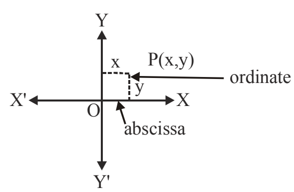
9

POINT AND STRAIGHT LINES

POINT

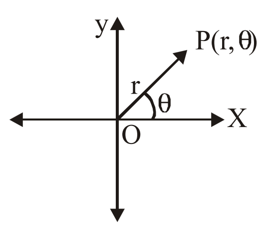
CO-ORDINATES SYSTEMS

* **Cartesian Co-ordinates :** Let XOX' and YOY' be two perpendicular straight lines drawn through point O in the plane of the paper. Then –
* **X–Axis:** The line XOX' is called X–axis
* **Y–Axis:** The line YOY' is called Y–axis
* **Co-ordinate axes:** x–axis and y–axis together are called axis of co-ordinates or axis of reference.
* **Origin :** The point 'O' is called the origin of co-ordinates or the Origin.
* **Oblique axis :** If both the axes are not perpendicular then they are called as Oblique axes.
* **Cartesian Co-ordinates :** The ordered pair of perpendicular distance from both axis of a point P lying in the plane is called Cartesian co-ordinates of P.



If the Cartesian co-ordinates of a point P are (x, y) then x is called abscissa or x co-ordinate of P and y is called the ordinate or y co-ordinate of point P.

* **Polar Co-ordinates :** Let OX be any fixed line which is usually called the initial line and O be a fixed point on it. If distance of any point P from the pole O is 'r' and $\angle XOP = \theta$, then (r, θ) are called the polar co-ordinates of a point P.
If (x, y) are the Cartesian co-ordinates of a point P, then
$x = r\cos\theta$; $y = r\sin\theta$
and $r = \sqrt{x^2 + y^2}$
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$



DISTANCE FORMULA

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Distance of point P(x,y) from the origin $= \sqrt{x^2 + y^2}$

Distance between two polar co-ordinates $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$ is given by $AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$

Example 1 :

If distance between the point (x, 2) and (3, 4) is 2, then find the value of x.

Sol. $2 = \sqrt{(x-3)^2 + (2-4)^2} \Rightarrow 2 = \sqrt{(x-3)^2 + 4}$

Squaring both sides, $4 = (x-3)^2 + 4 \Rightarrow x - 3 = 0 \Rightarrow x = 3$

Example 2 :

If polar co-ordinates of any points are $(2, \pi/3)$ then find its Cartesian co-ordinates.

Sol- $x = 2\cos\pi/3 = 1$, $y = 2\sin\pi/3 = \sqrt{3} \Rightarrow (1, \sqrt{3})$)

Example 3 :

If Cartesian co-ordinates of any point are $(\sqrt{3}, 1)$ then find its polar co-ordinates.

Sol- $\sqrt{3} = r\cos\theta$, $1 = r\sin\theta$

$r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6 \Rightarrow (2, \pi/6)$

APPLICATIONS OF DISTANCE FORMULA

(i) For given three points A, B, C to decide whether they are collinear or vertices of a particular triangle. After finding AB, BC, and CA we shall find that the point are

(a) Collinear, if the sum of any two distances is equal to the third.
(b) Vertices of an equilateral triangle if AB = BC = CA
(c) Vertices of an isosceles triangle if AB = BC or BC = CA or CA = AB
(d) Vertices of a right angled triangle if $AB^2 + BC^2 = CA^2$ etc.

(ii) For given four points A, B, C, D

(a) AB = BC = CD = DA ; AC = BD $\Rightarrow$ ABCD is a square
(b) AB = BC = CD = DA ; AC $\neq$ BD $\Rightarrow$ ABCD is a rhombus
(c) AB = CD , BC = DA ; AC = BD $\Rightarrow$ ABCD is a rectangle
(d) AB = CD , BC = DA ; AC $\neq$ BD $\Rightarrow$ ABCD is a parallelogram

S.No.	Quadrilateral	Diagonals	Angle between diagonals
(i)	Parallelogram	Not equal	$\theta \neq \pi/2$
(ii)	Rectangle	Equal	$\theta \neq \pi/2$
(iii)	Rhombus	Not equal	$\theta = \pi/2$
(iv)	Square	Equal	$\theta = \pi/2$

Example 4 :

If A(0, –1), B(6, 7), C(–2, 3) and D(λ, 3) forms a rectangle then find the value of λ.

Sol. AB = CD and AC = BD.

$$AB = \sqrt{6^2 + 8^2} = 10$$

$$CD = \sqrt{(\lambda+2)^2} = |\lambda + 2|$$

$\lambda + 2 = 10$ or $\lambda + 2 = -10$

$\lambda = 8$ or $\lambda = -12$ ... (1)

Now AC = BD

$4 + 16 = (6 - \lambda)^2 + 4^2$

$4 = 36 + \lambda^2 - 12\lambda \Rightarrow \lambda^2 - 12\lambda + 32 = 0 \Rightarrow \lambda = 4, 8$

Hence from (1) and (2), $\lambda = 8$

Example 5 :

Prove that the four points A(0, 0), B (2, 2), C ($2(\sqrt{2}+1)$, 2) and D ($2\sqrt{2}$, 0) form a Rhombus but not a rectangle.

Sol. Sides are

$AB = 2\sqrt{2}$, $BC = 2\sqrt{2}$, $CD = 2\sqrt{2}$, $DA = 2\sqrt{2}$

Diagonals, $AC = \sqrt{2^2(\sqrt{2}+1)^2 + 4}$

$BD = \sqrt{2^2(\sqrt{2}-1)^2 + 4}$. Since, AC ≠ BD

and all sides are equal hence given points from a Rhombus but not a rectangle.

SECTION FORMULA

Co-ordinates of a point which divides the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio $m_1 : m_2$ are :

(i) For internal division $= \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

(ii) For external division $= \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$

(iii) Co-ordinates of mid point of PQ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

[put $m_1 = m_2$]

NOTE

(i) Co-ordinates of any point on the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are

$\left(\frac{x_1 + \lambda x_2}{1+\lambda}, \frac{y_1 + \lambda y_2}{1+\lambda}\right), (\lambda \neq -1)$

(ii) Division by axes : Lines joins (x_1, y_1) and (x_2, y_2) is divided by

(a) x–axis in the ratio $\rightleftharpoons -y_1 / y_2$

(b) y–axis in the ratio $= -x_1 / x_2$

If ratio is positive divides internally; if ratio is negative divides externally.

(iii) Division by line : Line ax + by + c = 0 divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $-\left(\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right)$

Example 6 :

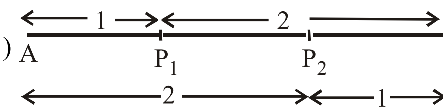
Find the ratio in which the line 3x + 4y = 7 divides the line segment joining the points (1, 2) and (–2, 1).

Sol- $\text{Ratio} = -\frac{3(1) + 4(2) - 7}{3(-2) + 4(1) - 7} = -\frac{4}{-9} = \frac{4}{9}$

Example 7 :

Find the points of trisection of line joining the points A (2, 1) and B (5, 3).

Sol-



$$P_1(x, y) = \left(\frac{1\times5 + 2\times2}{1+2}, \frac{1\times3 + 2\times1}{1+2}\right) = \left(3, \frac{5}{3}\right)$$

$$P_2(x, y) = \left(\frac{2\times5 + 1\times2}{2+1}, \frac{2\times3 + 1\times1}{2+1}\right) = \left(4, \frac{7}{3}\right)$$

Example 8 :

Prove that points A (1, 1), B (–2, 7) and C (3, –3) are collinear.

Sol- $AB = \sqrt{(1+2)^2 + (1-7)^2} = \sqrt{9+36} = 3\sqrt{5}$

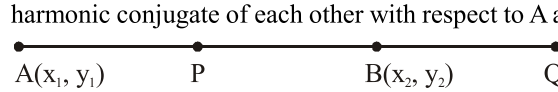
$BC = \sqrt{(-2-3)^2 + (7+3)^2} = \sqrt{25+100} = 5\sqrt{5}$

$CA = \sqrt{(3-1)^2 + (-3-1)^2} = \sqrt{4+16} = 2\sqrt{5}$

Clearly BC = AB + AC. Hence A, B, C are collinear

HARMONIC CONJUGATES

If two points P and Q divides the line A B internally and externally in the same ratio m : n, then P and Q are said to be harmonic conjugate of each other with respect to A and B.



i.e. $\frac{AP}{PB} = \frac{AQ}{BQ} = \lambda$ .....(1)

Also, AP, AB and AQ are in H.P. ie. $\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$

Proof : From (1), $\frac{AP}{AB - AP} = \frac{AQ}{AQ - AB}$

$$\frac{AB - AP}{AP} = \frac{AQ - AB}{AQ}$$

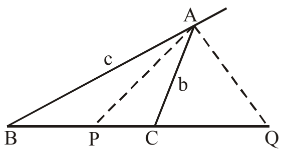
$$\frac{AB}{AP}-1=1-\frac{AB}{AQ}\ ;\ 2=\frac{AB}{AQ}+\frac{AB}{AP}$$

$$\frac{2}{AB}=\frac{1}{AP}+\frac{1}{AQ}$$

Examples:

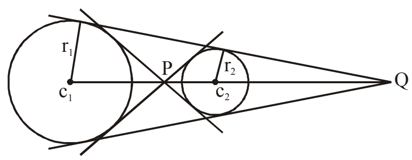
(i) Internal & exeternal angle bisector of an angle of a triangle divide the opposite base harmonically.

$$\frac{BP}{PC}=\frac{BQ}{CQ}=\frac{c}{b}$$



(ii) External and internal common tangents divide the line joining the centres of the two circles externally and internally in the ratio of the their radii.

$$\frac{C_1P}{PC_2}=\frac{C_1Q}{C_2Q}=\frac{r_1}{r_2}$$



Example 9 :

Find the harmonic conjugate of point R (2, 4) with respect to the points P (2, 2) and Q (2, 5).

Sol. Let R divides the PQ in ratio k : 1.

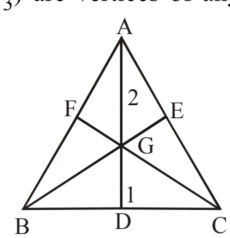
$$\frac{5k+2}{k+1}=4\ ;\ 5k+2=4k+4\ ;\ k=2$$

Harmonic conjugate is $\left(\frac{2\times2-1\times2}{2-1},\frac{2\times5-1\times2}{2-1}\right)=(2,8)$

CO-ORDINATE OF SOME PARTICULAR POINTS

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of any triangle ABC, then

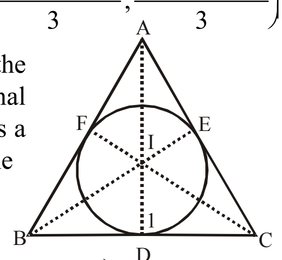
* **Centroid :** The centroid is the point of intersection of the medians (Line joining the mid point of sides and opposite vertices) Centroid divides the median in the ratio of 2 : 1.



Co-ordinates of centroid G $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

* **Incentre :** The incentre is the point of intersection of internal bisector of the angle. Also it is a centre of circle touching all the sides of a triangle.

Co-ordinates of incentre



$$I\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

where a, b, c are the sides of triangle ABC

NOTE

(i) Angle bisector divides the opposite sides in the ratio of remaining sides.

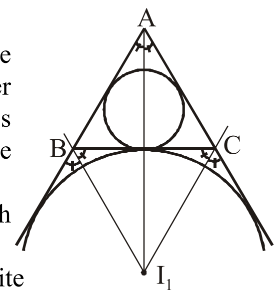
Ex. $\frac{BD}{DC}=\frac{AB}{AC}=\frac{c}{b}$

(ii) Incentre divides the angle bisectors in the ratio $(b+c):a,\ (c+a):b$ and $(a+b):c$

* **Excentre :**

Point of intersection of one internal angle bisector and other two external angle bisector is called as excentre. There are three excentre in a triangle.

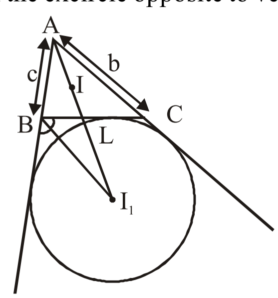
There are three excentres with respect to a given triangle.



I_1 : Centre of the excircle opposite to vertex A (as shown in figure)

I_2 : Centre of the excircle opposite to vertex B

I_3 : Centre of the excircle opposite to vertex C



The coordinates of I_1 are given by

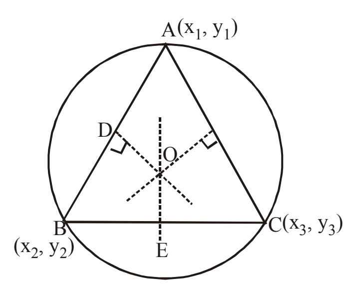
$$I_1=\left(\frac{-ax_1+bx_2+cx_3}{-a+b+c},\frac{-ay_1+by_2+cy_3}{-a+b+c}\right)$$

$$I_2=\left(\frac{ax_1-bx_2+cx_3}{a-b+c},\frac{ay_1-by_2+cy_3}{a-b+c}\right)$$

$$I_3=\left(\frac{ax_1+bx_2-cx_3}{a+b-c},\frac{ay_1+by_2-cy_3}{a+b-c}\right)$$

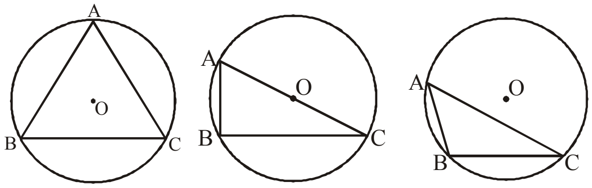
* **Circumcentre :**

It is the point of intersection of perpendicular bisectors of the sides of a triangle. It is also the centre of a circle passing vertices of the triangle. If O is the circumcentre of any triangle ABC, then $OA^2=OB^2=OC^2$



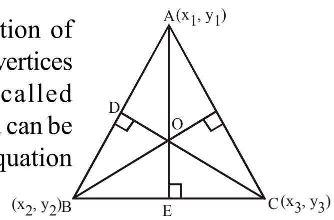
In case of acute angle triangle circumcentre lies inside the triangle.

For right angle triangle it lies on mid-point of hypotenuse and in case of obtuse angle triangle it lies outside the triangle.

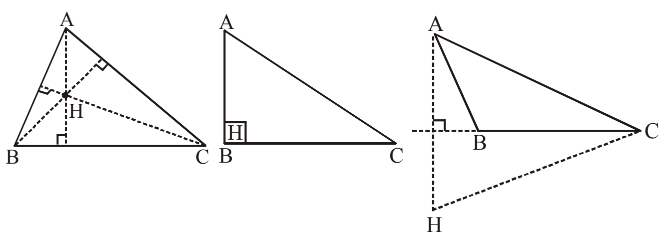


* **Orthocentre :**

It is the point of intersection of perpendicular drawn from vertices on opposite sides (called altitudes) of a triangle and can be obtained by solving the equation of any two altitudes.



In case of acute angle triangle orthocentre lies inside the triangle. For right angle triangle orthocentre lies at the vertex where it is right angled and in case of obtuse triangle orthocentre lies outside the triangle.



Note :

(i) If the triangle is equilateral, the centroid, incentre, orthocentre, circumcentre, coincides

(ii) Ortho centre, centroid and circumcentre are always colinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.

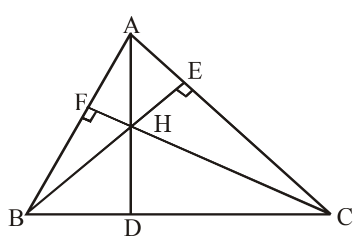
(iii) In an isosceles triangle centroid, orthocentre, incentre, circumcentre lies on the same line.

(iv) Important ratios which are useful to determine the coordinates of orthocentre are :

$$BD : DC = c\cos B : b\cos C$$

and $AH : HD = 2R\cos A : 2R\cos B\cos C$

where, R is the circumradius of the triangle.



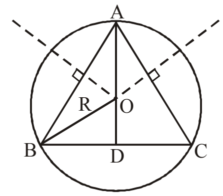
Coordinates of orthocentre are

$$\left(\frac{x_1\tan A + x_2\tan B + x_3\tan C}{\tan A + \tan B + \tan C}, \frac{y_1\tan A + y_2\tan B + y_3\tan C}{\tan A + \tan B + \tan C}\right)$$

(v) Important ratios which are useful to determine the coordinates of circumcentre are

$$BD : DC = \sin 2C : \sin 2B$$

and $AO : OD = \sin 2B + \sin 2C : \sin 2A$



R = Radius of circumcircle

Coordinates of circumcentre are

$$\left(\frac{x_1\sin 2A + x_2\sin 2B + x_3\sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1\sin 2A + y_2\sin 2B + y_3\sin 2C}{\sin 2A + \sin 2B + \sin 2C}\right)$$

Example 10 :

Find incentre of triangle whose vertices are A (–36, 7), B (20, 7), C (0, – 8).

Sol. Using distance formula

$$a = BC = \sqrt{20^2 + (7+8)^2} = 25$$

$$b = CA = \sqrt{36^2 + (7+8)^2} = 39$$

$$c = AB = \sqrt{(36+20)^2 + (7-7)^2} = 56$$

$$I = \left(\frac{25(-36) + 39(20) + 56(0)}{25 + 39 + 56}, \frac{25(7) + 39(7) + 56(-8)}{25 + 39 + 56}\right)$$

$$I = (-1, 0)$$

Example 11 :

If (0, 1), (1, 1) and (1, 0) are middle points of the sides of a triangle then find its incentre.

Sol- Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are vertices of a triangle, then $x_1 + x_2 = 0,\ x_2 + x_3 = 2,\ x_3 + x_1 = 2$

$$y_1 + y_2 = 2,\ y_2 + y_3 = 2,\ y_3 + y_1 = 0$$

Solving these equations, we get

A (0, 0), B (0, 2) and C (2, 0)

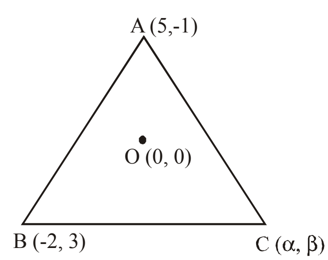
Now, $a = BC = 2\sqrt{2}$, $b = CA = 2$, $c = AB = 2$

Thus incentre of a Δ ABC is $(2 - \sqrt{2},\ 2 - \sqrt{2})$

Example 12 :

Two vertices of a triangle are (5, –1) and (–2, 3). If origin is the orthocentre, then find the third vertex of the triangle.

Sol- Let C (α, β) be the third vertex



$\overline{AO} \perp \overline{BC} = \left(\frac{\beta-3}{\alpha+2}\right)\left(\frac{-1}{5}\right) = -1 \Rightarrow 5\alpha - \beta = -13 ...(1)$

$\overline{BO} \perp \overline{AC} \Rightarrow \left(\frac{\beta+1}{\alpha-5}\right)\left(\frac{3}{-2}\right) = -1 \Rightarrow 2\alpha - 3\beta = 13 ...(2)$

Solving (1) and (2), $(\alpha, \beta) = (-4, -7)$

Example 13 :

Find the distance between the orthocentre and circumcentre of a triangle whose vertices are P(3, 0), Q (0, 0) and $R\left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$

Sol. side $PQ = \sqrt{(3-0)^2 + (0-0)^2} = 3$

$QR = \sqrt{\left(\frac{3}{2}-0\right)^2 + \left(\frac{-3\sqrt{3}}{2}-0\right)^2} = 3$

$PR = \sqrt{\left(3-\frac{3}{2}\right)^2 + \left(0+\frac{3\sqrt{3}}{2}\right)^2} = 3$

Hence, PQ = QR = PR

Hence, the triangle is equilateral.

Now, since in an equilateral triangle orthocentre and circumcentre coincides therefore distance between them is zero.

Example 14 :

If α, β and γ are the roots of equation $x^3 - 12x^2 + 44x - 48 = 0$. Find the centroid of the Δ whose co-ordinates are $A\left(\alpha, \frac{1}{\alpha}\right), B\left(\beta, \frac{1}{\beta}\right)$ and $C\left(\gamma, \frac{1}{\gamma}\right)$.

Sol. Centroid $= \left(\frac{\alpha+\beta+\gamma}{3}, \frac{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}}{3}\right)$; $\frac{\alpha+\beta+\gamma}{3} = \frac{12}{3} = 4$

$\frac{1}{3}\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right) = \frac{1}{3}\left(\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}\right) = \frac{1}{3}\left(\frac{44}{48}\right) = \frac{11}{36}$

Centroid is (4, 11/36)

Example 15 :

Find the co-ordinates of circumcentre of the triangle whose vertices are (8, 6), (8, –2) and (2, –2).

Sol. Let A (8, 6) and B (8, –2) and C (2, –2)

P(h, k) be the circumcentre

PA = PB = PC

$\Rightarrow PA^2 = PB^2$

$(h-8)^2 + (k-6)^2 = (h-8)^2 + (k+2)^2$

$16k = 32 \Rightarrow k = 2$

$PB^2 = PC^2$

$(h-8)^2 + (k+2)^2 = (h-2)^2 + (k+2)^2$

$12h = 60 \Rightarrow h = 5$

Hence the co-ordinate of the circumcentre is (5, 2).

AREA OF TRIANGLE AND QUADRILATERAL

Area of Triangle : Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of a triangle, then area of a triangle ABC

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

NOTE

1. If the area of a triangle is zero, then its points are collinear.
2. When one vertex is origin i.e. if the vertices are (0,0), (x_1, y_1) and (x_2, y_2) then its area $\Delta = \frac{1}{2}|x_1y_2 - x_2y_1|$
3. When abscissa or ordinate of all vertices are equal then its area is zero.
4. When two vertices be on x - axis say (a, 0), (b, 0) and third vertex be (h,k) then its area $= \frac{1}{2}|a-b|k$
5. Area of the triangle formed by coordinates axes and the line $ax + by + c = 0$ is $\left|\frac{c^2}{2ab}\right|$
6. When ABC is right angled triangle and $\angle B = 90°$, then $\Delta = \frac{1}{2}(AB \times BC)$
7. When D, E, F are the mid-points of the sides AB, BC, CA of the triangle ABC, then its are a $\Delta = 4$ (ΔDEF)
8. A triangle having vertices $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ and $(at_3^2, 2at_3)$ then area is $\Delta = a^2[(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)]$
9. Three points $(x_1, y_1), (x_2, y_2)$ (x_3, y_3) are collinear if $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$
10. In an equilateral triangle
    (a) having sides 'a' area is $= \frac{\sqrt{3}}{4}a^2$
    (b) having length of perpendicular as 'p' area $= \frac{p^2}{\sqrt{3}}$
11. If a triangle has polar coordinates $(r_1, \theta_1), (r_2, \theta_2)$ & (r_3, θ_3) then its area

$$\Delta = \frac{1}{2}[r_1r_2\sin(\theta_2 - \theta_1) + r_2r_3\sin(\theta_3 - \theta_2) + r_3r_1\sin(\theta_1 - \theta_3)]$$

12. Area of triangle when equations of its sides are given

If $a_r x + b_r y + c_r = 0$ $(r = 1, 2, 3)$ are sides of a triangle then its area is given by $\Delta = \frac{1}{2C_1C_2C_3}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2$

where C_1, C_2, C_3 are cofactors of c_1, c_2, c_3 in the determinant.

Area of Quadrilateral : If (x_1,y_1), (x_2,y_2), (x_3,y_3) and (x_4,y_4) are vertices of a quadrilateral then its area

$$= \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)]$$

NOTE

1. If the area of quadrilateral joining four points is zero then those four points are collinear.
2. If two opposite vertex of rectangle are (x_1, y_1) and (x_2, y_2) then its area may be $|(y_2 - y_1)(x_2 - x_1)|$
3. If two opposite vertex of a square are $A(x_1,y_1)$ and $C(x_2,y_2)$ then its area is $= \frac{1}{2}AC^2 = \frac{1}{2}[(x_2 - x_1)^2 + (y_2 - y_1)^2]$
4. Quadrilateral containing two sides parallel is called as Trapezium whose area is given by $\frac{1}{2}$ (sum of parallel sides) (distance between parallel sides)
5. Area of Rhombus formed by $ax \pm by \pm c = 0$ is $\left|\frac{2c^2}{ab}\right|$

Area of polygon : Area of polygon whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3), ... (x_n, y_n) is

$$\frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} + ... + \begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right|$$

Example 16 :

If the coordinates of two opposite vertex of a square are (a, b) and (b, a) then find the area of square.

Sol- We know that area of square

$$= \frac{1}{2}d^2 = \frac{1}{2}[(a - b)^2 + (b - a)^2] = (a - b)^2$$

Example 17 :

If A(1, 1), B(3, 4), C(5, –2) and D(4, –7) in order are the vertices of a quadrilateral. Find its area.

Sol. **Method 1 :** To find the area of quadrilateral ABCD, we can calculate area of ΔABC & ΔADC

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \frac{18}{2}$$

$$\text{Area of } \Delta ADC = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 4 & -7 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \frac{23}{2}$$

Hence Area of quadrilateral ABCD

$$= \Delta\, ABC + \Delta\, ADC = \frac{18}{2} + \frac{23}{2} = \frac{41}{2}$$

Method 2 : $\frac{1}{2}[1 \times 4 - 3 \times 1 + 3 \times (-2) - 5(4) + 5(-7) - 4(-2) + 4(1) - 1(-7)]$

$$= \frac{1}{2}[4 - 3 - 6 - 20 - 35 + 8 + 4 + 7] = 41/2 \text{ units}$$

Example 18 :

Find the area of a triangle with vertices (a, θ) ; $\left(2a, \theta + \frac{\pi}{3}\right)$; $\left(3a, \theta + \frac{2\pi}{3}\right)$

Sol. $= \frac{1}{2}\left[2a^2 \sin\frac{\pi}{3} + 6a^2 \sin\frac{\pi}{3} + 3a^2 \sin\left(\frac{-2\pi}{3}\right)\right] = \frac{5\sqrt{3}}{4}a^2$

Example 19 :

If the area of the triangle formed by the points (1, 2), (2, 3) and (x, 4) is 40 sq. units then find x.

Sol. Let A (1, 2), B (2, 3) & C (x, 4) be three points.

$$\text{Area (D)} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ x & 4 & 1 \end{vmatrix} = 40$$

$= |1(3 - 4) - 2(2 - 4) + x(2 - 3)| = 80$

$= |-1 + 4 - x| = 80$

$= |3 - x| = 80$

$3 - x = 80$ or $3 - x = -80$; $x = -77$ or $x = 83$

Example 20 :

Find the value of k for which points (2, 3), (3, 5) and (5, k) are collinear.

Sol. If the points are collinear then, Area $(\Delta) = 0$

$$\frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ 3 & 5 & 1 \\ 5 & k & 1 \end{vmatrix} = 0$$

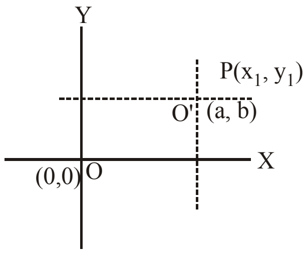
$|5(3 - k) - k(2 - 3) + 1(10 - 9)| = 0$

$|-10 + k + 1| = 0$; $|k - 9| = 0$; $k = 9$

TRANSFORMATION OF AXES

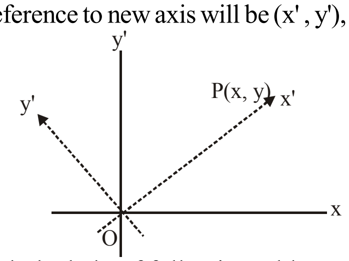
Parallel transformation : Let origin O(0, 0) be shifted to a point (a, b) by moving the x–axis and y–axis parallel to themselves. If the co-ordinate of point P with reference to old axis are (x_1, y_1) then coordinate of this point with respect to new axis will be $(x_1 - a, y_1 - b)$

$P(x', y') = P(x_1 - a, y_1 - b)$



Rotational transformation : Let OX and OY be the old axis and OX' and OY' be the new axis obtained by rotating the old OX and OY through an angle θ Again, if co-ordinates of any point P(x, y) with reference to new axis will be (x' , y'), then

$x' = x \cos\theta + y\sin\theta$
$y' = -x \sin\theta + y\cos\theta$
$x = x' \cos\theta - y'\sin\theta$
$y = x' \sin\theta + y'\cos\theta$



The above relation between (x,y) and (x', y') can be easily obtained with the help of following table

	$x\downarrow$	$y\downarrow$
$x' \rightarrow$	$\cos\theta$	$\sin\theta$
$y' \rightarrow$	$-\sin\theta$	$\cos\theta$

Note : To remove the term of xy in the equation $ax^2 + 2hxy + by^2 = 0$, the angle θ through which the axis must be turned (rotated) is given by $\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$

Reflection (Image) of a point : Let (x, y) be any point, then its image with respect to

(i) x– axis ⇒ (x, –y) (ii) y– axis ⇒ (–x, y)
(iii) origin ⇒ (–x, –y) (iv) line y = x ⇒ (y, x)

Example 21 :

Keeping the origin constant axis are rotated at an angle 30° in negative direction then find new coordinate of (2, 1) with respect to old axis.

Sol-

	2	1
x'	cos (–30°)	sin (–30°)
y'	–sin (–30°)	cos(–30°)

$$x = 2\cos30° + \sin 30° = \frac{2\sqrt{3}+1}{2}$$

$$y = -2\sin30° + \cos 30° = \frac{-2+\sqrt{3}}{2}$$

Example 22 :

Find the new coordinates of point (3, –4) if the origin is shifted to (1, 2) by translation.

Sol. Since origin is shifted to x = 1 and y = 2

Hence x – 1 = X and y – 2 = Y

Point (3, –4) is shifted to

X = 3 – 1, Y = – 4 – 2
X = 2, Y = –6

Hence new coordinates are (2, –6)

LOCUS

A locus is the curve traced out by a point which moves under certain geometrical conditions. To find a locus of a point first we assume the co-ordinates of the moving point as (h, k) then try to find a relation between h and k with the help of the given conditions of the problem. In the last we replace h by x and k by y and get the locus of the point which will be an equated between x and y.

NOTE

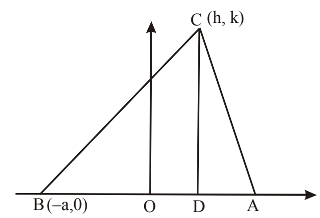
1. Locus of a point P which is equidistant from the two point A & B is straight line is a perpendicular bisector of line AB
2. In above case if PA = kPB where k ≠ 1 then the locus of P is a circle
3. Locus of P A and B is fixed
   (a) Circle if ∠APB = constant
   (b) Circle with diameter AB if ∠APB = π/2
   (c) Ellipse if PA + PB = constant
   (d) Hyperbola if PA – PB = constant

Example 23 :

A (a, 0) and B (–a, 0) are two fixed points of Δ ABC. If its vertex C moves in such way that cot A + cot B = λ , where λ is a constant, then find the locus of the point C.

Sol. We may suppose that coordinates of two fixed points A, B are (a, 0) and (–a, 0) and variable points C is (h, k)

From the adjoining figure



$$\cot A = \frac{DA}{CD} = \frac{a-h}{k}$$

$$\cot B = \frac{BD}{CD} = \frac{a+h}{k}$$

But cot A + cot B = λ

so we have $\frac{a-h}{k} + \frac{a+h}{k} = \lambda \Rightarrow \frac{2a}{k} = \lambda$.

Hence locus of C is y = 2a / λ.

Example 24 :

Find the locus of a point such that the sum of its distance from the points (0, 2) and (0,–2) is 6.

Sol. Let P (h, k) be any point on the locus and let A (0, 2) and B (0,–2) be the given points by the given condition PA + PB = 6

$$\Rightarrow \sqrt{(h-0)^2 + (k-2)^2} + \sqrt{(h-0)^2 + (k+2)^2} = 6$$

$$\Rightarrow \sqrt{h^2 + (k-2)^2} = 6 - \sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow h^2 + (k+2)^2 = 36 - 12\sqrt{h^2 + (k+2)^2} + h^2 + (k+2)^2$$

$$\Rightarrow -8k - 36 = -12\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow (2k + 9)^2 = 9(h^2 + (k+2)^2)$$

$$\Rightarrow 4k^2 + 36k + 81 = 9h^2 + 9k^2 + 36k + 36$$

$$\Rightarrow 9h^2 + 5k^2 = 45$$

Hence, locus of (h, k) is $9x^2 + 5y^2 = 45$

TRY IT YOURSELF-1

Q.1 Find the distance between P $(2, -\pi/6)$ and Q $(3, \pi/6)$

Q.2 Find the area of a triangle whose vertices are A (3, 2), B (11, 8) and C (8, 12).

Q.3 In what ratio does the x-axis divide the line segment joining the points (2, –3) and (5, 6) ?

Q.4 If a vertex, the circumcentre, and the centroid of a triangle are (0,0), (3,4), & (6, 8) respectively, then the triangle must be
(A) a right-angled triangle
(B) an equilaterial triangle
(C) an isosceles triangle
(D) a right-angled isosceles triangle

Q.5 A triangle with vertices (4, 0), (–1, –1), (3, 5) is –
(A) isosceles and right-angled
(B) isosceles but not right-angled
(C) right-angled but not isosceles
(D) neigher right-angled nor isosceles

Q.6 If the coordinates of a variable point P is $(a \cos \theta, b \sin \theta)$, where θ is a variable quantity, then find the locus of P.

Q.7 At what point the origin be shifted, if the coordinates of a point (4, 5) become (–3, 9) ?

Q.8 If P(1, 2), Q(4, 6), R(5, 7) & S(a, b) are the vertices of parallelogram PQRS, then :
(A) a = 2, b = 4 (B) a = 3, b = 4
(C) a = 2, b = 3 (D) a = 3, b = 5

Q.9 If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/are always rational point(s) ?
(A) centroid (B) incenter
(C) circumcenter (D) orthocenter

Q.10 Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are
(A) (4/3, 3) (B) (3, 2/3)
(C) (3, 4/3) (D) (4/3, 2/3)

ANSWERS

(1) $\sqrt{7}$ **(2)** 25 sq. units **(3)** 1 : 2

(4) (C) **(5)** (A) **(6)** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(7) (7, – 4) **(8)** (C) **(9)** (A) **(10)** (C)

STRAIGHT LINE

EQUATION OF STRAIGHT LINE

A relation between x and y which is satisfied by co-ordinates of every point lying on a line is called the equation of Straight Line. Every linear equation in two variable x and y always represents a straight line.
Ex. $4x + 9y = 40$, $-10x + 6y = 8$ etc.
General form of straight line is given by
$ax + by + c = 0$

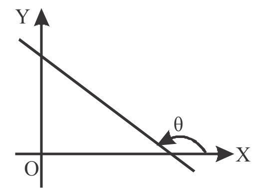
EQUATION OF STRAIGHT LINE PARALLEL TO AXES

(i) Equation of x axis $\Rightarrow y = 0$
Equation a line parallel to x axis (or perpendicular to y axis) at a distance 'a' from it $\Rightarrow y = a$

(ii) Equation of y axis $\Rightarrow x = 0$
Equation of a line parallel to y axis (or perpendicular to x axis) at a distance 'a' from it $\Rightarrow x = a$
Ex. Equation of a line which is parallel x axis and at a distance of 4 units in the negative direction is $y = -4$

SLOPE OF A LINE

The slope of a line is equal to the tangent of the angle which it makes with the positive side of x-axis and it is generally denoted by m.



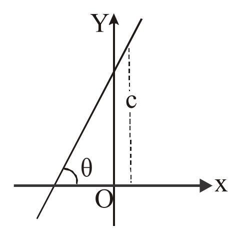
If a line makes an angle θ with x-axis then its slope $= m = \tan\theta$

The slope of a line joining two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

DIFFERENT FORMS OF THE EQUATION OF STRAIGHT LINE

General form : $ax + by + c = 0$
where a, b, c are any real numbers and a , b are not zero both at a time.
Particular case ; in $ax + by + c = 0$
$a = 0 \Rightarrow by + c = 0$, which is a line parallel to x- axis
$b = 0 \Rightarrow ax + c = 0$, which is a line parallel to y- axis
$c = 0 \Rightarrow ax + by = 0$, which is a line passing through origin.

Slope form : $y = mx + c$
Where 'm' is the slope of the line and 'c' is the length of the intercept made by it on y-axis for general form $ax + by + c = 0$



Slope$= \frac{-a}{b} = \frac{\text{coefficient of } x}{\text{coefficient of } y}$

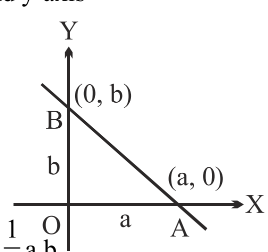
Slope point form : The equation of a line with slope m and passing through a point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

Two point form : The equation of a line passing through two given points (x_1, y_1) and (x_2, y_2) is –

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Intercept form : The equation of a line which makes intercept a and b on the x-axis and y-axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$.

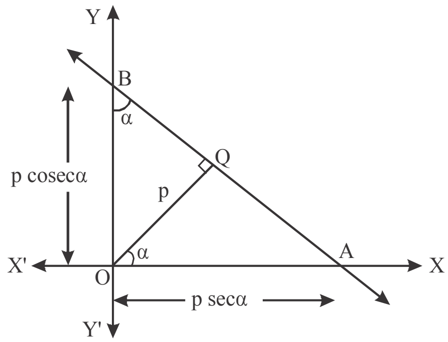
Here, the length of intercept between the co-ordinates axis $= \sqrt{a^2 + b^2}$



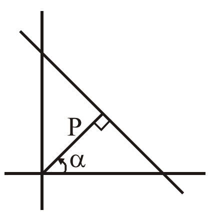
Area of Δ OAB $= \frac{1}{2}$OA. OB $= \frac{1}{2}$a.b.

Normal (Perpendicular) Form of a Line :

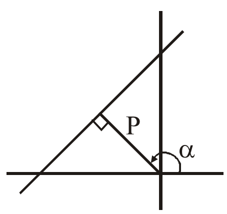
If p is the length of perpendicular on a line from the origin and α is the inclination of perpendicular with x axis then equation on this line is $x\cos\alpha + y\sin\alpha = p$



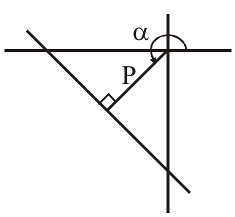
(i) $0 < \alpha < \pi/2$



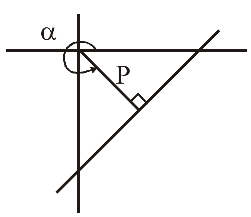
(ii) $\pi/2 < \alpha < \pi$



(iii) $\pi < \alpha < 3\pi$



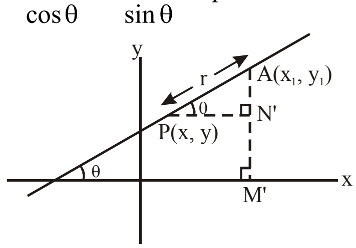
(iv) $3\pi/2 < \alpha < 2\pi$



Parametric Form (distance Form) :

If θ be the angle made by a straight line with x axis which is passing through the point (x_1, y_1) and r be the distance of any point (x, y) on the line from the point (x_1, y_1) then its

equation. $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$



$x = x_1 \pm r\cos\theta,\ y = y_1 \pm r\sin\theta$ are the co-ordinates of the points situated on the line at a distance r from the given point A (x_1, y_1).

REDUCTION OF GENERAL FORM OF EQUATIONS INTO STANDARD FORMS

General Form of equation $ax + by + c = 0$ then its –

(i) Slope intercept Form is

$y = -\dfrac{a}{b}x - \dfrac{c}{b}$, here slope $m = -\dfrac{a}{b}$, Intercept $c = -\dfrac{c}{b}$

(ii) Intercept Form is

$\dfrac{x}{-c/a} + \dfrac{y}{-c/b} = 1$, here x intercept is $= -c/a$,

y intercept is $= -c/b$

(iii) Normal Form is –

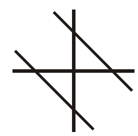
To change the general form of a line into normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2 + b^2}$ like.

$-\dfrac{ax}{\sqrt{a^2+b^2}} - \dfrac{by}{\sqrt{a^2+b^2}} = \dfrac{c}{\sqrt{a^2+b^2}}$, Here

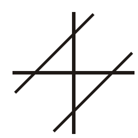
$\cos\alpha = -\dfrac{a}{\sqrt{a^2+b^2}}, \sin\alpha = -\dfrac{b}{\sqrt{a^2+b^2}}$ & $p = \dfrac{c}{\sqrt{a^2+b^2}}$

NOTE

(i) Line having equal interecept then $m = -1$.



(ii) Line having intercept equal in magnitude but opposite in sign then $m = +1$



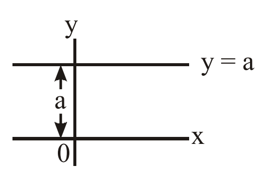
(iii) Line equally inclined with coordinate axes then $m = \pm 1$

(iv) Line cutting of equal non zero distance from origin then $m = \pm 1$.

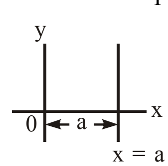
(v) $0.x + 0.y + c = 0$ $(c \neq 0)$ represents a straight line with x and y intercept both infinity

$\Rightarrow$ Straight line approaches at infinity.

(vi) Equations of lines parallel to x-axis are of form $y = a$.



(vii) Equations of lines parallel to y-axis are of form $x = a$.



Example 25 :

A line passes through the point (1, 0). Find the equation of line if

(a) It is inclined at an angle of $3\pi/8$ with positive x-axis.

(b) It passes through (2, 1).

(c) It passes through the point of intersection of lines $y = x$ and $y = 2x + 1$.

(d) It has equal non zero intercepts.

(e) It has intercepts equal in magnitude but opposite in sign.

(f) It cuts an intercept of 4 units on x-axis.

(g) It cuts an intercept of –3 units on y-axis.

(h) It cuts equal non zero distances on co-ordinate axes from origin.

(i) It is equally inclined with co-ordinate axes.

(j) It has an angle of 30º with positive y-axis.

Sol. Since equation passes through (1, 0)

(a) $\theta = 3\pi/8$; $m = \tan\theta = \sqrt{2} + 1$

$y - 0 = m(x - 1)$; $y = (\sqrt{2} + 1)(x - 1)$

(b) If it passes through (2, 1) & (1, 0)

$$\text{slope} = \frac{1-0}{2-1} = 1$$

Equation of line $y - 0 = 1(x - 1)$

$y = x - 1$

(c) Point of intersection of $y = x$ and $y = 2x + 1$ is $x = -1$, $y = -1$

Line passing through (–1, –1) & (1, 0)

$$\text{Slope (m)} = \frac{0-(-1)}{1-(-1)} = \frac{1}{2}$$

Equation of line $(y - 0) = \frac{1}{2}(x - 1)$; $2y = x - 1$

(d) If it has equal non zero intercepts then slope (m) = –1

Equation of line $(y - 0) = -1(x - 1)$; $x + y = 1$

(e) If it has intercepts equal in magnitude but opposite in sign then, m = 1

$y - 0 = 1(x - 1)$; $y = x - 1$

(f) It cuts an intercept of 4 units on x-axis then it passes through (4, 0)

Slope of line through (1, 0) and (4, 0) is

$$m = \frac{0-0}{4-1} = 0$$

Equation of line $y - 0 = 0(x - 1)$; $y = 0$

(g) If cuts an intercept of –3 on y-axis, then it passes through (0, –3) & (1, 0)

$$\text{slope (m)} = \frac{0-(-3)}{1-0} = 3$$

Equation of line $(y - 0) = 3(x - 1) \Rightarrow 3x - y = 3$

(h) If it cuts equal non zero distances then slope (m) = ± 1

Equation of lines are

$(y - 0) = 1(x - 1)$ or $(y - 0) = -1(x - 1)$

$x - y = 1$ or $x + y = 1$

(i) If it is equally inclined with co-roordinate axes then $m = \pm 1$

Equation of lines are $x - y = 1$ or $x + y = 1$

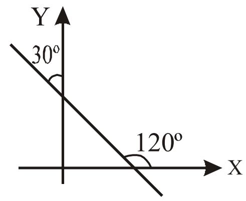
(j) If angle with y-axis is 30º then angle with positive x-axis = 120º

slope (m) = tan 120º = $-\sqrt{3}$

Equation of line is

$y - 0 = -\sqrt{3}(x - 1)$

$\sqrt{3}x + y = \sqrt{3}$



Example 26 :

Find the equation of a line which is passing through (3,– 4) and making an angle of 45° with x-axis.

Sol. $y - (-4) = \tan 45°(x - 3) \Rightarrow y + 4 = x - 3$

$\Rightarrow x - y - 7 = 0$

Example 27 :

Find the equation of a line which makes intercepts 3 and 4 on x axis and y axis respectively

Sol. $\frac{x}{3} + \frac{y}{4} = 1 \Rightarrow 4x + 3y = 12$

Example 28 :

Equation of a line which passes through point A (2, 3) and makes an angle of 45° with x-axis. If this line meet the line $x + y + 1 = 0$ at point P then find the distance of AP.

Sol. Here $x_1 = 2$, $y_1 = 3$ and $\theta = 45°$

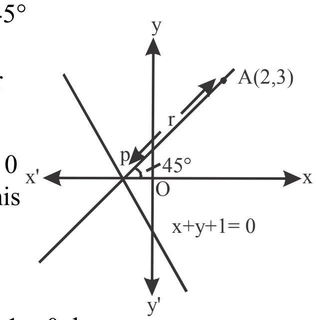
hence $\frac{x-2}{\cos 45°} = \frac{y-3}{\sin 45°} = r$

from first two parts

$\Rightarrow x - 2 = y - 3 \Rightarrow x - y + 1 = 0$

Co-ordinate of point P on this

line is $\left(2 + \frac{r}{\sqrt{2}}, 3 + \frac{r}{\sqrt{2}}\right)$



If this point is on line x + y +1 = 0 then

$$\left(2 + \frac{r}{\sqrt{2}}\right) + \left(3 + \frac{r}{\sqrt{2}}\right) + 1 = 0$$

$\Rightarrow r = -3\sqrt{2}$; $|r| = 3\sqrt{2}$

Example 29 :

Find the standard forms of a line 3x + 4y = 5

Sol. (i) Slope intercept form is $y = -\frac{3}{4}x + \frac{5}{4}$

Here m = 3/4, c = 5/4

(ii) Intercept form $\frac{x}{5/3} + \frac{y}{5/4} = 1$. Here a = 5/3, b = 5/4

(iii) Normal form $\frac{3x}{\sqrt{3^2+4^2}} + \frac{4y}{\sqrt{3^2+4^2}} = \frac{5}{\sqrt{3^2+4^2}}$

$\Rightarrow \frac{3x}{5} + \frac{4y}{5} = 1$, Here $p = 1$, $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$

Example 30 :

Find the equation to the straight line cutting off an intercept 3 from the negative direction of the axis of y and inclined at 120º to the positive direction of x-axis.

Sol. Slope of line (m) = tan 120º = $-\sqrt{3}$

y intercept (c) = –3

Equation of line is

$y = mx + c$

$y = -\sqrt{3}x + (-3)$

$y + \sqrt{3}x + 3 = 0$

Example 31 :

Find the equation to the straight line passing through the point (3, – 4) and cutting off intercepts, equal but of opposite signs from the two axes.

Sol. Let the intercepts cut off from the two axes are a & –a, then equation of straight line is given by

$\frac{x}{a} + \frac{y}{-a} = 1$; $x - y = a$

Since it passes through (3, –4)

hence, $3 - (-4) = a$; $a = 7$

Required equation is $x - y = 7$

Example 32 :

Find equation of a straight line on which length of perpendicular from the origin is four units and the line makes an anlge of 120º with the positive direction of x-axis.

Sol. $\alpha = 30º$, P = 4

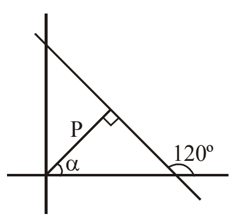
Equation line is given by

$x\cos\alpha + y\sin\alpha = P$

$x\cos 30º + y\sin 30º = 4$

$x\frac{\sqrt{3}}{2} + \frac{y}{2} = 4$

$\sqrt{3}x + y = 8$



TRY IT YOURSELF-2

Q.1 Find the coordinates of a point which is at +3 distance from points (1, –3) of line $2x + 3y + 7 = 0$

Q.2 Equation of line which cuts off an intercepts of 4 units on the x-axis & pass through (2, –3).

(A) $5x - 3y = 0$ (B) $x + y = 1$
(C) $3x - 2y = 12$ (D) $x + y = 7$

Q.3 Equation of line which cuts off equal non-zero intercepts on co-ordinate axes and pass through (2, 5).

(A) $5x - 3y = 0$ (B) $x + y = 1$
(C) $3x - 2y = 12$ (D) $x + y = 7$

Q.4 Equation of line passing through (0, 0) & (3, 5).

(A) $5x - 3y = 0$ (B) $x + y = 1$
(C) $3x - 2y = 12$ (D) $x + y = 7$

Q.5 Equation of line making an angle 135° with positive x-axis and pass through (1, 0)

(A) $5x - 3y = 0$ (B) $x + y = 1$
(C) $3x - 2y = 12$ (D) $x + y = 7$

Q.6 Equation of line passing through (1, 0) and equally inclined with co-ordinate axes.

(A) $5x - 3y = 0$ (B) $x + y = 1$
(C) $3x - 2y = 12$ (D) $x - y = 1$

ANSWERS

(1) $x = 1 - \frac{9}{\sqrt{13}}$, $y = -3 + \frac{6}{\sqrt{13}}$ **(2)** (C)
(3) (D) **(4)** (A) **(5)** (B)
(6) (D)

POSITION OF A POINT RELATIVE TO A LINE

(i) The point (x_1, y_1) lies on the line $ax + by + c = 0$ if, $ax_1 + by_1 + c = 0$

(ii) If $P(x_1, y_1)$ and $Q(x_2, y_2)$ do not lie on the line $ax + by + c = 0$ then they are on the same side of the line if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign and they lie on the opposite sides of line if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the opposite sign.

(iii) (x_1, y_1) is on origin or non origin sides of the line $ax + by + c = 0$ if $ax_1 + by_1 + c$ and c are of the same or opposite signs.

Example 33 :

Prove that Point (3, 4) and (–9, 6) lies on the opposite side of line $7x + 5y - 9 = 0$

Sol. Putting both the points in the given equation

$\Rightarrow 7 \times 3 + 5 \times 4 - 9 = 32$ and $7 \times (-9) + 5(6) - 9 = -42$

as both are of opposite sign so they lies opposite side of the given line.

Example 34 :

If the point (a, a^2) lies between the lines $x + y - 2 = 0$ and $4x + 4y - 3 = 0$ then find the range of values of a.

Sol. If (a, a^2) lies between the lines $x + y - 2 = 0$ and $4x + 4y - 3 = 0$ then sign of $a + a^2 - 2$ and $4a + 4a^2 - 3$ should be opposite hence $(a + a^2 - 2)(4a + 4a^2 - 3) < 0$

$\Rightarrow (a - 1)(a + 2)(2a + 3)(2a - 1) < 0$

$\Rightarrow a \in \left(-2, \frac{-3}{2}\right) \cup \left(\frac{1}{2}, 1\right)$

ANGLE BETWEEN TWO STRAIGHT LINES

The angle between two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

If any one line is parallel to y-axis then the angle between two straight line is given by $\tan\theta = \pm 1/m$

where m is the slope of other straight line

If the equation of lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then above formula would be

$$\tan\theta = \left|\frac{a_1b_2 - b_1a_2}{a_1a_2 + b_1b_2}\right|$$

Here two angles between two lines, but generally we consider the acute angle as the angle between them, so in all the above formula we take only positive value of $\tan\theta$.

Parallel Lines : Two lines are parallel, then angle between them is $0 \Rightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = \tan 0° = 0 \Rightarrow m_1 = m_2$

Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

Perpendicular lines : Two lines are perpendicular, then angle between them is 90°.

$$\Rightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = \tan 90° = \infty \Rightarrow m_1 m_2 = -1$$

Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular then $a_1a_2 + b_1b_2 = 0$.

Coincident Lines : Two lines $a_1x + b_1y + c_1 = 0$ and $a_1x + b_2y + c_2 = 0$ are coincident only and only if

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Example 35 :

Find the angle between $y = x + 6$ and $y = \sqrt{3}\, x + 7$.

Sol. Here, $m_1 = 1, m_2 = \sqrt{3}$.

$$\tan\theta = \left|\frac{1-\sqrt{3}}{1+\sqrt{3}}\right| \Rightarrow \theta = \tan^{-1}\frac{1-\sqrt{3}}{1+\sqrt{3}} = 15°$$

Example 36 :

If $7x + 3y + 9 = 0$ and $y = kx + 7$ are two parallel lines then find the value of k.

Sol. $m_1 = -7/3,\ m_2 = k$

Two lines are parallel if $m_1 = m_2$; $k = -7/3$

EQUATION OF PARALLEL & PERPENDICULAR LINES

(i) Equation of a line which is parallel to $ax + by + c = 0$ is $ax + by + k = 0$.

(ii) Equation of a line which is perpendicular to $ax + by + c = 0$ is $bx - ay + k = 0$.

The value of k in both cases is obtained with the help of additional information given in the problem.

Example 37 :

Find the equation of straight line parallel to $3x + 2y + 4 = 0$ and passing through (1, 1).

Sol. Let the line || to $3x + 2y + 4 = 0$ be

$3x + 2y + c = 0$

Since it passes through (1, 1)

$3(1) + 2(1) + c = 0$

$c = -5$

Equation of line $3x + 2y = 5$

Example 38 :

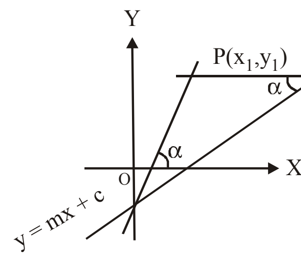
Find the equation of a line which passes through (–3, 2) and perpendicular to the $3x + 4y = 5$.

Sol. Let the eq. is $4x - 3y + k = 0$, this line passes through (–3, 2)

Hence $4(-3) - 3(2) + k = 0 \Rightarrow k = 18$

equation is $4x - 3y + 18 = 0$

EQUATION OF STRAIGHT LINES THROUGH (x_1, y_1) MAKING AN ANGLE α WITH $y = mx + c$



$$y - y_1 = \frac{m \mp \tan\alpha}{1 \pm m\tan\alpha}(x - x_1)$$

Example 39 :

One vertex of an equilateral triangle is (2,3) and the equation of line opposite to the vertex is $x + y = 2$, then find the equation of remaining two sides.

Sol. Since the two sides make an angle of 60° each with side $x + y = 2$. Therefore equations of these sides will be

$$y - 3 = \frac{-1 \pm \tan 60°}{1 \mp (-1)\tan 60°}(x-2) = \frac{-1 \pm \sqrt{3}}{1 \pm \sqrt{3}}(x-2)$$

$$\Rightarrow y - 3 = (2 \pm \sqrt{3})(x - 2)$$

Example 40 :

Find the equation of a line passing through (1, 2) and making an angle of 45° with the line $2x + 3y = 10$.

Sol. Slope of $2x + 3y = 10$ is $-2/3$

Let the slope of the required line is m, then

$$\tan 45° = \left|\frac{m - \left(-\frac{2}{3}\right)}{1 + m\left(-\frac{2}{3}\right)}\right|$$

$$1 = \left|\frac{3m+2}{3-2m}\right|$$

$3m + 2 = \pm(3 - 2m)$

$\Rightarrow$ $3m + 2 = 3 - 2m$ or $3m + 2 = -(3 - 2m)$

$m = 1/5$ or $m = -5$

Equation of newly formed lines

$y - 2 = -5(x - 1)$ or $y - 2 = \frac{1}{5}(x - 1)$

$y + 5x = 7$ or $5y - x = 9$

CONDITION OF CONCURRENCY

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, and $a_3x + b_3y + c_3 = 0$ are said to be concurrent, if they passes through a same point. The condition for their concurrency

is $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

Again, to test the concurrency of three lines, first find out the point of intersection of any two of the three lines. If this point lies on the remaining lines then the three lines are concurrent.

Example 41 :

Let λ & $\alpha \in R$. If lines

$\left.\begin{array}{l} \lambda x + (\sin\alpha)y + \cos\alpha = 0 \\ x + (\cos\alpha)y + \sin\alpha = 0 \\ -x + (\sin\alpha)y - (\cos\alpha) = 0 \end{array}\right\}$ are concurrent, then find the set of values of λ.

Sol. $\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_3$

$\begin{vmatrix} \lambda+1 & 0 & 2\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$

$\Rightarrow$ $(\lambda + 1)[-\cos^2\alpha - \sin^2\alpha] + 2\cos\alpha[\sin\alpha + \cos\alpha] = 0$

$\Rightarrow$ $\lambda + 1 = 2\cos\alpha[\sin\alpha + \cos\alpha]$

$\Rightarrow$ $\lambda = \sin 2\alpha + \cos 2\alpha$

$\Rightarrow$ $\lambda \in [-\sqrt{2}, \sqrt{2}]$

LENGTH OF PERPENDICULAR

The length P of the perpendicular from the point (x_1, y_1) on the line $ax + by + c = 0$ is given by

$$P = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Length of perpendicular from the point (x_1, y_1) on the line $x\cos\alpha + y\sin\alpha = p$ is $x_1\cos\alpha + y_1\sin\alpha = p$.

Distance between two parallel lines :

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

Example 42 :

Find the point on y-axis whose perpendicular distance from the line $4x - 3y - 12 = 0$ is 3

Sol. Let the point on y-axis be P(0, k)

Distance of P (0, k) from $4x - 3y - 12 = 0$ is

$$\left|\frac{4(0) - 3(k) - 12}{\sqrt{4^2 + (-3)^2}}\right| = 3$$

$|3k + 12| = 15$; $|k + 4| = 5$; $k = 1, -9$

Hence the points are (0, 1) & (0, –9)

Example 43 :

Find the distance between $3x + 2y + 7 = 0$ and $6x + 4y + 3 = 0$.

Sol. $\dfrac{\left|7 - \frac{3}{2}\right|}{\sqrt{3^2 + 2^2}} = \dfrac{\frac{11}{2}}{\sqrt{13}} = \dfrac{11}{2\sqrt{13}}$

BISECTORS OF ANGLES BETWEEN TWO LINES

A bisector is the locus of a point, which moves such that the length of perpendicular drawn from it to the two given lines, are equal.

(i) Equation of the bisector of angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

(ii) To discriminate between the acute angle bisector and the obtuse angle bisector : If θ be the angle between one of the lines and one of the bisector, find $\tan\theta$. If $|\tan\theta| < 1$ then $2\theta < 90°$ so that this bisector is the acute angle bisector. If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(iii) First write the equation of the lines so that the constant terms are positive. Then

(a) If $a_1a_2 + b_1b_2 > 0$ then on taking positive sign in the above bisectors equation we shall get the obtuse angle bisector and on taking negative sign we shall get the acute angle bisector.

(b) If $a_1a_2 + b_1b_2 < 0$, the positive sign give the acute angle and negative sign gives the obtuse angle bisector.

(c) On taking positive sign we shall get equation of the bisector of the angle which contains the origin and negative sign gives the equation of the bisector which does not contain origin.

Example 44 :

Find the equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$

Sol. The given equation can be written as

$3x - 4y + 7 = 0$ and $-12x - 5y + 2 = 0$

Here $a_1a_2 + b_1b_2 = 3(-12) - 4(-5) < 0$, so positive sign gives the acute angle bisector which is

$$\frac{3x - 4y + 7}{5} = \frac{-12x - 5y + 2}{13}$$

$\Rightarrow$ $99x - 27y + 81 = 0 \Rightarrow 11x - 3y + 9 = 0$

LINES PASSING THROUGH THE POINT OF INTERSECTION OF TWO LINES

If equation of two lines $P = a_1x + b_1y + c_1 = 0$ and $Q = a_2x + b_2y + c_2 = 0$, then the equation of the lines passing through the point of intersection of these lines is $P + \lambda Q = 0$ or $(a_1x + b_1y + c = 0) + \lambda (a_2x + b_2y + c_2 = 0) = 0$. Value of λ is obtained with the help of the additional information given in the problem.

Example 45 :

Find the equation of a line passing through the point of intersection of $x + y - 3 = 0$ & $2x - y + 1 = 0$ & a point $(2,-3)$

Sol. $(x + y - 3) + \lambda (2x - y + 1) = 0$

as this line also passes through $(2, -3)$

Hence $(2 - 3 - 3) + \lambda (2 \times 2 - (-3) + 1) = 0$

$-4 + 8\lambda = 0$; $\lambda = 1/2$

Equation is $(x + y - 3) + 1/2\ (2x - y + 1) = 0$

$4x + y - 5 = 0$

Example 46 :

It the family of straight lines $x(a + 2b) + y(a + 3b) = a + b$ passes through a fixed point for all values of a and b. Find the point.

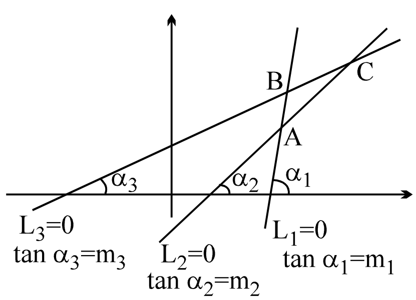
Sol. $x (a + 2b) + y (a + 3b) = a + b$

$\Rightarrow\ a (x + y - 1) + b (2x + 3y - 1) = 0$

This equation will always be satisfied for $x + y - 1 = 0$ & $2x + 3y - 1 = 0$ solving these equation we get

$x = 2, y = -1$

THE TANGENTS OF THE INTERIOR ANGLES OF A TRIANGLE FORMED BY 3 GIVEN LINES



Arrange the lines L_1, L_2 and L_3 in their descending order of slopes (as $m_1 > m_2 > m_3$) then tangents of interior angles of ΔABC can be written directly as

$$\tan A = \frac{m_1 - m_2}{1 + m_1m_2},\ \tan B = \frac{m_2 - m_3}{1 + m_2m_3},\ \tan C = \frac{m_3 - m_1}{1 + m_3m_1}$$

Explanation : $A = \alpha_1 - \alpha_2$ (from the figure)

$$\tan A = \tan (\alpha_1 - \alpha_2) = \frac{\tan\alpha_1 - \tan\alpha_2}{1 + \tan\alpha_1\ \tan\alpha_2} = \frac{m_1 - m_2}{1 + m_1m_2}$$

$B = \alpha_2 - \alpha_3$

$$\tan B = \tan(\alpha_2 - \alpha_3) = \frac{\tan\alpha_2 - \tan\alpha_3}{1 + \tan\alpha_2\ \tan\alpha_3} = \frac{m_2 - m_3}{1 + m_2m_3}$$

$$\pi - C = \alpha_1 - \alpha_3 \quad \therefore\ C = \pi + \alpha_3 - \alpha_1$$

$$\tan C = \tan (\alpha_3 - \alpha_1) = \frac{\tan\alpha_3 - \tan\alpha_1}{1 + \tan\alpha_3\ \tan\alpha_1} = \frac{m_3 - m_1}{1 + m_3m_1}.$$

Example 47 :

If a ΔABC is formed by the lines $2x + y - 3 = 0$; $x - y + 5 = 0$ and $3x - y + 1 = 0$, then obtain a cubic equation whose roots are the tangent of the interior angles of the triangle.

Sol. Given lines are

$2x + y - 3 = 0$ ...(i)

$x - y + 5 = 0$ ...(ii)

$3x - y + 1 = 0$ ...(iii)

Slope of line (i) = –2, Slope of line (ii) = 1

Slope of line (iii) = 3

Arranging the lines in descending order of their slopes

$\Rightarrow\ m_1 = 3,\ m_2 = 1, m_3 = -2$

$$\Rightarrow\ \tan A = \frac{m_1 - m_2}{1 + m_1m_2} = \frac{3 - 1}{1 + (3)(1)} = \frac{1}{2}$$

$$\tan B = \frac{m_2 - m_3}{1 + m_2m_3} = \frac{1 - (-2)}{1 + 1(-2)} = -3$$

$$\tan C = \frac{m_3 - m_1}{1 + m_3m_1} = \frac{(-2) - (3)}{1 + (-2)\,3} = 1$$

Roots of cubic are –3, 1/2 and 1.

$$\Rightarrow\ \text{Equation of cubic is } (x + 3)\left(x - \frac{1}{2}\right)(x - 1) = 0$$

$\Rightarrow\ 2x^3 + 3x^2 - 8x + 3 = 0$

HOMOGENEOUS EQUATION OF SECOND DEGREE

A Homogeneous equation of degree two of the type $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin and if.

(i) $h^2 > ab \Rightarrow$ Lines are real and distinct

(ii) $h^2 = ab \Rightarrow$ Lines are real and coincident

(iii) $h^2 < ab \Rightarrow$ Lines are imaginary with real point of intersection i. e. (0, 0)

If $y = m_1x$ and $y = m_2x$ be the two equation represented by

$$ax^2 + 2hxy + by^2 = 0 \text{ then } m_1 + m_2 = -\frac{2h}{b},\ m_1m_2 = \frac{a}{b}$$

If θ is the acute angle between the pair of straight lines represented by $ax^2 + 2hxy + by^2 = 0$, then

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right| \text{ and if}$$

(a) $a + b = 0 \Rightarrow$ Lines are perpendicular

(b) $h = 0 \Rightarrow$ Lines are equally inclined to the axis

The equation of the straight lines bisecting the angles between the straight lines, $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

GENERAL EQUATION OF SECOND DEGREE

An Equation of the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is called the general equation of second degree in x and y it represent a pair of two straight lines if

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

The angle θ between the two lines representing by a general equation is the same as that between the lines represented by its Homogeneous part only i.e. $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

EQUATION OF LINES JOINING THE INTERSECTION POINTS OF A LINE & CURVE TO THE ORIGIN

Let $\ell x + my + n = 0$ ..... (1)

and the second degree curve

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ..... (2)

then their joint equation is

$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{\ell x + my}{-n}\right) + 2fy\left(\frac{\ell x + my}{-n}\right) + c\left(\frac{\ell x + my}{-n}\right) = 0$$

i.e. making the equation (2) homogeneous using eq. (1)

Example 48 :

Lines represented by $2x^2 - 7xy + 3y^2 = 0$ are

(1) $x + 2y = 0, x - 3y = 0$ (2) $x - 2y = 0, x + 3y = 0$

(3) $2x - y = 0, 3x - y = 0$ (4) $2x - y = 0, x - 3y = 0$

Sol. (4). $2x^2 - 7xy + 3y^2 = 0 = (2x - y)(x - 3y)$

$\therefore$ $2x - y = 0, x - 3y = 0$

Example 49 :

For what value of k, the equation $kx^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines

Sol. Here $a = k, b = 12, c = -3, h = -5, g = 5/2, f = -8$

$\therefore \Delta = k(12),(-3) + 2(-8)(5/2)(-5) - k(-8)^2 - 12(5/2)^2 - (-3)(-5)^2 = 0$

$\Rightarrow k = 2$

Example 50 :

Find the angle and equation of bisectors of angle between the lines represented by the equation $2x^2 - 7xy + 3y^2 = 0$.

Sol. Since $2x^2 - 7xy + 3y^2 = (2x - y)(x - 3y)$

$\therefore$ Given lines are $2x - y = 0$ and $x - 3y = 0$

Since $a = 2, h = -7/2, b = 3$

Therefore angle between lines given by

$$\tan\theta = \frac{2\sqrt{(-7/2)^2 - 6}}{2+3} = 1 \Rightarrow \theta = 45°$$

Again equations of bisectors are given by

$$\frac{x^2 - y^2}{2-3} = \frac{xy}{-7/2} \Rightarrow 7x^2 - 2xy - 7y^2 = 0$$

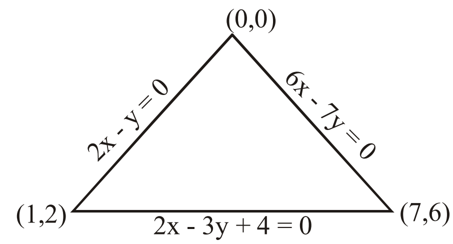
Example 51 :

Find the centroid of the triangle the equation of whose sides $12x^2 - 20xy + 7y^2 = 0$ and $2x - 3y + 4 = 0$.

Sol. $12x^2 - 20xy + 7y^2 = 0$

$12x^2 - 14xy - 6xy + 7y^2 = 0$

$2x(6x - 7y) - y(6x - 6y) = 0 \Rightarrow (2x - y)(6x - 7y) = 0$



So three lines are

$2x - y = 0$ .... (i)

$6x - 7y = 0$ .... (ii)

$2x - 3y + 4 = 0$ .... (iii)

Point of intersection of lines (i) and (ii) is (0, 0).

Point of intersection of lines (i) and (iii) is (1, 2).

and Point of intersection of lines (ii) and (iii) is (7, 6).

Centroid is $\left(\frac{0+1+7}{3}, \frac{0+2+6}{3}\right) = \left(\frac{8}{3}, \frac{8}{3}\right)$.

Example 52 :

Find the distance between the parallel lines

$4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$

Sol. Given lines is $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$

Again, $4x^2 + 4xy + y^2 = 0$

$(2x + y)^2 = 0$

$\Rightarrow$ $(2x + y + A)(2x + y + B) = 4x^2 + 4xy + y^2 - 6x - 3y - 4$

$\Rightarrow$ $4x^2 + 4xy + y^2 + (2A + 2B)x + (A + B)y + AB$

$= 4x^2 + 4xy + y^2 - 6x - 3y - 4$

Comparing both sides, we get

$A + B = -3$ ... (i)

$AB = -4$ ... (ii)

By solving (i) and (ii) we get, $A = -4, B = 1$

Two parallel lines are $2x - y - 4 = 0$ and $2x + y + 1 = 0$

Distance $= \left|\frac{-4-1}{\sqrt{2^2 + 1^2}}\right| = \sqrt{5}$.

Example 53 :

Find the equation of the line pair joining origin and the point of intersections of the line $2x - y = 3$ and the curve $x^2 - y^2 - xy + 3x - 6y + 18 = 0$. Also find the angle between these two lines.

Sol. Given curve : $x^2 + y^2 - xy + 3x - 6y + 18 = 0$

and line : $2x - y = 3$

Covert the curve in homogeneous equation with the help of line

$$x^2 - y^2 - xy + 3x\left(\frac{2x - y}{3}\right) - 6y\left(\frac{2x - y}{3}\right) + 18\left(\frac{2x - y}{3}\right)^2 = 0$$

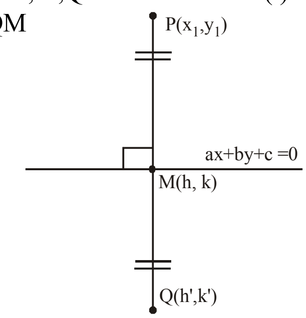
$\Rightarrow x^2 - y^2 - xy + x(2x - y) - 2y(2x - y) + 2(2x - y)^2 = 0$
$\Rightarrow x^2 - y^2 - xy + 2x^2 - xy - 4xy + 2y^2 + 8x^2 + 2y^2 - 8xy = 0$
$\Rightarrow 11x^2 + 3y^2 - 14xy = 0$
Angle between them is

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} \text{ where } a = 11, b = 3 \text{ and } h = -7$$

$$= \frac{2\sqrt{49 - 33}}{14} \text{ or } \tan\theta = 4/7 \Rightarrow \theta = \tan^{-1}(4/7)$$

FOOT OF THE PERPENDICULAR AND REFLECTION OF A POINT WITH RESPECT TO A LINE IS :

If M (h, k) be the foot of perpendicular from a point $P(x_1, y_1)$ on the line ax+by+c = 0 and Q(h',k') be the reflection of P in this line, the P,M,Q are collinear and (i) PM ⊥ the given line (ii) PM = QM



Further M. Q can be determine using following formula

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

$$\frac{h' - x_1}{a} = \frac{k' - y_1}{b} = -\frac{2(ax_1 + by_1 + c)}{a^2 + b^2}$$

Example 54 :
Find the foot of perpendicular drawn from the origin and the reflection of the origin on the line x + y = 2.

Sol. Let foot of the perpendicular be (h, k), and reflection point be (h′ k′) . Then

$$\frac{h-0}{1} = \frac{k-0}{1} = -\frac{0+0-2}{1+1} \Rightarrow h = 1, k = 1$$

∴ Perpendicular = (1, 1)

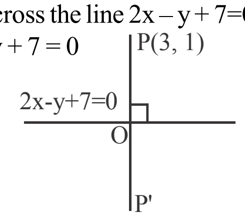
Also $\frac{h'-0}{1} = \frac{k'-0}{1} = -\frac{2(0+0-2)}{1+1}$

$\Rightarrow$ h′ = 2 , k′ = 2

∴ Reflection point = (2, 2)

Example 55 :
Find the image of (3, 1) across the line y = 2x + 7.

Sol. Let the point P(3, 1) has image P' across the line 2x – y + 7=0
Now PP' is perpendicular to 2x – y + 7 = 0
Slope of PP' = – 1/2
Equation of PP' is



$$y - 1 = -\frac{1}{2}(x - 3)$$

2y – 2 = –x + 3
x + 2y = 5
Point of intersection of lines 2x – y + 7 = 0 and x + 2y = 5 is

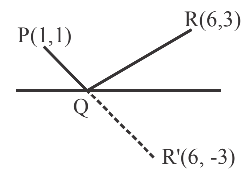
$O\left(\frac{-9}{5}, \frac{17}{5}\right)$. O is the mid point of PP'. Let P' (h, k)

$$\frac{h+3}{2} = \frac{-9}{5} \;;\; \frac{k+1}{2} = \frac{17}{5}$$

$$h = \frac{-33}{5}, k = \frac{29}{5} \Rightarrow \text{Image}\left(-\frac{33}{5}, \frac{29}{5}\right)$$

Example 56 :
A ray starts from point (1, 1) and is reflected by x-axis and then it passes through the point (6, 3). Find the equation of
(i) Incident ray (ii) Reflected ray

Sol. If the ray started from (1, 1) and after reflection it passes through (6, 3) then incident ray was supposed to pass from the image of (6, 3) across x-axis.
Image of R(6, 3) in x-axis is R'(6, –3)
Equation of PQ



$$y - 1 = \left(\frac{-3-1}{6-1}\right)(x - 1)$$

5y – 5 = –4x + 4
4x + 5y – 9 = 0
to find equaiton of reflected ray we have to find Q.

Q is point of intersection of ray with x-axis $Q\left(\frac{9}{4}, 0\right)$

Equation of QR $y - 3 = \left(\frac{3-0}{6-9/4}\right)(x - 6)$

$$y - 3 = \frac{4}{5}(x - 6);\; 5y - 15 = 4x - 24;\; 4x - 5y - 9 = 0$$

TRY IT YOURSELF-3

Q.1 If $\frac{a}{\sqrt{bc}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}}$, where a, b, c > 0, then family of lines $\sqrt{a}x + \sqrt{b}y + \sqrt{c} = 0$ passes through the fixed point given by –
(A) (1, 1) (B) (1, –2)
(C) (–1, 2) (D) (–1, 1)

Q.2 If it is possible to draw a line which belongs to all the given family of lines $y - 2x + 1 + \lambda_1(2y - x - 1) = 0$, $3y - x - 6 + \lambda_2(y - 3x + 6) = 0$, $ax + y - 2 + \lambda_3(6x + ay - a) = 0$, then –
(A) a = 4 (B) a = 3
(C) a = –2 (D) a = 2

Q.3 A light ray coming along the line 3x + 4y = 5 gets reflected from the line ax+by=1 & goes along the line 5x – 12y=10 then
(A) a = 64/115, b = 112/15 (B) a = 14/15, b = –8/115
(C) a = 64/115, b = –8/115 (D) a = 64/15, b = 14/15

Q.4 The coordinates of the foot of the perpendicular from the point (2, 3) on the line $-y + 3x + 4 = 0$ are given by –
(A) (37/10, –1/10) (B) (–1/10, 37/10)
(C) (10/37, –10) (D) (2/3, –1/3)

Q.5 If the family of straight lines $x(a + 2b) + y(a + 3b) = a + b$ passes through a fixed point for all values of a and b. Find the point.
(A) (2, –2) (B) (2, 1)
(C) (2, –1) (D) (3, –1)

Q.6 Find the image of (3, 1) across the line $y = 2x + 7$.
(A) $\left(\frac{33}{5}, \frac{29}{5}\right)$ (B) $\left(\frac{-33}{5}, \frac{-29}{5}\right)$
(C) $\left(\frac{33}{5}, \frac{-29}{5}\right)$ (D) $\left(\frac{-33}{5}, \frac{29}{5}\right)$

Q.7 Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is
(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$
(C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

Q.8 Area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pairs of straight lines $x^2 - y^2 + 2y = 1$ is
(A) 2 sq. units (B) 4 sq. units
(C) 6 sq. units (D) 8 sq.units

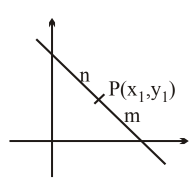
Q.9 A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is –
(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

ANSWERS

(1) (D)	**(2)** (A)	**(3)** (C)
(4) (B)	**(5)** (C)	**(6)** (D)
(7) (C)	**(8)** (A)	**(9)** (B)

SOME IMPORTANT POINTS

(i) A line passes through (x_1, y_1) if intercept between the axes divides in the ratio m : n at this point then the equation is



$$\frac{nx}{x_1} + \frac{my}{y_1} = m + n$$

(ii) A line of gradient m is equally inclined with the two lines of gradient m_1 and m_2 then $\frac{m_1 - m}{1 + m_1 m} = -\frac{m_2 - m}{1 + m_2 m}$

(iii) If a + b + c then line ax + by + c = 0 passes through (1, 1)

(iv) If $y = m_1x + c_1$, $y = m_1x + c_2$, $y = m_2x + d_1$ and $y = m_2x + d_2$ are the sides of a parallelogram then its area is

$$\left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$$

ADDITIONAL EXAMPLES

Example 1 :
The vertices of a triangle are A (0, – 6), B (– 6, 0) & C (1, 1) respectively, then find the coordinates of the ex-centre opposite to vertex A.

Sol. $a = BC = \sqrt{(-6-1)^2 + (0-1)^2} = \sqrt{50} = 5\sqrt{2}$

$b = CA = \sqrt{(1-0)^2 + (1+6)^2} = \sqrt{50} = 5\sqrt{2}$

$c = AB = \sqrt{(0+6)^2 + (-6-0)^2} = \sqrt{72} = 6\sqrt{2}$

Coordinates of ex-centre opposite to vertex A are

$$x = \frac{-ax_1 + bx_2 + cx_3}{-a + b + c}$$

$$= \frac{-5\sqrt{2}.0 + 5\sqrt{2}(-6) + 6\sqrt{2}(1)}{-5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} = \frac{-24\sqrt{2}}{6\sqrt{2}} = -4$$

$$y = \frac{-ay_1 + by_2 + cy_3}{-a + b + c}$$

$$= \frac{-5\sqrt{2}(-6) + 5\sqrt{2}.0 + 6\sqrt{2}(1)}{-5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} = \frac{-36\sqrt{2}}{6\sqrt{2}} = -6$$

Hence coordinates of ex-centre are (–4, – 6)

Example 2 :
If A (1,4) ; B (3, 0) and C (2,1) are vertices of a triangle then find the median through C.

Sol. The mid point D of sides AB = (2,2)

$\therefore$ the length of the median CD = $\sqrt{(2-2)^2 + (2-1)^2} = 1$

Example 3 :
Find the distance between the point P (a cosα, a sinα) and Q (a cos β, a sin β).

Sol. $d^2 = (a\cos\alpha - a\cos\beta)^2 + (a\sin\alpha - a\sin\beta)^2$
$= a^2(\cos\alpha - \cos\beta)^2 + a^2(\sin\alpha - \sin\beta)^2$
=

$$a^2\left\{2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}\right\}^2 + a^2\left\{2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right\}^2$$

$$= 4a^2\sin^2\frac{\alpha-\beta}{2}\left\{\sin^2\frac{\alpha+\beta}{2} + \cos^2\frac{\alpha+\beta}{2}\right\}$$

$$= 4a^2\sin^2\frac{\alpha-\beta}{2} \Rightarrow d = 2a\sin\frac{\alpha-\beta}{2}$$

Example 4 :

Find the condition that the three points (a, 0), $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are collinear.

Sol. Here the points are collinear if the are of the triangle is zero, hence, $1/2\,[a(t_1^2-1)\,2at_2 - 2at_1(at_2^2 - a)] = 0$

or $t_2(t_1^2-1) - t_1(t_2^2-1) = 0$

$\Rightarrow t_2t_1^2 - t_2 - t_1t_2^2 + t_1 = 0 \Rightarrow (t_1-t_2)(t_1t_2+1) = 0,\ t_1 \neq t_2$

$\therefore t_1t_2 + 1 = 0 \Rightarrow t_1t_2 = -1$

Example 5 :

Find the locus of the point of intersection of the line $x\sin\theta + (1-\cos\theta)\,y = a\sin\theta$ and $x\sin\theta - (1+\cos\theta) + y + a\sin\theta = 0$.

Sol. From the given equations we have

$$\frac{1-\cos\theta}{\sin\theta} = \frac{a-x}{y} \text{ and } \frac{1+\cos\theta}{\sin\theta} = \frac{x+a}{y}$$

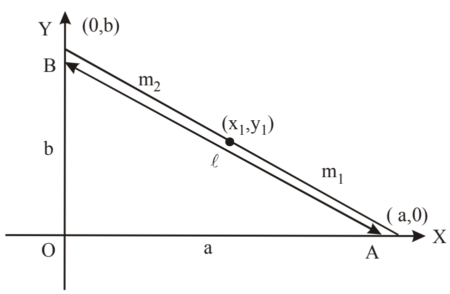
Multiplying we get

$$\frac{1-\cos^2\theta}{\sin^2\theta} = \frac{a^2-x^2}{y^2} \Rightarrow x^2+y^2 = a^2$$

Example 6 :

The ends of the rod of length ℓ moves on two mutually perpendicular lines, find the locus of the point on the rod which divides it in the ratio $m_1 : m_2$.

Sol. Let (x_1, y_1) be the point that divide the rod AB = ℓ, in the ratio $m_1 : m_2$ and OA = a, OB = b



$\therefore a^2 + b^2 = \ell^2$ ....(1)

$$\text{Now } x_1 = \left(\frac{m_2 a}{m_1+m_2}\right) \Rightarrow a = \left(\frac{m_1+m_2}{m_2}\right) x_1$$

$$y_1 = \left(\frac{m_1 b}{m_1+m_2}\right) \Rightarrow b = \left(\frac{m_1+m_2}{m_1}\right) y_1$$

These putting in (1)

$$\frac{(m_1+m_2)^2}{m_2^2}x_1^2 + \frac{(m_1+m_2)^2}{m_1^2}y_1^2 = \ell^2$$

$$\therefore \text{Locus of } (x_1, y_1) \text{ is } m_1^2x^2 + m_2^2y^2 = \left(\frac{m_1\,m_2\ell}{m_1+m_2}\right)^2$$

Example 7 :

Find the number of points on x-axis which are at a distance c (c < 3) from the point (2, 3).

Sol. Let a point on x-axis is $(x_1, 0)$, then its distance form the point $(2, 3) = \sqrt{(x_1-2)^2+9} = c$ or $(x_1-2)^2 = c^2 - 9$

$\therefore\quad x_1 - 2 = \sqrt{c^2-9}$

But $C < 3 \Rightarrow c^2 - 9 < 0 \quad \therefore x_1$ will be imaginary.

Example 8 :

Find the distance of the point (2, 3) from the line 2x–3y+9=0 measured along a line x – y + 1 = 0.

Sol. The slope of the line x – y + 1 = 0 is 1. So it makes an angle of 45° with x axis. The equation of a line passing through (2, 3) and making an angle of 45° is

$$\frac{x-2}{\cos 45°} = \frac{y-3}{\sin 45°} = r \qquad \left[\text{using } \frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r\right]$$

Coordinates of any point on this line are

$$(2 + r\cos 45°, 3 + r\sin 45°) \text{ or } \left(2+\frac{r}{\sqrt{2}}, 3+\frac{r}{\sqrt{2}}\right)$$

If this point lies on this line 2x – 3y + 9 = 0, then

$$4 + r\sqrt{2} - 9 - \frac{3r}{\sqrt{2}} + 9 = 0 \Rightarrow 4\sqrt{2}$$

So the required distance = $4\sqrt{2}$

Example 9 :

If A (–2, 1), B (2, 3) and C (–2, –4) are three points, then find the angle between BA and BC.

Sol. Let m_1 and m_2 be the slopes of BA and BC respectively.

$$\text{Then } m_1 = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2} \text{ and } m_2 = \frac{-4-3}{-2-2} = \frac{7}{4}$$

Let θ be the angle between BA and BC. Then

$$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right| = \left|\frac{\frac{7}{4}-\frac{1}{2}}{1+\frac{7}{4}\times\frac{1}{2}}\right| = \left|\frac{\frac{10}{8}}{\frac{15}{8}}\right| = \pm\frac{2}{3}$$

$\Rightarrow \theta = \tan^{-1}(2/3)$

Example 10 :

A straight line L perpendicular to the line 5x - y = 1 .The area of the triangle formed by the line L and co-ordinates axes is 5, then find the equation of line

Sol. Let the line L cuts the axes at A and B say. OA= a ,OB = b

$\therefore$ Area $\Delta OAB = \frac{1}{2}ab = 5$ ....(1)

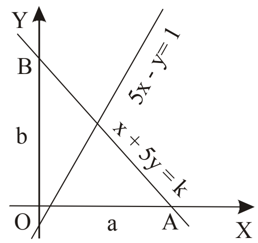
Now equation of line perpendicular to lines $5x - y = 1$ is $x + 5y = k$

Putting $x = 0, y = \frac{k}{5} = b,$

$y = 0, x = k = a$

$\therefore \frac{1}{2}k \cdot \frac{k}{5} = 5$ from (i)

$k^2 = 50 \Rightarrow k = 5\sqrt{2}$



Hence the required line is $x + 5y = \pm 5\sqrt{2}$

Example 11 :

The sides AB, BC, CD and DA of a quadrilateral have the equations $x + 2y = 3, x = 1, x - 3y = 4, 5x + y + 12 = 0$ respectively, then find the angle between the diagonals AC and BD.

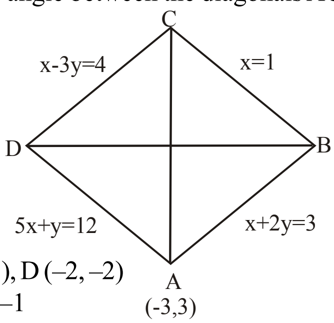
Sol. Solving for A,

$x + 2y - 3 = 0$

$5x + y + 12 = 0$

$$\frac{x}{+24+3} = \frac{y}{-15+3} = \frac{1}{-9}$$

$\therefore A(-3, 3)$



Similarly B (1, 1), C (1, – 1), D (–2, –2)

m_1 = slope of AC = –1

m_2 = slope of BD = 1

$m_1 m_2 = -1$

$\therefore$ the angle required is 90°

Example 12 :

The equation of two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side is passes through the point (1,– 10). Find the equation of the third side.

Sol. Third side passes through (1, – 10) so let its equation be

$y + 10 = m(x - 1)$

If it makes equal angle, say θ with given two sides, then

$$\tan\theta = \frac{m-7}{1+7m} = \frac{m-(-1)}{1+m(-1)} \Rightarrow m = -3 \text{ or } 1/3$$

Hence possible equations of third side are

$$y + 10 = -3(x - 1) \text{ and } y + 10 = \frac{1}{3}(x - 1)$$

or $3x = y + 7 = 0$ and $x - 3y - 31 = 0$

Example 13 :

Find the area of the parallelogram formed by the lines $4y - 3x = 1, 4y - 3x - 3 = 0, 3y - 4x + 1 = 0, 3y - 4x + 2 = 0$ is

Sol. Let the equation of sides AB, BC CD and DA of parallelogram ABCD are respectively

$$y = \frac{3}{4}x + \frac{1}{4} \quad \text{..........(1)}; \quad y = \frac{3}{4}x + \frac{3}{4} \quad \text{..........(2)}$$

$$y = \frac{4}{3}x - \frac{1}{3} \quad \text{..........(3)}; \quad y = \frac{4}{3}x - \frac{2}{3} \quad \text{..........(4)}$$

Here $m = \frac{3}{4}, n = \frac{4}{3}, a = \frac{1}{4}, b = \frac{3}{4}, c = -\frac{1}{3}, d = -\frac{2}{3}$

$\therefore$ Area of parallelogram ABCD

$$= \left|\frac{(a-b)(c-d)}{m-n}\right| = \left|\frac{\left(\frac{1}{4}-\frac{3}{4}\right)\left(-\frac{1}{3}+\frac{2}{3}\right)}{\frac{3}{4}-\frac{4}{3}}\right| = \left|\frac{-\frac{1}{2}\times\frac{1}{3}}{-\frac{7}{12}}\right| = \frac{2}{7}$$

Example 14 :

The equation of a line through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$.

Sol. Let the required line by method

$P + \lambda Q = 0$ be $(x - 3y + 1) + \lambda(2x + 5y - 9) = 0$

$\therefore$ Perpendicular from $(0, 0) = \sqrt{5}$ gives

$$\frac{1-9\lambda}{\sqrt{(1+2\lambda)^2 + (5-3\lambda)^2}} = \sqrt{5}$$

squaring and simplifying $(8\lambda - 7)^2 = 0 \Rightarrow \lambda = 7/8$

Hence the line required is

$(x - 3y + 1) + 7/8(2x + 5y - 9) = 0$

or $22x + 11y - 55 = 0 \Rightarrow 2x + y - 5 = 0$

Example 15 :

Find the distance between the lines represented by the equation $x^2 + 2\sqrt{2}xy + 2y^2 + 4x + 4\sqrt{2}y + 1 = 0$.

Sol. The given equation can be written as

$x^2 + 2(\sqrt{2}y + 2)x + (2y^2 + 4\sqrt{2}y + 1) = 0$

$$\therefore x = \frac{-2(\sqrt{2}y+2) \pm \sqrt{4(\sqrt{2}y+2)^2 - 4(2y^2 + 4\sqrt{2}y + 1)}}{2}$$

$= -(\sqrt{2}y + 2) \pm \sqrt{3}$

Therefore given lines are $x + \sqrt{2}y + 2 + \sqrt{3} = 0$

and $x + \sqrt{2}y + 2 - \sqrt{3} = 0$ which are parallel lines.

Therefore distance between them

$$\left|\frac{(2+\sqrt{3})-(2-\sqrt{3})}{\sqrt{1+2}}\right| = 2$$

Example 16 :

If sum and product of the slopes of lines represented by $4x^2 + 2hxy - 7y^2 = 0$ is equal then find the value of h.

Sol. Let the equations are $y = m_1 x$ and $y = m_2 x$

$\therefore 4x^2 + 2hxy - 7y^2 = (y - m_1 x)(y - m_2 x)$

$$\therefore m_1 m_2 = -\frac{4}{7}, m_1 + m_2 = \frac{2h}{7}$$

Given : $m_1 + m_2 = m_1 m_2 \Rightarrow \frac{2h}{7} = -\frac{4}{7} \Rightarrow h = -2$

Example 17 :

Find the condition that the pair of straight lines joining the origin to the intersections of the line y = mx + c and the circle $x^2 + y^2 = a^2$ may be at right angles.

Sol. The equation of the given straight line and the given curve

are $y = mx + c \Rightarrow \frac{y - mx}{c} = 1$ ....(1)

and $x^2 + y^2 = a^2$ ....(2)

The combined equation of the straight lines joining the origin to the point of intersection of (1) and (2) is

$$x^2 + y^2 = \left(\frac{y - mx}{c}\right)^2 . a^2$$

$\Rightarrow x^2 (c^2 - a^2 m^2) + 2a^2 mxy + y^2 (c^2 - a^2) = 0$ ....(3)

The lines given by (3) are at right angles, if

Coeff. of x^2 + Coeff. of $y^2 = 0$

$\Rightarrow (c^2 - a^2m^2) + (c^2 - a^2) = 0$

$\Rightarrow 2c^2 = a^2 (1 + m^2)$, which is the required condition.

Example 18 :

Find the area of the pentagon whose vertices are A (1, 1), B (7, 21), C (7, –3), D (12, 2) and E (0, –3).

Sol. The required area

$$= \frac{1}{2}\left|\begin{vmatrix} 1 & 1 \\ 7 & 21 \end{vmatrix} + \begin{vmatrix} 7 & 21 \\ 7 & -3 \end{vmatrix} + \begin{vmatrix} 7 & -3 \\ 12 & 2 \end{vmatrix} + \begin{vmatrix} 12 & 2 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 0 & -3 \\ 1 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2} |(21 - 7) + (-21 - 147) + (14 + 36) + (-36 - 0) + (0 + 3)|$$

$$= \frac{1}{2} |-137| = \frac{137}{2} \text{ sq. units.}$$

Example 19 :

For every value of p and q, the line (p + 2q) x + (p – 3q) y = p – q passes through the fixed point (a, b) then find a, b.

Sol. The given equation may be written as

$p (x + y - 1) + q (2x - 3y + 1) = 0$

$\Rightarrow (x + y - 1) + (q/p) (2x - 3y + 1) = 0$

which is of the form L + λ L' = 0. So it passes through the point of intersection of L = 0 and L' = 0 i.e., $x + y - 1 = 0$ and $2x - 3y + 1 = 0$

Solving these two we get the required point as (2/5, 3/5)

Example 20 :

If the equations of the pairs of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$, then find equations of its diagonals.

Sol. Equations of the sides of the parallelogram are

$(x - 3) (x - 2) = 0$ and $(y - 5) (y - 1) = 0$

i.e. x = 3, x = 2 ; y = 5, y = 1

Hence its vertices are : A (2 ,1); B (3, 1) ; C (3, 5) ; D (2, 5)

Equation of the diagonal AC is

$$y - 1 = \frac{4}{1} (x - 2) \Rightarrow y = 4x - 7$$

Equation of the diagonal BD is

$$y - 1 = \frac{4}{-1} (x - 3) \Rightarrow 4x + y = 13$$

Example 21 :

If the lines x + 2ay + a = 0, x + 3by + b = 0 and x + 4cy+c = 0 are concurrent, then a, b and c are in

(1) A.P. (2) G.P.

(3) H.P. (4) None of these

Sol. **(3).** Given lines will be concurrent if

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \Rightarrow -bc + 2ac - ab = 0 \Rightarrow b = \frac{2ac}{a + c}$$

$\Rightarrow$ a, b, c are in H.P.

Example 22 :

If $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, then find the value of λ

Sol. Here = a = λ, b = 12, c = – 3, f = – 8, g = 5/2, h = – 5

Using condition $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, we have

$\lambda (12) (-3) + 2 (-8) (5/2) (-5) - \lambda (64) - 12 (25/4) + 3 (25) = 0$

$\Rightarrow -36 \lambda + 200 - 64\lambda - 75 + 75 = 0$

$\Rightarrow 100\lambda = 200 \quad \therefore \lambda = 2$

Example 23 :

If the sides of triangle are x + y – 5 = 0, x – y + 1 = 0 and y – 1 = 0, then find its circumcentre.

Sol. Here the sides x + y – 5 = 0 and x – y + 1 = 0 are perpendicular to each other, therefore y = 1 will be hypotenuse of the triangle. Now its middle point will be the circumcentre.

Now solving the pair of equations

x + y – 5 = 0, y –1 = 0

and x – y + 1 = 0, y – 1 =0, we get

P ≡ (4, 1), Q ≡ (0, 1)

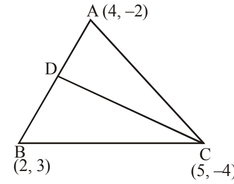
Mid point of PQ or circumcentre = (2, 1)

Example 24 :

The vertices of a triangle are A (4, –2), B (2, 3) and C (5, – 4). Find the equation of the median through C.

Sol. Coordinates of the mid point D (fig) of AB are

$$\left(\frac{2 + 4}{2}, \frac{3 - 2}{2}\right) \text{ i.e. } \left(3, \frac{1}{2}\right)$$



Thus, the median through C is

$$y - (-4) = \frac{\frac{1}{2} - (-4)}{3 - 5} (x - 5) \text{ (Two point form)}$$

or $y + 4 = \frac{9/2}{-2} (x - 5)$ or, $9 (x - 5) = -4 (y + 4)$

or $9x + 4y - 29 = 0$

Example 25 :

Find the point of intersection of lines represented by $2x^2 - 7xy - 4y^2 - x + 22y - 10 = 0$.

Sol. Let $\phi = 2x^2 - 7xy - 4y^2 - x + 22y - 10 = 0$

$$\frac{\partial \phi}{\partial x} = 4x - 7y - 1 = 0 \text{ and } \frac{\partial \phi}{\partial y} = -7x - 8y + 22 = 0$$

Solving above equations the point of intersection is $(x, y) = (2, 1)$.

Example 26 :

Find number of straight lines passing through (2, 4) & forming a triangle of 16 sq. cm with the co-ordinate axes.

Sol. Let the co-ordinates on x-axis be (h, 0)

Equation of line becomes $y - 0 = \frac{4}{2-h}(x-h)$

y-intercept $= \frac{4h}{h-2}$

area of triangle $= \frac{1}{2}\left|h \times \frac{4h}{h-2}\right| = 16$; $\left|\frac{h^2}{h-2}\right| = 8$

$h^2 = 8h - 16$ or $h^2 = -8h + 16$

$h^2 - 8h + 16 = 0$ or $h^2 + 8h - 16 = 0$

$(h-4)^2 = 0$

Three values of h are possible hence three equations are possible.

Example 27 :

Three lines $x + 2y + 3 = 0$, $x + 2y - 7 = 0$ and $2x - y - 4 = 0$ form three sides of two squares find the equations to the fourth sides of squares.

Sol. Distance between the lines $x + 2y + 3 = 0$ & $x + 2y - 7 = 0$ is

$$d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right| = \left|\frac{3-(-7)}{\sqrt{1+4}}\right| = \frac{10}{\sqrt{5}}$$

The fourth side is parallel to $2x - y - 4 = 0$

Let the fourth side be $2x - y + k = 0$

Distance between two sides $2x - y - 4 = 0$ and $2x - y + k = 0$

should be $\frac{10}{\sqrt{5}}$. $\left|\frac{k+4}{\sqrt{4+1}}\right| = \frac{10}{\sqrt{5}}$; $|k+4| = 10$

$k = 6$ or $k = -14$

hence the 4^{th} sides of squares are

$2x - y + 6 = 0$ or $2x - y - 14 = 0$

Example 28 :

A variable line $ax + by + c = 0$ passes through a fixed point if a, b, c are in arithmetic progression. Find the fixed pont.

Sol. If a, b, c are in A.P. then $2b = a + c$; $ax + by + c = 0$

$$ax + \left(\frac{a+c}{2}\right) y + c = 0$$

$$a\left(x + \frac{y}{2}\right) + c\left(\frac{y}{2} + 1\right) = 0$$

Solving $x + \frac{y}{2} = 0$ & $\frac{y}{2} + 1 = 0$,

we get $y = -2, x = 1$

Example 29 :

Find the point at which the origin be shifted so that the equation $x^2 + y^2 - 5x + 2y - 5 = 0$ has no first degree terms.

Sol. $x^2 + y^2 - 5x + 2y - 5 = 0$

$$\Rightarrow \left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + (y^2 + 2y + 1) - 1 - 5 = 0$$

$$\Rightarrow \left(x - \frac{5}{2}\right)^2 + (y+1)^2 - \frac{49}{4} = 0$$

$$\Rightarrow X^2 + Y^2 = \frac{49}{4}$$

If new equation to be formed has no first degree terms then shift the origin to $x = 5/2, y = -1 = (5/2, -1)$.

Example 30 :

Find the value of 'm' if the lines joining the origin to the points common to $x^2 + y^2 + x - 2y - m = 0$ & $x + y = 1$ are at right angles.

Sol. Given : $x^2 + y^2 + x - 2y - m = 0$

and a line : $x + y = 1$

Homogenize the curve with the help of line

$\Rightarrow x^2 + y^2 + x(x+y) - 2y(x+y) - m(x+y)^2 = 0$

$\Rightarrow x^2 + y^2 + x^2 + xy - 2xy - 2y^2 - mx^2 - my^2 - 2mxy = 0$

$\Rightarrow x^2(2-m) - y^2(1+m) - xy(1+2m) = 0$

If lines are at right angle then

$(2-m) - (1+m) = 0 \Rightarrow m = 1/2$

QUESTION BANK

CHAPTER 9 : POINT AND STRAIGHT LINES

EXERCISE - 1 [LEVEL-1]

PART - 1 : DISTANCE AND SECTION FORMULA

Q.1 If O be the origin and if the coordinates of any two points Q_1 and Q_2 be (x_1, y_1) and (x_2, y_2) respectively, then $OQ_1.OQ_2 \cos Q_1OQ_2 =$

(A) $x_1x_2 - y_1y_2$ (B) $x_1y_1 - x_2y_2$
(C) $x_1x_2 + y_1y_2$ (D) $x_1y_1 + x_2y_2$

Q.2 The distance between the points (7, 5) and (3, 2) is equal to

(A) 2 units (B) 3 units
(C) 4 units (D) 5 units

Q.3 The points which trisect the line segment joining the points (0, 0) and (9, 12) are

(A) (3,4), (6,8) (B) (4,3), (6,8)
(C) (4,3), (8,6) (D) (3,4), (8,6)

Q.4 Point $\left(\frac{1}{2}, \frac{-13}{4}\right)$ divides the line joining the points (3, –5) and (–7, 2) in the ratio of

(A) 1 : 3 internally (B) 3 : 1 internally
(C) 1 : 3 externally (D) 3 : 1 externally

Q.5 Three vertices of a parallelogram taken in order are (–1,–6), (2, –5) and (7, 2). The fourth vertex is –

(A) (1, 4) (B) (4, 1)
(C) (1, 1) (D) (4, 4)

Q.6 If P (1, 2), Q (4, 6) R (5, 7) and S (a, b) are the vertices of a parallelogram PQRS, then –

(A) a = 2, b = 4 (B) a = 3, b = 4
(C) a = 2, b = 3 (D) a = 3, b = 5

Q.7 The quadrilateral formed by the vertices (–1,1), (0,–3), (5,2) and (4,6) will be

(A) Square (B) Parallelogram
(C) Rectangle (D) Rhombus

Q.8 The triangle joining the points P (2, 7), Q (4, –1), R (–2, 6) is

(A) Equilateral triangle (B) Right-angled triangle
(C) Isosceles triangle (D) Scalene triangle

Q.9 If mid point of (1, a) & (a + b, 2) is (–1, –1) then value of $a^2 + b^2$ is –

(A) 25 (B) 21
(C) 20 (D) 17

Q.10 The line joining A (2, –7) and B (6, 5) is divided into 4 equal parts by the points P, Q and R such that AQ = RP = QB. The midpoint of PR is –

(A) (4, 12) (B) (–8, 1)
(C) (4, –1) (D) (8, –2)

Q.11 The points A (1, 2), B (2, 4) and C (4, 8) form a/an –

(A) right angled triangle (B) straight line
(C) equilateral triangle (D) isosceles triangle

PART - 2 : CO-ORDINATE OF PARTICULAR POINTS, AREA

Q.12 If the vertices of a triangle have integral coordinates, then the triangle is –

(A) Equilateral (B) Never equilateral
(C) Isosceles (D) None of these

Q.13 If the vertices of a triangle be (2,1), (5,2) and (3,4), then its circumcentre is

(A) $\left(\frac{13}{2}, \frac{9}{2}\right)$ (B) $\left(\frac{13}{4}, \frac{9}{4}\right)$
(C) $\left(\frac{9}{4}, \frac{13}{4}\right)$ (D) None

Q.14 A pole stands vertically inside a triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is the same, then in the Δ ABC, the foot of the pole is at the

(A) Centroid (B) Circumcentre
(C) Incentre (D) Orthocentre

Q.15 The point A divides the join of the points (–5,1) and (3,5) in the ratio k :1 and the coordinates of the points B and C are (1,5) and (7,–2) respectively. If the area of the triangle ABC be 2 units, then k =

(A) 6, 7 (B) 31/9, 9
(C) 7, 31/9 (D) 7, 9

Q.16 Area of the triangle with vertices $(a, b), (x_1, y_1)$ and (x_2, y_2), where a, x_1 and x_2 are in G.P. with common ratio r and b, y_1 & y_2 are in G.P. with common ratio s, is given by

(A $ab\,(r-1)(s-1)(s-r)$ (B) $\frac{1}{2}ab\,(r+1)(s+1)(s-r)$
(C) $\frac{1}{2}ab\,(r-1)(s-1)(s-r)$ (D) $ab\,(r+1)(s+1)(r-s)$

Q.17 The length of altitude through A of the triangle ABC, where $A \equiv (-3, 0); B \equiv (4, -1); C \equiv (5, 2)$, is

(A) $\frac{2}{\sqrt{10}}$ (B) $\frac{4}{\sqrt{10}}$ (C) $\frac{11}{\sqrt{10}}$ (D) $\frac{22}{\sqrt{10}}$

Q.18 If (x_i, y_i), i = 1, 2, 3 are vertices of equilateral triangle such that $(x_1 + 2)^2 + (y_1 - 3)^2 = (x_2 + 2)^2 + (y_2 - 3)^2 = (x_3 + 2)^2 + (y_3 - 3)^2$ then value of $\frac{x_1 + x_2 + x_3}{y_1 + y_2 + y_3}$ is –

(A) 2/3 (B) –2/3
(C) 3/2 (D) –3/2

Q.19 The area of the parallelogram formed by the lines $y = mx, y = mx + 1, y = nx$ and $y = nx + 1$ equals

(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$

(C) $\frac{1}{|m+n|}$ (D) $\frac{1}{|m-n|}$

Q.20 If the vertices of a triangle be $(am_1^2, 2am_1), (am_2^2, 2am_2)$ and $(am_3^2, 2am_3)$, then the area of the triangle is

(A) $a(m_2 - m_3)(m_3 - m_1)(m_1 - m_2)$

(B) $(m_2 - m_3)(m_3 - m_1)(m_1 - m_2)$

(C) $a^2(m_2 - m_3)(m_3 - m_1)(m_1 - m_2)$

(D) None of these

Q.21 Three points are $A(6, 3), B(-3, 5), C(4, -2)$ and $P(x, y)$ is a point, then the ratio of area of Δ PBC and Δ ABC

(A) $\left|\frac{x+y-2}{7}\right|$ (B) $\left|\frac{x-y+2}{2}\right|$

(C) $\left|\frac{x-y-2}{7}\right|$ (D) None of these

Q.22 If the area of the triangle with vertices (x, 0), (1, 1) and (0, 2) is 4 square units then a value of x is

(A) –2 (B) –4
(C) –6 (D) 8

Q.23 If the points $(x+1, 2), (1, x+2), \left(\frac{1}{x+1}, \frac{2}{x+1}\right)$ are collinear, then x is

(A) 4 (B) 0
(C) –4 (D) Both (B) and (C)

Q.24 The number of possible straight lines, passing through (2, 3) and forming a triangle with coordinate axes, whose area is 12 sq. units, is –

(A) 1 (B) 2
(C) 3 (D) 4

Q.25 If lines represented by $x + 3y - 6 = 0$, $2x + y - 4 = 0$ and $kx - 3y + 1 = 0$ are concurrent, then the value of k is –

(A) –6/19 (B) –19/6
(C) 19/6 (D) 6/19

Q.26 The points (11, 9), (2, 1) and (2, –1) are the midpoints of the sides of the triangle. Then the centroid is –

(A) (5, 3) (B) (–5, –3)
(C) (5, –3) (D) (3, 5)

PART - 3 : TRANSFORMATION OF AXES, LOCUS

Q.27 A (a, 0) and B (–a, 0) are two fixed points of triangle ABC. The vertex C moves in such a way that $\cot A + \cot B = \lambda$, where λ is a constant. Then the locus of the point C is

(A) $y\lambda = 2a$ (B) $ya = 2\lambda$
(C) $y = \lambda a$ (D) None of these

Q.28 A stick of length ℓ rests against the floor and a wall of a room. If the stick begins to slide on the floor, then the locus of its middle point is

(A) A straight line (B) Circle
(C) Parabola (D) Ellipse

Q.29 The point (4, 1) undergoes the following three transformations successively

(i) Reflection about the line $y = x$
(ii) Translation through a distance 2 units along the positive direction of x-axis
(iii) Rotation through an angle $\pi/4$ about the origin in the anti clockwise direction.

The final position of the point is given by the coordinates

(A) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (B) $(-\sqrt{2}, 7\sqrt{2})$

(C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D) $(\sqrt{2}, 7\sqrt{2})$

Q.30 A point starts moving from (1, 2) and its projections on x and y -axes are moving with velocities of 3m/sand 2m/s respectively. Its locus is –

(A) $2x - 3y + 4 = 0$ (B) $3x - 2y + 1 = 0$
(C) $3y - 2x + 4 = 0$ (D) $2y - 3x + 1 = 0$

Q.31 The equation $\sqrt{(x-2)^2 + y^2} + \sqrt{(x+2)^2 + y^2} = 4$ represents a –

(A) Circle (B) Pair of straight lines
(C) Parabola (D) Ellipse

Q.32 A line cuts intercepts a and b on the coordinate axes, if after rotating the axes at the constant angle without changing the origin it makes intercepts p and q on the new axes then –

(A) $a^2 + b^2 = p^2 + q^2$ (B) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

(C) $a^2 + p^2 = b^2 + q^2$ (D) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

Q.33 A ray of light passing through at point (1, 2) is reflected on the x-axis at point Q and passes through the point (5, 8). Then the abscissa of the point Q is –

(A) –3 (B) 9/5
(C) 13/5 (D) None of these

Q.34 A point moves in such a way that the sum of square of its distance from the points A (2, 0) and B (–2, 0) is always equal to the square of the distance between A and B. The locus of the point is

(A) $x^2 + y^2 - 2 = 0$ (B) $x^2 + y^2 + 2 = 0$
(C) $x^2 + y^2 + 4 = 0$ (D) $x^2 + y^2 - 4 = 0$

Q.35 A ray light passing through point A (–2, 3) and get reflected at point B on x-axis and then passes through point (3, 2) then equation of line AB ?

(A) $y - x = 5$ (B) $y = -x + 1$
(C) $x + y = 2$ (D) None of these

Q.36 Locus of a point which moves such that its distance from the X-axis is twice its distance from the line $x - y = 0$ is –
(A) $x^2 - 4xy - y^2 = 0$ (B) $x^2 - 4xy + y^2 = 0$
(C) $2x^2 - 4xy + y^2 = 0$ (D) $x^2 + 4xy - y^2 = 0$

Q.37 $A = (\cos\theta, \sin\theta)$, $B = (\sin\theta, -\cos\theta)$ are two points. The locus of the centroid of Δ OAB, where O is the origin is –
(A) $x^2 + y^2 = 3$ (B) $9x^2 + 9y^2 = 2$
(C) $2x^2 + 2y^2 = 9$ (D) $3x^2 + 3y^2 = 2$

PART - 4 : EQUATION OF STRAIGHT LINE

Q.38 Equation of a line which is making an angle of 60º with the x axis and an intercept of 5 unit length in negative direction of y axis is –
(A) $y = \sqrt{3}\,x - 5$ (B) $y = \sqrt{3}\,x + 5$
(C) $y = 2\sqrt{3}\,x - 5$ (D) $y = -\sqrt{3}\,x - 5$

Q.39 Equation of a line which is passing through origin and making an angle of 45º with x–axis is –
(A) $2x - y = 0$ (B) $x + y = 0$
(C) $x + 2y = 0$ (D) $x - y = 0$

Q.40 Equation of a line passing through (3, –4) and (4, 3) is –
(A) $y = 7x - 25$ (B) $y = 7x + 25$
(C) $y = 3x - 27$ (D) $y = 2x - 25$

Q.41 Equation of a line on which length of perpendicular from origin is 4 and inclination of this perpendicular is 60º with the positive direction of x– axis is –
(A) $2x + y\sqrt{3} = 8$ (B) $x - y\sqrt{3} = 8$
(C) $x - y\sqrt{3} = 7$ (D) $x + y\sqrt{3} = 8$

Q.42 Equation of a line which passes through (1, –2) and makes equal intercept on axes is
(A) $x - y - 3 = 0$ (B) $x + y + 2 = 0$
(C) $x + y - 1 = 0$ (D) $x + y + 1 = 0$

Q.43 Slope of a Line which is made by joining the point (1, 0) and $(-2, \sqrt{3})$ is –
(A) 120º (B) 60º
(C) 150º (D) 135º

Q.44 x and y intercepts of line $2x - 3y = 6$ are –
(A) 1/2, 1/3 (B) 2, –3
(C) 3, –2 (D) 1/3, 1/2

Q.45 If the intercept made by the line between the axes is bisected at the point (5, 2) then its equation is –
(A) $5x + 2y = 20$ (B) $2x + 5y = 20$
(C) $2x - 5y = 20$ (D) $5x - 2y = 20$

Q.46 The equation of the line which passes through the point (3, 4) and the sum of its intercept on the axes is 14, is –
(A) $4x - 3y = 24$, $x - y = 7$
(B) $4x + 3y = 24$, $x + y = 7$
(C) $4x + 3y + 24 = 0$, $x + y + 7 = 0$
(D) $4x - 3y + 24 = 0$, $x - y + 7 = 0$

Q.47 The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150º with the positive direction of y-axis. The equation of the line is –
(A) $\sqrt{3}\,x + y = 14$ (B) $\sqrt{3}\,x - y = 14$
(C) $\sqrt{3}\,x + y + 14 = 0$ (D) $\sqrt{3}\,x - y + 14 = 0$

Q.48 If the intercept made by the line between the axes is bisected at the point (x_1, y_1), then its equation is –
(A) $\frac{x}{x_1} + \frac{y}{y_1} = 2$ (B) $\frac{x}{x_1} + \frac{y}{y_1} = 1$
(C) $\frac{x}{x_1} + \frac{y}{y_1} = \frac{1}{2}$ (D) None

Q.49 A line L is perpendicular to the line $5x - y = 1$ and the area of the triangle formed by the line L and coordinate axes is 5. The equation of the line L is
(A) $x + 5y = 5$ (B) $x + 5y = \pm 5\sqrt{2}$
(C) $x - 5y = 5$ (D) $x - 5y = 5\sqrt{2}$

Q.50 If the coordinates of the points A, B, C be (– 1, 5), (0, 0) and (2, 2) respectively and D be the middle point of BC, then the equation of the perpendicular drawn from B to the line AD is
(A) $x + 2y = 0$ (B) $2x + y = 0$
(C) $x - 2y = 0$ (D) $2x - y = 0$

Q.51 The equation of the line which cuts off an intercept 3 units on OX and an intercept – 2 unit on OY, is
(A) $\frac{x}{3} - \frac{y}{2} = 1$ (B) $\frac{x}{3} + \frac{y}{2} = 1$
(C) $\frac{x}{2} + \frac{y}{3} = 1$ (D) $\frac{x}{2} - \frac{y}{3} = 1$

Q.52 The point P (a, b) lies on the straight line $3x + 2y = 13$ and the point Q (b, a) lies on the straight line $4x - y = 5$, then the equation of line PQ is –
(A) $x - y = 5$ (B) $x + y = 5$
(C) $x + y = -5$ (D) $x - y = -5$

Q.53 A line AB makes zero intercepts on x–axis and y–axis and it is perpendicular to another line CD, $3x + 4y + 6 = 0$. The equation of line AB is
(A) $y = 4$ (B) $4x - 3y + 8 = 0$
(C) $4x - 3y = 0$ (D) $4x - 3y + 6 = 0$

Q.54 The intercept of a line between the coordinate axes is divided by point (– 5, 4) in the ratio 1 : 2. The equation of the line will be
(A) $5x - 8y + 60 = 0$ (B) $8x - 5y + 60 = 0$
(C) $2x - 5y + 30 = 0$ (D) None of these

Q.55 Equation of one of the sides of an isosceles right angled triangle whose hypotenuse is $3x + 4y = 4$ and the opposite vertex of the hypotenuse is (2, 2), will be
(A) $x - 7y + 12 = 0$ (B) $7x + y - 12 = 0$
(C) $x - 7y + 16 = 0$ (D) $7x + y + 16 = 0$

Q.56 Slope of line whose parametric equation is given by

$$x = -2 + \frac{r}{\sqrt{10}},\ y = 1 + \frac{3r}{\sqrt{10}}$$ is –

(A) –1 (B) 1
(C) 1/3 (D) 3

PART - 5 : POINT RELATIVE TO LINE, PARALLEL AND PERPENDICULAR LINES

Q.57 If $x + 2y = 3$ is a line and $A(-1, 3)$; $B(2, -3)$; $C(4, 9)$ are three points, then –
(A) A is on one side and B, C are on other side of the line
(B) A, B are on one side and C is on other side of the line
(C) A, C on one side and B is no other side of the line
(D) All three points are on one side of the line

Q.58 Length of perpendicular from the origin to the line $3x + 4y = 10$ is –
(A) 5 (B) 1
(C) 2 (D) 4

Q.59 Normal form of line $x + y - 4 = 0$ is –
(A) $x \cos \pi/3 - y \sin \pi/3 = 2$ (B) $x \cos \pi/6 - y \sin \pi/6 = 2$
(C) $x \cos \pi/3 + y \sin \pi/3 = 2$ (D) $x \cos \pi/6 + y \sin \pi/6 = 2$

Q.60 If $3x + 4y - 5 = 0$ and $4x + ky - 8 = 0$ are two perpendicular lines then k is –
(A) 3 (B) 4
(C) –3 (D) –4

Q.61 Equation of a line which passes through (4, 6) and parallel to $3x - 7y + 2 = 0$ is -
(A) $3x - 7y + 30 = 0$ (B) $3x + 7y + 30 = 0$
(C) $2x + 4y + 30 = 0$ (D) $3x - 8y + 15 = 0$

Q.62 Length of the perpendicular from the point (3,4) on the line $7x + 9y + 6 = 0$ is -
(A) $\frac{63}{\sqrt{130}}$ (B) $\frac{56}{\sqrt{130}}$
(C) $\frac{56}{\sqrt{120}}$ (D) None

Q.63 Distance between $3x + 4y + 7 = 0$ and $3x + 4y + 22 = 0$ is -
(A) 4 (B) 5
(C) 3 (D) 1

Q.64 The points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$ are -
(A) (–3, 1), (–7, 11) (B) (3, 1), (7, 11)
(C) (3, 1), (–7, 11) (D) (1, 3), (–7, 11)

Q.65 The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is
(A) 133 (B) 190
(C) 233 (D) 105

Q.66 If p_1, p_2 and p_3 be the perpendiculars from the points $(m^2, 2m)$, $(mm', m + m')$ and $(m'^2, 2m')$ respectively on the line $x \cos\alpha + y \sin\alpha + \frac{\sin^2\alpha}{\cos\alpha} = 0$, then p_1, p_2 & p_3 are in
(A) A. P. (B) G. P.
(C) H. P. (D) None of these

Q.67 Nearest point on line $x - 3y = 5$ from point (1, 2) is –
(A) (2, –1) (B) (3, –2/3)
(C) (0, 0) (D) (5, 0)

Q.68 Line, any point on which is equidistant from the lines $x + y = 1$, $7x - y = 6$
(A) $2x - 6y + 1 = 0$ (B) $2x + 6y - 1 = 0$
(C) $12x + 4y - 11 = 0$ (D) None of these

Q.69 The reflection of the point (1, 1) along the line $y = -x$ is –
(A) (1, –1) (B) (0, 0)
(C) (–1, 1) (D) (–1, –1)

Q.70 If the line $6x - 7y + 8 + \lambda(3x - y + 5) = 0$ is parallel to y-axis, then λ =
(A) –7 (B) –2
(C) 7 (D) 2

Q.71 A line passes through (2, 2) and is perpendicular in the line $3x + y = 3$ its y-intercepts is –
(A) 1/3 (B) 2/3
(C) 4/3 (D) 1

Q.72 If the straight lines $2x + 3y - 3 = 0$ and $x + ky + 7 = 0$ are perpendicular, then the value of k is –
(A) 3/2 (B) –3/2
(C) 2/3 (D) –2/3

PART - 6 : ANGLE BISECTOR

Q.73 Angle between $x = 9$ and $x - \sqrt{3}\,y + 7 = 0$ is
(A) 15º (B) 30º
(C) 45º (D) 60º

Q.74 The equation of line passing through point of intersection of lines $3x - 2y - 1 = 0$ and $x - 4y + 3 = 0$ and the point $(\pi, 0)$ is
(A) $x - y = \pi$ (B) $x - y = \pi(y + 1)$
(C) $x - y = \pi(1 - y)$ (D) $x + y = \pi(1 - y)$

Q.75 Equations of lines which passes through the points of intersection of the lines $4x - 3y - 1 = 0$ and $2x - 5y + 3 = 0$ and are equally inclined to the axes are
(A) $y \pm x = 0$ (B) $y - 1 = \pm 1(x - 1)$
(C) $x - 1 = \pm 2(y - 1)$ (D) None of these

Q.76 The equation to the straight line passing through the point of intersection of the lines $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ and perpendicular to the line $3x - 5y + 11 = 0$ is
(A) $5x + 3y + 8 = 0$ (B) $3x - 5y + 8 = 0$
(C) $5x + 3y + 11 = 0$ (D) $3x - 5y + 11 = 0$

Q.77 The base BC of a triangle ABC is bisected at the point (p, q) and the equations to the sides AB and AC are respectively $px + qy = 1$ and $qx + py = 1$. Then the equation to the median through A is
(A) $(2pq - 1)(px + qy - 1) = (p^2 + q^2 - 1)(qx + py - 1)$
(B) $(p^2 + q^2 - 1)(px + qy - 1) = (2p - 1)(qx + py - 1)$
(C) $(pq - 1)(px + qy - 1) = (p^2 + q^2 - 1)(qx + py - 1)$
(D) None of these

Q.78 P is a point on either of the two lines $y-\sqrt{3}|x|=2$ at a distance of 5 units from their point of intersection. The coordinates of the foot of the perpendicular from P on the bisector of the angle between them are –

(A) $\left(0,\frac{4+5\sqrt{3}}{2}\right)$ or $\left(0,\frac{4-5\sqrt{3}}{2}\right)$ depending on which the point P is taken

(B) $\left(0,\frac{4+5\sqrt{3}}{2}\right)$ (C) $\left(0,\frac{4-5\sqrt{3}}{2}\right)$ (D) $\left(\frac{5}{2},\frac{5\sqrt{2}}{2}\right)$

Q.79 A straight line $(\sqrt{3}-1)x=(\sqrt{3}+1)y$ makes an angle 75° with another straight line which passes through origin. Then the equation of the line is

(A) $x=0$ (B) $y=0$
(C) $x+y=0$ (D) $x-y=0$

Q.80 Equation of angle bisector between the lines $3x+4y-7=0$ and $12x+5y+17=0$ are

(A) $\frac{3x+4y-7}{\sqrt{25}}=\pm\frac{12x+5y+17}{\sqrt{169}}$

(B) $\frac{3x+4y+7}{\sqrt{25}}=\frac{12x+5y+17}{\sqrt{169}}$

(C) $\frac{3x+4y+7}{\sqrt{25}}=\pm\frac{12x+5y+17}{\sqrt{169}}$

(D) None of these

Q.81 Let P ≡ (–1, 0), Q ≡ (0, 0) and R ≡ $(3, 3\sqrt{3})$ be three points. The eq. of the bisector of the angle PQR is

(A) $x-\sqrt{3}y=0$ (B) $\sqrt{3}x-y=0$
(C) $x+\sqrt{3}y=0$ (D) $\sqrt{3}x+y=0$

Q.82 The angle between the lines $\sin^2\alpha \cdot y^2-2xy\cdot\cos^2\alpha+(\cos^2\alpha-1)x^2=0$ is –

(A) 90° (B) α
(C) α/2 (D) 2α

Q.83 A straight lien passes through the points (5, 0) and (0, 3). The length of perpendicular form the point (4, 4) on the line is –

(A) $15/\sqrt{34}$ (B) $\sqrt{17}/2$
(C) 17/2 (D) $\sqrt{17/2}$

PART - 7 : GENERAL EQUATION OF SECOND DEGREE

Q.84 Equation of pair of straight lines which are formed by joining the origin and the points of intersection of a circle $x^2+y^2=a^2$ and a line $y=mx+c$ is -

(A) $x^2(c^2-a^2m^2)+y(c^2-a^2)-2ma^2xy=0$
(B) $x^2(c^2-a^2m^2)-y(c^2-a^2)-2ma^2xy=0$
(C) $x^2(c^2-a^2m^2)-y(c^2+a^2)+2ma^2xy=0$
(D) None of these

Q.85 If the lines $(p-q)x^2+2(p+q)xy+(q-p)y^2=0$ are perpendicular to each other then -

(A) p = q (B) q = 0
(C) p = 0 (D) p & q can take any value

Q.86 If the equation $x^2+y^2+2gx+2fy+1=0$ represents a pair of lines, then

(A) $g^2-f^2=1$ (B) $f^2-g^2=1$
(C) $g^2+f^2=1$ (D) $f^2+g^2=\frac{1}{2}$

Q.87 Let PQR be a right angled isosceles triangle, right angled at P (2, 1). If the equation of the line QR is $2x+y=3$, then the equation representing the pair of lines PQ and PR is

(A) $3x^2-3y^2+8xy+20x+10y+25=0$
(B) $3x^2-3y^2+8xy-20x-10y+25=0$
(C) $3x^2-3y^2+8xy+10x+15y+20=0$
(D) $3x^2-3y^2-8xy-10x-15y-20=0$

Q.88 Mixed term *xy* is to be removed from the general equation $ax^2+by^2+2hxy+2fy+2gx+c=0$. One should rotate the axes through an angle θ given by tan 2θ equal to

(A) $\frac{a-b}{2h}$ (B) $\frac{2h}{a+b}$ (C) $\frac{a+b}{2h}$ (D) $\frac{2h}{(a-b)}$

Q.89 If the equation $y^3-3x^2y+m(x^3-3xy^2)=0$ represents the three lines passing through origin, then

(A) Lines are equally inclined to each other
(B) Two lines makes equal angle with x-axis
(C) All three lines makes equal angle with x-axis
(D) None of these

Q.90 If pair of straight lines $x^2-2mxy-y^2=0$ and $x^2-2nxy-y^2=0$ be such that each pair bisects the angle between the other pair, then mn =

(A) 1 (B) – 1
(C) 0 (D) – 1/2

Q.91 Find the angle between the lines represented by the equation $x^2-2pxy+y^2=0$ –

(A) $\sec^{-1}(p)$ (B) $\tan^{-1}(p)$
(C) $\cos^{-1}(p)$ (D) None of these

Q.92 If the pairs of lines $x^2+2xy+ay^2=0$ and $ax^2+2xy+y^2=0$ have exactly one line in common, then the joint equation of the other two lines is given by –

(A) $3x^2+8xy-3y^2=0$ (B) $3x^2+10xy+3y^2=0$
(C) $y^2+2xy-3x^2=0$ (D) $x^2+2xy-3y^2=0$

Q.93 If m is the slope of one of the lines represented by $ax^2+2hxy+by^2=0$, then $(h+bm)^2=$

(A) $(a+b)^2$ (B) $(a-b)^2$
(C) h^2+ab (D) h^2-ab

Q.94 If one of the slopes of the pair of lines $ax^2+2hxy+by^2=0$ is n times the other then –

(A) $4ab=(n+1)^2h$ (B) $4(n+1)^2ab=nab$
(C) $4h^2=(n+1)^2ab$ (D) $4nh^2=(n+1)^2ab$

PART - 8 : MISCELLANEOUS

Q.95 If the sum of the distance of a point from two perpendicular lines in a plane is 1, then its locus is –
(A) square (B) circle
(C) a straight line (D) two intersecting lines

Q.96 Locus of a point that is equidistant from the lines $x + y - 2\sqrt{2} = 0$ and $x + y - \sqrt{2} = 0$ is
(A) $x + y - 5\sqrt{2} = 0$ (B) $x + y - 3\sqrt{2} = 0$
(C) $2x + 2y - 3\sqrt{2} = 0$ (D) $2x + 2y - 5\sqrt{2} = 0$

Q.97 Let the coordinates of the two points A and B be (1, 2) and (7, 5) respectively. The line AB is rotated through 45° in anti clockwise direction about the point of trisection of AB which is nearer to B. The equation of the line in new position is
(A) $2x - y - 6 = 0$ (B) $x - y - 1 = 0$
(C) $3x - y - 11 = 0$ (D) none of these

Q.98 If one vertex of equilateral Δ is at A (3, 4) and the base BC is $x + y - 5 = 0$, then the length of each side of the Δ is –
(A) $3\sqrt{3}$ (B) $4\sqrt{3}$
(C) $\frac{2\sqrt{2}}{\sqrt{3}}$ (D) $2\sqrt{2}$

Q.99 Given the family of lines, $a(3x + 4y + 6) + b(x + y + 2) = 0$. The line of the family situated at the greatest distance from the point P (2, 3) has equation –
(A) $4x + 3y + 8 = 0$ (B) $5x + 3y + 10 = 0$
(C) $15x + 8y + 30 = 0$ (D) None of these

Q.100 The diagonals AC and BD of a rhombus intersect at (5, 6). If A ≡ (3, 2) then equation of diagonal BD is
(A) $y - x = 1$ (B) $2y - x = 17$
(C) $y - 2x + 4 = 0$ (D) $2y + x = 17$

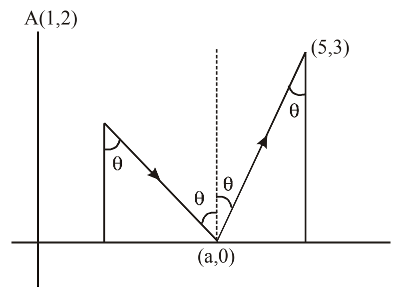
Q.101 The family of straight lines $(2a + 3b)x + (a - b)y + 2a - 4b = 0$ is concurrent at the point
(A) $\left(\frac{2}{5}, \frac{-14}{5}\right)$ (B) $\left(\frac{-2}{5}, \frac{-14}{5}\right)$
(C) $\left(\frac{-2}{5}, \frac{14}{5}\right)$ (D) $\left(\frac{2}{5}, \frac{14}{5}\right)$

Q.102 If $3a - 2b + 5c = 0$, family of straight lines $ax + by + c = 0$ are always concurrent at a point whose coordinate is -
(A) $\left(\frac{3}{5}, \frac{2}{5}\right)$ (B) $\left(-\frac{3}{5}, \frac{2}{5}\right)$
(C) $\left(\frac{3}{5}, -\frac{2}{5}\right)$ (D) $\left(\frac{-3}{5}, \frac{-2}{5}\right)$

Q.103 D is a point on AC of the triangle with vertices A(2, 3), B(1, –3), C(–4, –7) and BD divides ABC into two triangles of equal area. The equation of the line drawn through B at right angles to BD is
(A) $y - 2x + 5 = 0$ (B) $2y - x + 5 = 0$
(C) $y + 2x - 5 = 0$ (D) $2y + x - 5 = 0$

Q.104 R is the reflection of the point (3, 4) in the line $y = x$. R_1 is the reflection of R in y-axis. The point R_2 which is the reflection of R_1 in the x-axis has the coordinates
(A) (4, – 3) (B) (– 4, 3)
(C) (– 3, – 4) (D) None of these

Q.105 A ray of light passing through the point A (1, 2) is reflected at a point B on the x-axis and then passes through (5, 3). Then the equation of AB is–



(A) $5x - 4y = 13$ (B) $5x - 4y = -3$
(C) $4x + 5y = 14$ (D) $4x - 5y = -6$

Q.106 If a, b, c are in A.P. then $ax + by + c = 0$ represents
(A) a single line
(B) a family of concurrent lines
(C) a family of parallel lines
(D) none of these

Q.107 Image of the point P (1, 5) with respect to the line $4x + 3y + 6 = 0$ is -
(A) $\left(\frac{19}{25}, \frac{13}{25}\right)$ (B) $\left(\frac{13}{25}, \frac{19}{25}\right)$
(C) (–7, –1) (D) $\left(\frac{191}{25}, \frac{113}{25}\right)$

Q.108 The circumradius of the triangle formed by the three lines $y + 3x - 5 = 0$, $y = x$, $3y - x + 10 = 0$ is
(A) $\frac{25}{4\sqrt{2}}$ (B) $\frac{25}{3\sqrt{2}}$
(C) $\frac{25}{2\sqrt{2}}$ (D) $\frac{25}{\sqrt{2}}$

Q.109 A point moves in the xy plane, such that the sum of its distance from the two axes is always equal to 2. The area enclosed by the locus of this points is
(A) 16 units (B) 9/2 units
(C) 8 units (D) 9 units

Q.110 Consider the family of lines $5x + 3y - 2 + \lambda_1(3x - y - 4) = 0$ and $x - y + 1 + \lambda_2(2x - y - 2) = 0$. Equation of a straight line that belong to both families is -
(A) $25x - 62y + 86 = 0$ (B) $62x - 25y + 86 = 0$
(C) $25x - 62y = 86$ (D) $5x - 2y - 7 = 0$

Q.111 If one line of the pair of lines $ax^2 + 2hxy + by^2 = 0$ bisects the angle between coordinate axes in positive quadrant, then–
(A) $a + b = 2|h|$ (B) $a + b = -2h$
(C) $a - b = 2|h|$ (D) $(a - b)^2 = 4h^2$

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 OAB is an equilateral triangle of side 2 units and one vertex at origin. If OA is inclined at 60° to the positive x-axis, then the mid point of AB has coordinates

(A) $\left(\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right)$ (B) $\left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}+1}{2}\right)$

(C) $(1-\sqrt{3}, 1+\sqrt{3})$ (D) $\left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}-1}{2}\right)$

Q.2 S_1 and S_2 are two points on AB of a ΔABC with vertices (–2, 3), (4, –6) and (1, 1). CS_1 and CS_2 divide the triangle into three of equal area. The equation of the lines through the origin drawn parallel to CS_1 and CS_2 is-

(A) $y^2 + 4xy - 3x^2 = 0$ (B) $3y^2 + 4xy + x^2 = 0$

(C) $y^2 + 3xy - 4x^2 = 0$ (D) $y^2 + 5xy + 4x^2 = 0$

Q.3 If the lines $\ell x + my + n = 0$, $mx + ny + \ell = 0$ and $nx + \ell y + m = 0$ are concurrent then

(A) $\ell + m + n = 0$ (B) $\ell - m - n = 0$

(C) $\ell + m - n = 0$ (D) $m + n - \ell = 0$

Q.4 If the quadrilateral formed by the lines $ax + by + c = 0$, $ax + by + c_1 = 0$ $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c = 0$ has perpendicular diagonals, then

(A) $a^2 + b^2 = a_1^2 + b_1^2$ (B) $b^2 + c^2 = b_1^2 + c_1^2$

(C) $a^2 + c^2 = a_1^2 + c_1^2$ (D) $a + b = a_1 + b_1$

Q.5 The graph of the function
$y = \sin x \sin (x + 2) + \cos^2 (x + 1)$ is

(A) a straight line parallel to the x-axis

(B) a parabola

(C) a straight line parallel to the y-axis

(D) a circle

Q.6 A variable point $\left(1+\frac{\lambda}{\sqrt{2}}, 2+\frac{\lambda}{\sqrt{2}}\right)$ lies in between two parallel lines $x + 2y = 1$ and $2x + 4y = 15$, then the range of λ is given by –

(A) $0 < \lambda < \frac{5\sqrt{2}}{6}$ (B) $-\frac{4\sqrt{2}}{5} < \lambda < \frac{5\sqrt{2}}{6}$

(C) $-\frac{4\sqrt{2}}{5} < \lambda < 0$ (D) $-\sqrt{2} < \lambda < \frac{5}{3\sqrt{2}}$

Q.7 If the lines $\left.\begin{array}{l}\lambda x + (\sin\alpha)\, y + \cos\alpha = 0 \\ x + (\cos\alpha)\, y + \sin\alpha = 0 \\ x - (\sin\alpha)\, y + \cos\alpha = 0\end{array}\right]$ pass through the same point where $\alpha \in R$ then λ lies in the interval.

(A) $[-1, 1]$ (B) $[-\sqrt{2}, -\sqrt{2}]$

(C) $[-2, 2]$ (D) $(-\infty, \infty)$

Q.8 Consider the straight line $ax + by = c$ where $a, b, c \in R^+$. This line meets the coordinate axes at 'P' and 'Q' respectively. If the area of triangle OPQ, 'O' being origin, does not depend upon a, b and c, then

(A) a, b, c are in G.P. (B) a, c, b are in G.P.

(C) a, b, c are in A.P. (D) a, c, b are in A.P.

Q.9 The limiting position of the point of intersection of the straight lines, $3x + 5y = 1$ and $(2 + c)\, x + 5c^2 y = 1$ as c tends to one is –

(A) $\left(\frac{2}{5}, -\frac{1}{25}\right)$ (B) $\left(\frac{1}{2}, -\frac{1}{10}\right)$

(C) $\left(\frac{3}{8}, -\frac{1}{40}\right)$ (D) None of these

Q.10 A line intersects the straight lines $5x - y - 4 = 0$ and $3x - 4y - 4 = 0$ at A and B respectively. If a point P (1, 5) on the line AB is such that AP : PB = 2:1 (internally), find the point A.

(A) $\left(\frac{75}{17}, \frac{304}{17}\right)$ (B) $\left(\frac{65}{17}, \frac{304}{17}\right)$

(C) $\left(\frac{75}{17}, \frac{104}{17}\right)$ (D) $\left(\frac{75}{17}, \frac{180}{17}\right)$

Q.11 The nearest point on the line $3x + 4y = 12$ from the origin is

(A) $\left(\frac{36}{25}, \frac{48}{25}\right)$ (B) $\left(3, \frac{3}{4}\right)$ (C) $\left(2, \frac{3}{2}\right)$ (D) none of these

Q.12 A pair of perpendicular straight lines drawn through the origin form an isosceles triangle with line $2x + 3y = 6$, then area of the triangle so formed is –

(A) 36/13 (B) 12/17

(C) 13/5 (D) 17/13

Q.13 A variable line moves in such way that the product of the perpendiculars from (a, 0) and (0, 0) is equal k^2. The locus of the feet of the perpendicular from (0, 0) upon the variable line is a circle, the square of whose radius is (Given : $|a| < 2|k|$)

(A) $\frac{a^2}{4} + k^2$ (B) $\frac{a^2 + k^2}{4}$

(C) $a^2 + \frac{k^2}{4}$ (D) $\frac{a^2 + k^2}{2}$

Q.14 The sides AB, BC, CD and DA of a quadrilateral have the equations $x + 2y = 3$, $x = 1$, $x - 3y = 4$, $5x + y + 12 = 0$ respectively, then the angle between the diagonals AC and BD is –

(A) 60° (B) 45°

(C) 90° (D) None of these

Q.15 Among the lines passing through C (3, 1) BA is farthest from the origin and cuts the x-axis and y- axis at A and B respectively. Then BC:CA is

(A) 9 : 1 (B) 1 : 9

(C) 3 : 1 (D) $\sqrt{3}$: 1

Q.16 If the points P(a^2 , a) lies in the region corresponding to the acute angle between the lines 2y = x and 4y = x, then
(A) $a \in (2, 6)$ (B) $a \in (4, 6)$
(C) $a \in (2, 4)$ (D) none of these

Q.17 If the straight lines ax + may + 1 = 0, bx + (m + 1) by + 1=0 and cx + (m + 2) cy + 1 = 0, $m \neq 0$, are concurrent then a, b, c are in
(A) A.P. only for m = 1 (B) A.P. for all m
(C) G.P. for all m (D) H.P. for all

Q.18 The lines x + 2y + 3 = 0, x + 2y – 7 = 0 and 2x – y – 4 = 0 are the sides of a square. Equation of the remaining side of the square can be -
(A) 2x – y – 14 = 0 (B) 2x – y + 8 = 0
(C) 2x – y – 10 = 0 (D) 2x – y – 6 = 0

Q.19 The straight lines 7x – 2y + 10 = 0 and 7x + 2y – 10 = 0 forms an isosceles triangle with the line y = 2. Area of this triangle is equal to
(A) 15/7 sq. units (B) 10/7 sq. units
(C) 18/7 sq. units (D) none of these

Q.20 L_1 and L_2 are two lines. If the reflection of L_1 in L_2 and the reflection of L_2 in L_1 coincide, then the angle between the lines is
(A) 30° (B) 45°
(C) 60° (D) 90°

Q.21 If p_1, p_2, p_3 are the length of the perpendiculars from the point points (2m, m^2) (m + m', mm') and (2m', m'^2) respectively on the line $x \sin\alpha + y\cos\alpha + \dfrac{\sin^2\alpha}{\cos^2\alpha} = 0$, then p_1, p_2, p_3 are in –
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

Q.22 The equation of the line which passes through the point of intersection of the lines 3y + 2x – 5 = 0 and 2y – 3x + 7 = 0 , and is at right angles the to the line y + 5x + 10 = 0 is
(A) 5y – x + 2 = 0 (B) 5y – x + 10 = 0
(C) 5y – x – 2 = 0 (D) 5y – x + 1 = 0

Q.23 The vertices of a triangle ABC are A (p^2, –p), B (q^2, q), C (r^2, –r). The area of the triangle ABC is –
(A) $\frac{1}{2}$ (p + q) (q + r) (r + p) (B) $\frac{1}{2}$ (p – q) (q + r) (r + p)
(C) $\frac{1}{2}$ (p + q) (q – r) (r – p) (D) $\frac{1}{2}$ (p + q) (q + r) (p – r)

Q.24 Consider the equation, $3x^2 + 4y^2 – 18x + 16y + 43 + C = 0$, then which of the following is incorrect.
(A) cannot represent a real pair of straight lines for any value of C.
(B) represents an ellipse, if C < 0.
(C) represents a hyperbola, if C > 0.
(D) a point, if C = 0.

Q.25 The origin, the intersection of the lines $2x^2 + 5xy – 3y^2 + 3x – 5y – 2 = 0$ and the points in which these lines are cut by the line 3x – 5y = 2, are the vertices of a –
(A) parallelogram (B) rectangle
(C) rhombus (D) square

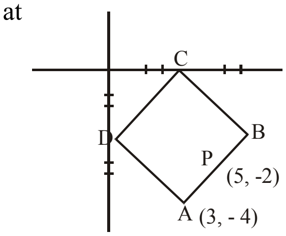
Q.26 Line ax + by + p = 0 makes angle π/4 with x cos α + y sinα = p, $p \in R^+$. If these lines and the line x sin α – y cosα = 0 are concurrent then -
(A) $a^2 + b^2 = 1$ (B) $a^2 + b^2 = 2$
(C) $2(a^2 + b^2) = 1$ (D) none of these

Q.27 If $4a^2 + b^2 + 2c^2 + 4ab – 6ac – 3bc = 0$, the family of lines ax + by + c = 0 is concurrent at one or the other of the two points-
(A) (–1, –1/2), (–2, –1) (B) (–1, –1), (–2, –1/2)
(C) (–1, 2), (1/2, –1) (D) (1, 2), (1/2, –1)

Q.28 Point 'P' lies on the line 3x + 5y = 15. If 'P' is also equidistant from the coordinate axes, then P can be located in which of the four quadrants –
(A) I only (B) II only
(C) I or IV only (D) IV only

Q.29 If A (1, 7) , B (4, 5), C (3h, – 2h) are the vertices of a triangle whose area is 23/2 square units, then
(A) there is no solution for h.
(B) h can not be found, since data is insufficient.
(C) h can be any real number.
(D) there is some conceptual mistake in the question.

Q.30 (3,–4) and (5, –2) are two consecutive vertices of a square in which (2, –2) is an interior point. The centre of the square is at



(A) (1, 2) (B) (3, – 2)
(C) (0, 2) (D) (4, – 2)

Q.31 The polar coordinates of the vertices of a triangle are (0 , 0) (3, π/6) and (3, π/2). The area of the triangle is
(A) $\frac{9\sqrt{3}}{4}$ (B) $\frac{9\sqrt{3}}{2}$ (C) $9\sqrt{3}$ (D) $\frac{9\sqrt{3}}{8}$

Q.32 If in triangle ABC, A ≡ (1, 10), circumcentre ≡ (–1/3, 2/3) and orthocentre ≡ (11/3, 4/3) then the coordinates of mid-point of side opposite to A is –
(A) (1, –11/3) (B) (1, 5)
(C) (1, – 3) (D) (1, 6)

Q.33 The orthocentre of the triangle formed by the lines 2x + 3y = 1, x + y = 1 and 3x – 2y = 2 is
(A) (–8/13, –1/13) (B) (1/13, –8/13)
(C) (8/13, –1/13) (D) (–1/13, 8/13)

Q.34 The range of value of β such that (0, β) lie on or inside the triangle formed by the lines y + 3x + 2 = 0, 3y – 2x – 5 = 0, 4y + x – 14 = 0 is
(A) $5 < \beta \le 7$ (B) $\frac{1}{2} \le \beta \le 1$ (C) $\frac{5}{3} \le \beta \le \frac{7}{2}$ (D) none

Q.35 The two of the straight lines represented by the equation $ax^3 + bx^2y + cxy^2 + dy^3 = 0$ will be at right angle if –
(A) $a^2 + c^2 = 0$ (B) $a^2 + ac + bd + d^2 = 0$
(c) $a^2c^2 + bd + d^2 = 0$ (D) None of these

Q.36 The incentre of a ΔABC with vertices (x_A, y_A), (x_B, y_B), (x_C, y_C) is the point with the coordinates-
(A) $\left(\frac{\Sigma ax_A}{\Sigma a}, \frac{\Sigma ay_A}{\Sigma a}\right)$ (B) $\left(\frac{\Sigma bc_A}{\Sigma bc}, \frac{\Sigma bcy_A}{\Sigma bc}\right)$
(C) $\left(\frac{\Sigma a^2x_A}{\Sigma a^2}, \frac{\Sigma a^2y_A}{\Sigma a^2}\right)$ (D) None of these

ASSERTION AND REASON QUESTIONS

Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 5 choices (A), (B), (C), (D) & (E) out of which ONLY ONE is correct.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.
(E) Statement -1 is False, Statement-2 is False.

Q.37 **Statement-1:** The lines $(a + b)x + (a - 2b)\, y = a$ are concurrent at the point (1/3, 2/3).
Statement-2: : The lines $x + y - 1 = 0$ and $x - 2y = 0$ intersect at the point (1/3, 2/3).

Q.38 **Statement 1 :** In a Δ ABC, equation of the side BC is $3x + 4y + 5 = 0$. If the perpendicular drawn from A to BC meets the circumcircle of the Δ ABC in the point (2, 1), which is different from A, then the point (–8/5, –19/5) is the orthocentre of the ΔABC.
Statement 2 : Image of the orthocentre of a Δ ABC in each of its sides lie on the circumcircle of the Δ ABC.

Q.39 **Statement-1:** The three non-parallel lines $a_1x + b_1y + c_1=0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$
Statement-2: The area of the triangle formed by three concurrent lines must be zero.

Q.40 Let P be the point lying inside the acute triangle ABC such that angles subtended by each side at P is 120°. Equilateral triangles AFB, BDC, CEA are constructed outwardly on sides AB, BC, CA of Δ ABC.
Statement 1 : Lines AD, BE, CF are concurrent at P.
Statement 2 : P is the radical centre of circumcircles of triangles ABC, BDC, CEA.

Q.41 **Statement-1:** If A(2a, 4a) and B(2a, 6a) are two vertices of a equilateral triangle ABC and the vertex C is given by $(2a + a\sqrt{3}, 5a)$.
Statement-2: In an equilateral triangle all the coordinates of three vertices can be rational.

Q.42 **Statement-1:** If a, b, c are in A.P. then every line of the form of $ax + by + c = 0$ where a, b, c are arbitrary constants pass through the point (1,-2)
Statement-2: Every line of the form of $ax + by + c = 0$ where a, b, c are arbitrary constants pass through a fixed point if their exist a linear relation between a, b & c.

MATCH THE COLUMN TYPE QUESTIONS

Q.43 Match the column –

Column I	Column II
(a) If orthocentre of the triangle ABC is $(a + ib)$, where $A(i + 8i)$, $B(2i)$ and $C(6 + i)$, then $(2a + b)$ is equal to	(p) 1
(b) The reflection of the point $(t - 1, 2t + 2)$ in a line is $(2t + 1, t)$, then slope of the line is equal to	(q) 3
(c) If line $y = 3x$ bisects the angle between the lines $ax^2 + 2axy + y^2 = 0$, then value of 11a is equal to	(r) 2
(d) Lines $x - 2y - 6 = 0$, $3x + y - 4 = 0$, $\lambda x + 4y + \lambda^2 = 0$ are concurrent, then positive value of λ is equals to	(s) 4

Code :
(A) a-r, b-p, c-q, d-r (B) a-p, b-q, c-r, d-s
(C) a-r, b-q, c-s, d-p (D) a-r, b-s, c-p, d-q

Q.44 Match the column –

Column I	Column II
(a) The lines $y = 0$, $y = 1$, $x - 6y = 4$ and $x + 6y - 9 = 0$ constitute a figure which is	(p) a cyclic quadrilateral
(b) The points A (a, 0), B (0, b), C (c, 0) and D (0, d) are such that ac = bd and a, b, c, d are all non-zero. The points A, B, C and D always constitute	(q) a rhombus
(c) The figure formed by the four lines $ax \pm by \pm c = 0$ $(a \neq b)$ is	(r) a square
(d) The line pairs $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ constitute a figure which is	(s) a trapezium

Code :
(A) a-qs, b-pqr, c-p, d-ps (B) a-ps, b-p, c-q, d-pqr
(C) a-r, b-pqr, c-ps, d-pq (D) a-qr, b-ps, c-pr, d-rs

Q.45 Match the column correctly –

Column I	Column II
(a) Lines $x - 2y - 6 = 0$, $3x + y - 4 = 0$, $\lambda x + 4y + \lambda^2 = 0$ are concurrent, then λ =	(p) 2
(b) The points $(\lambda + 1, 1)$, $(2\lambda + 1, 3)$ and $(2\lambda + 2, 2\lambda)$ are collinear then λ =	(q) 4
(c) If line $x + y - 1 - \lambda = 0$ passing through the intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$ is perpendicular to one of them, then λ =	(r) –1/2

(d) If line $y - x - 1 + \lambda = 0$ is equally inclined to axes and equidistant from the points (1, –2) and (3, 4) then λ = (s) – 4

Code :

(A) a-qs, b-pq, c-p, d-ps (B) a-ps, b-p, c-q, d-qr
(C) a-r, b-pr, c-ps, d-pq (D) a-ps, b-pr, c-p, d-p

PASSAGE BASED QUESTIONS

Passage 1- (Q.46-Q.48)

Consider a Δ ABC in which sides AB and AC are perpendicular to $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively vertex A is (–2, 3) & circumcentre of the ΔABC is (3/2,5/2).

Q.46 Equation of perpendicular bisector of side AB will be
(A) $x - y + 1 = 0$ (B) $x - y + 4 = 0$
(C) $x - y + 2 = 0$ (D) none of these

Q.47 Equation of perpendicular bisector of side AC will be
(A) $2x - y + 1 = 0$ (B) $2x - y - (1/2) = 0$
(C) $2x - y + (1/2) = 0$ (D) none of these

Q.48 The coordinate of B will be
(A) (2, –1) (B) (1, –2)
(C) (2, 3) (D) none of these

Passage 2- (Q.49-Q.51)

In a triangle ABC, if the equations of sides AB, BC and CA are $2x - y + 4 = 0$, $x - 2y - 1 = 0$ and $x + 3y - 3 = 0$ respectively, then–

Q.49 Tangent of internal angle A is equal to –
(A) 1/2 (B) –7
(C) –3 (D) 7

Q.50 The equation of external bisector of angle B is –
(A) $x - y + 1 = 0$ (B) $x + y + 5 = 0$
(C) $x - y - 1 = 0$ (D) $x + y - 5 = 0$

Q.51 The image of the point B w.r.t. the side CA is –
(A) (–3/5, –26/5) (B) (–3/5, 26/5)
(C) (3/5, –26/5) (D) (3/5, 26/5)

Passage 3- (Q.52-Q.54)

Consider a Δ ABC whose sides AB, BC and CA are represented by the straight lines $2x + y = 0$, $x + py = q$ and $x - y = 3$ respectively. The point P is (2, 3).

Q.52 If P is the centroid, then (p + q) equals –
(A) 47 (B) 50
(C) 65 (D) 74

Q.53 If P is the orthocentre, then (p + q) equals –
(A) 47 (B) 50
(C) 65 (D) 74

Q.54 If P is the circumcentre, then (p + q) equals –
(A) 47 (B) 50
(C) 65 (D) 74

Passage 4- (Q.55-Q.57)

Consider a line pair $ax^2 + 3xy - 2y^2 - 5x + 5y + c = 0$ representing perpendicular lines intersecting each other at C and forming a triangle ABC with the x-axis.

Q.55 If x_1 and x_2 are intercepts on the x-axis and y_1 and y_2 are the intercepts on the y-axis then the sum $(x_1 + x_2 + y_1 + y_2)$ is equal to–
(A) 6 (B) 5
(C) 4 (D) 3

Q.56 Distance between the orthocentre and circumcentre of the triangle ABC is –
(A) 2 (B) 3
(C) 7/4 (D) 9/4

Q.57 If the circle $x^2 + y^2 - 4y + k = 0$ is orthogonal with the circumcentre of the triangle ABC then k equals –
(A) 1/2 (B) 1
(C) 2 (D) 3/2

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 The origin is shifted to a point (3/A, – 2) so that the equation $y^2 + 4y + 8x - 2 = 0$ will not contain term in y and the constant. Find the value of A.

Q.2 The acute angle between the lines $2x + y + 11 = 0$, $x - 6y + 7 = 0$ is $\tan^{-1}(A/4)$. Find the value of A.

Q.3 The locus of a variable point whose distance from (1, 0) is half its distance from the line x = 4 is $Ax^2 + By^2 = C$. Find the value of A + B + C.

Q.4 Vertices of a triangle are $(1, \sqrt{3}), (2\cos\theta, 2\sin\theta)$ and $(2\sin\theta, -2\cos\theta)$. Then locus of orthocentre of the triangle is $(x-1)^2 + (y-\sqrt{3})^2 = A$. Find the value of A.

Q.5 Through a point A on the x-axis a straight line is drawn parallel to y-axis so as to meet the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ in B and C. If AB = BC and $Ph^2 = 9ab$ then find the value of P.

Q.6 Given A(0, 0) and B(x, y) with $x \in (0, 1)$ and $y > 0$. Let the slope of the line AB equals m_1. Point C lies on the line x=1 such that the slope of BC equals m_2 where $0<m_2<m_1$. If the area of the triangle ABC can be expressed as $(m_1 - m_2) f(x)$, then the largest possible value of f (x) is 1/P. Find the value of P.

Q.7 Let ABC be a triangle. Let A be the point (1, 2), y = x is the perpendicular bisector of AB and $x - 2y + 1 = 0$ is the angle bisector of ∠ C. If equation of BC is given by $ax + by - 5 = 0$, then find the value of a + b.

Q.8 The number of integer values of m, for which the x coordinate of the point of intersection of the lines 3x + 4y = 9 and y = mx + 1 is also an integer, is

Q.9 A straight line through the origin O meets the parallel lines 4x + 2y = 9 and 2x + y + 6 = 0 at points P and Q respectively. Then the point O divides the segment PQ in the ratio 3 : A. Find the value of A.

Q.10 Area of the triangle formed by the line x + y = 3 and angle bisectors of the pairs of straight lines $x^2 - y^2 + 2y = 1$ is

Q.11 Find the value of x for which the points (x, – 1), (2, 1) and (4, 5) are collinear.

Q.12 Find the distance of the point (–1, 1) from the line 12 (x + 6) = 5 (y – 2).

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 The points (– a, –b), (0, 0), (a, b) and (a^2, ab) are- **[AIEEE-2002]**
(A) collinear
(B) concyclic
(C) vertices of a rectangle
(D) vertices of a parallelogram

Q.2 The centroid of a triangle is (2, 3) and two of its vertices are (5, 6) and (– 1, 4). The third vertex of the triangle is **[AIEEE-2002]**
(A) (2, 1) (B) (2, – 1)
(C) (1, 2) (D) (1, – 2)

Q.3 The angle between the straight lines $x^2 + 4xy + y^2 = 0$ is **[AIEEE 2002]**
(A) 30° (B) 45°
(C) 60° (D) 90°

Q.4 The distance between a pair of parallel lines $9x^2 – 24xy + 16y^2 – 12x + 16y – 12 = 0$ **[AIEEE 2002]**
(A) 5 (B) 8
(C) 8/5 (D) 5/8

Q.5 A square of side a lies above the x- axis and has one vertex at the origin. The side passing through the origin makes an angle α ($0 < \alpha < \pi/4$) with the positive direction of x- axis. The equation of its diagonal not passing through the origin is- **[AIEEE 2003]**
(A) $y (\cos\alpha + \sin\alpha) + x (\cos \alpha – \sin\alpha) = a$
(B) $y (\cos\alpha – \sin\alpha) –x (\sin\alpha – \cos\alpha) = a$
(C) $y (\cos \alpha + \sin \alpha) + x (\sin \alpha – \cos \alpha) = a$
(D) $y (\cos \alpha + \sin \alpha) + x (\sin \alpha + \cos \alpha) = a$

Q.6 If the pair of straight lines $x^2 – 2pxy – y^2 = 0$ and $x^2 – 2pxy – y^2 = 0$ be such that each pair bisects the angle between the other pair, then – **[AIEEE 2003]**
(A) pq = –1 (B) p = q
(C) p = –q (D) pq = 1

Q.7 Locus of centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, – b cos t) and (1, 0), where t is a parameter, is- **[AIEEE 2003]**
(A) $(3x + 1)^2 + (3y)^2 = a^2 – b^2$
(B) $(3x– 1)^2 + (3y)^2 = a^2 – b^2$
(C) $(3x – 1)^2 + (3y)^2 = a^2 + b^2$
(D) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

Q.8 If the equation of the locus of a point equidistant from the points (a_1,b_1) and (a_2, b_2) is $(a_1 – a_2) x + (b_1 – b_2) y + c = 0$, then the value of c is - **[AIEEE-2003]**
(A) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$ (B) $\frac{1}{2}(a_2^2+b_2^2– a_1^2 – b_1^2)$
(C) $\frac{1}{2}(a_1^2 + a_2^2 +b_1^2+ b_2^2)$ (D) $a_1^2 – a_2^2 + b_1^2 – b_2^2$

Q.9 Let A (2, – 3) and B (– 2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line 2x+3y= 1, then the locus of the vertex C is the line **[AIEEE 2004]**
(A) 2x + 3y = 9 (B) 2x – 3y = 7
(C) 3x + 2y = 5 (D) 3x – 2y = 3

Q.10 The equation of the straight line passing through the point (4, 3) and making intercepts on the coordinate axes whose sum is – 1 is- **[AIEEE 2004]**
(A) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$
(B) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$
(C) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{2}+\frac{y}{1}=1$
(D) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

Q.11 If the sum of the slopes of the lines given by $x^2 – 2cxy – 7y^2 = 0$ is four times their product, then c has the value – **[AIEEE 2004]**
(A) 1 (b) –1
(C) 2 (D) –2

Q.12 If one of the lines given by $6x^2 – xy + 4cy^2 = 0$ is 3x + 4y = 0, then c equals – **[AIEEE 2004]**
(A) 1 (B) –1
(C) 3 (D) –3

Q.13 The line parallel to the x-axis and passing through the intersection of the lines ax + 2by + 3b = 0 and bx – 2ay–3a= 0, where (a, b) $\neq$ (0, 0) is - **[AIEEE-2005]**
(A) below the x-axis at a distance of 3/2 from it
(B) below the x-axis at a distance of 2/3 from it
(C) above the x-axis at a distance of 3/2 from it
(D) above the x-axis at a distance of 2/3 from it

Q.14 If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point that point is – **[AIEEE-2005]**
(A) (–1, 2) (B) (–1, –2)
(C) (1, –2) (D) (1, –1/2)

Q.15 Let P be the point (1, 0) and Q be a point on the curve $y^2 = 8x$. The locus of mid point of PQ is – **[AIEEE-2005]**
(A) $y^2 – 4x + 2 = 0$ (B) $y^2 + 4x + 2 = 0$
(C) $x^2 + 4y + 2 = 0$ (D) $x^2 – 4y + 2 = 0$

Q.16 If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2), then the centroid of the triangle is - **[AIEEE-2005]**
(A) (–1, 7/3) (B) (–1/3, 7/3)
(C) (1, 7/3) (D) (1/3, 7/3)

Q.17 A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is **[AIEEE 2006]**
(A) 3x – 4y + 7 = 0 (B) 4x + 3y = 24
(C) 3x + 4y = 25 (D) x + y = 7

Q.18 If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x, x > 0$, then a belongs to **[AIEEE 2006]**
(A) $(3, \infty)$ (B) $(1/2, 3)$
(C) $(-3, -1/2)$ (D) $(0, 1/2)$

Q.19 Let A (h, k), B (1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by **[AIEEE 2007]**
(A) {1, 3} (B) {0, 2}
(C) {–1, 3} (D) {–3, –2}

Q.20 Let P (–1, 0) Q = (0, 0) and R (3, $3\sqrt{3}$) be three points. The equation of the bisector of the angle PQR is - **[AIEEE 2007]**
(A) $\sqrt{3}x + y = 0$ (B) $x + \frac{\sqrt{3}}{2}y = 0$
(C) $\frac{\sqrt{3}}{2}x + y = 0$ (D) $x + \sqrt{3}y = 0$

Q.21 If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines xy = 0, then m is -
(A) – 1/2 (B) –2 **[AIEEE 2007]**
(C) 1 (D) 2

Q.22 The perpendicular bisector of the line segment joining P (1, 4) and Q (k, 3) has y-intercept-4. Then a possible value of k is -
(A) 2 (B) –2 **[AIEEE 2008]**
(C) –4 (D) 1

Q.23 The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for – **[AIEEE 2009]**
(A) more than two values of p (B) no value of p
(C) exactly one value of p (D) exactly two values of p

Q.24 Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to 1/3. Then the circumcentre of the triangle ABC is at the point –
(A) (5/3, 0) (B) (0, 0) **[AIEEE 2009]**
(C) (5/4, 0) (D) (5/2, 0)

Q.25 The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is –
(A) $\sqrt{17}$ (B) $\frac{17}{\sqrt{15}}$ **[AIEEE 2010]**
(C) $\frac{23}{\sqrt{17}}$ (D) $\frac{23}{\sqrt{15}}$

Q.26 The line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE 2011]**
Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.
Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.
(A) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

Q.27 If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals – **[AIEEE 2012]**
(A) 29/5 (B) 5
(C) 6 (D) 11/5

Q.28 A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. if the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE 2012]**
(A) –1/4 (b) – 4
(C) – 2 (D) –1/2

Q.29 A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is – **[JEE MAIN 2013]**
(A) $y = x + \sqrt{3}$ (B) $\sqrt{3}y = x - \sqrt{3}$
(C) $y = \sqrt{3}x - \sqrt{3}$ (D) $\sqrt{3}y = x - 1$

Q.30 The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is – **[JEE MAIN 2013]**
(A) $2 + \sqrt{2}$ (B) $2 - \sqrt{2}$
(C) $1 + \sqrt{2}$ (D) $1 - \sqrt{2}$

Q.31 Let a, b, c and d be non-zero numbers. If the point of intersection of the lines 4ax + 2ay + c = 0 and 5bx + 2by + d = 0 lies in the fourth quadrant and is equidistant from the two axes then – **[JEE MAIN 2014]**
(A) 2bc – 3ad = 0 (B) 2bc + 3ad = 0
(C) 3bc – 2ad = 0 (D) 3bc + 2ad = 0

Q.32 Let PS be the median of the triangle with vertices P (2, 2), Q (6, –1) and R (7, 3). The equation of the line passing through (1, –1) and parallel to PS is **[JEE MAIN 2014]**
(A) 4x – 7y – 11 = 0 (B) 2x + 9y + 7 = 0
(C) 4x + 7y + 3 = 0 (D) 2x – 9y – 11 = 0

Q.33 Two sides of a rhombus are along the lines, x – y + 1 = 0 and 7x – y – 5 = 0 . If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus **[JEE MAIN 2016]**
(A) (–3, –8) (B) (1/3, –8/3)
(C) (–10/3, –7/3) (D) (–3, –9)

Q.34 Let k be an integer such that the triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[JEE MAIN 2017]**
(A) (1, –3/4) (B) (2, 1/2)
(C) (2, –1/2) (D) (1, 3/4)

Q.35 A straight line through a fixed point (2, 3) intersects the coordinates axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is : **[JEE MAIN 2018]**
(A) $3x+2y=xy$ (B) $3x+2y=6xy$
(C) $3x+2y=6$ (D) $2x+3y=xy$

Q.36 Let the orthocentre and centroid of a triangle be A (–3, 5) and B (3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is : **[JEE MAIN 2018]**
(A) $3\sqrt{\frac{5}{2}}$ (B) $\frac{3\sqrt{5}}{2}$
(C) $\sqrt{10}$ (D) $2\sqrt{10}$

Q.37 Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true ? **[JEE MAIN 2019 (JAN)]**
(A) The lines are all parallel.
(B) Each line passes through the origin.
(C) The lines are not concurrent
(D) The lines are concurrent at the point (3/4, 1/2)

Q.38 A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in **[JEE MAIN 2019 (APRIL)]**
(A) 1st and 2nd quadrants (B) 4th quadrant
(C) 1st, 2nd and 4th quadrant (D) 1st quadrant

Q.39 Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of Δ AOP is 4, is : **[JEE MAIN 2019 (APRIL)]**
(A) $8x^2 - 9y^2 + 9y = 18$ (B) $9x^2 + 8y^2 - 8y = 16$
(C) $8x^2 + 9y^2 - 9y = 18$ (D) $9x^2 - 8y^2 + 8y = 16$

Q.40 Suppose that the points (h,k), (1, 2) and (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then (k/h) equals : **[JEE MAIN 2019 (APRIL)]**
(A) 3 (B) –1/7
(C) 1/3 (D) 0

Q.41 Slope of a line passing through P (2, 3) and intersecting the line, $x + y = 7$ at a distance of 4 units from P, is **[JEE MAIN 2019 (APRIL)]**
(A) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (B) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (C) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ (D) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

Q.42 Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance 3/5 from the origin. Then which one of the following points lies on any of these lines ? **[JEE MAIN 2019 (APRIL)]**
(A) (–1/4, 2/3) (B) (1/4, 2/3)
(C) (–1/4, –2/3) (D) (1/4, –1/3)

Q.43 A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is **[JEE MAIN 2019 (APRIL)]**
(A) $(\sqrt{3}+1)\,x + (\sqrt{3}-1)\,y = 8\sqrt{2}$
(B) $(\sqrt{3}-1)\,x + (\sqrt{3}+1)\,y = 8\sqrt{2}$
(C) $\sqrt{3}x + y = 8$
(D) $x + \sqrt{3}y = 8$

Q.44 A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2,3). Then the centroid of this triangle is : **[JEE MAIN 2019 (APRIL)]**
(A) (1/3, 1) (B) (1/3, 2)
(C) (1, 7/3) (D) (1/3, 5/3)

Q.45 Let A (1, 0), B (6, 2) and C (3/2, 6) be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point (–7/6, –1/3) is ________. **[JEE MAIN 2020 (JAN)]**
(A) 2 (B) 3
(C) 4 (D) 5

Q.46 From any point P on the line $x = 2y$ perpendicular is drawn on $y = x$. Let foot of perpendicular is Q. Find the locus of mid point of PQ. **[JEE MAIN 2020 (JAN)]**
(A) $2x = 3y$ (B) $5x = 7y$
(C) $3x = 2y$ (D) $7x = 5y$

Q.47 Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point **[JEE MAIN 2020 (JAN)]**
(A) (7, 6) (B) (–9, –6)
(C) (–9, –7) (D) (9, 7)

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	D	A	A	B	C	B	B	D	C	B	B	B	B	C	C	D	B	D	C	A	A	D	C	C
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	A	A	B	C	A	B	B	B	D	B	C	B	A	D	A	D	D	C	C	B	B	A	A	B	C
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	A	B	C	B	A	D	C	C	C	C	A	A	C	C	B	B	A	C	D	A	C	D	D	C	B
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
A	A	A	B	A	A	D	A	D	A	D	C	B	D	A	B	A	B	D	D	A	C	C	C	A	D
Q	101	102	103	104	105	106	107	108	109	110	111														
A	A	C	A	D	A	B	C	A	C	D	B														

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	C	A	A	A	D	B	A	A	A	A	A	A	C	A	C	D	A	C	C	B	A	D	C	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	B	A	C	C	B	A	A	C	C	B	A	A	A	A	B	C	A	A	B	D	A	B	A	B	B
Q	51	52	53	54	55	56	57																		
A	B	D	B	A	B	C	D																		

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12
A	4	13	19	8	8	8	2	2	4	2	1	5

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	A	B	C	C	A	A	C	B	A	D	C	D	A	C	A	C	B	B	C	A
Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	C	C	C	C	C	C	C	C	B	B	C	B	B	B	A	A	D	A	B	C
Q	41	42	43	44	45	46	47													
A	C	A	A	B	D	B	B													

CHAPTER- : 9
POINT AND STRAIGHT LINES
SOLUTIONS TO TRY IT YOURSELF
TRY IT YOURSELF-1

(1) $PQ = \sqrt{2^2+3^2-2.2.3\cos\left(-\frac{\pi}{6}-\frac{\pi}{6}\right)}$

$= \sqrt{13-12\times\cos\left(-\frac{\pi}{3}\right)} = \sqrt{13-12\times\frac{1}{2}} = \sqrt{7}$

(2) Let $A=(x_1, y_1)=(3, 2), B=(x_2, y_2)=(11, 8)$ and $C=(x_3, y_3)=(8, 12)$

Then area of Δ ABC

$= \frac{1}{2}|\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}|$

$= \frac{1}{2}|\{3(8-12)+11(12-2)+8(2-8)\}|$

$= \frac{1}{2}|\{-12+110-48\}| = 25$ sq. units

(3) Let the required ratio be $\lambda : 1$

Then, the coordinates of the point of division are

$\left(\frac{5\lambda+2}{\lambda-1}, \frac{6\lambda-3}{\lambda+1}\right)$

But it is a point on x-axis on which y-coordinates of every point is zero.

$\Rightarrow \frac{6\lambda-3}{\lambda+1} = 0 \Rightarrow \lambda = \frac{1}{2}$

Thus the required ratio is (1/2) : 1 or 1 : 2

(4) **(C).** Clearly, (0, 0), (3, 4) and (6, 8) are collinear. So, the circumcentre M and centoid G are on the median which is also the perpendicular bisector of the side, So, the Δ must be isosceles.

(5) **(A).** Let A (4, 0), B (–1, –1), and C (3, 5) be the given points. Then, we get

$|AB| = \sqrt{(-1-4)^2+(-1-0)^2} = \sqrt{25+1} = \sqrt{26}$

$|BC| = \sqrt{(3+1)^2+(5+1)^2} = \sqrt{16+36} = \sqrt{52}$

$|CA| = \sqrt{(4-3)^2+(0-5)^2} = \sqrt{1+25} = \sqrt{26}$

Clearly, $|AB| = |CA|$

$\Rightarrow$ Triangle is isosceles.

And $BC^2 = AB^2 + CA^2$ $[\because 52 = 26+26]$

$\Rightarrow$ Triangle is right angled.

(6) Let $P \equiv (x, y)$.

According to the question,

$x = a\cos\theta$ .... (1) $\quad y = b\sin\theta$ .... (2)

Squaring and adding eq. (1) and (2), we get

$\frac{x^2}{a^2}+\frac{y^2}{b^2} = \cos^2\theta+\sin^2\theta$ or $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$

(7) Let (h, k) be the point to which the origin is shifted. Then,

$x = 4, y = 5, X = -3, Y = 9$

$\therefore$ $x = X + h$ and $y = Y + k$

$\Rightarrow$ $4 = -3 + h$ and $5 = 9 + k \Rightarrow h = 7$ and $k = -4$

Hence, the origin must be shifted to (7, – 4)

(8) **(C).** PQRS will represent a parallelogram if and only if the mid point of PR is same as that of QS. That is, if and only

if $\frac{1+5}{2} = \frac{4+a}{2}$ and $\frac{2+7}{2} = \frac{6+b}{2}$

$\Rightarrow a = 2$ and $b = 3$

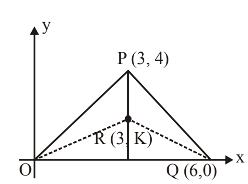
(9) **(A).** Since the coordinates of the centroid are

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$, the centroid is always a rational point, $x_1, x_2, x_3, y_1, y_2, y_3$ being rational.

(10) **(C).** Here OP = PQ = 5

$\Rightarrow$ Δ OPQ is an isosceles triangle.

$\therefore$ R must lie on angle bisector of $\angle$ P.



Let R (3, k)

Now, Ar (Δ ORQ) $= \frac{1}{3}$ Ar (Δ OPQ)

$\Rightarrow \frac{1}{2}\times 6\times k = \frac{1}{2}\times\frac{1}{3}\times 6\times 4 \Rightarrow k = \frac{4}{3} \Rightarrow R(3, 4/3)$

TRY IT YOURSELF-2

(1) Slope of given line is $= -\frac{2}{3}$ $\quad \therefore \tan\theta = -\frac{2}{3}$

Hence $90° < \theta < 180°$

$\therefore \sin\theta = \frac{2}{\sqrt{13}}$, $\cos\theta = -\frac{3}{\sqrt{13}}$

Distance from of line $2x + 3y + 7 = 0$ is

$$\frac{x-1}{\left(-\frac{3}{\sqrt{13}}\right)} = \frac{y+3}{\left(\frac{2}{\sqrt{3}}\right)} = r$$

Putting r = 3 we get the co-ordinate of desired point as

$$x - 1 = -\frac{9}{\sqrt{13}}, y + 3 = \frac{6}{\sqrt{13}} \text{ or } x = 1 - \frac{9}{\sqrt{13}}, y = -3 + \frac{6}{\sqrt{13}}$$

(2) **(C).** Line cutting off intercept 4 on the x-axis, then line passing through (4, 0). Equation of line passing through (4, 0) & (2, –3)

$$y + 3 = \frac{3}{2}(x - 2)$$

$2y + 6 = 3x - 6$; $3x - 2y = 12$

(3) **(D).** Line cutting off equal intercepts = a

Let the line be $\frac{x}{a} + \frac{y}{a} = 1$; $x + y = a$

It passes through (2, 5), $\quad 2 + 5 = a \Rightarrow a = 7$

Equation of line $x + y = 7$

(4) **(A).** Equation of line passing through (0, 0) & (3, 5)

$$y - 0 = \frac{5}{3}(x - 0);\ 3y = 5x;\ 5x - 3y = 0$$

(5) **(B).** Slope of line = tan 135° = –1

Equation of line through (1, 0)

$y - 0 = -1(x - 1)$; $y + x = 1$

(6) **(D).** If line is equally inclined then slope (m) = ±1

Equation of lines are $y - 0 = \pm 1(x - 1)$

$x + y = 1$ or $x - y = 1$

TRY IT YOURSELF-3

(1) **(D).** $\frac{a}{\sqrt{bc}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}}$

$\Rightarrow \quad a = b + c + 2\sqrt{bc} \Rightarrow a = (\sqrt{b} + \sqrt{c})^2$

$\Rightarrow \quad (\sqrt{a} - \sqrt{b} - \sqrt{c})(\sqrt{a} + \sqrt{b} + \sqrt{c}) = 0$

$\Rightarrow \quad \sqrt{a} - \sqrt{b} - \sqrt{c} = 0$

Since, $\sqrt{a} + \sqrt{b} + \sqrt{c} \neq 0$ $(\because a, b, c > 0)$

Comparing with $\sqrt{a}x + \sqrt{b}y + \sqrt{c} = 0$

We have, $x = -1, y = 1$.

(2) **(A).** First two family of lines passes through (1, 1) and (3, 3) respectively. The point of intersection of lines belonging to third family of lines will lie on line y = x. Hence,

$ax + x - 2 = 0$ and $6x + ax - a = 0$,

or $\frac{2}{a+1} = \frac{a}{6+a} \Rightarrow a^2 - a - 12 = 0 \Rightarrow (a-4)(a+3) = 0$

(3) **(C).** ax + by = 1 will be one of the bisectors of the given line. Equation of bisectors of the given lines are

$$\frac{3x + 4y - 5}{5} = \pm\left(\frac{5x - 12y - 10}{13}\right)$$

$\Rightarrow \quad 64x - 8y = 115$ or $14x + 112y = 15$

$\Rightarrow \quad a = \frac{64}{115}, b = -\frac{8}{115}$ or $a = \frac{14}{15}, b = \frac{12}{115}$

(4) **(B).** The line passing through (2, 3) and perpendicular to

$-y + 3x + 4 = 0$ is $\frac{y-3}{x-2} = -\frac{1}{3}$ or $3y + x - 11 = 0$

Therefore, foot is x = –1/10, y = 37/10

(5) **(C).** $x(a + 2b) + y(a + 3b) = a + b$

$\Rightarrow \quad a(x + y - 1) + b(2x + 3y - 1) = 0$

This equation will always be satisfied for

$x + y - 1 = 0$ & $2x + 3y - 1 = 0$ solving these equation

we get $x = 2, y = -1$.

(6) **(D).** Let the point P (3, 1) has image P' across the line

$2x - y + 7 = 0$

Now, PP' is perpendicular to $2x - y + 7 = 0$

Slope of PP' = –1/2

Equation of PP' is $y - 1 = -\frac{1}{2}(x - 3)$

$2y-2=-x+3$

$x+2y=5$

Point of intersection of lines

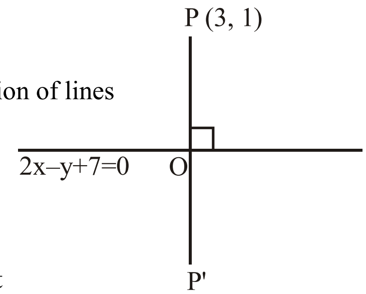
$2x-y+7=0$ and

$x+2y=5$ is

O (–9/5, 17/5)

O is the mid point of PP'

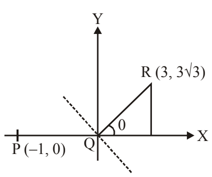
Let P' (h, k)



$$\frac{h+3}{2}=\frac{-9}{5},\ \frac{k+1}{2}=\frac{17}{5}$$

$$h'=\frac{-33}{5},\ k=\frac{29}{5}\Rightarrow \text{Image}\left(\frac{-33}{5},\frac{29}{5}\right)$$

(7) **(C).** Equation of PQ is $y=0$



Equation of QR is $y=\frac{3\sqrt{3}x}{3}$

$\therefore$ Angle bisector of $\angle$ PQR is given by

$$y=\pm\left(\frac{y-\sqrt{3}x}{\sqrt{1^2+\sqrt{3}^2}}\right)=\pm\left(\frac{y-\sqrt{3}x}{2}\right)$$

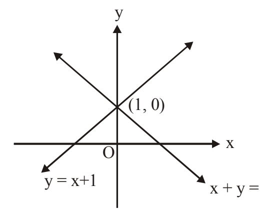
$\Rightarrow$ $y+\sqrt{3}x=0$ or $\sqrt{3}y-x=0$

But from figure $\angle$ PQR is obtuse angle.

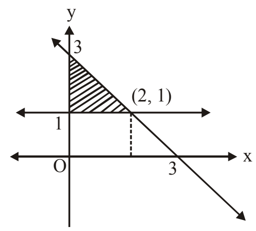
$\therefore$ Its bisector is the one obtained taking +ve sign

i.e., $\sqrt{3}x+y=0$

(8) **(A).** $x^2-y^2+2y=1\Rightarrow x=\pm(y-1)$



Bisectors of above lines are $x=0$ and $y=1$.



So, area between $x=0$, $y=1$ and $x+y=3$ is shaded reagion shown in figure.

$$\text{Area}=\frac{1}{2}\times 2\times 2=2\text{ sq. units}$$

(9) **(B).** Let slope of line L = m

$$\therefore\ \left|\frac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}\right|=\tan 60°=\sqrt{3}\ \Rightarrow\ \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right|=\sqrt{3}$$

Taking positive sign, $m+\sqrt{3}=\sqrt{3}-3m$; $m=0$

Taking negative sign,

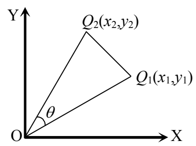
$m+\sqrt{3}+\sqrt{3}-3m=0$; $m=\sqrt{3}$

As L cuts x-axis $\Rightarrow$ $m=\sqrt{3}$

So, L is $y+2=\sqrt{3}\ (x-3)$

CHAPTER-: 9
POINT AND STRAIGHT LINES
EXERCISE-1

(1) **(C).** From triangle OQ_1Q_2, by applying cosine formula



$Q_1Q_2^2 = OQ_1^2 + OQ_2^2 - 2OQ_1.OQ_2 \cos Q_1OQ_2$

or $(x_1 - x_2)^2 + (y_1 - y_2)^2$

$= x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2OQ_1.OQ_2 \cos\theta$

or $x_1x_2 + y_1y_2 = OQ_1.OQ_2 \cos Q_1OQ_2$.

(2) **(D).** Distance between points (7, 5) and (3, 2)

$= \sqrt{(3-7)^2 + (2-5)^2} = \sqrt{16+9} = 5$ unit.

(3) **(A).** Let the point be (x, y)

(i) Point B (x, y) divides AD in 1 : 2

$\therefore\ x = \frac{0+9}{3} = 3$ and $y = \frac{0+12}{3} = 4$

(ii) Now point C (x, y) divides AD in 2 : 1,

Then $x = \frac{0+18}{3} = 6$ and $y = \frac{0+24}{3} = 8$.

(4) **(A).** Let the ratio be k : 1

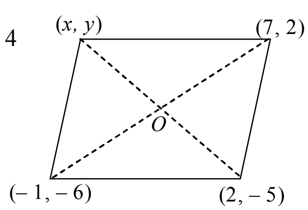
Therefore, $\frac{-7k+3}{k+1} = \frac{1}{2} \Rightarrow k = \frac{1}{3}$.

Hence ratio is 1 : 3 internally.

(5) **(B).** $\frac{-1+7}{2} = \frac{2+x}{2} \Rightarrow x = 4$

$\frac{-6+2}{2} = \frac{-5+y}{2} \Rightarrow y = 1$

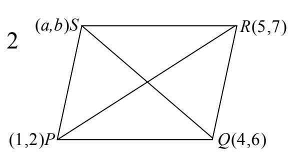
$\therefore$ Fourth vertex (x, y) is (4, 1).



(6) **(C).** Diagonals cut each other at middle points.

Hence, $\frac{a+4}{2} = \frac{1+5}{2} \Rightarrow a = 2$

$\frac{b+6}{2} = \frac{2+7}{2} \Rightarrow b = 3$.



(7) **(B).** Let $A(-1, 1)$, $B(0, -3)$, $C(5, 2)$ and $D(4, 6)$

$\Rightarrow AB = \sqrt{17}$, $CD = \sqrt{17}$, $BC = \sqrt{50}$,

$AD = \sqrt{50}$, $AC = \sqrt{37}$ and $BD = \sqrt{97}$

Obviously, $AB = CD$ and $BC = AD$. Also diagonal $AC \neq BD$. Therefore, quadrilateral is parallelogram.

(8) **(B).** Since $PQ = \sqrt{68}$, $PR = \sqrt{17}$, $QR = \sqrt{85}$

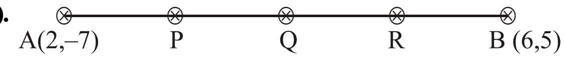
$\therefore PQ^2 + PR^2 = QR^2$, i.e. right angled triangle.

(9) **(D).** Mid point is $\left(\frac{1+a+b}{2}, \frac{a+2}{2}\right)$

But given (–1, 1)

$\Rightarrow$ a = – 4, b = 1

(10) **(C).**



Mid point $\equiv Q \equiv \left(\frac{6+2}{2}, \frac{-7+5}{2}\right) \equiv (4, -1)$

(11) **(B).** $\left.\begin{matrix}\text{Slope of AB} = 2\\ \text{Slope of BC} = 2\end{matrix}\right\} \Rightarrow$ A, B, C represent straight line

(12) **(B).** Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the coordinates of the vertices of a triangle and let $x_1, x_2, x_3, y_1, y_2, y_3$ be all integers.

Then the area of triangle

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= Some rational numbers, because x's and y's are integers. Also, if the triangle is equilateral and a be the length of its side, then $a^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ = A positive integer. The area of the triangle

$= \frac{1}{2} bc \sin A = \frac{1}{2}.a.a \sin 60^o = \frac{\sqrt{3}}{4} a^2$

{$\because$ every angle is of 60^o}

Which is irrationals, because a^2 is positive integer. But earlier we have calculated that the area of the triangle is rational number. Hence it is a contradiction. Therefore, if the vertices of a triangle are integers, the triangle cannot be equilateral.

(13) **(B).** Let circumcentre be $O(x, y)$ and given points are $A(2, 1)$, $B(5, 2)$, $C(3, 4)$. Hence $OA^2 = OB^2 = OC^2$

$\therefore\ (x-2)^2 + (y-1)^2 = (x-5)^2 + (y-2)^2$ ......(i)

and $(x-2)^2 + (y-1)^2 = (x-3)^2 + (y-4)^2$ .....(ii)

On solving (i) and (ii), we get $x = \frac{13}{4}$ and $y = \frac{9}{4}$.

(14) **(B).** If P be the foot, then $PA = PB = PC$, i.e. P is the circumcentre.

(15) **(C).** The coordinates of the point A are $\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$

and that of B and C are (1, 5) and (7, – 2). Thus area of triangle ABC is

$\frac{1}{2}\left[\frac{3k-5}{k+1}(5+2) + 1\left(-2 - \frac{5k+1}{k+1}\right) + 7\left(\frac{5k+1}{k+1} - 5\right)\right] = \pm 2$

$\Rightarrow 21k - 35 - 7k - 3 - 28 = \pm 4(k+1) \Rightarrow k = 7$ or $\frac{31}{9}$.

(16) **(C).** We have $x_1 = ar$, $x_2 = ar^2$, $y_1 = bs$, $y_2 = bs^2$

The area of the triangle is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ ar & bs & 1 \\ ar^2 & bs^2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}ab\begin{vmatrix} 1 & 1 & 1 \\ r & s & 1 \\ r^2 & s^2 & 1 \end{vmatrix} = \frac{1}{2}ab\begin{vmatrix} 1 & 1 & 1 \\ r-1 & s-1 & 0 \\ r^2-1 & s^2-1 & 0 \end{vmatrix}$$

(Applying $R_2 - R_1$, $R_3 - R_1$)

$$= \frac{1}{2}ab(r-1)(s-1)(s-r).$$

(17) **(D).** In Δ ABC, $A \equiv (-3, 0)$; $B \equiv (4, -1)$ and $C \equiv (5, 2)$

We know that $BC = \sqrt{(5-4)^2 + (2+1)^2}$

$= \sqrt{1+9} = \sqrt{10}$

and area of Δ ABC

$$= \frac{1}{2}[-3(-1-2) + 4(2-0) + 5(0+1)] = 11$$

Therefore, altitude $AL = \frac{2\Delta ABC}{BC} = \frac{2\times 11}{\sqrt{10}} = \frac{22}{\sqrt{10}}$.

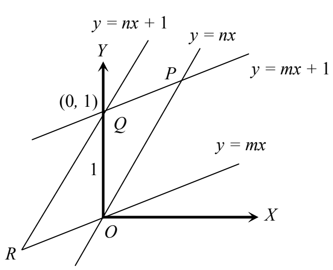
(18) **(B).** From given condition we can say circumcentre is point (–2, 3).
But triangle is equilateral so centroid is (–2, 3)

$$\Rightarrow \frac{x_1 + x_2 + x_3}{3} = -2,\ \frac{y_1 + y_2 + y_3}{3} = 3$$

$$\Rightarrow \frac{x_1 + x_2 + x_3}{y_1 + y_2 + y_3} = -\frac{2}{3}$$

(19) **(D).** Solving $y = nx$ and $y = mx + 1$, we get

$$P = \left(\frac{1}{n-m}, \frac{n}{n-m}\right)$$



∴ Area of parallelogram = 2 × (area of Δ POQ)

$$= 2\times\left|\frac{1}{2}\times OQ \times \frac{1}{n-m}\right| = \frac{1}{|n-m|} = \frac{1}{|m-n|}.$$

(20) **(C).** Area $= \frac{1}{2}\begin{vmatrix} am_1^2 & 2am_1 & 1 \\ am_2^2 & 2am_2 & 1 \\ am_3^2 & 2am_3 & 1 \end{vmatrix} = \frac{1}{2}a^2 \times 2\begin{vmatrix} m_1^2 & m_1 & 1 \\ m_2^2 & m_2 & 1 \\ m_3^2 & m_3 & 1 \end{vmatrix}$

$$= a^2\begin{vmatrix} m_1^2 - m_2^2 & m_1 - m_2 & 0 \\ m_2^2 - m_3^2 & m_2 - m_3 & 0 \\ m_3^2 & m_3 & 1 \end{vmatrix}, \text{ by } \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$$

$$= a^2 (m_2^2 - m_3^2)(m_1 - m_2) - (m_2 - m_3)(m_1^2 - m_2^2)$$

$$= a^2 (m_1 - m_2)(m_2 - m_3)(m_3 - m_1).$$

Let $a = 2$, $m_1 = 0$, $m_2 = 1$, $m_3 = 2$, then the coordinates are (0, 0), (2, 4), (8, 8).

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2 & 8 & 1 \\ 4 & 8 & 1 \end{vmatrix} = \frac{1}{2}(16-32) = 8 \ sq.\ units.$$

(21) **(A).** $\frac{\Delta PBC}{\Delta ABC} = \left|\frac{[-3(-2-y) + 4(y-5) + x(5+2)]}{[6(5+2) - 3(-2-3) + 4(3-5)]}\right|$

$$= \left|\frac{7x + 7y - 14}{49}\right| = \left|\frac{x+y-2}{7}\right|.$$

(22) **(A).** Given, area of triangle = 4

$$\therefore \begin{vmatrix} x & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 4 \Rightarrow x = -2.$$

(23) **(D).** Let $A \equiv (x+1, 2)$, $B \equiv (1, x+2)$, $C \equiv \left(\frac{1}{x+1}, \frac{2}{x+1}\right)$

then A, B, C are collinear if area of Δ ABC = 0

$$\Rightarrow \begin{vmatrix} x+1 & 2 & 1 \\ 1 & x+2 & 1 \\ \frac{1}{x+1} & \frac{2}{x+1} & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & -x & 0 \\ 1 & x+2 & 1 \\ \frac{1}{x+1} & \frac{2}{x+1} & 1 \end{vmatrix} = 0$$

$(R_1 \to R_1 - R_2)$

$$\Rightarrow \begin{vmatrix} x & 0 & 0 \\ 1 & x+3 & 1 \\ \frac{1}{x+1} & \frac{3}{x+1} & 1 \end{vmatrix} = 0 \qquad (C_2 \to C_2 + C_1)$$

$$\Rightarrow x\left(x + 3 - \frac{3}{x+1}\right) = 0 \Rightarrow x(x^2 + 3 + 4x - 3) = 0$$

$$\Rightarrow x^2(x+4) = 0 \Rightarrow x = 0, -4.$$

(24) **(C).** Let line be $y - 3 = m(x - 2)$

y intercept $3 - 2m$, x intercept $2 - \frac{3}{m}$

Area $12 = \frac{1}{2}\left|2 - \frac{3}{m}\right||3 - 2m| \Rightarrow 12 - \frac{9}{m} - 4m = 24$

$\Rightarrow 4m^2 + 12m + 9 = 0 \Rightarrow m = -3/2$

$\Rightarrow 12 - \frac{9}{m} - 4m = -24 \Rightarrow 4m^2 - 36m + 9 = 0; D > 0$

⇒ two values of m. Hence total 3 values.

(25) **(C).** Solving $x+3y=6$

$2x+y=4$

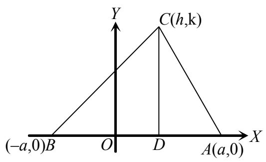
$\Rightarrow (x, y) = (6/5, 8/5)$

Sub in $kx - 3y + 1 = 0$

$$\frac{6k}{5}-\frac{24}{5}+1=0 \Rightarrow k=\frac{19}{6}$$

(26) **(A).** Centroid $=\left[\frac{11+2+2}{3},\frac{9+1-1}{3}\right]=(5,3)$

(27) **(A).** Let the coordinates of A and B are $A(a,0)$ and $B(-a,0)$ and variable point is $C(h,k)$.



In figure, $\cot A=\frac{a-h}{k}$ and $\cot B=\frac{a+h}{k}$

According to the condition, $\cot A+\cot B=\lambda$

$$\Rightarrow \frac{a-h}{k}+\frac{a+h}{k}=\lambda \Rightarrow \frac{2a}{k}=\lambda \Rightarrow k\lambda=2a.$$

Hence locus is $y\lambda=2a$.

(28) **(B).** Let middle point of stick AB is (h, k) therefore

$$\left(\frac{a+0}{2},\frac{0+b}{2}\right)\equiv(h,k)\Rightarrow h=\frac{a}{2},\ k=\frac{b}{2}$$

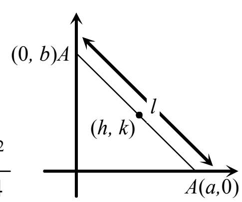
$\Rightarrow a=2h,\ b=2k$

But we know from figure that

$a^2+b^2=l^2 \Rightarrow 4h^2+4k^2=l^2$

Hence the locus is $x^2+y^2=\frac{l^2}{4}$

which is obviously a circle.



(29) **(C).** $(x_1,y_1)\to\left(\frac{y_1-1}{x_1-4}\right)=-1$ and $\frac{x_1+4}{2}=\frac{y_1+1}{2}$

$\Rightarrow x_1+y_1=5$ and $x_1-y_1=-3 \Rightarrow x_1=1, y_1=4$

2^{nd} operation $\Rightarrow$ (3,4)

3^{rd} operation $\Rightarrow \left(\frac{3}{\sqrt{2}}-\frac{4}{\sqrt{2}},\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{2}}\right)=\left(\frac{-1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$.

(30) **(A).** After time 't' secs.

Abcissa, $x=1+3t$ and ordinate, $y=2+2t$

Eliminating t, $\frac{x-1}{3}=\frac{y-2}{2}$ or $2x-2=3y-6$

or $2x-3y+4=0$.

(31) **(B).** $\sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}=4$

$\Rightarrow \sqrt{x^2+y^2+4-4x}+\sqrt{x^2+y^2+4+4x}=4$

$\Rightarrow \sqrt{k-4x}+\sqrt{k+4x}=4$, (where $k=x^2+y^2+4$)

$\Rightarrow \sqrt{k-4x}=4-\sqrt{k+4x}$

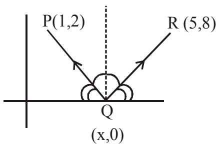
Squaring and on simplification it reduces to $y^2=0$.

Hence equation is a pair of two coincident straight lines.

(32) **(B).** The perpendicular distance of line from origin will remain same in both cases

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$$

(33) **(B).**

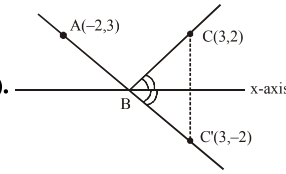


$m_{PQ}=-m_{RQ}$

$$\frac{0-2}{x-1}=-\left(\frac{8-0}{5-x}\right);\ x=\frac{9}{5}$$

(34) **(D).** $(x-2)^2+y^2+(x+2)^2+y^2=16 \Rightarrow x^2+y^2=4$

(35) **(B).**



Equation of line AB is same as line AC'

$$(y-3)=\frac{-2-3}{3+2}(x+2)$$

$y-3=-x-2 \Rightarrow x+y=1$

(36) **(C).** Let P (x, y) be a point on locus cond

$$|y|=2\left|\frac{x-y}{\sqrt{2}}\right| \Rightarrow |y|=\sqrt{2}\,|x-y|$$

Squaring, $y^2=2(x^2+y^2-2xy)$

$2x^2+y^2-4xy=0$

(37) **(B).** Take $\theta=\pi/4$

$$A=\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right), B=\left(\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right), O\ (0,0)$$

$$\therefore \text{ Centroid } \left(\frac{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+0}{3},\frac{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}+0}{3}\right)=\left(\frac{\sqrt{2}}{3},0\right)$$

Only equation $9x^2+9y^2=2$ holds.

(38) **(A).** Let line is $y=mx+c$

Here $m=\tan 60^\circ=\sqrt{3}$, $c=-5$

Equation is $y=\sqrt{3}x-5$

(39) **(D).** Here $m=\tan 45^\circ=1$ so the equation is $y=x$ or $x-y=0$

(40) **(A).** $y+4=\frac{3+4}{4-3}(x-3)$ or $y=7x-25$

(41) **(D).** $x\cos 60^\circ+y\sin 60^\circ=4$

$$\frac{x}{2}+\frac{y\sqrt{3}}{2}=4 \Rightarrow x+y\sqrt{3}=8$$

(42) **(D).** Let the equation is = 1, given a = b

$\because$ Line passes through (1, 2)

$$\therefore \frac{1}{a}+\frac{-2}{a}=1\ ;\ a=-1$$

Equation is $\frac{x}{-1}+\frac{y}{-1}=1$; $x+y+1=0$

(43) **(C).** $m=\frac{\sqrt{3}-0}{-2-1}=-\frac{1}{\sqrt{3}}$; $\tan\theta=-\frac{1}{\sqrt{3}} \Rightarrow \theta=150^\circ$

(44) **(C).** $2x-3y=6$

$$\frac{2x}{6}-\frac{3y}{6}=1 \Rightarrow \frac{2x}{6}-\frac{3y}{6}=1 \Rightarrow \frac{x}{3}+\frac{y}{-2}=1$$

$a=3, b=-2$

(45) **(B).** Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$ then the co-ordinates of point of intersection of this line and x axis and y axis are (a, 0) and (0, b) respectively (by putting x=and y = 0) hence mid point of the intercept is (a/2, b/2)

$\because a/2=5,\ b/2=2 \Rightarrow a=10,\ b=4$

hence required equation $\frac{x}{10}+\frac{y}{4}=1$ of the line is

$\Rightarrow 2x+5y=20$

(46) **(B).** Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$ ...(1)

This passes through (3, 4), therefore $\frac{3}{a}+\frac{4}{b}=1$ ...(2)

It is given that $a+b=14 \Rightarrow b=14-a$.

Putting $b=14-a$ in (2), we get

$$\frac{3}{a}+\frac{4}{14-a}=1 \Rightarrow a^2-13a+42=0$$

$\Rightarrow (a-7)(a-6)=0 \Rightarrow a=7,\ 6$

$a=7, b=14-7=7$

and for $a=6, b=14-6=8$.

Putting the values of a and b in (1),

we get the equations of the lines

$$\frac{x}{7}+\frac{y}{7}=1\ \&\ \frac{x}{6}+\frac{y}{8}=1 \text{ or } x+y=7 \text{ and } 4x+3y=24$$

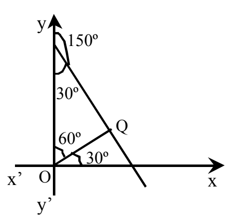
(47) **(A).** Here $p=7$ and $\alpha=30^\circ$

Equation of the required line is

$x\cos 30^\circ+y\sin 30^\circ=7$

or $x\frac{\sqrt{3}}{2}+y\times\frac{1}{2}=7$

or $\sqrt{3}x+y=14$



(48) **(A).** Let the equations of the line be $\frac{x}{a}+\frac{y}{b}=1$, then the coordinates of point of intersection of this line and x-axis and y–axis are respectively (a, 0), (0, b). Hence mid point of the intercept is (a/2, b/2).

$\therefore a/2=x_1 \Rightarrow a=2x_1$ & $b/2=y_1 \Rightarrow b=2y_1$

Hence required equation of the line is

$$\frac{x}{2x_1}+\frac{y}{2y_1}=1 \Rightarrow \frac{x}{x_1}+\frac{y}{y_1}=2$$

(49) **(B).** A line perpendicular to the line $5x-y=1$ is given by $x+5y-\lambda=0=L$, (given)

In intercept form $\frac{x}{\lambda}+\frac{y}{\lambda/5}=1$

So, area of triangle is $\frac{1}{2}\times$ (Multiplication of intercepts)

$$\Rightarrow \frac{1}{2}(\lambda)\times\left(\frac{\lambda}{5}\right)=5 \Rightarrow \lambda=\pm 5\sqrt{2}$$

Hence the equation of required straight line is

$x+5y=\pm 5\sqrt{2}$.

(50) **(C).** Here D(1, 1) therefore equation of line AD is given by $2x+y-3=0$. Thus the line perpendicular to AD is $x-2y+k=0$ and it passes through B, so k = 0. Hence required equation is $x-2y=0$.

(51) **(A).** Here intercept on x-axis is 3 and intercept on y-axis is –2. So using intercept form of the equation of line, the required line is $\frac{x}{3}-\frac{y}{2}=1$.

(52) **(B).** Point $P(a,b)$ is on $3x+2y=13$

So, $3a+2b=13$ .....(i)

Point $Q(b,a)$ is on $4x-y=5$

So, $4b-a=5$ .....(ii)

By solving (i) and (ii), $a=3, b=2$

$P(a,b)\to(3,2)$ and $Q(b,a)\to(2,3)$

Now, equation of PQ

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1) \Rightarrow y-2=\frac{3-2}{2-3}(x-3)$$

$\Rightarrow y-2=-(x-3) \Rightarrow x+y=5$.

(53) **(C).** Given, line AB making 0 intercepts on x-axis and y-axis or $(x_1,y_1)\equiv(0,0)$ and the line is perpendicular to line $CD, 3x+4y+6=0$. We know that standard equation of a line is $y=ax+b$. Comparing given equation of line CD with the standard equation, we get a = 3 and b = 4. We also know that slope of the given line $CD=-\frac{a}{b}=\frac{-3}{4}$. Since the line AB is perpendicular to the line CD, therefore slope of the line $AB(m)=\frac{4}{3}$.

Thus relation for the equation of the line AB will be

$(y-y_1)=m(x-x_1)$ or $y-0=\frac{4}{3}(x-0)$

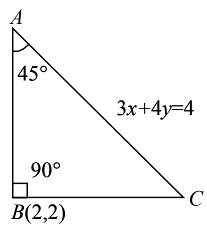
or $3y=4x$ or $4x-3y=0$.

(54) **(B).** Let the coordinates of axes are A (a, 0) and $B(0,b)$, but the point (–5,4) divides the line AB in the ratio of 1 : 2. Therefore, the coordinates of axes are $\left(\frac{-15}{2},0\right)$ and (0, 12). Therefore, the equation of line passing through these coordinate axes is given by $8x-5y+60=0$.

(55) **(A).** Since $\angle A=\angle C=45^o$. We have to find equation of AB. Here let gradient of AB be m, then equation of AB is

$y-2=m(x-2)$ ……(i)

But angle between $3x+4y=4$ and (i) is 45°



So, $\tan 45^o=\frac{m+\frac{3}{4}}{1-\frac{3m}{4}}\Rightarrow m=\frac{1}{7}$

Hence, the required equation is $x-7y+12=0$ {By putting the value of m in (i)}.

(56) **(D).** We have $(x+2)=\frac{r}{\sqrt{10}}$, $(y-1)=\frac{3r}{\sqrt{10}}$

Cartesian equation $(y-1)=3(x+2)$; Slope = 3

(57) **(C).** Substituting the coordinates of points A, B and C in the expression $x+2y-3$, we get

The value of expression for A = –1 + 6 – 3 = 2 > 0

The value of expression for B = 2 – 6 – 3 = – 7 < 0

The value of expression for C = 4 + 18 – 3 = 19 > 0

∵ Signs of expressions for A, C are same while for B, the sign of expression is different A, C are on one side and B is on other side of the line

(58) **(C).** Converting the given equation into normal form by dividing $\sqrt{3^2+4^2}=5$ is

$\frac{3x}{5}+\frac{4y}{5}=\frac{10}{5}\Rightarrow\frac{3x}{5}+\frac{4y}{5}=2$

Comparing with $x\cos\alpha+y\sin\alpha=p\Rightarrow p=2$

(59) **(C).** Given line x + y = 4 dividing both side by $\sqrt{(\sqrt{3})^2+1^2}=2$

we get $\frac{x}{2}+\frac{\sqrt{3}\,y}{2}=2$ or $x\cos\pi/3+y\sin\pi/3=2$

(60) **(C).** $m_1=-\frac{3}{4}$; $m_2=-\frac{4}{k}$

Two lines are perpendicular if $m_1m_2=-1$

$\Rightarrow\left(-\frac{3}{4}\right)\times\left(-\frac{4}{k}\right)=-1\Rightarrow k=-3$

(61) **(A).** Let the equation is 3x – 7y + k = 0 this line passes through (4, 6)

Hence $3(4)-7(6)+k=0\Rightarrow k=30$

The required equation is $3x-7y+30=0$

(62) **(A).** $=\frac{7\times3+9\times4+6}{\sqrt{7^2+9^2}}=\frac{63}{\sqrt{130}}$

(63) **(C).** Distance $=\frac{22-7}{\sqrt{3^2+4^2}}=\frac{15}{5}=3$

(64) **(C).** Let required point be (x_1, y_1), then

$x_1+y_1=4$ ...(1)

Also as given $\pm\frac{4x_1+3y_1-10}{\sqrt{16+9}}=1$

$\Rightarrow 4x_1+3y_1=15$ ...(2)

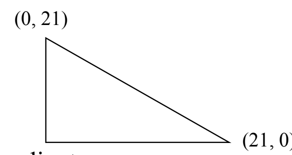
and $4x_1+3y_1=5$ ...(3)

Now (1) and (2) give $x_1=3, y_1=1$

(1) and (3) gives $x_1=-7, y_1=11$

∴ required points are (3,1), (–7,11)

(65) **(B).** $x+y=21$



The number of integral solutions to the equation $x+y<21$ i.e., $x<21-y$

∴ Number of integral co-ordinate

$=19+18+....+1=\frac{19\times20}{2}=190$.

(66) **(B).** $p_1=m^2\cos\alpha+2m\sin\alpha+\frac{\sin^2\alpha}{\cos\alpha}$

or $p_1=\left(\frac{m\cos\alpha+\sin\alpha}{\sqrt{\cos\alpha}}\right)^2$ and $p_3=\left(\frac{m'\cos\alpha+\sin\alpha}{\sqrt{\cos\alpha}}\right)^2$

$p_2=\frac{mm'\cos^2\alpha+(m+m')\sin\alpha\cos\alpha+\sin^2\alpha}{\cos\alpha}$

$=\frac{(m\cos\alpha+\sin\alpha)}{\sqrt{\cos\alpha}}\left(\frac{m'\cos\alpha+\sin\alpha}{\sqrt{\cos\alpha}}\right)=\sqrt{p_1}.\sqrt{p_3}$

Since $p_2^2=p_1.p_3$ and hence G.P.

(67) **(A).** Point must be point of intersection of given line and perpendicular line passing through (1, 2).

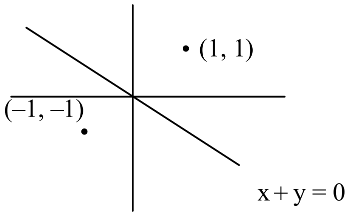
i.e. $3x+y=5\Rightarrow(2,-1)$

(68) **(C).** Line must be bisector lines

i.e., $\frac{x+y-1}{\sqrt{2}}=\pm\frac{7x-y-6}{\sqrt{50}}$

$\Rightarrow 2x-6y-1=0$ or $12x+4y-11=0$

(69) **(D).** Clearly from the diagram (–1, –1) is the reflection.



(70) **(A).** $m = -\dfrac{[6+3\lambda]}{-7-\lambda} = \infty \Rightarrow \lambda = -7$

(71) **(C).** Required line : x – 3y = 2 – 6

Put x = 0 : – 3y = – 4 $\therefore$ y = 4/3 = y-intercept

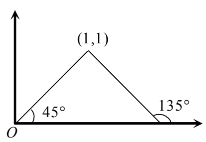
(72) **(D).** $m_1 = \dfrac{-2}{3},\ m_2 = \dfrac{-1}{k}\ ;\ \dfrac{-2}{3} \times \dfrac{-1}{k} = -1 \Rightarrow k = \dfrac{-2}{3}$

(73) **(D).** Here one line is parallel to y axis and slope of second line is $\dfrac{1}{\sqrt{3}}$. Hence $\tan\theta = \dfrac{1}{1/\sqrt{3}} = \sqrt{3}$, $\theta = 60^\circ$

(74) **(C).** The point of intersection is (1, 1) Therefore the equation of the line passing through (1, 1) and $(\pi, 0)$ is

$y - 1 = \dfrac{-1}{\pi - 1}(x-1) \Rightarrow x - y = \pi(1-y)$.

(75) **(B).** Slopes of the lines are 1 and –1



Since the point of intersection is (1, 1)

Hence the required equations are $y - 1 = \pm 1(x-1)$.

(76) **(A).** The point of intersection of $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ is $(-1, -1)$. Now the line perpendicular to $3x - 5y + 11 = 0$ is $5x + 3y + k = 0$, but it passes through (–1,–1) $\Rightarrow -5 - 3 + k = 0 \Rightarrow k = 8$

Hence required line is $5x + 3y + 8 = 0$.

(77) **(A).** Since the median passes through A, the intersection of the given lines. Its equation is given by $(px + qy - 1) + \lambda(qx + py - 1) = 0$, where λ is some real number. Also, since the median passes through the point (p, q), we have $(p^2 + q^2 - 1) + \lambda(qp + pq - 1) = 0$

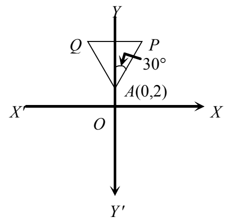
$\Rightarrow \lambda = -\dfrac{p^2 + q^2 - 1}{2pq - 1}$ and the equation of median through

A is $(px + qy - 1) - \dfrac{p^2 + q^2 - 1}{2pq - 1}(qx + py - 1) = 0$

$\Rightarrow (2pq - 1)(px + qy - 1) = (p^2 + q^2 - 1)(qx + py - 1)$.

(78) **(B).** Since $|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$, therefore the equations of two lines are $y = \sqrt{3}x + 2, x \ge 0$ and $y = -\sqrt{3}x + 2, x < 0$.

Clearly, y-axis the only bisector of the angle between these two lines. There are two points P and Q on these lines at a distance of 5 units from A. Clearly, M is foot of the perpendicular from P and Q on y-axis (bisector)



$AM = AP\cos 30^o = \dfrac{5\sqrt{3}}{2}$

Hence, co-ordinate of M are $\left(0,\ 2 + \dfrac{5\sqrt{3}}{2}\right)$.

(79) **(A).** We know that $\dfrac{\sqrt{3}+1}{\sqrt{3}-1} = \tan 75^o$

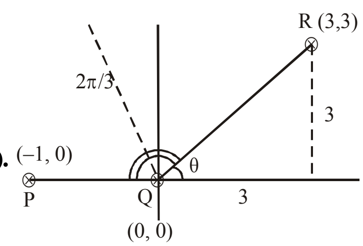
Hence the line makes an angle of 75^o with y- axis, so the equation of y-axis is x = 0.

(80) **(A).** By direct formulae $\dfrac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\dfrac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$

$$\frac{3x + 4y - 7}{\sqrt{3^2 + 4^2}} = \pm\frac{12x + 5y + 17}{\sqrt{(12)^2 + (5)^2}}$$

$$\frac{3x + 4y - 7}{5} = \pm\frac{12x + 5y + 17}{13}$$

(81) **(D).**



$\tan\theta = \sqrt{3} \Rightarrow \theta = \dfrac{\pi}{3}$; Angle $\left(\dfrac{\pi}{3} + \dfrac{\pi}{3}\right) = \dfrac{2\pi}{3}$

Slope = $\tan\dfrac{2\pi}{3} = -\sqrt{3} \Rightarrow \sqrt{3}x + y = 0$

(82) **(A).** Clearly $a + b = \sin^2\alpha + (\cos^2\alpha - 1) = 0 \Rightarrow \alpha = 90^\circ$

(83) **(D).** $y - 0 = \left(\dfrac{3-0}{-5}\right)(x-5)$

$-5y = 3x - 15$

$d = \left|\dfrac{3(4) + 5(4) - 15}{\sqrt{3^2 + 5^2}}\right| = \dfrac{17}{\sqrt{34}} = \sqrt{\dfrac{17}{2}}$

(84) **(A).** $x^2 + y^2 = a^2\left(\frac{y - mx}{c}\right)^2$

$\Rightarrow x^2(c^2 - a^2m^2) + y(c^2 - a^2) - 2ma^2xy = 0$

(85) **(D).** The condition of perpendicularity for two lines represented by homogeneous equation of second degree is coeff. of x^2 +coeff. of $y^2 = 0 \Rightarrow p - q + q - p = 0$

$\therefore$ p and q can take any value.

(86) **(C).** Comparing the given equation with the standard equation, we get a = 1, b = 1, h = 0 and c = 1. We also know that condition for the general equation

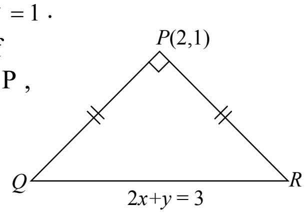
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\therefore (1 \times 1 \times 1) + (2 \times f \times g \times 0) - (1 \times f^2) - (1 \times g^2) - (1 \times 0) = 0$

or $1 - f^2 - g^2 = 0$ or $f^2 + g^2 = 1$.

(87) **(B).** Parametric equation of line passing through point P ,



$\frac{x-2}{\cos\theta} = \frac{y-1}{\sin\theta} = r$

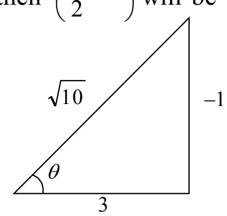
Hence the point is $r\cos\theta + 2,\ r\sin\theta + 1$. If this point lie on QR, then

$2(r\cos\theta + 2) + r\sin\theta + 1 = 3$

$r(2\cos\theta + \sin\theta) = -2$

$\Rightarrow 2\cos\theta + \sin\theta = \frac{-2}{r}$ ......(i)

If θ be the inclination of PQ, then $\left(\frac{\pi}{2} + \theta\right)$ will be the inclination of PR. Hence



$2\cos\left(\frac{\pi}{2} + \theta\right) + \sin\left(\frac{\pi}{2} + \theta\right) = \frac{-2}{r}$

$\Rightarrow -2\sin\theta + \cos\theta = \frac{-2}{r}$ .....(ii)

From (i) and (ii), we get

$2\cos\theta + \sin\theta = -2\sin\theta + \cos\theta$

$\Rightarrow \cos\theta = -3\sin\theta \Rightarrow \tan\theta = -\frac{1}{3}$

Hence the gradients of PQ and PR are –1/3 and 3. Therefore the equations of PQ and PR are

$y - 1 = -\frac{1}{3}(x - 2)$; $y - 1 = 3(x - 2)$

and their combine equation is

$[3(y-1) + (x-2)][(y-1) - 3(x-2)] = 0$

or $3(y-1)^2 - 8(y-1)(x-2) - 3(x-2)^2 = 0$

or $3(x-2)^2 + 8(x-2)(y-1) - 3(y-1)^2 = 0$

or $3(x^2 - 4x + 4) + 8(xy - x - 2y + 2) - 3(y^2 - 2y + 1) = 0$

or $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$.

(88) **(D).** Let (x', y') be the coordinates on new axes, then put $x = x'\cos\theta - y'\sin\theta,\ y = x'\sin\theta + y'\cos\theta$ in the equation. Then the coefficient of xy in the transformed equation is 0. So $2(b - a)\sin\theta\cos\theta + 2h\cos 2\theta = 0$

$\Rightarrow \tan 2\theta = \frac{2h}{a - b}$.

(89) **(A).** Factorize the given equation and then find the equation of lines. Now find their tangents and angles between them. The lines are equally inclined to each other.

(90) **(B).** Here equations of pair of straight lines are

$x^2 - 2mxy - y^2 = 0$ .....(i)

$x^2 - 2nxy - y^2 = 0$ .....(ii)

Therefore, equations of bisectors of these lines are

$mx^2 + 2xy - my^2 = 0$ .....(iii)

$nx^2 + 2xy - ny^2 = 0$ .....(iv)

But according to the condition (i) and (iv), and (ii) and (iii) must be coincident, $\therefore \frac{n}{1} = \frac{2}{-2m} = \frac{-n}{-1} \Rightarrow mn = -1$.

(91) **(A).** Let θ be the angle between the lines represented by $x^2 - 2pxy + y^2 = 0$.

Comparing the given equation with

$ax^2 + 2hxy + by^2 = 0$,

we get a = 1, b = 1 and 2h = –2p.

$\therefore \tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b} \Rightarrow \tan\theta = \frac{2\sqrt{p^2 - 1}}{1 + 1} = \sqrt{p^2 - 1}$

$\Rightarrow \tan^2\theta = p^2 - 1 \Rightarrow p^2 = 1 + \tan^2\theta \Rightarrow p^2 = \sec^2\theta$

$\Rightarrow \sec\theta = \pm p \Rightarrow \theta = \sec^{-1}(\pm p)$

Hence, the required acute angle is $\sec^{-1}(p)$

(92) **(B).** Let y = mx be a line common to the given pairs of lines. Then $am^2 + 2m + 1 = 0$ and $m^2 + 2m + a = 0$

$\Rightarrow \frac{m^2}{2(1-a)} = \frac{m}{a^2 - 1} = \frac{1}{2(1-a)}$

$\Rightarrow m^2 = 1$ and $m = -\frac{a+1}{2}$

$\Rightarrow (a+1)^2 = 4 \Rightarrow a = 1$ or -3

But for a = 1, the two pairs have both the lines common. So, a = – 3 and the slope m of the line common to both the pairs is 1.

Now, $x^2 + 2xy + ay^2 = x^2 + 2xy - 3y^2 = (x - y)(x + 3y)$

and $ax^2 + 2xy + y^2 = -3x^2 + 2xy + y^2 = -(x - y)(3x + y)$

So, the equation of the required lines is

$(x + 3y)(3x + y) = 0 \Rightarrow 3x^2 + 10xy + 3y^2 = 0$

(93) **(D).** $m + m_2 = -2h/b,\ m\,m_2 = a/b$

$m + \frac{a}{mb} = \frac{-2h}{b} \Rightarrow \frac{m^2b + a}{mb} = \frac{-2h}{b}$

$m^2b + 2hm = -a \Rightarrow m^2b^2 + 2hmb = -ab$

$\Rightarrow m^2b^2 + 2hmb + h^2 = h^2 - ab \Rightarrow (h + bm)^2 = h^2 - ab$

(94) **(D).** $m_1 + m_2 = -\dfrac{2h}{b}$

$$\alpha + n\alpha = -\frac{2h}{b},\ \alpha = -\frac{2h}{b(1+n)} \qquad \text{........ (1)}$$

$$m_1 m_2 = \frac{a}{b}\ ;\ \alpha.(n\alpha) = \frac{a}{b} \Rightarrow n\alpha^2 = \frac{a}{b} \qquad \text{......... (2)}$$

$$\text{Sq (1). } \alpha^2 = \frac{4h^2}{b^2(1+n^2)} = \frac{a}{nb}$$

$\therefore\ 4h^2nb = ab^2(1+n)^2\ \therefore\ 4nh^2 = ab(1+n)^2$

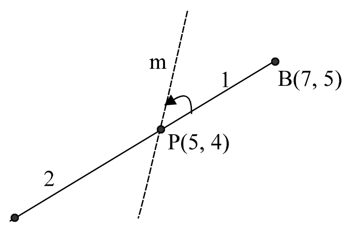
(95) **(A).** $|x| + |y| = 1$

$\therefore$ locus of (x, y) is a square.

(96) **(C).** For any point P(x, y) that is equidistant from given line, we have

$x + y - \sqrt{2} = -(x + y - 2\sqrt{2}) \Rightarrow 2x + 2y - 3\sqrt{2} = 0.$

(97) **(C).** $\dfrac{m-(1/2)}{1+(m/2)} = 1$



$\Rightarrow m - \dfrac{1}{2} = \dfrac{m}{2} + 1$

$\Rightarrow \dfrac{m}{2} = \dfrac{3}{2} \Rightarrow m = 3$

So, equation of line in new form will be

$y - 4 = 3(x-5)$ or $3x - y - 11 = 0$

(98) **(C).** If AD the altitude from A,

$$AD = \frac{|3+4-5|}{\sqrt{2}} = \sqrt{2}\ ;$$

$\therefore$ side of the Δ = AD cosec 60° = $\sqrt{2} \times \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{2}}{\sqrt{3}}$

(99) **(A).** Point of intersection is A (–2, 0).

The required line will be one which passes through (–2, 0) and is perpendicular to the line joining (–2, 0) and (2, 3).

(100) $m_{AC} = \dfrac{6-2}{5-3} = 2 \Rightarrow m_{BD} = -\dfrac{1}{2}$

Thus equation of BD is

$(y-6) = -\dfrac{1}{2}(x-5)$ i.e., $2y + x - 17 = 0.$

(101) **(A).** Rewriting the equation $(2x+y+2)a + (3x-y-4)b = 0$ and for all a, b the straight lines pass through the intersection of

$2x + y + 2 = 0$ and $3x - y - 4 = 0$ i.e., the point $\left(\dfrac{2}{5}, \dfrac{-14}{5}\right)$.

(102) **(C).** $3a - 2b + 5c = 0 \Rightarrow \dfrac{3}{5}a - \dfrac{2}{5}b + c = 0$

Thus the line ax + by + c = 0 passes through the point $\left(\dfrac{3}{5}, -\dfrac{2}{5}\right)$.

(103) **(A).** Since BD divides the triangle into two of equal area, BD is a median and D ≡ (–1, –2).

Slope of BD = $-\dfrac{1}{2}$. The equation of the required line is

$y + 3 = 2(x-1)$ i.e., $y - 2x + 5 = 0.$

(104) **(D).** R = (4, 3) as it is the reflection of (3, 4) in y = x.

$R_1 = (-4, 3)$ as it is the reflection of R in y-axis.

$R_2 = (-4, -3)$ as it is the reflection of R_1 in x-axis.

(105) **(A).** $\dfrac{a-1}{2} = \dfrac{5-a}{3} \Rightarrow a = \dfrac{13}{5}$

(106) **(B).** $ax + by + c = 0$ but $xb = \dfrac{a+c}{2}$

$\Rightarrow ax + \dfrac{(a+c)}{2}y + c = 0 \Rightarrow 2ax + ay + cy + c = 0$

$\Rightarrow a(y + 2x) + c(1 + y) = 0$ family of concurrent lines.

(107) **(C).** If Q (h, k) is the image then

$\dfrac{h-1}{4/5} = \dfrac{k-5}{3/5} = -10 \Rightarrow h = -7, k = -1$

(108) **(A).** $y + 3x - 5 = 0$ and $3y - x + 10 = 0$ are perpendicular to each other so the triangle is right angled and its circumradius is half the hypotenuse. y = x intersects these lines in $\left(\dfrac{5}{4}, \dfrac{5}{4}\right)$ and (–5, –5). Circumradius = $\dfrac{25}{4\sqrt{2}}$.

(109) **(C)** The lines are $|x| + |y| = 2$.

These lines form a rhombus.

Area of the rhombus = 4 x area of triangle in 1 quadrant

$= 4 \times \dfrac{1}{2} \times 2 \times 2 = 8.$

(110) **(D).** Lines $5x + 3y - 2 + \lambda_1(3x - y - 4) = 0$ are concurrent at (1, –1) and lines $x - y + 1 + \lambda_2(2x - y - 2) = 0$ are concurrent at (3, 4). Thus equation of line common to both family is,

$y - 4 = \dfrac{-1-4}{1-3}(x-3)$ i.e., $5x - 2y - 7 = 0.$

(111) **(B).** Since y = x is a member of $ax^2 + 2hxy + by^2 = 0$

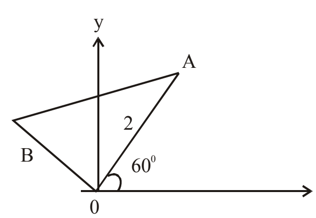
$\therefore\ ax^2 2 + 2hx^2 + bx^2 = 0$ for all x

$\therefore\ a + 2h + b = 0$

EXERCISE-2

(1) **(A)** Coordinates of A are (2cos 60°. 2 sin 60°)= (1, $\sqrt{3}$).

Coordinates of B are ($-\sqrt{3}$, 1)



Mid point of AB $= \left(\dfrac{1-\sqrt{3}}{2}, \dfrac{1+\sqrt{3}}{2}\right)$

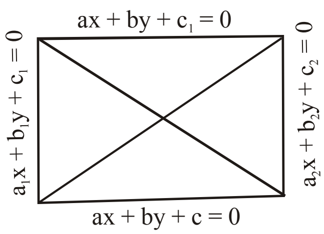
(2) **(C).** CS_1 and CS_2 divide the triangle into three of equal area so that S_1 and S_2 are points of trisection of AB.
Thus $S_1 = (0, 0)$, $S_2 = (2, -3)$. Equation of CS_1 is $y - x = 0$. Slope of $CS_2 = -4$. The line through (0, 0) drawn parallel to CS_2 is $y + 4x = 0$.
The combined equation is
$(y - x)(y + 4x) \equiv y^2 + 3xy - 4x^2 = 0$.

(3) **(A).** Since the lines are concurrent, so $\begin{vmatrix} \ell & m & n \\ m & n & \ell \\ n & \ell & m \end{vmatrix} = 0$

$\Rightarrow 3\ell mn - \ell^3 - m^3 - n^3 = 0$
$\Rightarrow (\ell + m + n)(\ell^2 + m^2 + n^2 - \ell m - mn - n\ell) = 0$
$\Rightarrow \ell + m + n = 0 \; [\because \ell^2 + m^2 + n^2 > \ell m + mn + n]$

(4) **(A)** The figure is evidently a parallelogram in which the diagonals are given to be at right angles.
Therefore, it is a rhombus and the distance between the opposite pairs of sides is the same.

$$\therefore \frac{|c - c_1|}{\sqrt{a^2 + b^2}} = \frac{|c - c_1|}{\sqrt{a_1^2 + b_1^2}} \Rightarrow a^2 + b^2 = a_1^2 + b_1^2$$



(5) **(A).** $\sin x \sin(x + 2) + \cos^2(x + 1)$

$$= \frac{1}{2}\{\cos 2 - \cos 2(x + 1)\} + \cos^2(x + 1)$$

$$= \frac{1}{2}\{\cos 2 - 2\cos^2(x + 1) + 1\} + \cos^2(x + 1)$$

$$= \frac{1}{2}(\cos 2 + 1) = \cos^2 1$$

$\therefore$ Given graph is $y = \cos^2 1$ which is a straight line parallel to the x-axis.

(6) **(D).** For $x = 1 + \frac{\lambda}{\sqrt{2}}$ $\qquad$ $x + 2y = 1$ gives

$$2y = 1 - \left(1 + \frac{\lambda}{\sqrt{2}}\right) = -\frac{\lambda}{\sqrt{2}} \quad ; \quad y = -\frac{\lambda}{2\sqrt{2}}$$

for $x = 1 + \frac{\lambda}{\sqrt{2}}$, $2x + 4y = 15$ gives

$$y = \frac{15 - 2 - \sqrt{2}\lambda}{4} = \frac{13 - \sqrt{2}\lambda}{4}$$

$$\therefore -\frac{\lambda}{\sqrt{2}} < 2 + \frac{\lambda}{\sqrt{2}} < \frac{13 - \sqrt{2}\lambda}{4} \Rightarrow -\sqrt{2} < \lambda < \frac{5}{3\sqrt{2}}$$

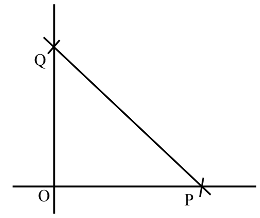
(7) **(B).** $D = \begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix} = 0$

$$= \lambda[\cos^2\alpha + \sin^2\alpha] - \sin\alpha[\cos\alpha - \sin\alpha] + \cos\alpha[-\sin\alpha - \cos\alpha] = 0$$

$$D = \lambda - \sin\alpha.\cos\alpha + \sin^2\alpha - \cos\alpha.\sin\alpha - \cos^2\alpha = \lambda - [\sin 2\alpha + \cos 2\alpha]$$

$\therefore \lambda = \sin 2\alpha + \cos 2\alpha \qquad \therefore \lambda \in [-\sqrt{2}, \sqrt{2}]$

(8) **(A).** $P \equiv \left(\frac{c}{a}, 0\right)$, $Q \equiv \left(0, \frac{c}{b}\right)$



$$\Delta_{OPQ} = \frac{1}{2}(OP)(OQ) = \frac{1}{2}\frac{c^2}{ab}$$

clearly Δ_{OPQ} will not depend upon a, b and c if $c^2 = ab$,
i.e. a, c, b are in G.P.

(9) **(A).** $x = \lim\limits_{c \to 1} \frac{5(c - 1)(c + 1)}{5(c - 1)(3c + 2)} = \frac{2}{5}$

$$y = \lim_{c \to 1} \frac{1 - c}{5(c - 1)(3c + 2)} = -\frac{1}{25}$$

(10) **(A).** Any point A on the first line is $(t, 5t - 4)$.

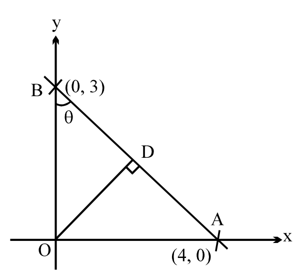
Any point B on the second line is $\left(r, \frac{3r - 4}{4}\right)$.

Hence $1 = \frac{2r + t}{3}$ and $5 = \frac{\frac{3r - 4}{2} + 5t - 4}{3}$

$\Rightarrow 2r + t = 3$ and $3r + 10t = 42$.

on solving, we get $t = \frac{75}{17}$. Hence A is $\left(\frac{75}{17}, \frac{304}{17}\right)$

(11) **(A).** If 'D' be the foot of altitude, drawn from origin to the given line, then 'D' is the required point.
Let $\angle OBA = \theta \Rightarrow \tan\theta = 4/3 \Rightarrow \angle DOA = \theta$

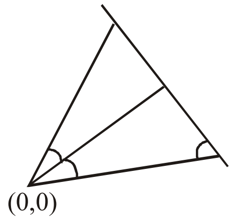


we have $OD = 12/5$.
If D is (h, k) then $h = OD\cos\theta$, $k = OD\sin\theta$
$\Rightarrow h = 36/25$, $k = 48/25$.

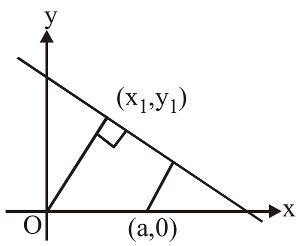
(12) **(A).** Distance of (0, 0) from the line $2x+3y-6=0$

$$\frac{6}{\sqrt{4+9}}=\frac{6}{\sqrt{13}}$$

$\therefore$ area of the Δ is $\left(\frac{6}{\sqrt{13}}\right)^2=\frac{36}{13}$



(13) **(A).**



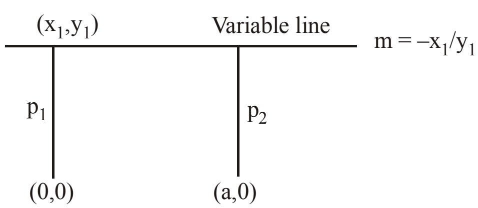
Let the equation of the variable line is
$xx_1+yy_1-(x_1^2+y_1^2)=0$

$$p_1p_2=\left|\frac{x_1^2+y_1^2}{\sqrt{x_1^2+y_1^2}}\right|\left|\frac{ax_1-(x_1^2+y_1^2)}{\sqrt{x_1^2+y_1^2}}\right|=k^2$$

$|x_1^2+y_1^2-ax_1|=k^2$
locus is $x^2+y^2-ax\pm k^2=0$

$$\therefore\ r^2=\frac{a^2}{4}-(\pm k^2)$$

+ve sign $r^2=\frac{a^2}{4}-k^2$ (not possible as r^2 become –ve)

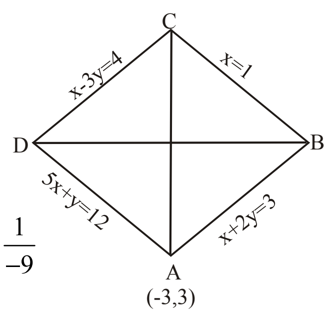


–ve sign $r^2=\frac{a^2}{4}+k^2$

(14) **(C).** Solving for A,
$x+2y-3=0$
$5x+y+12=0$

$$\Rightarrow\frac{x}{+24+3}=\frac{y}{-15+3}=\frac{1}{-9}$$

$\therefore$ A(–3, 3)
Similarly B (1, 1), C (1, – 1), D (–2, –2)
Now, m_1 = slope of AC = –1 ; m_2 = slope of BD = 1
$m_1m_2=-1$
$\therefore$ the angle required is 90°

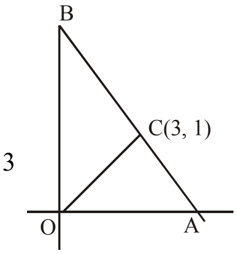


(15) **(A)** If BA is farthest from the origin, then OC must be perpendicular to BA and has a slope = – 3.
Equation of BA is $y-1=-3(x-3)$
$3x+y=10$

$\therefore A=\left(\frac{10}{3},0\right)$ and B = (0, 10).

If $\frac{BC}{CA}=\frac{\lambda}{1}$, then $\frac{\lambda.\frac{10}{3}+1.0}{\lambda+1}=3$

$\therefore\ \lambda=9$

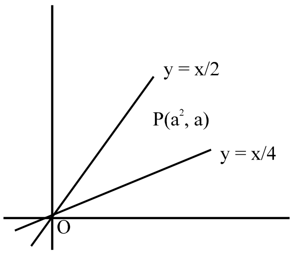


(16) **(C).** We have $a-\frac{a^2}{4}>0$

and $a-\frac{a^2}{2}<0$

$$\Rightarrow\left(a-\frac{a^2}{4}\right)\left(a-\frac{a^2}{2}\right)<0$$

$\Rightarrow a\in(2,4)$



(17) **(D).** The determinant of the coefficients of the equations

$$\begin{vmatrix}a & ma & 1\\ b & (m+1)b & 1\\ c & (m+2)c & 1\end{vmatrix}=\begin{vmatrix}1 & m & \frac{1}{a}\\ 1 & m+1 & \frac{1}{b}\\ 1 & m+2 & \frac{1}{c}\end{vmatrix}=\begin{vmatrix}1 & m & \frac{1}{a}\\ 0 & 1 & \frac{1}{b}-\frac{1}{a}\\ 1 & 1 & \frac{1}{c}-\frac{1}{b}\end{vmatrix}$$

If follows that for concurrency $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ should be in A.P.

(18) **(A).** Distance between $x+2y+3=0$ and $x+2y-7=0$ is $\frac{10}{\sqrt{5}}$. Let the remaining side be $2x-y+\lambda=0$.

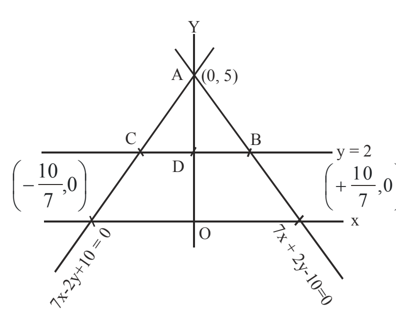
We have, $\frac{|\lambda+4|}{\sqrt{5}}=\frac{10}{\sqrt{5}}\Rightarrow\lambda=5,-14,$

thus remaining side is $2x-y+6=0$ or $2x-y-14=0$

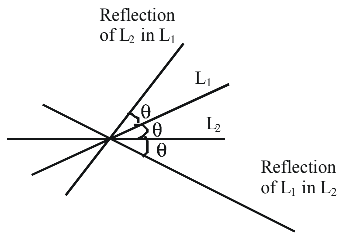
(19) **(C).** We have, B ≡ (6/7, 2),

$C\equiv\left(-\frac{6}{7},2\right)\Rightarrow BC=\frac{12}{7}, AD=3$

$$\Rightarrow\Delta_{ABC}=\frac{1}{2}.\frac{12}{7}.3=\frac{18}{7}\text{ sq. units}$$



(20) **(C).**



(21) **(B).**

$$p_1 = \frac{2m\sin\alpha + m^2\cos\alpha + \dfrac{\sin^2\alpha}{\cos^2\alpha}}{\sqrt{\sin^2\alpha + \cos^2\alpha}} = \frac{(m\cos\alpha + \sin\alpha)^2}{\cos\alpha}$$

Similarly, $p_2 = \dfrac{(m'\cos\alpha + \sin\alpha)^2}{\cos\alpha}$

$$p_2 = (m + m')\sin\alpha + mm'\cos\alpha + \frac{\sin^2\alpha}{\cos\alpha}$$

$$= \frac{(m\cos\alpha + \sin\alpha)\,(m'\cos\alpha + \sin\alpha)}{\cos\alpha}$$

Hence, $p_2^2 = p_1 p_3$

(22) **(A).** The required equation is of the form
$3y + 2x - 5 + \lambda(2y - 3x + 7) = 0$ and λ is to be chosen such that this line is at right angles to $y + 5x + 10 = 0$

i.e., $\dfrac{3\lambda - 2}{3 + 2\lambda}(-5) = -1$ which gives $\lambda = 1$ and the equation is $5y - x + 2 = 0$.

(23) **(D).** $D = \dfrac{1}{2}\begin{vmatrix} p^2 & -p & 1 \\ q^2 & q & 1 \\ r^2 & -r & 1 \end{vmatrix} = \dfrac{1}{2}\begin{vmatrix} p^2 - q^2 & -(p+q) & 0 \\ q^2 - r^2 & q + r & 0 \\ r^2 & -r & 1 \end{vmatrix}$

$$= \frac{1}{2}(p+q)(q+r)\begin{vmatrix} p - q & -1 & 0 \\ q - r & 1 & 0 \\ r^2 & -r & 1 \end{vmatrix}$$

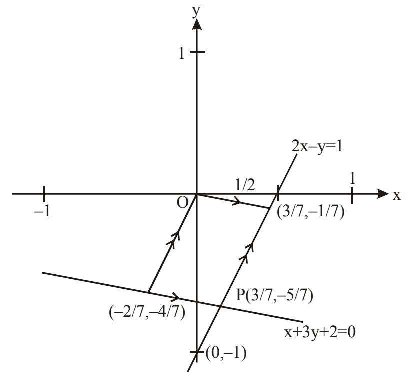
$$= \frac{1}{2}(p+q)(q+r)[(p-q) + (q-r)] = \frac{1}{2}(p+q)(q+r)(p-r)$$

(24) **(C).** $\Delta = \begin{vmatrix} 3 & 0 & -9 \\ 0 & 4 & 8 \\ -9 & 8 & c \end{vmatrix} = 12(c - 43)$

Also, $h^2 - ab = 0 - 12 < 0$

(25) **(A).** $(2x - y - 1)(x + 3y + 2) = 0$

hence the lines are $\left.\begin{matrix} 2x - y - 1 = 0 \\ x + 3y + 2 = 0 \end{matrix}\right] \Rightarrow P\left(\dfrac{1}{7}, -\dfrac{5}{7}\right)$



Equation of the two lines joining origin and the point of intersection of $3x - 5y = 2$ and $f(x, y) = 0$ is

$$2x^2 + 5xy - 3y^2 + \frac{(3x - 5y)(3x - 5y)}{2} - \frac{2(3x - 5y)^2}{4} = 0$$

$2x^2 + 5xy - 3y^2 + (3x - 4y)^2 - (3x - 5y)^2 = 0$

$2x^2 + 5xy - 3y^2 = 0$

(26) **(B).** Lines $x\cos\alpha + y\sin\alpha = p$ and $x\sin\alpha - y\cos\alpha = 0$ are mutually perpendicular.
Thus $ax + by + p = 0$ will be equally inclined to these line and would be the angle bisector of these lines.
Now equations of angle bisectors is ,
$x\sin\alpha - y\cos\alpha = \pm(x\cos\alpha + y\sin\alpha - p)$
$\Rightarrow x(\cos\alpha - \sin\alpha) + y(\sin\alpha + \cos\alpha) = p$
or $x(\sin\alpha + \cos\alpha) - y(\cos\alpha - \sin\alpha) = p$
Comparing these lines with $ax + by + p = 0$, we get

$$\frac{a}{\cos\alpha - \sin\alpha} = \frac{b}{\sin\alpha + \cos\alpha} = 1 \Rightarrow a^2 + b^2 = 2$$

or $\dfrac{a}{\sin\alpha + \cos\alpha} = \dfrac{b}{\sin\alpha - \cos\alpha} = 1 \Rightarrow a^2 + b^2 = 2$

(27) **(A).** $4a^2 + b^2 + 2c^2 + 4ab - 6ac - 3bc$
$\equiv (2a + b)^2 - 3(2a + b)c + 2c^2 = 0$

$\Rightarrow (2a + b - 2c)(2a + b - c) = 0 \Rightarrow c = 2a + b$ or $c = a + \dfrac{1}{2}b$

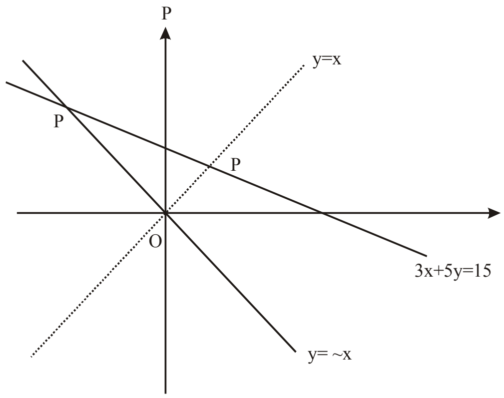
The equation of the family of lines is

$a(x + 2) + b(y + 1) = 0$ or $a(x + 1) + b\left(y + \dfrac{1}{2}\right) = 0$

giving the point of concurrence $(-2, -1)$ or $\left(-1, -\dfrac{1}{2}\right)$.

(28) **(C).** Let (x, y) lies in the 1^{st} quadrant
Hence $x = y = a > 0$
$3a + 5a = 15$

$a = \dfrac{15}{8}$ Hence $P\left(\dfrac{15}{8}, \dfrac{15}{8}\right)$



Note that P can lie either on y = x or y = – x in the 4^{th} quadrant only.
Solving $y = -x$ and the line $x = -15/2$.

Hence $P = \left(-\frac{15}{2}, \frac{15}{2}\right)$

(29) **(C).** Let centre of circle be P (h, k). So, that $PA^2 = PB^2$ where A = (2, 4) and B = (0, 1)
and (slope of OA) (slope of tangent at A) = –1

$\Rightarrow h^2 + (k-1)^2 = (h-2)^2 + (k-4)^2$

or $4h + 6k - 19 = 0$ .... (1)

also slope of OA = $\frac{k-4}{h-2}$ and slope of tangent at (2, 4) to $y = x^2$ is 4

$\therefore \frac{k-4}{h-2}.4 = -1$ or $4k - 16 + 2$

$\Rightarrow h + 4k = 18$ .... (2)
solving (1) and (2), we get $k = 53/10$ and $h = -16/5$
$\therefore$ Centre co-ordinates are (–16/5, 53/10)

(30) **(B).** Slope of AB = 1 and hence AB is inclined at 45° to the x-axis. $AB = 2\sqrt{2}$
$\therefore$ diagonal AC = 4 and AC is inclined at 90° to the x – axis. Hence the centre of the square is at E,2 units above A and its coordinates are (3 –2) A second square possible is ABC_1D_1 on the side of AB opposite to CD but (2, –2) will not be an interior point.

(31) **(A)** The cartesian coordinates are (0, 0) $\left(3\cos\frac{\pi}{6}, 3\sin\frac{\pi}{6}\right)$,

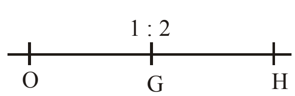
$\left(3\cos\frac{\pi}{2}, 3\sin\frac{\pi}{2}\right)$ i.e., (0, 0) $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ and (0, 3).

They form an equilateral triangle of side 3 units.

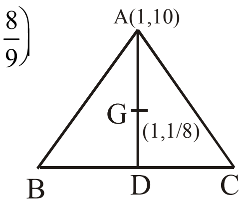
$$\text{Area} = \frac{a^2\sqrt{3}}{4} = \frac{3^2\sqrt{3}}{4} = \frac{9\sqrt{3}}{4}$$

(32) **(A).** Circumcentre orthocentre O is $\left(-\frac{1}{3}, \frac{2}{3}\right)$,

H is $\left(\frac{11}{3}, \frac{4}{3}\right)$



$\therefore$ Coordinate of G are $\left(1, \frac{8}{9}\right)$



A(1,10), G $\left(1, \frac{8}{9}\right)$

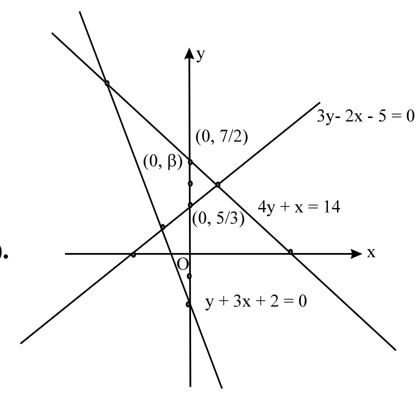
AD : DG = 3 : – 1

$$D_x = \frac{3-1}{2} = 1,\ D_y = \frac{\frac{8}{3} - 10}{2} = -\frac{11}{3}$$

Coordinate of the mid point is BC are (1, –11/3)

(33) **(C).** $2x + 3y = 1$ and $3x - 2y = 2$ intersect at right angles so that the triangle is a right-angled triangle. The orthocentre is the right-angled vertex i.e., the point of intersection of the lines $2x + 3y = 1$ and $3x - 2y = 2$ which is $\left(\frac{8}{13}, \frac{-1}{13}\right)$.

(34) **(C).**



$\Rightarrow \frac{5}{3} \le \beta \le \frac{7}{2}$

(35) **(B).** Let y = mx be any line represented by the equation
$ax^3 + bx^2y + cxy^2 + dy^3 = 0$
$\Rightarrow ax^3 + bx^2(mx) + cx(m^2x^2) + dm^3x^3 = 0$
$\Rightarrow a + bm + cm^2 + dm^3 = 0$ which is a cubic equation.
It represents three lines out of which two are perpendicular

hence $m_1m_2 = -1$ and $m_1m_2m_3 = -\frac{a}{d} \Rightarrow m_3 = \frac{a}{d}$

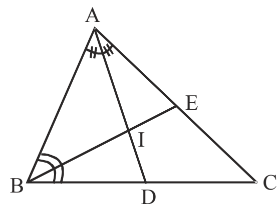
and m_3 is the root of the given equation

$$a + b\left(\frac{a}{d}\right) + c\left(\frac{a}{d}\right)^2 + d\left(\frac{a}{d}\right)^3 = 0 \Rightarrow d^2 + bd + ca + a^2 = 0$$

(36) **(A).** $\frac{BD}{DC} = \frac{b}{c} \Rightarrow x_D = \frac{bx_B + cx_C}{b+c}$

$$BD = \frac{c}{b+c}a \Rightarrow \frac{Al}{Dl} = c\frac{b+c}{ac} = \frac{b+c}{a}$$

$$\therefore x_1 = \frac{(b+c)x_D + ax_A}{a+b+c} \Rightarrow \frac{\Sigma ax_A}{\Sigma a}$$



Similarly $y_1 = \dfrac{\Sigma a y_A}{\Sigma a}$ $\therefore$ the coordinates $= \left(\dfrac{\Sigma a x_A}{\Sigma a}, \dfrac{\Sigma a y_A}{\Sigma a}\right)$

(37) **(A).** The Statement-1 is true and follows from reason R.
Since the family of lines can be written as
$a(x + y - 1) + b(x - 2y) = 0$.

(38) **(A).** Statement-2 is true (standard results)
Statement-2 is image of (2, 1) in the line $3x + 4y + 5 = 0$ is

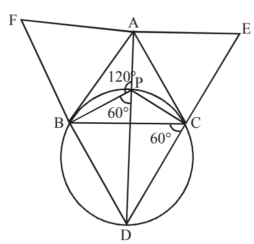
given by $\dfrac{x-2}{3} = \dfrac{y-1}{4} = -2\dfrac{3.2+4.1+5}{25} = -\dfrac{6}{5}$

i.e., $x = -\dfrac{8}{5}$ and $y = -\dfrac{19}{5}$

$\therefore$ Statement-1 is true and is explained by statement-2.

(39) **(A).** Both Statement-1 and Statement-2: are true and Statement-2: is the correct explanation of Statement-1.

(40) $\angle BPD = \angle BCD = 60°$ ($\because$ chord BD subtends equal angle in same segment)
$\therefore \angle APB + \angle DPB = 180°$
$\therefore$ A, P, D are collinear
Similarly B, P, E and C, P, F are also collinear



hence AD, BE, CF are concurrent at P.

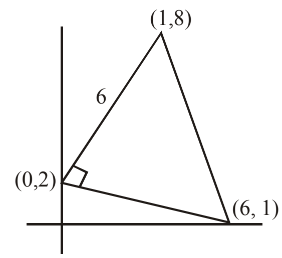
(41) **(C).** Let $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ are all rational coordinates

$$ar(\Delta ABC) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{\sqrt{3}}{4}\left[(x_1-x_2)^2 + (y_1-y_2)^2\right]$$

LHS = rational, RHG = irrational
Hence (x_1, y_1) (x_2, y_2) & (x_3, y_3) cannot be all rational

(42) **(A).** $2b = a + c$
$a - 2b + c = 0$
$ax + by + c = 0$
On comparing $x = 1, y = -2$
So (1, -2) point lies on these lines.

(43) **(A).**
(a) A (1 + 8i), B (2i) and C (6 + i)
Δ ABC is right angled at B



$\therefore$ Orthocentre is B (2i) $\therefore$ a = 0, b = 2 $\therefore$ 2a + b = 2
(b) Slope of line joining (t – 1, 2t + 2) and (2t + 1, t) is

$\dfrac{t+2}{-t-2} = -1$

$\therefore$ Slope of the line of symmetry = 1
(c) $ax^2 + 2axy + y^2 = 0$

Equation of the angle bisectors $= \dfrac{x^2 - y^2}{a-1} = \dfrac{xy}{a}$

If y = 3x is one of the bisecting lines, then

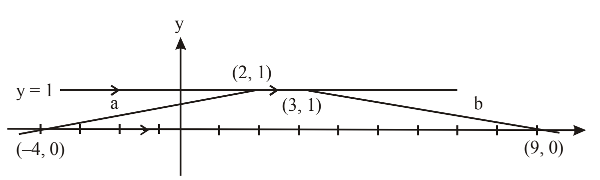
$\dfrac{x^2 - 9x^2}{a-1} = \dfrac{3x^2}{a}$ i.e., $\dfrac{-8}{a-1} = \dfrac{3}{a}$ i.e., $-8a = 3a - 3$

$\therefore a = \dfrac{3}{11}$ $\therefore 11a = 3$

(d) $0 = \begin{vmatrix} 1 & -2 & -6 \\ 3 & 1 & -4 \\ \lambda & 4 & \lambda^2 \end{vmatrix}$

$= (\lambda^2 + 16) + 2(3\lambda^2 + 4\lambda) - 6(12 - \lambda)$
i.e., $7\lambda^2 + 14\lambda - 56 = 0$ i.e. $\lambda^2 + 2\lambda - 8 = 0$
i.e., $(\lambda + 4)(\lambda - 2) = 0$
$\therefore \lambda = -4, 2$

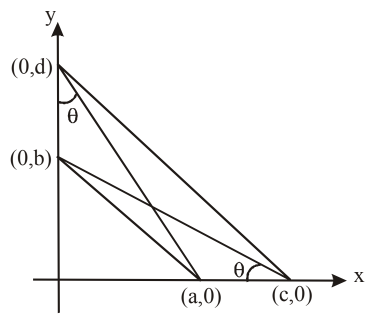
(44) **(B).**
(a) Obviously trapezium



$\left.\begin{array}{l} a = \sqrt{37} \\ b = \sqrt{37} \end{array}\right] \Rightarrow a = b$. hence isosceles trapezium

$\Rightarrow$ a cyclic quadrilateral also $\Rightarrow$ P, S

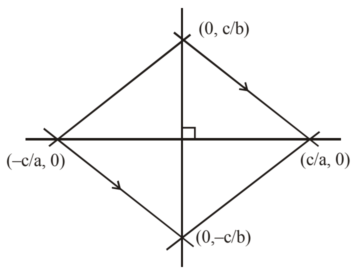
(b) $ac = bd \Rightarrow \dfrac{b}{c} = \dfrac{a}{d}$; $\left.\begin{array}{l} \tan\theta = \dfrac{b}{c} \\ \tan\phi = \dfrac{a}{d} \end{array}\right] \Rightarrow \theta = \phi$



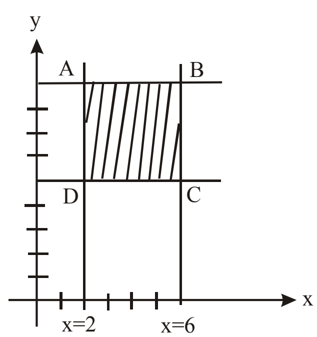
hence cyclic quadrilateral ⇒ P

(c) $ax \pm by \pm c = 0$ if $y = 0$, $x = \pm\frac{c}{a}$

if $x = 0$, $y = \pm\frac{c}{b}$ ⇒ rhombus ⇒ Q



(d) $(x-6)(x-2)=0$
$x = 6$ and $x = 2$
$y^2 - 14y + 45 = 0$
$(y-9)(y-5)=0$ ⇒ a square ⇒ P, Q, R



(45) **(D).**
(A) Lines are concurrent

$$\Rightarrow \begin{vmatrix} 1 & -2 & -6 \\ 3 & 1 & -4 \\ \lambda & 4 & \lambda^2 \end{vmatrix} = 0 \Rightarrow \lambda^2 + 2\lambda - 8 = 0 \Rightarrow \lambda = 2, -4$$

(B) Points are collinear

$$\Rightarrow \begin{vmatrix} \lambda+1 & 1 & 1 \\ 2\lambda+1 & 3 & 1 \\ 2\lambda+2 & 2\lambda & 1 \end{vmatrix} = 0$$

$\Rightarrow 2\lambda^2 - 3\lambda - 2 = 0 \Rightarrow \lambda = 2, -1/2$
(C) Point of intersection $x - y + 1 = 0$
and $3x + y - 5 = 0$ is (1, 2).
It will satisfy $x + y - 1 - \lambda = 0 \Rightarrow \lambda = 2$
(D) Midpoint of (1, –2) and (3, 4) will saifsfy
$y - x + 1 + \lambda = 0 \Rightarrow \lambda = 2$

(46) **(A).** P.B of AB is parallel to $x - y - 4 = 0$, $m = 1$
Passing through circumcentre = (3/2, 5/2)
$2x - 2y + 2 = 0 \Rightarrow x - y + 1 = 0$

(47) **(B).** P.B of AC is parallel to $2x - y - 5 = 0$, $m = 2$
Passing through circumcentre = (3/2, 5/2)
$4x - 2y - 1 = 0 \Rightarrow 2x - y - 1/2 = 0$

(48) **(A).** Taking image of vertex A is bisector of AB

$$\Rightarrow \frac{x+2}{1} = \frac{y-3}{-1} = -2\left(\frac{-2-3+1}{1+1}\right) \Rightarrow \frac{y-3}{1} = 4,\ y = -1$$

⇒ B is (2, –1)

(49) (B), (50) (B), (51) (B)

Slopes of the sides AB, BC and AB are $-\frac{1}{3}, \frac{1}{2}, 2$ respectively.

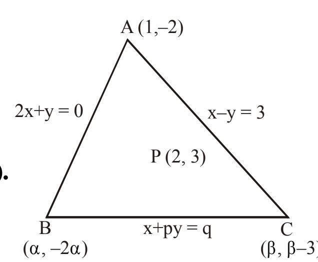
$$\tan A = \frac{-\frac{1}{3} - 2}{1 + \left(-\frac{1}{3}\right)2} = -7, \quad \tan B = \frac{2 - \frac{1}{2}}{1 + 2\left(\frac{1}{2}\right)} = \frac{3}{4}$$

∴ angle B is acute. Since $(-1)\,2 + 2\,(-1) < 0$
∴ origin lies in the acute angle
obtuse angle bisector of $-x + 2y + 1 = 0$ and $2x - y + 4 = 0$
is $-x + 2y + 1 = -(2x - y + 4)$ i.e. $x + y + 5 = 0$
Point B is (–3, –2)

its image in CA is $\frac{x+3}{1} = \frac{y+2}{3} = -2\frac{-3-6-3}{10} = \frac{12}{5}$

$$\therefore\ x = -\frac{3}{5},\ y = \frac{26}{5}$$

(52) **(D).**



A (1, –2), B (α, –2α), C (β, β – 3)
$1 + \alpha + \beta = 6,\ -2 - 2\alpha + \beta - 3 = 9 \Rightarrow \alpha = -3, \beta = 8$
B = (–3, 6), C = (8, 5)
Equation of BC is $x + 11y = 63$
∴ $p + q = 63 + 11 = 74$

(53) **(B).** Slope of BP = – 1 $\Rightarrow \frac{3+2\alpha}{2-\alpha} = -1 \Rightarrow \alpha = -5$

Slope of CP = $\frac{1}{2} \Rightarrow \frac{\beta-6}{\beta-2} = \frac{1}{2} \Rightarrow \beta = 10$

B (–5, 10), C (10, 7)

Equation of BC : x + 5y = 45 $\therefore$ p + q = 50

(54) **(A).** $PA^2 = 26 = PB^2 = PC^2$

$\Rightarrow (\alpha-2)^2 + (3+2\alpha)^2 = (\beta-2)^2 + (\beta-6)^2 = 26$

$\Rightarrow \alpha = -13/5$ or 1 (rejected because vertices A and B coincide). Similarly other equation gives β = 7 or 1 (rejected because vertices A and C coincide)

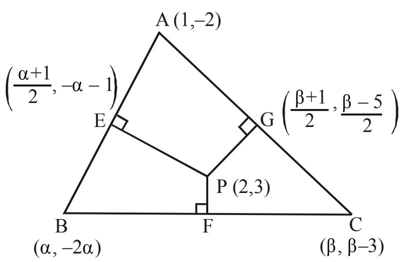
Hence $\alpha = -13/5$ and $\beta = 7$

$B = \left(\frac{-13}{5}, \frac{26}{5}\right), C = (7,4)$

The equation of BC is x + 8y = 39 $\Rightarrow$ (p + q) = 47

Alternatively for (iii): Slope of line EP = 1/2

$\frac{1}{2} = \frac{3+\alpha+1}{2-\frac{\alpha+1}{2}}; \ \frac{1}{2} = 2\left(\frac{4+\alpha}{3-\alpha}\right)$



$3 - \alpha = 16 + 4\alpha \Rightarrow 5\alpha = -13 \Rightarrow \alpha = -13/5$

Slope of line PG = – 1

$-1 = \frac{\frac{\beta-5}{2}-3}{\frac{\beta+1}{2}-2} = \frac{\beta-11}{\beta-3}$;

$-\beta + 3 = \beta - 11$; $2\beta = 14$; $\beta = 7$

$B = \left(\frac{-13}{5}, \frac{26}{5}\right), C = (7,4)$

The equation of BC is x + 8y = 39 $\Rightarrow$ (p + q) = 47

(55) **(B).** As lines are perpendicular

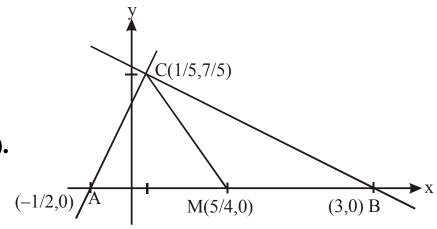
$\therefore$ a – 2 = 0 $\Rightarrow$ a = 2 (coefficient of x^2 + coefficient of y^2 = 0)

using $\Delta = 0 \Rightarrow c = -3$ ($D \equiv abc + 2fgh - af^2 - bg^2 - ch^2$)

hence the two lines are x + 2y – 3 = 0 and 2x – y + 1 = 0

$\left.\begin{array}{l}\text{x-intercepts } x_1 = 3, x_2 = -1/2 \\ \text{y-intercepts } y_1 = 3/2, y_2 = 1\end{array}\right] \Rightarrow x_1 + x_2 + y_1 + y_2 = 5$

(56) **(C).**



$(CM)^2 = \left(\frac{5}{4} - \frac{1}{5}\right)^2 + \frac{49}{25} = \left(\frac{25-4}{20}\right)^2 + \frac{49}{25}$

$= \frac{441}{400} + \frac{49}{25} = \frac{441+784}{400} = \frac{1225}{400} = \frac{49}{16} \Rightarrow CM = \frac{7}{4}$

(57) **(D).** Circumcircle of ABC

$\left(x + \frac{1}{2}\right)(x-3) + y^2 = 0 \Rightarrow (2x+1)(x-3) + 2y^2 = 0$

$\Rightarrow 2(x^2+y^2) - 5x - 3 = 0 \Rightarrow x^2 + y^2 - \frac{5}{2}x - \frac{3}{2} = 0$ ..... (1)

Given $x^2 + y^2 - 4y + k = 0$ which is orthogonal to (1) using the condition of orthogonality

we get, $0 + 0 = k - \frac{3}{2} \Rightarrow k = \frac{3}{2}$

EXERCISE-3

(1) **4.** Let the origin be shifted to the point (h, k) and let P(x, y) be any point on the curve and (x_1, y_1) be the coordinates of P with respect to new axes then

$x = x_1 + h$ and $y = y_1 + k$

Hence, new equation will be

$(y_1 + k)^2 + 4(y_1 + k) + 8(x_1 + h) - 2 = 0$

$y_1^2 + (2k+4)y_1 + 8x_1 + (k^2 + 4k + 8h - 2) = 0$

Thus new equation of the curve will be

$y^2 + (2k+4)y + 8x + (k^2 + 4k + 8h - 2) = 0$

Since this equation is required to be free from the term containing y and the constant, we have

$2k + 4 = 0$ and $k^2 + 4k + 8h - 2 = 0$

$\therefore$ k = –2 and h = 3/4

Hence, the point to which the origin be shifted is (3/4,–2)

(2) **13.** The gradients are – 2 and 1/6 ; the angle of slope of the first line is in the second quadrant while that of the second line is in the first quadrant ; accordingly, we write

$m_2 = -2, m_1 = 1/6, \tan\phi = \frac{-2-1/6}{1+(-2)(1/6)} = \frac{13}{4}$

Hence ϕ is an obtuse angle, If α is the acute angle between the lines then $\phi = 180° - \alpha$, from which

$\tan\phi = \tan(180° - \alpha)$. But $\tan(180° - \alpha) = -\tan\alpha$.

Hence $\tan\theta = -\tan\alpha$ and, by (i),

$\tan\alpha = \frac{13}{4} \Rightarrow \alpha = \tan^{-1}(13/4)$

(3) **19.** Let S be the given point, RT the line $x = 4$ and P (x, y) any point on the locus PQ is perpendicular to RT.
The given condition is equivalent to : $PQ^2 = 4PS^2$.
Now $PQ^2 = (4 - x)^2$ and $PS^2 = (x - 1)^2 + y^2$;
hence $(4 - x)^2 = 4[(x - 1)^2 + y]^2$ or, on simplification,
$3x^2 + 4y^2 = 12$

(4) **8.** Distance of all vertices from origin is 2.
So, circum-centre is origin.

$$\text{Centroid} = \left(\frac{1 + 2\cos\theta + 2\sin\theta}{3}, \frac{\sqrt{3} + 2\sin\theta - 2\cos\theta}{3}\right)$$

Let orthocentre is (h, k)

$h = 1 + 2(\cos\theta + \sin\theta)$; $k = \sqrt{3} + 2(\sin\theta - \cos\theta)$

$$\left(\frac{h-1}{2}\right)^2 + \left(\frac{k-\sqrt{3}}{2}\right)^2 = 2$$

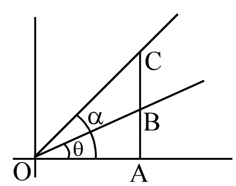
locus is $(x - 1)^2 + (y - \sqrt{3})^2 = 8$

(5) **8.** Given AB = BC

$$\tan\theta = \frac{AB}{OA} = m_1 \quad ; \tan\alpha = \frac{2AB}{OA} = m_2$$

$$\frac{m_2}{m_1} = 2 \; ; \; \frac{m_2 + m_1}{m_2 - m_1} = \frac{2+1}{2-1} = 3$$

$$\Rightarrow -\frac{\frac{2h}{b}}{\sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}} = 3$$



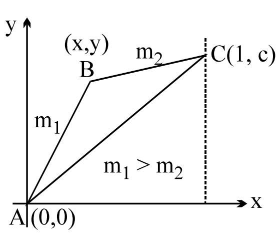
$$\Rightarrow \frac{4h^2}{b^2} - \frac{4a}{b} = \frac{4h^2}{9b^2}$$

$$\Rightarrow \frac{4h^2}{b^2} \times \frac{8}{9} = \frac{4a}{b} \Rightarrow 8h^2 = 9ab$$

(6) **8.** Let the coordinates of C be (1, c)

$$m_2 = \frac{c-y}{1-x} \; ; \qquad m_2 = \frac{c - m_1 x}{1-x}$$

$m_2 - m_2 x = c - m_1 x$



$(m_1 - m_2)x = c - m_2$
$c = (m_1 - m_2)x + m_2$ ....(1)

$$\text{now area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & m_1 x & 1 \\ 1 & c & 1 \end{vmatrix}$$

$$= \frac{1}{2}[cx - m_1 x] = \frac{1}{2}\left|[((m_1 - m_2)x + m_2)x - m_1 x]\right|$$

$$= \frac{1}{2}\left|[(m_1 - m_2)x^2 + m_2 x - m_1 x]\right|$$

$$= \frac{1}{2}(m_1 - m_2)(x - x^2) \quad (x > x^2 \text{ in } (0, 1))$$

Hence, $f(x) = \frac{1}{2}(x - x^2)$; $f(x)]_{max} = \frac{1}{8}$ when $x = \frac{1}{2}$

(7) **2.** The point B is (2, 1)
Image of A (1, 2) is the line $x - 2y + 1 = 0$ is given by

$$\frac{x-1}{1} = \frac{y-2}{-2} = \frac{4}{5}$$

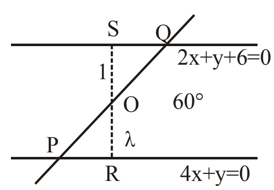
$\therefore$ Coordinate of the point are $\left(\frac{9}{5}, \frac{2}{5}\right)$

Since this point lies on BC.
$\therefore$ Equation of BC is $3x - y - 5 = 0$
$\therefore$ $a + b = 2$

(8) **2.** On solving equations $3x + 4y = 9$ and $y = mx + 1$, we get $x = \frac{5}{3+4m}$

Now, for x to be an integer $3 + 4m = \pm 5$ or ± 1
The integral values of m satisfying these conditions are –2 and –1.

(9) **4.** OP : OQ = OR : OS



The equation of SR is $y = \frac{1}{2}x$

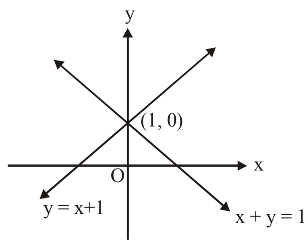
$$\therefore \; R = \left(\frac{9}{5}, \frac{9}{10}\right) \text{ and } S = \left(\frac{-12}{5}, \frac{-6}{5}\right)$$

$$\therefore \; O = (0, 0) = \left(\frac{\frac{-12}{5}\lambda + \frac{9}{5}}{\lambda + 1}, \frac{\frac{-6}{5}\lambda + \frac{9}{10}}{\lambda + 1}\right)$$

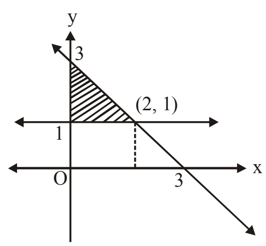
$\therefore \quad \frac{-12}{5}\lambda + \frac{9}{5} = 0,\ \frac{-6}{5}\lambda + \frac{9}{10} = 0 \Rightarrow 12\lambda = 9$

$\therefore \quad \lambda = 3/4$. So, OP : OQ = 3 : 4

(10) **2.** $x^2 - y^2 + 2y = 1 \Rightarrow x = \pm (y - 1)$



Bisectors of above lines are x = 0 and y = 1.



So, area between x = 0, y = 1 and x + y = 3 is shaded reagion shown in figure.

Area $= \frac{1}{2} \times 2 \times 2 = 2$ sq. units

(11) **1.** Let A (x, – 1), B (2, 1) and C (4, 5) be three collinear points.

$\therefore \quad$ Slope of AB $= \frac{1-(-1)}{2-x} = \frac{1+1}{2-x} = \frac{2}{2-x}$

Slope of BC $= \frac{5-1}{4-2} = \frac{4}{2} = 2$

Now, slope of AB = Slope of BC

$\therefore \quad \frac{2}{2-x} = 2 \Rightarrow 2 = 4 - 2x \Rightarrow 2x = 2 \Rightarrow x = 1.$

(12) **5.** We have, $12(x+6) = 5(y-2)$.

$\Rightarrow \quad 12x - 5y + 82 = 0$ .... (1)

Comparing (1) with general equation of the line

$Ax + By + C = 0$, we get

$A = 12,\ B = -5$ and $C = 82$

Given point is $(x_1, y_1) \equiv (-1, 1)$.

The distance of the given point from given line is

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|12(-1) + (-5)(1) + 82|}{\sqrt{144 + 25}}$$

$$= \frac{|-12 - 5 + 82|}{\sqrt{169}} = \frac{65}{13} = 5 \text{ units}$$

EXERCISE-4

(1) **(A).** Let coordinate of points A, B, C, and D are (–a, –b), (0, 0), (a, b) and (a^2, ab) respectively.

$\therefore$ Slope of line joining A and B is $\frac{0-(-b)}{0-(-a)} = \frac{b}{a}$

Slope of BC $= \frac{b-0}{a-0} = \frac{b}{a}$; Slope of CD $= \frac{ab-b}{a^2-a} = \frac{b}{a}$

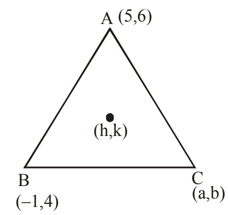
Slope of AD $= \frac{ab-(-b)}{a^2-(-a)} = \frac{b}{a}$

$\therefore$ Slope of AB = BC = CD = AD

$\therefore$ All four points are collinear.

(2) **(B).** Let centroid is (h, k) = (2, 3) (given)

and third vertex is (a, 3)



Now from definition of centroid

$\frac{a+5+(-1)}{3} = 2 \Rightarrow a + 4 = 6 \Rightarrow a = 2$

and $\frac{b+6+4}{3} = 3 \Rightarrow b + 10 = 9 \Rightarrow b = -1$

(3) **(C).** Combined equation of two lines is $x^2 + 4xy + y^2 = 0$ angle between then is defined as

$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$, here a = 1, b = 1, h = 2

$\therefore$ Standard equation is $ax^2 + 2hxy + by^2 = 0$

$\therefore \tan\theta = \frac{2\sqrt{4-1}}{1+1} = \sqrt{3} \Rightarrow \theta = 60°$

(4) **(C).** Equation of pair of parallel lines is

$9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$

$\Rightarrow \frac{9}{16}x^2 - \frac{24}{16}xy + y^2 - \frac{12}{16}x + y - \frac{12}{16} = 0$ .... (1)

Let two parallel lines are $y = mx + c_1$ and $y = mx + c_2$

Now their combine equation will be

$(mx - y + c_1)(mx - y + c_2) = 0$

$\Rightarrow m^2x^2 - 2mxy + y^2 + x(mc_1 + mc_2) - y(c_1 + c_2) + c_1c_2 = 0$ .... (2)

If we compare (1) and (2) we get, $m^2 = \frac{9}{16}$

$c_1 + c_2 = -1$; $c_1 c_2 = \frac{-12}{16}$

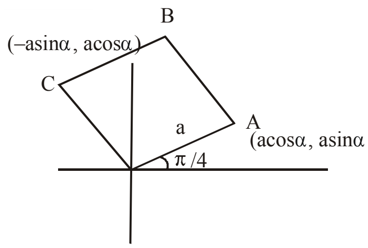
$\therefore\ c_1 - c_2 = \sqrt{(c_1+c_2)^2 - 4c_1c_2} = \sqrt{1-4\left(\frac{-12}{16}\right)} = \sqrt{4}$

$\Rightarrow |c_1 - c_2| = 2$

$\therefore$ distance between two parallel lines is

$$\left|\frac{|c_1-c_2|}{\sqrt{1+m^2}}\right| = \frac{2}{\sqrt{1+\frac{9}{16}}} = \frac{2\times 4}{5} = \frac{8}{5}$$

(5) (A).



$A = (a\cos\alpha, a\sin\alpha)$

and $C = (a\cos(\pi/2 + 2), a\sin(\pi/2+\alpha))$

$= (-a\sin\alpha, a\cos\alpha)$

$\therefore$ slope of AC is $= \dfrac{a[\cos\alpha - \sin\alpha]}{a[-\sin\alpha - \cos\alpha]}$

$= \dfrac{\cos\alpha - \sin\alpha}{-\sin\alpha - \cos\alpha} = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

Equation AC is

$$y - a\sin\alpha = \frac{\sin\alpha - \cos\alpha}{\sin\alpha+\cos\alpha}(x - a\cos\alpha)$$

i.e. $y(\cos\alpha + \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

$y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

(6) (A). Equation of the bisector of the angles between $x^2 - 2pxy - y^2 = 0$ is

$$\frac{x^2-y^2}{1-(-1)} = \frac{xy}{-1} \quad \left[\frac{x^2-y^2}{a-b} = \frac{xy}{h}\right]$$

$\Rightarrow -p(x^2-y^2) = 2xy \Rightarrow px^2 + 2xy - py^2 = 0$

This is same as $x^2 - 2qxy - y^2 = 0$

$$\therefore \frac{p}{1} = \frac{1}{-q} = \frac{p}{1} \Rightarrow pq = -1$$

(7) (C). Let $A(a\cos t, a\sin t)$, $B(b\sin t, -b\cos t)$, $C(1, 0)$

Let centroid of ΔABC is (h, k)

$$\therefore h = \frac{a\cos t + b\sin t + 1}{3}$$

$\Rightarrow 3h - 1 = a\cos t + b\sin t$ .... (1)

and $k = \dfrac{a\sin t - b\cos t}{3}$

$\Rightarrow 3k = a\sin t - b\cos t$ .... (2)

Squaring eq. (1) and eq. (2) and adding

$(3h-1)^2 + (3k)^2 = a^2 + b^2$

$\therefore$ Locus is $(3x-1)^2 + (3y)^2 = a^2 + b^2$

(8) (B). Let coordinate of point equidistant from (a_1, b_1) and (a_2, b_2) is (h, k)

$\therefore \sqrt{(h-a_1)^2 + (k-b_1)^2} = \sqrt{(h-a_2)^2 + (k-b_2)^2}$

$\Rightarrow (h-a_1)^2 + (k-b_1)^2 = (h-a_2)^2 + (k-b_2)^2$

$\Rightarrow h^2 - 2a_1h + a_1^2 + k^2 + b_1^2 - 2kb_1$

$= h^2 + a_2^2 - 2a_2h + k^2 + b_2^2 - 2kb_2$

$\Rightarrow h(2a_2 - 2a_1) + k(2b_2 - 2b_1) + a_1^2 + b_1^2 - a_2^2 - b_2^2 = 0$

$\Rightarrow h(a_1 - a_2) + k(b_1 - b_2) + \frac{1}{2}[a_2^2 + b_2^2 - a_1^2 - b_1^2] = 0$

$\therefore$ Locus is

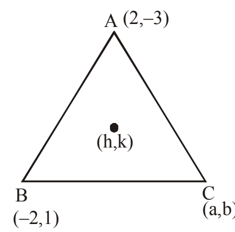
$x(a_1 - a_2) + y(b_1 - b_2) + \frac{1}{2}[a_2^2 + b_2^2 - a_1^2 - b_1^2] = 0$

Compare it with given equation

$(a_1 - a_2)x + (b_1 - b_2)y + c = 0$

We see, $c = \frac{1}{2}[a_2^2 + b_2^2 - a_1^2 - b_1^2]$

(9) (A). Let third vertex is (a, b) and centroid is (h, k)



$\therefore\ h = \dfrac{2 + (-2) + a}{3} \Rightarrow 3h = a \Rightarrow h = a/3$ .... (1)

and $k = \dfrac{b + (-3) + 1}{3} \Rightarrow 3k = b - 2 \Rightarrow k = \dfrac{b-2}{3}$ .... (2)

$\therefore$ (h, k) moves on line $2x + 3y - 1 = 0$

$\therefore$ they will satisfy it

$\therefore 2h + 3k - 1 = 0 = 2\dfrac{a}{3} + 3\left(\dfrac{b-2}{3}\right) - 1 = 0$

$2a + 3b - 6 - 3 = 0 \Rightarrow 2a + 3b - 9 = 0$

$\therefore$ locus of (a, b) is $2x + 3y - 9 = 0$

(10) (D). Let intercept of co-ordinate axis by a line is a and b.

$\therefore a + b = -1$ (from question) .... (1)

$\therefore$ equation of line is $\dfrac{x}{a} + \dfrac{y}{b} = 1 \Rightarrow bx + ay = ab$

$\because$ line passes through (4, 3)

$\because 4b + 3a = ab$

$\therefore 4b + 3a = ab$

$\Rightarrow 4(-a-1) + 3a = a(-a-1)$ {From (1)}

$\Rightarrow -4a - 4 + 3a = -a^2 - a \Rightarrow a^2 = 4 \Rightarrow a = \pm 2$

$\therefore a = 2,\ b = -3$ and $a = -2,\ b = 1$

$\Rightarrow$ equation of lines are $\dfrac{x}{2} + \dfrac{y}{-3} = 1$ and $\dfrac{x}{-2} + \dfrac{y}{1} = 1$

$\Rightarrow \dfrac{x}{2} - \dfrac{y}{3} = 1$ and $\dfrac{x}{-2} + \dfrac{y}{1} = 1$

(11) **(C).** $\because m_1 + m_2 = 4m_1m_2$

$\Rightarrow \frac{-2h}{b} = \frac{4a}{b} \Rightarrow \frac{-2(-c)}{-7} = \frac{4.1}{-7} \qquad \therefore c = 2$

(12) **(D).** $3x + 4y = 0 \Rightarrow y = \frac{-3}{4}x = mx$, where $m = \frac{-3}{4}$

Since, $6x^2 - xy + 4cy^2 = 0$

$\Rightarrow 4c\left(\frac{y}{x}\right)^2 - \frac{y}{x} + 6 = 0 \Rightarrow 4cm^2 - m + 6 = 0$

$\Rightarrow 4c\left(\frac{-3}{4}\right)^2 - \left(\frac{-3}{4}\right) + 6 = 0 \Rightarrow 4c\frac{9}{16} + \frac{3}{4} + 6 = 0$

$\Rightarrow \frac{9}{4}c + \frac{27}{4} = 0 \Rightarrow c = -3$

(13) **(A).** Line passing through the intersection of $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$

$\therefore$ equation of line is

$(ax + 2by + 3b) + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow x(a + \lambda b) + y(2b - 2a\lambda) + 3b - 3a\lambda = 0 \qquad \text{..... (1)}$

Slope of this line is $\frac{-(a + \lambda b)}{2b - 2a\lambda}$

$\therefore$ line is parallel to axis $\therefore$ its slope is zero

$\Rightarrow -\left(\frac{a + \lambda b}{2b - 2a\lambda}\right) = 0 \Rightarrow a + \lambda b = 0 \Rightarrow \lambda = \frac{-a}{b}$

Put this in (1) we get

$x\left[a + \left(\frac{-a}{b}\right)b\right] + y\left[2b - 2a\left(\frac{-a}{b}\right)\right] + 3b - 3a\left(\frac{-a}{b}\right) = 0$

$\Rightarrow y(2b^2 + 2a^2) + 3(b^2 + 3a^2) = 0 \Rightarrow y = -3/2$

$\Rightarrow$ Required line is below the x-axis and at a distance of 3/2 from it.

(14) **(C).** $\because$ a, b, c are in H.P. $\therefore b = \frac{2ac}{a + c} \Rightarrow \frac{1}{b} = \frac{a + c}{2ac}$

$\Rightarrow \frac{1}{b} = \frac{1}{2a} + \frac{1}{2c} \Rightarrow \frac{1}{2a} - \frac{1}{b} + \frac{1}{2c} = 0$

$\Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0 \qquad \text{........ (1)}$

Given equation of line is $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0 \qquad \text{........ (2)}$

If we put (1, –2) in (2) we get

$\frac{1}{a} - \frac{2}{b} + \frac{1}{c}$ which is true from (1)

$\therefore$ given line passes through fixed point (1, –2)

(15) **(A).** P (1, 0) (given)

Let coordinate of Q is (a, b) and mid point of PQ is (h, k)

$\therefore h = \frac{a + 1}{2} \Rightarrow a = 2h - 1 \qquad \text{....... (1)}$

and $k = \frac{b + 0}{2} \Rightarrow b = 2k \qquad \text{....... (2)}$

$\because$ Q is on curve $y^2 = 8x$

$\therefore$ It will satisfy it $\therefore b^2 = 8y$

$\Rightarrow 4k^2 = 8(2h - 1) \Rightarrow k^2 = 2(2h - 1) \Rightarrow k^2 = 4h - 2$

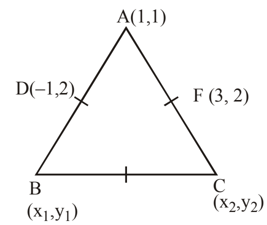
$\therefore$ locus of (h, k) is $y^2 = 4x - 2 \Rightarrow y^2 - 4x + 2 = 0$

(16) **(C).** Let other two vertices are $B = (x_1, y_1)$ and $C = (x_2, y_2)$ and mid point of AB is (–1, 2)

$\therefore -1 = \frac{1 + x_1}{2} \Rightarrow -2 = 1 + x_1 \Rightarrow x_1 = -3$ and

$2 = \frac{1 + y_1}{2} \Rightarrow 4 = 1 + y_1 \Rightarrow y_1 = 3$

$\therefore (x_1, y_1) = (-3, 3)$ and mid point of AC is (3, 2)



$\therefore 3 = \frac{1 + x_2}{2} \Rightarrow x_2 = 5$ and $2 = \frac{1 + y_2}{2} \Rightarrow y_2 = 3$

$\therefore (x_2, y_2) = (5, 3)$

$\therefore$ Centroid of Δ ABC is $\left(\frac{1 + (-3) + 5}{3}, \frac{1 + 3 + 3}{3}\right) = (1, 7/3)$

(17) **(B).** Let the straight line be $\frac{x}{a} + \frac{y}{b} = 1$

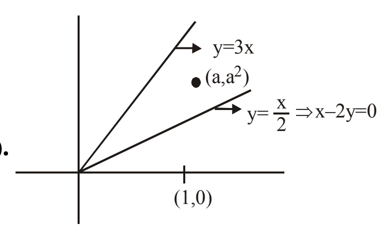
It meet x-axis at P (a, 0) and y-axis at Q (0, b)

Mid point of PQ is A (3, 4)

$\therefore \frac{a + 0}{2} = 3 \Rightarrow a = 6$ and $\frac{0 + b}{2} = 4 \Rightarrow b = 8$

Required equation of line is $\frac{x}{6} + \frac{y}{8} = 1 \Rightarrow 4x + 3y = 24$

(18) **(B).**



$\because (a, a^2)$ and (1, 0) lie on opposite side of line, $x - 2y = 0$

$\therefore$ at (1, 0), $x - 2y = 1$ (+ve)

$\Rightarrow (a, a^2)$ on satisfying gives $x - 2y < 0$

{$\because$ both points are on opposite side of line}

$\Rightarrow a - 2a^2 < 0$

$\Rightarrow 2a^2 - a > 0$

$\Rightarrow a(2a - 1) > 0$

$\Rightarrow a < 0$ or $a > 1/2 \qquad \text{......... (1)}$

Also, (a, a^2) and (1, 0) lie of same side of $3x - y = 0$

$\therefore$ at (1, 0), $3x - y$ gives 3 (+ve)

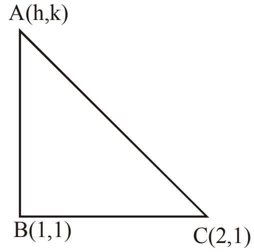
$\therefore (a, a^2)$ will also given on putting in $3x - y$

$\therefore 3a - a^2 > 0 \Rightarrow a^2 - 3a < 0$

$\Rightarrow a(a-3) < 0 \Rightarrow 0 < a < 3$ .... (2)

From (1) and (2) we get, $\frac{1}{2} < a < 3$

(19) **(C).** $\Delta = \frac{1}{2}\begin{vmatrix} h & k & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$



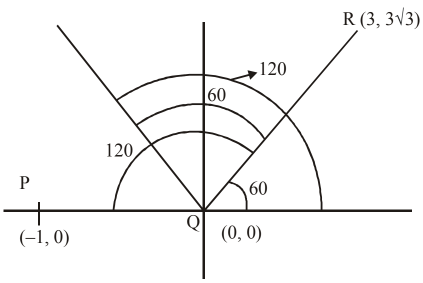
$= \frac{1}{2}[0 + k - 1] = \left|\frac{k-1}{2}\right|$

$\because \Delta = 1$ (given)

$\Rightarrow \left|\frac{k-1}{2}\right| = 1 \Rightarrow \frac{k-1}{2} = \pm 1 \Rightarrow k - 1 = \pm 2$

$\Rightarrow k = \pm 2 + 1 \Rightarrow k = 3, -1$

(20) **(A).** $P(-1, 0), Q(0, 0), R(3, 3\sqrt{3})$



$\because$ Slope of QR is $\frac{3\sqrt{3}}{3-0} = \sqrt{3}$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

$\therefore$ Angle between P, Q, R is 120° and it bisector makes angle 120° with positive direction of x-axis.

$\therefore$ its slope is $\tan 120° = -\sqrt{3}$

$\therefore$ bisector passes through (0, 0).

$\therefore$ it equation will be $y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

(21) **(C).** $xy = 0 \Rightarrow x = 0$ or $y = 0$

represents equation of co-ordinate axes.

$\therefore$ equation of angle bisector of co-ordinate axis is

$y = x$ or $y = -x \Rightarrow \frac{y}{x} = 1$ .... (1)

$\because$ One of the line $my^2 + (1 - m^2)xy - mx^2 = 0$ is bisector of the angle between the $xy = 0$

$\therefore$ $y = x$ will satisfy

$my^2 + (1 - m^2)xy - mx^2 = 0$

$\Rightarrow my^2 + (1 - m^2)yy - my^2 = 0 \Rightarrow m + 1 - m^2 - m = 0$

$\Rightarrow m^2 = 1 \Rightarrow m = \pm 1 \Rightarrow m = 1$

(22) **(C).** P (1, 4) and Q (k, 3) are given

Coordinate of mid point of PQ is

$\left(\frac{k+1}{2}, \frac{4+3}{2}\right) = \left(\frac{k+1}{2}, \frac{7}{2}\right)$

Now, slope of PQ is $\frac{3-4}{k-1} = \frac{-1}{k-1}$

$\therefore$ slope of line $\perp$ to PQ is $(k - 1)$

$\because$ $\perp$ bisector of PQ passes through $\left(\frac{k+1}{2}, \frac{7}{2}\right)$ and it slope is $k - 1$

$\therefore$ its equation is $\left(y - \frac{7}{2}\right) = (k-1)\left\{x - \frac{k+1}{2}\right\}$

$\Rightarrow (k-1)x - y = \frac{k^2-1}{2} - \frac{7}{2}$

$\Rightarrow \frac{(k-1)x}{\left[\frac{k^2-1}{2} - \frac{7}{2}\right]} + \frac{y}{-\left[\frac{k^2-1}{2} - \frac{7}{2}\right]} = 1$

Now y intercept of this line is $-\left[\frac{k^2-1}{2} - \frac{7}{2}\right]$

according to question

$-\left[\frac{k^2-1}{2} - \frac{7}{2}\right] = -4 \Rightarrow \frac{k^2-1}{2} = 4 + \frac{7}{2}$

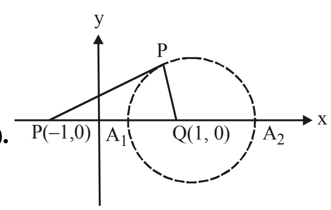
$\Rightarrow \frac{k^2-1}{2} = \frac{15}{2} \Rightarrow k^2 = 16 \Rightarrow k = \pm 4$

(23) **(C).** $p(p^2+1) = -\frac{(p^2+1)^2}{(p^2+1)}$

as the given lines will be parallel

$\Rightarrow p = -1$

(24) **(C).**



$A_1 \equiv \left(\frac{1}{2}, 0\right), A_2 \equiv (2, 0)$; Circumcentre $\equiv \left(\frac{5}{4}, 0\right)$

(25) **(C).** Slope of line L $= -\frac{b}{5}$; Slope of line K $= -\frac{3}{c}$

Line L is parallel to line k $\Rightarrow \frac{b}{5} = \frac{3}{c} \Rightarrow bc = 15$

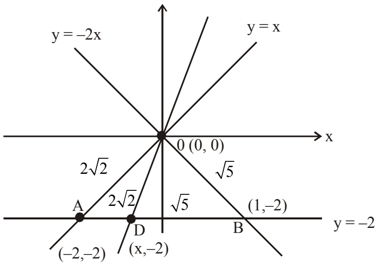
(13, 32) is a point on L.

$\Rightarrow \frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5} \Rightarrow b = -20 \Rightarrow c = -\frac{3}{4}$

Equation of K: $y - 4x = 3$

Distance between L and K = $\dfrac{|52-32+3|}{\sqrt{17}} = \dfrac{23}{\sqrt{17}}$

(26) **(C).**



$\therefore$ AD : DB = $2\sqrt{2} : \sqrt{5}$

$\because$ OD is angle bisector of angle AOB

$\therefore$ S : 1 true, S : 2 false (obvious)

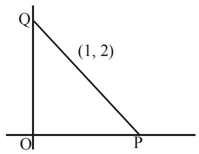
(27) **(C).**

$\therefore \; C\left(\dfrac{8}{5}, \dfrac{14}{5}\right)$

Line 2x + y = k passes $C\left(\dfrac{8}{5}, \dfrac{14}{5}\right)$

$\dfrac{2\times 18}{5} + \dfrac{14}{5} = k \;;\; k = 6$

(28) **(C).** $(y-2) = m(x-1)$



$OP = 1 - \dfrac{2}{m}$

$OQ = 2 - m$

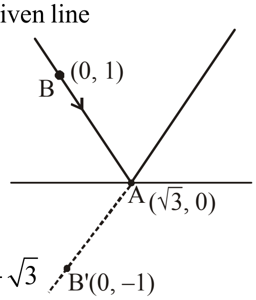
Area of Δ POQ = $\dfrac{1}{2}(OP)(OQ) = \dfrac{1}{2}\left(1-\dfrac{2}{m}\right)(2-m)$

$= \dfrac{1}{2}\left[2 - m - \dfrac{4}{m} + 2\right] = \dfrac{1}{2}\left[4 - \left(m + \dfrac{4}{m}\right)\right]$

$m = -2$

(29) **(B).** Take any point B (0, 1) on given line

Equation of AB'

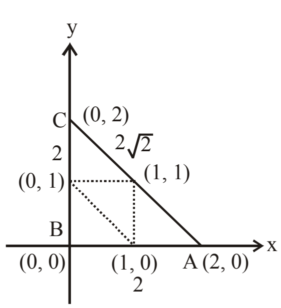


$y - 0 = \dfrac{-1-0}{0-\sqrt{3}}(x - \sqrt{3})$

$-\sqrt{3}y = -x + \sqrt{3}$

$x - \sqrt{3}y = \sqrt{3} \Rightarrow \sqrt{3}y = x - \sqrt{3}$

(30) **(B).**



x-coordinate of incentre

$= \dfrac{2\times 0 + 2\sqrt{2}.0 + 2.2}{2+2+2\sqrt{2}} = \dfrac{2}{2+\sqrt{2}} = 2 - \sqrt{2}$

(31) **(C).** Let point of intersection is (h, –h)

$\Rightarrow \begin{cases} 4ah - 2ah + c = 0 \\ 5b - 2bh + d = 0 \end{cases}$. So, $-\dfrac{c}{2a} = -\dfrac{d}{3b}$

$3bc - 2ad = 0$

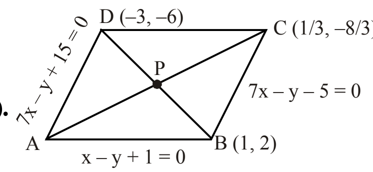
(32) **(B).** S (13/2, 1), P (2, 2) ; Slope = –2/9

Equation will be $\dfrac{y+1}{x-1} = -\dfrac{2}{9}$

$9y + 9 + 2x - 2 = 0$

$2x + 9y + 7 = 0$

(33) **(B).**



(34) **(B).** $\dfrac{1}{2}\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = 28$

$\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = 56$

$|k(k-2) + 3k(5+k) + 1(10+k^2)| = 56$

$|5k^2 + 3k + 10| = 56$

$5k^2 + 3k + 10 = \pm 56$

or $5k^2 + 3k + 66 = 0$ or $5k^2 + 3k - 46 = 0$

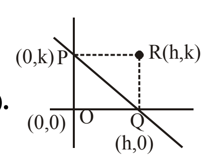
$D < 0$

Solving we get k = 2

Hence, the vertices are (2, –6) , (5, 2) , (–2, 2)

Solving the equation of two altitudes we get orthocente as (2, 1/2).

(35) **(A).**



Let, R ≡ (h, k) $\therefore$ P ≡ (0, k) Q ≡ (h, 0)

$\therefore$ Equation of line would be,

$\dfrac{x}{h} + \dfrac{y}{k} = 1 \Rightarrow \dfrac{2}{h} + \dfrac{3}{k} = 1 \Rightarrow 2k + 3h = hk$

Locus of (h, k) is 2y + 3x = xy

(36) **(A).** $\dfrac{2a-3}{3} = 3 \;;\; \dfrac{2b+5}{3} = 3$

$AC = \sqrt{(6+3)^2 + 3^2}$

Diameter = AC = $\sqrt{81+9} = \sqrt{90}$

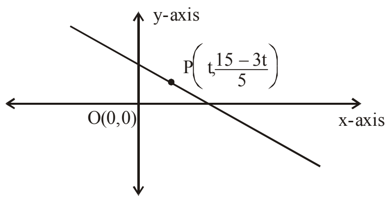
Radius $= \dfrac{3\sqrt{10}}{2} = \dfrac{3\times\sqrt{10}}{\sqrt{2}\times\sqrt{2}} = 3\sqrt{\dfrac{5}{2}}$

(37) **(D).** Given set of lines $px + qy + r = 0$
Given condition $3p + 2q + 4r = 0$

$\Rightarrow \dfrac{3}{4}p + \dfrac{1}{2}q + r = 0$

$\Rightarrow$ All lines pass through a fixed point (3/4, 1/2).

(38) **(A).**

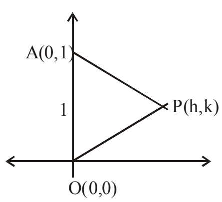


$\left|\dfrac{15-3t}{5}\right| = |t|$; $\dfrac{15-3t}{5} = t$ or $\dfrac{15-3t}{5} = -t$

$t = \dfrac{15}{8}$ or $t = \dfrac{-15}{2}$. So, $P\left(\dfrac{15}{8}, \dfrac{15}{8}\right) \in I^{st}$ quadrant

or $P\left(\dfrac{-15}{2}, \dfrac{15}{2}\right) \in II^{nd}$ quadrant

(39) **(B).**



$AP + OP + AO = 4$

$\sqrt{h^2 + (k-1)^2} + \sqrt{h^2 + k^2} + 1 = 4$

$\sqrt{h^2 + (k-1)^2} + \sqrt{h^2 + k^2} = 3$

$h^2 + (k-1)^2 = 9 + h^2 + k^2 - 6\sqrt{h^2 + k^2}$

$-2k - 8 = -6\sqrt{h^2 + k^2}$

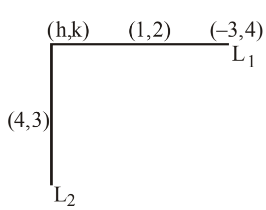
$k + 4 = 3\sqrt{h^2 + k^2}$; $k^2 + 16 + 8k = 9(h^2 + k^2)$

$9h^2 + 8k^2 - 8k - 16 = 0$

Locus of P is $9x^2 + 8y^2 - 8y - 16 = 0$

(40) **(C).** Equation of L_1 is

$y = -\dfrac{1}{2}x + \dfrac{5}{2}$ ...(1)



Equation of L_2 is

$y = 2x - 5$ ...(2)

By (1) and (2), $x = 3$

$y = 1 \Rightarrow h = 3, k = 1$

$\dfrac{k}{h} = \dfrac{1}{3}$

(41) **(C).** $x = 2 + r\cos\theta$

$y = 3 + r\sin\theta$

$\Rightarrow 2 + r\cos\theta + 3 + r\sin\theta = 7 \Rightarrow r(\cos\theta + \sin\theta) = 2$

$\Rightarrow \sin\theta + \cos\theta = \dfrac{2}{r} = \dfrac{2}{\pm 4} = \pm\dfrac{1}{2}$

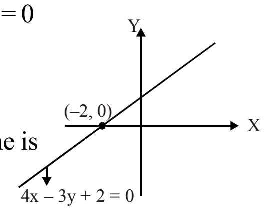
$\Rightarrow 1 + \sin 2\theta = \dfrac{1}{4} \Rightarrow \sin 2\theta = -\dfrac{3}{4}$

$\Rightarrow \dfrac{2m}{1+m^2} = -\dfrac{3}{4} \Rightarrow 3m^2 + 8m + 3 = 0$

$\Rightarrow m = \dfrac{-4 \pm \sqrt{7}}{1-7}$

$\dfrac{1-\sqrt{7}}{1+\sqrt{7}} = \dfrac{(1-\sqrt{7})^2}{1-7} = \dfrac{8-2\sqrt{7}}{-6} = \dfrac{-4+\sqrt{7}}{3}$

(42) **(A).** Required line is $4x - 3y + \lambda = 0$



$\left|\dfrac{\lambda}{5}\right| = \dfrac{3}{5} \Rightarrow \lambda = \pm 3$

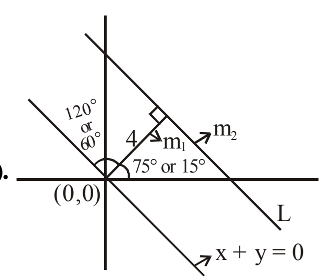
So, required equation of line is

$4x - 3y + 3 = 0$

and $4x - 3y - 3 = 0$

(1) $4\left(-\dfrac{1}{4}\right) - 3\left(\dfrac{2}{3}\right) + 3 = 0$

(43) **(A).**



$m_1 = \tan 75° = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ or $m = \tan 15° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$m_2 = \dfrac{-1}{m_1} = \dfrac{-(\sqrt{3}+1)}{\sqrt{3}-1}$ or $m_2 = \dfrac{-1}{m_1} = \dfrac{-(\sqrt{3}+1)}{\sqrt{3}-1}$

$y = m_2 x + C \Rightarrow y = \dfrac{-(\sqrt{3}-1)\,x}{\sqrt{3}+1} + C \Rightarrow L$

or $y = \dfrac{-(\sqrt{3}+1)\,x}{\sqrt{3}-1} + C \Rightarrow L$

Distance from origin $= 4$

$$\left|\frac{C}{\sqrt{1+\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)^2}}}\right| = 4 \text{ or } \left|\frac{C}{\sqrt{1+\frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)^2}}}\right| = 4$$

$$C = \frac{8\sqrt{2}}{(\sqrt{3}+1)} \text{ or } C = \frac{8\sqrt{2}}{(\sqrt{3}-1)}$$

$(\sqrt{3}-1)\,y + (\sqrt{3}-1)\,x = 8\sqrt{2}$

or $(\sqrt{3}-1)\,x + (\sqrt{3}+1)\,y = 8\sqrt{2}$

(44) **(B).** Let B (α, β) and C (γ, δ)

$$\frac{\alpha+1}{2} = -1 \Rightarrow \alpha = -3$$

$$\frac{\beta+2}{2} = 1 \Rightarrow \beta = 0 \Rightarrow B(-3, 0)$$

Now, $\frac{\gamma+1}{2} = 2 \Rightarrow \gamma = 3$

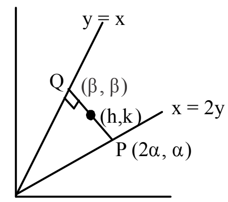
$$\frac{\delta+2}{2} = 3 \Rightarrow \delta = 4 \Rightarrow C(3, 4)$$

$\therefore$ Centroid of triangle is G (1/3, 2)

(45) **(D).** P will be centroid of Δ ABC

$$P\left(\frac{17}{6}, \frac{8}{3}\right) \Rightarrow PQ = \sqrt{\left(\frac{24}{6}\right)^2 + \left(\frac{9}{3}\right)} = 5$$

(46) **(B).** Slope of PQ $= \frac{k-\alpha}{h-2\alpha} = -1$



$\Rightarrow k - \alpha = -h + 2\alpha$

$\Rightarrow \alpha = \frac{h+k}{3}$ ......(1)

Also, $2h = 2\alpha + \beta$

$2k = \alpha + \beta$

$2h = \alpha + 2k$

$\alpha = 2h - 2k$ .....(2)

From (1) & (2), $\frac{h+k}{3} = 2(h-k)$

So locus is $6x - 6y = x + y \Rightarrow 5x = 7y$

(47) **(B).** Centroid of $\Delta = (2, 2)$

line passing through intersection of $x + 3y - 1 = 0$ and $3x - y + 1 = 0$, be given by

$(x + 3y - 1) + \lambda(3x - y + 1) = 0$

$\because$ It passes through (2, 2)

$\Rightarrow 7 + 5\lambda = 0 \Rightarrow \lambda = -7/5$

$\therefore$ Required line is $8x - 11y + 6 = 0$

$\because$ (–9, –6) satisfies this equation.