d_A > d_B$ hence $I_A < I_B$

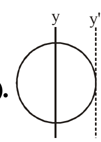
**(20)** **(D).**




Moment of inertia about X-axis will be zero because all points lie on the X-axis.

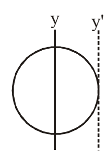
**(21)** **(A).** The radius of gyration about PQ axis is more than that about QR axis and the radius of gyration about QR axis is more than that about PR axis.
i.e, $K_{PQ} > K_{QR} > K_{PR}$
So, $I_1 > I_2 > I_3$

**(22)** **(D).**




Circular disc [1] $I_{y_1} = \frac{MR^2}{4}$

$\therefore \; I'_{y_1} = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$




Circular ring [2] $I_{y_2} = \frac{MR^2}{2}$

$\therefore \; I'_{y_2} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

$I'_{y_1} = MK_1^2, \; I'_{y_2} = MK_2^2$

$\therefore \; \frac{K_1^2}{K_2^2} = \frac{I'_{y_1}}{I'_{y_2}} \Rightarrow K_1 : K_2 = \sqrt{5} : \sqrt{6}$

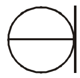
**(23)** **(A).** Mass of the whole disc = 4M
Moment of inertia of the disc about the given axis

$= \frac{1}{2}(4M)R^2 = 2MR^2$

∴ Moment of inertia of quarter section of the disc

$$= \frac{1}{4}(2MR^2) = \frac{1}{2}MR^2$$

**(24)** **(D).** The moment of inertia of a disc which is a tangent and parallel to its $\frac{5}{4}MR^2$



**(25)** **(A).** The moment of inertial of all seven rods parallel to AB and not lying on AB = $7 \times (1\,\ell)\,\ell^2 = 71\,\ell^3$

The moment of inertia of all five rods lying on AB = 0

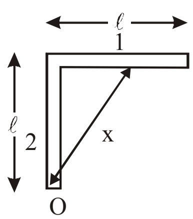
The moment of inertia of all 18 rods perpendicular to AB is = $18(1\,\ell)\frac{\ell^2}{3} = 61\,\ell^3$

Hence net MI of rod about AB = $71\,\ell^3 + 61\,\ell^3 = 131\,\ell^3$

**(26)** **(D).** Moment of inertia of rod 2 about an axis passing through O perpendicular to both roots, $I_2 = \frac{m\ell^2}{3}$

The distance from O to the parallel axis through the centre of mass of rod 2 is

$$\left[\ell^2 + \left(\frac{1}{2}\ell\right)^2\right]^{\frac{1}{2}}$$

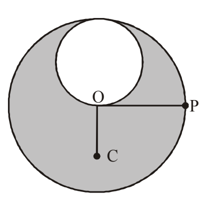



Moment of inertia of rod 1 about an axis passing through O perpendicular to both rods,

$$I_1 = \frac{m\,\ell^2}{12} + mx^2 = \frac{m\,\ell^2}{12} = \frac{5\,m\ell^2}{4} = \frac{4}{3}\,m\ell^2$$

$$I = I_1 + I_2 = \frac{4\,m\,\ell^2}{3} + \frac{m\,\ell^2}{3} = \frac{5\,m\,\ell^2}{3}$$

**(27)** **(C).** Let mass of compelte disc of radius 2R is 4M & mass of removed part is M.



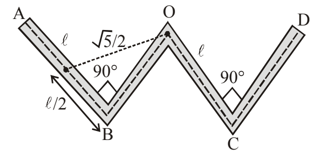

$$I_O = \frac{4M\,(2R)^2}{2} - \left[\frac{MR^2}{2} + MR^2\right] = \frac{13}{2}MR^2$$

$$I_P = \left[\frac{4M\,(2R)^2}{2} + 4M\,(2R)^2\right] - \left[\frac{M\,(R)^2}{2} + M\,(5R)^2\right] = \frac{37}{2}MR^2$$

**(28)** **(B).** The given structure can be broken into 4 parts.

For AB : $I = I_{CM} + m \times d^2 = \frac{m\ell^2}{12} + \frac{5m}{4}\ell^2$

For BO : $I = \frac{m\ell^2}{3}$




For composite frame : (by symmetry)

$$I = 2\,[I_{AB} + I_{OB}] = 2\left[\frac{4m\ell^2}{3} + \frac{m\ell^2}{3}\right] = \frac{10}{3}m\ell^2$$

**(29)** **(B).** Let $m_1$ = mass of the square plate of side 'a' and $m_2$ = mass of the square of side 'a/2'

Then $m_1 = \sigma\left(\frac{a}{2}\right)^2$, $m_2 = \sigma\,(a)^2$,

($\sigma$ along the area density) and $m_2 - m_1 = M$

$$I = \frac{m_2 a^2}{6} - \left\{\frac{m_1 (a/2)^2}{6} + m_1\left(\frac{a}{4}\right)^2\right\}$$

$$= \frac{\sigma a^4}{6} - \left\{\frac{\sigma\,(a/2)^4}{6} + \sigma\left(\frac{a}{2}\right)^2\left(\frac{a}{4}\right)^2\right\}$$

$$= \sigma a^4\left\{\frac{1}{6} - \frac{1}{16\times 6} - \frac{1}{4\times 16}\right\} = \sigma a^4\left\{\frac{(2\times 16) - 2 - 3}{16\times 12}\right\}$$

$$I = \sigma a^4\left\{\frac{27}{16\times 12}\right\}$$

Also, $M = \sigma\left\{1 - \frac{1}{4}\right\}a^2 \Rightarrow \sigma = \frac{4}{3}\frac{M}{a^2}$ ;

$$I = \left(\frac{4}{3}\frac{M}{a^2}\right)a^4\left\{\frac{27}{12\times 16}\right\} \Rightarrow I = \frac{3Ma^2}{16}$$

**(30)** **(B).** $K = \sqrt{\frac{\Sigma mr^2}{\Sigma m}} = \sqrt{\frac{2(3)^2 + 4(1)^2 + 5(2)^2 + 1(2)^2}{2+4+5+1}}$

$$= \sqrt{\frac{18+4+20+4}{12}} \quad ; \quad K = \sqrt{\frac{46}{12}} = \sqrt{\frac{23}{6}}\ \text{m}$$

**(31)** **(C).** We know that, $\vec{\tau} = \vec{r} \times \vec{F}$

$$\Rightarrow \vec{\tau} = (2\hat{i} + 3\hat{j}) \times (2\hat{i} + 6\hat{k}) = 12(-\hat{j}) + 6(-\hat{k}) + 18\hat{i}$$

$$= -12\hat{j} - 6\hat{k} + 18\hat{i}$$

[Note : $\hat{i} \times \hat{i} = 0$, $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{i} = -\hat{k}$ etc]

Now, $|\vec{\tau}| = \sqrt{(-12)^2 + (-6)^2 + (18)^2}$

$$= \sqrt{144 + 36 + 324} = \sqrt{504}$$

**(32)** **(D).** Torque, $\tau$ = rate of change of angular momentum

$$= \frac{dJ}{dt} = \frac{4A_0 - A_0}{4} = \frac{3A_0}{4}$$

**(33)** **(D).** $\tau \times \Delta t = L_0 \Rightarrow \tau \times \Delta t = I\omega$ ($\because$ since $L_f = 0$)

or $\tau \times 60 = 2 \times 2 \times 60\pi/60$ or $\tau = \frac{\pi}{15}$ N-m

**(34)** **(D).** Given, $I = 4$ kg-m$^2$, $\tau = 8$N-m and $t = 20$s

$$\tau = I\,\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{8}{4} = 2$$

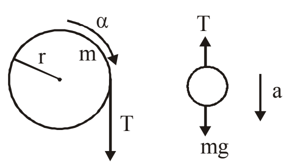
$$\theta = \frac{1}{2}\alpha\, t^2 = \frac{1}{2} \times 2 \times 20 \times 20 = 400$$

$$\omega = \tau\,\theta = 8 \times 400 = 3200 \text{ J}$$

**(35)** **(A).** The rotational analogue of force in linear motion is torque.

**(36)** **(D).** $\vec{\tau} = \vec{r} \times \vec{F} = (-1\hat{i} + 1\hat{j}) \times (-10\hat{k}) = -10\hat{i} - 10\hat{j}$

**(37)** **(C).** There is no slipping between pulley and thread.




So, $(a = \alpha r)$ ....... (1)

For point mass : $mg - T = ma$ ....... (2)

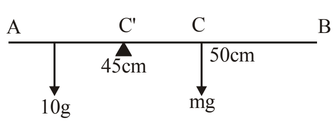
Equation of torque for disc $Tr = I\,\alpha$

$$Tr = \frac{mr^2}{2}\alpha \text{ or } T = \frac{mr\alpha}{2} = \left(\frac{mg}{2}\right) \text{ ....... (3)}$$

$$mg - \frac{mg}{2} = ma \;;\; mg = \frac{3mg}{2} \;;\; a = \frac{2g}{3}$$

**(38)** **(C).** Because you have not changed the moment arm about the center of mass and you are pulling with the same force, the torque is the same.

**(39)** **(A).** Let m be the mass of the stick which acts at C, (centre point).




For equilibrium about C' i.e., 45 cm mark

$10g\,(45 - 12) = mg\,(50 - 45)$

$$10g \times 33 = mg \times 5 \;;\; m = \frac{10 \times 33}{5} = 66\text{gm}$$

**(40)** **(C).**

(1) Disk $\tau = I\alpha$

$$\text{F.R.} = \frac{MR^2}{2}.\alpha_1$$

(2) Ring

$$\text{F.R.} = MR^2.\alpha_2$$

(3) Solid sphere

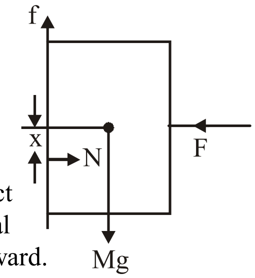
$$\text{F.R.} = \frac{2}{5}MR^2.\alpha_3$$

Hollow sphere

$$\text{F.R.} = \frac{2}{5}MR^2.\alpha_4$$

$$\alpha_3 > \alpha_1 > \alpha_4 > \alpha_2$$

**(41)** **(C).** $Mg\frac{L}{2} = T_A\,\frac{3L}{4}$




**(42)** **(D).** For equilibrium, $f = Mg$

So answer (A) is also correct

Also, $F = N$

Hence, answer (B) is also correct

For rotational equilibrium normal reactional force will shift downward.

Hence, torque due to friction about centre of mass is balanced by torque due to normal reaction about centre of mass. Hence, answer (D) is incorrect.

**(43)** **(C).** Immediately after string connected to end B is cut, the rod has tendency to rotate about point A.

Torque on rod AB about axis passing through A and normal to plane of paper is $\frac{m\ell^2}{3}a = mg\frac{\ell}{2}$ Þ $a = \frac{3g}{2\ell}$

**(44)** **(A).** $J_z = xp_y - yp_x$, because $\vec{J} = \vec{r} \times \vec{p}$

**(45)** **(C).** Mass of the body, $m = 1.0$ kg

The distance of the body from the axis of rotation,

$$r = \frac{2.0}{2} = 1.0\text{ m}$$

$\therefore$ Moment of inertia of the body about the axis of rotation is , $I = mr^2 = (A).(A)^2 = 1$ kg-m$^2$

Angular velocity of the body, $\omega = 2\pi n$, where n is the number of rotations per/sec

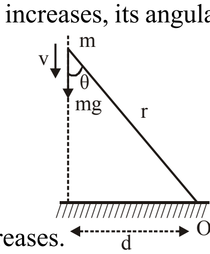
Here, $n = \frac{10}{31.4}$ $\therefore$ $\omega = 2 \times 3.14 \times \frac{10}{31.4} = 2$ rad/s

$\therefore$ Angular momentum, $J = I\omega = 1 \times 2 = 2$ kg-m$^2$/s

**(46)** **(C).** $|L_1| = mvd = mv4$ ; $|L_2| = 0$

$|L_3| = mv3$ ; $|L_4| = 0$

**(47)** **(B).** The magnitude of angular momentum of particle about O = mvd. Since speed v of particle increases, its angular momentum about O increases.




Magnitude of torque of gravitational force about O = mgd $\Rightarrow$ constant

Moment of inertia of particle about O = $mr^2$

Hence, MI of particle about O decreases.

Angular velocity of particle about O = $\frac{v\sin\theta}{r}$

$\because$ v and sin $\theta$ increase and r decreases.

$\therefore$ angular velocity of particle about O increases

**(48)** **(A).** Given, $M = 5.98 \times 10^{27}$ gm $= 5.98 \times 10^{24}$ kg and $R = 9.37 \times 10^6$ m

Angular velocity,

$$\omega = \frac{2\pi \text{ radian}}{1 \text{ day}} = \frac{2\pi}{24 \times 60 \times 60} \text{ rad/sec}$$

Moment of inertia of earth

$$I = \frac{2}{5}MR^2 = \frac{2}{5} \times (5.98 \times 10^{24})(9.37 \times 10^6)^2$$

$\therefore$ $J = I\omega = 1.53 \times 10^{34}$ kg m$^2$/sec
(Putting the values of $\omega$ and I)

**(49)** **(A).** We know that angular momentum of a particle

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = m(\vec{r} \times \vec{v})$$

$$= m\begin{vmatrix} i & j & k \\ x & y & z \\ V_x & V_y & V_z \end{vmatrix} = m\begin{vmatrix} i & j & k \\ x & y & 0 \\ V_x & V_y & 0 \end{vmatrix} = m\,\hat{k}\left(xV_y - yV_X\right)$$

**(50)** **(C).** According to law of conservation of angular momentum.
Angular momentum before the cockroach stops
= Angular momentum after the cockroach stops.

$$\Rightarrow \; I\omega - mvr = (I + mr^2)\,\omega' \quad \therefore \quad \omega' = \frac{I\omega - mvr}{I + mr^2}$$

**(51)** **(D).** As $\Sigma\tau = 0$, Angular momentum, linear momentum remains conserved.
As the two balls will move radially out, I changes. In order to keep the angular momentum ($L = I\omega$) conserved, angular speed ($\omega$) should change.

**(52)** **(B).** If external torque is zero, angular momentum remains conserved. [External torque is zero because the weight of child acts downward]
$L = I\omega$ = constant

**(53)** **(A).** During collapse of star no external torque acts on it, it undergoes redistribution of mass, resulting in change of moment of inertia. From conservation of angular momentum, $\vec{L}_i = \vec{L}_f$

$$\frac{2}{5}MR_i^2\left(\frac{2\pi}{T_i}\right) = \frac{2}{5}MR_f^2\left(\frac{2\pi}{T_f}\right)$$

or $\quad T_f = T_i\left(\frac{R_f}{R_i}\right)^2 = 2.7 \times 10^{-6}$ days

**(54)** **(A).** The M.I. of a rod about an axis passing through its one ends and perpendicular its to axis $I = \frac{M\ell^2}{3}$

Now rotational kinetic energy

$$K_r = (1/2)\,I\omega^2 = (1/2)\,\frac{M\ell^2}{3}\,.\,(2\pi f)^2 = (2/3)\,M\ell^2.\pi^2 f^2$$

**(55)** **(B).** The kinetic energy of a rotating body is, $K = 1/2\,I\omega^2$ and the angular momentum is, $J = I\omega$

$$\therefore \quad K = \frac{J^2}{2I}$$

Let $K_A$ and $K_B$ be the kinetic energies of A and B. The angular momentum of each is J. Then

$$\frac{K_A}{K_B} = \frac{J^2/2I_A}{J^2/2I_B} = \frac{I_B}{I_A}$$

But $I_A > I_B$ (given), $\therefore K_B > K_A$

**(56)** **(B).** K.E. $= (1/2)\,I\omega^2 = (1/2)\,I\,(\alpha t)^2 = (1/2)\,I\alpha^2 t^2$
$\therefore \quad 1500 = (1/2)\,(1.2)\,(25)^2\,t^2$ or $t = 2$ s

**(57)** **(A).** Here, $\omega = \frac{300 \times 2\pi}{60} = 31.4$ rad/sec
$I = mR^2 = 20\,(1/2)^2 = 5$ kg m$^2$

$$\therefore \quad K.E. = \frac{1}{2}\,I\omega^2 = \frac{1}{2} \times 5 \times (31.4)^2 = 2465 \text{ J}$$

**(58)** **(B).** $mgh = (1/2)\,I\omega^2 + (1/2)\,mv^2$
$= (1/2)\,I\omega^2 + (1/2)\,mr^2\omega^2$

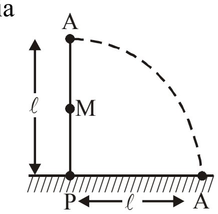
or $\quad 2mgh = [I + mr^2]\omega^2, \qquad \omega = \left[\frac{2mgh}{I + mr^2}\right]^{1/2}$

**(59)** **(B).** When rod fall on horizontal surface then rod will rotate about point P then its potential energy change into rotational kinetic energy.

From law of conservation of energy, $Mg\frac{\ell}{2} = \frac{1}{2}I\omega^2$

Since, moment of inertia of rod about point P is

$$I = \frac{M\ell^2}{3}$$




$$\therefore \quad Mg\cdot\frac{\ell}{2} = \frac{1}{2}\left(\frac{M\ell^2}{3}\right)\omega^2;$$

$$g\ell = \frac{\ell^2}{3}\left(\frac{v^2}{\ell^2}\right) \Rightarrow v = \sqrt{3g\ell}$$

**(60)** **(B).** Rotational kinetic energy $= \frac{1}{2}I\omega^2 = 8$J

$$\frac{1}{2} \times \frac{1}{2}mr^2\omega^2 = 8 \text{ or } \frac{1}{4} \times 2 \times (1)^2\omega^2 = 8$$

or $\quad \omega^2 = 16 \Rightarrow \omega = 4$ rad/s
Angular momentum,

$$L = I\omega = \frac{1}{2}mr^2\omega = \frac{1}{2} \times 2 \times (1)^2 \times 4 = 4 \text{ J-s}$$

**(61)** **(D).** Applying principle of conservation of total mechanical energy $mgh = \frac{1}{2}I\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2 \;\; \therefore \;\; v = \sqrt{\frac{4}{3}gh}$

**(62)** **(C).** Total energy, $E = (1/2)\,I\omega^2 + (1/2)\,mv^2$
$= (1/2)\,(2/5\,mr^2)\,\omega^2 + (1/2)\,mr^2\omega^2$
$= (1/5)\,mr^2\omega^2 + (1/2)\,mr^2\omega^2 = (7/10)\,mr^2\omega^2$
Rotational energy $= (1/5)\,mr^2\omega^2$

$$\therefore \quad \frac{\text{Rotational energy}}{\text{Total energy}} = \frac{\frac{1}{5}mr^2\omega^2}{\frac{7}{10}mr^2\omega^2} = \frac{2}{7}$$

**(63)** **(A).** We know that, $a = \frac{g\sin\theta}{(1 + k^2/R^2)}$

Here, $a_1 = \frac{5g\sin\theta}{7}$ and $a_2 = \frac{2g\sin\theta}{3}$,

$\therefore\ a_1 : a_2 = 15 : 14$

**(64)** **(B).** $\frac{E_{rotational}}{E_{Total}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)} = \frac{\frac{K^2}{R^2}}{1+\frac{K^2}{R^2}}$

For ring $\frac{K^2}{R^2} = 1$ ; $\frac{E_{rotational}}{E_{Total}} = \frac{1}{2} = 50\%$

**(65)** **(C).** When the sphere reaches the ground, its P.E. is converted into its. K.E.

$\therefore$ K.E. $= (1/2)\,mv^2 + (1/2)\,I\omega^2 = (1/2)\,mv^2 + (1/2)\,(mk^2)\,\omega^2$

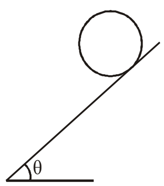
$mgh = (1/2)\,m\,(v^2 + k^2\omega^2)$

or $2gh = \omega^2\,(r^2 + k^2)$ ($\because v = r\omega$)

$\therefore\ \omega = \sqrt{\frac{2gh}{r^2+k^2}}$, $v = \sqrt{\frac{2gh}{1+k^2/r^2}}$

**(66)** **(D).** $a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$

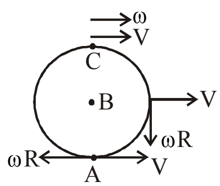
(where k is radius of curvature); $k_P > k_Q$)




**(67)** **(D).** $\vec{V}_A = V(\hat{i}) + \omega R\,(-\hat{i})$,

$\vec{V}_B = V\hat{i}$, $\vec{V}_C = V\hat{i} + \omega R\hat{i}$

$\vec{V}_C - \vec{V}_A = 2\omega R\hat{i}$




$2\,[\vec{V}_B - \vec{V}_C] = 2\,[V(\hat{i}) - V(\hat{i}) - \omega R(\hat{i})] = -2\omega R(\hat{i})$

Hence, $\vec{V}_C - \vec{V}_A = -2\,(\vec{V}_B - \vec{V}_C)$

so $|\vec{V}_C - \vec{V}_A| = |2\,(\vec{V}_B - \vec{V}_C)|$

$\vec{V}_C - \vec{V}_B = \omega R\,(\hat{i})$, $\vec{V}_B - \vec{V}_A = \omega R\,(\hat{i})$

$\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

Hence, $\vec{V}_C - \vec{V}_A = 2\omega R(\hat{i})$, $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

$4\vec{V}_B = 4V(\hat{i}) = 4\omega R(\hat{i})$ Hence, $\vec{V}_C - \vec{V}_A = 2\,(\vec{V}_B)$

**(68)** **(D).** As the inclined plane is smooth, the sphere can never roll rather it will just slip down.
Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

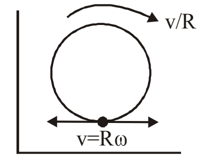
**(69)** **(B).** Here, $u = V_0$, $\omega_0 = -\frac{V_0}{2R}$

At pure rolling, $V = V_0 - \left(\frac{F_f}{m}\right)t$ ; $\frac{V}{R} = -\frac{V_0}{2R} + \left(\frac{F_f}{mR}\right)t$

(In pure $V = R\omega$) $\left(\alpha = \frac{\tau}{I} = \frac{F_f.R}{mR^2}\right)$

$\Rightarrow\ V_0 - V = V + \frac{V_0}{2}\ \Rightarrow\ 2V = \frac{V_0}{2}\ \Rightarrow\ V = \frac{V_0}{4}$

**(70)** **(D).** As the disc in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.




From figure, $v_{net}$ (for lowest point) $= v - R\omega = v - v = 0$

and Acceleration $= \frac{v^2}{R} + 0 = \frac{v^2}{R}$

(Since linear speed is constant)

**(71)** **(C).** If the track is smooth (case A), only translational kinetic energy changes to the gravitational potential energy. But, if the track is rough (case B), both translational and rotational kinetic energy changes to potential energy. Therefore, potential energy (= mgh) will be more in case B than in case A. Hence $h_1 > h_2$.

**(72)** **(C).** For sliding, $a = g\sin\theta$. Hence the velocity v is given by, $v^2 = 0 + 2\,(g\sin\theta)\,x\,\ell$ .......(i)
For rolling, acceleration down the inclined plane is given

by, $a = \frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)} = \frac{1}{2}\,g\sin\theta$ ($\because k^2 = R^2$)

In case of ring,

$V_r^2 = 2 \times (1/2\,g\sin\theta) \times \ell = \frac{v^2}{2}$ $\therefore\ V_r = \frac{v}{\sqrt{2}}$

**(73)** **(D).** By conservation of mechanical energy

$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mgh$

For dise $\frac{1}{2}mv^2\left(1+\frac{1}{2}\right) = mgh$ ; $h = \frac{3v^2}{4g}$

**(74)** **(B).** If we treat the train as a ring of mass M then its COM will be at a distance $\frac{2R}{\pi}$ from the centre of the circle.
Velocity of centre of mass is :

$V_{CM} = R_{CM}.\omega = \frac{2R}{\pi}\omega = \frac{2R}{\pi}\left(\frac{V}{R}\right)$ $\left(\because \omega = \frac{V}{R}\right)$

$\Rightarrow\ V_{CM} = \frac{2V}{\pi} \Rightarrow MV_{CM} = \frac{2MV}{\pi}$

As the linear momentum of any system $= MV_{CM}$.

$\therefore$ The linear momentum of the train $= \dfrac{2MV}{p}$

**(75)** **(C).** Let the tube displaced by x towards left, then

$$mx = m(R - x) \Rightarrow x = \frac{R}{2}$$

**(76)** **(C).** When two particles of masses $m_1$ and $m_2$ are moving under the action of their internal forces $\vec{f}_1$ and $\vec{f}_2$. Their relative motion can be obtained by assuming one of the particles to have an infinite mass and by replacing the mass of the second particle by the reduced mass of the system, the force between them of the same as before.

**(77)** **(B).** If the external force acting on a system has zero resultant the centre of mass must not accelerate

**(78)** **(C).** A loaded spring gun of mass M fires a shot of mass m with a velocity V at an angle of elevation θ. The gun is initially at rest on a horizontal frictionless surface. Just after firing, the centre of mass of the gun-shot system remains at rest.

**(79)** **(D).** If centre of mass of the system remains at rest then it is sure that Linear momentum of the system remains constant but if momentum is constant then centre of mass may be moving with constant velocity.

**(80)** **(C).** A particle at rest suddenly disintegrates into two particles of equal masses which start moving The two fragments will move in opposite directions with equal speeds (From momentum conservation).

**(81)** **(B).** In a free space, a rifle of mass ' M' shoots a bullet of mass 'm' at a stationary block of mass M distance 'D' away from it. When the bullet has moved through a distance 'd' towards the block, the centre of mass of the bullet block system is at a distance of (i) $\dfrac{(D-d)m}{M+m}$ from the block (ii) $(D-d)\dfrac{M}{M+m}$ from the bullet

**(82)** **(B).** A body has its centre of mass at the origin. The x-coordinates of the particles may be all negative.

**(83)** **(B).** Two particles A and B initially at rest move towards each other under a mutual force of attraction. The speed of centre of mass at the instant when the speed of A is v and the speed of B is 2v is zero (From conservation of momentum).

**(84)** **(D).** A bomb travelling in a parabolic path under the effect of gravity. explodes in mid air. The centre of mass of fragments will move in the parabolic path which the unexploded bomb would have travelled.

**(85)** **(D).** Two balls are thrown in air. The acceleration of the centre of mass of the two balls while in air (neglect air resistance) is equal to g.

**(86)** **(C).** As $\vec{V}_{cm} = 0 \therefore \vec{V} = -\vec{u}_1$ and $\vec{V}_2 = -\vec{u}_2$

**(87)** **(A).** For vertical equilibrium, $mg = N_2$ ............ (1)

For horizontal equilibrium, $\mu N_2 = N_1$............ (2)

Torque about A :

$N_1 \times 5\sin 37° = mg \times 2.5\cos 37°$ ............ (3)

**(88)** **(A).** Moment of inertia of the rod w.r.t. the axis through centre of the disc is (by parallel axis theorem)

$$I = \frac{ML^2}{12} + mR^2 \text{ and K.E. of rod w.r.t. disc}$$

$$= \frac{1}{2}I\omega^2 = \frac{1}{2}m\omega^2\left[R^2 + \frac{L^2}{12}\right]$$

**(89)** **(B).** The two forces along y-direction balance each other. Hence, the resultant force is 2F along x-direction.

Let the point of application of force be at (0, y).

(By symmetry x-coordinate will be zero).

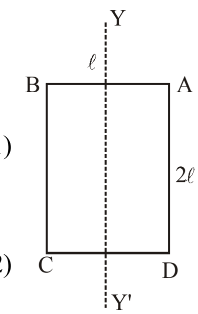
For rotational equilibrium :

$$F(a) + F(a) + F(a + y) - F(a - y) = 0 \Rightarrow y = -a$$

**(90)** **(C).** I (about YY') $= \dfrac{m\ell^2}{12}$

Using parallel axis theorem :

$$I\,(\text{about AD}) = \frac{m\ell^2}{12} + \frac{m\ell^2}{4} = \frac{m\ell^2}{3}$$




**(91)** **(D).** $F + f = ma$ ........... (1)

Also, $Fr - fr = I\dfrac{a}{r}$

$F - f = ma$ ........... (2)

$\Rightarrow f = 0$

**(92)** **(D).** Given $a_A = 2a = 5\,m/s^2$

$\Rightarrow a = 5/2\ rad/s^2 \Rightarrow a_B = 1(a) = (5/2)m/s^2$

**(93)** **(B).** Balancing torque about the centre of the rod :

$$N_1\frac{\ell}{4} - N_2\frac{\ell}{4} = 0 \Rightarrow N_1 = N_2$$

**(94)** **(C).** Lowest point of 'C' is at rest but the top most point has a velocity v $\therefore v_{CM}$ of C is v/2

For A : The lowest point has a velocity v and highest point 2V. $\therefore V_{CM}$ of A is 3v/2

**(95)** **(C).** Let us assume, surface is frictionless

$5F = ma,\ 2FR = I\alpha \Rightarrow 5F = mR\alpha$

Solving these equations, $a = \alpha R$

$\Rightarrow$ Acceleration of contact point along the surface is zero.

Thus, friction force = 0

**(96)** **(B).** Taking moment about A,

$$F \times \frac{3a}{4} = mg \times \frac{a}{2} \Rightarrow F = \frac{2mg}{3}$$

**(97)** **(D).** $mg\ell + \dfrac{3mg}{8}\left(\ell + \dfrac{\ell}{3}\right) = \left[m\left(\dfrac{4\ell^2}{3}\right) + \dfrac{3}{8}m\left(\dfrac{4\ell}{3}\right)^2\right]\alpha$

$$\alpha = \frac{3g}{4\ell},\ a = g$$

**(98)** **(C).** For toppling about edge xx'

At the moment of toppling the normal force pass through axis xx'. $F_{min}\frac{3a}{4} = mg\frac{a}{2}$ or $F_{min} = \frac{2mg}{3}$

## EXERCISE-2

**(1)** **(B).** $T = mg$

$\tau_R = T_R = I\alpha$ ; $\omega^2 = 2\alpha\theta$

$$\theta = \frac{\omega^2}{2\alpha} = \frac{\omega^2 I}{2TR} = \frac{\omega^2 mR^2}{2\times 2\times mgR} = \frac{\omega^2 R}{4g}$$

arc length $\theta R = \frac{\omega^2 R^2}{4g}$

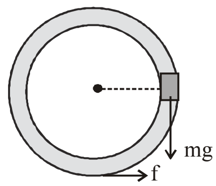
**(2)** **(B).** $f = 4\,ma$ ......... (1)

$(mg - f)\,r = (3mr^2 + mr^2)\,a$

$mg - f = 4ma$ ......... (2)

From (1) and (2)

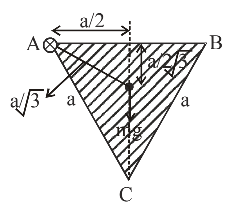
$8ma = mg \Rightarrow a = \frac{9}{8} \Rightarrow a = \frac{g}{8r}$




**(3)** **(C).** Torque about A :

$mg\frac{a}{2} = I\alpha \Rightarrow \alpha = \frac{mga}{2I}$

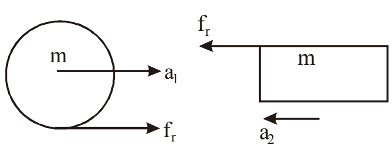
acceleration $= \frac{a}{\sqrt{3}}\alpha = \frac{mga^2}{2\sqrt{3}\,I}$




**(4)** **(C).** FBD for sphere and block

$a_1 = \frac{f_r}{m} = \frac{\mu mg}{m}$ , $a_2 = \frac{f_r}{m} = \frac{\mu mg}{m}$

$\vec{a}_1 = \mu g\hat{i}$ , $\vec{a}_2 = -\mu g\hat{i}$




**(5)** **(C).** The area of the spool occupied by the wound thick tape is $S_1 = r\,( r_f^2{}' - r_f^2 ), = 8\pi\ r_i^2$ .

Then the length of the would tape is $l = S_1/d = 8\pi\,( r_i^2/d)$,

Where d is the thickness of the thick tape.

The area of the spool occupied by the wound thin tape is $S_2 = \pi\,( r_f'^2 - r_i^2 )$, where $r_f'$ is the final radius of the wound part in the latter case. Since the lengths of the tapes are equal, and the tape thickness in the latter case is half that in the former case, we can write

$$l = \frac{2\pi\,(r_f'^2 - r_i^2)}{d},\ r'^2_f - r_i^2 = 4\,r_i^2$$

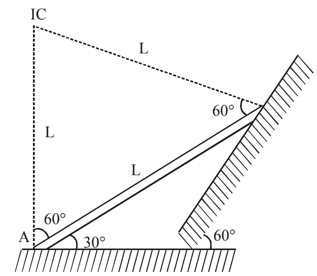
Consequently, the final radius $r'_f$ of the wound part in the latter case is $r'_f = \sqrt{5}r_1$.

The numbers of turns $N_1$ and $N_2$ of the spool for the former and latter winding can be written as

$N_1 = \frac{2r_1}{d}, N_2 = \frac{(\sqrt{5}-1)\,r_1}{d/2}$ whence $t_2 = \sqrt{(5-1)}t_1$.

**(6)** **(B).** Draw a normal at A and B to locate IC.

$\omega = \frac{V}{L}$




**(7)** **(D).** As torque = change in angular momentum

$\therefore$ $F.\Delta t = mv$ (linear) ........... (1)

and $\left(F.\frac{\ell}{2}\right)\Delta t = \frac{m\ell^2}{12}.\omega$ (angular) ........... (2)

Diving (1) and (2), $2 = \frac{12v}{\omega\ell} \Rightarrow \omega = \frac{6v}{\ell}$

Using $S = ut$ :

Displacement of COM is $\frac{\pi}{2} = \omega t = \left(\frac{6v}{\ell}\right)t$ and $x = vt$

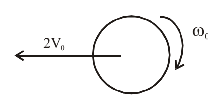
Diving : $\frac{2x}{\pi} = \frac{\ell}{6} \Rightarrow x = \frac{\pi\ell}{12}$

$\Rightarrow$ Coordinate of A will be $\left[\frac{\pi\ell}{12} + \frac{\ell}{2}, 0\right]$

**(8)** **(C).** As they are in pure rolling condition, we have [before collision]

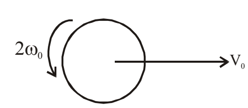
$V_0 = k\omega_0$ and $2v_0 = 2d\omega_0$

Immediately after collision the translational velocities are changed and the angular velocities will remain uneffected for A, the situation after collision will be




The velocity at lowest pain is $2v_0 + R\omega_0 = 3V_0$

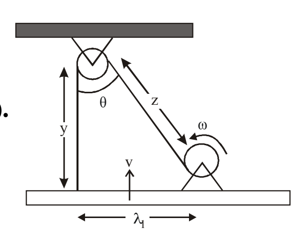
For B the velocities after collision are




The velocity at the lowest point is

$V_0 + 2R\omega_0 = 3V_0$

**(9)** **(A).**

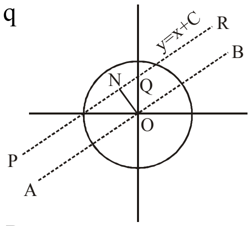


$\ell = y + z$ (total length at any time t) $= y + \sqrt{y^2 + \ell_1^2}$

$$\frac{d\ell}{dt} = \frac{dy}{dt} + \frac{y}{\sqrt{y^2+\ell_1^2}}\frac{dy}{dt}$$

$$wR = v\,(1+\cos q)\,,\quad v = \frac{wR}{1+\cos q}$$

**(10)** **(B).** $I_{PQR} = I_{AOB} + M.(ON)^2$

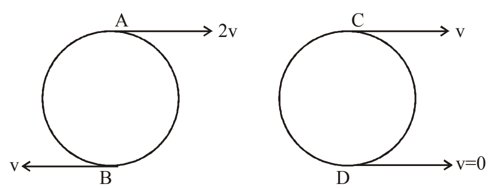



$$I_{PQR} = \frac{1}{4}MR^2 + M.\left(\frac{C}{\sqrt{2}}\right)^2$$

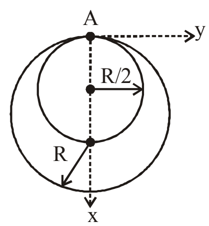
But $I_{PQR} = \frac{1}{2}MR^2 \therefore C = \pm\frac{R}{\sqrt{2}}$

**(11)** **(B).** In the absence of slipping, velocities of contact points of upper cylinders and lower cylinders are respectively.

$$\omega_{up} = \frac{3v}{2R} = \frac{v_{AB}}{AB}\ ;\ \omega_{lower} = \frac{v_{CD}}{CD} = \frac{v}{2R}\ ;\ \frac{\omega_{up}}{\omega_{lower}} = 3$$




**(12)** **(C).** $x_{cm} = \dfrac{m\left(\frac{R}{2}\right) + 2mR}{m+2m} = \dfrac{5}{6}R$




$I_A = I_1 + I_2,$

$$I_1 = m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2 = \frac{mR^2}{2}$$

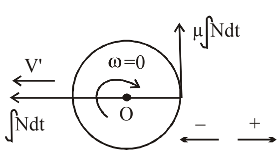
$$I_2 = 2mR^2 + 2mR^2 = 4mR^2 \Rightarrow I_A = \frac{9}{2}mR^2$$

For compound pendulum $T = 2\pi\sqrt{\dfrac{I}{Mgd}}$

Here $M = 3m,\ d = \frac{5}{6}R \Rightarrow T = 2\pi\sqrt{\dfrac{9R}{5g}}$

**(13)** **(A).** $-\int N dt = -mV' - (mV)$ ........ (1)

$\mu R\int N dt = \frac{2}{5}mR^2\left(\frac{V}{R}\right)$ ......... (2)




From eq. (1) and (2), we get $\int N dt = 2mV$ and $V' = V$

**(14)** **(B).** By conservation of mechanical energy

$$\frac{1}{2}\frac{M\ell^2}{3}\omega_0^2 = \frac{1}{2}m(\omega^2\ell^2 + v^2) + \frac{1}{2}\frac{M\ell^2}{3}\omega^2$$

By conservation of angular momentum

$$\frac{M\ell^2}{3}\omega_0 = \frac{M\ell^2}{3}\omega + m\ell^2\omega$$

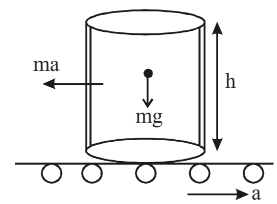
Solving above two equations : $v = \dfrac{\omega_0\ell}{\sqrt{1+\frac{3m}{M}}}$

**(15)** **(D).** WRT of belt, pseudo force ma acts on cylinder at COM as shown about to cylinder will be just about to topple when torque to weight w.r.t. P.

$$\frac{dv}{dt} = 2bt$$

$$m.2bt.\frac{h}{2} = mgr$$

$$\Rightarrow t = \frac{rg}{bh}$$



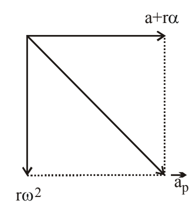

**(16)** **(A).** $\vec{a}_P = \vec{a}_{P_0} + \vec{a}_0$

Here, $\vec{a}_{P_0}$ = acceleration of P with respect to O

$= \vec{a}_{P_{0t}} + \vec{a}_{P_{0n}} \quad \therefore \quad \vec{a}_P = \vec{a}_{P_{0t}} + \vec{a}_{P_{0n}} + \vec{a}_0$

Here, $\vec{a}_{P_{0t}}$ = tangential component of $\vec{a}_{P_0}$ and

$\vec{a}_{P_{0x}}$ = normal component of $\vec{a}_{P_0}$.




$|\vec{a}_0 + \vec{a}_{P_{0t}}| = a + r\alpha$

$|\vec{a}_{P_{0x}}| = r\omega^2 \quad \therefore \quad |\vec{a}_P| = \sqrt{(a+r\alpha)^2 + (r\omega^2)^2}$

**(17)** **(A).** The tension at joint is due to force exerted by the rod of linear density $\mu_2$.

$$\text{So,}\quad F = \int_{\ell}^{2\ell}\mu_2 dx\omega^2 x = \frac{3\mu_2\omega^2\ell^2}{2}$$

**(18)** **(A).** Let F be the magnitude of force exerted on the rod due to the collision.

Then $F = ma$ and $F\frac{\ell}{4} = \frac{m\ell^2}{12}a$ (about O)

Þ $a = \frac{\ell}{3}a$ .......... (1)

Using, $S = ut + \frac{1}{2}at^2$ and $q = w_0 t + \frac{1}{2}a t^2$

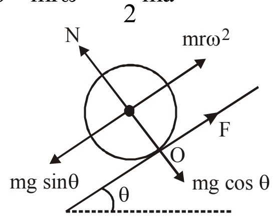
$S = \frac{1}{2}at^2$ and $6p = \frac{1}{2}a t^2$ Þ $\frac{6p}{s} = \frac{a}{a} = \frac{3}{\ell}$ [From (1)]

Þ $S = 2p\ell$

**(19)** **(A).** Free body diagram of the cylinder w.r.t. the incline plane. Torque about point of contact O.

$$(mg\sin\theta - mr\omega^2)\,R = I\alpha = \frac{3}{2}mR^2\,\frac{a}{R}$$

$$\therefore\ mg\sin\theta - mr\omega^2 = \frac{3}{2}ma$$




$$\therefore\ a = \frac{2}{3}(g\sin\theta - r\omega^2) = \frac{2}{3}\left[10\times\frac{1}{2} - 6\times\frac{1}{4}\right] = 2.33\ \text{m/s}^2$$

**(20)** **(B).** Consider a differential circular strip of the disc of radius x and thickness dx. Mass of this strip is
$dm = 2\rho\pi x\,dx$.

where $\rho = \frac{M}{\pi R^2}$. Frictional force on this strip is along the tangent and is equal to $dF = \mu\rho 2\pi\ x dx\ g$

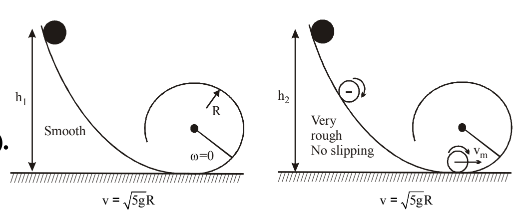
Torque on the strip due to frictional force is equal to $d\tau = \mu\rho g 2\pi x^2 dx$ disc is supposed to be the combination of number of such strips hence torque on the disc is given by $\tau = \int d\tau = \mu\rho\, g 2\pi \int_0^R x^2\,dx = \mu\rho g\, 2\pi\,\frac{R^3}{3}$

$$\Rightarrow \tau = \mu Mg\,(2/3)R \ \Rightarrow\ \alpha = \frac{2\mu MgR}{3\left(\frac{MR^2}{2}\right)} = \frac{4}{3}\frac{\mu g}{R}$$

The $\alpha$ is opposite to the $\omega$

$$\because\ \alpha(t) = \alpha_0 + \alpha t \Rightarrow\ 0 = \omega_0 - \frac{4\mu g}{3R}t \ \Rightarrow\ t = \frac{3\omega_0 R}{4\mu g}$$

**(21)** **(B).**




Case-1 : $mg h_1 = (K.E.)_{translation} = K_{trans.} = \frac{1}{2}mv^2$

Whole gravitational potential energy is converted into translational K.E. only

Case-2 : $mgh_2 = K_{trans} + K_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I_{cm}\omega^2$

Some part of gravitational potential energy will be converted into rotational K.E. also.
Hence in second case, greater height is required.

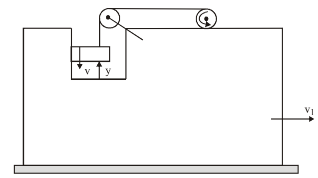
**(22)** **(C).** $Mg\,(H-y) = \frac{1}{2}M(v^2 + v_1^2) + \frac{1}{2}Mv_1^2$

$+\frac{1}{2}M(v_c)^2 + \frac{1}{2}I_c\omega^2$ ....... (i)

$2Mv_1 = Mv_c$ ....... (ii)

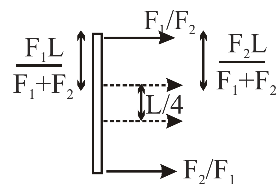
$2v_c = v$ ....... (iii)

Solving (i), (ii) and (iii), $\frac{3}{4}v^2 = g\,(H-y)$




$$\Rightarrow\ v = \frac{-dy}{dt} = \sqrt{\frac{4g}{3}(H-y)} \Rightarrow t = \sqrt{\frac{3H}{g}}$$

**(23)** **(B).**




$$\frac{F_1L}{F_1+F_2} - \frac{F_2L}{F_1+F_2} = \frac{L}{4} \Rightarrow \frac{F_1}{F_2} = \frac{5}{3}$$

**(24)** **(A).** For (a) : Take moment about O

$m_2 gr\sin\theta = m_1 g\, r\cos\theta$ (r = radius of cylinder)

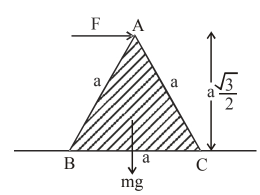
$$\tan\theta = \left(\frac{m_1}{m_2}\right)\ ;\ \ \theta = \tan^{-1}\left(\frac{m_1}{m_2}\right)$$

**For (b)** : Take moment about O

$m_2 gr\sin\theta = m_1 g\, r$

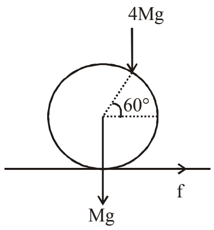
$$\sin\theta = \frac{m_1}{m_2}\ ;\ \ \theta = \sin^{-1}\left(\frac{m_1}{m_2}\right)$$

**(25)** **(A).** The tendency of rotating will be about the point C. For minimum force, the torque of F about C has to be equal to the torque of mg about C.



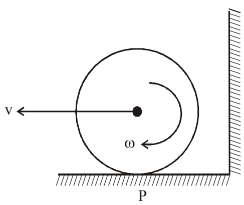

$$\therefore\ F\left(a\frac{\sqrt{3}}{2}\right) = mg\left(\frac{a}{2}\right) \Rightarrow F = \frac{mg}{\sqrt{3}}$$

**(26)** **(C).** $4mg\times\frac{R}{2} - f\times R = I\alpha\ ;\ f = Ma$ and $N = 5mg$




$\therefore \mu_{min} = \frac{2}{7}$

**(27)** **(B).** Before collision, pure uniform rolling therefore f = 0.
After collision, point P is having net velocity along -x axis



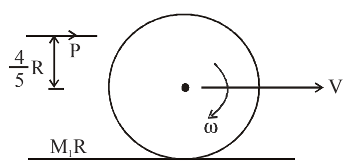

Therefore $f = \mu mg\,(\hat{i})$

**(28)** **(A).** Using impulse-momentum equation

$$P = MV \Rightarrow V = \frac{P}{M} \qquad \text{..........(1)}$$

Using angular impulse-momentum equation, w.r.t. centre

$$P\frac{4}{5}R = \frac{2}{5}mR^2\omega$$



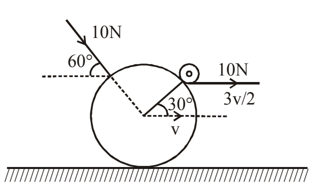

$$\omega = \frac{2P}{mR}$$

Total K.E. = Trans KE + Rotational KE

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m \times \frac{P^2}{m^2} + \frac{1}{2}.\frac{2}{5}mR^2\frac{4P^2}{m^2R^2} = \frac{13P^2}{10m}$$

**(29)** **(A).** Work done = change in K.E.

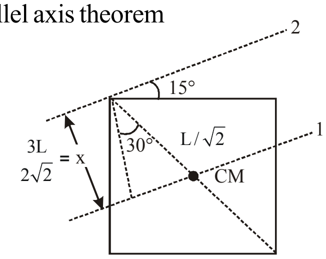



$$10 \times 60 \times \frac{3}{2} + 10 \times \cos 60° \times 60 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$v = r\omega$ ; $v = 40$ m/s

**(30)** **(B).** From parallel axis theorem

$$I_1 = \frac{ML^2}{12}$$



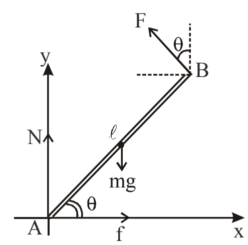

$$I_2 = I_1 + Mx^2 = \frac{ML^2}{12} + M\left(\frac{3L}{2\sqrt{2}}\right)^2 = \frac{11ML^2}{24}$$

**(31)** **(A).** Consider a general position of rod as shown in figure. Let applied force be F.
As the end is moved slowly so the rod has no acceleration. Applying Newton's law in x-direction

$$f - F\sin\theta = 0 \Rightarrow f = F\sin\theta \qquad \text{.........(1)}$$

In y-direction, $F\cos\theta + N = mg$ .........(2)

Making $\tau_A = 0 \Rightarrow F\ell - mg\frac{\ell}{2}\cos\theta = 0$



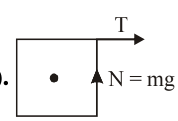

$$\Rightarrow F = \frac{mg\cos\theta}{2} \qquad \text{.........(3)}$$

For no slipping

$$f \le \mu N \Rightarrow \mu \ge \frac{f}{N} \Rightarrow \mu \ge \frac{F\sin\theta}{mg - F\cos\theta} \Rightarrow \mu \ge \frac{\sin\theta\cos\theta}{2 - \cos^2\theta}$$

$$\therefore \mu_{min} = \left(\frac{\sin\theta\cos\theta}{2 - \cos^2\theta}\right)_{max} = \frac{1}{2\sqrt{2}}$$

**(32)** **(A).**




$mg - T = ma \Rightarrow T = Ma$

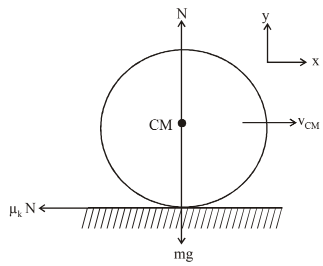
$$a = \frac{mg}{M+m} \Rightarrow T = Ma = \frac{Mmg}{M+m}$$

$$T\frac{h}{2} = Mg\frac{h}{4} \Rightarrow \frac{Mmg}{M+m}\frac{h}{2} = Mg\frac{h}{4}$$

$$2 = 1 + \frac{M}{m} \Rightarrow \frac{M}{m} = 1$$

**(33)** **(A).** Equations of motion for translation of sphere are :

$\Sigma f_x = -f = Ma$ ........... (1)




$\Sigma f_y = N - Mg = 0$ ........... (2)

From eq. (1) and (2)

$a = -\mu_k g$ ........... (3)

The velocity of the centre of mass at time is

$v = v_0 + a_x t = v_0 - \mu_k gt$ ........... (4)

Equation of motion for rotation is

$\Sigma\tau = \mu_k MgR = \frac{2}{5}MR^2\alpha$ ........... (5)

or $\alpha = \frac{5}{2}\frac{\mu_k g}{R}$ ........... (6)

Then the angular velocity of the ball at time t is

$\omega = \omega_0 + \alpha t = 0 + \frac{5\mu_k gt}{2R}$ ........... (7)

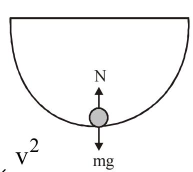
When the ball comes into contact with the horizontal surface, the friction force has two effects. Friction force acts to decrease the linear velocity of the CM (slow down the translation motion).

Secondly, friction tends to rotate the ball clockwise. The sphere starts rotating immediately after it touches the ground, but it rolls with slipping. It eventually stops slipping at the instant the contact point has zero velocity w.r.t. ground and centre of mass has velocity $v_{CM} = R\omega$.

The condition for pure rolling is that $v_{CM} = \omega R$. From eq. (4) and (7), $v_0 - \mu_k g t_0 = \frac{5\mu_k g t_0}{2R}R$, where $t_0$ is the time at which pure rolling starts.

So, $t_0 = \frac{2v_0}{7\mu_k g}$

**(34)** **(B).** $mgR = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$




$\omega = \frac{v}{r}$ ; $mgR = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mr^2 \times \frac{v^2}{r^2}$

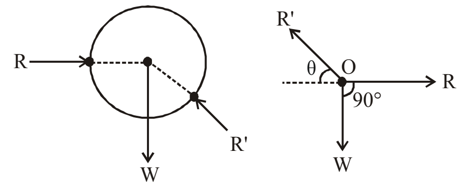
$gR = \frac{7}{10}mv^2 \Rightarrow v = \sqrt{\frac{10gR}{7}}$

$N - mg = \frac{mv^2}{R} \Rightarrow N = \frac{17mg}{7}$

**(35)** **(C).** Consider the free body diagram of the cylinder.

The forces acting on it are (i) Weight W vertically downward ; (ii) reaction R due to the wall at P and perpendicular to wall, (iii) reaction R' due to the rod at Q and perpendicular to rod.

Since three forces keep the cylinder in equilibrium so these must form a concurrent force system and Lami's theorem will be applicable.




$\therefore \frac{R'}{\sin 90°} = \frac{W}{\sin(180° - \theta)} = \frac{W}{\sin\theta}$ or $R' = \frac{W}{\sin\theta}$

Now the rod in equilibrium under the action of the forces:

(i) Tension T in string :

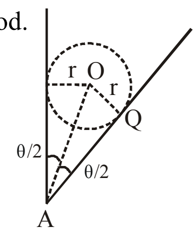
(ii) Reaction R" at Q due to the cylinder. R" will be equal and opposite to R'

(iii) Reaction $R_A$ at the hinge. Its direction is not known.

Taken moment about A (to eliminate $R_A$)

$\Sigma M_A = -T(AC) + R''(AQ) = 0$

AC = $\ell\cos\theta$ where $\ell$ is length of rod.




$AQ = r\cot\frac{\theta}{2}$, from triangle AOQ

$\therefore T = R''\frac{(AQ)}{(AC)} = \frac{W}{\sin\theta}\frac{r\cot\theta/2}{\ell\cos\theta}$

$= \frac{Wr}{\ell}\frac{\cos\frac{\theta}{2}/\sin\frac{\theta}{2}}{\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\cos\theta} = \frac{Wr}{\ell}\frac{1}{2\sin^2\frac{\theta}{2}\cos\theta}$

For T to be minimum, the term in denominator $2\sin^2\frac{\theta}{2}\cos\theta$ must be maximum.

$\therefore \frac{d}{d\theta}\left(2\sin^2\frac{\theta}{2}\cos\theta\right) = 0$

or $\frac{2}{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \sin^2\frac{\theta}{2}.\sin\theta = 0$

or $\frac{1}{2}\sin\theta\cos\theta - \sin^2\frac{\theta}{2}.\sin\theta = 0$

or $\sin\theta\left[\frac{1}{2}\cos\theta - \sin^2\frac{\theta}{2}\right] = 0$

$\therefore$ Either $\sin\theta = 0$, i.e. $\theta = 0$ which is not possible.

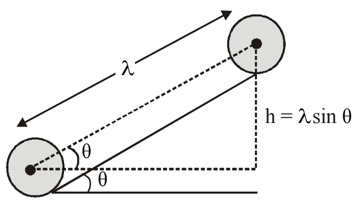
or $\left(\frac{1}{2}\cos\theta - \sin^2\frac{\theta}{2}\right) = 0$

$\therefore \frac{1}{2}\left(1-2\sin^2\frac{\theta}{2}\right)-\sin^2\frac{\theta}{2}=0 \left[\cos\theta = 1-2\sin^2\frac{\theta}{2}\right]$

$\therefore \sin^2\frac{\theta}{2}=\frac{1}{4}$ or $\sin\frac{\theta}{2}=\frac{1}{2}$ i.e., $\frac{\theta}{2}=30°$ or $\theta = 60°$

**(36)** **(B).** For pure rolling

Since the inclined surface is smooth, therefore the rotational kinetic energy of the ring is not converted into potential energy.



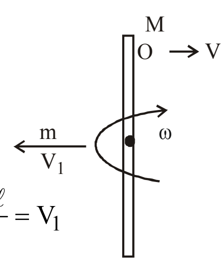

If $\ell$ = distance moved along the inclined plane then

$-\Delta K_{translational} = \Delta U$

$\frac{1}{2}mv^2 = mg\ell\sin\theta$ ; $\ell = \frac{\omega_0^2R^2}{2g\sin\theta}$

## EXERCISE-3

**(1)** **12.** (i) $L_i = L_f$




$\frac{m\ell^2}{12}\omega = MV\times\frac{\ell}{2} \Rightarrow V = \frac{m\omega\ell}{6M}$

(ii) $P_i = P_f$

$0 = MV - mV_1 \Rightarrow V = \frac{mV_1}{M} \Rightarrow \frac{\omega\ell}{6} = V_1$

(iii) $e = 1 = \frac{V-(-V_1)}{\omega\frac{\ell}{2}-0} \Rightarrow V + V_1 = \frac{\omega\ell}{2}$ ;

$\frac{m\omega\ell}{6M}+\frac{\omega\ell}{6}=\frac{\omega\ell}{2}$

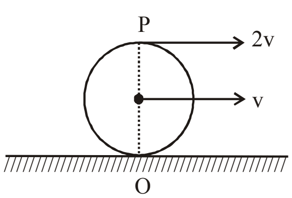
$\left(\frac{m}{M}+1\right)=3 \Rightarrow \frac{m}{M}=2 \Rightarrow M=\frac{m}{2}=12\text{ kg}$

**(2)** **4.** Since centre is non-accelerated thus acceleration of highest point w.r.t. centre of mass as well as from ground is same : $\frac{(2v)^2}{r}=\frac{v^2}{R} \therefore r = 4R$

**(3)** **0.** Taking torque about contact point,

$\tau = 4\times R - 2\times 2R = 0$

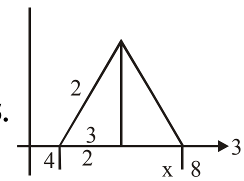
**(4)** **2.**




$v_r = v_{PQ} = 2v \therefore a_r = \frac{v_r^2}{2R}=\frac{4v^2}{2R}=\frac{2v^2}{R}$

**(5)** **3.** $2M\frac{a^2}{3}\omega - Mv\frac{a}{2}=0, \omega = \frac{3V}{4a}$

**(6)** **5.**




y-distance of point A and B will be same from instantaneous axis of rotation so x-velocity will be same.

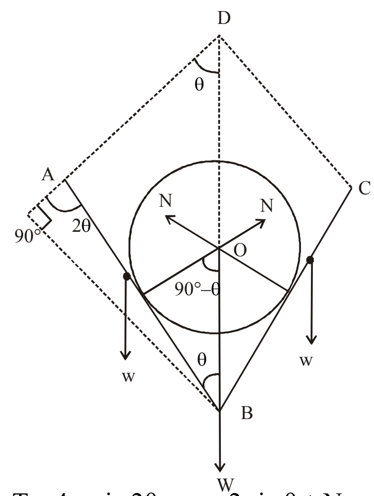
For B let x-coordinate $2\times x = 8$, $x = 4$ so $\omega\times 2 = 4$

**(7)** **12.** $I_0 = \frac{m\ell^2}{12}+\frac{m\ell^2}{12}=\frac{m\ell^2}{6}$ ; $I_{(x,y)} = \frac{m\ell^2}{12}$

**(8)** **3.** $2T\cos\theta = W + 2w$ ; $2N\cos(90-\theta) = W$ ; $2N\sin\theta = W$

Taking torque about B

$T\times AB\sin 2\theta = w\frac{1}{2}AB\sin\theta + N\times OB\cos\theta$

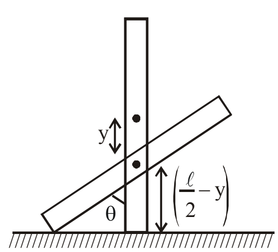



$T\times 4\times\sin 2\theta = w\times 2\sin\theta + N\times r\cot\theta$

$(AB = 4m,\ OB = r\,\text{cosec}\,\theta)$

Solving from above r = 3m

**(9)** **10.** $\cos\theta = \frac{\left(\frac{\ell}{2}-y\right)}{\ell/2}$




$1-\frac{2y}{\ell}=\cos\theta \Rightarrow y = \frac{\ell}{2}(1-\cos\theta)$

$V_y = +\frac{\ell}{2}\omega\sin\theta$ ......... (1)

Here, $\cos\theta = \frac{\ell/4}{\ell/2}=\frac{1}{2}\Rightarrow\theta = 60°$

Conservation of mechanical energy

$$mg\frac{\ell}{2} = mg\frac{\ell}{4} + \frac{1}{2}mv_y^2 + \frac{1}{2}\frac{m\ell^2}{12}\omega^2$$

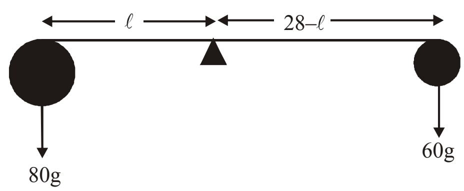
$$mg\frac{\ell}{4} = \frac{1}{2}mv_y^2 + \frac{1}{2}\frac{m\ell^2}{12}\omega^2 \qquad \text{........ (2)}$$

Solving eq. (1) and (2), $\omega = \sqrt{\frac{24g}{13\ell}}$ ; $\omega = 10$ rad/sec.

(10) **14.** $mgH_B = \frac{1}{2}mv^2$ ; $mgH_S = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

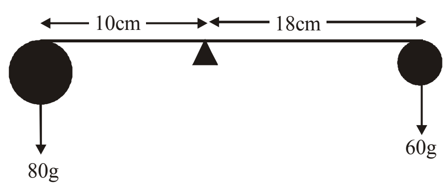
$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mr^2\omega^2 = \frac{1}{2}\times\frac{7}{5}mv^2 \; ; \; \frac{H_S}{H_B} = \frac{7}{5} = 1.4$$

(11) **10.** $80g \times \ell = 60g \times (25 - \ell)$




$8\ell = 168 - 6\ell$

$14\ell = 168$ ; $\ell = 12$cm.




$$-80g \times 10 + 60g \times 18 = (60 \times 18^2 + 80 \times 10^2)\,\alpha$$

$$\alpha = \frac{280g}{8000 + 19440} = \frac{280 \times 9.80}{27440} = 10 \text{ rad/s}^2$$

(12) **50.** By energy conservation,

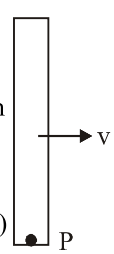
$$mg\frac{R}{\sqrt{2}} = \frac{1}{2}\frac{m(\sqrt{2}R)^2\omega^2}{3} \Rightarrow \omega^2 = \frac{3g}{\sqrt{2}R}$$

$$2N\cos 45° - mg = m \times \frac{3g}{\sqrt{2}R} \times \frac{R}{\sqrt{2}} \Rightarrow N = \frac{5mg}{2\sqrt{2}} = 50$$

(13) **6.**

After collision let velocity of centre rod is v' and angular velocity ω.

Applying conservation of angular momentum about point 'P' on ground (see figure) we get




$$\frac{MvL}{2} = \frac{Mv'L}{2} + \left(\frac{ML^2}{12}\right)\omega \qquad \text{........ (1)}$$

From conservation of momentum we get

$$Mv = \underbrace{Mv'}_{\text{momentum of rod}} + \underbrace{m(v' + L/2\omega)}_{\text{momentum of particle}} \qquad \text{........ (2)}$$

On solving (1) and (2) we get ω = 6 rad/sec.

## EXERCISE-4

(1) **(B).** By conservation of angular momentum.

$I_1\omega_1 = I_2\omega_2$

$MR^2\omega = [MR^2 + 2mR^2]\omega'$

$$\omega' = \frac{M}{M + 2m}\omega$$

(2) **(D).** In case of sliding acceleration = g sin θ same for all.

(3) **(B).** $I_{dia} + I_{dia} = MR^2$ ; $I_{dia} = MR^2/2$

(4) **(B).** Angular momentum J = mv (⊥ distance)

K = mv (ℓ)

(5) **(C).** Momentum of inertia $I = \frac{mR^2}{2}$

$m = Vd = (At)d = (\pi R^2)td$. So $I = \frac{1}{2}(\pi R^2 td)R^2$

$I \propto R^4 t$

$$\frac{I_X}{I_Y} = \frac{R_X^4}{R_Y^4}\times\frac{t_X}{t_Y} \; ; \; \frac{I_X}{I_Y} = \frac{R^4}{(4R)^4}\times\frac{t}{t/4}$$

$$\frac{I_X}{I_Y} = \frac{1}{4^4}\times 4 = \frac{1}{64} \; ; \qquad I_Y = 64\, I_X$$

(6) **(D).** $K = \frac{1}{2}I\omega^2$ ; $\frac{K}{2} = \frac{1}{2}I'(2\omega)^2 \Rightarrow I' = \frac{I}{8}$

So, $L' = I'\omega'$ ; $L' = \left(\frac{I}{8}\right)(2\omega) = \frac{L}{4}$

(7) **(B).** Torque is zero so any momentum will be constant.

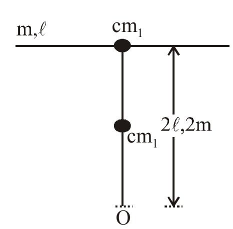
(8) **(C).** Solid sphere $I_A = \frac{2}{5}mR^2$; Hollow sphere $I_B = \frac{2}{3}mR^2$

$I_B > I_A$

(9) **(D).** Force $F = m\omega^2 r$ ; $\frac{F_1}{F_2} = \frac{R_1}{R_2}$

(10) **(D).** $\frac{1}{2}Mr^2$

(11) **(C).** Position of cm from O



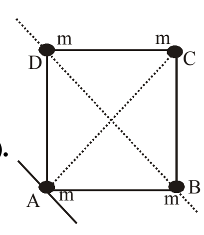

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{m(2\ell) + (2m)\ell}{m + 2m} = \frac{4}{3}\ell$$

(12) **(B).** $I_1\omega_1 = I_2\omega_2$ ; $mR^2\omega = (MR^2 + 2mR^2)\omega'$

(13) **(A).** $\vec{\tau} = \vec{r}\times\vec{F} = (\hat{i} - \hat{j})\times(-F\hat{k})$

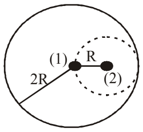
(14) **(A).**



$I = I_A + I_B + I_C + I_D$

$= 0 + m\left(\frac{\ell}{\sqrt{2}}\right)^2 + \left(\ell\sqrt{2}\right)^2 + m\left(\frac{\ell}{\sqrt{2}}\right)^2 = 3m\ell^2$

**(15)** **(C).** $I_Z = I_X + I_Y$ So $I_{EF} = I_{AC}$

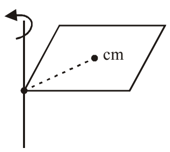
**(16)** **(A).** 


$$r_{cm} = \frac{A_1r_1 - A_2r_2}{A_1 - A_2} = \frac{0 - \pi R^2(R)}{\pi(2R)^2 - \pi R^2} = \frac{-R}{3}$$

**(17)** **(A).** Acceleration of particle rolling on inclined plane.

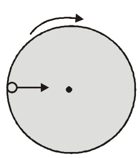
$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

**(18)** **(C).** Zero torque

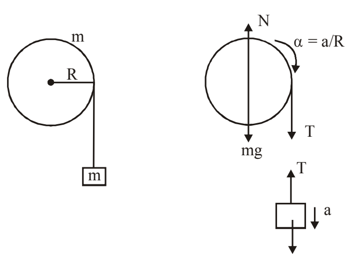
**(19)** **(C).** 


$$I = I_{cm} + md^2 = \frac{ma^2}{6} + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2}{3}ma^2$$

**(20)** **(C).** $\frac{1}{2}\frac{M\ell^2}{3}\omega^2 = Mg\Delta h$ ; $\Delta h = \frac{\omega^2\ell^2}{6g}$

**(21)** **(D).** 

From angular momentum conservation about vertical axis passing through centre.When insect is coming from circumference to center. Moment of inertia first decrease then increase. So angular velocity inecrease then decrease.

**(22)** **(C).** 


$mg - T = ma$

$TR = \frac{mR^2\alpha}{2}$ ; $T = \frac{mR\alpha}{2} = \frac{ma}{2}$

$mg - \frac{ma}{2} = ma$ ; $\frac{3ma}{2} = mg$ ; $a = \frac{2g}{3}$

**(23)** **(B).** To reverse the direction $\int \tau \, d\theta = 0$ (work done is zero)

$\tau = (20t - 5t^2)\,2 = 40t - 10t^2$

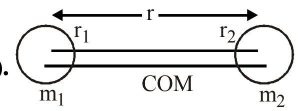
$\alpha = \frac{\tau}{I} = \frac{40t - 10t^2}{10} = 4t - t^2$ ; $\omega = \int_0^t \alpha\, dt = 2t^2 - \frac{t^3}{3}$

$\omega$ is zero at $2t^2 - \frac{t^3}{3} = 0$ ; $t^3 = 6t^2$ ; $t = 6$ sec.

$\theta = \int \omega\, dt = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right) dt$

$\left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6 = 216\left[\frac{2}{3} - \frac{1}{2}\right] = 36$ rad.

No. of revolution $\frac{36}{2\pi}$ less than 6.

**(24)** **(D).** 


$m_1r_1 = m_2r_2$ ; $r_1 + r_2 = r$

$\therefore\ r_1 = \frac{m_2r}{m_1 + m_2}$, $r_2 = \frac{m_1r}{m_1 + m_2}$

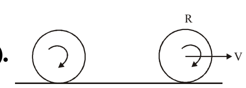
$\therefore\ \varepsilon = \frac{1}{2}I\omega^2 = \frac{1}{2}(m_1r_1^2 + m_2r_2^2).\omega^2$ ......... (1)

$mvr = \frac{h}{2\pi} = I\omega$ ; $\omega = \frac{nh}{2\pi I}$

$\therefore\ \varepsilon = \frac{1}{2}I.\frac{n^2h^2}{4\pi^2I^2} = \frac{n^2h^2}{8\pi^2}\left(\frac{1}{m_1r_1^2 + m_2r_2^2}\right)$

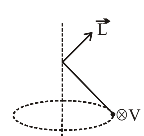
$= \frac{n^2h^2}{8\pi^2}\frac{1}{m_1\frac{m_2^2r_0^2}{(m_1+m_2)^2} + m_2\frac{m_1^2r^2}{(m_1+m_2)^2}}$

$= \frac{n^2h^2}{8\pi^2r^2}\frac{(m_1+m_2)^2}{m_1m_2(m_1+m_2)} = \frac{(m_1+m_2)\,n^2h^2}{8\pi^2r^2m_1m_2}$

**(25)** **(C).** 


$mr^2\omega_0 = mvr + mr^2 \times \frac{v_0}{r} \Rightarrow v = \frac{\omega_0 r}{2}$

**(26)** **(A).** $\vec{L}$ changes in direction not in magnitude.

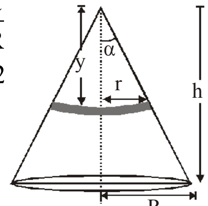


**(27)** **(D).** For the mass m, $mg - T = ma$

For the cylinder, $TR = mR^2 \frac{a}{R}$

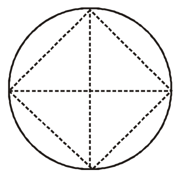
$\Rightarrow T = ma \Rightarrow mg = 2ma \Rightarrow a = g/2$

**(28)** **(A).** $dm = \pi r^2 \, dy \, \rho$




$$y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int_0^h \pi r^2 dy \times \rho \times y}{\frac{1}{3}\pi R^2 h\rho} = \frac{3h}{4}$$

**(29)** **(B).** $d = 2R = a\sqrt{3} \Rightarrow a = \frac{2}{\sqrt{3}}R$



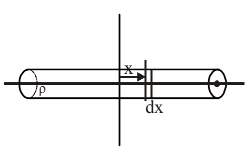
$$\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi \Rightarrow M' = \frac{2M}{\sqrt{3}\pi}$$

$$I = \frac{M'a^2}{6} = \frac{2M}{\sqrt{3}\pi} \times \frac{4}{3}R^2 \times \frac{1}{6} \; ; \quad I = \frac{4MR^2}{9\sqrt{3}\pi}$$

**(30)** **(AC).** Angular momentum of a particle moving in a straight line is $\vec{L} = m(\vec{r} \times \vec{v})$ . Hence, A and C are false.

**(31)** **(D).** As the wheel rolls forward the radius of the wheel decreases along AB hence for the same number of rotations it moves less distance along AB, hence it turns left.

**(32)** **(D).**




$dm = \rho\pi R^2 dx$

$$dI = \frac{dmR^2}{4} + dm\,x^2 \text{ (Parallel axis theorem)}$$

$$I = \int_{-\ell/2}^{\ell/2} dI = \int_{-\ell/2}^{\ell/2} \rho\frac{\pi R^2 \times R^2}{4}dx + \int_{-\ell/2}^{\ell/2} \rho\pi R^2 x^2 dx$$

$$= \rho\frac{\pi R^2 \times R^2}{4}[x]_{-\ell/2}^{\ell/2} + \rho\pi R^2\left(\frac{x^3}{3}\right)_{-\ell/2}^{\ell/2}$$

$$= \rho\frac{\pi R^2 \times R^2}{4} \times \ell + \rho\frac{\pi R^2}{3} \times \frac{\ell^3}{4} = \frac{mR^2}{4} + \frac{m\ell^2}{12}$$

$$I = \frac{mR^2}{4} + \frac{m\ell^2}{2} = \frac{m^2}{4\pi\rho\ell} + \frac{m\ell^2}{12} \quad (\rho\pi r^2\ell = m)$$

For I to be max.

$$\frac{dI}{d\ell} = -\frac{m^2}{4\pi\rho}\left(\frac{1}{\ell^2}\right) + \frac{m\ell}{6} = 0 \Rightarrow \frac{m^2}{4\mu\pi\rho} = \frac{m\ell^3}{6}$$

$$\ell^3 = \frac{3m}{2\pi\rho} \Rightarrow \ell = \left(\frac{3}{2}\right)^{1/3}\left(\frac{m}{\pi\rho}\right)^{1/3}$$

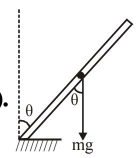
$$\rho = \frac{m}{\pi R^2 \ell} \Rightarrow R^2 = \frac{m}{\pi\rho\ell}$$

$$R^2 = \frac{m}{\pi\rho}\left(\frac{2}{3}\right)^{1/3}\left(\frac{\pi\rho}{n}\right)^{1/3} = \left(\frac{n}{\pi\rho}\right)^{2/3}\left(\frac{2}{3}\right)^{1/3}$$

$$\Rightarrow R = \left(\frac{m}{\pi\rho}\right)^{1/3}\left(\frac{2}{3}\right)^{1/6}$$

$$\frac{\ell}{R} = \frac{\left(\frac{3}{2}\right)^{1/3}\left(\frac{m}{\pi\rho}\right)^{1/3}}{\left(\frac{m}{\pi\rho}\right)^{1/3}\left(\frac{2}{3}\right)^{1/6}} = \left(\frac{3}{2}\right)^{1/3} + \left(\frac{3}{2}\right)^{1/6} = \sqrt{\frac{3}{2}}$$

**(33)** **(D).**




$$mg \times \frac{\ell}{2}\sin\theta = \frac{m\ell^2}{3}\alpha \; ; \quad \alpha = \frac{3g}{2\ell}\sin\theta$$

**(34)** **(B).** $I_0 = \frac{MR^2}{2} + 6\left[\frac{MR^2}{2} + M(2R)^2\right]$

$$= MR^2\left[\frac{1}{2} + 3 + 24\right] = \frac{55}{2}MR^2$$

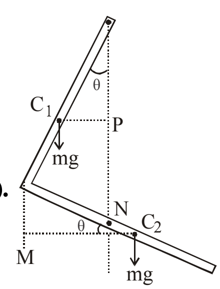
O is CM of the system. Applying parallel axis theorem between O & P. $I_P = I_0 + 7M(3R)^2$

$$= \frac{55}{2}MR^2 + 63MR^2 = \frac{181}{2}MR^2$$

**(35)** **(C).** $I = \frac{(9M)R^2}{2} - \left\{\frac{M(R/3)^2}{2} + M\left(\frac{2R}{3}\right)^2\right\}$

$$I = \frac{9MR^2}{2} - \left\{\frac{MR^2}{18} + \frac{4MR^2}{9}\right\} = 4MR^2$$

**(36)** **(B).**




Let mass of one rod is m.

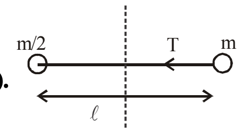
Balancing torque about hinge point.

$mg(C_1P) = mg(C_2N)$

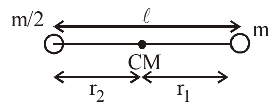
$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$

$$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta \Rightarrow \tan\theta = \frac{1}{3}$$

**(37) (D).**




$$\frac{r_1}{r_2} = \frac{1}{2} \Rightarrow r_1 = \frac{\ell}{3}$$




$$\frac{1}{2}k\theta_0^2 = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{m\ell^2}{9} + \frac{m}{2}\frac{4\ell^2}{9}\right)\omega^2$$

$$\omega^2 = \frac{3k\theta_0^2}{m\ell^2}\ ;\ T = m\omega^2\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

**(38) (A).** $\vec{a}_A = -a\,\hat{i},\ \vec{a}_B = a\,\hat{j},\ \vec{a}_C = a\,\hat{i},\ \vec{a}_D = -a\,\hat{j}$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2ma\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m} = \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j} = \frac{a}{5}(\hat{i} - \hat{j})$$

**(39) (D).** $M = \int_0^R \rho_0 r\,(2\pi r\,dr) = \frac{\rho_0 \times 2\pi \times R^3}{3}$

$$\underset{\text{(MOI about COM)}}{I_0} = \int_0^R \rho_0 r\,(2\pi r\,dr) \times r^2 = \frac{\rho_0 \times 2\pi R^5}{5}$$

By parallel axis theorem

$$I = I_0 + MR^2 = \frac{\rho_0 \times 2\pi R^5}{5} + \frac{\rho_0 \times 2\pi R^3}{5} \times R^2$$

$$= \rho_0 \times 2\pi R^5 \times \frac{8}{15} = MR^2 \times \frac{8}{5}$$

**(40) (A).** For solid sphere $\frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mR^2\cdot\frac{v^2}{R^2} = mgh_{sph}$

For solid cylinder $\frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{1}{2}mR^2\cdot\frac{v^2}{R^2} = mgh_{cyl}$

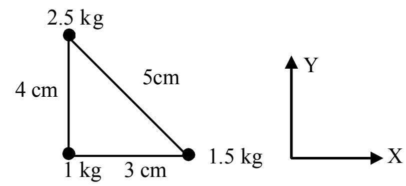
$$\Rightarrow \frac{h_{sph}}{h_{cyl}} = \frac{7/5}{3/2} = \frac{14}{15}$$

**(41) (A).** $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\ ;\ v = \omega R$ (no slipping)

$$mgh = \frac{1}{2}m\omega^2R^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2;\quad mgh = \frac{3}{4}m\omega^2R^2$$

$$\omega = \sqrt{\frac{4gh}{3R^2}} = \frac{1}{R}\sqrt{\frac{4gh}{3}}$$

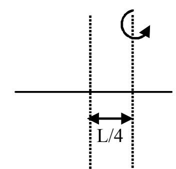
**(42) (B).** Take 1 kg mass at origin




$$X_{cm} = \frac{1\times0 + 1.5\times3 + 2.5\times0}{5} = 0.9\text{ cm}$$

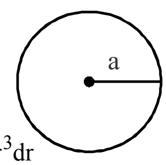
$$Y_{cm} = \frac{1\times0 + 1.5\times0 + 2.5\times4}{5} = 2\text{ cm}$$

**(43) (A).** $\frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 = MK^2$




$$\frac{L^2}{12} + \frac{L^2}{16} = K^2$$

**(44) (A).** $\sigma = A + Br$




$$\int dm = \int (A + Br)\,2\pi r\,dr$$

$$I = \int dm\,r^2 = \int_0^a (A + Br)\,2\pi r^3 dr$$

$$= 2\pi\left(A\frac{a^4}{4} + B\frac{a^5}{5}\right) = 2\pi a^3\left(\frac{A}{4} + \frac{Ba}{5}\right)$$

**(45) 50.00.** For no toppling,

$$F\left(\frac{a}{2} + b\right) \le mg\frac{a}{2}\ ;\quad \mu\frac{a}{2} + \mu b \le \frac{a}{2}$$

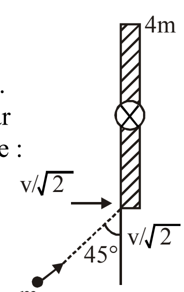
$0.2a + 0.4b \le 0.5a\ ;\quad 0.4b \le 0.3a$

$b \le 3a/4\ ;\ b \le 0.75a$ (in limiting case)

But it is not possible as b can maximum be equal to 0.5a

$$\therefore \left(100\frac{b}{a}\right)_{max} = 50.00$$

**(46) (B).** Let angular velocity of the system after collision be $\omega$.
By conservation of angular momentum about the hinge :

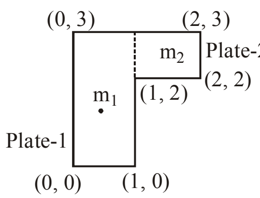



$$m\left(\frac{v}{\sqrt{2}}\right)\left(\frac{\ell}{2}\right)$$

$$= \left[\frac{4m\ell^2}{12} + \frac{m\ell^2}{4}\right]\omega$$

On solving, $\omega = \frac{3\sqrt{2}}{7}\left(\frac{v}{\ell}\right)$

**(47)** **(D).** $m_1 = 3$ kg, $m_2 = 1$ kg




Mass of plate-1 is assumed to be concentrated at (0.5, 1.5).
Mass of plate-2 is assumed to be concentrated at (1.5, 2.5).

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{3\times0.5 + 1\times1.5}{4} = 0.75$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{3\times1.5 + 1\times2.5}{4} = 1.75$$

## EXERCISE-5

**(1)** **(D).** $K = \frac{L^2}{2I} \Rightarrow L^2 = 2KI \Rightarrow L = \sqrt{2KI}$

$$\frac{L_1}{L_2} = \sqrt{\frac{K_1}{K_2}\cdot\frac{I_1}{I_2}} = \sqrt{\frac{K}{K}\cdot\frac{I}{2I}} = \frac{1}{\sqrt{2}}\ ;\ L_1 : L_2 = 1 : \sqrt{2}$$

**(2)** **(D).** Net work done by frictional force when drum rolls down without slipping is zero.
$W_{net} = 0$
$W_{trans} + W_{rot} = 0$
$\Delta K_{trans} + \Delta K_{rot} = 0$
$\Delta K_{trans} = -\Delta K_{rot}$
i.e., converts translation energy to rotational energy.

**(3)** **(B).** Moment of inertia of a disc about an axis perpendicular to its face and passing through its centre is $I_{CM} = \frac{1}{2}MR^2$

Now, given axis is parallel to the axis given above, so, applying theorem of parallel axis.

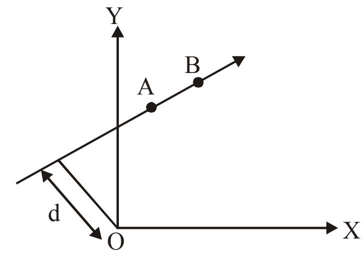
$$I = I_{CM} + Md^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

**(4)** **(D).** Torque due to gravitational force $= mg \times \frac{\ell}{2}$

Angular acceleration

$$= \frac{\text{Torque}}{\text{Moment of inertia}} = \frac{mg\frac{\ell}{2}}{m\frac{\ell^2}{3}} = \frac{3}{2}\frac{g}{\ell}$$

**(5)** **(A).** Angular momentum
= Linear momentum × distance of line of action of linear momentum about the origin.



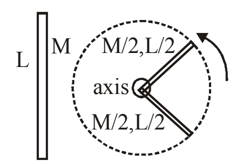

$L_A = P_A \times d,\ L_B = P_B \times d$
As linear momentum are equal, therefore, $L_A = L_B$

**(6)** **(A).** Since, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Where $\alpha$ is angular acceleration, $\omega_0$ is the initial angular speed $\theta = 2\times2 + \frac{1}{2}\times3\,(2)^2 = 4 + 6 = 10$ rad.

**(7)** **(C).** $\frac{K_{disc}}{K_{ring}} = \sqrt{\frac{I_{disc}}{I_{ring}}} = \sqrt{\frac{\frac{1}{2}mr^2}{mr^2}} = \frac{1}{\sqrt{2}}$

**(8)** **(C).** $I = 2\times\frac{1}{3}\frac{M}{2}\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12}$




**(9)** **(A).** $I_1\omega_1 = I_2\omega_2,\ I_1 = MR^2,\ I_2 = MR^2 + 2mR^2$

$\therefore\ \omega_2 = \frac{I_1}{I_2}\omega = \frac{M}{M+2m}\omega$

**(10)** **(B).** $\vec{\tau} = \vec{r}\times\vec{F} \Rightarrow \vec{r}.\vec{\tau} = 0\ ;\ \vec{F}.\vec{\tau} = 0$

**(11)** **(D).** Moment due to single rod about given axis of inertia

$$= \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{4ML^2}{12} = \frac{ML^2}{3}$$

Total M.I. $= 4\times\frac{ML^2}{3}$

**(12)** **(A).** $\vec{R} = \frac{m_1\vec{R}_1 + m_2\vec{R}_2}{(m_1+m_2)} = \frac{1}{4}[-8\hat{i} - 4\hat{j} + 4\hat{k}] = -2\hat{i} - \hat{j} + \hat{k}$

**(13)** **(D).** As no external torque is applied to the system, the angular momentum of the system remains conserved.
$\therefore\ L_i = L_f$
According to given problem, $I_t\omega_i = (I_t + I_b)\,\omega_f$

or $\omega_f = \frac{I_t\omega_i}{(I_t + I_b)}$ ........(i)

Initial energy, $E_i = \frac{1}{2}I_t\omega_i^2$ .........(ii)

Final energy, $E_f = \frac{1}{2}(I_t + I_b)\omega_f^2$ .........(iii)

Substituting the value of $\omega_f$ from equation (i) in equation (iii) we get Final energy,

$$E_f = \frac{1}{2}(I_t + I_b)\left(\frac{I_t\omega_i}{I_t + I_b}\right)^2 = \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)} \quad .........(iv)$$

Loss of energy, $\Delta E = E_i - E_f$

$$= \frac{1}{2}I_t^2\omega_i^2 - \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)} \text{ (Using (ii) \& (iv))}$$

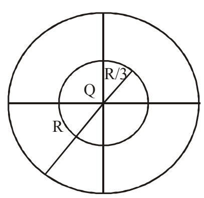
$$= \frac{\omega_i^2}{2}\left(I_t - \frac{I_t^2}{(I_t + I_b)}\right)$$

$$= \frac{\omega_i^2}{2}\left(\frac{I_t^2 + I_bI_t - I_t^2}{(I_t + I_b)}\right) = \frac{1}{2}\frac{I_bI_t}{(I_t + I_b)}\omega_i^2$$

**(14)** **(B).** As no external force is acting on the system, the centre of mass must be at rest i.e. $v_{CM} = 0$.

**(15)** **(A).** Mass of the disc = 9M

Mass of removed portion of disc = M




The moment of inertia of the complete disc about an axis passing through its centre O and perpendicular to its plane is $I_1 = \frac{9}{2}MR^2$

Now, the moment of inertia of the disc with removed portion $I_2 = \frac{1}{2}M\left(\frac{R}{3}\right)^2 = \frac{1}{18}MR^2$

Therefore, moment of inertia of the remaining portion of disc about O is

$$I = I_1 - I_2 = 9\frac{MR^2}{2} - \frac{MR^2}{18} = \frac{40MR^2}{9}$$

**(16)** **(D).** Time taken to reach the bottom of inclined plane

$$t = \sqrt{\frac{2\ell\left(1 + \frac{K^2}{R^2}\right)}{g\sin\theta}}$$

Here, $\ell$ is length of incline plane

For solid cylinder $K^2 = \frac{R^2}{2}$

For hollow cylinder $= K^2 = R^2$

Hence, solid cylinder will reach the bottom first.

**(17)** **(A).**

**(18)** **(D).** As no external toruqe is acting about the axis, angular momentum of system remains conserved.

$$I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{Mr^2\omega}{(M + 2m)\,r^2} = \frac{M\omega}{(M + 2m)}$$

**(19)** **(C).** $I = I_{CM} + Mh^2$ (Parallel axis theorem)

**(20)** **(B).** Torque zero means, $\alpha$ zero

$$\therefore \quad \frac{d^2\theta}{dt^2} = 0 \Rightarrow 12t - 12 = 0 \therefore t = 1 \text{ second}$$

**(21)** **(C).** $K.E. = \frac{L^2}{2I}$

From angular momemtum conservation about centre.

$L \to$ constant ; $I = mr^2$

$$K.E.' = \frac{L^2}{2\,(mr'^2)}; \quad r' = \frac{r}{2}; \quad K.E.' = 4\,K.E.$$

K.E. is increased by a factor of 4.

**(22)** **(A).** It's always in axial direction.

**(23)** **(B).** At maximum compression the solid cylinder will stop, so loss in K.E. of cylinder = gain in P.E. of spring

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \frac{3}{4}mv^2 = \frac{1}{2}kx^2 \Rightarrow \frac{3}{4} \times 3 \times (4)^2 = \frac{1}{2} \times 200x^2$$

$$\Rightarrow \frac{36}{100} = x^2 \Rightarrow x = 0.6m$$

**(24)** **(A).** $F_1x + F_2x = F_3x$ ; $F_3 = F_1 + F_2$

**(25)** **(C).** There is no external force so centre of mass will not shift.

**(26)** **(C).** Using angular momentum conservation

$L_i = 0$, $L_t = mvR - I\omega$

$mvR = I\omega$ ; $50 \times 1 \times 2 = 200\omega$ ; $\omega = 1/2$

$(v + \omega R)\,t = 2\pi R$

$$\left(1 + \frac{1}{2} \times 2\right)t = 2\pi \times 2 \; ; \; t = 2\pi \text{ sec.}$$

**(27)** **(A).** $I = I_{cm} + md^2$

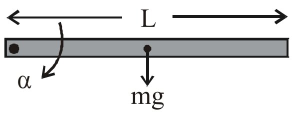
d is maximum for point B so $I_{max}$ about B

**(28)** **(A).** $X_{cm} = \frac{300 \times (0) + 500\,(40) + 400 \times 70}{300 + 500 + 400}$

$$X_{cm} = \frac{500 \times 40 + 400 \times 70}{1200}$$

$$X_{cm} = \frac{50 + 70}{3} = \frac{120}{3} = 40\text{cm.}$$

**(29)** **(B).**




$$\tau = I\alpha \Rightarrow mg\left(\frac{L}{2}\right) = \left(\frac{mL^2}{3}\right)\alpha \Rightarrow \alpha = \frac{3g}{2L}$$

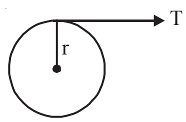
**(30)** **(A).** From conservation of mechanical energy

$$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mgh$$

$$\Rightarrow \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mg\left(\frac{3v^2}{4g}\right) \Rightarrow \frac{K^2}{R^2} = \frac{1}{2}$$

$\Rightarrow$ The object is disc.

**(31)** **(D).** $Tr = I\alpha$



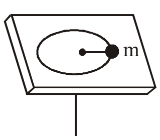

$$T = \frac{I\alpha}{r} = \frac{mr^2}{2} \times \frac{\alpha}{r} = \frac{mr\alpha}{2}$$

$$= \frac{50 \times 0.5 \times 2 \times 2\pi}{2} N = 157\ N$$

**(32)** **(A).** $a_{slipping} = g \sin\theta$

$$a_{rolling} = \frac{g\sin\theta}{1+\frac{K^2}{r^2}} = \frac{5}{7}g\sin\theta\ ; \qquad \frac{a_{rolling}}{a_{slipping}} = \frac{5}{7}$$

**(33)** **(B).**

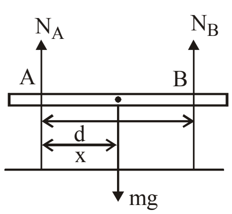



Applying angular momentum conservation

$$mv_0R_0 = (m)(v_1)\left(\frac{R_0}{2}\right)\ ; \quad v_1 = 2v_0$$

$$\text{New KE} = \frac{1}{2}m(2v_0)^2 = 2mv_0^2$$

**(34)** **(C).** Equating torque about center of mass

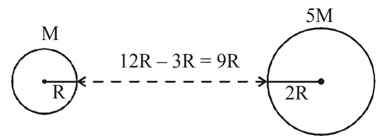


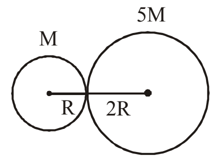

$N_A x = N_B(d-x)$

$N_A + N_B = mg$

$$\text{Solving, } N_A = \frac{W(d-x)}{d}$$

**(35)** **(B).** As their C.M. will remain stationary



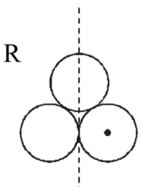




so, $(M)(x_1) = (5M)(x_2)$

and for touching

$x_1 + x_2 = 9R$, so $x_1 = 7.5\ R$

**(36)** **(C).** $I_{diameter} = \frac{2}{3}MR^2$



$$I_{tangential} = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$$

$$I_{Total} = \frac{2}{3}MR^2 + \left(\frac{5}{3}MR^2\right) \times 2 = 4MR^2$$

**(37)** **(B).** Velocity of the automobile

$$v = 54 \times \frac{5}{18} = 15\ m/s$$

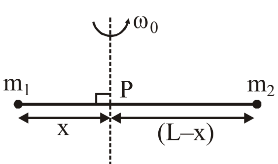
$$\omega_0 = \frac{v}{R} = \frac{15}{0.45} = \frac{100}{3}\ rad/s$$

So, angular acceleration,

$$\alpha = \frac{\Delta\omega}{t} = \frac{\omega_f - \omega_0}{t} = -\frac{100}{45}\ rad/s^2$$

$$\text{Torque} = I\alpha = 3 \times \frac{100}{45} = 6.66\ kg\ m^2 s^{-2}$$

**(38)** **(A).**



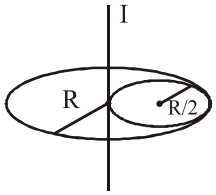

The position of point P on rod through which the axis should pass so that the work required to set the rod rotating with minimum angular velocity $\omega_0$ is their centre of mass, so $m_1x = m_2(L-x)$

$$x = \frac{m_2L}{m_1+m_2}$$

**(39)** **(B).** For conservation of angular momentum about origin.

$$\sum \vec{\tau}_{ext} = 0 \Rightarrow \vec{r} \times \vec{F} = 0 \Rightarrow \alpha = -1$$

**(40) (B).**




$I = I_{remain} + I_{(R/2)}$

$\Rightarrow I_{remain} = I - I_{(R/2)}$

$$= \frac{MR^2}{2} - \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2\right]$$

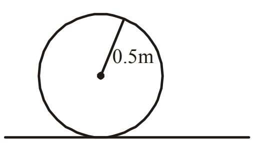
$$= \frac{MR^2}{2} - \left[\frac{MR^2}{32} + \frac{MR^2}{16}\right]$$

$$= \frac{MR^2}{2} - \left[\frac{MR^2 + 2MR^2}{32}\right]$$

$$= \frac{MR^2}{2} - \frac{3MR^2}{32}$$

$$= \frac{16MR^2 - 3MR^2}{32} = \frac{13MR^2}{32}$$

**(41) (A).**




Angular acceleration $\alpha = 2 \text{ rad s}^{-2}$

Angular speed $\omega = \alpha t = 4 \text{ rad s}^{-1}$

$a_c = r\omega^2 = 0.5 \times 16 = 8 \text{ m/s}^2$

$a_t = \alpha r = 1$ rad/s

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{8^2 + 1^2} = 8 \text{ m/s}^2$$

**(42) (B).** $a_{sphere} > a_{disc}$

Acceleration $(a) = \dfrac{g\sin\theta}{1 + K^2/r^2}$,

independent of mass and radius.

**(43) (C).** $E = \dfrac{L^2}{2I} \Rightarrow E_A = E_B$

$$\Rightarrow \frac{L_A^2}{2I_A} = \frac{L_B^2}{2I_B}$$

$I_B > I_A \Rightarrow L_B > L_A$

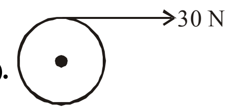
**(44) (B).** $$\frac{E_{sphere}}{E_{cylinder}} = \frac{\frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2}{\frac{1}{2}\left(\frac{1}{2}mR^2\right)(2\omega)^2} = \frac{1}{5}$$

**(45) (A).**


$$x_1 = \frac{m_2\ell}{m_1 + m_2} \text{ and } x_2 = \frac{m_1\ell}{m_1 + m_2}$$

$$I = m_1x_1^2 + m_2x_2^2 = \frac{m_1m_2}{m_1 + m_2}\ell^2$$

**(46) (B).**




$\tau = I\alpha$

$RF = mR^2\alpha$

$$\alpha = \frac{F}{mR} = \frac{30}{3 \times \frac{40}{100}} = 25 \text{ rad/s}^2$$

**(47) (A).** COAM : $I\omega_1 + I\omega_2 = 2I\omega$

$$\Rightarrow \omega = \frac{\omega_1 + \omega_2}{2}$$

$$(K.E.)_f = \frac{1}{2} \times 2I\omega^2 = I\left(\frac{\omega_1 + \omega_2}{2}\right)^2$$

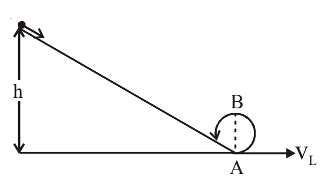
$$\text{Loss in K.E.} = (KE)_i - (KE)_f = \frac{1}{4}I(\omega_1 - \omega_2)^2$$

**(48) (D).** Centre of mass may lie on centre of gravity net torque of gravitational pull is zero about centre of mass.

$$\text{Mechanical advantage} = \frac{\text{Load}}{\text{Effort}} > 1$$

$\Rightarrow$ Load > Effort

**(49) (D).** As track is frictionless, so total mechanical energy will remain constant




$T.M.E_I = T.M.E_F$

$$0 + mgh = \frac{1}{2}mv_L^2 + 0$$

For completing the vertical circle, $v_L \geq \sqrt{5gR}$

$$h = \frac{5gR}{2g} = \frac{5}{2}R = \frac{5}{4}D$$

**(50)** **(C).** Work done required to bring them rest

$\Delta W = \Delta KE$ ; $\Delta W = \frac{1}{2} I\omega^2$

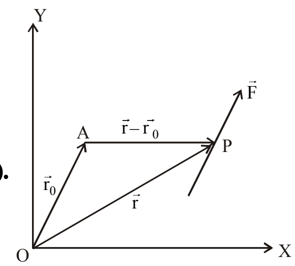
$\Delta W \propto I$ for same $\omega$

$$W_A : W_B : W_C = \frac{2}{5}MR^2 : \frac{1}{2}MR^2 : MR^2$$

$$= \frac{2}{5} : \frac{1}{2} : 1 = 4 : 5 : 10$$

$\Rightarrow W_C > W_B > W_A$

**(51)** **(D).**




$\vec{\tau} = (\vec{r} - \vec{r}_0) \times \vec{F}$ ........ (i)

$\vec{r} - \vec{r}_0 = (2\hat{i} + 0\hat{j} - 3\hat{k}) - (2\hat{i} - 2\hat{j} - 2\hat{k})$

$= 0\hat{i} + 2\hat{j} - \hat{k}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -1 \\ 4 & 5 & -6 \end{vmatrix} = -7\hat{i} - 4\hat{j} - 8\hat{k}$$

**(52)** **(D).** $\tau_{ext} = 0$. So, $\frac{dL}{dt} = 0$ i.e. $L$ = constant

So angular momentum remains constant.

**(53)** **(B).** $K_t = \frac{1}{2}mv^2$

$K_t + K_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{7}{10}mv^2$

So, $\frac{K_t}{K_t + K_r} = \frac{5}{7}$

**(54)** **(A).** Work required = change in kinetic energy

Final KE = 0

Initial KE $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2$

$= \frac{3}{4} \times 100 \times (20 \times 10^{-2})^2 = 3J$

$|\Delta KE| = 3$ J

**(55)** **(A).** Work energy theorem, $W = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$

$\theta = 2\pi$ revolution $= 2\pi \times 2\pi = 4\pi^2$ rad

$W_i = 3 \times \frac{2\pi}{60}$ rad / s

$-\tau\,\theta = \frac{1}{2} \times \frac{1}{2} mr^2 (0^2 - \omega_i^2)$

$$-\tau = \frac{\frac{1}{2} \times \frac{1}{2} \times 2 \times (4 \times 10^{-2})\left(-3 \times \frac{2\pi}{60}\right)^2}{4\pi^2}$$

$\tau = 2 \times 10^{-6}$ N m