Contents

MATHEMATICS CURRICULUM FOR CLASS VII

TEACHING POINTS **KEY CONCEPTS**

1. NUMBERS SYSTEM

- Multiplication and division of integers
- Properties of operations on integers: Commutativity, associativity, existence of identity and inverse and distributivity
- Problem solving using operations on integers
- Solution of word problems involving integers (all operations)
- Introduction to rational numbers (with representation on number line)
- Word problems on rational numbers (all operations)
- Decimal representation of rational numbers
- Problem solving using operations on rational numbers and decimal fractions
- Fraction as an operator
- Reciprocal of a fraction
- Multiplication and division of decimal fractions
- Exponents only natural numbers.
- Laws of exponents (through observing patterns to arrive at generalisation.)
- Equal, equivalent, universal sets
- Cardinal property of sets

2. RATIO AND PROPORTION

- Ratio and proportion (revision)
- Unitary method continued, consolidation, general expression for unitary method
- Percentage — an introduction.
- Understanding percentage as a fraction with denominator 100
- Converting fractions and decimals into percentage and vice-versa.
- Application to profit and loss (single transaction only)
- Application to simple interest (time period in complete years).
- Speed, distance and time.

3. ALGEBRA

- Terms related to algebra like constants, variable, terms, coefficient of terms, like and unlike terms etc.
- Generate algebraic expressions
- Performs operations on algebraic expressions
- Simple linear equations in one variable (in contextual problems) with two operations.
- Inequalities and solution of simple inequalities in one variable.

4. GEOMETRY

Understanding shapes:

- Pairs of angles (linear, supplementary, complementary, adjacent, vertically opposite)
- Properties of parallel lines with transversal (alternate, corresponding, interior, exterior angles)

Properties of triangles:

- Angle sum property
- Exterior angle property
- Pythagoras Theorem (Verification only)

Symmetry:
- Recalling reflection symmetry
- Idea of rotational symmetry, observations of rotational symmetry of 2-D objects. (90°, 120°, 180°)

Representing 3-D in 2-D:

- Identification and counting of vertices, edges, faces, nets (for cubes, cuboids, and cylinders, cones)
- Mapping the space around approximately through visual estimation.

Congruence :

- Congruence through superimposition
- Extend congruence to simple geometrical shapes e.g. triangles, circles.
- Criteria of congruence

Construction:

- Construction of a line parallel to a given line from a point outside it
- Construction of simple triangles.

5. MENSURATION

Revision of perimeter and idea of Circumference of Circle

Area
- Concept of measurement using a basic unit area of a square, rectangle, triangle, parallelogram and circle

6. DATA HANDLING

- Collection and organisation of data – choosing the data to collect for a hypothesis testing
- Mean, median and mode of ungrouped data – understanding what they represent, constructing bar graphs.
- Feel of probability using data through experiments. Notion of chance in events like tossing coins, dice etc. Tabulating and counting occurrences of 1 through 6 in a number of throws. Comparing the observation with that for a coin. Observing strings of throws, notion of randomness.

Theme – 1
NUMBERS SYSTEM

INTEGERS 1

1.1 INTRODUCTION

Integers (I) = ..., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...
= ...., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...

Negatives of natural numbers ← | Zero | → Natural Numbers

Positive integers = I^+
= 1, 2, 3, 4, 5, ...

Negative integers = I^-
= ...., –8, –7, –6, –5, –4, –3, –2, –1
= Negatives of natural numbers

1. Positive integers go upto infinity on the right side of 0 (zero) *i.e.* on positive side.
2. Negative integers go upto infinity on the left side of 0 (zero) *i.e.* on negative side.
3. Zero (0) is simply an integer. It is neither negative nor positive.

1.2 MULTIPLICATION OF INTEGERS

1. *The product of two integers of the same sign is always positive.*
   i.e. (one positive integer) × (another positive integer) = a positive integer
   and (one negative integer) × (another negative integer) = a positive integer

(+ve) × (+ve) = + ve
and (–ve) × (–ve) = +ve.

For example :

(i) 5 × 4 = 20 and (–5) × (–4) = 20
(ii) 7 × 6 = 42 and –7 × – 6 = 42
(iii) 15 × 8 = 120 and (–15) × (–8) = 120 and so on.

2. *The product of one positive integer and one negative integer is always negative.*
   i.e. (one positive integer) × (one negative integer) = a negative integer
   and (one negative integer) × (one positive integer) = a negative integer

(+ve) × (–ve) = – ve
and (–ve) × (+ve) = – ve.

For example :

(i) 5 × (–4) = –20 and (–5) × 4 = –20
(ii) 7 × (–6) = –42 and (–7) × 6 = –42
(iii) 15 × (–8) = –120 and (–15) × 8 = –120 and so on.

1.3 PROPERTIES OF MULTIPLICATION OF INTEGERS

1. Closure Property :

The multiplication (product) *of two integers is always an integer.*

That is : If m and n are integers, then $m \times n$ *i.e.*, mn is also an integer.

For example :

(i) 5 is an integer and 6 is an integer, then their multiplication *i.e.*, $5 \times 6 = 30$ is also an integer.

(ii) 8 is an integer and –12 is an integer, then their multiplication $8 \times -12 = -96$ is also an integer.

(iii) –15 and –6 are integers, then $(-15) \times (-6) = 90$ is also an integer.

(iv) –13 and 8 are integers and so $(-13) \times 8 = -104$ is also an integer and so on.

2. Commutative Property *i.e.* Commutativity :

According to this property, if m and n are two integers, then $\boldsymbol{m \times n = n \times m}$

Consider the following table :

	m	n	$m \times n$	$n \times m$	Is $m \times n = n \times m$?
1.	6	7	$6 \times 7 = 42$	$7 \times 6 = 42$	Yes
2.	–10	8	$(-10) \times 8 = -80$	$8 \times (-10) = -80$	Yes
3.	–15	–7	$(-15) \times (-7) = 105$	$(-7) \times (-15) = 105$	Yes
4.	21	–6	$21 \times (-6) = -126$	$(-6) \times 21 = -126$	Yes

3. Associative Property *i.e.* Associativity :

According to this property, if l, m and n are three integers, then $\boldsymbol{l \times (m \times n) = (l \times m) \times n}$

Consider the following table :

	l	m	n	$l \times (m \times n)$	$(l \times m) \times n$	Is $l \times (m \times n) = (l \times m) \times n$?
1.	5	3	2	$5 \times (3 \times 2) = 5 \times 6 = 30$	$(5 \times 3) \times 2 = 15 \times 2 = 30$	Yes
2.	–5	3	2	$-5 \times (3 \times 2) = -5 \times 6 = -30$	$(-5 \times 3) \times 2 = -15 \times 2 = -30$	Yes
3.	8	–5	4	$8 \times (-5 \times 4) = 8 \times (-20) = -160$	$(8 \times -5) \times 4 = -40 \times 4 = -160$	Yes
4.	4	–3	–5	$4 \times (-3 \times -5) = 4 \times 15 = 60$	$(4 \times -3) \times -5 = -12 \times -5 = 60$	Yes

4. Distributive Property (Distributivity) :

According to this property, if l, m and n are three integers, then

$$\boldsymbol{l \times (m + n) = l \times m + l \times n}$$

i.e. multiplication is distributive over addition.

Consider the following table :

	l	m	n	
1.	5	7	2	$l \times (m + n) = 5 \times (7 + 2) = 5 \times 9 = \mathbf{45}$ and $l \times m + l \times n = 5 \times 7 + 5 \times 2 = 35 + 10 = \mathbf{45}$ $\therefore \boldsymbol{l \times (m + n) = l \times m + l \times n}$

2.	8	−9	5	$l \times (m + n) = 8 \times (-9 + 5) = 8 \times -4 = \mathbf{-32}$ and, $l \times m + l \times n = 8 \times -9 + 8 \times 5 = -72 + 40 = \mathbf{-32}$ $\therefore\ \boldsymbol{l \times (m + n) = l \times m + l \times n}$
3.	−6	−15	8	$l \times (m + n) = -6 \times (-15 + 8) = -6 \times -7 = \mathbf{42}$ $l \times m + l \times n = -6 \times -15 + (-6) \times 8 = 90 - 48 = \mathbf{42}$ $\therefore\ \boldsymbol{l \times (m + n) = l \times m + l \times n}$

Note 1 : Distributivity of multiplication over addition can also be stated as :
$(l + m) \times n = l \times n + m \times n.$

Note 2 : Distributivity of multiplication over subtraction is also true :
i.e. $l \times (m - n) = l \times m - l \times n$
and $(l - m) \times n = l \times n - m \times n.$

For example :

If $l = 8$, $m = 13$ and $n = 7$, then

$$l \times (m - n) = 8 \times (13 - 7) = 8 \times 6 = \mathbf{48}$$

and

$$l \times m - l \times n = 8 \times 13 - 8 \times 7 = 104 - 56 = \mathbf{48}$$

$$\therefore\ \boldsymbol{l \times (m - n) = l \times m - l \times n}$$

5. Existence of multiplicative identity :

For every integer ***a***, we have

$\boldsymbol{a} \times 1 = \boldsymbol{a}$ and $1 \times \boldsymbol{a} = \boldsymbol{a}$

i.e. $\boldsymbol{a} \times 1 = 1 \times \boldsymbol{a} = \boldsymbol{a}$

$\because$ Multiplication of every integer ***a*** with integer one (1) gives the integer ***a*** itself.

Integer 1(one) is called the multiplicative identity.

For example :

$5 \times 1 = 5$, $1 \times 5 = 5$, $-12 \times 1 = -12$, $1 \times (-12) = -12$ and so on.

6. Existence of multiplicative inverse :

For any integer ***a***, its multiplicative inverse will be $\frac{1}{a}$ so that $a \times \frac{1}{a} = 1$, the multiplicative identity.

Thus, the multiplicative inverse of an integer exists if :

the integer × its multiplicative inverse = 1, the multiplicative identity.

Out of all the integers :

(i) multiplicative inverse of 1 is 1 itself as $1 \times \frac{1}{1} = 1$

(ii) multiplicative inverse of −1 is −1 itself as $-1 \times \frac{1}{-1} = 1$

7. (i) $-2 \times -3 \times -4$
$= -24$ [Product of *three* negative integers]

(ii) $-2 \times -3 \times -4 \times -5 \times -6$
$= -720$ [Product of *five* negative integers]

(iii) $-2 \times -3 \times -4 \times -5 \times -6 \times -7 \times -8$
$= -40320$ [Product of *seven* negative integers]

In (i), given above, we have multiplied three negative integers and we found that the product is a negative integer.

In (ii), given above, multiplication of 5-negative integers gives a negative integer.
In (iii), given above, multiplication of 7-negative integers gives a negative integer.
Thus, we conclude that *the proudct of odd number of negative integers is always negative.*

8. (i) $-2 \times -3 = 6$
(ii) $-2 \times -3 \times -4 \times -5 = 120$
(iii) $-2 \times -3 \times -4 \times -5 \times -6 \times -7 = 5040$

In (i), given above, the product of two negative integers gives a positive integer.
In (ii), given above, the product of four negative integers gives a positive integer.
In (iii), given above, the product of *six* negative integers gives a positive integer.
Thus, we conclude that *the proudct of even number of negative integers is always positive.*

1. $(-a_1) \times (-a_2) \times (-a_3) \times (-a_4) \times (-a_5) \times$ .... upto an odd number of negative integers
$= -(a_1 \times a_2 \times a_3 \times a_4 \times a_5 \times ....)$
= a negative integer
2. $(-a_1) \times (-a_2) \times (-a_3) \times (-a_4) \times$ .... upto an even number of negative integers
$= a_1 \times a_2 \times a_3 \times a_4 \times ....$
= a positive integer.

Example 1 :

Evaluate :

(i) $23 \times -3 + (-23) \times 97$ (ii) $897 \times 99 + 897$ (iii) $1389 \times 450 - 389 \times 450$

Solution :

(i) **$23 \times -3 + (-23) \times 97$**
$= (-23) \times 3 + (-23) \times 97$
$= (-23) \times (3 + 97) = -23 \times 100 =$ **-2300** **(Ans.)**

(ii) **$897 \times 99 + 897$**
$= 897 \times 99 + 897 \times 1$
$= 897 \times (99 + 1) = 897 \times 100 =$ **89700** **(Ans.)**

(iii) **$1389 \times 450 - 389 \times 450$**
$= (1389 - 389) \times 450$
$= 1000 \times 450 =$ **450000** **(Ans.)**

Example 2 :

Evaluate :

(i) $3 \times 5 \times 6$ (ii) $3 \times (-5) \times 6$
(iii) $3 \times (-5) \times (-6)$ (iv) $(-3) \times (-5) \times (-6)$

Solution :

(i) **$3 \times 5 \times 6 = 90$** **(Ans.)**

(ii) $3 \times -5 \times 6$ has only one negative integer
$\therefore$ **$3 \times -5 \times 6 = -90$** **(Ans.)**

(iii) $3 \times (-5) \times (-6)$ has two negative integers and the product of two negative integers is always positive, therefore :
$\therefore$ **$3 \times (-5) \times (-6)$** $= 3 \times 30 =$ **90** **(Ans.)**

(iv) $(-3) \times (-5) \times (-6)$ has three negative integers and the product of three negative integers is always negative.
$\therefore$ **$(-3) \times (-5) \times (-6)$** $= -(3 \times 5 \times 6) =$ **-90** **(Ans.)**

Example 3 :

Evaluate :

(i) $(-2) \times (-4) \times (-6) \times (-5)$ (ii) $(-1) \times (-4) \times (-5) \times (-7) \times (-8)$

(iii) $(-1) \times (-1) \times (-1) \times$ ...... 20 times (iv) $(-1) \times (-1) \times (-1) \times$ ...... 25 times

Solution :

(i) Since, the number of negative integers in the product is even, therefore the result of this product will be positive.

Hence, $(-2) \times (-4) \times (-6) \times (-5) = 2 \times 4 \times 6 \times 5 = 240$ **(Ans.)**

(ii) Since, the number of negative integers in the product is odd, therefore the result of this product will be negative.

Hence, $(-1) \times (-4) \times (-5) \times (-7) \times (-8) = -(1 \times 4 \times 5 \times 7 \times 8) = -1120$ **(Ans.)**

(iii) Since, the number of negative integers in the product is even, therefore the result of this product will be positive.

Hence, $(-1) \times (-1) \times (-1) \times (-1) \times$ ...... 20 times

$= 1 \times 1 \times 1 \times 1 \times$ ...... 20 times $= 1$ **(Ans.)**

(iv) Since, the number of negative integers in the product is odd, therefore the result of this product will be negative.

Hence, $(-1) \times (-1) \times (-1) \times (-1) \times$ ...... 25 times

$= -(1 \times 1 \times 1 \times 1 \times$ ...... 25 times$) = -1$ **(Ans.)**

Example 4 :

Complete the adjoining multiplication table :

X	–3	–2	–1	0	1	2	3
–3							
–2							
–1							
0							
1							
2							
3							

Is the multiplication table symmetrical about the diagonal joining the upper left corner to the lower right corner ?

Solution :

The required multiplication table will be as given below :

A

X	–3	–2	–1	0	1	2	3
–3	9	6	3	0	–3	–6	–9
–2	6	4	2	0	–2	–4	–6
–1	3	2	1	0	–1	–2	–3
0	0	0	0	0	0	0	0
1	–3	–2	–1	0	1	2	3
2	–6	–4	–2	0	2	4	6
3	–9	–6	–3	0	3	6	9

B

Yes, the multipication table is symmetric about the diagonal joining the upper left corner to the lower right corner.

AB is the diagonal joining the upper left corner to the lower right corner.

EXERCISE 1(A)

1. Evaluate :

(i) $427 \times 8 + 2 \times 427$ (ii) $394 \times 12 + 394 \times (-2)$ (iii) $558 \times 27 + 3 \times 558$

2. Evaluate :

(i) $673 \times 9 + 673$ (ii) $1925 \times 101 - 1925$

3. Verify :

(i) $37 \times \{8 + (-3)\} = 37 \times 8 + 37 \times (-3)$

(ii) $(-82) \times \{(-4) + 19\} = (-82) \times (-4) + (-82) \times 19$

(iii) $\{7 - (-7)\} \times 7 = 7 \times 7 - (-7) \times 7$

(iv) $\{(-15) - 8\} \times -6 = (-15) \times (-6) - 8 \times (-6)$

4. Evaluate :

(i) 15×8 (ii) $15 \times (-8)$

(iii) $(-15) \times 8$ (iv) $(-15) \times -8$

5. Evaluate :

(i) $4 \times 6 \times 8$ (ii) $4 \times 6 \times (-8)$ (iii) $4 \times (-6) \times 8$

(iv) $(-4) \times 6 \times 8$ (v) $4 \times (-6) \times (-8)$ (vi) $(-4) \times (-6) \times 8$

(vii) $(-4) \times 6 \times (-8)$ (viii) $(-4) \times (-6) \times (-8)$

6. Evaluate :

(i) $2 \times 4 \times 6 \times 8$ (ii) $2 \times (-4) \times 6 \times 8$ (iii) $(-2) \times 4 \times (-6) \times 8$

(iv) $(-2) \times (-4) \times 6 \times (-8)$ (v) $(-2) \times (-4) \times (-6) \times (-8)$

7. Determine the integer whose product with '–1' is :

(i) –47 (ii) 63

(iii) –1 (iv) 0

8. Eighteen integers are multiplied together. What will be the sign of their product, if :

(i) 15 of them are negative and 3 are positive ?

(ii) 12 of them are negative and 6 are positive ?

(iii) 9 of them are positive and the remaining are negative ?

(iv) all are negative ?

9. Find which is greater ?

(i) $(8 + 10) \times 15$ or $8 + 10 \times 15$ (ii) $12 \times (6 - 8)$ or $12 \times 6 - 8$

(iii) $\{(-3) - 4\} \times (-5)$ or $(-3) - 4 \times (-5)$

10. State, **true** or **false** :

(i) product of two different integers can be zero.

(ii) product of 120 negative integers and 121 positive integers is negative.

(iii) $a \times (b + c) = a \times b + c$

(iv) $(b - c) \times a = b - c \times a$.

1.4 DIVISION OF INTEGERS :

Division is an inverse process of multiplication.

To divide 36 by 9 means to find an integer which on multiplying with 9 gives 36. Such an integer is 4. This fact is expressed as :

$36 \div 9 = 4 \Rightarrow 4 \times 9 = 36$

Similarly, $75 \div 15 = 5 \Rightarrow 5 \times 15 = 75$

Remember :

1. *Division of an integer by an integer of same sign is always positive.*

   i.e. $\frac{\text{one integer}}{\text{an integer with same sign}}$ = a positive number.

For example :

(i) $\frac{-24}{-6} = 4$ (ii) $\frac{24}{6} = 4$

(iii) $\frac{32}{20} = \frac{8 \times 4}{5 \times 4} = \frac{8}{5}$ (iv) $\frac{-38}{-57} = \frac{-(2 \times 19)}{-(3 \times 19)} = \frac{2}{3}$ and so on

$\frac{+\text{ve integer}}{+\text{ve integer}}$ = a positive number

$\frac{-\text{ve integer}}{-\text{ve integer}}$ = a positive number

2. *Division of an integer by an integer of opposite sign is always negative.*

   i.e. $\frac{\text{one integer}}{\text{an integer with opposite sign}}$ = a negative number.

For example :

(i) $\frac{-24}{6} = -4$ (ii) $\frac{24}{-6} = -4$

(iii) $\frac{32}{-20} = \frac{8 \times 4}{-5 \times 4} = -\frac{8}{5}$ (iv) $\frac{-38}{57} = \frac{-2 \times 19}{3 \times 19} = -\frac{2}{3}$

$\frac{+\text{ve integer}}{-\text{ve integer}}$ = a negative number

$\frac{-\text{ve integer}}{+\text{ve integer}}$ = a negative number

Note 1 : In a division,

- the number to be divided is called **dividend**, thus in $65 \div 13 = 5$; dividend = 65.
- the number which divides is called **divisor**; thus in $65 \div 13 = 5$, divisor = 13.
- the result of division is called **quotient**; thus in $65 \div 13 = 5$, quotient = 5.

Note 2 : Dividend ÷ divisor = quotent *i.e.* $\frac{\text{dividend}}{\text{divisor}}$ = quotient

- if dividend is positive and divisor is negative, the quotient is negative.
- if dividend is negative and divisor is positive, the quotient is negative.
- if dividend and divisor both are positive or both are negative, the quotient is positive.

The following table will make the above concept more clear :

Dividend	Divisor	Dividend ÷ Divisor	Quotient
40	8	$40 \div 8 = \frac{40}{8}$	5
–40	8	$(-40) \div 8 = \frac{-40}{8}$	–5
–40	–8	$(-40) \div (-8) = \frac{-40}{-8}$	5
40	–8	$40 \div -8 = \frac{40}{-8}$	–5
Remember : $\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$			

1.5 PROPERTIES OF DIVISION OF INTEGERS :

1. *If m and n are two integers, then m ÷ n is not necessarily an integer.*

e.g. $18 \div 5 = \frac{18}{5}$, *which is not an integer.*

$-14 \div 4 = \frac{-14}{4} = -\frac{7}{2}$, *which is not an integer.*

2. *If m is a non-zero integer then m ÷ m = 1.*

e.g. $12 \div 12 = \frac{12}{12} = 1$

$17 \div 17 = \frac{17}{17} = 1$, $(-25) \div (-25) = \frac{-25}{-25} = 1$.

3. *For every non-zero integer m, m ÷ 1 = m.*

e.g. $8 \div 1 = \frac{8}{1} = 8$, $(-16) \div 1 = \frac{-16}{1} = -16$

$1 \div 1 = \frac{1}{1} = 1$

4. *For every non-zero integer m, 0 ÷ m = 0.*

e.g. $0 \div 6 = \frac{0}{6} = 0$, $0 \div (-11) = \frac{0}{-11} = 0$

$0 \div 1 = \frac{0}{1} = 0$

5. *For every integer m, m ÷ 0 is not meaningful* (not defined).

e.g. $8 \div 0 = \frac{8}{0}$, *which is not meaningful*

$(-23) \div 0 = \frac{-23}{0}$, *which is not meaningful*

6. *If l, m and n are non-zero integers, then*
$(l \div m) \div n \neq l \div (m \div n)$ unless $n = 1$.

When $n = 1$, $(l \div m) \div n = (l \div m) \div 1 = l \div m$
and, $l \div (m \div n) = l \div (m \div 1) = l \div m$

7. If l, m and n are integers, then

(i) $l > m$ and n is positive $\Rightarrow (l \div n) > (m \div n)$

e.g. $15 > 6$ and 3 is positive $\Rightarrow (15 \div 3) > (6 \div 3)$ *i.e.* $5 > 2$

(ii) $l > m$ and n is negative $\Rightarrow (l \div n) < (m \div n)$

e.g. $18 > 9$ and -3 is negative $\Rightarrow \{18 \div (-3)\} < \{9 \div (-3)\}$

$$\Rightarrow \frac{18}{-3} < \frac{9}{-3} \text{ i.e. } -6 < -3$$

An integer is negative means it is less than 0.
$(-5) < 0, (-12) < 0, (-23) < 0$ and so on.
An integer is positive means it is greater than 0.
$5 > 0, 12 > 0, 23 > 0$ and so on.

Example 5 :

Divide :

(i) 132 by 11
(ii) (–132) by 11
(iii) 132 by (–11)
(iv) (–132) by (–11)

Solution :

(i) $132 \div 11 = \frac{132}{11} = \frac{12 \times 11}{11} = 12$ **(Ans.)**

(ii) $(-132) \div 11 = \frac{-132}{11} = \frac{-12 \times 11}{11} = -12$ **(Ans.)**

(iii) $132 \div (-11) = \frac{132}{-11}$

$= -\frac{132}{11}$ $\left[\because \frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}\right]$

$= -\frac{12 \times 11}{11} = -12$ **(Ans.)**

(iv) $(-132) \div (-11) = \frac{-132}{-11} = \frac{132}{11} = \frac{12 \times 11}{11} = 12$ **(Ans.)**

Example 6 :

Evaluate :

(i) $(-91) \div 13$ (ii) $-98 \div 7$ (iii) $162 \div (-27)$

Solution :

(i) $(-91) \div 13 = \frac{-91}{13} = \frac{-7 \times 13}{13} = -7$ **(Ans.)**

(ii) $-98 \div 7 = \frac{-98}{7} = \frac{-14 \times 7}{7} = -14$ **(Ans.)**

(iii) $162 \div (-27) = \frac{162}{-27} = \frac{-162}{27} = \frac{-6 \times 27}{27} = -6$ **(Ans.)**

Example 7 :

Find the quotient in each of the following :

(i) $1872 \div 12$ (ii) $3159 \div (-13)$ (iii) $-17000 \div 125$

(iv) $-1352 \div (-26)$

Solution :

(i) **Required quotient** $= \frac{1872}{12} = \frac{156 \times 12}{12} = 156$ **(Ans.)**

(ii) **Required quotient** $= \frac{3159}{-13} = -\frac{3159}{13} = -\frac{243 \times 13}{13} = -243$ **(Ans.)**

(iii) **Required quotient** $= \frac{-17000}{125} = \frac{-136 \times 125}{125} = -136$ **(Ans.)**

(iv) **Required quotient** $= \frac{-1352}{-26} = \frac{1352}{26} = 52$ **(Ans.)**

1.6 USING DMAS

In DMAS,

D stands for **division**

M stands for **multiplication**

A stands for **addition**

and **S** stands for **subtraction**.

1. **DAMS** represents four fundamental operations **D** (division), **M** (multiplication), **A** (addition) and **S** (subtraction).
2. If an expression has more than one fundamental operations, we perfrom operations using the rule of DMAS *i.e.* **first of all** we perform **D** (division), **then M** (multiplication), **then A** (addition) and **in the last S** (subtraction).

Example 8 :

Evaluate : $18 - 6 \div 3 \times 4$.

Solution :

$18 - 6 \div 3 \times 4$

$= 18 - 2 \times 4$

$= 18 - 8$

$=$ **10** **(Ans.)**

Using DMAS

Division (D) : $6 \div 3 = 2$

Multiplication (M) : $2 \times 4 = 8$

Subtraction (S) : $18 - 8 = 10$

Example 9 :

Evaluate : $(-10) + (-4) \div (-2) \times 3$

Solution :

$(-10) + (-4) \div (-2) \times 3$

$= -10 + 2 \times 3$

$= -10 + 6$

$=$ **-4** **(Ans.)**

Using DMAS

Division (D) : $(-4) \div (-2) = 2$

Multiplication (M) : $2 \times 3 = 6$

Addition (A) : $-10 + 6 = -4$

Example 10 :

Evaluate : $5 + (-48) \div (-16) + (-2) \times 6$

Solution :

$5 + (-48) \div (-16) + (-2) \times 6$

$= 5 + 3 + (-2) \times 6$

$= 5 + 3 + (-12)$

$= 8 - 12$

$=$ **-4** **(Ans.)**

Using DMAS

Division (D) : $(-48) \div (-16) = 3$

Multiplication (M) : $(-2) \times 6 = -12$

Addition (A) : $5 + 3 = 8$

Subtraction (S) : $8 - 12 = -4$

EXERCISE 1(B)

1. Divide :

   (i) 117 by 9 (ii) (–117) by 9 (iii) 117 by (–9)

   (iv) (–117) by (–9) (v) 225 by (–15) (vi) $(-552) \div 24$

   (vii) (–798) by (–21) (viii) $(-910) \div 26$

2. Evaluate :

   (i) $(-234) \div 13$ (ii) $234 \div (-13)$ (iii) $(-234) \div (-13)$

   (iv) $374 \div (-17)$ (v) $(-374) \div 17$ (vi) $(-374) \div (-17)$

   (vii) $(-728) \div 14$ (viii) $272 \div (-17)$

3. Find the quotient in each of the following divisions :

   (i) $299 \div 23$ (ii) $299 \div (-23)$ (iii) $(-384) \div 16$

   (iv) $(-572) \div (-22)$ (v) $408 \div (-17)$

4. Divide :

   (i) 204 by 17 (ii) 152 by – 19 (iii) 0 by 35

   (iv) 0 by (–82) (v) 5490 by 10 (vi) 762800 by 100

5. State, true or false :

(i) $0 \div 32 = 0$ (ii) $0 \div (-9) = 0$

(iii) $(-37) \div 0 = 0$ (iv) $0 \div 0 = 0$

6. Evaluate :

(i) $42 \div 7 + 4$ (ii) $12 + 18 \div 3$ (iii) $19 - 20 \div 4$

(iv) $16 - 5 \times 3 + 4$ (v) $6 - 8 - (-6) \div 2$ (vi) $13 - 12 \div 4 \times 2$

(vii) $16 + 8 \div 4 - 2 \times 3$ (viii) $16 \div 8 + 4 - 2 \times 3$ (ix) $16 - 8 + 4 \div 2 \times 3$

(x) $(-4) + (-12) \div (-6)$ (xi) $(-18) + 6 \div 3 + 5$ (xii) $(-20) \times (-1) + 14 \div 7$

1.7 REMOVAL OF BRACKETS

In removal of brackets, operations within the bracket are performed before the operations outside it.

For example :

(i) $(24 \div 3) \times 2 = \frac{24}{3} \times 2$

$= 8 \times 2 = 16$ [$\because$ $(24 \div 3) = 8$]

(ii) $24 \div (3 \times 2) = 24 \div 6$

$= \frac{24}{6} = 4$ [$\because$ $24 \div 6 = 4$]

(iii) $18 \div (2 \times 18 \div 6 - 5) = 18 \div (2 \times 3 - 5)$ [$18 \div 6 = 3$]

$= 18 \div (6 - 5)$ [$2 \times 3 = 6$]

$= 18 \div 1$ [$6 - 5 = 1$]

$= \frac{18}{1} = 18$

Important : Whenever a number is written just before a bracket, with numbers inside the bracket and separated by **plus** or **minus** signs; the bracket is removed and at the same time each number inside the bracket is multiplied by the number written outside it.

For example :

(i) $3(a - b) = 3a - 3b$

(ii) $5(3a + 2b) = 15a + 10b$

(iii) $2(5 + 4 - 3) = 10 + 8 - 6 = 18 - 6 = 12$

(iv) $5(3 - 8 + 10) = 15 - 40 + 50 = 65 - 40 = 25$

(v) $-4(a + b - 3) = -4a - 4b + 12$

(vi) $-3(15 - 18 + 2) = -3(17 - 18) = -3 \times -1 = 3$

In a complex expression, many types of brackets are used. The most commonly used brackets are :

	Bracket's symbol	Bracket's name
1.	( )	Small bracket or Parentheses
2.	{ }	Curly (middle) bracket
3.	[ ]	Square bracket
4.	—	Vinculum.

Note : If in an expression, all the four types of brackets, as discussed above, are used, first of all we simplify expression below vinculum, then inside small bracket, then inside curly bracket and lastly inside square bracket.

Example 11 :

Evaluate : $30 - [26 - \{15 + (8 - \overline{6 - 3})\}]$.

Solution :

$30 - [26 - \{15 + (8 - \overline{6 - 3})\}]$

$= 30 - [26 - \{15 + (8 - 3)\}]$ [Removing vinculum]

$= 30 - [26 - \{15 + 5\}]$ [Removing small bracket]

$= 30 - [26 - 20]$ [Removing curly bracket]

$= 30 - 6$ [Removing square bracket]

$= 24$ **(Ans.)**

Order in which the brackets must be removed :

1. **When brackets used are :** $[\{(\overline{\quad})\}]$

   order of removing the brackets is : firstly '—', then ( ), then { } and finally [ ].

2. **When brackets used are :** [{( )}]

   order of removing the brackets is : firstly ( ), then { } and finally [ ].

3. **When brackets used are :** {( )}

   order of removing the brackets is : firstly ( ) and finally { }.

1. If there is a **plus sign** before a bracket, the bracket is removed without changing the sign of any term inside the bracket.

   i.e. (i) $+(a - b + c) = a - b + c$

   (ii) $+5(2a + b - 3c) = 10a + 5b - 15c$

2. If there is a **minus sign** before a bracket, the bracket is removed by changing the sign of each term inside the bracket.

   i.e. (i) $-(a - b + c) = -a + b - c$

   (ii) $-5(2a + b - 3c) = -10a - 5b + 15c$

Example 12 :

Evaluate : 30 – [8 + {31 – (32 – 10)}].

Solution :

30 – [8 + {31 – (32 – 10)}]

= 30 – [8 + {31 – 22}]

= 30 – [8 + 9] = 30 – 17 = **13** **(Ans.)**

Example 13 :

Evaluate : $55 - \left[25 - \{23 - (12 - \overline{11 - 8})\}\right]$

Solution :

$55 - \left[25 - \{23 - (12 - \overline{11 - 8})\}\right]$

= 55 – [25 – {23 – (12 – 3)}]

= 55 – [25 – {23 – 9}]

= 55 – [25 – 14] = 55 – 11 = **44** **(Ans.)**

Example 14 :

Evaluate : $45 - \left[28 - \{34 - (24 - \overline{14 - 8})\}\right]$

Solution :

$45 - \left[28 - \{34 - (24 - \overline{14 - 8})\}\right]$

= 45 – [28 – {34 – (24 – 6)}]

= 45 – [28 – {34 – 18}]

= 45 – [28 – 16] = 45 – 12 = **33** **(Ans.)**

Example 15 :

Evaluate : – {4 – (5 – 2)}.

Solution :

– {4 – (5 – 2)}

= – {4 – 5 + 2} [– (5 – 2) = – 5 + 2]

= – 4 + 5 – 2 = – 6 + 5 = **–1** **(Ans.)**

Alternative method :

= – {4 – (5 – 2)} = – {4 – 3} = **–1** **(Ans.)**

Example 16 :

Evaluate : 3{17 – 4(8 – 6)}.

Solution :

3{17 – 4(8 – 6)}

= 3{17 – 32 + 24} [–4(8 – 6) = –4 × 8 + 4 × 6 = –32 + 24]

= 51 – 96 + 72 = 123 – 96 = **27** **(Ans.)**

Alternative method :

$$3\{17 - 4(8 - 6)\} = 3\{17 - 4 \times 2\}$$
$$= 3\{17 - 8\} = 3 \times 9 = 27 \quad \textbf{(Ans.)}$$

EXERCISE 1(C)

Evaluate :

1. $18 - (20 - 15 \div 3)$.

2. $-15 + 24 \div (15 - 13)$.

3. $35 - \{15 + 14 - (13 + \overline{2 - 1 + 3})\}$

4. $27 - \{13 + 4 - (8 + 4 - \overline{1 + 3})\}$

5. $32 - [43 - \{51 - (20 - \overline{18 - 7})\}]$

6. $46 - [26 - \{14 - (15 - 4 \div 2 \times 2)\}]$

7. $45 - [38 - \{60 \div 3 - (6 - 9 \div 3) \div 3\}]$

8. $17 - [17 - \{17 - (17 - \overline{17 - 17})\}]$

9. $2550 - [510 - \{270 - (90 - \overline{80 + 7})\}]$

10. $30 + [\{-2 \times (25 - \overline{13 - 3})\}]$

11. $88 - \{5 - (-48) \div (-16)\}$

12. $9 \times (8 - \overline{3 + 2}) - 2(2 + \overline{3 + 3})$

13. $2 - [3 - \{6 - (5 - \overline{4 - 3})\}]$

EXERCISE 1(D)

1. The sum of two integers is –15. If one of them is 9, find the other.

2. The difference between integers x and –6 is –5. Find the values of x.

$x - (-6) = -5$ or $-6 - x = -5$

3. The sum of two integers is 28. If one intger is –45, find the other.

4. The sum of two integers is –56. If one integer is –42, find the other.

5. The difference between an integer x and (–9) is 6. Find all possible values of x.

6. Evaluate :

(i) $(-1) \times (-1) \times (-1) \times$ .... 60 times.

(ii) $(-1) \times (-1) \times (-1) \times (-1) \times$ .... 75 times.

7. Evaluate :

(i) $(-2) \times (-3) \times (-4) \times (-5) \times (-6)$

(ii) $(-3) \times (-6) \times (-9) \times (-12)$

(iii) $(-11) \times (-15) + (-11) \times (-25)$

(iv) $10 \times (-12) + 5 \times (-12)$

8. (i) If $x \times (-1) = -36$, is x positive or negative ?

(ii) If $x \times (-1) = 36$, is x positive or negative ?

9. Write all the integers between –15 and 15, which are divisible by 2 and 3.

10. Write all the integers between –5 and 5, which are divisible by 2 or 3.

11. Evaluate :

(i) $(-20) + (-8) \div (-2) \times 3$

(ii) $(-5) - (-48) \div (-16) + (-2) \times 6$

(iii) $16 + 8 \div 4 - 2 \times 3$

(iv) $16 \div 8 \times 4 - 2 \times 3$

(v) $27 - [5 + \{28 - (29 - 7)\}]$

(vi) $48 - [18 - \{16 - (5 - \overline{4 + 1})\}]$

(vii) $-8 - \{-6\,(9 - 11) + 18 \div -3\}$

(viii) $(24 \div \overline{12 - 9} - 12) - (3 \times 8 \div 4 + 1)$

12. Find the result of subtracting the sum of all integers between 20 and 30 from the sum of all integers from 20 to 30.

13. Add the product of (–13) and (–17) to the quotient of (–187) and 11.

14. The product of two integers is –180. If one of them is 12, find the other.

15. (i) A number changes from –20 to 30. What is the increase or decrease in the number ?

(ii) A number changes from 40 to –30. What is the increase or decrease in the number ?

RATIONAL NUMBERS 2

2.1 INTRODUCTION

If a and b are two integers, then each of $a + b$, $a - b$ and $a \times b$ is also an integer. However, it is not necessary that $a \div b$ or $b \div a$ is also an integer.

For example :

Consider two integers 8 and 5. Clearly :

(i) $8 + 5 = 13$ is an integer.

(ii) $8 - 5 = 3$ is an integer

$5 - 8 = -3$ is also an integer.

(iii) $8 \times 5 = 40$ is an integer

But $8 \div 5 = \frac{8}{5}$ is not an integer and

$5 \div 8 = \frac{5}{8}$ is also not an integer.

Here, $\frac{5}{8}$ and $\frac{8}{5}$ are fractions and are said to form the system of rational numbers.

2.2 RATIONAL NUMBERS

If p is an integer, q is an integer and $q \neq 0$, then the number of the form $\frac{p}{q}$ is called a rational number.

Infact, the number having a value that can be expressed as the result of dividing an integer by a non-zero integer, is defined as rational number.

For example :

(i) $\frac{3}{7}$ is a rational number as 3 and 7 both are integers and $7 \neq 0$.

(ii) $-\frac{15}{19}$ is a rational number as -15 is an integer, 19 is an integer and $19 \neq 0$.

In the same way, each of the following numbers is a rational number.

$\frac{3}{4}, \frac{-4}{7}, \frac{-12}{-5}, \frac{8}{-9}$, ...

Remember :

1. *Every integer is a rational number but the converse is not ture.*

   (i) $5 = \frac{5}{1}$ is a rational number as 5 and 1 are integers and $1 \neq 0$.

   (ii) $-8 = \frac{-8}{1}$ is a rational number as -8 and 1 are integers and $1 \neq 0$.

   (iii) $0 = \frac{0}{1}$ is a rational number as 0 and 1 are integers and $1 \neq 0$.

1. $\frac{3}{5}$ is a rational number as 3 and 5 are integers and $5 \neq 0$, but $\frac{3}{5}$ is not an integer.

Every natural number is an integer and so a rational number.

Every whole number is also an integer and so a rational number.

2. $0 = \frac{0}{1}$, $0 = \frac{0}{5}$, $0 = \frac{0}{-8}$, etc.

Zero is a rational number.

2. *Every fraction is a rational number but the converse is not rue.*

3. In the rational number in the form of a fraction $\frac{p}{q}$, p is called the numerator and q is called the denominator.

   (i) In $\frac{7}{11}$, numerator = 7 and denominator = 11.

   (ii) $\because 13 = \frac{13}{1}$, numerator = 13 and denominator = 1.

4. *Positive rational number :* A rational number is said to be positive, if its numerator and denominator both are either positive or negative.

5. *Negative rational number :* A rational number is said to be negative, if its numerator and denominator are with opposite signs *i.e.* if numerator is positive, denominator is negative and if numerator is negative, denominator is positive.

   Thus each of $\frac{-5}{7}$, $\frac{8}{-11}$, $\frac{15}{-13}$, $\frac{-23}{25}$, etc. is a negative rational number.

Remember : (i) $\frac{-5}{7} = \frac{5}{-7} = -\frac{5}{7}$

(ii) $\frac{8}{-11} = \frac{-8}{11} = -\frac{8}{11}$

6. (i) *Every positive integer (i.e. every natural number) is a positive rational number.*

   e.g. For natural number $5 = \frac{5}{1}$ is a positive rational number.

   Similarly, $10 = \frac{10}{1}$ is a positive rational number.

   (ii) *Every negative integer is a negative rational number.*

   e.g. $-5 = \frac{-5}{1}$ which is a negative rational number.

   In the same way, each of $-12 = \frac{-12}{1}$, $-28 = -\frac{28}{1}$, ... is a negative rational number.

Zero (0) is a rational number but it is neither positive nor negative

EXERCISE 2(A)

1. Write down a rational number whose numerator is the largest number of two digits and denominator is the smallest number of four digits.

2. Write the numerator of each of the following rational numbers :

(i) $\frac{-125}{127}$ (ii) $\frac{37}{-137}$ (iii) $\frac{-85}{93}$

(iv) 2 (v) 0

3. Write the denominator of each of the following rational numbers :

(i) $\frac{7}{-15}$ (ii) $\frac{-18}{29}$ (iii) $\frac{-3}{4}$

(iv) −7 (v) 0

4. Write down a rational number with numerator (−5) × (−4) and with denominator (28 − 27) × (8 − 5).

5. (i) $\frac{-15}{1}$ in integer form is .... (ii) $\frac{23}{-1}$ in integer form is ....

(iii) If $18 = \frac{18}{a}$ then a = ... (iv) If $-57 = \frac{57}{a}$ then a = ...

6. Separate positive and negative rational numbers from the following :

$\frac{-3}{5}, \frac{3}{-5}, \frac{-3}{-5}, \frac{3}{5}, 0, \frac{-13}{-3}, \frac{15}{-8}, \frac{-15}{8}$

7. Find three rational numbers equivalent to

(i) $\frac{3}{5}$ (ii) $\frac{4}{-7}$ (iii) $\frac{-5}{9}$ (iv) $\frac{8}{-15}$

(i) $\frac{4}{-7} = \frac{4\times2}{-7\times2} = \mathbf{\frac{8}{-14}}, \frac{4\times3}{-7\times3} = \mathbf{\frac{12}{-21}}$ and $\frac{4\times4}{-7\times4} = \mathbf{\frac{16}{-28}}$

⇒ Rational numbers $\frac{8}{-14}, \frac{12}{-21}$ and $\frac{16}{-28}$ are equivalent to the given rational number $\frac{4}{-7}$.

8. Which of the following are not rational numbers :

(i) −3 (ii) 0 (iii) $\frac{0}{4}$

(iv) $\frac{8}{0}$ (v) $\frac{0}{0}$

9. Express each of the following integers as a rational number with denominator 7 :

(i) 5 (ii) −8 (iii) 0

(iv) −16 (v) 7

(ii) $-8 = \frac{-8\times7}{7} = -\frac{56}{7}$ and (iii) $0 = \frac{0\times7}{7} = \frac{0}{7}$

10. Express $\frac{3}{5}$ as a rational number with denominator :

(i) 20 (ii) –20 (iii) 45 (iv) 25 (v) –35

(i) $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$ (v) $\frac{3}{5} \times \frac{-7}{-7} = \frac{-21}{-35}$

11. Express $\frac{4}{7}$ as a rational number with numerator :

(i) 12 (ii) –12 (iii) –16 (iv) –20 (v) 20

(i) $\frac{4}{7} = \frac{4}{7} \times \frac{3}{3} = \mathbf{\frac{12}{21}}$ (iii) $\frac{4}{7} = \frac{4 \times -4}{7 \times (-4)} = \mathbf{\frac{-16}{-28}}$

12. Find *x*, such that :

(i) $-\frac{2}{3} = \frac{6}{x}$ (ii) $\frac{7}{-4} = \frac{x}{8}$ (iii) $\frac{3}{7} = \frac{x}{-35}$

(iv) $\frac{-48}{x} = 6$ (v) $\frac{36}{x} = 3$ (vi) $\frac{-27}{x} = 9$

$\frac{3}{2} = \frac{x}{14} \Rightarrow \frac{3 \times 7}{2 \times 7} = \frac{x}{14} \Rightarrow \frac{21}{14} = \frac{x}{14} \Rightarrow x = 21$

$\frac{-5}{3} = \frac{-15}{x} \Rightarrow \frac{-5}{3} \times \frac{3}{3} = \frac{-15}{x} \Rightarrow \frac{-15}{9} = \frac{-15}{x} \Rightarrow x = 9$

13. Express each of the following rational numbers to the lowest terms :

(i) $\frac{12}{15}$ (ii) $\frac{-120}{144}$ (iii) $\frac{-48}{-72}$ (iv) $\frac{14}{-56}$

(iv) $\mathbf{\frac{14}{-56}} = \frac{2 \times 7}{-2 \times 2 \times 2 \times 7}$

$= \frac{1}{-4} = -\mathbf{\frac{1}{4}}$

2	14
	7

2	56
2	28
2	14
	7

Alternative method :

Find H.C.F. of numerator 14 and denominator 56 which is 14.

Now divide each term of the given rational number by their H.C.F. 14 (obtained above) to get the required answer.

Clearly, $\mathbf{\frac{14}{-56}} = \frac{14 \div 14}{-56 \div 14} = \frac{1}{-4} = -\mathbf{\frac{1}{4}}$

14. Express each of the following rational numbers in the standard form.

(i) $\frac{-7}{-8}$ (ii) $\frac{5}{-12}$ (iii) $\frac{-7}{-20}$ (iv) $\frac{4}{-9}$

A rational number is said to be in standard form, if its denominator is positive in lowest term.

2.3 REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE

1. Draw a straight line. Mark a point O on this line and name it 0 (zero).
2. On the right side of O, mark points A, B, C, D, E, ... on the line drawn such that the distance between consecutive points is same

   i.e. OA = AB = BC = CD = ...
3. In the same way, on the same line mark points A′, B′, C′, D′, etc.

   on the left side of O such that :

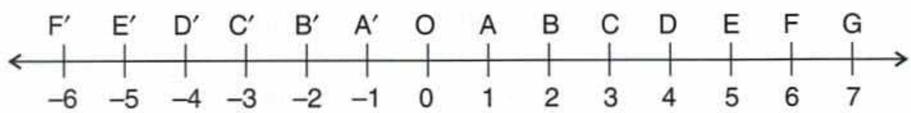
   OA′ = A′B′ = B′C′ = ...

   On the whole, we must have ;

   .... B′C′ = A′B′ = OA′ = OA = AB = BC = CD = ...

   Clearly, if the points A, B, C, D, E, ..., which are on the right side of zero (0); represent positive integers 1, 2, 3, 4, 5, ..., the points A′, B′, C′, D′, E′, ... will represent negative integers –1, –2, –3, –4, –5, ...

   See the number line, given below :

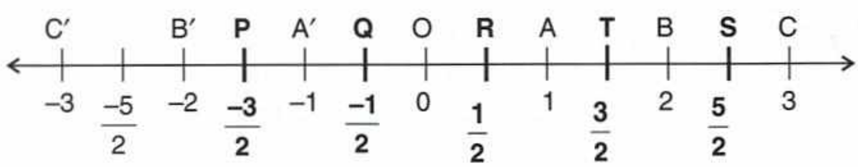


Example 1 :

Represent rational numbers $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ and $\frac{5}{2}$ on the same number line :

Solution :

Draw a number line as shown below :



Consider C′B′ = B′A′ = A′O = OA = AB = BC = one unit length

Clearly, A represents 1, B represents 2 and C represents 3.

In the same way, A′ represents –1, B′ represents = –2 and C′ represents –3.

Since, denominator of each given rational number is 2, mark middle points of each of OA, AB, BC and OA′, A′B′, B′C′ to divide each of OA, AB, BC and OA′, A′B′, B′C′ into two equal parts. Now the given points are marked on this number line.

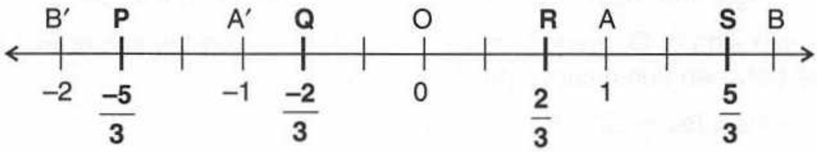
In the number line now obtained, P represents $-\frac{3}{2}$, Q represents $-\frac{1}{2}$, R represents $\frac{1}{2}$, T represents $\frac{3}{2}$ and S represents $\frac{5}{2}$.

Example 2 :

Represent rational numbers $-\frac{5}{3}, -\frac{2}{3}, \frac{2}{3}$ and $\frac{5}{3}$ on the same number line :

Solution :

Draw a number line of suitable length.



On this line, first of all mark the point O (representing zero) and then the points A, B, A′, B′, etc., such that OA = AB = OA′ = A′B′ = ..., = one unit.

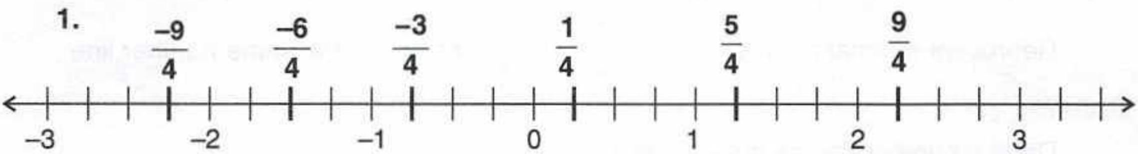
Since, the denominator of each given rational number is 3, divide each of OA, AB, OA′, A′B′, etc. into three equal parts. Now, each smaller part so obtained is equal to $\frac{1}{3}$.

For $-\frac{5}{3}$, mark the point P at the 5th part on the left side of 0, for $-\frac{2}{3}$, mark the point Q at the 2nd part on the left side of 0, for $\frac{2}{3}$ mark R at the 2nd part on the right side of 0 and for $\frac{5}{3}$, mark the 5th part on the right side of zero (0)..

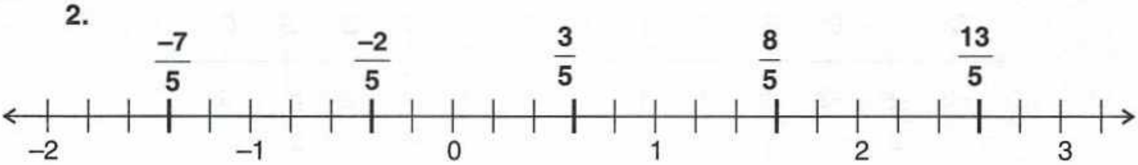
Clearly, P represents $-\frac{5}{3}$, Q represents $-\frac{2}{3}$, R represents $\frac{2}{3}$ and S represents $\frac{5}{3}$.

The following number lines will make the concept more clear :

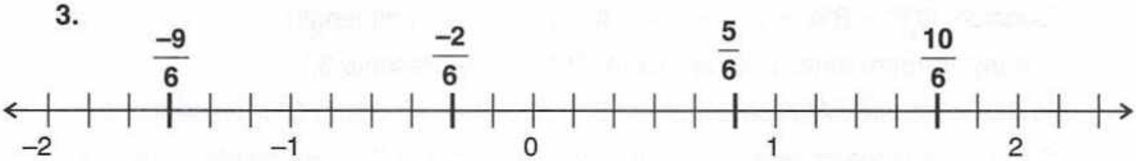
1.



2.



3.



2.4 COMPARING RATIONAL NUMBERS

1. Every positive rational number is greater than 0 (zero) and is greater than every negative number :
   e.g. 8 > 0, 8 > –5, 8 > –93, 8 > –1235, etc.
2. Every negative rational number is less than 0 (zero) and is less than every positive number :
   e.g. –8 < 0, –8 < 5, –8 < 93, –8 < 123, etc.

3. Zero (0) is greater than every negative number and is smaller than every positive number :

e.g. $0 > -5$, $0 > -\frac{3}{20}$, $0 < 5$, $0 < \frac{3}{20}$, etc.

Example 3 :

Compare $\frac{5}{7}$ and $-\frac{3}{5}$.

Solution :

$\frac{5}{7}$ is a positive rational number and $-\frac{3}{5}$ is a negative rational number.

Since, a positive rational number is always greater than 0 and greater than every negative number.

$\therefore$ **$\frac{5}{7}$ is greater than $-\frac{3}{5}$** **(Ans.)**

Example 4 :

Compare $\frac{3}{5}$ and $\frac{5}{7}$.

Solution :

First method :

1. Find the L.C.M. of denominators 5 and 7

   L.C.M. of 5 and 7 = 35

2. Make denominator of each rational number equal to L.C.M. obtained above *i.e.* equal to 35

   $$\frac{3}{5} = \frac{3\times7}{5\times7} = \frac{21}{35}$$

   and $$\frac{5}{7} = \frac{5\times5}{7\times5} = \frac{25}{35}$$

3. For the same denominator, the rational number with greater numerator is greater

   $\Rightarrow$ $\frac{25}{35}$ is greater than $\frac{21}{35}$

   $\Rightarrow$ **$\frac{5}{7}$ is greater than $\frac{3}{5}$** **(Ans.)**

Second method :

Cross-multiply $\frac{a}{b}$ and $\frac{c}{d}$; we get $a \times d$ and $b \times c$

1. If $a \times d$ is greater than $b \times c \Rightarrow \frac{a}{b}$ is greater than $\frac{c}{d}$ *i.e.* $\frac{a}{b} > \frac{c}{d}$.
2. If $a \times d$ is less than $b \times c \Rightarrow \frac{a}{b}$ is less than $\frac{c}{d}$ *i.e.* $\frac{a}{b} < \frac{c}{d}$.

For $\frac{3}{5}$ and $\frac{5}{7}$

we get : 3×7 and 5×5

$\Rightarrow$ 21 and 25

$\because$ $21 < 25$

$\Rightarrow$ **$\frac{3}{5}$ is smaller than $\frac{5}{7}$** **(Ans.)**

Example 5 :

Compare $\frac{-3}{7}$ and $\frac{8}{-15}$.

Solution :

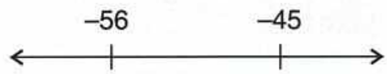
Express each given rational number with a positive denominator

$\therefore \frac{-3}{7}$ and $\frac{8}{-15} \Rightarrow \frac{-3}{7}$ and $\frac{-8}{15}$

Now, make the denominators of both the rational numbers the same. For this

$\frac{-3}{7}$ and $\frac{-8}{15} = \frac{-3\times15}{7\times15}$ and $\frac{-8\times7}{15\times7}$ [$\because$ L.C.M. of 7 and 15 = 7 × 15 = 105]

$= \frac{-45}{105}$ and $\frac{-56}{105}$



Since, –45 is greater than –56

$\Rightarrow \frac{-45}{105}$ is greater than $\frac{-56}{105}$

$\Rightarrow$ **$\frac{-3}{7}$ is greater than $\frac{-8}{15}$**. **(Ans.)**

Example 6 :

Arrange the rational numbers $-\frac{9}{10}, \frac{7}{-8}$ and $\frac{-3}{4}$ in ascending order.

Solution :

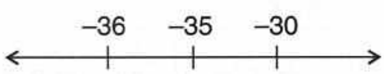
$-\frac{9}{10}, \frac{7}{-8}$ and $\frac{-3}{4}$ $= \frac{-9}{10}, \frac{-7}{8}$ and $\frac{-3}{4}$

$= \frac{-9\times4}{10\times4}, \frac{-7\times5}{8\times5}$ and $\frac{-3\times10}{4\times10}$ [L.C.M. of 10, 8 and 4 = 40]

$= \frac{-36}{40}, \frac{-35}{40}$ and $\frac{-30}{40}$

Since, $-36 < -35 < -30$

$\therefore \frac{-36}{40} < \frac{-35}{40} < \frac{-30}{40}$



$\Rightarrow$ **$\frac{-9}{10} < \frac{7}{-8} < \frac{-3}{4}$** **(Ans.)**

Example 7 :

Arrange the rational numbers $\frac{5}{-8}, \frac{-7}{12}$ and $\frac{13}{-24}$ in descending order.

Solution :

$$\frac{5}{-8}, \frac{-7}{12} \text{ and } \frac{13}{-24} = \frac{-5}{8}, \frac{-7}{12} \text{ and } \frac{-13}{24}$$

$$= \frac{-5\times3}{8\times3}, \frac{-7\times2}{12\times2} \text{ and } \frac{-13}{24} \quad \text{[L.C.M. of 8, 12 and 24 is 24]}$$

i.e., $\frac{-15}{24}, \frac{-14}{24}$ and $\frac{-13}{24}$ in which $-13 > -14 > -15$

$\Rightarrow \frac{-13}{24} > \frac{-14}{24} > \frac{-15}{24}$

$\Rightarrow \frac{13}{-24} > -\frac{7}{12} > \frac{5}{-8}$ **(Ans.)**

Important :

For any two rational numbers *a* and *b*, marked on a number line, if :

1. *a* is on the left of *b*

   then *a* is smaller than *b* *i.e.* $\boldsymbol{a < b}$

2. *a* is on the right of *b*

   then *a* is greater than *b* *i.e.* $\boldsymbol{a > b}$

Example 8 :

Using number line, compare the numbers $\frac{3}{5}$ and $-\frac{4}{7}$.

Solution :

Mark given rational numbers roughly on a number line as shown below :

Since, P is on the right of Q.

$\Rightarrow$ **$\frac{3}{5}$ is greater than $-\frac{4}{7}$** **(Ans.)**

EXERCISE 2(B)

1. Mark the following pairs of rational numbers on the separate number lines :

(i) $\frac{3}{4}$ and $-\frac{1}{4}$ (ii) $\frac{2}{5}$ and $\frac{-3}{5}$ (iii) $\frac{5}{6}$ and $-\frac{2}{3}$

(iv) $\frac{2}{5}$ and $-\frac{4}{5}$ (v) $\frac{1}{4}$ and $-\frac{5}{4}$

2. Compare :

(i) $\frac{3}{5}$ and $\frac{5}{7}$ (ii) $\frac{-7}{2}$ and $\frac{5}{2}$ (iii) -3 and $2\frac{3}{4}$

(iv) $-1\frac{1}{2}$ and 0 (v) 0 and $\frac{3}{4}$ (vi) 3 and -1

3. Compare :

(i) $-\frac{1}{4}$ and 0 (ii) $\frac{1}{4}$ and 0 (iii) $-\frac{3}{8}$ and $\frac{2}{5}$

(iv) $\frac{-5}{8}$ and $\frac{7}{-12}$ (v) $\frac{5}{-9}$ and $\frac{-5}{-9}$ (vi) $\frac{-7}{8}$ and $\frac{5}{-6}$

(vii) $\frac{2}{7}$ and $\frac{-3}{-8}$

4. Arrange the given rational numbers in ascending order :

(i) $\frac{7}{10}, \frac{-11}{-30}$ and $\frac{5}{-15}$ (ii) $\frac{4}{-9}, \frac{-5}{12}$ and $\frac{2}{-3}$

5. Arrange the given rational numbers in descending order :

(i) $\frac{5}{8}, \frac{13}{-16}$ and $\frac{-7}{12}$ (ii) $\frac{3}{-10}, \frac{-13}{30}$ and $\frac{8}{-20}$

6. Fill in the blanks :

(i) $\frac{5}{8}$ and $\frac{3}{10}$ are on the .... side of zero.

(ii) $-\frac{5}{8}$ and $\frac{3}{10}$ are on the .... sides of zero.

(iii) $-\frac{5}{8}$ and $-\frac{3}{10}$ are on the .... side of zero.

(iv) $\frac{5}{8}$ and $-\frac{3}{10}$ are on the .... sides of zero.

2.5 PROBLEMS ON RATIONAL NUMBERS (All operations) :

1. Addition of Rational Numbers :

Case 1 : When denominators are equal :

- Keeping the denominator same, add the numerators.
- If required, express the rational number obtained in its lowest terms.

Thus :

(a) $\frac{4}{15} + \frac{8}{15} = \frac{4+8}{15} = \frac{12}{15} = \frac{4}{5}$

(b) $\frac{-4}{15} + \frac{8}{15} = \frac{(-4)+8}{15} = \frac{4}{15}$

(c) $\frac{-4}{15} + \frac{8}{-15} = \frac{-4}{15} + \frac{-8}{15} = \frac{(-4)+(-8)}{15} = \frac{-12}{15} = -\frac{4}{5}$

(d) $\frac{4}{15} + \frac{8}{-15} = \frac{4}{15} + \frac{-8}{15} = \frac{4+(-8)}{15} = \frac{-4}{15}$

Case 2 : When denominators are unequal :

Make the denominators of all the given rational numbers the same and then proceed as case 1, given above. Thus :

(a) $\frac{-2}{3} + \frac{3}{4} = \frac{-2\times4}{3\times4} + \frac{3\times3}{4\times3}$ [∵ L.C.M. of 3 and 4 is 12 So, make each denominator equal to 12]

$= \frac{-8}{12} + \frac{9}{12}$

$= \frac{(-8)+9}{12} = \frac{-8+9}{12} = \frac{1}{12}$

(b) $\frac{-3}{5} + \frac{7}{-10} = \frac{-3}{5} + \frac{-7}{10}$

$= \frac{-3\times2}{5\times2} + \frac{-7}{10}$ [∵ L.C.M. of 5 and 10 is 10]

$= \frac{-6}{10} + \frac{-7}{10}$

$= \frac{(-6)+(-7)}{10} = \frac{-13}{10}$

(c) $\frac{11}{18} + \frac{7}{-27} = \frac{11}{18} + \frac{(-7)}{27}$

$= \frac{11\times3}{18\times3} + \frac{-7\times2}{27\times2}$ [∵ L.C.M. of 18 and 27 is 54]

$= \frac{33}{54} + \frac{-14}{54}$

$= \frac{33+(-14)}{54} = \frac{33-14}{54} = \frac{19}{54}$

(d) $\frac{9}{-16} + \frac{-5}{-12} = \frac{-9}{16} + \frac{5}{12}$

$= \frac{-9\times3}{16\times3} + \frac{5\times4}{12\times4}$ [∵ L.C.M. of 16 and 12 is 48]

$= \frac{-27}{48} + \frac{20}{48}$

$= \frac{-27+20}{48} = \frac{-7}{48}$

(e) $\frac{-4}{9} + \frac{7}{-12} + \frac{7}{18} = \frac{-4}{9} + \frac{-7}{12} + \frac{7}{18}$

$= \frac{-4\times4}{9\times4} + \frac{-7\times3}{12\times3} + \frac{7\times2}{18\times2}$ [∵ L.C.M. of 9, 12 and 18 is 36]

$= \frac{-16}{36} + \frac{-21}{36} + \frac{14}{36}$

$= \frac{-16+(-21)+14}{36} = \frac{-16-21+14}{36} = \frac{-37+14}{36} = \frac{-23}{36}$

(f) $\frac{5}{-27} + \frac{8}{9} + \frac{-7}{18} = \frac{-5}{27} + \frac{8}{9} + \frac{-7}{18}$

$= \frac{-5\times2}{27\times2} + \frac{8\times6}{9\times6} + \frac{-7\times3}{18\times3}$ [∵ L.C.M. of 27, 9 and 18 is 54]

$= \frac{-10}{54} + \frac{48}{54} + \frac{-21}{54}$

$= \frac{-10+48-21}{54} = \frac{-31+48}{54} = \frac{17}{54}$

2. Subtraction of Rational Numbers :

Case 1 : When denominators are equal :

(a) $\frac{5}{7} - \frac{4}{7} = \frac{5-4}{7} = \frac{1}{7}$

(b) $\frac{9}{13} - \frac{6}{13} = \frac{9-6}{13} = \frac{3}{13}$

(c) $\frac{12}{25} - \frac{8}{25} = \frac{12-8}{25} = \frac{4}{25}$

In general, for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$.

$\frac{a}{b} - \frac{c}{d} = \frac{a}{b}$ + (additive inverse of $\frac{c}{d}$)

$= \frac{a}{b} + \frac{-c}{d}$

Additive inverse of $\frac{8}{9}$ is $\frac{-8}{9}$, additive inverse of $\frac{-4}{7}$ is $\frac{4}{7}$.

(a) $\frac{3}{4} - \frac{2}{5} = \frac{3}{4}$ + (additve inverse of $\frac{2}{5}$)

$= \frac{3}{4} + \frac{-2}{5}$

$= \frac{3\times5}{4\times5} + \frac{-2\times4}{5\times4}$ [∵ L.C.M. of 4 and 5 is 20]

$= \frac{15}{20} + \frac{-8}{20} = \frac{15+(-8)}{20} = \frac{15-8}{20} = \frac{7}{20}$

(b) $-\frac{2}{3} - \left(-\frac{3}{5}\right) = -\frac{2}{3}$ + (additve inverse of $-\frac{3}{5}$)

$= -\frac{2}{3} + \frac{3}{5}$

$= \frac{-2\times5}{3\times5} + \frac{3\times3}{5\times3}$ [∵ L.C.M. of 3 and 5 is 15]

$= \frac{-10}{15} + \frac{9}{15} = \frac{-10+9}{15} = \frac{-1}{15}$

(c) $\frac{-2}{1} - \frac{7}{12} = \frac{-2}{1} + (\text{additive inverse of } \frac{7}{12})$

$= \frac{-2}{1} + \left(\frac{-7}{12}\right)$

$= \frac{-2 \times 12}{1 \times 12} + \frac{-7}{12}$ [∵ L.C.M. of 1 and 12 is 12]

$= \frac{-24}{12} + \frac{-7}{12} = \frac{-24-7}{12} = \frac{-31}{12}$

Direct method :

(a) $-3 - \frac{4}{7} = \frac{-3}{1} - \frac{4}{7}$

$= \frac{-3 \times 7}{1 \times 7} - \frac{4}{7}$

$= \frac{-21}{7} - \frac{4}{7} = \frac{-21-4}{7} = \frac{-25}{7}$

(b) $-\frac{5}{6} - \frac{5}{-3} = -\frac{5}{6} + \frac{5}{3}$

$= -\frac{5}{6} + \frac{5 \times 2}{3 \times 2} = \frac{-5}{6} + \frac{10}{6} = \frac{-5+10}{6} = \frac{5}{6}$

(c) $-\frac{5}{8} - (-3) = -\frac{5}{8} + 3 = -\frac{5}{8} + \frac{3 \times 8}{8}$

$= -\frac{5}{8} + \frac{24}{8} = \frac{-5+24}{8} = \frac{19}{8}$

Example 9 :

The sum of two rational numbers is –4. If one of them is $\frac{-13}{25}$, find the other.

Solution :

∵ Sum of two rational numbers $= -4$

and, one of them $= \frac{-13}{25}$

∴ **The other rational number** $= -4 - \left(\frac{-13}{25}\right)$

$= -4 + \frac{13}{25}$

$= \frac{-4 \times 25}{25} + \frac{13}{25}$

$= \frac{-100}{25} + \frac{13}{25} = \frac{-100+13}{25} = \frac{-87}{25}$ **(Ans.)**

Example 10 :

What should be added to $\frac{-5}{9}$ to get $\frac{-5}{6}$?

Solution :

Let the required rational number be x

$$\therefore \quad \frac{-5}{9} + x = \frac{-5}{6} \Rightarrow x = \left(\frac{-5}{6}\right) - \left(\frac{-5}{9}\right)$$

$$= -\frac{5}{6} + \frac{5}{9}$$

$$= \frac{-5\times3}{6\times3} + \frac{5\times2}{9\times2} \qquad [\because \text{ L.C.M. of 6 and 9 is 18}]$$

$$= \frac{-15}{18} + \frac{10}{18}$$

$$= \frac{-15+10}{18} = \mathbf{\frac{-5}{18}}$$ **(Ans.)**

Example 11 :

What sould be subtracted from $-\frac{5}{6}$ to get 1?

Solution :

The required rational number

$$= -\frac{5}{6} - 1$$

$$= -\frac{5}{6} - \frac{6}{6}$$

$$= \frac{-5-6}{6} = \mathbf{\frac{-11}{6}}$$ **(Ans.)**

Let x be subtracted

$$\therefore \quad -\frac{5}{6} - x = 1$$

$$\Rightarrow \quad -\frac{5}{6} - 1 = x$$

i.e. $$x = -\frac{5}{6} - 1$$

$$= -\frac{5}{6} - \frac{6}{6} = \frac{-5-6}{6} = \mathbf{\frac{-11}{6}}$$

EXERCISE 2(C)

1. Add :

(i) $\frac{7}{5}$ and $\frac{2}{5}$ (ii) $\frac{-4}{9}$ and $\frac{2}{9}$ (iii) $\frac{5}{-12}$ and $\frac{1}{12}$

(iv) $\frac{4}{-15}$ and $\frac{-7}{-15}$ (v) $\frac{-7}{25}$ and $\frac{9}{-25}$ (vi) $\frac{-7}{26}$ and $\frac{7}{-26}$

2. Add :

(i) $\frac{-2}{5}$ and $\frac{3}{7}$ (ii) $\frac{-5}{6}$ and $\frac{4}{9}$ (iii) -3 and $\frac{2}{3}$

(iv) $\frac{-5}{9}$ and $\frac{7}{18}$ (v) $\frac{-7}{24}$ and $\frac{-5}{48}$ (vi) $\frac{1}{-18}$ and $\frac{5}{-27}$

(vii) $\frac{-9}{25}$ and $\frac{1}{-75}$ (viii) $\frac{13}{-16}$ and $\frac{-11}{24}$ (ix) $\frac{-9}{-16}$ and $\frac{-11}{8}$

3. Evaluate :

(i) $\frac{-2}{5}+\frac{3}{5}+\frac{-1}{5}$ (ii) $\frac{-8}{9}+\frac{4}{9}+\frac{-2}{9}$ (iii) $\frac{5}{-24}+\frac{-1}{8}+\frac{3}{16}$

(iv) $\frac{-7}{6}+\frac{4}{-15}+\frac{-4}{-30}$ (v) $-2+\frac{2}{5}+\frac{-2}{15}$ (vi) $\frac{-11}{12}+\frac{5}{16}+\frac{-3}{8}$

4. Evaluate :

(i) $-\frac{11}{18}+\frac{-3}{9}+\frac{2}{-3}$ (ii) $\frac{-9}{4}+\frac{13}{3}+\frac{25}{6}$ (iii) $-5+\frac{5}{-8}+\frac{-5}{-12}$

(iv) $-\frac{2}{3}+\frac{5}{2}+2$ (v) $5+\frac{-3}{4}+\frac{-5}{8}$

5. Subtract :

(i) $\frac{2}{9}$ from $\frac{5}{9}$ (ii) $\frac{-6}{11}$ from $\frac{-3}{-11}$ (iii) $\frac{-2}{15}$ from $\frac{-8}{15}$

(iv) $\frac{11}{18}$ from $\frac{-5}{18}$ (v) $\frac{-4}{11}$ from -2.

6. Subtract :

(i) $-\frac{3}{10}$ from $\frac{1}{5}$ (ii) $\frac{-6}{25}$ from $\frac{-8}{5}$ (iii) $\frac{-7}{4}$ from -2

(iv) $\frac{-16}{21}$ from 1 (v) $\frac{-8}{15}$ from 0 (vi) 0 from $\frac{-3}{8}$

(vii) -2 from $\frac{-3}{10}$ (viii) $\frac{5}{8}$ from $\frac{-5}{16}$ (ix) 4 from $-\frac{3}{13}$

7. The sum of two rational numbers is $\frac{11}{24}$. If one of them is $\frac{3}{8}$, find the other.

8. The sum of two rational numbers is $\frac{-7}{12}$. If one of them is $\frac{13}{24}$, find the other.

9. The sum of two rational numbers is -4. If one of them is $-\frac{13}{12}$, find the other.

10. What should be added to $-\frac{3}{32}$ to get $\frac{53}{96}$?

11. What should be added to $\frac{-3}{20}$ to get $2\frac{9}{20}$?

12. What should be subtracted from $\frac{-4}{5}$ to get 1 ?

13. The sum of two numbers is $-\frac{6}{5}$. If one of them is -2, find the other.

14. What should be added to $\frac{-7}{12}$ to get $\frac{3}{8}$?

15. What should be subtracted from $\frac{5}{9}$ to get $\frac{9}{5}$?

3. Multiplication of Rational Numbers

Product (multiplication) of two or more rational numbers

$$= \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

For example :

(a) $\frac{5}{6} \times \left(\frac{-3}{4}\right) = \frac{5 \times (-3)}{6 \times 4} = \frac{-15}{24} = \frac{-5}{8}$

(b) $\left(\frac{-3}{8}\right) \times 3 = \frac{-3}{8} \times \frac{3}{1} = \frac{-3 \times 3}{8 \times 1} = \frac{-9}{8}$

(c) $\left(\frac{-36}{7}\right) \times \left(\frac{28}{-9}\right) = \frac{-36 \times 28}{7 \times -9} = \frac{-36 \times 28}{-7 \times 9} = \frac{\cancel{36}^{4} \times \cancel{28}^{4}}{\cancel{7} \times \cancel{9}} = 4 \times 4 = 16$

(d) $\frac{-8}{7} \times \frac{14}{5} = \frac{-8 \times \cancel{14}^{2}}{\cancel{7} \times 5} = \frac{-8 \times 2}{5} = \frac{-16}{5}$

(e) $\frac{4}{9} \times -3 = \frac{4}{9} \times \frac{-3}{1} = \frac{4 \times (-3)}{9 \times 1} = \frac{-\cancel{12}^{4}}{\cancel{9}_{3}} = -\frac{4}{3}$

(f) $-24 \times \frac{-5}{8} = \frac{-24}{1} \times \frac{-5}{8} = \frac{-24 \times -5}{8} = \frac{120}{8} = 15$

(g) $\frac{-4}{9} \times \frac{13}{-20} = \frac{-4}{9} \times \frac{-13}{20} = \frac{-4 \times -13}{9 \times 20} = \frac{\cancel{52}^{13}}{\cancel{180}_{45}} = \frac{13}{45}$

(h) $\left(\frac{-3}{2} \times \frac{4}{5}\right) + \left(\frac{9}{5} \times \frac{-10}{3}\right) - \left(\frac{1}{2} \times \frac{3}{4}\right)$

$$= \frac{-12}{10} + \frac{-90}{15} - \frac{3}{8}$$

$$= -\frac{6}{5} + (-6) - \frac{3}{8}$$

$$= -\frac{6}{5} - \frac{6}{1} - \frac{3}{8}$$

$$= -\frac{6 \times 8}{5 \times 8} - \frac{6 \times 40}{40} - \frac{3 \times 5}{8 \times 5}$$ [L.C.M. of 5, 1 and 8 is 40]

$$= -\frac{48}{40} - \frac{240}{40} - \frac{15}{40}$$

$$= \frac{-48 - 240 - 15}{40} = \frac{-303}{40}$$

Example 12 :

A cyclist moves with a speed of $7\frac{2}{5}$ km per hour. How much distance will he cover in $2\frac{1}{3}$ hours?

Solution :

$\because$ Speed $= 7\frac{2}{5}$ km per hour

$= \frac{37}{5}$ km per hour

And, time taken $= 2\frac{1}{3}$ hours $= \frac{7}{3}$ hours

$\therefore$ **Distance covered** = Speed × time

$= \frac{37}{5} \times \frac{7}{3}$ km

$= \frac{259}{15}$ km $=$ **$17\frac{4}{15}$ km** **(Ans.)**

4. Multiplicative Inverse (Reciprocal)

(a) Multiplicative inverse of $\frac{3}{4}$ is $\frac{4}{3} \Rightarrow$ Reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

(b) Multiplicative inverse of $\frac{-4}{5}$ is $\frac{5}{-4} \Rightarrow$ Reciprocal of $\frac{-4}{5}$ is $\frac{5}{-4}$.

(c) Multiplicative inverse (reciprocal) of $\frac{3}{-8}$ is $\frac{-8}{3}$.

(d) Multiplicative inverse (reciprocal) of $\frac{-6}{13}$ is $\frac{13}{-6} = \frac{-13}{6}$.

5. Division of Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then,

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times$ (multiplicative inverse of $\frac{c}{d}$)

$= \frac{a}{b} \times \frac{d}{c}$

For example :

(a) $\frac{4}{25} \div \frac{3}{5} = \frac{4}{25} \times \frac{5}{3}$ [Reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$]

$= \frac{4}{5 \times 3} = \frac{4}{15}$ **(Ans.)**

(b) $\frac{2}{7} \div \frac{-8}{35} = \frac{2}{7} \times \frac{35}{-8} = \frac{2 \times 5}{-8} = -\frac{10}{8} = -\frac{5}{4}$ **(Ans.)**

(c) $-\frac{4}{3} \div \frac{16}{21} = -\frac{4}{3} \times \frac{21}{16} = -\frac{1 \times 7}{1 \times 4} = -\frac{7}{4}$ **(Ans.)**

(d) $-\frac{4}{3} \div \frac{-16}{21} = -\frac{4}{3} \times \frac{21}{-16} = \frac{1 \times 7}{1 \times 4} = \frac{7}{4}$ **(Ans.)**

(e) $-\frac{3}{10} \div \frac{-9}{25} = -\frac{3}{10} \times \frac{25}{-9} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$ **(Ans.)**

Example 14 :

The product of two rational numbers is $\frac{12}{5}$. If one of the numbers is $-\frac{6}{7}$, find the other.

Solution :

$\because$ Product of two rational numbers $= \frac{12}{5}$

and, one of these two numbers $= -\frac{6}{7}$

$\therefore$ **The other number** $= \frac{12}{5} \div \left(-\frac{6}{7}\right)$

$$= \frac{12}{5} \times \left(-\frac{7}{6}\right) = -\frac{12}{5} \times \frac{7}{6} = -\frac{2 \times 7}{5 \times 1} = -\frac{14}{5} \quad \textbf{(Ans.)}$$

Example 15 :

Evaluate : $\left(3\frac{2}{5} + 2\frac{1}{10}\right) \div \left(4\frac{3}{5} - 3\frac{3}{10}\right)$.

Solution :

$$\left(3\frac{2}{5} + 2\frac{1}{10}\right) \div \left(4\frac{3}{5} - 3\frac{3}{10}\right) = \left(\frac{17}{5} + \frac{21}{10}\right) \div \left(\frac{23}{5} - \frac{33}{10}\right)$$

$$= \left(\frac{17 \times 2 + 21}{10}\right) \div \left(\frac{23 \times 2 - 33}{10}\right)$$

$$= \frac{34 + 21}{10} \div \frac{46 - 33}{10}$$

$$= \frac{55}{10} \div \frac{13}{10} = \frac{55}{10} \times \frac{10}{13} = \frac{55}{13} = 4\frac{3}{13} \quad \textbf{(Ans.)}$$

EXERCISE 2(D)

1. Evaluate :

(i) $\frac{5}{4} \times \frac{3}{7}$ (ii) $\frac{2}{3} \times -\frac{6}{7}$ (iii) $\left(\frac{-12}{5}\right) \times \left(\frac{10}{-3}\right)$

(iv) $-\frac{45}{39} \times \frac{-13}{15}$ (v) $3\frac{1}{8} \times \left(-2\frac{2}{5}\right)$ (vi) $2\frac{14}{25} \times \left(\frac{-5}{16}\right)$

(vii) $\left(\frac{-8}{9}\right) \times \left(\frac{-3}{16}\right)$ (viii) $\left(\frac{5}{-27}\right) \times \left(\frac{-9}{20}\right)$

2. Multiply :

(i) $\frac{3}{25}$ and $\frac{4}{5}$ (ii) $1\frac{1}{8}$ and $10\frac{2}{3}$ (iii) $6\frac{2}{3}$ and $\frac{-3}{8}$

(iv) $\frac{-13}{15}$ and $\frac{-25}{26}$ (v) $1\frac{1}{6}$ and 18 (vi) $2\frac{1}{14}$ and -7

(vii) $5\frac{1}{8}$ and -16 (viii) 35 and $\frac{-18}{25}$ (ix) $6\frac{2}{3}$ and $-\frac{3}{8}$

(x) $3\frac{3}{5}$ and -10 (xi) $\frac{27}{28}$ and -14 (xii) -24 and $\frac{5}{16}$

3. Evaluate :

(i) $\left(-6\times\frac{5}{18}\right)-\left(-4\frac{2}{9}\right)$ (ii) $\left(\frac{7}{8}\times\frac{8}{7}\right)+\left(\frac{-5}{9}\right)\times\left(\frac{6}{-25}\right)$

(iii) $\left(\frac{11}{-9}\times\frac{21}{44}\right)+\left(\frac{-5}{9}\right)\times\left(\frac{63}{-100}\right)$ (iv) $\left(\frac{-5}{9}\times\frac{6}{-25}\right)+\left(\frac{24}{21}\times\frac{7}{8}\right)$

(v) $\left(\frac{-35}{39}\times\frac{-13}{7}\right)-\left(\frac{7}{90}\times\frac{-18}{14}\right)$ (vi) $\left(\frac{-4}{5}\times\frac{3}{2}\right)+\left(\frac{9}{-5}\times\frac{10}{3}\right)-\left(\frac{-3}{2}\times\frac{-1}{4}\right)$

4. Find the cost of $3\frac{1}{2}$ m cloth, if one metre cloth costs ₹ $325\frac{1}{2}$.

5. A bus is moving with a speed of $65\frac{1}{2}$ km per hour. How much distance will it cover in $1\frac{1}{3}$ hours.

6. Divide :

(i) $\frac{15}{28}$ by $\frac{3}{4}$ (ii) $\frac{-20}{9}$ by $\frac{-5}{9}$ (iii) $\frac{16}{-5}$ by $\frac{-8}{7}$

(iv) -7 by $\frac{-14}{5}$ (v) -14 by $\frac{7}{-2}$ (vi) $\frac{-22}{9}$ by $\frac{11}{18}$

(vii) 35 by $\frac{-7}{9}$ (viii) $\frac{21}{44}$ by $-\frac{11}{9}$

7. Evaluate :

(i) $3\frac{5}{12}+1\frac{2}{3}$ (ii) $3\frac{5}{12}-1\frac{2}{3}$ (iii) $\left(3\frac{5}{12}+1\frac{2}{3}\right)\div\left(3\frac{5}{12}-1\frac{2}{3}\right)$

8. The product of two numbers is 14. If one of the numbers is $\frac{-8}{7}$, find the other.

9. The cost of 11 pens is ₹ $24\frac{3}{4}$. Find the cost of one pen.

10. If 6 identical articles can be bought for ₹ $2\frac{6}{17}$. Find the cost of each article.

11. By what number should $\frac{-3}{8}$ be multiplied so that the product is $\frac{-9}{16}$?

12. By what number should $\frac{-5}{7}$ be divided so that the result is $\frac{-15}{28}$?

13. Evaluate : $\left(\frac{32}{15}+\frac{8}{5}\right)\div\left(\frac{32}{15}-\frac{8}{5}\right)$.

14. Seven equal pieces are made out of a rope of $21\frac{5}{7}$ m. Find the length of each piece.

EXERCISE 2(E)

1. Evaluate :

(i) $\frac{-2}{3}+\frac{3}{4}$ (ii) $\frac{7}{-27}+\frac{11}{18}$ (iii) $\frac{-3}{8}+\frac{-5}{12}$

(iv) $\frac{9}{-16}+\frac{-5}{-12}$ (v) $\frac{-5}{9}+\frac{-7}{12}+\frac{11}{18}$ (vi) $\frac{7}{-26}+\frac{16}{39}$

(vii) $-\frac{2}{3}-\left(\frac{-5}{7}\right)$ (viii) $-\frac{5}{7}-\left(-\frac{3}{8}\right)$ (ix) $\frac{7}{26}+2+\frac{-11}{13}$

(x) $-1+\frac{2}{-3}+\frac{5}{6}$

2. The sum of two rational numbers is $\frac{-3}{8}$. If one of them is $\frac{3}{16}$, find the other.

3. The sum of two rational numbers is –5. If one of them is $\frac{-52}{25}$, find the other.

4. What rational number should be added to $-\frac{3}{16}$ to get $\frac{11}{24}$?

5. What rational number should be added to $-\frac{3}{5}$ to get 2 ?

6. What rational number should be subtracted from $-\frac{5}{12}$ to get $\frac{5}{24}$?

7. What rational number should be subtracted from $\frac{5}{8}$ to get $\frac{8}{5}$?

8. Evaluate :

(i) $\left(\frac{7}{8}\times\frac{24}{21}\right)+\left(\frac{-5}{9}\times\frac{6}{-25}\right)$ (ii) $\left(\frac{8}{15}\times\frac{-25}{16}\right)+\left(\frac{-18}{35}\times\frac{5}{6}\right)$

(iii) $\left(\frac{18}{33}\times\frac{-22}{27}\right)-\left(\frac{13}{25}\times\frac{-75}{26}\right)$ (iv) $\left(\frac{-13}{7}\times\frac{-35}{39}\right)-\left(\frac{-7}{45}\times\frac{9}{14}\right)$

9. The product of two rational numbers is 24. If one of them is $\frac{-36}{11}$, find the other.

10. By what rational number should we multiply $\frac{20}{-9}$, so that the product may be $\frac{-5}{9}$?

FRACTION
(Including Problems)

3

3.1 BASIC CONCEPT

If an apple is divided into five equal parts; each part is said to be one-fifth $\left(\frac{1}{5}\right)$ of the whole apple. And, if out of these five equal parts, 2 parts are eaten; we say two-fifths $\left(\frac{2}{5}\right)$ of the apple is eaten or three-fifths $\left(\frac{3}{5}\right)$ of the apple is left.

The numbers $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{3}{5}$ used in the statement, given above, are called **fractions**. *Each* of these fractions *indicates a part of the whole.*

In fraction $\frac{a}{b}$, ***a*** is called the **numerator** and ***b*** is called the **denominator** of the fraction.

$\therefore$ FRACTION = $\frac{\text{Numerator}}{\text{Denominator}}$

Every fraction can be expressed as $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$ *i.e.* denominator is not equal to zero.

3.2 CLASSIFICATION OF FRACTIONS

Types of fractions	Condition	Examples
1. ***Decimal fraction***	denominator is 10 or higher power of 10.	$\frac{1}{10}, \frac{3}{100}, \frac{15}{1000}, \frac{8}{10^5}, ...$
2. ***Vulgar fraction***	denominator is other than 10, 100, 1000, etc.	$\frac{2}{5}, \frac{4}{7}, \frac{8}{19}, \frac{23}{107}, ...$
3. ***Proper fraction***	denominator is greater than its numerator.	$\frac{4}{5}, \frac{3}{7}, \frac{101}{235}, ...$
4. ***Improper fraction***	denominator is less than its numerator.	$\frac{7}{5}, \frac{18}{13}, \frac{181}{60}, ...$
5. ***Mixed fraction***	consists of a natural number and a proper fraction.	$2\frac{5}{7}, 1\frac{3}{5}, 10\frac{1}{9}, ...$

If the numerator is equal to the denominator, the fraction is equal to **unity** (one).

e.g. $\frac{4}{4} = 1, \frac{3}{3} = 1, \frac{49}{49} = 1$ and so on.

Important : (a) $\mathbf{\frac{7}{20}} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100}$, **a decimal fraction.**

(b) $\mathbf{\frac{81}{500}} = \frac{81 \times 2}{500 \times 2} = \frac{162}{1000}$, **a decimal fraction.**

$\therefore$ If the denominator of a fraction can be expressed as 10 or as some higher power of 10, it is a decimal fraction.

Example 1 :

(a) Convert : (i) $3\frac{2}{7}$ (ii) $2\frac{5}{8}$ into improper fractions.

(b) Convert : (i) $\frac{11}{4}$ (ii) $\frac{19}{5}$ into mixed fractions.

Solution :

(a) (i) $3\frac{2}{7} = \frac{3\times7+2}{7} = \frac{23}{7}$ **(Ans.)**

$$\text{Given mixed fraction} = \frac{\text{Integral part} \times \text{Denominator} + \text{Numerator}}{\text{Denominator}}$$

(ii) $2\frac{5}{8} = \frac{2\times8+5}{8} = \frac{16+5}{8} = \frac{21}{8}$ **(Ans.)**

(b) (i) $\frac{11}{4} = \frac{2\times4+3}{4}$ $\because$ 4) 11 (2; 8; 3

$= 2 + \frac{3}{4} = 2\frac{3}{4}$ **(Ans.)**

(ii) $\frac{19}{5} = \frac{3\times5+4}{5}$ $\because$ 5) 19 (3; 15; 4

$= 3 + \frac{4}{5} = 3\frac{4}{5}$ **(Ans.)**

1. The value of a fraction remains the same if both its numerator and denominator are (i) multiplied or (ii) divided by the same non-zero number.

   e.g. (i) $\frac{5}{8} = \frac{5\times2}{8\times2} = \frac{10}{16}$; $\frac{3}{7} = \frac{3\times5}{7\times5} = \frac{15}{35}$ and so on.

   (ii) $\frac{10}{16} = \frac{10\div2}{16\div2} = \frac{5}{8}$; $\frac{15}{35} = \frac{15\div5}{35\div5} = \frac{3}{7}$ and so on.

2. *A fraction must always be expressed in its lowest term.*

3.3 REDUCING A GIVEN FRACTION TO ITS LOWEST TERM

Steps : First of all find H.C.F. of both the terms (numerator and denominator) of the given fraction. Then divide both terms by their H.C.F.

Example 2 :

Reduce : (i) $\frac{48}{60}$ (ii) $\frac{18}{27}$ to their lowest terms.

Solution :

(i) Since, H.C.F. of terms 48 and 60 = 12.

$\therefore \frac{48}{60} = \frac{48\div12}{60\div12}$ [Dividing each term by 12]

$= \frac{4}{5}$ **(Ans.)**

(ii) Since, H.C.F. of 18 and 27 is 9 $\therefore \frac{18}{27} = \frac{18 \div 9}{27 \div 9} = \frac{2}{3}$ **(Ans.)**

Alternative Method :

Resolve both the numerator and the denominator into prime factors, then cancel out the common factors among both.

Since, $48 = 2 \times 2 \times 2 \times 2 \times 3$ and $60 = 2 \times 2 \times 3 \times 5$

$\therefore \frac{48}{60} = \frac{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times 5}$ [Cancelling out the common factors]

$= \frac{2 \times 2}{5} = \frac{4}{5}$ **(Ans.)**

3.4 EQUIVALENT (EQUAL) FRACTIONS

Fractions having the same value are called equivalent fractions.

e.g., Since, $\frac{20}{25} = \frac{20 \div 5}{25 \div 5} =$ and $\frac{28}{35} = \frac{28 \div 7}{35 \div 7} = \frac{4}{5}$

$\therefore$ Fractions $\frac{20}{25}$ and $\frac{28}{35}$ are equivalent, *i.e.*, $\frac{20}{25} = \frac{28}{35} = \frac{4}{5}$.

3.5 SIMPLE AND COMPLEX FRACTIONS

A fraction, whose *numerator and denominator both are whole numbers with denominator ≠ 0,* is called a *simple fraction;* whereas a fraction, whose *numerator or denominator or both contain a fraction* is called a *complex fraction.*

e.g. (i) Each of $\frac{3}{8}, \frac{10}{17}, \frac{8}{15}$, etc., is a simple fraction.

(ii) Each of $\frac{5}{2/3}, \frac{1{\cdot}4}{8}, \frac{9/14}{2\frac{3}{7}}$, etc., is a complex fraction.

EXERCISE 3(A)

1. Classify each fraction given below as decimal or vulgar fraction, proper or improper fraction and mixed fraction :

(i) $\frac{3}{5}$ (ii) $\frac{11}{10}$ (iii) $\frac{13}{20}$ (iv) $\frac{18}{7}$ (v) $3\frac{2}{9}$

2. Express the following improper fractions as mixed fractions :

(i) $\frac{18}{5}$ (ii) $\frac{7}{4}$ (iii) $\frac{25}{6}$ (iv) $\frac{38}{5}$ (v) $\frac{22}{5}$

3. Express the following mixed fractions as improper fractions :

(i) $2\frac{4}{9}$ (ii) $7\frac{5}{13}$ (iii) $3\frac{1}{4}$ (iv) $2\frac{5}{48}$ (v) $12\frac{7}{11}$

4. Reduce the given fractions to lowest terms :

(i) $\frac{8}{18}$ (ii) $\frac{27}{36}$ (iii) $\frac{18}{42}$ (iv) $\frac{35}{75}$ (v) $\frac{18}{45}$

5. State *true* or *false* :

(i) $\frac{30}{40}$ and $\frac{12}{16}$ are equivalent fractions.

(ii) $\frac{10}{25}$ and $\frac{25}{10}$ are equivalent fractions.

(iii) $\frac{35}{49}, \frac{20}{28}, \frac{45}{63}$ and $\frac{100}{140}$ are equivalent fractions.

6. Distinguish each of the fractions, given below, as *a simple fraction* or a *complex fraction* :

(i) $\frac{0}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{5}{7}$ (iv) $\frac{3\frac{3}{5}}{18}$

(v) $\frac{6}{2\frac{2}{5}}$ (vi) $\frac{3\frac{1}{3}}{7\frac{2}{7}}$ (vii) $\frac{5\frac{2}{9}}{5}$ (viii) $\frac{8}{0}$

Remember : Each of the numbers of the form $\frac{5}{0}, \frac{7}{0}, \frac{8}{0}$, etc., is neither a simple fraction nor a complex fraction, as division by '0' is not defined.

3.6 LIKE AND UNLIKE FRACTIONS

Fractions having the *same denominators* are called *like fractions,* whereas the fractions *with different denominators* are called *unlike fractions.*

e.g. (i) $\frac{3}{8}, \frac{5}{8}, \frac{9}{8}$, etc., are *like fractions.*

(ii) $\frac{2}{7}, \frac{5}{9}, \frac{15}{23}, \frac{24}{37}$, etc., are *unlike fractions.*

3.7 CONVERTING UNLIKE FRACTIONS INTO LIKE FRACTIONS

Steps : 1. Find the L.C.M. of the denominators of all the given fractions.

2. For each given fraction, multiply its denominator by a suitable number so that the product obtained is equal to the L.C.M. obtained in Step 1.
3. Multiply the numerator also by the same number.

Example 3 :

Change $\frac{3}{4}, \frac{3}{5}, \frac{7}{8}$ and $\frac{9}{16}$ to like fractions.

Solution :

Since, L.C.M. of the denominators 4, 5, 8 and 16 is 80.

$\therefore \quad \frac{3}{4} = \frac{3\times20}{4\times20} = \frac{60}{80}; \qquad \frac{3}{5} = \frac{3\times16}{5\times16} = \frac{48}{80}$

$\frac{7}{8} = \frac{7 \times 10}{8 \times 10} = \frac{70}{80}$; $\frac{9}{16} = \frac{9 \times 5}{16 \times 5} = \frac{45}{80}$

∴ **Required like fractions are : $\frac{60}{80}, \frac{48}{80}, \frac{70}{80}$ and $\frac{45}{80}$** **(Ans.)**

3.8 COMPARING FRACTIONS

Steps : Convert all the given fractions *into like fractions,* then *the* ***fraction with the greater numerator is greater.***

Example 4 :

Compare the fractions : $\frac{2}{3}, \frac{3}{4}, \frac{5}{12}$ and $\frac{9}{16}$.

Solution :

∵ L.C.M. of the denominators 3, 4, 12 and 16 = 48.

∴ $\frac{2}{3} = \frac{2 \times 16}{3 \times 16} = \frac{32}{48}$; $\frac{3}{4} = \frac{3 \times 12}{4 \times 12} = \frac{36}{48}$

$\frac{5}{12} = \frac{5 \times 4}{12 \times 4} = \frac{20}{48}$ and $\frac{9}{16} = \frac{9 \times 3}{16 \times 3} = \frac{27}{48}$

] Converting into like fractions

Since, the biggest numerator is 36, thus the biggest fraction is $\frac{36}{48}$ (*i.e.,* $\frac{3}{4}$).

Next one is $\frac{32}{48}$ (*i.e.,* $\frac{2}{3}$) and the smallest fraction is $\frac{20}{48}$ (*i.e.,* $\frac{5}{12}$)

∴ **Fractions in ascending order** of values are : $\frac{5}{12}, \frac{9}{16}, \frac{2}{3}$ and $\frac{3}{4}$. **(Ans.)**

i.e. $\frac{5}{12} < \frac{9}{16} < \frac{2}{3} < \frac{3}{4}$

And, **fractions in descending order** of values are : $\frac{3}{4}, \frac{2}{3}, \frac{9}{16}$ and $\frac{5}{12}$. **(Ans.)**

i.e. $\frac{3}{4} > \frac{2}{3} > \frac{9}{16} > \frac{5}{12}$

Ascending order means arranging the numbers from **smallest to greatest** and **descending order** means arranging the numbers from **greatest to smallest**.

Alternate Method (By making numerators equal) **:**

Steps : 1. Convert all the given fractions into fractions of equal numerators.
2. The fraction which has a smaller denominator is greater.

Example 5 :

Compare : $\frac{2}{3}, \frac{3}{4}, \frac{5}{12}$ and $\frac{9}{16}$ by making their numerators equal.

Solution :

Step 1 : Since, L.C.M. of numerators 2, 3, 5 and 9 is 90

∴ $\frac{2}{3} = \frac{2 \times 45}{3 \times 45} = \frac{90}{135}$; $\frac{3}{4} = \frac{3 \times 30}{4 \times 30} = \frac{90}{120}$

$\frac{5}{12} = \frac{5 \times 18}{12 \times 18} = \frac{90}{216}$ and $\frac{9}{16} = \frac{9 \times 10}{16 \times 10} = \frac{90}{160}$

Step 2 : Since, $\frac{90}{120}$ has the smallest denominator, the biggest fraction is $\frac{90}{120}$ (*i.e.,* $\frac{3}{4}$).

As, $\frac{90}{216}$ has the biggest denominator, the smallest fraction is $\frac{90}{216}$ (*i.e.,* $\frac{5}{12}$).

$\therefore$ **Fractions in ascending order** are : $\frac{5}{12}, \frac{9}{16}, \frac{2}{3}$ and $\frac{3}{4}$. **(Ans.)**

And, **in descending order** they are : $\frac{3}{4}, \frac{2}{3}, \frac{9}{16}$ and $\frac{5}{12}$. **(Ans.)**

In order to compare two fractions, say : $\frac{a}{b}$ and $\frac{c}{d}$, find their cross-product, i.e., find $a \times d$ and $b \times c$. Then, if :

(i) $a \times d$ is greater than $b \times c \Rightarrow \frac{a}{b} > \frac{c}{d}$, (ii) $a \times d$ is less than $b \times c \Rightarrow \frac{a}{b} < \frac{c}{d}$,

(iii) $a \times d$ is equal to $b \times c \Rightarrow \frac{a}{b} = \frac{c}{d}$.

Example 6 :

Compare the fractions : $\frac{3}{13}$ and $\frac{7}{18}$.

Solution :

Taking the cross multiplication we get : $3 \times 18 = 54$ and $7 \times 13 = 91$

Since, 3×18 (*i.e.,* 54) is *smaller* than 7×13 (*i.e.,* 91) $\therefore \frac{3}{13} < \frac{7}{18}$ **(Ans.)**

3.9 TO INSERT A FRACTION BETWEEN THE TWO GIVEN FRACTIONS

Steps : Add numerators of the given fractions to get the numerator of required fraction. Similarly, add their denominators to get denominator of the required fraction. Then simplify, if required.

Example 7 :

Insert one fraction between : (i) $\frac{1}{2}$ and $\frac{3}{5}$ (ii) 2 and $3\frac{1}{2}$

Solution :

(i) **A fraction between $\frac{1}{2}$ and $\frac{3}{5}$** $= \frac{1+3}{2+5}$ [Adding numerators and denominators]

$= \frac{4}{7}$ **(Ans.)**

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions then fraction $\frac{a+c}{b+d}$ lies between $\frac{a}{b}$ and $\frac{c}{d}$.

Also, 1. If $\frac{a}{b} > \frac{c}{d}$, then $\frac{a}{b} > \frac{a+c}{b+d} > \frac{c}{d}$. **2.** If $\frac{a}{b} < \frac{c}{d}$, then $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$.

(ii) **A fraction between 2 and $3\frac{1}{2}$** = A fraction between $\frac{2}{1}$ and $\frac{7}{2}$

$= \frac{2+7}{1+2} = \frac{9}{3} = 3$ **(Ans.)**

Example 8 :

Insert three fractions between $\frac{1}{2}$ and $\frac{3}{5}$.

Solution :

A fraction between $\frac{1}{2}$ and $\frac{3}{5}$

$$= \frac{1+3}{2+5} = \frac{4}{7}$$

Now a fraction between $\frac{1}{2}$ and $\frac{4}{7} = \frac{1+4}{2+7} = \frac{5}{9}$

and a fraction between $\frac{4}{7}$ and $\frac{3}{5} = \frac{4+3}{7+5} = \frac{7}{12}$

$\therefore$ **Three fractions between $\frac{1}{2}$ and $\frac{3}{5} = \frac{5}{9}, \frac{4}{7}$ and $\frac{7}{12}$** **(Ans.)**

EXERCISE 3(B)

1. For each pair, given below, state whether it forms *like fractions* or *unlike fractions* :

(i) $\frac{5}{8}$ and $\frac{7}{8}$ (ii) $\frac{8}{15}$ and $\frac{8}{21}$ (iii) $\frac{4}{9}$ and $\frac{9}{4}$

2. Convert given fractions into fractions with *equal denominators* :

(i) $\frac{5}{6}$ and $\frac{7}{9}$ (ii) $\frac{2}{3}, \frac{5}{6}$ and $\frac{7}{12}$ (iii) $\frac{4}{5}, \frac{17}{20}, \frac{23}{40}$ and $\frac{11}{16}$

3. Convert given fractions into fractions with *equal numerators* :

(i) $\frac{8}{9}$ and $\frac{12}{17}$ (ii) $\frac{6}{13}, \frac{15}{23}$ and $\frac{12}{17}$ (iii) $\frac{15}{19}, \frac{25}{28}, \frac{9}{11}$ and $\frac{45}{47}$

4. Put the given fractions in ascending order by making denominators equal :

(i) $\frac{1}{3}, \frac{2}{5}, \frac{3}{4}$ and $\frac{1}{6}$ (ii) $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}$ and $\frac{3}{10}$ (iii) $\frac{5}{7}, \frac{3}{8}, \frac{9}{14}$ and $\frac{20}{21}$

5. Arrange the given fractions in descending order by making numerators equal :

(i) $\frac{5}{6}, \frac{4}{15}, \frac{8}{9}$ and $\frac{1}{3}$ (ii) $\frac{3}{7}, \frac{4}{9}, \frac{5}{7}$ and $\frac{8}{11}$ (iii) $\frac{1}{10}, \frac{6}{11}, \frac{8}{11}$ and $\frac{3}{5}$

6. Find the greater fraction :

(i) $\frac{3}{5}$ and $\frac{11}{15}$ (ii) $\frac{4}{5}$ and $\frac{3}{10}$ (iii) $\frac{6}{7}$ and $\frac{5}{9}$

7. Insert one fraction between :

(i) $\frac{3}{7}$ and $\frac{4}{9}$ (ii) 2 and $\frac{8}{3}$ (iii) $\frac{9}{17}$ and $\frac{6}{13}$

8. Insert three fractions between :

(i) $\frac{2}{5}$ and $\frac{4}{9}$ (ii) $\frac{1}{2}$ and $\frac{5}{7}$ (iii) $\frac{3}{8}$ and $\frac{6}{11}$

9. Insert two fractions between :

(i) 1 and $\frac{3}{11}$ (ii) $\frac{5}{9}$ and $\frac{1}{4}$ (iii) $\frac{5}{6}$ and $1\frac{1}{5}$

3.10 OPERATIONS ON FRACTIONS

1. ***Addition and Subtraction :***

(i) *For like fractions, add* or subtract (as required) *their numerators, keeping the denominator same :*

$\therefore \quad \frac{1}{8} + \frac{5}{8} = \frac{1+5}{8} = \frac{6}{8} = \frac{3}{4}$ and $\frac{9}{10} - \frac{3}{10} = \frac{9-3}{10} = \frac{6}{10} = \frac{3}{5}$

(ii) *For unlike fractions,* first of all change given fractions into like fractions and then do the addition or subtraction as above :

$\therefore \quad \frac{5}{7} - \frac{1}{4} = \frac{5 \times 4}{7 \times 4} - \frac{1 \times 7}{4 \times 7}$ [L.C.M. of 7 and 4 is 28]

$= \frac{20}{28} - \frac{7}{28} = \frac{20-7}{28} = \frac{13}{28}$

or, simply : $\frac{5}{7} - \frac{1}{4} = \frac{5 \times 4 - 1 \times 7}{28} = \frac{20-7}{28} = \frac{13}{28}$

And, $\frac{3}{4} + \frac{2}{5} - \frac{1}{3} = \frac{3 \times 15}{4 \times 15} + \frac{2 \times 12}{5 \times 12} - \frac{1 \times 20}{3 \times 20}$ [L.C.M. of 4, 5 and 3 = 60]

$= \frac{45}{60} + \frac{24}{60} - \frac{20}{60} = \frac{45+24-20}{60} = \frac{49}{60}$

or, simply : $\frac{3}{4} + \frac{2}{5} - \frac{1}{3} = \frac{3 \times 15 + 2 \times 12 - 1 \times 20}{60}$

$= \frac{45 + 24 - 20}{60} = \frac{49}{60}$.

2. ***Multiplication :***

(i) *To multiply a fraction with an integer, multiply its numerator with the integer.*

$\therefore \quad 5 \times \frac{3}{8} = \frac{5 \times 3}{8} = \frac{15}{8} = 1\frac{7}{8}$ and $\frac{4}{15} \times 7 = \frac{4 \times 7}{15} = \frac{28}{15} = 1\frac{13}{15}$.

(ii) *To multiply two or more fractions, multiply their numerators together and their denominators separately together.*

$\therefore \quad \frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$ and $\frac{3}{8} \times \frac{4}{5} \times \frac{2}{3} = \frac{3 \times 4 \times 2}{8 \times 5 \times 3} = \frac{1}{5}$.

3. ***Division :***

To divide one quantity (fraction or integer) *by some other quantity (fraction or integer),* multiply the first by the reciprocal of the second.

> Reciproal of 8 = $\frac{1}{8}$, reciprocal of $\frac{1}{5}$ = 5, reciprocal of $\frac{2}{7} = \frac{7}{2}$,
> reciprocal of $\frac{12}{25} = \frac{25}{12}$ and so on.

e.g. (i) $\frac{5}{8} \div 2 = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$ [Reciprocal of 2 is $\frac{1}{2}$]

(ii) $2 \div \frac{5}{8} = 2 \times \frac{8}{5} = \frac{16}{5} = 3\frac{1}{5}$ [Reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$]

(iii) $\frac{7}{10} \div \frac{3}{4} = \frac{7}{10} \times \frac{4}{3} = \frac{28}{30} = \frac{14}{15}$ and so on.

3.11 USING "OF"

The word "of" between any two fractions, is to be used as multiplication.

e.g. (i) $\frac{3}{16}$ **of 2** $= \frac{3 \times 2}{16} = \frac{3}{8}$

(ii) $\frac{1}{3}$ **of 18 kg** $= \frac{1 \times 18}{3}$ kg $=$ **6 kg**

(iii) $\frac{3}{4}$ **of ₹ 16** $=$ ₹ $\frac{3 \times 16}{4} =$ **₹ 12** and so on.

3.12 USING "BODMAS" :

The word **'BODMAS'** is the abbreviation formed by taking the initial letters of six operations; '**B**racket', '**O**f', '**D**ivision', '**M**ultiplication', '**A**ddition' and '**S**ubtraction'.

According to the rule of BODMAS, working must be done in the order corresponding to the letters appearing in the word, *i.e.,* first of all the terms inside **B**racket must be simplified; then **O**f must be simplied and then **D**ivision, **M**ultiplication, **A**ddition and finally **S**ubtraction.

e.g. $\left(\frac{1}{3} + \frac{2}{9}\right)$ **of** $\frac{8}{15} \div \frac{4}{9} \times \frac{3}{4} - \frac{1}{2} + 1$

$= \left(\frac{3+2}{9}\right)$ of $\frac{8}{15} \div \frac{4}{9} \times \frac{3}{4} - \frac{1}{2} + 1$ **First step (B) :** Simplifying the **Bracket**.

$= \frac{5}{9} \times \frac{8}{15} \div \frac{4}{9} \times \frac{3}{4} - \frac{1}{2} + 1$ **Second step (O) :** Removal of **'Of'**

$= \frac{8}{27} \times \frac{9}{4} \times \frac{3}{4} - \frac{1}{2} + 1$ **Third step (D) : Division**, *i.e.,* multiply by reciprocal.

$= \frac{8 \times 9 \times 3}{27 \times 4 \times 4} - \frac{1}{2} + 1$ **Fourth step (M) : Multiplication.**

$= \frac{1}{2} - \frac{1}{2} + 1$ **Fifth step (A and S) : Addition and Subtraction**

$= 1$ **(Ans.)**

Example 9 :

Evaluate :

(i) $2\frac{1}{4} \div \frac{5}{7} \times 1\frac{1}{3}$ (ii) $\frac{1}{4}$ of $2\frac{2}{7} \div \frac{4}{15}$

Solution :

> If required, convert the mixed fraction / fractions into improper fraction / fractions, then apply BODMAS and simplify.

(i) $2\frac{1}{4} \div \frac{5}{7} \times 1\frac{1}{3} = \frac{9}{4} \div \frac{5}{7} \times \frac{4}{3}$

$= \frac{9}{4} \times \frac{7}{5} \times \frac{4}{3} = \frac{9 \times 7 \times 4}{4 \times 5 \times 3} = \frac{21}{5} = 4\frac{1}{5}$ **(Ans.)**

(ii) $\frac{1}{4}$ of $2\frac{2}{7} \div \frac{4}{15} = \frac{1}{4}$ of $\frac{16}{7} \div \frac{4}{15}$

$= \frac{4}{7} \div \frac{4}{15}$ $\left[\because \frac{1}{4} \text{ of } \frac{16}{7} = \frac{1}{4} \times \frac{16}{7} = \frac{4}{7}\right]$

$= \frac{4}{7} \times \frac{15}{4} = \frac{15}{7} = 2\frac{1}{7}$ **(Ans.)**

Example 10 :

Evaluate :

(i) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9}$ (ii) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9}$ (iii) $\frac{5}{6}$ of $\frac{5}{13} \div \frac{15}{16} \times 1\frac{1}{2}$

Solution :

Remember : BODMAS

(i) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9} = \frac{4}{5} \div \frac{56}{135}$ $\left[\frac{7}{15} \text{ of } \frac{8}{9} = \frac{56}{135}\right]$

$= \frac{4}{5} \times \frac{135}{56} = \frac{27}{14} = 1\frac{13}{14}$ **(Ans.)**

(ii) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9} = \frac{4}{5} \times \frac{15}{7} \times \frac{8}{9}$ [Division (÷) first]

$= \frac{4 \times 15 \times 8}{5 \times 7 \times 9} = \frac{32}{21} = 1\frac{11}{21}$ **(Ans.)**

(iii) $\frac{5}{6}$ of $\frac{5}{13} \div \frac{15}{16} \times 1\frac{1}{2} = \frac{25}{78} \div \frac{15}{16} \times \frac{3}{2}$

$= \frac{25}{78} \times \frac{16}{15} \times \frac{3}{2} = \frac{25 \times 16 \times 3}{78 \times 15 \times 2} = \frac{20}{39}$ **(Ans.)**

EXERCISE 3(C)

1. Reduce to a single fraction :

(i) $\frac{1}{2} + \frac{2}{3}$ (ii) $\frac{3}{5} - \frac{1}{10}$ (iii) $\frac{2}{3} - \frac{1}{6}$

(iv) $1\frac{1}{3} + 2\frac{1}{4}$ (v) $\frac{1}{4} + \frac{5}{6} - \frac{1}{12}$ (vi) $\frac{2}{3} - \frac{3}{5} + 3 - \frac{1}{5}$

(vii) $\frac{2}{3} - \frac{1}{5} + \frac{1}{10}$ (viii) $2\frac{1}{2} + 2\frac{1}{3} - 1\frac{1}{4}$ (ix) $2\frac{5}{8} - 2\frac{1}{6} + 4\frac{3}{4}$

2. Simplify :

(i) $\frac{3}{4} \times 6$ (ii) $\frac{2}{3} \times 15$ (iii) $\frac{3}{4} \times \frac{1}{2}$

(iv) $\frac{9}{12} \times \frac{4}{7}$ (v) $45 \times 2\frac{1}{3}$ (vi) $36 \times 3\frac{1}{4}$

(vii) $2 \div \frac{1}{3}$ (viii) $3 \div \frac{2}{5}$ (ix) $1 \div \frac{3}{5}$

(x) $\frac{1}{3} \div \frac{1}{4}$ (xi) $-\frac{5}{8} \div \frac{3}{4}$ (xii) $3\frac{3}{7} \div 1\frac{1}{14}$

(xiii) $3\frac{3}{4} \times 1\frac{1}{5} \times \frac{20}{21}$

3. Subtract :

(i) 2 from $\frac{2}{3}$ (ii) $\frac{1}{8}$ from $\frac{5}{8}$ (iii) $-\frac{2}{5}$ from $\frac{2}{5}$

(iv) $-\frac{3}{7}$ from $\frac{3}{7}$ (v) 0 from $-\frac{4}{5}$ (vi) $\frac{2}{9}$ from $\frac{4}{5}$

(vii) $-\frac{4}{7}$ from $-\frac{6}{11}$

4. Find the value of :

(i) $\frac{1}{2}$ of 10 kg (ii) $\frac{3}{5}$ of 1 hour (iii) $\frac{4}{7}$ of $2\frac{1}{3}$ kg

(iv) $3\frac{1}{2}$ times of 2 metre (v) $\frac{1}{2}$ of $2\frac{2}{3}$ (vi) $\frac{5}{11}$ of $\frac{4}{5}$ of 22 kg

5. Simplify and reduce to a simple fraction :

(i) $\dfrac{3}{3\frac{3}{4}}$ (ii) $\dfrac{\frac{3}{5}}{7}$ (iii) $\dfrac{3}{\frac{5}{7}}$ (iv) $\dfrac{2\frac{1}{5}}{1\frac{1}{10}}$

(v) $\frac{2}{5}$ of $\frac{6}{11} \times 1\frac{1}{4}$ (vi) $2\frac{1}{4} \div \frac{1}{7} \times \frac{1}{3}$ (vii) $\frac{1}{3} \times 4\frac{2}{3} \div 3\frac{1}{2} \times \frac{1}{2}$

(viii) $\frac{2}{3} \times 1\frac{1}{4} \div \frac{3}{7}$ of $2\frac{5}{8}$ (ix) $0 \div \frac{8}{11}$ (x) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9}$

(xi) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9}$ (xii) $\frac{4}{5}$ of $\frac{7}{15} \div \frac{8}{9}$ (xiii) $\frac{1}{2}$ of $\frac{3}{4} \times \frac{1}{2} \div \frac{2}{3}$

6. A bought $3\frac{3}{4}$ kg of wheat and $2\frac{1}{2}$ kg of rice. Find the total weight of wheat and rice bought.

7. Which is greater, $\frac{3}{5}$ or $\frac{7}{10}$ and by how much ?

8. What number should be added to $8\frac{2}{3}$ to get $12\frac{5}{6}$?

9. What should be subtracted from $8\frac{3}{4}$ to get $2\frac{2}{3}$?

10. A rectangular field is $16\frac{1}{2}$ m long and $12\frac{2}{5}$ m wide. Find the perimeter of the field.

11. Sugar costs ₹ $37\frac{1}{2}$ per kg. Find the cost of $8\frac{3}{4}$ kg sugar.

12. A motor cycle runs $31\frac{1}{4}$ km consuming 1 litre of petrol. How much distance will it run consuming $1\frac{3}{5}$ litre of petrol ?

13. A rectangular park has length = $23\frac{2}{5}$ m and breadth = $16\frac{2}{3}$ m. Find the area of the park.

14. Each of 40 identical boxes weighs $4\frac{4}{5}$ kg. Find the total weight of all the boxes.

15. Out of 24 kg of wheat, $\frac{5}{6}$th of wheat is consumed. Find, how much wheat is still left ?

16. A rod of length $2\frac{2}{5}$ metre is divided into five equal parts. Find the length of each part so obtained.

17. If $A = 3\frac{3}{8}$ and $B = 6\frac{5}{8}$, find : (i) $A \div B$ (ii) $B \div A$.

18. Cost of $3\frac{5}{7}$ litres of oil is ₹ $83\frac{1}{2}$. Find the cost of one litre oil.

19. The product of two numbers is $20\frac{5}{7}$. If one of these numbers is $6\frac{2}{3}$, find the other.

20. By what number should $5\frac{5}{6}$ be multiplied to get $3\frac{1}{3}$?

3.13 USING BRACKETS

The types of brackets used, in general, are :

(i) ( ) are known as *Circular brackets* or *Parentheses* or *simply small brackets.*

(ii) { } are known as *Curly (middle) brackets.*

(iii) [ ] are known as *Square brackets* or *Box brackets or big brackets.*

Sometimes a **bar** is drawn above some terms which we want to treat as a single quantity.

e.g., (i) $\overline{4+5}$ means $(4 + 5) = 9$ (ii) $8 - \overline{3+2} = 8 - 5 = 3$

(iii) $3 + \overline{8-6} = 3 + 2 = 5$ and so on.

This "——" is known as **Bar bracket** or **Vinculum.**

Note : Multiplication sign is often omitted before a bracket and between the brackets.

e.g., (i) $4(9 - 3) = 4 \times (9 - 3) = 4 \times 6 = 24$

(ii) $(2 + 8)(7 - 3) = (2 + 8) \times (7 - 3) = 10 \times 4 = 40$

3.14 REMOVAL OF BRACKETS

The brackets are removed in the order given below :

(i) ——; bar or vinculum, (ii) ( ) ; parentheses,

(iii) { } ; curly brackets, (iv) [ ] ; square brackets.

Example 11 :

Simplify : $10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - \overline{6-4})\}\right]$

Solution :

$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - 2)\}\right]$ $[\because \overline{6-4} = 2]$

$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - 5\}\right]$ $[\because (7-2) = 5]$

$= 10\frac{1}{2} - \left[8\frac{1}{2} + 1\right]$ $[\because \{6-5\} = 1]$

$= 10\frac{1}{2} - 9\frac{1}{2}$ $\left[\because 8\frac{1}{2} + 1 = 9\frac{1}{2}\right]$

$= 1$ **(Ans.)**

1. *Whenever there is a positive (+) sign before a bracket, the bracket is removed without any change in the signs of its terms.*
   e.g., $8 + (3 - 1 + 5) = 8 + 3 - 1 + 5 = 16 - 1 = 15$
2. *Whenever there is a negative (–) sign before a bracket, the bracket is removed by changing the signs of all the terms inside the bracket (i.e., by changing every positive sign into negative and every negative sign into positive)*
   e.g., $8 - (3 - 1 + 5) = 8 - 3 + 1 - 5 = 9 - 8 = 1$

EXERCISE 3(D)

Simplify :

1. $6 + \left\{\frac{4}{3} + \left(\frac{3}{4} - \frac{1}{3}\right)\right\}$
2. $8 - \left\{\frac{3}{2} + \left(\frac{3}{5} - \frac{1}{2}\right)\right\}$
3. $\frac{1}{4}\left(\frac{1}{4} + \frac{1}{3}\right) - \frac{2}{5}$
4. $2\frac{3}{4} - \left[3\frac{1}{8} \div \left\{5 - \left(4\frac{2}{3} - \frac{11}{12}\right)\right\}\right]$
5. $12\frac{1}{2} - \left[8\frac{1}{2} + \left\{9 - \left(5 - \overline{3-2}\right)\right\}\right]$
6. $1\frac{1}{5} \div \left\{2\frac{1}{3} - \left(5 + \overline{2-3}\right)\right\} - 3\frac{1}{2}$
7. $\left(\frac{1}{2} + \frac{2}{3}\right) \div \left(\frac{3}{4} - \frac{2}{9}\right)$
8. $\frac{6}{5}$ of $\left(3\frac{1}{3} - 2\frac{1}{2}\right) \div \left(2\frac{5}{21} - 2\right)$
9. $10\frac{1}{8}$ of $\frac{4}{5} \div \frac{35}{36}$ of $\frac{20}{49}$
10. $5\frac{3}{4} - \frac{3}{7} \times 15\frac{3}{4} + 2\frac{2}{35} \div 1\frac{11}{25}$
11. $\frac{3}{4}$ of $7\frac{3}{7} - 5\frac{3}{5} \div 3\frac{4}{15}$

3.15 PROBLEMS INVOLVING FRACTIONS

Example 12 :

What fraction is 6 bananas of four dozen bananas ?

Solution :

Here 6 bananas are to be compared with 4 dozens *i.e.,* $4 \times 12 = 48$ bananas.

$\therefore$ **Required fraction** $= \frac{6}{48} = \frac{1}{8}$ **(Ans.)**

Example 13 :

Write all the natural numbers that lie between 5 and 15 ?

(i) How many of these natural numbers are odd ?

(ii) What fraction of these natural numbers are even ?

Solution :

Since, natural numbers between 5 and 15 are : 6, 7, 8, 9, 10, 11, 12, 13 and 14.

$\therefore$ **There are 9 natural numbers between 5 and 15.** **(Ans.)**

(i) Out of these natural numbers, odd natural numbers are : 7, 9, 11 and 13.

$\therefore$ **There are 4 odd natural numbers between 5 and 15.** **(Ans.)**

(ii) Out of all the given 9 natural numbers, 4 are odd.

$\therefore$ Remaining $9 - 4 = 5$ numbers are even.

So, the required fraction $= \frac{5}{9}$ **(Ans.)**

Example 14 :

The monthly income of a man is ₹ 18,000. He gives one-third of it to his wife and one-third of the remaining he spends on his children's education. Find :

(i) the money he gave to his wife.

(ii) the money he spends on his children's education.

(iii) the money still left with him.

Solution :

(i) **The man gives to his wife** $= \frac{1}{3}$ of ₹ 18,000

$= \frac{1}{3} \times$ ₹ 18,000 = **₹ 6,000** **(Ans.)**

(ii) Since, remaining money = ₹ 18,000 − ₹ 6,000 = ₹ 12,000

He spends on his children's education $= \frac{1}{3} \times$ ₹ 12,000 $= \frac{1}{3} \times$ ₹ 12,000 = **₹ 4,000** **(Ans.)**

(iii) **The money still left with the man**

= ₹ 12,000 − ₹ 4,000 = **₹ 8,000** **(Ans.)**

Example 15 :

Subtract the sum of $\frac{1}{4}$ and $\frac{3}{8}$ from the sum of $\frac{2}{3}, \frac{3}{4}$ and $\frac{7}{12}$.

Solution :

$\because$ Sum of $\frac{1}{4}$ and $\frac{3}{8} = \frac{1}{4} + \frac{3}{8} = \frac{2+3}{8} = \frac{5}{8}$

And, sum of $\frac{2}{3}, \frac{3}{4}$ and $\frac{7}{12} = \frac{2}{3} + \frac{3}{4} + \frac{7}{12} = \frac{8+9+7}{12} = \frac{24}{12} = 2$

$\therefore$ **Required number** $= 2 - \frac{5}{8} = \frac{2}{1} - \frac{5}{8} = \frac{16-5}{8} = \frac{11}{8} = 1\frac{3}{8}$ **(Ans.)**

or, directly, $\left(\frac{2}{3} + \frac{3}{4} + \frac{7}{12}\right) - \left(\frac{1}{4} + \frac{3}{8}\right) = \left(\frac{8+9+7}{12}\right) - \left(\frac{2+3}{8}\right)$

$= \frac{24}{12} - \frac{5}{8}$

$= \frac{48-15}{24} = \frac{33}{24} = \frac{11}{8} = 1\frac{3}{8}$ **(Ans.)**

Example 16 :

A man spent $\frac{2}{7}$ of his savings and still has ₹ 1,000 left with him. How much were his savings ?

Solution :

The man spent $\frac{2}{7}$ of his money.

$\therefore$ He still has $1 - \frac{2}{7} = \frac{5}{7}$ of his savings

Note : In fractions, the whole quantity is always taken as 1.

Since, $\frac{5}{7}$ of his savings = ₹ 1,000

$\therefore$ His savings = ₹ 1,000 $\div \frac{5}{7}$ = ₹ 1,000 $\times \frac{7}{5}$ = **₹ 1,400** **(Ans.)**

Example 17 :

$\frac{4}{7}$ of a pole is in the mud. When $\frac{1}{3}$ of it is pulled out, 250 cm of the pole is still in the mud. What is the full length of the pole ?

Solution :

$\frac{4}{7}$ of the pole $- \frac{1}{3}$ of the pole = 250 cm

$\Rightarrow$ $\left(\frac{4}{7} - \frac{1}{3}\right)$ of the pole = 250 cm

$\Rightarrow$ $\frac{5}{21}$ of the pole = 250 cm $\left[\frac{4}{7} - \frac{1}{3} = \frac{12-7}{21} = \frac{5}{21}\right]$

$\Rightarrow$ **Length of the pole** $= 250 \times \frac{21}{5}$ cm = **1050 cm** **(Ans.)**

EXERCISE 3(E)

1. A line AB is of length 6 cm. Another line CD is of length 15 cm. What fraction is :
   (i) the length of AB to that of CD ?
   (ii) $\frac{1}{2}$ the length of AB to that of $\frac{1}{3}$ of CD ?
   (iii) $\frac{1}{5}$ of CD to that of AB ?
2. Subtract $\left(\frac{2}{7} - \frac{5}{21}\right)$ from the sum of $\frac{3}{4}$, $\frac{5}{7}$ and $\frac{7}{12}$.
3. From a sack of potatoes weighing 120 kg, a merchant sells portions weighing 6 kg, $5\frac{1}{4}$ kg, $9\frac{1}{2}$ kg and $9\frac{3}{4}$ kg respectively.
   (i) How many kg did he sell ? (ii) How many kg are still left in the sack ?
4. If a boy works for six consecutive days for 8 hours, $7\frac{1}{2}$ hours, $8\frac{1}{4}$ hours, $6\frac{1}{4}$ hours, $6\frac{3}{4}$ hours and 7 hours respectively, how much money will he earn at the rate of ₹ 36 per hour ?
5. A student bought $4\frac{1}{3}$ m of yellow ribbon, $6\frac{1}{6}$ m of red ribbon and $3\frac{2}{9}$ m of blue ribbon for decorating a room. How many metres of ribbon did he buy ?
6. In a business, Ram and Deepak invest $\frac{3}{5}$ and $\frac{2}{5}$ of the total investment. If ₹ 40,000 is the total investment, calculate the amount invested by each.
7. Geeta had 30 problems for home work. She worked out $\frac{2}{3}$ of them. How many problems were still left to be worked out by her ?
8. A picture was marked at ₹ 90. It was sold at $\frac{3}{4}$ of its marked price. What was the sale price ?
9. Mani had sent fifteen parcels of oranges. What was the total weight of the parcels, if each weighed $10\frac{1}{2}$ kg ?
10. A rope is $25\frac{1}{2}$ m long. How many pieces each of $1\frac{1}{2}$ m length can be cut out from it ?
11. The heights of two vertical poles, above the earth's surface, are $14\frac{1}{4}$ m and $22\frac{1}{3}$ m respectively. How much higher is the second pole as compared with the height of the first pole ?
12. Vijay weighed $65\frac{1}{2}$ kg. He gained $1\frac{2}{5}$ kg during the first week, $1\frac{1}{4}$ kg during the second week, but lost $\frac{5}{16}$ kg during the third week. What was his weight after the third week ?

13. A man spends $\frac{2}{5}$ of his salary on food and $\frac{3}{10}$ on house rent, electricity, etc. What fraction of his salary is still left with him ?

14. A man spends $\frac{2}{5}$ of his salary on food and $\frac{3}{10}$ of the remaining on house rent, electricity, etc. What fraction of his salary is still left with him ?

15. Shyam bought a refrigerator for ₹ 5,000. He paid $\frac{1}{10}$ of the price in cash and the rest in 12 equal monthly instalments. How much had he to pay each month ?

16. A lamp post has half of its length in mud and $\frac{1}{3}$ of its length in water.

(i) What fraction of its length is above the water ?

(ii) If $3\frac{1}{3}$ m of the lamp post is above the water, find the whole length of the lamp post.

17. I spent $\frac{3}{5}$ of my savings and still have ₹ 2,000 left. What were my savings ?

18. In a school $\frac{4}{5}$ of the children are boys. If the number of girls is 200, find the number of boys.

19. If $\frac{4}{5}$ of an estate is worth ₹ 42,000, find the worth of the whole estate.

Also, find the value of $\frac{3}{7}$ of it.

20. After going $\frac{3}{4}$ of my journey, I find that I have covered 16 km. How much journey is still left ?

21. When Krishna travelled 25 km, he found that $\frac{3}{5}$ of his journey was still left. What was the length of the whole journey?

22. From a piece of land, one-third is bought by Rajesh and one-third of remaining is bought by Manoj. If 600 m^2 land is still left unsold, find the total area of the piece of land.

23. A boy spent $\frac{3}{5}$ of his money on buying cloth and $\frac{1}{4}$ of the remaining on buying shoes. If initially he has ₹ 2,400; how much did he spend on shoes ?

24. A boy spent $\frac{3}{5}$ of his money on buying cloth and $\frac{1}{4}$ of his money on buying shoes. If initially he has ₹ 2,400 how much did he spend on shoes.

DECIMAL FRACTIONS 4

(Decimals)

4.1 DEFINITION

A decimal fraction or a decimal number ***is a fraction whose denominator can be expressed as 10 or some higher power of 10.***

(i) $\frac{7}{10}, \frac{13}{100}, \frac{851}{1000}, \frac{79}{10^4}, \frac{2547}{10^7}$, etc., are all decimal fractions.

(ii) Since, $\frac{3}{5} = \frac{3\times2}{5\times2} = \frac{6}{10}$ *i.e.*, $\frac{3}{5}$ can be expressed as a fraction with denominator 10, therefore $\frac{3}{5}$ is a decimal fraction.

(iii) $\frac{7}{8} = \frac{7\times125}{8\times125} = \frac{875}{1000}$ *i.e.*, $\frac{7}{8}$ can be expressed as a fraction with denominator 1000 (*i.e.* 10^3, which is higher power of 10), therefore $\frac{7}{8}$ is also a decimal fraction.

For the same reason, each of $\frac{7}{20}, \frac{16}{25}, \frac{33}{50}, \frac{64}{125}$, etc., is also a decimal fraction.

In order to express a given decimal fraction in shorter form, *the denominator is not written, but its absence is shown by a dot* called *a decimal point, inserted in a proper place.*

e.g., $\frac{3}{10} = 0{\cdot}3$; $\frac{213}{100} = 2{\cdot}13$; $\frac{7}{100} = 0{\cdot}07$; $\frac{59}{10^4} = \frac{59}{10000} = 0{\cdot}0059$, etc.

1. *When there is no number to the left of the decimal point, generally, a zero is written.*
   i.e. (i) *·72 is written as 0·72*
   (ii) *·004 is written as 0·004 and so on.*
2. *2·4 means, 2 + 0·4.*
   Here, 2 is the integral part and 0·4 is the decimal part of the number 2·4.
3. *Any extra zero or zeroes written after the decimal part of a number does not change its value.*
   e.g., value of 3·5 is the same as 3·50 or 3·500 or 3·5000 and so on.

4.2 READING DECIMAL NUMBERS

The integral part is read according to its value and decimal part is read by naming each digit, in order, separately.

e.g., (i) 21·45 will be read as : Twenty one ***point*** four-five.

(ii) 152·639 will be read as : One fifty two ***point*** six-three-nine.

(iii) 0·08 will be read as : ***Point*** zero-eight or ***zero-point*** zero-eight.

In **decimal system**, the **first place** on the right of the decimal point is called **tenths'** place, **second place** to the right of decimal is called **hundredths'** place and so on.

Similarly, the **first place on the left of decimal** is the **units'** place, the **second place on the left of decimal** is the **tens'** place, and so on.

e.g., in number 5·46; 5 is at units' place, 4 is at tenths' place and 6 is at hundredths' place.

The following table shows the place values of different digits in a decimal number :

Number	Thousands (1000)	Hundreds (100)	Tens (10)	Units (1)	Decimal point	Tenths (1/10)	Hundredths (1/100)	Thousandths (1/1000)	Ten thousandths (1/10000)
(i) 45·986			4	5	·	9	8	6	
(ii) 936·4527		9	3	6	·	4	5	2	7
(iii) 7042·93	7	0	4	2	·	9	3		

4.3 CONVERTING A DECIMAL NUMBER INTO A FRACTION

Remove decimal point from the given decimal number. And, in its denominator write as many zeroes, as the number of digits in the decimal part, to the right of 1. Then simplify, if possible, to get the fraction obtained to its lowest terms.

Thus, (i) $0{\cdot}27 = \frac{27}{100}$, (ii) $3{\cdot}64 = \frac{364}{100} = \frac{91}{25} = 3\frac{16}{25}$,

(iii) $5{\cdot}750 = \frac{5750}{1000} = \frac{23}{4} = 5\frac{3}{4}$

(iv) $0{\cdot}0244 = \frac{00244}{10000} = \frac{61}{2500}$ and so on.

4.4 CONVERTING A GIVEN FRACTION INTO A DECIMAL FRACTION

1. When the denominator of the given fraction is 10, 100, 1000, etc. :

   Counting from extreme right to left, mark the decimal point after as many digits of the numerator as there are zeroes in the denominator.

   Thus, (i) $\frac{259}{10} = 25{\cdot}9$, (ii) $\frac{259}{100} = 2{\cdot}59$, (iii) $\frac{259}{1000} = 0{\cdot}259$,

   (iv) $\frac{259}{10000} = 0{\cdot}0259$ and so on.

2. When the denominator of the given fraction is other than 10, 100, 1000, etc. :

   Divide in an ordinary way and mark the decimal point in the quotient just after the division of unit digit is completed. After this, any number of zeroes (one by one) can be placed to complete the division.

Thus, (i) $\frac{15}{4}$ =

```
    3·75
 4 ) 15
     12
     ---
      30
      28
      ---
       20
       20
       ---
        ×
```

Ans. = 3·75

(ii) $\frac{27}{5}$ =

```
    5·4
 5 ) 27
     25
     ---
      20
      20
      ---
       ×
```

Ans. = 5·4

4.5 DECIMAL PLACES

The number of figures that follow the decimal point is called the number of decimal places.

Thus, 28·497 has 3 decimal places, 153·46 has 2 decimal places, 0·5497 has 4 decimal places and so on.

1. **Like decimals :** Decimals having the same number of decimal places are called **like decimals**.

   e.g. (i) 3·9, 8·7, 12·9, 132·4, etc.

   (ii) 9·45, 8·37, 25·03, 79·56, etc.

   (iii) 2·013, 56·826, 293·422, etc.

2. **Unlike decimals :** Decimals having unequal number of decimal places are called **unlike decimals**.

   e.g. Fraction 5·97 has two decimal places

   and 42·038 has three decimal places

   so 5·97 and 42·038 are unlike fractions.

   Similarly,

   (i) 4·308 and 0·8007 are unlike fractions.

   (ii) 80·76 and 235·8062 are unlike fractions and so on.

Any number of zeros put at the end (*i.e.* on the right side) of a decimal number does not change its value.

e.g. (i) 5·4 = 5·40 = 5·400 = 5·4000 = ... and so on.

(ii) 0·07 = 0·070 = 0·0700 = 0·07000 = ... and so on.

Important :

0·9, 5·32, 17·054 and 232·47703 are unlike decimals whereas

0·90000, 5·32000, 17·05400 and 232·47703 are like decimals.

Example 1 :

Convert each of the following decimal fractions into vulgar fraction in lowest terms :

(i) 0·125 (ii) 5·08 (iii) 26·25

Solution :

(i) $\mathbf{0{\cdot}125} = \frac{125}{1000} = \frac{1}{8}$ **(Ans.)**

(ii) $\mathbf{5{\cdot}08} = \frac{508}{100} = \frac{127}{25} = 5\frac{2}{25}$ **(Ans.)**

OR, $\mathbf{5{\cdot}08} = 5 + 0{\cdot}08$

$= 5 + \frac{8}{100} = 5 + \frac{2}{25} = 5\frac{2}{25}$ **(Ans.)**

(iii) $\mathbf{26{\cdot}25} = \frac{2625}{100} = \frac{105}{4} = 26\frac{1}{4}$ **(Ans.)**

OR, $\mathbf{26{\cdot}25} = 26 + 0.25$

$= 26 + \frac{25}{100} = 26 + \frac{1}{4} = 26\frac{1}{4}$ **(Ans.)**

Example 2 :

Convert each of the following into decimal fraction :

(i) $5\frac{3}{8}$ (ii) $\frac{2}{25}$ (iii) $2\frac{7}{100}$

Solution :

(i) $5\frac{3}{8} = \frac{5 \times 8 + 3}{8} = \frac{43}{8} = 5{\cdot}375$ **(Ans.)**

OR, $5\frac{3}{8} = 5 + \frac{3}{8} = 5 + 0.375 = 5{\cdot}375$ **(Ans.)**

(ii) $\frac{2}{25} = 0{\cdot}08$ **(Ans.)**

OR, $\frac{2}{25} = \frac{2 \times 4}{25 \times 4} = \frac{8}{100} = 0{\cdot}08$ **(Ans.)**

(iii) $2\frac{7}{100} = \frac{2 \times 100 + 7}{100} = \frac{207}{100} = 2{\cdot}07$ **(Ans.)**

OR, $2\frac{7}{100} = 2 + \frac{7}{100} = 2 + 0{\cdot}07 = 2{\cdot}07$ **(Ans.)**

EXERCISE 4(A)

1. Convert the following into fractions in their lowest terms :
   (i) 3·75 (ii) 0·5 (iii) 2·04 (iv) 0.65
   (v) 2.405 (vi) 0·085 (vii) 8·025
2. Convert into decimal fractions :
   (i) $2\frac{4}{5}$ (ii) $\frac{79}{100}$ (iii) $\frac{37}{10,000}$ (iv) $\frac{7543}{10^4}$
   (v) $\frac{3}{4}$ (vi) $9\frac{3}{5}$ (vii) $8\frac{5}{8}$ (viii) $5\frac{7}{8}$
3. Write the number of decimal places in :
   (i) 0·4762 (ii) 7·00349 (iii) 8235·403
   (iv) 35·4 (v) 2·608 (vi) 0·000879
4. Write the following decimals as word statements :
   (i) 0·4, 0·9, 0·1 (ii) 1·9, 4·4, 7·5 (iii) 0·02, 0·56, 13·06
   (iv) 0·005, 0·207, 111·519 (v) 0·8, 0·08, 0·008, 0·0008 (vi) 256·1, 10·22, 0·634
5. Convert the given fractions into like fractions :
   (i) 0·5, 3·62, 43·981 and 232·0037
   (ii) 215·78, 33·0006, 530·3 and 0·03569

4.6 ADDITION OF DECIMALS

Write the given decimal numbers in such a way, that the decimal points of all the numbers fall in the same vertical line. Digits with the same place value are placed one below the other, *i.e.*, units are written below units, tens below tens and so on.

Addition is started from the right side, as done in the usual addition (empty places may be filled up by zeroes). In the result (total), the decimal point is placed under decimal points of the numbers added.

Example 3 :

Add : (i) 3·92, 11·057 and 236·84 (ii) 2·8, 104·91 and 37·056

Solution :

[Converting into like decimals]

(i)
```
   3.92
  11.057
 236.84
--------
 251.817   (Ans.)
--------
```

```
    3·920
 + 11·057
 + 236·840
----------
  251·817   (Ans.)
----------
```

(ii)
```
     2.8
 + 104.91
 +  37.056
----------
   144.766   (Ans.)
----------
```

```
     2·800
 + 104·910
 +  37·056
----------
   144·766   (Ans.)
----------
```

Note : A whole number can also be expressed as a decimal number by putting a decimal after its last (unit) digit and after it as many zeroes as are required.

e.g. 15 = 15·0 = 15·00 = 15·0000, etc.

4.7 SUBTRACTION OF DECIMALS

In subtraction also, the numbers are written in such a way that their decimals are in the same vertical line. Digits with the same place value are placed one below the other empty places may be filled by zeros to get the like decimals.

Subtraction is started from the right side, as in the case of normal subtraction.

In the result, decimal point is placed just under the other decimal points.

Example 4 :

Subtract : (i) 5·27 from 13·89 (ii) 0·283 from 2
(iii) 0·45 from 4·5 (iv) 0·5 from 0·84

Solution :

(i)
```
   13·89
 −  5·27
--------
    8·62   (Ans.)
--------
```

(ii)
```
   2·000
 − 0·283
--------
   1·717   (Ans.)
--------
```

(iii)
```
   4·50
 − 0·45
-------
   4·05   (Ans.)
-------
```

(iv)
```
   0·84
 − 0·50
-------
   0·34   (Ans.)
-------
```

Example 5 :

Evaluate : 11·349 – 5·57 + 9·28 – 12·6.

Solution :

11·349 – 5·57 + 9·28 – 12·6

= 11·349 – 5·570 + 9·280 – 12·600 [converting into like decimals]

= (11·349 + 9·280) – (5·570 + 12·600)

= 20·629 – 18·170 = **2·459** **(Ans.)**

EXERCISE 4(B)

1. Add :

(i) 0·5 and 0·37
(ii) 3·8 and 8·7
(iii) 0·02, 0·008 and 0·309
(iv) 0·4136, 0·3195 and 0·52
(v) 9·25, 3·4 and 6·666
(vi) 3·007, 0·587 and 18·341
(vii) 0·2, 0·02 and 2·0002
(viii) 6·08, 60·8, 0·608 and 0·0608
(ix) 29·03, 0·0003, 0·3 and 7·2
(x) 3·4, 2·025, 9·36 and 3·6221

2. Subtract the first number from the second :

(i) 5·4, 9·8
(ii) 0·16, 4·3
(iii) 0·82, 8·6
(iv) 0·07, 8·43
(v) 2·237, 9·425
(vi) 41·03, 59·46
(vii) 3·92, 26·86
(viii) 4·73, 8·5
(ix) 12·63, 36·2
(x) 0·845, 3·71

3. Simplify :

(i) 28·796 – 13·42 – 2·555
(ii) 93·354 – 62·82 – 13·045
(iii) 36 – 18·59 – 3·2
(iv) 86 + 16·95 – 3·0042
(v) 32·8 – 13 – 10·725 + 3·517
(vi) 4000 – 30·51 – 753·101 – 69·43
(vii) 0·1835 + 163·2005 – 25·9 – 100
(viii) 38·00 – 30 + 200·200 – 0·230
(ix) 555·555 + 55·555 – 5·55 – 0·555

4. Find the difference between 6·85 and 0·685.

5. Take out the sum of 19·38 and 56·025, then subtract it from 200·111.

6. Add 13·95 and 1·003, and from the result, subtract the sum of 2·794 and 6·2.

7. What should be added to 39·587 to give 80·375 ?

8. What should be subtracted from 100 to give 19·29 ?

9. What is the excess of 584·29 over 213·95 ?

10. Evaluate :

(i) (5·4 – 0·8) + (2·97 – 1·462)
(ii) (6·25 + 0·36) – (17·2 – 8·97)
(iii) 9·004 + (3 – 2·462)
(iv) 879·4 – (87·94 – 8·794)

11. What is the excess of 75 over 48·29 ?

12. If A = 237·98 and B = 83·47. Find : (i) A – B (ii) B – A.

13. The cost of one kg of sugar increases from ₹ 28·47 to ₹ 32·65. Find the increase in cost.

4.8 MULTIPLICATION OF DECIMAL NUMBERS

1. ***Multiplication by 10, 100, 1000, etc. :***

Shift the decimal point, in the multiplicand, to the right by as many digits as there are zeroes in the multiplier.

Thus, (i) $3{\cdot}2985 \times 10 = 32{\cdot}985$

(ii) $3{\cdot}2985 \times 100 = 329{\cdot}85$

(iii) $3{\cdot}2985 \times 1000 = 3298{\cdot}5$ and so on.

[Here, multiplicand is 3·2985 and multipliers are 10, 100, 1000, etc.]

2. ***Multiplication by a whole number :***

Multiply in an ordinary way, without considering the decimal point.

In the product, the decimal point should be fixed by counting as many digits from the right as there are decimal places in the multiplicand.

Thus, (i) $0{\cdot}3 \times 6 = 1{\cdot}8$

(ii) $0{\cdot}26 \times 18 = 4{\cdot}68$ and so on.

3. ***Multiplication of a decimal number by another decimal number :***

Multiply the two numbers in a normal way, ignoring their decimals.

In the product, decimal point is fixed counting from right, the digits equal to the sum of decimal places in the multiplicand and the multiplier.

Thus, $32{\cdot}5 \times 0{\cdot}07 = 2{\cdot}275$

Since, the multiplicand (32·5) has one decimal place and the multiplier (0·07) has two decimal places, their product will have $1 + 2 = 3$ decimal places.

Similarly : (i) $0{\cdot}2 \times 0{\cdot}0004 = 0{\cdot}00008$

(ii) $2{\cdot}895 \times 1{\cdot}1 = 3{\cdot}1845$ and so on.

4.9 DIVISION OF DECIMALS

1. ***Division by 10, 100, 1000, etc. :***

Shift the decimal point to the left by as many digits as there are zeroes in the divisor.

Thus, (i) $623{\cdot}42 \div 10 = 62{\cdot}342$ [Shifting decimal point, one digit to the left]

(ii) $623{\cdot}42 \div 100 = 6{\cdot}2342$ [Shifting decimal point, two digits to the left]

(iii) $623{\cdot}42 \div 10000 = 0{\cdot}062342$ and so on.

2. ***Division by a whole number :***

Divide in the normal manner, ignoring the decimal, and mark the decimal point in the quotient, while just crossing over the decimal point in the dividend.

Thus, (i) **16·952 ÷ 8** (ii) **0·945 ÷ 9**

$$
\begin{array}{r}
2.119 \\
= 8 \overline{)16.952} \\
\underline{16} \\
9 \\
\underline{8} \\
15 \\
\underline{8} \\
72 \\
\underline{72} \\
\times
\end{array}
\qquad\qquad
\begin{array}{r}
0.105 \\
= 9 \overline{)0.945} \\
\underline{9} \\
45 \\
\underline{45} \\
\times
\end{array}
$$

$\therefore$ **16·952 ÷ 8 = 2·119 (Ans.)** $\qquad$ $\therefore$ **0·945 ÷ 9 = 0·105 (Ans.)**

3. ***Division of a decimal number by another decimal number :***

Shift the decimal points of the dividend and the divisor both by as many equal number of digits, so that the divisor converts into a whole number.

The division is then carried out as in Case 2 discussed above.

Thus, (i) **4·8 ÷ 0·8** $= \dfrac{4 \cdot 8}{0 \cdot 8} = \dfrac{48}{8} =$ **6**

(ii) **5·625 ÷ 1·25** $= \dfrac{5 \cdot 625}{1 \cdot 25}$

$= \dfrac{562 \cdot 5}{125}$

$=$ **4·5**

$$
\begin{array}{r}
4.5 \\
125 \overline{)562.5} \\
\underline{500} \\
625 \\
\underline{625} \\
\times
\end{array}
$$

Now consider $182 \cdot 37 \div 2 \cdot 3 = \dfrac{182 \cdot 37}{2 \cdot 3} = \dfrac{1823 \cdot 7}{23}$

$$
\begin{array}{r}
79.291 \\
23 \overline{)1823.7} \\
\underline{161} \\
213 \\
\underline{207} \\
67 \\
\underline{46} \\
210 \\
\underline{207} \\
30 \\
\underline{23} \\
7
\end{array}
\text{ and so on.}
$$

Here, the division can not be completed, *i.e.*, all the digits in the dividend are exhausted and still there is some remainder left. So, we go on writing zeroes (one by one) with the remainder and continue the division process. *We can write zeroes, because adding zero at the extreme right of a decimal number does not change the number.* Therefore, division can be continued for as many decimal places as we like to.

$\therefore$ **182·37 ÷ 2·3 = 79·291** *upto 3 decimal places.*

EXERCISE 4(C)

1. Multiply :

(i) 0·87 by 10 (ii) 2·948 by 100 (iii) 6·4 by 1000 (iv) 5·8 by 4

(v) 16·32 by 28 (vi) 5·037 by 8 (vii) 4·6 by 2·1 (viii) 0·568 by 6·4

2. Multiply each number by 10, 100 and 1000 :

(i) 0·5 (ii) 0·112 (iii) 4·8 (iv) 0·0359
(v) 16·27 (vi) 234·8

3. Evaluate :

(i) $5·897 \times 2·3$ (ii) $0·894 \times 87$ (iii) $0·01 \times 0·001$
(iv) $0·84 \times 2·2 \times 4$ (v) $4·75 \times 0·08 \times 3$ (vi) $2·4 \times 3·5 \times 4·8$
(vii) $0·8 \times 1·2 \times 0·25$ (viii) $0·3 \times 0·03 \times 0·003$

4. Divide :

(i) 54·9 by 10 (ii) 7·8 by 100 (iii) 324·76 by 1000
(iv) 12·8 by 4 (v) 27·918 by 9 (vi) 4·672 by 8
(vii) 4·32 by 1·2 (viii) 7·644 by 1·4 (ix) 4·8432 by 0·08

5. Divide each of the given numbers by 10, 100, 1000 and 10000 :

(i) 2·1 (ii) 8·64 (iii) 5·01
(iv) 0·0906 (v) 0·125 (vi) 111·11

6. Evaluate :

(i) $9·75 \div 5$ (ii) $4·4064 \div 4$ (iii) $27·69 \div 30$ (iv) $19·25 \div 25$
(v) $20·64 \div 16$ (vi) $3·204 \div 9$ (vii) $0·125 \div 25$ (viii) $0·14616 \div 72$
(ix) $0·6227 \div 1300$ (x) $257·894 \div 0·169$ (xi) $6·3 \div (0·3)^2$

7. Evaluate :

(i) $4·3 \times 0·52 \times 0·3$ (ii) $3·2 \times 2·5 \times 0·7$ (iii) $0·8 \times 1·5 \times 0·6$
(iv) $0·3 \times 0·3 \times 0·3$ (v) $1·2 \times 1·2 \times 0·4$ (vi) $0·4 \times 0·04 \times 0·004$
(vii) $0·5 \times 0·6 \times 0·7$ (viii) $0·5 \times 0·06 \times 0·007$

8. Evaluate :

(i) $(0·9)^2$ (ii) $(0·6)^2 \times 0·5$ (iii) $0·3 \times (0·5)^2$ (iv) $(0·4)^3$
(v) $(0·2)^3 \times 5$ (vi) $(0·2)^3 \times 0·05$

9. Find the cost of 36·75 kg wheat at the rate of ₹ 12·80 per kg.

10. The cost of a pen is ₹ 56·15. Find the cost of 16 such pens.

11. Evaluate :

(i) $0·0072 \div 0·06$ (ii) $0·621 \div 0·3$ (iii) $0·0532 \div 0·005$
(iv) $0·01162 \div 0·14$ (v) $(7·5 \times 40·4) \div 25$ (vi) $2·1 \div (0·1 \times 0·1)$

12. Fifteen identical articles weigh 31·50 kg. Find the weight of each article.

13. The product of two numbers is 211·2. If one of these two numbers is 16·5, find the other number.

14. One dozen identical articles cost ₹ 45·96. Find the cost of each article.

4.10 TERMINATING DECIMALS

Sometimes, in a division, the dividend is exactly divisible and *no remainder is left* after certain number of steps. Such answers in the quotient are called *terminating decimals.*

Thus, $31·76 \div 4 = \dfrac{31·76}{4} = 7·94$ *is a terminating decimal.*

4.11 NON-TERMINATING DECIMALS

In certain cases, while dividing *the remainder does not finish* (terminate) no matter how long the division is continued.

In such cases the quotient is called *non-terminating decimal.*

Thus, $13{\cdot}78 \div 7 = \frac{13{\cdot}78}{7}$

$= 1{\cdot}9685$ ......

which *is a non-terminating decimal.*

The fact, that it is a non-terminating decimal is shown by writing the digits of the quotient till the division is carried out. After that few dots are put to show that this division continues.

1·9685
7) 13·78 (
7
67
63
48
42
60
56
40
35
5 and so on.

4.12 RECURRING (Repeating) DECIMALS

On performing a division, sometimes we find that the same remainder is left, no matter how long we continue the division process.

***Consider* : 2 ÷ 3**

Here, the remainder is always 2. For this reason, same digit 6 appears again and again in the quotient.

This fact is shown by putting a dot or a bar over the repeating digit (or digits) in the quotient, *i.e.*, **2 ÷ 3** = 0·666 ... = $\mathbf{0{\cdot}\dot{6}}$ or $\mathbf{0{\cdot}\overline{6}}$

Here, $0{\cdot}\dot{6}$ (or $0{\cdot}\overline{6}$), which is a non-terminating repeating decimal, is called a **recurring decimal**. The dot or bar over 6 shows that 6 repeats infinitely.

0·666........
3) 2·000 (
18
20
18
20
18
2 and so on.

Similarly, consider 15 ÷ 22.

In this division, the remainders 18 and 4 keep repeating alternately and so are the digits 8 and 1 in the quotient.

$\therefore$ **15 ÷ 22** = $\frac{15}{22}$ = 0·68181......

$= \mathbf{0{\cdot}6\dot{8}\dot{1}}$ **or** $\mathbf{0{\cdot}6\overline{81}}$

0·68181...
22) 15·0000... (
132
180
176
40
22
180
176
40
22
18 and so on.

1. **Pure recurring decimal :** It is a decimal number in which all the digits in the decimal part are repeating.

   e.g. (i) $0{\cdot}\overline{7}$ (ii) $6{\cdot}\overline{43}$ (iii) $0{\cdot}\overline{327}$, etc.

2. **Mixed recurring decimal :** It is a decimal number in which all the digits in the decimal part are not repeating.

   e.g. (i) $0{\cdot}2\overline{7}$ (ii) $8{\cdot}5\overline{31}$ (iii) $0{\cdot}39\dot{6}$, etc.

Example 6 :

Convert the mixed recurring decimal $0{\cdot}23\overline{4}$ into a vulgar fraction.

Solution :

Step 1 : From the number formed by all the digits in the decimal part, subtract the number formed by the digits that are not repeating.

For $0{\cdot}23\overline{4}$, we get : $234 - 23 = 211$.

Step 2 : Form the number with as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

For $0{\cdot}23\overline{4} = 900$.

$\therefore$ **The vulgar fraction for the given mixed recurring decimal $0{\cdot}23\overline{4}$**

$$= \frac{\text{Result of step 1}}{\text{Result of step 2}} = \mathbf{\frac{211}{900}} \qquad \textbf{(Ans.)}$$

Directed method :

$$\mathbf{0{\cdot}23\overline{4}} = \frac{234-23}{900} = \mathbf{\frac{211}{900}} \qquad \textbf{(Ans.)}$$

And,

$$\mathbf{0{\cdot}2\overline{34}} = \frac{234-2}{990} = \frac{232}{990} = \mathbf{\frac{116}{495}}$$

$$\mathbf{0{\cdot}\overline{234}} = \frac{234-0}{999} = \frac{234}{999} = \mathbf{\frac{26}{111}} \quad \text{and so on.}$$

In the same way :

(i)

$$\mathbf{3{\cdot}\overline{27}} = 3 + 0{\cdot}\overline{27} = 3 + \frac{27-0}{99} = 3 + \frac{3}{11} = \mathbf{3\frac{3}{11}}$$

(ii)

$$\mathbf{0{\cdot}23\overline{45}} = \frac{2345-23}{9900} = \frac{2322}{9900} = \mathbf{\frac{129}{550}}$$

(iii)

$$\mathbf{0{\cdot}\dot{4}} = \frac{4-0}{9} = \mathbf{\frac{4}{9}} \quad \text{and so on.}$$

EXERCISE 4(D)

1. Find, whether the given division forms a *terminating decimal* or a *non-terminating decimal:*

   (i) $3 \div 8$ (ii) $8 \div 3$ (iii) $6 \div 5$ (iv) $5 \div 6$

   (v) $12{\cdot}5 \div 4$ (vi) $23 \div 0{\cdot}7$ (vii) $42 \div 9$ (viii) $0{\cdot}56 \div 0{\cdot}11$

2. Express as recurring decimals :

   (i) $1\frac{1}{3}$ (ii) $\frac{10}{11}$ (iii) $\frac{5}{6}$ (iv) $\frac{2}{13}$

   (v) $\frac{1}{9}$ (vi) $\frac{17}{90}$ (vii) $\frac{5}{18}$ (viii) $\frac{7}{12}$

3. Convert into vulgar fraction :

   (i) $0{\cdot}\bar{3}$ (ii) $0{\cdot}\bar{8}$ (iii) $4{\cdot}\bar{4}$ (iv) $23{\cdot}\bar{7}$

4. Convert into vulgar fraction :

   (i) $0{\cdot}\overline{35}$ (ii) $2{\cdot}\overline{23}$ (iii) $1{\cdot}\overline{28}$ (iv) $5{\cdot}\overline{234}$

5. Convert into vulgar fraction :

   (i) $0{\cdot}3\bar{7}$ (ii) $0{\cdot}2\overline{45}$ (iii) $0{\cdot}68\bar{5}$ (iv) $0{\cdot}4\overline{42}$

4.13 ROUNDING-OFF OF DECIMAL NUMBERS

Sometimes, the numbers are found upto larger number of decimal places e.g., 3·481, 8·6672843, 5·36592, 9·8527, etc.

But, we need answers only upto a few decimal places. In such cases, the answers are rounded-off to the required number of decimal places.

4.14 ROUNDING-OFF :

(i) If the answer required is correct to two decimal places, we retain digits upto three decimal places.

(ii) *If the digit in the third decimal place is five or more than five, then the digit in the second decimal place is increased by one and, if the digit in the third decimal place is less than five, then the digit in the second decimal place is not altered.*

(iii) The third digit which was retained, is now omitted.

Thus, for getting 8·4813 correct to two decimal places,

write the given number upto three decimal places, *i.e.,* 8·481.

Since, the digit in the third decimal place is 1, *which is less than* 5.

∴ The digit in the second decimal place is not altered.

Here, **8·4813 = 8·48**, *correct to two decimal places.*

In the same way, to get 3·946824 correct to nearest thousandth, *i.e.,* correct to three decimal places, take the given number upto four decimal places, *i.e.,* take 3·9468.

Since, the digit at the fourth place is 8, which is greater than 5.

∴ According to the rule, the digit in the third place changes from 6 to 7.

Hence, **3·946824 = 3·947**, *correct to three decimal places.* **(Ans.)**

Similarly,

(i) 4·738 = 4·74; **correct to two decimal places.**

(ii) 4·738 = 4·7; **correct to one decimal place.**

(iii) 4·738 = 5; **correct to nearest unit.**

(iv) 536 = 540; **correct to nearest ten.**

(v) 0·0083 rounded to the nearest hundredth = 0·01

Example 7 :

Round off to the nearest whole number :

(i) 73·52 (ii) 38·327 (iii) 544·67

Solution :

Step 1 : Retain all the digits in the whole number part and the first digit on the right side of the decimal point.

Thus : (i) take 73·52 as 73·5

(ii) take 38·327 as 38·3

(iii) take 544·67 as 544·6

Step 2 : If the decimal digit retained (the first digit on the right side of the decimal point) is 5 or more than 5, increase the unit digit of the whole number retained by 1 (one), otherwise the unit digit of the whole number will be left as it is.

Thus : (i) **73·52** = 73·5 = **74** **(Ans.)**

(ii) **38·327** = 38·3 = **38** **(Ans.)**

(iii) **544·67** = 544·6 = **545** **(Ans.)**

4.15 SIGNIFICANT FIGURES (DIGITS)

Significant figures are the total number of digits present in a number except the zeroes preceding the first numeral.

In counting the number of significant digits, it should be noted that :

(i) the position of the decimal is disregarded.

(ii) all zeroes in between the numerals are counted.

(iii) all zeroes after the last numeral are counted.

(iv) the zeroes preceding the first numeral are not counted.

The table, given below, will make the concept more clear :

Numbers :	502	50·2	5·02	0·502	0·0502	0·05020	502·0	50·20
No. of significant digits :	3	3	3	3	3	4	4	4

Example 8 :

Round-off :

(i) 0·0506 and 0·36089 correct to one significant figure.

(ii) 0·6079 and 4·083 correct to two significant figures.

(iii) 7·04870 correct to three significant figures.

(iv) 14·08 correct to five significant figures.

Solution :

Proceed in a way similar to that used for decimal places.

(i) **0·0506** = **0·05**, correct to **one** significant figure.
and, **0·36089** = **0·4**, correct to **one** significant figure. **(Ans.)**

(ii) **0·6079** = **0·61**, correct to **two** significant figures.
and, **4·083** = **4·1**, correct to **two** significant figures. **(Ans.)**

(iii) **7·04870** = **7·05**, correct to **three** significant figures. **(Ans.)**

(iv) **14·08** = **14·080**, correct to **five** significant figures. **(Ans.)**

EXERCISE 4(E)

1. Round off :

(i) 0·07, 0·112, 3·59, 9·489 to the nearest tenths.

(ii) 0·627, 100·479, 0·065 and 0·024 to the nearest hundredths.

(iii) 4·83, 0·86, 451·943 and 9·08 to the nearest whole number.

2. Simplify, and write your answers correct to the nearest hundredths :

(i) 18·35 × 1·2 (ii) 62·89 × 0·02

3. Write the number of significant figures (digits) in :

(i) 35·06 (ii) 0·35 (iii) 7·0068 (iv) 19·0

(v) 0·0062 (vi) 4·2 × 0·6 (vii) 0·08 × 25 (viii) 3·6 ÷ 0·12

4. Write :

(i) 35·869, 0·008426, 4·952 and 382·7 correct to three significant figures.

(ii) 60·974, 2·8753, 0·001789 and 400·04 correct to four significant figures.

(iii) 14·29462, 19·2, 46356·82 and 69 correct to five significant figures.

EXERCISE 4(F)

1. The weight of an object is 3·06 kg. Find the total weight of 48 similar objects.

2. Find the cost of 17·5 m cloth at the rate of ₹ 112·50 per metre.

3. One kilogramme of oil costs ₹ 73·40. Find the cost of 9·75 kilogramme of the oil.

4. Total weight of 8 identical objects is 51·2 kg. Find the weight of each object.

5. 18·5 m of cloth costs ₹ 666. Find the cost of 3·8 m cloth.

6. Find the value of :

(i) 0·5 of ₹ 7·60 + 1·62 of ₹ 30 (ii) 2·3 of 7·3 kg + 0·9 of 0·48 kg

(iii) 6·25 of 8·4 – 4·7 of 3·24 (iv) 0·98 of 235 – 0·09 of 3·2

7. Evaluate :

(i) 5·6 – 1·5 of 3·4 (ii) 4·8 ÷ 0·04 of 5

(iii) 0·72 of 80 ÷ 0·2 (iv) 0·72 ÷ 80 of 0·2

(v) 6·45 ÷ (3·9 – 1·75) (vi) 0·12 of (0·104 – 0·02) + 0·36 × 0·5

EXPONENTS 5

(Including Laws of Exponents)

5.1 INDEX OR EXPONENT

In $3 \times 3 \times 3 \times 3 \times 3$, the factor 3 is being multiplied 5 times by itself and can also be written as 3^5, *i.e.,* $3 \times 3 \times 3 \times 3 \times 3 = 3^5$.

In 3^5, the **repeated factor 3** is called the **base** and the **number 5**, written slightly raised at the right of the factor 3, is called the **index** or **exponent**.

Thus, an index or an exponent is a number which indicates how many times the base is used as a repeated factor.

The plural of index is indices.

1. If n is a **whole number** and **a** is **any number**, then :

   $a^n = \underbrace{a \times a \times a \times a \times \ldots\ldots\ldots \times a}_{n \text{ factors}}$ ***(base = a** and **index = n.)***

2. ***In $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$, base = 5** and **index = 7.***
3. *If base = −3 and exponent = 8, the number = $(-3)^8$. Read −3 raised to the power 8*

5.2 EXPONENTIAL FORM OF A NUMBER

1. Since, $625 = 5 \times 5 \times 5 \times 5 = 5^4$; we say 5^4 is the exponential form of the number 625. Read 5^4 as 5 raised to the power 4.
2. Since, $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

   $\Rightarrow$ 2^5 is the exponential form of the number 32.
3. Since, $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

   $\Rightarrow$ Exponential form of the number 729 is 3^6 and so on.

Example 1 :

Find the value of :

(i) 2^6 (ii) 8^3 (iii) 5^6 (iv) 3^5

Solution :

(i) $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 64$ **(Ans.)**

(ii) $8^3 = 8 \times 8 \times 8$

$= 512$ **(Ans.)**

(iii) $5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

$= 15625$ **(Ans.)**

(iv) $3^5 = 3 \times 3 \times 3 \times 3 \times 3$

$= 243$ **(Ans.)**

Example 2 :

Evaluate : (i) $2^3 \times 5^2$ (ii) $3^2 \times 7^2$ (iii) 0×8^2 (iv) $2^3 \times 10^2$

Solution :

(i) $2^3 \times 5^2 = 2 \times 2 \times 2 \times 5 \times 5$

$= 8 \times 25 = 200$ **(Ans.)**

(ii) $3^2 \times 7^2 = 3 \times 3 \times 7 \times 7$

$= 9 \times 49 = 441$ **(Ans.)**

(iii) $0 \times 8^2 = 0$ **(Ans.)**

Zero (0) multiplied with any number gives zero.

(iv) $2^3 \times 10^2 = 2 \times 2 \times 2 \times 10 \times 10$

$= 8 \times 100 = 800$ **(Ans.)**

Example 3 :

Evaluate : (i) $\left(\frac{2}{5}\right)^3$ (ii) $\left(\frac{-3}{4}\right)^4$ (iii) $\left(\frac{-2}{-3}\right)^5$

Solution :

(i) $\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$

$= \frac{2 \times 2 \times 2}{5 \times 5 \times 5} = \frac{8}{125}$ **(Ans.)**

(ii) $\left(\frac{-3}{4}\right)^4 = \frac{-3}{4} \times \frac{-3}{4} \times \frac{-3}{4} \times \frac{-3}{4}$

$= \frac{-3 \times -3 \times -3 \times -3}{4 \times 4 \times 4 \times 4} = \frac{81}{256}$ **(Ans.)**

(iii) $\left(\frac{-2}{-3}\right)^5 = \frac{-2}{-3} \times \frac{-2}{-3} \times \frac{-2}{-3} \times \frac{-2}{-3} \times \frac{-2}{-3}$

$= \frac{-2 \times -2 \times -2 \times -2 \times -2}{-3 \times -3 \times -3 \times -3 \times -3} = \frac{-32}{-243} = \frac{32}{243}$ **(Ans.)**

Alternative method :

$\left(\frac{-2}{-3}\right)^5 = \left(\frac{2}{3}\right)^5$

$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} = \frac{32}{243}$ **(Ans.)**

Example 4 :

Find the value of :

(i) $3^2 \times 5^3$ (ii) $2^5 \times 5^2$ (iii) $3^2 \times 10^3$ (iv) $2^4 \times 5^3$

Solution :

(i) $3^2 \times 5^3 = 3 \times 3 \times 5 \times 5 \times 5$

$= 9 \times 125 = 1125$ **(Ans.)**

(ii) $2^5 \times 5^2 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$= 32 \times 25 = 800$ **(Ans.)**

(iii) $3^2 \times 10^3 = 3 \times 3 \times 10 \times 10 \times 10$

$= 9 \times 1000 = 9000$ **(Ans.)**

(iv) $2^4 \times 5^3 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

$= 16 \times 125 = 2000$ **(Ans.)**

Example 5 :

Evaluate :

(i) $\left(\frac{3}{4}\right)^2 \times \left(\frac{4}{5}\right)^3$ (ii) $\left(-\frac{2}{3}\right)^3 \times \left(\frac{3}{5}\right)^2$

Solution :

(i) $\left(\frac{3}{4}\right)^2 \times \left(\frac{4}{5}\right)^3 = \frac{3}{4} \times \frac{3}{4} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

$= \frac{3 \times 3 \times 4 \times 4 \times 4}{4 \times 4 \times 5 \times 5 \times 5} = \frac{3 \times 3 \times 4}{5 \times 5 \times 5} = \frac{36}{125}$ **(Ans.)**

(ii) $\left(-\frac{2}{3}\right)^3 \times \left(\frac{3}{5}\right)^2 = -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times \frac{3}{5} \times \frac{3}{5}$

$= \frac{-2 \times -2 \times -2}{3 \times 5 \times 5} = \frac{-8}{75} = -\frac{8}{75}$ **(Ans.)**

Example 6 :

Which is greater :

(i) 5^3 or 3^5 ? (ii) 2^8 or 8^2 ?

Solution :

(i) Since, $5^3 = 5 \times 5 \times 5 = 125$

and, $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

$\because$ 243 is greater than 125 $\Rightarrow$ $3^5 > 5^3$ **(Ans.)**

(ii) Since, $2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

and, $8^2 = 8 \times 8 = 64$

$\because$ 256 is greater than 64 $\Rightarrow$ $2^8 > 8^2$ **(Ans.)**

Example 7 :

Find the product of the cube of 6 and the square of 3.

Solution :

Required = (cube of 6) × (square of 3)

$= 6^3 \times 3^2$

$= 6 \times 6 \times 6 \times 3 \times 3$

$= 216 \times 9 = 1944$ **(Ans.)**

Example 8 :

Evaluate : $2^3 \times 3^2 \times 5^2$.

Solution :

$2^3 \times 3^2 \times 5^2 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$= 8 \times 9 \times 25 = 1800$ **(Ans.)**

Example 9 :

Evaluate : $(4^2 - 2^3) \times 3^2$.

Solution :

$$(4^2 - 2^3) \times 3^2 = (4 \times 4 - 2 \times 2 \times 2) \times 3 \times 3$$
$$= (16 - 8) \times 9$$
$$= 8 \times 9 = \mathbf{72}$$ **(Ans.)**

Example 10 :

If $a = 3$ and $b = 2$; find the value of :

(i) $a^b + b^a$ (ii) $a^a + b^b$

Solution :

(i) $a^b + b^a = 3^2 + 2^3$
$= 3 \times 3 + 2 \times 2 \times 2$
$= 9 + 8 = \mathbf{17}$ **(Ans.)**

(ii) $a^a + b^b = 3^3 + 2^2$
$= 3 \times 3 \times 3 + 2 \times 2$
$= 27 + 4 = \mathbf{31}$ **(Ans.)**

Example 11 :

Evaluate :

(i) $2^3 \times 6^2 \times 3$ (ii) $2^3 \times 5^2 \times 7 + 4^3 \times 5^3 \times 8$

Solution :

(i) $2^3 \times 6^2 \times 3 = 2 \times 2 \times 2 \times 6 \times 6 \times 3$
$= 8 \times 36 \times 3 = \mathbf{864}$ **(Ans.)**

(ii) $2^3 \times 5^2 \times 7 + 4^3 \times 5^3 \times 8 = 2 \times 2 \times 2 \times 5 \times 5 \times 7 + 4 \times 4 \times 4 \times 5 \times 5 \times 5 \times 8$
$= 8 \times 25 \times 7 + 64 \times 125 \times 8$
$= 200 \times 7 + 64 \times 1000$
$= 1400 + 64000 = \mathbf{65400}$ **(Ans.)**

Example 12 :

Express each of the following in exponential form :

(i) 288 (ii) 972 (iii) 1125

Solution :

(i) $288 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= \mathbf{2^5 \times 3^2}$ **(Ans.)**

2	288
2	144
2	72
2	36
2	18
3	9
	3

(ii) $972 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 2^2 \times 3^5$ **(Ans.)**

2	972
2	486
3	243
3	81
3	27
3	9
	3

(iii) $1125 = 5 \times 5 \times 5 \times 3 \times 3$

$= 5^3 \times 3^2$ **(Ans.)**

5	1125
5	225
5	45
3	9
	3

Example 13 :

Express each of the following in the exponential form :

(i) 864 (ii) 1080 (iii) 16000

Solution :

(i) $864 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 2^5 \times 3^3$ **(Ans.)**

(ii) $1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^3 \times 3^3 \times 5$ **(Ans.)**

(iii) $16000 = 16 \times 10 \times 10 \times 10$

$= (2 \times 2 \times 2 \times 2) \times (2 \times 5) \times (2 \times 5) \times (2 \times 5)$

$= 2^7 \times 5^3$ **(Ans.)**

Example 14 :

Express : (i) $\frac{64}{125}$ (ii) $\frac{-32}{243}$ in exponential form.

(i) $\frac{64}{125} = \frac{4 \times 4 \times 4}{5 \times 5 \times 5} = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \left(\frac{4}{5}\right)^3$ **(Ans.)**

(ii) $\frac{-32}{243} = \frac{-2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3}$

$= \frac{-2 \times -2 \times -2 \times -2 \times -2}{3 \times 3 \times 3 \times 3 \times 3} = \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} = \left(\frac{-2}{3}\right)^5$ **(Ans.)**

Alternative method :

$\frac{-32}{243} = \frac{-2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} = -\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

$= -\left(\frac{2}{3}\right)^5 = \left(\frac{-2}{3}\right)^5$ **(Ans.)**

If n is an odd number.
$(-2)^n = -2^n$ and if n is even $(-2)^n = 2^n$.

Thus, $(-4)^5 = -4^5$, $(-4)^6 = 4^6$, $(-4)^9 = -4^9$ and so on.
$(-5)^8 = 5^8$, $(-5)^3 = -5^3$, $(-5)^4 = 5^4$ and so on.

EXERCISE 5(A)

1. Find the value of :

(i) 6^2 (ii) 7^3 (iii) 4^4
(iv) 5^5 (v) 8^3 (vi) 7^5

2. Evaluate :

(i) $2^3 \times 4^2$ (ii) $2^3 \times 5^2$ (iii) $3^3 \times 5^2$
(iv) $2^2 \times 3^3$ (v) $3^2 \times 5^3$ (vi) $5^3 \times 2^4$
(vii) $3^2 \times 4^2$ (viii) $(4 \times 3)^3$ (ix) $(5 \times 4)^2$

3. Evaluate :

(i) $\left(\frac{3}{4}\right)^4$ (ii) $\left(-\frac{5}{6}\right)^5$ (iii) $\left(\frac{-3}{-5}\right)^3$

4. Evaluate :

(i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^2$ (ii) $\left(-\frac{3}{4}\right)^3 \times \left(\frac{2}{3}\right)^4$ (iii) $\left(\frac{3}{5}\right)^2 \times \left(-\frac{2}{3}\right)^3$

5. Which is greater :

(i) 2^3 or 3^2 (ii) 2^5 or 5^2 (iii) 4^3 or 3^4
(iv) 5^4 or 4^5

6. Express each of the following in exponential form :

(i) 512 (ii) 1250 (iii) 1458
(iv) 3600 (v) 1350 (vi) 1176

7. If $a = 2$ and $b = 3$, find the value of :

(i) $(a + b)^2$ (ii) $(b - a)^3$ (iii) $(a \times b)^a$ (iv) $(a \times b)^b$

8. Express : (i) 1024 as a power of 2. (ii) 343 as a power of 7.
(iii) 729 as a power of 3.

9. If $27 \times 32 = 3^x \times 2^y$; find the values of x and y.

10. If $64 \times 625 = 2^a \times 5^b$; find : (i) the values of a and b. (ii) $2^b \times 5^a$

5.2 LAWS OF EXPONENTS

First Law (Product Law) :

$$a^m \times a^n = a^{m+n}$$

When numbers are in the exponent form with the same base, to get their product (multiplication), add their powers (indices) keeping the base same.

For example :

(i) $\boldsymbol{a^3 \times a^7} = a^{3+7} = \boldsymbol{a^{10}}$

$\boldsymbol{a^3 \times a^7} = (a \times a \times a)\ (a \times a \times a \times a \times a \times a \times a)$

$= a \times a \times a \times a \times a \times a \times a \times a \times a \times a = \boldsymbol{a^{10}}$

(ii) $\mathbf{x^2y^3 \times x^4y^2} = (x^2 \times x^4) \times (y^3 \times y^2)$

$= x^{2+4} \times y^{3+2} = \mathbf{x^6y^5}$

(iii) $\mathbf{4a^2b^3c^2 \times 8a^9b^6c \times 3ab^{10}c^5} = 4 \times 8 \times 3 \times a^{2+9+1} \times b^{3+6+10} \times c^{2+1+5}$

$= \mathbf{96a^{12}b^{19}c^8}$

Second Law (Quotient Law) :

$$\frac{a^m}{a^n} = a^{m-n}; \text{ if } m > n \quad \text{and} \quad \frac{a^m}{a^n} = \frac{1}{a^{n-m}}; \text{ if } m < n.$$

When a number in exponent form is divided by another number in the exponent form (both the numbers having the same base), the smaller index (power) *is subtracted from the bigger index* (power) *and the base is kept the same.*

For example :

(i) $\mathbf{x^5 \div x^3} = \frac{x^5}{x^3} = x^{5-3} = \mathbf{x^2}$ (ii) $\mathbf{15a^2 \div 5a^{10}} = \frac{15a^2}{5a^{10}} = \frac{3}{a^{10-2}} = \mathbf{\frac{3}{a^8}}$

Third Law (Power Law) :

$$(a^m)^n = a^{mn}$$

when a number in the index form is raised to another index, the base is raised to the product of these two indices.

For example :

(i) $\mathbf{(a^3)^6} = a^{3 \times 6} = \mathbf{a^{18}}$ (ii) $\mathbf{(x^6)^{3/2}} = x^{6 \times 3/2} = \mathbf{x^9}$

22.3 MORE ABOUT INDICES

1. $(ab)^m = a^mb^m$ And $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

e.g. (i) $(xy)^3 = x^3y^3$ (ii) $(2a^3)^2 = 2^2(a^3)^2 = 4a^6$

(iii) $\left(\frac{3x}{4y^2}\right)^4 = \frac{3^4 \cdot x^4}{4^4 \cdot (y^2)^4} = \frac{81x^4}{256\,y^8}$ and so on.

2. Any non-zero base raised to the power **zero** is equal to unity (*i.e.,* 1).

i.e. $\mathbf{a^0 = 1}$, if $a \neq 0$

e.g. (i) $5^0 = 1$ (ii) $(-3)^0 = 1$ (iii) $\left(\frac{x^2}{2y}\right)^0 = 1$ and so on.

3. Negative Index : If $a \neq 0$ then : $\mathbf{a^{-m} = \frac{1}{a^m}}$ and $\mathbf{\frac{1}{a^{-m}} = a^m}$

e.g. (i) $a^{-4} = \frac{1}{a^4}$ (ii) $\frac{1}{x^{-7}} = x^7$ and so on.

Also note that :

(i) $\sqrt{a} = a^{1/2}$, e.g. $\sqrt{3} = 3^{1/2}$ (ii) $\sqrt[3]{a} = a^{1/3}$, e.g. $\sqrt[3]{3} = 3^{1/3}$

(iii) $\sqrt{a^5} = a^{5/2}$, e.g. $\sqrt{3^5} = 3^{5/2}$ (iv) $\sqrt[n]{a} = a^{1/n}$, e.g. $\sqrt[n]{3} = 3^{1/n}$

EXERCISE 5(B)

1. Fill in the blanks :

   (i) In $5^2 = 25$, base = ... and index = ...

   (ii) If index = $3x$ and base = $2y$, the number = ...

2. Evaluate :

   (i) $2^8 \div 2^3$ (ii) $2^3 \div 2^8$ (iii) $(2^6)^0$

   (iv) $(3^0)^6$ (v) $8^3 \times 8^{-5} \times 8^4$ (vi) $5^4 \times 5^3 \div 5^5$

   (vii) $5^4 \div 5^3 \times 5^5$ (viii) $4^4 \div 4^3 \times 4^0$ (ix) $(3^5 \times 4^7 \times 5^8)^0$

3. Simplify, giving answers with positive index :

   (i) $2b^6 \cdot b^3 \cdot 5b^4$ (ii) $x^2y^3 \cdot 6x^5y \cdot 9x^3y^4$

   (iii) $(-a^5)(a^2)$ (iv) $(-y^2)(-y^3)$

   (v) $(-3)^2(3)^3$ (vi) $(-4x)(-5x^2)$

   (vii) $(5a^2b)(2ab^2)(a^3b)$ (viii) $x^{2a+7} \cdot x^{2a-8}$

   (ix) $3^y \cdot 3^2 \cdot 3^{-4}$ (x) $2^{4a} \cdot 2^{3a} \cdot 2^{-a}$

   (xi) $4x^2y^2 \div 9x^3y^3$ (xii) $(10^2)^3(x^8)^{12}$

   (xiii) $(a^{10})^{10}(1^6)^{10}$ (xiv) $(n^2)^2(-n^2)^3$

   (xv) $-(3ab)^2(-5a^2bc^4)^2$ (xvi) $(-2)^2 \times (0)^3 \times (3)^3$

   (xvii) $(2a^3)^4(4a^2)^2$ (xviii) $(4x^2y^3)^3 \div (3x^2y^3)^3$

   (xix) $\left(\frac{1}{2x}\right)^3 \times (6x)^2$ (xx) $\left(\frac{1}{4ab^2c}\right)^2 \div \left(\frac{3}{2a^2bc^2}\right)^4$

   (xxi) $\frac{(5x^7)^3 \cdot (10x^2)^2}{(2x^6)^7}$ (xxii) $\frac{(7p^2q^9r^5)^2\ (4pqr)^3}{(14p^6q^{10}r^4)^2}$

4. Simplify and express the answer in the positive exponent form :

   (i) $\frac{(-3)^3 \times 2^6}{6 \times 2^3}$ (ii) $\frac{(2^3)^5 \times 5^4}{4^3 \times 5^2}$ (iii) $\frac{36 \times (-6)^2 \times 3^6}{12^3 \times 3^5}$

   (iv) $-\frac{128}{2187}$ (v) $\frac{a^{-7} \times b^{-7} \times c^5 \times d^4}{a^3 \times b^{-5} \times c^{-3} \times d^8}$ (vi) $(a^3b^{-5})^{-2}$

5. Evaluate :

   (i) $6^{-2} \div (4^{-2} \times 3^{-2})$ (ii) $\left[\left(\frac{5}{6}\right)^2 \times \frac{9}{4}\right] \div \left[\left(-\frac{3}{2}\right)^2 \times \frac{125}{216}\right]$

   (iii) $5^3 \times 3^2 + (17)^0 \times 7^3$ (iv) $2^5 \times 15^0 + (-3)^3 - \left(\frac{2}{7}\right)^{-2}$

   (v) $(2^2)^0 + 2^{-4} \div 2^{-6} + \left(\frac{1}{2}\right)^{-3}$ (vi) $5^n \times 25^{n-1} \div (5^{n-1} \times 25^{n-1})$

6. If $m = -2$ and $n = 2$; find the value of :

   (i) $m^2 + n^2 - 2mn$ (ii) $m^n + n^m$

   (iii) $6m^{-3} + 4n^2$ (iv) $2n^3 - 3m$

Theme – 2
COMMERCIAL ARITHMETIC

RATIO AND PROPORTION 6

(Including Sharing in a Ratio)

6.1 DEFINITION

A ratio is a relationship between two quantities of the same kind with same unit and is obtained on dividing first quantity by the second.

The symbol for ratio is " : " and it is put in between the two quantities to be compared.

Thus, the ratio between 15 kg and 20 kg = 15 kg : 20 kg = $\frac{15}{20} = \frac{3}{4}$ = **3 : 4.**

1. *The two quantities must be of the same kind.*

   Thus, there can be a ratio between ₹50 and ₹80, but there can be no ratio between ₹50 and 80 kg.
2. *The ratio between 3 and 4 is written as* **3 : 4** *(read as 3 is to 4) or* $\frac{3}{4}$.
3. *In the ratio 3 : 4, the first term (i.e., 3) is called* ***antecedent*** *and the second term (i.e., 4) is called* ***consequent.*** In a ratio antecedent and consequent are co-prime.

   Let for a ratio, antecedent = 5 and consequent = 8; the corresponding ratio = 5 : 8.
4. *A ratio is a pure number.*
5. *In order to find the ratio between two quantities, both the quantities must be in the same unit, e.g., ratio between 30 cm and 2 metre*

   = 30 cm : 200 cm [As, 2 metre = 200 cm]

   $= \frac{30}{200} = \frac{3}{20}$ = **3 : 20**
6. *A ratio must always be expressed in its lowest terms in simplest form.*

   A ratio is said to be in simplest form, if both its terms (antecedent and consequent) are co-prime *i.e.* their H.C.F. is 1.

   e.g. (i) The ratio 4 : 5 is in simplest form as H.C.F. of 4 and 5 = 1.

   (ii) The ratio 6 : 8 is not in simplest form as H.C.F. of 6 and 8 is 2 and not 1.

   When the given ratio is not in simplest form, divide its each term by their H.C.F.

   ∵ H.C.F. of 6 and 8 is 2.

   ∴ **6 : 8 in simplest form** = $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$ = **3 : 4**
7. *A ratio has no unit because it is simply a number.*

6.2 TO CONVERT A FRACTIONAL RATIO INTO A WHOLE NUMBER RATIO

Example 1 :

Convert the ratio $\frac{1}{3} : \frac{1}{4}$ into simplest form.

Solution :

$$\frac{1}{3} : \frac{1}{4} = \frac{1}{3} \times \frac{4}{1}$$ [Dividing 1st quantity by the 2nd]

$$= \frac{4}{3} = \mathbf{4 : 3}$$ **(Ans.)**

Alternative method :

Steps : 1. Find the L.C.M. of denominators 3 and 4 of the given ratio $\frac{1}{3} : \frac{1}{4}$.

2. Multiply each term of the ratio by this L.C.M. and simplify.

> If each term of a ratio is multiplied or divided by the same non-zero number (quantity), the ratio remains the same.

Thus, $$\frac{1}{3} : \frac{1}{4} = \frac{1}{3} \times 12 : \frac{1}{4} \times 12$$ [Since, L.C.M. of 3 and 4 = 12]

$$= \mathbf{4 : 3}$$ **(Ans.)**

In the same way

(i) $$\frac{1}{9} : \frac{1}{12} = \frac{1}{9} \times 36 : \frac{1}{12} \times 36$$ [L.C.M. of 9 and 12 = 36]

$$= \mathbf{4 : 3}$$ **(Ans.)**

(ii) $$4\frac{1}{4} : 1\frac{1}{12} = \frac{17}{4} : \frac{13}{12}$$

$$= \frac{17}{4} \times 12 : \frac{13}{12} \times 12$$ [∵ L.C.M. of 4 and 12 = 12]

$$= \mathbf{51 : 13}$$ **(Ans.)**

(iii) $$1\frac{1}{3} : 2\frac{1}{4} : 1\frac{5}{6} = \frac{4}{3} : \frac{9}{4} : \frac{11}{6}$$

$$= \frac{4}{3} \times 12 : \frac{9}{4} \times 12 : \frac{11}{6} \times 12$$ [∵ L.C.M. of 3, 4 and 6 = 12]

$$= \mathbf{16 : 27 : 22}$$ **(Ans.)**

6.3 TO DIVIDE A GIVEN QUANTITY INTO A GIVEN RATIO

Example 2 :

20 sweets are distributed between A and B in the ratio 2 : 3. How many does each get ?

Solution :

A and B get sweets in the ratio 2 : 3 ⇒ If A gets 2 parts, then B gets 3 parts.

In other words, if we make (2 + 3) = 5 equal parts, then A should get 2 parts out of these 5 equal parts.

⇒ A gets = $\frac{2}{5}$ of the total number of sweets = $\frac{2}{5}$ of 20 = $\frac{2}{5} \times 20$ = 8 sweets

Similarly, B gets 3 parts out of 5 equal parts

$\Rightarrow$ B gets = $\frac{3}{5}$ of the total number of sweets = $\frac{3}{5}$ of 20 = $\frac{3}{5} \times 20 = 12$ sweets

Thus, **A gets 8 sweets** and **B gets 12 sweets.** **(Ans.)**

Direct method :

Since, the given ratio = 2 : 3 and 2 + 3 = 5

$\therefore$ **A gets** = $\frac{2}{5}$ of the total no. of sweets

= $\frac{2}{5} \times 20$ sweets = **8 sweets** **(Ans.)**

and, **B gets** = $\frac{3}{5} \times 20$ sweets = **12 sweets** **(Ans.)**

Example 3 :

Divide ₹ 260 among A, B and C in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.

Solution :

First of all convert the given ratio into its simple form.

Since, L.C.M. of denominators 2, 3 and 4 is 12.

$$\therefore \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = \frac{1}{2} \times 12 : \frac{1}{3} \times 12 : \frac{1}{4} \times 12 = 6 : 4 : 3$$

And, 6 + 4 + 3 = 13.

$\therefore$ A's share = $\frac{6}{13}$ of ₹ 260 = ₹ $\frac{6}{13} \times 260$ = ₹ 120

B's share = $\frac{4}{13}$ of ₹ 260 = ₹ $\frac{4}{13} \times 260$ = ₹ 80

and C's share = $\frac{3}{13}$ of ₹ 260 = ₹ $\frac{3}{13} \times 260$ = ₹ 60

Thus, **A gets ₹ 120, B gets ₹ 80** and **C gets ₹ 60.** **(Ans.)**

Example 4 :

Two numbers are in the ratio 10 : 13. If the difference between the numbers is 48, find the numbers.

Solution :

Let the two numbers be 10 and 13.

$\therefore$ The difference between these numbers = 13 – 10 = 3

Applying unitary method **:**

When difference between the numbers = 3, 1st number = 10

$\Rightarrow$ " " " " = 1, 1st number = $\frac{10}{3}$

$\Rightarrow$ " " " " = 48, 1st number = $\frac{10}{3} \times 48 = \mathbf{160}$

In the same way,

when difference between the numbers = 3, 2nd number = 13

$\Rightarrow$ " " " " = 48, 2nd number = $\frac{13}{3} \times 48 = \mathbf{208}$

$\therefore$ **Required numbers** are **160** and **208** **(Ans.)**

***Alternative method* (algebraic method) :**

Since, the ratio between the required numbers = 10 : 13

Let the numbers be $10x$ and $13x$.

$\therefore\ 13x - 10x = 48 \quad \Rightarrow\ 3x = 48$ and $x = \frac{48}{3} = 16$

$\therefore$ **Required numbers** = $10x$ and $13x$

= 10×16 and 13×16 = **160** and **208** **(Ans.)**

Example 5 :

The ratio between two numbers is 4 : 5. If their L.C.M. is 180, find the numbers.

Solution :

Let the numbers be $4x$ and $5x$

L.C.M. of $4x$ and $5x = 4 \times 5 \times x$

$= 20x$

Given L.C.M. = 180

$\Rightarrow\ 20x = 180$ and $x = \frac{180}{20} = 9$

$\therefore$ **Required numbers** = $4x$ and $5x$

= 4×9 ano 5×9 = **36 and 45** **(Ans.)**

Example 6 :

The ratio between two numbers is 3 : 4. If their H.C.F. is 15, find the numbers.

Solution :

Let the two numbers be $3x$ and $4x$

H.C.F. of $3x$ and $4x$ = Largest number common to $3x$ and $4x$

$= x$

Given H.C.F. = 15 $\Rightarrow x = 15$

$\therefore$ **Required numbers** = $3x$ and $4x$

= 3×15 and 4×15 = **45 and 60** **(Ans.)**

Example 7 :

A certain quantity of wheat is divided into two parts in the ratio 5 : 8. If the second part is 112 kg, find :

(i) the first part (ii) the total quantity of the wheat.

Solution :

(i) $\because$ Ratio between the two parts is 5 : 8

$\Rightarrow$ If second part = 8 kg, the first part = 5 kg

$\Rightarrow$ If second part = 1 kg, the first part = $\frac{5}{8}$ kg

$\Rightarrow$ If second part = 112 kg, the first part = $\frac{5}{8} \times 112$ kg = **70 kg** **(Ans.)**

(ii) The total quantity of wheat = Weight of the first part + weight of the second part

= 112 kg + 70 kg = **182 kg** **(Ans.)**

Alternative method :

Since, two parts of wheat are in the ratio 5 : 8

$\Rightarrow$ If the first part = $5x$ kg, the second part = $8x$ kg

Given, the second part = 112 kg

$\Rightarrow 8x = 112$ and $x = \frac{112}{8} = 14$

(i) **The first part** = $5x$ kg = 5×14 kg = **70 kg** **(Ans.)**

(ii) **The total quantity of wheat** = $8x$ kg + $5x$ kg

= $13x$ kg = 13 kg × 14 kg = **182 kg** **(Ans.)**

EXERCISE 6(A)

1. Express each of the given ratios in its simplest form :

(i) 22 : 66 (ii) 1·5 : 2·5 (iii) $6\frac{1}{4} : 12\frac{1}{2}$

(iv) 40 kg : 1 quintal (v) 10 paise : ₹ 1 (vi) 200 m : 5 km

(vii) 3 hours : 1 day (viii) 6 months : $1\frac{1}{3}$ years (ix) $1\frac{1}{3} : 2\frac{1}{4} : 2\frac{1}{2}$

2. Divide 64 cm long string into two parts in the ratio 5 : 3.

3. ₹ 720 is divided between x and y in the ratio 4 : 5. How many rupees will each get ?

4. The angles of a triangle are in the ratio 3 : 2 : 7. Find each angle.

5. A rectangular field is 100 m by 80 m. Find the ratio of :

(i) length to its breadth (ii) breadth to its perimeter.

6. The sum of three numbers, whose ratios are $3\frac{1}{3} : 4\frac{1}{5} : 6\frac{1}{8}$ is 4917. Find the numbers.

7. The ratio between two quantities is 3 : 4. If the first is ₹ 810, find the second.

8. Two numbers are in the ratio 5 : 7. Their difference is 10. Find the numbers.

9. Two numbers are in the ratio 10 : 11. Their sum is 168. Find the numbers.

10. A line is divided into two parts in the ratio 2·5 : 1·3. If the smaller one is 35·1 cm, find the length of the line.

11. In a class, the ratio of boys to the girls is 7 : 8. What part of the whole class are girls ?

12. The population of a town is 180,000, out of which males are $\frac{1}{3}$ of the whole population. Find the number of females. Also, find the ratio of the number of females to the whole population.

13. Ten gram of an alloy of metals A and B contains 7·5 gm of metal A and the rest is metal B. Find the ratio between :

(i) the weights of metals A and B in the alloy.

(ii) the weight of metal B and the weight of the alloy.

14. The ages of two boys A and B are 6 years 8 months and 7 years 4 months respectively. Divide ₹ 3,150 in the ratio of their ages.

15. Three persons start a business and spend ₹ 25,000, ₹ 15,000 and ₹ 40,000 respectively. Find the share of each out of a profit of ₹ 14,400 in a year.

16. A plot of land, 600 sq m in area, is divided between two persons such that the first person gets three-fifths of what the second gets. Find the share of each.

17. Two poles of different heights are standing vertically on a horizontal field. At a particular time, the ratio between the lengths of their shadows is 2 : 3. If the height of the smaller pole is 7·5 m, find the height of the other pole.

18. Two numbers are in the ratio 4 : 7. If their L.C.M. is 168, find the numbers.

19. ₹ 300 is divided between A and B in such a way that A gets half of B. Find :

   (i) the ratio between the shares of A and B.

   (ii) the share of A and the share of B.

20. The ratio between two numbers is 5 : 9. Find the numbers, if their H.C.F. is 16.

21. A bag contains ₹ 1,600 in the form of ₹ 10 and ₹ 20 notes. If the ratio between the numbers of ₹ 10 and ₹ 20 notes is 2 : 3; find the total number of notes in all.

22. The ratio between the prices of a scooter and a refrigerator is 4 : 1. If the scooter costs ₹ 45,000 more than the refrigerator, find the price of the refrigerator.

6.4 PROPORTION

When *four quantities* are so related that the *ratio between the first and the second quantities* is *equal* to the *ratio between the third and the fourth quantities,* the quantities are said to be in **proportion**.

Thus, ***proportion is equality of two ratios.***

In order to represent a proportion, either put the sign of equality (=) between the two ratios or put a double colon (: :).

Consider four quantities (numbers) 15, 20, 9 and 12.

The ratio between the first and the second quantities is $15 : 20 = \frac{15}{20} = \frac{3}{4} = 3 : 4$.

Whereas, the ratio between the third and the fourth quantities is $9 : 12 = \frac{9}{12} = \frac{3}{4} = 3 : 4$.

Since, the ratio between the first and the second quantities is same as the ratio between the third and the fourth quantities, we say the four quantities 15, 20, 9 and 12 are in proportion and we write : 15 : 20 = 9 : 12 *or* 15 : 20 : : 9 : 12.

(i) Each quantity in a proportion is called its **term** or its **proportional.**

(ii) In a proportion, the **first** and the **last terms** are called the **extremes,** whereas the **second** and the **third terms** are called the **means.**

(iii) For every proportion, **the product of the extremes is always equal to the product of the means.**

e.g., In proportion 15 : 20 : : 9 : 12; product of extremes = $15 \times 12 = 180$
and, product of means = $20 \times 9 = 180$.

(iv) The **fourth term** of a proportion is called **fourth proportional.**

Example 8 :

Find the fourth proportional of 3, 4 and 18.

Solution :

Let the fourth proportional be x $\Rightarrow$ 3, 4, 18 and x are in proportion.

$\therefore \quad 3 : 4 = 18 : x$

$\Rightarrow \quad 3 \times x = 4 \times 18$ [Product of extremes = Product of means]

$\Rightarrow \quad x = \frac{72}{3} = 24$ **(Ans.)**

6.5 CONTINUED PROPORTION

Three quantities are said to be in *continued* proportion, if the ratio between the first and the second quantities is equal to the ratio between the second and the third quantities.

i.e., a, b and *c* are in continued proportion, if $a : b = b : c$.

The ***second quantity*** is called the ***mean proportional*** between the first and the third.

i.e., in $a : b = b : c$, b is the *mean proportional* between a and c.

When b is mean proportion between a and c

$\Rightarrow$ *a, b* and *c* are in continued proportion

$\Rightarrow a : b = b : c$

$\Rightarrow \frac{a}{b} = \frac{b}{a}$ *i.e.* $b \times b = a \times c \quad \Rightarrow b^2 = ac$ and $b = \sqrt{ac}$

Thus mean proportion between 2 and 8

$= \sqrt{2 \times 8} = \sqrt{16} = 4$

The ***third quantity*** is called the ***third proportional*** to the first and the second.

i.e., in $a : b = b : c$, c is the *third proportional* to a and b.

Example 9 :

(i) Find the mean proportion between 4 and 9.

(ii) Find the third proportional to 12 and 30.

Solution :

(i) Let the mean proportion be x $\Rightarrow$ 4, x and 9 are in continued proportion.

$\therefore \quad 4 : x = x : 9 \Rightarrow x \times x = 4 \times 9 \Rightarrow x^2 = 36 \Rightarrow x = 6$ **(Ans.)**

(ii) Let x be the third proportional $\Rightarrow$ 12, 30 and x are in continued proportion.

$\therefore \quad 12 : 30 = 30 : x \Rightarrow 12 \times x = 30 \times 30 \Rightarrow x = \frac{900}{12} = 75 \Rightarrow x = 75$ **(Ans.)**

Example 10 :

If $a : b = 4 : 5$ and $b : c = 6 : 7$, find $a : c$.

Solution :

Since, $a : b = 4 : 5 \Rightarrow \frac{a}{b} = \frac{4}{5}$ and $b : c = 6 : 7 \Rightarrow \frac{b}{c} = \frac{6}{7}$

$\therefore \quad \frac{a}{b} \times \frac{b}{c} = \frac{4}{5} \times \frac{6}{7} \Rightarrow \frac{a}{c} = \frac{24}{35}$ *i.e.,* $a : c = 24 : 35$ **(Ans.)**

Example 11 :

If $a : b = 4 : 5$ and $b : c = 6 : 7$, find $a : b : c$.

Solution :

$$a : b = 4 : 5 = \frac{4}{5} : \frac{5}{5} \text{ or } \frac{4}{5} : 1$$ [Dividing each term by 5]

$$b : c = \frac{6}{6} : \frac{7}{6} \text{ or } 6 : 7 = 1 : \frac{7}{6}$$ [Dividing each term by 6]

In both the given ratios, the quantity b is common, so we have made the value of b same *i.e.*, one (1).

Clearly, $\boldsymbol{a : b : c} = \frac{4}{5} : 1 : \frac{7}{6} = \frac{4}{5} \times 30 : 1 \times 30 : \frac{7}{6} \times 30$ [L.C.M. of 5 and 6 = 30]

$= \mathbf{24 : 30 : 35}$ **(Ans.)**

Alternative method :

We know that if both the terms of a ratio are multiplied by the same number, the ratio remains the same.

So, multiply each ratio by such a number that the value of b (the common term in both the given ratios) acquires the same value.

$\therefore\ a : b = 4 : 5 = 24 : 30$ [Multiplying both the terms by 6]

and, $b : c = 6 : 7 = 30 : 35$ [Multiplying both the terms by 5]

Clearly, $\boldsymbol{a : b : c} = \mathbf{24 : 30 : 35}$ **(Ans.)**

EXERCISE 6(B)

1. Check whether the following quantities form a proportion or not :

(i) $3x$, $7x$, 24 and 56 (ii) 0·8, 3, 2·4 and 9 (iii) $1\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{1}{2}$ and $9\frac{3}{4}$

(iv) 0·4, 0·5, 2·9 and 3·5 (v) $2\frac{1}{2}$, $5\frac{1}{2}$, 3·0 and 6·0

(i) Ratio between first two quantities $= 3x : 7x = \frac{3x}{7x} = \frac{3}{7} = 3 : 7$ and,

ratio between last two quantities $= 24 : 56 = \frac{24}{56} = \frac{3}{7} = 3 : 7$

$\Rightarrow$ **the given four quantities are in proportion.**

2. Find the fourth proportional of :

(i) 3, 12 and 4 (ii) 5, 9 and 45 (iii) 2·1, 1·5 and 8·4

(iv) $\frac{1}{3}$, $\frac{2}{5}$ and 8·4 (v) 4 hours 40 minutes, 1 hour 10 minutes and 16 hours

3. Find the third proportional of :

(i) 27 and 9 (ii) 2m 40cm and 40cm (iii) 1·8 and 0·6

(iv) $\frac{1}{7}$ and $\frac{3}{14}$ (v) 1·6 and 0·8

4. Find the mean proportional between :

(i) 16 and 4 (ii) 3 and 27 (iii) 0·9 and 2·5

(iv) 0·6 and 9·6 (v) $\frac{1}{4}$ and $\frac{1}{16}$

5. (i) If A : B = 3 : 5 and B : C = 4 : 7, find A : B : C.

(ii) If $x : y = 2 : 3$ and $y : z = 5 : 7$, find $x : y : z$.

(iii) If $m : n = 4 : 9$ and $n : s = 3 : 7$, find $m : s$.

(iv) If P : Q = $\frac{1}{2} : \frac{1}{3}$ and Q : R = $1\frac{1}{2} : 1\frac{1}{3}$, find P : R.

(v) If $a : b = 1{\cdot}5 : 3{\cdot}5$ and $b : c = 5 : 6$, find $a : c$.

(vi) If $1\frac{1}{4} : 2\frac{1}{3} = p : q$ and $q : r = 4\frac{1}{2} : 5\frac{1}{4}$, find $p : r$.

6. If $x : y = 5 : 4$ and $2 : x = 3 : 8$, find the value of y.

7. Find the value of x, when $2{\cdot}5 : 4 = x : 7{\cdot}5$.

8. Show that 2, 12 and 72 are in continued proportion.

UNITARY METHOD

(Including Time and Work)

7

7.1 BASIC CONCEPT

Consider the following examples :

1. If the cost of 15 m cloth = ₹ 300

   ⇒ **The cost of 1 m cloth** = $\frac{₹\ 300}{15}$ = **₹ 20**

   and, **the cost of 10 m cloth** = 10 × ₹ 20
   = **₹ 200**

2. If 15 men can do some work in 300 days

   ⇒ **1 man will do the same work in** 300 × 15 days = **4500 days**

   and, **10 men will do it in** $\frac{4500}{10}$ days = **450 days**

In part 1, given above, the cost of 1 m cloth is first obtained from the given cost of 15 m and then the cost of 10 m is found.

In the same way, in part 2, given above, the number of days taken by 1 man is first calculated from the number of days by 15 men and then the number of days taken by 10 men is obtained.

> The method in which the value of a unit quantity is first calculated to get the value of given quantity is called the unitary method.

In unitary method, we come across two types of variations :

Type of variations	Condition	Examples
1. Direct variation	*Increase* in one quantity *causes increase* in the other and *decrease* in one quantity *causes decrease* in the other.	(i) With *more money, more articles* can be bought. (ii) With *a greater speed a larger distance* can be covered in the same time. (iii) With *less number of men, less work* can be done in same time, etc.
2. Inverse variation	*Increase* in one quantity *causes decrease* in the *other* and *decrease* in one quantity *causes increase* in the other.	(i) With *a greater speed, less time* will be taken to cover the same distance. (ii) With *less number of men, more days* are required for the *same work*, etc.

7.2 EXAMPLES OF DIRECT VARIATION

Example 1 :

A man earns ₹ 400 in 10 days. How much will he earn in 28 days ?

Solution :

∵ In 10 days, the man earns = ₹ 400

∴ In 1 day, he will earn = ₹ $\frac{400}{10}$ = ₹ 40

[*Less money* is earned in 1 day, so *divide*]

⇒ **In 28 days, he will earn** = 28 × ₹ 40

= **₹ 1,120** **(Ans.)**

Note : For solving problems (using unitary method), the sentences (statements) should be framed in such a way that, whatever is to be found is written at the end of the statement.

Arrow method :

Steps :

1. Form two columns, one heading earnings and the other heading no. of days (as shown alongside). [The quantity to be obtained must be written at the extreme right column. Here, quantity to be obtained is earnings of 28 days].

No. of days	**Earnings**
....	....
....	....

2. Let earnings of 28 days be ₹ x. Write the no. of days and corresponding earnings as shown alongside :

No. of days	**Earnings**
10	₹ 400
28	₹ x

3. For the column on the extreme right, mark an arrow in the downward direction.

No. of days	**Earnings**
10	₹ 400 ↓
28	₹ x ↓

4. If it is the case of direct variation, the arrow for the column (headed : no. of days) must be in the same direction as that for earnings.

No. of days	**Earnings**
10 ↓	₹ 400 ↓
28 ↓	₹ x ↓

In the case of inverse variation, the arrow for this column must be in the direction opposite to the direction of the first arrow.

Since, here we have the case of direct variation, therefore, for both the columns arrows must be in the same direction.

5. Now according to the arrows marked, take :

$\frac{\text{value on the head}}{\text{value on the tail}}$ for one arrow = $\frac{\text{value on the head}}{\text{value on the tail}}$ for the other arrow.

Thus, we get :

$$\frac{₹\ x}{₹\ 400} = \frac{28}{10} \quad \Rightarrow x = \frac{28}{10} \times 400 = 1,120$$

∴ **28 men will earn ₹ 1,120** **(Ans.)**

Example 2 :

0·75 metre cloth costs ₹ 45. What will be the cost of 0·6 metre of same cloth ?

Solution :

Given : the cost of 0·75 m cloth = ₹ 45

∴ The cost of 1 m cloth = ₹ $\frac{45}{0.75}$

= ₹ 60

And, **the cost of 0·6 m cloth** = 0·6 × ₹ 60

= **₹ 36 (Ans.)**

Arrow method :

Cloth (m)	Cost (₹)
0·75 ↓	45 ↓
0·6	x

$\Rightarrow \quad \frac{x}{45} = \frac{0.6}{0.75}$

$\Rightarrow \quad x = \frac{0.6}{0.75} \times 45 = 36$

∴ **Cost of 0·6 m cloth = ₹ 36 (Ans.)**

7.3 EXAMPLES OF INVERSE VARIATION

Example 3 :

4 men can do a piece of work in 5 days. How many men will do it in 4 days ?

Solution :

∵ In 5 days, the work is done by 4 men

∴ In 1 day, the work will be done by 4 × 5 = 20 men

[*More* number of men are required to do the work in 1 day, so *multiply*]

In 4 days, the work will be done by $\frac{20}{4}$ men

= **5 men (Ans.)**

Arrow method :

Days	No. of men
5 ↑	4 ↓
4	x

$\Rightarrow \quad \frac{x}{4} = \frac{5}{4}$ and $x = 5$

∴ **5 men will do the work in 4 days** **(Ans.)**

Example 4 :

With a speed of 60 km/h, it takes 4 hours to cover a certain distance. What should be the speed, if the same journey is to be completed in 3 hours ?

Solution :

∵ To cover a certain distance in 4 hours,

speed required = 60 km/h

∴ To cover the same distance in 1 hour,

speed required = 60 km/h × 4

= 240 km/h

And, **to cover the same distance in 3 hours,**

speed required = $\frac{240}{3}$ km/h

= **80 km/h (Ans.)**

Arrow method :

Time (hrs)	Speed (km/h)
4 ↑	60 ↓
3	x

$\Rightarrow \quad \frac{x}{60} = \frac{4}{3}$

$\Rightarrow \quad x = \frac{4}{3} \times 60 = 80$

∴ **Required speed = 80 km h^{-1} (Ans.)**

> ***Remember :*** (i) For getting **more, multiply.**
> (ii) For getting **less, divide.**

EXERCISE 7(A)

1. Weight of 8 identical articles is 4·8 kg. What is the weight of 11 such articles ?
2. 6 books weigh 1·260 kg. How many books will weigh 3·150 kg ?
3. 8 men complete a work in 6 hours. In how many hours will 12 men complete the same work ?
4. If a 25 cm long candle burns for 45 minutes, how long will another candle of the same material and same thickness but 5 cm longer than the previous one burn ?
5. A typist takes 80 minutes to type 24 pages. How long will he take to type 87 pages ?
6. ₹ 750 support a person for 15 days. For how many days will ₹ 2,500 support the same person ?
7. 400 men have provisions for 23 weeks. They are joined by 60 men. How long will the provisions last ?

Hint : For 400 men, the provisions are sufficient for 23 weeks

$\Rightarrow$ For 1 man, the provision will be sufficient for 23×400 weeks.

8. 200 men have provisions for 30 days. If 50 men have left, for how many days the same provisions would last for the remaining men ?
9. 8 men can finish a certain amount of provisions in 40 days. If 2 more men join them, find for how many days will the same amount of provisions be sufficient.
10. If the interest on ₹ 200 be ₹ 25 in a certain time, what will be the interest on ₹ 750 for the same time ?
11. If 3 dozen eggs cost ₹ 90, find the cost of 3 scores of eggs. [1 score = 20]
12. If the fare for 48 km is ₹ 288, what will be the fare for 36 km ?
13. What will be the cost of 3.20 kg of an item, if 3 kg of it costs ₹ 360 ?
14. If 9 lines of a print, in a column of a book, contain 36 words, how many words will a column of 51 lines contain ?
15. 125 students have food sufficient for 18 days. If 25 more students join them, how long will the food last now ?

    What assumption have you made to come to your answer ?
16. A carpenter prepares a new chair in 3 days, working 8 hours a day.

    At least how many hours per day must he work in order to make the same chair in 4 days ?
17. A man earns ₹ 5,800 in 10 days. How much will he earn in the month of Feburary of a leap year ?
18. A machine makes 500 rubber balls in 30 minutes. How many rubber balls will it make in $3\frac{1}{2}$ hours ?
19. In a school's hostel mess, 20 children consume a certain quantity of ration in 6 days. However, 5 children did not return to the hostel after holidays. How long will the same amount of ration last now ?

Example 5 :

If cost of $\frac{8}{15}$ of a certain cargo is ₹ 2,000. Find the cost of $\frac{3}{5}$ of the same cargo.

Solution :

Cost of $\frac{8}{15}$ of a cargo = ₹ 2,000

⇒ Cost of whole (one) cargo = ₹ 2,000 ÷ $\frac{8}{15}$

= ₹ 2,000 × $\frac{15}{8}$

= ₹ 3,750

∴ **Cost of $\frac{3}{5}$ of the same cargo** = ₹ 3,750 × $\frac{3}{5}$

= **₹ 2,250 (Ans.)**

Arrow method :

Cargo	**Cost**
$\frac{8}{15}$ ↓	₹ 2,000 ↓
$\frac{3}{5}$	₹ x

$$\Rightarrow \frac{x}{2{,}000} = \frac{3/5}{8/15}$$

$$\Rightarrow x = \frac{3}{5} \times \frac{15}{8} \times 2{,}000$$

$$= 2{,}250$$

∴ **Required cost = ₹ 2,250 (Ans.)**

Example 6 :

A watch gains 42 sec in 3 days and 8 hours. How long will it take to gain 2 min 6 sec ?

Solution :

Since, 3 days + 8 hours = (3 × 24 + 8) hours

= 80 hours

and, 2 min 6 sec = (2 × 60 + 6) sec

= 126 sec

Given : The watch gains 42 sec in 3 days 8 hours *i.e.* in 80 hours

⇒ The watch gains 1 sec in $\frac{80}{42}$ hours

and, it will gain 126 sec in $\frac{80}{42}$ × 126 hours = 240 hours.

∴ **Time taken by watch to gain 2 min 6 sec** = 240 hours = **10 days (Ans.)**

Arrow method :

Gain in time	**hrs**
42 ↓	80 ↓
126	x

$$\Rightarrow \frac{x}{80} = \frac{126}{42}$$

⇒ x = 240 hrs

∴ **Required time** = 240 hrs

= **10 days (Ans.)**

Example 7 :

If 135 kg of corn feeds 45 horses for 8 days, for how long will the same quantity of corn feed 24 horses ?

Solution :

Here, in both the cases, the quantity of corn remains the same.

Given : For 45 horses, the given quantity of corn is sufficient for 8 days.

⇒ For 1 horse, the same quantity of corn will be sufficient for 8 × 45 days

⇒ **For 24 horses, it will be sufficient for** $\frac{8 \times 45}{24}$ days = **15 days (Ans.)**

Arrow method :

No. of horses	**No. of days**
45 ↑	8 ↓
24	x

$$\Rightarrow \frac{x}{8} = \frac{45}{24} \Rightarrow x = 15$$

∴ **Required time**

= **15 days (Ans.)**

EXERCISE 7(B)

1. The cost of $\frac{3}{5}$ kg of ghee is ₹ 96, find the cost of : (i) one kg ghee. (ii) $\frac{5}{8}$ kg ghee.
2. $3\frac{1}{2}$ m of cloth costs ₹ 168, find the cost of $4\frac{1}{3}$ m of the same cloth.
3. A wrist-watch loses 10 sec in every 8 hours. In how much time will it lose 15 sec ?
4. In 2 days and 20 hours a watch gains 20 sec. Find, how much time the watch will take to gain 35 sec.
5. 50 men mow 32 hectares of land in 3 days. How many days will 15 men take to mow it ?
6. The wages of 10 workers for a six days week are ₹ 1,200. What are the one day wages:
   (i) of one worker ? (ii) of 4 workers ?
7. If 32 apples weigh 2 kg 800 g, how many apples will there be in a box, containing 35 kg of apples ?
8. A truck uses 20 litres of diesel for 240 km. How many litres will be needed for 1200 km ?
9. A garrison of 1200 men has provisions for 15 days. How long will the provisions last if the garrison be increased by 600 men ?
10. A camp has provisions for 60 pupils for 18 days. In how many days, the same provisions will finish off if the strength of the camp is increased to 72 pupils ?

7.4 TIME AND WORK

For solving problems on time and work, make the following simple facts clear :

1. If a man does a work in 50 days, his one day's work = $\frac{1}{50}$.

   Conversely, if one day's work of a man is $\frac{1}{50}$, he will complete the work in 50 days.

Thus, one day's work = $\frac{1}{\text{No. of days required to complete the work}}$

And, number of days required to complete a work = $\frac{1}{\text{One day work}}$

2. Time required to do a certain work = $\frac{\text{Work to be done}}{\text{Work done in unit time}}$

Example 8 :

A completes a piece of work in 4 days and B completes it in 6 days. How long will it take to complete the same work, if they both work on it together ?

Solution :

Given, A does the work in 4 days, $\therefore$ A's 1 day work = $\frac{1}{4}$

And, B does the work in 6 days, $\therefore$ B's 1 day work = $\frac{1}{6}$

$\Rightarrow$ (A + B)'s 1 day's work = $\frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12}$

Hence, **A and B together will complete the work in** $\frac{12}{5}$ days = $2\frac{2}{5}$ **days** **(Ans.)**

Example 9 :

Ajay and Vijay together can paint a hall in 6 days. Ajay alone can paint it in 8 days. In how many days can Vijay alone paint it ?

Solution :

Given, Ajay and Vijay together paint a hall in 6 days

$\therefore$ Ajay and Vijay in 1 day can paint $\frac{1}{6}$ of the hall.

Since, Ajay alone can paint the wall in 8 days

$\therefore$ Ajay alone in 1 day can paint $\frac{1}{8}$ of the hall.

$\Rightarrow$ Vijay alone in 1 day can paint $\frac{1}{6} - \frac{1}{8}$ of the hall $= \frac{4-3}{24} = \frac{1}{24}$ of the hall

$\therefore$ **Vijay alone can paint the hall in 24 days** **(Ans.)**

Example 10 :

A tap fills a cistern in 4 hours. Another tap empties the full tank in 6 hours. How long will it take to fill the tank, if the tank is empty and both the taps are open ?

Solution :

Since, one tap in 1 hour fills $\frac{1}{4}$ of the cistern

And, the second tap in 1 hour empties $\frac{1}{6}$ of the cistern

$\therefore$ Both the taps in 1 hour will fill $= \left(\frac{1}{4} - \frac{1}{6}\right)$ of the cistern

$= \frac{3-2}{12} = \frac{1}{12}$ of the cistern.

$\therefore$ **The cistern will be full in = 12 hours** **(Ans.)**

Example 11 :

A and B can do a piece of work in 15 days and 20 days respectively.

Find : (i) the work done by A in 3 days.

(ii) the work left after A has worked for 3 days.

(iii) the number of days that B will take to complete the remaining work.

Solution :

(i) $\because$ A's 1 day work $= \frac{1}{15}$

$\therefore$ **Work done by A in 3 days** $= \frac{1}{15} \times 3 = \frac{1}{5}$ **(Ans.)**

(ii) **Work left** after A has worked for 3 days $= 1 - \frac{1}{5} = \frac{4}{5}$ **(Ans.)**

(iii) Remaining work $= \frac{4}{5}$ and B's 1 day's work $= \frac{1}{20}$

$\therefore$ Number of days taken by B to complete the remaining work

$$= \frac{\text{work to be done}}{\text{B's 1 day's work}} = \frac{\frac{4}{5}}{\frac{1}{20}} = \frac{4}{5} \times \frac{20}{1} = 16 \text{ days}$$ **(Ans.)**

EXERCISE 7(C)

1. A can do a piece of work in 6 days and B can do it in 8 days. How long will they take to complete it together ?
2. A and B working together can do a piece of work in 10 days. B alone can do the same work in 15 days. How long will A alone take to do the same work ?
3. A can do a piece of work in 4 days and B can do the same work in 5 days. Find, how much work can be done by them working together in : (i) one day (ii) 2 days.

   What part of work will be left, after they have worked together for 2 days ?
4. A and B take 6 hours and 9 hours respectively to complete a work. A works for 1 hour and then B works for two hours.
   (i) How much work is done in these 3 hours ?
   (ii) How much work is still left ?
5. A, B and C can do a piece of work in 12, 15 and 20 days respectively. How long will they take to do it working together ?
6. Two taps can fill a cistern in 10 hours and 8 hours respectively. A third tap can empty it in 15 hours. How long will it take to fill the empty cistern, if all of them are opened together?
7. Mohit can complete a work in 50 days, whereas Anuj can complete the same work in 40 days.

   Find : (i) work done by Mohit in 20 days.
   (ii) work left after Mohit has worked on it for 20 days.
   (iii) time taken by Anuj to complete the remaining work.
8. Joseph and Peter can complete a work in 20 hours and 25 hours respectively.

   Find : (i) work done by both together in 4 hrs.
   (ii) work left after both worked together for 4 hrs.
   (iii) time taken by Peter to complete the remaining work.
9. A is able to complete $\frac{1}{3}$ of a certain work in 10 hrs and B is able to complete $\frac{2}{5}$ of the same work in 12 hrs.

   Find : (i) how much work can A do in 1 hour ?
   (ii) how much work can B do in 1 hour ?
   (iii) in how much time will the work be completed, if both work together?
10. Shaheed can prepare one wooden chair in 3 days and Shaif can prepare the same chair in 4 days. If they work together, in how many days will they prepare :
    (i) one chair ?
    (ii) 14 chairs of the same kind ?
11. A, B and C together finish a work in 4 days. If A alone can finish the same work in 8 days and B in 12 days, find how long will C take to finish the work.

PERCENT AND PERCENTAGE 8

8.1 BASIC CONCEPT

The word **CENT** means ***hundred***.

Hence, the word *percent* means, *per hundred* or *out of hundred*.

The notation for percent is "%".

Thus, 5 percent = 5%. [Read 5% as five percent].

8.2 TO EXPRESS AN ORDINARY GIVEN STATEMENT AS PERCENT

Steps :
1. Express the given statement as a fraction.
2. Convert this fraction into an equivalent fraction with denominator 100.

Example 1 :

7 out of 35 children in a class are absent. Express this statement as a percent.

Solution :

7 out of 35 means $\frac{7}{35} = \frac{1}{5}$ [Step 1]

$= \frac{1}{5} \times \frac{20}{20}$ [Step 2]

$= \frac{20}{100} = 20\% \Rightarrow$ **20% children are absent.** **(Ans.)**

OR, directly : **7 out of 35** means $\frac{7}{35} = \frac{7}{35} \times 100\% =$ **20%** **(Ans.)**

Therefore, to express a fraction or a decimal as percent, multiply it by 100 and in the same step write the sign of percent (%).

For example :

(i) $\frac{4}{10} = \frac{4}{10} \times 100\% = 40\%$ (ii) $0{\cdot}3 = 0{\cdot}3 \times 100\% = \frac{3}{10} \times 100\% = 30\%$

Conversely, to change a percent to a fraction or to a decimal, divide it by 100 and at the same time remove percent sign.

For example :

(i) $\frac{3}{4}\% = \frac{3}{4 \times 100} = \frac{3}{400}$ (as fraction) $= 0{\cdot}0075$ (as decimal)

(ii) $12{\cdot}5\% = \frac{12{\cdot}5}{100} = \frac{1}{8}$ (as fraction) $= 0{\cdot}125$ (as decimal) and so on.

More practice :

(i) 120% as fraction $= \frac{120}{100} = \frac{6}{5}$.

(ii) 3·6% as fraction = $\frac{3\cdot6}{100} = \frac{36}{100\times10} = \frac{9}{250}$.

(iii) $8\frac{1}{3}$% as fraction = $\frac{25}{3\times100} = \frac{1}{12}$.

(iv) 37% as decimal = $\frac{37}{100}$ = 0·37.

(v) 5% as decimal = $\frac{5}{100}$ = 0·05.

(vi) $18\frac{3}{4}$% as decimal = $\frac{75}{4\times100} = \frac{3}{16}$ = 0·1875.

(vii) 3 : 5 = $\frac{3}{5} = \frac{3}{5}$ × 100% = 60%.

(viii) 80% = $\frac{80}{100} = \frac{4}{5}$ = 4 : 5.

(ix) 24% as fraction = $\frac{24}{100} = \frac{6}{25}$.

24% as decimal = $\frac{24}{100}$ = 0·24.

24% as ratio = $\frac{24}{100} = \frac{6}{25}$ = 6 : 25.

(x) 0·27 as percent = 0·27 × 100% = 27%.

(xi) $3\frac{4}{5}$ as percent = $\frac{19}{5}$ × 100% = 380%.

Percentages, fractions and decimals are all different ways of representing values.

8.3 TO EXPRESS ONE QUANTITY AS A PERCENT OF THE OTHER

1. If necessary, convert the quantities into the same units.
2. Form the fraction with the number to be compared as numerator and the number with which it is to be compared as denominator.
3. Multiply the fraction obtained by 100 and at the same time write the percent sign (%).

Example 2 :

Express 40 p as a percent of ₹ 6.

Solution :

Fraction = $\frac{40\text{ p}}{600\text{ p}} = \frac{1}{15}$ [₹ 6 = 600 p]

Hence, required percent = $\frac{1}{15}$ × 100% = $\frac{20}{3}$% = $6\frac{2}{3}$% **(Ans.)**

Direct method :

40 p as percent of ₹ 6 = $\frac{40}{600}$ × 100% [∵ ₹ 6 = 600 p]

= $\frac{20}{3}$% = $6\frac{2}{3}$%

∴ If two quantities x and y are in the same unit, then

$$x \text{ as percent of } y = \frac{x}{y} \times 100\%$$

$$\text{and, } y \text{ as percent of } x = \frac{y}{x} \times 100\%$$

Example 3 :

A pudding is made of 400 g sugar, 200 g of eggs, 800 g of flour and 100 g of dry fruits. What percent of sugar is present in the whole pudding ?

Solution :

Here, the total weight of the pudding = (400 + 200 + 800 + 100) g = 1500 g

Weight of sugar = 400 g

∴ **Percentage of sugar in the pudding** = $\frac{400}{1500} \times 100\% = 26\frac{2}{3}\%$ **(Ans.)**

8.4 TO FIND PERCENTAGE OF A QUANTITY

1. $20\% \text{ of } 60 = \frac{20}{100} \times 60 = 12$
2. $40\% \text{ of } 7{\cdot}5 = \frac{40}{100} \times 7{\cdot}5 = 3$ and so on.

Example 4 :

In a class of 50 children, 10% are taking part in dramatics. How many children are not taking part ?

Solution :

Since, $10\% \text{ of } 50 = \frac{10}{100} \times 50 = 5$

Hence, 5 children are taking part and 50 – 5 = **45 are not taking part.** **(Ans.)**

Alternative method :

If 10% of the children are taking part

⇒ (100 – 10)% = 90% are not taking part

∴ **Number of children not taking part** = 90% of 50

$= \frac{90}{100} \times 50 = 45$ **(Ans.)**

EXERCISE 8(A)

1. Express each of the following as percent :

   (i) $\frac{3}{4}$ (ii) $\frac{2}{3}$ (iii) 0·025 (iv) 0·125

2. Express the following percentages as fractions and as decimal numbers :

   (i) $7\frac{1}{2}\%$ (ii) 2·50% (iii) 0·02% (iv) 175%

3. What percent is :

(i) 16 hours of 2 days ?
(ii) 40 paisa of ₹ 2 ?
(iii) 25 cm of 4 metres ?
(iv) 600 gm of 5 kg ?

4. Find the value of :

(i) 5% of ₹ 350
(ii) 10% of ₹ 400·40
(iii) 1% of ₹ 500
(iv) $12\frac{1}{2}$% of 80 kg
(v) $\frac{5}{8}$% of ₹ 600
(vi) $33\frac{1}{3}$% of 27 m

5. In a class of 60 children, 30% are girls. How many boys are there ?

6. In an election, two candidates A and B contested. A got 60% of the votes. The total votes polled were 8000. How many votes did each get ?

7. A person saves 12% of his salary every month. If his salary is ₹ 2,500, find his expenditure.

8. Seeta got 75% marks out of a total of 800. How many marks did she lose ?

9. A shop worth ₹ 25,000 was insured for 95% of its value. How much would the owner get in case of any mishappening ?

10. A class has 30 boys and 25 girls. What is the percentage of boys in the class ?

11. Express :

(i) $3\frac{2}{5}$ as percent
(ii) 0·0075 as percent
(iii) 3 : 20 as percent
(iv) 60 cm as percent of 1 m 25 cm
(v) 9 hours as percent of 4 days.

12 (i) Find 2% of 2 hours 30 min.

(ii) What percent of 12 kg is 725 gm ?

Example 5 :

Out of 50 apples, 20% were eaten and 20% of the remaining were rotten. Find the number of apples left.

Solution :

∵ Total number of apples = 50

∴ No. of apples eaten = 20% of 50 = $\frac{20}{100} \times 50 = 10$

No. of remaining apples = 50 − 10 = 40

No. of apples rotten = 20% of 40 = $\frac{20}{100} \times 40 = 8$

∴ **No. of apples left** = 40 − 8 = **32** **(Ans.)**

Example 6 :

The salary of a person is ₹ 2,000. Provident fund deducted is 8% of the salary. Of the remaining salary, he spends 10% on house rent and 20% on education of the children.

How much is the provident fund deducted ?

How much does he spend on the house rent and on education ?

Solution :

$\because$ Total Salary = ₹ 2,000

And, provident fund = 8% of ₹ 2,000 = $\frac{8}{100}$ × ₹ 2,000 = ₹ 160.

$\therefore$ Money left after deduction of provident fund = ₹ 2,000 – ₹ 160 = ₹ 1,840.

$\therefore$ Money spent on house rent = 10% of ₹ 1,840 = $\frac{10}{100}$ × ₹ 1,840 = ₹ 184

And, Money spent on education = 20% of ₹ 1,840 = $\frac{20}{100}$ × ₹ 1,840 = ₹ 368

$\therefore$ **Provident fund** = **₹ 160**, money spent on **house rent** = **₹ 184**

and **money spent on education** = **₹ 368** **(Ans.)**

Example 7 :

A girl does 25% of her home work in the morning and 45% of the home work in the evening. What percent of the work is still left ?

Solution :

	Home work done in the morning	=	25%
and,	home work done in the evening	=	45%
$\therefore$	Total home work done	=	25% + 45% = 70%
	Hence, percentage of **home work left**	=	(100 – 70)% = **30%** **(Ans.)**

The whole quantity, whole work, etc., are always taken as 100%

Example 8 :

20% of a number is 80. Find the number.

Solution :

Let the number be x.

$$\therefore 20\% \text{ of } x = 80 \quad \Rightarrow \quad \frac{20}{100} \times x = 80$$

$$\Rightarrow \quad x = \frac{80 \times 100}{20} = 400$$

Hence, **the required number** = **400** **(Ans.)**

EXERCISE 8(B)

1. Deepak bought a basket of mangoes containing 250 mangoes. 12% of these were found to be rotten. Of the remaining, 10% got crushed. How many mangoes were in good condition ?
2. In a Maths Quiz of 60 questions, Chandra got 90% correct answers and Ram got 80% correct answers. How many correct answers did each give ? What percent is Ram's correct answers to Chandra's correct answers ?
3. In an examination, the maximum marks are 900. A student gets 33% of the maximum marks and fails by 45 marks. What is the passing mark ? Also, find the pass percentage.

4. In a train, 15% people travel in first class and 35% travel in second class and the remaining travel in the A.C. class. Calculate the percentage of A.C. class travellers.

5. A boy eats 25% of the cake and gives away 35% of it to his friends. What percent of the cake is still left with him ?

6. What is the percentage of vowels in the English alphabet ?

7. (i) $6\frac{1}{4}\%$ of what number is 375 ?

   (ii) 0·2% of a number is 5. Find the number.

   (iii) 30 is $16\frac{2}{3}\%$ of a number. Find the number.

8. The money spent on the repairs of a house was 1% of its value. If the repair costs ₹ 5,000, find the cost of the house.

9. In a school, out of 300 students, 70% are girls and 30% are boys. If 30 girls leave and no new boy is admitted, what is the new percentage of girls in the school ?

10. Kumar bought a transistor for ₹ 960. He paid $12\frac{1}{2}\%$ cash money. The rest he agreed to pay in 12 equal monthly instalments. How much will he pay each month ?

11. An ore contains 20% zinc. How many kg of ore will be required to get 45 kg of zinc ?

8.5 PERCENTAGE CHANGE

1. Percentage change = $\frac{\text{Decrease (or increase) in the value}}{\text{Original value}} \times 100\%$

2. The change percent is always calculated on the original value.

Example 9 :

A bicycle costs ₹ 800. After six months its value became ₹ 650. By what percent has the price decreased ?

Solution :

Original price = ₹ 800 and reduced price = ₹ 650

∴ Decrease in price = ₹ 800 − ₹ 650 = ₹ 150

And, **percentage decrease** = $\frac{\text{Decrease in price}}{\text{Original price}} \times 100\%$

$= \frac{150}{800} \times 100\% = \frac{75}{4}\% = 18\frac{3}{4}\%$ **(Ans.)**

Example 10 :

A line of length 1·5 metres was measured 1·55 metres by mistake. Find the error percent.

Solution :

Actual length = 1·5 m and wrong length = 1·55 m

∴ Error = 1·55 m − 1·5 m = 0·05 m

And, **error %** = $\frac{\text{Error}}{\text{Actual length}} \times 100\% = \frac{0 \cdot 05}{1 \cdot 5} \times 100\% = \frac{10}{3}\% = 3\frac{1}{3}\%$ **(Ans.)**

Example 11 :

(i) Increase 80 by 25% (ii) Decrease 60 by 10%.

Solution :

(i) Since, original number = 80

Increase = 25% of 80 = $\frac{25}{100} \times 80 = 20$

$\therefore$ **new** (increased) **number** = 80 + 20 = **100** **(Ans.)**

(ii) Since, original number = 60

Decrease = 10% of 60 = $\frac{10}{100} \times 60 = 6$

$\therefore$ **new** (decreased) **number** = 60 − 6 = **54** **(Ans.)**

Example 12 :

What number when increased by 25% becomes 150 ?

Solution :

Let the number be 100.

$\because$ Increase in number = 25% of 100 = $\frac{25}{100} \times 100 = 25$

$\therefore$ Increased number = 100 + 25 = 125

When increased no. = 125, the original no. = 100

When increased no. = 1, the original no. = $\frac{100}{125}$

When increased no. = 150, the **original no.** = $\frac{100}{125} \times 150 =$ **120** **(Ans.)**

Alternative method :

Let the original number be x.

$\therefore$ Increase in number = 25% of $x = \frac{25}{100} \times x = \frac{x}{4}$

And, so $x + \frac{x}{4} = 150 \Rightarrow \frac{4x + x}{4} = 150$

$\Rightarrow 5x = 150 \times 4$

$\Rightarrow x = \frac{600}{5} = 120$ **(Ans.)**

Example 13 :

A number first dercreases by 60%, then again decreases by 80%. Find the percentage decrease on the whole.

Solution :

Let the number be 100.

In 1st case : Decrease = 60% of 100

$= \frac{60}{100} \times 100 = 60$

$\therefore$ Number after this decrease = 100 − 60 = 40

In 2nd case : Decrease = 80% of 40

$$= \frac{80}{100} \times 40 = 32$$

$\therefore$ Number after the 2nd decrease = 40 − 32 = 8

$\Rightarrow$ Total decrease on the whole = 60 + 32 = 92

and, **the percentage decrease on the whole**

$$= \frac{92}{100} \times 100\% = 92\%$$ **(Ans.)**

EXERCISE 8(C)

1. The salary of a man is increased from ₹ 600 per month to ₹ 850 per month. Express the increase in salary as percent.
2. Increase :
   (i) 60 by 5% (ii) 20 by 15% (iii) 48 by $12\frac{1}{2}$%
   (iv) 80 by 140% (v) 1000 by 3·5%
3. Decrease :
   (i) 80 by 20% (ii) 300 by 10% (iii) 50 by 12·5%
4. What number :
   (i) when increased by 10% becomes 88 ?
   (ii) when increased by 15% becomes 230 ?
   (iii) when decreased by 15% becomes 170 ?
   (iv) when decreased by 40% becomes 480 ?
   (v) when increased by 100% becomes 100 ?
   (vi) when decreased by 50% becomes 50 ?
5. The price of a car is lowered by 20% to ₹ 40,000. What was the original price ? Also, find the reduction in price.
6. If the price of an article is increased by 25%, the increase is ₹ 10. Find the new price.
7. If the price of an article is reduced by 10%, the reduction is ₹ 40. What is the old price?
8. The price of a chair is reduced by 25%. What is the ratio of :
   (i) change in price to the old price. (ii) old price to the new price.
9. If x is 20% less than y, find :
   (i) $\frac{x}{y}$ (ii) $\frac{y-x}{y}$ (iii) $\frac{x}{y-x}$

(i) Given : x = y − 20% of y

$$\Rightarrow \quad x = y - \frac{20y}{100} = \frac{100y - 20y}{100} = \frac{80y}{100} = \frac{4y}{5}$$

$$\therefore \quad x = \frac{4y}{5} \Rightarrow 5x = 4y \Rightarrow \frac{x}{y} = \frac{4}{5}$$ **(Ans.)**

(ii) $\frac{x}{y} = \frac{4}{5} \Rightarrow$ if $x = 4$, $y = 5$

$\therefore \frac{y - x}{y} = \frac{5 - 4}{5} = \frac{1}{5}$ **(Ans.)**

(iii) Again, $\frac{x}{y} = \frac{4}{5} \Rightarrow$ if $x = 4$, $y = 5 \Rightarrow \frac{x}{y - x} = \frac{4}{5 - 4} = 4$ **(Ans.)**

10. If x is 30% more than y; find :

(i) $\frac{x}{y}$ (ii) $\frac{y + x}{x}$ (iii) $\frac{y}{y - x}$

11. The weight of a machine is 40 kg. By mistake, it was weighed as 40·8 kg. Find the error percent.

12. From a cask, containing 450 litres of petrol, 8% of the petrol was lost by leakage and evaporation. How many litres of petrol were left in the cask ?

13. An alloy consists of 13 parts of copper, 7 parts of zinc and 5 parts of nickel. What is the percentage of each metal in the alloy ?

14. In an examination, first division marks are 60%. A student secures 538 marks and misses the first division by 2 marks. Find the total marks of the examination.

15. Out of 1200 pupils in a school, 900 are boys and the rest are girls. If 20% of the boys and 30% of the girls wear spectacles, find :

(i) how many pupils in all wear spectacles.

(ii) what percent of the total number of pupils wear spectacles.

16. Out of 25 identical bulbs, 17 are red, 3 are black and the remaining are yellow. Find the difference between the numbers of red and yellow bulbs and express this difference as percent.

17. A number first increases by 20% and then decreases by 20%. Find the percentage increase or decrease on the whole.

18. A number is first decreased by 40% and then again decreased by 60%. Find the percentage increase or decrease on the whole.

19. If 150% of a number is 750, find 60% of this number.

EXERCISE 8(D)

1. 28% of a number is 84. Find the number.

2. Every month, a man spends 72% of his income and saves ₹ 12,600. Find :

(i) his monthly income (ii) his monthly expenses

3. 1800 boys and 900 girls appeared for an examination. If 42% of the boys and 30% of the girls passed, find

(i) number of boys passed (ii) number of girls passed

(iii) total number of students passed (iv) number of students failed

(v) percentage of students failed.

4. $6\frac{1}{4}$% of a weight is 0·25 kg. What is 45% of this weight ?

5. An alloy consists of 13 parts of copper, 7 parts of zinc and 5 parts of nickel. Find the percentage of copper in the alloy.

6. An ore contains 15% of iron. How much ore will be required to get 36 kg of iron ?

7. Find the number which when increased by 6% becomes 424.

8. Find the number which when decreased by 15% becomes 1360.

9. The cost of an article decreases from ₹ 17,000 to 15,980. Find the percentage decrease.

10. Actual length of a rope is 22·5 m but it is wrongly measured as 21·6 m. Find the percentage error.

PROFIT, LOSS AND DISCOUNT 9

9.1 PROFIT AND LOSS

1. *Cost price (C.P.) of an article is the price at which the article is purchased.*
2. *Selling price (S.P.) of an article is the price at which the article is sold.*
3. *If **selling price** of an article is **more** than its **cost price;** it is sold at a **profit** (gain).*

   Profit = Selling Price – Cost Price

   i.e., **Profit** (gain) **= S.P. – C.P.** and **S.P. = C.P. + Gain**
4. *If **selling price** of an article is **less** than its **cost price;** it is sold at a **loss.***

   Loss = Cost Price – Selling Price

   i.e., **Loss = C.P. – S.P.** and **S.P. = C.P. – Loss**
5. *Profit percent and loss percent are always calculated on cost price (C.P.) only.*

   i.e., (i) Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100\%$ and (ii) Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100\%$

Example 1 :

(i) An article, bought for ₹ 120, is sold for ₹ 150. Find the gain or loss percent.

(ii) A bicycle, bought for ₹ 600, is sold for ₹ 550. Find gain or loss percent.

Solution :

(i) Given : C.P. = ₹ 120 and S.P. = ₹ 150

∴ Gain = ₹ 150 – ₹ 120 = ₹ 30

And, **gain %** = $\frac{\text{Gain}}{\text{C.P.}} \times 100\% = \frac{30}{120} \times 100\% = 25\%$ **(Ans.)**

(ii) Given : C.P. = ₹ 600 and S.P. = ₹ 550

∴ Loss = C.P. – S.P. = ₹ 600 – ₹ 550 = ₹ 50.

And, **loss percent** = $\frac{\text{Loss}}{\text{C.P.}} \times 100\% = \frac{50}{600} \times 100\% = \frac{25}{3}\% = 8\frac{1}{3}\%$ **(Ans.)**

9.2 TO FIND SELLING PRICE *(WHEN C.P. AND PROFIT% OR LOSS% ARE KNOWN)*

Example 2 :

(i) Geeta bought a watch for ₹ 450. For how much should she sell it to gain 10% ?

(ii) Rahul bought an article for ₹ 800 and sold it at 25% loss. Find the selling price.

Solution :

(i) Given : C.P. = ₹ 450

∴ Gain = 10% of C.P. = $\frac{10}{100} \times$ ₹ 450 = ₹ 45 [Gain % is always on C.P]

And, Selling Price = Cost price + Gain

= ₹ 450 + ₹ 45 = ₹ 495 **(Ans.)**

Alternative method *(Direct method)* **:**

$$\text{Gain} = 10\% \Rightarrow \text{S.P.} = 110\% \text{ of C.P.}$$

$$\therefore \quad \textbf{S.P.} = 110\% \text{ of ₹ } 450 = \frac{110}{100} \times ₹\ 450 = ₹\ 495 \quad \textbf{(Ans.)}$$

(ii) $\because$ C.P. = ₹ 800 and loss = 25% of ₹ 800 = $\frac{25}{100} \times$ ₹ 800 = ₹ 200

$\therefore$ **Selling price** = Cost price − Loss = ₹ 800 − ₹ 200 = **₹ 600** **(Ans.)**

Alternative method :

$\because$ Loss = 25% $\Rightarrow$ S.P. = 75% of C.P.

i.e., **selling price** = $\frac{75}{100} \times$ ₹ 800 = **₹ 600** **(Ans.)**

Remember :

1. If gain = 20%, S.P. = 120% of C.P., *i.e.,* S.P. = $\frac{120}{100} \times$ C.P.
2. If loss = 20%, S.P. = 80% of C.P., *i.e.,* S.P. = $\frac{80}{100} \times$ C.P.

Note : Some times, the selling price and the cost price are given for different number of articles, then to find loss or gain percent, first of all, find the cost price and the selling price of equal number of articles.

Examples 3 :

A shopkeeper buys 50 pencils for ₹ 80 and sells them at 40 pencils for ₹ 90. Find his gain or loss percent.

Solution :

Since, C.P. of 50 pencils = ₹ 80 $\therefore$ C.P. of 1 pencil = ₹ $\frac{80}{50}$ = ₹ 1.60

Since, S.P. of 40 pencils = ₹ 90 $\therefore$ S.P. of 1 pencil = ₹ $\frac{90}{40}$ = ₹ 2.25

Hence, gain = ₹ 2.25 − ₹ 1.60 = ₹ 0.65

And, **Gain %** = $\frac{0.65}{1.60} \times 100\% = 40\frac{5}{8}\%$ **(Ans.)**

EXERCISE 9(A)

1. Find the gain or loss percent, if :
   (i) C.P. = ₹ 200 and S.P. = ₹ 224
   (ii) C.P. = ₹ 450 and S.P. = ₹ 400
   (iii) C.P. = ₹ 550 and gain = ₹ 22
   (iv) C.P. = ₹ 216 and loss = ₹ 72
   (v) S.P. = ₹ 500 and loss = ₹ 100
2. Find the selling price, if :
   (i) C.P. = ₹ 500 and gain = 25%
   (ii) C.P. = ₹ 60 and loss = $12\frac{1}{2}\%$
3. Rohit bought a tape-recorder for ₹ 1,500 and sold it for ₹ 1,800. Calculate his profit or loss percent.

4. An article bought for ₹ 350 is sold at a profit of 20%. Find its selling price.
5. An old machine is bought for ₹ 1,400 and is sold at a loss of 15%. Find its selling price.
6. Oranges are bought at 5 for ₹ 10 and sold at 6 for ₹ 15. Find profit or loss as percent.
7. A certain number of articles are bought at 3 for ₹ 150 and all of them are sold at 4 for ₹ 180. Find the loss or gain as percent.
8. A vendor bought 120 sweets at 20 p each. In his house, 18 were consumed and he sold the remaining at 30 p each. Find his profit or loss as percent.

   C.P. of all the sweets = 120×20 p and S.P. of all the sweets = $(120 - 18) \times 30$ p.

9. The cost price of an article is ₹ 1,200 and selling price is $\frac{5}{4}$ times of its cost price.
   Find : (i) selling price of the article, (ii) profit or loss as percent.
10. The selling price of an article is ₹ 1,200 and cost price is $\frac{5}{4}$ times of its selling price.
    Find : (i) cost price of the article, (ii) profit or loss as percent.

9.3 TO FIND COST PRICE

Example 4 :

By selling an article for ₹ 550, a profit of 10% is made. Find its cost price.

Solution :

Let C.P. = ₹ 100

∴ Profit = 10% of ₹ 100 = ₹ 10

and S.P. = ₹ 100 + ₹ 10 = ₹ 110 [S.P. = C.P. + Profit]

When S.P. = ₹ 110, C.P. = ₹ 100

∴ When S.P. = ₹ 1, C.P. = ₹ $\frac{100}{110}$

And, when S.P. = ₹ 550, **C.P.** = ₹ $\frac{100}{110} \times 550$ = **₹ 500** **(Ans.)**

Alternative method :

∵ Profit = 10% ⇒ S.P. = 110% of C.P.

∴ ₹ 550 = $\frac{110}{100} \times$ C.P. ⇒ ₹ 550 × $\frac{100}{110}$ = C.P.

Hence, **C.P.** = ₹ 550 × $\frac{100}{110}$ = **₹ 500** **(Ans.)**

Algebraic method :

Let C.P. = ₹ x;

∴ Profit = 10% of ₹ x = $\frac{10}{100}$ x ₹ x = ₹ $\frac{x}{10}$

C.P. + Gain = S.P. ⇒ $x + \frac{x}{10} = 550$

$\Rightarrow \quad \frac{10x + x}{10} = 550 \quad \Rightarrow \quad \frac{11x}{10} = 550$

$\Rightarrow \quad x = 550 \times \frac{10}{11} \quad \Rightarrow \quad$ **C.P. = ₹ 500** **(Ans.)**

Example 5 :

By selling an article for ₹ 270, a loss of 10 percent is suffered.

Find : (i) the cost price of the article.

(ii) the price at which the article must be sold in order to gain 12%.

Solution :

(i) Let C.P. = ₹ 100

∴ Loss = 10% of ₹ 100 = ₹ 10

and S.P. = ₹ 100 – ₹ 10 = ₹ 90

∴ When S.P. = ₹ 90, C.P. = ₹ 100

When S.P. = ₹ 1, C.P. = ₹ $\frac{100}{90}$

And, when S.P. = ₹ 270, **C.P.** = ₹ $\frac{100}{90} \times 270$ = **₹ 300** **(Ans.)**

Alternative method :

Loss = 10% ⇒ S.P. = 90% of C.P.

∴ ₹ 270 = $\frac{90}{100}$ × C.P. ⇒ ₹ 270 × $\frac{100}{90}$ = C.P.

C.P. = ₹ 270 × $\frac{100}{90}$ = **₹ 300** **(Ans.)**

Algebraic method :

Let C.P. = ₹ x

Loss = 10% of ₹ x = $\frac{10}{100}$ × ₹ x = ₹ $\frac{x}{10}$

∵ C.P. – Loss = S.P. $\Rightarrow x - \frac{x}{10} = 270$

$\Rightarrow \quad \frac{10x - x}{10} = 270 \quad \Rightarrow \quad \frac{9x}{10} = 270$

$\Rightarrow \quad x = 270 \times \frac{10}{9} = 300 \quad \Rightarrow$ **C.P. = ₹ 300** **(Ans.)**

(ii) C.P. = ₹ 300

and gain = 12% of ₹ 300 = $\frac{12}{100}$ × ₹ 300 = ₹ 36.

∴ **New selling price** = ₹ 300 + ₹ 36 = **₹ 336** **(Ans.)**

Alternative method :

$$\text{Since, gain} = 12\%$$

$$\text{S.P.} = \frac{112}{100} \times \text{C.P.}$$

$$= \frac{112}{100} \times ₹\,300 = ₹\,336 \qquad \textbf{(Ans.)}$$

Example 6 :

A trader bought 12 eggs for ₹16. For how much should he sell one egg to gain 50 percent ?

Solution :

$\because$ C.P. of 12 eggs = ₹ 16

and, gain = 50% of ₹ 16 = $\frac{50}{100} \times ₹\,16 = ₹\,8$

$\therefore$ S.P. of 12 eggs = ₹ 16 + ₹ 8 = ₹ 24

and, **S.P. of 1 egg** = ₹ $\frac{24}{12}$ = **₹ 2** **(Ans.)**

Alternative method :

Since, C.P. of 12 eggs = ₹ 16 $\Rightarrow$ C.P. of 1 egg = ₹ $\frac{16}{12}$ = ₹ $\frac{4}{3}$

Gain = 50% $\Rightarrow$ S.P. = 150% of C.P.

$\Rightarrow$ **S.P.** = $\frac{150}{100} \times ₹\,\frac{4}{3}$ = **₹ 2** **(Ans.)**

EXERCISE 9(B)

1. Find the cost price, if :
   (i) S.P. = ₹ 21 and gain = 5%
   (ii) S.P. = ₹ 22 and loss = 12%
   (iii) S.P. = ₹ 340 and gain = ₹ 20
   (iv) S.P. = ₹ 200 and loss = ₹ 50
   (v) S.P. = ₹ 1 and loss = 5 p
2. By selling an article for ₹ 810, a loss of 10 percent is suffered. Find its cost price.
3. By selling a scooter for ₹ 9,200, a man gains 15%. Find the cost price of the scooter.
4. On selling an article for ₹ 2,640, a profit of 10 percent is made.
   Find : (i) cost price of the article.
   (ii) new selling price of it, in order to gain 15%.
5. A T.V. set is sold for ₹ 6,800 at a loss of 15%.
   Find : (i) cost price of the T.V. set.
   (ii) new selling price of it, in order to gain 12%.
6. A fruit seller bought mangoes at ₹ 90 per dozen and sold them at a loss of 8 percent. How much will a customer pay for :
   (i) one mango
   (ii) 40 mangoes

7. By selling two transistors for ₹ 600 each, a shopkeeper gains 20 percent on one transistor and loses 20 percent on the other.

Find : (i) C.P. of each transistor.

(ii) total C.P. and total S.P. of both the transistors.

(iii) profit or loss percent on the whole.

8. Mangoes are bought at 20 for ₹ 60. If they are sold at a profit of $33\frac{1}{3}$ percent, find : (i) selling price of each mango. (ii) S.P. of 8 mangoes.

9. Find the cost price of an article, which is sold for ₹ 4,050 at a loss of 10%. Also, find the new selling price of the article which must give a profit of 8%.

10. By selling an article for ₹ 825, a man loses an amount equal to $\frac{1}{3}$ of its selling price.

Find : (i) the cost price of the article.

(ii) the profit percent or the loss percent made, if the same article is sold for ₹ 1,265.

11. Find the loss or gain as percent, if the C.P. of 10 articles, all of the same kind, is equal to S.P. of 8 articles.

12. Find the loss or gain as percent, if the C.P. of 8 articles, all of the same kind, is equal to S.P. of 10 articles.

13. The cost price of an article is 96% of its selling price. Find the loss or the gain as percent on the whole.

Let S.P. = ₹ 100 $\Rightarrow$ C.P. = ₹ $\frac{96}{100} \times 100$ = ₹ 96

$\therefore$ Profit = ₹ 100 − ₹ 96 = ₹ 4

and, **profit as percent** = $\frac{4}{96} \times 100\% = 4\frac{1}{6}\%$ **(Ans.)**

14. The selling price of an article is 96% of its cost price. Find the loss or the gain as percent on the whole.

15. Hundred oranges are bought for ₹ 350 and all of them are sold at the rate of ₹ 48 per dozen. Find the profit percent or loss percent made.

$\because$ C.P. of one orange = ₹ $\frac{350}{100}$ = ₹ 3·50

and S.P. of one orange = ₹ $\frac{48}{12}$ = ₹ 4

Clearly, profit = ₹ 4 − ₹ 3·50 = ₹ 0·50

and, **profit percent** = $\frac{0{\cdot}50}{3{\cdot}50} \times 100\% = 14\frac{2}{7}\%$ **(Ans.)**

16. Oranges are bought at 100 for ₹ 80 and all of them are sold at 80 for ₹ 100. Find the loss or gain as percent in this transaction.

17. An article is bought for ₹ 5,700 and ₹ 1,300 is spent on its repairing, transportation, etc. For how much should this article be sold in order to gain 20% on the whole.

9.4 DISCOUNT

Students must have seen price-chits on the articles, which are kept in shops for sale.

The price written on a chit is the *Marked Price* (M.P.) or *List Price* or *Advertised Price of the article on which the chit is attached.*

Often the shopkeepers say, they will give a discount.

What does this discount mean ?

It means, a reduction in the price, which is always calculated on the marked price of the article.

> Selling price = Marked price – Discount
> *i.e.,* S.P. = M.P. – Discount

Example 7 :

A dealer marks a T.V. set for ₹ 9,000 but agrees to give a discount of 20%. Find the selling price of the T.V. set.

Solution :

Here, marked price (M.P.) = ₹ 9,000

Discount = 20% of ₹ 9,000 = ₹ $\frac{20}{100} \times 9{,}000$ = ₹ 1,800

∴ **S.P.** = ₹ 9,000 – ₹ 1,800 = **₹ 7,200** **(Ans.)**

Example 8 :

An article, marked at ₹ 155, is sold for ₹ 124. Find the discount and discount percent ?

Solution :

M.P. = ₹ 155 and S.P.= ₹ 124

∴ **Discount** = ₹ 155 – ₹ 124 = **₹ 31** **(Ans.)**

And, **discount percent** = $\frac{\text{Discount}}{\text{M.P.}} \times 100\%$

= $\frac{31}{155} \times 100\%$ = **20%** **(Ans.)**

EXERCISE 9(C)

1. A machine is marked at ₹ 5,000 and is sold at a discount of 10%. Find the selling price of the machine.
2. A shopkeeper marked a dinner set for ₹ 1,000. He sold it at ₹ 900. What percent discount did he give ?
3. A pair of shoes, marked at ₹ 320, are sold at a discount of 15 percent.
   Find : (i) the discount, (ii) the selling price of the shoes.
4. The list price of an article is ₹ 450 and it is sold for ₹ 360.
   Find : (i) the discount, (ii) the discount percent.

5. A shopkeeper buys an article for ₹ 300. He increases its price by 20% and then gives 10% discount on the new price.

Find : (i) the new price (marked price) of the article.

(ii) the discount given by the shopkeeper.

(iii) the selling price.

(iv) the profit percent made by the shopkeeper.

6. A car is marked at ₹ 50,000. The dealer gives 5% discount on first ₹ 20,000 and 2% discount on the remaining ₹ 30,000.

Find : (i) the total discount. (ii) the price charged by the dealer.

7. A dealer buys a T.V. set for ₹ 2,500. He marks it at ₹ 3,200 and then gives a discount of 10% on it.

Find : (i) the selling price of the T.V. set (ii) the profit percent made by the dealer.

8. A sells his goods at 15% discount. Find the price of an article which is sold for ₹ 680.

9. A shopkeeper allows 20% discount on the marked price of his articles. Find the marked price of an article for which he charges ₹ 560.

10. An article is bought for ₹ 1,200 and ₹ 100 is spent on its transportation, etc.

Find : (i) the total C.P. of the article.

(ii) the selling price of it in order to gain 20% on the whole.

11. 40 pens are bought at 4 for ₹ 50 and all of them are sold at 5 for ₹ 80.

Find : (i) C.P. of one pen.

(ii) S.P. of one pen.

(iii) Profit made by selling one pen.

(iv) Profit percent made by selling one pen.

(v) C.P. of 40 pens.

(vi) S.P. of 40 pens.

(vii) Profit made by selling 40 pens.

(viii) Profit percent made by selling 40 pens.

Are the results of parts (iv) and (viii) same ?

What conclusion do you draw from the above result ?

12. The C.P. of 5 identical articles is equal to S.P. of 4 articles. Calculate the profit percent or loss percent made if all the articles bought have been sold.

Let C.P. of 5 articles = S.P. of 4 articles = ₹ 100

$\Rightarrow$ C.P. of 1 article $= \frac{₹ 100}{5} = ₹ 20$

and, S.P. of 1 article $= \frac{₹ 100}{4} = ₹ 25$

$\therefore$ Profit on 1 article = ₹ 25 − ₹ 20 = ₹ 5

Profit % $= \frac{\text{Profit}}{\text{C.P.}} \times 100\%$

$= \frac{₹ 5}{₹ 20} \times 100\% = $ **25%** **(Ans.)**

Alternative method

$$\text{Let C.P. of 1 article} = ₹\ 100$$

$\Rightarrow$ C.P. of 5 articles = ₹ 100 × 5 = ₹ 500

$\Rightarrow$ S.P. of 4 articles = ₹ 500

and S.P. of 1 article = $\frac{₹\ 500}{4}$ = ₹ 125

Profit on 1 article = ₹ 125 − ₹ 100 = ₹ 25

Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100\%$

$= \frac{25}{100} \times 100\% = 25\%$ **(Ans.)**

13. The C.P. of 12 pens is the same as the S.P. of 15 pens. Calculate the profit or loss percent made, if all the pens bought are considered to be sold.

14. A certain number of articles are bought at ₹ 450 per dozen and all of them are sold at a profit of 20%. Find the S.P. of :
(i) one article (ii) seven articles.

15. An article is marked 60% above the cost price and sold at 20% discount. Find the profit percent made.

Let C.P. = ₹ 100

$\Rightarrow$ M.P. = ₹ 100 + 60% of ₹ 100

= ₹ 100 + ₹ 60 = ₹ 160

Now, M.P. = ₹ 160 and discount = 20%, so we can find the S.P.

With the known S.P. and the known C.P., we can find the profit percent.

SIMPLE INTEREST 10

10.1 TERMS USED

1. *Simple Interest :*

It is the extra money, which the lender gets from a borrower, in consideration of the sum (money borrowed) *used by the borrower.*

The **simple interest** (S.I.) and the **interest** (I) mean the same.

2. *Principal* *(P) :*

It is the sum (money) *which the lender gives to a borrower.*

3. *Rate* or ***Rate of Interest*** (R) :

It is the interest for a fixed period on every ₹ 100.

e.g. (i) Rate of interest is 18% per year means, on ₹ 100 the interest in one year is ₹ 18.

(ii) Rate is 1.5% per month means, the interest of one month on ₹ 100 is ₹ 1.5, *i.e.,* ₹ 1.50.

4. *Time Period* (T) :

It is the time for which the sum (principal) *is borrowed or lent.*

1. *Questions on interest involve four quantities : the Principal* (**P**), *the Rate of Interest* (**R**), *the Period* (**T**) *and the Interest* (**I**) *of that period.*

   And, all these are related to each other as :

$$\text{Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} \quad \textit{i.e.,} \quad \mathbf{I} = \frac{\mathbf{P \times R \times T}}{\mathbf{100}}$$

2. *The formula* $I = \frac{P \times R \times T}{100}$ *can also be expressed as :*

   (i) $\mathbf{P} = \frac{\mathbf{I \times 100}}{\mathbf{R \times T}}$ (ii) $\mathbf{R} = \frac{\mathbf{I \times 100}}{\mathbf{P \times T}}$ (iii) $\mathbf{T} = \frac{\mathbf{I \times 100}}{\mathbf{P \times R}}$

5. *Amount* (A) *:*

It is the sum of the Principal and the Interest on it.

∴ Amount = Principal + Interest *i.e.,* **A = P + I**

Example 1 :

Find the amount of a loan of ₹ 3,000 at 4% per year and for 5 years.

Solution :

Given : P = ₹ 3,000, R = 4% and T = 5 years.

$$\therefore \quad \text{Interest, } I = \frac{P \times R \times T}{100} = ₹\ \frac{3{,}000 \times 4 \times 5}{100} = ₹\ 600.$$

And, **Amount = P + I** = ₹ 3,000 + ₹ 600 = **₹ 3,600** **(Ans.)**

Example 2 :

Find the interest on ₹ 800 at 6 percent per month for 9 months.

Solution :

> **Note :** Time is always taken according to the percentage rate, *i.e.*, if the rate is given per annum, *i.e.*, per year, the time must be taken in years and if the rate is per month, the time must be taken in months.

Since, in this example, Rate = 6% per month.

∴ Time should be taken in months.

Now, P = ₹ 800, R = 6% per month and T = 9 months.

$$\Rightarrow \quad I = \frac{P \times R \times T}{100} = ₹\ \frac{800 \times 6 \times 9}{100} = ₹\ 432 \qquad \textbf{(Ans.)}$$

Example 3 :

Find the S.I. on ₹ 900 lent on August 10 and received back on October 22 of the same year, the rate being $8\frac{3}{4}$% p.a. (per annum).

Solution :

Given : P = ₹ 900, R = $8\frac{3}{4}$% p.a. = $\frac{35}{4}$% p.a.

	August		September		October
T =	21	+	30	+	22
	(31 − 10)				

$$= 73 \text{ days} = \frac{73}{365} \text{ years} = \frac{1}{5} \text{ years} \qquad \text{[As rate is p.a.]}$$

$$\therefore \quad \text{S.I.} = \frac{P \times R \times T}{100} = ₹\ \frac{900 \times 35 \times 1}{100 \times 4 \times 5} = ₹\ 15.75 \qquad \textbf{(Ans.)}$$

> For finding time, the starting date is not included.
> In example 3, given above, the starting date is 10th August. Therefore, from total number of days in August, 10 days are subtracted. But the last date, *i.e.*, October 22 is included.

Example 4 :

(i) What sum will earn an interest of ₹ 480 in 3 years, at 16% per year ?

(ii) In what time will ₹ 2,100 fetch an interest of ₹ 525 at 5% p.a. ?

(iii) At what rate percent per year will a sum of money double itself in 10 years ?

Solution :

(i) Given : I = ₹ 480, T = 3 years and R = 16%

$$\therefore \quad \textbf{Sum } (P) = \frac{I \times 100}{R \times T} = ₹\ \frac{480 \times 100}{16 \times 3} = ₹\ 1{,}000 \qquad \textbf{(Ans.)}$$

(ii) Given : P = ₹ 2,100, I = ₹ 525 and R = 5%

$$\therefore \quad \textbf{Time } (T) = \frac{I \times 100}{P \times R} = \frac{525 \times 100}{2{,}100 \times 5} \text{ years} = 5 \text{ years} \qquad \textbf{(Ans)}$$

(iii) Given : $P = ₹\ 100, A = ₹\ 200$

Then, $I = ₹\ 200 - ₹\ 100 = ₹\ 100$

$\therefore$ Rate% (R) $= \dfrac{I \times 100}{P \times T} = \dfrac{100 \times 100}{100 \times 10} = 10\%$ **(Ans)**

Example 5 :

What sum of money will amount to ₹ 992 at 4% in 6 years ?

Solution :

Let the sum (principal) be ₹ 100.

$\therefore$ Interest $= \dfrac{P \times R \times T}{100} = ₹\ \dfrac{100 \times 4 \times 6}{100} = ₹\ 24$

And, amount (A) $= P + I = ₹\ 100 + ₹\ 24 = ₹\ 124$

$\therefore$ When A = ₹ 124; P = ₹ 100 [Applying Unitary method]

$\Rightarrow$ When A = ₹ 992; **P** $= ₹\ \dfrac{100}{124} \times 992 =$ **₹ 800** **(Ans.)**

Alternative method :

$A = P + I \Rightarrow A = P + \dfrac{P \times R \times T}{100}$ *i.e.* $₹\ 992 = P + \dfrac{P \times 4 \times 6}{100}$

$\Rightarrow ₹\ 992 = \dfrac{100P + 24P}{100}$ *i.e.* $₹\ 992 \times 100 = 124P$

$\Rightarrow$ **P** $= ₹\ \dfrac{992 \times 100}{124} =$ **₹ 800** **(Ans.)**

Example 6 :

How long will it take ₹ 1,500 to become ₹ 2,040 at 8% per annum simple interest ?

Solution :

Given : $P = ₹\ 1{,}500$; $A = ₹\ 2{,}040$ and $R = 8\%$

$\therefore I = A - P = ₹\ 2{,}040 - ₹\ 1{,}500 = ₹\ 540$

And, so, Time (T) $= \dfrac{I \times 100}{P \times R}$

$= \dfrac{540 \times 100}{1{,}500 \times 8}$ years $= \dfrac{9}{2}$ years $= 4\dfrac{1}{2}$ **years** **(Ans.)**

$4\dfrac{1}{2}$ years $= 4$ years $+ \dfrac{1}{2}$ year

$= 4$ years $+ \dfrac{1}{2} \times 12$ months

$= 4$ years $+ 6$ months

or 4 years and 6 months

Example 7 :

A invests ₹ 8,000 and B invests ₹ 11,000 at the same rate of interest per annum. If at the end of 3 years, B gets ₹ 720 more interest than A; find the rate of interest.

Solution :

Let the rate of interest = R% per annum.

For A : P = ₹ 8,000 and T = 3 years

$\therefore \quad I = \frac{₹\,8{,}000 \times R \times 3}{100} = ₹\,240\,R \qquad \left[\because I = \frac{P \times R \times T}{100}\right]$

For B : P = ₹ 11,000 and T = 3 Years

$\therefore \quad I = \frac{₹\,11{,}000 \times R \times 3}{100} = ₹\,330\,R$

$\therefore \quad 330\,R - 240\,R = 720$ [∵ B gets ₹ 720 more interest than A]

$\Rightarrow \quad 90\,R = 720$ and $R = \frac{720}{90} = 8$

⇒ **Rate of interest = 8%** **(Ans.)**

Alternative method :

Since, A invests ₹ 8,000 and B invests ₹ 11,000

∴ B invests ₹ (11,000 – 8,000) = ₹ 3,000 more than A

⇒ Interest of 3 years on ₹ 3,000 = ₹ 720

$\therefore \quad \frac{3000 \times R \times 3}{100} = 720 \qquad \left[\because \frac{P \times R \times T}{100} = I\right]$

$\Rightarrow \quad 90\,R = 720$ and $R = \frac{720}{90} = 8$

∴ **Rate of interest = 8%** **(Ans.)**

Example 8 :

A sum of money lent on simple interest, becomes $\frac{7}{5}$ of itself in 4 years. Find the rate of interest per annum.

Solution :

Let the sum of money (Principal) = ₹ x

$\therefore \quad \text{Amount (A)} = \frac{7}{5} \times ₹\,x = ₹\,\frac{7x}{5}$

Simple interest (I) earned = A – P

$= ₹\,\frac{7x}{5} - ₹\,x = ₹\,\frac{2x}{5}$

Rate% $= \frac{100 \times I}{P \times T}\,\%$

$= \frac{100 \times \frac{2x}{5}}{x \times 4}\,\% = \frac{100 \times 2x}{x \times 4 \times 5}\,\% = 10\%$ **(Ans.)**

Alternative method :

$$\text{Let sum (P)} = ₹\ 100$$

$$\therefore \quad \text{Amount (A)} = \frac{7}{5} \times ₹\ 100 = ₹\ 140$$

$$\text{Interest (I)} = A - P$$

$$= ₹\ 140 - ₹\ 100 = ₹\ 40$$

$$\text{Rate\%} = \frac{I \times 100}{P \times T}\%$$

$$= \frac{40 \times 100}{100 \times 4}\% = 10\% \qquad \textbf{(Ans.)}$$

EXERCISE 10

1. Find the S.I. and the amount on :
   (i) ₹ 150 for 4 years at 5% per year.
   (ii) ₹ 350 for $3\frac{1}{2}$ years at 8% p.a.
   (iii) ₹ 620 for 4 months at 8 p per rupee per month.
   (iv) ₹ 3,380 for 30 months at $4\frac{1}{2}$% p.a.
   (v) ₹ 600 from July 12 to Dec. 5 at 10% p.a.
   (vi) ₹ 850 from 10th March to 3rd August at $2\frac{1}{2}$% p.a.
   (vii) ₹ 225 for 3 years 9 months at 16% p.a.
2. On what sum of money does the S.I. for 10 years at 5% become ₹ 1,600 ?
3. Find the time in which ₹ 2,000 will amount to ₹ 2,330 at 11% p.a.
4. In what time will a sum of money double itself at 8% p.a.
5. In how many years will ₹ 870 amount to ₹ 1,044, the rate of interest being $2\frac{1}{2}$% p.a. ?
6. Find the rate percent, if the S.I. on ₹ 275 in 2 years is ₹ 22.
7. Find the sum which will amount to ₹ 700 in 5 years at 8% p.a.
8. What is the rate of interest, if ₹ 3,750 amounts to ₹ 4,650 in 4 years ?
9. In 4 years, ₹ 6,000 amounts to ₹ 8,000. In what time will ₹ 525 amount to ₹ 700 at the same rate ?
10. The interest on a sum of money at the end of $2\frac{1}{2}$ years is $\frac{4}{5}$ of the sum. What is the rate percent ?

11. What sum of money lent out at 5% for 3 years will produce the same interest as ₹ 900 lent out at 4% for 5 years ?

12. A sum of ₹ 1,780 becomes ₹ 2,136 in 4 years.

Find : (i) the rate of interest.

(ii) the sum that will become ₹ 810 in 7 years at the same rate of interest.

13. A sum amounts to ₹ 2,652 in 6 years at 5% p.a. simple interest.

Find : (i) the sum

(ii) the time in which the same sum will double itself at the same rate of interest.

14. P and Q invest ₹ 36,000 and ₹ 25,000 respectively at the same rate of interest per year. If at the end of 4 years, P gets ₹ 3,080 more interest than Q, find the rate of interest.

15. A sum of money is lent for 5 years at R% simple interest per annum. If the interest earned be one-fourth of the money lent, find the value of R.

16. The simple interest earned on a certain sum in 5 years is 30% of the sum. Find the rate of interest.

Theme – 3
ALGEBRA

11 FUNDAMENTAL CONCEPTS

(Including Fundamental Operations)

11.1 ELEMENTARY TREATMENT

1. Constants and Variables :

In Arithmetic, we use digits (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) each of which has a fixed value and so these digits and the numbers formed by these digits are called ***constants,*** whereas *in Algebra,* we use letters of English alphabet which can be assigned any value according to the requirement. So the letters used in Algebra are called ***variables.***

(i) A combination of two or more than two constants is always a constant.

e.g. 3 is a constant and 8 is also a constant, therefore each of $3 + 8$, $3 - 8$, $8 - 3$, $8 \div 3$, $3 \div 8$, 3×8, 38, 83, etc., is also a constant.

Similarly, 5 is a constant, 3 is a constant and 2 is also a constant; so each of $5 \times 3 \div 2$, $3 \div 5 \times 2$, $3 + 5 \times 2$, 352, $5 \times 3 - 2$, 532, etc., is also a constant.

(ii) A combination of two or more variables is always a variable.

e.g. x, y and z are variables and so each of $x + y - z$, $x - yz$, $x - y + z$, $x \div y \times z$, etc., is also a variable.

(iii) A combination of one or more constants and one or more variables is also a variable.

e.g. Each of $6 + x$, $x - 8$, $10z$, $8x \div 3y$, $2x - 3y + 4z$, etc., is a variable.

2. Term :

A term is *a number* (constant), *a variable* or *a combination* (product or quotient) *of numbers and variables.*

e.g. 5, $3a$, x, ax, xy, $-4xy$, $\frac{4x}{3y}$, $\frac{8pq}{7a}$, $\frac{8}{z}$, $\frac{5ax}{12}$, etc.

3. Algebraic Expression :

An algebraic expression is *a collection of one or more terms*, which are *separated* from each other *by addition* (+) *or subtraction* (–) sign*(s).*

e.g. $4x$, $3xy - 7$, $a + 2b$, $5x - 7y$, $2x^2 + 5xy + 3$, $ax - by - cz$, etc.

Only plus (+) and minus (–) signs separate the terms, whereas the product (×) and division (÷) do not separate the terms.

For example :

The expression $3x + 4y$ has two terms, the expression $3x - 4y$ also has two terms but each of $3x \times 4y$ and $3x \div 4y$ has only one term.

4. Types of algebraic expressions :

Name	Condition	Examples
1. Monomial	has **only one term**	x, $5xy$, $\frac{-7x}{4}$, $\frac{ax^2}{7}$, etc.
2. Binomial	has **two terms**	$2a + x$, $\frac{7x}{4} - 8$, $2x^2 - y^2$, etc.
3. Trinomial	has **three terms**	$ax^2 + bx + c$, $a^2 - 4x + 8x^2$, $xy^2 - xy + \frac{x}{4}$, etc.
4. Multinomial	has **more than three terms**	$4 - a + ax + by$, $x^2 - 4x - xy + 8y + a$, etc.
5. Polynomial	has **two or more than two terms**	every binomial, every trinomial, every multinomial, etc.

An expression of the type $\frac{4}{x}$ does not form a monomial unless x is not equal to zero (0).

Reason : *Since x is a variable, it can take any value. If it takes the value zero i.e., if $x = 0$, the expression $\frac{4}{x}$ is equal to $\frac{4}{0}$, which is not defined.*

Similarly, $\frac{7}{y}$ *is not a monomial unless* $y \neq 0$,

$\frac{15}{xy}$ *is not a monomial unless* $x \neq 0$ *and* $y \neq 0$.

5. Product :

When two or more quantities (constants, variables or both) *are multiplied together, the result is called their* ***product.***

For example :

(i) 4ay is the product of 4, a and y.

(ii) 8b is the product of 8 and b and so on.

6. Factor :

Each of the quantities (constants or variables) *multiplied together to form a term is called a factor of the term.*

For example :

(i) 3, a and y are the factors of the term 3ay.

(ii) 2, a and b are the factors of the term 2ab and so on.

In fact, factor of a quantity is each and every constant, variable, combination of constant and variable, etc., by which the given quantity is completely divisible.

For example :

Quantity $3ay$ is completely divisible by each of 1, 3, a, y, $3a$, $3y$, ay and $3ay$, so the factors of $3ay$ are : 1, 3, a, y, $3a$, $3y$, ay and $3ay$.
Similarly, the factors of $2ab$ are : 1, 2, a, b, $2a$, $2b$, ab and $2ab$.

7. Co-efficient :

In a monomial, any factor or group of factors of a term is called the coefficient of the remaining part of the monomial.

For example :

In $5xyz$, 5 is the co-efficient of xyz, x is the co-efficient of $5yz$, y is the coefficient of $5xz$, $5x$ is the co-efficient of yz, xy is the co-efficient of $5z$ and so on.

> *If a factor is a numerical quantity (i.e., constant), it is called* **numerical co-efficient,** *while the factor involving letter(s) is called the* **literal co-efficient.**
>
> *Thus in 5xyz, 5 is numerical co-efficient and each of x, y, z, 5x, 5y, 5z, xy, yz, xz, 5xy, 5yz, 5xz, xyz and 5xyz are the literal co-efficients.*

8. Degree of a monomial :

The degree of a monomial is the exponent of its variable or the sum of the exponents of its variables.

For example :

(i) The degree of $4x^2 = 2$ [Since, exponent of x^2 is 2]

(ii) The degree of $7x = 1$ [Since, $x = x^1$]

(iii) The degree of $8x^2y^3 = 2 + 3 = 5$ [Sum of the exponents of the variables x and y]

(iv) The degree of $\frac{2}{7}xy^4 = 1 + 4 = 5$

(v) The degree of $2 = 0$ [Since, it has no variable]

9. Degree of a polynomial :

The degree of a polynomial is the degree of its highest degree term.

e.g. (i) In expression $5x^4 + 7x^3y^2 + 2xy^2$, the degree of term $5x^4 = 4$, the degree of term $7x^3y^2 = 3 + 2 = 5$ and the degree of term $2xy^2 = 1 + 2 = 3$.

Since, the highest degree term is $7x^3y^2$ and its degree is 5, therefore, degree of the given polynomial is 5.

(ii) The degree of polynomial $x^5 - x^2y^4 + x^3y$ is $2 + 4 = 6$.

10. Like and unlike terms :

Terms having the *same literal coefficients* or alphabetic letters are called ***like terms,*** whereas the terms with *different literal co-efficients* are called ***unlike terms***.

For example :

(i) $5x$ and $8x$ are *like terms,* whereas $5x$ and $8y$ are *unlike terms.*

(ii) $7x^2$ and $2x^2$ are *like terms,* whereas $7x^2$ and $2x$ are *unlike terms.*

(iii) $3xy^2$ and $4xy^2$ are *like terms,* whereas $3xy^2$ and $4x^2y$ are *unlike terms.*

Example 1 :

For algebraic expression $5 - 8xy + 6x^2y^3 + 5xy^2 - 8x^3y^4$, find :

(i) number of terms

(ii) degree of the expression

(iii) coefficient of x^3 in $-8x^3y^4$

(iv) coefficient of x in $6x^2y^3$

(v) constant term

(vi) numerical coefficient of $-8xy$

(vii) coefficient of x^2 in $-8x^3y^4$

(viii) all the factors of $5xy^2$.

Solution :

(i) **No. of terms = 5** **(Ans.)**

(ii) **Degree of the expression = 3 + 4 = 7** **(Ans.)**

(iii) **Coefficient of x^3 in $-8x^3y^4 = -8y^4$** **(Ans.)**

(iv) **Coefficient of x in $6x^2y^3 = 6xy^3$** **(Ans.)**

(v) **Constant term = 5** **(Ans.)**

(vi) **Numerical coefficient of $-8xy = -8$** **(Ans.)**

(vii) **Coefficient of x^2 in $-8x^3y^4 = -8xy^4$** **(Ans.)**

(viii) **All the factors of $5xy^2$ = 1, 5, x, y, xy, y^2, xy^2, 5x, 5xy, 5y, $5y^2$ and $5xy^2$** **(Ans.)**

EXERCISE 11(A)

1. Separate *constant terms* and *variable terms* from the following :

$8,\quad x,\quad 6xy,\quad 6 + x,\quad -5xy^2,\quad 15az^2,\quad \frac{32z}{xy},\quad \frac{y^2}{3x}$

2. For each expression, given below, state whether it is a *monomial, binomial* or *trinomial* :

(i) $2x \div 15$ (ii) $ax + 9$ (iii) $3x^2 \times 5x$

(iv) $5 + 2a - 3b$ (v) $2y - \frac{7}{3}z \div x$ (vi) $3p \times q \div z$

(vii) $12z \div 5x + 4$ (viii) $12 - 5z - 4$ (ix) $a^3 - 3ab^2 \times c$

3. Write the coefficient of :

(i) xy in $-3axy$ (ii) z^2 in p^2yz^2

(iii) mn in $-mn$ (iv) 15 in $-15p^2$

4. For each of the following monomials, write its *degree* :

(i) $7y$ (ii) $-x^2y$ (iii) xy^2z

(iv) $-9y^2z^3$ (v) $3m^3n^4$ (vi) $-2p^2q^3r^4$

5. Write the *degree of* each of the following *polynomials* :

(i) $3y^3 - x^2y^2 + 4x$ (ii) $p^3q^2 - 6p^2q^5 + p^4q^4$ (iii) $-8mn^6 + 5m^3n$

(iv) $7 - 3x^2y + y^2$ (v) $3x - 15$ (vi) $2y^2z + 9yz^3$

6. Group the *like terms* together :

(i) $9x^2$, xy, $-3x^2$, x^2 and $-2xy$ (ii) ab, $-a^2b$, $-3ab$, $5a^2b$ and $-8a^2b$.

(iii) $7p$, $8pq$, $-5pq$, $-2p$ and $3p$

7. Write the *numerical coefficient of each of the following* :

(i) y (ii) $-y$ (iii) $2x^2y$

(iv) $-8xy^3$ (v) $3py^2$ (vi) $-9a^2b^3$

8. In $-5x^3y^2z^4$; write the coefficient of :

(i) z^2 (ii) y^2 (iii) yz^2

(iv) x^3y (v) $-xy^2$ (vi) $-5xy^2z$

Also, write the degree of the given algebraic expression.

11.2 ADDITION AND SUBTRACTION

Method : *Add or subtract* (as required) *the numerical coefficients of like terms.*

For example :

(i) Addition of $8xy$, $15xy$ and $3xy$ $= 8xy + 15xy + 3xy$
$= (8 + 15 + 3)xy = \mathbf{26xy}$

(ii) Subtraction of $8xy$ from $15xy$ $= 15xy - 8xy$
$= (15 - 8)xy = \mathbf{7xy}$

Similarly :

(iii) $12x - 3x + 4x = (12 - 3 + 4)x = (16 - 3)x = \mathbf{13x}$

(iv) $23m^2n - 15m^2n - 20m^2n = (23 - 15 - 20)\ m^2n = (23 - 35)\ m^2n = \mathbf{-12m^2n}$

(v) $6a + 8b - 3a - 2b = 6a - 3a + 8b - 2b$ [Grouping like terms]
$= (6 - 3)\ a + (8 - 2)\ b = \mathbf{3a + 6b}$

(vi) $-3x^2y + 15xy^2 - 4x^2y - 6xy^2 = -3x^2y - 4x^2y + 15xy^2 - 6xy^2$
$= \mathbf{-7x^2y + 9xy^2}$

Example 2 :

Add : (i) $3a + 2b - 4c$ and $2c - 5b + 8a$.

(ii) $2ax + 3by + 4cy$, $5by - 3cy - ax$ and $6cy + 4ax - 9by$.

Solution :

Column method : Re-write the given expressions in such a way that their like terms are one below the other, then operate (add or subtract, as the case may be) like terms column-wise. Thus :

(i)
$$\begin{array}{r} 3a + 2b - 4c \\ 8a - 5b + 2c \\ \hline 11a - 3b - 2c \\ \hline \end{array}$$
(Ans.)

(ii)
$$\begin{array}{r} 2ax + 3by + 4cy \\ -\ ax + 5by - 3cy \\ 4ax - 9by + 6cy \\ \hline 5ax -\ by + 7cy \\ \hline \end{array}$$
(Ans.)

Row method :

Steps :

1. In a single row, write each of the given polynomials (expressions) in a bracket with plus sign between the consecutive brackets.
2. Remove the brackets without changing the sign of any term.
3. Group the like terms and add.

Thus for Example 2, given above, we have :

(i) $(3a + 2b - 4c) + (2c - 5b + 8a)$ [Step 1]
$= 3a + 2b - 4c + 2c - 5b + 8a$ [Step 2]
$= 3a + 8a + 2b - 5b - 4c + 2c$ [Step 3]
$= \mathbf{11a - 3b - 2c}$ **(Ans.)**

(ii) $(2ax + 3by + 4cy) + (5by - 3cy - ax) + (6cy + 4ax - 9by)$
$= 2ax + 3by + 4cy + 5by - 3cy - ax + 6cy + 4ax - 9by$
$= 2ax - ax + 4ax + 3by + 5by - 9by + 4cy - 3cy + 6cy$
$= \mathbf{5ax - by + 7cy}$ **(Ans.)**

Example 3 :

Add : $4x^2 + 3xy - 8y^2$, $2xy - 6x^2 + 7y^2$ and $y^2 - xy + 5x^2$

Solution :

Row Method :

$(4x^2 + 3xy - 8y^2) + (2xy - 6x^2 + 7y^2) + (y^2 - xy + 5x^2)$

$= 4x^2 + 3xy - 8y^2 + 2xy - 6x^2 + 7y^2 + y^2 - xy + 5x^2$

$= 4x^2 - 6x^2 + 5x^2 + 3xy + 2xy - xy - 8y^2 + 7y^2 + y^2$

$= 3x^2 + 4xy$ **(Ans.)**

Column method :

$$\begin{array}{r} 4x^2 + 3xy - 8y^2 \\ -\ 6x^2 + 2xy + 7y^2 \\ +\ 5x^2 - xy + y^2 \\ \hline 3x^2 + 4xy \\ \hline \end{array}$$ **(Ans.)**

Rules of addition of like terms :

1. If the signs of both the terms are same, add their numerical coefficients and affix the same sign. e.g.

(i) $\begin{array}{r} 8xy^2 \\ 5xy^2 \\ \hline 13xy^2 \\ \hline \end{array}$ or $\begin{array}{r} +\ 8xy^2 \\ +\ 5xy^2 \\ \hline +\ 13xy^2 \\ \hline \end{array}$ and (ii) $\begin{array}{r} -\ 8xy^2 \\ -\ 5xy^2 \\ \hline -\ 13xy^2 \\ \hline \end{array}$

2. If the signs of both the terms are different, subtract the smaller numerical coefficient from the greater and affix the sign of the greater coefficient. e.g.

(i) $\begin{array}{r} +\ 8xy^2 \\ -\ 5xy^2 \\ \hline +\ 3xy^2 \\ \hline \end{array}$ or $\begin{array}{r} 8xy^2 \\ -\ 5xy^2 \\ \hline 3xy^2 \\ \hline \end{array}$ and (ii) $\begin{array}{r} -\ 8xy^2 \\ 5xy^2 \\ \hline -\ 3xy^2 \\ \hline \end{array}$

Example 4 :

Subtract : (i) $2a + 3b - c$ from $4a + 5b + 6c$.

(ii) $2x^2 + 5x - 6$ from $5x^2 - 3x + 1$.

Solution :

Column method :

1. Write the given expressions in two rows in such a way that the like terms are written one below the other, taking care that the expression to be subtracted is written in the second row.
2. Change the sign of each term in the second row (lower row).
3. With these new signs of the terms in lower row, add the like terms columnwise.

Thus :

(i)

Step 1 :	$4a + 5b + 6c$
	$2a + 3b - c$
Step 2 :	$- \quad - \quad +$
Step 3 :	$2a + 2b + 7c$ **(Ans.)**

(ii)

$$\begin{array}{r} 5x^2 - 3x + 1 \\ 2x^2 + 5x - 6 \\ - \quad\ - \quad\ + \\ \hline 3x^2 - 8x + 7 \\ \hline \end{array}$$ **(Ans.)**

Row method :

1. Write both the expressions in a single row with the expression to be subtracted in a bracket and put a *minus* sign before (outside) this bracket (minus sign is for subtraction).
2. Open the bracket by changing the sign of each term inside the bracket.
3. Add the like terms.

Thus, for Example 4, given above, we have :

(i) $4a + 5b + 6c - (2a + 3b - c)$ **[Step 1]**

$= 4a + 5b + 6c - 2a - 3b + c$ **[Step 2]**

$= 4a - 2a + 5b - 3b + 6c + c$

$= 2a + 2b + 7c$ **[Step 3]** **(Ans.)**

(ii) $5x^2 - 3x + 1 - (2x^2 + 5x - 6)$

$= 5x^2 - 3x + 1 - 2x^2 - 5x + 6$

$= 5x^2 - 2x^2 - 3x - 5x + 1 + 6 = 3x^2 - 8x + 7$ **(Ans.)**

Example 5 :

Subtract : $a^2 + 2ab$ from $b^2 + 4ab$.

Solution :

Column method :

$$\begin{array}{rr} b^2 + & 4ab \\ + & 2ab + a^2 \\ & - \quad - \\ \hline b^2 + & 2ab - a^2 \\ \hline \end{array}$$ **(Ans.)**

Row method :

$b^2 + 4ab - (a^2 + 2ab)$

$= b^2 + 4ab - a^2 - 2ab$

$= b^2 + 2ab - a^2$ **(Ans.)**

The statement for subtracting $x + 2y$ from $2x + 5y$ can be given in many ways, such as :

1. Take away $x + 2y$ from $2x + 5y$.
2. What is the excess of $2x + 5y$ over $x + 2y$?
3. By how much does $2x + 5y$ exceed $x + 2y$?
4. How much is $x + 2y$ less than $2x + 5y$?
5. What must be subtracted from $2x + 5y$ to get $x + 2y$?
6. What should be added to $x + 2y$ to get $2x + 5y$?

11.3 ADDITION OR SUBTRACTION OF UNLIKE TERMS

As seen above, the two like terms can be added or subtracted to get a single like term, but two unlike terms cannot be added or subtracted together to get a single term. All that can be done is to connect them by the sign as required.

For example :

(i) Addition of $5x$ and $7y = 5x + 7y$

(ii) Addition of $8xy^2$ and $5x^2y = 8xy^2 + 5x^2y$

(iii) Subtraction of $9y$ from $3xy = 3xy - 9y$ and so on.

Example 6 :

Subtract : $5x^2 + 9y^2 - 3z^2$ from the sum of $6x^2 - 5y^2$ and $y^2 + 5z^2$.

Solution :

Column method :

Sum of $6x^2 - 5y^2$ and $y^2 + 5z^2$.

$$
\begin{aligned}
= \quad & 6x^2 - 5y^2 \\
& \quad\quad\; y^2 + 5z^2 \\
\hline
= \quad & 6x^2 - 4y^2 + 5z^2
\end{aligned}
$$

Required :

$$
\begin{aligned}
= \quad & 6x^2 - 4y^2 + 5z^2 \\
& 5x^2 + 9y^2 - 3z^2 \\
& - \quad\; - \quad\; + \\
\hline
= \quad & x^2 - 13y^2 + 8z^2
\end{aligned}
$$
(Ans.)

Row method :

Required : $= (6x^2 - 5y^2) + (y^2 + 5z^2) - (5x^2 + 9y^2 - 3z^2)$

$= 6x^2 - 5y^2 + y^2 + 5z^2 - 5x^2 - 9y^2 + 3z^2$

$= 6x^2 - 5x^2 - 5y^2 + y^2 - 9y^2 + 5z^2 + 3z^2 = x^2 - 13y^2 + 8z^2$ **(Ans.)**

EXERCISE 11(B)

1. Fill in the blanks :
   - (i) $8x + 5x =$ ....
   - (ii) $8x - 5x =$ ....
   - (iii) $6xy^2 + 9xy^2 =$ ....
   - (iv) $6xy^2 - 9xy^2 =$ ....
   - (v) The sum of $8a$, $6a$ and $5b =$ ....
   - (vi) The addition of 5, $7xy$, 6 and $3xy =$ ....
   - (vii) $4a + 3b - 7a + 4b =$ ....
   - (viii) $-15x + 13x + 8 =$ ....
   - (ix) $6x^2y + 13xy^2 - 4x^2y + 2xy^2 =$ ....
   - (x) $16x^2 - 9x^2 =$ .... and $25xy^2 - 17xy^2 =$ ....
2. Add :
   - (i) $-9x$, $3x$ and $4x$
   - (ii) $23y^2$, $8y^2$ and $-12y^2$
   - (iii) $18pq$, $-15pq$ and $3pq$
3. Simplify :
   - (i) $3m + 12m - 5m$
   - (ii) $7n^2 - 9n^2 + 3n^2$
   - (iii) $25zy - 8zy - 6zy$
   - (iv) $-5ax^2 + 7ax^2 - 12ax^2$
   - (v) $-16am + 4mx + 4am - 15mx + 5am$
4. Add :
   - (i) $a + b$ and $2a + 3b$
   - (ii) $2x + y$ and $3x - 4y$
   - (iii) $-3a + 2b$ and $3a + b$
   - (iv) $4 + x$, $5 - 2x$ and $6x$
5. Find the sum of :
   - (i) $3x + 8y + 7z$, $6y + 4z - 2x$ and $3y - 4x + 6z$
   - (ii) $3a + 5b + 2c$, $2a + 3b - c$ and $a + b + c$
   - (iii) $4x^2 + 8xy - 2y^2$ and $8xy - 5y^2 + x^2$
   - (iv) $9x^2 - 6x + 7$, $5 - 4x$ and $6 - 3x^2$
   - (v) $5x^2 - 2xy + 3y^2$, $-2x^2 + 5xy + 9y^2$ and $3x^2 - xy - 4y^2$

(vi) $a^2 + b^2 + 2ab$, $2b^2 + c^2 + 2bc$ and $4c^2 - a^2 + 2ac$

(vii) $9ax - 6bx + 8$, $4ax + 8bx - 7$ and $-6ax - 4bx - 3$

(viii) $abc + 2ba + 3ac$, $4ca - 4ab + 2bca$ and $2ab - 3abc - 6ac$

(ix) $4a^2 + 5b^2 - 6ab$, $3ab$, $6a^2 - 2b^2$ and $4b^2 - 5ab$

(x) $x^2 + x - 2$, $2x - 3x^2 + 5$ and $2x^2 - 5x + 7$

(xi) $4x^3 + 2x^2 - x + 1$, $2x^3 - 5x^2 - 3x + 6$, $x^2 + 8$ and $5x^3 - 7x$

6. Find the sum of :

(i) x and $3y$ (ii) $-2a$ and $+5$ (iii) $-4x^2$ and $+7x$

(iv) $+4a$ and $-7b$ (v) x^3, $3x^2y$ and $2y^2$ (vi) 11 and $-by$

7. The sides of a triangle are $2x + 3y$, $x + 5y$ and $7x - 2y$. Find its perimeter.

8. The two adjacent sides of a rectangle are $6a + 9b$ and $8a - 4b$. Find its perimeter.

9. Subtract the second expression from the first :

(i) $2a + b$, $a + b$ (ii) $-2b + 2c$, $b + 3c$

(iii) $5a + b$, $-6b + 2a$ (iv) $a^3 - 1 + a$, $3a - 2a^2$

(v) $p + 2$, 1 (vi) $x + 2y + z$, $-x - y - 3z$

(vii) $3a^2 - 8ab - 2b^2$, $3a^2 - 4ab + 6b^2$ (viii) $4pq - 6p^2 - 2q^2$, $9p^2$

(ix) $10abc$, $2a^2 + 2abc - 4b^2$ (x) $a^2 + ab + c^2$, $a^2 - d^2$

10. Subtract :

(i) $4x$ from $8 - x$ (ii) $-8c$ from $c + 3d$

(iii) $-5a - 2b$ from $b + 6c$ (iv) $4p + p^2$ from $3p^2 - 8p$

(v) $5a - 3b + 2c$ from $4a - b - 2c$

(vi) $-xy + yz - zx$ from $xy - yz + xz$

(vii) $2x^2 - 7xy - y^2$ from $3x^2 - 5xy + 3y^2$

(viii) $a^2 - 3ab - 6b^2$ from $2b^2 - a^2 + 2ab$

(ix) $4x^2 - 5x^2y + y^2$ from $-3y^2 + 5xy^2 - 7x^2 - 9x^2y$

(x) $6m^3 + 4m^2 + 7m - 3$ from $3m^3 + 4$

11. Subtract $-5a^2 - 3a + 1$ from the sum of $4a^2 + 3 - 8a$ and $9a - 7$.

12. By how much does $8x^3 - 6x^2 + 9x - 10$ exceed $4x^3 + 2x^2 + 7x - 3$?

13. What must be added to $2a^3 + 5a - a^2 - 6$ to get $a^2 - a - a^3 + 1$?

14. What must be subtracted from $a^2 + b^2 + 2ab$ to get $-4ab + 2b^2$?

15. Find the excess of $4m^2 + 4n^2 + 4p^2$ over $m^2 + 3n^2 - 5p^2$.

16. By how much is $3x^3 - 2x^2y + xy^2 - y^3$ less than $4x^3 - 3x^2y - 7xy^2 + 2y^3$?

17. Subtract the sum of $3a^2 - 2a + 5$ and $a^2 - 5a - 7$ from the sum of $5a^2 - 9a + 3$ and $2a - a^2 - 1$.

18. The perimeter of a rectangle is $28x^3 + 16x^2 + 8x + 4$. One of its sides is $8x^2 + 4x$. Find the other side.

19. The perimeter of a triangle is $14a^2 + 20a + 13$. Two of its sides are $3a^2 + 5a + 1$ and $a^2 + 10a - 6$. Find its third side.

20. If $x = 4a^2 + b^2 - 6ab$, $y = 3b^2 - 2a^2 + 8ab$ and $z = 6a^2 + 8b^2 - 6ab$, find :

(i) $x + y + z$ (ii) $x - y - z$

21. If $m = 9x^2 - 4xy + 5y^2$ and $n = -3x^2 + 2xy - y^2$, find :

(i) $2m - n$ (ii) $m + 2n$ (iii) $m - 3n$.

22. Simplify :

(i) $3x + 5(2x + 6) - 7x$ (ii) $3(4y - 10) + 2(y - 1)$

(iii) $-(7 + 6x) - 7(x + 2)$ (iv) $x - (x - y) - y - (y - x)$

(v) $4x + 7y - [5y - 8] - 2x$ (vi) $-2m + 5 + 4(m - 3)$

(vii) $2x - y + 5 - (x - y)$ (viii) $2(x - y) - (x - 8)$

(ix) $4(3x - 8) - 3(5x + 3) - 2(6x - 8)$ (x) $5(x - 4) - 3(x - 4) + 7(x - 4)$

11.4 MULTIPLICATION

1. *Multiplication of monomials :*

Steps : **1.** Multiply the numerical coefficients together.

2. Multiply the literal coefficients separately together.

For example :

(i) $8x \times 3y = (8 \times 3) \times (x \times y)$

$= 24 \times xy = \mathbf{24xy}$

(ii) $5a \times 3b \times 6c = (5 \times 3 \times 6) \times (a \times b \times c) = \mathbf{90\ abc}$

1. *If both the terms (monomials) have same signs, the sign of the product is always positive.*

$\therefore$ $(5a) \times (4b) = 20ab$ *and* $(-5a) \times (-4b) = 20ab$

2. *If both the terms (monomials) have opposite signs, the sign of the product is always negative.*

$\therefore$ $(-5a) \times 4b = -20ab$ *and* $(5a) \times (-4b) = -20ab$

2. *Multiplication of a polynomial by a monomial :*

Steps : **1.** Write the given polynomial inside a bracket and the monomial outside it.

2. Multiply the monomial with each term of the polynomial and simplify.

For example :

(i) Multiplication of $2x + y - 8$ and $4x = 4x(2x + y - 8)$

$= 4x \times 2x + 4x \times y - 4x \times 8$

$= \mathbf{8x^2 + 4xy - 32x}$

(ii) Multiplication of $-2a^2$ with $6a + 2b - 3c = -2a^2(6a + 2b - 3c)$

$= -2a^2 \times 6a - 2a^2 \times 2b + (-2a^2) \times (-3c)$

$= \mathbf{-12a^3 - 4a^2b + 6a^2c}$

Alternative method :

Instead of multiplying the polynomial by the monomial horizontally, as done above, these may be multiplied vertically as shown below :

(i)
$$\begin{array}{r} 2x + y - 8 \\ 4x \\ \hline 8x^2 + 4xy - 32x \\ \hline \end{array}$$

$\because$ $4x \times 2x = 8x^2$

$4x \times y = 4xy$

and $4x \times -8 = -32x$

(ii)
$$\begin{array}{r} 6a + 2b - 3c \\ -2a^2 \\ \hline -12a^3 - 4a^2b + 6a^2c \\ \hline \end{array}$$

$\because$ $-2a^2 \times 6a = -12a^3$

$-2a^2 \times 2b = -4a^2b$

and $-2a^2 \times -3c = 6a^2c$

3. *Multiplication of a polynomial by a polynomial :*

Steps : **1.** Multiply each term of one polynomial by each term of other polynomial.

2. Combine (add or subtract) the like terms.

Thus for the multiplication of $a + b$ and $2a + 3b$, we have :

Column method :

$$\begin{array}{r} a + b \\ 2a + 3b \\ \hline 2a^2 + 2ab \quad\quad\; \\ 3ab + 3b^2 \\ \hline 2a^2 + 5ab + 3b^2 \\ \hline \end{array}$$

$[2a(a + b) = 2a^2 + 2ab]$

$[3b(a + b) = 3ab + 3b^2]$

[Adding like terms]

Row method :

Multiplication of $a + b$ and $2a + 3b$ $= (a + b)(2a + 3b)$

$= a(2a + 3b) + b(2a + 3b)$

$= a(2a) + a(3b) + b(2a) + b(3b)$

$= 2a^2 + 3ab + 2ab + 3b^2$

$= 2a^2 + 5ab + 3b^2$

Similarly;

(i) $(x - 2y)(3x - 5y)$

$= x(3x - 5y) - 2y(3x - 5y)$

$= 3x^2 - 5xy - 6xy + 10y^2$

$= 3x^2 - 11xy + 10y^2$

(ii) $(4x + 8)(3x^3 - x + 1)$

$= 4x(3x^3 - x + 1) + 8(3x^3 - x + 1)$

$= 12x^4 - 4x^2 + 4x + 24x^3 - 8x + 8$

$= 12x^4 + 24x^3 - 4x^2 + 4x - 8x + 8$

$= 12x^4 + 24x^3 - 4x^2 - 4x + 8$

Example 7 :

Evaluate : $(2x - 5y)(3x + 7y)(8x - 9y)$.

Solution :

$(2x - 5y)(3x + 7y)(8x - 9y)$

$= [2x(3x + 7y) - 5y(3x + 7y)](8x - 9y)$

$= [6x^2 + 14xy - 15xy - 35y^2]\ (8x - 9y)$
$= [6x^2 - xy - 35y^2]\ (8x - 9y)$
$= 6x^2\ (8x - 9y) - xy\ (8x - 9y) - 35y^2\ (8x - 9y)$
$= 48x^3 - 54x^2y - 8x^2y + 9xy^2 - 280xy^2 + 315y^3$
$= \mathbf{48x^3 - 62x^2y - 271xy^2 + 315y^3}$ **(Ans.)**

EXERCISE 11(C)

1. Multiply :
   (i) $3x$, $5x^2y$ and $2y$
   (ii) 5, $3a$ and $2ab^2$
   (iii) $5x + 2y$ and $3xy$
   (iv) $6a - 5b$ and $-2a$
   (v) $4a + 5b$ and $4a - 5b$
   (vi) $9xy + 2y^2$ and $2x - 3y$
   (vii) $-3m^2n + 5mn - 4mn^2$ and $6m^2n$
   (viii) $6xy^2 - 7x^2y^2 + 10x^3$ and $-3x^2y^3$
2. Copy and complete the following multiplications :
   (i) $\begin{array}{r} 3a + 2b \\ \times\ -3xy \\ \hline \end{array}$
   (ii) $\begin{array}{r} 9x - 5y \\ \times\ -3xy \\ \hline \end{array}$
   (iii) $\begin{array}{r} 3xy - 2x^2 - 6x \\ \times\ -5x^2y \\ \hline \end{array}$
   (iv) $\begin{array}{r} a + b \\ \times\ a + b \\ \hline \end{array}$
   (v) $\begin{array}{r} ax - b \\ \times\ 2ax + 2b^2 \\ \hline \end{array}$
   (vi) $\begin{array}{r} 2a - b + 3c \\ \times\ 2a - 4b \\ \hline \end{array}$
   (vii) $\begin{array}{r} 3m^2 + 6m - 2n \\ \times\ 5n - 3m \\ \hline \end{array}$
   (viii) $\begin{array}{r} 6 - 3x + 2x^2 \\ \times\ 1 + 5x - x^2 \\ \hline \end{array}$
   (ix) $\begin{array}{r} 4x^3 - 10x^2 + 6x - 8 \\ \times\ 3 + 2x - x^2 \\ \hline \end{array}$
3. Evaluate :
   (i) $(c + 5)\ (c - 3)$
   (ii) $(3c - 5d)\ (4c - 6d)$
   (iii) $\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a - \frac{1}{2}b\right)$
   (iv) $(a^2 + 2ab + b^2)\ (a + b)$
   (v) $(3x - 1)\ (4x^3 - 2x^2 + 6x - 3)$
   (vi) $(4m - 2)\ (m^2 + 5m - 6)$
   (vii) $(8 - 12x + 7x^2 - 6x^3)\ (5 - 2x)$
   (viii) $(4x^2 - 4x + 1)\ (2x^3 - 3x^2 + 2)$
   (ix) $(6p^2 - 8pq + 2q^2)\ (-5p)$
   (x) $-4y\ (15x + 12y - 8z)\ (x - 2y)$
   (xi) $(a^2 + b^2 + c^2 - ab - bc - ca)\ (a + b + c)$
4. Evaluate :
   (i) $(a + b)\ (a - b)$.
   (ii) $(a^2 + b^2)\ (a + b)\ (a - b)$, using the result of (i).
   (iii) $(a^4 + b^4)\ (a^2 + b^2)\ (a + b)\ (a - b)$, using the result of (ii).
5. Evaluate :
   (i) $(3x - 2y)\ (4x + 3y)$.
   (ii) $(3x - 2y)\ (4x + 3y)\ (8x - 5y)$.

(iii) $(a + 5)(3a - 2)(5a + 1)$

(iv) $(a + 1)(a^2 - a + 1)$ and $(a - 1)(a^2 + a + 1)$;
and then : $(a + 1)(a^2 - a + 1) + (a - 1)(a^2 + a + 1)$.

(v) $(5m - 2n)(5m + 2n)(25m^2 + 4n^2)$.

6. Multiply :

(i) mn^4, m^3n and $5m^2n^3$

(ii) $2mnpq$, $4mnpq$ and $5mnpq$

(iii) $pq - pm$ and p^2m

(iv) $x^3 - 3y^3$ and $4x^2y^2$

(v) $a^3 - 4ab$ and $2a^2b$

(vi) $x^2 + 5yx - 3y^2$ and $2x^2y$.

7. Multiply :

(i) $(2x + 3y)(2x + 3y)$

(ii) $(2x - 3y)(2x + 3y)$

(iii) $(2x + 3y)(2x - 3y)$

(iv) $(2x - 3y)(2x - 3y)$

(v) $(-2x + 3y)(2x - 3y)$

(vi) $(xy + 2b)(xy - 2b)$

(vii) $(x - a)(x + 3b)$

(viii) $(2x + 5y + 6)(3x + y - 8)$

(ix) $(3x - 5y + 2)(5x - 4y - 3)$

(x) $(6x - 2y)(3x - y)$

(xi) $(1 + 6x^2 - 4x^3)(-1 + 3x - 3x^2)$

11.5 DIVISION

1. *Dividing a monomial by a monomial :*

Steps : **1.** Write the dividend in numerator and divisor in denominator.

2. Simplify the fraction obtained in Step (1).

For example :

(i) Division of $15xy$ by $5x = \frac{15xy}{5x} = \mathbf{3y}$

(ii) $(6x^2y) \div (2xy) = \frac{6x^2y}{2xy} = \mathbf{3x}$

(iii) $(-12a^2bc) \div (9ab^2c^2) = \frac{-12a^2bc}{9ab^2c^2} = \frac{-3 \times 2 \times 2 \times a \times a \times b \times c}{3 \times 3 \times a \times b \times b \times c \times c} = -\frac{\mathbf{4a}}{\mathbf{3bc}}$

(iv) $\frac{3ab}{5} \div \frac{-4a}{5} = \frac{3ab}{5} \times \frac{5}{-4a} = -\frac{\mathbf{3b}}{\mathbf{4}}$

(v) $(72x^5y^2z^3) \div 54x^2y^5z^7$

$$= \frac{72x^5y^2z^3}{54x^2y^5z^7} = \frac{4 \times 18x^{5-2}}{3 \times 18y^{5-2}z^{7-3}} = \frac{4\boldsymbol{x}^3}{3\boldsymbol{y}^3\boldsymbol{z}^4}$$

2. *Dividing a polynomial by a monomial :*

Divide each term of the polynomial by the monomial :

For example :

(i) Division of $4x^2 + 5x$ by $2x$

$$= (4x^2 + 5x) \div 2x = \frac{4x^2}{2x} + \frac{5x}{2x} = \mathbf{2x + \frac{5}{2}}$$

(ii) $(8a^2b^3c + 4ab^2c^2 - 6abc) \div (-2a^2bc)$

$$= \frac{8a^2b^3c}{-2a^2bc} + \frac{4ab^2c^2}{-2a^2bc} - \frac{6abc}{-2a^2bc} = \mathbf{-4b^2 - \frac{2bc}{a} + \frac{3}{a}}$$

3. *Dividing a polynomial by a polynomial :*

Example 8 :

Divide : $6x^2 + 19x + 10$ by $3x + 2$.

Step 1 : Set the two expressions as : $3x + 2\,\overline{)\,6x^2 + 19x + 10}$

Step 2 : Divide first term of the dividend by the first term of divisor to get the first term of quotient. Here, $\frac{6x^2}{3x} = 2x$, which is the first term of the quotient.

$$\begin{array}{r}
2x \\
3x+2\,\overline{)\,6x^2 + 19x + 10}
\end{array}$$

Step 3 : Multiply quotient ($2x$) with each term of divisor ($3x + 2$) to get :

$$2x(3x + 2) = 6x^2 + 4x$$

$$\begin{array}{r}
2x \\
3x+2\,\overline{)\,6x^2 + 19x + 10} \\
\underline{6x^2 + 4x}
\end{array}$$

Step 4 : Subtract the result of Step 3 and take the next term/ terms of the dividend down :

$$\begin{array}{r}
2x \\
3x+2\,\overline{)\,6x^2 + 19x + 10} \\
6x^2 + 4x \\
\underline{-\quad -} \\
15x + 10
\end{array}$$

Step 5 : Repeat the process from Step 2 to Step 4 taking remainder $15x + 10$ as new dividend :

$$\because\; 5(3x + 2) = 15x + 10$$

$$\begin{array}{r}
2x + 5 \\
3x+2\,\overline{)\,6x^2 + 19x + 10} \\
6x^2 + 4x \\
\underline{-\quad -} \\
15x + 10 \\
15x + 10 \\
\underline{-\quad -} \\
0
\end{array}$$

Thus, $\mathbf{(6x^2 + 19x + 10) \div (3x + 2)} = \mathbf{2x + 5}$ **(Ans.)**

> *While dividing one polynomial by another polynomial, arrange the terms of both the dividend and the divisor in descending or in ascending order of their powers.*
>
> *In Example 8, given above, both dividend $6x^2 + 19x + 10$ and divisor $3x + 2$ are in descending order of their powers.*

Example 9 :

Divide : $6x^2 + 7xy - 3y^2$ by $2x + 3y$.

Solution :

$$\begin{array}{r|l} & 3x - y \\ \hline 2x + 3y & 6x^2 + 7xy - 3y^2 \\ & 6x^2 + 9xy \\ & - \quad - \\ \hline & -2xy - 3y^2 \\ & -2xy - 3y^2 \\ & + \quad + \\ \hline & 0 \\ \hline \end{array}$$

$\because$ $6x^2$ divided by $2x$ gives $3x$

and, $3x(2x + 3y) = 6x^2 + 9xy$

$\because$ $-2xy$ divided by $2x$ gives $-y$

and, $-y(2x + 3y) = -2xy - 3y^2$

$\therefore$ $(6x^2 + 7xy - 3y^2) \div (2x + 3y) = 3x - y$ **(Ans.)**

EXERCISE 11(D)

1. Divide :
   (i) $-16ab^2c$ by $6abc$ (ii) $25x^2y$ by $-5y^2$ (iii) $8x + 24$ by 4
   (iv) $4a^2 - a$ by $-a$ (v) $8m - 16$ by -8 (vi) $-50 + 40p$ by $10p$
   (vii) $4x^3 - 2x^2$ by $-x$ (viii) $10a^3 - 15a^2b$ by $-5a^2$
   (ix) $12x^3y - 8x^2y^2 + 4x^2y^3$ by $4xy$ (x) $9a^4b - 15a^3b^2 + 12a^2b^3$ by $-3a^2b$

2. Divide :
   (i) $n^2 - 2n + 1$ by $n - 1$ (ii) $m^2 - 2mn + n^2$ by $m - n$
   (iii) $4a^2 + 4a + 1$ by $2a + 1$ (iv) $p^2 + 4p + 4$ by $p + 2$
   (v) $x^2 + 4xy + 4y^2$ by $x + 2y$ (vi) $2a^2 - 11a + 12$ by $a - 4$
   (vii) $6x^2 + 5x - 6$ by $2x + 3$ (viii) $8a^2 + 4a - 60$ by $2a - 5$
   (ix) $9x^2 - 24xy + 16y^2$ by $3x - 4y$ (x) $15x^2 + 31xy + 14y^2$ by $5x + 7y$
   (xi) $35a^3 + 3a^2b - 2ab^2$ by $5a - b$ (xii) $6x^3 + 5x^2 - 21x + 10$ by $3x - 2$

3. The area of a rectangle is $6x^2 - 4xy - 10y^2$ square unit and its length is $2x + 2y$ unit. Find its breadth.

4. The area of a rectangular field is $25x^2 + 20xy + 3y^2$ square unit. If its length is $5x + 3y$ unit, find its breadth. Hence, find its perimeter.

5. Divide :
   (i) $2m^3n^5$ by $-mn$ (ii) $5x^2 - 3x$ by x
   (iii) $10x^3y - 9xy^2 - 4x^2y^2$ by xy (iv) $3y^3 - 9ay^2 - 6ab^2y$ by $-3y$
   (v) $x^5 - 15x^4 - 10x^2$ by $-5x^2$ (vi) $12a^2 + ax - 6x^2$ by $3a - 2x$
   (vii) $6x^2 - xy - 35y^2$ by $2x - 5y$ (viii) $x^3 - 6x^2 + 11x - 6$ by $x^2 - 4x + 3$
   (ix) $m^3 - 4m^2 + m + 6$ by $m^2 - m - 2$

11.6 COMBINING ALGEBRAIC EXPRESSIONS WITH INTEGRAL DENOMINATORS

In pure arithmetic, the simplification of fractions is already done.

To simplify algebraic expressions (fractions), the same rules and methods are used.

For example :

1. $$\frac{x}{2}+\frac{x}{3}=\frac{3x+2x}{6}=\frac{5x}{6}$$ [L.C.M. of the denominators 2 and 3 is 6]

2. $$\frac{y}{4}+\frac{y+2}{3}=\frac{3y+4(y+2)}{12}=\frac{3y+4y+8}{12}=\frac{7y+8}{12}$$ [L.C.M. of denominators 4 and 3 is 12]

3. $$\frac{a}{5}-\frac{a-2}{2}+a=\frac{a}{5}-\frac{a-2}{2}+\frac{a}{1}$$
$$=\frac{2a-5(a-2)+10a}{10}$$ [L.C.M. of 5, 2 and 1 is 10]
$$=\frac{2a-5a+10+10a}{10}=\frac{7a+10}{10}$$

4. $$\frac{3}{5}b \text{ of } \left(\frac{2b+3b}{4}\right)=\frac{3}{5}b \text{ of } \frac{5b}{4}$$ [Terms inside the bracket are simplified first]
$$=\frac{3b}{5}\times\frac{5b}{4}=\frac{3b^2}{4}$$

5. $$\left(\frac{m}{2}\times2\frac{1}{3}\right)\div\left(4\frac{1}{2}\times\frac{p}{3}\right)=\left(\frac{m}{2}\times\frac{7}{3}\right)\div\left(\frac{9}{2}\times\frac{p}{3}\right)$$
$$=\frac{7m}{6}\div\frac{9p}{6}=\frac{7m}{6}\times\frac{6}{9p}=\frac{7m}{9p}$$

6. $$\frac{2\frac{1}{5}x}{1\frac{1}{10}}+x=\frac{\frac{11x}{5}}{\frac{11}{10}}+x=\frac{11x}{5}\times\frac{10}{11}+x=2x+x=3x$$

EXERCISE 11(E)

Simplify :

1. $\frac{x}{2}+\frac{x}{4}$
2. $\frac{a}{10}+\frac{2a}{5}$
3. $\frac{y}{4}+\frac{3y}{5}$
4. $\frac{x}{2}-\frac{x}{8}$

5. $\frac{3y}{4} - \frac{y}{5}$

6. $\frac{2p}{3} - \frac{3p}{5}$

7. $\frac{k}{2} + \frac{k}{3} + \frac{2k}{5}$

8. $\frac{2x}{5} + \frac{3x}{4} - \frac{3x}{5}$

9. $\frac{4a}{7} - \frac{2a}{3} + \frac{a}{7}$

10. $\frac{2b}{5} - \frac{7b}{15} + \frac{13b}{3}$

11. $\frac{6k}{7} - \left(\frac{8k}{9} - \frac{k}{3}\right)$

12. $\frac{3a}{8} + \frac{4a}{5} - \left(\frac{a}{2} + \frac{2a}{5}\right)$

13. $x + \frac{x}{2} + \frac{x}{3}$

14. $\frac{y}{5} + y - \frac{19y}{15}$

15. $\frac{x}{5} + \frac{x+1}{2}$

16. $x + \frac{x+2}{3}$

17. $\frac{3y}{5} - \frac{y+2}{2}$

18. $\frac{2a+1}{3} + \frac{3a-1}{2}$

19. $\frac{k+1}{2} + \frac{2k-1}{3} - \frac{k+3}{4}$

20. $\frac{m}{5} - \frac{m-2}{3} + m$

21. $\frac{5(x-4)}{3} + \frac{2(5x-3)}{5} + \frac{6(x-4)}{7}$

22. $\left(p + \frac{p}{3}\right)\left(2p + \frac{p}{2}\right)\left(3p - \frac{2p}{3}\right)$

23. $\frac{7}{30}$ of $\left(\frac{p}{3} + \frac{7p}{15}\right)$

24. $\left(2p + \frac{p}{7}\right) \div \left(\frac{9p}{10} + 4p\right)$

25. $\left(\frac{5k}{8} - \frac{3k}{5}\right) \div \frac{k}{4}$

26. $\left(\frac{y}{6} + \frac{2y}{3}\right) \div \left(y + \frac{2y-1}{3}\right)$

11.7 USING BRACKETS

The brackets are used to combine the terms in different situations.

For example :

(i) For addition of $2a + 7b$ and $9a - 3b + 8$;

we write : $(2a + 7b) + (9a - 3b + 8)$

(ii) For subtraction of $2b - 7a + 5$ from $3a - 8 + 4b$;

we write : $(3a - 8 + 4b) - (2b - 7a + 5)$

(iii) For multiplication of $4x$ and $x - 8y$;

we write : $(4x) \times (x - 8y)$ or, simply : $(4x)\,(x - 8y)$ or, $4x(x - 8y)$

Similarly, $(3x^2 + 7x + 9)\,(2x^2 + 3x)$ shows the multipication of $3x^2 + 7x + 9$ and $2x^2 + 3x$.

(iv) For addition of $8x$ and 12 multiplied by $5y$;

we write : $(8x + 12) \times 5y$ or, $5y(8x + 12)$

11.8 REMOVAL OF BRACKETS

For removal of brackets :

1. *When there is positive (+) sign before the brackets, remove the brackets without changing the sign of any term inside it.*

   e.g. $4x + (2y + 7z - 9) = 4x + 2y + 7z - 9.$

2. *When there is negative (–) sign before the brackets, remove the brackets and at the same time change the sign of each term inside it.*

   e.g. $3a^2 - (2a + 4b - 7) = 3a^2 - 2a - 4b + 7$

3. *When a term is written just before the brackets, it is said to be in multiplication with the brackets.*

   Remove the brackets by multiplying this term with each term inside the brackets.

   e.g. (i) $\mathbf{3x\,(x^2 - 5x - 7)} = 3x \times x^2 - 3x \times 5x - 3x \times 7 = \mathbf{3x^3 - 15x^2 - 21x}$

   (ii) $\mathbf{-5\,(-8a^2 + 3ab - 1)} = \mathbf{40a^2 - 15ab + 5}$ and so on.

11.9 TYPES OF BRACKETS

The names of different types of brackets and the order in which they are removed is shown below :

(i) $\overline{\quad\quad}$; bar (vinculum) bracket, (ii) ( ) ; circular brackets,

(iii) { } ; curly brackets and then, (iv) [ ] ; square brackets.

Example 10 :

Simplify : (i) $a - [\,2b + \{c - (2a - b)\}\,]$ (ii) $4x - [\,2y - \{2x + (x - \overline{y - x})\,\}\,]$

Solution :

(i) $\mathbf{a - [\,2b + \{c - (2a - b)\}\,]}$

$= a - [2b + \{c - 2a + b\}\,]$ [Removing the circular brackets]

$= a - [2b + c - 2a + b\,]$ [Removing the curly brackets]

$= a - 2b - c + 2a - b$ [Removing the square brackets]

$= \mathbf{3a - 3b - c}$ **(Ans.)**

(ii) $\mathbf{4x - [\,2y - \{2x + (x - \overline{y - x})\,\}\,]}$

$= 4x - [\,2y - \{2x + (x - y + x)\,\}\,]$ [Removing the bar brackets]

$= 4x - [\,2y - \{2x + (2x - y)\,\}\,]$

$= 4x - [\,2y - \{2x + 2x - y\}\,]$ [Removing the circular brackets]

$= 4x - [\,2y - \{4x - y\}\,]$

$= 4x - [\,2y - 4x + y]$ [Removing the curly brackets]

$= 4x - 2y + 4x - y$ [Removing the square brackets]

$= \mathbf{8x - 3y}$ **(Ans.)**

EXERCISE 11(F)

Enclose the given terms in brackets as required :

1. $x - y - z = x -$ (........................)
2. $x^2 - xy^2 - 2xy - y^2 = x^2 -$ (........................)
3. $4a - 9 + 2b - 6 = 4a -$ (........................)
4. $x^2 - y^2 + z^2 + 3x - 2y = x^2 -$ (........................)
5. $-2a^2 + 4ab - 6a^2b^2 + 8ab^2 = -2a$ (........................)

Simplify :

6. $2x - (x + 2y - z)$
7. $p + q - (p - q) + (2p - 3q)$
8. $9x - (-4x + 5)$
9. $6a - (-5a - 8b) + (3a + b)$
10. $(p - 2q) - (3q - r)$
11. $9a\,(2b - 3a + 7c)$
12. $-5m\,(-2m + 3n - 7p)$
13. $-2x\,(x + y) + x^2$
14. $b\left(2b - \frac{1}{b}\right) - 2b\left(b - \frac{1}{b}\right)$
15. $8\,(2a + 3b - c) - 10\,(a + 2b + 3c)$
16. $a\left(a + \frac{1}{a}\right) - b\left(b - \frac{1}{b}\right) - c\left(c + \frac{1}{c}\right)$
17. $5x\,(2x + 3y) - 2x\,(x - 9y)$
18. $a + (b + c - d)$
19. $5 - 8x - 6 - x$
20. $2a + (b - \overline{a - b})$
21. $3x + [4x - (6x - 3)]$
22. $5b - \{6a + (8 - b - a)\}$
23. $2x - [5y - (3x - y) + x]$
24. $6a - 3\,(a + b - 2)$
25. $8\,[m + 2n - p - 7\,(2m - n + 3p)]$
26. $\{9 - (4p - 6q)\} - \{3q - (5p - 10)\}$
27. $2\,[a - 3\,\{a + 5\,(a - 2) + 7\}]$
28. $5a - [6a - \{9a - (10a - \overline{4a - 3a})\}]$
29. $9x + 5 - [4x - \{3x - 2\,(4x - 3)\}]$
30. $(x + y - z)\,x + (z + x - y)\,y - (x + y - z)z$
31. $-1\,[a - 3\,\{b - 4\,(a - b - 8) + 4a\} + 10]$
32. $p^2 - [x^2 - \{x^2 - (q^2 - \overline{x^2 - q^2}) - 2y^2\}]$
33. $10 - \{4a - (7 - \overline{a - 5}) - (5a - \overline{1 + a})\}$
34. $7a - [8a - \{11a - (12a - \overline{6a - 5a})\}]$
35. $8x - [4y - \{4x + (2x - \overline{2y - 2x})\}]$
36. $x - (3y - \overline{4z - 3x} + 2z - \overline{5y - 7x})$

SIMPLE LINEAR EQUATIONS 12

(Including Word Problems)

12.1 DEFINITION OF AN EQUATION

An equation is a statement which states that the two expressions are equal.

For example :

(i) If the expressions $3x + 2$ and $x - 7$ are equal, then $3x + 2 = x - 7$ forms an equation.

(ii) The expressions $8y - 15$ and $\frac{y}{2}$ are equal $\Rightarrow 8y - 15 = \frac{y}{2}$ is an equation.

To solve *an equation means to find the value of the variable used in it.*

e.g., in equation $x + 2 = 6$ [*where x is a variable*]

$\Rightarrow x = 6 - 2 \quad \Rightarrow x = 4$

$\therefore$ **x = 4 is the solution** *(root)* **of the equation x + 2 = 6**

Important note : An equation remains unchanged if :

1. The same number is added to each side of the equation.

   i.e. $\mathbf{x - 5 = 6} \Rightarrow x - 5 + 5 = 6 + 5$ [Adding 5 on both the sides]

   $\Rightarrow \mathbf{x = 11}$

2. The same number is subtracted from each side of the equation.

   i.e. $\mathbf{x + 5 = 6} \Rightarrow x + 5 - 5 = 6 - 5$ [Subtracting 5 from both the sides]

   $\Rightarrow \mathbf{x = 1}$

3. The same number is multiplied to each side of the equation.

   i.e. $\mathbf{\frac{x}{3} = 2} \Rightarrow \frac{x}{3} \times 3 = 2 \times 3$ [Multiplying both the sides by 3]

   $\Rightarrow \mathbf{x = 6}$

4. Each side of the equation is divided by the same non-zero number.

   i.e. $\mathbf{4x = 12} \Rightarrow \frac{4x}{4} = \frac{12}{4}$ [Dividing both the sides by 4]

   $\Rightarrow \mathbf{x = 3}$

Example 1 :

Solve the equation $x + 3 = 9$.

Solution :

$\mathbf{x + 3 = 9} \Rightarrow x + 3 - 3 = 9 - 3$ [Subtracting 3 from both the sides]

$\Rightarrow \mathbf{x = 6}$ **(Ans.)**

Example 2 :

Solve : (i) $2x + 5 = 11$ (ii) $\frac{x}{3} - 8 = 10$

Solution :

(i) $2x + 5 = 11$ $\Rightarrow$ $2x + 5 - 5 = 11 - 5$ [Subtracting 5 from both the sides]

$\Rightarrow$ $2x = 6$

$\Rightarrow$ $\frac{2x}{2} = \frac{6}{2}$ [Dividing each side by 2]

$\Rightarrow$ $x = 3$ **(Ans.)**

(ii) $\frac{x}{3} - 8 = 10$ $\Rightarrow$ $\frac{x}{3} - 8 + 8 = 10 + 8$ [Adding 8 on both the sides]

$\Rightarrow$ $\frac{x}{3} = 18$

$\Rightarrow$ $\frac{x}{3} \times 3 = 18 \times 3$ [Multiplying both the sides by 3]

$\Rightarrow$ $x = 54$ **(Ans.)**

12.2 SHORT-CUT METHOD (SOLVING AN EQUATION BY TRANSPOSING TERMS)

1. In an equation, if a + ve (positive) term is transposed (taken) from one side to the other, its sign is reversed, *i.e.*, it becomes – ve (negative).

   i.e., $x + 3 = 6$ $\Rightarrow$ $x = 6 - 3$ [Transposing + 3]

   $\Rightarrow$ $x = 3$

2. In an equation, if a – ve (negative) term is transposed from one side to the other, its sign becomes + ve (positive).

   i.e., $x - 3 = 6$ $\Rightarrow$ $x = 6 + 3$ [Transposing – 3]

   $\Rightarrow$ $x = 9$

3. In an equation, if a term in multiplication is transposed to the other side, its sign is reversed, *i.e.*, it goes in division.

   i.e., $3x = 6$ $\Rightarrow$ $x = \frac{6}{3}$ [Transposing 3, which is in multiplication with x]

   $\Rightarrow$ $x = 2$

4. In an equation, if a term is in division, it is transposed to the other side in multiplication.

   i.e., $\frac{x}{3} = 6$ $\Rightarrow$ $x = 6 \times 3$ [Transposing 3, which is in division with x]

   $\Rightarrow$ $x = 18$

Although it is not a rule, the variable (x, y or z, etc.) is prefered to be kept on the left hand side of the equation.

Example 3 :

Solve : (i) $\frac{2}{3}x = 16$ (ii) $\frac{3}{4}x + 5 = 8$ (iii) $5x - \frac{1}{2} = \frac{3}{4}$

Solution :

(i) $\frac{2}{3}x = 16$ $\Rightarrow$ $2x = 16 \times 3$ [Transposing 3]

$\Rightarrow$ $x = \frac{48}{2}$ [Transposing 2]

$\Rightarrow$ $x = 24$ **(Ans.)**

(ii) $\frac{3}{4}x + 5 = 8$ $\Rightarrow$ $\frac{3}{4}x = 8 - 5$

$\Rightarrow$ $\frac{3}{4}x = 3$ $\Rightarrow$ $3x = 3 \times 4$

$\Rightarrow$ $x = \frac{12}{3}$ $\Rightarrow$ $x = 4$ **(Ans.)**

(iii) $5x - \frac{1}{2} = \frac{3}{4}$ $\Rightarrow$ $5x = \frac{3}{4} + \frac{1}{2}$

$\Rightarrow$ $5x = \frac{5}{4}$ $\left[\frac{3}{4}+\frac{1}{2}=\frac{3+2}{4}=\frac{5}{4}\right]$

$\Rightarrow$ $x = \frac{5}{4 \times 5}$ $\Rightarrow$ $x = \frac{1}{4}$ **(Ans.)**

EXERCISE 12(A)

Solve the following equations :

1. $x + 5 = 10$
2. $2 + y = 7$
3. $a - 2 = 6$
4. $x - 5 = 8$
5. $5 - d = 12$
6. $3p = 12$
7. $14 = 7m$
8. $2x = 0$
9. $\frac{x}{9} = 2$
10. $\frac{y}{-12} = -4$
11. $8x - 2 = 38$
12. $2x + 5 = 5$
13. $5x - 1 = 74$
14. $14 = 27 - x$
15. $10 + 6a = 40$
16. $c - \frac{1}{2} = \frac{1}{3}$
17. $\frac{a}{15} - 2 = 0$
18. $12 = c - 2$
19. $4 = x - 2.5$
20. $y + 5 = 8\frac{1}{4}$
21. $x + \frac{1}{4} = -\frac{3}{8}$
22. $p + 0.02 = 0.08$
23. $p - 12 = 2\frac{2}{3}$
24. $-3x = 15$
25. $1.3b = 39$
26. $\frac{5}{8}n = 20$
27. $\frac{3}{16}m = 21$
28. $2a - 3 = 5$
29. $3p - 1 = 8$
30. $9y - 7 = 20$
31. $2b - 14 = 8$
32. $\frac{7}{10}x + 6 = 41$
33. $\frac{5}{12}m - 12 = 48.$

12.3 SOLVING AN EQUATION WITH THE VARIABLE ON BOTH THE SIDES

Transpose the terms, containing the variable, to one side and the constants (*i.e.*, terms without the variable) on the other side.

Example 4 :

Solve : $8x - 3 = 5x + 9$

Solution :

$8x - 5x = 9 + 3$ [Transposing + $5x$ to left and -3 to the right side]

$\Rightarrow \quad 3x = 12$

$\Rightarrow \quad x = \frac{12}{3} \Rightarrow x = 4$ **(Ans.)**

Example 5 :

Solve : $7 + 4x = 9x - 13$

Solution :

$7 + 4x = 9x - 13$

$\Rightarrow \quad 7 + 13 = 9x - 4x$

$\Rightarrow \quad 20 = 5x$

$\Rightarrow \quad \frac{20}{5} = x$

$\Rightarrow \quad 4 = x$ **(Ans.)**

OR

$7 + 4x = 9x - 13$

$\Rightarrow \quad 4x - 9x = -13 - 7$

$\Rightarrow \quad -5x = -20$

$\Rightarrow \quad x = \frac{-20}{-5}$

$\Rightarrow \quad x = 4$ **(Ans.)**

Example 6 :

Solve : $2(x - 5) + 3(x - 2) = 8 + 7(x - 4)$

Solution :

$2x - 10 + 3x - 6 = 8 + 7x - 28$ [On removing the brackets]

$\Rightarrow \quad 5x - 16 = 7x - 20$

$\Rightarrow \quad 5x - 7x = -20 + 16$

$\Rightarrow \quad -2x = -4 \quad \Rightarrow \quad x = \frac{-4}{-2} \quad \Rightarrow x = 2$ **(Ans.)**

EXERCISE 12(B)

Solve :

1. $8y - 4y = 20$
2. $9b - 4b + 3b = 16$
3. $5y + 8 = 8y - 18$
4. $6 = 7 + 2p - 5$
5. $8 - 7x = 13x + 8$
6. $4x - 5x + 2x = 28 + 3x$
7. $9 + m = 6m + 8 - m$
8. $24 = y + 2y + 3 + 4y$
9. $19x + 13 - 12x + 3 = 23$
10. $6b + 40 = -100 - b$
11. $6 - 5m - 1 + 3m = 0$
12. $0{\cdot}4x - 1{\cdot}2 = 0{\cdot}3x + 0{\cdot}6$
13. $6(x + 4) = 36$
14. $9(a + 5) + 2 = 11$
15. $4(x - 2) = 12$
16. $-3(a - 6) = 24$
17. $7(x - 2) = 2(2x - 4)$
18. $(x - 4)(2x + 3) = 2x^2$
19. $21 - 3(b - 7) = b + 20$
20. $x(x + 5) = x^2 + x + 32$

12.4 EQUATIONS INVOLVING FRACTIONS

Example 7 :

Solve : (i) $\frac{x}{5} + x = 12$ (ii) $\frac{x-1}{3} - \frac{2x-3}{5} = 1$

Solution :

(i) $\frac{x}{5} + x = 12 \Rightarrow \frac{x}{5} + \frac{x}{1} = 12$

$\Rightarrow \frac{x+5x}{5} = 12$ [L.C.M. of 5 and 1 is 5]

$\Rightarrow \frac{6x}{5} = 12$

$\Rightarrow 6x = 12 \times 5 \Rightarrow x = \frac{60}{6} \Rightarrow x = 10$ (Ans.)

(ii) $\frac{x-1}{3} - \frac{2x-3}{5} = 1$

$\Rightarrow \frac{5(x-1)-3(2x-3)}{15} = 1$ [L.C.M. of 3 and 5 = 15]

$\Rightarrow \frac{5x-5-6x+9}{15} = 1$

$\Rightarrow -x + 4 = 15 \times 1$

$\Rightarrow -x = 15 - 4$

$\Rightarrow -x = 11 \Rightarrow x = -11$ (Ans.)

Example 8 :

Solve : $y + 15\%$ of $y = 46$

Solution :

$y + \frac{15}{100} \times y = 46 \Rightarrow y + \frac{3}{20}y = 46$

$\Rightarrow \frac{20y+3y}{20} = 46$

$\Rightarrow \frac{23y}{20} = 46$

$\Rightarrow 23y = 46 \times 20 \Rightarrow y = \frac{46 \times 20}{23} \Rightarrow y = 40$ (Ans.)

EXERCISE 12(C)

Solve :

1. $\frac{x}{2} + x = 9$
2. $\frac{x}{5} + 2x = 33$
3. $\frac{3x}{4} + 4x = 38$
4. $\frac{x}{2} + \frac{x}{5} = 14$
5. $\frac{x}{3} - \frac{x}{4} = 2$
6. $y + \frac{y}{2} = \frac{7}{4} - \frac{y}{4}$

7. $\frac{4x}{3} - \frac{7x}{3} = 1$

8. $\frac{1}{2}m + \frac{3}{4}m - m = 2\cdot5$

9. $\frac{2x}{3} + \frac{x}{2} - \frac{3x}{4} = 1$

10. $\frac{3a}{4} + \frac{a}{6} = 66$

11. $\frac{2p}{3} - \frac{p}{5} = 35$

12. $0\cdot6a + 0\cdot2a = 0\cdot4a + 8$

13. $p + 1\cdot4p = 48$

14. 10% of $x = 20$

15. $y + 20\%$ of $y = 18$

16. $x - 30\%$ of $x = 35$

17. $\frac{x+4}{2} + \frac{x}{3} = 7$

18. $\frac{y+2}{3} + \frac{y+5}{4} = 6$

19. $\frac{3a-2}{7} - \frac{a-2}{4} = 2$

20. $\frac{1}{2}(x+5) - \frac{1}{3}(x-2) = 4$

21. $\frac{x-1}{2} - \frac{x-2}{3} - \frac{x-3}{4} = 0$

22. $\frac{x+1}{3} + \frac{x+4}{5} = \frac{x-4}{7}$

23. $15 - 2(5 - 3x) = 4(x - 3) + 13$

24. $\frac{2x+1}{3x-2} = 1\frac{1}{4}$

25. $21 - 3(x - 7) = x + 20$

26. $\frac{3x-2}{7} - \frac{x-2}{4} = 2$

27. $\frac{2x-3}{3} - (x-5) = \frac{x}{3}$

28. $\frac{x-4}{7} = \frac{x+3}{7} + \frac{x+4}{5}$

29. $\frac{x-1}{5} - \frac{x}{3} = 1 - \frac{x-2}{2}$

30. $2x + 20\%$ of $x = 12\cdot1$

12.5 WORD PROBLEMS

Example 9 :

Five added to twice a whole number gives 35. Find the whole number.

Solution :

Let the required whole number be x.

$\therefore \quad 2x + 5 = 35$

$\Rightarrow \quad 2x = 35 - 5 = 30$

and, $\quad x = \frac{30}{2} = 15$

$\therefore$ **Required whole number = 15** **(Ans.)**

Example 10 :

One-fifth of a number is 4. Find the number.

Solution :

Let the number be x.

$\therefore$ One-fifth of the number $= \frac{1}{5}x$

Given : $\frac{1}{5}x = 4 \Rightarrow x = 4 \times 5 = 20$

$\therefore$ **Required number = 20** **(Ans.)**

Example 11 :

Divide 80 into two parts, so that the greater part is 4 times the smaller.

Solution :

Let the greater part be x.

$\therefore$ The smaller part $= 80 - x$

Given : Greater part $= 4 \times$ smaller part

$\Rightarrow x = 4 \times (80 - x)$

$\Rightarrow x = 320 - 4x$

i.e. $x + 4x = 320 \Rightarrow 5x = 320$

and, $x = \frac{320}{5} = 64$

$\therefore$ **Greater number** $= x =$ **64**

and **smaller number** $= 80 - x = 80 - 64 =$ **16** **(Ans.)**

Example 12 :

A number is multiplied by 9 and then 11 is added to the product. If the final result is 56, find the number.

Solution :

Let the number be x.

$\therefore 9x + 11 = 56$

$\Rightarrow 9x = 56 - 11 = 45$

and, $x = \frac{45}{9} = 5$

$\therefore$ **The required number = 5** **(Ans.)**

Example 13 :

The price of 2 tables and 3 chairs is ₹ 680. If a table costs ₹ 40 more than a chair, find the cost of each.

Solution :

Let the cost of each chair = ₹ x

$\therefore$ The cost of each table = ₹ $(x + 40)$

Since, cost of 2 tables + cost of 3 chairs = 680

$\Rightarrow 2(x + 40) + 3x = 680$

$\Rightarrow 2x + 80 + 3x = 680$

$\Rightarrow 5x = 680 - 80 = 600$

$\Rightarrow x = \frac{600}{5} = 120$

$\therefore$ **Cost of one chair** = ₹ x = **₹ 120**

and **cost of one table** = ₹ $(x + 40)$

= ₹ $(120 + 40)$ = **₹ 160** **(Ans.)**

Example 14 :

A father is 3 times as old as his son. In 10 years time, his age will be double his son's age. Find their present ages.

Solution :

Let the present age of the son = x years

$\therefore$ Present age of the father = $3x$ years

After 10 years :

Son's age will be $(x + 10)$ years

and father's age will be $(3x + 10)$ years

According to given condition :

$$3x + 10 = 2(x + 10)$$

$$\Rightarrow \quad 3x + 10 = 2x + 20$$

$$\Rightarrow \quad 3x - 2x = 20 - 10 \quad i.e. \quad x = 10$$

$\therefore$ **Present age of son** = **10 years** and

present age of father = $3x = 3 \times 10$ years = **30 years** **(Ans.)**

Example 15 :

The permeter of a rectangle is 50 cm. If its length exceeds its breadth by 5 cm, find the length and the breadth of the rectangle.

Solution :

Let the breadth of the rectangle = x cm

$\therefore$ Length of the rectangle = $(x + 5)$ cm

$\therefore$ Perimeter of the rectangle

= 2(length + breadth)

= $2(x + 5 + x)$ cm

= $2(2x + 5)$ cm = $(4x + 10)$ cm

Given : $4x + 10 = 50$ *i.e.* $4x = 50 - 10 = 40$

$$\Rightarrow \quad x = \frac{40}{4} = 10$$

$\therefore$ **Length of the rectangle** = $(x + 5)$ cm

= $(10 + 5)$ cm = **15 cm**

and, **breadth of the rectangle** = x cm = **10 cm** **(Ans.)**

Example 16 :

A certain number of ₹10 notes and a certain number of ₹ 50 notes are kept in a purse so that there are 60 notes in the purse and their total value is ₹ 1,400. Find the number of each type of notes.

Solution :

Let the number of ₹ 10 notes = x

$\therefore$ The number of ₹ 50 notes = $60 - x$

Value of ₹ 10 notes = $x \times$ ₹ 10 = ₹ $10x$

and value of ₹ 50 notes = $(60 - x) \times$ ₹ 50 = ₹ $(3000 - 50x)$

∵ Total value of all the notes = ₹ 1,400

$\therefore \quad 10x + (3000 - 50x) = 1400$

$\Rightarrow \quad 10x + 3000 - 50x = 1400$

$\Rightarrow \quad -40x = 1400 - 3000$

$\Rightarrow \quad -40x = -1600$

and, $x = \dfrac{-1600}{-40} = 40$

∴ **The number of ₹ 10 notes** = x = **40**

and, **the number of ₹ 50 notes** = $60 - x$

$= 60 - 40 =$ **20** **(Ans.)**

EXERCISE 12(D)

1. One-fifth of a number is 5, find the number.
2. Six times a number is 72, find the number.
3. If 15 is added to a number, the result is 69, find the number.
4. The sum of twice a number and 4 is 80, find the number.
5. The difference between a number and one-fourth of itself is 24, find the number.
6. Find a number whose one-third part exceeds its one-fifth part by 20.
7. A number is as much greater than 35 as is less than 53. Find the number.
8. The sum of two numbers is 18. If one is twice the other, find the numbers.
9. A number is 15 more than the other. The sum of the two numbers is 195. Find the numbers.
10. The sum of three consecutive even numbers is 54. Find the numbers.
11. The sum of three consecutive odd numbers is 63. Find the numbers.
12. A man has ₹ x from which he spends ₹ 6. If twice of the money left with him is ₹ 86, find x.
13. A man is four times as old as his son. After 20 years, he will be twice as old as his son at that time. Find their present ages.
14. If 5 is subtracted from three times a number, the result is 16. Find the number.
15. Find three consecutive natural numbers such that the sum of the first and the second is 15 more than the third.
16. The difference between two numbers is 7. Six times the smaller plus the larger is 77. Find the numbers.
17. The length of a rectangular plot exceeds its breadth by 5 metre. If the perimeter of the plot is 142 metres, find the length and the breadth of the plot.
18. The numerator of a fraction is four less than its denominator. If 1 is added to both, its numerator and denominator, the fraction becomes $\frac{1}{2}$. Find the fraction.
19. A man is thrice as old as his son. After 12 years, he will be twice as old as his son at that time. Find their present ages.
20. A sum of ₹ 500 is in the form of notes of denominations of ₹ 5 and ₹ 10. If the total number of notes is 90, find the number of notes of each type.

SET CONCEPTS 13

13.1 BASIC CONCEPT

In our day to day life, different collective nouns are used to describe collection of objects, such as : *a group of students playing cricket, a pack of cards, a bunch of flowers,* etc.

In mathematics, *such collections of objects are named as* **sets**.

A set is a collection of well-defined objects, things or symbols, etc.

The phrase 'well-defined' means, it must be possible to know, without any doubt, whether a given object, thing or symbol belongs to the set under consideration or not.

For example :

"The set of tall boys of Class 10" is not *well-defined,* since it is not possible to know which boys are to be included and exactly what is the limit.

But when we say, "The set of boys of Class 10, which are taller than Peter", now we can compare the heights of different boys with the height of Peter and can know exactly which boys are to be included in the required set. Thus, *the objects are well-defined.*

13.2 ELEMENTS OF A SET

The objects (things, symbols, etc.) *used to form a set are called elements* or *members of the set.*

In general, a set is denoted by a capital letter of English alphabet with its elements written inside curly braces and separated by commas.

e.g., Set A = {5, 10, 12, 15}

13.3 USE OF SYMBOL '$\in$' OR SYMBOL '$\notin$'

The symbol '$\in$' stands for *'belongs to'* or *'is an element of'* or *'is a member of'*, whereas the symbol '$\notin$' stands for *'does not belong to'* or *'is not an element of'* or *'is not a member of'.*

e.g., For set P = {3, 6, 8, 13, 18}, $3 \in P$, $5 \notin P$ and so on.

1. ***The elements in a set can be written in any order.***
   Thus, {*a, b, c, d*} is the same set as {*b, d, a, c*} or {*c, b, d, a*}, etc.
2. ***The elements in a set should not be repeated,*** *i.e., if any element occurs many times, it should be written only once.*
   Thus, set of letters of the word 'crook' = {c, r, o, k}.
   There are **two o's** *in the given word "crook", but in the set, it is* **written only once.**

13.4 REPRESENTATION OF A SET

A set, in general, is represented in :

(i) Description method (form)
(ii) Tabular or Roster method (form)
(iii) Set-builder or Rule method

For example :

N is the set of natural numbers [Description method]

$N = \{1, 2, 3, 4, 5, ...\}$ [Roster or Tabular method]

$N = \{x : x$ is a natural number$\}$, or $\{x : x \in N\}$ [Set-builder or Rule method]

[The symbol ' : ' stands for **such that** and the set $\{x : x \in N\}$ is read as, "the set of x *such that* x is a natural number"].

It is clear from the example given above that :

(i) in *description method* a well-defined description about the set is given.

(ii) in *roster* or *tabular method* the elements of the set are written inside a pair of curly braces and are separated by commas.

(iii) in *set-builder* or *rule method* the actual elements of the set are not written, but *a rule* or *a statement* or *a formula is written* in the briefest possible way.

More examples :

1. Z is the set of integers [Description method]

   $Z = \{...., -3, -2, -1, 0, 1, 2, 3,.....\}$ [Tabular or Roster method]

   $= \{x : x \in Z\}$ [Rule or Set-builder method]

2. A is the set of whole numbers less than 8 [Description method]

   $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ [Tabular or Roster method]

   $A = \{x : x \in W$ and $x < 8\}$ [Rule or Set-builder method]

Example 1 :

Express each of the given sets as required :

(i) The set of integers between –3 and 5, *in roster form.*

(ii) The set of even natural numbers, *in roster form.*

(iii) Set $A = \{1, 3, 5, 7, 9,......\}$, *in set-builder form.*

(iv) Set $B = \{0, 3, 6, 9, 12,....\}$, *in set-builder notation.*

(v) Set $C = \{y : y = 2x + 1$ and $x \in N\}$, *in roster form.*

Solution :

(i) **The set of integers between –3 and 5** = **$\{-2, -1, 0, 1, 2, 3, 4\}$** **(Ans.)**

(ii) **The set of even natural numbers** = **$\{2, 4, 6, 8, 10, 12, ...\}$** **(Ans.)**

(iii) **Set A** = $\{1, 3, 5, 7, 9,......\}$ = **$\{x : x$ is an odd natural number$\}$** **(Ans.)**

(iv) **Set B** = $\{0, 3, 6, 9, 12,....\}$ = **$\{x : x$ is a whole number divisible by 3$\}$** **(Ans.)**

(v) Since, $x \in N$ (the set of natural numbers)

$\therefore$ x can be 1, 2, 3, 4, ...

And so, **set *C*** $= \{y : y = 2x + 1$ and $x \in N\}$

$= \{2 \times 1 + 1, 2 \times 2 + 1, 2 \times 3 + 1, 2 \times 4 + 1, ...\}$

$=$ **$\{3, 5, 7, 9, ...\}$** **(Ans.)**

EXERCISE 13(A)

1. Find, whether or not, each of the following collections represent a set :
   (i) The collection of good students in your school.
   (ii) The collection of the numbers between 30 and 45.
   (iii) The collection of fat-people in your colony.
   (iv) The collection of interesting books in your school library.
   (v) The collection of books in the library and are of your interest.

2. State whether **true** or **false** :
   (i) Set {4, 5, 8} is same as the set {5, 4, 8} and the set {8, 4, 5}.
   (ii) Sets {*a, b, m, n*} and {*a, a, m, b, n, n*} are same.
   (iii) Set of letters in the word 'suchismita' is {*s, u, c, h, i, m, t, a*}.
   (iv) Set of letters in the word 'MAHMOOD' is {M, A, H, O, D}.

3. Let set A = {6, 8, 10, 12} and set B = {3, 9, 15, 18}.
   Insert the symbol '$\in$' or '$\notin$' to make each of the following true :
   (i) 6 .... A (ii) 10 .... B (iii) 18 .... B (iv) (6 + 3) .... B
   (v) (15 – 9) .... B (vi) 12 .... A (vii) (6 + 8) .... A (viii) 6 and 8 .... A

4. Express each of the following sets in **roster form** :
   (i) Set of odd whole numbers between 15 and 27.
   (ii) A = Set of letters in the word "CHITAMBARAM"
   (iii) B = {All even numbers from 15 to 26}
   (iv) P = {x : x is a vowel used in the word 'ARITHMETIC'}
   (v) S = {Squares of first eight whole numbers}
   (vi) Set of all integers between 7 and 94 which are divisible by 6.
   (vii) C = {All composite numbers between 2 and 20}
   (viii) D = Set of prime numbers from 2 to 23.
   (ix) E = Set of natural numbers below 30 which are divisible by 2 or 5.
   (x) F = Set of factors of 24.
   (xi) G = Set of names of three closed figures in Geometry.
   (xii) H = {$x : x \in$ W and $x < 10$}
   (xiii) J = {$x : x \in$ N and $2x - 3 \leq 17$}
   (xiv) K = {x : x is an integer and $-3 < x < 5$}

5. Express each of the following sets in **set-builder notation** (form) :
   (i) {3, 6, 9, 12, 15} (ii) {2, 3, 5, 7, 11, 13, ...}
   (iii) {1, 4, 9, 16, 25, 36} (iv) {0, 2, 4, 6, 8, 10, 12, ...}
   (v) {Monday, Tuesday, Wednesday} (vi) {23, 25, 27, 29, ...}
   (vii) $\left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}\right\}$ (viii) {42, 49, 56, 63, 70, 77}

6. Given : A = { x : x is a multiple of 2 and is less than 25 }
   B = { x : x is a square of a natural number and is less than 25 }
   C = { x : x is a multiple of 3 and is less than 25 }
   D = { x : x is a prime number less than 25 }
   Write the sets A, B, C and D in roster form.

13.5 CARDINAL NUMBER

The cardinal number of a set is the number of elements in it.

Thus, if a set A has 5 elements; its cardinal number is 5 and we represent it by writing $n(A) = 5$.

Similarly, if set B = Set of even natural numbers less than 10

then, B = {2, 4, 6, 8} and $n(B) = 4$.

If B = {0}, *then* n(B) = 1. Since, 0 *is an element of set* B.

13.6 TYPES OF SETS

1. *Finite set :*

A set is said to be a finite set, if it has a limited (countable) number of elements in it.

For example :

(i) S = Set of natural numbers between 10 and 15 = {11, 12, 13, 14}

(ii) P = {0, 1, 2, ..., 20} = $\{x : x \in W \text{ and } x \leq 20\}$ and so on.

2. *Infinite set :*

A set is said to be an infinite set, if it has an unlimited (uncountable) number of elements in it.

For example :

(i) P = Set of prime numbers = {2, 3, 5, ...}

(ii) B = $\{x : x \in N \text{ and } x \geq 21\}$ = {21, 22, 23, ...} and so on.

3. *Empty set* or *Null set* :

The set, with no element in it, is called the empty set or the null set.

The empty set is represented by a pair of braces with no element in it or by the Danish letter ϕ, which is pronounced as *'oe'*.

Thus, the empty set = { } = ϕ.

For empty set, it is wrong to call ***'an empty set'*** *or* ***'a null set'*** *as there is one and only one empty set though it may have many descriptions.*

Therefore, it is always called ***"the empty set"*** *or* ***"the null set"***.

Some examples of the empty set **:**

(i) Let A = {a man of age more than 400 years}.

Since, there can not be any man with the age more than 400 years, the set A will have no element in it, *i.e.,* it is the empty set. And we write : A = { } or ϕ.

(ii) If B = {Triangles with 4 sides}, it is clear that B = ϕ.

1. *$\phi \neq \{0\}$, since {0} is a set with 0 as its element whereas ϕ has no element.*
2. *$\{\phi\} \neq \{0\}$, since both the sets have different elements.*
3. *The cardinal number of the empty set is 0, i.e., $n(\phi) = 0$.*

4. ***Disjoint sets :***

Sets having no element in common are called disjoint sets.

For example :

Sets P = {5, 7, 9} and Q = {4, 6, 10, 12} *are disjoint,* as they do not have any element in common.

5. ***Joint** (overlapping) **sets :***

Sets having at least one element in common are called joint or overlapping sets.

For example :

Set B = {4, 6, 8, 10, 12} and set C = {3, 6, 9, 12, 15} are *joint sets,* as they have elements 6 and 12 common.

6. ***Equal sets :***

Two sets are said to be equal, if the elements of both the sets are the same.

For example :

If set A = {x, y, z} and set B = {last three letters of English alphabet}.

Clearly, sets A and B have the same elements and so **set A = set B.**

7. ***Equivalent sets :***

Two sets are said to be equivalent, if they have equal number of elements in them, i.e., the cardinal numbers of both the sets are equal.

For example :

Let A = {3, 6, 9} and B = {a, b, c}.

Since, set A has 3 elements and set B also has 3 elements, *i.e.*, n(A) = n(B). Therefore, sets A and B are equivalent and for this, we write : A ↔ B.

1. *Equal sets are always equivalent, but the converse is not always true* (*i.e.*, it is not necessary that equivalent sets are equal also).
2. *In equivalent sets, the number of elements* (cardinal number) *are equal, whereas in equal sets the elements are the same.*
3. *Two infinite sets are always equivalent.*

EXERCISE 13(B)

1. Write the **cardinal number** of each of the following sets :
   (i) A = Set of days in a leap year.
   (ii) B = Set of numbers on a clock-face.
   (iii) C = {x : $x \in$ N and $x \leq 7$}
   (iv) D = Set of letters in the word "PANIPAT".
   (v) E = Set of prime numbers between 5 and 15.
   (vi) F = {x : $x \in Z$ and $-2 < x \leq 5$}
   (vii) G = {x : x is a perfect square number, $x \in$ N and $x \leq 30$}.
2. For each set, given below, state whether it is **finite set, infinite set** or **the null set :**
   (i) {natural numbers more than 100}.

(ii) A = {x : x is an integer between 1 and 2}.

(iii) B = {x : $x \in$ W; x is less than 100}.

(iv) Set of mountains in the world.

(v) {multiples of 8}.

(vi) {even numbers not divisible by 2}.

(vii) {squares of natural numbers}.

(viii) {coins used in India}

(ix) C = {x | x is a prime number between 7 and 10}.

(x) Planets of the solar system.

3. State, which of the following pairs of sets are **disjoint** :

(i) {0, 1, 2, 6, 8} and {odd numbers less than 10}.

(ii) { birds } and { trees }

(iii) {x : x is a fan of cricket} and {x : x is a fan of football}.

(iv) A = {natural numbers less than 10} and B = {x : x is a multiple of 5}.

(v) {people living in Calcutta} and {people living in West Bengal}

4. State whether the given pairs of sets are **equal** or **equivalent :**

(i) A = {first four natural numbers} and B = {first four whole numbers}.

(ii) A = Set of letters of the word "FOLLOW" and
B = Set of letters of the word "WOLF".

(iii) E = {even natural numbers less than 10} and
O = {odd natural numbers less than 9}.

(iv) A = {days of the week starting with letter S} and
B = {days of the week starting with letter T}.

(v) M = {multiples of 2 and 3 between 10 and 20} and
N = {multiples of 2 and 5 between 10 and 20}.

(vi) P = {prime numbers which divide 70 exactly} and
Q = {prime numbers which divide 105 exactly}.

(vii) A = {0^2, 1^2, 2^2, 3^2, 4^2} and B = {16, 9, 4, 1, 0}.

(viii) E = {8, 10, 12, 14, 16} and F = {even natural numbers between 6 and 18}.

(ix) A = {letters of the word SUPERSTITION} and
B = {letters of the word JURISDICTION}.

5. Examine which of the following sets are the **empty sets :**

(i) The set of triangles having three equal sides.

(ii) The set of lions in your class.

(iii) {x : $x + 3 = 2$ and $x \in$ N}

(iv) P = {x : $3x = 0$}

6. State **true** or **false** :

(i) All examples of the empty set are equal.

(ii) All examples of the empty set are equivalent.

(iii) If two sets have the same cardinal number, they are equal sets.

(iv) If n(A) = n(B), then A and B are equivalent sets.

(v) If B = $\{x : x + 4 = 4\}$, then B is the empty set.

(vi) The set of all points in a line is a finite set.

(vii) The set of letters in your Mathematics book is an infinite set.

(viii) If M = {1, 2, 4, 6} and N = $\{x : x$ is a factor of 12$\}$, then M = N.

(ix) The set of whole numbers greater than 50 is an infinite set.

(x) If A and B are two different infinite sets, then $n(A) = n(B)$.

7. Which of the following represents the **null set** ?

ϕ, { 0 }, 0, { }, { ϕ }.

13.7 SUBSET

If each element of set A is also present in set B, then set A is said to be the subset of set B. Conversely, if B is subset of A, then each element of set B is present in set A.

For example :

(i) A = {2, 4, 6} and B = {1, 2, 3, 4, 5, 6, 7} implies that A is subset of B, as each element of set A is in set B also.

(ii) A = {6, 7, 8, 9, 10} and B = {7, 9, 10} $\Rightarrow$ B is subset of A and so on.

Symbolically, if A is subset of B, we write : **A $\subseteq$ B**.

and, if B is subset of A, we write : **B $\subseteq$ A**.

13.8 SUPER-SET

When set A is the subset of set B, then set B is called the super-set of A.

Symbolically, we write : **B $\supseteq$ A**.

1. A $\subseteq$ B is read as : "A is subset of B" or "A is contained in B".
2. B $\supseteq$ A is read as : "B is super-set of A" or "B contains A".
3. A $\not\subseteq$ B means 'A is not a subset of B'. And B $\not\supseteq$ A means "B is not a super-set of A".
4. Every set is a subset of itself, *i.e.*, A $\subseteq$ A, B $\subseteq$ B and so on.
5. Empty set is subset of every set, *i.e.*, $\phi \subseteq$ A, $\phi \subseteq$ B and so on.

Example 2 :

Find subsets of : (i) { } (ii) $\{a, b\}$

Solution :

Since, every set is subset of itself and the empty set is subset of every set :

$\therefore$ (i) **Subset of { }** = { } **(Ans.)**

(ii) **Subsets of $\{a, b\}$** = { }, $\{a\}$, $\{b\}$ and $\{a, b\}$ **(Ans.)**

13.9 PROPER SUBSET

All the subsets, other than the set itself, are called proper subsets.

The symbol for proper subset is '$\subset$', *i.e.*, if A is proper subset of B, we write : A $\subset$ B.

1. No set is proper subset of itself.
2. When set A is proper subset of set B :
   (i) each element of set A is in set B,
   (ii) number of elements in set A is less than the number of elements in set B.

Example 3 :

Find, if possible, all proper subsets of : (i) { } (ii) {a} (iii) {a, b}.

Solution :

Since, no set is proper subset of itself, therefore :

(i) **{ } has no proper subset.** **(Ans.)**

(ii) **Proper subset of {a} = { }** **(Ans.)**

(iii) **Proper subsets of {a, b} = { }, {a} and {b}.** **(Ans.)**

13.10 NUMBER OF SUBSETS AND NUMBER OF PROPER SUBSETS OF A GIVEN SET

If a set has 'n' elements in it,

(i) the number of its subsets = 2^n and

(ii) the number of its proper subsets = $2^n - 1$ [No set is proper subset of itself]

e.g., Number of elements in {a, b} = 2 $\Rightarrow$ $n = 2$

$\therefore$ No. of its subsets = $2^n = 2^2 = 4$

and no. of its proper subsets = $2^n - 1 = 2^2 - 1 = 3$

13.11 UNIVERSAL SET

Any set, which contains all the elements of various sets under discussion, is called the universal set.

For example :

If the sets under discussion are :

A = {2, 3, 5}, B = {5, 6, 9, 12} and C = {3, 6, 9, 12}, then form a set which contains every element of sets A, B and C.

Clearly, the set obtained is {2, 3, 5, 6, 9, 12}. So this set is called the *universal set* for the sets A, B and C under discussion.

A universal set is represented by the symbol ξ (read pxi) or U.

Thus, ξ = {2, 3, 5, 6, 9, 12}

Note : *Universal set for the sets under consideration is not unique, i.e., we may have more than one universal set for the same sets under consideration.*

Thus, for the sets A, B and C, given above,

(i) {1, 2, 3, 4, ..., 15} can be taken as universal set, since it contains every element of the sets under discussion.

(ii) Set N, the set of natural numbers, can also be taken as universal set, since each element of the sets under discussion is a natural number and so on.

1. *Every set under discussion is a subset of the universal set.*
2. *$\xi \subseteq \xi$, because every set is a subset of itself.*

EXERCISE 13(C)

1. Fill in the blanks :
   (i) If each element of set P is also an element of set Q, then P is said to be .... of Q and Q is said to be .... of P.
   (ii) Every set is a .... of itself.
   (iii) The empty set is a .... of every set.
   (iv) If A is proper subset of B, then n(A) .... n(B).
2. If A = {5, 7, 8, 9}, then which of the following are subsets of A ?
   (i) B = {5, 8} (ii) C = {0} (iii) D = {7, 9, 10}
   (iv) E = { } (v) F = {8, 7, 9, 5}
3. If P = {2, 3, 4, 5}, then which of the following are proper subsets of P ?
   (i) A = {3, 4} (ii) B = { } (iii) C = {23, 45}
   (iv) D = {6, 5, 4} (v) E = {0}
4. If A = {even numbers less than 12},
   B = {2, 4}, C = {1, 2, 3}, D = {2, 6} and E = { 4 }
   State, which of the following statements are **true** :
   (i) $B \subset A$ (ii) $C \subseteq A$ (iii) $D \subset C$
   (iv) $D \not\subset A$ (v) $E \supseteq B$ (vi) $A \supseteq B \supseteq E$
5. Given A = {a, c}, B = {p, q, r} and C = Set of digits used to form the number 1351. Write all the subsets of sets A, B and C.
6. (i) If A = {p, q, r}, then number of subsets of A = ...
   (ii) If B = {5, 4, 6, 8}, then number of proper subsets of B = ...
   (iii) If C = {0}, then number of subsets of C = ...
   (iv) If M = {x : $x \in N$ and $x < 3$}, then M has .... proper subsets.
7. For the universal set {4, 5, 6, 7, 8, 9, 10, 11, 12, 13}, find its subsets A, B, C and D such that :
   (i) A = {even numbers} (ii) B = {odd numbers greater than 8}
   (iii) C = {prime numbers} (iv) D = {even numbers less than 10}.

13.12 OPERATIONS ON SETS

1. **Union of sets :**

The union of two sets A and B is the set which contains elements either from set A or from set B.

Union of sets A and B is denoted by $A \cup B$ and is read as **A union B.**

Let Set A = {5, 6, 7, 8, 9} and set B = {4, 6, 8, 10}

∴ **A union B** = $A \cup B$

= {elements from set A or from set B}

= **{4, 5, 6, 7, 8, 9, 10}**

For any two sets A and B :

A union B = $A \cup B$

= Set containing all the elements of set A and all the elements of set B.

2. **Intersection of sets :**

The intersection of two sets A and B is the set which contains elements common to both A and B.

Intersection of sets A and B is denoted by $A \cap B$ and is read **A intersection B**.

Let Set A = {5, 6, 7, 8, 9} and set B = {4, 6, 8, 10}

∴ **A intersection B** = $A \cap B$

= {elements common to sets A and B}

= **{6, 8}**

For any two sets A and B :

A intersection B = $A \cap B$

= Set containing elements that are common to set A and set B both.

3. Difference of sets :

The difference between two sets A and B is taken as A – B or B – A, where

(i) A – B = {elements in set A and not in set B}

(ii) B – A = {elements in set B and not in set A}

Let set A = {5, 6, 7, 8, 9} and set B = {4, 6, 8, 10}

Then : (i) **A – B** = {elements from set A which do not belong to set B}

= **{5, 7, 9}**

(ii) **B – A** = {elements from set B which are not in set A}

= **{4, 10}**

1. $A \cup B$ = {all the elements from set A and all the elements from set B with no repetition of any element}

2. $A \cap B$ = {all the elements that are common to sets A and B}

3. (i) A – B = (elements of set A that are not in set B)

(ii) B – A = {elements of set B that are not in set A}.

Example 4 :

If set A = {4, 5, 6, 7, 8} and set B = {5, 7, 9, 11}, find :

(i) $A \cup B$ (ii) $A \cap B$ (iii) A – B (iv) B – A

Solution :

(i) **$A \cup B$** = {all the elements from set A and all the elements from set B}

= **{4, 5, 6, 7, 8, 9, 11}** **(Ans.)**

(ii) **$A \cap B$** = {elements common to both the sets A and B}

= **{5, 7}** **(Ans.)**

(iii) **A – B** = {elements of set A which are not in set B}

= **{4, 6, 8}** **(Ans.)**

(iv) **B – A** = {elements of set B which are not in set A}

= **{9, 11}** **(Ans.)**

Example 5 :

If set A = {*b, c, d, e*}, set B = {*a, c, d*} and set C = {*d, e, m, n*}, find :

(i) $A \cup B$ (ii) $B \cup C$ (iii) $A \cap C$ (iv) $C \cap B$

(v) A – B (vi) B – C

Solution :

(i) **A∪B** = {*b, c, d, e*} ∪ {*a, c, d*}

= **{*a, b, c, d, e*}** **(Ans.)**

(ii) **B∪C** = {*a, c, d*} ∪ {*d, e, m, n*}

= **{*a, c, d, e, m, n*}** **(Ans.)**

(iii) **A∩C** = {*b, c, d, e*} ∩ {*d, e, m, n*}

= **{*d, e*}** **(Ans.)**

(iv) **C∩B** = {*d, e, m, n*} ∩ {*a, c, d*}

= **{*d*}** **(Ans.)**

(v) **A – B** = {elements belonging to set A, but not belonging to set B}

= **{*b, e*}** **(Ans.)**

(vi) **B – C** = {elements belonging to set B, but not beloning to set C}

= **{*a, c*}** **(Ans.)**

13.13 CARDINAL PROPERTIES OF SETS

If a set A has 10 elements in it, we say its cardinal number is 10 and write it as **n(A) = 10**.

If set B = {2, 4, 6, 8, 10, 12}

⇒ Set B has 6 elements *i.e.* **n(B) = 6**.

For any two sets A and B

(i) $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

(ii) $n(A\cap B) = n(A) + n(B) - n(A\cup B)$

(iii) If A and B are disjoint, $A\cap B = \phi$ and $n(A\cap B) = 0$

∴ $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

⇒ $n(A\cup B) = n(A) + n(B) - 0$

⇒ $\mathbf{n(A\cup B) = n(A) + n(B)}$

(iv) $n(A - B) = n(A) - n(A\cap B)$ and $n(B - A) = n(B) - n(A\cap B)$

Example 6 :

Let A {2, 3, 4, 5, 6} and B = {4, 6, 8, 10}. Find

(i) A∪B (ii) A∩B (iii) n(A) (iv) n(B)

(v) n(A∪B) (vi) n(A∩B) (vii) n(A – B) (viii) n(B – A)

Solution :

(i) **A∪B** = {2, 3, 4, 5, 6} ∪ {4, 6, 8, 10}

= **{2, 3, 4, 5, 6, 8, 10}** **(Ans.)**

(ii) **A∩B** = {elements common to A and B both}

= **{4, 6}** **(Ans.)**

(iii) **n(A)** = Number of elements in set A

= **5** **(Ans.)**

(iv) **n(B)** = Number of elements in set B.

= **4** **(Ans.)**

(v) **n(A∪B) = 7** **(Ans.)**

(vi) $n(A \cap B) = 2$ **(Ans.)**

(vii) $n(A - B) = n(A) - n(A \cap B)$

$= 5 - 2 = 3$ **(Ans.)**

(viii) $n(B - A) = n(B) - n(B \cap A)$ $[A \cap B = B \cap A]$

$= 4 - 2 = 2$ **(Ans.)**

Example 7 :

If $A = \{a, b, c, d\}$ and $B = \{b, c, e\}$, verfiy that : $n(A \cup B) + n(A \cap B) = n(A) + n(B)$.

Solution :

$\because$ $A \cup B = \{a, b, c, d, e\}$ $\Rightarrow$ $n(A \cup B) = 5$,

$A \cap B = \{b, c\}$ $\Rightarrow$ $n(A \cap B) = 2$,

$n(A) = 4$ and $n(B) = 3$

$n(A \cup B) + n(A \cap B) = 5 + 2 = 7$

and $n(A) + n(B) = 4 + 3 = 7$

$\Rightarrow$ $n(A \cup B) + n(A \cap B) = n(A) + n(B)$ **(Ans.)**

EXERCISE 13(D)

1. If A = {4, 5, 6, 7, 8} and B = {6, 8, 10, 12}, find :
   (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$.
2. If A = {3, 5, 7, 9, 11} and B = {4, 7, 10}, find :
   (i) $n(A)$ (ii) $n(B)$ (iii) $A \cup B$ and $n(A \cup B)$ (iv) $A \cap B$ and $n(A \cap B)$
3. If A = {2, 4, 6, 8} and B = {3, 6, 9, 12}, find :
   (i) $(A \cap B)$ and $n(A \cap B)$ (ii) $(A - B)$ and $n(A - B)$
   (iii) $n(B)$
4. If P = {*x* : *x* is a factor of 12} and Q = {*x* : *x* is a factor of 16}, find :
   (i) $n(P)$ (ii) $n(Q)$ (iii) $Q - P$ and $n(Q - P)$
5. M = {*x* : *x* is a natural number between 0 and 8} and N = {*x* : *x* is a natural number from 5 to 10}. Find :
   (i) $M - N$ and $n(M - N)$ (ii) $N - M$ and $n(N - M)$
6. If A = {*x* : *x* is natural number divisible by 2 and $x < 16$} and B = {*x* : *x* is a whole number divisible by 3 and $x < 18$}, find :
   (i) $n(A)$ (ii) $n(B)$ (iii) $A \cap B$ and $n(A \cap B)$ (iv) $n(A - B)$
7. Let A and B be two sets such that $n(A) = 75$, $n(B) = 65$ and $n(A \cap B) = 45$, find :
   (i) $n(A \cup B)$ (ii) $n(A - B)$ (iii) $n(B - A)$
8. Let A and B be two sets such that $n(A) = 45$, $n(B) = 38$ and $n(A \cup B) = 70$, find :
   (i) $n(A \cap B)$ (ii) $n(A - B)$ (iii) $n(B - A)$
9. Let $n(A) = 30$, $n(B) = 27$ and $n(A \cup B) = 45$, find :
   (i) $n(A \cap B)$ (ii) $n(A - B)$
10. Let $n(A) = 31$, $n(B) = 20$ and $n(A \cap B) = 6$, find :
    (i) $n(A - B)$ (ii) $n(B - A)$ (iii) $n(A \cup B)$

Theme – 4
GEOMETRY

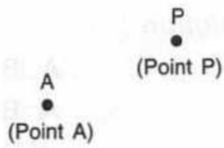
LINES AND ANGLES

(Including Construction of Angles)

14

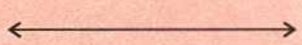
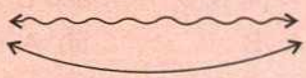
14.1 REVIEW

1. **POINT :** *A point is a mark of position, which has no length, no breadth and no thickness.* In general, a fine dot marked with a very short edged pencil represents a point. It is represented by a capital letter as shown alongside.



2. **LINE :** *A line has length,* but *no breadth* or *thickness.*

The given figure shows a line $\overleftrightarrow{AB}$ in which two arrow-heads in opposite directions show that it can be extended infinitely in both the directions.

> 1. *A line may be straight or curved but when we say 'a line' it means a straight line only.*
>
> (i)  (ii) 
>
> (A line or a straight line) (Curved lines)
>
> 2. *Each line, whatever be its length, has an infinite number of points in it.*

3. **RAY :** *It is a straight line which starts from a fixed point and moves in the same direction.*

The given figure shows a ray $\overrightarrow{AB}$ with fixed initial point A and moving in the direction AB.

- A ray is a line whose one end is fixed and the other end is variable which extends infinitely in one direction of the line.

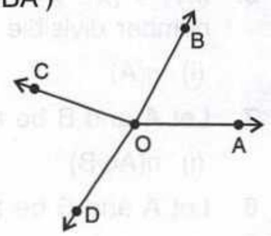
(i) 

(Ray $\overrightarrow{AB}$)

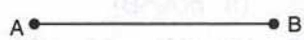
(ii) 

(Ray $\overleftarrow{AB}$ or ray $\overrightarrow{BA}$)

- Any number of rays can be drawn through the same fixed point.

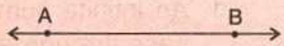


4. **LINE SEGMENT :** *It is a straight line with its both ends fixed.* The given figure shows a line segment, whose both the ends A and B are fixed.

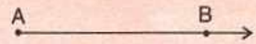


- A line segment has a definite length, which can be measured.
- Through any two fixed points, one and only one line segment can be drawn.
- If two line segments have equal lengths, the line segments are said to be equal.

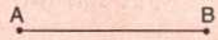
1. The adjoining figure shows a line AB which can be extended upto infinity on its both sides.



2. The adjoining figure shows a ray AB with fixed end as point A and which can be extended upto infinity through point B. It is clear from the figure, that a ray is a part of a line.

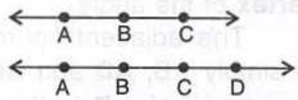


3. The adjoining figure shows a line-segment AB with fixed ends A and B.



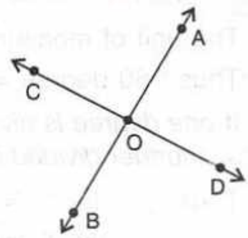
It is clear from the different figures, that a line-segment is a part of a ray as well as of a line. Also, a line segment is the shortest distance between two fixed points.

1. **Collinear points :** If three or more points lie on the same straight line (as shown alongise), the points are said to be **collinear points**.

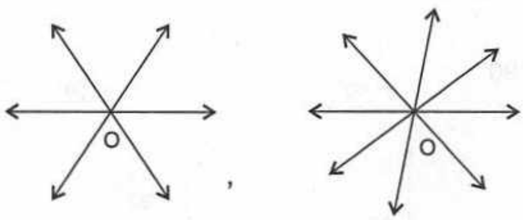


2. **Intersecting lines :** When two lines intersect (meet) each other at one point only, the lines are called **intersecting lines** and the point at which these lines intersect is called their **point of intersection**.

In the given figure, lines AB and CD are intersecting lines and the common point O is the **point of intersection**.

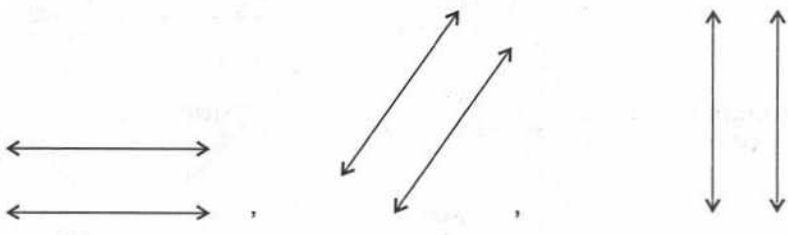


3. **Concurrent lines :** When three or more coplanar lines pass through the same fixed point, the lines are said to be **concurrent** and the fixed point O, through which the lines pass, is called **point of concurrence**.



, etc.

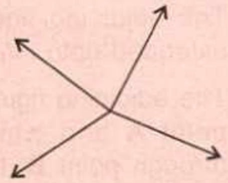
4. **Parallel lines :** Two lines in the same plane are said to be parallel to each other, if they do not meet (intersect) with each other no matter upto what length they are produced.



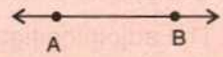
, etc.

The distance between two parallel lines always remains the same.

1. An infinite number of lines can be drawn to pass through a given fixed point.

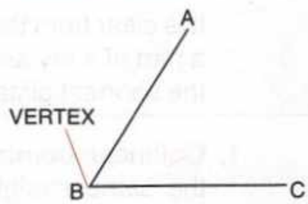


2. One and only one line can be drawn through two fixed points.



5. **ANGLE :** *An angle is formed when two line segments or two rays have a common end-point.*

The *two line segments, forming an angle,* are called the **arms** of the angle whereas their *common end-point* is called the **vertex** of the angle.



The adjacent figure represents an angle ABC or ∠ABC or simply ∠B. **AB** and **BC** are the **arms of the angle** and their **common point B is the vertex.**

14.2 MEASUREMENT OF AN ANGLE

The unit of measuring an angle is ***degree***. The symbol for degree is °.

Thus : 60 degree = 60°, 87 degree = 87° and so on.

If *one degree is divided into 60 equal parts,* each part is called a *minute* (′) and if *one minute is further divided into 60 equal parts, each part* is called *a second* (″).

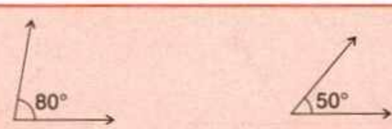
Thus, (i) 1° = 60′ and 1′ = 60″

(ii) 8 minutes 45 seconds = 8′ 45″

(iii) 25 degrees 30 minutes 15 seconds = 25° 30′ 15″ and so on.

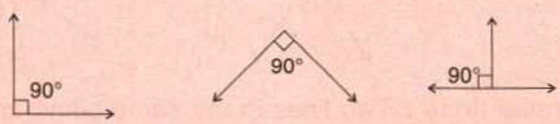
14.3 TYPES OF ANGLES

1. ***Acute angle :*** measures less than 90°



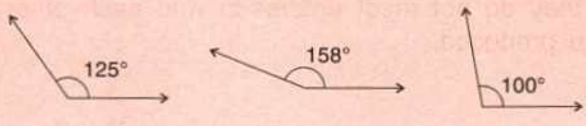
etc.

2. ***Right angle :*** measures 90°



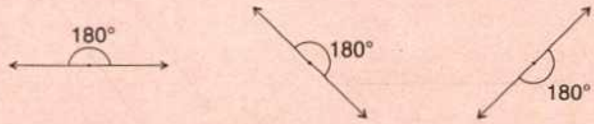
etc.

3. ***Obtuse angle :*** measures between 90° and 180°



etc.

4. ***Straight angle :*** measures 180°

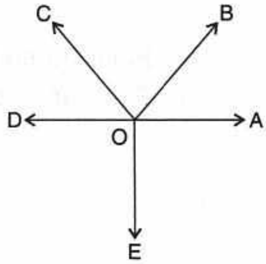


etc.

5. ***Reflex angle :*** measures between 180° and 360°

etc.

14.4 MORE ABOUT ANGLES



1. *Angles about a point* : If a number of angles are formed about a point, their sum is always 360°.

In the adjoining figure :

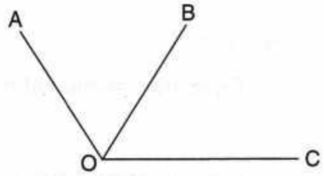
$\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$.

2. *Adjacent angles* : Two angles are said to be adjacent angles, if :

(i) they have a common vertex,

(ii) they have a common arm and

(iii) the other arms of the two angles lie on opposite sides of the common arm.



The adjoining figure shows a pair of adjacent angles :

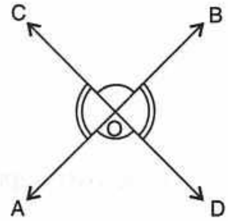
because (i) they have a common vertex (O),

(ii) they have a common arm (OB) and

(iii) the other arms OA and OC of the two angles are on opposite sides of the common arm OB.

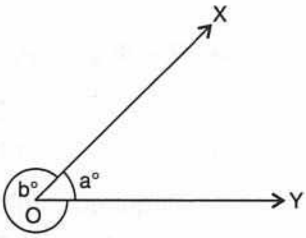
3. *Vertically opposite angles* : When two straight lines intersect each other four angles are formed.

The pair of angles which lie on the opposite sides of the point of intersection are called **vertically opposite angles**.



In the adjoining figure, two straight lines AB and CD intersect each other at a point O. Angles AOD and BOC form one pair of vertically opposite angles, whereas angles AOC and BOD form another pair of vertically opposite angles.

> *Vertically opposite angles are always equal.*
> *i.e.* $\angle AOD = \angle BOC$
> *and* $\angle AOC = \angle BOD$.



Important : In the adjoining figure, rays OX and OY meet at O to form $\angle XOY$ (*i.e.,* $\angle a$) and reflex $\angle XOY$ (*i.e.,* $\angle b$). It must be noted that $\angle XOY$ represents the smaller angle only, unless it is mentioned otherwise.

14.5 COMPLEMENTARY AND SUPPLEMENTARY ANGLES

1 *Two angles are called* ***complementary angles,*** *if their sum is one right angle, i.e.,* 90°. Each angle is called the **complement** of the other.
e.g., 20° and 70° are complementary angles, because 20° + 70° = 90°.
Clearly, 20° is the complement of 70° and 70° is the complement of 20°.
Thus, the **complement of angle 53°** = 90° − 53° = **37°.**

2. *Two angles are called* ***supplementary angles,*** *if their sum is two right angles, i.e.,* 180°. Each angle is called the **supplement of the other.**
e.g., 30° and 150° are supplementary angles because 30° + 150° = 180°.
Clearly, 30° is the supplement of 150° and vice-versa.
Thus, the **supplement of 105°** = 180° − 105° = **75°.**

Example 1 :

(i) Find the complement of the angle 47° 36′.

(ii) Find the supplement of the angle 93° 28′ 43″.

Solution :

(i) $90° = 89°\ 60'$

$\therefore$ **Complement of 47° 36′** $= 90° - 47°\ 36'$

$= 89°\ 60' - 47°\ 36' =$ **42° 24′** **(Ans.)**

(ii) $180° = 179°\ 60'$

$= 179°\ 59'\ 60''$

$\therefore$ **Supplement of 93° 28′43″** $= 180° - 93°\ 28'\ 43''$

$= 179°\ 59'\ 60'' - 93°\ 28'\ 43''$

$=$ **86° 31′ 17″** **(Ans.)**

Example 2 :

Find the angle which is five times of its complement.

Solution :

Let the angle be $x°$.

Its complement $= (90 - x)°$

$\Rightarrow \quad x° = 5 \times (90 - x)°$

$\Rightarrow \quad x = 450 - 5x$

$\Rightarrow \quad 6x = 450 \text{ and } x = \frac{450}{6} = 75$

$\therefore$ **Required angle = 75°** **(Ans.)**

Example 3 :

Find two supplementary angles which are in the ratio 13 : 5.

Solution :

Let the angles be $13x°$ and $5x°$

$\therefore \quad 13x° + 5x° = 180°$

$\Rightarrow \quad 18x° = 180° \text{ i.e. } x = \frac{180}{18} = 10$

$\therefore$ Required angles are :

$13x°$ and $5x° = 13 \times 10°$ and $5 \times 10°$

= **130° and 50°** **(Ans.)**

EXERCISE 14(A)

1. State, **true** or **false** :

(i) A line segment 4 cm long can have only 2000 points in it.

(ii) A ray has one end point and a line segment has two end-points.

(iii) A line segment is the shortest distance between any two given points.

(iv) An infinite number of straight lines can be drawn through a given point.

(v) Write the number of end points in

(a) a line segment AB (b) a ray AB (c) a line AB

(vi) Out of $\overleftrightarrow{AB}$, $\overrightarrow{AB}$, $\overleftarrow{AB}$ and $\overline{AB}$, which one has a fixed length ?

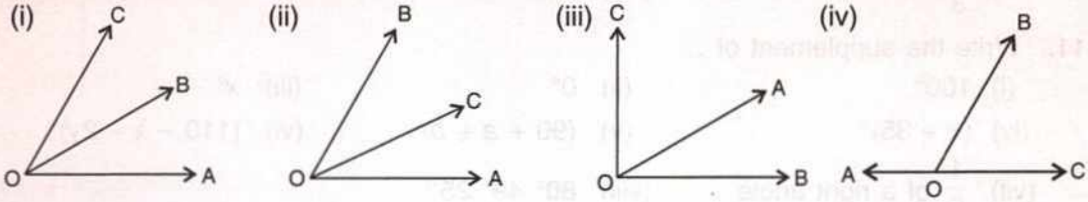
(vii) How many rays can be drawn through a fixed point O ?

(viii) How many lines can be drawn through three

(a) collinear points ? (b) non-collinear points ?

(ix) Is 40° the complement of 60° ? (x) Is 45° the supplement of 45° ?

2. In which of the following figures, are ∠AOB and ∠AOC adjacent angles ? Give, in each case, reason for your answer.

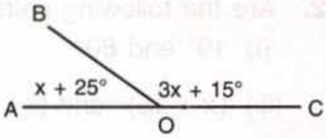


3. In the given figure, AC is a straight line. Find :

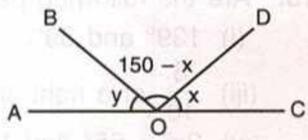
(i) x,

(ii) ∠AOB,

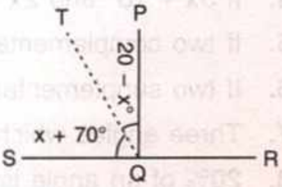
(iii) ∠BOC.



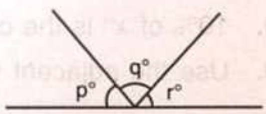
4. Find y in the given figure.



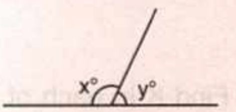
5. In the given figure, find ∠PQR.



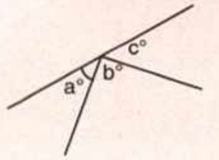
6. In the given figure, $p° = q° = r°$, find each.



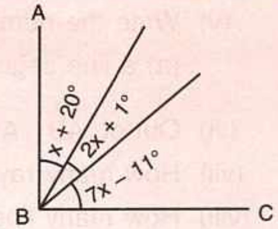
7. In the given figure, if $x = 2y$, find x and y.



8. In the adjoining figure, if $b° = a° + c°$, find b.



9. In the given figure, AB is perpendicular to BC at B.



Find : (i) the value of x,

(ii) the complement of angle x.

10. Write the complement of :

(i) 25° (ii) 90° (iii) $a°$

(iv) $(x + 5)°$ (v) $(30 - a)°$ (vi) $\frac{1}{2}$ of a right angle

(vii) $\frac{1}{3}$ of 180° (viii) 21° 17′

11. Write the supplement of :

(i) 100° (ii) 0° (iii) $x°$

(iv) $(x + 35)°$ (v) $(90 + a + b)°$ (vi) $(110 - x - 2y)°$

(vii) $\frac{1}{5}$ of a right angle (viii) 80° 49′ 25″

12. Are the following pairs of angles complementary ?

(i) 10° and 80° (ii) 37° 28′ and 52° 33′

(iii) $(x + 16)°$ and $(74 - x)°$ (iv) 54° and $\frac{2}{5}$ of a right angle.

13. Are the following pairs of angles supplementary ?

(i) 139° and 39°. (ii) 26° 59′ and 153° 1′.

(iii) $\frac{3}{10}$ of a right angle and $\frac{4}{15}$ of two right angles.

(iv) $2x° + 65°$ and $115° - 2x°$.

14. If $3x + 18°$ and $2x + 25°$ are supplementary, find the value of x.

15. If two complementary angles are in the ratio 1 : 5, find them.

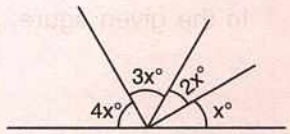
16. If two supplementary angles are in the ratio 2 : 7, find them.

17. Three angles which add upto 180° are in the ratio 2 : 3 : 7. Find them.

18. 20% of an angle is the supplement of 60°. Find the angle.

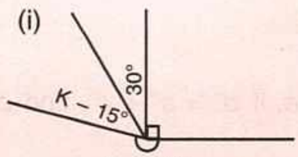
19. 10% of $x°$ is the complement of 40% of $2x°$. Find x.

20. Use the adjacent figure, to find angle x and its supplement.

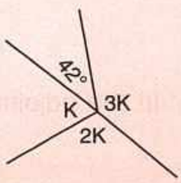


21. Find K in each of the given figures.

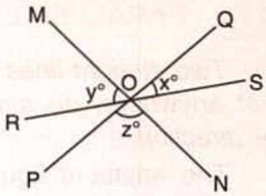
(i)



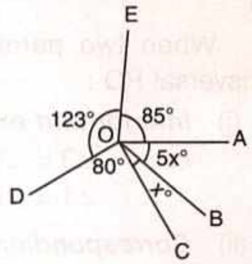
(ii)



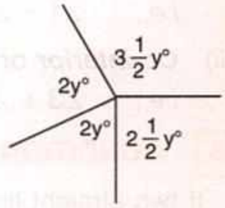
22. In the given figure, lines PQ, MN and RS intersect at O. If $x : y = 1 : 2$ and $z = 90°$, find ∠ROM and ∠POR.



23. In the given figure, find ∠AOB and ∠BOC.



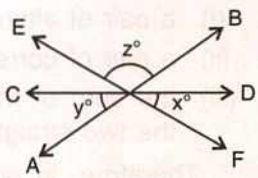
24. Find each angle shown in the figure.



25. *AB, CD* and *EF* are three lines intersecting at the same point.

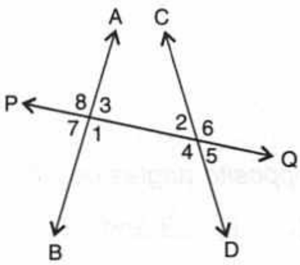
(i) Find x, if $y = 45°$ and $z = 90°$.

(ii) Find a, if $x = 3a$, $y = 5x$ and $z = 6x$.



14.6 CONCEPT OF A TRANSVERSAL

A straight line which cuts two or more given straight lines is called a **transveral**.



In the adjoining figure, PQ cuts straight lines AB and CD, and so it is a transversal.

When a transversal cuts two given straight lines (refer the adjoining figure), the following pairs of angles are formed.

1. **Two pairs of *interior alternate angles* :**

   Angles marked 1 and 2 form one pair of interior alternate angles, while angles marked 3 and 4 form another pair of interior alternate angles.

2. **Two pairs of *exterior alternate angles* :**

   Angles marked 5 and 8 form one pair, while angles marked 6 and 7 form the other pair of exterior alternate angles.

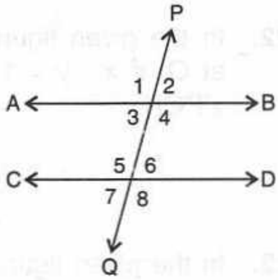
3. **Four pairs of *corresponding angles* :**

   Angles marked 3 and 6, 1 and 5, 8 and 2, 7 and 4 form the four pairs of corresponding angles.

4. **Two pairs of *allied* or *co-interior or conjoined angles* :**

   Angles marked 3 and 2 form one pair and angles marked 1 and 4 form another pair of allied angles.

14.7 PARALLEL LINES



Two straight lines are said to be parallel, if they do not meet anywhere, no matter how long are they produced in any direction.

The adjacent figure shows two parallel lines AB and CD.

When two parallel lines AB and CD are cut by a transversal PQ :

(i) ***Interior and exterior alternate angles are equal*** **:**

i.e., $\angle 3 = \angle 6$ and $\angle 4 = \angle 5$ [Interior alternate angles]

$\angle 1 = \angle 8$ and $\angle 2 = \angle 7$ [Exterior alternate angles]

(ii) ***Corresponding angles are equal*** **:**

i.e., $\angle 1 = \angle 5$; $\angle 2 = \angle 6$; $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$

(iii) ***Co-interior*** **or** ***allied angles are supplementary*** **:**

i.e., $\angle 3 + \angle 5 = 180°$ and $\angle 4 + \angle 6 = 180°$.

14.8 CONDITIONS OF PARALLELISM

If two straight lines are cut by a transversal such that :

(i) a pair of alternate angles are equal, or

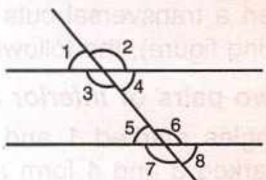
(ii) a pair of corresponding angles are equal, or

(iii) the sum of the interior angles on the same side of the transversal is 180°, then the two straight lines are parallel to each other.

Therefore, in order to prove that the given lines are parallel; show either *alternate angles* are *equal* or, *corresponding angles are equal* or, the *co-interior angles are supplementary.*

EXERCISE 14(B)

In questions 1 and 2, given below, identify the given pairs of angles as *corresponding angles, interior alternate angles, exterior alternate angles, adjacent angles, vertically opposite angles* or *allied angles* :



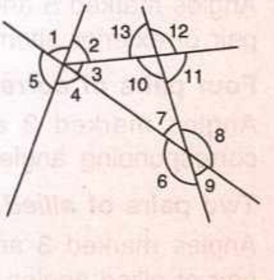
1. (i) $\angle 3$ and $\angle 6$ (ii) $\angle 2$ and $\angle 4$

(iii) $\angle 3$ and $\angle 7$ (iv) $\angle 2$ and $\angle 7$

(v) $\angle 4$ and $\angle 6$ (vi) $\angle 1$ and $\angle 8$

(vii) $\angle 1$ and $\angle 5$ (viii) $\angle 1$ and $\angle 4$

(ix) $\angle 5$ and $\angle 7$



2. (i) $\angle 1$ and $\angle 4$ (ii) $\angle 4$ and $\angle 7$

(iii) $\angle 10$ and $\angle 12$ (iv) $\angle 7$ and $\angle 13$

(v) $\angle 6$ and $\angle 8$ (vi) $\angle 11$ and $\angle 8$

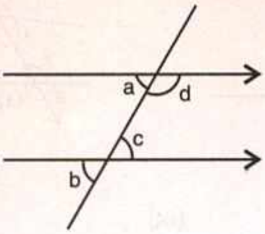
(vii) $\angle 7$ and $\angle 9$ (viii) $\angle 4$ and $\angle 5$

(ix) $\angle 4$ and $\angle 6$ (x) $\angle 6$ and $\angle 7$

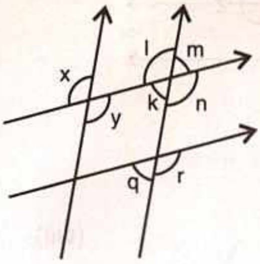
(xi) $\angle 2$ and $\angle 13$.

3. In the following figures, the arrows indicate parallel lines.
State which angles are equal. Give reasons.

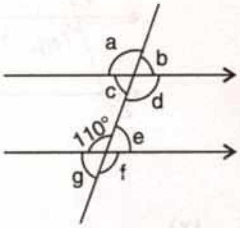
(i)



(ii)

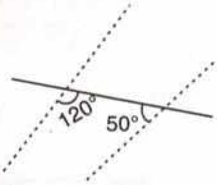


4. In the given figure, find the measure of the unknown angles :

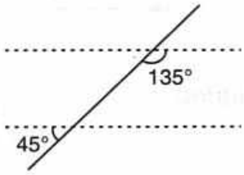


5. Which pair of the dotted line segments, in the following figures, are parallel. Give reason :

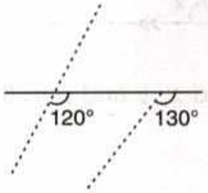
(i)



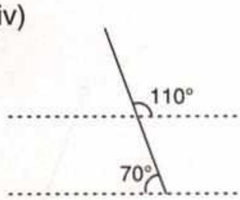
(ii)



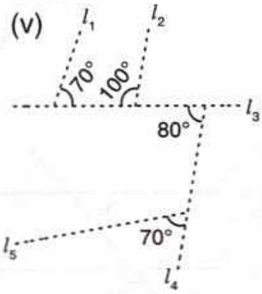
(iii)



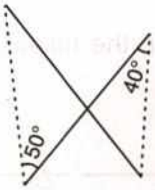
(iv)



(v)

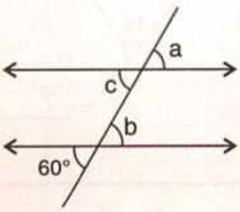


(vi)

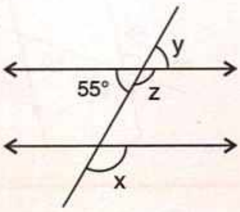


6. In the given figures, the directed lines are parallel to each other. Find the unknown angles.

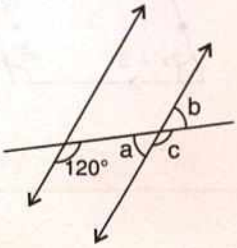
(i)



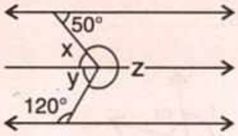
(ii)



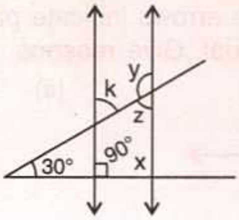
(iii)



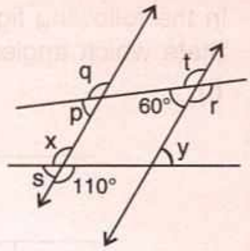
(iv)



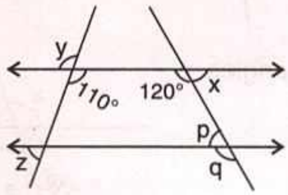
(v)



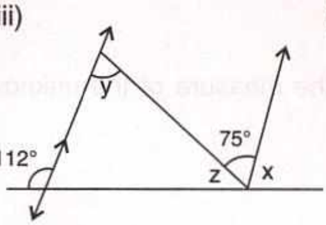
(vi)



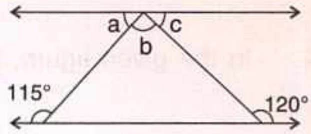
(vii)



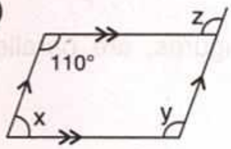
(viii)



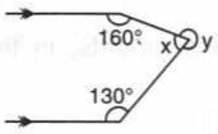
(ix)



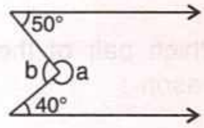
(x)



(xi)

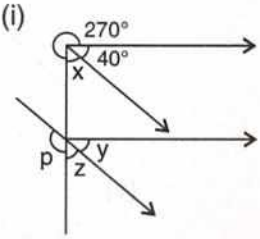


(xii)

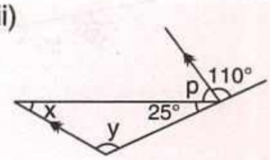


7. Find *x*, *y* and *p* in the given figures :

(i)

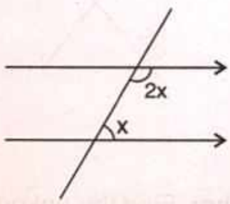


(ii)

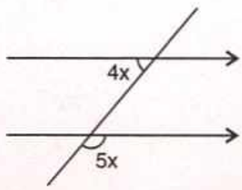


8. Find *x* in the following cases :

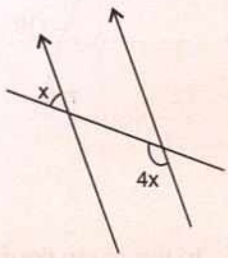
(i)



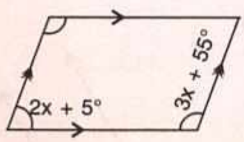
(ii)



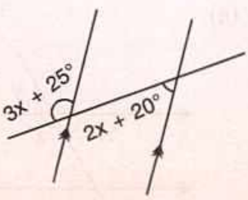
(iii)



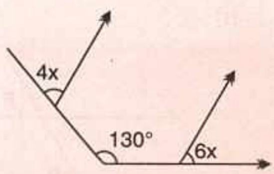
(iv)



(v)



(vi)

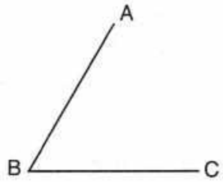


14.9 CONSTRUCTIONS OF ANGLES

(i) ***Construction of an angle congruent** (equal) **to a given angle :***

Let the given angle be angle ABC as shown in Fig. (a) alongside and, we are to construct an angle congruent (equal) to it.

(a)

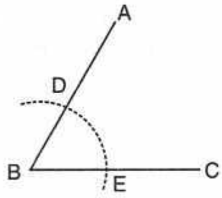


Steps :

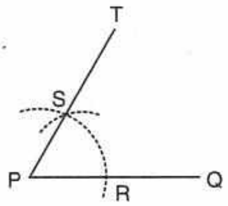
1. With any suitable radius, draw an arc with point B as centre, which meets AB at point D and BC at point E.
2. Draw a line segment PQ of any suitable length as shown in Fig. (c)
3. With the same radius as taken in Step 1, draw one more arc with point P as centre, which meets PQ at point R.
4. With R as centre and radius equal to DE, draw an arc which cuts first arc at point S.
5. Join PS and produce it upto point T.

   ∠TPQ is the required angle such that : **∠TPQ = ∠ABC**

(b)

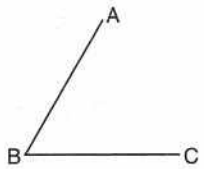


(c)



(ii) ***Bisecting a given angle :***

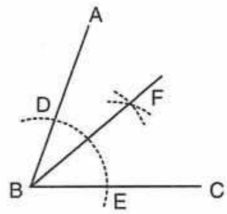
Let the given angle be ∠ABC which is to be bisected, *i.e.,* to be divided into two equal parts.



Steps :

1. With B as centre and any suitable radius, draw an arc which meets AB at point D and BC at point E.
2. With E as centre and radius equal to more than half of DE, draw an arc.
3. With D as centre and with same radius as taken in Step 2, draw another arc which meets the first arc, with centre E, at point F.
4. Join B and F. **Then BF is the required bisector of given angle ABC.**

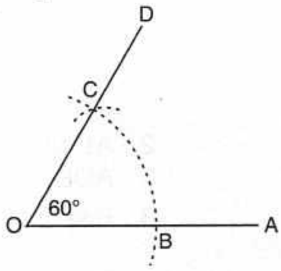
   [Since, BF bisects angle ABC, ∠ABF = ∠FBC = $\frac{1}{2}$ ∠ABC]



(iii) ***Constructing an angle of* 60° :**

Steps :

1. Draw a line segment OA of any suitable length.
2. With O as centre, draw an arc of any size to cut *OA* at B.
3. With B as centre draw another arc of same size to cut the previous arc at C.
4. Join OC and produce it to any point D. Then, **∠DOA = 60°.**

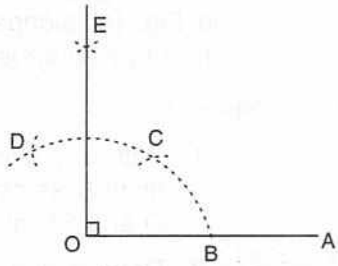


(iv) *Constructing an angle of* 90° :

Let OA be the line and at point O, angle of 90° is to be drawn.

Steps :

1. With O as centre, draw an arc of any suitable radius to cut OA at B.
2. With B as centre and radius as taken in Step 1, draw the arc to cut the previous arc at C.
3. Again with C as centre and with the same radius, draw one more arc to cut the first arc at D.
4. With C and D as centres, draw two arcs of equal radii to cut each other at *E*.
5. Join OE. Then, **∠AOE = 90°.**



(v) *Constructing an angle of* 45° :

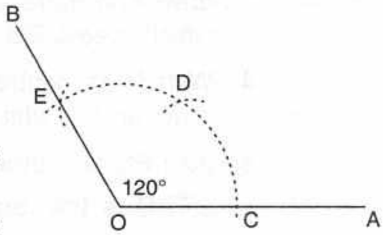
Draw an angle of 90° as above and bisect it. Each angle so obtained will be 45°.

(vi) *Constructing an angle of* 120° :

Steps :

1. Draw a line segment OA of any suitable length.
2. With O as centre, draw an arc to cut OA at C.
3. With C as centre, draw an arc of the same radius (as taken in Step 2) to cut first arc at D.
4. With D as centre, draw one more arc of the same radius which cuts the first arc at E.

   Join OE and produce it upto any point *B*. Then, **∠AOB = 120°**.



(vii) *To construct an angle of* 135° :

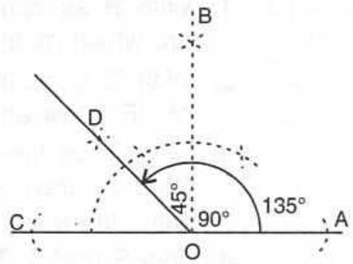
Steps :

1. Draw an angle BOA = 90° at the point O on the given line segment AC.
2. Construct OD to bisect the angle BOC on the other side of OB.

   Since, ∠BOC = ∠BOA = 90°

   ∴ ∠BOD = ∠COD = 45°

   and, **∠AOD** = 90°+ 45° = **135°**

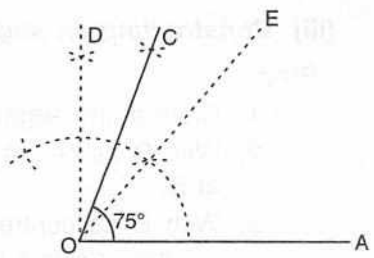


(viii) *To construct an angle of* 75° :

Steps :

1. Draw an angle AOD = 90° at a point O on the line segment OA.
2. At the same point O, draw an angle AOE = 60°.
3. Bisect the ∠DOE, so that

   ∠EOC = ∠DOC = 15°.



In the given figure : **∠AOC** = ∠AOE + ∠EOC = 60° + 15° = **75°.**

Note : Making similar combinations, many other angles can be drawn,

e.g., 105° = 90° + 15°, 165° = 180° − 15°, etc.

14.10 SOME SPECIAL CONSTRUCTIONS

(i) *To construct the bisector of a given line segment :*

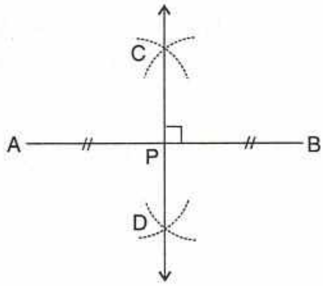
Let the given line segment be AB as shown alongside.

Steps :

1. Taking any suitable radius, which must be more than half of given line segment AB, draw arcs with centres at points A and B.

   These arcs must be drawn on both the sides of AB.

   Let the arcs drawn, intersect each other at points C and D as shown alongside.

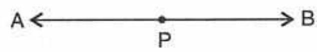


2. Draw a line through the points C and D.

> 1. Line *CD*, as drawn above, bisects the given line segment AB perpendicularly.
>   *i.e.,* if CD cuts AB at point P, then
> $AP = PB = \frac{1}{2}$ AB and $\angle APC = 90°$
> 2. Every line segment has one and only one perpendicular bisector of it.

(ii) *To construct a perpendicular to a line through a point in it*

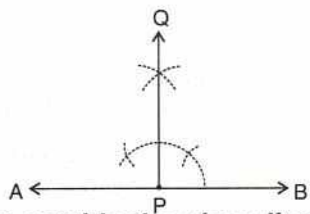
Let the given line be line AB and P be a point in it through which a perpendicular is to be drawn.



Steps :

At point P, in line AB, construct an angle of 90° *i.e.,* draw PQ so that $\angle QPB = \angle QPA = 90°$.

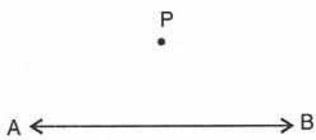
Then, **PQ is the required perpendicular to** the given line AB at point P on it.



(iii) *To construct a perpendicular to a line through a point outside the given line.*

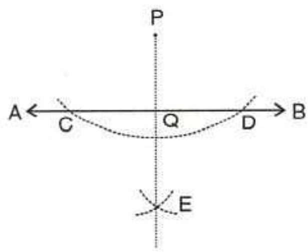
Let AB be the given line and P be the point outside AB.

Required to draw perpendicular to AB from point P.



Steps :

1. Taking P as centre and with a suitable radius, draw an arc which meets AB at points C and D.
2. Taking C and D as centres, draw arcs of equal radii which cut each other at point E, below the line AB.
   [The radii of arcs in this step must be of lengths more than half of CD].
3. Join P and E.



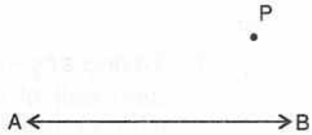
4. Let PE intersects AB at point Q.

   ∴ **PQ is the required perpendicular.**

(iv) ***To construct a line parallel to the given line and passing through the given fixed point.***

Let AB be the given line and P be the given fixed point.

Required to draw a line through the point P and parallel to the given line AB.

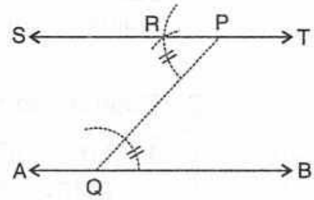


Steps :

1. Mark a point Q in the given line AB.
2. Join P and Q.
3. At point P, draw (copy) angle QPR equal to angle PQB.

   i.e., draw ∠QPR = ∠PQB.
4. Draw line ST through points R and P.

   ∴ **Line ST is the required line through the given point P and parallel to the given line *AB*.**



EXERCISE 14(C)

1. Using ruler and compasses, construct the following angles :

(i) 30° (ii) 15° (iii) 75° (iv) 180°

(v) 165° (vi) 22.5° (vii) 37.5° (viii) 67.5°.

2. Draw ∠ABC = 120°. Bisect the angle using ruler and compasses only. Measure each angle so obtained and check whether the angles obtained on bisecting ∠ABC are equal or not.

3. Draw a line segment PQ = 6 cm. Mark a point A in PQ so that AP = 2 cm. At point A, construct angle QAR = 60°.

4. Draw a line segment AB = 8 cm. Mark a point P in AB so that AP = 5 cm. At P, construct angle APQ = 30°.

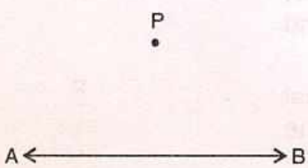
5. Construct an angle of 75° and then bisect it.

6. Draw a line segment of length 6·4 cm. Draw its perpendicular bisector.

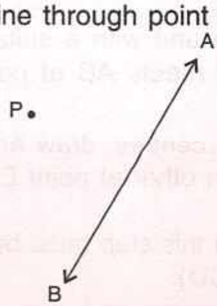
7. Draw a line segment AB = 5·8 cm. Mark a point *P* in AB such that PB = 3·6 cm. At P, draw perpendicular to AB.

8. In each case, given below, draw a line through point P and parallel to AB :

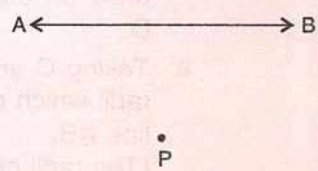
(i)



(ii)

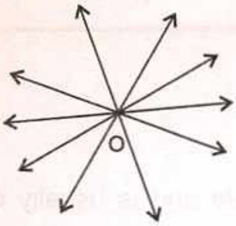


(iii)



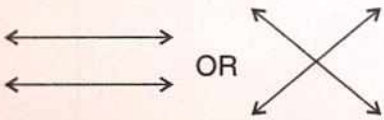
Do you know :

1. Through any given point (say, point O) an infinite number of lines can be drawn :



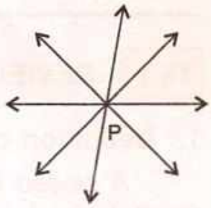
2. Through any two given points (say, points A and B) one and only one straight line can be drawn :

3. If two straight lines are drawn on a plane surface (say, the page of your note-book), the lines are either parallel to each other or intersect each other exactly at one point.



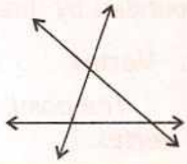
4. When several (three or more than three) lines are drawn in a plane (say, page of your note-book) such that all the lines drawn pass through the same point; the lines are called **concurrent lines**.

The point through which the lines drawn pass, is called the **point of concurrence.**

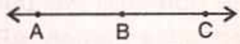


In the given figure, P is the point of concurrence.

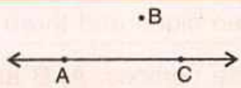
The adjoining figure shows **non-concurrent lines**.



5. If three or more points lie on the same straight line, the points are called **collinear points**.



[A, B and C are collinear points]



[A, B and C are non-collinear points]

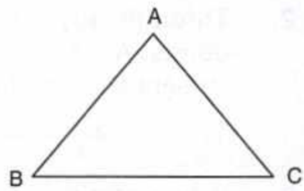
TRIANGLES 15

15.1 REVIEW

1. Definition of a triangle :

A closed figure, having 3 sides, is called ***a triangle*** and is usually denoted by the Greek letter Δ (delta).

The figure, given alongside, shows a triangle ABC (Δ ABC) bounded by three sides AB, BC and CA.



2. Vertex :

The point, where any two sides of a triangle meet, is called a ***vertex.***

Clearly, the given triangle ABC has three vertices, namely : A, B and C.

[Vertices is the plural of vertex]

Triangle ABC has :

1. three non-collinear points A, B and C.
2. three sides namely AB, BC and CA.
3. three angles ∠ABC, ∠BCA and ∠CAB.

Three sides and three angles of a triangle are called its six parts or six elements.

4. Three vertices, A, B and C.

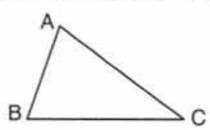
A is vertex opposite to side BC, B is vertex opposite to side AC and C is vertex opposite to side AB.

3. Interior angles :

In Δ ABC (given above), the angles BAC, ABC and ACB are called its interior angles as they lie inside the Δ ABC.

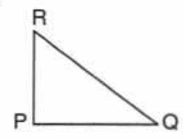
The sum of interior angles of a triangle is always 180°.

(i)



[∠A + ∠B + ∠C = 180°]

(ii)

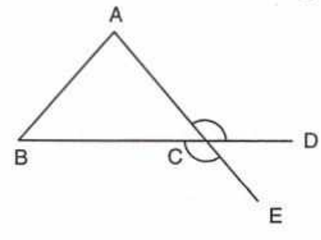


[∠P + ∠Q + ∠R = 180°]

4. Exterior angles :

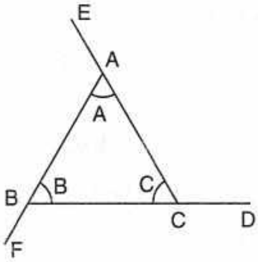
When any side of a triangle is produced the angle so formed, outside the triangle and at its vertex, is called its ***exterior angle.***

For a given triangle ABC, if side BC is produced to the point D, then ∠ACD is its exterior angle. And, if side AC is produced to the point E, then the exterior angle would be ∠BCE.



Thus, at every vertex, two exterior angles can be formed and that these two angles being vertically opposite angles, are always equal.

Also, at each vertex of a triangle, the sum of the exterior angle and its corresponding interior angle is 180°.



In ΔABC, given alongside,

Exterior angle + Interior angle = 180°

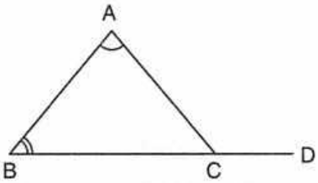
$\Rightarrow$ At vertex A : $\angle BAE + \angle A = 180°$

At vertex B : $\angle CBF + \angle B = 180°$ and

At vertex C : $\angle ACD + \angle C = 180°$

5. Interior opposite angles :

When any side of a triangle is produced, an exterior angle is formed. The two interior angles of this triangle, that are opposite to the exterior angle formed, are called its ***interior opposite angles.***

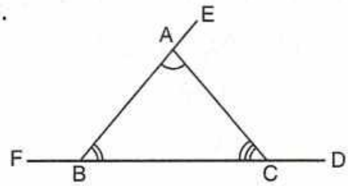


In the given figure, side BC of Δ ABC is produced to the point D, so that the exterior $\angle ACD$ is formed. Then the two interior opposite angles are $\angle BAC$ and $\angle ABC$.

6. Relation between exterior angle and interior opposite angles :

Exterior angle of a triangle is always equal to the sum of its two interior opposite angles.

Thus in the figure, given above, $\angle ACD = \angle BAC + \angle ABC$.



Similarly; in the triangle ABC, drawn alongside,

Exterior angle CAE = $\angle B + \angle C$

and exterior angle ABF = $\angle A + \angle C$.

Example 1 :

(i) Can a triangle have angles 60°, 70° and 70° ?

(ii) Two angles of a triangle are 48° and 73°, find its third angle.

(iii) Three angles of a triangle are $(2x + 20)°$, $(x + 30)°$ and $(2x - 10)$. Find the angles.

Solution :

(i) Since, 60° + 70° + 70° = 200°

$\Rightarrow$ **A triangle can not have angles 60°, 70° and 70°** **(Ans.)**

[**Remember :** Sum of the angles of a triangle is always 180°]

(ii) Sum of two given angles = 48° + 73° = 121°

$\Rightarrow$ **The third angle** = 180° − 121° = 59° **(Ans.)**

(iii) Since, the sum of the interior angles of a triangle = 180°

$\therefore \quad (2x + 20) + (x + 30) + (2x - 10) = 180°$

$\Rightarrow \quad 5x + 40 = 180°$

i.e. $\quad 5x = 180 - 40 = 140$ and $x = \frac{140}{5} = 28$

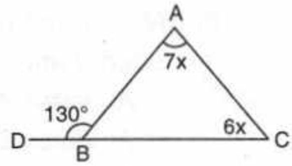
$\therefore$ **Required angles** $= (2x + 20)^\circ, (x + 30)^\circ$ and $(2x - 10)^\circ$

$= (2 \times 28 + 20)^\circ, (28 + 30)^\circ$ and $(2 \times 28 - 10)^\circ$

$=$ **76°, 58°** and **46°** **(Ans.)**

Example 2 :

Use the figure, given alongside, to find the value of :



(i) x,

(ii) $\angle BAC$,

(iii) $\angle ACB$.

Solution :

(i) Since, the exterior angle of a Δ = sum of its two interior opposite angles

$\therefore$ $130^\circ = 7x + 6x$

$\Rightarrow$ $13x = 130^\circ$

$\Rightarrow$ $x = \frac{130^\circ}{13} = 10^\circ$ **(Ans.)**

(ii) $\angle$**BAC** $= 7x = 7 \times 10^\circ =$ **70°** **(Ans.)**

(iii) $\angle$**ACB** $= 6x = 6 \times 10^\circ =$ **60°** **(Ans.)**

EXERCISE 15(A)

1. State, if the triangles are possible with the following angles :

(i) 20°, 70° and 90°

(ii) 40°, 130° and 20°

(iii) 60°, 60° and 50°

(iv) 125°, 40° and 15°

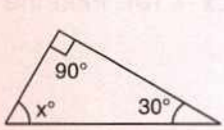
2. If the angles of a triangle are equal, find its angles.

3. In a triangle ABC, $\angle A = 45^\circ$ and $\angle B = 75^\circ$, find $\angle C$.

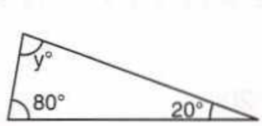
4. In a triangle PQR, $\angle P = 60^\circ$ and $\angle Q = \angle R$, find $\angle R$.

5. Calculate the unknown marked angles in each figure :

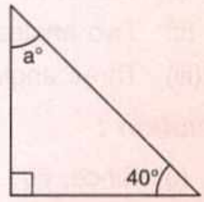
(i)



(ii)

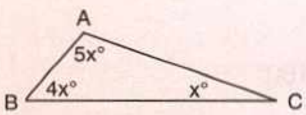


(iii)

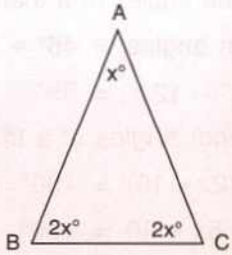


6. Find the value of each angle in the given figures :

(i)

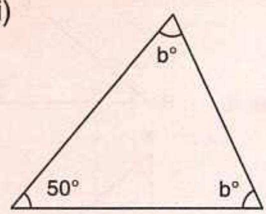


(ii)

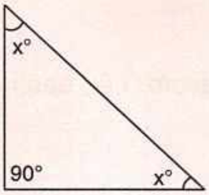


7. Find the unknown marked angles in the given figures :

(i)



(ii)



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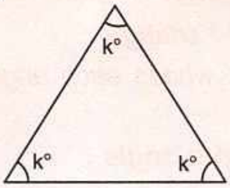
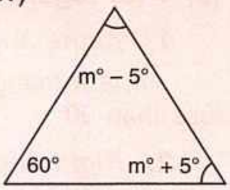


(iv)

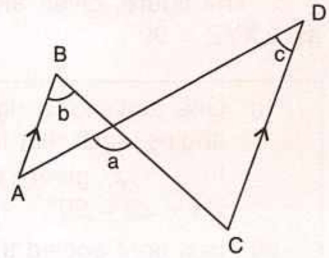


8. In the given figure, show that : $\angle a = \angle b + \angle c$.

(i) If $\angle b = 60°$ and $\angle c = 50°$, find $\angle a$.

(ii) If $\angle a = 100°$ and $\angle b = 55°$, find $\angle c$.

(iii) If $\angle a = 108°$ and $\angle c = 48°$, find $\angle b$.



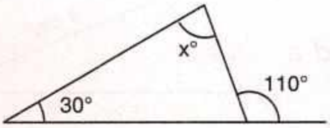
9. Calculate the angles of a triangle, if they are in the ratio 4 : 5 : 6.

10. One angle of a triangle is 60°. The other two angles are in the ratio of 5 : 7. Find the two angles.

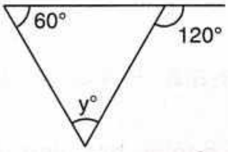
11. One angle of a triangle is 61° and the other two angles are in the ratio $1\frac{1}{2} : 1\frac{1}{3}$. Find these angles.

12. Find the unknown marked angles in the given figures.

(i)



(ii)



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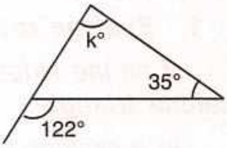

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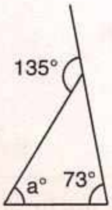

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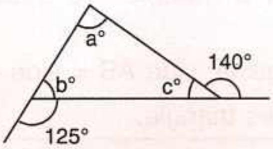

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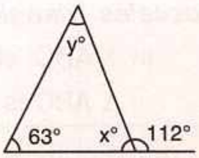
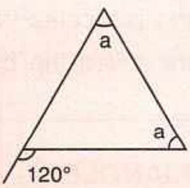


(vii)



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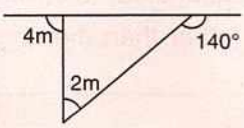

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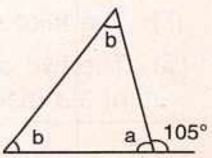
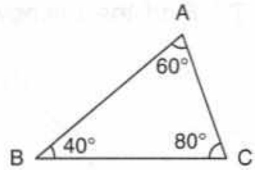


15.2 CLASSIFICATION OF TRIANGLES

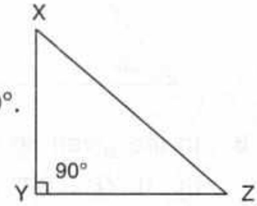
(a) With regard to their angles :

1. ***Acute angled triangle :***



It is a triangle, whose each angle is acute, i.e., each angle is less than 90°.

2. ***Right angled triangle :***



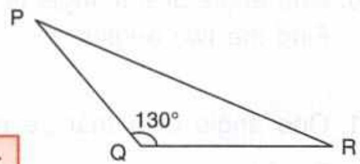
It is a triangle, whose one angle is a right angle, i.e., equal to 90°.

The figure, given alongside, shows a right angled triangle XYZ as ∠XYZ = 90°.

(i) One angle of a right angled triangle is 90° and the other two angles of it are acute angles, such that their sum is always 90°.
In Δ *xyz*, given above, ∠*y* = 90° and each of ∠*x* and ∠*z* is acute such that ∠*x* + ∠*z* = 90°.

(ii) In a right angled triangle, the side opposite to the right angle is largest of all its sides and is called the **hypotenuse.** In given right angled Δ XYZ, side XZ is the hypotenuse.

3. ***Obtuse angled triangle :***

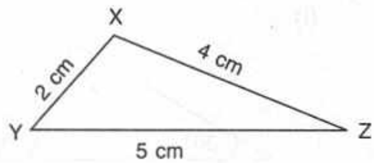


If one angle of a triangle is more than 90°, it is called an obtuse angled triangle.

In case of an obtuse angled triangle, each of the other two angles is always acute and their sum is less than 90°.

(b) With regard to their sides :

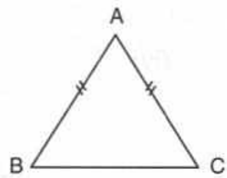
1. ***Scalene triangle :***



If all the sides of a triangle are unequal, it is called a ***scalene triangle.***

In a scalene triangle, all its angles are also unequal.

2. ***Isosceles triangle :***



If atleast two sides of a triangle are equal, it is called an ***isosceles triangle.***

In Δ ABC, shown alongside, side AB = side AC.

∴ Δ ABC is an isosceles triangle.

(i) The angle contained by equal sides *i.e.* ∠BAC is called the **vertical angle** or the **angle of vertex**.

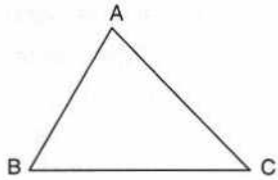
(ii) The third side (the unequal side) is called the **base** of the isosceles triangle.

(iii) The two other angles (other than the angle of vertex) are called the **base angles** of the triangle.

15.3 IMPORTANT PROPERTIES OF AN ISOSCELES TRIANGLE

The base angles, *i.e.*, the angles opposite to equal sides of an isosceles triangle are always equal.

In given triangle ABC,



(i) if side AB = side BC, then angle opposite to AB = angle opposite to BC, *i.e.,* $\angle C = \angle A$.

(ii) if side BC = side AC, then angle opposite to BC = angle opposite to AC, *i.e.* $\angle A = \angle B$ and so on.

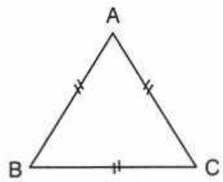
Conversely **:** If any two angles of a triangle are equal, the sides opposite to these angles are also equal, *i.e.,* the triangle is isosceles.

Thus in Δ ABC,

(i) if $\angle B = \angle C \Rightarrow$ side opposite to $\angle B$ = side opposite to $\angle C \Rightarrow$ side AC = side AB.

(ii) if $\angle A = \angle B \Rightarrow$ side BC = side AC and so on.

3. *Equilateral triangle :*



If all the three sides of a triangle are equal, it is called an ***equilateral triangle****.*

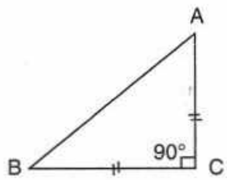
In the given figure, Δ ABC is equilateral, because AB = BC = CA

Also, all the angles of an equilateral triangle are equal to each other and so each angle = 60° [$\because 60° + 60° + 60° = 180°$]

Since, all the angles of an equilateral triangle are equal, it is also known as **equiangular triangle.**

> *An equilateral triangle is always an isosceles triangle, but its converse is not always true.*

4. *Isosceles right angled triangle :*



If one angle of an isosceles triangle is 90°, it is called an Isosceles right angled triangle.

In the given figure, Δ ABC is an isosceles right angled triangle, because : $\angle ACB = 90°$ and AC = BC.

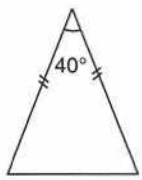
Here, the base is AB, the vertex is C and the base angles are $\angle BAC$ and $\angle ABC$, which are equal.

Since, the sum of the angles of a triangle = 180°

$\therefore \angle ABC = \angle BAC = 45°$ [45° + 45° + 90° = 180°]

Example 3 :

In the given isosceles triangle, find the base angles.



Solution :

Let each of the base angles be x.

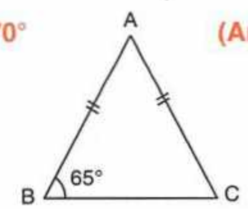
$\therefore \quad x + x + 40° = 180°$ [Sum of the angles of a Δ = 180°]

$\Rightarrow \quad 2x + 40° = 180°$

$\Rightarrow \quad 2x = 180° - 40°$

$\Rightarrow \quad x = \frac{140°}{2} = 70°$ $\therefore$ **Each base angle is 70°** **(Ans.)**

Example 4 :



One base angle of an isosceles triangle is 65°.

Find its angle of vertex.

Solution :

Since, the base angles of an isosceles triangle are equal.

$\therefore$ Other base angle is also 65°.

Let the angle of vertex be x.

$\therefore \quad x + 65° + 65° = 180°$ [Sum of the angles of a Δ = 180°]

$\Rightarrow \quad x + 130° = 180°$

$\Rightarrow \quad x = 180° - 130° = 50°$ **(Ans.)**

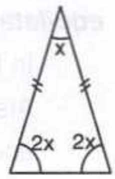
Example 5 :

If one base angle of an isosceles triangle is double of the vertical angle, find all its angles.

Solution :

Draw an isosceles triangle in which mark the vertical angle as x.

$\therefore$ The two base angles will be $2x$ each.



Hence, $x + 2x + 2x = 180°$

$\Rightarrow \quad 5x = 180°$ and $x = \frac{180°}{5} = 36°$

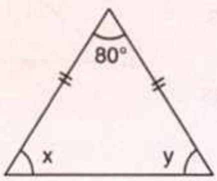
$\Rightarrow \quad 2x = 2 \times 36° = 72°$

$\therefore$ **Vertical angle = 36° and each base angle = 72°** **(Ans.)**

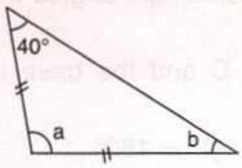
EXERCISE 15(B)

1. Find the unknown angles in the given figures :

(i)



(ii)



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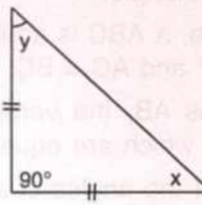
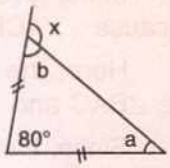
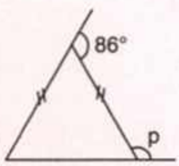


(iv)



(v)



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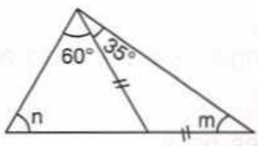

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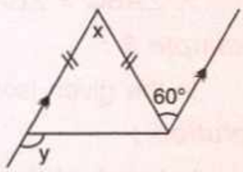
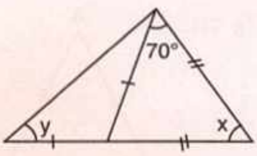
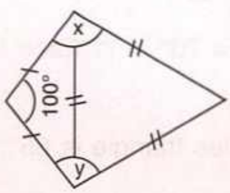


2. Apply the properties of isosceles and equilateral triangles to find the unknown angles in the given figures :

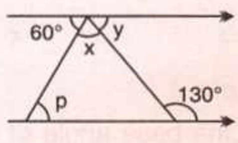
(i)



(ii)



(iii)



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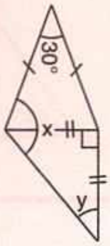
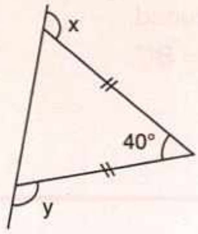
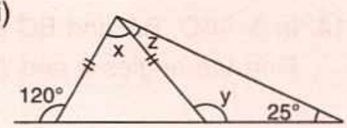


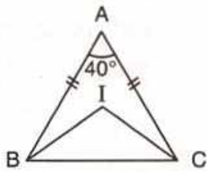
(v)



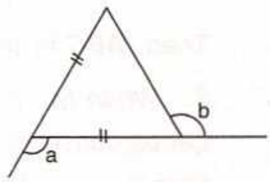
(vi)



3. The angle of vertex of an isosceles triangle is 100°. Find its base angles.
4. One of the base angles of an isosceles triangle is 52°. Find its angle of vertex.
5. In an isosceles triangle, each base angle is four times of its vertical angle. Find all the angles of the triangle.
6. The vertical angle of an isosceles triangle is 15° more than each of its base angles. Find each angle of the triangle.
7. The base angle of an isosceles triangle is 15° more than its vertical angle. Find its each angle.
8. The vertical angle of an isosceles triangle is three times the sum of its base angles. Find each angle.
9. The ratio between a base angle and the vertical angle of an isosceles triangle is 1 : 4. Find each angle of the triangle.
10. In the given figure, BI is the bisector of ∠ABC and CI is the bisector of ∠ACB. Find ∠BIC.

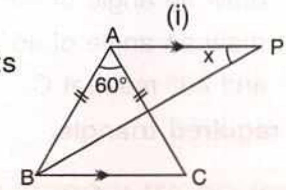


11. In the given figure, express *a* in terms of *b*.

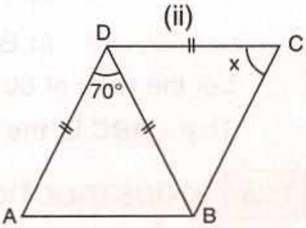


12. (a) In Figure (i) BP bisects ∠ABC and AB = AC. Find *x*.
    (b) Find x in Figure (ii).
    Given : DA = DB = DC, BD bisects ∠ABC and ∠ADB = 70°.

(i)

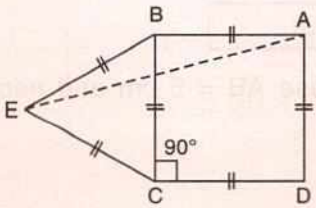


(ii)

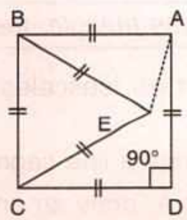


13. In each figure, given below, ABCD is a square and Δ BEC is an equilateral triangle.

(i)

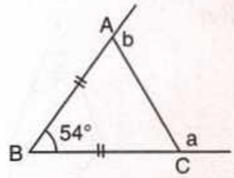


(ii)



Find, in each case : (i) ∠ABE (ii) ∠BAE

14. In Δ ABC, BA and BC are produced. Find the angles a and b, if AB = BC.

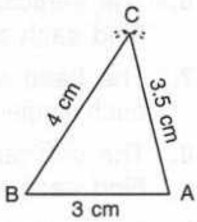


15.4 CONSTRUCTION OF TRIANGLES

1. *When the lengths of three sides are given :*

Let us construct a Δ ABC, such that AB = 3 cm, BC = 4 cm and CA = 3·5 cm.

Steps :
1. Draw a line segment *AB* = 3 cm.
2. With A as centre, draw an arc of radius 3·5 cm and with B as centre draw another arc with radius 4 cm. Let these arcs meet at C.
3. Join BC and AC.

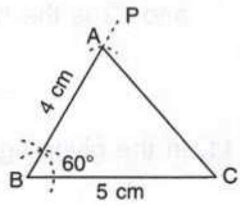


Then, **triangle ABC so obtained is the required triangle.**

2. *When the lengths of two sides and the included angle are given :*

Let us construct a Δ ABC, such that AB = 4 cm, BC = 5 cm and $\angle$ABC = 60°.

Steps :
1. Draw a line segment BC = 5 cm.
2. At B, draw an angle PBC = 60°.
3. With B as centre, draw an arc of 4 cm radius, which cuts PB at *A*. Join AC.



Then, ***ABC* is the required triangle.**

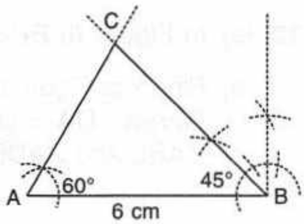
3. *When two angles and the included side are given :*

Let us construct a Δ ABC, such that AB = 6 cm, $\angle$A = 60° and $\angle$B = 45°.

Steps :
1. Draw a line segment AB = 6 cm.
2. At A, draw an angle of 60°.
3. At B, draw an angle of 45°.

Let the lines of 60° and 45° meet at C.

Then, ***ABC* is the required triangle.**



15.5 CONSTRUCTION OF AN ISOSCELES TRIANGLE

1. *When base and one of the base angles are given :*

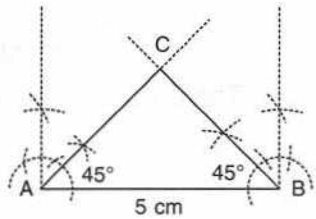
In an isosceles triangle, the base angles are equal.

Let us construct an isosceles Δ ABC such that, base AB = 5 cm and each base angle = 45°.

Steps :
1. Draw a line segment AB = 5 cm.
2. At A, draw an angle of 45°.

3. At B also, draw an angle of 45°. Let these 45° lines meet at C.

Then, ***ABC* is the required triangle.**



2. *When one of the equal sides and the vertex angle are given :*

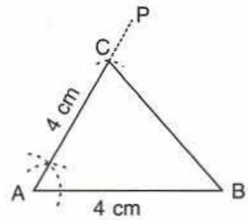
Vertex angle is the angle between two equal sides.

Let us construct an isosceles triangle ABC such that :
AB = AC = 4 cm and ∠BAC = 60°.

Steps : 1. Draw a line segment AB = 4 cm.

2. At A, draw AP so that angle BAP = 60°.

3. With A as centre draw an arc of 4 cm radius, which cuts AP at C. Join C and *B*.



Then, **ABC is the required isosceles triangle.**

15.6 CONSTRUCTION OF AN EQUILATERAL TRIANGLE

An equilateral triangle can be drawn when one of its sides is given.

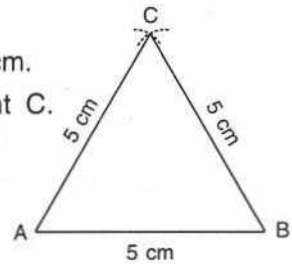
Let us construct an equilateral Δ ABC with each side equal to 5 cm.

Steps : 1. Draw a line AB = 5 cm.

2. With A as centre, draw an arc of radius 5 cm.

3. With B as centre, draw another arc of radius 5 cm.
Let, the two arcs intersect each other at point C. Join AC and BC.

Then, **ABC is the required triangle.**



15.7 CONSTRUCTION OF A RIGHT ANGLED TRIANGLE

1. *When the lengths of the sides, containing the right angle, are given :*

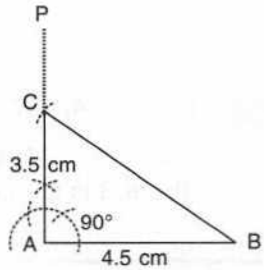
Let us construct a right angled Δ ABC such that,

AB = 4·5 cm, AC = 3·5 cm and ∠A = 90°.

Steps : 1. Draw a line segment AB = 4·5 cm.

2. At A, draw AP so that angle PAB = 90°.

3. With A as centre, draw an arc of radius 3·5 cm which cuts AP at point C. Join BC.



Then, ***ABC* is the required triangle.**

2. *When the lengths of one side and the hypotenuse are given :*

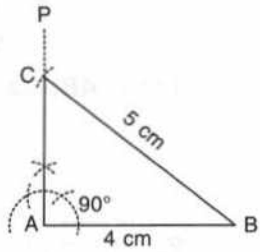
Let us construct a right angled Δ ABC such that AB = 4 cm, ∠A = 90° and BC (hypotenuse) = 5 cm.

Steps : 1. Draw a line segment AB = 4 cm.

2. At A, draw AP so that angle PAB = 90°.

3. With B as centre and radius = 5 cm, draw an arc which cuts AP at point C. Join BC.

Then, **ABC is the required triangle.**



15.8 CIRCUMCIRCLE AND INCIRCLE

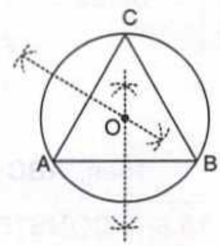
1. **Circumcircle :** *If a circle passes through all the three vertices of a triangle, it is called the* ***circumcircle*** *of the triangle.* Its centre is called **circumcentre** and its radius is called **circumradius.**

To construct the circumcircle of a triangle :

Steps : 1. Construct the ΔABC with the given measurements.

2. Draw the perpendicular bisectors of any two sides of the triangle.

   Here, the perpendicular bisectors of the sides AB and AC are drawn. These bisectors intersect each other at point O.



3. Taking O as centre and OA or OB or OC as radius, draw a circle.

   The circle so drawn passes through the vertices A, B and C.

[Here, the **centre O** is the **circumcentre** of Δ ABC, whereas **OA** = **OB** = **OC** = its **circumradius**].

2. **Incircle :** *If a circle is drawn, inside a triangle, such that it touches all the three sides of the triangle, it is called the* ***incircle*** *of that triangle.*

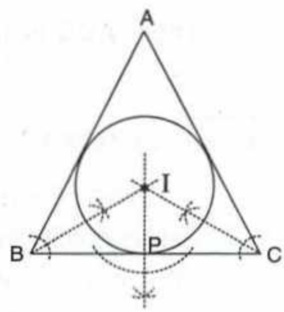
The centre of this circle is called the **incentre** of the triangle.

To construct the incircle of a triangle :

Steps : 1. Construct the Δ *ABC* according to the given measurements.

2. Bisect any two of its angles. Let these two bisectors meet at *I*.

3. From *I*, draw a perpendicular on *BC*. This perpendicular meets *BC* at point *P*.



4. With *I* as centre and *IP* as radius, draw a circle.

   The circle so drawn will touch the sides *BC*, *AB* and *AC*.

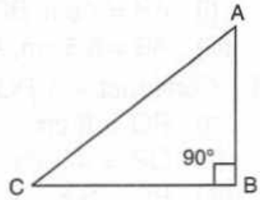
[Here, I is the **incentre** of the triangle].

EXERCISE 15(C)

1. Construct a Δ ABC such that :
   (i) AB = 6 cm, BC = 4 cm and CA = 5·5 cm (ii) CB = 6·5 cm, CA = 4·2 cm and BA = 5·1 cm
   (iii) BC = 4 cm, AC = 5 cm and AB = 3·5 cm
2. Construct a Δ ABC such that :
   (i) AB = 7 cm, BC = 5 cm and ∠ABC = 60° (ii) BC = 6 cm, AC = 5·7 cm and ∠ACB = 75°
   (iii) AB = 6·5 cm, AC = 5·8 cm and ∠A = 45°
3. Construct a Δ PQR such that :
   (i) PQ = 6 cm, ∠Q = 60° and ∠P = 45°. Measure ∠R.
   (ii) QR = 4·4 cm, ∠R = 30° and ∠Q = 75°. Measure PQ and PR.
   (iii) PR = 5·8 cm, ∠P = 60° and ∠R = 45°. Measure ∠Q and verify it by calculations.
4. Construct an isosceles Δ ABC such that :
   (i) base BC = 4 cm and base angle = 30° (ii) base AB = 6·2 cm and base angle = 45°
   (iii) base AC = 5 cm and base angle = 75°. Measure the other two sides of the triangle.
5. Construct an isosceles Δ ABC such that :
   (i) AB = AC = 6·5 cm and ∠A = 60°
   (ii) One of the equal sides = 6 cm and vertex angle = 45°. Measure the base angles.
   (iii) BC = AB = 5·8 cm and ∠B = 30°. Measure ∠A and ∠C.
6. Construct an equilateral Δ ABC such that :
   (i) AB = 5 cm. Draw the perpendicular bisectors of BC and AC. Let P be the point of intersection of these two bisectors. Measure PA, PB and PC.
   (ii) Each side is 6 cm.
7. (i) Construct a Δ ABC such that AB = 6 cm, BC = 4·5 cm and AC = 5·5 cm. Construct a circumcircle of this triangle.
   (ii) Construct an isosceles Δ PQR such that PQ = PR = 6·5 cm and ∠PQR = 75°. Using ruler and compasses only construct a circumcircle to this triangle.
   (iii) Construct an equilateral triangle ABC such that its one side = 5·5 cm. Construct a circumcircle to this triangle.
8. (i) Construct a Δ ABC such that AB = 6 cm, BC = 5·6 cm and CA = 6·5 cm. Inscribe a circle to this triangle and measure its radius.
   (ii) Construct an isosceles Δ MNP such that base MN = 5·8 cm, base angle MNP = 30°. Construct an incircle to this triangle and measure its radius.
   (iii) Construct an equilateral Δ DEF whose one side is 5·5 cm. Construct an incircle to this triangle.
   (iv) Construct a Δ PQR such that PQ = 6 cm, ∠QPR = 45° and angle PQR = 60°. Locate its incentre and then draw its incircle.

PYTHAGORAS THEOREM 16

16.1 INTRODUCTION



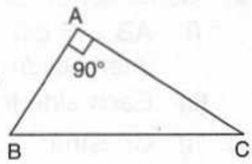
In a right-angled triangle, the side opposite to the angle of 90° is called hypotenuse. It is the largest side of the triangle.

The adjoining figure shows a right-angled triangle ABC in which angle ABC is 90°. The side opposite to angle ABC (∠ABC = 90°) is the hypotenuse *i.e.* side AC is hypotenuse and is the largest side of triangle ABC.

An **Indian Mathematician, Buddhayan** and then a **Greek Mathematician, Pythagoras** developed the relation between the square on hypotenuse of a right-angled triangle and the sum of the squares on the remaining two sides of the triangle. This relationship is known as **Pythagoras theorem**.

16.2 PYTHAGORAS THEOREM

In a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



The adjoining figure shows a triangle ABC with angle BAC = 90°. The side opposite to angle BAC is BC. Therefore, BC is hypotenuse whereas AB and AC are the remaining two sides.

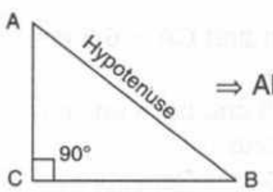
According to Pythagoras theorem :

Square on the hypotenuse = Sum of the squares on the remaining two sides

$\Rightarrow \qquad BC^2 = AB^2 + AC^2$

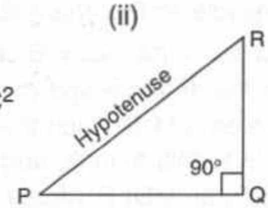
The following figures will make this concept more clear :

(i)



$\Rightarrow AB^2 = AC^2 + BC^2$

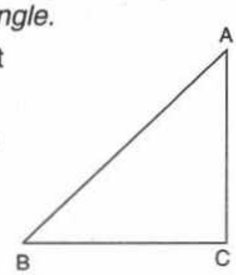
(ii)



$\Rightarrow PR^2 = PQ^2 + QR^2$

16.3 CONVERSE OF PYTHAGORAS THEOREM

In a triangle, if the square of one side (largest side) is equal to the sum of the squares on the remaining two sides, then the angle opposite to the first side (largest side) is a right angle and so the triangle under consideration is a right-angled triangle.



The adjoining figure shows a triangle in which AB is the largest side and

the square on AB = Sum of the squares on the sides AC and BC.

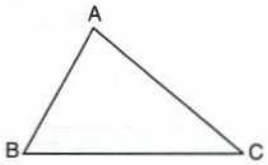
i.e. $AB^2 = AC^2 + BC^2$

⇒ angle opposite to largest side AB is 90°

⇒ ∠ACB = 90° and so the triangle ABC is a right-angled triangle.

Make the following figures clear :

(i)

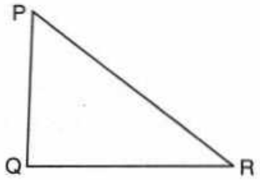


If $BC^2 = AB^2 + AC^2$

$\Rightarrow \angle A = 90°$

and, so the given triangle ABC is a right-angled triangle.

(ii)



If $PR^2 = PQ^2 + QR^2$

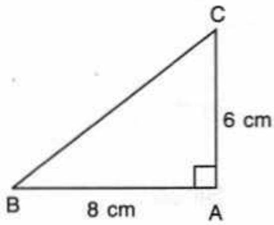
$\Rightarrow \angle Q = 90°$

and, so the given triangle PQR is a right-angled triangle.

Example 1 :

Triangle ABC is right-angled at vertex A. Calculate the length of side BC, if AB = 8 cm and AC = 6 cm.

Solution :



The given triangle is right-angled at vertex A means angle A = 90° and so BC is hypotenuse.

According to Pythagoras theorem :

$BC^2 = AB^2 + AC^2$

$= 8^2 + 6^2 = 64 + 36 = 100$

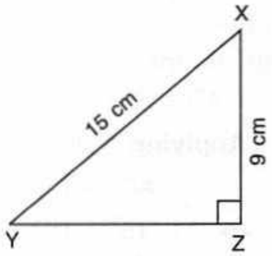
$\therefore$ **BC** = $\sqrt{100}$ cm = **10 cm** **(Ans.)**

Example 2 :

The triangle XYZ is right-angled at vertex Z. Calculate the length of YZ, if :

XY = 15 cm and XZ = 9 cm.

Solution :



Given $\angle Z = 90°$

$\Rightarrow$ side XY is the hypotenuse.

According to Pythagoras theorem :

$XY^2 = XZ^2 + YZ^2$

$\Rightarrow 15^2 = 9^2 + YZ^2$

$\Rightarrow 225 = 81 + YZ^2$

$\Rightarrow 225 - 81 = YZ^2$

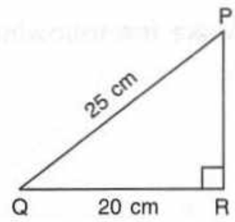
$\Rightarrow 144 = YZ^2$

i.e. **YZ** = $\sqrt{144}$ cm = **12 cm** **(Ans.)**

Example 3 :

Triangle PQR is right-angled at vertex R. Calculate the length of side PR, if PQ = 25 cm and QR = 20 cm.

Solution :



According to Pythagoras theorem :

$PR^2 + QR^2 = PQ^2$

$\Rightarrow \quad PR^2 + 20^2 = 25^2$

$\Rightarrow \quad PR^2 = 625 - 400$

$= 225$

$\Rightarrow \quad$ **PR** $= \sqrt{225}$ cm = **15 cm** **(Ans.)**

Example 4 :

The sides of a triangle are 20 cm, 9 cm and 12 cm. Is this triangle a right-angled triangle.

Solution :

The given triangle will be a right-angled triangle if square on its largest side is equal to the sum of the squares on the other two sides.

i.e. if $(20)^2 = (9)^2 + (12)^2$

$\because \quad (20)^2 = 400$

and, $(9)^2 + (12)^2 = 81 + 144$

$= 225$

Since, $400 \neq 225$

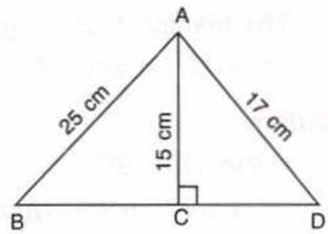
$\Rightarrow \quad (20)^2 \neq (9)^2 + (12)^2$

And, so **the triangle is not a right-angled triangle.** **(Ans.)**

Example 5 :

In the given figure, angle ACB = angle ACD = 90°, AB = 25 cm, AD = 17 cm and AC = 15 cm. Find :

(i) BC (ii) CD (iii) BD



Solution :

(i) In right-angled triangle ABC,
AB = 25 cm and AC = 15 cm

Applying Pythagoras theorem, we get :

$AC^2 + BC^2 = AB^2$

$\Rightarrow \quad 15^2 + BC^2 = 25^2$ *i.e.* $225 + BC^2 = 625$

$\Rightarrow \quad BC^2 = 625 - 225 = 400$

$\therefore \quad$ **BC** $= \sqrt{400}$ cm = **20 cm** **(Ans.)**

(ii) In right-angled triangle ACD,
AD = 17 cm and AC = 15 cm

Applying Pythagoras theorem, we get :

$AC^2 + CD^2 = AD^2$

$\Rightarrow \quad 15^2 + CD^2 = 17^2$ *i.e.* $225 + CD^2 = 289$

$\Rightarrow$ $CD^2 = 289 - 225 = 64$

$\therefore$ **CD** $= \sqrt{64}$ cm = **8 cm** **(Ans.)**

(iii) **BD** = BC + CD

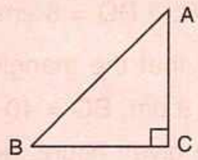
= 20 cm + 8 cm = **28 cm** **(Ans.)**

Extra information :

Consider a triangle ABC in which side AB is the largest side. Then, there can be the following three possible cases.

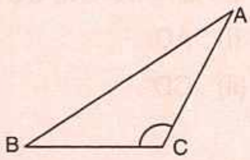
1. $AB^2 = BC^2 + AC^2$

$\Rightarrow \Delta ABC$ is a right-angled triangle with $\angle C = 90°$.



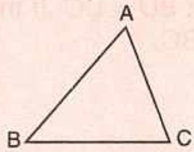
2. $AB^2 > BC^2 + AC^2$

$\Rightarrow \Delta ABC$ is an obtuse-angled triangle with $\angle C > 90°$.



3. $AB^2 < BC^2 + AC^2$

$\Rightarrow \Delta ABC$ is an acute-angled triangle with $\angle C < 90°$.

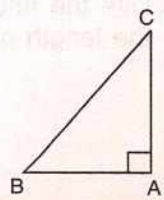


EXERCISE 16

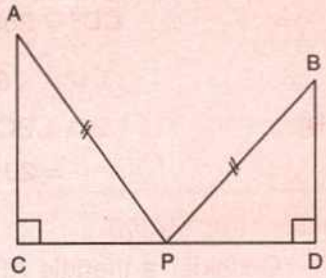
1. Triangle ABC is right-angled at vertex A. Calculate the length of BC, if AB = 18 cm and AC = 24 cm.
2. Triangle XYZ is right-angled at vertex Z. Calculate the length of YZ, if XY = 13 cm and XZ = 12 cm.
3. Triangle PQR is right-angled at vertex R. Calculate the length of PR, if : PQ = 34 cm and QR = 33·6 cm.
4. The sides of a certain triangle are given below. Find, which of them is right-triangle

   (i) 16 cm, 20 cm and 12 cm (ii) 6 m, 9 m and 13 m

5. In the adjoining figure, angle BAC = 90°, AC = 400 m and AB = 300 m. Find the length of BC



6. In the given figures, angle ACP = ∠BDP = 90°, AC = 12 m, BD = 9 m and PA = PB = 15 m. Find :



(i) CP

(ii) PD

(iii) CD

7. In triangle PQR, angle Q = 90°, find :

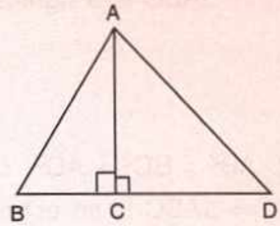
(i) PR, if PQ = 8 cm and QR = 6 cm (ii) PQ, if PR = 34 cm and QR = 30 cm

8. Show that the triangle ABC is a right-angled triangle; if :

AB = 9 cm, BC = 40 cm and AC = 41 cm.

9. In the given figure, angle ACB = 90° = angle ACD.

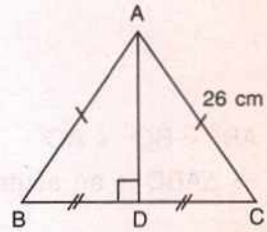
If AB = 10 cm, BC = 6 cm and AD = 17 cm, find :



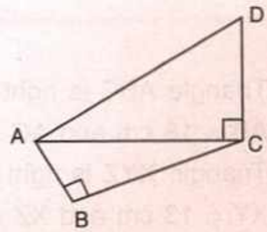
(i) AC

(ii) CD

10. In the given figure, angle ADB = 90°, AC = AB = 26 cm and BD = DC. If the length of AD = 24 cm; find the length of BC.



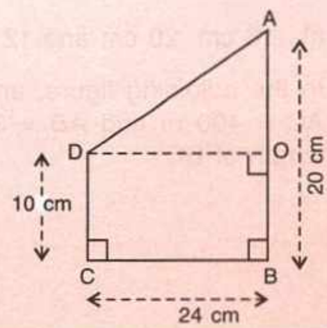
11. In the given figure, AD = 13 cm, BC = 12 cm, AB = 3 cm and angle ACD = angle ABC = 90°. Find the length of DC.



12. A ladder, 6·5 m long, rests against a vertical wall. If the foot of the ladder is 2·5 m from the foot of the wall, find upto how much height does the ladder reach ?

13. A boy first goes 5 m due north and then 12 m due east. Find the distance between the initial and the final positions of the boy.

14. Use the information given in the adjoining figure to find the length of AD.



SYMMETRY 17

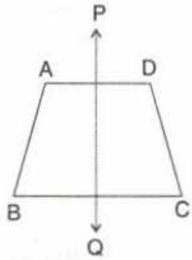
(Including Reflection and Rotation)

17.1 INTRODUCTION

A geometrical figure is said to be symmetric about a line in it, if

(i) this line divides the figure into two identical parts

(ii) on folding the figure about this line, the two parts of the figure exactly coincide.



The adjoining figure shows a quadrilateral ABCD and a line PQ in it which divides the quadrilateral ABCD into two identical parts. If the figure is folded about the line PQ and the two parts of the figure exacity coincide, *i.e.,* A and D coincide, B and C coincide, AB and DC coincide and vice-versa, then the whole figure is said to be *symmetric* about the line PQ.

If the figure is symmetric about a line in it, the line is said to be a **line of symmetry** or an **axis of symmetry.** Thus, in the figure ABCD, discussed above, line PQ is the line of symmetry of ABCD.

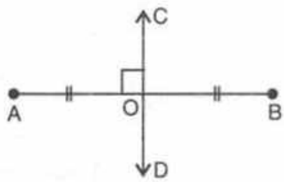
Designers of clothing, jewellery, etc. use the idea of symmetry to make things look gorgeous. Human body and leaves of the plants also exhibit symmetry.

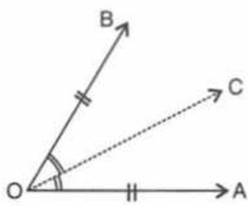
17.2 LINES OF SYMMETRY OF GIVEN GEOMETRICAL FIGURES

It is not necessary that every figure under consideration will definitely have a line of symmetry. If we consider different types of figures, we find :

1. A line segment is symmetrical about its perpendicular bisector.

   Here, the line segment AB is symmetrical about its perpendicular bisector COD.





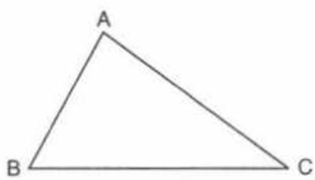
2. A given angle with equal arms is symmetrical about the bisector of the angle.

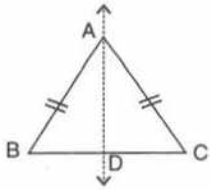
   In the given figure, in angle AOB, OA = OB and OC is the bisector of angle AOB.

   ∴ Angle AOB is symmetrical about line OC which is the bisector of angle AOB.

3. A **scalene triangle** has **no line of symmetry :**

   We can not have a line of symmetry in a scalene triangle. In whatever manner we fold the scalene triangle ABC, the two parts of the figure do not coincide.



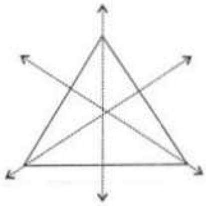


4. An **isosceles triangle** has only **one line of symmetry.**

   The bisector of the angle of vertex which is also the perpendicular bisector of its base.

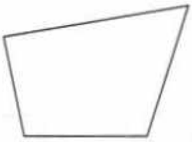
5. An **equilateral triangle** has **three lines of symmetry.**

   The ***bisectors of the angle of vertices*** which are also the ***perpendicular bisectors of*** its ***sides.***



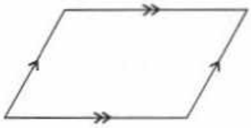
6. Line(s) of symmetry of different types of quadrilaterals are shown below by dotted lines :

(i)



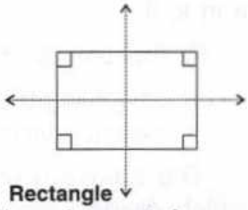
[No line of symmetry]

(ii)



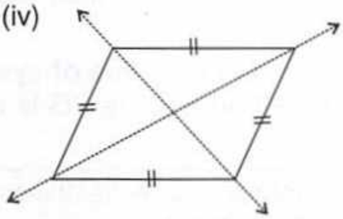
Parallelogram
[No line of symmetry]

(iii)



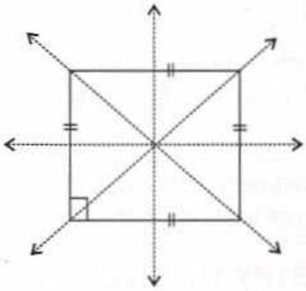
Rectangle
[Two lines of symmetry]

(iv)



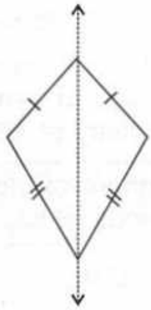
Rhombus
[Two lines of symmetry]

(v)



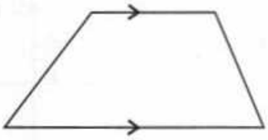
Square
[Four lines of symmetry]

(vi)



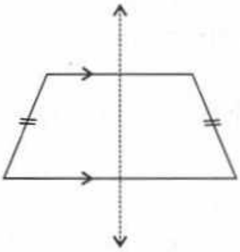
Kite-shaped figure
[One line of symmetry]

(vii)



Trapezium
[No line of symmetry]

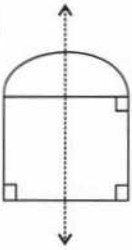
(viii)



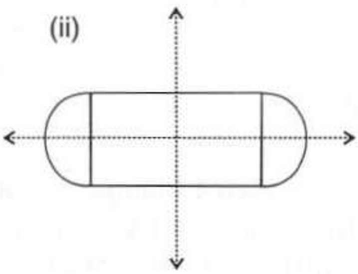
Isosceles trapezium
[One line of symmetry]

7. In each of the following, the dotted line/lines are the line(s) of symmetry of the given figure :

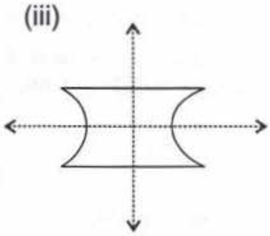
(i)



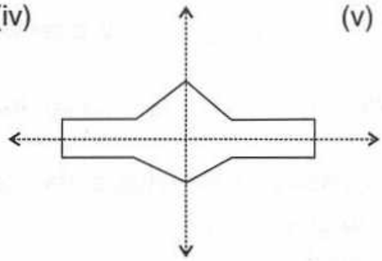
(ii)



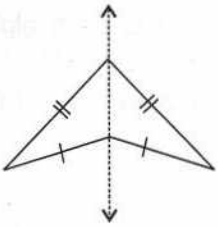
(iii)



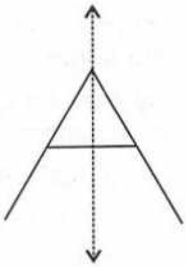
(iv)



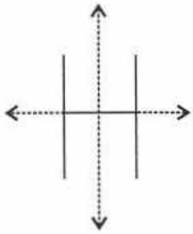
(v)



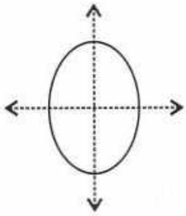
(vi)



(vii)

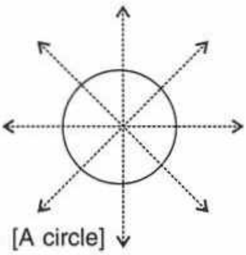


(viii)



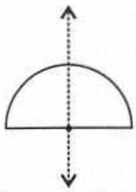
8. As shown below,
   (i) a circle has infinite lines of symmetry; every line through its centre.
   (ii) a semi-circle has one line of symmetry.
   (iii) a quadrant (one-fourth) of a circle has one line of symmetry and so on.

(i)



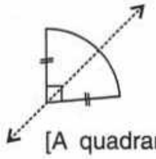
[A circle]

(ii)



[A semi-circle]

(iii)

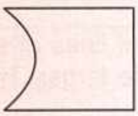


[A quadrant]

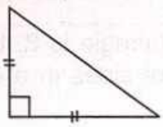
EXERCISE 17(A)

1. For each figure, given below, draw the line(s) of symmetry, if possible :

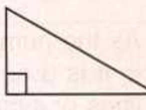
(i)



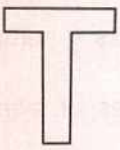
(ii)



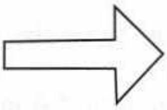
(iii)



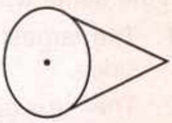
(iv)



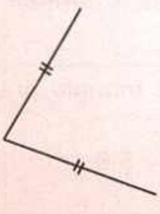
(v)



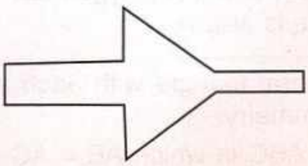
(vi)



(vii)



(viii)



2. Write capital letters A to Z of English alphabet and in each case, if possible, draw the largest number of lines of symmetry.

3. By drawing a free hand sketch of each of the following, draw in each case, the line(s) of symmetry, if any :

(i) a scalene triangle
(ii) an isosceles right angled triangle
(iii) a rhombus
(iv) a kite shaped figure
(v) a rectangle
(vi) a square
(vii) an isosceles triangle.

4. Draw a triangle with :

(i) no line of symmetry,
(ii) only one line of symmetry,
(iii) exactly two lines of symmetry,
(iv) exactly three lines of symmetry,
(v) more than three lines of symmetry.

In each case, if possible, represent the line(s) of symmetry by dotted lines. Also, write the specific name of the triangle drawn.

5. Draw a quadrilateral with :

(i) no line of symmetry.
(ii) only one line of symmetry.
(iii) exactly two lines of symmetry.
(iv) exactly three lines of symmetry.
(v) exactly four lines of symmetry.
(vi) more than four lines of symmetry.

In each case, if possible, represent the line(s) of symmetry by dotted lines. Also, write the specific name of the quadrilateral drawn.

It is clear from the question numbers 4 and 5, given above, that :

1. The *largest number* of *lines of symmetry* of a triangle is *three* (3).
2. The largest number of lines of symmetry of a quadrilateral is four (4).

> As the number of sides in a triangle is 3, the largest number of lines of symmetry in it is 3 and as the number of sides in a quadrilateral is 4, the largest number of lines of symmetry in it is 4.

In the same way :

1. The largest number of lines of symmetry of a pentagon is 5, as a pentagon has 5 sides.
2. The hexagon has 6 sides and so the largest number of lines of symmetry of a hexagon is 6.

In general, we can say, that if a polygon has ***n* sides, the largest number of lines of symmetry** it can have is also ***n***.

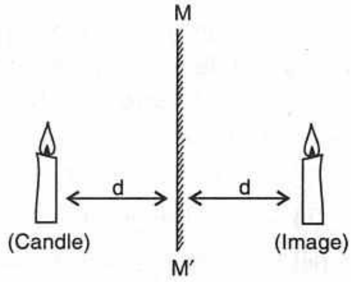
6. Construct an equilateral triangle with each side 6 cm. In the triangle drawn, draw all possible lines of symmetry.

7. Construct a triangle ABC in which AB = AC = 5 cm and BC = 5·6 cm.
If possible, draw its line(s) of symmetry.

8. Construct a triangle PQR such that PQ = QR = 5·5 cm and angle PQR = 90°. If possible, draw its lines of symmetry.
9. If possible, draw a rough sketch of a quadrilateral which has exactly two lines of symmetry.
10. A quadrilateral ABCD is symmetric about its diagonal AC. Name the sides of this quadrilateral which are equal.

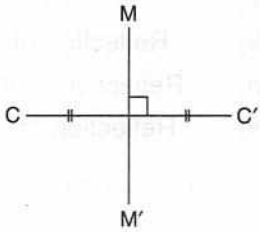
17.3 REFLECTION

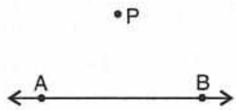
In physics, we have studied that if any object is placed at a certain distance before a plane mirror, its image is formed at the same distance behind the mirror.



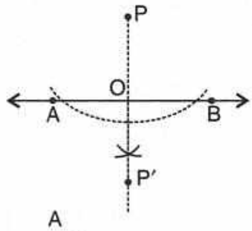
The given figure shows a candle placed at a distance *'d'* before a plane mirror MM′. The image of the candle is obtained in the mirror at the same distance *'d'* behind the mirror.

If we see, geometrically, the line joining the candle (C) and its image (C′) has perpendicularly bisected by the mirror line MM′.





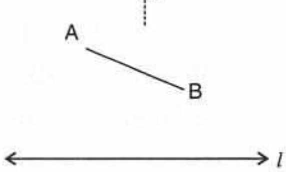
Now if we want to find the image of a point P in line AB, we consider the point P as an object, the line AB as plane mirror and we find point P′ on the other side of AB so that PP′ is perpendicularly bisected by AB.



For this, from the given point P, draw perpendicular to AB which meets AB at point O. From PO produced, cut OP′ = OP.

P′ is the reflection (image) of the given point P in the line AB.

Example 1 :



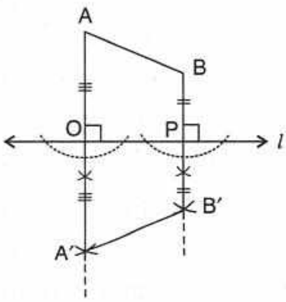
The given figure shows a line segment AB and a line *l*. Find, geometrically, the reflection of AB in the line *l*.

Solution :

From the point A, draw AO perpendicular to the line *l* and from AO produced cut OA′ such that OA′ = OA.

Similarly from the point B, draw *BP* perpendicular to the line *l* and from BP produced cut PB′ such that PB′ = PB.

Join A′ and B′.

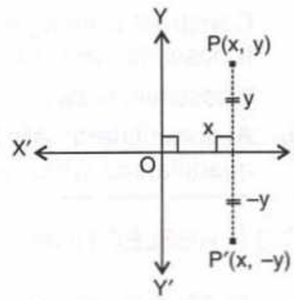


∴ A′ B′ is the required reflection of AB in the line *l*.

17.4 REFLECTON IN *X*-AXIS

Reflection in *x*-axis means the *x*-axis is considered as the plane mirror, the given point as the object and then we find (calculate) its image.

Let P (x, y) be a point as shown in the figure. When it is reflected in x-axis to point P′, the co-ordinates of image point P′ are (x, –y).



Thus, reflection of P (x, y) in x-axis = P′ (x, –y)

In other words :

Image of P (x, y) in x-axis = P′ (x, –y)

We can say, when a point (x, y) is reflected in x-axis, the sign of its second component (ordinate) changes, *i.e.,* the sign of y changes and so the image of (x, y) in x-axis is (x, –y).

∴ (i) Reflection of (5, 4) in x-axis = (5, –4)

(ii) Image of (–5, 4) in x-axis = (–5, –4)

(iii) Image of (–5, –4) in x-axis = (–5, 4)

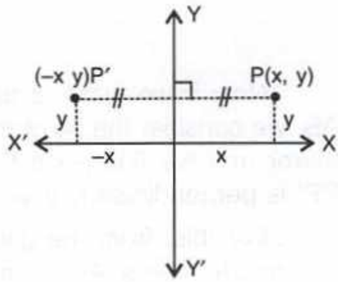
(iv) Image of (–8, 5) in x-axis = (–8, –5)

(v) Reflection of (3, 0) in x-axis = (3, 0)

(vi) Reflection of (0, –6) in x-axis = (0, 6)

(vii) Reflection of (0, 0) in x-axis = (0, 0) and so on.

17.5 REFLECTON IN Y-AXIS



As it is clear from the figure, given alongside, the reflection P (x, y) in y-axis is point P′ (–x, y).

We can say, when a point (x, y) is reflected in y-axis, the sign of its first component (abscissa) changes, *i.e.,* the sign of x changes and so the image of (x, y) in y-axis is (–x, y).

∴ (i) Image of (5, 4) in y-axis = (–5, 4)

(ii) Reflection of (5, – 4) in y-axis = (–5, – 4)

(iii) Reflection of (–5, – 4) in y-axis = (5, – 4)

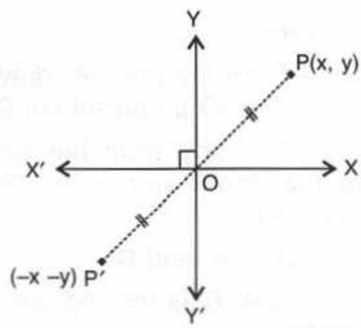
(iv) Image of (–8, 5) in y-axis = (8, 5)

(v) Image of (3, 0) in y-axis = (–3, 0)

(vi) Reflection of (0, – 6) in y-axis = (0, – 6)

(vii) Reflection of (0, 0) in y-axis = (0, 0) and so on.

17.6 REFLECTON IN ORIGIN



When point P (x, y) is reflected in origin, the signs of both of its components change, *i.e.,* the image of P (x, y) is P′ (–x, –y) as shown alongside.

∴ (i) Image of (5, 4) in origin = (–5, –4)

(ii) Reflection of (5, – 4) in origin = (–5, 4)

(iii) Image of (–5, – 4) in origin = (5, 4)

(iv) Reflection of (–8, 5) in origin = (8, –5)

(v) Image of (3, 0) in origin = (–3, 0)

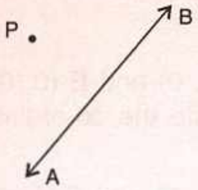
(vi) Reflection of (0, – 6) in origin = (0, 6)

(vii) Reflection of (0, 0) in origin = (0, 0) and so on.

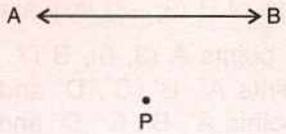
EXERCISE 17(B)

1. In each figure, given below, find the image of the point P in the line AB :

(i)

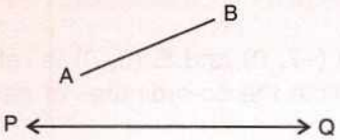


(ii)

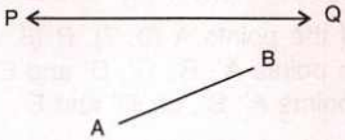


2. In each figure, given below, find the image of the line segment AB in the line PQ :

(i)



(ii)



3. Complete the following table :

Point	Reflection in		
	x-axis	*y*-axis	origin
(i) (8, 2)	....	....	....
(ii) (5, 6)	....	....	....
(iii) (4, –5)	....	....	....
(iv) (6, –2)	....	....	....
(v) (–3, 7)	....	....	....
(vi) (–4, 5)	....	....	....
(vii) (–2, –7)	....	....	....
(viii) (–6, –3)	....	....	....
(ix) (4, 0)	....	....	....
(x) (–7, 0)	....	....	....
(xi) (0, –6)	....	....	....
(xii) (0, 8)	....	....	....
(xiii) (0, 0)	....	....	....

4. A point P (7, 3) is reflected in *x*-axis to point P′. The point P′ is further reflected in *y*-axis to point P″. Find :

(i) the co-ordinates of P′ (ii) the co-ordinates of P″

(iii) the image of *P* (7, 3) in origin.

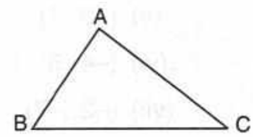
5. A point A (–5, 4) is reflected in *y*-axis to point B. The point B is further refelcted in origin to point C. Find :

(i) the co-ordinates of B (ii) the co-ordinates of C

(iii) the image of A (–5, 4) in *x*-axis.

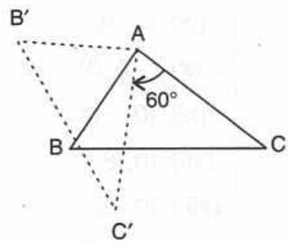
6. The point P (3, –8) is reflected in origin to point Q. The point Q is further reflected in *x*-axis to point R. Find :
   (i) the co-ordinates of Q (ii) the co-ordinates of R
   (iii) the image of P (3, –8) in *y*-axis.
7. Each of the points A (3, 0), B (7, 0), C (–8, 0), D (–7, 0) and E (0, 0) is reflected in *x*-axis to points A′, B′, C′, D′ and E′ respectively. Write the co-ordinates of each of the image points A′, B′, C′, D′ and E′.
8. Each of the points A (0, 4), B (0, 10), C (0, –4), D (0, –6) and *E* (0, 0) is reflected in *y*-axis to points A′, B′, C′, D′ and E′ respectively. Write the co-ordinates of each of the image points A′, B′, C′, D′ and E′.
9. Each of the points A (0, 7), B (8, 0), C (0, –5), D (–7, 0) and E (0, 0) is reflected in origin to points A′, B′, C′, D′ and E′ respectively. Write the co-ordinates of each of the image points A′, B′, C′, D′ and E′.
10. Mark points A (4, 5) and B (–5, 4) on a graph paper. Find A′, the image of A in *x*-axis and B′, the image of B in *x*-axis. Mark A′ and B′ also on the same graph paper. Join AB and A′ B′ and find if AB = A′ B′ ?
11. Mark points A (6, 4) and B (4, –6) on a graph paper. Find A′, the image of A in *y*-axis and B′, the image of B in *y*-axis. Mark A′ and B′ also on the same graph paper.
12. Mark points A (–6, 5) and B (–4, –6) on a graph paper. Find A′, the image of A in origin and B′, the image of B in origin. Mark A′ and B′ also on the same graph paper. Join AB and A′ B′ and find if AB = A′ B′ ?

17.6 ROTATION



Consider a triangle ABC which is free to rotate about its vertex A.

1. Let a rotation of 60° be given to this triangle about vertex A, in the clockwise direction so that the new position of the triangle is Δ AB′C′ as shown alongside:

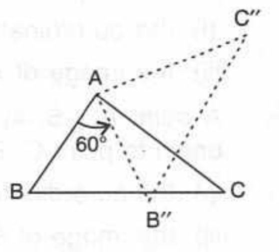
   

   Each and every part of the given triangle will get rotated through 60° in the clockwise direction in such a way that the shapes and sizes of both the triangles remain same.

i.e., ∠B′AC′ = ∠BAC, ∠AB′C′ = ∠ABC and ∠AC′B′ = ∠ACB

And, AB′ = AB, B′C′ = BC and AC′ = AC.

2. Now, let the given triangle ABC be rotated about vertex A through an angle of 60° in the anticlockwise direction and the resulting position of it be triangle AB″C″.

   Clearly, the shapes and sizes of the two triangles are the same and so :

   ∠B″AC″ = ∠BAC ; ∠AB″C″ = ∠ABC and ∠AC″B″ = ∠ACB

   And, AB″ = AB, AC″ = AC and B″C″ = BC.

1. Whatever be the angle of rotation of the figure, the resulting figure and the given figure are always same in shape and size.
2. The point, about which the figure is rotated, is called the **centre of rotation**.

17.7 ROTATIONAL SYMMETRY

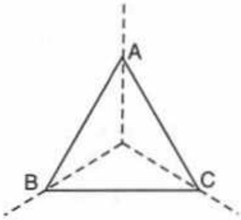
If a figure is given one complete rotation and during this rotation the figure fits onto itself more than once, then the figure is said to have **rotational symmetry**.

The number of times a figure fits onto itself during one complete rotation is called the order of rotational symmetry.

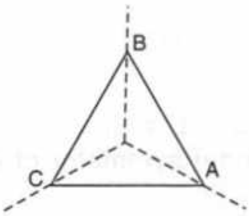
17.8 ROTATION OF AN EQUILATERAL TRIANGLE

Let us rotate an equilateral triangle ABC through 120°, 240° and 360° in the clockwise direction to attain the positions shown below :

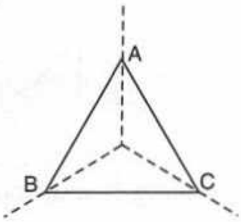
1.



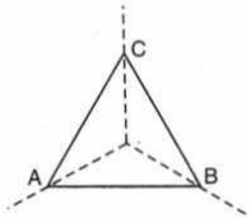
On giving rotation of 120°.



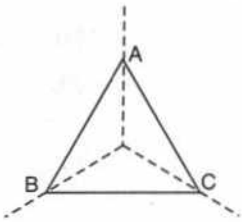
2.



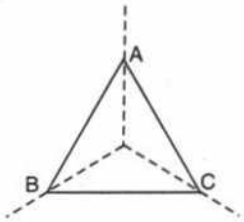
On giving rotation of 240°.



3.



On giving rotation of 360°.

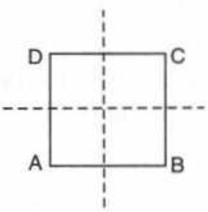


Since, each of the three times, the figure (equilaterial triangle) fits onto itself, we say, it has a **rotational symmetry of order 3.**

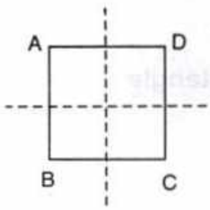
17.9 ROTATION OF A SQUARE

Let us rotate a square ABCD through 90°, 180°, 270° and 360° in the clockwise direction to attain the positions shown below :

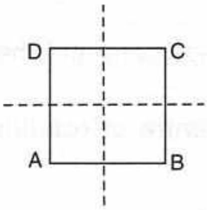
1.



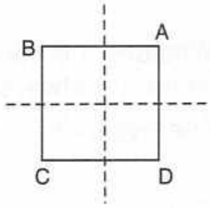
On giving rotation of 90°.



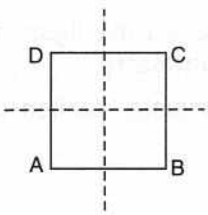
2.



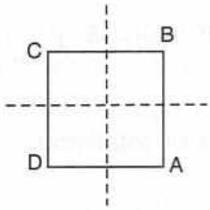
On giving rotation of 180°.



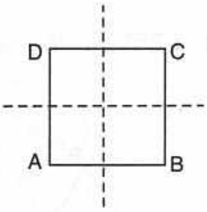
3.



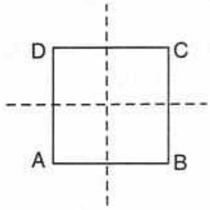
On giving rotation of 270°.



4.

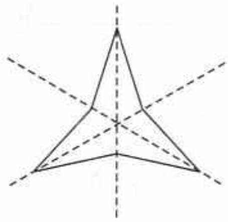


On giving rotation of 360°.



Since, each of the four times, the figure (square) fits onto itself, we say, it has a **rotational symmetry of order 4.**

Note : The adjoining figure when rotated through 120°, 240° and 360° (clockwise or anticlockwise) will fit exactly onto itself each time. So, it has a rotational symmetry of order 3.



Rotating a figure through 90° clockwise is the same as rotating it anticlocwise through 270° (360° – 90° = 270°).

(i) Rotation of 120° clockwise = Rotation of 240° anticlockwise (360° – 120° = 240°).

(ii) Rotation of 180° clockwise = Rotation of 180° anticlockwise and so on.

The following table that shows line symmetry and rotational symmetry of different figures will make the concept of symmetry more clear.

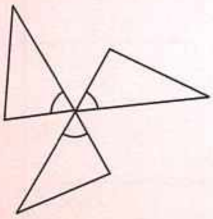
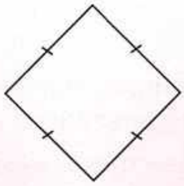
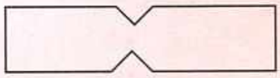
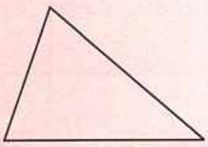
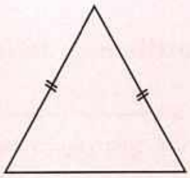
Name of the figures	Line symmetry	Rotational symmetry
1. Equilateral triangle	3 lines of symmetry; the bisectors of its interior angles.	Rotational symmetry of order 3; when rotated through 120°, 240° and 360°.
2. Square	4 lines of symmetry the diagonals and lines through mid-points of opposite sides.	Rotational symmetry of order 4; when rotated through 90°, 180°, 270° and 360°.
3. Rectangle	2 lines of symmetry the lines through the mid-points of opposite sides.	Rotational symmetry of order 2; when rotated through 180° and 360°.

EXERCISE 17(C)

1. How many lines of symmetry does a rhombus have ?
2. What is the order of rotational symmetry of a rhombus ?
3. Show that each of the following figures has two lines of symmetry and a rotational symmetry of order 2.

   (i) H (ii) I (iii) O (iv) X

4. Name a figure that has a line of symmetry but does not have any rotational symmetry.
5. In each of the following figures, draw all possible lines of symmetry and also write the order of rotational symmetry :

   (i)

   (ii)

   (iii)

   (iv)

   (v)


RECOGNITION OF SOLIDS 18

(Representing 3-D in 2-D)

18.1 IDENTIFICATION OF 3D SHAPES

[Cube, cuboid, cylinder and cone.]

Solid : An object, of fixed shape, that occupies space is called a **solid**. A book, a brick, a ball, etc., are some examples of a solid.

1. A thin straight line drawn on paper, *i.e.* a line drawn on a plane, has only length.

   Thus, we say that a **straight line** has **only one dimension**, namely, a **length**.

Straight line

2. A rectangle, drawn on paper, has length and breadth.

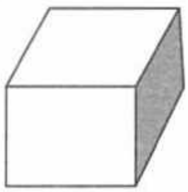
   Thus, we say that a **rectangle** has **two dimensions**, namely, **length** and **breadth**.

   In fact, every rectilinear figure (such as : square, parallelogram, trapezium, etc.) is a **two-dimensional** figure.

3. **Solids** have **length**, **breadth** and **height**. For this reason, every solid is a **three-dimensional** figure.

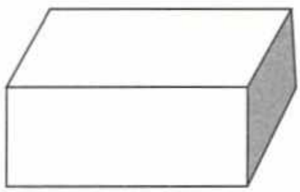
Some three dimensional shapes are given below :

(i)



(Cube)

(ii)



(Cuboid)

(iii)

(Cylinder)

(iv)

(Cone)

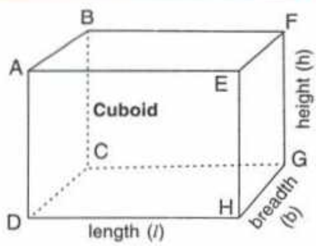
18.2 RECOGNIZING FACES, EDGES AND VERTICES (CORNERS) OF A POLYHEDRON AND COUNTING THEM

In elementary geometry, a **polyhedron** (plural : polyhedra or polyhedrons) is a solid in three dimensions with flat polygonal faces, straight edges and sharp corners (or, vertices.)

(a) Cuboid (a rectangular solid) **:**

Cuboid is a solid or hollow body which has six rectangular faces at right angles to each other.

It is a three-dimensional solid all of whose sides are not necessarily equal. That is, in general, a cuboid has length, breadth and height of different values (sizes).



The given figure shows a cuboid. It is clear from the figure that a cuboid has :

(i) ***six faces,*** namely, ABCD, ABFE, AEHD, CGHD, CGFB and EFGH.

Each face of a cuboid is a rectangle.

(ii) ***twelve edges,*** namely, AB, BC, CD, DA, AE, EH, HD, EF, FG, GH, BF and CG.

(iii) ***eight vertices*** (corners), namely, A, B, C, D, E, F, G and H.

Also, (i) length (l) of the cuboid = AE = DH = CG = BF

(ii) breadth (b) of the cuboid = AB = DC = HG = EF

(iii) height (h) of the cuboid = AD = BC = EH = FG

(b) Cube :

Cube is a symmetrical three-dimensional shape, either solid or hollow, contained by six equal squares.

A cube is a cuboid with all sides equal, i.e. length = breadth = height.

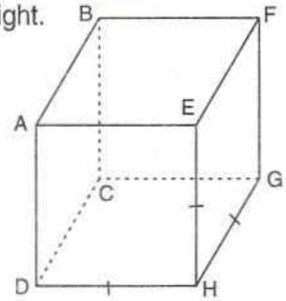
The adjoining figure shows a cube.

Since a cube is a cuboid, it also has :

(i) ***six faces :*** ABCD, ABFE, AEHD, CGHD, CGFB and EFGH.

(ii) ***twelve edges :*** AB, BC, CD, DA, AE, EH, HD, EF, FG, GH, BF and CG.

(iii) ***eight corners :*** A, B, C, D, E, F, G and H.

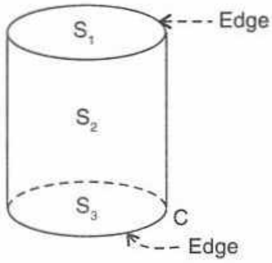


Each face of a cube is a square in shape and all the six faces of a cube are congruent (equal).

(c) Cylinder :

A **cylinder** is a solid or hollow geometrical figure with a curved side and two identical circular flat ends.

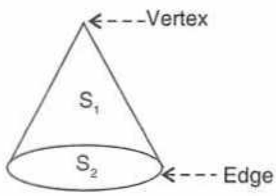
A cylinder is a 3D figure with no vertex, **two edges** (shown in the adjoining figure) and **three faces** shown by S_1, S_2 and S_3, where S_1 and S_3 are circular in shape and S_2 is a curved surface.



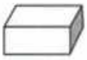
(d) Cone :

A **cone** is a solid or hollow object which tapers from a circular base to a point.

A cone is a 3D figure with **one vertex**, **one edge** and **two surfaces** represented by S_1 and S_2, where S_1 is a curved surface and S_2 is a circle.



The number of vertices (V), no. of edges (E) and the no. of faces (F) of different 3D figures are as given below :

Figure		V	E	F
1. *Cube*		8	12	6
2. *Cuboid*		8	12	6
3. *Cylinder*		0	2	3
4. *Cone*		1	1	2

18.3 EULER'S FORMULA

Euler's formula deals with shapes called **Polyhedra**.

For a 3D solid, if

1. **V** stands for **number of vertices**,
2. **E** stands for **number of edges** and
3. **F** stands for **number of faces**, then the relation between V, E and F is

$V + F - E = 2$, which is called **Euler's formula**.

For example :

For a cube, V = 8, E = 12 and F = 6

$\therefore$ $\mathbf{V + F - E} = 8 + 6 - 12$

$= \mathbf{2}$

18.4 NETS OF 3D FIGURES (Representing 3D figures into 2D)

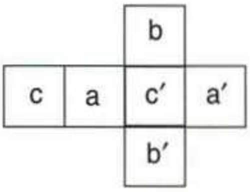
[For cube, cuboid, cylinder and cone]

A pattern, that can be cut and folded to make a model of a solid shape, is called a **net**.

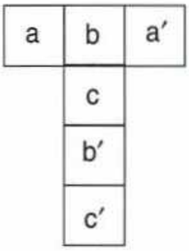
1. Nets of a cube :

Some of the nets of a cube are shown below :

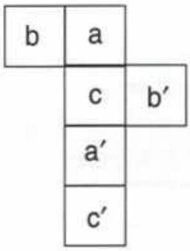
(i)



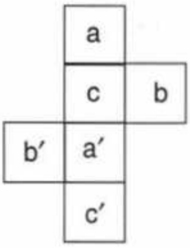
(ii)



(iii)



(iv)



On folding each of these patterns to get the model of a cube.

(i) face **a** comes opposite to face **a′**.

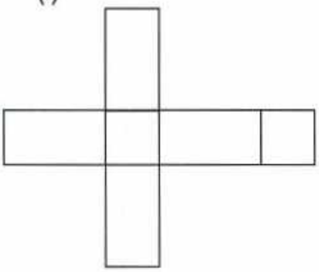
(ii) face **b** comes opposite to face **b′**.

and (iii) face **c** comes opposite to face **c′**.

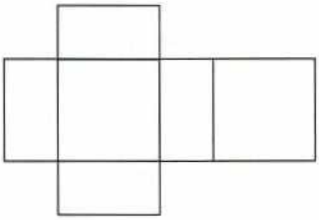
2. Nets of a cuboid :

Some of the nets of a cuboid are shown below :

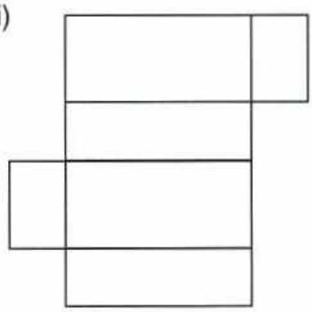
(i)



(ii)

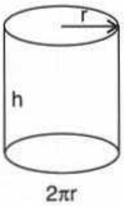


(iii)



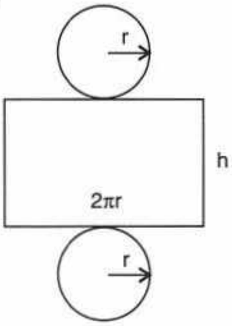
3. Nets of a cylinder :

Let the radius of the base (cross-section) be *r* unit and height be *h* unit.

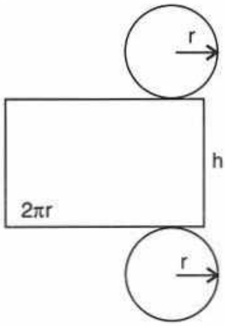


⇒

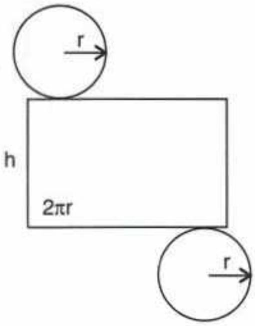
(i)



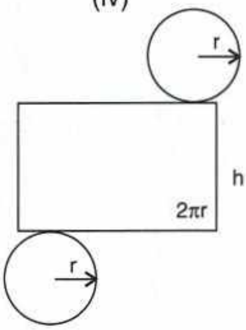
(ii)



(iii)



(iv)

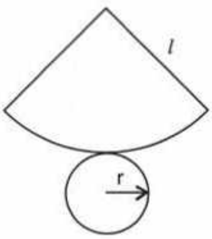


, etc.

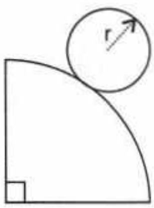
4. Nets of a cone :

⇒

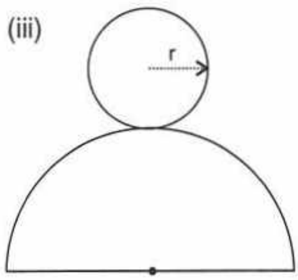
(i)



(ii)



(iii)



Figures given above are not drawn to scale.

18.5 MAPPING SPACE AROUND US

Maps are different from pictures

Usually, maps are used to represent large/different regions such as cities, towns, rivers, mountains, etc.

Through maps, we can locate places like your school play round, mountain, etc.

The map given below shows the location of Geeta's house (A) and her school (B). It also shows the route from Geeta's house to her school.

Looking at the picture, can you tell the actual distance between Geeta's house and her school ?

No. It is impossible to get the actual distance between the two places just by looking at the picture *i.e.* by estimation.

But if the map is drawn to scale and the scale at which it is drawn is mentioned on the map, one can easily find the distance between any two places in the map. If the above map is drawn to a scale of 1 cm = 100 metre and the distance between any two places is 6 cm on the map, then we can easily say that the actual distance between the two places is 6 X 100 metre = 600 metre.

Maps are used to find the actual distance between two places.

For example :

Let the scale for a map is given to be 1 cm = 10 km.

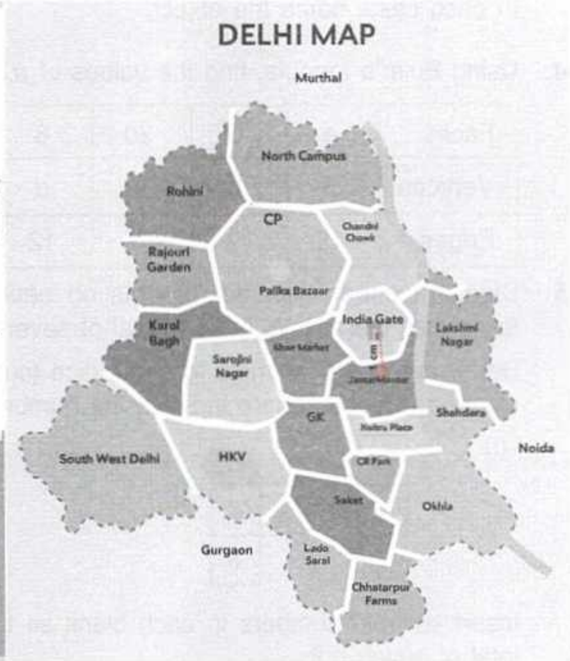
$\Rightarrow$ if the distance between two places is 1 cm on the map, then the actual distance between the two places is 10 km.

$\Rightarrow$ if the distance between any two places is 5 cm on the map, then the actual distance between the two places is 5×10 km = 50 km.

Thus, we can say that while drawing (or reading) a map one must know to what scale it is drawn *i.e.* how much actual distance is to be denoted by one unit length (1 mm or 1 cm etc.) in the map. The scale can vary from map to map but not within the same map. The different scales of different maps do not change the distance between the places.

For example, look at the maps of Delhi and India given below. The two maps are same in size but the scale of distance used in both the maps is different. 1 cm of space in the map of Delhi represents smaller distances as compared to 1 cm space in the map of India.

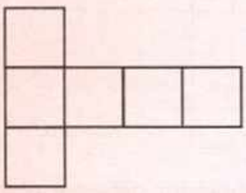
On a map, the distances shown are proportional to the actual distances on the ground.



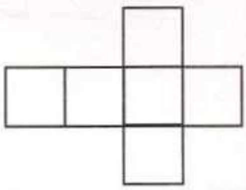
EXERCISE 18

1. Identify the nets which can be used to form cubes :

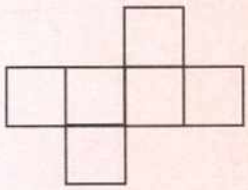
(i)



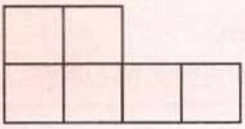
(ii)



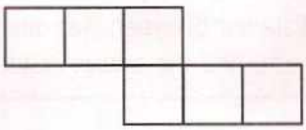
(iii)



(iv)



(v)



2. Draw at least three different nets for making a cube.

3. The figure, given below, shows shadows of some 3D objects, when seen under the lamp of an overhead projector :

(i)

A square

(ii)

A rectangle

(iii)

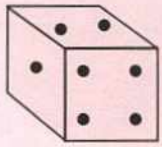
a triangle

In each case, name the object

4. Using Euler's formula, find the values of *a*, *b*, *c* and *d*.

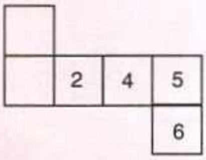
Faces	a	5	20	6
Vertices	6	b	12	d
Edges	12	9	c	12

5. Dice are cubes with dot or dots on each face. Opposite faces of a die always have a total of seven on them.

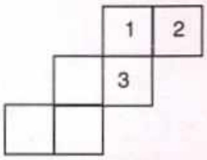


Below are given two nets to make dice (cube), the numbers inserted in each square indicate the number of dots in it.

(i)



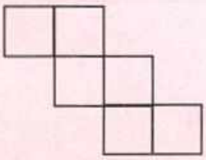
(ii)



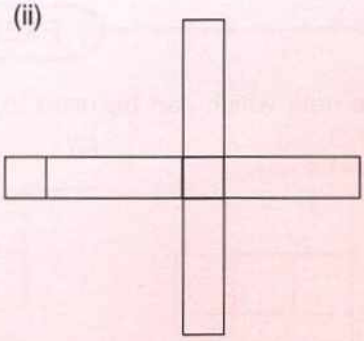
Insert suitable numbers in each blank so that numbers in opposite faces of the die have a total of seven dots.

6. The following figures represent nets of some solids. Name the solids.

(i)



(ii)



7. Draw a map of your class room using proper scale and symbols for different objects.

8. Draw a map of your school compound using proper scale and symbols for various features like play ground, main building, garden, etc.

9. In the map of India, the distance between two cities is 13·8 cm.

Taking scale : 1 cm = 12 km, find the actual distance between these two cities.

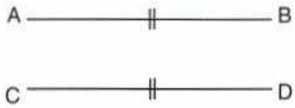
CONGRUENCY : CONGRUENT TRIANGLES 19

19.1 MEANING OF CONGRUENCY

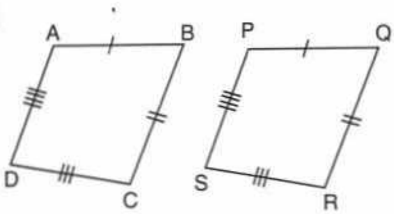
If two geometrical figures coincide exactly, by placing one over the other, the figures are said to be congruent to each other.

Congruent figures are exactly the same both in shape and size.

1. Two lines AB and CD are said to be congruent if, on placing AB on CD, or CD on AB, the two lines AB and CD exactly coincide.
   It is possible only when AB and CD are equal in length.

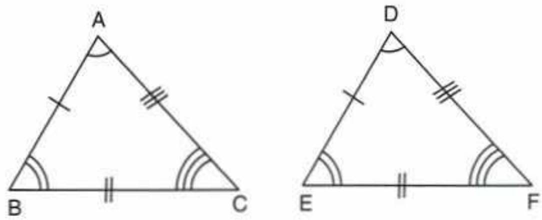


2. Two figures ABCD and PQRS are said to be congruent if, on placing ABCD on PQRS or PQRS on ABCD, the two figures exactly coincide, *i.e.,* A and P coincide, B and Q coincide, C and R coincide and D and *S* coincide.
   It is possible only when :
   AB = PQ, BC = QR, CD = RS and AD = PS
   Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R and ∠D = ∠S.



19.2 CONGRUENCY IN TRIANGLES

Let triangle ABC be placed over triangle DEF; such that, vertex A falls on vertex D and side AB falls on side DE, then if the two triangles coincide with each other in such a way that B falls on E, C falls on F; side BC coincides with side EF and side AC coincides with side DF, then the two triangles are congruent to each other.



The symbol used for congruency is "≡" or "≅".

∴ **Δ ABC is congruent to Δ DEF** is written as : **Δ ABC ≡ Δ DEF or Δ ABC ≅ Δ DEF.**

19.3 CORRESPONDING SIDES AND CORRESPONDING ANGLES

In case of congruent triangles ABC and DEF, as given above, *the sides* of the two triangles, *which coincide with each other, are called corresponding sides.*

Thus, the sides AB and DE are corresponding sides, sides BC and EF are corresponding sides and sides AC and DF are also corresponding sides.

In the same way, *the angles* of the two triangles *which coincide with each other, are called corresponding angles.* Thus, three pairs of corresponding angles are ∠A and ∠D, ∠B and ∠E and also ∠C and ∠F.

The corresponding parts of congruent triangles are always equal (congruent).

∴ (i) AB = DE, BC = EF and AC = DF, *i.e., corresponding sides are equal.*

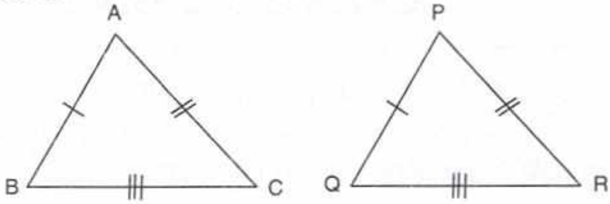
Also, (ii) ∠A = ∠D, ∠B = ∠E and ∠C = ∠F, *i.e., corresponding angles are equal.*

19.4 CONDITIONS OF CONGRUENCY

1. *If three sides of one triangle are equal to three sides of the other triangle, each to each, then the two triangles are congruent.*

This condition is known as : **side, side, side** and is abbreviated as **S.S.S.**

In triangles ABC and PQR, given alongside :



AB = PQ,

BC = QR and AC = PR

So Δ ABC is congruent to Δ PQR, *i.e.,* Δ ABC ≅ Δ PQR by S.S.S.

Similarly, in congruent triangles, corresponding angles are equal.

∴ ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R

2. *If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, each to each, then the triangles are congruent.*

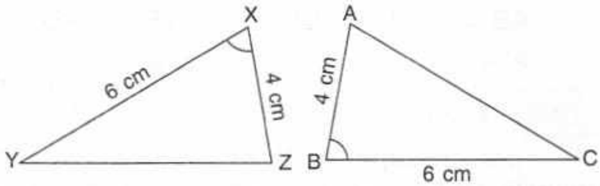
This condition is known as : **side, angle, side** and is abbreviated as **S.A.S.**

In the given triangles,

AB = XZ,

BC = XY and ∠ABC = ∠ZXY

∴ Δ ABC ≡ Δ ZXY (by S.A.S.)



Triangles will be congruent by S.A.S., only when the angles included by the corresponding equal sides are equal.

The pairs of corresponding sides of these two congruent triangles are :

AB and ZX, BC and XY, AC and ZY

The pairs of corresponding angles are :

∠B and ∠X, ∠A and ∠Z, ∠C and ∠Y.

3. *If two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle, then the triangles are congruent.*

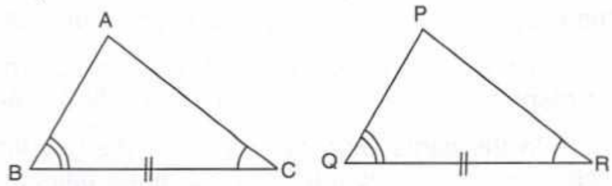
This condition is known as : **angle, side, angle** and is abbreviated as **A.S.A.**

In the given figure :

BC = QR,

∠B = ∠Q and ∠C = ∠R

∴ Δ ABC ≡ Δ PQR. (by A.S.A.)



4. *If any two angles and a side (not the included side) of one triangle are equal to two angles and the corresponding side of the other triangle; then the two triangles are congruent.*

This condition is known as : **angle, angle, side** and is abbreviated as : **A.A.S.**

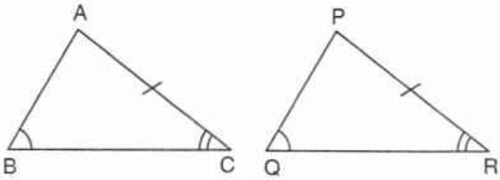
In the given figure :

$\angle B = \angle Q$,

$\angle C = \angle R$

and, $AC = PR$

$\therefore \ \Delta ABC \cong \Delta PQR$. (by A.S.A.)



5. *If the hypotenuse and one side of a right angled triangle are equal to the hypotenuse and one side of another right angled triangle, then the two triangles are congruent.*

This condition is known as : **right angle, hypotenuse, side** and is abbreviated as **R.H.S.**

In the given figure :

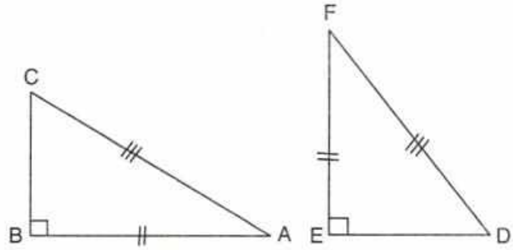
$\angle B = \angle E = 90°$, $AB = FE$

and hypotenuse AC = hypotenuse FD

$\therefore \ \Delta ABC \equiv \Delta FED$ (by R.H.S.)

The corresponding angles in this case are :

$\angle A$ and $\angle F$, $\angle B$ and $\angle E$, $\angle C$ and $\angle D$,



and the corresponding sides are :

AB and EF, AC and FD, BC and ED.

Since the triangles are congruent, therefore all its corresponding sides are equal and corresponding angles are also equal.

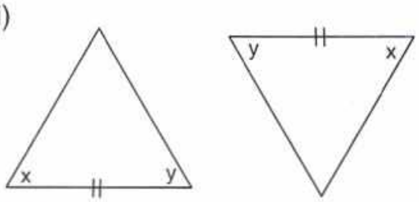
$\therefore \ BC = ED$, $\angle A = \angle F$ and $\angle C = \angle D$

> *If three angles of a triangle are equal to the three angles of the other triangle, then the triangles are* **not necessarily congruent.**
>
> *For congruency at least* ***one pair of corresponding sides must be equal.***
>
> $\therefore$ *A.A.A. is not a test of congruency.*

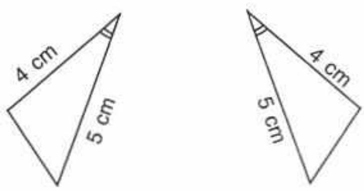
Example 1 :

State, by what test the following triangles are congruent.

(i)



(ii)



Solution :

(i) Here, two angles of one triangle are equal to the two angles of the other triangle and the included sides are equal.

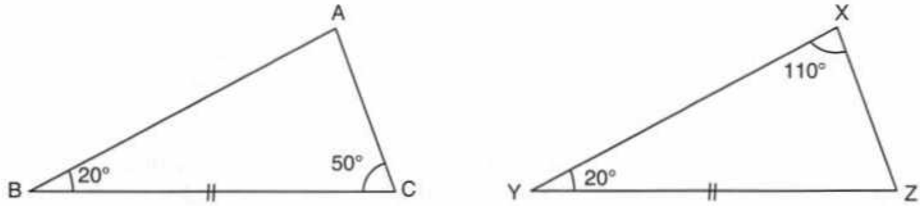
$\therefore$ The two **triangles are congruent by A.S.A.** **(Ans.)**

(ii) Since, two sides of one triangle are equal to the two sides of the other triangle and the included angles are equal.

$\therefore$ The **triangles are congruent by S.A.S.** **(Ans.)**

Example 2 :

State, whether or not, the following triangles are congruent.



Solution :

Here, $\angle A = 180° - (20° + 50°) = 110°$

Also, $\angle Z = 180° - (110° + 20°) = 50°$

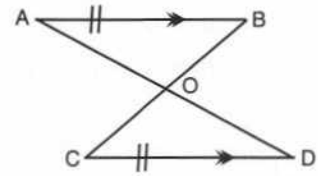
We see that in Δ ABC and Δ XYZ,

$\angle C = \angle Z = 50°$

$\angle B = \angle Y = 20°$ and BC = YZ and they are included sides.

$\therefore$ **Δ ABC $\cong$ Δ XYZ** by A.S.A. **(Ans.)**

Example 3 :



In the given figure, AB // CD and AB = CD.

Prove that : (i) Δ AOB $\cong$ Δ DOC

(ii) AO = DO

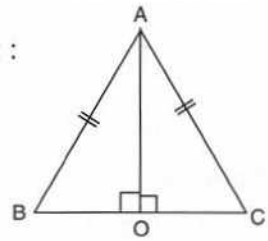
(iii) BO = CO

Solution :

Statement	Reason
In triangles AOB and COD	
AB = CD	Given
$\angle$BAO = $\angle$CDO	Alternate angles, as AB // CD
$\angle$ABO = $\angle$DCO	Alternate angles, as AB // CD
$\therefore$ (i) Δ AOB $\cong$ Δ DOC	A.S.A.
(ii) AO = DO	Corresponding sides of congruent triangles.
also, (iii) BO = CO	Corresponding sides of congruent triangles.

Hence proved.

Example 4 :



In the given figure, AB = AC, $\angle$BOA = $\angle$COA = 90°. Prove that :

(i) Δ AOB $\cong$ Δ AOC (ii) $\angle B = \angle C$ (iii) BO = CO

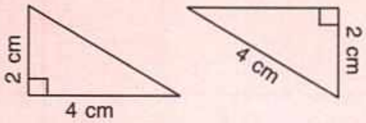
Solution :

	Statement	Reason
(i)	In Δ AOB and Δ AOC,	
	AB = AC	Given
	AO = AO	Common
and	$\angle$BOA = $\angle$COA	Each 90°
$\therefore$	**Δ AOB $\cong$ Δ AOC**	R.H.S.
(ii)	Δ AOB = Δ AOC	Proved above
$\Rightarrow$	**$\angle$B = $\angle$C**	Corresponding angles of congruent triangles
(iii) and	BO = CO	Corresponding sides of congruent triangles

EXERCISE 19

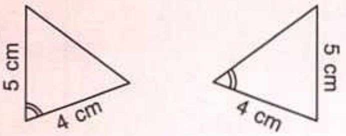
1. State, whether the pairs of triangles given in the following figures are congruent or not :

(i)

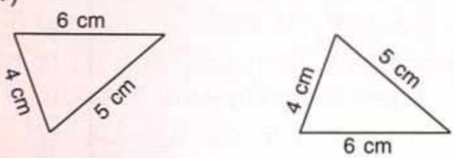


(ii)

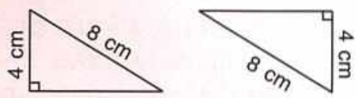
(iii)



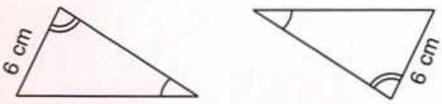
(iv)



(v)

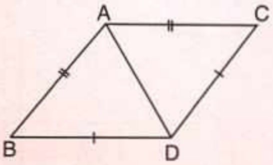


(vi)

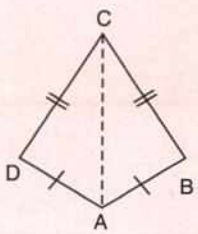


(vii) Δ ABC in which AB = 2 cm, BC = 3.5 cm and $\angle C = 80°$.
and, Δ DEF in which DE = 2 cm, DF = 3.5 cm and $\angle D = 80°$.

2. In the given figure, prove that :
$\Delta ABD \cong \Delta ACD$

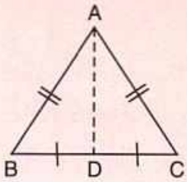


3. Prove that :
(i) $\Delta ABC \cong \Delta ADC$
(ii) $\angle B = \angle D$
(iii) AC bisects angle DCB

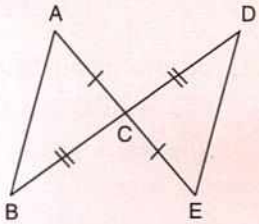


(iii) As $\Delta ABC \cong \Delta ADC$
$\Rightarrow \angle BCA = \angle DCA$
and so AC bisects angle DCB.

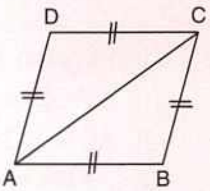
4. Prove that :
(i) $\Delta ABD \cong \Delta ACD$
(ii) $\angle B = \angle C$
(iii) $\angle ADB = \angle ADC$
(iv) $\angle ADB = 90°$



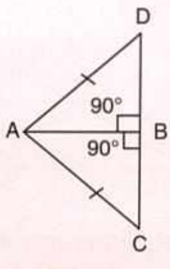
5. In the given figure, prove that :
(i) $\Delta ACB \cong \Delta ECD$
(ii) AB = ED



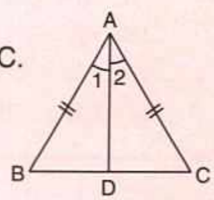
6. Prove that :
(i) $\Delta ABC \cong \Delta ADC$
(ii) $\angle B = \angle D$



7. In the given figure, prove that :
BD = BC.



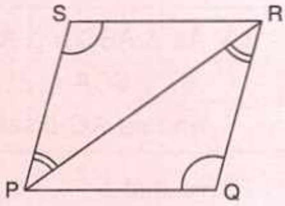
8. In the given figure,
$\angle 1 = \angle 2$ and AB = AC.
Prove that :
(i) $\angle B = \angle C$
(ii) BD = DC
(iii) AD is perpendicular to BC.



9. In the given figure, prove that :

(i) PQ = RS

(ii) PS = QR

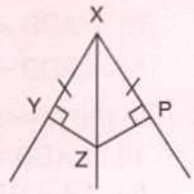


10. In the given figure, prove that :

(i) Δ XYZ ≅ Δ XPZ

(ii) YZ = PZ

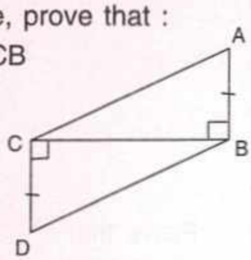
(iii) ∠YXZ = ∠PXZ



11. In the given figure, prove that :

(i) Δ ABC ≅ Δ DCB

(ii) AC = DB



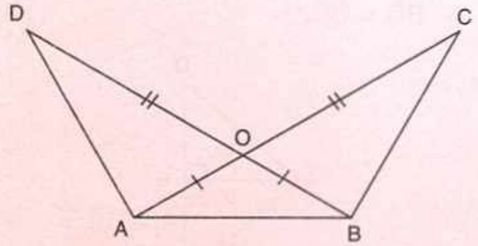
12. In the given figure, prove that :

(i) Δ AOD ≅ Δ BOC

(ii) AD = BC

(iii) ∠ADB = ∠ACB

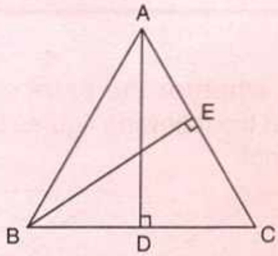
(iv) Δ ADB ≅ Δ BCA



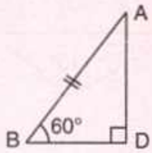
13. ABC is an equilateral triangle, AD and BE are perpendiculars to BC and AC respectively. Prove that :

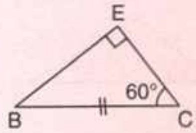
(i) AD = BE

(ii) BD = CE



Consider Δ ADB and Δ CEB :





- AB = BC (sides of same equilateral triangle)
- ∠ADB = ∠BEC (each 90°)
- ∠ABD = ∠BCE (each 60°)

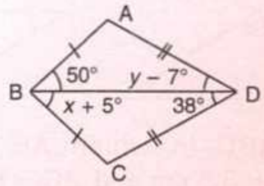
∴ Δ ADB ≅ Δ BEC by A.A.S.

Since, the corresponding parts of the triangles are congruent, therefore:

(i) AD = BE and (ii) BD = CE

14. Use the informations given in the following figure to prove that triangles ABD and CBD are congruent.

Also, find the values of x and y.

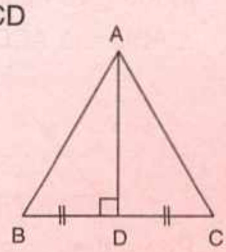


15. The given figure shows a triangle ABC in which AD is perpendicular to side BC and BD = CD. Prove that :

(i) Δ ABD ≅ Δ ACD

(ii) AB = AC

(iii) ∠B = ∠C.



MENSURATION

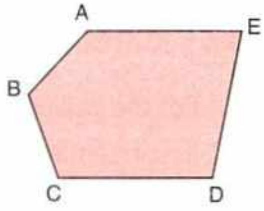
(Perimeter and Area of Plane Figures)

20

20.1 INTRODUCTION

Perimeter is the total length of the boundary of a closed plane figure and the region bounded by the perimeter of the figure is called its area.

The adjoining diagram shows a closed plane figure.



Its **perimeter** = length of its boundary

= AB + BC + CD + DE + EA

And, its **area** = The interior part of the figure along with its boundary (perimeter).

20.2 RELATION BETWEEN THE UNIT OF LENGTH AND UNIT OF AREA

Unit of length	Unit of area
1 cm = 10 mm	$1\ cm^2 = (10\ mm) \times (10\ mm) = 100\ mm^2$
1dm = 10 cm	$1\ dm^2 = (10\ cm) \times (10\ cm) = 100\ cm^2$
1 m = 10 dm	$1\ m^2 = (10\ dm) \times (10\ dm) = 100\ dm^2$
1 dam = 10 m	$1\ dam^2 = (10\ m) \times (10\ m) = 100\ m^2$
1 hm = 10 dam	$1\ hm^2 = (10\ dam) \times (10\ dam) = 100\ dam^2$
1 km = 10 hm	$1\ km^2 = (10\ hm) \times (10\ hm) = 100\ hm^2$

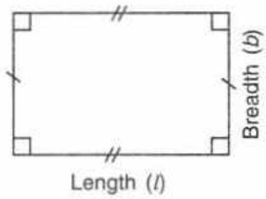
Also, 1m = 100 cm $\Rightarrow$ $1\ m^2 = (100\ cm) \times (100\ cm) = 10000\ cm^2 = 10^4\ cm^2$

1hm = 100 m $\Rightarrow$ $1\ hm^2 = (100\ m) \times (100\ m) = 10000\ m^2 = 10^4\ m^2$

and, 1 km = 1000 m $\Rightarrow$ $1\ km^2 = (1000\ m) \times (1000\ m) = 10^6\ m^2$

20.3 PERIMETER OF SOME PLANE FIGURES

1. Rectangle



Perimeter = 2(length + breadth)

$\Rightarrow$ $P = 2(l + b)$

$$P = 2(l + b)$$

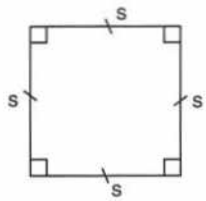
$$\Rightarrow \quad \text{length } (l) = \frac{P}{2} - b$$

$$\text{and, breadth } (b) = \frac{P}{2} - l$$

2. Square

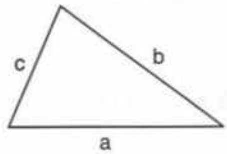
Perimeter = 4 × side

⇒ P = 4 s



Side of the square (s) = $\frac{P}{4}$

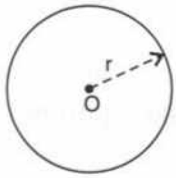
3. Triangle



Perimeter = Sum of lengths of all the three sides

= ***a + b + c***

20.4 CIRCUMFERENCE (perimeter) OF A CIRCLE



Let the radius of a circle be *r*

its circumference = $2\pi r$

1. When a wire of certain length is bent to form the largest circle of radius *r*, **the length of the wire**

   = circumference of the circle formed

   = $\mathbf{2\pi r}$.

2. $\pi = \frac{22}{7}$

3. If *d* is the diameter of the circle, $\mathbf{d = 2r}$ and $\mathbf{r = \frac{d}{2}}$.

Example 1 :

The length and the breadth of a rectangular field are 230 m and 85 m respectively, find its perimeter.

Solution :

Given : length (l) = 230 m and breadth (b) = 85 m

⇒ **Perimeter** = $2(l + b)$

= 2(230 m + 85 m) = **630 m** **(Ans.)**

Example 2 :

If P = perimeter of a rectangle, l = its length and b = its breadth, find :

(i) P, if l = 32 cm and b = 16 cm.

(ii) l, if P = 80 cm and b = 18 cm.

(iii) b, if P = 120 m and l = 35 m.

Solution :

(i) **P** = $2(l + b)$

= 2(32 cm + 16 cm)

= 2 x 48 cm = **96 cm** **(Ans.)**

(ii) $l = \frac{P}{2} - b$ **(Ans.)**

= $\frac{80 \text{ cm}}{2}$ − 18 cm

= 40 cm − 18 cm = **22 cm** **(Ans.)**

(iii) $b = \frac{P}{2} - l$

$= \frac{120 \text{ m}}{2} - 35 \text{ m}$

$= 60 \text{ m} - 35 \text{ m} = \mathbf{25 \text{ m}}$ **(Ans.)**

Example 3 :

The length and the breadth of a rectangular plot are in the ratio 5 :3. If perimeter of the plot is 480 m, find its length and breadth.

Solution :

Let the length = 5x m and breadth = 3x m

$2(l + b) = \text{perimeter}$

$\Rightarrow 2(5x + 3x) = 480$ *i.e.* $2 \times 8x = 480$

$\Rightarrow x = \frac{480}{16} = 30$

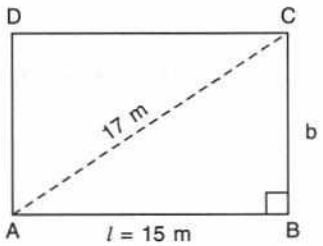
$\therefore$ **Length** $= 5x = 5 \times 30 \text{ m} = \mathbf{150 \text{ m}}$

and, **breadth** $= 3x = 3 \times 30 \text{ m} = \mathbf{90 \text{ m}}$ **(Ans.)**

Example 4 :

Find the perimeter of a rectangle if its length is 15 m and one of the diagonals is 17 m.

Solution :



Let breadth of the rectangle = *b* m.

Applying Pythagoras theorem in triangle ABC, we get:

$AC^2 = AB^2 + BC^2$

$\Rightarrow 17^2 = 15^2 + b^2$ *i.e.* $289 = 225 + b^2$

$\Rightarrow 289 - 225 = b^2$ *i.e.* $b^2 = 64$

$\Rightarrow b = \sqrt{64} \text{ m} = 8 \text{ m}$

Now, **perimeter of the rectangle** $= 2(l + b)$

$= 2(15 \text{ m} + 8 \text{ m})$

$= 2 \times 23 \text{ m} = \mathbf{46 \text{ m}}$ **(Ans.)**

Example 5 :

The cost of fencing a square field at the rate of ₹ 35 per meter is ₹ 14,000. Find the perimeter and the side of the square field.

Solution :

Length of the fence × its rate = ₹ 14,000

$\Rightarrow$ Length of the fence = ₹ $\frac{14,000}{35}$ = 400 m

$\therefore$ **Perimeter of a square field** = length of its fence = **400 m** **(Ans.)**

Since, perimeter of a square = 4 × length of its side

$\Rightarrow$ **Length of the side of the square** $= \dfrac{\text{Perimeter}}{4} = \dfrac{400\text{ m}}{4} =$ **100 m** **(Ans.)**

Example 6 :

The length and the breadth of a rectangle are 40 m and 30 m respectively. If its perimeter is equal to the perimeter of a square, find the side of the square.

Solution :

Perimeter of the rectangle = 2(length + breadth)

= 2(40 m + 30m) = 2 x 70 m = 140 m

Given, perimeter of the square = perimeter of the rectangle

= 140 m

$\therefore$ **Side of the square** $= \dfrac{\text{Perimeter}}{4}$

$= \dfrac{140\text{ m}}{4} =$ **35 m** **(Ans.)**

Example 7 :

The radius of a circle is 14 cm. Find its circumference (perimeter). Take $\pi = 3\frac{1}{7}$.

Solution :

Given; radius (r) = 14 cm and $\pi = \dfrac{22}{7}$

Circumference of the circle $= 2\pi r$

$= 2 \times \dfrac{22}{7} \times 14$ cm = **88 cm** **(Ans.)**

Example 8 :

Find the radius of the circle whose circumference is equal to the sum of the circumferences of two smaller circles having radii 6 cm and 9 cm respectively.

Solution :

For circle with radius = 6 cm

circumference $= 2\pi r$

$= 2 \times \dfrac{22}{7} \times 6\text{ cm} = \dfrac{264}{7}\text{ cm}$

For circle with radius = 9 cm

circumference $= 2\pi r$

$= 2 \times \dfrac{22}{7} \times 9\text{ cm} = \dfrac{396}{7}\text{ cm}$

Sum of the circumferences of these two circles

$$= \frac{264}{7} \text{ cm} + \frac{396}{7} \text{ cm} = \frac{660}{7} \text{ cm}$$

If the required radius = R cm

$\Rightarrow$ its circumference = $2\pi R$

$$= 2 \times \frac{22}{7} \times R \text{ cm} = \frac{44}{7} R \text{ cm}$$

Given $\frac{44}{7} R = \frac{660}{7} \Rightarrow R = \frac{7}{44} \times \frac{660}{7} \text{ cm} = 15 \text{ cm}$

$\therefore$ **Required radius = 15 cm** **(Ans.)**

Alternative method :

If the required radius = R cm

Circumference of the circle with radius R cm

= Sum of the circumferences of the circles with radii 6 cm and 9 cm

$\Rightarrow$ $2\pi R = 2\pi \times 6 + 2\pi \times 9$

Dividing each term by 2π, we get :

$$R = 6 + 9 = 15$$

$\therefore$ **Required radius = 15 cm** **(Ans.)**

Example 9 :

Find the diameter of the circle whose circumference is equal to the sum of the circumferences of the circles with radii 6 cm, 10 cm and 15 cm respectively.

Solution :

Let the radius of the circle = R cm

$\therefore$ $2\pi R = 2\pi \times 6 + 2\pi \times 10 + 2\pi \times 15$

On dividing each term by 2π, we get :

$$R = 6 + 10 + 15 = 31$$

$\therefore$ Radius of the circle obtained = 31 cm

And, its **diameter** = 2 × radius

= 2 × 31 cm = **62 cm** **(Ans.)**

Example 10 :

Find the radius of the circle whose circumference is five times the circumference of the circle with diameter 18 cm.

Solution :

Diameter of the given circle = 18 cm

$\therefore$ Radius of the given circle = $\frac{18}{2}$ cm [Diameter = 2 × radius]

circumference of the given circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 9 \text{ cm} = \frac{396}{7} \text{ cm}$$

circumference of the required circle $= 5 \times \frac{396}{7}$ cm $= \frac{1980}{7}$ cm

If the radius of the required circle = R cm

Its circumference $= 2\pi R$

$= 2 \times \frac{22}{7} \times R \text{ cm} = \frac{44}{7} \times R \text{ cm}$

Given : $\frac{44}{7} \times R = \frac{1980}{7}$

$\Rightarrow \quad R = \frac{1980}{7} \times \frac{7}{44} = 45$

$\therefore$ **Required radius = 45 cm** **(Ans.)**

Alternative method :

Circumference of the required circle = 5 × circumference of the given circle

$\Rightarrow \quad 2\pi R = 5 \times 2\pi \times 9$ *i.e.* $R = 5 \times 9$ cm = **45 cm** **(Ans.)**

EXERCISE 20(A)

1. The length and the breadth of a rectangular plot are 135 m and 65 m. Find, its perimeter and the cost of fencing it at the rate of ₹ 60 per m.

2. The length and breadth of a rectangular field are in the ratio 7 : 4. If its perimeter is 440 m, find its length and breadth.
Also, find the cost of fencing it @ ₹ 150 per m.

3. The length of a rectangular field is 30 m and its diagonal is 34 m. Find the breadth of the field and its perimeter.

4. The diagonal of a square is $12\sqrt{2}$ cm. Find its perimeter.

5. Find the perimeter of a rectangle whose length = 22.5 m and breadth = 16 dm.

6. Find the perimeter of a rectangle with length = 24 cm and diagonal = 25 cm

7. The length and breadth of a rectangular piece of land are in the ratio of 5 : 3. If the total cost of fencing it at the rate of ₹ 48 per metre is ₹ 19,200, find its length and breadth.

8. A wire is in the shape of a square of side 20 cm. If the wire is bent into a rectangle of length 24 cm, find its breadth

9. If P = perimeter of a rectangle, l = its length and b = its breadth; find :

(i) P, if $l = 38$ cm and $b = 27$ cm

(ii) b, if P = 88 cm and $l = 24$ cm

(iii) l, if P = 96 m and $b = 28$ m

10. The cost of fencing a square field at the rate of ₹ 75 per meter is ₹ 67,500. Find the perimeter and the side of the square field.

11. The length and the breadth of a rectangle are 36 cm and 28 cm. If its perimeter is equal to the perimeter of a square, find the side of the square.

12. The radius of a circle is 21 cm. Find its circumference (Take $\pi = 3\frac{1}{7}$).

13. The circumference of a circle is 440 cm. Find its radius and diameter. (Take $\pi = \frac{22}{7}$)

14. The diameter of a circular field is 56 m. Find its circumference and cost of fencing it at the rate of ₹ 80 per m. (Take $\pi = \frac{22}{7}$).

15. The radii of two circles are 20 cm and 13 cm. Find the difference between their circumferences. (Take $\pi = \frac{22}{7}$).

16. The diameter of a circle is 42 cm, find its perimeter. If the perimeter of the circle is doubled, what will be the radius of the new circle. (Take $\pi = \frac{22}{7}$).

17. The perimeter of a square and the circumference of a circle are equal. If the length of each side of the square is 22 cm, find :
    (i) perimeter of the square. (ii) circumference of the circle.
    (iii) radius of the circle.

18. Find the radius of the circle whose circumference is equal to the sum of the circumferences of the circles having radii 15 cm and 8 cm.

19. Find the diameter of a circle whose circumference is equal to the sum of circumferences of circles with radii 10 cm, 12 cm and 18 cm.

20. The circumference of a circle is eight times the circumference of the circle with radius 12 cm. Find its diameter.

21. The radii of two circles are in the ratio 3 : 5, find the ratio between their circumferences.

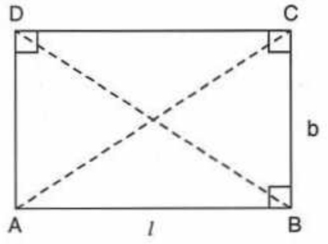
22. The circumferences of two circles are in the ratio 5 :7, find the ratio between their radii.

23. The perimeters of two squares are in the ratio 8 : 15, find the ratio between the lengths of their sides.

24. The lengths of the sides of two squares are in the ratio 8 :15, find the ratio between their perimeters.

25. Each side of a square is 44 cm. Find its perimeter. If this perimeter is equal to the circumference of a circle, find the radius of the circle.

20.4 AREA OF SOME PLANE FIGURES



1. Rectangle

(i) **Area** = length × breadth

$= AB \times BC = l \times b$

(ii) $AC^2 = AB^2 + BC^2$

$= l^2 + b^2 = BD^2$

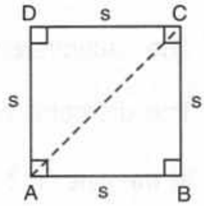
$\therefore$ Diagonal AC = diagonal BD = $\sqrt{l^2 + b^2}$.

2. Square

(i) **Area** = $(\text{side})^2 = s^2$

(ii) Diagonal AC = diagonal BD = $s\sqrt{2}$.

Also, (iii) **Area** = $\frac{1}{2} \times (\text{diagonal})^2$.



3. Triangle

(i) If a, b and c are the sides of a triangle and s is its semi-perimeter,

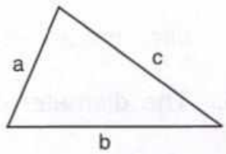
perimeter of the triangle = a + b + c

i.e. $2s = a + b + c$

$\Rightarrow$ Semi-perimeter of the triange is :

$$s = \frac{a+b+c}{2}$$

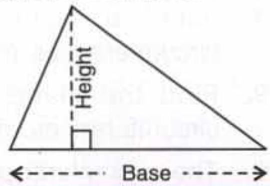
And, area of this triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

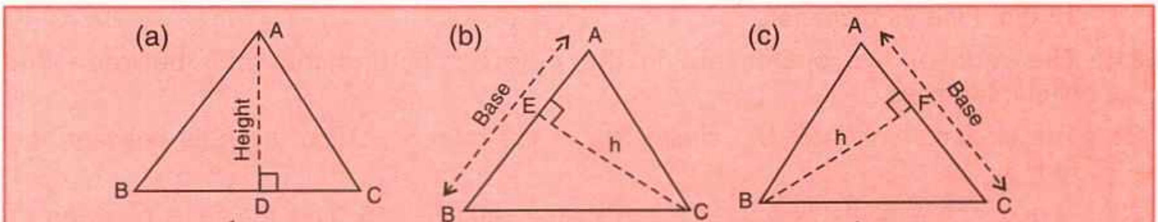


This formula of finding the area of the triangle is known as **Heron's formula**.

(ii) When base and height (altitude) of a triangle are known then :

its area = $\frac{1}{2} \times \text{base} \times \text{height}$



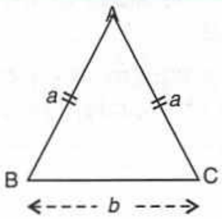


Area = $\frac{1}{2} \times BC \times AD$

Area = $\frac{1}{2} \times AB \times CE$

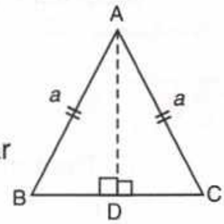
Area = $\frac{1}{2} \times AC \times BF$

(iii) In case of an isosceles triangle ABC, if side AB = side AC = a and base BC = b; its area = $\frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$.



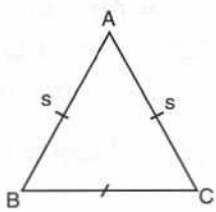
In an isosceles triangle ABC, if AB = AC and AD is perpendicular to base BC,

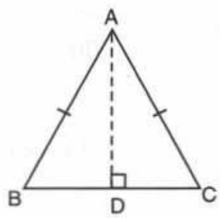
AD bisects BC *i.e.* BD = CD.



(iv) In case of an equilateral triangle ABC, its area = $\frac{\sqrt{3}}{4}\ (\text{side})^2 = \frac{\sqrt{3}}{4} \times s^2$.

If AD is perpendicular to BC, D is mid-point of BC *i.e.* BD = CD.





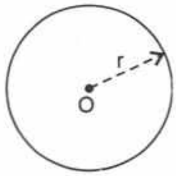
4. Circle

(i) Diameter of the circle = 2 × its radius

(ii) Radius = $\frac{\text{Diameter}}{2}$

(iii) Area = $\pi \times (\text{radius})^2$

$= \pi r^2$, where $\pi = 3\frac{1}{7} = \frac{22}{7}$.



Example 11 :

The length and breadth of a rectangular field are 240 m and 150 m respectively. Find the area of the field and cost of turfing it at ₹ 30 per square meter (m^2).

Solution :

For the field : length = 240 m and breadth = 150 m

∴ **Area of the field** = length × breadth

= 240 m × 150 m = **36000 m^2** **(Ans.)**

Cost of turfing the field = Area of the field x rate of turfing

= 36000 m^2 x ₹ 30 per m^2

= **₹ 1080000** **(Ans.)**

Example 12 :

The perimeter of a rectangle is 240 cm and its length and breadth are in the ratio 5 : 3, find the area of the rectangle.

Solution :

Let length = 5x cm and breadth = 3x cm

2(length + breadth) = perimeter

⇒ 2 x (5x + 3x) = 240

⇒ $2 \times 8x = 240$ and $x = \frac{240}{16} = 15$

∴ Length = 5x = 5 X 15 cm = 75 cm and

breadth = 3x

= 3 x 15 cm = 45 cm

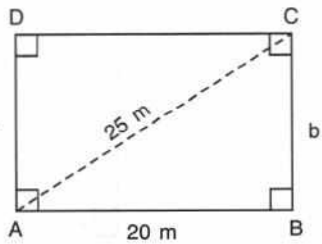
∴ Area of the rectangle = length × breadth

= 75 cm × 45 cm= **3375 cm^2** **(Ans.)**

Example 13 :

A rectangular field has length = 20m and diagonal = 25 m. Find the area of the field.

Solution :



In right-angled triangle ABC,

$AB^2 + BC^2 = AC^2$

⇒ $(20)^2 + BC^2 = (25)^2$

⇒ $400 + BC^2 = 625$ i.e. $BC^2 = 625 - 400 = 225$

∴ $BC = \sqrt{225}$ m = 15 m

Area of the rectangular field = length × breadth

= AB × BC

= 20 m × 15 m = **300 m²** **(Ans.)**

Example 14 :

The length of the diagonal of a square is $12\sqrt{2}$ cm. Find its area.

Solution :

Diagonal of a square = its side × $\sqrt{2}$

$\Rightarrow$ side $\sqrt{2}$ = $12\sqrt{2}$ i.e. side = 12 cm

$\therefore$ **Area of the square** = $(\text{side})^2$

= $(12 \text{ cm})^2$ = **144 cm^2** **(Ans.)**

Example 15 :

The cost of fencing a square plot is ₹ 32400 at the rate of ₹ 90 per meter. Find the cost of levelling the surface of the plot at the rate of ₹ 20 per square meter.

Solution :

Length of the fence = $\frac{\text{Cost of the fence}}{\text{Rate of the fencing}}$

= $\frac{32400}{90}$ m = 360 m

$\Rightarrow$ length of the boundary = 360 m

$\therefore$ Perimeter of the plot = 4 × side

$\Rightarrow$ 360 m = 4 × side

$\Rightarrow$ side of the square = $\frac{360}{4}$ m = 90 m

Area of the plot = $(\text{side})^2$

= $(90 \text{ m})^2$ = **8100 m^2**

$\Rightarrow$ **Cost of levelling the surface of the plot**

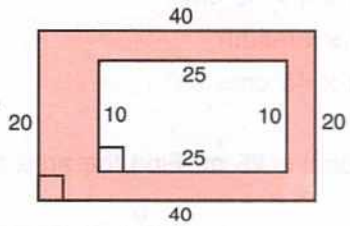
= Area of the plot × rate of levelling

= 8100 × ₹ 20 = **₹ 162000** **(Ans.)**

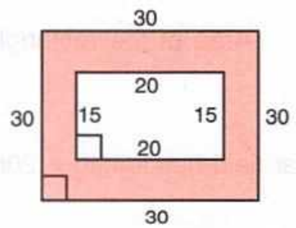
Example 16 :

In each of the following figures, find the area of the shaded portion. (all measurements are in cm).

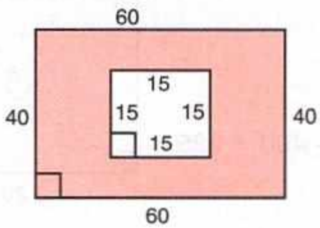
(i)



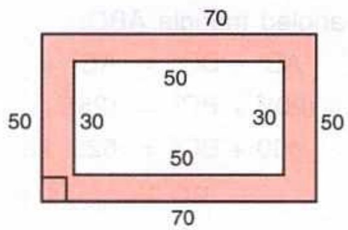
(ii)



(iii)



(iv)



Solution :

(i) **Area of the outer rectangle** = length x breadth

$= 40 \text{ cm} \times 20 \text{ cm} = 800 \text{ cm}^2$

And, **area of the inner rectangle** = $25 \text{ cm} \times 10 \text{ cm}$

$= \mathbf{250 \text{ cm}^2}$ **(Ans.)**

$\therefore$ **Area of the shaded portion** = Outer area – inner area

$= 800 \text{ cm}^2 - 250 \text{ cm}^2 = \mathbf{550 \text{ cm}^2}$ **(Ans.)**

(II) **Area of the shaded portion** = Area of outer figure – area of inner figure

$= 30 \times 30 \text{ cm}^2 - 20 \times 15 \text{ cm}^2$

$= 900 \text{ cm}^2 - 300 \text{ cm}^2 = \mathbf{600 \text{ cm}^2}$ **(Ans.)**

(III) **Area of the shaded portion** = $60 \times 40 \text{ cm}^2 - 15 \times 15 \text{ cm}^2$

$= 2400 \text{ cm}^2 - 225 \text{ cm}^2 = \mathbf{2175 \text{ cm}^2}$ **(Ans.)**

(IV) **Area of the shaded portion** = $70 \times 50 \text{ cm}^2 - 50 \times 30 \text{ cm}^2$

$= 3500 \text{ cm}^2 - 1500 \text{ cm}^2 = \mathbf{2000 \text{ cm}^2}$ **(Ans.)**

Example 17 :

Find the area of a triangle with base = 32 cm and height = 20 cm.

Solution :

Area of the triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 32 \text{ cm} \times 20 \text{ cm} = \mathbf{320 \text{ cm}^2}$ **(Ans.)**

Example 18 :

The base of a triangular plot is twice of it height . If the cost of levelling it at the rate of ₹ 25 per square meter is ₹ 11,025 find the base and the height of the plot.

Solution :

Area of the plot = $\frac{\text{Total cost of levelling}}{\text{Rate of levelling}}$

$= \frac{11,025}{25} \text{ m}^2 = 441 \text{ m}^2$

Let height of the triangular plot = x m

$\therefore$ Its base = 2x m

and, its area = $\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 2x \times x \text{ m}^2 = x^2 \text{ m}^2$

Given $x^2 = 441 \Rightarrow x = \sqrt{441} \text{ m} = 21 \text{ m}$

$\therefore$ **Required base** = $2x \text{ m} = 2 \times 21 \text{ m} = \mathbf{42 \text{ m}}$ **(Ans.)**

and, **required height** = $x \text{ m} = \mathbf{21 \text{ m}}$ **(Ans.)**

Example 19 :

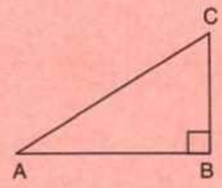
Find the area of a right-angled triangle whose legs are 20 cm and 28 cm.

Solution :

Consider a right-angled triangle ABC,as given alongside in which $\angle B = 90°$, AC = hypotenuse whereas AB and BC are its two legs (sides).

Area of such a triangle $= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times$ product of legs.



Given, legs (*i.e.*, sides of the triangle, other than hypotenuse) of the right-angled triangle are 20 cm and 28 cm

$\therefore$ Area $= \frac{1}{2} \times$ product of legs

$= \frac{1}{2} \times 20 \text{ cm} \times 28 \text{ cm} = $ **280 cm²** **(Ans)**

Example 20 :

The sides of a triangle are 39 cm, 25 cm and 56 cm Find its area.

Solution :

Let a, b and c represent sides of the triangle so that a = 39 cm, b = 25 cm and c = 56 cm

$\therefore a + b + c = 39 \text{ cm} + 25 \text{ cm} + 56 \text{ cm} = 120 \text{ cm}$

and $s = \frac{a+b+c}{2} = \frac{120}{2} \text{ cm} = 60 \text{ cm}$

Area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-39)(60-25)(60-56)} \text{ cm}^2$

$= \sqrt{60 \times 21 \times 35 \times 4} \text{ cm}^2$

$= \sqrt{2 \times 2 \times 3 \times 5 \times 3 \times 7 \times 5 \times 7 \times 2 \times 2} \text{ cm}^2$

$= 2 \times 3 \times 5 \times 7 \times 2 \text{ cm}^2 = $ **420 cm²** **(Ans.)**

Example 21 :

Find the area of an equilateral triangle whose each side is 20 cm (Take $\sqrt{3} = 1.73$).

Solution :

Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{1 \cdot 73}{4} \times 20 \times 20 \text{ cm}^2 = $ **173 cm²** **(Ans.)**

Example 22 :

The base of an isosceles triangle is 12 cm and its perimeter is 32 cm. Find its area.

Solution :

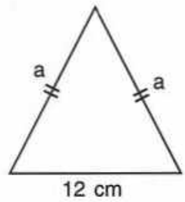
Let each of the equal sides be *a* cm

$\therefore$ Perimeter = $(a + a + 12)$ cm

= $(2a + 12)$ cm

Given : Perimeter = 32 cm

$\Rightarrow$ $2a + 12 = 32$



$\Rightarrow 2a = 32 - 12 = 20$ and $a = \frac{20}{2}$ cm = 10 cm

Given : Base, $b = 12$ cm

$\therefore$ **Area of the triangle** $= \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$

$= \frac{1}{4} \times 12 \times \sqrt{4 \times 10^2 - 12^2}$ cm²

$= 3 \times \sqrt{400 - 144}$ cm²

$= 3 \times \sqrt{256}$ cm² $= 3 \times 16$ cm² $=$ **48 cm²** **(Ans.)**

Example 23 :

(i) Find the area of the circle of radius 20 cm (Take $\pi = 3.14$)

(II) Find the area of a circle of radius 21 cm (Take $\pi = \frac{22}{7}$)

Solution :

(i) **Area of the circle** $= \pi r^2$ (Given : r = 20 cm)

$= 3.14 \times 20 \times 20$ cm² $=$ **1256 cm²** **(Ans.)**

(ii) Area of the circle $= \pi r^2$

$= \frac{22}{7} \times (21)^2$ cm² $= \frac{22}{7} \times 21 \times 21$ cm² $=$ **1386 cm²** **(Ans.)**

Example 24 :

The circumference of a circle is 88 cm. Find its area. (Take $\pi = \frac{22}{7}$)

Solution :

Let radius of the circle be *r* cm

$\therefore$ $2\pi r = 88$ *i.e.* $2 \times \frac{22}{7} \times r = 88$

$\Rightarrow$ $r = 88 \times \frac{7}{2 \times 22}$ cm $= 14$ cm

Hence, **area of the circle**

$$= \pi r^2$$

$$= \frac{22}{7} \times 14 \times 14 \text{ cm}^2 = \mathbf{616 \text{ cm}^2} \quad \textbf{(Ans.)}$$

Example 25 :

The area of a circular disc is 38.5 cm². Find it circumference (Take $\pi = \frac{22}{7}$).

Solution :

Let the radius of the circular disc = r cm

$\therefore \quad \pi r^2 = 38.5$

$\Rightarrow \quad \frac{22}{7} \times r^2 = 38.5$

$\Rightarrow \quad r^2 = 38.5 \times \frac{7}{22} = \frac{49}{4}$

$\Rightarrow \quad r = \sqrt{12.25}$ cm = 3·5 cm

Now , **circumference of the circle**

$$= 2\pi r$$

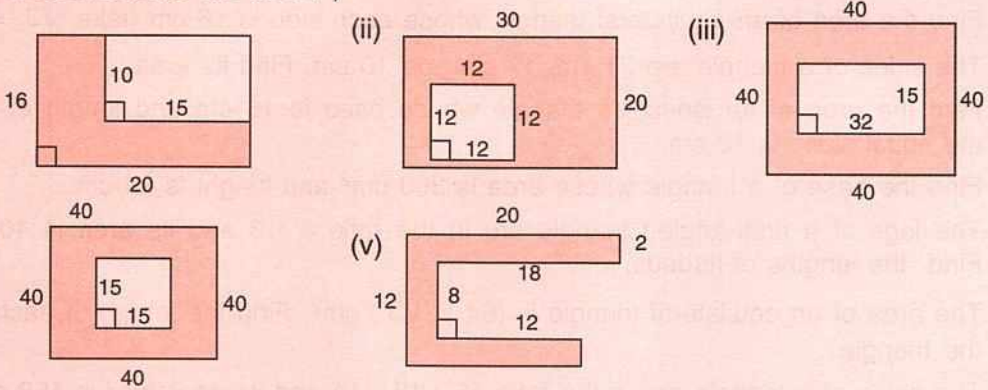
$$= 2 \times \frac{22}{7} \times 3.5 \text{ cm} = \mathbf{22 \text{ cm}} \quad \textbf{(Ans.)}$$

EXERCISE 20(B)

1. Find the area of a rectangle whose length and breadth are 25 cm and 16 cm.
2. The diagonal of a rectangular board is 1 m and its length is 96 cm. Find the area of the board.
3. The sides of a rectangular park are in the ratio 4 : 3. If its area is 1728 m^2, find :
   (i) its perimeter
   (ii) cost of fencing it at the rate of ₹ 40 per metre.
4. A floor is 40 m long and 15 m broad. It is covered with tiles, each measuring 60 cm by 50 cm .Find the number of tiles required to cover the floor.
5. The length and breadth of a rectangular piece of land are in the ratio 5 : 3. If the total cost of fencing it at the rate of ₹ 24 per metre is ₹ 9600, find its :
   (i) length and breadth
   (ii) area
   (iii) cost of levelling at the rate of ₹ 60 per m^2.
6. Find the area of the square whose perimeter is 56 cm.
7. A square lawn is surrounded by a path 2.5 m wide. If the area of the path is 165 m^2, find the area of the lawn.

8. For each figure ,given below ,find the area of shaded region:

(All measurements are in cm)

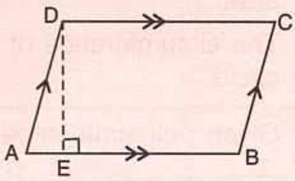


9. One side of a parallelogram is 20 cm and its distance from the opposite side is 16 cm. Find the area of the parallelogram.

Area of the parallelogram

= base x height

= AB x DE



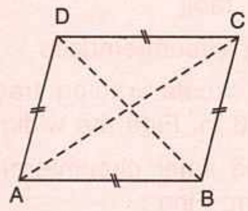
10. The base of a parallelogram is thrice its height. If its area is 768 cm^2, find the base and the height of the parallelogram.

11. Find the area of the rhombus, if its diagonals are 30 cm and 24 cm.

If AC and BD are the diagonals of a rhombus, its

$$\text{Area} = \frac{1}{2} \times \text{product of it diagonals}$$

$$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times d_1 \times d_2$$



12. If the area of a rhombus is 112 cm^2 and one of its diagonals is 14 cm , find its other diagonal.

13. One side of a parallelogram is 18 cm and its area is 153 cm^2. Find the distance of the given side from its opposite side.

14. The adjacent sides of a parallelogram are 15 cm and 10 cm. If the distance between the longer sides is 6 cm, find the distance between the shorter sides.

15. The area of a rhombus is 84 cm^2 and its perimeter is 56 cm. Find its height.

16. Find the area of a triangle whose base is 30 cm and height is 18 cm.

17. Find the height of a triangle whose base is 18 cm and area is 270 cm^2.

18. The area of a right-angled triangle is 160 cm^2. If its one leg is 16 cm long, find the length of the other leg.

19. Find the area of a right-angled triangle whose hypotenuse is 13 cm long and one of its legs is 12 cm long.

20. Find the area of an equilateral triangle whose each side is 16 cm (take $\sqrt{3}$ =1·73).

21. The sides of a triangle are 21 cm, 17 cm and 10 cm. Find its area.

22. Find the area of an isosceles triangle whose base is 16 cm and length of each of the equal sides is 10 cm.

23. Find the base of a triangle whose area is 360 cm^2 and height is 24 cm.

24. The legs of a right-angled triangle are in the ratio 4 : 3 and its area is 4056 cm^2. Find the lengths of its legs.

25. The area of an equilateral triangle is $(64 \times \sqrt{3})$ cm^2. Find the length of each side of the triangle.

26. The sides of a triangle are in the ratio 15 : 13 : 14 and its perimeter is 168 cm. Find the area of the triangle.

27. The diameter of a circle is 20 cm. Taking π = 3.14, find its circumference and its area.

28. The circumference of a circle exceeds its diameter by 18 cm. Find the radius of the circle.

> Given : circumference − diameter =18
>
> $\Rightarrow \quad 2\pi r - 2r = 18$

29. The ratio between the radii of two circles is 5 : 7. Find the ratio between their:
    (i) circumferences (ii) areas

30. The ratio between the areas of two circles is 16 : 9. Find the ratio between their:
    (i) radii (ii) diameters
    (iii) circumferences

31. A circular racing track has inner circumference 528 m and outer circumference 616 m. Find the width of the track.

32. The inner circumference of a circular track is 264 m and the width of the track is 7 m. Find :
    (i) the radius of the inner track.
    (ii) the radius of the outer circumference.
    (iii) the length of the outer circumference.
    (iv) the cost of fencing the outer circumference at the rate of ₹ 50 per m.

33. The diameter of the wheel of a car is 63 cm. How much distance will the car move during 2000 revolutions of its wheel (in kilometres).

> In one revolution of the wheel, car moves = circumference of the wheel
>
> $= 2\pi r$
>
> In n revolutions, car will move = $2\pi r \times n$

34. The diameter of the wheel of a car is 70 cm. How many revolutions will it make to travel one kilometre ?

35. A metal wire, when bent in the form of a square of largest area, encloses an area of 484 cm^2. Find the length of the wire.

If the same wire is bent to a largest circle, find :

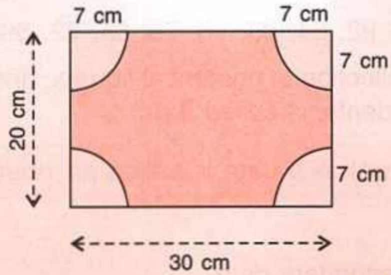
(i) radius of the circle formed.

(ii) area of the circle.

36. A wire is along the boundary of a circle with radius 28 cm. If the same wire is bent in the form of a square, find the area of the square formed.

37. The length and the breadth of a rectangular paper are 35cm and 22cm .Find the area of the largest circle which can be cut out of this paper.

38. From each corner of a rectangular paper (30 cm x 20 cm) a quadrant of a circle of radius 7 cm is cut. Find the area of the remaining paper *i.e.*, shaded portion



Theme – 5
DATA HANDLING
(Statistics)

DATA HANDLING 21

21.1 DATA

Statistics is the science that deals with the collection, presentation, analysis and interpretation of numerical information. This helps to organise our experiences and draw inferences from them.

Consider the marks (out of 100) scored by 20 students of a class as given below :

55, 65, 20, 40, 35, 70, 90, 92, 84, 85, 70, 75, 65, 72, 80, 78, 64, 88, 78 and 76.

When organised, such a collection of numerical figures, giving some specific information about the performance of 20 students, is called **data**.

Each figure (numerical figure) in a data is called an **observation**.

Data are of two types :

(i) Primary data (ii) Secondary data

- **Primary data :** These are the data collected by a person directly without taking the help of any source for a specific purpose.
- **Secondary data :** These are the data borrowed by a person from some other source to be used further.

The collected data in its original form is called **raw data**.

Let the marks obtained by 30 students of class VII in a class test, out of 50 marks, be :

43, 29, 9, 37, 23, 25, 16, 45, 16, 25, 23, 5, 14, 12, 16, 21, 21, 23, 21, 44, 45, 16, 45, 37, 23, 25, 37, 9, 5 and 25.

The data in this form is called raw data or ungrouped data.

When the above data are written in ascending order, we get :

5, 5, 9, 9, 12, 14, 16, 16, 16, 16, 21, 21, 21, 23, 23, 23, 23, 25, 25, 25, 25, 29, 37, 37, 37, 43, 44, 45, 45 and 45.

And, if written in descending order, we get :

45, 45, 45, 44, 43, 37, 37, 37, 29, 25, 25, 25, 25, 23, 23, 23, 23, 21, 21, 21, 16, 16, 16, 16, 14, 12, 9, 9, 5 and 5.

The raw data when put in ascending or descending order of magnitude is called an **array** or **arrayed data**.

21.2 TABULATION OF DATA

Arranging the data in a systematic form, in the form of a table, is called tabulation of data. For tabulating the given data in the form of a table, we adopt the following steps :

Step 1 : Make three columns; **first column** headed by **marks**, **second column** headed by **tally marks** and **third column** headed by **number of students**.

Marks	Tally marks	No. of students
5	II	2
9	II	2
12	I	1
14	I	1
16	IIII	4
21	III	3
23	IIII	4
25	IIII	4
29	I	1
37	III	3
43	I	1
44	I	1
45	III	3

Step 2 : In the first column write marks from lowest to highest.

Step 3 : Since, 5 appears 2 times in the given data, mark two strokes in the second column against the mark 5. Since16 appears 4 times in the given data , mark four strokes in the second column against the mark 16. Continue in the similar manner till the second column of the table formed is completed as shown above.

Step 4 : Count the number of tally marks and write their number in the third column.

This way of representation of data is known as frequency distribution. The number of times an observation occurs in the given data is called its **frequency**.

For example, in the above table

for mark 5, frequency is 2

for mark 16 , frequency is 4

for mark 21, frequency is 3 and so on.

The table, as obtained above, is called a **frequency distribution table**.

Using such a table, one can easily understand the information contained in the raw data.

The raw data, given below shows the number of members in 20 families.

4, 4, 5, 6, 7, 6, 5, 4, 4, 7, 8, 5, 7, 5, 5, 4, 6, 6, 5 and 8.

The frequency distribution for these data will be as shown below :

Numbers of members	Tally marks	Frequency
4	~~IIII~~	5
5	~~IIII~~ I	6
6	IIII	4
7	III	3
8	II	2

Example 1 :

Given below are the ages (in years) of 25 persons in a small locality. Prepare a frequency distribution table.

40, 41, 41, 39, 42, 42, 41, 40, 40, 41, 41, 42, 40, 41, 41, 39, 41, 40, 39, 40, 41, 41, 40, 39, and 40.

Solution :

Required frequency distribution table will be as shown below:

Age (in years)	Tally marks	Frequency
39	IIII	4
40	~~IIII~~ III	8
41	~~IIII~~ ~~IIII~~	10
42	III	3
Total		**25**

EXERCISE 21(A)

1. Consider the following numbers :
   68, 76, 63, 75, 93, 83, 70, 115, 82, 105, 90, 103, 92, 52, 99, 73, 75, 63, 77 and 71.
   (i) Arrange these numbers in ascending order.
   (ii) What is the range of these numbers ?

   Range of given numbers = Largest number – smallest number

2. Represent the following data in the form of a frequency distribution table :
   16, 17, 21, 20, 16, 20, 16, 18, 17, 21, 17, 18, 19, 17, 15, 15, 19, 19, 18, 17, 17, 15, 15, 16, 17, 17, 19, 18, 17, 16, 15, 20, 16, 17, 19, 18, 19, 16, 21 and 17.

3. A die was thrown 20 times and following scores were recorded.
   2, 1, 5, 2, 4, 3, 6, 1, 4, 2, 5, 1, 6, 2, 6, 3, 5, 4, 1 and 3.
   Prepare a frequency table for the scores .

4. Following data shows the weekly wages (in ₹) of 10 workers in a factory.
   3500, 4250, 4000, 4250, 4000, 3750 , 4750 , 4000, 4250 and 4000
   (i) Prepare a frequency distribution table. (ii) What is the range of wages (in ₹) ?
   (iii) How many workers are getting the maximum wages ?

5. The marks obtained by 40 students of a class are given below :
   80, 10, 30, 70, 60, 50, 50, 40, 40, 20, 40, 90, 50, 30, 70, 10, 60, 50, 20, 70, 70, 30, 80, 40 , 20, 80, 90, 50, 80, 60, 70, 40, 50, 60, 90, 60, 40, 40, 60 and 60
   (i) Construct a frequency distribution table.
   (ii) Find how many students have marks equal to or more than 70 ?
   (iii) How many students obtained marks below 40 ?

6. Arrange the following data in descending order :
   3·3, 3·2, 3·1, 3·7, 3·6, 4·0, 3·5, 3·9, 3·8, 4·1, 3·5, 3·8, 3·7, 3·9 and 3·4.
   (i) Determine the range. (ii) How many numbers are less than 3·5 ?
   (iii) How many numbers are 3·8 or above ?

Choosing the data to collect for a hypothesis testing : In this topic, we test whether the information obtained from a sample of data is applicable to the entire data or not. For hypothesis testing, we generally find mean, median and mode.

21.3 MEAN (Arithmetic Mean or Average)

The mean of a group of observations is the value which is equally shared out among all the observations. The value of the mean is not necessarily from the observations under consideration.

The mean of a group of observations is :

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

For example :

1. For the mean of 26, 25, 40 , 36, 50 and 45.

$$\text{Sum of all observations} = 26 + 25 + 40 + 36 + 50 + 45 = 222$$

and, number of observations = 6

$$\therefore \quad \textbf{Required mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}} = \frac{222}{6} = \textbf{37}$$ **(Ans.)**

> 1. Mean 37 does not belong to the given observations.
> 2. Observations with values less than mean (37) are 25, 26 and 36. Number of these observations is 3. Similarly, observations with values greater than mean (37) are 40, 50 and 45. Number of these observations is also 3.
>
> Thus, mean (37) divides the given observations equally.

2. For the mean of 01, 06, 16, 10, 09, 23, 18, 27, 20, 17, 02, 22, 15, 12, 21, 25 and 11.

$$\text{Sum of all observations} = 01 + 06 + 16 + 10 + 09 + 23 + 18 + 27 + 20 + 17 + 02 + 22 + 15 + 12 + 21 + 25 + 11 = 255$$

and, number of observations = 17

$$\therefore \quad \textbf{Required mean} = \frac{255}{17} = \textbf{15}$$ **(Ans.)**

3. The mean of 55, 56, 70, 66 , 80 and 75

$$= \frac{55+56+70+66+80+75}{6} = \frac{402}{6} = \textbf{67}$$ **(Ans.)**

4. The mean of 124 cm, 132 cm, 131 cm, 138 cm and 135 cm.

$$= \frac{124+132+131+138+135}{5} \text{ cm} = \frac{660}{5} \text{ cm} = \textbf{132 cm}$$ **(Ans.)**

5. The mean of first 8 natural numbers.

$$= \frac{1+2+3+4+5+6+7+8}{8} = \frac{36}{8} = \textbf{4·5}$$ **(Ans.)**

6. The mean of first seven whole numbers.

$$= \frac{0+1+2+3+4+5+6}{7}$$

$$= \frac{21}{7} = 3$$ **(Ans.)**

7. The mean of 42, 51, 38, 64, 45, 36, 33, 43, 48 and 50.

$$= \frac{1}{10}(42 + 51 + 38 + 64 + 45 + 36 + 33 + 43 + 48 + 50)$$

$$= \frac{450}{10} = 45$$ **(Ans.)**

Example 2 :

If the mean of 8, 6, 9, $p + 15$ and $p + 2$ is 10, find the value of p.

Solution :

$$\frac{8+6+9+(p+15)+(p+2)}{5} = 10$$

$\Rightarrow$ $40 + 2p = 50$ *i.e.* $2p = 50 - 40 \Rightarrow p = 5$ **(Ans.)**

21.4 SOME IMPORTANT PROPERTIES OF MEAN

Property 1 :

Let the mean of n observations be m. If each observation under consideration is increased by number x, the new mean of observations will be $m + x$.

Let the mean of 20 observations is 17. If each observation is increased by 8.

new mean $= 17 + 8 = 25$

Property 2 :

Let the mean of n observations be M. If each observation under consideration is decreased by number x, the mean of the resulting observations will be $M - x$,

Let the mean of 35 observations is 23. If 5 is subracted from each observation.

new mean $= 23 - 5 = 18$

Property 3 :

Let the mean of n observations be m. If each observation is multiplied by x, the new mean of the observations will be $m \times x$.

Let the mean of 7 observations be 54. If each observation is multiplied by 6,

new mean $= 54 \times 6 = 324$

Property 4 :

Let the mean of n observations be m. If each observation is divided by x, the new mean of observations will be $m \div x$.

Let the mean of 15 observations is 54. If each observation is divided by 6.

new mean $= 54 \div 6 = \frac{54}{6} = 9$

Property 5 :

If mean of n observations is M, the sum (total) of all the observations = $M \times n$.

Let the mean of 30 observations is 12.

Then the sum of all the observations = 30 x 12 = **360**

Example 3 :

The mean of 5 observations $x + 3$, $x + 5$, $x + 7$, $x + 9$ and $x + 11$ is 14. Find :

(i) the value of x.

(ii) the mean of last 3 observations.

Solution :

(i) Given : $\frac{(x+3)+(x+5)+(x+7)+(x+9)+(x+11)}{5} = 14$

$\Rightarrow \quad 5x + 35 = 14 \times 5$

$\Rightarrow \quad 5x = 70 - 35 = 35 \quad \Rightarrow x = \frac{35}{5} = 7$ **(Ans.)**

(ii) Since last 3 observations are

$x + 7$, $x + 9$ and $x + 11$

$= 7 + 7$, $7 + 9$ and $7 + 11 = 14$, 16 and 18

$\therefore$ **Required mean** $= \frac{14+16+18}{3} = \frac{48}{3} = 16$ **(Ans.)**

Example 4 :

The mean of 40 observations is 160. It is found that the value of 165 is wrongly copied as 125. Find the correct mean.

Solution :

Mean of 40 observations = 160

$\Rightarrow$ Total of 40 observations = 160 x 40 = 6400

$\Rightarrow$ Incorrect total of 40 observations = 6400

$\Rightarrow$ Correct total of 40 observations = 6400 − 125 + 165 = 6440

$\therefore$ **Correct mean** $= \frac{6440}{40} = 161$ **(Ans.)**

Example 5 :

The mean of 15 observations is 200. If one observation is excluded, the mean of remaining observations is 198. Find the value of the excluded observation.

Solution :

Mean of 15 observations = 200

$\Rightarrow$ Total sum of 15 observations = 200 x 15 = 3000

On excluding an observation, the mean of remaining 14 observations = 198

$\Rightarrow$ Total of remaining 14 observations = $198 \times 14 = 2772$

$\Rightarrow$ **Excluded observation** = Total of 15 observations − Total of 14 observations

= 3000 − 2772 = **228** **(Ans.)**

21.5 MEDIAN

Median is the value which lies in the middle of the arranged data (data written in ascending or descending order) with half of the observations below it and the other half above it. In other words, median of a group of observations is the value of the arrayed data which divides the group into two equal parts.

Thus for the ungrouped data, 5, 8 , 7 , 6 and 4, on writing them in ascending order we get 4, 5, 6, 7 and 8. Clearly, the value of the middle term is 6 *i.e.* median is 6, and we see that the median 6 divides the given observations into two equal groups, 4, 5 and 7, 8.

Example 6 :

Find the median of the following data :

(i) 15, 12, 10, 9, 8, 13, 17 (ii) 3, 5, 9, 10, 11, 4, 5, 8, 12, 15.

Solution :

Let the number of terms be n

1. If number of terms n is odd,

   $$\text{Median} = \left(\frac{n+1}{2}\right) \text{ term}$$

2. If number of terms n is even,

   $$\text{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{ term}\right]$$

(i) On arranging the given data in ascending order of their magnitudes, we get :
8, 9, 10, 12, 13, 15, 17

Since, the number of terms (n) is 7 and 7 is odd

$$\textbf{Median} = \left(\frac{n+1}{2}\right) \text{ term} = \left(\frac{7+1}{2}\right) \text{ term} = \text{4th term} = \mathbf{12}$$ **(Ans.)**

1. For finding the median of given data, arrange them in ascending or descenting order of their values.
2. Whether the given data be arranged in ascending or descending order, the value of their median is unique.

(ii) On arranging the given data in descending order of their magnitudes, we get :
15, 12, 11, 10, 9, 8, 5, 5, 4, 3

As the number of terms (n) is 10 and 10 is even :

$$\therefore \quad \textbf{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{ term}\right]$$

$$= \frac{1}{2}\left[\left(\frac{10}{2}\right)^{\text{th}} \text{ term} + \left(\frac{10}{2}+1\right)^{\text{th}} \text{ term}\right] = \frac{1}{2}[5^{\text{th}} \text{ term} + 6^{\text{th}} \text{ term}]$$

$$= \frac{1}{2}(9+8) = \frac{1}{2} \times 17 = \mathbf{8{\cdot}5}$$ **(Ans.)**

21.6 MODE

Mode of a set of observations is the value of the observation that occurs most frequently.

In a set of observations : 8, 3, 5, 6, 7, 8, 7, 5, 8, 8, 10, 12 and 15
8 occurs most frequently.

Therefore, for the given observations : **Mode = 8**

The number which appears maximum times in the given statistical data is called ***mode.***

In other words, mode is the number whose frequency is maximum.

Example 7 :

Find the mode of following data : 15, 20, 15, 30, 20, 20, 30, 15, 20, 20.

Solution :

Since, in the given data, the number 20 appears maximum times. ∴ **Mode = 20** **(Ans.)**

Example 8 :

Find the mode from the following frequency distribution :

Numbers	10	11	12	13	14	15	16	17	18
Frequency	7	15	20	25	30	9	8	12	24

Solution :

Since, the frequency of number 14 is maximum. ∴ **Mode = 14** **(Ans.)**

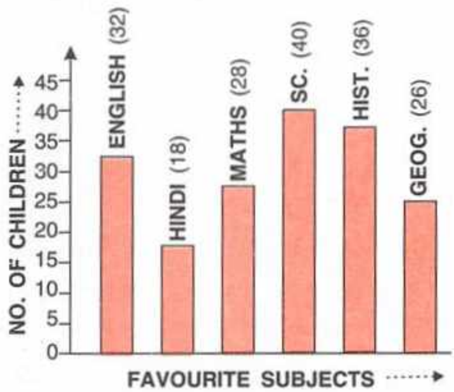
21.7 BAR GRAPH (COLUMN GRAPH) :

It is the simplest and most widely used graph in which the numerical data is represented by the height of rectangular bars of equal widths.

For example :

The following table gives the number of children classified according to their favourite subjects and the corresponding bar graph is drawn alongside :

Subject	No. of Children
English	32
Hindi	18
Mathematics	28
Science	40
History	36
Geography	26



For drawing this bar graph :

1. Names of the different subjects are taken along horizontal axis and number of children is taken along vertical axis.
2. The height of a bar is proportionally drawn according to the number it represents.
3. Bars drawn can be of any suitable width, but widths of all the bars must be the same.
4. The space between consecutive bars must also be same. It is *not necessary that this space between the consecutive bars is same as the width of the bars.*
5. These bars are normally shaded for better effect.

EXERCISE 21(B)

1. Find the mean of 53, 61, 60, 67 and 64.
2. Find the mean of first six natural numbers.
3. Find the mean of first ten odd natural numbers.
4. Find the mean of all factors of 10.
5. Find the mean of $x + 3$, $x + 5$, $x + 7$, $x + 9$ and $x + 11$.
6. If different values of variable x are 19·8, 15·4, 13·7, 11·71, 11·8, 12·6, 12·8, 18·6, 20·5 and 21·1, find the mean.

7. The mean of a certain number of observations is 32. Find the resulting mean, if each observation is,

(i) increased by 3 (ii) decreased by 7

(iii) multiplied by 2 (iv) divided by 0.5

(v) increased by 60% (vi) decreased by 20%

8. The pocket expenses (per day) of Anuj, during a certain week, from monday to saturday were ₹ 85.40, ₹ 88.00, ₹ 86.50, ₹ 84.75, ₹ 82.60 and ₹ 87.25. Find the mean pocket expenses per day.

9. If the mean of 8, 10, 7, $x + 2$ and 6 is 9, find the value of x.

10. Find the mean of first six multiples of 3.

11. Find the mean of first five prime numbers .

12. The mean of six numbers : $x - 5$, $x - 1$, x, $x + 2$, $x + 4$ and $x + 12$ is 15. Find the mean of first four numbers.

13. Find the mean of squares of first five whole numbers.

14. If the mean of 6, 4, 7, p and 10 is 8, find the value of p

15. Find the mean of first six multiples of 5.

16. The rainfall (in mm) in a city on 7 days of a certain week is recorded as follows :

Day :	Mon	Tue	Wed	Thus	Fri	Sat	Sun
Rainfall (in mm) :	0·5	2·7	2·6	0·5	2	5·8	1·5

Find the total and average (mean) rainfall for the week

17. The mean of marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find the correct mean.

18. The mean of five numbers is 27.If one number is excluded, the mean of remaining numbers is 25. Find the excluded number.

19. The mean of 5 numbers is 27. If one new number is included, the new mean is 25. Find the included number.

20. Mean of 5 numbers is 20 and mean of other 5 numbers is 30. Find the mean of all the 10 numbers taken together.

21. Find the median of :

(i) 5, 7, 9, 11, 15, 17, 2, 23 and 19

(ii) 9, 3, 20, 13, 0, 7 and 10

(iii) 18, 19, 20, 23, 22, 20, 17, 19, 25 and 21

(iv) 3·6, 9·4, 3·8, 5·6, 6·5, 8·9, 2·7, 10·8, 15·6, 1·9 and 7·6.

22. Find the mean and the mode for the following data :

Term	18	22	26	30	34	38
Frequency	3	5	10	2	8	2

23. Find the mode of :

(i) 5, 6, 9, 13, 6, 5, 6, 7, 6, 6, 3 (ii) 7, 7, 8, 10, 10, 11, 10, 13, 14

24. Find the mode of :

(i)

x	15	16	17	18	19	20	21	22	23
f	6	7	9	13	10	12	8	0	4

(ii)

Height (cm)	37	38	39	40	41
Number of plants	46	89	93	90	153

25. The heights (in cm) of 8 girls of a class are 140, 142, 135, 133, 137, 150, 148 and 138 respectively. Find the mean height of these girls and their median height.

26. Find the mean, the median and the mode of :

(i) 12, 24, 24, 12, 30 and 12

(ii) 21, 24, 21, 6, 15, 18, 21, 45, 9, 6, 27 and 15.

27. The following table shows the market positions of some brands of soap.

Draw a suitable bar graph :

Soap (brands) :	A	B	C	D	E
No. of buyers :	51	27	15	24	18

28. The birth rate per thousand of different countries over a particular period of time is shown below.

Draw a suitable bar graph :

INDIA	U.K.	CHINA	GERMANY	SWEDEN
35	22	42	13	8

PROBABILITY 22

22.1 INTRODUCTION

Probability means **likelihood.** It means likely to be true or not true.

In our day to day life, we speak or we hear many of the statements such as :

1. Probably Azhar will play today.
2. Most probably Sachin is going abroad.
3. There is chance to have rain today.

In the same way, we say or hear :

1. Our school team has very good chances of winning.
2. It is most likely to happen, etc.

- The words like probably, chance, chances, most-likely, etc. as used above have almost the same sense and involve an **element of uncertainity**.
- The measure of uncertainly of an event is called '**Theory of Probability**.'

For example :

If a coin is tossed once, the experiment will give either a head or a tail which is a well defined outcome.

Each outcome head or tail is an event.

1. **Outcomes :** When a coin is tossed once, we get either **head** (H) or **tail** (T). Here, head and tail are called **outcomes** or possible outcomes.

   In a single throw of a coin, outcomes are : **head** (H) and **tail** (T).

   In a single throw of two coins, possible outcomes are :

   H H, H T, T H and T T.

   In throwing a dice, possible outcomes are 1, 2, 3, 4, 5 and 6.
2. **An event :** An event is the collection of some favourable outcomes.

27.2 AN EXPERIMENTAL APPROACH

Experiment 1 :

Toss a coin (1) 5 times; (2) 10 times and then (3) 20 times. Each time note down the number of times a head comes up and the number of times a tail comes up.

Record your observations in the following table :

Steps	(1)	(2)	(3)
1. Number of times the coin is tossed.	**5**	**10**	**20**
2. Number of times head comes up.	—	—	—
3. Number of times tail comes up.	—	—	—
4. $\frac{\text{Number of times a head comes up}}{\text{Total number of times the coin is tossed}}$	—	—	—
5. $\frac{\text{Number of times a tail comes up}}{\text{Total number of times the coin is tossed}}$	—	—	—

Now, fill the above table as shown below :

Steps	(1)	(2)	(3)
1. Number of times the coin is tossed.	**5**	**10**	**20**
2. Number of times head comes up.	3	7	11
3. Number of times tail comes up.	2	3	9
4. $\frac{\text{Number of times a head comes up}}{\text{Total number of times the coin is tossed}}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{11}{20}$
5. $\frac{\text{Number of times a tail comes up}}{\text{Total number of times the coin is tossed}}$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{9}{20}$

In column (1) of the above table :

$\frac{3}{5}$ is the probability of getting head and $\frac{2}{5}$ is the probability of getting tail.

In column (2) of the above table :

$\frac{7}{10}$ is probability of getting a head and $\frac{3}{10}$ is the probability of getting a tail.

In the same way, **in column (3) of the above table**,

$\frac{11}{20}$ is the probability of getting a head and $\frac{9}{20}$ is the probability of getting a tail.

Experiment 2 :

Take a pair of two coins and toss them simultaneously (1) 5 times (2) 10 times and then (3) 20 times. In each case; write down the number of times **no head** comes up, **exactly one head** comes up or **two heads** come up.

Record your observations in the following table :

Steps	(1)	(2)	(3)
1. Number of times the coins are tossed.	**5**	**10**	**20**
2. Number of times **no head** comes up.	1	3	5
3. Number of times **exactly one head** comes up.	2	2	7
4. Number of times **two heads** come up.	2	5	8
5. $\frac{\text{Number of times no head comes up}}{\text{Total number of times the two coins are tossed}}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{5}{20}$
6. $\frac{\text{Number of times exactly one head comes up}}{\text{Total number of times the two coins are tossed}}$	$\frac{2}{5}$	$\frac{2}{10}$	$\frac{7}{20}$
7. $\frac{\text{Number of times two heads come up}}{\text{Total number of times the two coins are tossed}}$	$\frac{2}{5}$	$\frac{5}{10}$	$\frac{8}{20}$

See the table, obtained above, carefully and make the following facts clear :

(i) probability of getting exactly one head out of 5 trials = $\frac{2}{5}$

(ii) probability of getting exactly two heads out of 10 trials = $\frac{5}{10} = \frac{1}{2}$.

(iii) probability of getting no head out of 20 trials = $\frac{5}{20} = \frac{1}{4}$ and so on.

22.3 SOME TERMS RELATED TO PROBABILITY

Experiment :

An operation which can produce some well-defined outcomes is called an experiment. Each outcome is called an event.

For example :

If a die is thrown to get an odd number, the event would consist of three outcomes 1, 3 and 5. Similarly, if the die is thrown to get a number greater than 4, the event would consist of two outcomes 5 and 6.

22.4 PROBABILITY (Empirical Probability) :

If an experiment consists of *n* trials out of which E events favour a particular outcome; the probability of event E is denoted by P(E); which means :

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$$

For example :

Let a coin be tossed 300 times out of which head is obained 186 times

Clearly, number of trials = 300

out of which 186 times head appears and

300 − 186 = 114 times tail appears

∴ **Probability of getting a head**

$$= \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$$

$$= \frac{186}{300} = \frac{62}{100} = 0{\cdot}62$$ **(Ans.)**

The statement for finding probability can also be taken as :

Probability of getting a head

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{186}{300} = \frac{62}{100} = 0{\cdot}62$$

And, **probability of getting a tail**

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{114}{300} = \frac{38}{100} = 0{\cdot}38$$ **(Ans.)**

Example 1 :

A coin is tossed 40 times and head is obtained 14 times. Find the probability of getting

(i) a head (ii) a tail

Solution :

(i) $\because$ Total number of possible outcomes = 40

and, the number of favourable outcomes = 14

$\therefore$ **Probability of getting a head**

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{14}{40} = \frac{7}{20}$$ **(Ans.)**

(ii) $\because$ Total number of possible outcomes = 40

and, the number of favourable outcomes = 40 – 14 = 26

$\therefore$ **Probability of getting a tail**

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{26}{40} = \frac{13}{20}$$ **(Ans.)**

Example 2 :

Out of 400 students, 165 like tea only, 120 like coffee only and the remaining like both. If a student is chosen at random, what is the probability that he likes :

(i) tea only (ii) coffee only (iii) both.

Solution :

Total number of students = 400

Number of students who like tea only = 165

Number of students who like coffee only = 120

and, number of students who like both = 400 – 165 – 120 = 115

(i) **P (chosen student likes tea only)** $= \frac{165}{400} = \frac{33}{80}$ **(Ans.)**

(ii) **P (chosen student likes coffee only)** $= \frac{120}{400} = \frac{3}{10}$ **(Ans.)**

(iii) **P (chosen student likes both)** $= \frac{115}{400} = \frac{23}{80}$ **(Ans.)**

Example 3 :

A dice is thrown 50 times and the outcomes are noted as shown below :

Outcomes	1	2	3	4	5	6
No. of times an outcome appears	5	8	10	9	8	10

When a dice is thrown at random, find the probability of getting a

(i) 1 (ii) 4 (iii) 5

Solution :

$\because$ Total number of outcomes = 50

(i) **P (getting 1)** = $\frac{5}{50} = \frac{1}{10}$ **(Ans.)**

(ii) **P (getting 4)** = $\frac{9}{50}$ **(Ans.)**

(iii) **P (getting 5)** = $\frac{8}{50} = \frac{4}{25}$ **(Ans.)**

Example 4 :

A dice is thrown 20 times and the outcomes are noted as shown below :

Outcomes	1	2	3	4	5	6
No. of times	4	3	3	4	4	2

If a dice is thrown at random, find the probability of getting,

(i) an even number (ii) an odd number (iii) a number less than 4.

Solution :

$\because$ Total number of outcomes = 20

(i) An even number (2, 4 or 6) will appear 3 + 4 + 2 = 9 times

$\therefore$ **Required probability** = $\frac{9}{20}$ **(Ans.)**

(ii) An odd number (1, 3 or 5) will appear 4 + 3 + 4 = 11 times

$\therefore$ **Required probability** = $\frac{11}{20}$ **(Ans.)**

(i) A number less than 4 (1, 2 or 3) will appear 4 + 3 + 3 = 10 times

$\therefore$ **Required probability** = $\frac{10}{20} = \frac{1}{2}$ **(Ans.)**

EXERCISE 22(A)

1. A coin is tossed once. Find the probability of :

(i) getting a head (ii) not getting a head.

2. A coin is tossed 80 times and the head is obtained 38 times. Now, if a coin is tossed once, what will the probability of getting :

(i) a tail (ii) a head

3. A dice is thrown 20 times and the outcomes are noted as shown below :

Outcomes	1	2	3	4	5	6
No. of times	2	3	4	4	3	4

Now a dice is thrown at random, find the probability of getting :

(i) 3 (ii) a number less than 3

(iii) a number greater than 3.

4. A survey of 50 boys showed that 21 like tea while 29 dislike it. Out of these boys, one boy is chosen at random. What is the probability that the chosen boy

(i) likes tea (ii) dislikes tea

5. In a cricket match, a batsman hits a boundary 12 times out of 80 balls he plays. Further, if he plays one ball more, what will be the probability that :

(i) he hits a boundary (ii) he does not hit a boundary

6. There are 8 marbles in a bag with numbers from 1 to 8 marked on each of them. What is the probability of drawing a marble with number

(i) 3 (ii) 7

7. Two coins are tossed simultaneously 100 times and the outcomes are as given below :

Outcomes	Two heads (H H)	Exactly one head (H T or T H)	No head (T T)
No. of times	21	55	24

If the same pair of coins is tossed again at random, find the probability of getting :

(i) two heads (ii) exactly one head (iii) no head.

8. A bag contains 4 white and 6 black balls, all of the same shape and same size. A ball is drawn from the bag without looking into the bag. Find the probability that the ball drawn is :

(i) a black ball (ii) a white ball (iii) not a black ball

9. In a single throw of a dice, find the probability of getting a number :

(i) 4 (ii) 6 (iii) greater than 4

10. Hundred identical cards are numbered from 1 to 100. The cards are well shuffled and then a card is drawn. Find the probability that the number on the card drawn is :

(i) 50 (ii) 80 (iii) 40

- Probability of an event is a measure of the chance or likelyhood of its occurring.
- Probability can be given as a percentage from 0% to 100%, or as proper fractions or decimal numbers from 0 to 1.
- Probability of an impossible event is 0 (*i.e.* 0%) and probability of a sure (certain) event is 1 (*i.e.* 100%).
- All other events have their probabilities between 0 and 1.
- Out of all given probabilities, the event with largest probability is said be most likely event and the event with smallest probability is said to be least likely event.

22.5 COMPLEMENTARY EVENTS

Two events are complementary, if exactly one of them occurs.

For an event E, its complementary event is denoted by E'.

E' is the event when E does not occur.

If $P(E)$ = Probability of occurence of event E

and, $P(E')$ = Probability of non-occurence of event E.

$P(E) + P(E') = 1$

i.e. $P(E) = 1 - P(E')$ and $P(E') = 1 - P(E)$

EXERCISE 22(B)

1. Suppose S is the event that it will snow tomorrow and $P(S) = 0{\cdot}03$.

(i) State, in words, the complementary event S'.

(ii) Find $P(S')$

2. Five students A, B, C, D and E are competing in a long distance race. Each student's probability of winning the race is given below :

$A \rightarrow 20\%$, $B \rightarrow 22\%$, $C \rightarrow 7\%$, $D \rightarrow 15\%$ and $E \rightarrow 36\%$

(i) Who is most likely to win the race ?

(ii) Who is least likely to win the race ?

(iii) Find the sum of probabilities given.

(iv) Find the probability that either A or D will win the race.

(v) Let S be the event that B will win the race.

(a) Find P(S) (b) State, in words, the complementary event S'.

(c) Find $P(S')$.

3. A ticket is randomly selected from a basket containing 3 green, 4 yellow and 5 blue tickets. Determine the probability of getting :

(a) a green ticket. (b) a green or yellow ticket.

(c) an orange ticket.

4. Ten cards with numbers 1 to 10 written on them are placed in a bag. A card is chosen from the bag at random. Determine the probability of choosing :

(a) 7 (b) 9 or 10

(c) a number greater than 4 (d) a number less than 6

5. A carton contains eight brown and four white eggs. Find the probability that an egg selected at random is :

(a) brown (b) white

6. A box contains 3 yellow, 4 green and 8 blue tickets. A ticket is chosen at random. Find the probability that the ticket is :

(a) yellow (b) green

(c) blue (d) red

(e) not yellow

7. The following table shows number of males and number of females of a small locality in different age groups.

Age in years	10-20	21-50	Above 50
Male	8	12	6
Female	6	10	4

If one of the persons, from this locality, is picked at random, what is the probability that

(a) the person picked is a male ?

(b) the person picked is a female ?

(c) the person picked is a female aged 21-50 ?

(d) the person is a male with age upto 50 years ?

ANSWERS

Exercise 1 (A)

1. (i) 4270 (ii) 3940 (iii) 16740 **2.** (i) 6730 (ii) 192500 **3.** (i) 185, correct (ii) –1230, correct (iii) 98, correct (iv) 138, correct **4.** (i) 120 (ii) –120 (iii) –120 (iv) 120 **5.** (i) 192 (ii) –192 (iii) –192 (iv) –192 (v) 192 (vi) 192 (vii) 192 (viii) –192 **6.** (i) 384 (ii) –384 (iii) 384 (iv) –384 (v) 384 **7.** (i) 47 (ii) –63 (iii) 1 (iv) 0 **8.** (i) –ve (ii) +ve (iii) –ve (iv) +ve **9.** (i) $(8 + 10) \times 15$ (ii) $12 \times 6 - 8$ (iii) $\{(-3) -4\} \times -5$ **10.** (i) true ($5 \times 0 = 0, 0 \times -8 = 0$ and so on) (ii) false (iii) false (iv) false

Exercise 1 (B)

1. (i) 13 (ii) –13 (iii) –13 (iv) 13 (v) –15 (vi) –23 (vii) 38 (viii) –35 **2.** (i) –18 (ii) –18 (iii) 18 (iv) –22 (v) –22 (vi) 22 (vii) –52 (viii) –16 **3.** (i) 13 (ii) –13 (iii) –24 (iv) 26 (v) –24 **4.** (i) 12 (ii) –8 (iii) 0 (iv) 0 (v) 549 (vi) 7628 **5.** (i) true (ii) true (iii) false (iv) false **6.** (i) 10 (ii) 18 (iii) 14 (iv) 5 (v) 1 (vi) 7 (vii) 12 (viii) 0 (ix) 14 (x) –2 (xi) –11 (xii) 22

Exercise 1 (C)

1. 3 **2.** –3 **3.** 23 **4.** 18 **5.** 31 **6.** 23 **7.** 26 **8.** 0 **9.** 2307 **10.** 0 **11.** 86 **12.** 11 **13.** 1

Exercise 1 (D)

1. –24 **2.** $x = -11$ and $x = -1$ **3.** 73 **4.** –14 **5.** –3 and –15 **6.** (i) 1 (ii) –1 **7.** (i) –720 (ii) 1944 (iii) 440 (iv) –180 **8.** (i) positive (ii) negative **9.** –12, –6, 0, 6 and 12 **10.** –4, –3, –2, 0, 2, 3, 4 **11.** (i) –8 (ii) –20 (iii) 12 (iv) 2 (v) 16 (vi) 46 (vii) –14 (viii) –11 **12.** Required = $(20 + 21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29 + 30) - (21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29) = 20 + 30 = 50$ **13.** 204 **14.** –15 **15.** (i) Increase = 50 (ii) Decrease = 70

Exercise 2 (A)

1. $\frac{99}{1000}$ **2.** (i) –125 (ii) 37 (iii) –85 (iv) 2 (v) 0 **3.** (i) –15 (ii) 29 (iii) 4 (iv) 1 (v) Every non-zero number **4.** $\frac{20}{3}$ **5.** (i) –15 (ii) –23 (iii) 1 (iv) –1 **6.** Positive rational numbers are $\frac{-3}{-5}$, $\frac{3}{5}$ and $\frac{-13}{-3}$. Negative rational numbers are : $\frac{-3}{5}$, $\frac{3}{-5}$, $\frac{15}{-8}$ and $\frac{-15}{8}$

7. (i) $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}$ (ii) $\frac{8}{-14}, \frac{12}{-21}, \frac{16}{-28}$ (iii) $\frac{-10}{18}, \frac{-15}{27}, \frac{-20}{36}$ (iv) $\frac{16}{-30}, \frac{24}{-45}, \frac{32}{-60}$

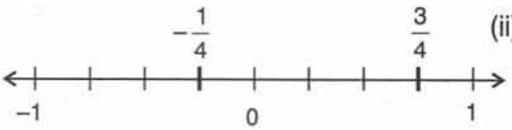
8. (iv) and (v) **9.** (i) $\frac{35}{7}$ (ii) $\frac{-56}{7}$ (iii) $\frac{0}{7}$ (iv) $\frac{-112}{7}$ (v) $\frac{49}{7}$

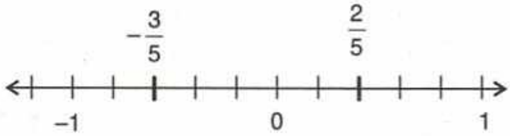
10. (i) $\frac{12}{20}$ (ii) $\frac{-12}{-20}$ (iii) $\frac{27}{45}$ (iv) $\frac{15}{25}$ (v) $\frac{-21}{-35}$ **11.** (i) $\frac{12}{21}$ (ii) $\frac{-12}{-21}$ (iii) $\frac{-16}{-28}$ (iv) $\frac{-20}{-35}$ (v) $\frac{20}{35}$

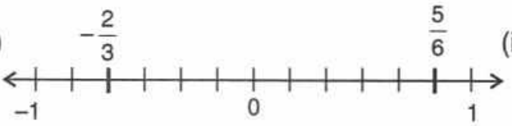
12. (i) –9 (ii) –14 (iii) –15 (iv) –8 (v) 12 (vi) –3 **13.** (i) $\frac{4}{5}$ (ii) $-\frac{5}{6}$ (iii) $\frac{2}{3}$ (iv) $-\frac{1}{4}$

14. (i) $\frac{7}{8}$ (ii) $\frac{-5}{12}$ (iii) $\frac{7}{20}$ (iv) $\frac{-4}{9}$

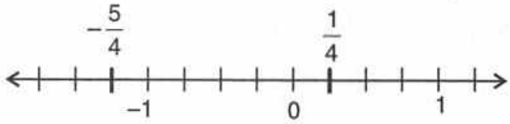
Exercise 2 (B)

1. (i) 

(ii) 

(iii) 

(iv) 

(iv) 

2. (i) $\frac{3}{5} < \frac{5}{7}$ (ii) $\frac{-7}{2} < \frac{5}{2}$ (iii) $-3 < 2\frac{3}{4}$ (iv) $-1\frac{1}{2} < 0$ (v) $0 < \frac{3}{4}$ (vi) $3 > -1$

3. (i) $-\frac{1}{4} < 0$ (ii) $\frac{1}{4} > 0$ (iii) $-\frac{3}{8} < \frac{2}{5}$ (iv) $\frac{-5}{8} < \frac{7}{-12}$ (v) $\frac{5}{-9} < \frac{-5}{-9}$ (vi) $\frac{-7}{8} < \frac{5}{-6}$ (vii) $\frac{2}{7} < \frac{-3}{-8}$ **4.** (i) $\frac{5}{-15} < \frac{-11}{-30} < \frac{7}{10}$ (ii) $\frac{2}{-3} < \frac{4}{-9} < \frac{-5}{12}$

5. (i) $\frac{5}{8} > \frac{-7}{12} > \frac{13}{-16}$ (ii) $\frac{3}{-10} > \frac{8}{-20} > \frac{-13}{30}$ **6.** (i) same (ii) opposite (iii) same (iv) opposite

Exercise 2 (C)

1. (i) $1\frac{4}{5}$ (ii) $\frac{-2}{9}$ (iii) $-\frac{1}{3}$ (iv) $\frac{1}{5}$ (v) $\frac{-16}{25}$ (vi) $\frac{-14}{26} = \frac{-7}{13}$ **2.** (i) $\frac{1}{35}$ (ii) $\frac{-7}{18}$ (iii) $-\frac{7}{3}$ (iv) $-\frac{3}{18} = -\frac{1}{6}$ (v) $\frac{-19}{48}$ (vi) $-\frac{13}{54}$ (vii) $\frac{-28}{75}$ (viii) $\frac{-61}{48}$ (ix) $\frac{-13}{16}$ **3.** (i) 0 (ii) $-\frac{2}{3}$ (iii) $\frac{-7}{48}$ (iv) $-\frac{13}{10}$ (v) $-\frac{26}{15}$ (vi) $-\frac{47}{48}$ **4.** (i) $-\frac{29}{18}$ (ii) $\frac{25}{4} = 6\frac{1}{4}$ (iii) $\frac{-125}{24}$ (iv) $\frac{23}{6} = 3\frac{5}{6}$ (v) $3\frac{5}{8}$ **5.** (i) $\frac{1}{3}$ (ii) $\frac{9}{11}$ (iii) $-\frac{2}{5}$ (iv) $\frac{-8}{9}$ (v) $\frac{-18}{11}$ **6.** (i) $\frac{1}{2}$ (ii) $\frac{-34}{25}$ (iii) $\frac{-1}{4}$ (iv) $\frac{37}{21} = 1\frac{16}{21}$ (v) $\frac{8}{15}$ (vi) $\frac{-3}{8}$ (vii) $\frac{17}{10} = 1\frac{7}{10}$ (viii) $\frac{-15}{16}$ (ix) $\frac{-55}{13}$ **7.** $\frac{1}{12}$ **8.** $\frac{-9}{8}$ **9.** $\frac{-35}{12}$ **10.** $\frac{31}{48}$ **11.** $\frac{13}{5} = 2\frac{3}{5}$ **12.** $\frac{-9}{5}$ **13.** $\frac{4}{5}$ **14.** $\frac{23}{24}$ **15.** $\frac{-56}{45}$

Exercise 2 (D)

1. (i) $\frac{15}{28}$ (ii) $\frac{-4}{7}$ (iii) 8 (iv) 1 (v) $\frac{-15}{2}$ (vi) $\frac{-4}{5}$ (vii) $\frac{1}{6}$ (viii) $\frac{1}{12}$

2. (i) $\frac{12}{125}$ (ii) 12 (iii) $-\frac{5}{2}$ (iv) $\frac{5}{6}$ (v) 21 (vi) $\frac{-29}{2}$ (vii) −82 (viii) $-\frac{126}{5}$ (ix) $\frac{-5}{2}$ (x) −36 (xi) $\frac{-27}{2}$ (xii) $\frac{-15}{2}$ **3.** (i) $2\frac{5}{9}$ (ii) $1\frac{2}{15}$ (iii) $-\frac{7}{30}$ (iv) $1\frac{2}{15}$ (v) $1\frac{23}{30}$ (vi) $-\frac{303}{40}$

4. ₹ $1139\frac{1}{4}$ **5.** $87\frac{1}{3}$ km **6.** (i) $\frac{5}{7}$ (ii) 4 (iii) $2\frac{4}{5}$ (iv) $2\frac{1}{2}$ (v) 4 (vi) −4 (vii) −45 (viii) $-\frac{189}{484}$

7. (i) $5\frac{1}{12}$ (ii) $1\frac{3}{4}$ (iii) $2\frac{19}{21}$ **8.** $-\frac{49}{4}$ **9.** ₹ $2\frac{1}{4}$ **10.** ₹ $\frac{20}{51}$ **11.** $1\frac{1}{2}$ **12.** $\frac{4}{3} = 1\frac{1}{3}$

13. 7 **14.** $3\frac{5}{49}$ m

Exercise 2 (E)

1. (i) $\frac{1}{12}$ (ii) $\frac{19}{54}$ (iii) $-\frac{19}{24}$ (iv) $-\frac{7}{48}$ (v) $-\frac{19}{36}$ (vi) $\frac{11}{78}$ (vii) $\frac{1}{21}$ (viii) $\frac{-19}{56}$ (ix) $\frac{37}{26} = 1\frac{11}{26}$ (x) $-\frac{5}{6}$ **2.** $-\frac{9}{16}$ **3.** $-\frac{73}{25}$ **4.** $\frac{31}{48}$ **5.** $2\frac{3}{5}$ **6.** $\frac{-5}{8}$ **7.** $-\frac{39}{40}$ **8.** (i) $1\frac{2}{15}$ (ii) $\frac{-53}{42}$ (iii) $1\frac{1}{18}$ (iv) $1\frac{23}{30}$ **9.** $\frac{-22}{3}$ **10.** $\frac{1}{4}$

Exercise 3 (A)

1. (i) Vulgar and proper (ii) Decimal and improper (iii) Decimal and proper (iv) Vulgar and improper (v) Mixed **2.** (i) $3\frac{3}{5}$ (ii) $1\frac{3}{4}$ (iii) $4\frac{1}{6}$ (iv) $7\frac{3}{5}$ (v) $4\frac{2}{5}$ **3.** (i) $\frac{22}{9}$ (ii) $\frac{96}{13}$ (iii) $\frac{13}{4}$ (iv) $\frac{101}{48}$ (v) $\frac{139}{11}$ **4.** (i) $\frac{4}{9}$ (ii) $\frac{3}{4}$ (iii) $\frac{3}{7}$ (iv) $\frac{7}{15}$ (v) $\frac{2}{5}$ **5.** (i) True (ii) False (iii) True **6.** (i) Simple (ii) Simple (iii) Simple (iv) Complex (v) Complex (vi) Complex (vii) Complex (viii) Neither

Exercise 3 (B)

1. (i) Like (ii) Unlike (iii) Unlike **2.** (i) $\frac{15}{18}$ and $\frac{14}{18}$ (ii) $\frac{8}{12}, \frac{10}{12}$ and $\frac{7}{12}$ (iii) $\frac{64}{80}, \frac{68}{80}, \frac{46}{80}$ and $\frac{55}{80}$ **3.** (i) $\frac{24}{27}$ and $\frac{24}{34}$ (ii) $\frac{60}{130}, \frac{60}{92}$ and $\frac{60}{85}$ (iii) $\frac{225}{285}, \frac{225}{252}, \frac{225}{275}$ and $\frac{225}{235}$ **4.** (i) 1/6, 1/3, 2/5, 3/4 (ii) 3/10, 5/6, 7/8, 11/12 (iii) 3/8, 9/14, 5/7, 20/21 **5.** (i) 8/9, 5/6, 1/3, 4/15 (ii) 8/11, 5/7, 4/9, 3/7 (iii) 8/11, 3/5, 6/11, 1/10 **6.** (i) 11/15 (ii) 4/5 (iii) 6/7 **7.** (i) $\frac{7}{16}$ (ii) $2\frac{1}{2}$ (iii) $\frac{1}{2}$

8. (i) 5/12, 3/7, 7/16 (ii) 3/5, 2/3, 7/10 (iii) 4/9, 9/19, 1/2 **9.** (i) $\frac{1}{3}, \frac{2}{7}$ (ii) $\frac{6}{13}, \frac{7}{17}$ (iii) $1, 1\frac{1}{6}$

Exercise 3 (C)

1. (i) $1\frac{1}{6}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{2}$ (iv) $3\frac{7}{12}$ (v) 1 (vi) $2\frac{13}{15}$ (vii) $\frac{17}{30}$ (viii) $3\frac{7}{12}$ (ix) $5\frac{5}{24}$ **2.** (i) $4\frac{1}{2}$ (ii) 10 (iii) $\frac{3}{8}$ (iv) $\frac{3}{7}$ (v) 105 (vi) 117 (vii) 6 (viii) $7\frac{1}{2}$ (ix) $1\frac{2}{3}$ (x) $1\frac{1}{3}$ (xi) $-\frac{5}{6}$ (xii) $3\frac{1}{5}$ (xiii) $4\frac{2}{7}$ **3.** (i) $-1\frac{1}{3}$ (ii) $\frac{1}{2}$ (iii) $\frac{4}{5}$ (iv) $\frac{6}{7}$ (v) $-\frac{4}{5}$ (vi) $\frac{26}{45}$ (vii) $\frac{2}{77}$

4. (i) 5 kg (ii) 36 minutes (iii) $1\frac{1}{3}$ kg (iv) 7 metres (v) $1\frac{1}{3}$ (vi) 8 kg **5.** (i) $\frac{4}{5}$ (ii) $\frac{3}{35}$ (iii) $4\frac{1}{5}$ (iv) 2 (v) $\frac{3}{11}$ (vi) $5\frac{1}{4}$ (vii) $\frac{2}{9}$ (viii) $\frac{20}{27}$ (ix) 0 (x) $1\frac{13}{14}$ (xi) $1\frac{11}{21}$ (xii) $\frac{21}{50}$ (xiii) $\frac{9}{32}$ **6.** $6\frac{1}{4}$ kg **7.** $\frac{7}{10}$ by $\frac{1}{10}$ **8.** $4\frac{1}{6}$ **9.** $6\frac{1}{12}$ **10.** $57\frac{4}{5}$ m **11.** ₹ $328\frac{1}{8}$ **12.** 50 km **13.** 390 m^2 14. 192 kg **15.** 4 kg **16.** $\frac{12}{25}$ m **17.** (i) $\frac{27}{53}$ (ii) $1\frac{26}{27}$ **18.** ₹ $\frac{1169}{52}$ = ₹ $22\frac{25}{52}$ **19.** $3\frac{3}{28}$ **20.** $\frac{4}{7}$

Exercise 3 (D)

1. $7\frac{3}{4}$ **2.** $6\frac{2}{5}$ **3.** $-\frac{61}{240}$ **4.** $\frac{1}{4}$ **5.** -1 **6.** $-4\frac{11}{50}$ **7.** $2\frac{4}{19}$ **8.** $4\frac{1}{5}$ **9.** $20\frac{103}{250}$ **10.** $\frac{3}{7}$ **11.** $3\frac{6}{7}$

Exercise 3 (E)

1. (i) 2/5 (ii) 3/5 (iii) 1/2 **2.** 2 **3.** (i) he sold $30\frac{1}{2}$ kg (ii) he is left with $89\frac{1}{2}$ kg

4. ₹ 1,575 **5.** $13\frac{13}{18}$ metres **6.** Ram's investment = ₹ 24,000 and Deepak's investment = ₹ 16,000 **7.** 10 **8.** ₹ 67·50 **9.** 157·5 kg **10.** 17 **11.** $8\frac{1}{12}$

12. $67\frac{67}{80}$ kg **13.** $\frac{3}{10}$ **14.** $\frac{21}{50}$ **15.** ₹ 375 **16.** (i) $\frac{1}{6}$ (ii) 20 m **17.** ₹ 5,000 **18.** 800

19. ₹ 52,500; ₹ 22,500 **20.** $5\frac{1}{3}$ km **21.** $62\frac{1}{2}$ km **22.** 1350 m^2 **23.** ₹ 240 **24.** ₹ 600

Exercise 4 (A)

1. (i) $\frac{15}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{51}{25}$ (iv) $\frac{13}{20}$ (v) $\frac{481}{200}$ (vi) $\frac{17}{200}$ (vii) $\frac{321}{40}$ **2.** (i) 2·8 (ii) 0·79 (iii) 0·0037 (iv) 0·7543 (v) 0·75 (vi) 9·6 (vii) 8·625 (viii) 5·875 **3.** (i) 4 (ii) 5 (iii) 3 (iv) 1 (v) 3 (vi) 6

4. (i) zero-point-four; zero-point nine; zero-point-one.
(ii) one-point-nine; four-point-four; seven-point-five.
(iii) zero-point-zero-two; zero-point-five-six; thirteen-point-zero-six.
(iv) zero-point-zero-zero-five; zero-point-two-zero-seven; one hundred eleven-point-five-one-nine.
(v) zero-point-eight; zero-point-zero-eight; zero-point-zero-zero-eight; zero-point-zero-zero-zero-eight.
(vi) two hundred fifty six-point-one; ten-point-two-two; zero-point-six-three-four.

5. (i) 0·5000, 3·6200, 43·9810 and 232·0037
(ii) 215·78000, 33·00060, 530·30000 and 0·03569

Exercise 4 (B)

1. (i) 0·87 (ii) 12·5 (iii) 0·337 (iv) 1·2531 (v) 19·316 (vi) 21·935 (vii) 2·2202 (viii) 67·5488 (ix) 36·5303 (x) 18·4071 **2.** (i) 4·4 (ii) 4·14 (iii) 7·78 (iv) 8·36 (v) 7·188 (vi) 18·43 (vii) 22·94 (viii) 3·77 (ix) 23·57 (x) 2·865 **3.** (i) 12·821 (ii) 17·489 (iii) 14·21 (iv) 99·9458 (v) 12·592 (vi) 3146·959 (vii) 37·484 (viii) 207·97 (ix) 605·005 **4.** 6·165 **5.** 124·706 **6.** 5·959 **7.** 40·788 **8.** 80·71 **9.** 370·34 **10.** (i) 6·108 (ii) −1·62 (iii) 9·542 (iv) 800·254 **11.** 26·71 **12.** (i) 154·51 (ii) −154·51 **13.** ₹ 4·18

Exercise 4 (C)

1. (i) 8·7 (ii) 294·8 (iii) 6400 (iv) 23·2 (v) 456·96 (vi) 40·296 (vii) 9·66 (viii) 3·6352 **2.** (i) 5, 50, 500 (ii) 1·12, 11·2, 112 (iii) 48, 480, 4800 (iv) 0·359, 3·59, 35·9 (v) 162·7, 1627, 16270 (vi) 2348, 23480, 234800 **3.** (i) 13·5631 (ii) 77·778 (iii) 0·00001 (iv) 7·392 (v) 1·14 (vi) 40·32

(vii) 0·24 (viii) 0·000027 **4.** (i) 5·49 (ii) 0·078 (iii) 0·32476 (iv) 3·2 (v) 3·102 (vi) 0·584 (vii) 3·6 (viii) 5·46 (ix) 60·54 **5.** (i) 0·21, 0·021, 0·0021, 0·00021 (ii) 0·864, 0·0864, 0·00864, 0·000864 (iii) 0·501, 0·0501, 0·00501, 0·000501 (iv) 0·00906, 0·000906, 0·0000906, 0·00000906 (v) 0·0125, 0·00125, 0·000125, 0.0000125 (vi) 11·111, 1·1111, 0·11111, 0·011111 **6.** (i) 1·95 (ii) 1·1016 (iii) 0·923 (iv) 0·77 (v) 1·29 (vi) 0·356 (vii) 0·005 (viii) 0·00203 (ix) 0·000479 (x) 1526 (xi) 70 **7.** (i) 0·6708 (ii) 5·6 (iii) 0·72 (iv) 0·027 (v) 0·576 (vi) 0·000064 (vii) 0·21 (viii) 0·00021 **8.** (i) 0·81 (ii) 0·18 (iii) 0·075 (iv) 0·064 (v) 0·04 (vi) 0·00040 **9.** ₹ 470·40 **10.** ₹ 898·40 **11.** (i) 0·12 (ii) 2·07 (iii) 10·64 (iv) 0·083 (v) 12·12 (vi) 210 **12.** 2·1 kg **13.** 12·8 **14.** ₹ 3·83

Exercise 4 (D)

1. (i) Terminating (ii) Non-terminating (iii) Terminating (iv) Non-terminating (v) Terminating (vi) Non-terminating (vii) Non-terminating (viii) Non-terminating **2.** (i) $1.\overline{3}$ (ii) $0.\overline{90}$ (iii) $0.8\overline{3}$ (iv) $0.\overline{153846}$ (v) $0.\overline{1}$ (vi) $0.1\overline{8}$ (vii) $0.2\overline{7}$ (viii) $0.58\overline{3}$ **3.** (i) $\frac{1}{3}$ (ii) $\frac{8}{9}$ (iii) $4\frac{4}{9}$ (iv) $23\frac{7}{9}$ **4.** (i) $\frac{35}{99}$ (ii) $2\frac{23}{99}$ (iii) $1\frac{28}{99}$ (iv) $5\frac{234}{999}$ **5.** (i) $\frac{17}{45}$ (ii) $\frac{27}{110}$ (iii) $\frac{617}{900}$ (iv) $\frac{219}{495}$

Exercise 4 (E)

1. (i) 0·1, 0·1, 3·6, 9·5 (ii) 0·63, 100·48, 0·07, 0·02 (iii) 5, 1, 452, 9 **2.** (i) 22·02 (ii) 1·26, **3.** (i) 4 (ii) 2 (iii) 5 (iv) 3 (v) 2 (vi) 3 (vii) 1 (viii) 2 **4.** (i) 35·9, 0·00843, 4·95, 383 (ii) 60·97, 2·875, 0·001789, 400.0 (iii) 14·295, 19·200, 46357, 69·000

Exercise 4 (F)

1. 146·88 kg **2.** ₹ 1968·75 **3.** ₹ 715·65 **4.** 6·4 kg **5.** ₹ 136·80 **6.** (i) ₹ 52·40 (ii) 17·222 kg (iii) 37·272 (iv) 230·012 **7.** (i) 0·5 (ii) 24 (iii) 288 (iv) 0·045 (v) 3 (vi) 0·19008

Exercise 5 (A)

1. (i) 36 (ii) 343 (iii) 256 (iv) 3125 (v) 512 (vi) $7^5 = 16807$ **2.** (i) 128 (ii) 200 (iii) 675 (iv) 108 (v) 1125 (vi) 2000 (vii) 144 (viii) 1728 (ix) 400 **3.** (i) $\frac{81}{256}$ (ii) $-\frac{3125}{7776}$ (iii) $\frac{27}{125}$ **4.** (i) $\frac{1}{6}$ (ii) $-\frac{1}{12}$ (iii) $-\frac{8}{75}$ **5.** (i) 3^2 (ii) 2^5 (iii) 3^4 (iv) 4^5 **6.** (i) 2^9 (ii) 2×5^4 (iii) 2×3^6 (iv) $2^4 \times 3^2 \times 5^2$ (v) $2 \times 3^3 \times 5^2$ (vi) $2^3 \times 3 \times 7^2$ **7.** (i) 25 (i) 1 (iii) 36 (iv) 216 **8.** (i) 2^{10} (ii) 3^6 (iii) 3^6 **9.** $x = 3$ and $y = 5$ **10.** (i) $a = 6$ and $b = 4$ (ii) 250000

Exercise 5 (B)

1. (i) 5 and 2 (ii) $(2y)^{3x}$ **2.** (i) $2^5 = 32$ (ii) $\frac{1}{2^5} = 2^{-5} = \frac{1}{32}$ (iii) 1 (iv) 1 (v) $8^2 = 64$ (vi) $5^2 = 25$ (vii) $5^6 = 15625$ (viii) 4 (ix) 1 **3.** (i) $10b^{13}$ (ii) $54x^{10}y^8$ (iii) $-a^7$ (iv) y^5 (v) 3^5 (vi) $20x^3$ (vii) $10a^6b^4$ (viii) x^{4a-1} (ix) 3^{y-2} (x) 2^{6a} (xi) $\frac{4}{9xy}$ (xii) 10^6x^{96} (xiii) a^{100} (xiv) $-n^{10}$ (xv) $-225a^6b^4c^8$ (xvi) 0 (xvii) $256a^{16}$ (xviii) $\frac{64}{27}$ (xix) $\frac{9}{2x}$ (xx) $\frac{a^6c^6}{81}$ (xxi) $\frac{3125}{32x^{17}}$ (xxii) $\frac{16qr^5}{p^5}$

4. (i) $-2^2 \times 3^2$ (ii) $2^9 \times 5^2$ (iii) $\left(\frac{3}{2}\right)^2$ (iv) $-\left(\frac{2}{3}\right)^7 = \left(-\frac{2}{3}\right)^7$ (v) $\frac{c^8}{a^{10}b^2d^4}$ (vi) $\frac{b^{10}}{a^6}$

5. (i) 4 (ii) $1\frac{1}{5}$ (iii) 1468 (iv) $\frac{-29}{4}$ (v) 13 (vi) 5 **6.** (i) 16 (ii) $4\frac{1}{4}$ (iii) $15\frac{1}{4}$ (iv) 22

Exercise 6 (A)

1. (i) 1 : 3 (ii) 3 : 5 (iii) 1 : 2 (iv) 2 : 5 (v) 1 : 10 (vi) 1 : 25 (vii) 1 : 8 (viii) 3 : 8 (ix) 16 : 27 : 30 **2.** 40 cm and 24 cm **3.** x gets ₹ 320 and y gets ₹ 400 **4.** 45°, 30°, 105° **5.** (i) 5 : 4 (ii) 2 : 9 **6.** 1200, 1512, 2205 **7.** ₹ 1080 **8.** 25, 35 **9.** 80, 88 **10.** 102·6 cm **11.** 8/15 **12.** (i) 120,000 (ii) 2 : 3 **13.** (i) 3 : 1 (ii) 1 : 4 **14.** ₹ 1,500 and ₹1,650 **15.** ₹ 4,500; ₹ 2,700 and ₹ 7,200 **16.** 225 sq. m and 375 sq. m **17.** 11·25 m **18.** 24 and 42 **19.** (i) 1 : 2 (ii) A = ₹ 100 and B = ₹ 200 **20.** 80 and 144 **21.** 100 **22.** ₹ 15,000

Exercise 6 (B)

1. (i) Yes (ii) Yes (iii) Yes (iv) No (v) No **2.** (i) 16 (ii) 81 (iii) 6·0 (iv) 10·08 (v) 4 hours **3.** (i) 3 (ii) $6\frac{2}{3}$ cm (iii) 0·2 (iv) $\frac{9}{28}$ (v) 0·4 **4.** (i) 8 (ii) 9 (iii) 1·5 (iv) 2·4 (v) $\frac{1}{8}$

5. (i) 12 : 20 : 35 (ii) 10 : 15 : 21 (iii) 4 : 21 (iv) 27 : 16 (v) 5 : 14 (vi) 45 : 98 **6.** $4\frac{4}{15}$ **7.** $4\frac{11}{16}$

Exercise 7 (A)

1. 6·6 kg **2.** 15 **3.** 4 hours **4.** 54 minutes **5.** 290 minutes **6.** 50 days **7.** 20 weeks **8.** 40 days **9.** 32 days **10.** ₹ 93·75 **11.** ₹ 150 **12.** ₹ 216 **13.** ₹ 384 **14.** 204 **15.** 15 days [All the pupil consume same amount of food every day] **16.** 6 hrs per day **17.** ₹ 16,820 **18.** 3500 **19.** 8 days

Exercise 7 (B)

1. (i) ₹ 160 (ii) ₹ 100 **2.** ₹ 208 **3.** 12 hours **4.** 4 days 23 hours **5.** 10 days **6.** (i) ₹ 20 (ii) ₹ 80 **7.** 400 apples **8.** 100 litres **9.** 10 days **10.** 15 days.

Exercise 7 (C)

1. $3\frac{3}{7}$ days **2.** 30 days **3.** (i) $\frac{9}{20}$ (ii) $\frac{9}{10}$; $\frac{1}{10}$ **4.** $\frac{7}{18}$, $\frac{11}{18}$ **5.** 5 days **6.** $6\frac{6}{19}$ **7.** (i) $\frac{2}{5}$ (ii) $\frac{3}{5}$ (iii) 24 days **8.** (i) $\frac{9}{25}$ (ii) $\frac{16}{25}$ (iii) 16 hrs **9.** (i) $\frac{1}{30}$ (ii) $\frac{1}{30}$ (iii) 15 hrs **10.** (i) $1\frac{5}{7}$ days (ii) 24 days **11.** 24 days

Exercise 8 (A)

1. (i) 75% (ii) $66\frac{2}{3}$ % (iii) 2·5% (iv) 12·5% **2.** (i) $\frac{3}{40}$, 0·075 (ii) $\frac{1}{40}$, 0·025 (iii) $\frac{1}{5000}$, 0·0002 (iv) $1\frac{3}{4}$, 1·75 **3.** (i) $33\frac{1}{3}$ % (ii) 20% (iii) $6\frac{1}{4}$ % (iv) 12% **4.** (i) ₹ 17·50 (ii) ₹ 40·04 (iii) ₹ 5 (iv) 10 kg (v) ₹ 3·75 (vi) 9m **5.** 42 **6.** A = 4800, B = 3200 **7.** ₹ 2,200 **8.** 200 **9.** ₹ 23,750 **10.** $54\frac{6}{11}$ % **11.** (i) 340% (ii) 0·75% (iii) 15% (iv) 48% (v) $9\frac{3}{8}$ % = 9·375% **12.** (i) 3 min. (ii) $6\frac{1}{24}$ %

Exercise 8 (B)

1. 198 **2.** Chandra got 54 and Ram got 48; $88\frac{8}{9}\%$ **3.** 342; 38% **4.** 50% **5.** 40% **6.** $19\frac{3}{13}\%$ **7.** (i) 6000 (ii) 2500 (iii) 180 **8.** ₹ 5,00,000 **9.** $66\frac{2}{3}\%$ **10.** ₹ 70 **11.** 225 kg

Exercise 8 (C)

1. $41\frac{2}{3}\%$ **2.** (i) 63 (ii) 23 (iii) 54 (iv) 192 (v) 1035 **3.** (i) 64 (ii) 270 (iii) 43·75 **4.** (i) 80 (ii) 200 (iii) 200 (iv) 800 (v) 50 (vi) 100 **5.** ₹ 50,000; ₹ 10,000 **6.** ₹ 50 **7.** ₹ 400 **8.** (i) 1 : 4 (ii) 4 : 3 **10.** (i) $\frac{13}{10}$ (ii) $\frac{23}{13}$ (iii) $-\frac{10}{3}$ **11.** 2% **12.** 414 litres **13.** copper 52%, zinc 28% and nickel 20% **14.** 900 **15.** (i) 270 (ii) 22·5% **16.** 12 and 48% **17.** 4% decrease **18.** 76% decrease **19.** 300

Exercise 8 (D)

1. 300 **2.** (i) ₹ 45,000 (ii) ₹ 32,400 **3.** (i) 756 (ii) 270 (iii) 1026 (iv) 1674 (v) 62% **4.** 1·8 kg **5.** 52% **6.** 240 kg **7.** 400 **8.** 1600 **9.** 6% 10. 4%

Exercise 9 (A)

1. (i) 12% gain (ii) Loss = $11\frac{1}{9}\%$ (iii) 4% gain (iv) $33\frac{1}{3}\%$ loss (v) Loss = $16\frac{2}{3}\%$ **2.** (i) ₹ 625 (ii) ₹ 52·50 **3.** Profit = 20% **4.** ₹ 420 **5.** ₹ 1,190 **6.** Profit = 25% **7.** Loss = 10% **8.** Profit = 27·5% **9.** (i) ₹ 1500 (ii) Profit = 25% **10.** (i) ₹ 1,500 (ii) Loss = 20%

Exercise 9 (B)

1. (i) ₹ 20 (ii) ₹ 25 (iii) ₹ 320 (iv) ₹ 250 (v) ₹ 1.05 **2.** ₹ 900 **3.** ₹ 8,000 **4.** (i) ₹ 2,400 (ii) ₹ 2,760 **5.** (i) ₹ 8,000 (ii) ₹ 8,960 **6.** (i) ₹ 6.90 (ii) ₹ 276 **7.** (i) ₹ 500 and ₹ 750 (ii) Total CP = ₹ 1,250; total S.P. = ₹ 1,200 (iii) Loss = 4% **8.** (i) ₹ 4 (ii) ₹ 32 **9.** C.P. = ₹ 4,500 ; S.P. = ₹ 4,860 **10.** (i) ₹ 1,100 (ii) Profit = 15% **11.** 25% gain **12.** 20% loss **14.** 4% loss **16.** 56·25% profit **17.** ₹ 8,400

Exercise 9 (C)

1. ₹ 4,500 **2.** 10% **3.** (i) ₹ 48 (ii) ₹ 272 **4.** (i) ₹ 90 (ii) 20% **5.** (i) ₹ 360 (ii) ₹ 36 (iii) ₹ 324 (iv) 8% **6.** (i) ₹ 1,600 (ii) ₹ 48,400 **7.** (i) ₹ 2,880 (ii) 15·2% **8.** ₹ 800 **9.** ₹ 700 **10.** (i) ₹ 1,300 (ii) ₹ 1,560 **11.** (i) ₹ 12.50 (ii) ₹ 16 (iii) ₹ 3.50 (iv) 28% (v) ₹ 500 (vi) ₹ 640 (vii) ₹ 140 (viii) 28% yes, results of parts (iv) and (viii) are same. If we know the C.P. and the S.P. of equal number of articles, which may be 1 (one), 40 or 100, etc.; the profit percent in all the cases will be the same. **13.** 20% loss **14.** (i) ₹ 45 (ii) ₹ 315 **15.** 28%

Exercise 10

1. (i) ₹ 30; ₹ 180 (ii) ₹ 98; ₹ 448 (iii) ₹ 198.40; ₹ 818.40 (iv) ₹ 380.25; ₹ 3,760.25 (v) ₹ 24; ₹ 624 (vi) ₹ 8.50; ₹ 858.50 (vii) ₹ 135; ₹ 360 **2.** ₹ 3,200 **3.** $1\frac{1}{2}$ years

4. $12\frac{1}{2}$ years **5.** 8 years **6.** 4% **7.** ₹ 500 **8.** 6% **9.** 4 years **10.** 32% **11.** ₹ 1,200 **12.** (i) 5% (ii) ₹ 600 **13.** (i) ₹ 2,040 (ii) 20 years **14.** 7% **15.** 5% **16.** 6%

Exercise 11 (A)

1. 8 is the only constant term and each of the remaining terms is variable. **2.** (i) Monomial (ii) Binomial (iii) Monomial (iv) Trinomial (v) Binomial (vi) Monomial (vii)Binomial (viii) Binomial (ix) Binomial **3.** (i) $-3a$ (ii) p^2y (iii) -1 (iv) $-p^2$ **4.** (i) 1 (ii) 3 (iii) 4 (iv) 5 (v) 7 (vi) 9 **5.** (i) 4 (ii) 8 (iii) 7 (iv) 3 (v) 1 (vi) 4 **6.** (i) $9x^2, -3x^2$ and x^2; xy and $-2xy$ (ii) ab and $-3ab$; $-a^2b$; $5a^2b$ and $-8a^2b$ (iii) $7p, -2p$ and $3p$; $8pq$ and $-5pq$ **7.** (i) 1 (ii) -1 (iii) 2 (iv) -8 (v) 3 (vi) -9 **8.** (i) $-5x^3y^2z^2$ (ii) $-5x^3z^4$ (iii) $-5x^3yz^2$ (iv) $-5yz^4$ (v) $5x^2z^4$ (vi) x^2z^3. The degree of given algebraic expression $= 3 + 2 + 4 = 9$.

Exercise 11 (B)

1. (i) $13x$ (ii) $3x$ (iii) $15xy^2$ (iv) $-3xy^2$ (v) $14a+5b$ (vi) $11+10xy$ (vii) $-3a+7b$ (viii) $-2x+8$ (ix) $2x^2y+15xy^2$ (x) $7x^2$ and $8xy^2$ **2.** (i) $-2x$ (ii) $19y^2$ (iii) $6pq$ **3.** (i) $10m$ (ii) n^2 (iii) $11yz$ (iv) $-10ax^2$ (v) $-7am-11mx$ **4.** (i) $3a+4b$ (ii) $5x-3y$ (iii) $3b$ (iv) $9+5x$ **5.** (i) $-3x+17y+17z$ (ii) $6a+9b+2c$ (iii) $5x^2+16xy-7y^2$ (iv) $6x^2-10x+18$ (v) $6x^2+2xy+8y^2$ (vi) $3b^2+2ab+2bc+2ac+5c^2$ (vii) $7ax-2bx-2$ (viii) ac (ix) $10a^2+7b^2-8ab$ (x) $-2x+10$ (xi) $11x^3-2x^2-11x+15$ **6.** (i) $x+3y$ (ii) $-2a+5$ (iii) $-4x^2+7x$ (iv) $4a-7b$ (v) $x^3+3x^2y+2y^2$ (vi) $11-by$ **7.** $10x+6y$ **8.** $28a+10b$ **9.** (i) a (ii) $-3b-c$ (iii) $7b+3a$ (iv) a^3+2a^2-2a-1 (v) $p+1$ (vi) $2x+3y+4z$ (vii) $-4ab-8b^2$ (viii) $4pq-15p^2-2q^2$ (ix) $-2a^2+8abc+4b^2$ (x) $ab+c^2+d^2$ **10.** (i) $8-5x$ (ii) $9c+3d$ (iii) $3b+5a+6c$ (iv) $2p^2-12p$ (v) $-a+2b-4c$ (vi) $2(xy-yz+xz)$ (vii) $x^2+2xy+4y^2$ (viii) $-2a^2+5ab+8b^2$ (ix) $-11x^2-4x^2y-4y^2+5xy^2$ (x) $-3m^3-4m^2-7m+7$ **11.** $9a^2+4a-5$ **12.** $4x^3-8x^2+2x-7$ **13.** $-3a^3+2a^2-6a+7$ **14.** $a^2+6ab-b^2$ **15.** $3m^2+n^2+9p^2$ **16.** $x^3-x^2y-8xy^2+3y^3$ **17.** 4 **18.** $14x^3+2$ **19.** $10a^2+5a+18$ **20.** (i) $8a^2+12b^2-4ab$ (ii) $-10b^2-8ab$ **21.** (i) $21x^2-10xy+11y^2$ (ii) $3x^2+3y^2$ (iii) $18x^2-10xy+8y^2$ **22.** (i) $6x+30$ (ii) $14y-32$ (iii) $-21-13x$ (iv) $x-y$ (v) $2x+2y+8$ (vi) $2m-7$ (vii) $x+5$ (viii) $x-2y+8$ (ix) $-15x-25$ (x) $9x-36$

Exercise 11 (C)

1. (i) $30x^3y^2$ (ii) $30a^2b^2$ (iii) $15x^2y+6xy^2$ (iv) $-12a^2+10ab$ (v) $16a^2-25b^2$ (vi) $18x^2y-23xy^2-6y^3$ (vii) $-18m^4n^2+30m^3n^2-24m^3n^3$ (viii) $-18x^3y^5+21x^4y^5-30x^5y^3$ **2.** (i) $-9axy-6bxy$ (ii) $-27x^2y+15xy^2$ (iii) $-15x^3y^2+10x^4y+30x^3y$ (iv) a^2+b^2+2ab (v) $2a^2x^2-2abx+2ab^2x-2b^3$ (vi) $4a^2-10ab+6ac+4b^2-12bc$ (vii) $15m^2n+36mn-10n^2-9m^3-18m^2$ (viii) $6+27x-19x^2+13x^3-2x^4$ (ix) $-14x^3-10x^2+2x-24-4x^5+18x^4$ **3.** (i) $c^2+2c-15$ (ii) $12c^2-38cd+30d^2$ (iii) $\frac{1}{4}a^2-\frac{1}{4}b^2$ (iv) $a^3+3a^2b+3ab^2+b^3$ (v) $12x^4-10x^3+20x^2-15x+3$ (vi) $4m^3+18m^2-34m+12$ (vii) $40-76x+59x^2-44x^3+12x^4$ (viii) $8x^5-20x^4+14x^3+5x^2-8x+2$ (ix) $-30p^3+40p^2q-10pq^2$ (x) $-60x^2y+72xy^2+32xyz+96y^3-64y^2z$ (xi) $a^3+b^3+c^3-3abc$ **4.** (i) a^2-b^2 (ii) a^4-b^4 (iii) a^8-b^8 **5.** (i) $12x^2+xy-6y^2$ (ii) $96x^3-52x^2y-53xy^2+30y^3$ (iii) $15a^3+68a^2-37a-10$ (iv) a^3+1; a^3-1 and $2a^3$ (v) $625m^4-16n^4$ **6.** (i) $5m^6n^8$ (ii) $40m^3n^3p^3q^3$ (iii) $p^3qm-p^3m^2$ (iv) $4x^5y^2-12x^2y^5$ (v) $2a^5b-8a^3b^2$ (vi) $2x^4y+10x^3y^2-6x^2y^3$ **7.** (i) $4x^2+12xy+9y^2$ (ii) $4x^2-9y^2$ (iii) $4x^2-9y^2$ (iv) $4x^2-12xy+9y^2$ (v) $12xy-4x^2-9y^2$

(vi) $x^2y^2 - 4b^2$ (vii) $x^2 + 3bx - ax - 3ab$ (viii) $6x^2 + 17xy + 2x + 5y^2 - 34y - 48$ (ix) $15x^2 - 37xy + x + 20y^2 + 7y - 6$ (x) $18x^2 - 12xy + 2y^2$ (xi) $-1 + 3x - 9x^2 + 22x^3 - 30x^4 + 12x^5$

Exercise 11 (D)

1. (i) $-\frac{8}{3}b$ (ii) $\frac{-5x^2}{y}$ (iii) $2x + 6$ (iv) $-4a + 1$ (v) $-m + 2$ (vi) $-\frac{5}{p} + 4$ (vii) $-4x^2 + 2x$ (viii) $-2a + 3b$ (ix) $3x^2 - 2xy + xy^2$ (x) $-3a^2 + 5ab - 4b^2$ **2.** (i) $n - 1$ (ii) $m - n$ (iii) $2a + 1$ (iv) $p + 2$ (v) $x + 2y$ (vi) $2a - 3$ (vii) $3x - 2$ (viii) $4a + 12$ (ix) $3x - 4y$ (x) $3x + 2y$ (xi) $7a^2 + 2ab$ (xii) $2x^2 + 3x - 5$ **3.** $3x - 5y$ **4.** $5x + y$; $20x + 8y$ **5.** (i) $-2m^2n^4$ (ii) $5x - 3$ (iii) $10x^2 - 9y - 4xy$ (iv) $-y^2 + 3ay + 2ab^2$ (v) $-\frac{1}{5}x^3 + 3x^2 + 2$ (vi) $4a + 3x$ (vii) $3x + 7y$ (viii) $x - 2$ (ix) $m - 3$

Exercise 11 (E)

1. $\frac{3x}{4}$ **2.** $\frac{a}{2}$ **3.** $\frac{17y}{20}$ **4.** $\frac{3x}{8}$ **5.** $\frac{11y}{20}$ **6.** $\frac{p}{15}$ **7.** $\frac{37k}{30}$ **8.** $\frac{11x}{20}$ **9.** $\frac{a}{21}$ **10.** $\frac{64b}{15}$ **11.** $\frac{19k}{63}$ **12.** $\frac{11a}{40}$ **13.** $\frac{11x}{6}$ **14.** $\frac{-y}{15}$ **15.** $\frac{7x+5}{10}$ **16.** $\frac{4x+2}{3}$ **17.** $\frac{y-10}{10}$ **18.** $\frac{13a-1}{6}$ **19.** $\frac{11k-7}{12}$ **20.** $\frac{13m+10}{15}$ **21.** $\frac{475x-1186}{105}$ **22.** $\frac{70p^3}{9}$ **23.** $\frac{14p}{75}$ **24.** $\frac{150}{343}$ **25.** $\frac{1}{10}$ **26.** $\frac{5y}{2(5y-1)}$

Exercise 11 (F)

1. $x - (y + z)$ **2.** $x^2 - (xy^2 + 2xy + y^2)$ **3.** $4a - (9 - 2b + 6)$ **4.** $x^2 - (y^2 - z^2 - 3x + 2y)$ **5.** $-2a(a - 2b + 3ab^2 - 4b^2)$ **6.** $x - 2y + z$ **7.** $2p - q$ **8.** $13x - 5$ **9.** $14a + 9b$ **10.** $p - 5q + r$ **11.** $18ab - 27a^2 + 63ca$ **12.** $10m^2 - 15mn + 35mp$ **13.** $-x^2 - 2xy$ **14.** 1 **15.** $6a + 4b - 38c$ **16.** $a^2 - b^2 - c^2 + 1$ **17.** $8x^2 + 33xy$ **18.** $a + b + c - d$ **19.** $-1 - 7x$ **20.** $a + 2b$ **21.** $x + 3$ **22.** $6b - 5a - 8$ **23.** $4x - 6y$ **24.** $3a - 3b + 6$ **25.** $-104m + 72n - 176p$ **26.** $p + 3q - 1$ **27.** $-34a + 18$ **28.** $-a$ **29.** 11 **30.** $x^2 - y^2 + z^2 + 2xy - 2xz$ **31.** $-a + 15b - 86$ **32.** $p^2 + x^2 - 2q^2 - 2y^2$ **33.** $21 - a$ **34.** $-a$ **35.** $16x - 6y$ **36.** $-9x + 2y + 2z$

Exercise 12 (A)

1. 5 **2.** 5 **3.** 8 **4.** 13 **5.** –7 **6.** 4 **7.** 2 **8.** 0 **9.** 18 **10.** 48 **11.** 5 **12.** 0 **13.** 15 **14.** 13 **15.** 5 **16.** $\frac{5}{6}$ **17.** 30 **18.** 14 **19.** 6·5 **20.** $3\frac{1}{4}$ **21.** $-\frac{5}{8}$ **22.** 0·06 **23.** $14\frac{2}{3}$ **24.** –5 **25.** 30 **26.** 32 **27.** 112 **28.** 4 **29.** 3 **30.** 3 **31.** $b = 11$ **32.** 50 **33.** 144

Exercise 12 (B)

1. 5 **2.** 2 **3.** $8\frac{2}{3}$ **4.** 2 **5.** 0 **6.** –14 **7.** $\frac{1}{4}$ **8.** 3 **9.** 1 **10.** –20 **11.** $2\frac{1}{2}$ **12.** 18 **13.** 2 **14.** –4 **15.** 5 **16.** –2 **17.** 2 **18.** $-2\frac{2}{5}$ **19.** $5\frac{1}{2}$ **20.** 8

Exercise 12 (C)

1. 6 **2.** 15 **3.** 8 **4.** 20 **5.** 24 **6.** 1 **7.** –1 **8.** 10 **9.** $2\frac{2}{5}$ **10.** 72 **11.** 75 **12.** 20 **13.** 20 **14.** 200 **15.** 15 **16.** 50 **17.** 6 **18.** 7 **19.** 10 **20.** 5 **21.** 11 **22.** $-4\frac{15}{41}$ **23.** –2 **24.** 2 **25.** $5\frac{1}{2}$ **26.** 10 **27.** 6 **28.** –9 **29.** 6 **30.** $5\frac{1}{2}$

Exercise 12 (D)

1. 25 **2.** 12 **3.** 54 **4.** 38 **5.** 32 **6.** 150 **7.** 44 **8.** 12 and 6 **9.** 105 and 90 **10.** 16, 18 and 20 **11.** 19, 21 and 23 **12.** 49 **13.** Man = 40 years and son = 10 years **14.** 7 **15.** 16, 17 and 18 **16.** 10 and 17 **17.** 38 m and 33 m **18.** $\frac{3}{7}$ **19.** 36 years and 12 years

20. 80 notes of ₹ 5 each and 10 notes of ₹ 10 each

Exercise 13 (A)

1. (i) Not a set (ii) Set (iii) Not a set (iv) Not a set (v) Set.

2. (i) True (ii) True (iii) True, as $\{s, u, c, h, i, s, m, i, t, a\} = \{s, u, c, h, i, m, t, a\}$ (iv) True.

3. (i) $\in$ (ii) $\notin$ (iii) $\in$ (iv) $\in$ (v) $\notin$ (vi) $\in$ (vii) $\notin$ (viii) $\in$

4. (i) {17, 19, 21, 23, 25} (ii) A = $\{c, h, i, t, a, m, b, r\}$ (iii) B = {16, 18, 20, 22, 24, 26}
(iv) P = $\{a, i, e\}$ (v) S = {0, 1, 4, 9, 16, 25, 36, 49}
(vi) {12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90}
(vii) C = {4, 6, 8, 9, 10, 12, 14, 15, 16, 18} (viii) D = {2, 3, 5, 7, 11, 13, 17, 19, 23}
(ix) E = {2, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18, 20, 22, 24, 25, 26, 28}
(x) F = {1, 2, 3, 4, 6, 8, 12, 24} (xi) G = {triangle, circle, square)
(xii) H = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} (xiii) J = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
(xiv) K = {–2, –1, 0, 1, 2, 3, 4}

5. (i) $\{x : x$ is a natural number divisible by 3; $x < 18\}$ (ii) $\{x : x$ is a prime number$\}$
(iii) $\{x : x$ is perfect square natural number; $x \le 36\}$ (iv) $\{x : x$ is a whole number divisible by 2$\}$
(v) $\{x : x$ is one of the first three days of the week$\}$
(vi) $\{x : x$ is an odd natural number; $x \ge 23\}$
(vii) $\{x : x = \frac{1}{n}$, where n is a natural number; $3 \le n \le 8\}$
(viii) $\{x : x$ is a natural number divisible by 7; $42 \le x \le 77\}$

6. A = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24}; B = {1, 4, 9, 16};
C = {3, 6, 9, 12, 15, 18, 21, 24}; D = {2, 3, 5, 7, 11, 13, 17, 19, 23}

Exercise 13 (B)

1. (i) 366 (ii) 12 (iii) 7 (iv) 5 (v) 3 (vi) 7 (vii) 5

2. (i) Infinite (ii) ϕ (iii) Finite (iv) Infinite (v) Infinite (vi) ϕ (vii) Infinite (viii) Finite (ix) ϕ (x) Finite **3.** (ii) **4.** (i) Equivalent (ii) Equal (iii) Equivalent (iv) Equivalent (v) None (vi) Equivalent (vii) Equal (viii) Equal (ix) None **5.** (ii) and (iii)

6. (i) True (ii) True (iii) False (iv) True (v) False (vi) False (vii) False (viii) False (ix) True (x) True **7.** ϕ and { }

Exercise 13 (C)

1. (i) subset; superset (ii) subset (iii) subset (iv) is less than **2.** (i), (iv) and (v)

3. (i) and (ii) **4.** (i) True (ii) False (iii) False (iv) False (v) False (vi) True

5. Subsets of A are : { }, $\{a\}$, $\{c\}$, $\{a, c\}$
Subsets of B are : { }, $\{p\}$, $\{q\}$, $\{r\}$, $\{p, q\}$, $\{p, r\}$, $\{q, r\}$, $\{p, q, r\}$
Subsets of C are : { }, {1}, {3}, {5}, {1, 3}, {3, 5}, {1, 5}, {1, 3, 5}

6. (i) 8 (ii) 15 (iii) 2 (iv) 3 **7.** (i) {4, 6, 8, 10, 12} (ii) {9, 11, 13} (iii) {5, 7, 11, 13} (iv) {4, 6, 8}

Exercise 13 (D)

1. (i) {4, 5, 6, 7, 8, 10, 12} (ii) {6, 8} (iii) {4, 5, 7} (iv) {10, 12} **2.** (i) 5 (ii) 3 (iii) {3, 4, 5, 7, 9, 10, 11} and 7 (iv) {7} and 1 **3.** (i) {6} and 1 (ii) {2, 4, 8} and 3 (iii) 4

4. (i) 6 (ii) 5 (iii) {8, 16} and 2 **5.** (i) {1, 2, 3, 4} and 4 (ii) {8, 9, 10} and 3

6. (i) 7 (ii) 6 (iii) {6, 12} and 2 (iv) 5 **7.** (i) 95 (ii) 30 (iii) 20 **8.** (i) 13 (ii) 32 (iii) 25

9. (i) 12 (ii) 18 **10.** (i) 25 (ii) 14 (iii) 45

Exercise 14 (A)

1. (i) False (ii) True (iii) True (iv) True (v) (a) 2 (b) 1 (c) 0 (vi) $\overline{AB}$ (vii) infinite (viii) (a) 1 (b) 3 (ix) No (x) No

2. (i) No; since ∠AOB and ∠AOC are not on opposite sides of the common arm OB.
(ii) No; since ∠AOB and ∠AOC are not on opposite sides of the common arm OC.
(iii) Yes; since ∠AOB and ∠AOC are on opposite sides of the common arm OA.
(iv) No; since ∠AOB and ∠AOC are not on opposite sides of the common arm OB.

3. (i) 35° (ii) 60° (iii) 120° **4.** 30° **5.** 90° **6.** $p° = q° = r° = 60°$ **7.** $x = 120°$ and $y = 60°$

8. $b = 90°$ **9.** (i) 8° (ii) 82° **10.** (i) 65° (ii) 0° (iii) $(90 - a)°$ (iv) $(85 - x)°$ (v) $(60 + a)°$ (vi) 45° (vii) 30° (viii) 68° 43′ **11.** (i) 80° (ii) 180° (iii) $(180 - x)°$ (iv) $(145 - x)°$ (v) $(90 - a - b)°$ (vi) $(70 + x + 2y)°$ (vii) 162° (viii) 99° 10′ 35″ **12.** (i) Yes (ii) No (iii) Yes (iv) Yes **13.** (i) No (ii) Yes (iii) No (iv) Yes **14.** (27·4)° = 27° 24′

15. 15° and 75° **16.** 40° and 140° **17.** 30°, 45° and 105° **18.** 600° **19.** 100°

20. x = 18° and its supplement = 162° **21.** (i) (52·5)° = 52° 30′ (ii) 53° **22.** 60° and 30°

23. 60° and 12° **24.** 72°, 72°, 90° and 126° **25.** (i) 45° (ii) 5°

Exercise 14 (B)

1. (i) Interior alternate angles (ii) Adjacent angles (iii) Corresponding angles (iv) Exterior alternate angles (v) Cointerior (allied) angles (vi) Exterior alternate angles (vii) Corresponding angles (viii) Vertically opposite angles (ix) Adjacent angles

2. (i) Vertically opposite angles (ii) Interior alternate angles (iii) Vertically opposite angles (iv) Corresponding angles (v) Vertically opposite angles (vi) Cointerior angles (vii) Vertically opposite angles (viii) Adjacent angles (ix) Cointerior angles (x) Adjacent angles (xi) Cointerior angles

3. (i) $\angle a = \angle b = \angle c$ (ii) $x = y = l = n = r$ and $k = m = q$ **4.** $a = 110°$, $b = 70°$; $c = 70°$, $d = 110°$, $e = 70°$, $f = 110°$, $g = 70°$ **5.** (ii), (iv) and in (v) l_2 and l_4 **6.** (i) $a = 60°$; $b = 60°$ and $c = 60°$ (ii) $x = 125° = z$ and $y = 55°$ (iii) $a = 60°$; $b = 60°$ and $c = 120°$
(iv) $x = 50°$; $y = 60°$ and $z = 250°$ (v) $k = 60°$; $x = 90°$; $y = 120°$ and $z = 60°$
(vi) $x = 110°$; $s = 70°$; $y = 70°$; $p = 60°$; $q = 120°$; $r = 120°$ and $t = 120°$
(vii) $y = 110°$; $z = 70°$; $x = 60°$; $p = 60°$ and $q = 120°$ (viii) $y = 75°$; $z = 37°$ and $x = 68°$
(ix) $a = 65°$; $b = 55°$ and $c = 60°$; (x) $z = 110°$; $x = 70°$ and $y = 110°$
(xi) $x = 70°$ and $y = 290°$ (xii) $a = 270°$ and $b = 90°$

7. (i) $x = 50°$; $y = 40°$; $z = 50°$ and $p = 130°$ (ii) $x = 45°$; $y = 110°$ and $p = 45°$

8. (i) $x = 60°$ (ii) $x = 20°$ (iii) $x = 36°$ (iv) $x = 24°$ (v) $x = 27°$ (vi) $x = 13°$

Exercise 15 (A)

1. (i) Yes (ii) No (iii) No (iv) Yes **2.** $60°$ each **3.** $C = 60°$ **4.** $60°$ **5.** (i) $60°$ (ii) $80°$ (iii) $50°$ **6.** (i) $x° = 18°$; $\angle A = 90°$; $\angle B = 72°$ and $\angle C = 18°$ (ii) $\angle A = 36°$; $\angle B = 72° = \angle C$
7. (i) $b = 65°$ (ii) $x = 45°$ (iii) $k = 60°$ (iv) $m = 60°$ **8.** (i) $\angle a = 110°$ (ii) $\angle c = 45°$ (iii) $\angle b = 60°$
9. $48°$, $60°$ and $72°$ **10.** $50°$ and $70°$ **11.** $63°$ and $56°$
12. (i) $x = 80°$ (ii) $y = 60°$ (iii) $k = 87°$ (iv) $a = 62°$ (v) $a = 85°$; $b = 55°$ and $c = 40°$ (vi) $x = 68°$ and $y = 49°$ (vii) $a = 60°$ (viii) $m = 20°$ (ix) $a = 75°$ and $b = 52{\cdot}5° = 52°30'$

Exercise 15 (B)

1. (i) $x = 50° = y$ (ii) $a = 100°$ and $b = 40°$ (iii) $y = 45° = x$ (iv) $x = 130°$ and $a = 50° = b$ (v) $p = 137°$ (vi) $n = 50°$ and $m = 35°$ (vii) $x = 60°$ and $y = 120°$ **2.** (i) $y = 35°$ and $x = 40°$ (ii) $x = y = 100°$ (iii) $x = 70°$; $p = 60°$ and $y = 50°$ (iv) $x = 120°$ and $y = 45°$ (v) $x = y = 110°$ (vi) $x = 60°$; $y = 120°$ and $z = 35°$ **3.** $40°$, $40°$ **4.** $76°$ **5.** $80°$, $80°$ and $20°$
6. $55°$, $55°$ and $70°$ **7.** $65°$, $65°$ and $50°$ **8.** $22°\ 30'$, $22°\ 30'$ and $135°$ **9.** $30°$, $30°$ and $120°$
10. $110°$ **11.** $a = 360° - 2b$ **12.** (i) $30°$ (ii) $55°$ **13.** (i) $\angle ABE = 150°$ and $\angle BAE = 15°$ (ii) $\angle ABE = 30°$ and $\angle BAE = 75°$ **14.** $a = b = 117°$

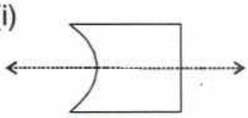
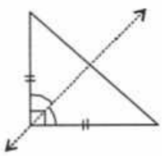
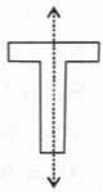
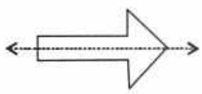
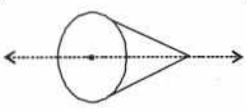
Exercise 15 (C)

3. (i)$\angle R = 75°$ (ii) PQ = 2·3 cm and PR = 4·4 cm (iii) $\angle Q = 75°$ **4.** AB = BC = 9·6 cm approx.
5. (i) Each base angle is $(67{\cdot}5)° = 67°\ 30'$ (ii) $\angle A = 75° = \angle C$ **6.** (i) PA = PB = PC = 2·9 cm approx.

Exercise 16

1. 30 cm **2.** 5 cm **3.** 5·2 cm **4.** (i) **5.** 500 m **6.** (i) 9 m (ii) 12 m (iii) 21 m
7. (i) 10 cm (ii) 16 cm **9.** (i) 8 cm (ii) 15 cm **10.** 20 cm **11.** 4 cm **12.** 6 m
13. 13 m **14.** 26 cm

Exercise 17 (A)

1. (i)

(ii)

(iii) not possible
(iv)

(v)

(vi)

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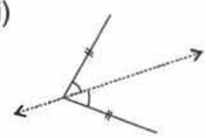
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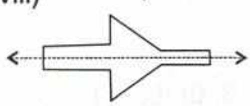
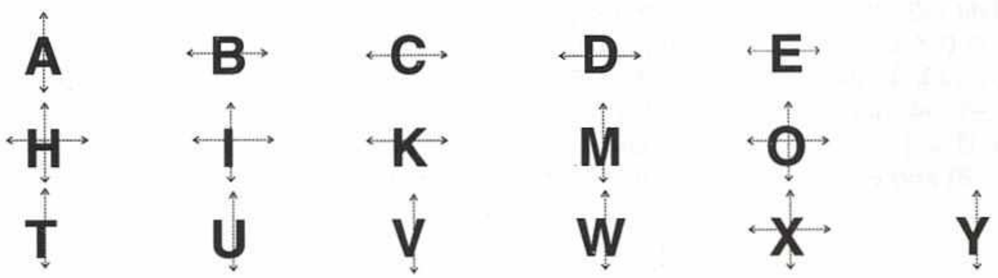
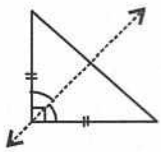
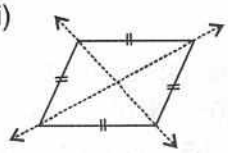
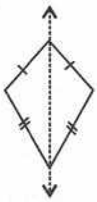


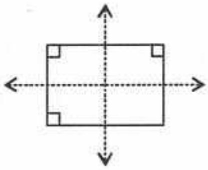
2.


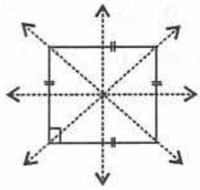
3. (i) not possible

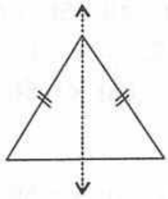
(ii) 

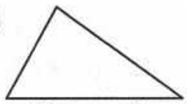
(iii) 

(iv) 

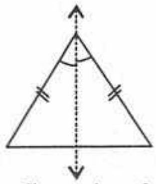
(v) 

(vi) 

(vii) 

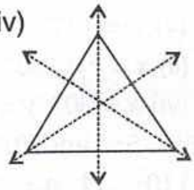
4. (i) 

[Scalene Δ]

(ii) 

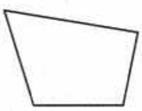
[Isosceles Δ]

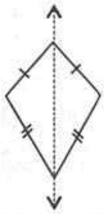
(iii) not possible

(iv) 

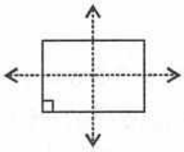
[Equilateral Δ]

(v) not possible

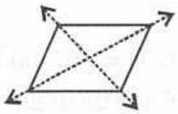
5. (i) 

(ii) 

[Kite-shaped]

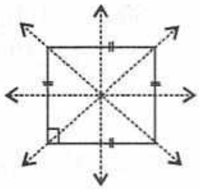
(iii) 

[Rectangle]



[Rhombus]

(iv) not possible

(v) 

(vi) not possible

6. Since, the triangle is equilateral; it will have three lines of symmetry.
7. AB = AC ⇒ the triangle ABC is isosceles and so it has only one line of symmetry.
8. PQ = QR ⇒ Δ is isosceles and it has only one line of symmetry.
9. The required figure is a rhombus or a rectangle.
10. AB = AD and CB = CD

Exercise 17 (B)

3. (i) (8, –2), (–8, 2) and (–8, –2) (ii) (5, –6), (–5, 6) and (–5, –6) (iii) (4, 5), (–4, –5) and (–4, 5) (iv) (6, 2), (–6, –2) and (–6, 2) (v) (–3, –7), (3, 7) and (3, –7) (vi) (–4, –5), (4, 5) and (4, –5) (vii) (–2, 7), (2, –7) and (2, 7) (viii) (–6, 3), (6, –3) and (6, 3) (ix) (4, 0), (–4, 0) and (–4, 0) (x) (–7, 0), (7, 0) and (7, 0) (xi) (0, 6), (0, –6) and (0, 6) (xii) (0, –8), (0, 8) and (0, –8) (xiii) (0, 0), (0, 0) and (0, 0) **4.** (i) P′ = (7, –3) and P′ = (–7, –3) (ii) (–7, –3) **5.** (i) B = (5, 4) (ii) C = (–5, –4) (iii)(–5, –4) **6.** (i) Q = (–3, 8) (ii) R = (–3, –8) (iii) (–3, –8) **7.** A′ = (3, 0), B′ = (7, 0), C′ = (–8, 0), D′ = (–7, 0) and E′ = (0, 0) **8.** A′ = (0, 4), B′ = (0, 10), C′ = (0, –4), D′ = (0, –6) and E′ = (0, 0) **9.** A′ = (0, –7), B′ = (–8, 0), C′ = (0, 5), D′ = (7, 0) and E′ = (0, 0) **10.** A′ = (4, –5), and B′ = (–5, –4). Yes : AB = A′B′ **11.** A′ = (–6, 4), and B′ = (–4, –6) **12.** A′ = (6, –5), and B′ = (4, 6). Yes : AB = A′B′

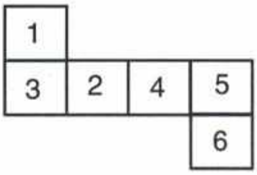
Exercise 17 (C)

1. 2 **2.** 2 **4.** A kite shaped figure **5.** (i) none and 3 (ii) 2 and 2 (iii) 2 and 2 (iv) none and none (v) one and none

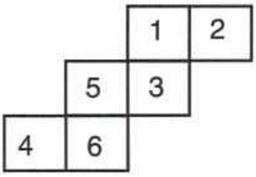
Exercise 18

1. (i), (ii) and (v) **3.** (i) cube (ii) cuboid (iii) prism 4. a = 8, b = 6 and c = 30

5. (i)



(ii)



6. (i) cube (ii) cuboid **9.** 165·6 km

Exercise 19

1. (i) no (ii) yes (by S.A.S.) (iii) no (iv) yes (by S.S.S.) (v) yes (by R.H.S.) (vi) yes (by A.A.S. or A.S.A.) (vii) no **14.** x = y = 45°

Exercise 20 (A)

1. 400 m and ₹ 24,000 **2.** 140 m and 80 m ; ₹ 66,000

3. Breadth = $\sqrt{34^2 - 30^2}$ m = $\sqrt{256}$ m = 16 m; 92 m **4.** 48 cm **5.** 48·2 m **6.** 62 cm

7. 125 m and 75 m **8.** 16 cm **9.** (i) 130 cm (ii) 20 cm (iii) 20 m **10.** 900 m and 225 m

11. 32 cm **12.** 132 cm **13.** 70 cm and 140 cm **14.** 176 m and ₹ 14,080

15. 44 cm **16.** 132 cm and 42 cm **17.** (i) 88 cm (ii) 88 cm (iii) 14 cm **18.** 23 cm

19. 80 cm **20.** 192 cm **21.** 3 : 5 **22.** 5 : 7 **23.** 8 : 15 **24.** 8 : 15

25. 176 cm and 28 cm

Exercise 20 (B)

1. 400 cm^2 **2.** 2688 cm^2 **3.** (i) 168 m (ii) ₹ 6,720 **4.** 2000 **5.** (i) 125 m and 75 m (ii) 9375 m^2 (iii) ₹ 5,62,500 **6.** 196 cm^2 **7.** 196 m^2 **8.** (i) 170 cm^2 (ii) 456 cm^2 (iii) 1120 cm^2 (iv) 1375 cm^2 (vi) 84 cm^2 **9.** 320 cm^2 **10.** 48 cm and 16 cm

11. 360 cm^2 **12.** 16 cm **13.** 8·5 cm **14.** 9 cm **15.** 6 cm **16.** 270 cm^2 **17.** 30 cm

18. 20 cm **19.** 30 cm^2 **20.** 110.72 cm^2 **21.** 84 cm^2 **22.** 48 cm^2 **23.** 30 cm

24. 104 cm and 78 cm **25.** 16 cm **26.** 1344 cm^2 **27.** 62.8 cm and 314 cm^2

28. 4.2 cm **29.** (i) 5 : 7 (ii) 25 : 49 **30.** (i) 4 : 3 (ii) 4 : 3 (iii) 4 : 3

31. 14 m **32.** (i) 42 cm (ii) 49 m (iii) 308 m (iv) ₹ 15,400 **33.** 3.96 km

34. $454\frac{6}{11}$ **35.** 88 cm (i) 14 cm (ii) 616 cm^2 **36.** 1936 cm^2 **37.** $380\frac{2}{7}$ cm^2 = 380·29 cm^2

38. 446 cm^2

Exercise 21 (A)

1. (i) 52, 63, 63, 68, 70, 71, 73, 75, 75, 76, 77, 82, 83, 90, 92, 93, 99, 103, 105, 115

(ii) 115 – 52 = 63

2.

Numbers	Tally-marks	Frequency
15	𝍸	5
16	𝍸 II	7
17	𝍸 𝍸 I	11
18	𝍸	5
19	𝍸 I	6
20	III	3
21	III	3
Total		**40**

3.

Numbers	Tally-marks	Frequency
1	IIII	4
2	IIII	4
3	III	3
4	III	3
5	III	3
6	III	3
Total		**20**

4.

Wages (in ₹)	Tally-marks	Frequency
3500	I	1
3750	I	1
4000	IIII	4
4250	III	3
4750	I	1
Total		**10**

(ii) Range = ₹ 4750 – 3500 = ₹ 1250

(iii) Only one

5.

Numbers	Tally-marks	Frequency
10	II	2
20	III	3
30	III	3
40	𝍸 II	7
50	𝍸 I	6
60	𝍸 II	7
70	𝍸	5
80	IIII	4
90	III	3
Total		**40**

(ii) 5 + 4 + 3 = 12 (iii) 2 + 3 + 3 = 8

6. 4.1, 4.0, 3.9, 3.9, 3.8, 3.8, 3.7, 3.7, 3.6, 3.5, 3.5, 3.4, 3.3, 3.2, 3.1 (i) 4.1 – 3.1 = 1 (ii) 4 (iii) 6

Exercise 21 (B)

1. 61 **2.** 3.5 **3.** 10 **4.** 4.5 **5.** x + 7 **6.** 15.801 **7.** (i) 35 (ii) 25 (iii) 64 (iv) 64 (v) 51.2 (vi) 25.6 **8.** ₹ 85.75 **9.** 12 **10.** 10.5 **11.** 5.6 **12.** 12 **13.** 6 **14.** p = 13 **15.** 17.5 **16.** 15.6 mm and 2.23 mm (approx.) **17.** 39.7 **18.** 35 **19.** 15 **20.** 25 **21.** (i) 11 (ii) 9 (iii) 20 (iv) 6·5 **22.** Mean = 27·73 (approx.) and mode = 26 **23.** (i) 6 (ii) 10 **24.** (i) 18 (ii) 41 **25.** Mean = 140·375 cm and median = 139 cm **26.** (i) 19, 18 and 12 (ii) 19, 19·5 and 21

Exercise 22 (A)

1. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ **2.** (i) $\frac{21}{40}$ (ii) $\frac{19}{40}$ **3.** (i) $\frac{1}{5}$ (ii) $\frac{1}{4}$ (iii) $\frac{11}{20}$ **4.** (i) $\frac{21}{50}$ (ii) $\frac{29}{50}$ **5.** (i) $\frac{3}{20}$ (ii) $\frac{17}{20}$ **6.** (i) $\frac{1}{8}$ (i) $\frac{1}{8}$ **7.** (i) $\frac{21}{100}$ (iii) $\frac{11}{20}$ (iii) $\frac{6}{25}$ **8.** (i) $\frac{6}{10} = \frac{3}{5}$ (ii) $\frac{4}{10} = \frac{2}{5}$ (iii) $\frac{2}{5}$ **9.** (i) $\frac{1}{6}$ (ii) $\frac{1}{6}$ (iii) $\frac{2}{6} = \frac{1}{3}$ **10.** (i) $\frac{1}{100}$ (ii) $\frac{1}{100}$ (iii) $\frac{1}{100}$

Exercise 22 (B)

1. (i) It will not snow tomorrow (ii) P(S′) = 1 – 0.03 = 0.97 **2.** (i) E (ii) C (iii) 100% *i.e.* 1 (iv) $\frac{35}{100} = \frac{7}{20}$ (v) (a) $\frac{11}{50}$ (b) S′ = B will not win the race. (c) P(S′) = 1 – P(S) $= 1 - \frac{11}{50} = \frac{39}{50}$ **3.** (a) $\frac{3}{12} = \frac{1}{4}$ (b) $\frac{7}{12}$ (c) 0 **4.** (a) $\frac{1}{10}$ (b) $\frac{2}{10} = \frac{1}{5}$ (c) $\frac{6}{10} = \frac{3}{5}$ (d) $\frac{5}{10} = \frac{1}{2}$ **5.** (a) $\frac{8}{12} = \frac{2}{3}$ (b) $\frac{4}{12} = \frac{1}{3}$ 6. (a) $\frac{3}{15} = \frac{1}{5}$ (b) $\frac{4}{15}$ (c) $\frac{8}{15}$ (d) 0 (e) $\frac{12}{15} = \frac{4}{5}$ **7.** (i) $\frac{13}{23}$ (ii) $\frac{10}{23}$ (iii) $\frac{5}{23}$ (iv) $\frac{10}{23}$

SPEED, DISTANCE AND TIME 10A

10.1 SPEED

*The distance covered by a moving object in unit time is called its **speed**.*

$$\therefore \text{ Speed} = \frac{\text{Distance}}{\text{Time}}$$

1. If the **distance** moved by an object is **in metres** (m) and **time** is **in seconds** (s), its speed is **in metres per second** (ms^{-1}).

   Thus, **speed** in metres per second (ms^{-1}) $= \dfrac{\text{Distance in metres (m)}}{\text{Time in seconds (s)}}$

2. If the **distance** covered by an object is **in kilometres** (km) and **time** is **in hours**, its speed is **in kilometres per hour** ($km\ h^{-1}$).

   Thus, **speed** in kilometres per hour $= \dfrac{\text{Distance in kilometres (km)}}{\text{Time in hours (h)}}$

Moreover :

	Distance	**Time**	**Speed = $\frac{\text{Distance}}{\text{Time}}$**
1.	Centimetres (cm)	Seconds (s)	Centimetres per second ($cm\ s^{-1}$)
2.	Metres (m)	Minutes (min)	Metres per minute (m/min)
3.	Kilometres (km)	minutes (min)	Kilometres per minute ($km\ min^{-1}$) and so on.

Example 1 :

A boy walks 300 metres in 2 minutes. Find his speed in :
(i) m/min ($m\ min^{-1}$) (ii) m/sec (ms^{-1}) (iii) km/h ($km\ h^{-1}$)

Solution :

(i) $\because$ Distance = 300 m and time = 2 min

$\therefore$ **Speed** $= \dfrac{\text{Distance}}{\text{Time}} = \dfrac{300\text{ m}}{2\text{ min}} = $ **150 m min^{-1}** **Ans.**

(ii) $\because$ Distance = 300 m and time = 2 min = 2 × 60 sec = 120 sec

$\therefore$ **Speed** $= \dfrac{300\text{ m}}{120\text{ sec}} = \dfrac{5}{2}\ ms^{-1} = $ **2·5 ms^{-1}** **Ans.**

(iii) $\because$ Distance $= 300\text{ m} = \dfrac{300}{1000}\text{ km} = 0{\cdot}3\text{ km}$

and, time $= 2\text{ min} = \dfrac{2}{60}\text{ h} = \dfrac{1}{30}\text{ hr}$

$\therefore$ **Speed** $= \dfrac{0{\cdot}3\text{ km}}{\frac{1}{30}\text{ h}} = 0{\cdot}3 \times 30\text{ km h}^{-1} = $ **9 km h^{-1}** **Ans.**

10.2 DISTANCE

$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}} \Rightarrow$ Distance covered by a moving body = Its speed × Time taken by it

1. If speed = 10 km h^{-1} and time = 2 h,

the distance covered = Speed × time = $10 \text{ km h}^{-1} \times 2 \text{ h} = 20 \text{ km}$

2. If speed = 4 km min^{-1} and time = 2 h,

the distance covered = Speed × time = $4 \text{ km min}^{-1} \times 2 \text{ h}$

$= 4 \text{ km min}^{-1} \times 120 \text{ min}$ [2 h = 2 × 60 min = 120 min]

= 480 km.

10.3 TIME

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \Rightarrow \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

1. If an object covers a distance of 800 m with a speed of 40 ms^{-1},

the time taken by the object $= \frac{\text{Distance covered}}{\text{Speed}} = \frac{800 \text{ m}}{40 \text{ ms}^{-1}} = \mathbf{20 \text{ s}}$

2. If a man covers a distance of 4 km with a speed of 12 km h^{-1}

the time taken by him $= \frac{\text{Distance}}{\text{Speed}} = \frac{4 \text{ km}}{12 \text{ km h}^{-1}} = \frac{1}{3} \text{ h} = \frac{1}{3} \times 60 \text{ min} = \mathbf{20 \text{ min}}$

Example 2 :

A man covers a distance of 3·6 km in 24 min. Find :

(i) his speed in km h^{-1}. (ii) his speed in ms^{-1}.

(iii) distance covered by him in three hours.

(iv) time taken by him to cover a distance of 4·8 km.

Solution :

(i) ∵ Distance covered = 3·6 km and time taken = 24 min = $\frac{24}{60}$ hour = $\frac{2}{5}$ hour

∴ **His speed** $= \frac{\text{Distance}}{\text{Time}} = \frac{3{\cdot}6 \text{ km}}{\frac{2}{5} \text{ h}} = 3{\cdot}6 \times \frac{5}{2} \text{ km h}^{-1} = \mathbf{9 \text{ km h}^{-1}}$ **Ans.**

(ii) ∵ Distance = 3·6 km = 3·6 × 1000 m = 3600 m and

time = 24 min = 24 × 60 sec = 1440 sec

∴ **Speed** $= \frac{\text{Distance}}{\text{Time}} = \frac{3600 \text{ m}}{1440 \text{ s}} = \frac{5}{2} \text{ ms}^{-1} = \mathbf{2{\cdot}5 \text{ ms}^{-1}}$ **Ans.**

(iii) ∵ Speed = 9 km h^{-1} and time = 3 hr

∴ **Distance covered** = Speed × time = $9 \text{ km h}^{-1} \times 3 \text{ hr} = \mathbf{27 \text{ km}}$ **Ans.**

(iv) Distance = 4·8 km and speed = 9 km h^{-1}

∴ **Time taken** $= \frac{\text{Distance}}{\text{Speed}} = \frac{4{\cdot}8 \text{ km}}{9 \text{ km h}^{-1}} = \frac{4{\cdot}8}{9} \text{ h} = \frac{4{\cdot}8}{9} \times 60 \text{ min} = \mathbf{32 \text{ min}}$ **Ans.**

10.4 CONVERSION OF UNITS

1. **From km h^{-1} to ms^{-1} :**

$\mathbf{1 \text{ km h}^{-1}} = \frac{1 \text{ km}}{1 \text{ hour}} = \frac{1000 \text{ m}}{60 \times 60 \text{ s}} = \mathbf{\frac{5}{18} \text{ ms}^{-1}}$

∴ *To convert km h^{-1} into ms^{-1}, multiply by* $\frac{5}{18}$.

For example :

(i) $\textbf{1 km h}^{-1} = 1 \times \frac{5}{18}\ ms^{-1} = \frac{5}{18}\ \textbf{ms}^{-1}$

(ii) $\textbf{72 km h}^{-1} = 72 \times \frac{5}{18}\ ms^{-1} = \textbf{20 ms}^{-1}$

(iii) $\textbf{14·4 km h}^{-1} = 14·4 \times \frac{5}{18}\ ms^{-1} = \textbf{4 ms}^{-1}$ and so on. **Ans.**

2. **From ms^{-1} to $km\ h^{-1}$:**

$$1\ ms^{-1} = \frac{1m}{1s} = \frac{\frac{1}{1000}km}{\frac{1}{60 \times 60}h} = \frac{1}{1000} \times 60 \times 60\ km\ h^{-1} = \frac{18}{5}\ km\ h^{-1}$$

∴ *To convert ms^{-1} into $km\ h^{-1}$, multiply by $\frac{18}{5}$.*

For example :

(i) $\textbf{15 ms}^{-1} = 15 \times \frac{18}{5}\ km\ h^{-1} = \textbf{54 km h}^{-1}$

(ii) $\textbf{4·5 ms}^{-1} = 4·5 \times \frac{18}{5}\ km\ h^{-1} = 0·9 \times 18\ km\ h^{-1} = \textbf{16·2 km h}^{-1}$

More conversions:

(i) $\textbf{27 km min}^{-1} = \frac{27\ km}{1 min} = \frac{27000\ m}{60\ sec} = \textbf{450 ms}^{-1}$

(ii) $\textbf{27 km min}^{-1} = \frac{27\ km}{1 min} = \frac{27000\ m}{\frac{1}{60}h} = 27000 \times 60\ m\ h^{-1} = \textbf{16,20,000 m h}^{-1}$

(iii) $\textbf{90 cm s}^{-1} = \frac{90\ cm}{1 s} = \frac{\frac{90}{100}m}{\frac{1}{60}min} = \frac{90}{100} \times \frac{60}{1}\ m\ min^{-1} = \textbf{54 m min}^{-1}$

Example 3 :

A distance of 3·6 km is covered in 20 minutes. Find the speed in :

(i) km per hour ($km\ h^{-1}$) (ii) metre per second (ms^{-1})

Solution :

(i) ∵ Distance covered = 3·6 km

and, time-taken = 20 minutes = $\frac{20}{60}$ h = $\frac{1}{3}$ h

∴ **Speed** $= \frac{\text{Distance}}{\text{Time}} = \frac{3·6\ km}{\frac{1}{3}h} = 3·6 \times 3\ km\ h^{-1} = \textbf{10·8 km h}^{-1}$ **Ans.**

(ii) ∵ Distance covered = 3·6 km = 3·6 × 1000 m = 3600 m

and, time taken = 20 min = 20 × 60 sec = 1200 sec

∴ **Speed** $= \frac{\text{Distance}}{\text{Time}} = \frac{3600\ m}{1200\ sec} = \textbf{3 ms}^{-1}$ **Ans.**

Direct method : Speed = 10·8 km h–1

$= 10·8 \times \frac{5}{18} ms^{-1} = 0·6 \times 5\ ms^{-1} = \textbf{3 ms}^{-1}$

Example 4 :

A car covers 375 m distance in 25 sec.

(i) Find its speed in km/h.

(ii) If the car covers a distance of 525 m with the same speed, find the time taken by it.

Solution :

(i) $\because$ Distance = 375 m and the time taken = 25 sec

$\therefore$ **Speed** $= \frac{\text{Distance}}{\text{Time}} = \frac{375\text{ m}}{25\text{ s}} = 15\text{ ms}^{-1} = 15 \times \frac{18}{5}\text{ km h}^{-1} =$ **54 km h^{-1} Ans.**

(ii) $\because$ Distance = 525 m and speed = 15 ms^{-1}

$\therefore$ **Time taken** $= \frac{\text{Distance}}{\text{Speed}} = \frac{525\text{ m}}{15\text{ ms}^{-1}} =$ **35 s** **Ans.**

Example 5 :

A certain distance is covered in 40 minutes with a speed of 15 km/h. If the same distance is covered in 30 minutes, find the increase in speed.

Solution :

In the first case : Time = 40 min $= \frac{40}{60}$ h $= \frac{2}{3}$ h and speed = 15 km h^{-1}

$\therefore$ Distance covered = Speed × time

$= 15\text{ km h}^{-1} \times \frac{2}{3}\text{ h} = 10\text{ km}$

In the second case : Distance = 10 km and time = 30 min $= \frac{30}{60}$ h $= \frac{1}{2}$ h

$\therefore$ **Speed** $= \frac{\text{Distance}}{\text{Time}} = \frac{10\text{ km}}{\frac{1}{2}\text{h}} = 10 \times 2\text{ km h}^{-1} = 20\text{ km h}^{-1}$

$\therefore$ **Increase in speed** = 20 km h^{-1} − 15 km h^{-1} = **5 km h^{-1}** **Ans.**

Example 6 :

A journey of 360 km is covered in 5 hours. If two-third of the journey is covered with a speed of 80 km/h, find the speed with which the remaining journey is covered.

Solution :

For two-third of the journey :

Distance $= \frac{2}{3} \times 360$ km = 240 km and speed = 80 km/h

$\therefore$ Time taken $= \frac{\text{Distance}}{\text{Speed}} = \frac{240\text{ km}}{80\text{ km h}^{-1}} = 3\text{ h}$

For the remaining journey :

Distance = 360 km − 240 km = 120 km

and, time = 5 h − 3 h = 2 h

$\therefore$ **Speed** $= \frac{\text{Distance}}{\text{Time}} = \frac{120\text{ km}}{2\text{ h}} =$ **60 km/h** **Ans.**

Example 7 :

A certain distance is covered in 5 hours at 60 km h^{-1}. If the speed is lowered by 10 km h^{-1}, find the extra time taken to cover the same distance.

Solution :

In the first case : Distance covered = Speed × time

$= 60 \text{ km h}^{-1} \times 5 \text{ h} = \mathbf{300 \text{ km}}$

In the second case : Distance = 300 km and speed = $60 \text{ km h}^{-1} - 10 \text{ km h}^{-1} = 50 \text{ km h}^{-1}$

$\therefore$ Time taken = $\frac{\text{Distance}}{\text{Speed}} = \frac{300 \text{ km}}{50 \text{ km h}^{-1}} = 6 \text{ h}$

$\therefore$ **Extra time taken** = 6 h − 5 h = **1 h** **Ans.**

EXERCISE 10(A)

1. Convert

   (i) 3·6 km/h to m/s (ii) 54 km/h to cm/s (iii) 60 m/s to km/h (iv) 540 cm/s to km/h

2. Fill in the blanks :

	Distance	Speed	Time
(i)	129 m	....	3 min
(ii)	....	40 km/h^{-1}	45 min
(iii)	720 m	48 km/h	....

3. Rohit covers a distance of 140 m with the speed of 75 km/h. How much extra time will he take to cover a distance of 400 m with the same speed ?
4. Which is greater : 72 km/h or 21 m/s ?
5. A journey is covered in 3 hours with a speed of 90 km h^{-1}. What must be the speed, if the same journey is to be completed in 5 hour ?
6. A certain distance is covered in 50 minutes with a speed of 60 km per hour. How much extra time will be needed, if the distance is kept same and the speed is halved?
7. Geeta covers 800 metres in 5 minutes.
   (i) How much distance will she cover in half an hour ?
   (ii) How much time will she take to cover 6·4 km ?
   Take the speed same for each case.
8. A bus, going at 60 km per hour, takes 4 hours to travel from Delhi to Agra.
   (i) How much time will the bus take to cover the same distance, if its speed is increased by 20 km per hour ?
   (ii) How much extra time will the bus take in going from Delhi to Agra, if its speed is reduced by 20 km per hour ?
9. An animal is walking at a speed of 6 km per hour.
   (i) How much distance will it walk in 8 minutes ?
   (ii) How much time will it take to cover 600 metres with the same speed ?
10. Distance between two stations A and B is 80 km. Manoj goes from station A to station B with a certain speed and returns back to station A, with the same speed, in 5 hours. Find the speed of Manoj.
11. The initial distance between two cars A and B is 220 km. At 6 a.m., car A starts moving towards car B at 40 km/h and at 7 a.m. car B starts moving towards car A at 50 km/h. Find the time when cars A and B will meet.

Let car A meets car B after x hrs.

$\Rightarrow$ If car A moves for x hrs, car B moves for (x − 1) hrs

$\Rightarrow$ Distance moved by car A in x hrs + distance moved by car B in $(x - 1)$ hrs = 220 km

$\Rightarrow 40 \times x + 50 \times (x - 1) = 220 \quad \Rightarrow \quad x = 3$

$\therefore$ A and B meet at (6 + 3) a.m. *i.e.* 9 a.m.

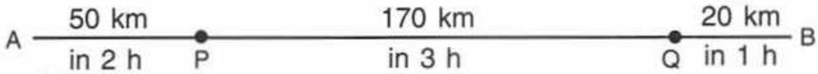
12. How much distance will be covered by a car in 24 seconds at 45 km/h ?

13. A car is moving with a speed of 72 km h^{-1} and another car is moving with a speed of 16 ms^{-1}. If both the cars start from the same place, in the same direction and at the same time, find the difference of distances covered by both the cars in :

(i) 1 second (ii) 15 seconds

14. A distance of 6 km is covered in 8 minutes. If the same distance is to be covered in 4 minutes, by how much must the speed be increased ?

10.5 AVERAGE SPEED



Suppose, a car goes from place A to place B through places P and Q. The car goes from A to P, a distance of 50 km in 2 h, then from P to Q, a distance of 170 km in 3 h and finally from Q to B, a distance of 20 km in 1 h.

Clearly, the total distance covered by the car in going from place A to place B

$= 50 \text{ km} + 170 \text{ km} + 20 \text{ km} = 240 \text{ km}$

and, the total time taken by the car in going from place A to place B

$= 2 \text{ h} + 3 \text{ h} + 1 \text{ h} = 6 \text{ h}$

$$\text{Speed} = \frac{240 \text{ km}}{6 \text{ h}} = 40 \text{ km per hour}$$

The speed so obtained is called **average speed** which is obtained on dividing the total distance covered by the total time taken.

$$\therefore \ \textbf{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

Example 8 :

A, B, C and D are four stations. A boy goes from A to B, a distance of 54 km, in three hours, from B to C, a distance of 18 km in 30 minutes and finally from C to D a distance of 45 km in one hour. Find the average speed of the boy.

Solution :

Clearly, total distance covered $= 54 \text{ km} + 18 \text{ km} + 45 \text{ km} = 117 \text{ km}$

and, total time taken $= 3 \text{ h} + 30 \text{ min} + 1 \text{ h}$

$$= 3\text{h} + \frac{1}{2}\text{h} + 1 \text{ h} = 4\frac{1}{2} \text{ h} = \frac{9}{2} \text{ h}$$

$$\therefore \quad \text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{117 \text{ km}}{\frac{9}{2}\text{h}} = 117 \times \frac{2}{9} \text{ km h}^{-1} = \mathbf{26 \text{ km h}^{-1}} \quad \textbf{Ans.}$$

Example 9 :

A car covers 250 km, 120 km and 50 km with the speeds 50 km/hr, 40 km/hr and 25 km/hr respectively. Find its average speed on the whole journey.

Solution :

$$\because \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\therefore \quad \text{Time taken in 1st part} = \frac{250}{50} \text{ hr} = 5 \text{ hr}$$

Time taken in 2nd part = $\frac{120}{40}$ hr = 3 hr

and, time taken in 3rd part = $\frac{50}{25}$ hr = 2 hr

Total time taken = 5 hr + 3 hr + 2 hr = 10 hr

And, total distance covered = 250 km + 120 km + 50 km = 420 km

$\therefore$ **Average speed** = $\frac{420 \text{ km}}{10 \text{ hr}}$ = **42 km/h** **Ans.**

10.6 RELATIVE SPEED

The relative speed between two moving objects is the speed of any one of these two objects with respect to the other.

In fact, the relative speed between two moving objects
= the increase or decrease of distance between them in unit time.

1. *When two objects are moving in opposite directions, the relative speed between them = sum of their speeds.*

   In the given diagram, two objects are moving with speeds x km/h and y km/h in opposite directions. The relative speed between them
   = sum of their speeds = $(x + y)$ km/h

   (a) $\overrightarrow{x \text{ km/h}}$ $\overleftarrow{y \text{ km/h}}$

   (b) $\overleftarrow{x \text{ km/h}}$ $\overrightarrow{y \text{ km/h}}$

For figure (a), the distance between the two objects is decreasing by $(x + y)$ km in every one hour.

For figure (b), the distance between the two objects is increasing by $(x + y)$ km in every one hour.

2. *When two objects are moving in the same direction, the relative speed between them = difference between their speeds.*

   In the given diagram, two objects are moving with speeds x km/h and y km/h in the same direction. The relative speed between them

   $\overrightarrow{x \text{ km/h}}$ $\overrightarrow{y \text{ km/h}}$

   = difference between their speeds
   = $(x - y)$ km/h, when x is greater than y.
   or, = $(y - x)$ km/h, when y is greater than x.

In this case, the distance between the two objects is decreasing either by $(x - y)$ km or by $(y - x)$ km in every one hour.

Further :

(i) $\overrightarrow{30 \text{ km/h}}$ $\overleftarrow{20 \text{ km/h}}$ $\Rightarrow$ Relative speed = 30 km/h + 20 km/h = 50 km/h

(ii) $\overleftarrow{50 \text{ km/h}}$ $\overrightarrow{40 \text{ km/h}}$ $\Rightarrow$ Relative speed = (50 + 40) km/h = 90 km/h

(iii) $\overrightarrow{30 \text{ km/h}}$ $\overrightarrow{20 \text{ km/h}}$ $\Rightarrow$ Relative speed = (30 – 20) km/h = 10 km/h

(iv) $\overleftarrow{70 \text{ km/h}}$ $\overleftarrow{85 \text{ km/h}}$ $\Rightarrow$ Relative speed = (85 – 70) km/h = 15 km/h.

The distance between the two moving objects at a given time
= Relative speed between the objects × time

Example 10 :

Two boys start from the same place. One of them walks at 8 km per hour and the other at 9 km per hour. Find the distance between the boys at the end of 5 hours, if they walk in the : (i) same direction (ii) opposite directions.

Solution :

(i) **When the boys walk in the same direction :**

Relative speed = 9 km/h − 8 km/h = 1 km/h

The distance between the two boys at the end of 5 hours

= Relative speed × time = 1 km/h × 5 h = **5 km** **Ans.**

(ii) **When the boys walk in opposite directions :**

Relative speed = 9 km/h + 8 km/h = 17 km/h

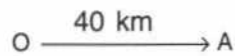
The distance between the two boys at the end of 5 hours

= Relative speed × time = 17 km/h × 5 h = **85 km** **Ans.**

Alternative method :

(i) **For 1st boy :**

Speed = 8 km/h and time = 5 h

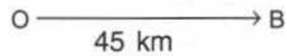


⇒ Distance covered = Speed × time

= 8 km/h × 5 h = 40 km

For 2nd boy :

Speed = 9 km/h and time = 5 h



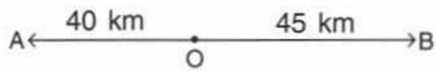
⇒ Distance covered = 9 km/h × 5 h = 45 km

∴ **The distance between the two boys at the end of 5 h**

= 45 km − 40 km = **5 km** **Ans.**

(ii) **Required distance** = 40 km + 45 km

= **85 km** **Ans.**



10.7 TRAIN

1. *In order to cross a pole or a stationary person by a moving train,* the distance covered by the train = the length of the train.
2. *In order to cross a platform* (or, a bridge, etc.) *by a moving train,* the distance covered by the train

= length of the train + length of the platform (or, a bridge, etc.).

Example 11 :

A 540 metre long train is running at a speed of 81 km/hr. Find the time taken by the train to cross a pole ?

Solution :

The distance covered by the train to cross a pole = Length of train = 540 metre

And, its speed = 81 km/h = $81 \times \frac{5}{18}$ m/s = $\frac{45}{2}$ m/s

∴ **Time taken** = $\frac{540 \text{ m}}{45/2 \text{ m/s}} = \frac{540 \times 2}{45}$ sec = **24 sec** **Ans.**

Example 12 :

A train, 160 m long, takes one minute in crossing a tunnel 840 m long. Find the speed of the train.

Solution :

Distance travelled by the train = Length of the train + length of the tunnel

= 160 m + 840 m = 1000 m = 1 km

and, time taken = 1 min = $\frac{1}{60}$ hr

$\therefore$ **Speed** = $\frac{1\,\text{km}}{\frac{1}{60}\,\text{hr}}$ = **60 km/hr** **Ans.**

Example 13 :

A train passes a platform 270 m long in 38 s and a man standing on the platform in 20 s. Find : (i) the speed of the train in km h^{-1} (ii) the length of the train.

Solution :

(i) $\because$ Distance covered in 38 s = Length of the train + 270 m

and, distance covered in 20 s = length of the train

On subtracting, we get : Distance covered in 18 s = 270 m

$\therefore$ **Speed** = $\frac{\text{Distance}}{\text{Time}} = \frac{270\text{ m}}{18\text{ s}}$ = **15 m/s** = $15 \times \frac{18}{5}$ km/h = **54 km/h** **Ans.**

(ii) **Length of the train** = Distance covered by the train in 20 s

= Speed × time = 15 m/s × 20 s = **300 m** **Ans.**

Alternative method :

(i) Let the length of the train = x m

$\because$ Distance covered in 38 sec = Length of the train + length of the platform

= $(x + 270)$ m

$\therefore$ Speed = $\frac{x+270}{38}$ m/s

$\because$ Distance covered in 20 sec = Length of the train = x m

$\therefore$ Speed = $\frac{x}{20}$ m/s

Since, speed in both the cases will be same,

$\therefore$ $\frac{x+270}{38} = \frac{x}{20} \Rightarrow 38x = 20x + 5400$

i.e. $18x = 5400 \Rightarrow x = \frac{5400}{18}$ m = 300 m

$\therefore$ (i) **Speed** = $\frac{x}{20}$ m/s = $\frac{300\text{ m}}{20\text{ s}}$ = **15 m/s**

= $15 \times \frac{18}{5}$ km h–1 = **54 km h^{-1}** **Ans.**

and, (ii) **The length of the train** = x m = **300 m** **Ans.**

EXERCISE 10(B)

1. A train covers 200 km in the first two hours, then 126 km in the next 2 hours and finally 143 km in the last 3 hours. Find the average speed of the train for the whole of this journey.
2. A bus travels at a speed of 72 km/h for 5 hours and at a speed of 90 km/h for 4 hours. Find the average speed of the bus for the whole journey.
3. Out of a distance of 80 km, the first 60 km is covered at a speed of 40 km/h and the remaining distance at a speed of 20 km/h. Calculate the average speed.

4. P and Q are two stations. A car goes from station P to station Q at a speed of 60 km/h and returns back at a speed of 30 km/h. Find the average speed for the entire journey.

> Let the distance between stations P and Q = x km.
>
> For the entire journey, distance = x km + x km = $2x$ km
>
> and, time taken = $\frac{x}{60}$ h + $\frac{x}{30}$ h = $\frac{3x}{60}$ h = $\frac{x}{20}$ h
>
> $\therefore$ Average speed = $\frac{2x}{x/20}$ km h^{-1} = 40 km h^{-1}.

5. Out of a journey of 300 km; the first part of distance 85 km is covered at a speed of 51 km/h, the second part of 90 km at a speed of 135 km/h and the remaining distance at a speed of 75 km/h. Find :
   (i) the distance covered at a speed of 75 km/h. (ii) the total time taken.
   (iii) the average speed for the whole journey.
6. A moter-cycle covers a distance of 72 km at a speed of 36 km/h and a distance of 135 km at a speed of 45 km/hr. Find the average speed of the motor-cycle.
7. Speed of car P is 120 km/h and speed of car Q is 75 km/h.
   (i) If both are moving in opposite directions, what is their relative speed ?
   (ii) What is their relative speed when they are moving in the same direction ?
8. A train 900 m long, crosses a pole in 45 sec. Find its speed in km per hour.
9. Find the length of the train moving at a speed of 90 km/h, if it passes a standing man in 8 seconds.
10. A 100 m long train passes a 200 m long platform in 20 seconds. Find the speed of the train.
11. Two cars start from the same place with speeds 80 km/h and 50 km/h. Find the distance between the two cars at the end of 3 hours, if :
    (i) they are going in the same direction. (ii) they are going in opposite directions.
12. A train, 80 m long, passes a platform 220 m long. If the speed of the train is 45 km/h, find the time taken by the train.
13. The speed of a bus is 90 km/h and the speed of a truck is 72 km/h. Both start from the same place. Find the distance between the two after 20 seconds, when they go in the : (i) same direction (ii) opposite directions.
14. A train passes a 50 m long railway platform in $4\frac{1}{2}$ seconds and a pole in 2 seconds. Find the length of the train and its speed.
15. A train passes a platform, 225 m length, in 21 sec and a man, standing on the platform, in 6 sec. Find : (i) the length of the train. (ii) the speed of the train.

ANSWER :

Exercise 10(A)

1. (i) 1 m/s (ii) 1500 cm/s (iii) 216 km/h (iv) 19·44 km/h **2.** (i) 43 m/min (ii) 30 km (iii) 54 sec **3.** 12·48 sec **4.** 21 m/s **5.** 54 km h^{-1} **6.** 50 minutes **7.** (i) 4800 m = 4·8 km (ii) 40 minutes **8.** (i) 3 hours (ii) 2 hours **9.** (i) 0·8 km = 800 m (ii) 6 min **10.** 32 km/h **11.** 9 a.m. **12.** 300 m **13.** (i) 4 m (ii) 60 m **14.** 0·75 km/min

Exercise 10(B)

1. 67 km h^{-1} **2.** 80 km h^{-1} **3.** 32 km h^{-1} **4.** 40 km h^{-1} **5.** (i) 125 km (ii) 4 hours (iii) 75 km h^{-1} **6.** 41·4 km h^{-1} **7.** (i) 195 km h^{-1} (ii) 45 km h^{-1} **8.** 72 km h^{-1} **9.** 200 m **10.** 15 ms^{-1} = 54 km h^{-1} **11.** (i) 90 km (ii) 390 km **12.** 24 sec **13.** (i) 100 m (ii) 900 m **14.** 40 m and 72 km h^{-1} **15.** (i) 90 m (ii) 54 km h^{-1}

INEQUALITIES
(Solution in one variable)

12A

12.1 INEQUALITY

A **mathematical statement**, which shows that two expressions are not equal, forms an inequality.

For example :

(i) $x > 8$, **Read as** : x is **greater than** 8.

(ii) $x \geq 8$, **Read as** : x is **greater than** or **equal** to 8.

(iii) $x < 8$, **Read as** : x is **less than** 8.

(iv) $x \leq 8$, **Read as** : x is **less than** or **equal** to 8.

12.2 INEQUATION (Linear inequation)

An **inequality** involving **atleast one variable** is known as an **inequation.**

For example :

(i) $3x > 5x + 7$ is an inequation involving only one variable *i.e.* x.

(ii) Inequation $3x \geq 5y + 7$ involves two variables *i.e.* x and y.

(iii) Inequation $5y - 15 < 0$ involves only one variable *i.e.* y.

(iv) $5y - 18x \leq 0$ is an inequation that involves two variables *i.e.* x and y.

An inequation, with only one variable, is a linear inequation.

For example :

(i) $7x < 8$ (ii) $5y \geq 18$

(iii) $3x + 5 \leq 24$ (iv) $5x - 24 \geq 3x$, etc.

12.3 REPLACEMENT SET AND SOLUTION SET

Replacement set :

Consider the inequation $x \leq 6$, where $x \in$ N (set of natural numbers).

Here, the set N (set of natural numbers) from which the values of x, satisfying the inequation $x \leq 6$ are to be chosen, is called **replacement set**.

Thus, solution of inequation $x \leq 6$, $x \in$ N will be the set of all values of x satisfying $x \leq 6$ and belonging to the set of natural numbers.

$x \leq 6$ and $x \in$ N (set of natural numbers)

= Set of natural numbers which are less than or equal to 6

= {1, 2, 3, 4, 5, 6}

= **Solution set** for $x \leq 6$ and $x \in$ N.

Consider the followng table :

	Inequation	Replacement set	Solution set
(i)	$x > 10,\ x \in N$	**N** (set of natural numbers)	{11, 12, 13, 14, ...}
(ii)	$y \leq 3,\ y \in W$	**W** (set of whole numbers)	{0, 1 , 2 , 3}
(iii)	$-2 \leq x < 2,\ x \in I$	**I** (set of integers)	{–2, –1, 0, 1}
(iv)	$-5 \leq x \leq 4,\ x \in N$	**N**	{1, 2, 3, 4}
	$-5 \leq x \leq 4,\ x \in W$	**W**	{0, 1, 2, 3, 4}
	$-5 \leq x \leq 4,\ x \in I$	**I**	{–5, –4, –3, –2, –1, 0, 1, 2, 3, 4}

12.4 PROPERTIES OF INEQUATIONS

Property 1 :

Adding the same number, to each side of an inequation, does not change the inequality.

For example :

(i) $x > 3 \Rightarrow x + 5 > 3 + 5 \Rightarrow x + 5 > 8$

(ii) $x \geq 8 \Rightarrow x + 5 \geq 8 + 5 \Rightarrow x + 5 \geq 13$

(iii) $x - 3 < 5 \Rightarrow x - 3 + 3 < 5 + 3 \Rightarrow x < 8$

(iv) $x - 8 \leq 3 \Rightarrow x - 8 + 8 \leq 3 + 8 \Rightarrow x \leq 11$

Property 2 :

Subtracting the same number from each side of an inequation does not change the inequality.

For example :

(i) $x + 5 \leq 8 \Rightarrow x + 5 - 5 \leq 8 - 5 \Rightarrow x \leq 3$

(ii) $x + 3 \geq 15 \Rightarrow x + 3 - 3 \geq 15 - 3 \Rightarrow x \geq 12$

(iii) $x + 2 < -6 \Rightarrow x + 2 - 2 < -6 - 2 \Rightarrow x < -8$

(iv) $x + 7 > 4 \Rightarrow x + 7 - 7 > 4 - 7 \Rightarrow x > -3$

Property 3 :

Multiplying each side of an inequation by the same positive number, does not change the inequality.

For example :

(i) $\frac{x}{3} > 2 \Rightarrow \frac{x}{3} \times 3 > 2 \times 3 \Rightarrow x > 6$

(ii) $\frac{x}{2} < 5 \Rightarrow \frac{x}{2} \times 2 < 5 \times 2 \Rightarrow x < 10$

(iii) $\frac{x}{4} \geq -6 \Rightarrow \frac{x}{4} \times 4 \geq -6 \times 4 \Rightarrow x \geq -24$

(iv) $\frac{x}{5} \leq -3 \Rightarrow \frac{x}{5} \times 5 \leq -3 \times 5 \Rightarrow x \leq -15$

Property 4 :

Multiplying each side of an inequation, by the same negative number, changes (reverses) the inequality.

For example :

(i) $x > 4 \Rightarrow x \times -3 < 4 \times -3 \Rightarrow -3x < -12$

(ii) $-x < -2 \Rightarrow -x \times -5 > -2 \times -5 \Rightarrow 5x > 10$

(iii) $3x \geq -5 \Rightarrow 3x \times -2 \leq -5 \times -2 \Rightarrow -6x \leq 10$

(iv) $-x \leq 4 \Rightarrow -x \times -4 \geq 4 \times -4 \Rightarrow 4x \geq -16$

Property 5 :

Dividing each side of an inequation, by the same positive number, does not change the inequality.

(i) $3x > 6 \Rightarrow \frac{3x}{3} > \frac{6}{3} \Rightarrow x > 2$

(ii) $-8x < -10 \Rightarrow \frac{-8x}{2} < \frac{-10}{2} \Rightarrow -4x < -5$

(iii) $5x \geq -15 \Rightarrow \frac{5x}{5} \geq \frac{-15}{5} \Rightarrow x \geq -3$

(iv) $-6x \leq 36 \Rightarrow \frac{-6x}{6} \leq \frac{36}{6} \Rightarrow -x \leq 6$

Property 6 :

Dividing each side of an inequation, by the same negative number, changes (reverses) *the inequality.*

For example :

(i) $-2x > 6 \Rightarrow \frac{-2x}{-2} < \frac{6}{-2} \Rightarrow x < -3$

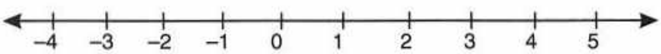
(ii) $3x < -15 \Rightarrow \frac{3x}{-3} > \frac{-15}{-3} \Rightarrow -x > 5$

(iii) $4x \geq -12 \Rightarrow \frac{4x}{-4} \leq \frac{-12}{-4} \Rightarrow -x \leq 3$

(iv) $-3x \leq -18 \Rightarrow \frac{-3x}{-3} \geq \frac{-18}{-3} \Rightarrow x \geq 6$

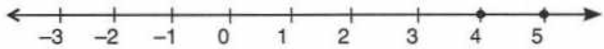
12.5 GRAPHICAL REPRESENTATION

The following figure shows a number line representing integers.



The dark arrows at both the ends of the number line show that integers go upto infinity on the positive side as well as on the negative side.

A number line can be used to represent the solution set of an inequation. The following illustrations will make the statement more clear :

(i) $x > 3$ and $x \in N$ $\Leftrightarrow$ 

[Dark marks at 4 and 5 show the elements of the solution set and the dark arrow at the right side of the number line shows that the solution set of the given inequation continues to the right side of 5].

(ii) $x < 3$ and $x \in N$ $\Leftrightarrow$ 

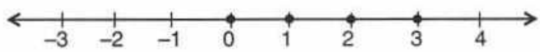
(iii) $x \leq 3$ and $x \in W$ $\Leftrightarrow$ 

(iv) $x \leq 3$ and $x \in I$ (integers) $\Leftrightarrow$

(v) $-2 \leq x \leq 3$ and

(a) $x \in N$ $\Leftrightarrow$ 

(b) $x \in I$ $\Leftrightarrow$ 

(c) $x \in W$ $\Leftrightarrow$ 

Example 1 :

Solve : (i) $x < 6$, $x \in$ N (ii) $x \leq 6$, $x \in$ W (iii) $x < 6$, $x \in$ I (iv) $-3 < x \leq 3$, $x \in$ I

Solution :

(i) $x < 6$, $x \in$ N = Set of all natural numbers which are less than 6

= {1, 2, 3, 4, 5} **Ans.**

(ii) $x \leq 6$, $x \in$ W = Set of all whole numbers which are less than or equal to 6

= {0, 1, 2, 3, 4, 5, 6} **Ans.**

(iii) $x < 6$, $x \in$ I = Set of integers which are less than 6

= {....., –2, –1, 0, 1, 2, 3, 4, 5} **Ans.**

(iv) $-3 < x \leq 3$, $x \in$ I = Set of integers greater than –3 and less than or equal to 3

= {–2, –1, 0, 1, 2, 3} **Ans.**

Example 2 :

Solve : (i) $x + 3 > 5$, $x \in$ N (ii) $x - 5 \leq 2$, $x \in$ W (iii) $2x - 3 \geq 7$, $x \in$ I

Solution :

(i) $x + 3 > 5 \Rightarrow x > 5 - 3$ *i.e.* $x > 2$

$\therefore$ $x > 2$, $x \in$ N = {3, 4, 5, 6, ...} **Ans.**

(ii) $x - 5 \leq 2 \Rightarrow x \leq 2 + 5$ *i.e.* $x \leq 7$

$\therefore$ $x \leq 7$, $x \in$ W = {0, 1, 2, 3, 4, 5, 6, 7} **Ans.**

(iii) $2x - 3 \geq 7 \Rightarrow 2x \geq 7 + 3$

$\Rightarrow 2x \geq 10$ *i.e.* $x \geq 5$

$x \geq 5$, $x \in$ I = {5, 6, 7, 8, ...} **Ans.**

1. For $-2 \leq x < 2$, $x \in N$; the solution set = {1}
2. For $-2 \leq x < 2$, $x \in W$; the solution set = {0, 1}
3. For $-2 \leq x < 2$, $x \in I$; the solution set = {–2, –1, 0, 1}

Example 3 :

Solve each of the following inequations and represent, in each case, the solution set on a number line :

(i) $3x - 8 < 7$, $x \in N$ (ii) $4x + 5 \geq 1$, $x \in W$ (iii) $5x - 12 \leq 8$, $x \in I$

Solution :

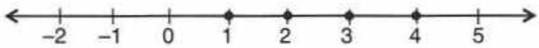
(i) $3x - 8 < 7 \Rightarrow 3x < 7 + 8$

$\Rightarrow 3x < 15 \Rightarrow x < \frac{15}{3}$ *i.e.* $x < 5$

$x < 5$ and $x \in N$

$\Rightarrow$ **The solution set = {1, 2, 3, 4}** **Ans.**

And the solution set on the number line is :



Ans.

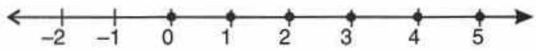
(ii) $4x + 5 \geq 1 \Rightarrow 4x \geq 1 - 5$

$\Rightarrow 4x \geq -4$ and $x \geq \frac{-4}{4}$ *i.e.* $x \geq -1$

$x \geq -1$, $x \in W$

$\Rightarrow$ **The solution set = {0, 1, 2, 3, 4, ...}** **Ans.**

And the solution set on the number line is :



Ans.

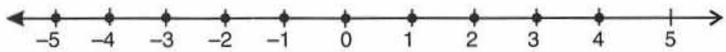
(iii) $5x - 12 \leq 8 \Rightarrow 5x \leq 8 + 12$

$\Rightarrow 5x \leq 20 \Rightarrow x \leq \frac{20}{5}$ *i.e.* $x \leq 4$

$x \leq 4$, $x \in I$

$\Rightarrow$ **The solution set = {....., –2, –1, 0, 1, 2, 3, 4}** **Ans.**

And the solution set on the number line is :



Ans.

Example 4 :

Solve and, in each case, represent the solution set on a number line :

(i) $2x - 5 < 3$; $x \in W$ (ii) $3x + 4 > 10$; $x \in N$

Solution :

(i) $2x - 5 < 3 \Rightarrow 2x < 3 + 5$

$\Rightarrow 2x < 8 \Rightarrow x < \frac{8}{2} \Rightarrow x < 4$

$x < 4$ and $x \in W \Rightarrow$ **The solution set = {0, 1, 2, 3}** **(Ans.)**

The solution set on the number line is : 

(Ans.)

(ii) $3x + 4 > 10 \Rightarrow 3x > 10 - 4$

$$\Rightarrow 3x > 6 \Rightarrow x > \frac{6}{3} \quad i.e. \quad x > 2$$

Now, $x > 2$ and $x \in N$ **(Ans.)**

$\Rightarrow$ **The solution set = {3, 4, 5, 6, ...}**

The solution set on the number line is :

(Ans.)

EXERCISE 12A

1. Find the resulting inequation, when :
   (i) 10 is added to each side of $4x - 3 < 2$.
   (ii) 7 is subtracted from each side of $2x + 5 \geq 8$.
   (iii) when each side of $3x - 5 \leq 2$ is multiplied by 4.
   (iv) when each side of $8x + 3 > 13$ is divided by 6.
2. If the replacement set = {–4, –3, –2, –1, 0, 1, 2, 3, 4, 5}, find the solution set, if :
   (i) $x < 4$ (ii) $x < -1$ (iii) $x - 3 \geq 0$
   (iv) $2x > 6$ (v) $\frac{3}{2} + x \leq \frac{1}{2}$.
3. If the replacement set = Set of whole numbers between – 2 and 6, find the solution set for each of the following inequations :
   (i) $-5 < x < 4$ (ii) $2x - 3 \geq 5$ (iii) $3x - 5 \leq 10$
   (iv) $2 < x \leq 7$ (v) $0 \leq x \leq 6$
4. Represent the solution set for each of the following inequations on the number line :
   (i) $x - 5 \leq 2$, $x \in W$ (ii) $2x - 3 < 7$, $x \in N$ (iii) $5x + 12 > -13$, $x \in I$
   (iv) $3x - 15 \geq 15$, $x \in N$ (v) $2x + 5 \leq -3$, $x \in I$

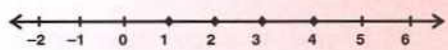
ANSWER : **Exercise 12A**

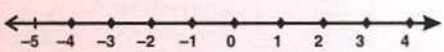
1. (i) $4x + 7 < 12$ (ii) $2x - 2 \geq 1$ (iii) $12x - 20 \leq 8$ (iv) $\frac{4x}{3} + \frac{1}{2} > \frac{13}{6}$

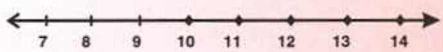
2. (i) {–4, –3, –2, –1, 0, 1, 2, 3} (ii) {–4, –3, – 2} (iii) {3, 4, 5} (iv) {4, 5} (v) {–4, –3, –2, –1}

3. (i) {0, 1, 2, 3} (ii) {4, 5} (iii) {0, 1, 2, 3, 4, 5} (iv) {3, 4, 5} (v) {0, 1, 2, 3, 4, 5}

4. (i) 

(ii) 

(iii) 

(iv) 

(v)