Chapter-10

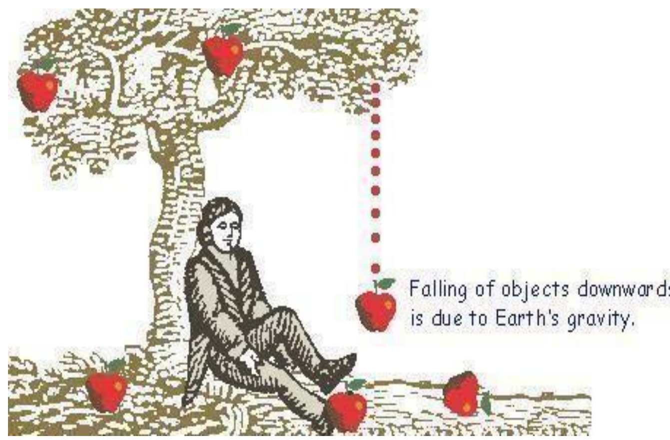
GRAVITATION

Sub-Topic Name:

Universal Law of Gravitation, Free Fall, Acceleration Due to Gravity (g), Difference between G and g, Mass and Weight, Weight of an Object on the Surface of Moon.

Newton's Observations

- Why does Apple fall on Earth from a tree? – Because the earth attracts it towards itself.
- Can Apple attract the earth? - Yes. It also attracts the earth as per Newton's third law(everyaction has an equal and opposite reaction). But the mass of the earth is much larger than Apple's mass thus the force applied by Apple appears negligible and Earth never moves towards it.



- Newton thus suggested that all objects in this universe attract each other. This force of attraction is called Gravitational Force.

What Is Gravity?

Force of attraction exerted by earth on nearby objects is called Force of Gravity.

Example

Moon revolves around earth because of force of gravity of earth on moon.

What Is Gravitational Force?

Every object in universe attracts every other object with a force

Example

Earth Revolves around Sun

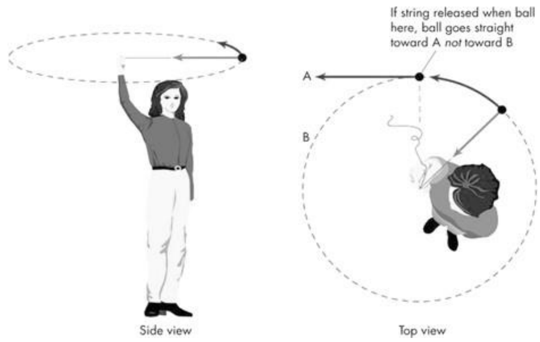
This is because of Gravitational Force of sun on earth.

Kepler (1571-1631) Studied the planetary motion in detail and formulated his three laws of planetary motion, which were available Universal law of gravitation.

What is the Centripetal Force?

- We know that an object in circular motion keeps on changing its direction.
- Due to this, the velocity of the object also changes.
- A force called **Centripetal Force** acts upon the object that keeps it moving in a circular path.
- The centripetal force is exerted from the center of the path.
- Without the Centripetal Force objects cannot move in circular paths, they would always travel straight.
- **For Example**, the rotation of Moon around the Earth is possible because of the centripetal force exerted by Earth.

Consider a person whirling a stone tied to a thread along a circular path as shown below in the figure

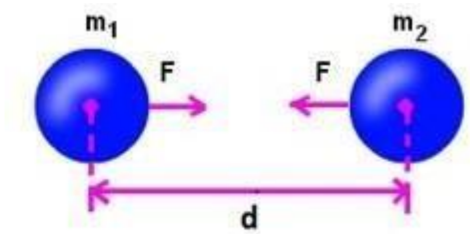


If he releases the stone then it flies along the tangent, at that point on the circular path.

- Before the release of thread, it is centripetal force responsible for the motion of stone in the circular path where the stone moves with a certain speed and changes direction at every point.
- During this motion the change in direction involves change in velocity which produces acceleration. This force which is called centripetal force, causes this acceleration and keeps the body moving along the circular path is acting towards the center.
- Now when the thread is released the stone does not experience this force and flies off along a straight line that is tangent to the circular path

Universal Law of Gravitation by Newton

- According to the universal law of gravitation, every object attracts every other



object with a force.

- This force is directly proportional to the product of their masses
- This force is inversely proportional to the square of distances between them.
- Consider the figure given below. It depicts the force of attraction between two objects with masses m_1 and m_2 respectively that are ‘d’ distance apart.

- The figure below describes how the universal law of gravitation is derived mathematically.
- From the above equation we can rewrite them as the following:
- If we remove the proportionality we get proportionality constant G as

the following:

$$F = G\frac{m_1 m_2}{r^2}$$

The above equation is the mathematical representation of Newton's

$$F \alpha \frac{m_1 m_2}{r^2} ------(iv)^{-}(i)$$

$$F \alpha m_2 ------(ii)$$

$$F \alpha \frac{1}{r^2} ------(iii)$$

universal Law of gravitation

G = $Fr^2 / m_1 m_2$

SI Unit: $Nm^2 kg^{-2}$

Value of ***G = $6.673 \times 10^{-11} Nm^2 kg^{-2}$*** (was found out by *Henry Cavendish* (1731-1810))

- The proportionality constant G is also known as the **Universal Gravitational Constant.**

<u>Important Characteristics of Gravitational forces</u>

- The gravitational force is a central force that is It acts along the line joining the centers of two bodies.
- it is a long range force. The gravitational force is effective even at large distances.
- It is a conservative force. This means that the work done by the gravitational force in displacing a body from one point to another is only dependent on the initial and final positions of the body and is independent of the path followed.
- Unlike electrostatic and magnetic forces, the gravitational force is always attractive.

Importance of The Universal Law of Gravitation

- It binds us to the earth.
- It is responsible for the motion of the moon around the earth.
- It is responsible for the motion of planets around the Sun.
- Gravitational force of moon causes tides in seas on earth.

<u>Free Fall</u>

- ***<u>Acceleration due to gravity</u>*** – Whenever an object falls towards the Earth there is an acceleration associated with the movement of the object. This acceleration is called acceleration due to gravity.
- Denoted by: g
- SI Unit: m s^{-2}
- We know that, F= ma
- Therefore, F = mg
- Following demonstrates the mathematical derivation of ‘g'

> Acceleration due to gravity doesn’t depends up on mass of object

The force (F) of gravitational attraction on a body of mass m due to earth of mass M and radius R is given by

$$F = G\frac{mM}{R^2} \qquad \dots (1)$$

We know from Newton’s second law of motion that the force is the product of mass and acceleration.

∴ F = ma

But the acceleration due to gravity is represented by the symbol g. Therefore, we can write

$$F = mg \qquad \text{..... (2)}$$

From the equation (1) and (2), we get

$$mg = G\frac{mM}{R^2} \quad \text{or} \quad g = \frac{GM}{R^2} \qquad \text{.....(3)}$$

When body is at a distance 'r' from the center of the earth then

$$g = \frac{GM}{r^2}$$

Value of 'g' may vary at different parts of earth –

- From the equation $g = GM/r^2$ It is clear that the value of 'g' depends upon the distance of the object from the earth's center.
- This is because the shape of the earth is not a perfect sphere. It is rather flattened at poles and bulged out at the equator.

Calculation of g

- $F = GMm/r^2$
- $g = F/m = GM/r^2$
- Mass of Earth = 6×10^{24} kg
- Radius of Earth = 6.4×10^6 m
- Calculate g on Earth
- **$g = GM/r^2$**
- **$= 6.67 \times 10^{-11} \times 6 \times 10^{24} / (6.4 \times 10^6)^2$**
- **$= 9.8\ Nkg^{-1}$**

Hence, the value of 'g' is **greater at the poles and lesser at the equator**. However, for our convenience, we take a constant value of 'g' throughout.

Difference between Gravitation Constant (G) and Gravitational Acceleration (g)

S. No.	Gravitation Constant (G)	Gravitational acceleration (g)
1.	Its value is $6.67 \times 10^{-11} Nm^2/kg^2$.	Its value is $9.8\ m/s^2$.
2.	It is a scalar quantity.	It is a vector quantity.
3.	Its value remains constant alwaysand everywhere.	Its value varies at various places.
4.	Its unit is Nm^2/kg^2.	Its unit is m/s^2.

What is Free Fall?

When an object falls towards the earth due to earth's gravity and no other force is acting upon it, the object is said to be in **free fall state**. Free falling objects are not even resisted by the air.

$g = 9.8\ m/s^2$ is also called the **Free-fall Acceleration**.

Value of 'g' is same on the earth, so the equations of motion for an object with uniform motion are valid where acceleration 'a' is replaced by 'g', as given under:

$V=u + gt$

$S= ut + (1/2)\ gt^2$

$2gs = v^2 – u^2$

Consider the equations of motion given in different scenarios

When an object at rest falls towards earth – its initial velocity is zero

$v = gt$

$s = t + (1/2)\ gt^2$

$2\ g\ s = v^2$

When an object with some initial velocity (u) falls towards earth –

$v = u + gt$

$s = ut + (1/2)\ gt^2$

$2\ g\ s = v^2 – u^2$

When an object is thrown upwards from earth – the gravitational force acts in opposite direction, hence g is negative

$v = u - gt$

$s = ut - (1/2)\ gt^2$

$-2\ g\ s = v^2 – u^2$

Mass & weight

Mass (m)

- The mass of a body is the quantity of matter contained in it.
- Mass is a scalar quantity which has only magnitude but no direction.
- Mass of a body always remains constant and does not change from place to place.
- SI unit of mass is kilogram (kg).
- Mass of a body can never be zero.

Weight (W)

- The force with which an object is attracted towards the centre of the earth, is called the weight of the object.
  Now, Force = $m \times a$
  But in case of earth, $a = g$
  $\therefore F = m \times g$
  But the force of attraction of earth on an object is called its weight (W).
  $\therefore W = mg$
- As weight always acts vertically downwards, therefore, weight has both magnitude and direction and thus it is a vector quantity.
- The weight of a body changes from place to place, depending on mass of object.
- The SI unit of weight is Newton.
- Weight of the object becomes zero if g is zero.

Difference between mass and weight

Mass	Weight
Mass is defined as the quantity of matter in an object.	The weight of an object is the force by which the gravitation pull of earth attracts the object.
Mass is a scalar quantity	Weight is a vector quantity
The mass of an object is always constant as it depends upon inertia of the object	The weight of an object can vary at different locations because change in gravitational force of the earth
Mass can never be zero	Weight can be zero at places there is no gravitational force
Denoted as: m	Denoted as W F = mg
Unit: kg	Unit: N

Weight of an object on the Moon

Just like the Earth, the Moon also exerts a force upon objects. Hence, objects on moon also have some weight. The weight will not be same as than on the earth. So, weight on the Moon can be calculated as –

$$W_M = \frac{GM_M m}{R_M^2}$$

Now,

$$\Rightarrow \frac{W_M}{W_E} = \frac{M_M R_E^{\,2}}{M_E R_M^2}$$

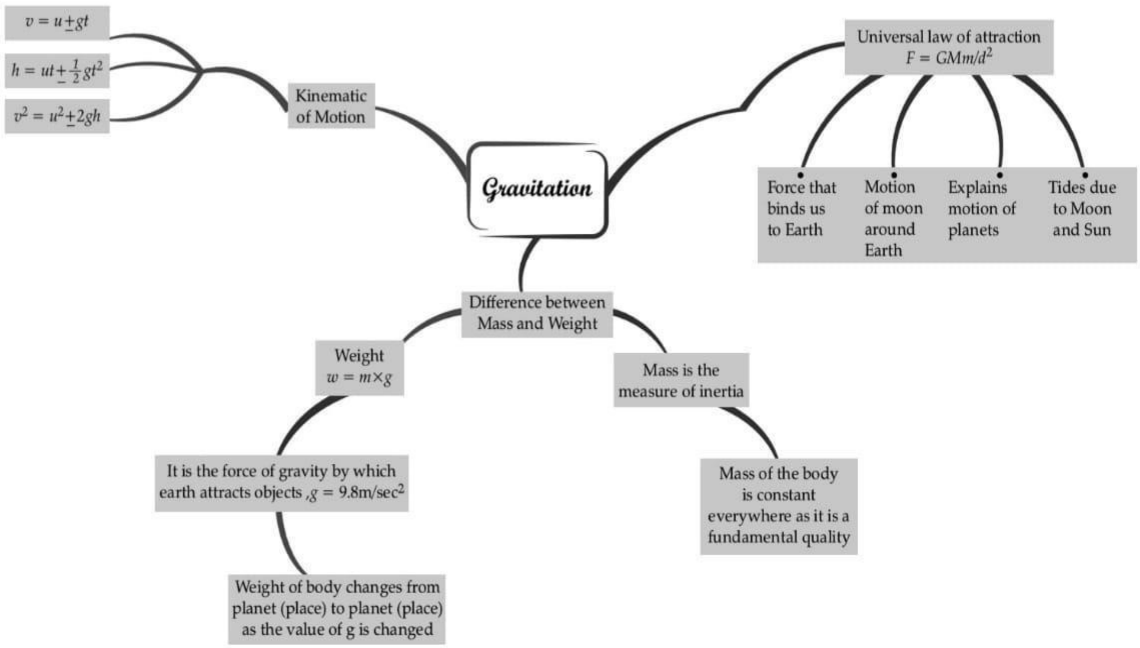
Where,

$M_E = 5.98 \times 10^{24}\ kg$
$M_M = 7.36 \times 10^{22}\ kg$
$R_E = 6.4 \times 10^{6}\ m$
$R_M = 1.74 \times 10^{6}\ m$

$$\Rightarrow \frac{W_M}{W_E} = \frac{7.36 \times 10^{22} \times (6.4 \times 10^{6})^2}{5.98 \times 10^{24} \times (1.74 \times 10^{6})^2} = 0.165 \approx \frac{1}{6}$$

Therefore, weight of an object on the moon is $\frac{1}{6}$ of its weight on the Earth.

MIND MAP



$v = u \pm gt$
$h = ut \pm \frac{1}{2}gt^2$
$v^2 = u^2 \pm 2gh$
Kinematic of Motion
Gravitation
Universal law of attraction $F = GMm/d^2$
Force that binds us to Earth
Motion of moon around Earth
Explains motion of planets
Tides due to Moon and Sun
Difference between Mass and Weight
Weight $w = m \times g$
Mass is the measure of inertia
It is the force of gravity by which earth attracts objects ,g = 9.8m/sec^2
Mass of the body is constant everywhere as it is a fundamental quality
Weight of body changes from planet (place) to planet (place) as the value of g is changed