9

GRAVITATION

GRAVITATIONAL AND INTERNAL MASS

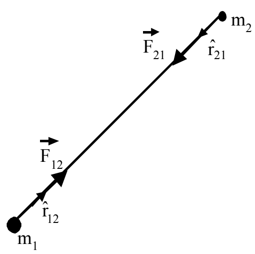
(a) **Inertial mass :** When mass is defined on the property of inertia, it is termed as inertial mass.

(b) **Gravitational mass :** When mass is defined on the property of gravity, it is called gravitational mass.

(c) **Properties of inertial mass :** It is equal to the ratio of magnitude of external force applied on the body to the acceleration produced in it by that force. $m = F/a$

NEWTON'S LAW OF GRAVITATION

On the basis of Kepler's laws of planetary motion, Newton stated his famous law of gravitation.



According to this law - Every two objects in the universe attract each other. The force of attraction is directly proportional to the product of masses and inversely proportional to the square of distance between the two masses, i.e., if two masses m_1 and m_2 are separated from each other by a distance r then

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = G\frac{m_1 m_2}{r^2}$$

In vector form of gravitational force is $\vec{F} = \frac{G.m_1 m_2}{r^3}\vec{r}$

where G = universal gravitational constant

$G = 6.67 \times 10^{-11}$ N-m^2 / kg^{-2} (unit) $M^{-1}L^3T^{-2}$ (dim.)

$G = 6.67 \times 10^{-8}$ dyne cm^{-2} gm^{-2}

Gravitation force is a mutual force hence it is action - reaction force i.e. $\vec{F}_{12} = -\vec{F}_{21}$

It acts along the line joining two masses. It is a weak intensity force but its range is infinite. This force is independent of charge etc., it depends only on mass. The motion of a satellite and motion of planets around the sun are consequences of this force.

Example 1 :

Two particles of masses 1 kg and 2 kg are placed at a separation of 50 cm. Assuming that the only forces acting on the particles are their mutual gravitation, find the initial acceleration of heavier particle.

Sol. Force exerted by one particle on another

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times1\times2}{(0.5)^2} = 5.3\times10^{-10}\text{ N}$$

Acceleration of heavier particle

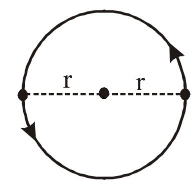
$$= \frac{F}{m_2} = \frac{5.3\times10^{-10}}{2} = 2.65\times10^{-10}\text{ ms}^{-2}$$

This example shows that gravitation is very weak but only this force keep bind our solar system and also this universe, all galaxies and other interstellar system.

Example 2 :

Two particles of equal mass (m) each move in a circle of radius (r) under the action of their mutual gravitational attraction. Find the speed of each particle.

Sol. $$\frac{mv^2}{r} = \frac{GMm}{(2r)^2}$$



$$\Rightarrow \quad v^2 = \frac{Gm}{4r} \quad \Rightarrow \quad v = \frac{1}{2}\sqrt{\frac{Gm}{r}}$$

GRAVITATIONAL FIELD

The gravitational field is the space around a mass or an assembly of masses over which it can exert gravitational forces on other masses. It is characterised by –

(a) gravitational field intensity (b) Gravitational potential.

Gravitational field strength or gravitational field intensity: Gravitational field strength at a point in the gravitational field is the force experienced by a unit mass placed at that point. It is directed towards the particle producing the field.

The gravitational field intensity is given by $\vec{E} = \frac{\vec{F}}{m}$

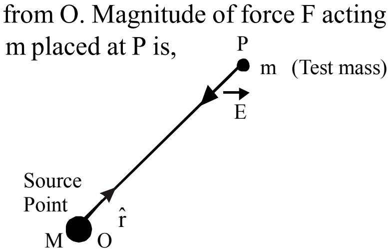
Acceleration due to gravity $\vec{g}$ is also $\frac{\vec{F}}{m}$. Hence, for the Earth's gravitational field $\vec{g}$ and $\vec{E}$ are same (Neglecting earth rotation).

Gravitational field due to a point mass :

Suppose, a particle of mass M is placed at point O. We want to find the intensity of gravitational field $\vec{E}$ at a point P, at distance r from O. Magnitude of force F acting on a particle of mass m placed at P is,

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}$$

$$\therefore \vec{E} = \frac{\vec{F}}{m} = -\frac{GM}{r^2}\hat{r}$$



or $\vec{E} = -\frac{GM}{r^2}\hat{r}$

The direction of the force F and hence of E is from P to O as shown in figure.

Gravitational field due to a uniform solid sphere :

Let mass of the solid sphere is M and its radius is R.

Field at an external point : For calculating the gravitational field at an external point an uniform solid sphere may be treated as a single particle of same mass placed at its centre. Let r = distance of the external point from the centre of the sphere.

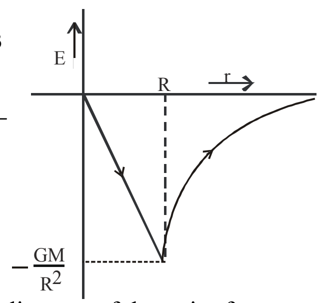
$$\vec{E}(r) = -\frac{GM}{r^2}\hat{r} \quad \text{or} \quad E(r) \propto \frac{1}{r^2} \qquad \text{for} \qquad r \geq R$$

At surface $\vec{E} = -\frac{GM}{R^2}\hat{r}$ for r = R

Field at an internal point : For calculating the gravitational field at an internal point at a distance r (r < R) from centre imagine a sphere of radius r, mass M' (uniform density) then value of gravitational field intensity

$$= \frac{GM'}{r^2} = \frac{G\dfrac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3}{r^2}$$

$$= \frac{GM}{R^3}r = G\rho\frac{4}{3}\pi r$$



i.e. it is proportional to the distance of the point from the centre of the sphere.

In vector notation :

$$\vec{E}(r) = -\frac{GM}{R^3}.\vec{r} \qquad \text{for} \qquad r \leq R$$

or E (r) $\propto$ r. Hence, E versus r graph is as shown in fig.

Field due to a uniform spherical shell

At an External Point

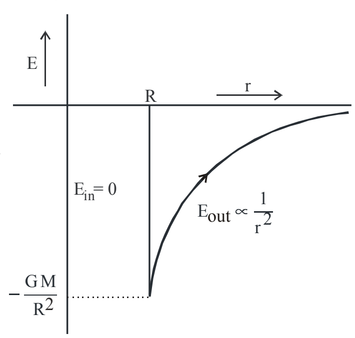
For an external point the shell may be treated as a single particle of same mass placed at its centre. Thus, at an external point the gravitational field is given by,

$$\vec{E}(r) = -\frac{GM}{r^2}\hat{r}$$

For $r \geq R$

At the surface of the shell

$$\vec{E} = -\frac{GM}{R^2}\hat{r} \text{ For } r = R$$



At an Internal Point

The field inside a uniform spherical shell

E = 0 For r < R

Newton's shell theorem : A uniform shell of matter exerts no net gravitational force on a particle located inside it. This statement does not mean that the gravitational forces on the particle from the various elements of the shell magically disappear. Rather, it means that the sum of the force vectors on the particle from all the elements is zero.

GRAVITATIONAL POTENTIAL (V)

At a point in a gravitational field potential V is defined as negative of the work done per unit mass in shifting a rest mass from some reference point (usually at infinity) to the given point,

i.e., $V = -\frac{W}{m}$ [dimensions $[L^2 T^{-2}]$ and unit J/kg.]

It is a scalar quantity. As by definition potential energy

$U = -W$. So, $V = \frac{U}{m}$, i.e., $U = mV$

i.e., physically potential at a point represents potential energy of a unit point mass at that point.

As by definition of work $W = \int \vec{F}.\vec{dr}$

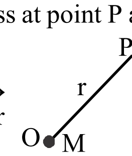
$$\text{So} \quad V = -\frac{1}{m}\int \vec{F}.\vec{dr} = -\int \vec{E}.\vec{dr} \qquad \left[\text{as } \frac{\vec{F}}{m} = \vec{E}\right]$$

i.e., $dV = -E\,dr$ or $E = -\frac{dV}{dr}$

Gravitational potential due to a point mass

Suppose a point mass M is situated at a point O.

The gravitational potential due to this mass at point P any distance r from O.



$$V = -\int_{\infty}^{r} \vec{E}.\vec{dr} = -\int_{\infty}^{r} -\frac{GM}{r^2}\hat{r}.\vec{dr}$$

$$= \int_{\infty}^{r} \frac{GM}{r^2}dr \; ; \; V = -\frac{GM}{r}$$

Gravitational potential due to a uniform solid sphere:

Potential at an external point : The gravitational potential due to a uniform sphere at an external point is same as that due to a single particle of same mass placed at its centre.

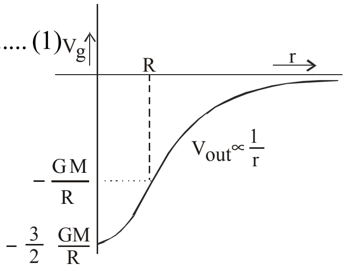
Thus, $V(r) = -\frac{GM}{r} \quad r \geq R$

At the surface, $r = R$ and $V = -\frac{GM}{R}$

To calculate potential at a point distant r from the centre of the sphere (r < R) we can use $E = -dV/dr$

$$-\frac{GM}{R^3}r = -\frac{dV}{dr} \;;\; \int dV = \int \frac{GM}{R^3} r\, dr$$

$$\Rightarrow \quad V = \frac{GM}{2R^3}r^2 + C \quad ....(1)$$



At $r = R$, $V = -\frac{GM}{R}$

$$-\frac{GM}{R} = \frac{GM}{2R} + C$$

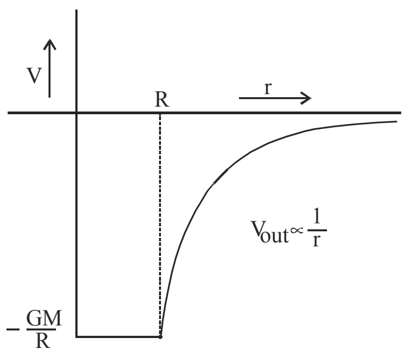
$$\Rightarrow \quad C = -\frac{3GM}{2R} \text{ Put in eq. (1)}$$

$$V = \frac{-GM(3R^2 - r^2)}{2R^3}. \quad \text{At centre, } r = 0,\ V = -\frac{3GM}{2R}$$

Potential due to a Uniform Thin Spherical Shell

For an external point, spherical shell behave as whole of its mass is concentrated at the centre, i.e.,

$$V(r) = -\frac{GM}{r} \qquad r \geq R$$



For an internal point ($r < R$) as gravitational intensity is zero the potential everywhere is same and equal to its value at the surface, i.e., $V = -\frac{GM}{R}$ = constant [for $r < R$]

GRAVITATIONAL POTENTIAL ENERGY

Assume a mass m_1 is fixed at a point and bring m_2 from infinity at a distance r from m_1 then work done in bringing m_2 will be stored in form of potential energy.

$$U = -W = m_2 V = m_2\left(\frac{-Gm_1}{r}\right) = \frac{-Gm_1 m_2}{r}$$

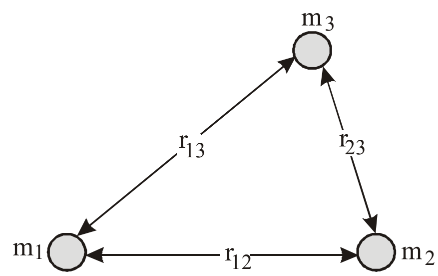
Hence, the gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by,

$$U = -\frac{Gm_1 m_2}{r} \qquad ....(1)$$

This is actually the negative of work done in bringing those masses from infinity to a distance r by the gravitational forces between them.

Gravitational potential energy of a three particle system

The gravitational potential energy of the system is the sum of the gravitational potential energies of all three pairs of particles.



If a system contains more than two particles, we consider each pair of particles in turn, calculate the gravitational potential energy of that pair with eq^n. (1) as if the other particles were not there, and then algebraically sum gives the results.

Applying eq^n. (1) to each of the three pairs (m_1, m_2), (m_1, m_3) and (m_2, m_3) gives the potential energy of the system as

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right)$$

Gravitational potential energy of a body on earth's surface:

The gravitational potential energy of mass m in the gravitational field of mass M at a distance r from it is, (r is greater than radius of the Earth i.e. $r > R$)

$$U_P = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -\int_{\infty}^{r}\left(-\frac{GMm}{r^2}\hat{r}\right).\vec{dr} = GMm\int_{\infty}^{r} r^{-2}dr$$

$$= GMm\left(\frac{r^{-2+1}}{-2+1}\right)_{\infty}^{r} \quad \text{or} \quad U_P = -\frac{GMm}{r}$$

The Earth behaves for all external points as if its mass M were concentrated at its centre. Therefore, a mass m near Earth's surface may be considered at a distance R (the radius of earth) from M.

The potential energy of m at the surface of the Earth is

$$U = -\frac{GMm}{R}$$

The gravitational potential energy of mass m at a height h above the surface of Earth is given by

$$U = -\frac{GMm}{R+h}$$

[$\because$ The distance between the mass m and the centre of Earth is $(R + h)$]

$$\therefore U = -\frac{GMm}{R\left(1+\frac{h}{R}\right)} = -\frac{GMm}{R}\left(1+\frac{h}{R}\right)^{-1} \text{ (For any height h)}$$

So, expanding the right hand side of the above equation by Binomial theorem and neglecting squares and higher powers of $\frac{h}{R}$, we get $U = -\frac{GMm}{R}\left(1-\frac{h}{R}\right)$ For h << R

or $U = -\frac{GMm}{R} + \frac{GMmh}{R^2}$

But $\frac{GM}{R^2} = g$ (acc. due to gravity) $\therefore$ $U = -\frac{GMm}{R} + mgh$

But $-\frac{GMm}{R}$ = gravitational potential energy of mass m at the surface of Earth.

If we set a reference such the gravitational potential energy at the surface of Earth is zero then U = mgh

BINDING ENERGY

Total mechanical energy (potential + kinetic) of a closed system is negative. The modulus of this total mechanical energy is known as the binding energy of the system. This is the energy due to which system is closed or different parts of the system are bound to each other.

Suppose the mass m is placed on the surface of earth.

The radius of the earth is R and its mass is M.

Then, the kinetic energy of the particle K = 0

and potential energy of the particle is $U = -\frac{GMm}{R}$.

Therefore, the total mechanical energy of the particle is,

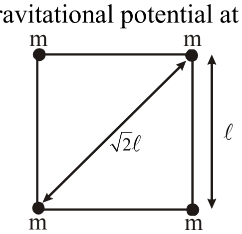
$$E = K + U = 0 - \frac{GMm}{R} \quad \text{or} \quad E = -\frac{GMm}{R}$$

$$\therefore \quad \text{Binding energy} = |E| = \frac{GMm}{R}$$

It is due to this energy, the particle is attached with the earth. If this much energy is supplied to the particle in any form (normally kinetic) the particle no longer remains bound to the earth. It goes out of the gravitational field of earth.

Example 3 :

Find the gravitational potential energy of a system of four particles, each having mass m, placed at the vertices of a square of side ℓ. Also obtain the gravitational potential at the centre of the square.



Sol. The system has four pairs with distance ℓ and two diagonal pairs with distance $\sqrt{2}\,\ell$.

$\therefore$

$$U = -4\frac{Gm^2}{\ell} - 2\frac{Gm^2}{\sqrt{2}\ell} = -\frac{2Gm^2}{\ell}\left(2+\frac{1}{\sqrt{2}}\right) = -5.41\frac{Gm^2}{\ell}$$

The gravitational potential at the centre of the square is $(r = \sqrt{2}\ell/2)$

V = Algebraic sum of potential due to each particle

$$\Rightarrow \quad V = -\frac{4\sqrt{2}Gm}{\ell}$$

Example 4 :

Two bodies of mass 10^2 Kg and 10^3 kg are lying 1m apart. The gravitational potential at the mid-point of the line joining them is -

(A) Zero (B) –1.47 Joule/Kg

(C) 1.47 Joule/Kg (D) -1.47×10^{-7} Joule /Kg

Sol. (D). $V_g = V_{g1} + V_{g2}$ or $V_g = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2}$

or $V_g = -1.47 \times 10^{-11}\left[\frac{10^2}{0.5} + \frac{10^3}{0.5}\right]$

or $V_g = -1.47 \times 10^{-7}$ Joule /Kg

ACCELERATION DUE TO GRAVITY

The acceleration produced in a body due to earth's gravitational force is called acceleration due to gravity.

$$g = \frac{GM_e}{R_e^2} = \frac{4}{3}\pi G R_e d \text{ m/s}^2 \text{ (unit) } M^0L^1T^{-2} \text{ (dim.)}$$

Weight : The gravitational force that a body experiences is called its weight. If a body of mass m experiences an acceleration due to gravity g, then weight of the body is W = mg

Variation of g :

The value of g depend on the following factors.

(a) Height above the earth surface

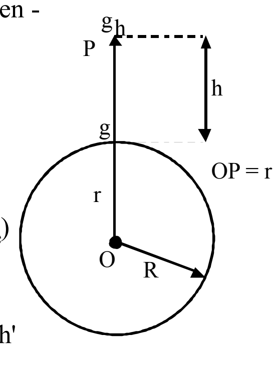
(b) Depth below the earth surface

(c) Shape of the earth

(d) Axial rotation of the earth.

(a) Height above the earth surface - If value of gravity at surface of earth is g and gravity at a height 'h' above the surface of earth is g' then -

$$g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$$

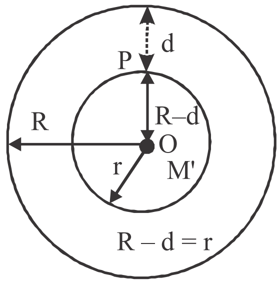


$$g' = g\left(1 - \frac{2h}{R_e}\right) \quad (\text{if } h << R_e)$$

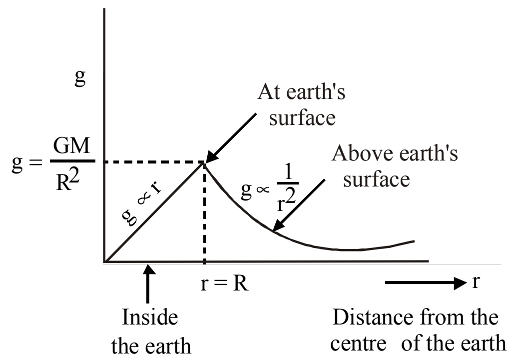
$\therefore$ The decrease in the value of g on going up a height 'h' above the surface of earth.

$$\Delta g = \frac{2gh}{R_e}$$

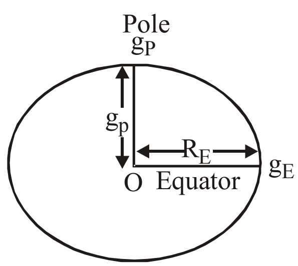
(b) Depth below the earth surface - if the value of gravity at surface of earth is g and the value of gravity at a distance h below earth surface is g' then



$$g' = g\left(1 - \frac{h}{R_e}\right) = \frac{4}{3}\pi G\,(R_w - h)\,d$$



(c) Shape of the earth - The earth is elliptical in shape. It is flatter at the poles and bulged out at the equator. Now, we know that $g \propto 1/R^2$, $R_P < R_e$; $g_E < g_P$



therefore the value of g at the equator is minimum and the value of g at the poles is maximum.

(d) Axial rotation of the earth - If the observed value of g at the latitude λ is represented by g_λ, then

$g_\lambda = g_e - R_e\omega^2 \cos^2\lambda$

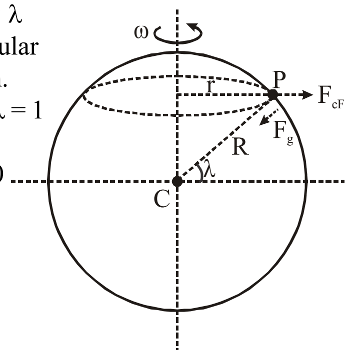
where ω is the angular velocity of the earth.

At equator $\lambda = 0^o$; $\cos\lambda = 1$

$g_\lambda = g_e - R_e\,\omega^2$

At poles $\lambda = 90^o$; $\cos\lambda = 0$

$g_\lambda = g_e$.



Example 5 :

On a planet whose size is the same and mass 4 times as that of the earth, find the amount of energy needed to lift a 2kg mass vertically upwards through 2 m distance on the planet. Value of g on the surface of the earth is 10 m/s^2.

Sol. On the surface of the earth $g = (GM/r^2)$

On the planet $g' = \frac{G \times 4M}{R^2}$ $\therefore \frac{g'}{g} = 4$ or $g' = 4\,g = 40$ m/s^2

Energy needed $= mg'h = 2 \times 40 \times 2 = 160$ Joule.

Example 6 :

If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same?

Sol. $g = \frac{GM}{R^2} = \frac{G\rho \times \frac{4}{3}\pi R^3}{R^2} = \frac{4}{3}\pi G R\rho$ .... (1)

$g' = \frac{4}{3}\pi G\,(5R)\,\rho'$ .... (2)

Given that, $g = g'$, so $\rho' = \rho/5$

Example 7 :

At what altitude will the acceleration due to gravity be 25% of that at the earth's surface (given radius of earth is R) ?

(A) R/4 (B) R

(C) 3R/8 (D) R/2

Sol. (B). $g = \frac{GM}{r^2} \Rightarrow g_0 = \frac{GM}{R^2}$ ...... (1) ; $g_h = \frac{GM}{(R+h)^2}$

$$\frac{g_h}{g_0} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{4} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{2}$$

$R + h = 2R \Rightarrow h = R$

TRY IT YOURSELF-1

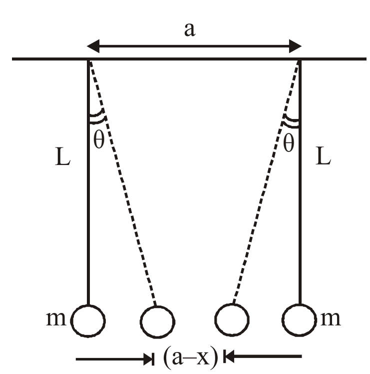
Q.1 Two small balls of mass m each are suspended side by side by two equal threads of length L as shown in the figure. If the distance between the upper ends of the threads be a, the angle θ that the threads will make with the vertical due to attraction between the balls is –

(A) $\tan^{-1}\frac{(a-x)\,g}{mG}$

(B) $\tan^{-1}\frac{mG}{(a-x)^2 g}$

(C) $\tan^{-1}\frac{(a-x)^2 g}{mG}$

(D) $\tan^{-1}\frac{(a^2-x^2)\,g}{mG}$



Q.2 In a spherical region, the density varies inversely with the distance from the centre. Gravitational field at a distance r from the centre is –
(A) proportional to r (B) proportional to 1/r
(C) proportional to r^2 (D) same everywhere

Q.3 In the above question, the gravitational potential is –
(A) linearly dependent on r (B) proportional to 1/r
(C) proportional to r^2 (D) same every where.

Q.4 A meteorite 80,000 km from the earth is moving towards the earth at 2000m/s. Ignoring air friction what will be its velocity on impact?

Q.5 A uniform ring of mass m and radius a is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is at a distance $\sqrt{3}$ a from the centre of the sphere. Find the gravitational force exerted by the sphere on the ring.

Q.6 Assuming that the moon is a sphere of the same mean density as that of the earth and one quarter of its radius, the length of a seconds pendulum on the moon (its length on the earth's surface is 99.2 cm) is
(A) 24 8 cm (B) 49.6 cm
(C) 99.2 cm (D) $\frac{99.2}{\sqrt{2}}$ cm

Q.7 Calculate the height above the Earth's surface at which the value of acceleration due to gravity reduces to half its value on the Earth's surface. Assume the Earth to be a sphere of radius 6400 km.

Q.8 Find the potential energy of gravitational interaction of a point mass m and a thin uniform rod of mass M and length ℓ, if they are located along a straight line at a distance a from each other

Q.9 Three particles each of mass m, are situated at the vertices of an equilateral triangle of side length a. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original separation a. Find the initial velocity that should be given to each particle for the circular motion.

Q.10 In the above questions, find the time period.

ANSWERS

(1) (B) **(2)** (D) **(3)** (A)

(4) 1.09×10^4 m/s. **(5)** $F = Mg = \frac{\sqrt{3}GMm}{8a^2}$

(6) **(A).** **(7)** 2649.6 km

(8) $U = -\frac{GmM}{\ell}\log_e\left(\frac{a+\ell}{a}\right)$ **(9)** $v = \sqrt{\frac{Gm}{a}}$

(10) $T = 2\pi\left(\frac{a^3}{3Gm}\right)^{1/2}$

ESCAPE VELOCITY

(i) Escape velocity is the minimum velocity that should be given to the body to enable it to escape away from the gravitational field of earth.
If the mass of the planet is M and its radius is R, then the escape velocity from its surface will be

$$V_e = \sqrt{(2GM/R)} = \sqrt{(8\pi R^2 dG/3)} = \sqrt{(2gR)}.$$

Escape velocity from the surface of earth is 11.2 km/sec.

(ii) The value of escape velocity does not depend upon the mass of the projected body, instead it depends on the mass and radius of the planet from which it is being projected.

(iii) The value of escape velocity does not depend on the angle and direction of projection.

(iv) The minimum energy needed for escape is = GMm/R.

(v) If the velocity of a satellite orbiting the earth is increased by 41.4% , then it will escape away from the gravitational field of the earth.

Example 8 :
If velocity given to an object from the surface of the earth is n times the escape velocity then what will be the residual velocity at infinity.

Sol. Let residual velocity be v then from energy conservation

$$\frac{1}{2}m(nv_e)^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 + 0$$

$$\Rightarrow v^2 = n^2v_e^2 - \frac{2GM}{R} = n^2v_e^2 - v_e^2 = (n^2-1)v_e^2$$

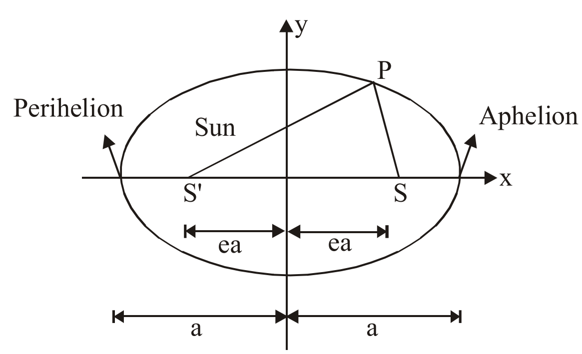
$$\Rightarrow v = (\sqrt{n^2-1})\, v_e$$

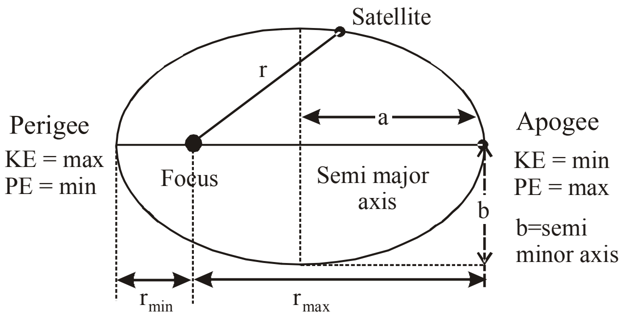
KEPLER'S LAWS FOR PLANETARY MOTION

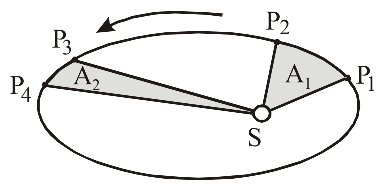
1. **First Law -** All planets revolve in elliptical orbits around the sun and the sun is situated at the focus of the elliptical path.

2. **Second law -** Areal velocity $\left(\frac{dA}{dt}\right)$ of a planet remains constant, i.e. the line joining the sun to planet covers equal areas in same intervals of time.

$$\frac{dA}{dt} = \text{constant} = \frac{mr^2}{2m}\frac{d\theta}{dt} = \frac{J}{2m}$$







3. **Third law -** The square of the period of revolution of the planet is directly proportional to the cube of semi major axis of its orbit. $T^2 \propto a^3$

* **Perigee :** the position of a planet nearest to the sun is known as perigee. In this position the speed of planet is maximum.

* **Apogee :** The position of a planet at the maximum distance from sun is known as apogee. In this position the speed of the planet is minimum.

Motion of planets in elliptical orbits - If r_1 is minimum distance (perigee) and r_2 is maximum distance (apogee) from sun which is at the focus, then from the law of conservation of angular momentum $mV_{max} r_1 = mV_{min} r_2$

$$\frac{V_{max}}{V_{min}} = \frac{r_2}{r_1}$$

$$\frac{1+e}{1-e} = \frac{\text{semi major axis}}{\text{semi min or axis}} \;;\; e = \frac{a-b}{a+b} \quad [e = \text{eccentricity}]$$

Semi-major axis $a = \dfrac{r_1 + r_2}{2}$

Semi-minor axis $b = a(1-e^2)^{1/2}$

Area of orbit $= \pi ab = \pi a^2 (1-e^2)^{1/2}$

Example 9 :

Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T. On what power of r, will the square of time period depend if the gravitational force of attraction between the planet and the star is proportional to $r^{-5/2}$.

Sol. As gravitation provides centripetal force

$$\frac{mv^2}{r} = \frac{K}{r^{5/2}}, \text{ i.e., } v^2 = \frac{K}{mr^{3/2}} ; T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{mr^{3/2}}{K}}$$

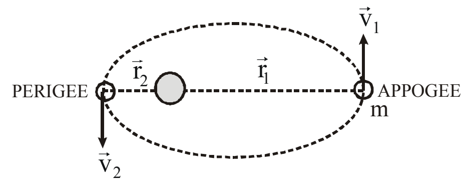
i.e., $T^2 = \dfrac{4\pi^2 m}{K} r^{7/2}$; so $T^2 \propto r^{7/2}$.

Example 10 :

A satellite revolves round a planet in an elliptical orbit. Its maximum and minimum distances from the planet are 1.5×10^7m and 0.5×10^7m respectively. If the speed of the satellite at the farthest point be 5×10^3m/s, calculate the speed at the nearest point.

Sol. From law of conservation of angular momentum

$$L = mv_1r_1 = mv_2r_2 \quad \text{or} \quad \frac{v_1}{v_2} = \frac{r_2}{r_1}$$



Substituting the given values, we get ,

$$\frac{5 \times 10^3}{v_2} = \frac{0.5 \times 10^7}{1.5 \times 10^7} \quad \text{or} \quad v_2 = 1.5 \times 10^4 \text{ m/sec}$$

Example 11 :

The Mars-sun distance is 1.524 times the Earth-sun distance. The period of revolution of Mars around the sun will be -

(1) 0.88 Years (2) 1.88 Years
(3) 2.88 Years (4) 3.88 Years

Sol. **(2).** According to third law of Kepler $T^2 \propto r^3$

$$\text{or} \quad \frac{T_{mars}}{T_{Earth}} = \left(\frac{r_{mars}}{r_{Earth}}\right)^{3/2} = (1.524)^{3/2} = 1.88 \text{ Years}$$

SATELLITE

(a) Orbital velocity of a satellite (v_0) :

In dynamic equilibrium gravitational force provides centripetal force.

$$\frac{GM_e m}{r^2} = \frac{mv_0^2}{r} \quad \text{or} \quad v_0 = \sqrt{\frac{GM_e}{r}} = \sqrt{\frac{GM_e}{R_e + h}}$$

For a satellite near the surface of the earth ($h << R_e$), neglecting h in comparison to R_e

$$v = \sqrt{gR_e} = 7.92 \text{ km/s.}$$

(b) Velocity of projection (v_P) :

To set up a satellite at a given height above the surface of earth it has to projected from the surface with a predetermined velocity, called velocity of projection.

Loss of KE = Gain in PE

$$\frac{1}{2} mv_P^2 = -\frac{GM_e m}{(R_e + h)} = -\left(-\frac{GM_e m}{R_e}\right)$$

$$v_P = \left[\frac{2GM_e h}{R_e (R_e + h)}\right]^{1/2} = \left(\frac{2gh}{(1 + h/R_e)}\right)^{1/2}$$

(c) Period of revolution (T) :

The time interval during which the satellite completes one revolution is called period of revolution of a satellite.

$$T = \frac{\text{circumference of an orbit}}{\text{velocity in orbit}}$$

$$= \frac{2\pi r}{v_0} = \frac{2\pi r^{3/2}}{\sqrt{GM}} = \frac{2\pi (R_e + h)^{3/2}}{\sqrt{gR_e^2}} = 2\pi\sqrt{\frac{R_e}{g}}\left(1+\frac{h}{R_e}\right)^{3/2}$$

For a satellite revolving very close to the surface of earth

$$(h << R_e) : \; T = 2\pi\sqrt{\frac{R_e}{g}} = 84.4 \text{ min.}$$

(d) Energy of satellite :

(i) Potential energy of satellite (P.E or U) -

$$U = -\frac{GM_e m}{r}$$

(ii) Kinetic energy of satellite (K.E.) -

$$KE = \frac{1}{2}mV_0^2 = \frac{GM_e m}{2r}$$

(iii) Total energy of satellite (T.E.) -

$$T.E. = K.E. + P.E. = -\frac{GM_e m}{2r}$$

(iv) Binding energy of satellite (B.E.) - Binding energy is the energy given to satellite in order that the satellite escape away form its orbit.

$$B.E. = -\text{Total Energy} = \frac{1}{2}\frac{GM_e m}{r}$$

Note :

Effective weight in a satellite : Satellite is an accelerated reference frame, with acceleration a = g towards centre of earth. $\therefore$ $W = m(g - a) = m(g - g) = 0$

Bound and Unbound trajectories :

Imagine a very tall tower on the surface of earth from where a projectile is fired with a velocity v parallel to the surface of earth. The trajectory of the projectile depends on its velocity.

Velocity	Effect
$v < \frac{v_e}{\sqrt{2}}$	Projectile does not orbit the earth. It falls back on the earth's surface.
$v = \frac{v_e}{\sqrt{2}}$	Projectile orbits the earth in a circular path.
$\frac{v_e}{\sqrt{2}} < v < v_e$	Projectile orbits in an elliptical path.
$v = v_e$	Projectile does not orbit. It escapes the gravitational field of earth in a parabolic path.
$v > v_e$	Projectile does not orbit. It escape the gravitational field of earth in a hyperbolic path.

GEOSTATIONARY SATELLITE (PARKING SATELLITE)

(i) If an artificial satellite revolves around the earth in an equatorial plane with a time period of 24 hrs. in the same sense as that of earth, then it will appear stationary to the observer on earth. Such a satellite is known as Geostationary satellite or Parking satellite.

(ii) Time period of Geostationary satellite is 24 hrs.

(iii) The radius of orbit of Geostationary satellite is 42,400 km and its height above the surface of earth is 36000 Km.

(iv) The relative angular velocity of the geostationary satellite is zero.

(v) The orbit velocity of a Geostationary satellite is 3.08 km/s

Example 12 :

Two satellites have their masses in the ratio of 3 : 1. The radii of their circular orbits are in the ratio of 1 : 4. What is the ratio of total mechanical energy of A and B?

Sol. $E = -\frac{GMm}{2r}$

$$\frac{E_1}{E_2} = \frac{m_1}{r_1} \times \frac{r_2}{m_2} = \left[\frac{m_1}{m_2}\right]\left[\frac{r_2}{r_1}\right] = \frac{3}{1} \times \frac{4}{1} = \frac{12}{1}$$

Example 13 :

A satellite of mass m, initially at rest on the earth, is launched into a circular orbit at a height equal to the radius of the earth. Find the minimum energy required.

Sol. We know that, $V_0 = \sqrt{\frac{GM}{r}}$ & $g = \frac{GM}{R^2}$

From energy conservation, $U_i + K_i = U_f + K_f$

$$-\frac{GMm}{R} + K_f = -\frac{GMm}{2R} + \frac{1}{2}mv_0^2$$

$$K_i = \frac{GMm}{2R} + \frac{1}{2}m\left(\sqrt{\frac{GM}{2R}}\right)^2 = \frac{3GMm}{4R} = \frac{3}{4}mgR$$

Example 14 :

A satellite can be in a geostationary orbit around earth at a distance r from the the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the centre is –

(A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$ (C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$

Sol. **(C).** Let angular velocity is $\omega \Rightarrow mr\omega^2 = \frac{GMm}{r^2}$

$$\omega^2 = \frac{GM}{r^3}, \text{ so } \omega_1^2 r_1^3 = \omega_2^2 r_2^3$$

$$r_2^3 = r_1^3\left(\frac{\omega_1}{\omega_2}\right)^2 = \frac{r_1^3}{4} \Rightarrow r_2 = \frac{r_1}{4^{1/3}}$$

TRY IT YOURSELF-2

Q.1 A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of V.

(A) $V_e = 2V$ (B) $V_e = V$

(C) $V_e = V/2$ (D) $V_e = \sqrt{3}V$

Q.2 Satellites A and B are orbiting around the earth in orbits of radio R and 4R respectively. The ratio of their areal velocities is –

(A) 1 : 2 (B) 1 : 4

(C) 1 : 8 (D) 1 : 16

Q.3 Which of the following are Kepler's laws?

(1) Each planet moves in an elliptical orbit, with the sun at the center of the ellipse.

(2) Each planet moves in an elliptical orbit, with the sun at the focus of the ellipse.

(3) A line from the sun to a given planet sweeps out equal areas in equal times.

(4) Planets move equal distances in equal times.

(5) The periods of the planets are proportional to the cube of the semi-major axis lengths of their orbits.

(6) The periods of the planets are proportional to the 3/2 power of the semi-major axis lengths of their orbits.

(A) 1, 3, 5 (B) 1, 3, 6

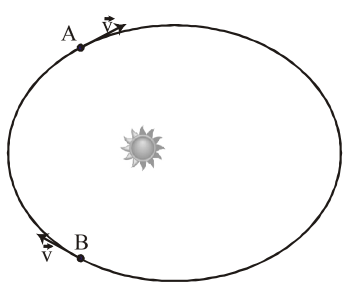
(C) 1, 4, 5 (D) 2, 3, 6

Q.4 A satellite following an elliptical path around a planet has an angular velocity ω_{far} when at its maximum distance d units from the planet's center. At its closest point, the distance between the satellite and planet's center is d/3. The satellite's angular velocity at that closest point is:

(A) $\omega_{far}/3$ (B) ω_{far}

(C) $3\omega_{far}$ (D) $9\,\omega_{far}$

Q.5 A planet orbits a sun in an elliptical orbit as shown. Which principles of physics most clearly and directly explain why the speed of the planet is the same at positions A and B?



(A) Conservation of Energy

(B) Conservation of Angular Velocity

(C) Conservation of Angular Momentum

(D) Conservation of Charge

Q.6 Find the period of the circular orbit of our sun around the center of our Galaxy (take as point mass 4×10^{41} kg) at a radius 3×10^4 light years.

Q.7 A satellite is in an elliptical orbit around the earth with altitudes ranging from 230 to 890 km. At the high point it is moving at 7.23 km/s. What is its speed at the low point?

Q.8 The planet Neptune travels around the Sun with a period of 165 year. Find the radius of Neptune orbit in terms of Earth's orbit, both being considered as circular.

Q.9 Suppose Earth's orbital motion around the Sun is suddenly stopped. What time will the Earth take to fall into the Sun?

Q.10 The period of lunar orbit around the Earth is 27.3 days and the radius of the orbit is 3.9×10^5 km. Using the measured value of $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, estimate the mass of Earth.

ANSWERS

(1) (A)	**(2)** (A)	**(3)** (D)	**(4)** (D)
(5) (AC)	**(6)** 200 Million years.		**(7)** 7.95 km/s
(8) $30\,R_1$	**(9)** 2 months		**(10)** 6.3×10^{24} kg

IMPORTANT POINTS

1. The acceleration due to gravity.

(a) At a height h above the Earth's surface

$$g(h) = \frac{GM_E}{(R_E + h)^2} = \frac{GM_E}{R_E^2}\left(1 - \frac{2h}{R_E}\right) \text{ ; for } h << R_E$$

$$g(h) = g(0)\left(1 - \frac{2h}{R_E}\right) \quad \text{where} \quad g(0) = \frac{GM_E}{R_E^2}$$

(b) At depth d below the Earth's surface is

$$g(d) = \frac{GM_E}{R_E^2}\left(1 - \frac{d}{R_E}\right) = g\,(0)\left(1 - \frac{d}{R_E}\right)$$

2. Kepler's laws of planetary motion state that –

(a) All planets move in elliptical orbits with the Sun at one of the focal points

(b) The radius vector drawn from the sun to a planet sweeps out equal areas in equal time intervals. This follows from the fact that the force of gravitation on the planet is central and hence angular momentum is conserved.

(c) The square of the orbital period of a planet is proportional to the cube of the semimajor axis of the elliptical orbit of the planet.

The period T and radius R of the circular orbit of a planet about the Sun are related by $T^2 = \left(\frac{4\pi^2}{GM_S}\right) R^3$

where M_S is the mass of the Sun. Most planets have nearly circular orbits about the Sun.

For elliptical orbits, the above equation is valid if R is replaced by the semi-major axis, a.

3. Angular momentum conservation leads to Kepler's second law. However, it is not special to the inverse square law of gravitation. It holds for any central force.

In Kepler's third law and $T^2 = K_s R^3$. The constant K_S is the same for all planets in circular orbits. This applies to satellites orbiting the Earth.

4. The escape speed from the surface of the Earth is

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E}$$ and has a value of 11.2 km s^{-1}.

5. A geostationary (geosynchronous communication) satellite moves in a circular orbit in the equatorial plane at a approximate distance of 4.22×10^4 km from the Earth's centre.

6. If $F \propto r^n$ then $T^2 \propto (r)^{1-n}$; If $U \propto r^m$ then $T^2 \propto (r)^{2-m}$

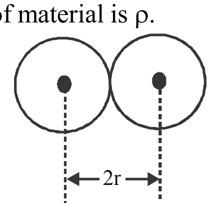
ADDITIONAL EXAMPLES

Example 1 :

Sphere of the same material and same radius r are touching each other. Show that gravitational force between them is directly proportional to r^4. Let mass of both sphere are m_1 and m_2 respectively. The density of material is ρ.

Sol. $m_1 = m_2 = $ (volume) (density)

$$= \left(\frac{4}{3}\pi r^3\right)\rho$$



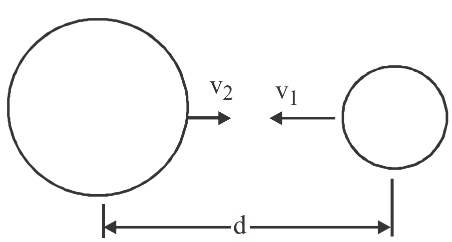
$$\therefore \quad F = \frac{Gm_1m_2}{r^2} = \frac{Gm\left(\frac{4}{3}\pi r^3\right)\left(\frac{4}{3}\pi r^3\right)\rho^2}{r^2} \text{ or } F \propto r^4$$

Example 2 :

Two hypothetical planets of masses m_1 and m_2 and radii r_1 and r_2, respectively, are nearly at rest when they are an infinite distance apart Because of their gravitational attraction, they head toward each other on a collision course.

(a) When their center-to-center separation is d, find expressions for the speed of each planet and their relative velocity.

(b) Find the kinetic energy of each planet just before they collide, if $m_1 = 2.00 \times 10^{24}$ kg, $m_2 = 8.00 \times 10^{24}$ kg, $r_1 = 3.00 \times 10^6$ and $r_2 = 5.00 \times 10^6$ m.
(Note : Both energy and momentum are conserved.)



Sol. (a) At infinite separation, U = 0; and at rest, K = 0. Since energy is conserved, we have

$$0 = \frac{1}{2}mv_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} \qquad \text{........ (1)}$$

The initial momentum is zero and momentum is conserved. Therefore,

$$_1v_1 - m_2v_2 \qquad \text{........ (2)}$$

Combine eq. (1) and eq. (2) tofind

$$v_1 = m_2\sqrt{\frac{2G}{d\,(m_1+m_2)}} \text{ and } v_2 = m_1\sqrt{\frac{2G}{d\,(m_1+m_2)}}$$

The relative velocity is

$$v_r = v_1 - (-v_2) = \sqrt{\frac{2G\,(m_1+m_2)}{d}}$$

(b) Substitute the given numerical values into the equation found for v_1 and v_2 in part (a) to find
$v_1 = 1.03 \times 10^4$ m/s and $v_2 = 2.58 \times 10^3$ m/s.

Therefore, $K_1 = \frac{1}{2}m_1v_1^2 = 1.07 \times 10^{32}$ J

and $K_2 = \frac{1}{2}m_2v_2^2 = 2.67 \times 10^{31}$ J

Example 3 :

An asteroid, whose mass is 2.0×10^{-4} times the mass of earth, revolves in a circular orbit around the Sun at a distance that is twice earth's distance from the Sun, (a) Calculate the period of revolution of the asteroid in years. (b) What is the ratio of the kinetic energy of the asteroid to that of Earth ?

Sol. (a) Use the law of periods : $T^2 = \frac{4\pi^2}{GM}r^3$, where M is the mass of the Sun (1.99×10^{30} kg) and r is the radius of the orbit. The radius of the orbit is twice the radius of earth's orbit :

$r = 2r_e = 2\,(150 \times 10^9 \text{ m}) = 300 \times 10^9$ m. Thus

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2(300\times10^9\text{ m})^3}{(6.67\times10^{-11}\text{m}^3/\text{s}^2.\text{kg})\,(1.99\times10^{30}\text{kg})}}$$

$= 8.96 \times 10^7$ s

Divide by (365 d/y) (24 h/d) (60 min/h) (60s/min) to obtain T = 2.8 y.

(b) The kinetic energy of any asteroid or planet in a circular orbit of radius r is given by K = GMm/2r, where m is the mass of the asteroid or planet. Notice that it is proportional to m and inversely proportional to r. The ratio of the kinetic energy of the asteroid to the kinetic energy of earth is $K/K_e = (m/m_e)\,(r_e/r)$.

Substitute $m = 2.0 \times 10^{-4}\, m_e$ and $r = 2r_e$ to obtain

$$\frac{K}{K_e} = 1.0 \times 10^{-4}.$$

Example 4 :

A satellite moves eastwards very near the surface of the earth in equatorial plane with speed (v_0). Another satellite moves at the same height with the same speed in the equatorial plane but westwards. If R = radius of the earth and ω be its angular speed of the earth about its own axis. Then find the approximate difference in the two time period as observed on the earth.

Sol. $T_{west} = \frac{2\pi R}{v_0 + R\omega}$ and $T_{east} = \frac{2\pi R}{v_0 - R\omega}$

$\Rightarrow \Delta T = T_{east} - T_{west}$

$$= 2\pi R\left[\frac{2R\omega}{v_0^2 - R^2\omega^2}\right] = \frac{4\pi\omega R^2}{v_0^2 - R^2\omega^2}$$

Example 5 :

If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth. Height of satellite above the earth's surface is equal to radius of earth.

Sol. From conservation of energy

Total energy at height h = Total energy at earth's surface

i.e., $0 - \frac{GMm}{R+h} = \frac{1}{2}mv^2 - \frac{GMm}{R}$

or $\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{R+h} = \frac{GMm}{R} - \frac{GMm}{2R}$ $\{\because h = R\}$

$\therefore \frac{1}{2}mv^2 = \frac{GMm}{2R}$

This gives $v = \sqrt{\left(\frac{GM}{R}\right)} = \sqrt{\left(\frac{R^2 g}{R}\right)}$

$$= \sqrt{(Rg)} = \sqrt{6400 \times 10^3 \times 9.8} = 7.919 \times 10^3 \text{ m/s.}$$

Example 6 :

At what height from the surface of earth the gravitational force will be reduced by 10% if the radius of earth is 6370 km.

Sol. Gravitational force on earth surface, $F_S = \frac{GMm}{R_e^2}$

Gravitational force at height h

$$F_h = \frac{GMm}{(R_e + h)^2} \quad \therefore \frac{F_s}{F_h} = \frac{(R_e + h)^2}{R_e^2} = \frac{10}{9}$$

$$\frac{R_e + h}{R_e} = \frac{3.162}{3}$$

$\therefore h = \frac{0.162 \times 6370}{3}$ or $h = 344$ Km

Example 7 :

At what height from the surface of earth is the acceleration due to gravity 1/100 of its value on earth surface ?

(A) 5.76×10^6 m (B) 5.76 m

(C) 0.576 m (D) 57.6×10^6

Sol. (A). $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \frac{1}{100}$

or $\frac{R}{R+h} = \frac{1}{10}$

or $h = 9R = 57.6 \times 10^6$ m

Example 8 :

Two spherical planets P and Q have the same uniform density r, masses M_P and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density r and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$

(C) $V_R/V_P = 3$ (D) $V_P/V_Q = 1/2$

Sol. (BD). Escape velocity $= \sqrt{\frac{2GM}{R}} \propto \sqrt{\frac{\frac{4}{3}\pi R^3}{R}} \propto \sqrt{\text{Area}}$

[since density of each planet is same]

Example 9 :

Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) –

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$.

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(D) The energy of the mass m remains constant.

Sol. (B). $\frac{-2GMm}{L} + \frac{1}{2}mv^2 = 0 \Rightarrow v = 2\sqrt{\frac{GM}{L}}$

The energy of mass 'm' means its kinetic energy (KE) only and not the potential energy of interaction between m and the two bodies (of mass M each)– which is the potential energy of the system.

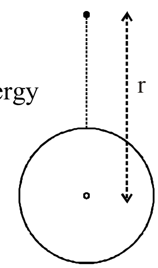
Example 10 :

A Bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $1/4^{th}$ of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{n}$, then the value of N is

(Ignore energy loss due to atmosphere)

Sol. **2.** When it reaches its maximum height, its acceleration due to the planet's gravity is $1/4^{th}$ of its value at the surface of the planet.

$$\frac{GM}{r^2} = \frac{1}{4}\frac{GM}{R^2} \ ; \ r = 2R$$



By conservation of mechanical energy

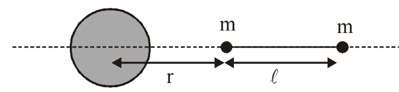
$$\frac{-GMm}{R} + \frac{1}{2}mv^2 = \frac{-GMm}{r} + 0$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2R}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}} = v\sqrt{N} \ ; \ N = 2$$

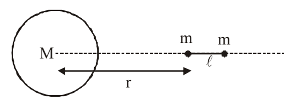
Example 11 :

A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k(M/288)$. The value of k is



Sol. **7.** For point mass at distance $r = 3\ell$

$$\frac{GMm}{(3\ell)^2} - \frac{Gm^2}{\ell^2} = ma$$



For point mass at distance $r = 4\ell$

$$\frac{GMm}{(4\ell)^2} + \frac{Gm^2}{\ell^2} = ma$$

$$\frac{GMm}{9\ell^2} - \frac{Gm^2}{\ell^2} = \frac{GMm}{16\ell^2} + \frac{Gm^2}{\ell^2}$$

QUESTION BANK

CHAPTER 9 : GRAVITATION

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : NEWTON'S LAW OF GRAVITATION

Q.1 Two stars of masses m_1 and m_2 are parts of a binary star system. The radii of their orbits are r_1 and r_2 respectively, measured from the centre of mass of the system. The magnitude of gravitational force m_1 exerts on m_2 is –

(A) $\frac{Gm_1m_2}{(r_1+r_2)^2}$ (B) $\frac{Gm_1}{(r_1+r_2)^2}$

(C) $\frac{Gm_2}{(r_1+r_2)^2}$ (D) $\frac{G(m_1+m_2)}{(r_1+r_2)^2}$

Q.2 The gravitational force between two stones of mass 1 kg each separated by a distance of 1 metre in vacuum is

(A) Zero (B) 6.675×10^{-5} N

(C) 6.675×10^{-11} N (D) 6.675×10^{-8} N

Q.3 Far from any other masses, two masses, m_1 and m_2, are interacting gravitationally. The value for the mass of m_1 suddenly doubles. What happens to the value of the gravitational force that mass m_2 exerts on mass m_1?

(A) It doubles.

(B) It decreases by a factor of 2.

(C) It quadruples.

(D) Nothing, because the mass m_2 did not change.

Q.4 Two identical solid copper spheres of radius R placed in contact with each other. The gravitational attraction between them is proportional to

(A) R^2 (B) R^{-2}

(C) R^4 (D) R^{-4}

Q.5 The Moon remains in its orbit around the Earth rather than falling to the Earth because –

(A) it is outside of the gravitational influence of the Earth.

(B) it is in balance with the gravitational forces from the Sun and other planets.

(C) the net force on the Moon is zero.

(D) none of these

Q.6 Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be –

(A) 3F (B) F

(C) F/3 (D) F/9

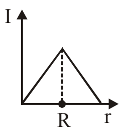
Q.7 Mass M is divided into two parts x M and (1 – x) M. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is

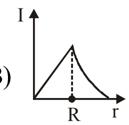
(A) 1/2 (B) 3/5

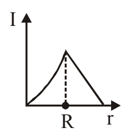
(C) 1 (D) 2

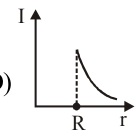
PART - 2 : GRAVITATIONAL FIELD INTENSITY

Q.8 Which one of the following groups represents correctly the variation of the gravitational intensity I with the distance r from the centre of a spherical shell of mass M and radius R?

(A)


(B)


(C)


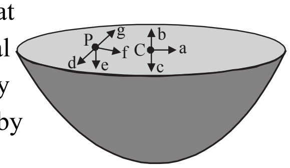
(D)


Q.9 A uniform ring of mass m and radius a is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is at a distance $\sqrt{3}\,a$ from the centre of the sphere. Find the gravitational force exerted by the sphere on the ring.

(A) $\frac{\sqrt{3}GMm}{8a^2}$ (B) $\frac{\sqrt{5}GMm}{8a^2}$

(C) $\frac{\sqrt{3}GMm}{5a^2}$ (D) $\frac{\sqrt{2}GMm}{5a}$

Q.10 The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig.)



(A) a (B) b

(C) c (D) 0

Q.11 For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow

(A) d (B) e

(C) f (D) g

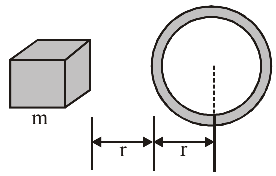
PART - 3 : GRAVITATIONAL POTENTIAL AND POTENTIAL ENERGY

Q.12 A particle of mass m is placed at the centre of a uniform spherical shell of mass 3m and radius R. The gravitational potential on the surface of the shell is –

(A) $-\frac{Gm}{R}$ (B) $-\frac{3Gm}{R}$

(C) $-\frac{4Gm}{R}$ (D) $-\frac{2Gm}{R}$

Q.13 A block of mass m is lying at a distance r from a spherical shell of mass m and radius r as shown in the figure. Then



(A) only gravitational field inside the shell is zero.
(B) gravitational field and gravitational potential both are zero inside the shell.
(C) gravitational potential as well as gravitational field inside the shell are not zero.
(D) can't be ascertained.

Q.14 Two bodies of mass 10^2 kg and 10^3 kg are lying 1m apart. The gravitational potential at the mid-point of the line joining them is -
(A) 0 (B) –1.47 Joule/kg
(C) 1.47 Joule/kg (D) -1.47×10^{-7} Joule /kg

Q.15 If g is the acceleration due to gravity on the earth's surface, the gain in potential energy of an object of mass m raised from the surface of the earth to a height of the radius R of the earth is -
(A) mgR (B) 2mgR
(C) $\frac{1}{2}$ mgR (D) $\frac{1}{4}$ mgR

Q.16 Four particles each of mass m are placed at the vertices of a square of side ℓ. The potential energy of the system is –
(A) $-\frac{\sqrt{2}Gm^2}{\ell}\left(2-\frac{1}{\sqrt{2}}\right)$ (B) $-\frac{2Gm^2}{\ell}\left(2+\frac{1}{\sqrt{2}}\right)$
(C) $-\frac{\sqrt{2}Gm^2}{\ell}\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)$ (D) $-\frac{2Gm^2}{\ell}\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)$

Q.17 The radius of the earth is R_e and the acceleration due to gravity at its surface is g. The work required in raising a body of mass m to a height h form the surface of the earth
(A) $\frac{mgh}{\left(1-\frac{h}{R_e}\right)}$ (B) $\frac{mgh}{\left(1+\frac{h}{R_e}\right)^2}$
(C) $\frac{mgh}{\left(1+\frac{h}{R_e}\right)}$ (D) $\frac{mg}{\left(1+\frac{h}{R_e}\right)}$

PART - 4 : ACCELERATION DUE TO GRAVITY

Q.18 Two planets have the same average density but their radii are R_1 and R_2. If acceleration due to gravity on these planets be g_1 and g_2 respectively, then –
(A) $\frac{g_1}{g_2}=\frac{R_1}{R_2}$ (B) $\frac{g_1}{g_2}=\frac{R_2}{R_1}$
(C) $\frac{g_1}{g_2}=\frac{R_1^2}{R_2^2}$ (D) $\frac{g_1}{g_2}=\frac{R_1^3}{R_2^3}$

Q.19 The diameters of two planets are in the ratio 4 : 1 and their mean densities are in the ratio 1 : 2. The acceleration due to gravity on the planets will be in the ratio –
(A) 1 : 2 (B) 2 : 3
(C) 2 : 1 (D) 4 : 1

Q.20 At what altitude in metre will the acceleration due to gravity be 25% of that at the earth's surface (Radius of earth = R m)
(A) R/4 (B) R
(C) 3R/8 (D) R/2

Q.21 If the angular speed of the earth is doubled, the value of acceleration due to gravity (g) at the north pole
(A) Doubles (B) Becomes half
(C) Remains same (D) Becomes zero

Q.22 The angular speed of earth, so that the object on equator may appear weightless, is
($g = 10$ m/s^2, radius of earth 6400km)
(A) 1.25×10^{-3} rad/sec (B) 1.56×10^{-3} rad/sec
(C) 1.25×10^{-1} rad/sec (D) 1.56 rad/sec

Q.23 At what distance from the centre of the earth, the value of acceleration due to gravity g will be half that on the surface (R = radius of earth)
(A) 2 R (B) R
(C) 1.414 R (D) 0.414 R

Q.24 If density of earth increased 4 times and its radius become half of that it is, our weight will –
(A) four times (B) Be doubled
(C) Remain same (D) Be halved

Q.25 If both the mass and the radius of the earth decrease by 1%, the value of the acceleration due to gravity will –
(A) Decrease by 1% (B) Increase by 1%
(C) Increase by 2% (D) Remain unchanged

Q.26 If the change in the value of g at a height h above the surface of the earth is the same as at a depth x below it, then (both x & h being much smaller than the radius of the earth)
(A) x = h (B) x = 2h
(C) x = h/2 (D) $x = h^2$

Q.27 A body of mass m is taken to the bottom of a deep mine. Then –
(A) Its mass increases (B) Its mass decreases
(C) Its weight increases (D) Its weight decreases

Q.28 A body weight W newton at the surface of the earth. Its weight at a height equal to half the radius of the earth will
(A) W/2 (B) 2W/3
(C) 4W/9 (D) 8W/27

Q.29 The depth at which the effective value of acceleration due to gravity is g/4 is
(A) R (B) 3R/4
(C) R/2 (D) R/4

Q.30 If the radius of the earth were to shrink by one percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would -
(A) decrease (B) remain unchanged
(C) increase (D) None of these

Q.31 Assuming that the moon is a sphere of the same mean density as that of the earth and one quarter of its radius, the length of a seconds pendulum on the moon (its length on the earth's surface is 99.2 cm) is
(A) 24.8 cm (B) 49.6 cm
(C) 99.2 cm (D) $\frac{99.2}{\sqrt{2}}$ cm

PART - 5 : ESCAPE VELOCITY

Q.32 Escape velocity will depend on –
(A) Mass of the planet
(B) Mass of the particle escaping
(C) Radius of the planet
(D) Both (A) and (C)

Q.33 v_e and v_p denotes the escape velocity from the earth and another planet having twice the radius and the same mean density as the earth. Then –
(A) $v_e = v_p$ (B) $v_e = v_p/2$
(C) $v_e = 2v_p$ (D) $v_e = v_p/4$

Q.34 The escape velocity of a sphere of mass m from earth having mass M and radius R is given by –
(A) $\sqrt{\frac{2GM}{R}}$ (B) $2\sqrt{\frac{GM}{R}}$
(C) $\sqrt{\frac{2GMm}{R}}$ (D) $\sqrt{\frac{GM}{R}}$

Q.35 There are two planets. The ratio of radius of the two planets is K but ratio of acceleration due to gravity of both planets is g. What will be the ratio of their escape velocity
(A) $(Kg)^{1/2}$ (B) $(Kg)^{-1/2}$
(C) $(Kg)^2$ (D) $(Kg)^{-2}$

Q.36 A missile is launched with a velocity less than the escape velocity. The sum of its kinetic and potential energy is
(A) Positive
(B) Negative
(C) Zero
(D) May be positive or negative depending upon its initial velocity.

Q.37 The escape velocity of a body on an imaginary planet which is thrice the radius of the earth and double the mass of the earth is (v_e is the escape velocity of earth)
(A) $\sqrt{\frac{2}{3}}v_e$ (B) $\sqrt{\frac{3}{2}}v_e$
(C) $\frac{\sqrt{2}}{3}v_e$ (D) $\frac{2}{\sqrt{3}}v_e$

Q.38 If the radius of a planet is four times that of earth and the value of g is same for both, the escape velocity on the planet will be –
(A) 11.2 km/s (B) 5.6 km/s
(C) 22.4 km/s (D) None

Q.39 A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of V.
(A) $V_e=2V$ (B) $V_e=V$
(C) $V_e=V/2$ (D) $V_e=\sqrt{3}V$

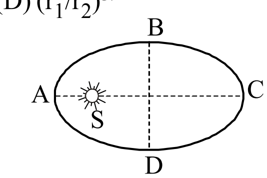
PART - 6 : KEPLER'S LAW OF PLANETARY MOTION

Q.40 A planet moves around the sun. At a given point P, it is closest from the sun at a distance d_1 and has a speed v_1. At another point Q, when it is farthest from the sun at a distance d_2, its speed will be –
(A) $\frac{d_1^2 v_1}{d_2^2}$ (B) $\frac{d_2 v_1}{d_1}$
(C) $\frac{d_1 v_1}{d_2}$ (D) $\frac{d_2^2 v_1}{d_1^2}$

Q.41 The orbital speed of Jupiter is –
(A) Greater than the orbital speed of earth.
(B) Less than the orbital speed of earth.
(C) Equal to the orbital speed of earth.
(D) Zero.

Q.42 Two planets move around the sun. The periodic times and the mean radii of the orbits are T_1, T_2 and r_1, r_2 respectively. The ratio T_1/T_2 is equal to
(A) $(r_1/r_2)^{1/2}$ (B) r_1/r_2
(C) $(r_1/r_2)^2$ (D) $(r_1/r_2)^{3/2}$

Q.43 A planet is revolving around the sun as shown in elliptical path The correct option is –



(A) The time taken in travelling DAB is less than that for BCD.
(B) The time taken in travelling DAB is greater than that for BCD.
(C) The time taken in travelling CDA is less than that for ABC.
(D) The time taken in travelling CDA is greater than that for ABC.

Q.44 In the previous question the orbital velocity of the planet will be minimum at –
(A) A (B) B
(C) C (D) D

Q.45 The radius of orbit of a planet is two times that of the earth. The time period of planet is –
(A) 4.2 years (B) 2.8 years
(C) 5.6 years (D) 8.4 years

Q.46 According to Kepler, the period of revolution of a planet (T) and its mean distance from the sun (r) are related by the equation
(A) T^3r^3 = constant (B) T^2r^{-3} = constant
(C) Tr^3 = constant (D) T^2r = constant

Q.47 The earth revolves round the sun in one year. If the distance between them becomes double, the new period of revolution will be
(A) 1/2 year (B) $2\sqrt{2}$ years
(C) 4 years (D) 8 years

Q.48 The mass of a planet that has a moon whose time period and orbital radius are T & R respectively can be written
(A) $4\pi^2R^3G^{-1}T^{-2}$ (B) $8\pi^2R^3G^{-1}T^{-2}$
(C) $12\pi^2R^3G^{-1}T^{-2}$ (D) $16\pi^2R^3G^{-1}T^{-2}$

Q.49 If a graph is plotted between T^2 and r^3 for a planet then its slope will be -
(A) $4\pi^2/GM$ (B) $GM/4\pi^2$
(C) $4\pi GM$ (D) 0

Q.50 A satellite following an elliptical path around a planet has an angular velocity ω_{far} when at its maximum distance d units from the planet's center. At its closest point, the distance between the satellite and planet's center is d/3. The satellite's angular velocity at that closest point is:
(A) $\omega_{far}/3$ (B) ω_{far}
(C) $3\omega_{far}$ (D) $9\ \omega_{far}$

Q.51 The planet Neptune travels around the Sun with a period of 165 year. Find the radius of Neptune orbit in terms of Earth's orbit, both being considered as circular.
(A) 30 R_1 (B) 25 R_1
(C) 15 R_1 (D) 10 R_1

PART - 7 : SATELLITE

Q.52 Consider a satellite going round the earth in an orbit. Which of the following statements is wrong
(A) It is a freely falling body
(B) It suffers no acceleration
(C) It is moving with a constant speed
(D) Its angular momentum remains constant.

Q.53 Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are revolving round the earth in circular orbits of radius r_1 and r_2 ($r_1 > r_2$) respectively. Which of the following statements is true regarding their speeds v_1 and v_2 ?
(A) $v_1 = v_2$ (B) $v_1 < v_2$
(C) $v_1 > v_2$ (D) $\frac{v_1}{r_1} = \frac{v_2}{r_2}$

Q.54 A geo-stationary satellite is orbiting the earth at a height of 6 R above the surface of earth, R being the radius of earth. The time period of another satellite at a height of 2.5R from the surface of earth is –
(A) 10 hr (B) $(6/\sqrt{2})$ hr
(C) 6 hr (D) $6\sqrt{2}$ hr

Q.55 Two satellites of mass m and 9m are orbiting a planet in orbits of radius R. Their periods of revolution will be in the ratio of –
(A) 1 : 3 (B) 1 : 1
(C) 3 : 1 (D) 9 : 1

Q.56 A geostationary satellite is revolving around the earth. To make it escape from gravitational field of earth, is velocity must be increased –
(A) 100% (B) 41.4%
(C) 50% (D) 59.6%

Q.57 The orbital speed of an artificial satellite very close to the surface of the earth is V_0. Then the orbital speed of another artificial satellite at a height equal to three times the radius of the earth is
(A) $4V_0$ (B) $2V_0$
(C) $0.5\ V_0$ (D) $4V_0$

Q.58 The period of a satellite in a circular orbit of radius R is T, the period of another satellite in a circular orbit of radius 4R is
(A) 4T (B) T/4
(C) 8T (D) T/8

Q.59 Which of the following statements is incorrect regarding the polar satellites?
(A) A polar satellites goes around the earth's pole in north-south direction.
(B) Polar satellites are used to study topography of Moon, Venus and Mars.
(C) A polar satellite is a high altitude satellite.
(D) The time period of polar satellite is about 100minutes.

Q.60 A communication satellite is in a circular orbit around Earth. If the speed of the satellite is constant, the force acting on the satellite
(A) is zero.
(B) is decreasing.
(C) points toward the center of Earth at all times.
(D) points in the direction that the satellite is moving.

PART - 8 : MISCELLANEOUS

Q.61 Mass of moon is 1/81 times that of earth and its radius is 1/4 the earth's radius. If escape velocity at surface of earth is
11.2 km/sec, then its value at surface of moon is:
(A) 0.14 km/sec (B) 0.5 km/sec
(C) 2.5 km/sec (D) 5 km/sec.

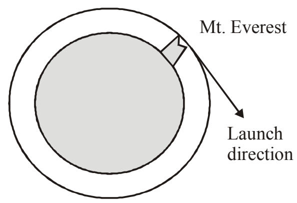
Q.62 Calculate the escape velocity for an atmospheric particle 1000 km above the Earth's surface, given that the radius of the Earth = 6.4×10^6 m and acceleration due to gravity on the surface of the Earth = 9.8 m s^{-2}.
(A) 12.42 km/s (B) 10.42 km/s
(C) 11.22 km/s (D) 9.42 km/s

Q.63 A particle is suspended from a spring and it stretches the spring by 1 cm on the surface of earth. The same particle will stretch the same spring at a placed 800 Km above earth surface by
(A) 0.39 cm (B) 0.79 cm
(C) 0.49 cm (D) 0.89 cm

Q.64 The little prince (the main character of the novel written by antoine de saint-Exupery) lives on the spherical planet named B-612, the density of which is 5200 kg/m^3. The Little Prince noticed that if he quickens his pace, he feels himself lighter. When he reached the speed of 2m/s he became weightless, and began to orbit about the planet as a satellite. What is escape speed on the surface of planet.

(A) $2\sqrt{2}$ m/s (B) 2 m/s
(C) $4\sqrt{2}$ m/s (D) $8\sqrt{2}$ m/s

Q.65 Suppose earth had no atmosphere and a ball was fired from the top of Mt. Everest in a direction tangent to the ground. If the initial speed is high enough to cause the ball to travel in a circular trajectory around earth, the ball's acceleration would. (g is value of acceleration due to gravity at the surface of earth)



(A) be much less than g (because the ball doesn't fall to the ground)
(B) be approximately g
(C) depend on the ball's speed
(D) much larger than g

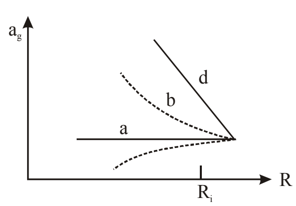
Q.66 The acceleration due to gravity on the surface of the moon is 1/6 that on the surface of earth and the diameter of the moon is one-fourth that of earth. The ratio of escape velocities on earth and moon will be –

(A) $\sqrt{6}/2$ (B) $\sqrt{24}$
(C) 3 (D) $\sqrt{3}/2$

Q.67 The cosmonauts who landed at the pole of a planet found that the force of gravity there is 0.01 of that on the Earth, while the duration of the day on the planet is the same as that on the Earth. It turned out besides that the force of gravity on the equator is zero. Determine the radius R of the planet.

(A) 18000km. (B) 9000km.
(C) 27000km. (D) 22000km.

Q.68 A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse –



(A) a (B) b
(C) c (D) d

Q.69 Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles. An object weighed at the equator gives the same reading as a reading taken at a depth d below earth's surface at a pole (d << R). The value of d is –

(A) $\frac{\omega^2R^2}{g}$ (B) $\frac{\omega^2R^2}{2g}$

(C) $\frac{2\omega^2R^2}{g}$ (D) $\frac{\sqrt{Rg}}{g}$

Q.70 The kinetic energy of a satellite in an orbit close to the surface of the earth is E. What should be its kinetic energy so that it escapes from the gravitational field of the earth ?

(A) $\sqrt{2}$E (B) 2E
(C) $2\sqrt{2}$E (D) 4E

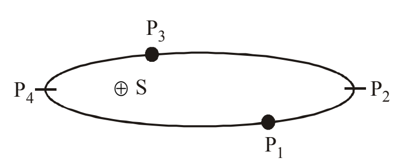
Q.71 A geo-stationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometers above the earth's surface (R_{earth}=6400 km) will approximately be

(A) 1/2 hr. (B) 1 hr.
(C) 2 hr. (D) 4 hr.

Q.72 A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is radius of earth. The value of T_2/T_1 is

(A) 1 (B) $\sqrt{2}$
(C) 4 (D) 2

Q.73 The figure shows a planet in elliptical orbit around the sun S. Where is the kinetic energy of the planet maximum



(A) P_1 (B) P_2
(C) P_3 (D) P_4

Q.74 A satellite is revolving round the earth in an orbit of radius r with time period T. If the satellite is revolving round the earth in an orbit of radius r + Δr (Δr << r) with time period T + ΔT then,

(A) $\frac{\Delta T}{T}=\frac{3}{2}\frac{\Delta r}{r}$ (B) $\frac{\Delta T}{T}=\frac{2}{3}\frac{\Delta r}{r}$

(C) $\frac{\Delta T}{T}=\frac{\Delta r}{r}$ (D) $\frac{\Delta T}{T}=-\frac{\Delta r}{r}$

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 In an experiment a boy draws graph between v^2 and a^2 (where v = velocity and a = acceleration) for a simple pendulum. The graph is found to be a straight line of negative slope making an angle of 30° when experiment was done on the ground and 60° when experiment was done at height h above the ground. Then h must be (R = radius of earth)

(A) 0.5 R (B) 0.24 R
(C) 0.73 R (D) R

Q.2 A magnetic storm from sun can disrupt a satellite as well as move it, either toward or away from Earth radially. Ground-based enginners start it back in a new circular orbit at the new position. Due to the storm –

(A) The period of a satellite displaced further away is more than the previous period.
(B) The mechanical energy of a satellite displaced towards earth is more than the previous energy.
(C) The speed of a satellite displaced further away is more than the previous speed.
(D) The angular momentum of a satellite displaced towards earth is more than the previous angular momentum

Q.3 A straight rod of length extends L from x = a to x = L + a. The gravitational force exerted on a point mass m at x = 0 if the mass per unit length of the rod is $A + Bx^2$, is

(A) $GmA\left[\frac{1}{a+L}-\frac{1}{a}+BL\right]$ (B) $Gm\left[\frac{A}{a}-\frac{A}{a+L}+BL\right]$

(C) $GmA\left[\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$ (D) $GmA\left[\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

Q.4 Two thin rings each of radius R are coaxially placed at a distance R. The rings have a uniform mass distribution and have mass m_1 and m_2 respectively. Then the work done in moving a mass m from centre of one ring to that of the other is

(A) zero (B) $\frac{Gm(m_1-m_2)(\sqrt{2}-1)}{\sqrt{2}R}$

(C) $\frac{Gm(\sqrt{2})(m_1-m_2)}{R}$ (D) $\frac{Gmm_1(\sqrt{2}+1)}{m_2R}$

Q.5 In older times, people used to think that the earth was flat. Imagine that the earth is indeed not a sphere of radius R, but an infinite plate of thickness H. What value of H is needed to allow the same gravitational acceleration to be experienced as on the surface of the actual earth ? (Assume that the earth's density is uniform and equal in the two models)

(A) 2R/3 (B) 4R/3
(C) 8R/3 (D) R/3

Q.6 Our Sun, with mass 2.0×10^{30} kg, revolves about the center of the Milky Way galaxy, which is 2.2×10^{20}m away, once every 2.5×10^8 years. Assuming that each of the stars in the galaxy has a mass equal to that of our Sun, that the stars are distributed uniformly in a sphere about the galactic center, and that our Sun is eccentially at the edge of that sphere, estimate roughly the number of stars in the galaxy.

(A) 2.1×10^{10} (B) 5.1×10^6
(C) 5.1×10^{10} (D) 5.1×10^{16}

Q.7 A planet revolves about the sun is elliptical orbit. The areal velocity (dA/dt) of the planet is 4.0×10^{16} m^2/s. The least distance between planet and the sun is 2×10^{12}m. Then the maximum speed of the planet in km/s is –

(A) 10 (B) 20 (C) 30 (D) 40

Q.8 In a certain region of space gravitational field is given by E = – (K/r). Taking the reference point to be at $r = r_0$ with $V = V_0$, the potential is –

(A) $V = V_0 \ln\left(\frac{r}{r_0}\right)$ (B) $V = V_0 e^{-r/r_0}$

(C) $V = V_0 + K\ln\frac{r}{r_0}$ (D) $V = V_0 e^{+r/r_0}$

Q.9 A planet of mass m moves along an ellipse around the sun so that its maximum and minimum distances from the sun are equal to r_1 and r_2 respectively. Find the angular momentum of this planet relative to the centre of the sun.

(A) $m\sqrt{\frac{2GMr_1r_2}{(r_1+r_2)}}$ (B) $2m\sqrt{\frac{2GMr_1r_2}{(r_1+r_2)}}$

(C) $m\sqrt{\frac{3GMr_1r_2}{(r_1+r_2)}}$ (D) $m\sqrt{\frac{GMr_1r_2}{2(r_1+r_2)}}$

Q.10 A particle is fired vertically upward with a speed of 9.8 km/s. Find the maximum height attained by the particle. Radius of earth = 6400 km and g at the surface = 9.8 m/s^2. Consider only earth's gravitation.

(A) 17900 km (B) 10900 km
(C) 15900 km (D) 20900 km

Q.11 A uniform sphere has a mass M and radius R. Find the pressure P inside the sphere, caused by gravitational compression, as a function of the distance r from the centre. Evaluate P at the centre of the Earth, assuming it to be a uniform sphere.

(A) 1.65×10^6 atm (B) 1.65×10^4 atm
(C) 4.65×10^6 atm (D) 1.65×10^3 atm

Q.12 The gravitational potential of two homogeneous spherical shells A and B of same surface density at their respective centres are in the ratio 3 : 4. If the two shells coalesce into single one such that surface charge density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to –

(A) 3 : 2 (B) 4 : 3
(C) 5 : 3 (D) 5 : 6

Q.13 A point P lies on the axis of a fixed ring of mass M and radius R, at a distance 2R from its centre O. A small particle starts from P and reaches O under gravitational attraction only. Its speed at O will be –

(A) zero (B) $\sqrt{\frac{2GM}{R}}$

(C) $\sqrt{\frac{2GM}{R}(\sqrt{5}-1)}$ (D) $\sqrt{\frac{2GM}{R}(1-\frac{1}{\sqrt{5}})}$

Q.14 Three identical stars, each of mass M, form an equilateral triangle (stars are positioned at the corners) that rotates around the centre of the triangle. The system is isolated and edge length of the triangle is L. The amount of work done, that is required to dismantle the system is :

(A) $\frac{3GM^2}{L}$ (B) $\frac{3GM^2}{2L}$ (C) $\frac{3GM^2}{4L}$ (D) $\frac{GM^2}{2L}$

Q.15 The eccentricity of the earth's orbit is 0.0167. The ratio of its maximum speed in its orbit to its minimum speed is

(A) 1.67 (B) 1.034
(C) 1 (D) 0.167

Q.16 Two bodies of masses M_1 and M_2 are placed at a distance d apart. What is the potential at the position where the gravitational field due to them is zero ?

(A) $-\frac{G}{d}\left(M_1+M_2+2\sqrt{M_1}\sqrt{M_2}\right)$

(B) $-\frac{G}{d}\left(M_1+M_2-2\sqrt{M_1}\sqrt{M_2}\right)$

(C) $-\frac{G}{d}\left(2M_1+M_2+2\sqrt{M_1}\sqrt{M_2}\right)$

(D) $-\frac{G}{2d}\left(M_1+M_2+2\sqrt{M_1}\sqrt{M_2}\right)$

Q.17 A cavity of radius R/2 is made inside a solid sphere of radius R. The centre of the cavity is located at a distance R/2 from the centre of the sphere. The gravitational force on a particle of mass 'm' at a distance R/2 from the centre of the sphere on the line joining both the centres of sphere and cavity is – (opposite to the centre of gravity)

[Here $g = GM/R^2$, where M is the mass of the sphere]

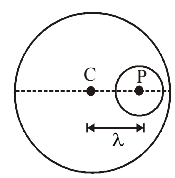
(A) $\frac{mg}{2}$ (B) $\frac{3mg}{8}$ (C) $\frac{mg}{16}$ (D) None of these

Q.18 A satellite of mass m is orbiting the earth in a circular orbit of radius R. It starts losing energy due to small air resistance at the rate of CJ/s. Find the time taken for the satellite to reach the earth.

(A) $\frac{GMm}{C}\left[\frac{1}{R}-\frac{1}{r}\right]$ (B) $\frac{GMm}{2C}\left[\frac{1}{R}+\frac{1}{r}\right]$

(C) $\frac{GMm}{2C}\left[\frac{1}{R}-\frac{1}{r}\right]$ (D) $\frac{2GMm}{C}\left[\frac{1}{R}+\frac{1}{r}\right]$

Q.19 Inside a uniform sphere of density ρ there is a spherical cavity whose centre is at a distance ℓ from the centre of the sphere. Find the strength F of the gravitational field inside the cavity at the point P.



(A) $\frac{4}{3}G\pi\rho\vec{\ell}$ (B) $\frac{1}{3}G\rho\vec{\ell}$ (C) $\frac{2}{3}G\pi\rho\vec{\ell}$ (D) $\frac{1}{2}G\pi\rho\vec{\ell}$

Q.20 The mass M of a planet-earth is uniformly distributed over a spherical volume of radius R. Calculate the energy needed to disassemble the planet against the gravitational pull amongst its constituent particles.

Given : $MR = 2.5 \times 10^{31}$ kg-m and $g = 10$ m/s^2

(A) 3.0×10^{32} J (B) 2.5×10^{32} J
(C) 0.5×10^{28} J (D) 1.5×10^{32} J

Q.21 A satellite is launched in the equatorial plane in such a way that it can transmit signals upto 60° latitude on the earth. The angular velocity of the satellite is –

(A) $\sqrt{\frac{GM}{8R^3}}$ (B) $\sqrt{\frac{GM}{2R^3}}$ (C) $\sqrt{\frac{GM}{4R^3}}$ (D) $\sqrt{\frac{3\sqrt{3}GM}{8R^3}}$

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 If the radius of the earth be increased by a factor of 5, its density should be changed by factor 1/A to keep the value of g same. Find the value of A.

Q.2 A man of mass m starts falling towards a planet of mass M and radius R. As he reaches near to the surface, he realizes that he will pass through a small hole in the planet. As he enters the hole, he sees that the planet is really made of two pieces a spherical shell of negligible thickness of mass 2M/3 and a point mass M/3 at the centre. Change in the force of gravity experienced by the man is $\frac{2}{A}\frac{GMm}{R^2}$. Find the value of A.

Q.3 Two planets A and B have the same material density. If the radius of A is twice that of B, then the ratio of the escape velocity v_A/v_B is

Q.4 The escape velocity of a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be $v_e/\sqrt{A}$. Find the value of A.

Q.5 An artificial satellite moving in a circular orbit around the earth has a total (K.E. + P.E.) = E_0. Its potential energy is AE_0. Find the value of A.

Q.6 Two point masses of mass 4m and m respectively separated by d distance are revolving under mutual force of attraction. Ratio of their kinetic energies is 1/A. Find the value of A.

Q.7 Satellites A and B are orbiting around the earth in orbits of ratio R and 4R respectively. The ratio of their areal velocities is (1/A). Find the value of A.

Q.8 A particle is projected vertically upward from the surface of the earth with a speed of $\sqrt{\frac{3}{2}gR}$, R being the radius of the earth and g is the acceleration due to gravity on the surface of the earth. Then the maximum height ascended is aR (neglect cosmic dust resistance). Find the value a.

Q.9 A missile, which missed its target when into orbit around the earth at a mean radius 3 times as great as the parking orbit of the satellite. The period of the missile is $X\sqrt{3}$ day. Find the value of X.

Q.10 Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is 2/3 times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be :

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 A mass m is raised from a distance 2 Re from surface of earth to 3Re. Work done to do so against gravity will be- **[AIEEE-2002]**

(A) $\frac{mg\,Re}{10}$ (B) $\frac{mg\,Re}{11}$ (C) $\frac{mg\,Re}{12}$ (D) $\frac{mg\,Re}{14}$

Q.2 The escape velocity of a body of mass m from earth depends on - **[AIEEE-2002]**

(A) m^2 (B) m^1
(C) m^0 (D) None of above

Q.3 If suddenly gravitational force on a satellite becomes zero it will – **[AIEEE-2002]**

(A) go in tangential direction of orbit
(B) fall on earth
(C) follow hellical path towards earth
(D) follow hellical path away from earth

Q.4 The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is

(A) mgR/2 (B) 2 mgR **[AIEEE-2002]**
(C) mgR (D) mgR/4

Q.5 The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be– **[AIEEE-2003]**

(A) 22 km/s (B) 11 km/s
(C) $11/\sqrt{2}$ km/s (D) $11\sqrt{2}$ km/s

Q.6 The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become – **[AIEEE-2003]**

(A) 80 hours (B) 40 hours (C) 20 hours (D) 10 hours

Q.7 Two spherical bodies of mass M and 5 M and redii R and 2R respectively are released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is – **[AIEEE-2003]**

(A) 4.5 R (B) 7.5 R
(C) 1.5 R (D) 2.5 R

Q.8 A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **[AIEEE-2004]**

(A) gx (B) $\frac{gR}{R-x}$ (C) $\frac{gR^2}{R+x}$ (D) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

Q.9 The time period of an earth satellite in circular orbit is independent of – **[AIEEE-2004]**

(A) The mass of the satellite.
(B) Radius of its orbit.
(C) Both the mass and radius of the orbit.
(D) Neither the mass of the satellite nor the radius of its orbit.

Q.10 If 'g' is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass 'm' raised from the surface of the earth to a height equal to the radius 'R' of the earth is– **[AIEEE-2004]**

(A) 2 mgR (B) 1/2 mgR
(C) 1/4 mgR (D) mgR

Q.11 Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius 'r' around the sun will be proportional to – **[AIEEE-2004]**

(A) $r^{\left(\frac{n+1}{2}\right)}$ (B) $r^{\left(\frac{n-1}{2}\right)}$ (C) r^n (D) $r^{\left(\frac{n-2}{2}\right)}$

Q.12 Average density of the earth **[AIEEE-2005]**

(A) does not depend on g
(B) is a complex function of g
(C) is directly proportional to g
(D) is inversely proportional to g

Q.13 The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct ? **[AIEEE-2005]**

(A) d = h/2 (B) d = 3h/2
(C) d = 2h (D) d = h

Q.14 A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere
(Take $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$) **[AIEEE-2005]**

(A) 13.34×10^{-10} J (B) 3.33×10^{-10} J
(C) 6.67×10^{-9} J (D) 6.67×10^{-10} J

Q.15 If g_E and g_m are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio (electronic charge on the moon/ electronic charge on the earth) to be

(A) 1 (B) 0 **[AIEEE-2007]**
(C) g_E/g_M (D) g_M/g_E

Q.16 **Statement-1 :** For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is 4 πGM. and **[AIEEE-2008]**

Statement-2 : If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(B) Statment-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true

Q.17 A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be **[AIEEE-2008]**

(A) 11 km s^{-1} (B) 110 km s^{-1}
(C) 0.11 km s^{-1} (D) 1.1 km s^{-1}

Q.18 The height at which the acceleration due to gravity becomes g/9(where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is - **[AIEEE-2009]**

(A) $R/\sqrt{2}$ (B) R/2
(C) $\sqrt{2}\,R$ (D) 2R

Q.19 Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is **[AIEEE-2011]**

(A) zero (B) $-\frac{4Gm}{r}$ (C) $-\frac{6Gm}{r}$ (D) $-\frac{9Gm}{r}$

Q.20 The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10m/s^2 and 6400 km respectively. The required energy for this work will be – **[AIEEE-2012]**

(A) 6.4×10^{11} Joules (B) 6.4×10^{8} Joules
(C) 6.4×10^{9} Joules (D) 6.4×10^{10} Joules

Q.21 What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? **[JEE MAIN 2013]**

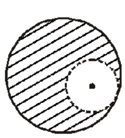
(A) $\frac{5GmM}{6R}$ (B) $\frac{2GmM}{3R}$ (C) $\frac{GmM}{2R}$ (D) $\frac{GmM}{3R}$

Q.22 Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is – **[JEE MAIN 2014]**

(A) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (B) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

(C) $\sqrt{\frac{GM}{R}}$ (D) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

Q.23 From a solid sphere of mass M and radius R, a spherical portion of radius (R/2) is removed, as shown in the figure.



Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is
(G = gravitational constant) **[JEE MAIN 2015]**

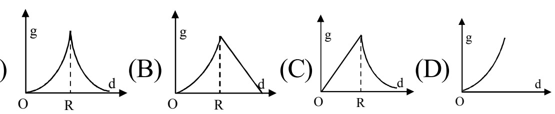
(A) $\frac{-GM}{R}$ (B) $\frac{-2GM}{3R}$ (C) $\frac{-2GM}{R}$ (D) $\frac{-GM}{2R}$

Q.24 A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius or earth R; h <<R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to: (Neglect the effect of atmosphere) **[JEE MAIN 2016]**

(A) $\sqrt{gR}$ (B) $\sqrt{gR/2}$

(C) $\sqrt{gR}\,(\sqrt{2}-1)$ (D) $\sqrt{2gR}$

Q.25 The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) : **[JEE MAIN 2017]**



Q.26 A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then: **[JEE MAIN 2018]**

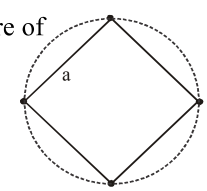
(A) $T \propto R^{(n+1)/2}$ (B) $T \propto R^{n/2}$

(C) $T \propto R^{3/2}$ for any n (D) $T \propto R^{\frac{n}{2}+1}$

Q.27 If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is : **[JEE MAIN 2019 (JAN)]**

(A) 4L / m (B) L / m
(C) L / 2m (D) 2L / m

Q.28 Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square? **[JEE MAIN 2019 (APRIL)]**



(A) $1.21\sqrt{\frac{GM}{a}}$ (B) $1.41\sqrt{\frac{GM}{a}}$

(C) $1.16\sqrt{\frac{GM}{a}}$ (D) $1.35\sqrt{\frac{GM}{a}}$

Q.29 A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon ? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon : **[JEE MAIN 2019 (APRIL)]**

(A) E / 4 (B) E / 16
(C) E / 32 (D) E / 64

Q.30 A satellite of mass M is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass M/10 so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M_e is the mass of the earth): **[JEE MAIN 2020 (JAN)]**

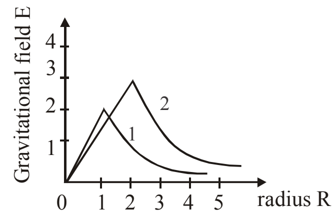
(A) $5M\left(u^2-\frac{119\,GM_e}{200R}\right)$ (B) $5M\left(u^2-\frac{113\,GM_e}{200R}\right)$

(C) $\frac{M}{20}\left(u^2-\frac{119\,GM_e}{100R}\right)$ (D) $\frac{M}{20}\left(u^2-\frac{113\,GM_e}{200R}\right)$

Q.31 A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to – (Take g = 10 ms^{-2} at the north pole and the radius of the earth = 6400 km): **[JEE MAIN 2020 (JAN)]**

(A) 194.32 N (B) 194.66 N
(C) 195.32 N (D) 195.66 N

Q.32 Consider two solid spheres of radii R_1 = 1m, R_2 = 2m and masses M_1 and M_2, respectively. The gravitational field due to sphere 1 and 2 are shown. The value of M_1 / M_2 is : **[JEE MAIN 2020 (JAN)]**



(A) 1/2 (B) 2/3
(C) 1/3 (D) 1/6

Q.33 An asteroid is moving directly towards the centre of the earth. When at a distance of 10R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s) ? Give your answer to the nearest integer in kilometer/s _____. **[JEE MAIN 2020 (JAN)]**

EXERCISE - 5 (PREVIOUS YEARS AIPMT/NEET EXAM QUESTIONS)

Choose one correct response for each question.

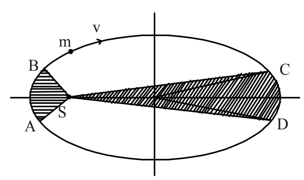
Q.1 Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then – **[AIPMT 2005]**
(A) g' = g/9 (B) g' = 27g
(C) g' = 9g (D) g' = 3g

Q.2 For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is **[AIPMT 2005]**
(A) 1/2 (B) $1/\sqrt{2}$
(C) 2 (D) $\sqrt{2}$

Q.3 The earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is fv, where v is its escape velocity from the surface of the earth. The value of f is – **[AIPMT 2006]**
(A) $1/\sqrt{2}$ (B) 1/3 (C) 1/2 (D) $\sqrt{2}$

Q.4 Two satellites of earth S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true **[AIPMT 2007]**
(A) The potential energies of earth satellites in the two cases are equal.
(B) S_1 and S_2 are moving with the same speed
(C) The kinetic energies of the two satellites are equal.
(D) The time period of S_1 is four times that of S_2.

Q.5 The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. if t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then **[AIPMT 2009]**



(A) $t_1 = 4t_2$ (B) $t_1 = 2t_2$
(C) $t_1 = t_2$ (D) $t_1 > t_2$

Q.6 The radii of circular orbits of two satellites A and B of the earth, are 4R and R, respectively. If the speed of satellite A is 3V, then the speed of satellite B will be – **[AIPMT (PRE) 2010]**
(A) 3V/4 (B) 6V
(C) 12V (D) 3V/2

Q.7 A particle of mass M is situated at the center of a spherical shell of same mass and radius a. The gravitational potential at a point situated at a/2 distance from the centre, will be – **[AIPMT (PRE) 2010]**
(A) $-\frac{3GM}{a}$ (B) $-\frac{2GM}{a}$ (C) $-\frac{GM}{a}$ (D) $-\frac{4GM}{a}$

Q.8 A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively. Then the ratio v_1/v_2 is **[AIPMT (PRE) 2011]**
(A) r_1/r_2 (B) $(r_1/r_2)^2$
(C) r_2/r_1 (D) $(r_2/r_1)^2$

Q.9 The height at which the weight of a body becomes 1/16th, its weight on the surface of earth (radius R), is : **[AIPMT (PRE) 2012]**
(A) 5R (B) 15R
(C) 3R (D) 4R

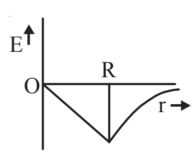
Q.10 A spherical planet has a mass M_P and diameter D_P. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity, equal **[AIPMT (PRE) 2012]**
(A) $4GM_P/D_P^2$ (B) GM_Pm/D_P^2
(C) GM_P/D_P^2 (D) $4GM_Pm/D_P^2$

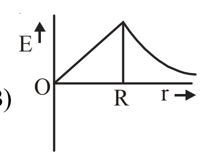
Q.11 A geostationary satellite is orbiting the earth at a height of 5R above that surface of the earth, R being the radius of the earth. The time period of another satellite in hours at a height of 2R from the surface of the earth is : **[AIPMT (PRE) 2012]**
(A) 5 (B) 10
(C) $6\sqrt{2}$ (D) $6/\sqrt{2}$

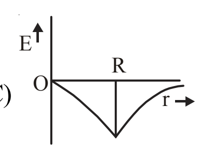
Q.12 Infinite number of bodies, each of mass 2 kg are situated on x-axis at distance 1m, 2m, 4m, 8m, ..., respectively, from the origin. The resulting gravitational potential due to this system at the origin will be – **[NEET 2013]**
(A) – 4G (B) –G
(C) (–8/3) G (D) (–4/3) G

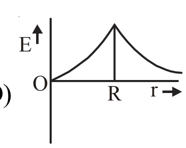
Q.13 A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (Mass = 5.98×10^{24} kg) have to be compressed to be a black hole? **[AIPMT 2014]**
(A) 10^{-9} m (B) 10^{-6} m
(C) 10^{-2} m (D) 100 m

Q.14 Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by – **[AIPMT 2014]**

(A)


(B)


(C)


(D)


Q.15 Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet i.e. $T^2 = Kr^3$, here K is constant. If the masses of sun and planet are M and m respectively then as per Newton's law of gravitation force of attraction between them is $F = \frac{GMm}{r^2}$, here G is gravitational constant, The relation between G and K is described as: **[AIPMT 2015]**
(A) $GMK = 4\pi^2$ (B) $K = G$
(C) $K = 1/G$ (D) $GK = 4\pi^2$

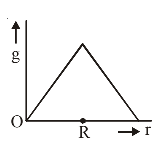
Q.16 A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then, **[RE-AIPMT 2015]**
(A) the acceleration of S is always directed towards the centre of the earth.
(B) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
(C) the total mechanical energy of S varies periodically with time.
(D) the linear momentum of S remains constant in magnitude.

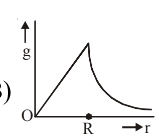
Q.17 A remote - sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and $g = 9.8\ ms^{-2}$, then the orbital speed of the satellite is **[RE-AIPMT 2015]**
(A) 6.67 km/s (B) 7.76 km/s
(C) 8.56 km/s (D) 9.13 km/s

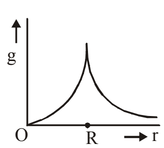
Q.18 At what height from the surface of earth the gravitation potential and the value of g are $-5.4 \times 10^7\ J\ kg^{-2}$ and 6.0 ms^{-2} respectively? Take the radius of earth as 6400 km **[NEET 2016 PHASE 1]**
(A) 2600 km (B) 1600 km
(C) 1400 km (D) 2000 km

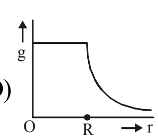
Q.19 The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is **[NEET 2016 PHASE 1]**
(A) 1 : 2 (B) $1 : 2\sqrt{2}$ (C) 1 : 4 (D) $1 : \sqrt{2}$

Q.20 Starting from the centre of the earth having radius R, the variation of g (acceleration due to gravity) is shown by **[NEET 2016 PHASE 2]**

(A)


(B)


(C)


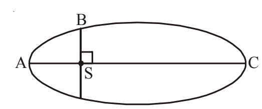
(D)


Q.21 A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface, is **[NEET 2016 PHASE 2]**
(A) $\frac{mg_0R^2}{2(R+h)}$ (B) $-\frac{mg_0R^2}{2(R+h)}$
(C) $\frac{2mg_0R^2}{R+h}$ (D) $-\frac{2mg_0R^2}{R+h}$

Q.22 The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then – **[NEET 2017]**
(A) d = 1 km (B) d = 2/3 km
(C) d = 2 km (D) d = 1/2 km

Q.23 Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will – **[NEET 2017]**
(A) Move towards each other.
(B) Move away from each other.
(C) Will become stationary
(D) Keep floating at the same distance between them.

Q.24 The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C, respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then **[NEET 2018]**



(A) $K_B < K_A < K_C$ (B) $K_A > K_B > K_C$
(C) $K_A < K_B < K_C$ (D) $K_B > K_A > K_C$

Q.25 If the mass of the Sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which of the following is not correct? **[NEET 2018]**
(A) Time period of a simple pendulum on the Earth would decrease.
(B) Walking on the ground would become more difficult.
(C) Raindrops will fall faster.
(D) 'g' on the Earth will not change.

Q.26 A body weighs 200 N on the surface of the earth. How much will it weigh half way down to the centre of the earth ? **[NEET 2019]**
(A) 150 N (B) 200 N
(C) 250 N (D) 100 N

Q.27 The work done to raise a mass m from the surface of the earth to a height h, which is equal to the radius of the earth, is: **[NEET 2019]**
(A) mgR (B) 2mgR
(C) $\frac{1}{2}$ mgR (D) $\frac{3}{2}$ mgR

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	C	A	C	B	B	A	D	A	C	B	C	C	D	C	B	C	A	C	B	C	A	D	B	B
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	B	D	C	B	C	A	D	B	A	A	B	A	C	A	C	B	D	A	C	B	B	B	A	A	D
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	
A	A	B	B	D	B	B	C	C	C	C	C	B	B	A	B	B	A	B	A	B	C	D	D	A	

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
A	C	A	B	B	A	C	D	C	A	D	A	C	D	B	B	A	B	C	A	D	A

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10
A	5	3	2	2	2	4	2	3	3	3

EXERCISE - 4

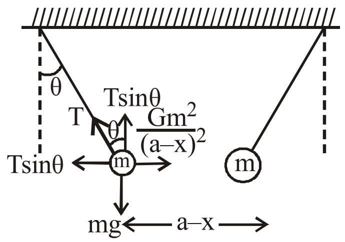
Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	C	A	C	B	B	B	D	A	B	A	A	C	D	A	A	B	D	D	D
Q	21	22	23	24	25	26	27	28	29	30	31	32	33							
A	A	B	A	C	C	A	C	C	B	A	C	D	16							

EXERCISE - 5

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
A	D	A	A	B	B	B	A	C	C	A	C	A	C	A	A	A	B	A	B	B	B	C	A	B	D	D	C

GRAVITATION
TRY IT YOURSELF-1

(1) (B).



$$T\sin\theta = \frac{Gm^2}{(a-x)^2}\ ;\ T\cos\theta = mg$$

Dividing we get, $\tan\theta = \dfrac{mG}{(a-x)^2 g}$

(2) (D). From modified Gauss's theorem for gravitation

$$\int \vec{E}.d\vec{s} = 4\pi G\left(\int_{r=0}^{r=r} \rho dv\right)$$

$$\Rightarrow\ E\,4\pi r^2 = 4\pi G\int_{r=0}^{r=r}\frac{k}{r}4\pi r^2 dr$$

Get E = constant

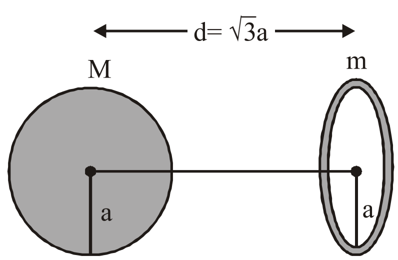
(3) (A). As E is constant, so the Potential ($V = -\int E\,dr$) will be proportional to r.

(4) Conservation of energy says

$$-\frac{GMm}{R}+\frac{1}{2}m(v_0)^2 = -\frac{GMm}{R_E}+\frac{1}{2}mv^2$$

where M is the mass of the earth and R_E is its radius, the initial separation is $R = 80,000\times10^3$m, and initial velocity is $v_0 = 2000$m/s from which we get $v = 1.09\times10^4$m/s.

(5) The gravitational field at any point on the ring due to the sphere is equal to the field due to single particle of mass M placed at the centre of the sphere.



Thus, the force on the ring due to the sphere is also equal to the force on it by particle of mass M placed at this point. By Newton's third law it is equal to the force on the particle by the ring. Now the gravitational field due to the ring at a distance $d = \sqrt{3}\,a$ on its axis is given as

$$g = \frac{Gmd}{(a^2+d^2)^{3/2}} = \frac{\sqrt{3}Gm}{8a^2}$$

The force on sphere of mass M placed here is

$$F = Mg = \frac{\sqrt{3}GMm}{8a^2}$$

(6) (A). $g = \dfrac{GM}{R^2} = \dfrac{G(\rho)\left(\frac{4}{3}\pi R^3\right)}{R^2} = \dfrac{4}{3}G\pi\rho R$

$g \propto R$

As Radius of the moon is one forth so g on moon is also one fourth.

Time period of a second pendulum on the earth.

$$T = 2\pi\sqrt{\frac{\ell}{g_{earth}}}\ ;\ \text{at moon}\ T = 2\pi\sqrt{\frac{\ell}{g_{moon}}}$$

Dividing, $\ell' = \ell\dfrac{g_{moon}}{g_{earth}} = \ell\left(\dfrac{1}{4}\right)$

$$\ell' = \frac{99.2}{4} = 24.8\text{cm}$$

(7) We know that $\dfrac{g_h}{g} = \left(\dfrac{R}{R+h}\right)^2$; But $g_h = \dfrac{g}{2}$

$$\therefore\ \frac{1}{2} = \left(\frac{R}{R+h}\right)^2 \quad \text{or} \quad \frac{R}{R+h} = \frac{1}{\sqrt{2}}$$

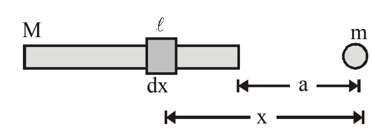
$$\text{or}\quad \frac{R+h}{R} = \sqrt{2} \quad \text{or} \quad \frac{h}{R} = \sqrt{2}-1 = 0.414$$

$h = 0.414\times R = 0.414\times 6400\,\text{km}$ or $h = 2649.6$ km

At a height of 2649.6 km from the Earth's surface, the acceleration due to gravity will be half its value on the surface.

(8) Consider small element dx of the rod whose mass

$$dm = \frac{M}{\ell}dx$$



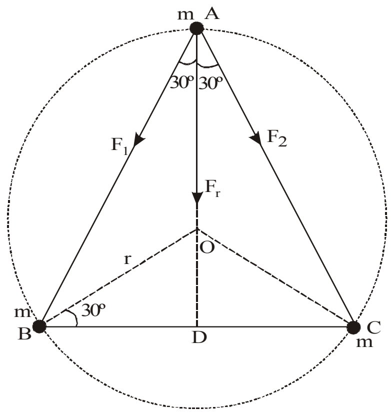
$$\Rightarrow\ U = \int dU = -\frac{GmM}{\ell}\int_a^{a+\ell}\frac{dx}{x} = -\frac{GmM}{\ell}[\ln x]_a^{a+\ell}$$

$$\Rightarrow U = -\frac{GmM}{\ell}\log_e\left(\frac{a+\ell}{a}\right)$$

(9) Fig. shows three particles located at vertices A, B and C of an equilateral triangle of sides AB = BC = CA = a. These particles move in a circle with O as the centre and radius r = OA = OB = OC

$$r = \frac{BD}{\cos 30°} = \frac{a/2}{\sqrt{3}/2} = \frac{a}{\sqrt{3}}$$

Let us find the net gravitational force acting on one particle, say at A, due to particles at B and C.



Particle at A is attracted

towards B with a force, $F_1 = \frac{Gmm}{a^2}$ and towards C with a

force, $F_2 = \frac{Gmm}{a^2}$

Notice that $F_1 = F_2 = F$ (say) $= \frac{Gm^2}{a^2}$.

The angle between these equal forces is $\theta = \angle BAC = 60°$. The resultant force on the particle at A is

$F_r = (F^2 + F^2 + 2F^2 \cos 60°)^{1/2}$

$\Rightarrow F_r = \sqrt{3}F$; $F_r = \sqrt{3}\frac{Gm^2}{a^2}$ directed along AO.

Thus the net force on particle at A is radial. Similarly, the net force on particle at B and at C is F_r, each directed towards centre O. This force provides the necessary centripetal force. If v is the required initial velocity of each particle,

then $\frac{mv^2}{r} = \sqrt{3}\frac{Gm^2}{a^2}$

or $v^2 = \sqrt{3}\frac{Gm}{\sqrt{3}a} = \frac{Gm}{a}$ $\left(r = \frac{a}{\sqrt{3}}\right)$ or $v = \sqrt{\frac{Gm}{a}}$

(10) Time period $T = \frac{2\pi r}{v} = \frac{2\pi \times \frac{a}{\sqrt{3}}}{\sqrt{\frac{Gm}{a}}} = 2\pi\left(\frac{a^3}{3Gm}\right)^{1/2}$

TRY IT YOURSELF-2

(1) **(A).** $g_e = g_p - R\omega^2 \Rightarrow \frac{g}{2} = g - R\omega^2 \Rightarrow R\omega^2 = \frac{g}{2}$

$\Rightarrow R^2\omega^2 = \frac{gR}{2} \Rightarrow V^2 = \frac{gR}{2}$ .... (1)

$V_e = \sqrt{2gR}$

From eq. (1) and (2), $V_e = \sqrt{2 \times 2V^2} \Rightarrow V_e = 2V$

(2) **(A).** $L = mvr \Rightarrow L = m\sqrt{\frac{GM}{r}}\,r$

$\Rightarrow L = m\sqrt{GMr}$ .... (1)

$L = 2m\frac{dA}{dt}$ .... (2)

From eq. (1) and (2),

$\frac{dA}{dt} \propto \sqrt{r} \Rightarrow \frac{(dA/dt)_1}{(dA/dt)_2} = \sqrt{\frac{4}{1}} = \frac{2}{1}$

(3) **(D).**

(4) **(D).** As there are no external torques acting on the system, angular momentum is conserved.

$I_{far}\,\omega_{far} = I_{near}\,\omega_{near}$,

or $(mR_{far}^2)\,\omega_{far} = (mR_{near}^2)\,\omega_{near}$.

Setting $R_{near} = d/3$ and $R_{far} = d$,

we get $\omega_{near} = 9\omega_{far}$.

An alternative is to look at the body's translational motion. We can write the conservation of angular momentum as $R_{far} \times p_{far} = R_{near} \times p_{near}$.

At the near and far points, the angle between R and p is 90°, so the cross products become (p)(R)sin 90° terms and we can write $(mv_{far})\,d = (mv_{near})\,(d/3)$, or $v_{near} = 3v_{far}$.

Noting that $v_{far} = d\omega_{far}$ and $v_{near} = (d/3)\,\omega_{near}$,

we can rewrite $v_{near} = 3v_{far}$ as $(d/3)\,\omega_{near} = 3(d)\,\omega_{far}$,

or $\omega_{near} = 9\omega_{far}$.

(5) **(AC).**

(A) Ultimately conservation of energy can lead you to this conclusion, though there are several steps analyzing kinetic and gravitational potential energy to get there.

(B) There is no such law as Conservation of Angular Velocity. This is silly.

(C) An analysis using conservation of Angular Momentum leads directly to $m_A v_A R_A \sin\theta_A = m_B v_B R_B \sin\theta_B$, and given that R, θ, and m are the same at positions A and B, you have a clear and direct path to $v_A = v_B$.

(D) Conservation of electrical charge, though true, does not help you with this problem.

(6) Start with $T = 2\pi\sqrt{R^3/GM}$ and plug in the numbers to get $T = 6 \times 10^{15}$s = 200 Million years.

(7) The law of conservation of energy says that at any two points on the orbit labelled 1 and 2,

$$-\frac{GMm}{r_1}+\frac{1}{2}mv_1^2=-\frac{GMm}{r_2}+\frac{1}{2}mv_2^2$$

Cancelling m and plugging in

$r_1 = R_E + 890 km = 6.37 \times 10^6 m + 8.9 \times 10^5 m$

and $v_1 = 7.23 \times 10^3 m/s$

and similarly for r_2, we find $v_2 = 7.95$ km/s.

(8) **30 R_1.**

$$T_1 = T_{Earth} = 1 \text{ year},\ T_2 = T_{Neptune} = 165 \text{ year}$$

Let R_1 and R_2 be the radii of the circular orbits of Earth and Neptune respectively.

$$\frac{T_1^2}{T_2^2}=\frac{R_1^3}{R_2^3}\quad \therefore\ R_2^3=\frac{R_1^3T_2^2}{T_1^2}\quad \text{or}\quad R_2^3=\frac{R_1^3\times 165^2}{1^2}$$

$$\therefore\ R_2^3 = 165^2 R_1^3 \quad \text{or} \quad R_2 \approx 30 R_1.$$

(9) When the Earth's motion is suddenly stopped, it would fall into the Sun and (suppose) it comes back. If the effect of temperature of Sun is ignored, we can say that the Earth would continue to move along a strongly extended flat ellipse whose extreme points are located at the Earth's orbit and at the centre of the Sun.

The semi major axis of such ellipse is R/2.

Now $\frac{T'^2}{T^2}=\left[\frac{R}{2}\right]^3\left[\frac{1}{R^3}\right]$

Where T is the time period of normal orbit of earth.

or $T'^2=\frac{T^2}{8}$ or $T'=\frac{T}{2\sqrt{2}}$

Now, time required to fall into the Sun,

$$t=\frac{T'}{2}=\frac{T}{4\sqrt{2}}=\frac{365}{4\sqrt{2}}\approx 65 \text{ day}$$

So, the Earth would take slightly more than 2 months to fall into the Sun.

(10) If R is the radius of the lunar orbit. T is the time period of the Moon around the Earth.

Mass of the Earth $M_e=\frac{4\pi^2R^3}{GT^2}$

Here, T = 27.3 days = 27.3 × 24 × 60 × 60 second; R = 3.9×10^5 km = 3.9×10^8 m, G = 6.67×10^{-11} N m^2 kg^{-2}

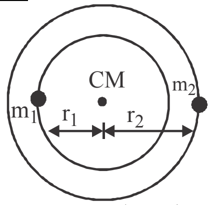
Substituting the values, we get

$$M_e=\frac{4\times(3.142)^2\times(3.9\times10^8)^3}{6.67\times10^{-11}\times(27.3\times24\times60\times60)^2}\ \text{kg}=6.3\times10^{24}\ \text{kg}$$

CHAPTER-9 : GRAVITATION

EXERCISE-1

(1) **(A).** The situation is as shown in the figure. According to Newton's law of gravitation, gravitational force between two bodies of masses m_1 and m_2 is



$F = \frac{Gm_1m_2}{r^2}$, where r is the distance between the two masses. Hence, $r = r_1 + r_2$ $\therefore$ $F = \frac{Gm_1m_2}{(r_1+r_2)^2}$

(2) **(C).** $F = G\frac{m_1m_2}{r^2} = 6.675 \times \frac{1\times 1}{1^2} \times 10^{-11} = 6.675 \times 10^{-11}\,N$

(3) **(A).** $F_G = \frac{Gm_1m_2}{r^2}$

Doubling the value of either one of the masses, while not changing anything else, will double the value of the force.

(4) **(C).** $F = \frac{G\times m\times m}{(2R)^2} = \frac{G\times\left(\frac{4}{3}\pi R^3\rho\right)^2}{4R^2} = \frac{4}{9}\pi^2\rho^2R^4$

$\therefore\ F \propto R^4$

(5) **(B).** The Moon remains in its orbit around the Earth because it is in balance with the gravitational forces from the Sun and other planets.

(6) **(B).** It will remain the same as the gravitational force is independent of the medium separating the masses.

(7) **(A).** $F \propto xm \times (1-x)m = xm^2(1-x)$

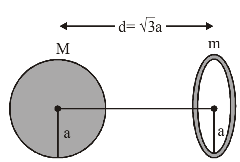
For maximum force $\frac{dF}{dx} = 0$

$\Rightarrow \frac{dF}{dx} = m^2 - 2xm^2 = 0 \Rightarrow x = 1/2$

(8) **(D).** We know that, for $r \le R$, $I = 0$ and $r \ge R$; $I \propto 1/r^2$. So the curve (D) gives the correct relationship.

(9) **(A).** The gravitational field at any point on the ring due to the sphere is equal to the field due to single particle of mass M placed at the centre of the sphere.

$g = \frac{Gmd}{(a^2+d^2)^{3/2}} = \frac{\sqrt{3}Gm}{8a^2}$



The force on sphere of mass M placed here is

$F = Mg = \frac{\sqrt{3}GMm}{8a^2}$

(10) **(C).** Gravitational intensity is zero at all points inside a hollow spherical shell. Thus, if we remove the upper hemispherical shell the net gravitational force acting on the particle at the centre and at any other point will be acting downward which will be direction of gravitational intensity.

(11) **(B).** The gravitational intensity at P will be along 'e'.

(12) **(C).** Gravitational potential on the surface of the shell is V = Gravitational potential due to particle (V_1) + Gravitational potential due to shell itself (V_2)

$= -\frac{Gm}{R} + \left(-\frac{G3m}{R}\right) = -\frac{4Gm}{R}$

(13) **(C).** Inside the shell gravitational field due to the shell will be zero but there will be some gravitational field due to the block.

(14) **(D).** $V_g = V_{g_1} + V_{g_2} = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2}$

$= -6.67\times 10^{-11}\left[\frac{10^2}{0.5}+\frac{10^3}{0.5}\right] = -1.47\times 10^{-7}$ Joule/kg

(15) **(C).** The P.E. of the object on the surface of earth is

$U_1 = -\frac{GMm}{R}$

The P.E. of object at a height R, $U_2 = -\frac{GMm}{(R+R)}$

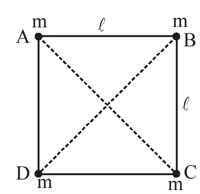
The gain in P E is $U_2 - U_1 = \frac{GMm}{2R} = \frac{1}{2}mgR$

$\left[\because g = \frac{GM}{R^2} \text{ on surface of earth}\right]$

(16) **(B).** From figure,

$AB = BC = CD = AD = \ell$

$\therefore\ AC = BD = \sqrt{\ell^2+\ell^2} = \ell\sqrt{2}$



Total potential energy of the system of four particles each of mass m placed at the vertices A, B, C and D of a square is

$U = \left(-\frac{G\times m\times m}{AB}\right) + \left(-\frac{G\times m\times m}{AC}\right)$

$+\left(-\frac{G\times m\times m}{AD}\right) + \left(-\frac{G\times m\times m}{BC}\right)$

$+\left(-\frac{G\times m\times m}{BD}\right) + \left(-\frac{G\times m\times m}{CD}\right)$

$= \left(-\frac{Gm^2}{\ell}\right) + \left(-\frac{Gm^2}{\ell\sqrt{2}}\right) + \left(-\frac{Gm^2}{\ell}\right)$

$+\left(-\frac{Gm^2}{\ell}\right) + \left(-\frac{Gm^2}{\ell\sqrt{2}}\right) + \left(-\frac{Gm^2}{\ell}\right)$

$$= -\frac{4Gm^2}{\ell} - \frac{2Gm^2}{\ell\sqrt{2}} = -\frac{2Gm^2}{\ell}\left(2+\frac{1}{\sqrt{2}}\right)$$

(17) **(C).** Let M_e be the mass of the earth.

The work required $W = GM_e m\left[\frac{1}{R_e} - \frac{1}{R_e + h}\right]$

$$= \frac{GM_e mh}{R_e(R_e+h)} = \frac{gR_e^2\,mh}{R_e(R_e+h)} = \frac{mgh}{\left(1+\frac{h}{R_e}\right)}$$

$[\therefore\ GM_e = gR_e^2]$

(18) **(A).** $g = \frac{4}{3}\pi\rho GR$. If ρ = constant then $\frac{g_1}{g_2} = \frac{R_1}{R_2}$

(19) **(C).** $g = \frac{4}{3}G\pi R\rho \Rightarrow \frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{2}\times\frac{4}{1} = \frac{2}{1}$

(20) **(B).** $g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{g}{4} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{2} = \frac{R}{R+h}$

$\Rightarrow\ R+h = 2R \therefore h = R$

(21) **(C).** Acceleration due to gravity at poles is independent of the angular speed of earth.

(22) **(A).** For condition of weightlessness of equator

$$\omega = \sqrt{\frac{g}{R}} = \frac{1}{800} = 1.25\times10^{-3}\,\frac{rad}{s}$$

(23) **(D).** $g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{\sqrt{2}} = \frac{R}{R+h}$

$\Rightarrow R+h = \sqrt{2}R \Rightarrow h = (\sqrt{2}-1)R = 0.414R$

(24) **(B).** $g \propto \rho R$

(25) **(B).** As $g = \frac{GM}{R^2}$ therefore 1% decrease in mass will decreases the value of g by 1%.
But 1% decrease in radius will increase the value of g by 2%. As a whole value of g increase by 1%.

(26) **(B).** The value of g at the height h from the surface of earth, $g' = g\left(1-\frac{2h}{R}\right)$

The value of g at depth x below the surface of earth,

$g' = g\left(1-\frac{x}{R}\right)$

These two are given equal, hence

$\left(1-\frac{2h}{R}\right) = \left(1-\frac{x}{R}\right)$

On solving, we get x = 2h

(27) **(D).** Because acceleration due to gravity decreases.

(28) **(C).** $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{4}{9}g \ \therefore W' = \frac{4}{9}W$

(29) **(B).** $g' = g\left(1-\frac{d}{R}\right) \Rightarrow \frac{g}{4} = g\left(1-\frac{d}{R}\right) \Rightarrow d = \frac{3}{4}R$

(30) **(C).** $g \propto \frac{1}{R^2}$; $R\downarrow\ g\uparrow$

(31) **(A).** $g = \frac{GM}{R^2} = \frac{G(\rho)\left(\frac{4}{3}\pi R^3\right)}{R^2} = \frac{4}{3}G\pi\rho R$; $g \propto R$

As radius of the moon is one forth so g on moon is also one fourth.
Time period of a second pendulum on the earth.

$$T = 2\pi\sqrt{\frac{\ell}{g_{earth}}} \ ; \ \text{at moon } T = 2\pi\sqrt{\frac{\ell'}{g_{moon}}}$$

Dividing, $\ell' = \ell\frac{g_{moon}}{g_{earth}} = \ell\left(\frac{1}{4}\right)$; $\ell' = \frac{99.2}{4} = 24.8cm$

(32) **(D).** $v_e = \sqrt{\frac{2GM}{R}}$ i.e. escape velocity depends upon the mass and radius of the planet.

(33) **(B).** $v_e = \sqrt{\frac{2GM}{R}} = R\sqrt{\frac{8}{3}\pi G\rho}$

If mean density is constant then $v_e \propto R$

$\frac{v_e}{v_p} = \frac{R_e}{R_p} = \frac{1}{2} \Rightarrow v_e = \frac{v_p}{2}$

(34) **(A).** Escape velocity does not depend on the mass of the projectile. $v_e = \sqrt{\frac{2GM}{R}}$

(35) **(A).** $v = \sqrt{2gR}\ \therefore \frac{v_1}{v_2} = \sqrt{\frac{g_1}{g_2}\times\frac{R_1}{R_2}} = \sqrt{g\times K} = (Kg)^{1/2}$

(36) **(B).** If missile launched with escape velocity then it will escape from the gravitational field and at infinity its total energy becomes zero.
But if the velocity of projection is less than escape velocity then sum of energies will be negative. This shows that attractive force is working on the satellite.

(37) **(A).** $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e}\times\frac{R_e}{R_p}} = \sqrt{2\times\frac{1}{3}} = \sqrt{\frac{2}{3}}\ \therefore\ v_p = \sqrt{\frac{2}{3}}v_e$

(38) **(C).** $v = \sqrt{2gR} \Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e}\times\frac{R_p}{R_e}} = \sqrt{1\times4} = 2$

$\Rightarrow\ v_p = 2\times v_e = 2\times11.2 = 22.4$ km/s

(39) **(A).** $g_e = g_p - R\omega^2 \Rightarrow \frac{g}{2} = g - R\omega^2$

$\Rightarrow\ R\omega^2 = \frac{g}{2} \Rightarrow\ R^2\omega^2 = \frac{gR}{2} \Rightarrow V^2 = \frac{gR}{2}$ ....(1)

$V_e = \sqrt{2gR}$

From eq. (1) and (2), $V_e = \sqrt{2 \times 2V^2} \Rightarrow V_e = 2V$

(40) **(C).** Angular momentum remains constant

$$mv_1d_1 = mv_2d_2 \Rightarrow v_2 = \frac{v_1d_1}{d_2}$$

(41) **(B).** Orbital radius of Jupiter > Orbital radius of Earth

$v = \sqrt{\frac{GM}{r}}$. As $r_J > r_e$ therefore $v_J < v_e$

(42) **(D).** $T^2 \propto r^3 \Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

(43) **(A).** During path DAB planet is nearer to sun as comparison with path BCD. So time taken in travelling DAB is less than that for BCD because velocity of planet will be more in region DAB.

(44) **(C).** Because distance of point C is maximum from the sun.

(45) **(B).** $T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 1 \times (2)^{3/2} = 2.8$ year

(46) **(B).** $\frac{T^2}{r^3} =$ constant $\Rightarrow T^2r^{-3} =$ constant

(47) **(B).** $\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = (2)^{3/2} = 2\sqrt{2} \Rightarrow T_2 = 2\sqrt{2}$ years.

(48) **(A).** $m\omega^2R = \frac{GMm}{R^2} \Rightarrow \left(\frac{2\pi}{T}\right)^2 R = \frac{GM}{R^2} \Rightarrow M = \frac{4\pi^2R^3}{GT^2}$

(49) **(A).** $T^2 = \frac{4\pi^2r^3}{GM}$

(50) **(D).** As there are no external torques acting on the system, angular momentum is conserved.

$I_{far}\ \omega_{far} = I_{near}\ \omega_{near}$,

or $(mR_{far}^2)\ \omega_{far} = (mR_{near}^2)\ \omega_{near}$.

Setting $R_{near} = d/3$ and $R_{far} = d$, we get $\omega_{near} = 9\omega_{far}$.

(51) **(A).** $T_1 = T_{Earth} = 1$ year, $T_2 = T_{Neptune} = 165$ year

Let R_1 and R_2 be the radii of the circular orbits of Earth and Neptune respectively.

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \quad \therefore R_2^3 = \frac{R_1^3T_2^2}{T_1^2} = \frac{R_1^3 \times 165^2}{1^2}$$

$\therefore\ R_2^3 = 165^2R_1^3$ or $R_2 \approx 30R_1$.

(52) **(B).** Centripetal acceleration works on it.

(53) **(B).** $v = \sqrt{\frac{GM}{r}}$ if $r_1 > r_2$ then $v_1 < v_2$

Orbital speed of satellite does not depends upon the mass of the satellite

(54) **(D).** Distances of the satellite from the centre are 7R and 3.5R respectively.

$$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2} \Rightarrow T_2 = 24\left(\frac{3.5R}{7R}\right)^{3/2} = 6\sqrt{2}\text{ hr}$$

(55) **(B).** Time period is independent of mass.

(56) **(B).** $v_e = \sqrt{2}v_0 = 1.414\ v_0$

Fractional increase in orbital velocity

$$\left(\frac{\Delta v}{v}\right) = \frac{v_e - v_0}{v_0} = 0.414$$

$\therefore$ Percentage increase = 41.4%

(57) **(C).** $v \propto \frac{1}{\sqrt{r}}$, If r = R then $v = V_0$

If $r = R + h = R + 3R = 4R$ then $v = \frac{V_0}{2} = 0.5V_0$

(58) **(C).** $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2} \Rightarrow T_2 = 8T_1$

(59) **(C).** A polar satellite is a low altitude satellite. Hence, option (C) is an incorrect statement. While all the other statement are correct.

(60) **(C).** A communication satellite is in a circular orbit around Earth. If the speed of the satellite is constant, the force acting on the satellite points toward the center of Earth at all times.

(61) **(C).** $V_e = 11.2 = \sqrt{\frac{2GM}{R}}$, $V_m = \sqrt{\frac{2GM_m}{R_m}}$

$$V_m = \sqrt{\frac{G \times M \times 4}{81 \times R}} = \frac{11.2 \times 2}{9} = 2.5\text{ km/sec}.$$

(62) **(B).** Escape velocity, $v_e = \sqrt{2gR}$

At a height h above the Earth's surface,

$$v'_e = \sqrt{2g_h(R+h)},\quad g_h = \frac{gR^2}{(R+h)^2}$$

$$v'_e = \sqrt{\frac{2 \times gR^2}{(R+h)^2}(R+h)} = \sqrt{\frac{2gR^2}{R+h}}$$

$g = 9.8\ m\,s^{-2}$, $R = 6.4 \times 10^6$ m, h = 1,000 km = 10^6 m

$$v'_e = \sqrt{\frac{2 \times 9.8 \times (6.4 \times 10^6)^2}{(6.4+1)10^6}}\ ms^{-1}$$

$= 10.42 \times 10^3\ m\,s^{-1} = 10.42\ km\,s^{-1}$.

$\therefore$ The escape velocity at a height 1,000 km from Earth's surface = $10.42\ km\,s^{-1}$.

(63) **(B).** The extension in the length of spring is

$$x_1 = \frac{mg}{k} = \frac{GMm}{R^2k} \qquad \therefore x \propto \frac{1}{R^2}$$

$$\therefore \frac{x_2}{x_1} = \frac{R^2}{(R+h)^2} \text{ or } x_2 = 1 \times \left(\frac{6400}{7200}\right)^2 = \frac{64}{81} = 0.79 \text{ cm}$$

(64) **(A).** Orbital speed $v_0 = \sqrt{\frac{Gm}{R}}$; $2 = \sqrt{\frac{Gm}{R}}$

Hence, escape speed $v_e = \sqrt{\frac{2Gm}{R}} = 2\sqrt{2}$ m/s

(65) **(B).** Acceleration will be due to gravitational pull only.

(66) **(B).** $v_e = \sqrt{2g_eR_e}$; $v_m = \sqrt{2g_mR_m}$

$$\frac{v_e}{v_m} = \sqrt{\frac{g_e}{g_m}\frac{R_e}{R_m}} = \sqrt{24}$$

(67) **(A).** For a body of mass m resting on the equator of a planet of radius R, which rotates at an angular velocity ω, the equation of motion has the form $m\omega^2R = mg' - N$.
Where N is the normal reaction of the planet surface, and $g' = 0.01g$ is the free- fall acceleration on the planet.
By hypothesis, the bodies on the equator are weightless, i.e. N = 0. Considering that $\omega = 2\pi/T$, where T is the period of rotation of the planet about its axis (equal to the solar day), we obtain $R = \frac{T^2}{4\pi^2}g'$
Substituting the values $T = 8.6 \times 10^4$ s and $g' \cong 0.1$ m/s^2, we get $R \cong 1.8 \times 10^7$ m = 18000km.

(68) **(B).** $g \propto \frac{1}{R^2}$. R decreasing g increase curve b represent variation.

(69) **(A).** $g\left(1-\frac{d}{R}\right) = g - \omega^2 R$; $d = \frac{\omega^2R^2}{g}$

(70) **(B).** Orbital velocity close to surface of earth is $\sqrt{gR}$.

So, $E = \frac{1}{2}m(\sqrt{gR})^2 \Rightarrow E = \frac{1}{2}mgR$
If the body is to escape, the velocity at surface of earth is $\sqrt{2gR}$. If E' is the kinetic energy corresponding to this velocity then $E' = \frac{1}{2}m(\sqrt{2gR})^2 \Rightarrow E' = 2E$

(71) **(C).** $T^2 \propto R^3$ or $(T_2/T_1) = (R_2/R_1)^{3/2}$

or $\frac{T_2}{T_1} = \left(\frac{6400}{36000}\right)^{3/2}$ or $T_2 = \left(\frac{6400}{36000}\right)^{3/2} \times 24 \approx 2$ hr.

(72) **(D).** The acceleration due to gravity at earth's surface is g and at a distance R from earth's surface it is g/4.

Hence, $\frac{T_2}{T_1} = 2$ $\left[\because T = 2\pi\sqrt{\ell/g}\right]$

(73) **(D).** Angular momentum of the planet about S is conserved. So, mvr = constant.
v is maximum when r is minimum. So, v is maximum at point P_4.

(74) **(A).** Since, $T^2 = kr^3$
Differentiating the above equation

$$\Rightarrow 2\frac{\Delta T}{T} = 3\frac{\Delta r}{r} \Rightarrow \frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta r}{r}$$

EXERCISE-2

(1) **(C).** $\omega = \frac{1}{\sqrt{\tan\theta}} \Rightarrow T = 2\pi\sqrt{\tan\theta} = 2\pi\sqrt{\frac{\ell}{g}}$

$$\therefore g = \frac{\ell}{\tan\theta} \quad \therefore \frac{g_{ground}}{g_h} = \frac{\tan\theta_h}{\tan\theta ground} = \frac{(R+h)^2}{R^2}$$

$$\frac{\tan 60°}{\tan 30°} = \left(\frac{R+h}{R}\right)^2 \Rightarrow h = 0.73\,R$$

(2) **(A).**

(A) $T^2 \propto r^3$ (B) $E = \frac{-GMm}{2r}$

(C) $v = \sqrt{\frac{GM}{r}}$ (D) $L = mvr = m\sqrt{GMr}$

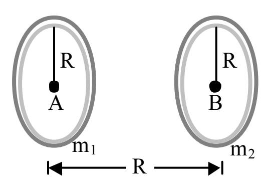
(3) **(B).** Mass per unit length $= A + Bx^2$. So the mass of length dx is $dM = dx\,(A+Bx^2)$

$$F = \int_a^{a+L} Gm\left(\frac{1}{x^2}\right)dx\left(A+Bx^2\right)$$

$$= \int_a^{a+L} Gm\left(\frac{A}{x^2}+B\right)dx = Gm\left[\frac{A}{a}-\frac{A}{a+L}+BL\right].$$

(4) **(B).** $V_A = \begin{pmatrix}\text{Potential at}\\ \text{A due to A}\end{pmatrix} + \begin{pmatrix}\text{Potential at}\\ \text{A due to B}\end{pmatrix}$

$\Rightarrow V_A = -\frac{Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R}$ and

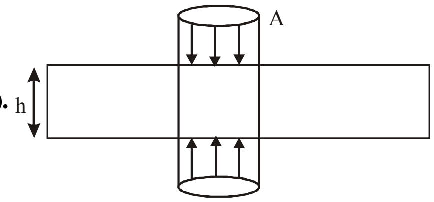


$V_B = \begin{pmatrix}\text{Potential at}\\ \text{B due to A}\end{pmatrix} + \begin{pmatrix}\text{Potential at}\\ \text{B due to B}\end{pmatrix}$

$\Rightarrow V_B = -\frac{Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$. Since $W_{A\to B} = m(V_B - V_A)$

$$\Rightarrow W_{A\to B} = \frac{Gm(m_1-m_2)(\sqrt{2}-1)}{\sqrt{2}R}$$

(5) **(A).**



Gauss law for gravitation

$$\int \vec{g}.d\vec{s} = -m_{in}.4\pi G \quad ; \quad g = \frac{GM}{R^2}$$

$$2 \times \frac{GM}{R^2} \times A = \frac{M}{\frac{4}{3}\pi R^3}(h \times A) \times 4\pi G \;\Rightarrow\; h = \frac{2R}{3}$$

(6) **(C).** Let N be the number of stars in the galaxy, M be the mass of the Sun, and R be the radius of the galaxy. The total mass in the galaxy is NM and the magnitude of the gravitational force acting on the Sun is $F = GNM^2/R^2$.

The force points toward the galactic center. The magnitude of the Sun's acceleration is $a = v^2/R$ where v is its speed. If T is the period of the Sun's motion around the galactic center then $v = \frac{2\pi R}{T}$ and $a = \frac{4\pi^2 R}{T^2}$.

Newton's second law yields $\frac{GNM^2}{R^2} = \frac{4\pi^2 MR}{T^2}$.

The solution for N is $N = \frac{4\pi^2 R^3}{GT^2 M}$

The period is 2.5×10^8 y, which is 7.88×10^{15} s, so

$$N = \frac{4\pi^2\,(2.2 \times 10^{20}\text{m})^3}{(6.67 \times 10^{-11}\text{m}^3/\text{s}^2.\text{kg})\,(7.88 \times 10^{15}\text{s})^2\,(2.0 \times 10^{30}\text{kg})} = 5.1 \times 10^{10}$$

(7) **(D).** $\frac{L}{2m} = \frac{dA}{dt}$ (L = angular momentum)

$$\frac{mv_{max} r_{min}}{2m} = \frac{dA}{dt} \;;\; v_{max} = \frac{2\,dA/dt}{r_{min}} = 40$$

(8) **(C).** $E = -\frac{K}{r}$, $\int_{V_0}^{V} dV = -\int_{r_0}^{r} \vec{E}.d\vec{r} = -\int_{r_0}^{r} \frac{K}{r}\,dr$

$$V - V_0 = K \ln\frac{r}{r_0}$$

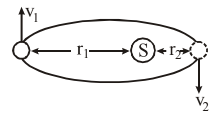
Potential at distance r, $V = V_0 + K\ln\frac{r}{r_0}$

(9) **(A).** As the angular momentum of the planet is constant, we have $mv_1r_1 = mv_2r_2$ or $v_1r_1 = v_2r_2$ ...(i)

Further, the total energy of the planet is also constant, hence $-\frac{GMm}{r_1} + \frac{1}{2}mv_1^2 = -\frac{GMm}{r_2} + \frac{1}{2}mv_2^2$

where M is the mass of the sun.

$$GM\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{v_2^2}{2} - \frac{v_1^2}{2}$$



or $GM\left(\frac{r_1 - r_2}{r_1 r_2}\right) = \frac{v_1^2 r_1^2}{2r_2^2} - \frac{v_1^2}{2}$ (as $v_2 = \frac{v_1 r_1}{r_2}$ from (i))

or $GM\left(\frac{r_1 - r_2}{r_1 r_2}\right) = \frac{v_1^2}{2}\left(\frac{r_1^2}{r_2^2} - 1\right) = \frac{v_1^2}{2}\left(\frac{r_1^2 - r_2^2}{r_2^2}\right)$

$$\therefore\; v_1^2 = \frac{2GM(r_1 - r_2)r_2^2}{r_1 r_2 (r_1^2 - r_2^2)} = \frac{2GMr_2}{r_1(r_1 + r_2)}$$

$$\Rightarrow\; v_1 = \sqrt{\left[\frac{2GMr_2}{r_1(r_1 + r_2)}\right]}$$

Now Angular momentum $= mv_1r_1 = m\sqrt{\left[\frac{2GMr_1r_2}{(r_1 + r_2)}\right]}$.

(10) **(D).** At the surface of the earth, the potential energy of the earth-particle system is $-\frac{GMm}{R}$

and the kinetic energy is $\frac{1}{2}mv_0^2$. At the maximum height the kinetic energy is zero.

If the maximum height reached is H, the potential energy of the earth-particle system at this instant is $-\frac{GMm}{R+H}$.

Using conservation of energy,

$$-\frac{GMm}{R} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R+H}$$

Writing $GM = gR^2$ and dividing by m,

$$-gR + \frac{v_0^2}{2} = \frac{-gR^2}{R+H} \quad \text{or} \quad \frac{R^2}{R+H} = R - \frac{v_0^2}{2g}$$

$$\text{or} \quad R + H = \frac{R^2}{R - \frac{v_0^2}{2g}}$$

Putting the values of R, v_0 and g on the right side,

$$R + H = \frac{(6400\text{km})^2}{6400\text{km} - \frac{(9.8\text{km/s})^2}{2 \times 9.8\text{m/s}^2}} = \frac{(6400\text{km})^2}{1500\text{km}}$$

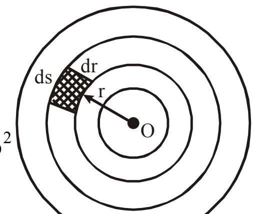
$$= 27300 \text{ km}$$

$$H = (27300 - 6400)\text{km} = 20900 \text{ km}.$$

(11) **(A).** Consider a strip of thickness dr and area ds at a distance r from the centre of the sphere as shown in figure.
Its mass dm = (ds) (dr) ρ
Inward gravitational pull dF on the element dr is due to the part of earth contained within radius r

$$= \frac{G(ds\ dr)\rho \times \left(\frac{4}{3}\pi r^3 \rho\right)}{r^2} \quad \ldots(i)$$

where ρ = density of sphere
For equilibrium of the element
dP . ds = – dF



$$dP.ds = -(G\,ds\,dr)\left(\frac{4}{3}\pi r\right)\rho^2$$

or $dP = -\frac{4}{3}\pi G\rho^2\ r\ dr$ …(ii)

In order to find P, we integrate this expression within proper limits. Thus $\int_0^P dP = -\frac{4}{3}\pi G\rho^2 \int_R^r r\,dr$

[Here at the outer end, the pressure P = 0]

$$\therefore\ P = -\frac{4}{3}\pi G\rho^2 \left[\frac{r^2}{2}\right]_R^r$$

$$= -\frac{4}{6}\pi G\rho^2 (r^2 - R^2) = +\frac{4}{6}\pi G\rho^2 R^2 \left(1 - \frac{r^2}{R^2}\right)$$

$$= \frac{3}{8}\left(1 - \frac{r^2}{R^2}\right)\frac{GM^2}{\pi R^4} \quad \ldots(iii)$$

Pressure at the centre of the earth:

$$r = 0,\ R = 6400\ \text{km},\ M = 6 \times 10^{24}\ \text{kg}$$

and $G = 6.6 \times 10^{-11}\ N - m^2/kg^2$

Substituting these value in eq. (iii), we get

$$P = 1.69 \times 10^{11}\ N/m^2 \simeq 1.65 \times 10^6\ \text{atmosphere.}$$

(12) **(C).** $M_A = \sigma 4\pi R_A^2, M_B = \sigma 4\pi R_B^2,$
where σ is surface density

$$V_A = \frac{-GM_A}{R_A},\ V_B = \frac{-GM_B}{R_B}$$

$$\frac{V_A}{V_B} = \frac{M_A}{M_B}\frac{R_B}{R_A} = \frac{\sigma 4\pi R_A^2}{\sigma 4\pi R_B^2}\frac{R_B}{R_A} = \frac{R_A}{R_B}$$

Given $\frac{V_A}{V_B} = \frac{R_A}{R_B} = \frac{3}{4}$ then $R_B = \frac{4}{3}R_A$

for new shell of mass M and radius R :

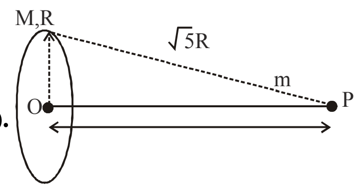
$$M = M_A + M_B = \sigma 4\pi R_A^2 + \sigma 4\pi R_B^2$$

$$\sigma 4\pi R^2 = \sigma 4\pi (R_A^2 + R_B^2) \text{ then}$$

$$\frac{V}{V_A} = \frac{M}{R}\frac{R_A}{M_A} = \frac{\sigma 4\pi (R_A^2 + R_B^2)}{(R_A^2 + R_B^2)^{1/2}}\frac{R_A}{\sigma 4\pi R_A^2}$$

$$= \frac{\sqrt{R_A^2 + R_B^2}}{R_A} = \frac{5}{3}$$

(13) **(D).**



Gravitational potential at P, $v_P = \frac{-GM}{\sqrt{5}R}$

Gravitational potential at O, $v_O = \frac{-GM}{R}$

Work KE theorem $W = \Delta K \Rightarrow m[v_P - v_O] = \frac{1}{2}mv^2$

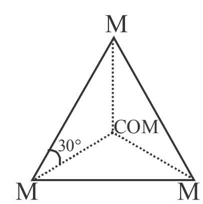
$$m\left[\frac{GM}{R} - \frac{GM}{\sqrt{5}R}\right] = \frac{1}{2}mv^2 \text{ or } \sqrt{\frac{2GM}{R}\left(1 - \frac{1}{\sqrt{5}}\right)}$$

(14) **(B).** Stars move around COM. Distance of each star from COM is $\frac{2}{3} \times L\cos 30° = \frac{L}{\sqrt{3}}$

Force on each star M due to the other two

$$\Rightarrow 2\frac{GMM}{L^2}\cos 30° = \sqrt{3}\frac{GM^2}{L^2}$$

This acts as centripetal force.



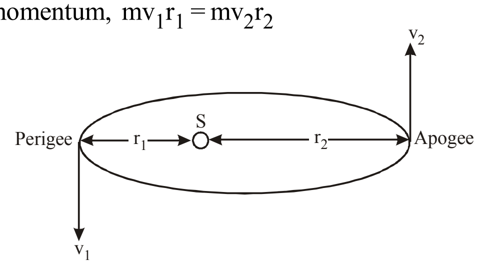
$$\sqrt{3}\frac{GM^2}{L^2} = \frac{mv^2}{(L/\sqrt{3})}$$

$$\Rightarrow v = \sqrt{\frac{GM}{L}}$$

To dismantle the system, energy equal to total energy of the system must be provided.

$$\text{i.e. } 3 \times \frac{1}{2}MV^2 + \left(3 \times \frac{GM^2}{L}\right) = -\frac{3}{2}\frac{GM^2}{L}$$

(15) **(B).** According to the law of conservation of angular momentum, $mv_1r_1 = mv_2r_2$



$\frac{v_1}{v_2} = \frac{r_2}{r_1} = \frac{a(1+e)}{a(1-e)}$, where e is eccentricity

of the earth's orbit= $\frac{(1+0.0167)}{(1-0.0167)} = 1.034$

(16) **(A).** Let the gravitational field be zero at a point distant x from M_1.

$$\frac{GM_1}{x^2} = \frac{GM_2}{(d-x)^2} \;;\; \frac{x}{d-x} = \sqrt{\frac{M_1}{M_2}}$$

$$x\sqrt{M_2} = \sqrt{M_1}d - x\sqrt{M_1} \;;\; x\left[\sqrt{M_1}+\sqrt{M_2}\right] = \sqrt{M_1}d$$

$$x = \frac{d\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}, \quad d-x = \frac{d\sqrt{M_2}}{\sqrt{M_1}+\sqrt{M_2}}$$

Potential at this point due to both the masses will be

$$-\frac{GM_1}{x} - \frac{GM_2}{(d-x)}$$

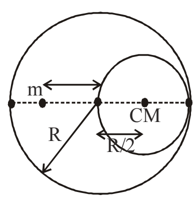
$$= -G\left[\frac{M_1\left(\sqrt{M_1}+\sqrt{M_2}\right)}{d\sqrt{M_1}} + \frac{M_2\left(\sqrt{M_1}+\sqrt{M_2}\right)}{d\sqrt{M_2}}\right]$$

$$= -\frac{G}{d}\left(\sqrt{M_1}+\sqrt{M_2}\right)^2 = -\frac{G}{d}\left(M_1+M_2+2\sqrt{M_1}\sqrt{M_2}\right)$$

(17) **(B).** Gravitational field at mass m due to full solid sphere

$I_1 = G\rho\frac{4}{3}\pi\frac{R}{2}$ (toward centre)

Gravitational field at mass m due to cavity



$I_2 = \frac{G\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{R^2}$ (toward centre)

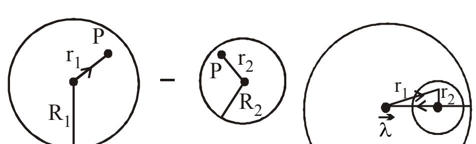
Net gravitational field = $G\rho\frac{4}{3}\pi\frac{3}{8}R$

Here, $\rho = \frac{M}{(4/3)\pi R^3}$ then $F = \frac{3mg}{8}$

(18) **(C).** $E = -\frac{GMm}{2r} \;;\; -\frac{dE}{dt} = \frac{GMm}{2r}\frac{1}{r^2}\frac{dr}{dt}$

$$\int_0^t dt = -\frac{GMm}{2C}\int_r^R \frac{dr}{r^2} \;;\; t = \frac{GMm}{2C}\left[\frac{1}{R} - \frac{1}{r}\right]$$

(19) **(A).** For calculation of gravitational field intensity inside the cavity.



Gravitational field intensity without cavity (I_1)   (I_2) $\vec{\lambda}+\vec{r}_1 = \vec{r}_2$

$$\vec{I}_1 = \frac{G\left(\frac{4}{3}\pi R_1^3\right)\rho(-\vec{r}_1)}{R_1^3}, \quad \vec{I}_2 = \frac{G\left(\frac{4}{3}\pi R_2^3\right)\rho(-\vec{r}_2)}{R_2^3}$$

$\vec{I} = \vec{I}_1 - \vec{I}_2$ ($\vec{I}$ - intensity inside the cavity)

$$= \frac{4}{3}G\pi\rho[-\vec{r}_1+\vec{r}_2] = \frac{4}{3}G\pi\rho\vec{\ell}$$

(20) **(D).** If M is the mass and R is the radius of earth, then the density $\rho = \frac{M}{\frac{4}{3}\pi R^3}$.

The spherical volume may be supposed to be formed by a large number of their concentric spherical shells.
Let the sphere be disassembled by removing such shells. When there is a spherical core of radius x the energy needed to disassemble a spherical shell of thickness dx is

$$dW = \frac{Gm_1m_2}{x} \;;\; m_1 = \frac{4}{3}\pi x^3\rho$$

m_2 = mass of spherical shell of radius x and thickness $\pi x^2 dx\,\rho$.

$$\therefore\; dW = \frac{G\left(\frac{4}{3}\pi x^3\rho\right)(4\pi x^2 dx\rho)}{x} = \frac{16}{3}\pi^2\rho^2 Gx^4 dx$$

$\therefore$ Total energy required

$$W = \int_0^R \frac{16}{3}\pi^2\rho^2 Gx^3 dx = \frac{16}{3}\pi^2\rho^2 G\left[\frac{x^5}{5}\right]_0^R$$

$$= \frac{16}{3}\pi^2\rho^2\frac{GR^5}{5} = \frac{16}{15}\pi^2\left(\frac{M}{\frac{4}{3}\pi R^3}\right)GR^5 = \frac{3}{5}\frac{GM^2}{R}$$

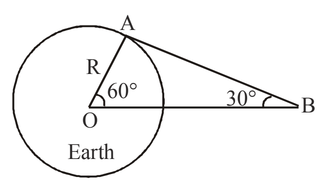
But $GM = gR^2$

$$\therefore\; W = \frac{3}{5}\frac{gR^2}{M}\frac{M^2}{R} = \frac{3}{5}gMR = \frac{3}{5}\times 10\times 2.5\times 10^{31}$$

$= 1.5\times 10^{32}$ J

(21) **(A).** In ΔAOB : $\cos 60° = \frac{R}{OB} \Rightarrow OB = 2R$

Here gravitational force will provide the required centripetal force. Hence, $\frac{GMm}{(OB)^2} = m(OB)\omega^2$



$$\Rightarrow \omega = \sqrt{\frac{GM}{(OB)^3}} = \sqrt{\frac{GM}{(2R)^3}} \Rightarrow \omega = \sqrt{\frac{GM}{8R^3}}$$

EXERCISE-3

(1) **5.** $g = \frac{GM}{R^2} = \frac{Gd\frac{4}{3}\pi R^3}{R^2}$ [d = density] ; $g = Gd\frac{4}{3}\pi R$

Since g is constant and R increased by factor 5,
d decreases by factor 1/5.

(2) **3.** Change in force of gravity $= \frac{GMm}{R^2} - \frac{G\frac{M}{3}m}{R^2}$

(only due to mass M/3 due to shell gravitational field is zero (inside the shell)).

(3) **2.** $V_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R^2}R} = \sqrt{\frac{2Gd\frac{4}{3}\pi R^3}{R^2}R}$

$= R\sqrt{2Gd\frac{4}{3}\pi}$ as $V_e \propto R$ for same density, $\frac{V_A}{V_B} = 2$

(4) **2.** $\frac{1}{2}mv_e^2 = \frac{GMm}{R}$; $v_e = \sqrt{\frac{GMm}{R}} = \sqrt{2gR}$

In tunnel body will perform SHM at centre
$V_{max} = A\omega$ (see chapter on SHM)

$$= \frac{R2\pi}{2\pi\sqrt{R/g}} = \sqrt{gR} = \frac{v_e}{\sqrt{2}}$$

(5) **2.** Total energy = kinetic energy + Potential energy

$$E_0 = \frac{1}{2}mv^2 - \frac{GMm}{r} \quad ...(i)$$

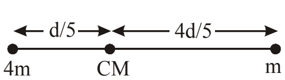
Further, $\frac{mv^2}{r} = \frac{GMm}{r^2}$ or $\frac{1}{2}mv^2 = \frac{GMm}{2r}$ ...(ii)

Substituting the value of $\frac{1}{2}mv^2$ in equation (i) from equation (ii), we get $E_0 = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$

Therefore, $P.E. = -\frac{GMm}{r} = 2E_0$

(6) **4.** Due to mutual force of attraction they will revolve about centre of mass with same ω.

$$\frac{KE_{4m}}{KE_m} = \frac{\frac{1}{2}4m\omega^2\left(\frac{d}{5}\right)^2}{\frac{1}{2}m\omega^2\left(\frac{4d}{5}\right)^2} = \frac{4\times\frac{1}{25}}{\frac{16}{25}} = \frac{1}{4}$$



(7) **2.** Remember Keplers law are applicable for satellite also.

Areal velocity $= \frac{dA}{dt} = \frac{J}{2m} = \frac{1}{2}vr = \frac{1}{2}\sqrt{\frac{GM}{r}}r$

$\left(\frac{mv^2}{r} = \frac{GMm}{r^2}\right)$

Areal velocity $\propto \sqrt{r}$. Hence ratio of areal velocity = 1 : 2

(8) **3.** $\frac{1}{2}mv_0^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$ and $g = \frac{GM}{R^2}$

Therefore $\frac{1}{2}\times\frac{3}{2}\times gR - Rg = -\frac{R^2g}{R+h}$

We get, h = 3R

(9) From Kepler's law, $T^2 \propto R^3$
Parking orbit, $1^2 \propto R^3$
After missing, $T^2 \propto (3R)^3$

$$\frac{T^2}{1} = \frac{27R^3}{R^3} = 27 \; ; \; T = \sqrt{27} = 3\sqrt{3} \text{ day}$$

(10) **3.** $g = \frac{GM}{R^2} = \frac{(G)\rho\left(\frac{4}{3}\pi R^3\right)}{R^2}$; $g \propto \rho R$

$$\frac{g'}{g} = \left(\frac{\rho'}{\rho}\right)\left(\frac{R'}{R}\right) = \left(\frac{2}{3}\right)\left(\frac{R'}{R}\right) = \frac{\sqrt{6}}{11}$$

Given, $\frac{R'}{R} = \frac{3\sqrt{6}}{22}$; $V_e = \sqrt{\frac{GM}{R}} = \sqrt{\frac{(G)\left((\rho)\left(\frac{4}{3}\pi R^3\right)\right)}{R}}$

$\Rightarrow V_e \propto R\sqrt{\rho}$; V_e = 3 km/hr.

EXERCISE-4

(1) **(C).** Work $= U_f - U_i$

$$= -\frac{GMm}{(R+3R)} + \frac{GMm}{(R+2R)}$$

$$= -\frac{GMm}{4R} + \frac{GMm}{3R} = -\frac{GMm}{12R} = \frac{mgR}{12}$$

(2) **(C).** $v_e = \sqrt{\frac{2GM}{R}}$

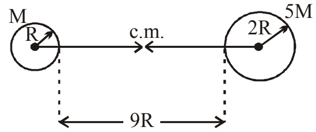
(3) **(A).** Satellite go in tangential direction.

(4) **(C).** Kinetic energy needed to project body so that it can escape out $K = \frac{1}{2}mv_e^2 = \frac{1}{2}m\left(\sqrt{2gR}\right)^2 = mgR$

(5) **(B).** Escape velocity does not depend on angle.

(6) **(B).** $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

(7) **(B).**



Both the particles will strike at centre of mass.
So, distance of c.m. from smaller body

$$r_1 = \frac{m_2 r}{m_1 + m_2} = \frac{5m(9R)}{m + 5m} = \frac{5m(9R)}{6m} = 7.5R$$

(8) **(D).** $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{(R+x)}}$

(9) **(A).** Time period does not depend on mass of satellite.

(10) **(B).** Gain in P.E. $= \frac{mgRh}{R+h}$

(11) **(A).** $F \propto \frac{1}{r^n} \Rightarrow F = \frac{k}{r^n}$

$$F = \frac{k}{r^n} = \frac{mv^2}{r} \ ; \ v = \sqrt{\frac{k}{m}\left(\frac{1}{r^{n-1}}\right)}$$

Time period $T = \frac{2\pi r}{v} \Rightarrow T = 2\pi r\sqrt{\frac{m}{k}(r^{n-1})} \ ; T \propto r^{\frac{n+1}{2}}$

(12) **(A).** g depends on density of earth, but the reverse is not true.

(13) **(C).** $g_h = \left(1 - \frac{2h}{R}\right); g_d = \left(1 - \frac{d}{R}\right)$

$g_h = g_d \ ; 2h = d$

(14) **(D).** $W = \frac{GMm}{R} = \frac{6.67 \times 10^{-11} \times 100 \times 10 \times 10^{-3}}{10 \times 10^{-2}}$

$W = 6.67 \times 10^{-10}$ J

(15) **(A).** Electron charge is same everywhere.

(16) **(A).** Gauss Theorem

(17) **(B).** Escape velocity $v_e = \sqrt{\frac{GM}{r}}$

Hence $M = 10 M_e \cdot r = \frac{r_e}{10}$

$$\therefore v_e = \sqrt{\frac{G \times 10 M_e}{r_e / 10}} = 110 \text{ km/s}$$

(18) **(D).** $g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} \ ; \ \frac{g}{9} = \frac{g}{\left(1 + \frac{h}{R}\right)^2} \ ; 1 + \frac{h}{R} = 3 \ ; h = 2R$

(19) **(D).** $\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$

$\frac{1}{x} = \frac{2}{r-x} \ ; \ r - x = 2x \ ; \ 3x = \frac{r}{3} \Rightarrow x = \frac{r}{3}$

$$V = -\frac{GM}{r/3} - \frac{G(4m)}{2r/3} = -\frac{3GM}{r} - \frac{6Gm}{r} = -\frac{9GM}{r}$$

(20) **(D).** $W = 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$

$= gR^2 \times \frac{m}{R} = mgR = 1000 \times 10 \times 6400 \times 10^3$

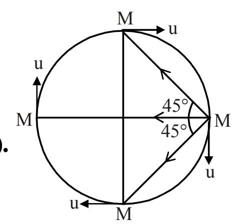
$= 64 \times 10^9 \text{ J} = 6.4 \times 10^{10}$

(21) **(A).** $E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R}$

$$= \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R} = \frac{GMm}{3R}\left(\frac{1}{2} - 1\right) = \frac{-GMm}{6R}$$

$$E_i = \frac{GMm}{R} + K \ ; \ E_i = E_f ; \ K = \frac{5GMm}{6R}$$

(22) **(B).**



Net force on any one particle

$$= \frac{GM^2}{(2R)^2} + \frac{GM^2}{(R\sqrt{2})^2}\cos 45° + \frac{GM^2}{(R\sqrt{2})^2}\cos 45°$$

$$= \frac{GM^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right]$$

This force will be equal to centripetal force so

$$\frac{Mu^2}{R} = \frac{GM^2}{R^2}\left[\frac{1 + 2\sqrt{2}}{4}\right]$$

$$u = \sqrt{\frac{GM}{4R}[1 + 2\sqrt{2}]} = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$$

(23) **(A).** $V = V_1 - V_2 \ ; \ V_1 = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$

$$V_2 = -\frac{3G(M/8)}{2(R/2)} \Rightarrow V = \frac{-GM}{R}$$

(24) **(C).** $V_0 = \sqrt{\frac{GM}{R+h}}, \ V_e = \sqrt{\frac{2GM}{R+h}}$

Increase required $= V_e - V_0$

$$= (\sqrt{2} - 1)\left(\sqrt{\frac{2GM}{R+h}}\right) = (\sqrt{2} - 1)\sqrt{gR}$$

(25) **(C).** $g = \frac{GM}{R^3}r, \; 0 \le r \le R \quad ; \; g = \frac{GM}{r^2}, r \ge R$

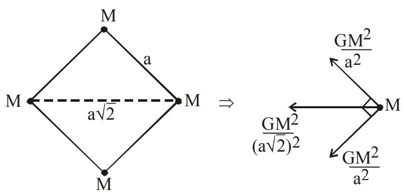
(26) **(A).** $F \propto \frac{1}{R^n} \; ; \; F = \frac{k}{R^n} = \frac{mV^2}{R}$

$\Rightarrow V^2 = \frac{k}{mR^{n-1}} \Rightarrow V \propto R^{\frac{(1-n)}{2}}$

Now, $T = \frac{2\pi R}{V} \propto \frac{R}{R^{\frac{(1-n)}{2}}} \propto R^{\frac{n+1}{2}}$

(27) **(C).** $\frac{dA}{dt} = \frac{L}{2m}$

(28) **(C).**



Net force on particle towards centre of circle is

$F_C = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\sqrt{2} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$

This force will act as centripetal force.

Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$

$r = \frac{a}{\sqrt{2}}, \; F_C = \frac{mv^2}{r} \; ; \; \frac{mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$

$v^2 = \frac{GM}{a}\left(\frac{1}{2\sqrt{2}} + 1\right) = \frac{GM}{a}(1.35) \; ; \quad v = 1.16\sqrt{\frac{GM}{a}}$

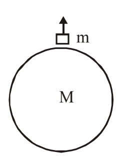
(29) **(B).** Minimum energy required (E)

= – (Potential energy of object at surface of earth)

Now $M_{earth} = 64\ M_{moon}$

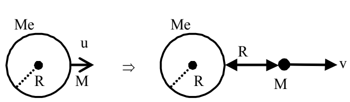
$\rho \cdot \frac{4}{3}\pi R_e^3 = 64 \cdot \frac{4}{3}\pi R_m^3 \Rightarrow R_e = 4R_m$

Now, $\frac{E_{moon}}{E_{earth}} = \frac{M_{moon}}{M_{earth}} \cdot \frac{R_{earth}}{R_{moon}} = \frac{1}{64} \times \frac{4}{1}$

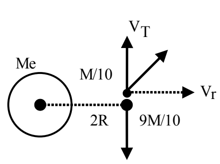


$\Rightarrow E_{moon} = \frac{E}{16}$

(30) **(A).** $\frac{-GM_eM}{R} + \frac{1}{2}Mu^2 = \frac{-GM_eM}{2R} + \frac{1}{2}Mv^2$



$v = \sqrt{u^2 - \frac{GM_e}{R}}$



$V_T \rightarrow$ Transverse velocity of rocket

$V_R \rightarrow$ Radial velocity of rocket

$\frac{M}{10}V_T = \frac{9M}{10}\sqrt{\frac{GM_e}{2R}} \; ; \quad \frac{M}{10}V_r = M\sqrt{u^2 - \frac{GM_e}{R}}$

Kinetic energy $= \frac{1}{2}\frac{M}{10}(V_T^2 + V_r^2)$

$= \frac{M}{20}\left(81\frac{GM_e}{2R} + 100u^2 - 100\frac{GM_e}{R}\right)$

$= \frac{M}{20}\left(100u^2 - \frac{119GM_e}{2R}\right) = 5M\left(u^2 - \frac{119GM_e}{200R}\right)$

(31) **(C).** At pole, weight = mg = 196

m = 19.6 kg

At equator, weight = $mg - m\omega^2R$

$= 196 - (19.6)\left[\frac{2\pi}{24 \times 3600}\right]^2 \times 6400 \times 10^3 = 195.33\ N$

(32) **(D).** Gravitational field on the surface of a solid sphere

$I_g = \frac{GM}{R^2}$. By the graph, $\frac{GM_1}{(1)^2} = 2$ and $\frac{GM_2}{(2)^2} = 3$

On solving, $\frac{M_1}{M_2} = \frac{1}{6}$

(33) **16.** $U_1 + K_1 = U_2 + K_2$

$-\frac{GM_em}{10R} + \frac{1}{2}mv_0^2 = -\frac{GM_em}{R} + \frac{1}{2}mv^2$

$+\frac{9}{10} \times \frac{GM_em}{R} + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$

$\frac{9}{10} \times \frac{1}{2}M \times v_e^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$

$v^2 = \frac{9}{10}v_e^2 + v_0^2 = \frac{9}{10} \times (11.2)^2 + (12)^2$

$v^2 = 112.896 + 144$

$v = 16.027$

$v = 16$ km/s

EXERCISE-5

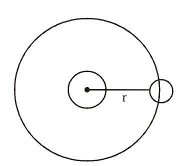
(1) **(D).** $g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2} = \frac{4}{3}\pi GR\rho$

$\frac{g'}{g} = \frac{R'}{R} = \frac{3R}{R} = 3 \qquad \therefore \; g' = 3g$

(2) **(A).** K.E. of satellite moving in an orbit around the earth is

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 = \frac{GMm}{2r}$

P.E. of satellite and earth system is



$U = \frac{GMm}{r} \Rightarrow \frac{K}{U} = \frac{\frac{GMm}{2r}}{\frac{GMm}{r}} = \frac{1}{2}$

(3) **(A).** Potential energy at height R $= -\frac{GMm}{2R}$

If m be the mass of a body which is thrown with velocity v_e so that it goes out of gravitational field from distance R, then

$$\frac{1}{2}mv_e^2 = \frac{GM}{2R}m \Rightarrow v_e = \sqrt{\frac{GM}{R}}$$

or $v_e = \sqrt{gR}$. Now, $v = \sqrt{2gR}$,

so, $v = \sqrt{2}v_e$ or $v_e = \frac{v}{\sqrt{2}}$

Comparing it with given equation, $f = 1/\sqrt{2}$

(4) **(B).** Since orbital velocity of satellite is

$v = \sqrt{\frac{GM}{r}}$, if does not depend upon the mass of the satellite. Therefore, both satellites will move with same speed.

(5) **(B).** SCD : $A_1 - t_1$ (areal velocity constant)

SAB : $A_2 - t_2$; $\frac{A_1}{t_1} = \frac{A_2}{t_2}$, $t_1 = t_2 . \frac{A_1}{A_2}$

$A_1 = 2A_2$ $\quad \therefore\ t_1 = 2t_2$

(6) **(B).** Orbit speed of the satellite around the

earth is $v = \sqrt{\frac{GM}{r}}$

where, G = Universal gravitational constant
M = Mass of earth, r = Radius of the orbit of the satellite.
For satellite A

$r_A = 4R, v_A = 3V;\ v_A = \sqrt{\frac{GM}{r_A}}$ .....(i)

For satellite B,

$r_B = R, v_B = ?,\ \ v_B = \sqrt{\frac{GM}{r_B}}$ .....(ii)

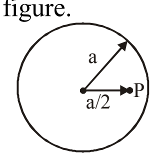
Dividing equation (ii) by equation (i),

$\therefore\ \frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}}$; $v_B = v_A\sqrt{\frac{r_A}{r_B}}$

Substituting the given values, we get

$v_B = 3V\sqrt{\frac{4R}{R}};\ v_B = 6V$

(7) **(A).** Here, Mass of the particle = M
Mass of the spherical shell = M
Radius of the spherical shell = a
Point P is at a distance a/2 from the centre of the shell as shown in figure.



Gravitational potential at point P due to particle at O is $V_1 = -\frac{GM}{(a/2)}$.

Gravitational potential at point P due to spherical shell is $V_2 = -\frac{GM}{a}$

Hence, total gravitational potential at the point P is
$V = V_1 + V_2$

$$= -\frac{GM}{(a/2)} + \left(-\frac{GM}{a}\right) = -\frac{2GM}{a} - \frac{GM}{a} = -\frac{3GM}{a}$$

(8) **(C).** $v_1r_1 = v_2r_2$ ($\because$ Angular momentum is constant)

(9) **(C).** $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$; $\frac{g}{16} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$$\left(1+\frac{h}{R}\right)^2 = 16\ ;\ 1+\frac{h}{R} = 4\ ;\ \frac{h}{R} = 3\ ;\ h = 3R$$

(10) **(A).** Gravitational attraction force on particle B

$$F_g = \frac{GM_pm}{(D_p/2)^2}$$

Acceleration of particle due to gravity

$$a = \frac{F_g}{m} = \frac{4GM_p}{D_p^2}$$

(11) **(C).** $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} = \frac{(6R)^3}{(3R)^3} = 8$; $T_2^2 = \frac{24\times 24}{8}$

$T_2{}^2 = 72$; $T_2{}^2 = 36\times 2$; $T_2 = 6\sqrt{2}$

(12) **(A).** $V = -G\,(2)\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+........\right]$

$$= -2G\left[\frac{1}{1-(1/2)}\right] = -4G$$

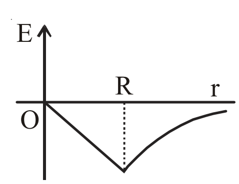
(13) **(C).** $V_e = \sqrt{\frac{2GM}{R}} = C$

$$R = \frac{2GM}{C^2} = \frac{2\times 6.67\times 10^{-11}\times 5.98\times 10^{24}}{(3\times 10^8)^2}$$

$$= \frac{2\times 6.67\times 5.98\times 10^{-3}\,m}{9} = 8.86\times 10^{-3}\,m \approx 10^{-2}\,m$$

(14) **(A).** $E_{in} = -\frac{GMr}{R^3}$

$E_{out} = -\frac{GM}{r^2}$



(15) **(A).** $T^2 = \frac{4\pi^2}{GM}r^3$. So, $K = \frac{4\pi^2}{GM}$

(16) **(A).** The gravitation force on the satellite will be aiming toward the centre of earth so acceleration of the satellite will also be aiming toward the centre of earth.

(17) **(B).** For the satellite revolving around earth

$$v_0 = \sqrt{\frac{GM_e}{(R_e+h)}} = \sqrt{\frac{GM_e}{R_e\left(1+\frac{h}{R_e}\right)}} = \sqrt{\frac{gR_e}{1+\frac{h}{R_e}}}$$

Substituting the values $v_0 = \sqrt{60\times10^6}$ m / s

$v_0 = 7.76 \times 10^3$ m/s = 7.76 km/s

(18) **(A).** $V = -\frac{GM}{(R+h)}$; $g' = \frac{GM}{(R+h)^2} \Rightarrow \frac{|V|}{g'} = R+h$

$$\frac{5.4\times10^7}{6.0} = R+h \Rightarrow 9\times10^6 = R+h$$

$\Rightarrow h = (9-6.4)\times10^6 = 2.6\times10^6 = 2600$ km

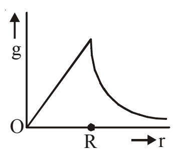
(19) **(B).** $v_e = \sqrt{2gR} = R\sqrt{\frac{8}{3}\pi G\rho}$

$$\Rightarrow \frac{v_e}{v_p} = \frac{R_e\sqrt{\rho_e}}{R_p\sqrt{\rho_p}} = \frac{1}{2\sqrt{2}} \quad \left[\because \begin{matrix} R_p = 2R_e \\ \rho_p = 2\rho_e \end{matrix}\right]$$

(20) **(B).** $g_{in} = \frac{GMr}{R^3} \Rightarrow g_{in} \propto r$

$g_{out} = \frac{GM}{r^2}$

$\Rightarrow g_{out} \propto 1/r^2$



(21) **(B).** Total energy $= -\frac{GMm}{2r}$

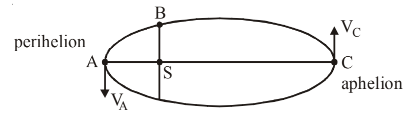
Here, r = R + h and $GM = g_0R^2$.

$$\Rightarrow E = -\frac{mg_0R^2}{2(R+h)}$$

(22 **(C).** $g_h = g_d$; $g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{d}{R}\right)$; d = 2h = 2 km

(23) **(A).** Astronauts move towards each other under mutual gravitational force.

(24) **(B).**



Point A is perihelion and C is aphelion.

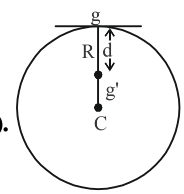
So, $V_A > V_B > V_C$. So, $K_A > K_B > K_C$

(25) **(D).** If Universal Gravitational constant becomes ten times, then G' = 10 G

So, acceleration due to gravity increases.

i.e. (D) is wrong option.

(26) **(D).**



Acceleration due to gravity at a depth d from surface of earth $g' = g\left(1-\frac{d}{R}\right)$ ...(1)

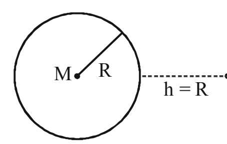
Where g = acceleration due to gravity at earth's surface.

Multiplying by mass 'm' on both sides of (1)

$$mg' = mg\left(1-\frac{d}{R}\right) \qquad \left(d = \frac{R}{2}\right)$$

$$= 200\left(1-\frac{R}{2R}\right) = \frac{200}{2} = 100 \text{ N}$$

(27) **(C).** Initial potential energy at earths surface is $U_i = -\frac{GMm}{R}$



Final potential energy at height h = R

$$U_f = -\frac{GMm}{2R}$$

As work done = Change in PE

$$\therefore \quad W = U_f - U_i = \frac{GMm}{2R} = \frac{gR^2m}{2R} = \frac{mgR}{2} \quad (\because GM = gR^2)$$