Contents

Theme – 1
NUMBERS

NUMBER SYSTEM 1

(Consolidating the Sense of Numberness)

1.1 INTRODUCTION

Consolidating means to combine or to unite into one.

So, different facts in number system will be combined in this chapter such as :

(i) Comparing numbers. (ii) Numbers in ascending and descending order.

(iii) Effect of shifting digits, etc.

1.2 COMPARING NUMBERS

In order to compare two numbers, follow the rules, given below :

Rule 1 : *If the two numbers have different number of digits; the number with larger number of digits is greater.*

For example :

Number 325 has larger number of digits as compared to 89; therefore 325 is greater than 89 *i.e.*, 325 > 89.

Consider the following table :

Numbers to be compared	Comparision
1. 5,293 and 62,000	62,000 is greater than 5,293
2. 24,522 and 8,899	24,522 is greater than 8,899
3. 23,92,460 and 3,46,87,300	3,46,87,300 is greater than 23,92,460

Rule 2 : *If the two numbers have equal numbers of digits, adopt the following steps :*

Step 1 : Compare the digits at the leftmost places in both the numbers. The number with larger digit will be greater than the number with smaller digit.

Thus (i) <u>8</u>7 is greater than <u>6</u>9. (as 8 > 6)

(ii) <u>3</u>297 is greater than <u>2</u>988. (as 3 > 2)

(iii) <u>8</u>,20,797 is greater than <u>6</u>,90,824. (as 8 > 6)

Step 2 : If both the numbers have same digits at the leftmost place, compare their second digits from the left.

In this case, if the second digits from the left are unequal in value, the number with greater digit is greater.

Thus (i) 4<u>5</u>37 is greater than 4<u>2</u>89. (as 5 > 2)

(ii) 5,<u>6</u>2,789 is greater than 5,<u>3</u>2,999. (as 6 > 3)

Step 3 : If the first digits as well as the second digits from the left of given numbers are equal in value, compare their third digits from the left, then proceed as discussed above.

Thus (i) 9832 is greater than 9823 (as 3 > 2)

(ii) 536250 is greater than 534798 (as 6 > 4)

(iii) 8224327 is smaller than 8254327 (as 2 < 5)

Similarly, (i) 76,234 is greater than 76218 (as 3 > 1)

(ii) 98375 is smaller than 98378 (as 5 < 8)

1.3 USING CHARTS FOR COMPARING NUMBERS

Example 1 :

Make a chart for comparing 62,36,489 and 62,53,278.

Solution :

The given numbers can be written in a chart as shown below :

6	2	3	6	4	8	9
6	2	5	3	2	7	8

Clearly, both the numbers have equal number of digits.

At the leftmost, both have the same digits *i.e.*, 6 each.

At the second place from the left, again they have the same digits *i.e.* 2 each.

At the third place from the left, the first number has digit 3 and the second number has digit 5.

Since, 3 < 5.

⇒ **62,36,489 < 62,53,278** **(Ans.)**

Now compare the numbers written in the table below :

4	9	3	2	7	5	1
	8	2	1	0	4	9
2	3	7	8	8	0	8
	8	2	0	1	4	4
4	5	7	6	3	2	1
	7	8	4	3	2	5

Ascending order means : smaller to greater.

The numbers, given in the above table, are written in ascending order as follows :

7,84,325 < 8,20,144 < 8,21,049 < 23,78,808 < 45,76,321 < 49,32,751.

Example 2 :

Use table form to compare the numbers 6,45,824, 23,78,926, 3,28,792 and 6,54,284 and write the given numbers in descending order.

Solution :

The required table is as given below :

	6	4	5	8	2	4
2	3	7	8	9	2	6
	3	2	8	7	9	2
	6	5	4	2	8	4

Descending order means : greater to smaller

Clearly, given numbers in descending order are as :

23,78,926 > 6,54,284 > 6,45,824 > 3,28,792. **(Ans.)**

Example 3 :

Find the greatest and the smallest numbers in each case given below :

(i) 573, 8294, 37, 54908 and 1036.

(ii) 2483, 79312, 103, 4078 and 573.

(iii) 2754, 63200, 321, 728 and 2134.

(iv) 3862, 7592, 8888, 13003 and 573.

Solution :

(i) Since, 54908 has maximum number of digits and 37 has the least number of digits.

∴ **54908 is the greatest and 37 is the smallest.** **(Ans.)**

(ii) 79312 has maximum number of digits whereas 103 and 573 have the least number of digits. Out of 103 and 573; 103 is smaller.

∴ Out of the given numbers

79312 is the greatest and 103 is the smallest. **(Ans.)**

In the same way.

(iii) **63200 is the greatest number and 321 is the smallest number.** **(Ans.)**

(iv) **13003 is the greatest number and 573 is the smallest number.** **(Ans.)**

1.4 TO FORM THE SMALLEST AND THE GREATEST NUMBERS USING GIVEN DIGITS

(a) When the given digits include digit 0.

Example 4 :

Form the smallest and the greatest 6-digit numbers using the digits 2, 0, 7, 8, 9 and 5 without repetition.

Solution :

To obtain the smallest number :

The smallest digit, other than zero, is put at the extreme left, then put zero, and then the remaining digits in ascending (increasing) order of their values.

Since, out of the given digits 2, 0, 7, 8, 9 and 5, the smallest digit other than 0 is 2, write 2 at the extreme left, then write 0 and then the remaining digits (7, 8, 9 and 5) in ascending order of their values, *i.e.*, 5, 7, 8 and 9.

Thus, the **required smallest number** is **205789.** **(Ans.)**

> The number 025789 is not a 6-digit number; it is a 5-digit number.
> **A number cannot begin with the digit 0.**

To obtain the greatest number :

Put the greatest digit at the extreme left, then put the remaining digits in descending order of their values with 0 at the end.

Thus, the **required greatest number** is 987520. **(Ans.)**

(b) When the given digits do not include digit 0 :

Example 5 :

Form the smallest and the greatest 4-digit numbers using the digits 3, 8, 5 and 2 without repetition.

Solution :

To obtain the smallest number :

Write the digit with smallest value at the extreme left, and then the remaining digits in ascending order of their values.

∴ **The required smallest number** = 2358 **(Ans.)**

To obtain the greatest number :

Write the digit with largest value at the extreme left, and then the remaining digits in descending order of values.

∴ **The required greatest number** = 8532 **(Ans.)**

Example 6 :

(i) What is the smallest number of five digits ?

(ii) What is the greatest number of five digits ?

Solution :

(i) To form the smallest number of five digits, place 1 (unity) at the extreme left, and then four zeroes to the right of 1.

∴ The required **smallest number of five digits** = **10000** **(Ans.)**

(ii) In forming the greatest number of five digits, we should have the greatest digit, *i.e.* 9, in all places.

∴ The required **greatest number of five digits** = **99999** **(Ans.)**

1.5 HOW MANY NUMBERS CAN YOU MAKE?

1. How many two digit numbers can you form using the digits 3 and 8 ?

   If digits are not repeated, the numbers formed are 38 and 83.

   If digits may repeat, the numbers 33 and 88 will also be formed.

   In all, four numbers 38, 83, 33 and 88 can be formed using the digits 3 and 8.

   [Clearly, 88 is the largest and 33 is the smallest of the numbers formed with digits 3 and 8].

2. Consider the three digit numbers which can be formed by using the digits 5, 2 and 7 without repetition of any digit.

   The numbers formed are : 527, 572, 257, 275, 752 and 725.

   Which of these numbers is smallest and which is largest ?

   Clearly, the smallest number is 257 and the largest number is 752.

To get the smallest number, using the given digits, the smallest digit is written at the leftmost and then the remaining digits in ascending order of their values to the right *e.g.* 257.

To get the largest number, using the given digits, the largest digit is written at the leftmost and then the remaining digits in descending order of their values to the right *e.g.* 752.

EXERCISE 1(A)

1. Which is greater ?
   (i) 537 or 98 (ii) 2428 or 529 (iii) 2,59,467 or 10,35,729
2. Which is smaller ?
   (i) 428 or 437 (ii) 2497 or 2597 (iii) 3297 or 3596
3. Which is greater ?
   (i) 45293 or 45427 (ii) 380362 or 381007 (iii) 63520 or 63250
4. By making a suitable chart, compare :
   (i) 540276 and 369998 (ii) 6983245 and 6893254
5. Compare the numbers written in the following table by writing them in ascending order :

	5	4	3	2	9	7	2
2	3	1	0	6	2	9	3
	5	2	2	3	7	9	1
2	3	1	8	2	6	3	4
5	4	3	4	4	7	8	2

6. Use table form to compare the numbers in descending order :
   5,43,287 ; 54,82,900 ; 27,32,940
   43,877 ; 78,396 and 4,999

7. Find the smallest and the greatest numbers in each case given below :
   (i) 983, 5754, 84 and 5942   (ii) 32849, 53628, 5499 and 54909.

8. Form the greatest and the smallest 4-digit numbers using the given digits, without repetition :
   (i) 3, 7, 2 and 5   (ii) 6, 1, 4 and 9   (iii) 7, 0, 4 and 2
   (iv) 1, 8, 5 and 3   (v) 9, 6, 0 and 7

9. Form the greatest and the smallest 3-digit numbers using any three different digits with the condition that digit 6 is always at the unit (one's) place.

The required **greatest number** is **986** and the required **smallest number** is **106**.

10. Form the greatest and the smallest 4-digit numbers using any four different digits, with the condition that digit 5 is always at ten's place.

11. Fill in the blanks :
   (i) The largest number of 5 digits is .... and the smallest number of 6-digits is ....
   (ii) The difference between the smallest number of four digits and the largest number of three digits = ... − .... = ...
   (iii) The sum (addition) of the smallest number of three digits and the largest number of two digits = ... + ... = ...
   (iv) On adding one to the largest five-digit number, we get ...., which is the smallest .... digit number.
   (v) On subtracting one from the smallest four-digit number, we get ...., which is the .... three digit number.

12. Form the largest number with the digits 2, 3, 5, 9, 6 and 0 without repetition of any digit.

13. Write the smallest and the greatest numbers of 4 digits without repetition of any digit.

14. Find the greatest and the smallest five-digit numbers with 8 in hundred's place and with all the digits different.

15. Find the sum of the largest and the smallest four-digit numbers.

16. Find the difference between the smallest and the greatest six-digit numbers.

17. (i) How many four-digit numbers are there between 999 and 3000 ?
   (ii) How many four-digit numbers are there between 99 and 3000 ?

18. How many four-digit numbers are there between 500 and 3000 ?

19. Write all the possible three-digit numbers using the digits 3, 6 and 8, if repetition of any digit is not allowed.

20. Make the greatest and the smallest 4-digit numbers using the digits 5, 4, 7 and 9 (without repeating the digits) and with the condition that :
   (i) 7 is at unit's place.   (ii) 9 is at ten's place.
   (iii) 4 is at hundred's place

1.6 OPERATIONS ON LARGER NUMBERS

Example 7 :

Three different tanks contain 54,937 litre, 49,328 litre and 1,07,244 litre of water. What is the total quantity of water in the three tanks ?

Solution :

∵ Water in one tank = 54,937 litre,

water in another tank = 49,328 litre

and, water in third tank = 1,07,244 litre

∴ **Water in all the three tanks = 2,11,509 litre** **(Ans.)**

Example 8 :

The weight of one bag of wheat is 98 kg 450 gm and the weight of another bag of wheat is 106 kg 928 gm. What is the total weight of both the bags ?

Solution :

∵ Weight of one bag of wheat = 98 kg 450 gm

and, weight of another bag of wheat = 106 kg 928 gm

∴ **Total weight of both the bags** = 204 kg 1378 gm

Since 1378 gm = 1 kg 378 gm, therefore total weight of both bags

= **205 kg 378 gm** **(Ans.)**

Example 9 :

In a departmental store, the sale during a particular week was ₹ 3,85,476 and the sale during the following week was ₹ 4,07,235.

(a) What was the total sale during these two weeks ?

(b) During which week was the sale greater and by how much ?

Solution :

(a) Sale during one week = ₹ 3,85,476

and, sale during the next week = ₹ 4,07,235

∴ **Total sale during two weeks = ₹ 7,92,711** **(Ans.)**

(b) Since, ₹ 4,07,235 is greater than ₹ 3,85,476

⇒ **Sale during second week was greater.** **(Ans.)**

Because : ₹ 4,07,235

− ₹ 3,85,476

₹ 21,759

∴ **Sale during the second week was greater by ₹ 21,759.** **(Ans.)**

Example 10 :

If A = 48,497, B = 7,328 and C = 1,33,224. Find :

(i) A + B (ii) C − A (iii) B + C

Solution :

(i) **A + B** = 48,497 + 7,328

= 48,497 + 7,328 = **55,825** **(Ans.)**

$$\begin{array}{r} 48{,}497 \\ +\ 7{,}328 \\ \hline 55{,}825 \end{array}$$

(ii) **C – A** = 1,33,224 – 48,497

$$\begin{array}{r} 1{,}33{,}224 \\ -\ 48{,}497 \\ \hline 84{,}727 \end{array}$$

(Ans.)

(iii) **B + C** =

$$\begin{array}{r} 7{,}328 \\ +\ 1{,}33{,}224 \\ \hline 1{,}40{,}552 \end{array}$$

(Ans.)

Example 11 :

Cost of one article is ₹ 57,390. What will be the cost of 275 such articles ?

Solution :

Cost of one article = ₹ 57,390

∴ **Cost of 275 such articles** = 275 × ₹57,390 = **₹ 1,57,82,250** **(Ans.)**

Example 12 :

In an examination, a student multiplied 40,327 by 46 instead of 64. By how much was his answer greater than or less than the correct answer ?

Solution :

The student's answer = 40,327 × 46

$$\begin{array}{r} 40{,}327 \\ \times\ 46 \\ \hline 241962 \\ +\ 161308 \\ \hline 18{,}55{,}042 \end{array}$$

Correct answer = 40,327 × 64

$$\begin{array}{r} 40{,}327 \\ \times\ 64 \\ \hline 161308 \\ +\ 241962 \\ \hline 25{,}80{,}928 \end{array}$$

Incorrect answer is less than the correct answer

by 25,80,928 – 18,55,042 = 25,80,928
– 18,55,042
7,25,886 **(Ans.)**

Example 13 :

₹ 82,804 is divided equally among 326 pupils. How much will each get ?

Solution :

Each will get = ₹ 82,804 ÷ 326
= **₹ 254** **(Ans.)**

```
        254
326 ) 82804 (
      - 652      (326 × 2)
       1760
      - 1630     (326 × 5)
        1304
      - 1304     (326 × 4)
           X
```

Example 14 :

(i) Add : 5,25,469 and 2,73,459

(ii) Subtract : 7,36,940 from 9,78,453

(iii) Multiply : 43,627 and 327

(iv) Divide : 21,945 by 385

Solution :

(i)
```
   5,25,469
 + 2,73,459
   7,98,928
```
7,98,928 **(Ans.)**

(ii)
```
   9,78,453
 – 7,36,940
   2,41,513
```
2,41,513 **(Ans.)**

(iii)
```
      43,627
       × 327
      305389
      87254
     130881
 1,42,66,029
```
1,42,66,029 **(Ans.)**

(iv)
```
          57
385 ) 21,945 (
      19 25
      26 95
      26 95
          X
```
∴ **21,945 ÷ 385 = 57** **(Ans.)**

Example 15 :

The sale receipt of a company during a certain year was ₹ 8,73,540. In the following year, it was decreased by ₹ 84,670. What was the sale receipt of the company during second year ?

Solution :

The sale receipt of the company during second year

= ₹ 8,73,540 – ₹ 84,670 = ₹ 8,73,540
– ₹ 84,670
₹ 7,88,870 **(Ans.)**

EXERCISE 1(B)

1. Population of a city was 3,54,976 in the year 2014. In the year 2015, it increased by 68,438. What was the population of the city at the end of the year 2015 ?
2. A = 7,43,000 and B = 8,00,100. Which is greater A or B ? And, by how much ?
3. A notebook has 56 pages. How many pages will 5326 such note-books have ?
4. A person has 75,000 sheets of paper. If each sheet makes 8 pages of a note book, how many note books of 200 pages can be made using the above sheets ?

> Total number of pages obtained = 8 × 75,000 = 6,00,000
> Then, number of notebooks made = 6,00,000 ÷ 200 = **3,000**

5. Add 1,76,209 ; 4,50,923 and 44,83,947.
6. A cricket player has so far scored 7,849 runs in test matches. He wishes to complete 10,000 runs; how many more runs does he need ?
7. In an election two candidates A and B are the only contestants. If candidate A scored 9,32,567 votes and candidate B scored 9,00,235 votes, by how much margin did A win or loose the election ?
8. Find the difference between the largest and the smallest five digit numbers that can be formed using the digits 5, 1, 6, 3 and 2 without repeating any digit.
9. A machine manufactures 5,782 screws every day. How many screws will it manufacture in the month of April ?
10. A man has ₹ 1,57,184 with him. He placed an order for purchasing 80 articles at ₹ 125 each. How much money will remain with him after the purchase ?
11. A student multiplied 8,035 by 87 instead of multiplying by 78. By how much was his answer greater than or less than the correct answer ?
12. Mohani has 30 m cloth and she wants to make some shirts for her son. If each shirt requires 2 m 30 cm cloth, how many shirts, in all, can be made and how much length of cloth will be left ?
13. The weight of a box is 4 kg 800 gm. What is the total weight of 150 such boxes ?
14. The distance between two places A and B is 3 km 760 m. A boy travels from A to B and then B to A every day. How much distance does he travel in 8 days ?
15. An oil-tin contains 6 litre and 60 ml oil. How many identical bottles can this oil fill, if capacity of each bottle is 30 ml ?
16. The sale receipt of a company in a certain year was ₹ 83,73,540. In the following year, it decreased by ₹ 7,84,670.
    (i) What was the sale receipt of the company during second year ?
    (ii) What was the total sale receipt of the company during these two years ?
17. A number exceeds 8,59,470 by 3,00,999. What is the number ?

ESTIMATION 2

2.1 INTRODUCTION

To organize an event in a school, such as sports day, annual function etc. a rough or an approximate estimate of expenditure is required or imagined. For example;

1.	Expenditure on invitation cards	=	₹ 1,530
2.	Expenditure on hiring the furniture	=	₹ 7,250
3.	Expenditure on dress of participants	=	₹ 11,638
4.	Expenses on transportation	=	₹ 4,547
5.	Expenditure on refreshments	=	₹ 2,590
6.	Miscellaneous expenses	=	₹ 6,240
	Total estimate (approximate expenses)	=	**₹ 33,795**

Thus, to conduct an event, such as sports day, annual function, etc. we make an estimate which may or may not be an exact figure.

Estimation means to make an idea about quantities, to judge approximate size, cost, population, etc.

For example :

(i) Cost of construction of a building does not exceed ₹ 5 crores.

(ii) Approximately 5,000 articles will be supplied every year.

(iii) Population of a town in year 2020 will be approximately 3·6 lac, etc.

(iv) The size of the school play-ground is approximately 1850 sq. m.

The estimation, is also known as rounding off numbers to the nearest ten or nearest hundred or nearest thousand, etc.

(a) Rounding off a number to the nearest ten :

Steps :

1. See the digit at unit (ones) place.
2. If the digit at ones place is less than 5, replace ones digit by 0, and keep the other digits as they are.

   Thus, 542 to the nearest ten is 540,
   934 to the nearest ten is 930,
   221 to the nearest ten is 220, etc.

3. If the digit at ones place is 5 or more than 5, increase tens digit by 1, and replace ones digit by 0.

   Thus, 586 to the nearest ten is 590,
   347 to the nearest ten is 350,
   455 to the nearest ten is 460, etc.

(b) Rounding off a number to the nearest hundred :

Steps :

1. See the digit at tens place of the given number.
2. If the tens digit is less than 5, replace each one of tens and ones digits by 0 and keep the other digits as they are.

   Thus, 549 to the nearest hundred is 500,

   634 to the nearest hundred is 600,

   926 to the nearest hundred is 900, etc.
3. If the tens digit is 5 or more than 5, increase the hundreds digit by 1 and replace each of tens digit and ones digit by 0.

   Thus, 564 to the nearest hundred is 600,

   852 to the nearest hundred is 900,

   360 to the nearest hundred is 400, etc.

(c) Rounding off a number to the nearest thousand :

Steps :

1. See the hundreds digit of the given number.
2. If hundreds digit is less than 5, replace each one of hundreds, tens and ones digits by 0 and keep the other digits as they are.

   Thus, 2394 to the nearest thousand is 2000,

   5432 to the nearest thousand is 5000, etc.
3. If, in the given number, hundreds digit is 5 or more than 5, increase thousands digit by 1 and replace each other digit on its right by 0.

   Thus, 5629 to the nearest thousand is 6000,

   7536 to the nearest thousand is 8000, etc.

EXERCISE 2(A)

1. Round off each of the following to the nearest ten :

   (i) 62 (ii) 265 (iii) 543 (iv) 8261

   (v) 6294 (vi) 3008 (vii) 72326
2. Round off each of the following to the nearest hundred :

   (i) 748 (ii) 784 (iii) 2667 (iv) 5432

   (v) 6388 (vi) 59237
3. Round off each of the following to the nearest thousand :

   (i) 6475 (ii) 6732 (iii) 25352 (iv) 32568

   (v) 9248 (vi) 83294
4. Round off

   (i) 578 to the nearest ten. (ii) 578 to the nearest hundred.

   (iii) 4327 to the nearest thousand. (iv) 32974 to the nearest ten-thousand.

   (v) 27487 to the nearest ten-thousand

5. Round off each of the following to the nearest ten, nearest hundred and nearest thousand.

(i) 864	(ii) 1249	(iii) 54,547	(iv) 68,076
(v) 56,293	(vi) 7,293	(vii) 89,24,379	

6. Round off the following to the nearest ten;

(i) ₹ 562	(ii) 837 m	(iii) 545 cm	(iv) ₹ 27

7. List all the numbers which can be rounded off to 30.
8. List all the numbers which can be rounded off to 50.
9. Write the smallest and the largest numbers which are rounded off to 90.
10. Write the smallest and the largest numbers which are rounded off to 130.

2.2 ESTIMATION OF NUMBERS

Consider the following example :

Example 1 :

In a cricket match between India and Pakistan; 63,000 spectators watched the match sitting in the stadium and 53 million television viewers world wide. Explain the statement.

Does the statement, given above, state that exactly 63,000 spectators were in the stadium or did exactly 53 million viewers watch the match on television ?

Explanation :

Certainly not. It states that approximately 63,000 spectators were in the stadium and approximately 53 million viewers watched the match on television.

63,000 could be 62,500 or 65,400 or 64,670, etc.; but in no case it can be taken as 70,000 or 75,000 or 50,000, etc. In the same way, 53 million may be 52·6 million or 53·8 million or 54 million, etc. and not 57 million or 48 million.

The quantities 63,000 and 53 million, given above, are infact not exact counts, but estimates to give an idea of these quantities.

2.3 APPROXIMATION

Approximation means : nearest to the actual amount or value.

Example :

(i) The area of a piece of land is half of an acre approximately.

(ii) My house is about 800 m away from Mohan's house.

(iii) In a marriage in your house, you can not know the exact number of visitors. Infact, in this case an approximate number can be taken out.

In the same way, we come across many situations in which the exact values can not be obtained. In such situations only approximate values are used.

2.4 MORE ABOUT ESTIMATION

There are many situations, where we have to estimate the sum or difference or product or quotient of numbers. There are no rigid rules for these operations (sum, difference, product and quotient). However, the procedure of estimation depends upon the following :

(i) Degree of accuracy required.

(ii) Simplicity of computation.

(iii) How quickly the estimation is completed ?

and, (iv) How quickly the guessed answer would be obtained ?

(a) To estimate the sum :

We already know the rounding off a number to any place.

For example :

(i) 38 rounded off to ten = 40

(ii) 32 rounded off to ten = 30

(iii) 247 round off to hundred = 200

(iv) 5,497 rounded off to ten = 5,500

5,497 rounded off to hundred = 5,500 and

5,497 rounded off to thousand = 5,000

Example 2 :

Estimate the sum (69 + 73) to the nearest ten.

Solution :

Round of each given number to the nearest ten

$\because$ 69 to the nearest ten = 70

and, 73 to the nearest ten = 70

$\therefore$ **Required sum** = 70 + 70 = **140** **(Ans.)**

Example 3 :

Estimate the sum of 576 and 383 to the (i) nearest ten (ii) nearest hundred.

Solution :

(i) 576 to the nearest ten = 580

and, 383 to the nearest ten = 380

$\therefore$ **Required sum** = 580 + 380 = **960** **(Ans.)**

(ii) 576 to the nearest hundred = 600

and, 383 to the nearest hundred = 400

$\therefore$ **Required sum** = 600 + 400 = **1,000** **(Ans.)**

Example 4 :

Estimate the sum of 598, 734 and 606 to the nearest ten.

Solution :

598 to the nearest ten = 600

734 to the nearest ten = 730

and, 606 to the nearest ten = 610

∴ **Required sum** = 600 + 730 + 610 = **1,940** **(Ans.)**

Example 5 :

Estimate the sum of 2,563 and 8,469 to the nearest thousand.

Solution :

2,563 to the nearest thousand = 3,000

and, 8,469 to the nearest thousand = 8,000

∴ **Required sum** = 3.000 + 8,000 = **11,000** **(Ans.)**

Example 6 :

Estimate the sum of 92,456; 80.326 and 4,555 to the nearest thousand.

Solution :

∵ 92,456 correct to nearest thousand = 92,000

80,326 correct to nearest thousand = 80,000

and, 4,555 correct to nearest thousand = 5,000

∴ **Required sum** = 92,000 + 80,000 + 5,000

= **1,77,000** **(Ans.)**

(b) To estimate the difference :

Example 7 :

Estimate the difference 537 – 382 correct to nearest ten.

Solution :

∵ 537 correct to nearest ten = 540

and, 382 correct to nearest ten = 380

∴ **Required difference** = 540 – 380 = **160** **(Ans.)**

Example 8 :

Estimate the difference 56,738 – 2,395.

(i) to the nearest hundred. (ii) to the nearest thousand.

Solution :

(i) 2,395 to the nearest hundred = 2,400

and, 56,738 to the nearest hundred = 56,700

∴ **Required difference** = 56,700 – 2,400 = **54,300** **(Ans.)**

(ii) 56,738 to the nearest thousand = 57,000

and, 2,398 to the nearest thousand = 2,000

∴ **Required difference** = 57,000 − 2,000 = **55,000** **(Ans.)**

(c) To estimate the product :

Example 9 :

Estimate the product of 63 and 79 correct to nearest ten.

Solution :

∵ 63 to the nearest ten = 60

and, 79 to the nearest ten = 80

∴ **Required product** = 60 × 80 = **4800** **(Ans.)**

Example 10 :

Estimate the product of 382 and 247 by rounding off each number to the nearest hundred.

Solution :

382 to the nearest hundred = 400

and, 247 to the nearest hundred = 200

∴ **Required product** = 400 × 200 = **80,000** **(Ans.)**

Example 11 :

Estimate the product of 5,836 and 428 by rounding off 5,836 correct to the nearest thousand and 428 correct to the nearest hundred.

Solution :

5,836 to the nearest thousand = 6,000

and, 428 to the nearest hundred = 400

∴ **Required product** = 6,000 × 400 = **24,00,000** **(Ans.)**

(d) To estimate the quotient :

Example 12 :

Find the estimated quotient for :

(i) 843 ÷ 26, taking each number correct to nearest 10.

(ii) 972 ÷ 462, taking each number correct to nearest hundred.

Solution :

(i) 843 ÷ 26 is approximately (to the nearest 10) equal to

$$840 \div 30 = \frac{840}{30} = \mathbf{28}$$ **(Ans.)**

(ii) 972 ÷ 462 is approximately (to the nearest hundred) is equal to

$$1000 \div 500 = \frac{1000}{500} = \mathbf{2}$$ **(Ans.)**

Important :

(i) $70 \div 30 = \frac{70}{30} = \frac{7}{3} = 2\frac{1}{3}$, it is nearest to 2

(ii) $70 \div 40 = \frac{70}{40} = \frac{7}{4} = 1\frac{3}{4}$, it is also nearest to 2.

Can you explain the difference between both of these parts ? If not, discuss with your teacher.

In the same way,

$67 \div 24$ is approximately equal to $70 \div 20 = 3{\cdot}5$, which is approximately equal to 4.

EXERCISE 2(B)

1. Estimate the sum of each pair of numbers to the nearest ten :
   (i) 67 and 44 (ii) 34 and 87 (iii) 23 and 66 (iv) 78 and 18
   (v) 96 and 55 (vi) 76 and 62 (vii) 457 and 175 (viii) 474 and 173
   (ix) 527 and 267
2. Estimate the sum of each pair of numbers to the nearest hundred :
   (i) 336 and 782 (ii) 546 and 342 (iii) 270 and 495 (iv) 4280 and 5295
   (v) 4230 and 2410 (vi) 30047 and 39287
3. Estimate the sum of the following pairs of numbers to the nearest thousand :
   (i) 53826 and 36455 (ii) 56802 and 22475
4. Estimate the following differences correct to the nearest ten :
   (i) 82 – 27 (ii) 96 – 36 (iii) 508 – 248
5. Estimate each difference to the nearest hundred :
   (i) 769 – 314 (ii) 856 – 687 (iii) 6352 – 2086
6. Estimate each difference to the nearest thousand :
   (i) 45974 – 38766 (ii) 76003 – 48399
7. Estimate each of the following products by rounding off each number to the nearest ten :
   (i) 49×52 (ii) 63×38 (iii) 27×54 (iv) 53×85
   (v) 74×67 (vi) 25×33
8. Estimate each of the following products by rounding off each number to the nearest hundred :
   (i) 477×213 (ii) 624×236 (iii) 333×247 (iv) 537×283
   (v) 382×127 (vi) 472×328
9. Estimate each of the following products by rounding off the first number correct to the nearest ten and the other number correct to the nearest hundred :
   (i) 28×287 (ii) 432×128 (iii) 48×165 (iv) 72×258
   (v) 83×664 (vi) 44×250
10. Estimate each of the following quotients by converting each number to the nearest ten :
    (i) $87 \div 28$ (ii) $84 \div 23$ (iii) $77 \div 22$ (iv) $198 \div 24$
    (v) $355 \div 26$ (vi) $444 \div 42$ (vii) $843 \div 33$

NUMBERS IN INDIAN AND INTERNATIONAL SYSTEMS 3

(With Comparison)

3.1 INTRODUCING UNIT, NUMBER, NUMERAL AND NUMERATION

1. **In mathematics unit means a single thing.**
   For example, a pen, a boy, a metre, a day, etc.

   > A unit is the first and the lowest natural number as a standard of measurement.

2. **The number written before the name of a unit indicates** how many times that unit is taken. e.g. 8 m means; the number 8 is written before the unit m $\Rightarrow$ unit **m** (metre) is taken 8 times.

For example :

(i) **Four litre** means one litre (unit of volume) is taken **4 times.**

(ii) **Length = 3 m** means unit of length is **m** (metre) and it is taken **3 times**.

(iii) **Weight = 63 kg** means unit of weight used is **kg** (kilogramme) and it is taken 63 times.

3.2 NUMERAL AND NUMERATION

A numeral is a **symbol representing** a given **number** and **numeration represents** that **number in words**.

Number	Numeral	Numeration
3	3	three
15	15	fifteen
72	72	seventy-two
0	0	zero

3.3 HINDU-ARABIC (INDIAN) SYSTEM OF NUMERATION

The Indian system of numeration is in fact the *decimal system* that is in use all over the world. This system was developed by the ancient Hindu-Mathematicians in India and was carried to the West by the Arabs. For this reason, it is called **the Hindu-Arabic system of numeration**.

In Indian number system (also known as denary system); the ten **symbols** 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used to write a numeral (number). Each of these ten symbols is called a digit. Out of these digits :

(i) 0, 2, 4, 6 and 8 are even numerals.

(ii) 1, 3, 5, 7 and 9 are odd numerals.

In Indian System, also called Hindu Arabic System, to read and write large quantities with ease, the groups are made with certain periods as shown below :

Periods →	Crores		Lakhs		Thousands		Ones		
↓ Places	Ten crores (T C)	Crore (C)	Ten lakhs (T L)	Lakh (L)	Ten thousands (T Th)	Thousand (Th)	Hundreds (H)	Tens (T)	Ones (0)
One									1
Ten								1	0
1 Hundred							1	0	0
1 Thousand						1	0	0	0
10 Thousands					1	0	0	0	0
1 Lakh				1	0	0	0	0	0
10 Lakhs			1	0	0	0	0	0	0
1 Crore		1	0	0	0	0	0	0	0
10 Crores	1	0	0	0	0	0	0	0	0

Clearly, counting from the right hand-side towards left hand-side, we have :

(i) **One's period :** First three digits *i.e.* ones, tens and hundreds.

(ii) **Thousand's period :** Next two digits *i.e.* thousand and ten thousands.

(iii) **Lakh's period :** Next two digits *i.e.* lakh and ten lakhs.

(iv) **Crore's period :** Next two digits *i.e.* crore and ten crores.

While reading a numeral, all the digits in the same period are read together and the name of the period (except the ones) is read along with it. Thus :

Periods →	Crores		Lakhs		Thousands		Ones		
Numbers ↓	T C	C	T L	L	T Th	Th	H	T	O
(i) 7456123			7	4	5	6	1	2	3
(ii) 225437				2	2	5	4	3	7
(iii) 61786425		6	1	7	8	6	4	2	5
(iv) 82854137		8	2	8	5	4	1	3	7
(v) 743813256	7	4	3	8	1	3	2	5	6

Clearly,

(i) **7456123** = 74,56,123

= Seventy-four lakh fifty-six thousand one hundred twenty three.

(ii) **225437** = 2,25,437

= Two lakh twenty-five thousand four hundred thirty-seven.

(iii) **61786425** = 6,17,86,425
= Six crore seventeen lakh eighty-six thousand four hundred twenty-five.

(iv) **82854137** = 8,28,54,137
= Eight crore twenty-eight lakh fifty-four thousand one hundred thirty-seven.

(v) **743813256** = 74,38,13,256
= Seventy-four crore thirty-eight lakh thirteen thousand two hundred fifty-six.

[Clearly, commas in numerals separate the groups].

1. **Do not write the periods in plural**
   Never write 5493 as five thousands four hudreds ninety three.
   Correct form is : Five thousand four hundred ninety three.
2. **Do not use the word 'and' before tens and ones**
   Do not read 5493 as five thousand four hundred and ninety three.
   Correct form is : Five thousand four hundred ninety three.
3. **Do not put commas when writing the numeral :**
   Never write 2468 as two thousand, four hundred, sixty eight.
   Correct way is : Two thousand four hundred sixty eight.

3.4 INTERNATIONAL SYSTEM OF NUMERATION

In this system, the groups are made with periods as shown below :

Periods →	Billions			Millions			Thousands			Ones		
↓ Places	Hundred billions (H B)	Ten billions (T B)	Billion (B)	Hundred millions (H M)	Ten millions (T M)	Million (M)	Hundred thousands (H Th)	Ten thousands (T Th)	Thousand (Th)	Hundreds (H)	Tens (T)	Ones (0)
One												1
Ten											1	0
1 Hundred										1	0	0
1 Thousand									1	0	0	0
10 Thousands								1	0	0	0	0
100 Thousands							1	0	0	0	0	0
1 Million						1	0	0	0	0	0	0
10 Millions					1	0	0	0	0	0	0	0
100 Millions				1	0	0	0	0	0	0	0	0
1 Billion			1	0	0	0	0	0	0	0	0	0
10 Billions		1	0	0	0	0	0	0	0	0	0	0
100 Billions	1	0	0	0	0	0	0	0	0	0	0	0

Counting from the right-hand side towards left-hand side, we have

(i) **One's period :** First three digits *i.e.* hundreds, tens and ones.

(ii) **Thousand's period :** Next three digits *i.e.* Hundred-thousands, Ten-thousands and Thousands.

(iii) **Million's period :** Next three digits *i.e.* Hundred-millions, Ten-millions and Million.

(iv) **Billion's period :** Next three digits *i.e.* Hundred-billions, Ten-billions and Billion.

In this system also, while reading a numeral, all digits in the same period are read together and the name of the period (except the ones) is read along with it. Thus:

Period →	Millions			Thousands			Ones		
Numbers ↓	H M	T M	M	H Th	T Th	Th	H	T	O
(i) 7456123			7	4	5	6	1	2	3
(ii) 225437				2	2	5	4	3	7
(iii) 61786425		6	1	7	8	6	4	2	5
(iv) 82854137		8	2	8	5	4	1	3	7
(v) 743813256	7	4	3	8	1	3	2	5	6

Clearly,

(i) **7456123** = 7,456,123
= Seven million four hundred-fifty-six thousand one hundred twenty three.

(ii) **225437** = 225,437
= Two hundred-twenty-five thousand four hundred thirty-seven.

(iii) **61786425** = 61,786,425
= Sixty-one million seven hundred eighty-six thousand four-hundred twenty-five.

(iv) **82854137** = 82,854,137
= Eighty-two million eight hundred-fifty-four thousand one hundred-thirty-seven.

(v) **743813256** = 743,813,256
= Seven hundred-forty-three million eight-hundred-thirteen thousand two hundred-fifty-six.

[Here also commas separate the periods].

Comparing the Indian place value chart with the International place value chart.

Indian	Crores		Lakhs		Thousands		Ones		
	T C	C	T L	L	T Th	Th	H	T	O
International	Millions			Thousands			Ones		
	H M	T M	M	H Th	T Th	Th	H	T	O

Thus, 100 thousands = 1 lakh

1 million = 10 lakhs

10 millions = 1 crore

100 millions = 10 crores, etc.

Example 1 :

Using Hindu-Arabic System of numeration, read the number 850746.

Solution :

Lakhs		Thousands		Ones		
T L	L	T Th	Th	H	T	O
	8	5	0	7	4	6

$\therefore$ **850746** = 8,50,746

= **Eight lakh fifty thousand seven hundred forty six.** **Ans.**

Example 2 :

Write four crore fifteen lakh fifty thousand five hundred twenty seven in numeral form using Hindu-Arabic System.

Solution :

Crores		Lakhs		Thousands		Ones		
TC	C	TL	L	T Th	Th	H	T	O
	4	1	5	5	0	5	2	7

$\therefore$ Given number = **4,15,50,527** **Ans.**

Make the relationship between Hindu-Arabic and International System of numerations clear, *as shown below :*

Hindu-Arabic System | International System

	TC	C	TL	L	TTh	Th	H	T	O		HM	TM	M	HTh	TTh	T	H	T	O
1.		5	0	8	1	6	4	0	7	1.		5	0	8	1	6	4	0	7
	= Five crore eight lakh sixteen thousand four hundred seven										= Fifty million eight hundred sixteen thousand four hundred seven								
2.	3	2	5	0	0	8	6	0	0	2.	3	2	5	0	0	8	6	0	0
	= Thirty two crore fifty lakh eight thousand six hundred										= Three hundred twenty five million eight thousand six hundred								
3.			7	6	5	0	3	9	0	3.			7	6	5	0	3	9	0
	= Seventy six lakh fifty thousand three hundred ninety										= Seven million six hundred fifty thousand three hundred ninety								

EXERCISE 3

1. Write the following numerals using Indian System or International system (as required) in words :
   - (i) 4,35,342 = ...
   - (ii) 36,71,430 = ...
   - (iii) 4,28,30,004 = ...
   - (iv) 75,132,684 = ...
   - (v) 815,906 = ...
   - (vi) 5,420,700 = ...

2. Write the following numbers, placing the commas, according to Indian system :
   - (i) 835629 = ... (ii) 35640254 = ...
   - (iii) 2826040 = ...

3. Write the following numbers, by placing the commas, according to International system :
   - (i) 6509820 = ... (ii) 428140584 = ...
   - (iii) 63560981 = ...

4. Fill in the blanks :
   - (i) Four lakh sixty seven thousand three hundred six.

     = ... (In numeral form)

     = ... (In International System)

     = ...

     .... (In International numeration)
   - (ii) Thirteen lakh forty five.

     = ... (In numeral form)

     = ... (In International System)

     = ... (In international numerals)

5. Fill in the blanks :
   - (i) Six hundred four thousand eight hundred forty seven.

     = ... (In numeral form)

     = ... (In Indian System)

     = ...

     .... (Number name in Indian System)
   - (ii) Two million three hundred ten thousand one hundred four.

     = ... (In numeral form)

     = ... (In Indian System)

     = ... (In Indian numeration)

     .... (In international numerals)

PLACE VALUE 4

4.1 RECAPITULATION

Consider a two digit number say 85.

85. in expanded form can be written as 85 = 80 + 5 = 8 × 10 + 5.

⇒ 8 is at tens place and 5 is at unit's place.

In other words, we can say, in 85 the place value of 8 is **80** (eight tens *i.e.*, 8 × 10) and the place value of 5 is **5** (5 ones *i.e.*, 5 × 1).

In the same way, consider a three digit number 457.

457 in expanded form can be written as :

$$457 = 400 + 50 + 7$$
$$= 4 \times 100 + 5 \times 10 + 7$$

⇒ 4 is at hundreds place, 5 is at tens place and 7 is at ones place.

⇒ Place value of 4 is 400, place value of 5 is 50 and the place value of 7 is 7.

4.2 PLACE VALUE (Local value)

In a number, the **place** (or local) **value** of a non-zero digit, is the value of this digit according to its position in the number.

Consider a four digit number, say 7289.

Clearly, 7289 = 7000 + 200 + 80 + 9

= 7 is at thousands place, 2 at hundred's place, 8 at ten's place and 9 at one's place

⇒ **In number 7289**

Place value of 7 is 7000, place value of 2 is 200,

place value of 8 is 80 and place value of 9 is 9.

In the same way, in number 26048 :

Place value of 2 is 20000, place value of 0 is 0,

place value of 4 is 40 and so on.

The **place value** of a digit depends upon the position it occupies in the number.

The place value of the digit 0 is always 0 regardless of the place it occupies in the given number.

For digit	**Place value** (*Local value*)
3 in 2305	300
0 in 907	0
7 in 472	70
5 in 1450	50
2 in 2000	2000
8 in 18605	8000

Example 1 :

Write the place values of the two 6s (sixes) used in the number 36268 and find the sum of these two values.

Solution :

In 36268, **one 6** occurs at thousand's place, so its **place value** = **6000** **(Ans.)**

The **other 6** occurs at ten's place, so its **place value** = **60** **(Ans.)**

The **sum of** these **two place values of 6** = 6000 + 60 = **6060** **(Ans.)**

Example 2 :

Write the place values of the two 5s in 9,45,582 and find the difference of these place values.

Solution :

In 9,45,582, **the first 5** occurs at thousand's place

$\Rightarrow$ **Its place value** = 5 thousand = **5000** **(Ans.)**

The **second 5** occurs at hundred's place

$\Rightarrow$ **Its place value** = 5 hundred = **500** **(Ans.)**

The **difference of the two place values of 5** = 5000 − 500 = **4500** **(Ans.)**

1. *A* **concrete number** *is a number which refers to a particular unit and is meaningful such as : 8 metre, 12 kg, 18 km, 36 cm, etc.*
2. *An* **abstract number** *is a number which does not refer to any particular unit; such as : 8, 12, 18, 36, etc.*

EXERCISE 4(A)

1. Fill in the blanks :
   (i) In 20 kg, the unit is ...., which is taken .... times.
   (ii) In 80 m, the unit is ...., which is taken .... times.
   (iii) If a unit cm (centimetre) is taken 5 times, the corresponding quantity is ....
   (iv) If a unit km (kilometre) is taken 24 times, the corresponding quantity is ....
   (v)

Number	Numeral	Numeration
(a) .....53.....	....	....
(b)	.....9.....	....
(c) ....240....	....	....

2. Fill in the blanks :
   (i) **In 24,673,** the place value of 6 is ....
   (ii) **In 8,039,** the place value of 8 is ....
   (iii) **In 3,25,648,** the local value of 5 is ....
   (iv) **In 6,439,** the local value of 6 is ....

3. Find the difference between the place values of 3 and 5 in the number 3945.
4. In the number 40562,
   (i) the local value of 5 = ...
   (ii) the place value of 6 = ...
   (iii) the sum of the place value of 5 and the place value of 6 = ...
5. Read and write the following numbers in words and also in expanded form :
   (i) 35,000 = Thirty five thousands
   = $3 \times 10000 + 5 \times 1000$
   (ii) 76,000 = ...
   = ...
   (iii) 6,23,000 = ...
   = ...
   (iv) 40,075 = ...
   = ...
   (v) 50,004 = ...
   = ...
6. Find the difference in the place values of two sevens in the number 8,72,574.

4.3 LARGEST AND THE SMALLEST NUMBERS

(i) What is the largest one digit number ? It is 9.
On adding 1 to 9, we get 10; which is the smallest two digit number.

(ii) What is the largest three digits number ? It is 999.
On adding 1 to 999, we get 1,000 and it is the smallest four digit number.

(iii) What is the largest seven digit number ? It is 9999999.
On adding one to 9999999, we get 10000000; which is the smallest eight digit number.

Pattern formed is :

Largest number of n digits + 1 = Smallest number of $(n + 1)$ digits

(i) Largest number of 3 digits + 1 = Smallest number of 4 digits
i.e. 999 + 1 = 1,000

(ii) Largest number of 5 digits + 1 = Smallest number of 6 digits
i.e. 99,999 + 1 = 1,00,000

Thus, 9 + 1 = 10 *i.e.* 10 = 9 + 1
99 + 1 = 100 *i.e.* 100 = 99 + 1
9,999 + 1 = 10,000 or 10,000 = 9,999 + 1
99,99,999 + 1 = 1,00,00,000 and so on.

Further :

(i) What is the smallest number of two digits ? It is 10.

On subtracting 1 from 10, we get 9; which is the largest number of one digit.

(ii) What is the smallest number of 5 digits ? It is 10000.

On subtracting 1 from 10000, we get 9999; which is the largest number of four digits.

Pattern formed is :

Smallest number of n digits − 1 = Largest number of $(n - 1)$ digits

(i) Smallest number of 3 digits − 1 = Largest number of 2 digits

i.e. 100 − 1 = 99

(ii) Smallest number of 6 digits − 1 = Largest number of 5 digits

i.e. 1,00,000 − 1 = 99,999

Thus, 1,000 − 1 = 999 *i.e.* 999 = 1,000 − 1

10,000 − 1 = 9,999 *i.e.* 9,999 = 10,000 − 1

10,00,000 − 1 = 9,99,999 and so on.

EXERCISE 4(B)

1. Fill in the blanks :
   (i) 999 + 1 = ...
   (ii) 10,000 − 1 = ...
   (iii) 10 coins − one coin = ...
   (iv) ₹ 99 + ₹ 1 = ...
   (v) 10,000 boys − 1 boy = ...
   (vi) 1000 toys − 1 toy = ...
2. Would the number of students in your school be a 3-digit number or a 4-digit number or a 5-digit number ?
3. Write the smallest number which is just more than 9,99,999.
4. Starting from the greatest 5-digit number, write the previous five numbers in descending order.
5. Starting from the smallest 7-digit number, write the next four numbers in ascending order.
6. How many numbers lie between the largest 3-digit number and the smallest 4-digit number ?
7. How many 5-digit numbers are there in all ?

NATURAL NUMBERS AND WHOLE NUMBERS

5

(Including Patterns)

5.1 NATURAL NUMBERS

Natural numbers are those numbers by which we can count things in nature like 4 trees, 6 houses, 5 cows, etc.

These are the numbers used for counting purpose.

∴ **Natural numbers = 1, 2, 3, 4, 5, 6, ...**

1. The first and the smallest natural number is 1 (one).
2. The last and the greatest natural number can not be obtained, [infact there are infinite natural numbers, like counting of stars in nature.]
3. The difference between any two consecutive natural numbers is 1 (one).
4. By adding 1 to any natural number its next natural number is obtained.

1. The fractions like : $\frac{3}{4}$, $-\frac{12}{17}$, $\frac{215}{317}$, etc. are not natural numbers.
2. The decimal numbers like : 3·4, 5·28, 2·227, etc. are not natural numbers.
3. 0 (zero) is not a natural number.
4. No natural number is negative *i.e.* none of –15, –3, –214, etc. is a natural number.
5. If n is any natural number : (i) 2n is an even natural number and
   (ii) (2n – 1) is an odd natural number.

5.2 WHOLE NUMBERS

The natural numbers along with 0 (zero) form a collection of whole numbers.

∴ **Whole numbers = 0, 1, 2, 3, 4, 5, 6, ...**

A whole number is either 0 or a natural number.

1. The first and the smallest whole number is 0 (zero).
2. The last and the greatest whole number can not be obtained.
   [Like natural numbers, whole numbers are also infinite].
3. The difference between two consecutive whole numbers is 1(one).
4. By adding 1 to any whole number, its next whole number is obtained.
5. All natural numbers are whole numbers, but all whole numbers are not natural numbers.

5.3 SUCCESSOR AND PREDECESSOR OF A WHOLE NUMBER

Successor of a whole or a natural number is the number obtained on adding 1 (one) to it.

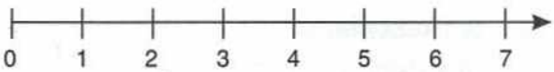
∴ Sucessor of 0 is 1; successor of 1 is 2; successor of 5 is 6 and so on.

Predecessor of a whole number is obtained on subtracting 1 (one) from it.

∴ Predecessor of 1 is 0; predecessor of 2 is 1; predecessor of 8 is 7 and so on.

1. Successor of a whole number is the whole number plus one and the predecessor of a whole number is the whole number minus one.
2. Predecessor of whole number 0 is not possible.
3. Predecessor of natural number 1 (one) is not possible.

Number line used to represent whole numbers is as given below :



The number line, representing whole numbers, starts from zero (0). The points are marked at equal distances to represent whole numbers 1, 2, 3, 4, 5, etc. The arrow, at the extreme right of the number line, shows that the whole numbers are infinite.

1. Zero (0) is the smallest whole number.
2. There is no whole number on the left side of 0.
3. Each whole number (other than 0) is on the right side of 0.
4. A whole number is :
   (a) greater than every whole number on its left.
   (b) smaller than every whole number on its right.

5.4 PROPERTIES OF WHOLE NUMBERS

The main properties for the operations of addition, subtraction, multiplication and division on whole numbers are :

1. Closure property
2. Commutative property
3. Associative property
4. Distributive property
5. Identity
6. Inverse

For Addition

1. Closure property :

If x and y are two whole numbers, then x + y is also a whole number.

In other words, whole numbers are said to be closed for addition if on adding any two whole numbers, we get a whole number only.

Consider the following table.

One whole number (x)	Another whole number (y)	Their sum (x + y)
5	8	5 + 8 = 13, *a whole number.*
4	0	4 + 0 = 4, *a whole number.*
7	3	7 + 3 = 10, *a whole number.*
15	18	15 + 18 = 33, *a whole number.*

2. **Commutative property (Commutativity) :**

If x and y are two whole numbers, then $x + y = y + x$.

In other words, the sum of two whole numbers is said to be commutative for addition if their sum remains the same even if the order of addition is changed.

Consider the following table :

x	y	x + y	y + x	x + y = y + x
4	3	4 + 3 = 7	3 + 4 = 7	4 + 3 = 3 + 4
52	27	52 + 27 = 79	27 + 52 = 79	52 + 27 = 27 + 52
0	28	0 + 28 = 28	28 + 0 = 28	0 + 28 = 28 + 0

3. **Associative property (Associativity) :**

For any three whole numbers x, y and z.

$x + (y + z) = (x + y) + z$.

In other words, for any set of three whole numbers if the sum of any two whole numbers is added to the third whole number, then whatever be their order, the sum will remain the same.

Consider the following table :

For 3, 5, 6	: **3 + (5 + 6)** = 3 + 11 = **14** and **(3 + 5) + 6** = 8 + 6 = **14**
For 8, 4, 0	: **8 + (4 + 0)** = 8 + 4 = **12** and **(8 + 4) + 0** = 12 + 0 = **12**
For 8, 4, 9	: **8 + (4 + 9)** = 8 + 13 = **21** and **(8 + 4) + 9** = 12 + 9 = **21**

4. **Existence of identity (Additive identity) :**

For any whole number x,

$x + 0 = x,\ 0 + x = x$ *i.e.* $x + 0 = 0 + x$

In other words, the addition (sum) of any whole number and whole number 0 (zero) is the number itself.

The whole number 0 for which $x + 0 = 0 + x$ is called **additive identity** or **identity element for addition**.

For whole number 8 : 8 + 0 = 8, 0 + 8 = 8 ∴ 8 + 0 = 0 + 8

For whole number 15 : 15 + 0 = 15, 0 + 15 = 15, therefore 0 + 15 = 15 + 0

5. **Additive inverse :**

If the sum of two whole numbers is 0 (additive identity), the two numbers are called additive inverse of each other.

Thus, if m and n are two whole numbers such that $m + n$ is equal to zero then, m and n are additive inverse of each other.

That is for any whole number x, its additive inverse will be $-x$, provided $-x$ is also a whole number and $x + (-x) = 0$.

Consider the whole number 5, its additive inverse will be –5 if

(i) –5 is also a whole number

and (ii) 5 + (–5) = 0

Here $5 + (-5) = 0$ but -5 is not a whole number, therefore -5 is not the additive inverse of 5.

The same fact can be shown for every other whole number other than 0, as $0 + (-0) = 0$

6. **Cancellation law of addition :**

If x, y and z are any three whole numbers then

$x + y = x + z \Rightarrow \cancel{x} + y = \cancel{x} + z$

$\Rightarrow y = z$

In the same way,

(i) $a + b = b + c \Rightarrow a + \cancel{b} = \cancel{b} + c$

$\Rightarrow a = c$

(ii) $x + 8 = 5 + 8 \Rightarrow x + \cancel{8} = 5 + \cancel{8}$

$\Rightarrow x = 5$, etc.

Example 1 :

By re-arranging the given numbers, evaluate :

(i) 463 + 826 + 337 + 274 (ii) 5 + 7 + 8 + 15 + 985 + 992 + 993 + 995

Solution :

(i) **463 + 826 + 337 + 274**

= (463 + 337) + (826 + 274)

= 800 + 1,100 = **1,900** **(Ans.)**

(ii) **5 + 7 + 8 + 15 + 985 + 992 + 993 + 995**

= (5 + 995) + (7 + 993) + (8 + 992) + (15 + 985)

= 1,000 + 1,000 + 1,000 + 1,000 = **4,000** **(Ans.)**

EXERCISE 5(A)

1. Fill in the blanks :
   (i) Smallest natural number is ....
   (ii) Smallest whole number is ....
   (iii) Largest natural number is ....
   (iv) Largest whole number is ....
   (v) All natural numbers are ....
   (vi) All whole numbers are not ....
   (vii) Successor of 4099 is ....
   (viii) Predecessor of 4330 is ....
2. Represent the following whole numbers on a number line :
   0, 3, 5, 8, 10.
3. State, true or false :
   (i) Whole numbers are closed for addition.
   (ii) If *a* and *b* are any two whole numbers, then *a* + *b* is not a whole number.
   (iii) If *a* and *b* are any two whole numbers, then *a* + *b* = *b* + *a*.
   (iv) 0 + 18 = 18 + 0.

(v) Addition of whole numbers is associative.

(vi) $10 + 12 + 16 = (10 + 12) + 16 = 10 + (12 + 16)$.

4. Fill in the blanks :

(i) $54 + 234 = 234 +$ .... (ii) $332 + 497 =$ .... $+ 332$

(iii) $286 + 0 =$ .... (iv) $286 \times 1 =$ ....

(v) $a + (b + c) = (a +$ ....$) + c$

5. By re-arranging the given numbers, evaluate :

(i) $237 + 308 + 163$

(ii) $162 + 253 + 338 + 47$

(iii) $21 + 22 + 23 + 24 + 25 + 75 + 76 + 77 + 78 + 79$

(iv) $1 + 2 + 3 + 4 + 596 + 597 + 598 + 599$

6. Is $a + b + c = a + (b + c)$

$= (b + a) + c$?

7. Which property of addition is satisfied by :

(i) $8 + 7 = 15$ (ii) $3 + (5 + 4) = (3 + 5) + 4$

(iii) $8 \times (8 + 0) = 8 \times 8 + 8 \times 0$ (iv) $(7 + 6) \times 10 = 7 \times 10 + 6 \times 10$

(v) $(15 - 12) \times 18 = 15 \times 18 - 12 \times 18$ (vi) $16 + 0 = 16$

(vii) $23 + (-23) = 0$

8. State, true or false :

(i) The sum of two odd numbers is an odd number.

(ii) The sum of two odd numbers is an even number.

(iii) The sum of two even numbers is an even number.

(iv) The sum of two even numbers is an odd number.

(v) The sum of an even number and an odd number is odd number.

(vi) Every whole number is a natural number.

(vii) Every natural number is a whole number.

(viii) Every whole number + 0 = The whole number itself.

(ix) Every whole number × 1 = The whole number itself.

(x) Commutativity and associativity are properties of natural numbers and whole numbers both.

(xi) Commutativity and associativity are properties of addition for natural numbers and whole numbers both.

(xii) If x is a whole number then $-x$ is also a whole number.

For Subtraction

1. Closure property :

If x and y are two whole numbers, then $x - y$ is not necessarily a whole number.

For example :

(i) If $x = 8$ and $y = 3$, $x - y = 8 - 3 = 5$; *which is a whole number.*

(ii) If $x = 6$ and $y = 6$, $x - y = 6 - 6 = 0$; *which is a whole number.*

(iii) If $x = 15$ and $y = 18$, $x - y = 15 - 18 = -3$, *which is not a whole number.*

(iv) If $x = 0$ and $y = 12$, $x - y = 0 - 12 = -12$, *which is not a whole number.*

In other words, the whole numbers are not closed for subtraction.

2. Commutative property :

If x and y are two whole numbers, then $x - y \neq y - x$.

This implies, the subtraction of two whole numbers is not commutative.

For example :

(i) If $x = 15$ and $y = 8$, $x - y = 15 - 8 = 7$ and $y - x = 8 - 15 = -7$

$\therefore\ x - y \neq y - x$

(ii) If $x = 8$ and $y = 10$, $x - y = 8 - 10 = -2$ and $y - x = 10 - 8 = 2$

$\therefore\ x - y \neq y - x$

(iii) If $x = 0$ and $y = 18$, $x - y = 0 - 18 = -18$ and $y - x = 18 - 0 = 18$

$\therefore\ x - y \neq y - x$

3. Associative property :

For any three whole numbers x, y and z,

$x - (y - z) \neq (x - y) - z$

That is subtraction of whole numbers does not satisfy associativity.

For example :

(i) **For $x = 15$, $y = 10$ and $z = 7$:**

$x - (y - z) = 15 - (10 - 7) = 15 - 3 = \mathbf{12}$

$(x - y) - z = (15 - 10) - 7 = 5 - 7 = \mathbf{-2}$

$\therefore\ x - (y - z) \neq (x - y) - z$

(ii) **For $x = 23$, $y = 16$ and $z = 10$:**

$x - (y - z) = 23 - (16 - 10) = 23 - 6 = \mathbf{17}$

$(x - y) - z = (23 - 16) - 10 = 7 - 10 = \mathbf{-3}$

$\therefore\ x - (y - z) \neq (x - y) - z$

4. Distributive property :

For any three whole numbers x, y and z :

$x \times (y - z) = x \times y - x \times z$ and

$(y - z) \times x = y \times x - z \times x$

Students are adviced to verify this property on their own.

5. Existence of identity :

For any whole number x, $x - 0 = x$ *but* $0 - x \neq x$

Thus for subtraction no identity number exists.

When 0 is subtracted from itself, we get $0 - 0 = 0$, so 0 is its own identity for subtraction.

6. Existence of inverse :

Since subtraction for every non-zero whole number does not have identity number, its inverse does not exist.

EXERCISE 5(B)

1. Consider two whole numbers *a* and *b* such that *a* is greater than *b*.
   (i) Is $a - b$ a whole number ? Is this result always true ?
   (ii) Is $b - a$ a whole number ? Is this result always true ?
2. Fill in the blanks :
   (i) $8 - 0 =$ .... and $0 - 8 =$ ....
   $8 - 0 \neq 0 - 8$, this shows subtraction of whole numbers is not ....
   (ii) $5 - 10 =$ ...., which is not a ....
   $\Rightarrow$ Subtraction of .... is not closed.
   (iii) $7 - 18 =$ .... and $(7 - 18) - 5 =$ ....
   $18 - 5 =$ .... and $7 - (18 - 5) =$ ....
   Is $(7 - 18) - 5 = 7 - (18 - 5)$?
   $\Rightarrow$ Subtraction of whole numbers is not ....
3. Write the identity number, if possible for subtraction.
4. Write the inverse, if possible for subtraction of whole numbers ?
5. $12 \times (9 - 6) =$ .... = ...
   $12 \times 9 - 12 \times 6 =$ .... = ...
   Is $12 \times (9 - 6) = 12 \times 9 - 12 \times 6$? ....
   Is this type of result always true ? ....
   Name the property used here ....
6. $(16 - 8) \times 24 =$ .... = ...
   $16 \times 24 - 8 \times 24 =$ .... − .... = ...
   Is $(16 - 8) \times 24 = 16 \times 24 - 8 \times 24$? ....
   Is the type of result always true ? ....
   Name the property used here. ....

For Multiplication

1. Closure property :

If x and y are two whole numbers, then x × y is also a whole number.

The following table makes it clear :

x	*y*	*x* × *y*
5	4	5 × 4 = 20; *which is a whole number*
12	0	12 × 0 = 0; *which is a whole number*
0	23	0 × 23 = 0; *which is a whole number*
15	6	15 × 6 = 90; *which is a whole number*

2. **Commutative property :**

If x and y are two whole numbers, then $x \times y = y \times x$.

Consider the following table.

x	y	x × y	y × x	x × y = y × x
4	5	$4 \times 5 = 20$	$5 \times 4 = 20$	$4 \times 5 = 5 \times 4$
3	0	$3 \times 0 = 0$	$0 \times 3 = 0$	$3 \times 0 = 0 \times 3$
8	12	$8 \times 12 = 96$	$12 \times 8 = 96$	$8 \times 12 = 12 \times 8$
0	28	$0 \times 28 = 0$	$28 \times 0 = 0$	$0 \times 28 = 28 \times 0$

3. **Associative property :**

For any three whole numbers x, y and z.

$x \times (y \times z) = (x \times y) \times z$.

Consider the following table.

For 4, 8, 10 : $\mathbf{4 \times (8 \times 10)} = 4 \times 80 = \mathbf{320}$ and $\mathbf{(4 \times 8) \times 10} = 32 \times 10 = \mathbf{320}$

For 7, 0, 12 : $\mathbf{7 \times (0 \times 12)} = 7 \times 0 = \mathbf{0}$ and $\mathbf{(7 \times 0) \times 12} = 0 \times 12 = \mathbf{0}$

For 3, 5, 9 : $\mathbf{3 \times (5 \times 9)} = 3 \times 45 = \mathbf{135}$ and $\mathbf{(3 \times 5) \times 9} = 15 \times 9 = \mathbf{135}$

4. **Distributive property :**

For any three whole numbers x, y and z.

$x \times (y + z) = x \times y + x \times z$.

In other words, the multiplication of whole numbers is distributive over their addition.

Consider the following examples :

(i) Let $x = 5$, $y = 3$ and $z = 4$

∴ $\mathbf{x \times (y + z)} = 5 \times (3 + 4) = 5 \times 7 = \mathbf{35}$

and, $\mathbf{x \times y + x \times z} = 5 \times 3 + 5 \times 4 = 15 + 20 = \mathbf{35}$

∴ $\mathbf{x \times (y + z) = x \times y + x \times z}$

(ii) Let $x = 8$, $y = 7$ and $z = 10$

∴ $\mathbf{(y + z) \times x} = (7 + 10) \times 8 = 17 \times 8 = \mathbf{136}$

and, $\mathbf{y \times x + z \times x} = 7 \times 8 + 10 \times 8 = 56 + 80 = \mathbf{136}$

∴ $\mathbf{(y + z) \times x = y \times x + z \times x}$

In the same way, take more sets of values of x, y and z and every time prove the above results verify the distributivity of multiplication over addition.

The multiplication of whole numbers is also distributive over their subtraction *i.e.*

(i) $x \times (y - z) = x \times y - x \times z$ and

(ii) $(y - z) \times x = y \times x - z \times x$, if y is greater than z.

Applications of distributive property :

1. $15 \times 135 = 15 \times (100 + 30 + 5)$
$= 1500 + 450 + 75 = 2025$

2. $27 \times 98 = 27 \times (100 - 2)$
$= 2700 - 54 = 2646$

3. $342 \times 197 = 342 \times (200 - 3)$
$= 68400 - 1026 = 67374$, etc.

5. Existence of identity :

For any whole number x,

$x \times 1 = x,\ 1 \times x = x$ i.e. $x \times 1 = 1 \times x$

In other words, the multiplication of any whole number with 1 (one), is the number itself.

The whole number 1 (one) for which $x \times 1 = 1 \times x = x$ is called **multiplicative identity** or **identity element for multiplication**.

For whole number 9 : $9 \times 1 = 9,\ 1 \times 9 = 9 \quad \therefore\ 9 \times 1 = 1 \times 9$

For whole number 15 : $15 \times 1 = 15,\ 1 \times 15 = 15 \quad \therefore\ 15 \times 1 = 1 \times 15$

Therefore identity element for multiplication is 1 (one).

6. Multiplicative inverse :

If x is any whole number ($x \neq 0$), then its multiplicative inverse will be $\frac{1}{x}$.

So that, $x \times \frac{1}{x} = 1$, but $\frac{1}{x}$ is a whole number only when $x = 1$.

For other values of whole number x ($x \neq 1$), $\frac{1}{x}$ is not a whole number.

$\therefore$ For any whole number x ($x \neq 1$), its multiplicative inverse does not exist.

> $\because 1 \times \frac{1}{1} = 1$ and $\frac{1}{1} \times 1 = 1$; therefore multiplicative inverse of 1 is $\frac{1}{1}$
> *i.e.* 1 itself.

7. Cancellation law of multiplication :

If x, y and z are three non-zero whole numbers, then

$x \times y = x \times z \Rightarrow \cancel{x} \times y = \cancel{x} \times z$
$\Rightarrow y = z$

In the same way,

(i) $3 \times a = 3 \times b \Rightarrow \cancel{3} \times a = \cancel{3} \times b$
$\Rightarrow a = b$

(ii) $m \times 7 = n \times 7 \Rightarrow m \times \cancel{7} = n \times \cancel{7}$
$\Rightarrow m = n$, etc.

Example 2 :

By re-aranging the given numbers, evaluate :

(i) $2 \times 9 \times 5$ (ii) $4 \times 398 \times 25$ (iii) $125 \times 483 \times 8$

Solution :

(i) $\because \quad 2 \times 5 = 10$

$\therefore \quad 2 \times 9 \times 5 = (2 \times 5) \times 9$

$= 10 \times 9 = 90$ **(Ans.)**

(ii) $\because \quad 4 \times 25 = 100$

$\therefore \quad 4 \times 398 \times 25 = (4 \times 25) \times 398$

$= 100 \times 398 = 39800$ **(Ans.)**

(iii) $\because \quad 125 \times 8 = 1000$

$\therefore \quad 125 \times 483 \times 8 = (125 \times 8) \times 483$

$= 1000 \times 483 = 483000$ **(Ans.)**

Example 3 :

By re-aranging the given numbers, evaluate :

(i) $4 \times 125 \times 8 \times 5$ (ii) $25 \times 50 \times 8$

Solution :

(i) $4 \times 125 \times 8 \times 5 = (4 \times 5) \times (125 \times 8)$

$= 20 \times 1000 = 20,000$ **(Ans.)**

(ii) $25 \times 50 \times 8 = 25 \times 50 \times 4 \times 2$

$= (25 \times 4) \times (50 \times 2)$

$= 100 \times 100 = 10,000$ **(Ans.)**

Example 4 :

Use various properties to evaluate :

(i) $397 \times 7 + 397 \times 3$ (ii) $479 \times 87 + 479 \times 13$

(iii) $828 \times 101 - 828$ (iv) $664 \times 32 - 664 \times 7$

Solution :

(i) $397 \times 7 + 397 \times 3 = 397 \times (7 + 3)$ [Using distributivity]

$= 397 \times 10$

$= 3970$ **(Ans.)**

(ii) $479 \times 87 + 479 \times 13 = 479 \times (87 + 13)$ [Using distributivity]

$= 479 \times 100$

$= 47900$ **(Ans.)**

(iii) **828 × 101 − 828** = 828 × 101 − 828 × 1

= 828 × (101 − 1) [Using distributivity]

= 828 × 100

= **82800** **(Ans.)**

(iv) **664 × 32 − 664 × 7** = 664 × (32 − 7) [Using distributivity]

= 664 × 25

= **16600** **(Ans.)**

Example 5 :

Using properties, evaluate : 497 × 351 × 10 − 251 × 4970.

Solution :

497 × 351 × 10 − 251 × 4970

= 497 × 10 × 351 − 4970 × 251 [Using commutativity]

= 4970 × 351 − 4970 × 251

= 4970 × (351 − 251) [Using distributivity]

= 4970 × 100 = **497000** **(Ans.)**

Example 6 :

Evaluate :

(i) 375 × 8 (ii) 2304 × 25 (iii) 35672 × 20

Solution :

(i) **375 × 8** = (300 + 70 + 5) × 8

= 300 × 8 + 70 × 8 + 5 × 8

= 2400 + 560 + 40 = **3000** **(Ans.)**

(ii) **2304 × 25** = (2000 + 300 + 4) × 25

= 2000 × 25 + 300 × 25 + 4 × 25

= 50000 + 7500 + 100 = **57600** **(Ans.)**

(iii) **35672 × 20** = (30000 + 5000 + 600 + 70 + 2) × 20

= 30000 × 20 + 5000 × 20 + 600 × 20 + 70 × 20 + 2 × 20

= 600000 + 100000 + 12000 + 1400 + 40

= **713440** **(Ans.)**

Example 7 :

Find the product of the greatest number of 3 digits and the greatest number of two digits.

Solution :

Greatest number of three digits = 999

and, greatest number of two digits = 99

$\therefore$ **Required product** $= 999 \times 99$

$= 999 \times (100 - 1)$

$= 999 \times 100 - 999 \times 1$ [Using distributivity]

$= (1000 - 1) \times 100 - (1000 - 1) \times 1$

$= 1000 \times 100 - 1 \times 100 - 1000 + 1$

$= 100000 - 100 - 1000 + 1$

$= 100001 - 1100 =$ **98901** **(Ans.)**

EXERCISE 5(C)

1. Fill in the blanks :
   (i) $42 \times 0 =$ .... (ii) $592 \times 1 =$ ....
   (iii) $328 \times 573 =$ .... $\times 328$ (iv) $229 \times$ .... $= 578 \times 229$
   (v) $32 \times 15 = 32 \times 6 + 32 \times 7 + 32 \times$ ....
   (vi) $23 \times 56 = 20 \times 56 +$ .... $\times 56$
   (vii) $83 \times 54 + 83 \times 16 = 83 \times$ (................) $= 83 \times$ .... $=$ ....
   (viii) $98 \times 273 - 75 \times 273 =$ (................) $\times 273 =$ .... $\times 273$
2. By re-arranging the given numbers, evaluate :
   (i) $2 \times 487 \times 50$ (ii) $25 \times 444 \times 4$ (iii) $225 \times 20 \times 50 \times 4$
3. Use distributive law to evaluate :
   (i) 984×102 (ii) 385×1004 (iii) 446×10002
4. Evaluate using properties :
   (i) 548×98 (ii) 924×988 (iii) 3023×723
5. Evaluate using properties :
   (i) $679 \times 8 + 679 \times 2$ (ii) $284 \times 12 - 284 \times 2$
   (iii) $55873 \times 94 + 55873 \times 6$ (iv) $7984 \times 15 - 7984 \times 5$
   (v) $8324 \times 1945 - 8324 \times 945$ (vi) $3333 \times 987 + 13 \times 3333$
6. Find the product of the :
   (i) greatest number of three digits and smallest number of five digits.
   (ii) greatest number of four digits and the greatest number of three digits.
7. Fill in the blanks :
   (i) $(437 + 3) \times (400 - 3) = 397 \times$ .... $=$ ....
   (ii) $66 + 44 + 22 = 11 \times$ (..........................) $= 11 \times$ .... $=$ ....
8. Evaluate :
   (i) 355×18 (ii) 6214×12
   (iii) 15×49372 (iv) 9999×8

For Division

1. **Closure property :** *I*

   if x and y are two whole numbers, then x ÷ y is not necessarily a whole number.

   For example :

   5 and 8 are whole numbers, but 5 ÷ 8 is not a whole number

   ∴ Closure property does not exist for division of whole numbers.

2. **Commutative property :**

   If x and y are two whole numbers, then $x \div y \neq y \div x$.

   For example :

   $3 \div 5 \neq 5 \div 3$, $8 \div 13 \neq 13 \div 8$ and so on.

   ∴ Division of whole numbers is not commutative.

3. **Associative property :**

   If x, y and z are any three whole numbers, then $x \div (y \div z) \neq (x \div y) \div z$

   ∴ Division of whole numbers is not associative.

4. **Existence of identity :**

   The identity element for division of whole numbers does not exist.

5. **Existence of inverse :**

   The inverse for division of whole numbers does not exist.

If a is any non-zero whole number, then

(i) $\mathbf{a \div a = 1}$ *i.e.* 5 ÷ 5 = 1, 12 ÷ 12 = 1, 28 ÷ 28 = 1, etc.

(ii) $\mathbf{a \div 1 = a}$ *i.e.* 5 ÷ 1 = 5, 16 ÷ 1 = 16, 28 ÷ 1 = 28, etc.

(iii) $\mathbf{0 \div a = 0}$ *i.e.* 0 ÷ 8 = 0, 0 ÷ 23 = 0, 0 ÷ 47 = 0, etc.

(iv) **$a \div 0$ is not defined** *i.e.* 8 ÷ 0 is not defined, 24 ÷ 0 is not defined, etc.

EXERCISE 5(D)

1. Show that :
   (i) division of whole numbers is not closed.
   (ii) any whole number divided by 1, always gives the number itself.
   (iii) every non-zero whole number divided by itself gives 1 (one).
   (iv) zero divided by any non-zero number is zero only.
   (v) a whole number divided by 0 is not defined.

   For each part, given above, give two suitable examples.
2. If x is a whole number such that $x \div x = x$; state the value of x.

3. Fill in the blanks :

(i) $987 \div 1 =$ .... (ii) $0 \div 987 =$ ....

(iii) $336 - (888 \div 888) =$ .... (iv) $(23 \div 23) - (437 \div 437) =$ ....

4. Which of the following statements are true ?

(i) $12 \div (6 \times 2) = (12 \div 6) \times (12 \div 2)$ (ii) $a \div (b - c) = \frac{a}{b} - \frac{a}{c}$

(iii) $(a - b) \div c = \frac{a}{c} - \frac{b}{c}$ (iv) $(15 - 13) \div 8 = (15 \div 8) - (13 \div 8)$

(v) $8 \div (15 - 13) = \frac{8}{15} - \frac{8}{13}$

Example 8 :

Take any three whole numbers a, b and c and show that $a - (b - c) \neq (a - b) - c$.

Solution :

Let $a = 30$, $b = 16$ and $c = 12$

$\therefore \quad a - (b - c) = 30 - (16 - 12) = 30 - 4 = 26$

and, $\quad (a - b) - c = (30 - 16) - 12 = 14 - 12 = 2$

$\therefore \quad 30 - (16 - 12) \neq (30 - 16) - 12$

This statement says that the subtraction of whole numbers is not associative.

Example 9 :

Take any three whole numbers a, b and c such that $a - b = c$, then $a = b + c$.

Solution :

Let $a = 50$, $b = 35$ and $c = 15$

Then : $a - b = c \Rightarrow 50 - 35 = 15$

and, $a = b + c \Rightarrow 50 = 35 + 15$

Example 10 :

Find two 2-digit numbers such that their product is 840, the prdouct of their unit's digits is 20 and the product of their ten's digits is 6.

Solution :

$\because$ Product of unit's digits $= 20 = 4 \times 5$

$\therefore$ Unit's digits are 4 and 5

The product of their ten's digits $= 6 = 2 \times 3$

$\therefore$ Ten's digits are 2 and 3.

Thus, the numbers are either 34 and 25 or, 35 and 24

$\because \quad 34 \times 25 = 850$

and, $35 \times 24 = 840$

Clearly, required numbers are 35 and 24. **(Ans.)**

Example 11 :

Find the number which on dividing by 13 gives quotient 9 and remainder 7.

Solution :

The number (dividend) = Divisior × Quotient + Remainder

Since, divisior = 13, quotient = 9 and remainder = 7

∴ **The required number** (Dividend)

= Divisor × Quotient + Remainder

= 13 × 9 + 7 = 117 + 7 = **124** **(Ans.)**

EXERCISE 5(E)

1. Find the difference between the largest number of four digits and the smallest number of six digits.
2. Find the difference between the smallest number of eight digits and the largest number of five digits.
3. The product of two numbers is 528. If the product of their unit's digits is 8 and the product of their ten's digits is 4; find the numbers.
4. Does there exist a number *a* such that $a \div a = a$?
5. Divide 5936 by 43 to find the quotient and remainder. Also, check your division by using the formula, dividend = divisor × quotient + remainder.

5.5 PATTERNS

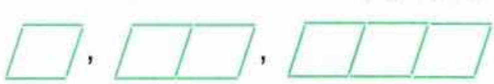
A pattern is an arrangement of numbers, shapes, etc.

By observing the patterns, we recognise and extend them.

Below are given some patterns using numbers and shapes :

(i) 1, 4, 7, 10, 13, ...

(ii) $2^2 + 4^2 + 6^2 +$ ....

(iii)  , ...

(iv) T, T⊥, T⊥T, ... and so on.

5.6 PATTERN IN WHOLE/NATURAL NUMBERS (Including magic square)

Observe the following patterns :

Pattern 1 :

$$1 + 3 = 4 = 2 \times 2 = 2^2$$

$$1 + 3 + 5 = 9 = 3 \times 3 = 3^2$$

$$1 + 3 + 5 + 7 = 16 = 4 \times 4 = 4^2$$

Clearly, next three steps of this pattern will be :

1. $1 + 3 + 5 + 7 + 9 = 25 = 5 \times 5 = 5^2$

2. $1 + 3 + 5 + 7 + 9 + 11 = 36 = 6 \times 6 = 6^2$

3. $1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7 \times 7 = 7^2$

According to this pattern, the sum of first n odd natural numbers = n^2.

i.e. $1 + 3 + 5 + 7 +$ .... upto n odd natural numbers = n^2.

For example :

1. $1 + 3$ is the sum of first two ($n = 2$) odd natural numbers,

   $\therefore\ 1 + 3 = 2^2$

2. $1 + 3 + 5 + 7 + 9$ is the sum of first five ($n = 5$) odd natural numbers,

   $\therefore\ 1 + 3 + 5 + 7 + 9 = 5^2$

3. $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$ is the sum of first eight ($n = 8$) odd natural numbers,

   $\therefore\ 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 8^2$ and so on.

In the same way,

1. The sum of first n natural numbers is $\frac{n(n+1)}{2}$,

   $\Rightarrow 1 + 2 + 3 + 4 + 5 = \frac{5 \times (5+1)}{2} = 15$

   $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = \frac{8 \times (8+1)}{2} = 36$ and so on.

2. The sum of first n even natural numbers is $n(n + 1)$

   $\Rightarrow 2 + 4 + 6 + 8 + 10 = 5 \times (5 + 1) = 30$

   $2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 = 9(9 + 1) = 90.$

Pattern 2 :

$1 + 2 + 3 + 4 + 5 = 15$

$2 + 3 + 4 + 5 + 6 = 20$

$3 + 4 + 5 + 6 + 7 = 25$

Clearly, next two steps of this pattern will be :

1. $4 + 5 + 6 + 7 + 8 = 30$

2. $5 + 6 + 7 + 8 + 9 = 35$

Pattern 3 :

$1 \times 1 = 1$

$11 \times 11 = 121$

$111 \times 111 = 12321$

Clearly, next two steps of this pattern will be :

1. $1111 \times 1111 = 1234321$
2. $11111 \times 11111 = 123454321$

5.7 MAGIC SQUARE

A magic square is an arrangment of different whole numbers in the form of square such that the sum of the numbers in each horizontal line, in each vertical line and in each diagonal is the same.

One magic square is shown alongside :

2	9	4
7	5	3
6	1	8

1. **Row-wise sum :**

   2 + 9 + 4 = **15**, 7 + 5 + 3 = **15** and 6 + 1 + 8 = **15**

2. **Column-wise sum :**

   2 + 7 + 6 = **15**, 9 + 5 + 1 = **15** and 4 + 3 + 8 = **15**

3. **Diagonal-wise sum :**

   2 + 5 + 8 = **15** and 4 + 5 + 6 = **15**

5.8 MATCHSTICK PATTERNS

Each of the patterns to be discussed below is called a matchstick pattern as different shapes or diagrams can be constructed with matchsticks.

Example 12 :

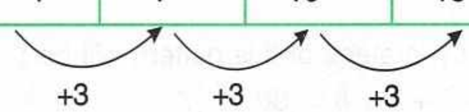
Consider the following pattern :

Fig. 1 Fig. 2 Fig. 3 Fig. 4

Let for getting a pattern with n squares, the number of matchsticks used is M.

From the figures, in the above pattern, we can create a summary table as shown below :

Figure number (n)	1	2	3	4
Number of matchsticks (M)	4	7	10	13



Solution :

It can easily be observed that each time the figure number (n) is increased by 1, the number of matchsticks (M) increases by 3.

In order to find M in terms of n, we should compare $3n$ with M.

Important :

(i) If for each increase of n by 1, M increases by 3, we compare $3n$ with M.

(ii) If for each increase of n by 1, M increases by 2, then compare $2n$ with M and so on.

3n	3	6	9	12
M	4	7	10	13

Clealry, **M is always one more than 3n.**

$\therefore$ M = 3n + 1

We can use the above pattern to predict that;

(i) 10^{th} figure has $3n + 1 = 3 \times 10 + 1 = 31$ matchsticks
i.e. for $n = 10$, M = 31.

(ii) 100^{th} figure has $3 \times 100 + 1 = 301$ matchsticks and so on.

Example 13 :

Consider the following pattern :

Fig. 1 Fig. 2 Fig. 3 etc.

Solution :

Let for the figure number n, the number of matchsticks used is T.

From the figures, in the above pattern, we can create a summary table as given below :

Figure number (n)	1	2	3
Number of matchsticks (T)	3	5	7

+2 +2

It can easily be observed that each time the figure number (n) is increased by 1, the number of matchsticks (T) increases by 2.

This suggests that we should compare 2n with T.

Now, we have :

2n	2	4	6
T	3	5	7

Clealry, **T is always one more than 2n.**

$\therefore$ T = 2n + 1

We can use the above pattern to predict that the;

(i) 5^{th} figure has $2 \times 5 + 1 = 11$ matchsticks

(ii) 20^{th} figure has $2 \times 20 + 1 = 41$ matchsticks and so on.

EXERCISE 5(F)

1. For each pattern, given below, write the next three steps :

(i)
$1 \times 9 + 1 = 10$
$12 \times 9 + 2 = 110$
$123 \times 9 + 3 = 1110$
....
....
....

(ii)
$9 \times 9 + 7 = 88$
$98 \times 9 + 6 = 888$
$987 \times 9 + 5 = 8888$
....
....
....

(iii)
$1 \times 8 + 1 = 9$
$12 \times 8 + 2 = 98$
$123 \times 8 + 3 = 987$
....
....
....

(iv)
$111 \div 3 = 37$
$222 \div 6 = 37$
$333 \div 9 = 37$
....
....
....

2. Complete each of the following magic squares :

(i)

6	7	....
....	5	9
8	....	4

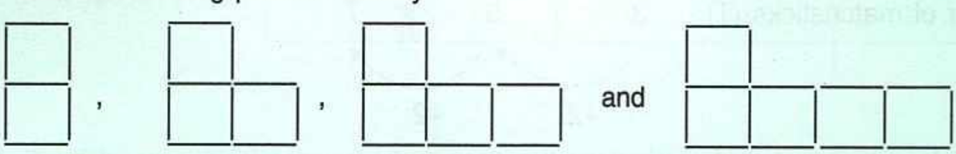
(ii)

4	....	8
....	7	....
....	....	10

(iii)

16	2	....
....	10	....
....	....	4

3. See the following pattern carefully :



(i) If n denotes the number of figures and S denotes the number of matchsticks; find S in terms of n.

(ii) Find how many matchsticks are required to make the :
(1) 15th figure (2) 40th figure

(iii) Write a discription of the pattern in words,

4. (i) In the following pattern, draw the next two figures.

(ii) Construct a table to describe the figures in the above pattern.

(iii) If n denotes the number of figures and L denotes the number of matchsticks, find L in terms of n.

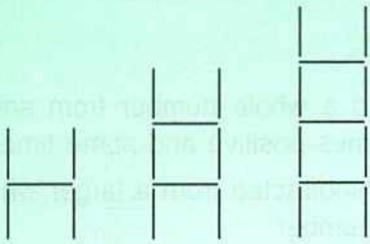
(iv) Find how many matchsticks are required to make the :
(1) 12th figure (2) 20th figure

5. In each of the following patterns, construct the next figure.

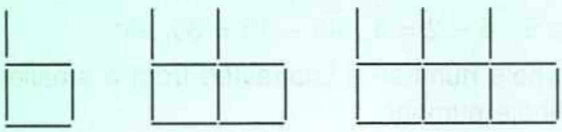
(i) In each case, if n denotes the number of figures and F denotes the number of matchsticks used, find F in terms of n.

(ii) Also find, in each case, how many matchsticks are required to make the : 16^{th} figure and 30^{th} figure.

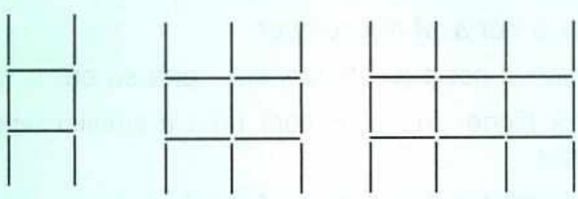
(a)



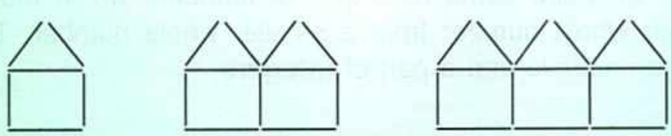
(b)



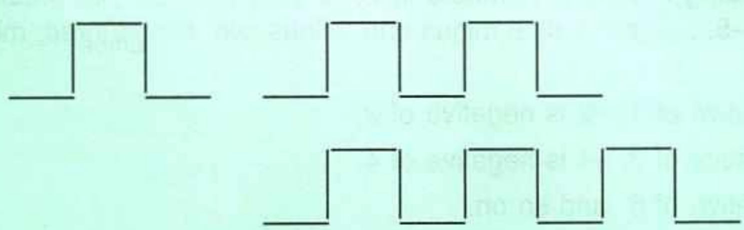
(c)



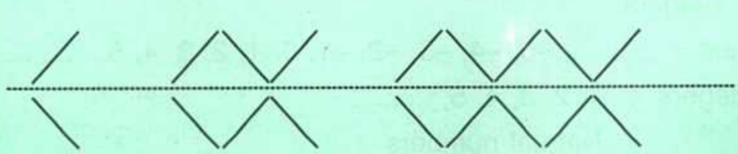
(d)



(e)



(f)



NEGATIVE NUMBERS AND INTEGERS 6

6.1 NEED FOR NEGATIVE NUMBERS

In the previous chapter, we subtracted a whole number from some other whole number and found that the result is some times positive and some times negative.

1. When a smaller whole number is subtracted from a larger whole number, the result is always a positive whole number.

   For example :

   15 – 6 = 9, 8 – 3 = 5, 6 – 2 = 4, 45 – 15 = 30, etc.

2. But when a larger whole number is subtracted from a smaller whole number, the result is not a whole number.

   For example :

   6 – 15 = –9, *which is not a whole number.*

   18 – 38 = –20, *which is not a whole number* and so on.

Clearly, on subtracting a bigger whole number from a smaller whole number, the result is never a whole number.

Thus, the set of whole numbers is not closed for subtraction.

For this reason, we need some new type of numbers which may represent the subtraction of a bigger whole number from a smaller whole number. These new type of numbers are always negative and a part of **integers**.

6.2 INTEGERS

Corresponding to natural numbers 1, 2, 3, 4, 5, ..., etc. we create new numbers –1, –2, –3, –4, –5, ... etc. called minus one, minus two, minus three, minus four, minus five etc.; where

–1 is negative of 1, –2 is negative of 2,

–3 is negative of 3, –4 is negative of 4,

–5 is negative of 5 and so on.

Combining these new numbers with whole numbers, we get a new collection of numbers, called integers.

Thus integers = ..., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...

Positive integers = 1, 2, 3, 4, 5, ...

= Natrual numbers

Negative integers = –1, –2, –3, –4, –5, ...

Also, 1 + (–1) = 0, 2 + (–2) = 0, 5 + (–5) = 0,

15 + (–15) = 0 and so on

⇒ (i) –1 and 1 are called opposites of each other.

(ii) –2 and 2 are opposites of each other.

(iii) –5 and 5 are opposites of each other and so on.

0 is neither positive nor negative. It is a neutral integer.

Note : The symbol '–' denotes negative integer or indicates subtraction. According to the context, it will be clear where the sign '–' is for negative integer or for subtraction.

6.3 CONNECTION OF NEGATIVE NUMBER IN DAILY LIFE

In our day to day life, we come across many situations in which if :

(i) profit is represented by a positive integer, then loss is represented by a negative integer.

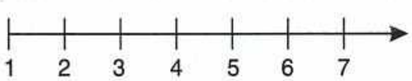
(ii) height above the sea is represented by a positive integer, the depth below sea level is represented by a negative integer.

(iii) +5 represents 5 m due north, –5 represents 5 m due south and so on.

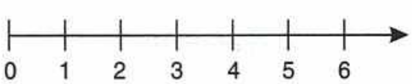
6.4 REPRESENTATION OF NEGATIVE NUMBERS ON NUMBER LINE

We know,

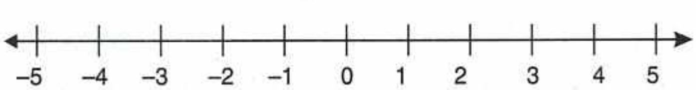
(i) a number line representing natural numbers is



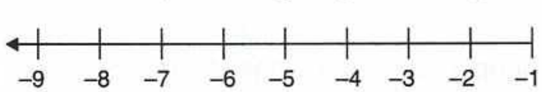
(ii) a number line representing whole numbers is



(iii) a number line representing integers is



(iv) a number line representing negative integers is



6.5 ORDERING OF INTEGERS

For any two integers on a number line :

1. The integers on the right side are greater than the integers on the left side.
2. The integers on the left side are smaller than the integers on the right side.

In the number line for integers, given above,

(i) 5 is on the right of 2 ⇒ 5 is greater than 2 *i.e.* 5 > 2.

(ii) 0 is on the right of –3 ⇒ 0 > –3.

(iii) 2 is on the right of –1 ⇒ 2 > –1.

(iv) –4 is on the left of –3 ⇒ –4 is less than –3 *i.e.* –4 < –3.

(v) –3 is on the left of 4 ⇒ –3 < 4.

We conclude :

1. Every positive integer is greater than 0.
2. Every positive integer is greater than every negative integer.
3. Zero is greater than every negative integer and is less than every positive integer.
4. Every negative integer is smaller than every positive integer.

Note : The greater is an integer, the lesser is its negative (opposite).

(i) Since, 10 > 6 ⇒ –10 < –6

(ii) Since, 8 > – 3 ⇒ –8 < 3

(iii) Since, –5 > –12 ⇒ 5 < 12 and so on.

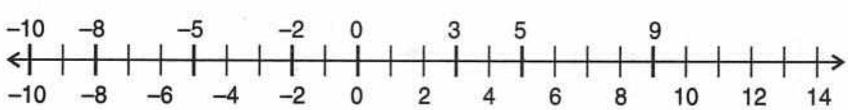
Example 1 :

Use a number line to write the following integers in ascending order :

5, –8, 0, 3, –5, –10, 9 and –2.

Solution :

Draw a number line for integers, as shown below, and mark on it all the given integers.



Clearly, the given integers in the ascending order are :

–10 < –8 < –5 < –2 < 0 < 3 < 5 < 9 **(Ans.)**

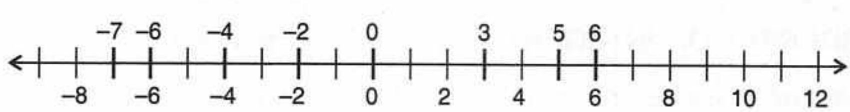
Example 2 :

Use a number line to write the following integers in descending order :

6, –4, 5, –7, 0, –6, –2 and 3.

Solution :

Draw a number line for integers, as shown below, and mark on it all the given integers.



Clearly, the given integers in descending order are :

6 > 5 > 3 > 0 > –2 > –4 > –6 > –7 **(Ans.)**

6.6 ADDITION OF INTEGERS

Rule 1 : *When both the integers are positive :*

Add them and assign plus sign to the result.

For example :

(i) +59 + +32 = +91 (ii) 58 + 27 = 85

(iii) 78 + 45 = 123 and so on.

Rule 2 : *When both the integers are negative :*

Add them and assign minus sign to the result.

For example :

(i) (–43) + (–55) = –98 (ii) (–12) + (–67) = –79

(iii) (–123) + (–507) = –630 and so on.

Rule 3 : *When both the integers are of opposite signs :*

From the integer with greater numerical value subtract the integer with smaller numerical value and then assign (to the result) the sign of the integer with greater numercial value.

For example :

(i) (–38) + 72,

Here both the integers are of opposite signs and the numerical value of 72 is greater than the numerical value of (–38) which is 38

∴ From 72 subtract 38 and assign plus sign to the result.

∴ (–38) + 72 = 34

Conversely, (–72) + 38 = **–34**

(ii) (–95) + 43,

∵ –98 has greater numercial vaue, so subtract 43 from 95 and assign minus sign to the result.

∴ –95 + 43 = –52 [As, 95 – 43 = 52]

(iii) (–65) + 84 = 19

(iv) 65 + (–84) = –19

(v) 159 + (–78) = 81

(vi) (–159) + 78 = –81 and so on.

6.7 SUBTRACTION OF INTEGERS

If the integer to be subtracted is positive, do the ordinary subtraction and if the integer to be subtracted is negative, change its sign and then add.

For example :

(i) 8 – (5) = 8 – 5 = 3 (ii) –8 – (5) = –8 – 5 = –13

(iii) 8 – (–5) = 8 + 5 = 13 (iv) –8 – (–5) = –8 + 5 = –3

More examples :

(i) To subtract 5 from 9, change the sign of 5 and then add it to 9.
Thus : $9 - 5 = 9 + (-5) = 4$

(ii) To subtract –5 from 9, change the sign of –5 and then add it to 9.
Thus : $9 - (-5) = 9 + 5 = 14$.

(iii) To subtract 5 from –8, change the sign of 5 and then add it to –8.
$\therefore\ -8 - 5 = -8 + (-5) = -13$.

(iv) To subtract –8 from –5, change the sign of –8 and then add it to –5.
$\therefore\ -5 - (-8) = -5 + 8 = 3$

EXERCISE 6

1. Fill in the blanks :
   (i) Negative of –20 is ....
   (ii) Negative of 0 is ....
   (iii) Negative of 8 is ....
   (iv) If 10 represents gain of ₹ 10, then –10 represents ....
   (v) If going south is negative then going north is ....
   (vi) Because 5 < 7, therefore –5 .... –7.
   (vii) If 3 > –2, then 3 is on the .... of –2.
   (viii) If –8 < –6, then –8 is on the .... of –6.
2. Use a number line to write the following integers in ascending (increasing) order :
   (i) –5, 8, 0, –9, 4, –14 and 12
   (ii) –6, 7 , 0, –9, 5 and 9
3. Use a number line to write the following integers in descending (decreasing) order :
   (i) –10, 0, 3, –4, 12, 11, –1 and 5
   (ii) –4, 3, –8, –12, –7 and 6.
4. Add :
   (i) 13 and 15
   (ii) –13 and 15
   (iii) 13 and –15
   (iv) –13 and –15
5. Add :
   (i) 259 and 214
   (ii) –528 and –243
   (iii) –623 and 326
   (iv) 258 and –473
   (v) –622 and –254
   (vi) 257 and –254
6. Subtract :
   (i) 5 from 8
   (ii) –5 from 8
   (iii) 4 from –7
   (iv) –8 from – 2
   (v) –3 from 12
   (vi) –6 from –3
7. Subtract :
   (i) –123 from 453
   (ii) –78 from –12
   (iii) 329 from –124
   (iv) –222 from 0

NUMBER LINE 7

7.1 NUMBER LINE

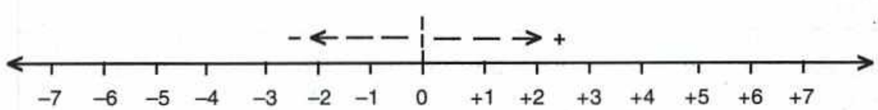
A number line can be used to represent all types of real numbers : –23, 0, 8, $\frac{3}{5}$, $2\frac{1}{5}$, $\sqrt{2}$, $\sqrt{5}$, etc. Here –23, 0 and 8 are integers and $\frac{3}{5}$, $2\frac{1}{5}$, $\sqrt{2}$ and $\sqrt{5}$ are non-integers. **But in the current chapter we shall be dealing with the number line representing integers only.**

Steps for drawing a number line :

1. Draw a straight line of any suitable length.
2. Mark points on the drawn line to divide it into the required number of equal parts.
3. Mark any one of the points, marked on the line in step 2, as zero.
4. Starting from zero and on the right hand side of it mark the positive integers +1, +2, +3, etc., at the points marked in step 2.

   Similarly, starting from zero, on the left hand side of it mark the negative integers –1, –2, –3, etc., at the points marked in step 2.

   The line so obtained will be a number line of the form shown below :

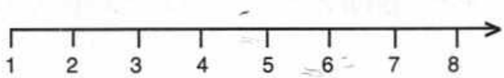


Arrow-heads at the two ends of the number line show that the line as well as the integers continue up to infinity on both the positive and the negative sides.

7.2 NUMBER LINES FOR NATURAL NUMBERS, WHOLE NUMBERS AND INTEGERS

1. Natural Numbers :

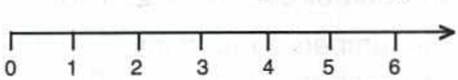
A number line starting from 1 (one) and marked 2, 3, 4, 5, ... at equal distances on the right hand side of 1 is called ***a number line representing the natural numbers*** (as shown below) :



The arrow-head on the right side shows that the natural numbers continue up to infinity.

2. Whole Numbers :

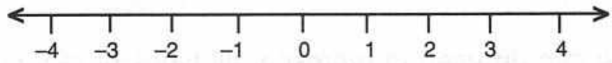
A number line starting from 0 (zero) and marked 1, 2, 3, 4, ... at equal distances on the right hand side of 0 is called ***a number line representing the whole numbers*** (as shown below) :



The arrow-head on the right side shows that the whole numbers continue up to infinity.

3. Integers :

Since integers = { ..., – 3, – 2, – 1, 0, 1, 2, 3, 4, ... }, a ***number line with zero* (0)**, marked any where on it, with positive numbers **1, 2, 3, ...** marked ***on the right hand side of* 0** at equal distances and negative numbers **–1, – 2, –3, ...** marked ***on the left hand side of* 0** (zero) at the same equal distances, is said to represent **integers** (as shown below) :



Arrow-heads on the two sides show that the integers continue up to infinity on the positive side as well as on the negative side.

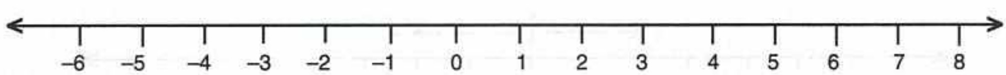
1. Integers = {........, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...}
2. Natural numbers = {1, 2, 3, 4, 5, 6, ...}
3. Whole numbers = {0, 1, 2, 3, 4, 5, ...}
4. Whether the number line is drawn for integers, natural numbers or whole numbers, the distance between any two consecutive numbers is always the same.

7.3 USING A NUMBER LINE TO COMPARE NUMBERS

Out of any two numbers, marked on a number line, *the **number*** which is

(i) ***to the right** is **greater***

(ii) ***to the left** is **smaller***.



Considering the number line drawn above :

(i) 6 is **greater** than 2 because it is **to the right** of 2

(ii) – 2 is **greater** than – 5 because it is **to the right** of – 5

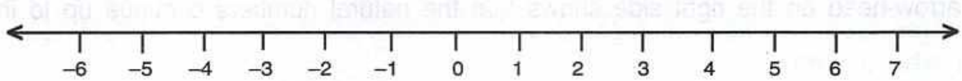
(iii) 3 is **greater** than – 1 because it is **to the right** of – 1

(iv) – 6 is **smaller** than – 2 because it is **to the left** of – 2

(v) – 4 is **smaller** than 1 because it is **to the left** of 1 and so on.

Thus, each number on a number line is always greater than each and every number to its left. Similarly, *each number on a number line is always smaller than each and every number to its right.*

For the following number line :



(i) 7 is greater than all numbers to its left
i.e. 7 is greater than each of 6, 5, 4, 3, 2, 1, 0, –1, –2, –3, *etc.*

(ii) –6 is smaller than all numbers to its right
i.e. –6 is smaller than each of –5, –4, –3, –2, –1, 0, 1, 2, *etc.*

Also, (i) *Every positive number is greater than every negative number.*

(ii) *Zero is smaller than every positive number but greater than every negative number.*

(iii) *The greater the number, the smaller is its opposite.*
viz. 8 is greater than 5 but – 8 is less than – 5
Similarly, $-9 > -15 \Rightarrow 9 < 15$ *and so on*

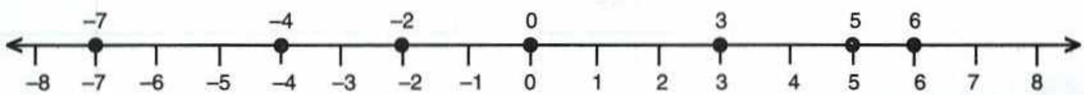
(iv) *The smaller the number, the greater is its opposite.*
viz. 6 is smaller than 7 but – 6 is greater than – 7
Similarly, $-8 < -5 \Rightarrow 8 > 5$ *and so on.*

Example 1 :

Using a number line, write the following numbers (integers) in ascending order of value : 3, –2, 5, 0, –7, 6 and –4.

Solution :

Draw a suitable number line and mark on it the given numbers, as shown below:



Since **ascending** order means **smaller to greater.**

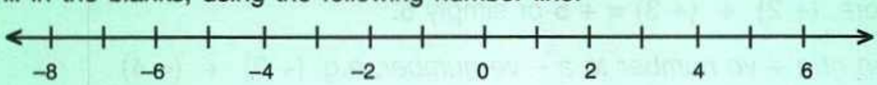
∴ The given **numbers in ascending order**
= **–7, –4, –2, 0, 3, 5 and 6** **(Ans.)**

Symbol '<' means 'is less than' and symbol '>' means 'is greater than.'

∴ Answer to Example 1, given above, can also be written as :
$-7 < -4 < -2 < 0 < 3 < 5 < 6$.
If required, the same numbers in descending (decreasing) order will be written as :
6, 5, 3, 0, –2, –4 and –7
or, $6 > 5 > 3 > 0 > -2 > -4 > -7$

EXERCISE 7(A)

1. Fill in the blanks, using the following number line:



(i) An integer, on the given number line, is .... than every number on its left.
(ii) An integer on the given number line is greater than every number to its ....
(iii) 2 is greater than –4 implies 2 is to the .... of –4.
(iv) –3 is .... than 2 and 3 is .... than –2.
(v) –4 is .... than –8 and 4 is .... than 8.
(vi) 5 is .... than 2 and –5 is .... than –2.
(vii) –6 is .... than 3 and the opposite of – 6 is .... than opposite of 3.
(viii) 8 is .... than –5 and –8 is .... than 5.

2. In each of the following pairs, state ***which*** integer ***is greater*** :
(i) – 15, –23 (ii) –12, 15 (iii) 0, 8 (iv) 0, – 3

3. In each of the following pairs, state ***which*** integer ***is smaller*** :
(i) 0, – 6 (ii) 2, –3 (iii) 15, –51 (iv) 13, 0

4. In each of the following pairs, replace * with < or > to make the statement true :
   (i) 3 * 0 (ii) 0 * – 8 (iii) –9 * –3 (iv) –3 * 3
   (v) 5 * –1 (vi) –13 * 0 (vii) –8 * –18

5. In each case, **arrange** the given integers **in ascending order**, using a number line :
   (i) –8, 0, –5, 5, 4, –1 (ii) 3, –3, 4, –7, 0, –6, 2

6. In each case, **arrange** the given integers **in descending order**, using a number line :
   (i) –5, –3, 8, 15, 0, –2 (ii) 12, 23, –11, 0, 7, 6

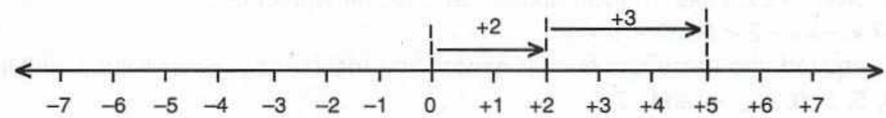
7. For each of the statements given below, state whether it is **true** or **false** :
   (i) The smallest integer is 0. (ii) The opposite of –17 is 17.
   (iii) The opposite of zero is zero. (iv) Every negative integer is smaller than 0.
   (v) 0 is greater than every positive integer.
   (vi) Since zero is neither negative nor positive, it is not an integer.

7.4 USING A NUMBER LINE

A number line can be used for addition and subtraction of numbers.

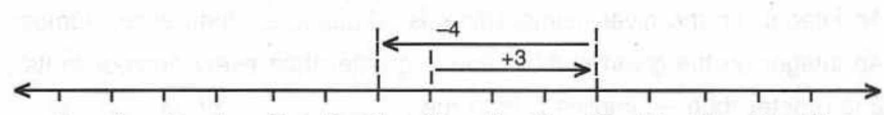
A. *For addition of numbers :*

1. *Addition of a + ve number to a + ve number,* e.g. (+ 2) + (+ 3)



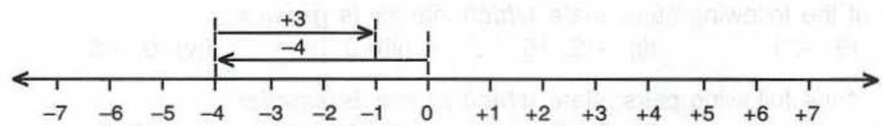
First of all, for + 2, count 2 units to the right of zero (because the right side is for the positive sign). Then for + 3, move three units to the right of + 2. You reach + 5. Therefore, **(+ 2) + (+ 3) = + 5** or simply **5.**

2. *Addition of a + ve number to a – ve number,* e.g. (+ 3) + (– 4)



For + 3, move 3 units to the right of zero, and then, for – 4, move 4 units to the left of +3. You reach – 1.
Therefore, **(+ 3) + (– 4) = – 1.**

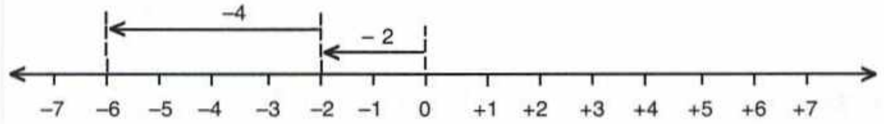
3. *Addition of a – ve number to a + ve number,* e.g. (– 4) + (+ 3)



For – 4, move 4 units to the left of zero, and then, for + 3, move 3 units to the right of – 4. You reach – 1.

Therefore, **(– 4) + (+ 3) = – 1.**

4. *Addition of a – ve number to a – ve number,* e.g. (– 2) + (– 4)



For – 2, start from zero and move two units to the left, and then, for – 4, move 4 units to the left of – 2. You reach – 6.

Therefore, **(– 2) + (– 4) = – 6.**

B. *For subtraction of numbers :*

To subtract a given number from another number :

1. *Mark the two given numbers on the same number line,* each starting from zero.
2. *Find out how many steps are needed to reach the position of the first number, from the position of the second number,* (*i.e.* the one which is to be subtracted).

   This number of steps is the required answer.

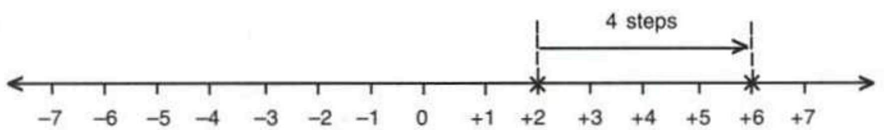
If the number of steps moved is ***towards the right,*** *the answer is a* ***positive number,*** *and if the number of steps moved is* ***towards the left,*** *the answer is a* ***negative number.***

Example 2 :

Using a number line, evaluate : (+ 6) – (+ 2).

Solution :

Mark the positions of the numbers + 6 and + 2 on the same number line.



Now count how many steps are needed from the position of number + 2 to reach the position of number + 6. We find that 4 steps are needed towards right.

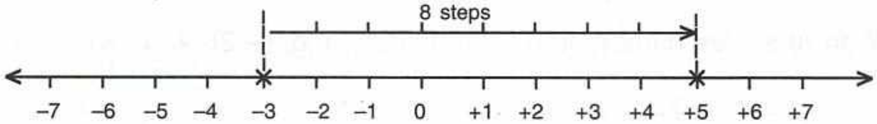
∴ **(+ 6) – (+ 2) = + 4** **(Ans.)**

Example 3 :

Using a number line, evaluate : (+5) – (–3).

Solution :

Mark the positions of + 5 and – 3 on the same number line.



Now, starting from the position of – 3, count the number of steps needed to reach + 5. Also, see the direction. We find that we have to move 8 steps to the right.

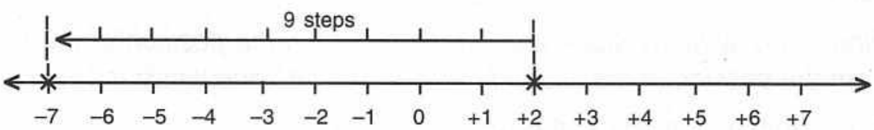
∴ **(+5) – (–3) = + 8** **(Ans.)**

Example 4 :

Using a number line, evaluate : (–7) – (+2).

Solution :

After marking the positions of – 7 and + 2 on the same number line, count from the position of + 2, both the number of steps and the direction needed to reach – 7.



We find that we have to move 9 steps to the left.

∴ **(–7) – (+2) = –9** **(Ans.)**

Example 5 :

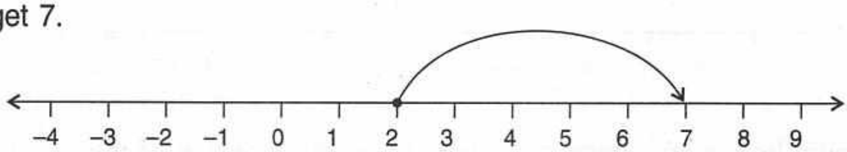
Using a number line, find the integer which is :

(i) 5 more than 2 (ii) 3 less than 4

(iii) 6 more than –8 (iv) 3 less than –4

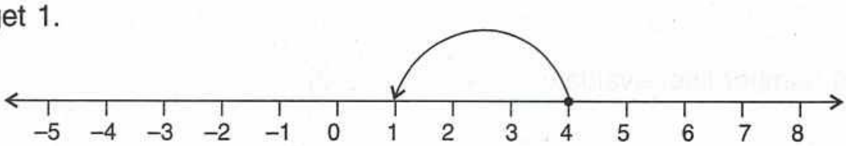
Solution :

(i) To get 5 more than 2, start from 2 and then move 5 units to the right of 2 to get 7.



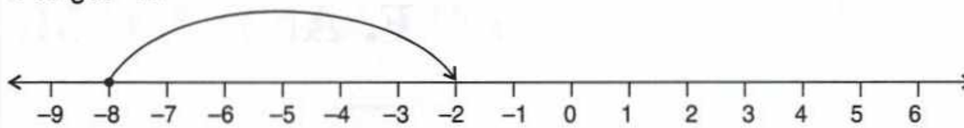
∴ **5 more than 2 is 7** **(Ans.)**

(ii) To get 3 less than 4, start from 4 and then move 3 units to the left of 4 to get 1.



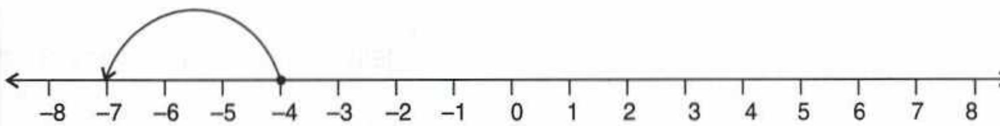
∴ **3 less than 4 is 1** **(Ans.)**

(iii) To get 6 more than –8, start from –8 and then move 6 units to the right of –8 to get –2.



∴ **6 more than –8 is –2** **(Ans.)**

(iv) To get 3 less than –4, start from –4 and then move 3 units to the left of –4 to get –7.



∴ **3 less than –4 is –7** **(Ans.)**

EXERCISE 7(B)

Use a number line to evaluate each of the following :

1. (i) (+7) + (+4) (ii) 0 + (+6) (iii) (+5) + 0
2. (i) (–4) + (+5) (ii) 0 + (–2) (iii) (–1) + (+4)
3. (i) (+4) + (–2) (ii) (+3) + (–6) (iii) 3 + (–7)
4. (i) (–1) + (–2) (ii) (–3) + (–4) (iii) (–2) + (–5)
5. (i) (+10) – (+2) (ii) (+8) – (–5) (iii) (–6) – (+2)
   (iv) (–7) – (+5) (v) (+4) – (–2) (vi) (–8) – (–4)
6. Using a number line, find the integer which is :
   (i) 3 more than –1 (ii) 5 less than 2
   (iii) 5 more than –9 (iv) 4 less than –4
   (v) 7 more than 0 (vi) 7 less than –8

H.C.F. AND L.C.M. 8

8.1 INTRODUCTION

HCF *i.e.* highest common factor of two or more numbers is the greatest number that divides each given number completely.

For example :

H.C.F. of 8 and 12 is 4 as 4 is the largest number that divides 8 and 12 both completely.

L.C.M. *i.e.* lowest common multiple of two or more numbers is the smallest number which is divisible by each given number.

For example :

L.C.M. of 8 and 12 is 24 as 24 is divisible completely by 8 and 12 both.

8.2 FACTORS

When two or more natural numbers are multiplied together, the result is referred to as their **product**, and each of the numbers multiplied is called a **factor** of this product.

For example :

(i) Product of 5 and 7 = 5 × 7 = 35; therefore 5 and 7 are factors of 35.

(ii) Product of 2, 3 and 7 = 2 × 3 × 7 = 42; therefore each of 2, 3 and 7 is a factor of 42.

In other words :

Any natural number that divides a given natural number completely is called a factor of the given number.

For example :

(i) 5 divides 20 completely ⇒ 5 is a factor of 20

(ii) 6 divides 12 completely ⇒ 6 is a factor of 12

(iii) 15 divides 30 completely ⇒ 15 is a factor of 30 and so on.

Now consider the following examples :

(i) 24 = 1 × 24 ⇒ 1 and 24 are factors of 24.

(ii) 24 = 2 × 12 ⇒ 2 and 12 are factors of 24.

(iii) 24 = 3 × 8 ⇒ 3 and 8 are factors of 24.

(iv) 24 = 4 × 6 ⇒ 4 and 6 are factors of 24.

Combining, we get :

1, 2, 3, 4, 6, 8, 12 and 24 are factors of 24.

Each of 1, 2, 3, 4, 6, 8, 12 and 24 divides 24 completely.

∴ Factors of 24 = F_{24}

= 1, 2, 3, 4, 6, 8, 12 and 24

In the same way,

(i) F_{30} = Factors of 30
= 1, 2, 3, 5, 6, 10, 15 and 30

(ii) F_{18} = 1, 2, 3, 6, 9 and 18

(iii) F_{45} = 1, 3, 5, 9, 15 and 45 and so on.

Factors of 6
= Each natural number that divides 6 completely
= 1, 2, 3 and 6

- 1 (one) is a factor of every number.
- Every number is a factor of itself.
- Zero (0) can not be a factor of any number.

Example 1 :

Write the factors of (i) 13 (ii) 25 (iii) 28

Solution :

(i) **Factors of 13** = F_{13} = **1 and 13** **(Ans.)**

(ii) F_{25} = **1, 5 and 25** **(Ans.)**

(iii) F_{28} = **1, 2, 4, 7, 14 and 28** **(Ans.)**

8.3 PRIME NUMBERS

*A natural number that is divisible only by 1 (one) and itself is called a **prime number**.*

For example :

(i) 2 is divisible only by 1 (one) and itself; therefore **2 is a prime number**.

(ii) 3 is divisible only by 1 and itself; therefore **3 is a prime number**.

(iii) 4 is divisible by 1 and itself as well as by 2; so **4 is not a prime number**.

In the same way; we find :

(iv) each of 5, 7, 11, 13, 17, ... is a prime number.

(v) none of 1, 4, 6, 8, 9, 10, 12, ... is a prime number.

1. *Every prime number is greater than 1 (one).*
2. *Two (2) is the smallest prime number.*
3. *All prime numbers except 2 are odd numbers.*

To be more clear, note :

If a natural number has **only two factors**, it is a **prime number**.

⇒ (i) **1** (one) is **not a prime number** as it has only **one factor**, which is one (1) itself.

(ii) 7 is a **prime number** as it has only **two factors** *i.e.*, 1 and 7.

(iii) **10** is **not a prime number** as it has **more than two** factors *i.e.*, 1, 2, 5 and 10.

Every natural number that has more than two factors is called a **composite number**.

Since, factors of 10 are 1, 2, 5 and 10

∴ **10 is a composite number**

Similarly, each of 4, 6, 8, 9, 10, 12, 14, ..., 24, 25, 26, 27, etc. is a composite number.

Every even number greater than 2 is a composite number.

8.4 PRIME NUMBERS FROM 1 TO 100

ERATOSTHENES, a Greek scholar, used the following method to distinguish the prime numbers from among the natural numbers. For this reason, this method is known as the **Sieve of Eratosthenes**.

Step 1 : Write the natural numbers 1 to 100 in rows of 10 as shown below :

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Step 2 : 1 (one) is not a prime number; strike it out.

Step 3 : The first, *i.e.* the smallest, prime number is 2; encircle 2 and strike out all other numbers which are divisible by 2.

Step 4 : The next prime number is 3, encircle 3 and strike out all other numbers that are divisible by 3.

(Some of these numbers are already cut as they are divisible by 2 as well).

Step 5 : The next prime number is 5, encircle 5 and strike out all other numbers that are divisible by 5.

(Some of these numbers are already cut as they are divisible by 2 and/or 3).

Step 6 : Adopt the same steps for prime numbers 7, 11, 13 and so on.

Finally, some numbers emerge encircled, and they are the prime numbers between 1 and 100.

8.5 PRIME FACTORS

Let us take a number, say, 24. The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24. Out of these factors, 2 and 3 are prime numbers and so the factors **2 and 3** are called the **prime factors of 24**.

Thus, factors of 24 = 1, 2, 3, 4, 6, 8, 12 and 24

$\Rightarrow$ Prime factors of 24 = 2 and 3

In other words, we write :

F_{24} = 1, 2, 3, 4, 6, 8, 12 and 24; and $P.F_{24}$ = 2 and 3

In the same way :

(i) F_{50} = 1, 2, 5, 10, 25 and 50 $\Rightarrow$ $P.F_{50}$ = 2 and 5

(ii) F_{64} = 1, 2, 4, 8, 16, 32 and 64 $\Rightarrow$ $P.F_{64}$ = 2 and so on.

Note : Every number can be written as the product of its prime factors.

e.g. (i) prime factors of 12 are 2 and 3, and $12 = 2 \times 2 \times 3$

(ii) prime factors of 54 are 2 and 3, and $54 = 2 \times 3 \times 3 \times 3$ and so on.

EXERCISE 8(A)

1. Write all the factors of :
   (i) 15 (ii) 55 (iii) 48 (iv) 36 (v) 84
2. Write all prime numbers :
   (i) less than 25 (ii) between 15 and 35 (iii) between 8 and 76
3. Write the prime numbers from :
   (i) 5 to 45 (ii) 2 to 32 (iii) 8 to 48 (iv) 9 to 59
4. Write the prime factors of :
   (i) 16 (ii) 27 (iii) 35 (iv) 49
5. If P_n means prime factors of n, find :
   (i) P_6 (ii) P_{24} (iii) P_{50} (iv) P_{42}

8.6 HIGHEST COMMON FACTOR

H.C.F. stands for **Highest Common Factor**, and the H.C.F. of two or more given numbers is the greatest number that completely divides each of the given numbers.

For example :

(i) The greatest number that can divide both 18 and 24 completely is 6; therefore, H.C.F. of 18 and 24 = 6.

(ii) H.C.F. of numbers 16, 24 and 32 is 8; this is because 8 is the greatest number that divides each of the given numbers 16, 24 and 32 completely.

8.7 METHODS OF FINDING H.C.F.

For finding the H.C.F. of two or more given numbers, any of the following three methods can by used :

1. *Common Factor Method*
2. *Prime Factor Method*
3. *Division Method*

8.8 COMMON FACTOR METHOD

1. Find all the possible factors of each given number.
2. From the factors obtained in Step 1, select the common factors.
3. Out of the common factors, obtained in Step 2, take the highest factor, which is the **Highest Common Factor** (H.C.F.) of the given numbers.

Example 2 :

Using the common factor method, find the H.C.F. of 36 and 48.

Solution :

Step 1 : Factors of 36, *i.e.* F_{36} = 1, 2, 3, 4, 6, 9, 12, 18 and 36

Similarly, F_{48} = 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48

Step 2 : Factors that are common to F_{36} and F_{48}

= 1, 2, 3, 4, 6 and 12

Step 3 : From the result of Step 2, the highest common factor = 12

$\Rightarrow$ **H.C.F.** of the given numbers

36 and 48 = **12** **(Ans.)**

12 is the largest number that divides both 36 and 48 completely.

Example 3 :

Using the common factor method, find the H.C.F. of :

(i) 18, 27 and 36 (ii) 16, 32 and 49

Solution :

(i) F_{18} = 1, 2, 3, 6, 9 and 18

F_{27} = 1, 3, 9 and 27

and F_{36} = 1, 2, 3, 4, 6, 9, 12, 18 and 36

$\therefore$ Common factors = 1, 3 and 9

$\Rightarrow$ Required **H.C.F. = 9** **(Ans.)**

(ii) F_{16} = 1, 2, 4, 8 and 16

F_{32} = 1, 2, 4, 8, 16 and 32

and F_{49} = 1, 7 and 49

$\therefore$ Common factor = 1

$\Rightarrow$ Required **H.C.F. = 1** **(Ans.)**

1 is the greatest number that divides each of 16, 32 and 49 completely.

8.9 PRIME FACTOR METHOD

Steps :

1. Split each given number into its prime factors.
2. Select the common prime factors.
3. Multiply the prime factors obtained in Step 2.

The product so obtained is the H.C.F. of the given numbers.

Example 4 :

Find the H.C.F. of 15 and 25.

5 and 3 are primes

Solution :

Prime factors of 15 are 5 and 3, since $15 = 5 \times 3$

Prime factors of 25 are 5 and 5, since $25 = 5 \times 5$

Since the common prime factor is 5 only,

$\therefore$ **H.C.F.** of 15 and 25 = **5** **(Ans.)**

Example 5 :

Find the H.C.F. of 24, 12, 36 and 60.

Solution :

Since $24 = 2 \times 2 \times 2 \times 3$; $12 = 2 \times 2 \times 3$;

$36 = 2 \times 2 \times 3 \times 3$ and $60 = 2 \times 2 \times 3 \times 5$.

The prime factors common to the given numbers are 2, 2 and 3.

$\therefore$ **H.C.F.** $= 2 \times 2 \times 3 =$ **12** **(Ans.)**

1. Any two numbers that do not have a common prime factor are called **co-prime numbers**.
   e.g. (i) 39 *and* 175 (ii) 15 *and* 16 (iii) 27 *and* 64 and so on

Reason :

$39 = 3 \times 13$ *and* $175 = 5 \times 5 \times 7$

$\Rightarrow$ *39 and 175 have no common factor.*

$\therefore$ *39 and 175 are co-prime numbers.*

2. ***The H.C.F. of two co-prime numbers is always 1.***
   Thus, H.C.F. of 15 and 16 = 1; H.C.F. of 27 and 64 = 1 and so on

8.10 DIVISION METHOD

Steps :

1. Divide the greater number by the smaller number.
2. By the remainder of division in Step 1, divide the smaller number.
3. By the remainder in Step 2, divide the remainder obtained in Step 1.
4. Continue in the same way till no remainder is left.

The last divisor is the required H.C.F.

Example 6 :

Find H.C.F. of 36 and 60.

Solution : **Step 1** 36) 60 (1 [Dividing the bigger number by the smaller one]
36
Step 2 24) 36 (1 [Dividing the smaller number by the remainder of step 1]
24
Step 3 12) 24 (2 [Continuing in a similar way]
24
×

Since the last divisor is 12, ∴ **H.C.F. = 12** **(Ans.)**

Example 7 :

Find the H.C.F. of 18, 24 and 32.

Solution :

> To find the H.C.F. of more than two numbers :
> (i) first find the H.C.F. of any two of the given numbers, then
> (ii) find the H.C.F. of the third given number and the H.C.F. obtained in (i).

Let us first find the H.C.F. of 18 and 24.

18) 24 (1
18
6) 18 (3
18
×

H.C.F. of 18 and 24 = 6

Since the third number is 32 and the H.C.F. obtained above is 6, find the H.C.F. of 32 and 6.

6) 32 (5
30
2) 6 (3
6
×

The H.C.F. of 32 and 6 is 2.

∴ **H.C.F.** of given numbers 18, 24 and 32 = **2** **(Ans.)**

Similarly, in order to find the H.C.F. of four numbers :

1. First of all find the H.C.F. of any three of the given four numbers.
2. Then find the H.C.F. of the fourth number and the H.C.F. obtained in Step 1.

EXERCISE 8(B)

1. Using the ***common factor method***, find the H.C.F. of :
   (i) 16 and 35 (ii) 25 and 20 (iii) 27 and 75
   (iv) 8, 12 and 18 (v) 24, 36, 45 and 60
2. Using the ***prime factor method***, find the H.C.F. of :
   (i) 5 and 8 (ii) 24 and 49 (iii) 40, 60 and 80
   (iv) 48, 84 and 88 (v) 12, 16 and 28

3. Using the ***division method***, find the H.C.F. of the following :

(i) 16 and 24 (ii) 18 and 30 (iii) 7, 14 and 24

(iv) 70, 80, 120 and 150 (v) 32, 56 and 46

4. Use a method of your own choice to find the H.C.F. of :

(i) 45, 75 and 135 (ii) 48, 36 and 96 (iii) 66, 33 and 132

(iv) 24, 36, 60 and 132 (v) 30, 60, 90 and 105

5. Find the greatest number that divides each of 180, 225 and 315 completely.
6. Show that 45 and 56 are co-prime numbers.
7. Out of 15, 16, 21 and 28, find out all the pairs of co-prime numbers.
8. Find the greatest number that will divide 93, 111 and 129, leaving remainder 3 in each case.

Since, 93 − 3 = 90, 111 − 3 = 108 and 129 − 3 = 126.

∴ Required number is H.C.F. of 90, 108 and 126.

8.11 MULTIPLES

Since $5 \times 1 = 5$, $5 \times 2 = 10$, $5 \times 3 = 15$

$5 \times 4 = 20$, $5 \times 5 = 25$, etc.

therefore, 5, 10, 15, 20, 25, etc. are multiples of 5.

i.e. Multiples of 5 = 5, 10, 15, 20, 25, ...

i.e. M_5 = 5, 10, 15, 20, 25, 30, ...

In the same way :

(i) Multiples of 3 = 3, 6, 9, 12, 15, 18, ...

i.e. M_3 = 3, 6, 9, 12, 15, 18, ...

$3 \times 1 = 3$, $3 \times 2 = 6$, $3 \times 3 = 9$, $3 \times 4 = 12$, $3 \times 5 = 15$ and so on

(ii) M_7 = Multiples of 7 = 7, 14, 21, 28, 35, ...

(iii) M_8 = Multiples of 8 = 8, 16, 24, 32, 40, ...

8.12 LOWEST COMMON MULTIPLE

L.C.M. stands for **Lowest Common Multiple**. The L.C.M. of two or more given numbers is the lowest (smallest) number that is a multiple of each of the given numbers. Thus, ***it is the smallest number which is exactly divisible by each of the given numbers.***

For example :

Multiples of 15 = 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, ...

Multiples of 25 = 25, 50, 75, 100, 125, 150, 175, ...

Common multiples of 15 and 25 = 75, 150, ...

Lowest common multiple of 15 and 25 = 75

∴ **L.C.M.** of 15 and 25 = **75**

If checked carefully, you will find that 75 is the smallest number which is exactly divisible by both 15 and 25.

8.13 METHODS OF FINDING L.C.M.

The following three methods are most commonly used to find the L.C.M. :

1. *Common Multiple Method* 2. *Prime Factor Method* 3. *Common Division Method*

8.14 COMMON MULTIPLE METHOD

Steps :

1. Find a few multiples of each given number.
2. From the multiples obtained in Step 1, select the common ones.
3. The lowest number (multiple) obtained in Step 2 is the required **lowest common multiple** (L.C.M.) of the given numbers.

Example 8 :

Using common multiple method, find the L.C.M. of :

(i) 4, 5 and 10 (ii) 12, 15 and 20

Solution :

(i) M_4 = Multiples of 4

= 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...

M_5 = Multiples of 5

= 5, 10, 15, 20, 25, 30, 35, 40, 45, ...

and, M_{10} = Multiples of 10

= 10, 20, 30, 40, 50, 60, ...

20 is the smallest number divisible by each of 4, 5 and 10.

⇒ Common multiples of 4, 5 and 10 = 20, 40, ...

⇒ **L.C.M.** of 4, 5 and 10 = **20** **(Ans.)**

(ii) ∵ M_{12} = 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, ...

M_{15} = 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, ...

and, M_{20} = 20, 40, 60, 80, 100, 120, 140, ...

⇒ Common multiples of 12, 15 and 20 = 60, 120, ...

⇒ **Required L.C.M. = 60** **(Ans.)**

8.15 PRIME FACTOR METHOD

Example 9 :

Use prime factor method to find L.C.M. of 18, 24 and 36.

Solution :

Step 1 :

Express each of the given numbers as a product of its prime factors and then in index form.

Clearly, $18 = 2 \times 3 \times 3$

$= 2^1 \times 3^2$ [Index form]

$24 = 2 \times 2 \times 2 \times 3$

$= 2^3 \times 3^1$ [Index form]

and, $36 = 2 \times 2 \times 3 \times 3$

$= 2^2 \times 3^2$ [Index form]

Step 2 :

L.C.M. = Product of all the prime factors obtained with highest power of each.

Since, the prime factors 2 and 3, obtained above, with highest power are 2^3 and 3^2 respectively.

$\therefore$ **Required L.C.M.** = $2^3 \times 3^2$

$= 2 \times 2 \times 2 \times 3 \times 3 =$ **72** **(Ans.)**

8.16 COMMON DIVISION METHOD

Example 10 :

Find the L.C.M. of 16, 20 and 24.

Solution :

Steps :

1. Write all the given numbers in a horizontal line, separating them by commas.

	16, 20, 24

2. Divide by a suitable number, that exactly divides at least two of the given numbers. And, write down the quotients and the undivided numbers obtained, below the first line.

2	16, 20, 24
	8, 10, 12

3. Repeat the process until we get a line of numbers that are prime to one-another.
4. The product of all the divisors and the numbers obtained in the last line will be the required L.C.M.

2	16, 20, 24
2	8, 10, 12
2	4, 5, 6
	2, 5, 3

$\therefore$ **Required L.C.M.** = $2 \times 2 \times 2 \times 2 \times 5 \times 3 =$ **240** **(Ans.)**

Example 11 :

Find the smallest number which, when divided by 8, 12, 16, 24 and 36, leaves no remainder.

Solution :

The smallest number that is exactly divisible by each of the given numbers is their L.C.M.

$\therefore$ Required number = L.C.M. of 8, 12, 16, 24 and 36.

Steps 1, 2 and 3 :

2	8, 12, 16, 24, 36
2	4, 6, 8, 12, 18
2	2, 3, 4, 6, 9,
3	1, 3, 2, 3, 9
	1, 1, 2, 1, 3

Step 4 :

L.C.M. $= 2 \times 2 \times 2 \times 3 \times 2 \times 3$

$= 144$

$\therefore$ **Req. no. = 144** **(Ans.)**

Example 12 :

Find the smallest number which, when :

(i) decreased by 1 (ii) increased by 3

is exactly divisible by the numbers 21, 45, 63, 81 and 210.

Solution :

First find the L.C.M. of the given numbers.

$\therefore$

3	21, 45, 63, 81, 210
3	7, 15, 21, 27, 70
5	7, 5, 7, 9, 70
7	7, 1, 7, 9, 14
	1, 1, 1, 9, 2

5670 is exactly divisible by each of 21, 45, 63, 81 and 210

$\therefore$ L.C.M. $= 3 \times 3 \times 5 \times 7 \times 9 \times 2 = 5670$

(i) **The required number** $= 5670 + 1 =$ **5671** **(Ans.)**

[**Reason** : On decreasing 5671 by 1, we get $5671 - 1 = 5670$; which is the L.C.M. of the given numbers, *i.e.* exactly divisible by each of them]

(ii) **The required number** $= 5670 - 3 =$ **5667** **(Ans.)**

An Important Result : *For any two numbers :*

The product of their L.C.M. and H.C.F. = The product of the numbers.

Consider the numbers 48 and 60.

Their H.C.F. = 12 and their L.C.M. = 240

$\therefore$ The **product of** their **H.C.F. and L.C.M.** $= 12 \times 240 =$ **2880**

Also, the **product of** the **given numbers** $= 48 \times 60 =$ **2880**

$\therefore$ H.C.F. $\times$ L.C.M. of any two numbers = Product of the two numbers.

$\Rightarrow$ (i) L.C.M. of two numbers $= \dfrac{\text{Their product}}{\text{Their H.C.F.}}$

(ii) H.C.F. of two numbers $= \dfrac{\text{Their product}}{\text{Their L.C.M.}}$

and (iii) $\dfrac{\text{Product of L.C.M. and H.C.F.}}{\text{One number}} =$ The other number

EXERCISE8(C)

1. Using the ***common multiple method***, find the L.C.M. of the following :
   (i) 8, 12, and 24 (ii) 10, 15 and 20 (iii) 3, 6, 9 and 12
2. Find the L.C.M. of each of the following groups of numbers, using (i) **the *prime factor method*** and (ii) **the *common division method* :**
   (i) 18, 24 and 96 (ii) 100, 150 and 200 (iii) 14, 21 and 98
   (iv) 22, 121 and 33 (v) 34, 85 and 51
3. The H.C.F. and the L.C.M. of two numbers are 50 and 300 respectively. If one of the numbers is 150, find the other one.
4. The product of two numbers is 432 and their L.C.M. is 72. Find their H.C.F.
5. The product of two numbers is 19,200 and their H.C.F. is 40. Find their L.C.M.
6. Find the smallest number which, when divided by 12, 15, 18, 24 and 36 leaves no remainder.
7. Find the smallest number which, when increased by one is exactly divisible by 12, 18, 24, 32 and 40.
8. Find the smallest number which, on being decreased by 3, is completely divisible by 18, 36, 32 and 27.

Revision Exercise (Chapter 8)

1. Find the H.C.F. of : (i) 108, 288 and 420 (ii) 36, 54 and 138
2. Find the L.C.M. of : (i) 72, 80 and 252 (ii) 48, 66 and 120
3. State **true** or **false** (Give an example in support of your answer in each case) :
   (i) H.C.F. of two prime numbers is 1.
   (ii) H.C.F. of two co-prime numbers is 1.
   (iii) L.C.M. of two prime numbers is equal to their product.
   (iv) L.C.M. of two co-prime numbers is equal to their product.
4. The product of two numbers is 12096 and their H.C.F. is 36. Find their L.C.M.
5. The product of the H.C.F. and the L.C.M. of two numbers is 1152. If one number is 48, find the other one.
6. (i) Find the smallest number that is completely divisible by 28 and 42.
   (ii) Find the largest number that can divide 28 and 42 completely.
7. Find the L.C.M. of 140 and 168. Use the L.C.M. obtained to find the H.C.F. of the given numbers.
8. Find the H.C.F. of 108 and 450 and use the H.C.F. obtained to find the L.C.M. of the given numbers.

PLAYING WITH NUMBERS 9

9.1 SIMPLIFICATION OF BRACKETS

Brackets are grouping symbols whcih are used to indicate a part of an expression which should be evaluated first.

- If an expression contains one set of brackets, evaluate that part first.
- If an expression contains two or more sets of brackets one inside the other, evaluate the innermost set first.

Example 1 :

(a) $24 - (6 + 8) - 3$

(b) $(90 \div 15) \div 3$

(c) $30 \div (15 \div 3)$

(d) $[12 + (9 \div 3)] - 4^2$

(e) $[21 - (4 + 5) + 2] \times 2$

Solution :

(a) $24 - (6 + 8) - 3$

$= 24 - 14 - 3$ [Evaluating the brackets first]

$= 10 - 3 = 7$ **(Ans.)**

(b) $(90 \div 15) \div 3$

$= \left(\frac{90}{15}\right) \div 3$

$= 6 \div 3 = 2$ **(Ans.)**

(c) $30 \div (15 \div 3)$

$= 30 \div \left(\frac{15}{3}\right)$

$= 30 \div 5 = \frac{30}{5} = 6$ **(Ans.)**

(d) $[12 + (9 \div 3)] - 4^2$

$= \left[12 + \left(\frac{9}{3}\right)\right] - 16$ [Evaluating the inner bracket first]

$= [12 + 3] - 16$

$= 15 - 16$ [Evaluating the outer bracket]

$= -1$ **(Ans.)**

(e) $[21 - (4 + 5) + 2] \times 2$

$= [21 - 9 + 2] \times 2$

$= [12 + 2] \times 2$

$= 14 \times 2 = 28$ **(Ans.)**

There are four kinds of brackets :

1. Bar or Vinculum —
2. Small brackets or Parenthesis ( )
3. Curly (middle) brackets { }
4. Square brackets or big brackets [ ]

The order of simplifying these brackets is :

1. Bar — 2. Parenthesis ( )
3. Curly brackets { } and 4. Square brackets [ ]

Example 2 :

Simplify : 28 – [19 – {14 – (10 – 2)}].

Solution :

28 – [19 – {14 – (10 – 2)}]

= 28 – [19 – {14 – 8}] [Simplifying small brackets *i.e.* ( )]

= 28 – [19 – 6] [Simplifying curly brackets *i.e.* { }]

= 28 – 13 [Simplifying square brackets *i.e.* [ ]]

= 15 **(Ans.)**

Example 3 :

Simplify : $14 - [7 - \{8 + 28 \div (3 - \overline{4 - 3})\}]$.

Solution :

$14 - [7 - \{8 + 28 \div (3 - \overline{4 - 3})\}]$

= 14 – [7 – {8 + 28 ÷ (3 – 1)}] [Simplifying bar bracket]

= 14 – [7 – {8 + 28 ÷ 2}] [Simplifying ( )]

= 14 – [7 – {8 + 14}] [28 ÷ 2 = 28/2 = 14]

= 14 – [7 – 22] [Simplifying { }]

= 14 + 15 [∵ – [7 – 22] = –7 + 22 = 15]

= 29 **(Ans.)**

EXERCISE9(A)

For simplifying a given expression involving various operations, the rule of BODMAS is used, where
B = Barackets, **O** = Of, **D** = Division, **M** = Multiplication,
A = Addition and **S** = Subtraction

1. 19 – (1 + 5) – 3

2. 30 × 6 ÷ (5 – 2)

3. 28 – (3 × 8) ÷ 6

4. 9 – [(4 – 3) + 2 × 5]

5. [18 – (15 ÷ 5) + 6]

6. [(4 × 2) – (4 ÷ 2)] + 8

7. 48 + 96 ÷ 24 – 6 × 18

8. 22 – [3 – {8 – (4 + 6)}]

9. $34 - [29 - \{30 + 66 \div (24 - \overline{28 - 26})\}]$

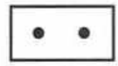
10. 60 – {16 ÷ (4 × 6 – 8)}

11. $25 - [12 - \{5 + 18 \div (4 - \overline{5 - 3})\}]$

12. 15 – [16 – {12 + 21 ÷ (9 – 2)}]

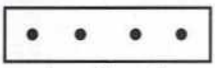
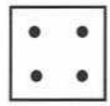
9.2 TO FIND DIVISORS

1. The number of ways in which 2 coins can be arranged to form a rectangle :

=  or 

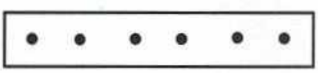
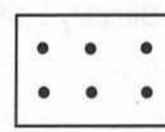
$1 \times 2 = 2$ $\quad$ $2 \times 1 = 2$

2. The number of ways in which 4 coins can be arranged to form a rectangle :

=  or  or 

$1 \times 4 = 4$ $\quad$ $2 \times 2 = 4$ $\quad$ $4 \times 1 = 4$

3. The number of ways in which 6 coins can be arranged to form a rectangle :

=  or  or  or 

$1 \times 6 = 6$ $\quad$ $3 \times 2 = 6$ $\quad$ $6 \times 1 = 6$ $\quad$ 2×3

In $1 \times 6 = 6$, we say 1 and 6 divide 6 exactly.

In $2 \times 3 = 6$ and $3 \times 2 = 6$, we say 2 and 3 are exact divisors of 6.

Combining we say that 6 is completely divisible by each of 1, 2 , 3 and 6.

Now consider 12 coins.

∵ **1** × 12 = 12, **2** × 6 = 12, **3** × 4 = 12, **4** × 3 = 12, **6** × 2 = 12 and **12** × 1 = 12

∴ 1, 2, 3, 4, 6 and 12 are divisors of 12.

In the same way,

$18 = 1 \times 18$, $18 = 2 \times 9$, $18 = 3 \times 6$, $18 = 6 \times 3$, $18 = 9 \times 2$ and $18 = 18 \times 1$

∴ 1, 2, 3, 6, 9 and 18 are exact divisors of 18.

Again consider the number 6

(i) On dividing 6 by 1, we get :
Quotient = 6 and remainder = 0

1) 6 (6
−6
0

(ii) On dividing 6 by 2, we get :
Quotient = 3 and remainder = 0

2) 6 (3
−6
0

(iii) On dividing 6 by 3, we get :
Quotient = 2 and remainder = 0

3) 6 (2
−6
0

(iv) On dividing 6 by 4, we get :
Quotient = 1 and remainder = 2

4) 6 (1
−6
2

(v) On dividing 6 by 5, we get :
Quotient = 1 and remainder = 1

5) 6 (1
−5
1

(vi) On dividing 6 by 6, we get :
Quotient = 1 and remainder = 0

6) 6 (1
−6
0

Here, we see that 1, 2, 3 and 6 are exact divisiors of 6 and are called factors of 6.

> **Factor** of a number is an exact divisor of that number

Since **5** × 1 = **5**, **5** × 2 = **10**, **5** × 3 = **15**, **5** × 4 = **20**,

We say 5, 10, 15, 20, ... are multiples of 5.

In the same way, multiples of 8 are :

$= 8 \times 1,\ 8 \times 2,\ 8 \times 3,\ 8 \times 4, \ldots\ldots$

$= 8, 16, 24, 32, \ldots\ldots$

> Multiples of a number are numbers obtained by multiplying it by natural numbers.

$\because$ $6 \times 4 = 24$ $\Rightarrow$ factors 6 and 4 are exact divisors of 24

$6 \times 4 = 24$ $\Rightarrow$ the number 24 is a multiple of each of the factors 6 and 4.

Thus, (i) A factor of a number is an exact divisor of that number.

(ii) A number is a multiple of its factors.

If we write 24 as $24 = 3 \times 8$, we say 3 and 8 are factors of 24 and 24 is a multiple of 3 and 8.

Since, $24 = 1 \times 24$, $24 = 2 \times 12$, $24 = 3 \times 8$, $24 = 4 \times 6$ and $24 = 12 \times 2$

$\Rightarrow$ Factors of 24 are : 1, 2, 3, 4, 6, 8, 12 and 24.

And, 24 is a multiple of each of 1, 2, 3, 4, 6, 8, 12 and 24.

$\Rightarrow$ A number is a multiple of each of its factors.

$\because$ $3 = 1 \times 3$, $6 = 2 \times 3$, $9 = 3 \times 3$, $12 = 3 \times 4$, $15 = 3 \times 5$, $18 = 3 \times 6$, etc.

$\Rightarrow$ Each of 3, 6, 9, 12, 15, 18, ..., etc. is a multiple of 3.

In the same way.

(i) Multiples of 4 are : 1×4, 2×4, 3×4, 4×4, ...

= 4, 8, 12, 16, ...

(ii) Multiples of 5 are : 5, 10, 15, 20, 25, ... and so on.

One (1) is a factor of every number.
Every number is a factor of itself.

Factors of 4 = 1, 2 and 4.

Factors of 15 = 1, 3, 5 and 15.

Factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18 and 36, etc.

See carefully. What do you find ?

For a given number, each of its factors is less than or equal to the number. Also, it can easily be seen that *the factors of a given number are finite* (countable).

As shown above, total number of factors of 4 is 3 (a finite number), number of factors of 36 is 7 (a finite number).

To be more clear,

factors of 32 = 1, 2, 4, 8, 16 and 32.

$\Rightarrow$ 1 is a factor of 32,

32 is a factor of itself. Every factor of a number is an exact divisor of the number.

Each factor of 32 is less than or equal to 32. Factors of 32 are finite in numbers.

In the same way,

multiples of 5 = 5, 10, 15, 20, 25, ...

$\Rightarrow$ (i) every multiple of a number is greater than or equal to that number.

(ii) every number is a multiple of itself.

(iii) the number of multiples of a given number is infinite.

Example 1 :

Write all the factors of 56.

Solution :

$56 = 1 \times 56$, $56 = 2 \times 28$, $56 = 4 \times 14$, $56 = 7 \times 8$,

$\therefore$ **Factors of 56 = 1, 2, 4, 7, 8, 14, 28 and 56.** **(Ans.)**

Example 1 :

Write the multiples of 7.

Solution :

Multiples of 7 = 1 × 7, 2 × 7, 3 × 7, 4 × 7, 5 × 7, ...

= **7, 14, 21, 28, 35, ...** **(Ans.)**

Example 3 :

Write the first six multiples of 8.

Solution :

Required multiples = 1 × 8, 2 × 8, 3 × 8, 4 × 8, 5 × 8 and 6 × 8

= **8, 16, 24, 32, 40 and 48** **(Ans.)**

EXERCISE 9(B)

1. Fill in the blanks :
   (i) On dividing 9 by 7, quotient = ... and remainder = ...
   (ii) On dividing 18 by 6, quotient = ... and remainder = ...
   (iii) Factor of a number is .... of ....
   (iv) Every number is a factor of ....
   (v) Every number is a multiple of ....
   (vi) .... is factor of every number.
   (vii) For every number, its factors are .... and its multiples are ....
   (viii) x is a factor of y, then y is a .... of x.
2. Write all the factors of :
   (i) 16 (ii) 21 (iii) 39 (iv) 48
   (v) 64 (vi) 98
3. Write the first six multiples of :
   (i) 4 (ii) 9 (iii) 11 (iv) 15
   (v) 18 (vi) 16
4. The product of two numbers is 36 and their sum is 13. Find the numbers.

   Since, 36 = 1 × 36, 2 × 18, 3 × 12, **4 × 9**, 6 × 6
   Clearly numbers are **4 and 9** as 4 × 9 = 36 and 4 + 9 = 13.

5. The product of two numbers is 48 and their sum is 16. Find the numbers.
6. Write two numbers which differ by 3 and whose product is 54.
7. Without making any actual division show that 7007 is divisible by 7.

   7007 = 7000 + 7
   = 7 × (1000 + 1) = 7 × 1001
   Clearly, 7007 is divisible by 7.

8. Without making any actual division, show that 2300023 is divisible by 23.

9. Without making any actual division, show that each of the following numbers is divisible by 11.

(i) 11011 (ii) 110011 (iii) 11000011

10. Without actual division, show that each of the following numbers is divisible by 8 :

(i) 1608 (ii) 56008 (iii) 240008

9.3 EVEN AND ODD NUMBERS

All multiples of two are called **even numbers**.

Since, each of 2, 4, 6, 8, 10, 12, 14, 16, ... is a multiple of 2.

$\therefore$ Each of 2, 4, 6, 8, 10, 12, 14, 16 .... is an even number.

Every even number is divisible by 2 or 2 is a factor of every even number.

Numbers which are not divisible by 2 are called ***odd numbers****.*

Since, none of 1, 3, 5, 7, 9, 11, 13, ... is divisible by 2; so each of 1, 3, 5, 7, 9, 11, 13, ... is an odd number.

1. Every number is either even or odd.
2. A number cannot be both even as well as odd.

9.4 DIVISIBILITY RULES

Rule 1 : Divisibility by 2 :

A number is divisible by 2, if its unit's digit is 0, 2, 4, 6 or 8.

For example, each of the following numbers is divisible by 2 :

8, 1**4**, 32**0**, 549**6**, 322**2**, 155**8**, etc.

Rule 2 : Divisibility by 4 :

A number is divisible by 4, if the number formed by its digits in ten's place and unit's place is divisible by 4.

In 2**32**, the number formed by its digits in ten's place and unit's place is 32 and 32 is divisible by 4.

$\therefore$ **232 is divisible by 4.**

In the same way, 524 is divisible by 4 as 24 is divisible by 4.

Each of the following numbers is divisible by 4

3**12**, 508, 12**72**, 193**16**, etc.

Rule 3 : Divisibility by 8 :

A number is divisible by 8, if the number formed by its digits in hundred's place, ten's place and units place is divisible by 8.

In 5**408**, the number formed by its digits in hundred's place, ten's place and unit's place is 408 and 408 is divisibly by 8.

∴ **5408 is divisible by 8.**

In the same way, 188**24** is divisible by 8 as **824 is divisible by 8**.

Each of following numbers is divisible by 8.

7**200**, 35**144**, 82**176**, 632**432**, 5**008**, 5**064** etc.

Rule 4 : Test of divisibility by 3 :

A number is divisible by 3, if the sum of its digits is divisible by 3.

Consider the number 4215.

∵ Sum of its digits = 4 + 2 + 1 + 5 = 12, which is divisible by 3

∴ **4215 is divisible by 3**.

Similarly, consider the number 32547.

Sum of its digits = 3 + 2 + 5 + 4 + 7

= 21, *which is divisible by 3.*

∴ **32547 is divisible by 3**.

In the same way each of the following numbers is divisible by 3.

123, 6243, 8031, 233454, etc.

Rule 5 : Test of divisibility by 6 :

A number is divisible by 6, if it is divisible by 2 as well as by 3 (as 6 = 2 × 3).

Consider the number 35556.

∵ The number is divisible by 2 as digit at its unit's place is 6 which is even.

Also, sum of its digits = 3 + 5 + 5 + 5 + 6 = 24, which is divisible by 3.

∴ **35556 is divisible by 6**

In the same way, each of the following numbers is divisible by 6 :

702, 2256, 88536, 524676, etc.

Rule 6 : Divisibility by 9 :

A number is divisible by 9, if sum of its digits is divisible by 9.

Consider the number 7236.

Sum of digits = 7 + 2 + 3 + 6

= 18, *which in divisible by* 9.

∴ 7236 is divisible by 9.

In the same way, each of the following numbers is divisible by 9.

927, 27171, 542970, 4035798, etc.

Rule 7 : Divisibility by 5 :

A number is divisible by 5, if its unit's digit is either 0 or 5.

Each of the following numbers is divisible by 5

250, 725, 4020, 602735, etc.

Rule 8 : Divisibility by 11 :

A number is divisible by 11, if the difference of sum of its digits in odd places from the right side and the sum of its digits in even places from the right side is divisible by 11.

Consider the number 90816

Sum of its digits in odd places from the right side

$= 6 + 8 + 9 = 23$

And, sum of its digits in even places from the right side = $1 + 0 = 1$

Difference of the two sums = $23 - 1 = 22$, *which is divisible by 11.*

$\therefore$ **90816 is divisible by 11**.

In the same way, **consider the number 1,27,446**.

Sum of its digits in even places from the right side = $4 + 7 + 1 = 12$

and, sum of its digits in odd places from the right side = $6 + 4 + 2 = 12$

Difference of the two sums = $12 - 12 = 0$, *which is divisible by 11.*

$\therefore$ **1,27,446 is divisible by 11.**

Remember that :

1. For any two numbers x and y, if x is divisible by y, x is divisible by each factor of y.

   For example :

   As 36 is divisible by 12, 36 is divisible by each factor of 12 (*i.e.* by 1, 2, 3, 4, 6 and 12).

2. For any three numbers x, y and z, if x is divisible by y and y is divisible by z, then x is divisible by z.

   For example :

   As 72 is divisible by 24 and 24 is diviisble by 4, therefore 72 is divisible by 4.

3. If a number is divisible by two co-prime numbers, it is divisible by their product.

   For example :

   48 is divisible by 2 and 3 both

   $\Rightarrow$ 48 is divisible by 2×3 *i.e.* by 6.

4. If a number is a factor of each of the two given numbers, it is a factor of their sum.

   i.e., for numbers x, y and z, x is factor of y and z both, then x is factor of $y + z$.

   For example :

   As 3 is a factor of 18 and 24; then 3 is factor of 18 + 24 *i.e.* 3 is a factor of 42.

5. If a number is a factor of each of the two given numbers, it is a factor of their difference.

   For example :

   As 3 is factor of 18 and 24, then 3 is factor of 24 – 18 *i.e.*, 3 is a factor of 6.

EXERCISE 9(C)

1. Find which of the following numbers are divisible by 2 :
   (i) 352 (ii) 523 (iii) 496 (iv) 649
2. Find which of the following numbers are divisible by 4 :
   (i) 222 (ii) 532 (iii) 678 (iv) 9232
3. Find the which of the following numbers are divisible by 8 :
   (i) 324 (ii) 2536 (iii) 92760 (iv) 444320
4. Find which of the following numbers are divisible by 3 :
   (i) 221 (ii) 543 (iii) 28492 (iv) 92349
5. Find which of the following numbers are divisible by 9 :
   (i) 1332 (ii) 53247 (iii) 4968 (iv) 200314
6. Find which of the following numbers are divisible by 6 :
   (i) 324 (ii) 2010 (iii) 33278 (iv) 15505
7. Find which of the following numbers are divisible by 5 :
   (i) 5080 (ii) 66666 (iii) 755 (iv) 9207
8. Find which of the following numbers are divisible by 10 :
   (i) 9990 (ii) 0 (iii) 847 (iv) 8976
9. Find which of the folloiwng numbers are divisible by 11 :
   (i) 5918 (ii) 68,717 (iii) 3882 (iv) 10857
10. Find which of the folloiwng numbers are divisible by 15 :
    (i) 960 (ii) 8295 (iii) 10243 (iv) 5013

> A number is divisible by 15, if it is divisible by both 3 and 5.
> In the same way, a number is divisible by 18, if it is divisible by both 2 and 9.
> It is because : $15 = 3 \times 5$, $18 = 2 \times 9$.

11. In each of the following numbers, replace M by the smallest whole number to make the resulting number divisible by 3 :
    (i) 64 M 3 (ii) 46 M 46 (iii) 27 M 53
12. In each of the following numbers, replace M by the smallest whole number to make the resulting number divisible by 9.
    (i) 76 M 91 (ii) 77548 M (iii) 627 M 9
13. In each of the following numbers, replace M by the smallest whole number to make the resulting number divisible by 11.
    (i) 39 M 2 (ii) 3 M 422 (iii) 70975 M (iv) 14 M 75
14. State, **true** or **false** :
    (i) If a number is divisible by 4. it is divisible by 8.
    (ii) If a number is a factor of 16 and 24, it is a factor of 48.
    (iii) If a number is divisible by 18, it is divisible by 3 and 6.
    (iv) If a divides both b and c completely, then a divides (i) $a + b$ (ii) $a - b$ also completely.

SETS 10

10.1 INTRODUCTION

In our day-to-day life we often speak or hear about different types of collections.

Such as :

1. A collection of stamps.
2. A collection of toys.
3. A collection of books, etc.

Each collection is well defined

In the same way, we have different types of groups made for different activities.

Such as :

1. A group of boys playing hockey.
2. A group of girls playing badminton.
3. A group of students going for picnic, etc.

Each group is well defined

In mathematics, a collection of particular things or *a group of particular objects is called a* **set.**

10.2 IDEA OF A SET

A set is a collection of well-defined objects.

Meaning of "WELL-DEFINED" :

Well-defined means, it must be absolutely clear which object belongs to the set and which does not.

For example :

(i) **A collection of "Lovely Flowers"** is not a set *because the objects* (flowers) *to be included are* ***not well-defined.***

Reason : The word "Lovely" is a relative term. What may be lovely to one person may not be the same to another person.

(ii) **A collection of "Red Flowers"** is a set because every red flower will be included in this set, *i.e.* ***the objects of the set are well-defined.***

(iii) **A group of "Young Players"** is not a set, as the range of the age for young players is not given *i.e.* it cannot be decided which player is to be considered young, *i.e.* ***the objects are not well-defined.***

(iv) **A group of "Players with ages between 14 years and 18 years"** is a set, because the range of age of the players is given such that it can be easily decided which players are to be included and which are to be excluded.

Hence, ***the objects are well-defined.***

Example 1 :

State, giving reason, whether or not the following objects form a set :

(i) All the problems in this book that are difficult to solve.

(ii) All the problems in this book that are difficult for Mohit to solve.

(iii) All objects heavier than 28 kg.

Solution :

(i) The given **objects do not form a set.** **(Ans.)**

Reason : Some problems may be difficult for one person but not so difficult for others. Hence the given **objects are not well-defined.**

Thus, the given objects do **not** form **a set.**

(ii) The given **objects form a set.** **(Ans.)**

Reason : It can be easily found which problems are difficult to solve for Mohit and which are not.

(iii) The given **objects form a set.** **(Ans.)**

Reason : Every object can be compared with certainity in relation to weight of 28 kg. Thus, it is very easy to select objects which are heavier than 28 kg, *i.e.* the **objects are well-defined,** so they **form a set.**

The members (objects) of each of the following collections form a set :

(i) students in a class room,

(ii) books in your school bag,

(iii) counting numbers between 10 and 20,

(iv) the students of your class, that are taller than you and so on.

EXERCISE 10(A)

1. State whether or not the following elements form a set; if not, give reason :
   (i) All the easy problems in your text book.
   (ii) All the three sided figures.
   (iii) The first five counting numbers.
   (iv) All the tall boys of your class.
   (v) The last three days of a week.
   (vi) All triangles that are difficult to draw.
   (vii) The first three letters of the English alphabet.
   (viii) All tasty fruits.
   (ix) All the clever boys of class 6.
   (x) All the good schools in Delhi.
   (xi) All the girls in your class whose heights are less than your height.
   (xii) All the boys in your class whose heights are more than your height.
   (xiii) All the problems in your Mathematics book that are difficult for Amit.

10.3 ELEMENTS (or MEMBERS) OF A SET

The objects used to form a set are called its **elements** or its **members**.

Generally, the elements of a set are written inside a pair of curly (middle) braces and separated by commas. *The name of the set is always written in capital letters.*

For example :

$$A = \{ p, q, r, s, t \}$$

Here, 'A' is the name of the set, whose elements (members) are p, q, r, s and t.

1. *The pair of curly braces { } denotes a set.*
2. *The Greek letter **Epsilon** '∈' is used for the words "belongs to", "is an element of", etc.*

   ∴ p ∈ A will be read as "p belongs to set A" or as "p is an element of set A".

   In the same way; q ∈ A, r ∈ A, s ∈ A and t ∈ A.

   The symbol '∉' stands for "does not belong to" or "is not an element of".

   ∴ a ∉ A will be read as "a does not belong to set A" or as "a is not an element of set A".

   In the same way b ∉ A, c ∉ A and so on.

Example 2 :

If set A = { 3, 6, 9, 12, 15, 18 }, state whether the following statements are true or false :

(i) 8 ∈ A (ii) 12 ∉ A (iii) 15 ∈ A

(iv) 9 and 12 ∈ A (v) 12, 14 and 15 ∈ A

Solution :

(i) False : as element 8 does not belong to the given set A. **(Ans.)**

(ii) False : as element 12 belongs to the given set A. **(Ans.)**

(iii) True : as element 15 belongs to the given set A. **(Ans.)**

(iv) True : as elements 9 and 12 both belong to the given set A. **(Ans.)**

(v) False : as element 14 does not belong to the given set A. **(Ans.)**

10.4 SOME PROPERTIES OF A SET

1. *A change in order of writing the elements does not make any change in the set.*

For example :

Set { 5, 6, 7, 8 } is same as set { 6, 8, 7, 5 }

i.e. { 5, 6, 7, 8 } = { 6, 8, 7, 5 }

Similarly, { b, d, n, y, z } = { y, z, b, d, n } = { z, d, y, b, n } and so on.

2. *If one or many elements of a set are repeated, the set remains the same.*

For example :

The set { a, b, d, a, c, b } is the same as the set { a, b, c, d }

i.e. { a, b, d, a, c, b } = { a, b, c, d }

In general, *the elements of a set are not repeated.* Thus,

(i) if Q is the set of letters of the word **book**; then Q = { b, o, k }.

There are two **o**s in the word **book** but in the set it is written only once.

(ii) if R = { letters of the word 'PUPPET' }; then R = { p, u, e, t }.

Review :

1. *The elements of a set are written inside a pair of curly braces and separated by commas.*
2. *The set is represented by a capital letter.*
3. *If the elements of a set are alphabets, these elements are written in small letters.*
4. *The elements of a set may be written in any order.*
5. *No element of a set must be repeated.*

Example 3 :

(i) Write the set of letters used in the word 'MEERUT'.

(ii) Write the set of vowels used in the word "MUSSOORIE".

(iii) Write the set of consonants used in the word "AMRITSAR".

Solution :

(i) **Required set = { *m, e, r, u, t* }** **(Ans.)**

(ii) **Required set = { *u, o, i, e* }** **(Ans.)**

(iii) **Required set = { *m, r, t, s* }** **(Ans.)**

Each element is written only once.

Example 4 :

For each statement, given below, state whether it is **true** or **false** :

(i) { 5, 5, 5, ... } = { 5 }

(ii) { 3, 4, 7 } = { 4, 3, 7 }

(iii) { 6 – 2, 5 – 2, 4 – 2, 3 – 2 } = { 4, 3, 2, 1 }

Solution :

(i) **True :** as repetition of elements does not change a set. **(Ans.)**

(ii) **True :** as the change in order of writing the elements does not change a set **(Ans.)**

(iii) **True.** **(Ans.)**

EXERCISE 10(B)

1. If set A = { 2, 3, 4, 5, 6 }, state which of the following statements are **true** and which are **false** :

(i) 2 ∈ A (ii) 5, 6 ∈ A (iii) 3, 4, 7 ∈ A (iv) 2, 8 ∈ A

2. If set B = { 4, 6, 8, 10, 12, 14 }, state which of the following statements are **correct** and which are **wrong** :
   (i) 5 ∈ B (ii) 12 ∈ B (iii) 14 ∈ B (iv) 9 ∈ B
   (v) B is the set of even numbers between 2 and 16.
   (vi) 4, 6 and 10 are the members of the set B.
   Also, write the wrong statements correctly.
3. State whether **true** or **false** :
   (i) Sets { 4, 9, 6, 2 } and { 6, 2, 4, 9 } are not the same.
   (ii) Sets { 0, 1, 3, 9, 4 } and { 4, 0, 1, 3, 9 } are the same.
   (iii) Sets { 5, 4 } and { 5, 4, 4, 5 } are not the same.
   (iv) Sets { 8, 3 } and { 3, 3, 8 } are the same.
   (v) Collection of vowels used in the word 'ALLAHABAD' forms a set.
   (vi) If P is the set of letters in the word 'ROOP'; then P = { p, o, r }
   (vii) If M is the set of letters used in the word 'MUMBAI', then :
   M = { m, u, b, a, i }
4. Write the set containing :
   (i) the first five counting numbers. (ii) the three types of angles.
   (iii) the three types of triangles. (iv) the members of your family.
   (v) the first six consonants of the English alphabet.
   (vi) the first four vowels of the English alphabet.
   (vii) the names of any three Prime Ministers of India.
5. (a) Write the members (elements) of each set given below :
   (i) { 3, 8, 5, 15, 12, 7 } (ii) { c, m, n, o, s }
   (b) Write the sets whose elements are :
   (i) 2, 4, 8, 16, 64 and 128 (ii) 3, 5, 15, 45, 75 and 90
6. (i) Write the set of letters used in the word 'BHOPAL'.
   (ii) Write the set of vowels used in the word 'BENGAL'.
   (iii) Write the set of consonants used in the word 'HONG-KONG'.

10.5 REPRESENTATION OF SETS

For representing a set, the following methods are commonly used :
(i) *Description Method* (ii) *Roster* or *Tabular Method*
(iii) *Rule* or *Set-builder Method*

1. Description Method :

In this method, a well-defined description of the elements of a set is made. Sometimes, the discription of elements is enclosed within the curly brackets.

For example :

(i) A set of cricket players with ages between 20 years and 28 years.
= {cricket players with ages between 20 years and 28 years}

(ii) A set of numbers greater than 40 and smaller than 75.

= {numbers greater than 40 and smaller than 75} and so on

2. Roster or Tabular Method :

In this method, the elements (members) of the set under consideration are written inside a pair of curly braces and are separated by commas.

For example :

(i) If P is the set of the last four months of the year. [Description Method]

then, P = {September, October, November, December}. [Roster Method]

(ii) If Q is the set of counting numbers less than 6. [Description Method]

then, Q = { 1, 2, 3, 4, 5 } [Roster Method]

3. Rule or Set-Builder Method :

In this method, *the actual elements of a set are not listed, rather a brief rule or statement or formula is written inside a pair of curly braces.*

For example :

If A is the set of counting numbers greater than 12,

Set A is written in set-builder form as :

A = { x : x is a counting number greater than 12 }

or

A = { x | x is a counting number greater than 12 }

This will be read as "A is the set of elements x such that x is a counting number greater than 12".

Each of the symbols ':' and '|' stands for "such that".

Thus, if B is the set of x such that x is a factor of 6, then

B = { x : x is a factor of 6 } or B = { x | x is a factor of 6 }

10.6 SOME IMPORTANT SETS :

N = Natural Numbers

= Set of all counting numbers starting from 1 [Description Method]

= Set of numbers 1, 2, 3, 4, ...

= { 1, 2, 3, 4, ... } [Roster Method]

= { x : x is a counting number starting from 1 } [Set-Builder Method]

W = Whole Numbers

= Set containing zero and all the natural numbers.

= { 0, 1, 2, 3, 4, 5, 6, ... }

Z or **I** = Integers

= Set containing the negatives of the natural numbers, zero and the natural numbers.

= { ..., –3, –2, –1, 0, 1, 2, 3, 4, ... }

E = Even Natural Numbers

= Set of natural numbers that are divisible by 2.

= { 2, 4, 6, 8, ... }

O = Odd Natural Numbers
= Set of natural numbers that are not divisible by 2.
= { 1, 3, 5, 7, 9, ... }

Almost every set can be expressed in all the three methods (forms) discussed above.

For example :

(i) If A = Set of odd natural numbers less than 15 [Description Form]
A = { 1, 3, 5, 7, 9, 11, 13 } [Roster Form]
and A = { x : x is an odd natural number less than 15 } [Set-Builder Form]

(ii) If B = { x : x is a letter in the word 'MEERUT' } [Set-Builder Form]
B = Set of letters used in the word 'MEERUT' [Description Form]
and B = { m, e, r, u, t } [Roster Form]

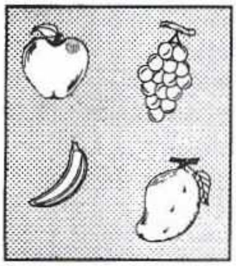
(iii) If P = { 5, 10, 15, 20, ... } [Roster Form]
P = { x : x is a natural number divisible by 5 } [Set-Builder Form]
and, P = {natural numbers divisible by 5} [Description Form]

Important :

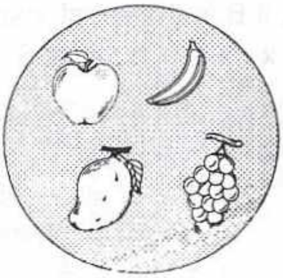
A set or a collection of well-defined objects may also be represented by pictures drawn inside closed figures, like a circle, a rectangle, a square, etc.

For example :

(i) A set of some fruits as :

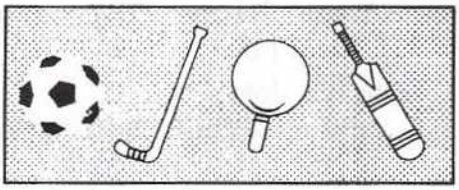


or

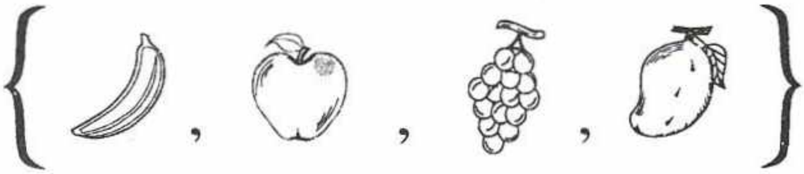


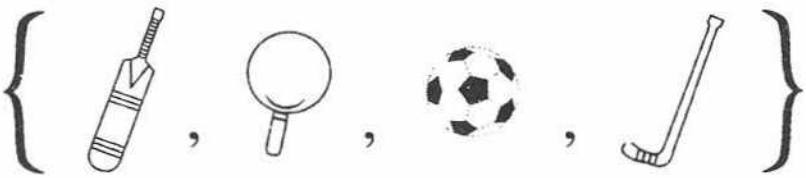
(ii) A set of some sports goods as :

or



Instead of showing (drawing) these pictures inside a circle, a rectangle, etc., we can also draw them within a pair of curly braces (brackets).

(i) 

(ii) 

The two sets shown above by pictures can also be expressed by writing their names as shown below :

(i) { banana, apple, grapes, mango }.

(ii) { cricket bat, tennis racket, football, hockey }.

EXERCISE 10(C)

1. Write each of the following sets in the **Roster Form** :
   (i) The set of five numbers each of which is divisible by 3.
   (ii) The set of integers between – 4 and 4.
   (iii) { x : x is a letter in the word 'SCHOOL' }
   (iv) { x : x is an odd natural number between 10 and 20 }
   (v) { Vowels used in the word 'AMERICA' }
   (vi) { Consonants used in the word 'MADRAS' }

2. Write each given set in the **Roster Form** :
   (i) All prime numbers between 1 and 20.
   (ii) The squares of the first four natural numbers.
   (iii) Even numbers between 1 and 9.
   (iv) The first eight letters of the English alphabet.
   (v) The letters of the word 'BASKET'.
   (vi) Four cities of India whose names start with the letter J.
   (vii) Any four closed geometrical figures.
   (viii) Vowels used in the word 'MONDAY'.
   (ix) Single digit numbers that are perfect squares as well.

3. Write each given set in the **Set-Builder Form** :
   (i) { 2, 4, 6, 8, 10 }
   (ii) { 2, 3, 5, 7, 11 }
   (iii) { January, June, July }
   (iv) { a, e, i, o, u }
   (v) { Tuesday, Thursday }
   (vi) { 1, 4, 9, 16, 25 }
   (vii) { 5, 10, 15, 20, 25, 30 }

4. Write each of the following sets in **Roster** (tabular) **Form** and also in **Set-Builder Form** :
   (i) Set of all natural numbers that can divide 24 completely.
   (ii) Set of odd numbers between 20 and 35.
   (iii) Set of letters used in the word 'CALCUTTA'.
   (iv) Set of names of the first five months of a year.
   (v) Set of all two-digit numbers that are perfect squares as well.

5. Write, in **Roster Form,** the set of :
   (i) the first four odd natural numbers each divisible by 5.
   (ii) the counting numbers between 15 and 35; each of which is divisible by 6.
   (iii) the names of the last three days of a week.
   (iv) the names of the last four months of a year.

10.7 TYPES OF SETS

In this part of set theory, we shall be finding whether :

(i) a given set has a countable number of elements or not,
(ii) a given set is empty or not,
(iii) two given sets have identical elements or not,
(iv) two given sets have an equal number of elements or not,
(v) two given sets have some elements in common or not, etc.

10.8 FINITE SET

A set is said to be a finite set if it has a limited number of elements, *i.e.* the number of elements in it can be counted.

For example :

Set P is finite as it has a countable number of elements.

1. P = { Natural numbers less than 50 }
   = { 1, 2, 3, ..., 49 }
2. Q = { x : x is number of students in your school }
3. R = { Whole numbers between 5 and 45 } and so on.

10.9 INFINITE SET

A set is said to be an infinite set if it has an unlimited number of elements *i.e.* the number of elements of such a set cannot be counted.

For example :

Each of these sets has an uncountable number of elements

1. Q = Set of whole numbers = { 0, 1, 2, 3, ... }
2. R = { Stars in the sky }
3. A = { x : x is a natural number greater than 32 }
   = { 33, 34, 35, 36, ... } and so on.

Such sets are expressed in the Roster Form by writing a few elements and then putting some dots to show that the elements continue till infinity.

10.10 THE EMPTY SET or THE NULL SET

The empty set is a finite set.

It is a set that has no elements.

The empty set is expressed by a pair of curly braces with no element written inside them, *i.e.* { } represents an empty set.

The empty set (the null set) is also represented by the Greek letter ∅, spelt as phi. The symbols { } and ∅ represent the same set, *i.e.* { } = ∅.

Examples :

(i) Let A = { Triangles with four sides }
then A = ∅, *i.e.* A = { }

There is no triangle with four sides.

(ii) Let B = { Natural numbers less than 1 }
then B = { }, *i.e.* B = ∅

There is no natural number less than 1.

10.11 EQUAL SETS

Two sets are said to be equal if the elements of the two sets are the same, i.e. the elements of the two sets are identical. The symbol used for equality of sets is the usual sign "=", *i.e.* "*equal to*".

For example :

Let A = { 1, 2, 3, 4 }
and B = { Natural numbers less than 5 }. Then, set A is equal to set B.

And so we write : **Set A = Set B,** or simply **A = B**

10.12 EQUIVALENT SETS

Two sets are said to be equivalent if the number of eiements in both the sets are equal. The elements may be the same or they may be different, but each set must contain the same number of elements.

For example :

Consider A = { x, y, z }
and B = { Patna, Calcutta, Delhi }

Here, A and B are equivalent sets because they have an equal number of elements (both A and B have three elements).

1. *Equal sets are equivalent but the converse is not always true.*
2. *Two empty sets are always equal.*
3. *Two infinite sets are always equivalent.*

10.13 DISJOINT SETS

If two given sets have no element in common, they are disjoint sets.

For example :

(i) Let A = Set of students of Class X

and B = Set of students of Class XII

Since, no student can be common to the two classes, **sets A and B are disjoint.**

(ii) Let P = { a, b, c, d } and Q = { 1, 2, 3, 4, 5 }

Clearly, sets P and Q have no element in common; therefore, **sets P and Q are disjoint sets.**

10.14 OVERLAPPING SETS

If two given sets have at least one element in common, they are said to be overlapping sets.

Overlapping sets are also known as joint sets.

For example :

(i) If set A = { 5, 6, 7, 8, 9, 10 } and set B = { 4, 6, 8, 10, 12 }, **sets A and B are overlapping** as they have elements 6, 8 and 10 in common.

(ii) Let P = Set of students of Class X

and Q = Set of students of Class X in Sophia School.

Clearly, the students of Class X in Sophia School are common to the two given sets; therefore **sets P and Q are overlapping sets.**

EXERCISE 10(D)

1. State whether the given set is **infinite** or **finite** :
   (i) { 3, 5, 7, ... }
   (ii) { 1, 2, 3, 4 }
   (iii) { ..., –3, –2, –1, 0, 1, 2 }
   (iv) { 20, 30, 40, 50, ..., 200 }
2. Which of the following sets is **empty** ?
   (i) Set of counting numbers between 5 and 6.
   (ii) Set of odd numbers between 7 and 19.
   (iii) Set of odd numbers between 7 and 9.
   (iv) Set of even numbers that are not divisible by 2.
   (v) { 0 }.
3. State which pair of sets given below are **equal sets** and which are **equivalent** :
   (i) { 3, 5, 7 } and { 5, 3, 7 }
   (ii) { 8, 6, 10, 12 } and { 3, 2, 4, 6 }
   (iii) { 7, 7, 2, 1, 2 } and { 1, 2, 7 }
   (iv) { 2, 4, 6, 8, 10 } and { a, b, d, e, m }
4. State which of the following are **finite** sets and which are **infinite :**
   (i) Set of integers
   (ii) { Multiples of 5 }
   (iii) { Fractions between 1 and 2 }
   (iv) { Number of people in India }
   (v) Set of trees in the world
   (vi) Set of leaves on a tree
   (vii) Set of children in all the schools of Delhi
   (viii) { ...., – 4, – 2, 0, 2, 4, 6, 8 }
   (ix) { – 12, – 9, – 6, – 3, 0, 3, 6, ... }
   (x) { Number of points in a line segment 4 cm long }.

5. State whether or not the following sets are **empty :**
   (i) { Prime numbers divisible by 2 } (ii) { Negative natural numbers }
   (iii) { Women with height 5 metre } (iv) { Integers less than 5 }
   (v) { Prime numbers between 17 and 23 }
   (vi) Set of even numbers not divisible by 2
   (vii) Set of multiples of 3 that are more than 9 and less than 15.
6. State if the given pairs of sets are **equal sets** or **equivalent sets :**
   (i) { Natural numbers less than five } and { Letters of the word 'BOAT' }.
   (ii) { 2, 4, 6, 8, 10 } and { even natural numbers less than 12 }.
   (iii) { 1, 3, 5, 7, ... } and set of odd natural numbers.
   (iv) { Letters of the word MEMBER } and { Letters of the word 'REMEMBER' }.
   (v) { Negative natural numbers } and { 50th day of a month }
   (vi) { Even natural numbers } and { Odd natural numbers }.
7. State whether the following are **finite** or **infinite sets :**
   (i) { 2, 4, 6, 8, ..., 800 } (ii) { ...., – 5, – 4, – 3, – 2 }
   (iii) { x : x is an integer between – 60 and 60 }
   (iv) { No. of electrical appliances working in your house }
   (v) { x : x is a whole number greater than 20 }
   (vi) { x : x is a whole number less than 20 }.
8. For each statement, given below, write **True** or **False :**
   (i) { ...., – 8, – 4, 0, 4, 8 } is a finite set.
   (ii) { – 32, – 28, – 24, – 20, ..., 0, 4, 8, 16 } is an infinite set.
   (iii) { x : x is a natural number less than 1 } is the empty set.
   (iv) { Whole numbers between 15 and 16 } = { Natural numbers between 5 and 6 }.
   (v) { Odd numbers divisible by 2 } is the empty set.
   (vi) { Even natural numbers divisible by 3 } is the empty set.
   (vii) { x : x is positive and x < 0 } is the empty set.
   (viii) { ...., –5, –3, –1, 1, 3, 5, ... } is a finite set.
9. State, giving reasons, which of the following pairs of sets are **disjoint sets** and which are **overlapping sets** :
   (i) A = { Girls with ages below 15 years } and
   B = { Girls with ages above 15 years }
   (ii) C = { Boys with ages above 20 years } and
   D = { Boys with ages above 27 years }
   (iii) A = { Natural numbers between 35 and 60 } and
   B = { Natural numbers between 50 and 80 }
   (iv) P = { Students of Class IX studying in I.C.S.E. Board } and
   Q = { Students of Class IX }
   (v) A = { Natural numbers that are multiples of 3 and less than 30 } and
   B = { Natural numbers divisible by 4 and lying between 20 and 45 }
   (vi) P = { Letters in the word 'ALLAHABAD' } and
   Q = { Letters in the word 'MUSSOORIE' }

10.15 CARDINALITY OF A SET

The number of elements in a set is called its ***cardinal number****.*

The elements in the set must not be repeated.

For example :

(i) Set P = { 2, 9, 11, 14 } has 4 elements ∴ Cardinal number of set P = 4.

(ii) Set M = { x, y, z } has 3 elements ∴ Cardinal number of set M = 3.

(iii) Set E = { } has no element ∴ Cardinal number of set E = 0 and so on.

The symbol used for showing the cardinal number is the small letter 'n' attached before the name of the set that is written inside brackets.

Thus, the cardinal number of set A is represented by n(A).

Conversely, n(B) represents the cardinal number of set B.

Thus, for the sets P, M and E given above :

1. Cardinal number of the empty set 0.
2. Cardinal number of infinite set is not defined.

(i) n(P) = 4, as set P has only four elements; 2, 9, 11 and 14.

(ii) n(M) = 3, as set M has only three elements; x, y and z.

(iii) n(E) = 0, as set E is the empty set and so on.

Example 5 :

Write the cardinal number of each of the following sets :

(i) A = { 2, 3, 5, 5, 3, 3 } (ii) B = { letters in the word "NOORJAHAN" }

(iii) P = { counting numbers between 10 and 30; that are divisible by 5 }

Solution :

(i) Since A = { 2, 3, 5, 5, 3, 3 } = { 2, 3, 5 }

∴ Cardinal number of set A = 3, *i.e.* **n(A) = 3** **(Ans.)**

(ii) Since B = { n, o, r, j, a, h }

∴ Cardinal number of set B = 6, *i.e.* **n(B) = 6** **(Ans.)**

(iii) Since P = { 15, 20, 25 }, **n(P) = 3** **(Ans.)**

EXERCISE 10(E)

1. Write the cardinal number of each of the following sets :

   (i) A = { 0, 1, 2, 4 } (ii) B = { − 3, − 1, 1, 3, 5, 7 } (iii) C = { }

   (iv) D = { 3, 2, 2, 1, 3, 1, 2 } (v) E = { Natural numbers between 15 and 20 }

   (vi) F = { Whole numbers from 8 to 14 }.

2. Given : A = { Natural numbers less than 10 }

   B = { Letters of the word 'PUPPET' }

   C = { Squares of the first four whole numbers }

   D = { Odd numbers divisible by 2 }

   Find :

   (i) n(A) (ii) n(B) (iii) n(C) (iv) n(D)

3. State **true** or **false** for each of the following. Correct the wrong statement.

   (i) If A = { 0 }, then n(A) = 0. (ii) n(∅) = 1.

   (iii) If T = { a, l, a, h, b, d, h }; then n(T) = 5

   (iv) If B = { 1, 5, 51, 15, 5, 1 }, then n(B) = 6.

Theme – 2
RATIO, PROPORTION AND ARITHMETIC PROBLEMS

RATIO 11

11.1 INTRODUCTION

Most of the time, we compare things, numbers, etc. (say, ***x*** and ***y***) by saying :

(i) ***x*** is *greater* than ***y***

(ii) ***x*** is *less* than ***y***

(iii) ***x*** is *double* of ***y***

(iv) ***x*** is *one-third* of ***y***

(v) $\frac{x}{y} = \frac{4}{5}$

(vi) $\frac{y}{x} = \frac{3}{2}$,

(vii) ***x*** is equal to ***y***, etc.

In general, for comparing two things (quantities) in our daily life, we use either subtraction method or division method.

1. **Comparision by subtraction method :** *In this method, we come to know how much is one quantity more than or less than the other.*

   For example :

   If the age of Mohit is 15 years and that of Saran is 10 years, how many years is Mohit older than Saran ?

   Clearly, Mohit is 15 years – 10 years = 5 years older than Saran.

   In the same way, if the weight of an object A is 40 kg and that of object B is 60 kg, which of these two objects is having more weight and by how much ?

   Here, the weight of B is greater than that of object A by 60 kg – 40 kg = 20 kg.

   We can also say that weight of object A is 20 kg less than that of object B.

   In the same way, many more pairs of objects can be compared for their heights, ages, weights, etc. by the method of subtraction.

2. **Comparision by division method :** *In this method, we come to know how many times is one quantity of the other.*

   For example :

   If the age of a boy is 7 years and that of his father is 35 years,

   Since, $\frac{\text{the father's age}}{\text{the son's age}} = \frac{35 \text{ years}}{7 \text{ years}} = 5.$

   We say that father's age is 5 times that of son's age.

In the same way, if the weight of an object A is 40 kg and that of object B is 60 kg :

$$\frac{\text{A's weight}}{\text{B's weight}} = \frac{40\text{ kg}}{60\text{ kg}} = \frac{2}{3}$$

> Ratio must always be expressed in the simplest form *i.e.* both of its terms must not have any common prime factor other than 1.

i.e., A's weight is $\frac{2}{3}$ times that of B.

Also, $\frac{\text{B's weight}}{\text{A's weight}} = \frac{60\text{ kg}}{40\text{ kg}} = \frac{3}{2}$

i.e., B's weight is $\frac{3}{2}$ times that of A's.

> Ratio is an ordered expression, that is, ratio between x and y is $\frac{x}{y}$ and not $\frac{y}{x}$.

The method of comparing two quantities (numbers, things, etc.) *by dividing one quantity by the other, is called* **ratio.**

Thus : $\frac{x}{y} = \frac{2}{3}$ represents the ratio of x to y.

and, $\frac{y}{x} = \frac{3}{2}$ represents the ratio of y to x.

11.2 RATIO

The relation of two quantities (both of the same kind and in the same unit) *obtained on dividing one quantity by the other is called their* ***ratio.***

∴ The **ratio** of two quantities x and y, both of the same kind and in the same unit, is $\frac{x}{y}$, and is often written as $x : y$ (read as ***x* to *y*** or ***x* is to *y***).

Meaning of the two quantities of the same kind and in the same unit :

1. **Both the quantities must be of the same kind, means :** If one quantity is length, the other quantity must also be length; if one quantity represents mass, the other quantity must also be representing mass and so on.

   The ratio between unlike quantities has no meaning.

   For example, the ratio of length to mass has no meaning.

2. **Both the quantities must be in the same unit, means :** The two quantities must have the same unit of measurement.

   For example, if the lengths of two objects are given to be 60 cm and 1·5 m; then before finding the ratio of one length to that of other, both of these lengths must either be converted into cm or into m.

Examples :

(i) The ratio of 5 kg to 15 kg = $\frac{5\text{ kg}}{15\text{ kg}} = \frac{1}{3}$ = **1 : 3**

(ii) The ratio of 800 gm to 1.2 kg

> 1.2 kg = 1.2 × 1000 gm = 1200 gm

$= \frac{800\text{ gm}}{1200\text{ gm}} = \frac{2}{3}$ = **2 : 3**

(iii) The ratio of 2 m to 80 cm

$= \frac{2\text{ m}}{80\text{ cm}} = \frac{200\text{ cm}}{80\text{ cm}} = \frac{5}{2} = \mathbf{5 : 2}$

$2\text{ m} = 2 \times 100\text{ cm} = 200\text{ cm}$

(iv) The ratio of $1\frac{1}{2}$ years to 10 months

$= \frac{18\text{ months}}{10\text{ months}} = \frac{9}{5} = \mathbf{9 : 5}$

$1\frac{1}{2}\text{ years} = \frac{3}{2} \times 12\text{ months} = 18\text{ months}$

A ratio is a pure number and so has no unit.

1. *The ratio of two numbers or quantities is denoted by the colon mark " : ".*
   Thus, the ratio of two quantities p and q = **p : q**
2. *The ratio of two quantities of same kind and in the same unit is obtained on dividing one quantity by the other.*
   Thus, *the ratio of* 20 kg *to* 80 kg $= \frac{20\text{ kg}}{80\text{ kg}} = \frac{1}{4} = \mathbf{1 : 4}$
3. *The* ***first term*** *of a ratio is called the* ***antecedent*** *and the* ***second term*** *is called the* ***consequent.***
   In the ratio 1 : 4; *antecedent* = 1 *and consequent* = 4.

   The value of a ratio remains unchanged, if its antecedent and consequent both are multiplied by the same non-zero number.
4. *A ratio must always be expressed in its lowest terms.*
5. *Whatever be the unit of the terms of a ratio, the ratio has no unit. The ratio of 15 km and 20 km* $= \frac{15\text{ km}}{20\text{ km}} = \frac{3}{4}$ *= 3 : 4. Here, the two quantities 15 km and 20 km have unit km, but their ratio 3:4 has no unit. On dividing, units cancel out.*
6. *The terms of a ratio are written in a definite order.*
   The ratio of 5 kg and 8 kg $= \frac{5\text{ kg}}{8\text{ kg}} = \frac{5}{8}$ *= 5 : 8 and*
   the ratio of 8 kg and 5 kg $= \frac{8\text{ kg}}{5\text{ kg}} = \frac{8}{5}$ *= 8 : 5*
   Remember : *5 : 8 and 8 : 5 are not equal to each other.*

Example 1 :

Find the ratio of : (i) 60 to 48 (ii) 3.75 kg to 750 gm

Solution :

(i) **Required ratio** $= \frac{60}{48}$

$= \frac{5}{4} = \mathbf{5 : 4}$ **(Ans.)**

(ii) Since 3.75 kg = 3.75 × 1000 gm = 3750 gm

Required ratio $= \frac{3.75\text{ kg}}{750\text{ gm}}$

$= \frac{3750\text{ gm}}{750\text{ gm}} = \frac{5}{1} = \mathbf{5 : 1}$ **(Ans.)**

11.3 CONVERTING INTO SIMPLE RATIO

Example 2 :

Express as simple ratio : (i) $3\frac{1}{2} : 2\frac{1}{3}$ (ii) $\frac{2}{3} : \frac{4}{5} : \frac{1}{2}$

Solution :

(i) Divide the first term of the ratio by its second term and then simplify.

Given ratio = $3\frac{1}{2} : 2\frac{1}{3} = \frac{7}{2} : \frac{7}{3}$

$= \frac{7}{2} \times \frac{3}{7} = \frac{3}{2} = \mathbf{3 : 2}$ **(Ans.)**

Alternative method :

Multiply each term of the ratio by the L.C.M. of their consequents and then simplify.

$\therefore$ Given ratio = $\frac{7}{2} : \frac{7}{3} = \frac{7}{2} \times 6 : \frac{7}{3} \times 6$ (L.C.M. of 2 and 3 is 6)

$= 21 : 14 = \frac{21}{14} = \frac{3}{2} = \mathbf{3 : 2}$ **(Ans.)**

(ii) Given ratio = $\frac{2}{3} : \frac{4}{5} : \frac{1}{2} = \frac{2}{3} \times 30 : \frac{4}{5} \times 30 : \frac{1}{2} \times 30$ (L.C.M. of 3,5 and 2 is 30)

$= \mathbf{20 : 24 : 15}$ **(Ans.)**

11.4 RATIO *a* : *b* : *c*

Three quantities *a*, *b* and *c*, all of same kind and with the same unit, are said to be in ratio *a* : *b* : *c*, if the quantities can be taken as *ak*, *bk* and *ck* respectively, where *k* is a positive number.

i.e., for ratio *a* : *b* : *c*.

The first quantity = *ak*, the second quantity = *bk* and the third quantity = *ck*.

Example :

If A : B : C = 3 : 5 : 4 and *k* = 7

A = 3*k* = 3 × 7 = 21, B = 5*k* = 5 × 7 = 35 and C = 4*k* = 4 × 7 = 28.

In the same way,

If the ages, in years, of Geeta, Harry and John are in the ratio 6 : 8 : 7 and *k* = 2.

The age of Geeta = 6*k* = 6 × 2 years = 12 years,

the age of Harry = 8*k* = 8 × 2 years = 16 years,

and the age of John = 7*k* = 7 × 2 years = 14 years.

8.5 TO SIMPLIFY THE RATIO *a* : *b* : *c*

Example 3 :

Simplify the ratios :

(i) $\frac{2}{3} : \frac{1}{2} : \frac{4}{5}$ (ii) $\frac{1}{6} : \frac{1}{3} : \frac{1}{8}$

Solution :

(i) Since, L.C.M. of consequents (denominators) 3, 2 and 5 is 30

$$\therefore \quad \frac{2}{3} : \frac{1}{2} : \frac{4}{5} = \frac{2}{3} \times 30 : \frac{1}{2} \times 30 : \frac{4}{5} \times 30$$

$$= \mathbf{20 : 15 : 24}$$ **(Ans.)**

(ii) Since, L.C.M. of consequents (denominators) 6, 3 and 8 is 24

$$\therefore \quad \frac{1}{6} : \frac{1}{3} : \frac{1}{8} = \frac{1}{6} \times 24 : \frac{1}{3} \times 24 : \frac{1}{8} \times 24$$

$$= \mathbf{4 : 8 : 3}$$ **(Ans.)**

Example 4 :

If the ratio between $x + 3$ and $2x - 3$ is 5 : 7; find x.

Solution :

Given : $\frac{x+3}{2x-3} = \frac{5}{7} \Rightarrow 7(x + 3) = 5(2x - 3)$ [By cross multiplication]

$\Rightarrow 7x + 21 = 10x - 15$

$\Rightarrow 21 + 15 = 10x - 7x$

$\Rightarrow 36 = 3x$

and, $x = \frac{36}{3} = 12$ **(Ans.)**

Example 5 :

If $x : y = 3 : 5$, find $2x : 3y$.

Solution :

$x : y = 3 : 5 \Rightarrow \frac{x}{y} = \frac{3}{5}$

$\Rightarrow 5x = 3y$

$\Rightarrow x = \frac{3y}{5}$

$\therefore$ **2x : 3y** $= \frac{2x}{3y} = \frac{2}{3y} \times x$

$= \frac{2}{3y} \times \frac{3y}{5} = \frac{2}{5} =$ **2 : 5** **(Ans.)**

Alternative method :

$x : y = 3 : 5$

$\Rightarrow \frac{x}{y} = \frac{3}{5}$

$\Rightarrow \frac{2}{3} \times \frac{x}{y} = \frac{2}{3} \times \frac{3}{5}$

$\Rightarrow \frac{2x}{3y} = \frac{2}{5}$

i.e. **2x : 3y = 2 : 5**

EXERCISE 11(A)

1. Express each of the following ratios in its simplest form :

(a) (i) 4 : 6 (ii) 48 : 54 (iii) 200 : 250

(b) (i) 5 kg : 800 gm (ii) 30 cm : 2 m (iii) 3 m : 90 cm

(iv) 2 years : 9 months (v) 1 hour : 45 min (vi) 4 min : 45 sec

(c) (i) $1\frac{1}{2} : 2\frac{1}{2}$ (ii) $3\frac{1}{2} : 7$ (iii) $2\frac{1}{3} : 3\frac{1}{2} : 1\frac{1}{4}$

(iv) $x^2 : 4x$ (v) 2.5 : 1.5 (vi) 2.5 : 5

2. A field is 80 m long and 60 m wide. Find the ratio of its width to its length.
3. State, **true** or **false** :
   (i) A ratio equivalent to 7 : 9 is 27 : 21.
   (ii) A ratio equivalent to 5 : 4 is 240 : 192.
   (iii) A ratio of 250 gm and 3 kg is 1 : 12.
4. Is the ratio of 15 kg and 35 kg same as the ratio of 6 years and 14 years ?
5. Is the ratio of 6 g and 15 g same as the ratio of 36 cm and 90 cm ?
6. Find the ratio between 3·5 m, 475 cm and 2·8 m.
7. Find the ratio between 5 dozen and 2 scores. [1 score = 20]

11.6 WORD PROBLEMS ON RATIO

Example 6 :

The strength of a class is 50 with 30 boys and the remaining girls. Find the ratio of the number of boys to the number of girls in the class.

Solution :

Since, the strength of the class = 50

and, the number of boys in the class = 30

⇒ The number of girls in the class = 50 − 30 = 20

∴ **Required ratio** = $\frac{\text{No. of boys in the class}}{\text{No. of girls in the class}}$

$= \frac{30}{20} = \frac{3}{2} = $ **3 : 2** **(Ans.)**

Example 7 :

A man's monthly income is ₹ 15,000, out of which he spends ₹ 12,500 every month. Find the ratio of his :

(i) savings to expenditure (ii) expenditure to income

(iii) income to savings

Solution :

Since the monthly income of the man = ₹ 15,000

And his monthly expenditure = ₹ 12,500

$\Rightarrow$ His savings per month = ₹ 15,000 − ₹ 12,500 = ₹ 2,500

(i) **Ratio of savings to expenditure** $= \frac{₹\,2,500}{₹\,12,500} = \frac{1}{5} =$ **1 : 5** **(Ans.)**

(ii) **Ratio of expenditure to income** $= \frac{₹\,12,500}{₹\,15,000} = \frac{5}{6} =$ **5 : 6** **(Ans.)**

(iii) **Ratio of income to savings** $= \frac{₹\,15,000}{₹\,2,500} = \frac{6}{1} =$ **6 : 1** **(Ans.)**

Example 8 :

The ages of A and B are in the ratio 5 : 4. If B's age is 16 years, find the age of A.

Solution :

A : B = 5 : 4 $\Rightarrow$ if A = 5, B is 4
and, if B = 4, A = 5

A's age : B's age = 5 : 4

$\Rightarrow$ if B's age = 4 years, A's age is 5 years

and, if B's age = 1 year, A's age is $\frac{5}{4}$ years

$\Rightarrow$ if B's age = 16 years, **A's age** is $\frac{5}{4} \times 16$ years = **20 years** **(Ans.)**

Example 9 :

Manisha bought 8 kg rice from the market and brought it to her home. On reaching home, she found that there is a hole in the bag containing rice because of which 800 g rice is lost. Find :

(i) the ratio of rice lost to the total rice bought.

(ii) the ratio of rice lost to the rice brought home.

(iii) the ratio of the rice brought home to the total rice bought.

Solution :

Total rice bought = 8 kg
= 8000 g,

and, rice lost = 800 g

$\therefore$ Rice brought home = 8000 g − 800 g
= 7200 g

(i) **Required ratio** $= \frac{\text{Quantity of rice lost}}{\text{Total quantity of rice bought}}$

$= \frac{800\text{ g}}{8000\text{ g}} = \frac{1}{10} =$ **1 : 10** **(Ans.)**

(ii) **Required ratio** $= \dfrac{\text{Quantity of rice lost}}{\text{Quantity of rice brought home}}$

$$= \frac{800\text{ g}}{7200\text{ g}} = \frac{1}{9} = \mathbf{1 : 9} \qquad \textbf{(Ans.)}$$

(iii) **Required ratio** $= \dfrac{\text{Quantity of rice brought home}}{\text{Total quantity of rice bought}}$

$$= \frac{7200\text{ g}}{8000\text{ g}} = \frac{9}{10} = \mathbf{9 : 10} \qquad \textbf{(Ans.)}$$

Example 10 :

Coffee costs ₹ 80 per 50 g and tea costs ₹ 440 per kg. Find the ratio of costs of 1 g coffee to 1 g tea.

Solution :

$\because$ Cost of 50 g coffee = ₹ 80

Cost of 1 g coffee = ₹ $\dfrac{80}{50}$ = ₹ $\dfrac{8}{5}$

$\because$ Cost of 1 kg (1000 g) tea = ₹ 440

Cost of 1 g tea = ₹ $\dfrac{440}{1000}$ = ₹ $\dfrac{11}{25}$

Required ratio $= \dfrac{\text{Cost of 1 g coffee}}{\text{Cost of 1 g tea}}$

$$= \frac{₹\ 8/5}{₹\ 11/25} = \frac{8}{5} \times \frac{25}{11} = \mathbf{40 : 11} \qquad \textbf{(Ans.)}$$

EXERCISE 11(B)

1. The monthly salary of a person is ₹ 12,000 and his monthly expenditure is ₹ 8,500. Find the ratio of his :
   (i) salary to expenditure
   (ii) expenditure to savings
   (iii) savings to salary
2. The strength of a class is 65, including 30 girls. Find the ratio of the number of :
   (i) girls to boys
   (ii) boys to the whole class
   (iii) the whole class to girls.
3. The weekly expenses of a boy have increased from ₹ 1,500 to ₹ 2,250. Find the ratio of :
   (i) increase in expenses to original expenses.
   (ii) original expenses to increased expenses.
   (iii) increased expenses to increase in expenses.

4. Reduce each of the following ratios to their lowest terms :
   (i) 1 hour 20 min : 2 hours
   (ii) 4 weeks : 49 days
   (iii) 3 years 4 months : 5 years 5 months
   (iv) 2 m 40 cm : 1 m 44 cm
   (v) 5 kg 500 gm : 2 kg 750 gm
5. Two numbers are in the ratio 9 : 2. If the smaller number is 320, find the larger number.
6. A bus travels 180 km in 3 hours and a train travels 450 km in 5 hours. Find the ratio of speed of train to speed of bus.
7. In winters, a school opens at 10 a.m. and closes at 3.30 p.m. If the lunch interval is of 30 minutes, find the ratio of lunch interval to total time of the class periods.
8. Rohit goes to his school by car at 60 km per hour and Manoj goes to the same school by scooty at 40 km per hour. If they both live in the same locality, find the ratio between the time taken by Rohit and Manoj to reach their school.
9. In a club having 360 members, 40 play carrom, 96 play table tennis, 144 play badminton and the remaining members play volley-ball. If no member plays two or more games, find the ratio of members who play :
   (i) carrom to the number of those who play badminton.
   (ii) badminton to the number of those who play table-tennis.
   (iii) table-tennis to the number of those who play volley-ball.
   (iv) volley-ball to the number of those who play other games.
10. The length of a pencil is 18 cm and its radius is 4 cm. Find the ratio of its length to its diameter.
11. Ratio of the distance of the school from A's home and from B's home is 2 : 1.
   (i) Who lives nearer to the school ?
   (ii) Complete the following table :

Distance (in km) from A's home to school	4	–	8	–	6
Distance (in km) from B's home to the same school	–	9	–	8	–

12. The student-teacher ratio in a school is 45 : 2. If there are 4050 students in the school, how many teachers are there in the school ?

11.7 TO DIVIDE A GIVEN QUANTITY IN A GIVEN RATIO

Example 11 :

12 sweets are to be divided between A and B in the ratio 1 : 3. Find how many sweets each gets ?

Solution :

Here, A and B get sweets in the ratio 1 : 3.

This means, if all the sweets are divided in 1 + 3 = 4 equal parts,

Then, **A gets** = One part out of the 4 equal parts made

$= \frac{1}{4}$ of the total number of sweets

$= \frac{1}{4} \times 12$ sweets = **3 sweets** **(Ans.)**

And **B gets** $= 3$ parts out of the 4 equal parts made

$= \frac{3}{4}$ of the total number of sweets

$= \frac{3}{4} \times 12$ sweets $=$ **9 sweets** **(Ans.)**

> Thus, if a whole quantity is divided into two parts in the ratio 3 : 4
>
> $\therefore$ The first part $= \frac{3}{7} \times$ the whole quantity [As, 3 + 4 = 7]
>
> and the second part $= \frac{4}{7} \times$ the whole quantity

Example 12 :

A pole of length 165 cm is divided into two parts such that their lengths are in the ratio 7 : 8. Find the length of each part of the pole.

Solution :

Here, 165 cm is to be divided into two lengths in the ratio 7 : 8 and 7 + 8 = 15.

$\therefore$ **Length of one** (shorter) **part** $= \frac{7}{15} \times 165$ cm $=$ **77 cm**

and **length of the other** (longer) **part** $= \frac{8}{15} \times 165$ cm $=$ **88 cm** **(Ans.)**

Example 13 :

Divide 99 into three parts in the ratio 2 : 4 : 5.

Solution :

Since $2 + 4 + 5 = 11$

$\therefore$ **1st part** $= \frac{2}{11} \times 99 =$ **18**,

2nd part $= \frac{4}{11} \times 99 =$ **36**

and **3rd part** $= \frac{5}{11} \times 99 =$ **45** **(Ans.)**

Example 14 :

Divide 268 into two parts in the ratio $2\frac{1}{3} : 3\frac{1}{4}$.

Solution :

Given ratio $= 2\frac{1}{3} : 3\frac{1}{4} = \frac{7}{3} : \frac{13}{4} = \frac{7}{3} \times \frac{4}{13} = \frac{28}{39} = 28 : 39$

Since $28 + 39 = 67$

$\therefore$ **1st part** $= \frac{28}{67} \times 268 = 28 \times 4 =$ **112** **(Ans.)**

And **2nd part** $= \frac{39}{67} \times 268 = 39 \times 4 =$ **156** **(Ans.)**

Example 15 :

The total weight of the mixture of two things A and B is 50 kg. If A and B are mixed in the ratio 3 : 7, find the quantity of B in the mixture.

Solution :

Since A and B are mixed in the ratio 3 : 7 and 3 + 7 = 10

$\therefore$ **Quantity of B in the mixture** = $\frac{7}{10} \times 50$ kg = **35 kg** **(Ans.)**

Example 16 :

420 articles are divided among A, B and C, such that A gets three times that of B and B gets five times that of C. Find the number of articles received by B.

Solution :

Let the number of articles C gets = 1

$\Rightarrow$ The number of articles that B gets = five times that of $C = 5 \times 1 = 5$

and the number of articles that A gets = three times that of $B = 3 \times 5 = 15$

$\therefore$ $A : B : C = 15 : 5 : 1$ and $15 + 5 + 1 = 21$

$\Rightarrow$ **The no. of articles received by B** = $\frac{5}{21} \times 420 =$ **100** **(Ans.)**

EXERCISE 11(C)

1. ₹ 120 is to be divided between Hari and Gopi in the ratio 5 : 3. How much does each get ?
2. Divide 72 in the ratio $2\frac{1}{2} : 1\frac{1}{2}$.
3. Divide 81 into three parts in the ratio 2 : 3 : 4.
4. Divide ₹ 10,400 among A, B and C in the ratio 1/2 : 1/3 : 1/4.
5. A profit of ₹ 2,500 is to be shared among three persons in the ratio 6 : 9 : 10. How much does each person get ?
6. The angles of a triangle are in the ratio 3 : 7 : 8. Find the greatest and the smallest angles.

   The sum of the angles of a triangle is 180°.

7. The sides of a triangle are in the ratio 3 : 2 : 4. If the perimeter of the triangle is 27 cm, find the length of each side.
8. An alloy of zinc and copper weighs $12\frac{1}{2}$ kg. If, in the alloy, the ratio of zinc and copper is 1 : 4, find the weight of copper in it.
9. How will ₹ 31,500 be shared between A, B and C, if A gets the double of what B gets, and B gets the double of what C gets ?
10. Mr. Gupta divides ₹ 81,000 among his three children, Ashok, Mohit and Geeta, in such a way that Ashok gets four times what Mohit gets and Mohit gets 2·5 times what Geeta gets. Find the share of each of them.

11.8 COMPARING THE RATIOS

For any two ratios $\frac{a}{b}$ and $\frac{c}{d}$, if :

(i) $a \times d = b \times c \Rightarrow \frac{a}{b} = \frac{c}{d}$ *i.e.* both the ratios are equal.

(ii) $a \times d > b \times c \Rightarrow \frac{a}{b} > \frac{c}{d}$ *i.e.* $\frac{a}{b}$ is greater than $\frac{c}{d}$.

(iii) $a \times d < b \times c \Rightarrow \frac{a}{b} < \frac{c}{d}$ *i.e.* $\frac{a}{b}$ is smaller than $\frac{c}{d}$.

Example 17 :

Which ratio is greater ?

(i) $\frac{5}{7}$ or $\frac{7}{9}$ (ii) $\frac{12}{17}$ or $\frac{15}{19}$.

Solution :

(i) $\frac{5}{7}$ or $\frac{7}{9}$ $\Rightarrow 5 \times 9$ or 7×7

$\Rightarrow 45$ or 49

Since, $49 > 45$ $\Rightarrow$ **$\frac{7}{9}$ is greater.** **(Ans.)**

(ii) $\frac{12}{17}$ or $\frac{15}{19}$ $\Rightarrow 12 \times 19$ or 17×15

$\Rightarrow 228$ or 255

Since, $255 > 228$ $\Rightarrow$ **$\frac{15}{19}$ is greater.** **(Ans.)**

Example 18 :

Which ratio is smaller ?

(i) $\frac{5}{8}$ or $\frac{8}{11}$ (ii) $\frac{7}{18}$ or $\frac{9}{20}$.

Solution :

(i) $\frac{5}{8}$ or $\frac{8}{11}$ $\Rightarrow 5 \times 11$ or 8×8

$\Rightarrow 55$ or 64

Since, $55 < 64$ $\Rightarrow$ **$\frac{5}{8}$ is smaller.** **(Ans.)**

(ii) $\frac{7}{18}$ or $\frac{9}{20}$ $\Rightarrow$ 7×20 or 9×18

$\Rightarrow$ 140 or 162

Since, $140 < 162$ $\Rightarrow$ $\frac{7}{18}$ **is smaller.** **(Ans.)**

11.9 INCREASE OR DECREASE IN A GIVEN RATIO

1. If a given quantity is increased in the ratio $a : b$ (where $b > a$);
   the new (resulting) quantity = $\frac{b}{a}$ × the given quantity.
2. If a given quantity is decreased in the ratio $a : b$ (where $b < a$);
   the new (resulting) quantity = $\frac{b}{a}$ × the given quantity.

Example 19 :

Increase 342 in the ratio 3 : 4.

Solution :

The increased quantity = $\frac{4}{3}$ × the given quantity

$= \frac{4}{3} \times 342 =$ **456** **(Ans.)**

Example 20 :

Decrease 575 in the ratio 5 : 2.

Solution :

The decreased quantity= $\frac{2}{5}$ × the given quantity

$= \frac{2}{5} \times 575 =$ **230** **(Ans.)**

Example 21 :

First of all decrease 800 in the ratio 5 : 3 and then increase the result in the ratio 2 : 5.

Solution :

The given quantity = 800 and decrease in the ratio is 5 : 3.

$\therefore$ The decreased quantity = $\frac{3}{5} \times 800 = 480$

Now, the quantity is increased in the ratio 2 : 5.

$\therefore$ **The resulting quantity** = $\frac{5}{2} \times 480 =$ **1200** **(Ans.)**

EXERCISE 11(D)

1. Which ratio is greater :

   (i) $\frac{8}{15}$ or $\frac{5}{9}$ (ii) $\frac{3}{7}$ or $\frac{6}{13}$

2. Which ratio is smaller :

   (i) $\frac{9}{17}$ or $\frac{8}{15}$ (ii) $\frac{7}{15}$ or $\frac{15}{32}$

3. Increase 95 in the ratio 5 : 8.
4. Decrease 275 in the ratio 11 : 7.
5. Decrease 850 in the ratio 17 : 6 and then increase the result in the ratio 5 : 9.
6. Decrease 850 in the ratio 17 : 6 and then decrease the resulting number in the ratio 4 : 3.
7. Increase 1200 in the ratio 2 : 3 and then decrease the resulting number in the ratio 10 : 3.
8. Increase 1200 in the ratio 3 : 7 and then increase the resulting number again in the ratio 4 : 7.
9. The number 650 is decreased to 500 in the ratio $a : b$; find the ratio $a : b$.
10. The number 800 is increased to 960 in the ratio $a : b$; find the ratio $a : b$.

PROPORTION 12

(Including Word Problems)

12.1 CONCEPT

Consider the following examples :

1. What is the ratio of the number of boys to the number of girls in a group of 8 boys and 12 girls ?

   The required ratio $= \frac{\text{Number of boys}}{\text{Number of girls}} = \frac{8}{12} = \frac{2 \times 4}{3 \times 4} = \mathbf{\frac{2}{3}}$

2. What is the ratio of ₹ 18 to ₹ 27 ?

   The required ratio $= \frac{₹\,18}{₹\,27} = \frac{2 \times 9}{3 \times 9} = \mathbf{\frac{2}{3}}$

It is observed in the examples given above that the ratios $\frac{8}{12}$ and $\frac{18}{27}$ are equal.

i.e. $\frac{8}{12} = \frac{18}{27}$ or **8 : 12 = 18 : 27**

Such an equality of two ratios is called a **proportion** and is read as :

"8 is to 12 as 18 is to 27".

This can be expressed by writing :

8 : 12 : : 18 : 27 or 8 : 12 = 18 : 27

Thus, a proportion is an expression which states that the two given ratios are equal.

The numbers 8, 12, 18 and 27 that are used in the proportion are called its **terms,** *i.e.* **8** is the **first term, 12** is the **second term, 18** is the **third term** and **27** is the **fourth term** of the proportion **8 : 12 = 18 : 27.**

In general, the symbol for representing a proportion is " : : ".

Example 1 :

Check whether or not the two ratios form a proportion.

(i) ₹ 6 : ₹ 8 and 12 kg : 16 kg (ii) 6 kg : 9 kg and 10 m : 16 m

Solution :

(i) Since ₹ 6 : ₹ 8 $= \frac{6}{8} = \frac{3}{4}$ and 12 kg : 16 kg $= \frac{12}{16} = \frac{3}{4}$

∴ **Ratios ₹6 : ₹8 and 12 kg : 16 kg** are **equal**, they **form a proportion.**

(Ans.)

(ii) Since, 6 kg : 9 kg $= \frac{6}{9} = \frac{2}{3}$ and 10 m : 16 m $= \frac{10}{16} = \frac{5}{8}$

∴ **Ratio 6 kg : 9 kg ≠ ratio 10 m : 16 m, they do not form a proportion.**

(Ans.)

1. *In a proportion, the* **first two terms** *(quantities) must be of the same kind and of the same unit, whereas the last* **two terms** *(quantities) must also be of the same kind and of the same unit.*

   [All the* four *quantities in a proportion may be of the same kind and the same unit.]

2. *In a proportion, the* **first** *and the* **fourth terms** *are called* **extremes** *whereas the* **second** *and the* **third terms** *are called* **means.**

   Thus, in 8 : 12 = 18 : 27; the terms **12** *and* **18** *are the* **means** *and* **8** *and* **27** *are the* **extremes.**

   Also **Product of extremes = Product of means**

3. *In a : b = c : d; d is known as* ***fourth proportional.***

Example 1 :

Check whether or not the given ratios form a proportion :

(i) 15 : 24 and 35 : 56 (ii) $2\frac{1}{4} : 5\frac{2}{5}$ and $3\frac{1}{3} : 4\frac{1}{6}$.

Solution :

(i) Product of extremes = 15 × 56 = 840

and product of means = 24 × 35 = 840

Since, product of extremes = product of means

⇒ **The given two ratios form a proportion** **(Ans.)**

(ii) Product of extremes $= 2\frac{1}{4} \times 4\frac{1}{6} = \frac{9}{4} \times \frac{25}{6} = \frac{75}{8}$

and product of means $= 5\frac{2}{5} \times 3\frac{1}{3} = \frac{27}{5} \times \frac{10}{3} = \frac{18}{1}$

⇒ product of extremes ≠ product of means

⇒ **The given two ratios do not form a proportion** **(Ans.)**

Example 3 :

(i) The numbers 8, x, 9 and 36 are in proportion. Find x.

(ii) If x : 15 = 8 : 12, find x.

Solution :

(i) The numbers 8, x, 9 and 36 are in proportion

⇒ 8 : x = 9 : 36

⇒ x × 9 = 8 × 36 (Product of means = Product of extremes)

⇒ $\mathbf{x} = \frac{8 \times 36}{9} = \mathbf{32}$ **(Ans.)**

(ii) x : 15 = 8 : 12 ⇒ x × 12 = 15 × 8

⇒ $\mathbf{x} = \frac{15 \times 8}{12} = \mathbf{10}$ **(Ans.)**

Example 4 :

The first, third and fourth terms of a proportion are 12, 8 and 14, respectively. Find the second term.

Solution :

Let the second term be x.

$\therefore$ 12, x, 8 and 14 are in proportion, *i.e.* $12 : x = 8 : 14$

$$\Rightarrow \quad x \times 8 = 12 \times 14$$

$$\Rightarrow \quad x = \frac{12 \times 14}{8} = 21$$

$\therefore$ **The second term** of the proportion **is 21**. **(Ans.)**

Example 5 :

The ratio of the length and the width of a sheet of paper is 3 : 2. If the length is 12 cm, find the width.

Solution :

Let width = x cm

The ratio of length to width = 12 : x [Given length = 12 cm]

According to the given statement, 12 : x = 3 : 2

$$\Rightarrow \quad x \times 3 = 12 \times 2$$

$$\Rightarrow \quad x = \frac{12 \times 2}{3} = 8$$

$\therefore$ **Width = 8 cm** **(Ans.)**

EXERCISE 12(A)

1. In each of the following, check whether or not the given ratios form a proportion :
   (i) 8 : 16 and 12 : 15 (ii) 16 : 28 and 24 : 42 (iii) 12 ÷ 3 and 8 ÷ 2
   (iv) 25 : 40 and 20 : 32 (v) $\frac{15}{18}$ and $\frac{10}{12}$ (vi) $\frac{7}{8}$ and 14 : 16
2. Find the value of x in each of the following proportions :
   (i) x : 4 = 6 : 8 (ii) 14 : x = 7 : 9 (iii) 4 : 6 = x : 18
   (iv) 8 : 10 = x : 25 (v) 5 : 15 = 4 : x (vi) 16 : 24 = 6 : x
3. Find the value of x so that the given four numbers are in proportion :
   (i) x, 6, 10 and 15 (ii) x, 4, 15 and 30 (iii) 2, x, 10 and 25
   (iv) 4, x, 6 and 18 (v) 9, 12, x and 8 (vi) 4, 10, 36 and x.
4. The first, second and the fourth terms of a proportion are 6, 18 and 75, respectively. Find its third term.
5. Find the second term of the proportion whose first, third and fourth terms are 9, 8 and 24, respectively.
6. Find the fourth term of the proportion whose first, second and third terms are 18, 27 and 32 respectively.
7. The ratio of the length and the width of a school ground is 5 : 2. Find the length, if the width is 40 metres.
8. The ratio of the sale of eggs on a Sunday and that of the whole week at a grocery shop was 2 : 9. If the total value of the sale of eggs in the same week was ₹ 360, find the value of the sale of eggs that Sunday.

9. The ratio of copper and zinc in an alloy is 9 : 8. If the weight of zinc in the alloy is 9·6 kg, find the weight of copper in the alloy.

10. The ratio of the number of girls to the number of boys in a school is 2 : 5. If the number of boys is 225, find :
    (i) the number of girls in the school.
    (ii) the number of students in the school.

11. In a class, one out of every 5 students passes. If there are 225 students in all the sections of a class, find how many have passed ?

12. Make set of all possible proportions from the numbers 15, 18, 35 and 42.

(i) 15 : 18 : : 35 : 42	(ii) 15 : 35 : : 18 : 42
(iii) 42 : 18 = 35 : 15	(iv) 42 : 35 = 18 : 15

12.2 CONTINUED PROPORTION

Three terms *a*, *b* and *c* (all of the same type and in the same unit) are said to be in continued proportion, if :

$$a : b = b : c$$

Here the **repeating term *b*** is called the **mean proportional** between *a* and *c* and the **third term *c*** is called the **third proportional**.

If *a*, *b* and *c* are in continued proportion

$$a : b = b : c \Rightarrow \frac{a}{b} = \frac{b}{c} \Rightarrow b \times b = a \times c$$

$$\Rightarrow \quad \boldsymbol{b^2 = ac} \text{ and } \boldsymbol{b = \sqrt{ac}}$$

Example 6 :

Find the mean proportional between

(i) $\frac{1}{18}$ and $\frac{1}{8}$ (ii) 2·8 and 0·7

Solution :

(i) Let the required mean proportional be *x*.

$\therefore$ $\frac{1}{18}$, x and $\frac{1}{8}$ are in continued proportion.

$\Rightarrow \frac{1}{18} : x : : x : \frac{1}{8}$

$\Rightarrow x \times x = \frac{1}{18} \times \frac{1}{8}$ i.e. $x^2 = \frac{1}{144}$

$\Rightarrow x = \sqrt{\frac{1}{144}} = \frac{1}{12}$

$\therefore$ **Required mean proportional** = $\frac{1}{12}$ **(Ans.)**

(ii) Let the required mean proportional be x.

$\therefore$ 2·8, x and 0·7 are in continued proportion

$\Rightarrow$ $2·8 : x = x : 0·7$

$\Rightarrow$ $x \times x = 2·8 \times 0·7$ *i.e.* $x^2 = 1·96$

$\Rightarrow$ $x = \sqrt{1·96} = 1·4$

$\therefore$ **Required mean proportional = 1·4** **(Ans.)**

Example 7 :

Find the third proportional to

(i) ₹ 15 and ₹ 45 (ii) 16 and 36.

Solution :

(i) Let the required third proportional be ₹ x.

$\therefore$ ₹ 15, ₹ 45 and ₹ x are in continued proportion

That is, $15 : 45 = 45 : x$

$\Rightarrow$ $15x = 45 \times 45$ *i.e.* $x = 135$

$\therefore$ **Required third proportional = ₹ 135** **(Ans.)**

(ii) Let the required third proportional be x.

$\therefore$ 16, 36 and x are in continued proportion

$\Rightarrow$ $16 : 36 = 36 : x$ *i.e.* $16x = 36 \times 36$

$\Rightarrow$ $x = \dfrac{36 \times 36}{16} = 81$

$\therefore$ **Required third proportional = 81** **(Ans.)**

EXERCISE 12(B)

1. If x, y and z are in continued proportion, then which of the following is true :
   (i) $x : y = x : z$ (ii) $x : x = z : y$
   (iii) $x : y = y : z$ (iv) $y : x = y : z$
2. Which of the following numbers are in continued proportion :
   (i) 3, 6 and 15 (ii) 15, 45 and 48
   (iii) 6, 12 and 24 (iv) 12, 18 and 27
3. Find the mean proportional between
   (i) 3 and 27 (ii) 0·06 and 0·96
4. Find the third proportional to :
   (i) 36 and 18 (ii) 5·25 and 7 (iii) ₹ 1.60 and ₹ 0.40
5. The ratio between 7 and 5 is same as the ratio between ₹ x and ₹ 20.50; find the value of x.

6. If $(4x + 3y) : (3x + 5y) = 6 : 7$, find :

(i) $x : y$ (ii) x, if $y = 10$ (iii) y, if $x = 27$

(i) $(4x + 3y) : (3x + 5y) = 6 : 7$

$\Rightarrow \quad \frac{4x+3y}{3x+5y} = \frac{6}{7}$

$\Rightarrow \quad 7 \times (4x + 3y) = 6 \times (3x + 5y)$

$\Rightarrow \quad 28x + 21y = 18x + 30y$

$\Rightarrow \quad 28x - 18x = 30y - 21y$

$\Rightarrow \quad 10x = 9y$

$\Rightarrow \quad \frac{x}{y} = \frac{9}{10}$ *i.e.* $\mathbf{x : y = 9 : 10}$ **(Ans.)**

7. If $\frac{2y+5x}{3y-5x} = 2\frac{1}{2}$, find :

(i) $x : y$ (ii) x, if $y = 70$ (iii) y, if $x = 33$

EXERCISE 12(C)

1. Are the following numbers in proportion ?

(i) 32, 40, 48 and 60 (ii) 12, 15, 18 and 20

2. Find the value of x in each of the following such that the given numbers are in proportion.

(i) 14, 42, x and 75 (ii) 45, 135, 90 and x

3. The costs of two articles are in the ratio 7 : 4. If the cost of the first article is ₹ 2,800, find the cost of the second article.

If the cost of the second article is ₹ x $\Rightarrow 7 : 4 = 2800 : x$

4. The ratio of the length and the width of a rectangular sheet of paper is 8 : 5. If the width of the sheet is 17·5 cm, find its length.

Let length = x cm $\Rightarrow 8 : 5 = x : 17{\cdot}5$

5. The ages of A and B are in the ratio 6 : 5. If A's age is 18 years, find the age of B.
6. A sum of ₹ 10,500 is divided among A, B and C in the ratio 5 : 6 : 4. Find the share of each.
7. Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion ?
8. Do the ratios 2 kg : 80 kg and 25 g : 625 g form a proportion ?
9. 10 kg sugar costs ₹ 350. If x kg sugar of the same kind costs ₹ 175, find the value of x.

10 kg : x kg = ₹ 350 : ₹ 175

$\Rightarrow \quad 10 \times 175 = x \times 350$

i.e. $\quad x = \frac{10 \times 175}{350} = 5$

10. The lengths of two ropes are in the ratio 7 : 5. Find the length of :

(i) shorter rope, if the longer one is 22·5 m (ii) longer rope, if the shorter is 9·8 m.

11. If 4, x and 9 are in continued proportion, find the value of x.
12. If 25, 35 and x are in continued proportion, find the value of x.

UNITARY METHOD 13

13.1 INTRODUCTION

Consider the following examples :

1. Cost of 12 pens = ₹ 96

$\Rightarrow$ Cost of 1 pen = ₹ $\frac{96}{12}$ = ₹ 8

and, cost of 5 pens = ₹ 8 × 5 = ₹ 40

2. Weight of 6 indentical articles = 2·4 kg

$\Rightarrow$ Weight of 1 article = $\frac{2 \cdot 4}{6}$ kg = 0·4 kg

and, weight of 8 same type of articles

= 0·4 kg × 8 = 3·2 kg.

> The method in which we first find the value of unit (single) quantity and then use it to find the value of any required quantity, is called the **unitary method**.

Example 1 :

Cost of 15 pens is ₹ 360; what is the cost of 8 such pens ?

Solution :

$\because$ Cost of 15 pens = ₹ 360

$\Rightarrow$ cost of 1 pen = ₹ $\frac{360}{15}$ = ₹ 24

And, **cost of 8 such pens** = 8 × ₹ 24 = **₹ 192** **(Ans.)**

Example 2 :

For ₹ 384, a man can buy 12 articles. How many articles can he buy for ₹ 512 ?

Solution :

For ₹ 384, the man can buy 12 articles

$\Rightarrow$ for ₹ 1, he can buy $\frac{12}{384}$ articles

And, for ₹ 512, he can buy $\frac{12}{384}$ × 512 articles = **16 articles** **(Ans.)**

1. While applying unitary method, statement is made in such a way that, whatever is required to find is written at the end of the statement.

   In example 1, given above, we are to find the cost of 8 pens, therefore cost must come at the end of the statement.

   In example 2, given above, we are to find the number of articles bought for ₹ 512; therefore number of articles must come at the end of the statement.

2. Now we find the unit of the other quantity which is written at the beginning of the statement.

   In example 1, given above, the cost of unit (one) pen is found to get the cost of total number of pens.

   Similarly, in example 2, given above, the number of articles bought for ₹ 1 is found to get the number of articles bought for the total money.

Example 3 :

If 25 identical articles weigh 275 g; find how many articles will weigh 990 g ?

Solution :

In this example, we have to find the number of articles, so the statement will be formed in such a way that number of articles comes at the end of the statement and the other quantity (weight of the articles) at the beginning of the statement.

Since, 275 g is the weight of 25 articles

$\Rightarrow$ 1 g is the weight of $\frac{25}{275}$ articles

And, **990 g is the weight of** $\frac{25}{275} \times 990$ articles = **90 articles** **(Ans.)**

Example 4 :

18 men can make 90 identical tables in one day. Find, how many men will make 20 such tables, in one day.

Solution :

In one day, 90 tables are made by 18 men

$\Rightarrow$ In one day, 1 table is made by $\frac{18}{90}$ men

And, in one day, **20 tables are made by** $\frac{18}{90} \times 20$ men = **4 men** **(Ans.)**

Unitary method consists of two types of variations :

1. Direct variation
2. Inverse variation

1. **Direct variation :** Two quantities are said to have **direct variation**, if on increasing one quantity, the other quantity also increases and on decreasing one quantity, the other quantity also decreases.

For example :

Number of articles and their cost. Clearly, if the number of articles is more, their cost is more and if the number of articles is less, their cost is less.

2. **Inverse variation :** Two quantities are said to have **inverse variation**, if :
   (i) on increasing one quantity, the other quantity decreases.
   (ii) on decreasing one quantity, the other quantity increases.

For example :

Number of men and the number of days taken by them to complete a certain work. Clearly, if the number of men is more (increased), the number of days taken by them to complete the same work, is less (decreased). And, if the number of men is less (decreased), the number of days, taken by them to complete the same work, is more (increased).

13.2 USING DIRECT VARIATION

Example 5 :

A car, running with uniform speed, covers a distance of 96 km in 3 hours. How much distance will the car cover in 5 hours, running with the same speed ?

Solution :

$\because$ In 3 hours, car covers 96 km

$\Rightarrow$ In 1 hour, car covers $\frac{96}{3}$ km = 32 km

And, **in 5 hours, car covers** = 32 km × 5 = **160 km** **(Ans.)**

Example 6 :

A car can travel 360 km, consuming 24 litres of petrol. How much petrol will it consume while travelling through a distance of 480 km ?

Solution :

$\because$ Car can travel 360 km consuming 24 litres of petrol

$\Rightarrow$ Car can travel 1 km consuming $\frac{24}{360}$ litres of petrol

And, **car can travel 480 km consuming** = $\frac{24}{360}$ × 480 litres of petrol

= **32 litres of petrol** **(Ans.)**

Example 7 :

A car covers a distance of 30 km consuming 2 litres of petrol, whereas a motorcycle covers a distance of 90 km consuming 1·2 litres of petrol.

(i) How much distance will each cover after consuming 3 litres of petrol ?

(ii) Which of these two will cover more distance (each consuming 3 litres of petrol) and by how much ?

Solution :

(i) Consuming 2 litres petrol, car covers 30 km

$\Rightarrow$ Consuming 1 litre petrol, car covers $\frac{30}{2}$ km = 15 km.

And, **consuming 3 litres petrol, car covers 15 km × 3 = 45 km** **(Ans.)**

Now, consuming 1·2 litres petrol, motorcycle covers 90 km

$\Rightarrow$ Consuming 1 litre petrol, motorcycle covers $\frac{90}{1 \cdot 2}$ km = 75 km

And, **consuming 3 litres of petrol, motorcycle covers 75 × 3 km = 225 km**

(ii) **Motorcycle covers more distance and by 225 km – 45 km = 180 km**

(Ans.)

EXERCISE 13(A)

1. The price of 25 identical articles is ₹ 1,750. Find the price of :
   (i) one article (ii) 13 articles
2. A motorbike travels 330 km consuming 5 litres of petrol. How much distance will it cover consuming :
   (i) one litre of petrol ? (ii) 2·5 litres of petrol ?
3. If the cost of one dozen identical soap-bars is ₹ 460.80, what will be the cost of :
   (i) each soap-bar ? (ii) 15 soap-bars ?
   (iii) 3 dozen soap-bars ?
4. The cost of 35 envelopes is ₹ 105. How many envelopes can be bought for ₹ 90 ?
5. If the cost of 8 cans of juice is ₹ 280, what will be the cost of 6 cans of juice ?.
6. For ₹ 378, 9 cans of juice can be bought. How many cans of juice can be bought for ₹ 504 ?
7. A motorbike travels 425 km in 5 hours. How much distance will it cover in 3·2 hours ?
8. If the cost of one dozen articles of the same kind is ₹ 672, what will be the cost of 18 such articles ?
9. A car covers a distance of 180 km in 5 hours.
   (i) How much distance will the car cover in 3 hours with the same speed ?
   (ii) How much time will the car take to cover 54 km with the same speed ?
10. If it has rained 276 cm in the last 3 days, how many cm of rain will fall in one week (7 days) ?
    Assume that the rain continues to fall at the same rate.
11. The cost of 10 kg of wheat is ₹ 180;
    (i) what will 18 kg of wheat cost ?
    (ii) what quantity of wheat can be purchased for ₹ 432 ?
12. Rohit buys 10 pens, all of one brand, for ₹ 150 and Manoj buys 14 pens, of some other brand, for ₹ 168. Who bought the pens cheaper ?
13. A tree 24 m high casts a shadow of 15 m. At the same time, another tree casts a shadow of 6 m. Find the height of the second tree.
14. A loaded truck travels 18 km in 25 minutes. If the speed remains the same, how far will it travel in 5 hours ?

EXERCISE 13(B)

1. Weight of 15 identical books is 6 kg. What is the weight of 45 such books ?
2. A made 84 runs in 6 overs and B made 126 runs in 7 overs. Who made more runs per over ?
3. Geeta types 108 words in 6 minutes. How many words would she type in half an hour ?
4. The temperature dropped 18 degree celsius in the last 24 days. If the rate of temperature drop remains the same, how many degrees will the temperature drop in the next 18 days ?
5. Mr. Chopra pays ₹ 12,000 as rent for 3 months. How much does he pay for a year, if the rent per month remains same ?
6. A truck requires 108 litres of diesel for covering a distance of 1188 km. How much diesel will be required by the truck to cover a distance of 3300 km ?
7. If a deposit of ₹ 2,000 earns an interest of ₹ 500 in 3 years, how much interest would a deposit of ₹ 36,000 earn in 3 years with the same rate of simple interest ?
8. If John walks 250 steps to cover a distance of 200 metres, find the distance covered by him in 350 steps.
9. 25 metres of cloth costs ₹ 1,012.50.
   (i) What will be the cost of 20 metres of cloth of the same type ?
   (ii) How many metres of cloth of the same kind can be bought for ₹ 1,620 ?
10. In a particular week, a man works for 48 hours and earns ₹ 4,320. But in the next week he worked 6 hours less, how much has he earned in this week ?

FRACTIONS 14

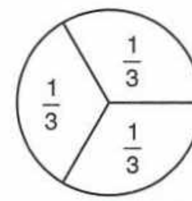
14.1 BASIC CONCEPT

If a certain quantity of rice is divided into four equal parts, each part so obtained is said to be one-fourth $\left(\frac{1}{4}\right)$ of the whole quantity of the rice.

Similarly, if an apple is divided into five equal parts, each part is one-fifth $\left(\frac{1}{5}\right)$ of the whole apple. Now, if two parts of these 5 equal parts are eaten, three parts are left and we say three-fifths $\left(\frac{3}{5}\right)$ of the apple is left.

The numbers $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{3}{5}$ discussed above, each representing a part of the whole quantity, are called **fractions**.

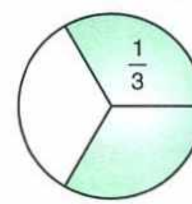
$\therefore$ ***A fraction is a quantity that expresses a part of the whole.***



To make the concept of fractions more clear :

Draw a circle with any suitable radius.

Divide the circle into three equal parts (sectors).



If two parts of the three equal parts be shaded, we say $\frac{2}{3}$ (two-thirds) of the circle is shaded and $\frac{1}{3}$ (one-third) of the circle is not.

In the fraction $\frac{a}{b}$, a is the **numerator** of the fraction and b is its **denominator**.

$$\therefore \text{ FRACTION} = \frac{\text{NUMERATOR}}{\text{DENOMINATOR}}$$

Thus, in fraction $\frac{7}{11}$, numerator = 7 and denominator = 11.

Important : In fraction $\frac{a}{b}$, the numerator a is always a whole number while the denominator b is a natural number. Fractions with denominator = 0 are not defined.

Note :

1. The **numerator** and the **denominator** are also known as the terms of a fraction.
2. Every fraction must be expressed in its lowest terms. In other words, the terms of a fraction must not have any common factor except 1(one).

   Fractions $\frac{3}{7}$, $\frac{15}{11}$ and $\frac{7}{10}$ are in their lowest terms, because the terms of each of these fractions have only 1 (one) as common factor.
3. **5 out of 7** means a given quantity is divided into seven equal parts and five of these equal parts are taken. Thus, 5 out of 7 = $\frac{5}{7}$.

4. The denominator of whole numbers is 1.

e.g. $0 = \frac{0}{1}$, $3 = \frac{3}{1}$, $105 = \frac{105}{1}$ and so on.

6. The value of a fraction is zero if its numerator is zero but the denominator is not equal to zero.

e.g. $\frac{0}{7} = 0$, $\frac{0}{29} = 0$, $\frac{0}{113} = 0$ and so on.

14.2 TYPES OF FRACTIONS

1. *Proper Fraction :*

A fraction whose ***numerator*** is ***less*** than its ***denominator*** is called a ***proper fraction***, e.g. $\frac{4}{5}, \frac{3}{7}, \frac{101}{235}, \frac{4}{7}, \frac{9}{14}$, etc.

2. *Improper Fraction :*

A fraction whose ***numerator*** is ***greater than or equal to*** its ***denominator*** is called an ***improper fraction***.

e.g. (i) $\frac{7}{5}, \frac{25}{12}, \frac{181}{62}$, etc. — Numerator is greater than denominator

(ii) $\frac{3}{3}, \frac{4}{4}, \frac{5}{5}$, etc. — Numerator is equal to denominator

If the numerator and the denominator of a fraction are equal, the value of the fraction is unity (1). e.g. $\frac{4}{4} = 1$, $\frac{3}{3} = 1$, etc.

3. *Mixed Fraction :*

A mixed fraction consists of *two* parts : (i) a natural number and (ii) a proper fraction.

e.g. $4\frac{2}{3}$ is a mixed fraction, consisting of a natural number (4) and a proper fraction $\left(\frac{2}{3}\right)$.

$3\frac{2}{5} = 3 + \frac{2}{5}$, $8\frac{5}{6} = 8 + \frac{5}{6}$, $2\frac{1}{8} = 2 + \frac{1}{8}$ *and so on.*

Conversely, $2 + \frac{3}{8} = 2\frac{3}{8}$, $7 + \frac{4}{9} = 7\frac{4}{9}$, $8 + \frac{5}{6} = 8\frac{5}{6}$ *and so on.*

4. *Like and Unlike Fractions :*

Two or more fractions with the ***same denominator*** but ***different numerators*** are called ***like fractions***.

e.g. $\frac{3}{5}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{7}{5}$, etc. are like fractions.

Two or more fractions with different ***denominators*** are called ***unlike fractions***.

e.g. $\frac{5}{9}, \frac{7}{8}, \frac{3}{4}, \frac{1}{3}$, etc.

5. *Equivalent Fractions :*

If two or more ***fractions*** have the ***same value***, they are called ***equivalent*** or ***equal fractions***.

e.g.the fractions $\frac{1}{3}, \frac{3}{9}, \frac{6}{18}$ and $\frac{9}{27}$ are equivalent fractions as $\frac{1}{3} = \frac{3}{9} = \frac{6}{18} = \frac{9}{27}$.

The value of a fraction does not change if its numerator and denominator are both *multiplied or divided by the same non-zero number.*

e.g. $\frac{4}{7}$ and $\frac{4\times2}{7\times2}$ *i.e.* $\frac{4}{7}$ and $\frac{8}{14}$ are equivalent fractions.

Also, $\frac{15}{20}$ and $\frac{15\div5}{20\div5}$ *i.e.* $\frac{15}{20}$ and $\frac{3}{4}$ are equivalent fractions.

14.3 CONVERTING A MIXED FRACTION INTO AN IMPROPER FRACTION

Multiply the natural number part by the denominator and add the numerator to the product. The result so obtained is the numerator of the required improper fraction.

The denominator of the required fraction will be the same as the denominator of the given mixed fraction.

Thus, for the mixed fraction $3\frac{7}{15}$,

the **required improper fraction** $= \frac{\text{Natural number part} \times \text{Denominator} + \text{Numerator}}{\text{Denominator}}$

$= \frac{(3\times15)+7}{15} = \frac{45+7}{15} = \frac{52}{15}$

Similarly, $5\frac{3}{4} = \frac{5\times4+3}{4} = \frac{20+3}{4} = \frac{23}{4}$

$7\frac{5}{6} = \frac{7\times6+5}{6} = \frac{42+5}{6} = \frac{47}{6}$ and so on.

14.4 CONVERTING AN IMPROPER FRACTION INTO A MIXED FRACTION

Divide the numerator by the denominator. The quotient of this division is the integral part and the remainder obtained is the numerator of the required mixed fraction.

Of course, the denominator will remain the same.

Thus, $\frac{23}{4}$ = Quotient $\frac{\text{Remainder}}{\text{Denominator}} = 5\frac{3}{4}$

4) 23 (5
−20
3

On dividing 23 by 4, quotient = 5 and remainder = 3.

Similarly, $\frac{37}{8}$ = Quotient $\frac{\text{Remainder}}{\text{Denominator}} = 4\frac{5}{8}$

8) 37 (4
−32
5

$\frac{41}{9} = 4\frac{5}{9}$, $\frac{73}{12} = 6\frac{1}{12}$ and so on.

14.5 CONVERTING UNLIKE FRACTIONS INTO LIKE FRACTIONS

Steps :

1. Find the L.C.M. of the denominators of all the given fractions.
2. Multiply the numerator and the denominator of each fraction by a same suitable number so that the denominator of each fraction becomes equal to the L.C.M. obtained in step 1.

Example 1 :

Convert $\frac{3}{7}$, $\frac{4}{5}$ and $\frac{1}{3}$ into like fractions.

Solution :

L.C.M. of denominators 7, 5 and 3 = 105 [Step 1]

Now, $\frac{3}{7} = \frac{3\times15}{7\times15} = \frac{45}{105}$, $\frac{4}{5} = \frac{4\times21}{5\times21} = \frac{84}{105}$ and $\frac{1}{3} = \frac{1\times35}{3\times35} = \frac{35}{105}$ [Step 2]

$\therefore$ $\frac{3}{7}$, $\frac{4}{5}$ and $\frac{1}{3} = \frac{45}{105}$, $\frac{84}{105}$ and $\frac{35}{105}$ respectively **(Ans.)**

EXERCISE 14(A)

1. For each expression, given below, **write a fraction :**
   (i) 2 out of 7 = ... (ii) 5 out of 17 = ... (iii) three-fifths = ...
2. Fill in the blanks :
   (i) $\frac{5}{8}$ is .... fraction. (ii) $\frac{8}{5}$ is .... fraction.
   (iii) $\frac{15}{15}$ is .... fraction. (iv) The value of $\frac{5}{5}$ = ...
   (v) The value of $\frac{5}{5}$ = ... (vi) $3\frac{3}{10}$ is .... fraction.
   (vii) $\frac{2}{15}$ and $\frac{7}{15}$ are .... fractions. (viii) $\frac{23}{12}$ and $\frac{23}{15}$ are .... fractions.
   (ix) $\frac{6}{15}$ and $\frac{28}{70}$ are .... fractions. (x) $\frac{8}{24}$ and $\frac{8}{32}$ are not .... fractions.
   (xi) $3\frac{2}{13} = \frac{3\times13+.......}{13}$ = ... (xii) $4\frac{3}{5}$ = ... = ...
3. From the following fractions, separate (i) **proper fractions** and (ii) **improper fractions :**
   $\frac{2}{9}$, $\frac{4}{3}$, $\frac{7}{15}$, $\frac{11}{20}$, $\frac{20}{11}$, $\frac{18}{23}$, $\frac{27}{35}$.
4. **Change** the following mixed fractions **to improper fractions :**
   (i) $2\frac{1}{5}$ (ii) $3\frac{1}{4}$ (iii) $7\frac{1}{8}$ (iv) $2\frac{1}{11}$
5. **Change** the following improper fractions **to mixed fractions :**
   (i) $\frac{100}{17}$ (ii) $\frac{81}{11}$ (iii) $\frac{209}{7}$ (iv) $\frac{113}{15}$
6. **Change** the following groups of fractions **to like fractions :**
   (i) $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{4}$, $\frac{1}{6}$ (ii) $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{3}{10}$ (iii) $\frac{2}{7}$, $\frac{7}{8}$, $\frac{5}{14}$, $\frac{9}{16}$

14.6 REDUCING A FRACTION TO ITS LOWEST TERMS

A fraction is said to be in its lowest terms if its numerator and denominator have no common factor other than 1, *i.e.* the numerator and the denominator are co-prime.

To reduce a fraction to its lowest terms :

(i) find the H.C.F. of its numerator and denominator.

(ii) divide each term of the fraction by the H.C.F. obtained in step (i).

For example :

Consider the fraction $\frac{48}{60}$.

As the H.C.F. of 48 and 60 is 12, divide both numerator and denominator by 12.

Thus, $\frac{48}{60} = \frac{48 \div 12}{60 \div 12} = \frac{4}{5}$, ***which is the fraction in its lowest terms.***

Similarly, $= \frac{45 \div 15}{75 \div 15} = \frac{3}{5}$

H.C.F. of 45 and 75 is 15

If both the terms of a fraction are divided or multiplied by the same number, the value of the fraction remains unchanged.

Alternative method :

First express each term of the given fraction as a product of prime factors; then cancel the common factors.

e.g. $\frac{48}{60} = \frac{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times 5} = \frac{2 \times 2}{5} = \frac{4}{5}$.

In the same way, $\frac{45}{75} = \frac{\cancel{3} \times 3 \times \cancel{5}}{\cancel{3} \times \cancel{5} \times 5} = \frac{3}{5}$ and so on.

14.7 COMPARING FRACTIONS

Comparing fractions means comparing their values to find out which of them is greater or smaller.

Example 2 :

Which fraction is greater $\frac{3}{8}$ or $\frac{5}{12}$?

Solution :

Step 1 :

1. Convert the given fractions to like fractions.
2. The fraction with the greater numerator is greater.

If two fractions have the same denominator, the greater fraction has the greater numerator

Since the L.C.M. of the denominators 8 and 12 = 24,

$\therefore \quad \frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$ and $\frac{5}{12} = \frac{5 \times 2}{12 \times 2} = \frac{10}{24}$.

Step 2 :

See the numerators of these like fractions. The numerator 10 is greater.

$\therefore \quad \frac{10}{24}$ *i.e.* $\frac{5}{12}$ **is greater.** **(Ans.)**

Alternative method :

Fractions can also be compared by making the numerators equal and then comparing the denominators. In this case, ***the fraction with the smaller denominator is greater.***

Since the L.C.M. of the numerators 3 and 5 = 15,

$\therefore \quad \frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$ and $\frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$.

For the same numerator, the fraction is greater if the denominator is smaller.

$\therefore \quad \frac{15}{36}$ *i.e.* $\frac{5}{12}$ **is greater.** **(Ans.)**

Example 3 :

Which of the given fractions is smaller, $\frac{8}{15}$ or $\frac{12}{25}$?

Solution :

First method : *By making the denominators equal :*

With equal denominators, the fraction with the smaller numerator is smaller.

Since, the L.C.M. of denominators 15 and 25 = 75.

$\therefore \quad \frac{8}{15} = \frac{8 \times 5}{15 \times 5} = \frac{40}{75}$ and $\frac{12}{25} = \frac{12 \times 3}{25 \times 3} = \frac{36}{75}$

Hence $\frac{36}{75}$ *i.e.* $\frac{12}{25}$ **is smaller.** **(Ans.)**

Second method : *By making numerators equal :*

With equal numerators, the fraction with the bigger denominator is smaller.

Since, the L.C.M. of numerators 8 and 12 = 24.

$\therefore \quad \frac{8}{15} = \frac{8 \times 3}{15 \times 3} = \frac{24}{45}$ and $\frac{12}{25} = \frac{12 \times 2}{25 \times 2} = \frac{24}{50}$

Hence $\frac{24}{50}$ *i.e.* $\frac{12}{25}$ **is smaller.** **(Ans.)**

Example 4 :

Compare the fractions $\frac{2}{3}, \frac{3}{4}, \frac{5}{12}$ and $\frac{9}{16}$ by writing them in the descending order.

Solution :

Step 1 :

Since the L.C.M. of the denominators 3, 4, 12 and 16 = 48,

$\therefore \quad \frac{2}{3} = \frac{2 \times 16}{3 \times 16} = \frac{32}{48}, \quad \frac{3}{4} = \frac{3 \times 12}{4 \times 12} = \frac{36}{48},$

$\frac{5}{12} = \frac{5 \times 4}{12 \times 4} = \frac{20}{48}$ and $\frac{9}{16} = \frac{9 \times 3}{16 \times 3} = \frac{27}{48}$.

Making the denominators equal.

Step 2 :

See the numerators of these like fractions. The fraction with the largest numerator is largest.

The largest numerator is 36. $\quad \therefore \quad \frac{36}{48}$ *i.e.* $\frac{3}{4}$ is the largest fraction.

The smallest numerator is 20. $\therefore$ $\frac{20}{48}$ *i.e.* $\frac{5}{12}$ is the smallest fraction.

Thus, the given fractions in descending order of value are :

$$\frac{36}{48}, \frac{32}{48}, \frac{27}{48} \text{ and } \frac{20}{48} \quad i.e. \quad \frac{3}{4}, \frac{2}{3}, \frac{9}{16} \text{ and } \frac{5}{12}$$

$$i.e. \quad \frac{3}{4} > \frac{2}{3} > \frac{9}{16} > \frac{5}{12}$$ **(Ans.)**

Alternative method : (Comparing fractions by making the numerators equal)

Taking the same fractions as given in the above example,

i.e. $\frac{2}{3}, \frac{3}{4}, \frac{5}{12}$ and $\frac{9}{16}$,

The L.C.M. of the numerators 2, 3, 5 and 9 = 90

$\therefore \quad \frac{2}{3} = \frac{2\times45}{3\times45} = \frac{90}{135}, \quad \frac{3}{4} = \frac{3\times30}{4\times30} = \frac{90}{120},$

$\frac{5}{12} = \frac{5\times18}{12\times18} = \frac{90}{216}$ and $\frac{9}{16} = \frac{9\times10}{16\times10} = \frac{90}{160}$.

Making the numerators equal.

We know that, with the numerators being the same, the fraction with the smallest denominator is the biggest fraction and the fraction with the largest denominator is the smallest fraction.

Thus, $\frac{90}{120}$ is the biggest fraction and $\frac{90}{216}$ is the smallest fraction.

And so, $\frac{90}{120}, \frac{90}{135}, \frac{90}{160}$ and $\frac{90}{216}$ are in the descending order of value,

i.e. $\frac{3}{4}, \frac{2}{3}, \frac{9}{16}$ and $\frac{5}{12}$ are in the descending order of value,

i.e. $\frac{3}{4} > \frac{2}{3} > \frac{9}{16} > \frac{5}{12}$ **(Ans.)**

For the example given above, the fractions can be written in ascending order as $\frac{5}{12} < \frac{9}{16} < \frac{2}{3} < \frac{3}{4}$.

EXERCISE 14(B)

1. **Reduce** the given fractions **to** their **lowest terms :**

   (i) $\frac{8}{10}$ (ii) $\frac{50}{75}$ (iii) $\frac{18}{81}$

   (iv) $\frac{40}{120}$ (v) $\frac{105}{70}$

2. State whether **true** or **false** :

   (i) $\frac{2}{5} = \frac{10}{15}$ (ii) $\frac{35}{42} = \frac{5}{6}$ (iii) $\frac{5}{4} = \frac{4}{5}$

   (iv) $\frac{7}{9} = 1\frac{1}{7}$ (v) $\frac{9}{7} = 1\frac{1}{7}$

3. **Which** fraction **is greater** ?

   (i) $\frac{3}{5}$ or $\frac{2}{3}$ (ii) $\frac{5}{9}$ or $\frac{3}{4}$ (iii) $\frac{11}{14}$ or $\frac{26}{35}$

4. **Which** fraction **is smaller** ?

(i) $\frac{3}{8}$ or $\frac{4}{5}$ (ii) $\frac{8}{15}$ or $\frac{4}{7}$ (iii) $\frac{7}{26}$ or $\frac{10}{39}$

5. Arrange the given fractions in **descending order** of magnitude :

(i) $\frac{5}{16}, \frac{13}{24}, \frac{7}{8}$ (ii) $\frac{4}{5}, \frac{7}{15}, \frac{11}{20}, \frac{3}{4}$ (iii) $\frac{5}{7}, \frac{3}{8}, \frac{9}{11}$

6. Arrange the given fractions in **ascending order** of magnitude :

(i) $\frac{9}{16}, \frac{7}{12}, \frac{1}{4}$ (ii) $\frac{5}{6}, \frac{2}{7}, \frac{8}{9}, \frac{1}{3}$ (iiii) $\frac{2}{3}, \frac{5}{9}, \frac{5}{6}, \frac{3}{8}$

7. I bought one dozen bananas and ate five of them. What fraction of the total number of bananas was left ?

8. Insert the symbol '=' or '>' or '<' between each of the pairs of fractions given below :

(i) $\frac{6}{11}$ ..... $\frac{5}{9}$ (ii) $\frac{3}{7}$ ..... $\frac{9}{13}$ (iii) $\frac{56}{64}$ ..... $\frac{7}{8}$ (iv) $\frac{5}{12}$ ..... $\frac{8}{33}$

9. Out of 50 identical articles, 36 are broken. Find the fraction of :

(i) the total number of articles and the articles broken.

(ii) the remaining articles and total number of articles.

14.8 FUNDAMENTAL OPERATIONS ON FRACTIONS

The four fundamental operations are *addition, subtraction, multiplication* and *division.*

14.9 ADDITION AND SUBTRACTION

Steps :

1. If any of the given fractions is in mixed form, convert it into an improper fraction.
2. Convert the fractions obtained in step 1 into like fractions.
3. Keeping the denominator same, as obtained in step 2, combine the numerators of the equivalent fractions and obtain a single fraction.
4. Reduce, if required, the fraction so obtained into its lowest terms and then into a mixed fraction.

For example :

(i) $\frac{3}{4} + \frac{2}{5}$

L.C.M. of 4 and 5 is 20

$= \frac{3\times5}{4\times5} + \frac{2\times4}{5\times4}$

$= \frac{15}{20} + \frac{8}{20}$

$= \frac{15+8}{20} = \frac{23}{20} = 1\frac{3}{20}$

(ii) $1\frac{5}{7} - \frac{5}{6}$

$1\frac{5}{7} = \frac{1\times7+5}{7} = \frac{7+5}{7} = \frac{12}{7}$

$= \frac{12}{7} - \frac{5}{6}$

L.C.M. of 7 and 6 is 42

$= \frac{12\times6}{7\times6} - \frac{5\times7}{6\times7}$

$= \frac{72}{42} - \frac{35}{42} = \frac{72-35}{42} = \frac{37}{42}$

(iii) $2\frac{2}{5} - 3\frac{3}{4} + 4\frac{1}{2} = \frac{12}{5} - \frac{15}{4} + \frac{9}{2}$ — Converting into improper fractions

$= \frac{12 \times 4}{5 \times 4} - \frac{15 \times 5}{4 \times 5} + \frac{9 \times 10}{2 \times 10}$ [L.C.M. of 5, 4 and 2 is 20]

$= \frac{48}{20} - \frac{75}{20} + \frac{90}{20} = \frac{48 - 75 + 90}{20} = \frac{63}{20} = 3\frac{3}{20}$

EXERCISE 14(C)

1. **Add** the following fractions :

(i) $1\frac{3}{4}$ and $\frac{3}{8}$ (ii) $\frac{2}{5}, 2\frac{3}{15}$ and $\frac{7}{10}$ (iii) $1\frac{7}{8}, 1\frac{1}{2}$ and $1\frac{3}{4}$

(iv) $3\frac{3}{4}, 2\frac{1}{6}$ and $1\frac{5}{8}$ (v) $2\frac{8}{9}, \frac{11}{18}$ and $3\frac{5}{6}$ (vi) $3\frac{1}{8}, 5\frac{5}{12}$ and $\frac{5}{16}$

2. **Simplify :**

(i) $1\frac{11}{12} - \frac{13}{16}$ (ii) $2\frac{3}{4} - 1\frac{5}{6}$ (iii) $2\frac{5}{7} + \frac{3}{14} - \frac{13}{21}$

(iv) $3\frac{5}{6} - \frac{1}{6} - 1\frac{1}{12}$ (v) $6 + \frac{3}{10} - 1\frac{8}{15}$ (vi) $1\frac{3}{4} + 2\frac{5}{7} - 1\frac{3}{14}$

(vii) $4 + 3\frac{1}{8} - 3\frac{1}{6}$ (viii) $6 - 3\frac{1}{2} - 2\frac{1}{5}$ (ix) $1\frac{5}{8} - 2\frac{1}{6} + 3\frac{3}{4}$

(x) $3\frac{1}{2} + 1\frac{2}{3} - 2\frac{1}{4}$ (xi) $4\frac{3}{5} - 2\frac{7}{9} - 1\frac{2}{15} - \frac{2}{45}$

14.10 MULTIPLICATION AND DIVISION

Steps for multiplication :

1. See that each given fraction is in proper form or improper form, *i.e.* no fraction is in mixed form.
2. Multiply the numerators of the fractions together to get the numerator of the resulting fraction and also multiply the denominators of the fractions together to get the denominator of the resulting fraction
3. If required, reduce the resulting fraction, obtained in step (2), into its lowest terms and then into a mixed fraction, if possible.

Example 5 :

Simplify : (i) $\frac{3}{4} \times 5$ (ii) $2\frac{2}{5} \times \frac{5}{18}$

Solution :

(i) $\frac{3}{4} \times 5 = \frac{3}{4} \times \frac{5}{1} = \frac{3 \times 5}{4 \times 1} = \frac{15}{4} = 3\frac{3}{4}$ **(Ans.)**

(ii) $2\frac{2}{5} \times \frac{5}{18} = \frac{12}{5} \times \frac{5}{18} = \frac{12 \times 5}{5 \times 18} = \frac{\cancel{2} \times 2 \times \cancel{3} \times \cancel{5}}{\cancel{5} \times \cancel{3} \times 3 \times \cancel{2}} = \frac{2}{3}$ **(Ans.)**

For division :

Multiply the dividend (the first fraction) *by the reciprocal of the divisor* (the second fraction).

1. Reciprocal of $\frac{5}{7} = \frac{7}{5}$, reciprocal of $\frac{8}{15} = \frac{15}{8}$ and so on.
2. Since, $5 = \frac{5}{1}$; reciprocal of $5 = \frac{1}{5}$. Similarly, reciprocal of $8 = \frac{1}{8}$, reciprocal of $15 = \frac{1}{15}$ and so on.

For example :

$\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{10}{9} = 1\frac{1}{9}$

Reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$

14.11 COMBINED OPERATIONS OF MULTIPLICATION AND DIVISION

In such cases, *first the operation of division is completed and then of multiplication.*

Example 6 :

Simplify : $\frac{3}{8} \div \frac{4}{7} \times \frac{1}{2}$

Reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$

Solution :

Since, $\frac{3}{8} \div \frac{4}{7} = \frac{3}{8} \times \frac{7}{4} = \frac{21}{32}$

$\therefore$ $\frac{3}{8} \div \frac{4}{7} \times \frac{1}{2} = \frac{21}{32} \times \frac{1}{2} = \frac{21 \times 1}{32 \times 2} = \frac{21}{64}$ **(Ans.)**

or, directly : $\frac{3}{8} \div \frac{4}{7} \times \frac{1}{2} = \frac{3}{8} \times \frac{7}{4} \times \frac{1}{2} = \frac{3 \times 7 \times 1}{8 \times 4 \times 2} = \frac{21}{64}$ **(Ans.)**

14.12 USING ' of ' ALONG WITH MULTIPLICATION AND DIVISION

Apart from Addition, Subtraction, Multiplication and Division, there is one more operation, termed 'of'.

The word 'of' written in between two fractions or numbers is to be worked out just as an ordinary multiplication. The multiplication for the word 'of' must be carried out before division and multiplication.

For example :

(i) $\frac{3}{2}$ of $\frac{3}{4} \div \frac{9}{2}$

$= \frac{9}{8} \div \frac{9}{2}$ [Operating 'of' before division, we get : $\frac{3}{2}$ of $\frac{3}{4} = \frac{9}{8}$]

$= \frac{9}{8} \times \frac{2}{9} = \frac{1}{4}$

(ii) $\frac{5}{6}$ of $\frac{3}{4} \div \frac{7}{8} \times 1\frac{1}{2}$

$$= \frac{5}{8} \div \frac{7}{8} \times \frac{3}{2}$$

$\left[\text{Operating 'of' before division and multiplication, we get: } \frac{5}{6} \text{ of } \frac{3}{4} = \frac{5}{6} \times \frac{3}{4} = \frac{15}{24} = \frac{5}{8}\right]$

$$= \frac{5}{8} \times \frac{8}{7} \times \frac{3}{2} = \frac{5 \times \cancel{8} \times 3}{\cancel{8} \times 7 \times 2} = \frac{15}{14} = 1\frac{1}{14}$$

14.13 USING 'BODMAS'

"BODMAS" is the acronym (abbreviation) formed by taking the initial letters of the six operations, where :

'B' stands for **"BRACKET"**

'O' stands for **"OF"**

'D' stands for **"DIVISION"**

'M' stands for **"MULTIPLICATION"**

'A' stands for **"ADDITION"**

'S' stands for **"SUBTRACTION"**

Fractions inside brackets are to be operated (combined) first

While simplifying an expression involving three or more operations, the order of operations must be the same as in the order of letters used in 'BODMAS'

For example :

(i) $1\frac{1}{2} \times \frac{1}{12} \div \frac{5}{4} = \frac{3}{2} \times \frac{1}{12} \times \frac{4}{5}$

Using BODMAS; division is done first

$$= \frac{3 \times 1 \times 4}{2 \times 12 \times 5} = \frac{1}{10}$$

Using BODMAS; bracket is simplified first

(ii) $\frac{1}{3} + \frac{7}{9} \div \left(\frac{7}{10} \times 1\frac{1}{4}\right)$

$$= \frac{1}{3} + \frac{7}{9} \div \left(\frac{7}{10} \times \frac{5}{4}\right)$$

$$= \frac{1}{3} + \frac{7}{9} \div \frac{7}{8}$$ $\left[\frac{7}{10} \times \frac{5}{4} = \frac{7 \times 5}{10 \times 4} = \frac{7}{8}\right]$

$$= \frac{1}{3} + \frac{7}{9} \times \frac{8}{7} = \frac{1}{3} + \frac{8}{9} = \frac{3+8}{9} = \frac{11}{9} = 1\frac{2}{9}$$

Example 7 :

Simplify : $\left(\frac{2}{3} + \frac{5}{9}\right)$ of $\frac{9}{22} \div \frac{2}{3} \times \frac{4}{5} - \frac{1}{5}$

Solution :

$$= \frac{11}{9} \text{ of } \frac{9}{22} \div \frac{2}{3} \times \frac{4}{5} - \frac{1}{5}$$ $\left[\text{Removing 'bracket', we get : } \frac{2}{3} + \frac{5}{9} = \frac{6+5}{9} = \frac{11}{9}\right]$

$= \frac{1}{2} \div \frac{2}{3} \times \frac{4}{5} - \frac{1}{5}$ [On operating 'of', we get: $\frac{11}{9}$ of $\frac{9}{22} = \frac{11}{9} \times \frac{9}{22} = \frac{1}{2}$]

$= \frac{1}{2} \times \frac{3}{2} \times \frac{4}{5} - \frac{1}{5} = \frac{1 \times 3 \times 4}{2 \times 2 \times 5} - \frac{1}{5} = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$ **(Ans.)**

EXERCISE 14(D)

1. **Simplify :**

(i) $\frac{3}{7} \times \frac{2}{5}$ (ii) $\frac{4}{9} \times \frac{3}{5}$ (iii) $\frac{5}{12} \times 8$

(iv) $\frac{7}{6}$ of $\frac{3}{14}$ (v) $3\frac{3}{8} \times 3\frac{6}{7}$ (vi) $\frac{1}{2}$ of $\frac{1}{3} \times \frac{3}{4}$

(vii) $\frac{3}{7} \times \frac{5}{9} \times 4\frac{1}{5}$ (viii) $1\frac{1}{3} \times 1\frac{2}{7}$ of $1\frac{1}{4}$

2. **Simplify :**

(i) $\frac{2}{3} \div 1\frac{1}{5}$ (ii) $4\frac{1}{2} \div \frac{4}{9}$ (iii) $1 \div \frac{2}{5}$

(iv) $\frac{4}{9} \div \frac{4}{9}$ (v) $2\frac{1}{3} \div 1\frac{3}{4}$ (vi) $2\frac{2}{3} \times 3\frac{1}{2} \div 2\frac{4}{9}$

3. **Simplify :**

(i) $\frac{1}{4}$ of $2\frac{2}{7} \div \frac{3}{5}$ (ii) $1\frac{1}{4} \times \frac{1}{2} \div 1\frac{1}{3}$ (iii) $6\frac{1}{7} \times 0 \times 5\frac{3}{8}$

(iv) $\frac{3}{4} \times 1\frac{1}{3} \div \frac{3}{7}$ of $2\frac{5}{8}$ (v) $2\frac{1}{4} \div \frac{2}{7}$ of $1\frac{1}{3} \times \frac{2}{3}$ (vi) $\left(\frac{3}{7} \div \frac{1}{2}\right)$ of $1\frac{1}{7}$

(vii) $\left(1\frac{7}{8} \div 1\frac{1}{2}\right)$ of $\left(8\frac{1}{3} \div 1\frac{1}{2}\right)$ (viii) $\frac{1}{3}$ of $60 \div 60$

4. **Simplify :**

(i) $5 - \left(\frac{8}{11} - 3\frac{3}{11}\right)$ (ii) $\frac{1}{2} \div \left(\frac{7}{8} - \frac{3}{5}\right)$

(iii) $2\frac{1}{3} \div \left(5\frac{1}{2} + 3\frac{3}{4}\right)$ (iv) $\left(3\frac{7}{8} - 3\frac{3}{5}\right) \div \frac{1}{2}$

(v) $\frac{4}{7} \div \left(\frac{1}{3} \times 2\frac{4}{5}\right)$ (vi) $\frac{3}{4} \div \left(\frac{1}{6} \div \frac{1}{2}\right)$

(vii) $\left(\frac{1}{4} - \frac{1}{6}\right)$ of $\left(\frac{2}{3} - \frac{5}{12}\right) \times \left(\frac{5}{8} - \frac{7}{12}\right)$

5. **Simplify :**

(i) $\left(\frac{1}{2} + \frac{1}{3}\right) \div \left(\frac{1}{4} - \frac{1}{6}\right)$ (ii) $\left(\frac{24}{35} \div \frac{6}{7} + \frac{5}{9}\right) \times \frac{3}{4}$

(iii) $\frac{3}{4}$ of $6\frac{1}{8} - \frac{2}{3}$ of $2\frac{1}{4}$ (iv) $\frac{7}{30}$ of $\left(\frac{1}{3} + \frac{7}{15}\right) \div \left(\frac{5}{6} - \frac{3}{5}\right)$

(v) $2\frac{1}{2} - 3\frac{1}{2} \times 1\frac{3}{4} + 2\frac{1}{2}$

(vi) $4\frac{5}{7}\left(3\frac{1}{8} \div \frac{11}{12}\right)$

(vii) $\frac{2}{5}$ of $\left(\frac{1}{7} - \frac{1}{12}\right)$ of $1\frac{2}{5}$

(viii) $\left(\frac{1}{2} - \frac{1}{3}\right)\left(\frac{3}{4} - \frac{4}{5}\right) \div \left(\frac{1}{2} - \frac{2}{5} + \frac{1}{7}\right)$

(ix) $\frac{5}{6} - \frac{3}{5}\left(\frac{1}{3} + \frac{2}{11}\right)$

(x) $4\frac{2}{3} \div \left(3 - \frac{1}{2}\right) + \left(\frac{2}{5} \div 1\frac{1}{5}\right)$

(xi) $\frac{1}{2}$ of $40 + 1\frac{3}{4}$ of $2\frac{2}{9} + 2\frac{1}{5} \times 0$

14.14 PROBLEMS INVOLVING FRACTIONS

Example 8 :

A man earns ₹ 7,500 per month. If he saves $\frac{1}{4}$ of his earning, find :

(a) his savings per month (b) his expenditure per month.

Solution :

(a) **Savings per month** = $\frac{1}{4}$ of his earning

= $\frac{1}{4}$ of ₹ 7,500 = ₹ $\frac{1}{4} \times 7{,}500$ = **₹ 1,875** **(Ans.)**

(b) His **expenditure per month** = ₹ 7,500 – ₹ 1,875 = **₹ 5,625** **(Ans.)**

Alternative method :

In fractions, the whole quantity is taken as 1.

Since the man saves $\frac{1}{4}$ of his earning,

his expenditure = $1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$ of his earning.

⇒ **Expenditure per month**

= $\frac{3}{4}$ of ₹ 7,500 = **₹ 5,625** **(Ans.)**

Example 9 :

There are 12 dozen bananas in a basket. $\frac{5}{24}$ of them are rotten and $\frac{1}{3}$ of them get eaten. How many bananas are left ?

Solution :

Total number of bananas = 12 dozen = $12 \times 12 = 144$

No. of rotten bananas = $\frac{5}{24}$ of 144 = $\frac{5}{24} \times 144 = 30$

No. of bananas eaten = $\frac{1}{3}$ of 144 = $\frac{1}{3} \times 144 = 48$

Since 30 + 48 = 78

∴ **No. of bananas left** = 144 – 78 = **66** **(Ans.)**

Example 10 :

A man spends $\frac{2}{5}$ of his money and is left with ₹ 30. How much did he initially have ?

Solution :

Remember, while solving problems on fractions, the whole quantity is always considered 1.

Since, the man spends $\frac{2}{5}$ of his money,

$\therefore$ Money left with him $= \left(1 - \frac{2}{5}\right)$ of his money $= \frac{3}{5}$ of his money

Given : $\frac{3}{5}$ of his initial money = ₹ 30

$\therefore$ **Initially he had** = ₹ $30 \times \frac{5}{3}$ = **₹ 50** **(Ans.)**

Example 11 :

After travelling 10 km, Dev found that $\frac{1}{3}$ of his journey was still left. How long was his total journey ?

Solution :

Since $\frac{1}{3}$ of the journey is left,

therefore, $1 - \frac{1}{3} = \frac{2}{3}$ of the journey is completed.

Given : $\frac{2}{3}$ of the total journey = 10 km

$\therefore$ **Total journey** = 10 km $\times \frac{3}{2}$ = **15 km** **(Ans.)**

EXERCISE 14(E)

1. From a rope $10\frac{1}{2}$ m long, $4\frac{5}{8}$ m is cut off. Find the length of the remaining rope.
2. A piece of cloth is 5 m long. After washing, it shrinks by $\frac{1}{25}$ of its length. What is the length of the cloth after washing ?
3. I bought wheat worth ₹ $12\frac{1}{2}$, rice worth ₹ $25\frac{3}{4}$ and vegetables worth ₹ $10\frac{1}{4}$. I gave a hundred-rupee note to the shopkeeper; how much money did he return to me ?
4. Out of 500 oranges in a box, $\frac{3}{25}$ are rotten and $\frac{1}{5}$ are kept for some guests. How many oranges are left in the box ?
5. An ornament piece is made of gold and copper. Its total weight is 96 g. If $\frac{1}{12}$ of the ornament is copper, find the weight of gold in it.
6. A girl did half of some work on Monday and one-third of it on Tuesday. How much will she have to do on Wednesday in order to complete the work ?
7. A man spends $\frac{3}{8}$ of his money and still has ₹ 720 left with him. How much money did he have at first ?

8. In a school, $\frac{4}{5}$ of the students are boys, and the number of girls is 100. Find the number of boys.

9. After finishing $\frac{3}{4}$ of my journey, I find that 12 km of my journey is covered. How much distance is still left to be covered ?

10. When Ajit travelled 15 km, he found that one-fourth of his journey was still left. What was the full length of the journey ?

11. In a particular month, a man earns ₹ 7,200. Out of this income, he spends $\frac{3}{10}$ on food, $\frac{1}{4}$ on house-rent, $\frac{1}{10}$ on insurance and $\frac{2}{25}$ on holidays. How much did he save in that month ?

DECIMAL FRACTIONS 15

15.1 BASIC CONCEPT

A fraction whose denominator is 10 or a higher power of 10, *i.e.*, 100, 1000, etc., is called a **decimal fraction.** Thus, each of $\frac{7}{10}, \frac{13}{10^2}, \frac{357}{1000}, \frac{29}{10^4}$ is a decimal fraction.

For such a fraction, the denominator is removed and its absence is shown by a small **dot** (called the ***decimal point***) inserted in its proper place.

For example :

$$\frac{2}{10} = 0 \cdot 2,\ \frac{24}{100} = 0 \cdot 24,\ \frac{3159}{1000} = 3 \cdot 159,\ \frac{31}{10} = 3 \cdot 1, \text{ etc.}$$

1. Since $\frac{2}{10}$ and $\frac{31}{10}$ have 10 as denominator; therefore, when 10 is removed, a dot representing a decimal point is placed just one digit from the right
   i.e. $\frac{2}{10} = \cdot 2 = 0 \cdot 2$ and $\frac{31}{10} = 3 \cdot 1$.
   In the same way, when denominator is 100 and it is removed, the decimal point is placed just after two digits from the right, at the same time; so $\frac{24}{100} = \cdot 24 = 0 \cdot 24$, $\frac{479}{100} = 4 \cdot 79$, etc.
   In the same way; $\frac{5278}{1000} = 5 \cdot 278$, $\frac{5278}{10000} = 0 \cdot 5278$, $\frac{5278}{100000} = 0 \cdot 05278$ and so on.
2. Also, $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0 \cdot 6$; $\frac{17}{20} = \frac{17 \times 5}{20 \times 5} = \frac{85}{100} = 0 \cdot 85$, etc.
   ⇒ A fraction whose denominator can be expressed as 10 or some higher power of 10 is also a decimal fraction.
3. **3·1** means 3 + 0·1. Here **3** is the **integral part** and **0·1** is the **decimal part**.

15.2 NUMBER OF DECIMAL PLACES

The number of digits in the decimal part of a number is the number of decimal places in it.

For example :

In 3·462, the decimal part is ·462, which contains *three digits.*

∴ The number **3·462** has ***3 decimal places.***

Similarly, **4·83** has ***2 decimal places;*** 0·0478 has *4 decimal places* and so on.

When a number has only the decimal part, such as ·7, ·83, ·403, etc., it is always advised to write a zero before the decimal point.
i.e. write ·7 as 0·7; ·83 as 0·83; ·403 as 0·403 and so on.

15.3 LIKE AND UNLIKE DECIMAL NUMBERS

The given decimal numbers are said to be **like decimal numbers**, if they **have the same number of decimal places**. Otherwise, they are called **unlike decimal numbers**.

For example :

(i) 5·7, 0·8, 329·2 and 50·6 are **like decimal numbers**.

(ii) 26·03, 8·87, 0·52 and 400·04 are **like decimal numbers**.

(iii) 2·6, 40·32, 0·009, 3·0728 and 328·2 are **unlike decimal numbers**.

Note : *Unlike decimal numbers can be converted into like decimal numbers.*

For example :

Consider the unlike decimal numbers :

5·8, 239·06 and 0·5497

In these numbers, 5·8 has one decimal place, 239·06 has two decimal places and 0·5497 has four decimal places.

Since 0·5497 has the maximum number of decimal places (four decimal places), make the decimal places in each given decimal number equal to four.

Thus, 5·8 = 5·8000,

239·06 = 239·0600

and 0·5497 = 0·5497

∴ The given unlike decimal numbers 5·8, 239·06 and 0·5497 are converted into the like decimal numbers 5·8000, 239·0600 and 0·5497.

> The value of a given decimal fraction does not change, if one or more zeroes are placed on the right side of it.

Similarly, the unlike decimal numbers 320·98, 0·07325 and 53·4 will be 320·98000, 0·07325 and 53·40000 as like decimal numbers.

15.4 CONVERSION OF A GIVEN FRACTION INTO A DECIMAL FRACTION

1. When the denominator is 10, 100, 1000, etc.

Steps :

1. Count the number of zeroes in the denominator of the given fraction.
2. In the numerator, mark the decimal point after as many digits (counting from extreme right to left) as the number of zeroes in the denominator. At the same time, remove the denominator.

For example :

In the fraction $\frac{327}{100}$, the denominator is 100, which has *two zeroes* in it. Therefore, in the numerator 327, mark the decimal point after two digits from right to left, giving 3·27.

Thus $\frac{327}{100}$ = 3·27. Similarly, $\frac{7}{10}$ = ·7 = 0·7, $\frac{14}{1000}$ = 0·014 and so on.

In $\frac{14}{1000}$, the denominator has three zeroes, so the decimal point is to be marked after 3 digits from the right of the numerator 14. Since 14 has only two digits, write one zero to the left of 14 and then place the decimal point.

In a similar manner,

(i) $\frac{17}{1000} = \cdot017 = 0\cdot017,\ \frac{5}{1000} = \cdot005 = 0\cdot005$ and so on.

(ii) $\frac{254}{10} = 25\cdot4,\ \frac{254}{100} = 2\cdot54,\ \frac{254}{1000} = \cdot254 = 0\cdot254,$

$\frac{254}{10000} = 0\cdot0254$ and so on.

(iii) $\frac{57}{10^2} = \frac{57}{100} = \cdot57 = 0\cdot57,\ \frac{34}{10^4} = \frac{34}{10000} = 0\cdot0034$ and so on.

Remember: $10^2 = 10 \times 10 = 100,\ 10^3 = 10 \times 10 \times 10 = 1000,\ 10^4 = 10 \times 10 \times 10 \times 10 = 10000$ and so on.

2. When the denominator can be expressed as 10, 100, 1000, etc.

Multiply both the numerator and the denominator of the given fraction by a suitable number so that the denominator becomes 10 or a power of 10. Then proceed as above.

Thus :

(i) $\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = \mathbf{0.25}$

$4 \times 25 = 100$, a power of ten

(ii) $\frac{73}{125} = \frac{73 \times 8}{125 \times 8} = \frac{584}{1000} = \mathbf{0.584}$

$125 \times 8 = 1000$, a power of ten

(iii) $\frac{6}{8} = \frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0\cdot75,$

$\frac{8}{250} = \frac{8 \times 4}{250 \times 4} = \frac{32}{1000} = 0\cdot032$ and so on.

Also, seven-tenths = $\frac{7}{10} = 0\cdot7$, 53-hundredths = $\frac{53}{100} = 0\cdot53$ and so on.

There are several fractions whose denominators are neither 10 nor some higher power of 10. Also, their denominators cannot be converted into 10 or some higher power of 10. Such fractions can also be expressed as decimal fractions. Conversion of such fractions into decimal fractions will be discussed in the next class, *i.e.* in class 7.

15.5 CONVERSION OF A GIVEN DECIMAL FRACTION INTO A NON-DECIMAL FRACTION

Remove the decimal point and at the same time, write in the denominator as many zeroes to the right of 1 (one) *as there are digits in the decimal part.* Then simplify.

For example :

(i) $0{\cdot}42 = \frac{42}{100} = \frac{21}{50}$ [Two digits in the decimal part]

(ii) $0{\cdot}021 = \frac{021}{1000} = \frac{21}{1000}$ [Three digits in the decimal part]

(iii) $1{\cdot}75 = \frac{175}{100} = \frac{7}{4} = 1\frac{3}{4}$ and so on.

EXERCISE 15(A)

1. Write the number of decimal places in each of the following :
   (i) 7·03 (ii) 0·509 (iii) 146·2
   (iv) 0·0065 (v) 8·03207
2. Convert the given unlike decimal fractions into like decimal fractions :
   (i) 1·36, 239·8 and 47·008
   (ii) 507·0752, 8·52073 and 0·808
   (iii) 459·22, 7·03093 and 0·200037
3. Change each of following fractions to a *decimal fraction* :
   (i) $\frac{7}{10}$ (ii) $\frac{47}{10}$ (iii) $\frac{343}{100}$ (iv) $\frac{3}{10^3}$
   (v) $\frac{7295}{10^5}$ (vi) $\frac{289}{10^6}$ (vii) 95-hundredths
4. Convert into a *decimal fraction* :
   (i) $\frac{3}{4}$ (ii) $\frac{3}{40}$ (iii) $\frac{1}{125}$ (iv) $\frac{7}{25}$
5. Change the given decimal fractions to fractions in their *lowest terms :*
   (i) 0·05 (ii) 3·95 (iii) 4·005 (iv) 0·876
   (v) 50·06 (vi) 0·01075 (vii) 4·8806

15.6 ADDITION OF DECIMAL NUMBERS

Steps :

1. Convert, if required, the given decimal numbers into like decimal numbers.
2. Write all the like decimal numbers, obtained in step 1, one below the other in such a way that their decimal points are in the same vertical line.
3. Add the numbers and in the result, mark the decimal point below the other decimal points.

Example 1 :

Add : (i) 2·7, 35·82 and 140·052 (ii) 8·09, 0·9273 and 233·4

Solution :

(i) *Step 1*

For the given numbers, the like decimal numbers are 2·700, 35·820 and 140·052

Steps 2 and 3

$$\begin{array}{r} 2{\cdot}700 \\ 35{\cdot}820 \\ 140{\cdot}052 \\ \hline 178{\cdot}572 \\ \hline \end{array}$$ **(Ans.)**

(ii)

$$\begin{array}{r} 8{\cdot}0900 \\ 0{\cdot}9273 \\ 233{\cdot}4000 \\ \hline 242{\cdot}4173 \\ \hline \end{array}$$ **(Ans.)**

15.7 SUBTRACTION

If required, convert the given decimal numbers into like decimal numbers, and then complete the required subtraction. Care must be taken that the decimal point in the given numbers and in the result must be in the same vertical line.

Example 2 :

Subtract : (i) 35·724 from 180·938 (ii) 72·385 from 85·4

Solution :

(i)
$$\begin{array}{r} 180{\cdot}938 \\ -\ 35{\cdot}724 \\ \hline 145{\cdot}214 \\ \hline \end{array}$$ **(Ans.)**

(ii)
$$\begin{array}{r} 85{\cdot}400 \\ -\ 72{\cdot}385 \\ \hline 13{\cdot}015 \\ \hline \end{array}$$ **(Ans.)**

Make the decimal places the same in both the given numbers

Example 3 :

Simplify : (i) 14·8 − 7·23 + 9·631 (ii) 3·241 − 0·53 + 6·6105 − 8·2413 + 5·2

Solution :

Add all the positive numbers together and all the negative numbers separately together as well. Finally, add or subtract as required :

(i) 14·8 − 7·23 + 9·631

= 24·431 − 7·23

= 17·201 **(Ans.)**

$$\begin{array}{r} 14{\cdot}800 \\ +\ 9{\cdot}631 \\ \hline 24{\cdot}431 \\ \hline \end{array}$$ and $$\begin{array}{r} 24{\cdot}431 \\ -\ 7{\cdot}230 \\ \hline 17{\cdot}201 \\ \hline \end{array}$$

(ii) 3·241 − 0·53 + 6·6105 − 8·2413 + 5·2

= 15·0515 − 8·7713 = 6·2802 **(Ans.)**

Here,
$$\begin{array}{r} 3{\cdot}241 \\ +\ 6{\cdot}6105 \\ +\ 5{\cdot}200 \\ \hline 15{\cdot}0515 \\ \hline \end{array}$$,
$$\begin{array}{r} -\ 0{\cdot}5300 \\ -\ 8{\cdot}2413 \\ \hline -\ 8{\cdot}7713 \\ \hline \end{array}$$ and, finally
$$\begin{array}{r} 15{\cdot}0515 \\ -\ 8{\cdot}7713 \\ \hline 6{\cdot}2802 \\ \hline \end{array}$$

EXERCISE 15(B)

1. **Add** the following :

(i) 0·243, 2·47 and 3·009

(ii) 0·0736, 0·6095 and 0·9107

(iii) 1·01, 257 and 0·200

(iv) 18, 200·35, 11·72 and 2·3

(v) 0·586, 0·0586 and 0·00586

2. Find the value of :

(i) 6·8 – 2·64 (ii) 2 – 1·0304 (iii) 0·1 – 0·08 (iv) 0·83 – 0·342

3. **Subtract :**

(i) 0·43 from 0·97 (ii) 2·008 from 22·1058 (iii) 0·18 from 0·6

(iv) 1·002 from 17 (v) 83 from 92·05

4. **Simplify :**

(i) 3·5 – 2·43 + 0·075 (ii) 7·84 + 0·3 – 4·016 (iii) 2·987 – 1·25 – 0·54

(iv) 52·9 – 231·666 + 204 (v) 8·57 – 6·4432 – 1·70 + 0·683

5. From the sum of 75·75 and 4·9 subtract 28·465.
6. Subtract the sum of 8·14 and 12·9 from 32·7.
7. Subtract the sum of 34·27 and 159·8 from the sum of 20·937 and 200·6.
8. From the sum of 2·43 and 4·349 subtract the sum of 0·8 and 3·15.
9. By how much does the sum of 18·0495 and 34·9644 exceed the sum of 7·6752 and 24·876 ?
10. What least number must be added to 89·376 to get 1000 ?

15.8 MULTIPLICATION

Steps :

1. Multiply the two given decimal numbers, ignoring their decimal points.
2. In the product, obtained in step 1, mark the decimal point such that the decimal places in it is equal to the sum of decimal places of the two given numbers.

Example 4 :

(i) Evaluate : 532·43 × 7 (ii) Multiply : 4·09 and 5·6

(iii) Evaluate : 0·856 × 12·39 (iv) Evaluate : 2·4 × 0·5 × 0·04

Solution :

(i) Since 53243 × 7 = 372701 [Step 1]

∴ **532·43 × 7 = 3727·01** [Step 2] **(Ans.)**

(ii) Since 409 × 56 = 22904 [Step 1]

∴ **4·09 × 5·6 = 22·904** [Step 2] **(Ans.)**

(iii) Since 856 × 1239 = 1060584

∴ **0·856 × 12·39 = 10·60584** **(Ans.)**

(iv) Since 24 × 5 × 4 = 480

And, the sum of decimal places in the given decimal numbers 2·4, 0·5 and 0·04 = 1 + 1 + 2 = 4. So, in the answer, the decimal point must be placed after 4 digits from the right.

∴ **2·4 × 0·5 × 0·04 = 0·0480** or **0·048** **(Ans.)**

Note : 0·0480 and 0·048 are the same.

Multiplication of a decimal number by 10 or higher powers of 10.

To multiply a decimal number by 10, 100, 1000, ... shift the decimal point to the right by as many digits as there are zeroes in 10, 100, 1000, etc.

For example :

(i) $43{\cdot}8725 \times 10 =$ **438·725** (ii) $43{\cdot}8725 \times 100 =$ **4387·25**

(iii) $43{\cdot}8725 \times 1000 =$ **43872·5**

Also,

(iv) $5{\cdot}7 \times 10 = 57$, $5{\cdot}7 \times 100 = 570$, $5{\cdot}7 \times 1000 = 5700$.

(v) $0{\cdot}008 \times 10 = 0{\cdot}08$, $0{\cdot}008 \times 100 = 0{\cdot}8$, $0{\cdot}008 \times 10000 = 80$.

15.9 DIVISION

(a) ***Division of a decimal number by a natural number*** (*i.e.* by a counting number) :

Divide in the ordinary way, and in the quotient obtained, place the decimal just after the division of the integral part of the given decimal number.

For example :

(i) $\frac{83{\cdot}6}{2} = \mathbf{41{\cdot}8}$

$$\begin{array}{r|l} & 41{\cdot}8 \\ 2 & 83{\cdot}6 \\ & 8 \\ \hline & 3 \\ & 2 \\ \hline & 16 \\ & 16 \\ \hline & \times \end{array}$$

(ii) $\frac{64{\cdot}56}{12} = \mathbf{5{\cdot}38}$

$$\begin{array}{r|l} & 5{\cdot}38 \\ 12 & 64{\cdot}56 \\ & 60 \\ \hline & 45 \\ & 36 \\ \hline & 96 \\ & 96 \\ \hline & \times \end{array}$$

(b) ***Division of a decimal number by a decimal number :***

Steps : **1.** Form a fraction with the decimal number to be divided as the numerator and the other decimal number (divisor) as the denominator.

2. Multiply both the terms of the fraction formed in step 1 by 10 or 100 or 1000, etc., so that the decimal point in the denominator is removed, and then divide.

For example :

(i) $\frac{36{\cdot}8}{1{\cdot}6} = \frac{36{\cdot}8 \times 10}{1{\cdot}6 \times 10}$

$= \frac{368}{16} = \mathbf{23}$

$1{\cdot}6 \times 10 = 16$

(ii) $\frac{5{\cdot}065}{0{\cdot}05} = \frac{5{\cdot}065 \times 100}{0{\cdot}05 \times 100} = \frac{506{\cdot}5}{5} = \mathbf{101{\cdot}3}$ and so on.

(c) ***Division of a decimal number by* 10, 100, 1000, etc.**

Shift the decimal point in the given number (dividend) to the left by as many digits as there are zeroes in the divisor : 10, 100, 1000, etc.

For example :

$\frac{48 \cdot 7}{10} = \mathbf{4 \cdot 87}$, $\frac{937 \cdot 3}{100} = \mathbf{9 \cdot 373}$, $\frac{520 \cdot 81}{1000} = \mathbf{0 \cdot 52081}$ and so on.

EXERCISE 15(C)

1. **Multiply :**
   (i) 5·6 and 8 (ii) 38·46 and 9 (iii) 0·943 and 62
   (iv) 0·0453 and 35 (v) 7·5 and 2·5 (vi) 4·23 and 0·8
   (vii) 83·54 and 0·07 (viii) 0·636 and 1·83

2. **Evaluate :**
   (i) $0{\cdot}0008 \times 26$ (ii) $0{\cdot}038 \times 95$ (iii) $1{\cdot}2 \times 2{\cdot}4 \times 3{\cdot}6$
   (iv) $0{\cdot}9 \times 1{\cdot}8 \times 0{\cdot}27$ (v) $1{\cdot}5 \times 1{\cdot}5 \times 1{\cdot}5$ (vi) $0{\cdot}025 \times 0{\cdot}025$
   (vii) $0{\cdot}2 \times 0{\cdot}002 \times 0{\cdot}001$

3. **Multiply** each of the following numbers by 10, 100 and 1000 :
   (i) 3·9 (ii) 2·89 (iii) 0·0829
   (iv) 40·3 (v) 0·3725

4. **Evaluate :**
   (i) $8{\cdot}64 \div 8$ (ii) $0{\cdot}0072 \div 6$ (iii) $20{\cdot}64 \div 16$
   (iv) $1{\cdot}602 \div 15$ (v) $13{\cdot}08 \div 4$ (vi) $3{\cdot}204 \div 9$
   (vii) $3{\cdot}024 \div 12$ (viii) $5{\cdot}15 \div 5$ (ix) $3 \div 5$

5. **Divide** each of the following numbers by 10, 100 and 1000 :
   (i) 49·79 (ii) 0·923 (iii) 0·0704

6. **Evaluate :**
   (i) $9{\cdot}4 \div 0{\cdot}47$ (ii) $6{\cdot}3 \div 0{\cdot}09$ (iii) $2{\cdot}88 \div 1{\cdot}2$
   (iv) $8{\cdot}64 \div 1{\cdot}6$ (v) $37{\cdot}188 \div 3{\cdot}6$ (vi) $16{\cdot}5 \div 0{\cdot}15$
   (vii) $3{\cdot}2 \div 0{\cdot}005$ (viii) $3{\cdot}24 \div 0{\cdot}0016$

7. **Fill in the blanks** with 10, 100, 1000 or 10000, etc. :
   (i) 7·85 × .... = 78·5 (ii) 0·442 × .... = 442 (iii) 0·0924 × .... = 9·24
   (iv) 0·00187 × .... = 18·7 (v) 2·6 × .... = 2600 (vi) 0·08 × .... = 80
   (vii) 96·7 ÷ .... = 0·967 (viii) 5·2 ÷ .... = 0·52 (ix) 33·15 ÷ .... = 0·03315
   (x) 0·7 ÷ .... = 0·007 (xi) 0·00672 × .... = 67·2

8. **Evaluate :**
   (i) $9{\cdot}32 - 28{\cdot}54 \div 10$ (ii) $0{\cdot}234 \times 10 + 62{\cdot}8$ (iii) $3{\cdot}06 \times 100 - 889{\cdot}4 \div 100$
   (iv) $2{\cdot}86 \times 7{\cdot}5 + 45{\cdot}4 \div 0{\cdot}2$ (v) $97{\cdot}82 \times 0{\cdot}03 - 0{\cdot}54 \div 0{\cdot}3$

15.10 USING DECIMALS IN

(a) *Denoting the value of currency :*

The currency of our country is *Rupee*, we write it as Re or Rs or ₹. When it is divided into hundred equal parts, each part is called a *paisa* (P).

We can express Rupees (₹) and paise together in decimal system, as shown below :

(i) ₹ 14 and 42 paise = ₹ 14·42

(ii) ₹ 3 and 8 paise = ₹ 3·08

(iii) 5 paise = ₹ 0·05

Rupee is written before the decimal point and paise after it.

(b) *Measuring lengths :*

The most commonly used unit of length is *metre* (m). When a length of one metre is divided into 100 equal parts, each part is called a *centimetre* (cm).

The units for measuring smaller lengths are *decimetre* (dm), *millimetre* (mm), etc. and these for bigger lengths are *decametre* (dam), *hectametre* (hm), *kilometre* (km), etc.

The relations between the different units used for measuring lengths are given below :

10 millimetre (mm) = 1 centimetre (cm)

10 centimetre (cm) = 1 decimetre (dm)

100 centimetre = 10 decimetre (dm) = 1 metre (m)

10 metre (m) = 1 decametre (dam)

10 dam = 1 hectametre (hm)

1000 metre = 10 hm = 1 kilometre (km)

In our daily life, we most commonly use *metre* (m) and *centimetre* (cm) for measuring lengths and the relation between these two units of length is :

$$1 \text{ m} = 100 \text{ cm} \quad \text{and} \quad 1 \text{ cm} = \frac{1}{100} \text{ m}$$

In the decimal system, the relation between these two units is as shown below :

(i) 3 m and 58 cm = 3·58 m (ii) 7 m and 8 cm = 7·08 m

(iii) 250 cm = 2·50 m (iv) 63 cm = 0·63 m and so on

(c) *Measuring weights* (mass) *:*

The most commonly used units for measuring weight are *kilogram* (kg) and *gram* (gm), and the relation between these two units of mass is :

$$1 \text{ kg} = 1000 \text{ gm} \quad \text{and} \quad 1 \text{ gm} = \frac{1}{1000} \text{ kg}$$

∴ (i) 1 kg 546 gm = 1·546 kg (ii) 5 kg 68 gm = 5·068 kg

(iii) 64 kg 5 gm = 64·005 kg (iv) 875 gm = 0·875 kg and so on

EXERCISE 15(D)

1. Express in *paise* :
   (i) ₹ 8·40 (ii) ₹ 0·97 (iii) ₹ 0·09 (iv) ₹ 62·35
2. Express in *rupees* :
   (i) 55 p (ii) 8 p (iii) 695 p (iv) 3279 p
3. Express in *centimetre* (cm) :
   (i) 6 m (ii) 8·54 m (iii) 3·08 m (iv) 0·87 m
   (v) 0·03 m (vi) 25·04 m
4. Express in *metre* (m) :
   (i) 250 cm (ii) 2328 cm (iii) 86 cm (iv) 4 cm
   (v) 107 cm
5. Express in *gram* (gm) :
   (i) 6 kg (ii) 5·543 kg (iii) 0·078 kg (iv) 3·62 kg
   (v) 4·5 kg
6. Express in *kilogram* (kg) :
   (i) 7000 gm (ii) 6839 gm (iii) 445 gm (iv) 93 gm
   (v) 8 gm (vi) 13545 gm
7. Add, giving answer in rupees :
   (i) ₹ 5·37 and ₹ 12 (ii) ₹ 24·03 and 532 paise
   (iii) 73 paise and ₹ 2·08 (iv) 8 paise and ₹ 15·36
8. Subtract :
   (i) ₹ 35·74 from ₹ 63·22 (ii) 286 paise from ₹ 7·02 (iii) ₹ 0·55 from 121 paise
9. Add, giving answer in metre :
   (i) 2·4 m and 1·78 m (ii) 848 cm and 2·9 m (iii) 0·93 m and 64 cm
10. Subtract, giving answer in metre :
   (i) 5·03 m from 19·6 m (ii) 428 cm from 1033 m (iii) 0·84 m from 122 cm
11. Add, giving answer in kg :
   (i) 2·06 kg and 57·864 kg (ii) 778 gm and 1·939 kg (iii) 0·065 kg and 4023 gm
12. Subtract, giving answer in kg :
   (i) 9·462 kg from 15·6 kg (ii) 4317 gm from 23 kg (iii) 0·798 kg from 4169 gm

15.11 WORD PROBLEMS BASED ON DECIMALS

Example 5 :

The cost of one metre cloth is ₹ 15·75. Find the cost of 2·4 m cloth.

Solution :

Since cost of 1 m cloth = ₹ 15·75

∴ **Cost of 2.4 m cloth** = ₹ 15·75 × 2·4 = **₹ 37·80** **(Ans.)**

Example 6 :

The length of a rod is 28·14 m. If it is divided into 3 equal pieces, find the length of each piece.

Solution :

Clearly, the length of 3 pieces = 28·14 m

[∵ The length of rod = the sum of the lengths of 3 pieces]

and **the length of each piece** = $\frac{28 \cdot 14}{3}$ m = **9·38 m** **(Ans.)**

Example 7 :

The total weight of 8 identical bricks is 20·4 kg.

Find : (i) the weight of each brick. (ii) the total weight of 5 bricks.

Solution :

(i) Given weight of 8 bricks = 20·4 kg,

∴ **Weight of each brick** = $\frac{20 \cdot 4}{8}$ kg = **2·55 kg** **(Ans.)**

(ii) Since weight of 1 brick = 2·55 kg,

∴ **Total weight of 5 bricks** = 2·55 kg × 5 = **12·75 kg** **(Ans.)**

EXERCISE 15(E)

1. The cost of a fountain pen is ₹ 13·25. Find the cost of 8 such pens.
2. The cost of 25 identical articles is ₹ 218·25. Find the cost of one article.
3. The length of an iron rod is 10·32 m. The rod is divided into 4 pieces of equal length. Find the length of each piece.
4. What will be the total length of cloth required to make 5 shirts if 2·15 m of cloth is needed for each shirt ?
5. Find the distance walked by a boy in $1\frac{1}{2}$ hours if he walks 2·150 km every hour.
6. 83 note-books are sold at ₹ 15·25 each. Find the total money (in rupees) obtained by selling these note-books.
7. If the length of one bed-cover is 2·1 m, find the total length of 17 bed-covers.
8. A piece of rope is 10 m 67 cm long. Another rope is 16 m 32 cm long. By how much is the second rope longer than the first one ?
9. 12 cakes of soap together weigh 5 kg and 604 gm. Find the weight of :
   (i) one cake in both kg and gramme (ii) 5 cakes in kg.
10. Three strings of lengths 50 m 75 cm, 68 m 58 cm and 121 m 3 cm, respectively, are joined together to get a single string of greatest length, find the length of the single string obtained. If this single string is then divided into 12 equal pieces, find the length of each piece.

PERCENT (Percentage) 16

16.1 IDEA OF A PERCENT

Per means ***'for every'*** or ***'out of'***, and **cent** means ***'hundred.'*** Thus, **percent** means ***'for every hundred'*** or ***'out of hundred.'***

For example :

If in an examination for 100 marks, Geeta secured 83 marks; that means Geeta scored 83 percent marks.

Conversely, if in the same examination, Rohit secured 67 percent marks that means Rohit has got 67 marks out of 100.

The symbol for percent is %.

Percent is the numerator of a fraction with denominator 100.

For example : *60 out of 100* $= \frac{60}{100}$ (As fraction with denominator 100)

= 60 as percent, written as 60%.

Thus, when a fraction is so expressed that its **denominator is 100**, the corresponding **numerator is** called **percent** (or percentage).

$\therefore$ (i) $\frac{3}{100} = 3\%$ (ii) $\frac{7}{50} = \frac{7 \times 2}{50 \times 2} = \frac{14}{100} = 14\%$ and so on

In fact, we do not follow any difference between percent and percentage strictly.

Example 1 :

Express each of the following statements in the percentage form :

(i) 5 out of 20 eggs are bad.

(ii) 3 children in a class of 30 are absent.

Solution :

(i) 5 out of 20 eggs are bad means $\frac{5}{20}$

And $\frac{5}{20} = \frac{5 \times 5}{20 \times 5} = \frac{25}{100} = 25\%$ $\therefore$ **25% eggs are bad.** **(Ans.)**

(ii) 3 children out of 30 are absent is written as $\frac{3}{30}$

And $\frac{3}{30} = \frac{1}{10} = \frac{10}{100} = 10\%$ $\therefore$ **10% children are absent.** **(Ans.)**

16.2 CONVERTING A GIVEN FRACTION OR DECIMAL INTO PERCENTAGE (PERCENT FORM) :

Multiply the given fraction or decimal by 100 and at the same time write the sign of percentage.

For example :

(i) $\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%$ (ii) $0{\cdot}225 = 0{\cdot}225 \times 100\% = 22{\cdot}5\%$ and so on.

In example 1 given above :

(i) **5 out of 20 eggs are bad** = $\frac{5}{20}$ eggs are bad = $\frac{5}{20} \times 100\%$ eggs are bad

= **25% eggs are bad.**

(ii) **3 children in a class of 30 are absent** = $\frac{3}{30}$ children are absent

= $\frac{3}{30} \times 100\%$ children are absent

= **10% children are absent**

16.3 CONVERTING A GIVEN PERCENTAGE INTO A FRACTION OR DECIMAL:

Remove the sign of percentage and at the same time divide by 100. Then reduce the resulting fraction obtained to its lowest terms or decimal as required.

For example :

(i) $25\% = \frac{25}{100} = \frac{1}{4}$ or, $25\% = \frac{25}{100} = 0{\cdot}25$

(ii) $37{\cdot}5\% = \frac{37 \cdot 5}{100} = \frac{375}{100 \times 10} = \frac{3}{8}$ or, $37{\cdot}5\% = \frac{37 \cdot 5}{100} = 0{\cdot}375$ and so on.

When a whole number is divided by 100, decimal point is placed just after two digits from the right. But when a decimal number is divided by 100, the decimal point is shifted two places to the left.

EXERCISE 16(A)

1. Express each of the following statements in percentage form :
   (i) 13 out of 20 (ii) 21 eggs out of 30 are good
2. Express the following fractions as percent :
   (i) $\frac{3}{200}$ (ii) $\frac{5}{6}$ (iii) $\frac{65}{80}$ (iv) $\frac{2}{3}$
3. Express as percent :
   (i) 0·10 (ii) 0·02 (iii) 0·7 (iv) 0·15 (v) 0·032

4. Convert into fractions in their lowest terms :

(i) 8% (ii) 20% (iii) 85% (iv) 250% (v) $12\frac{1}{2}\%$

5. Express as decimal fractions :

(i) 25% (ii) 108% (iii) 95% (iv) 4·5% (v) 29·2%

6. Express each of the following numbers as percent :

(i) 7 (ii) 2 (iii) 19·5 (iv) 5·37

16.4 EXPRESSING ONE QUANTITY (NUMBER) AS A PERCENTAGE OF THE OTHER :

Divide the first quantity by the second one and at the same time multiply the result by 100%.

For example :

(i) **20 kg as a percentage of 200 kg** $= \frac{20}{200} \times 100\% = 10\%$

(ii) **60 paise as a percent of ₹ 3** = 60 paise as a percent of 300 paise

$= \frac{60}{300} \times 100\% = 20\%$

1. *Percent / percentage has no unit.*
2. *In order to express one quantity as a percentage of another quantity, the two quantities must have the same unit.*

16.5 FINDING PERCENTAGE (PERCENT) OF A GIVEN QUANTITY :

Express the given percent as fraction and multiply by the given number.

For example :

25% of ₹ 500 $= \frac{25}{100} \times$ ₹ 500 = **₹ 125** and **30% of 400** $= \frac{30}{100} \times 400 = 120$

Example 2 :

In a class of 50 students, 40% are girls. Find the number of girls and number of boys in the class.

Solution :

No. of girls in the class = 40% of 50 $= \frac{40}{100} \times 50 = 20$ **(Ans.)**

No. of boys in the class = $50 - 20 = 30$ **(Ans.)**

Alternative method :

When a class has 40% girls, it has (100 – 40)% boys, *i.e.* 60% boys.

$\therefore$ **No. of girls** = 40% of 50 $= \frac{40}{100} \times 50 = 20$

and **no. of boys** = 60% of 50 $= \frac{60}{100} \times 50 = 30$ **(Ans.)**

Example 3 :

A girl scored 60 out of 75 in English, 60 out of 90 in Mathematics and 80 out of 100 in Science. Find the marks scored by her as percentage :

(i) in Mathematics (ii) in all the three subjects (as a whole)

Solution :

(i) **Percentage score in Mathematics**

$$= 60 \text{ out of } 90 = \frac{60}{90} \times 100\% = \frac{200}{3}\%$$

$$= 66\frac{2}{3}\%$$ **(Ans.)**

(ii) Sum of the maximum marks of all the three subjects = 75 + 90 + 100 = 265

and total score in the three subjects = 60 + 60 + 80 = 200

$\therefore$ **Percentage score as a whole** $= \frac{200}{265} \times 100\%$

$$= 75\frac{25}{53}\%$$ **(Ans.)**

Example 4 :

300 students appeared in an examination. Of these students, 28% got first division, 54% got second division and the remaining just passed. Assuming that no student failed, find the number of students who just passed.

Solution :

$\because$ Number of students with first division = 28% of 300

$= \frac{28}{100} \times 300 = 84$

And the number of students with second division = 54% of 300

$= \frac{54}{100} \times 300 = 162$

$\therefore$ **The number of students who just passed** = 300 − (84 + 162) = **54** **(Ans.)**

Alternative method :

Since the percentage of first divisioners = 28%

and the percentage of second divisioners = 54%

$\therefore$ The percentage of those who just passed = (100 − 28 − 54)%

= (100 − 82)% = 18%

$\Rightarrow$ **The number of those who just passed** = 18% of 300

$= \frac{18}{100} \times 300 =$ **54** **(Ans.)**

EXERCISE 16(B)

1. Express :
   (i) ₹ 5 as a percentage of ₹ 25. (ii) 80 paise as a percent of ₹ 4.
   (iii) 700 gm as a percentage of 2·8 kg. (iv) 90 cm as a percent of 4·5 m.
2. Express the first quantity as a percent of the second :
   (i) 40 p, ₹ 2 (ii) 500 gm, 6 kg (iii) 42 seconds, 6 minutes.
3. Find the value of each of the following :
   (i) 20% of ₹ 150 (ii) 90% of 130
   (iii) 15% of 2 minutes (iv) 7·5% of 500 kg.
4. If a man spends 70% of his income, what percent does he save ?
5. A girl gets 65 marks out of 80. What percentage of marks does she get ?
6. A class contains 25 children, of which 6 are girls. What is the percentage of boys in the class ?
7. A tin contains 20 litres of petrol. Due to leakage, 3 litres of petrol is lost. What is the percentage of petrol left in the tin ?
8. An alloy of copper and zinc contains 45% copper and the rest is zinc. Find the weight of zinc in 20 kg of the alloy.
9. A boy got 60 out of 80 in Hindi, 75 out of 100 in English, and 65 out of 70 in Arithmetic. In which subject is his percentage of marks the best ? Also, find his overall percentage.
10. 60 more soldiers joined a camp of 500 soldiers. What percentage of the earlier soliders do they make ?
11. In a plot of ground of area 6000 sq.m, only 4500 sq.m is allowed for construction. What percentage of area is to be left without construction ?
12. Mr. Sharma has a monthly salary of ₹ 8,000. If he spends ₹ 6,400 every month, find:
    (i) his monthly expenditure as percent. (ii) his monthly savings as percent.
13. The monthly salary of Rohit is ₹ 24,000. If his salary increases by 12%, find his new monthly salary.
14. In a sale, the price of an article is reduced by 30%. If the original price of the article is ₹ 1,800, find :
    (i) the reduction in the price of the article (ii) the reduced price of the article.
15. Evaluate :
    (i) 30% of 200 + 20% of 450 – 25% of 600
    (ii) 10% of ₹ 450 – 12% of ₹ 500 + 8% of ₹ 500.

16.6 TO FIND THE INCREASE OR DECREASE IN PERCENT :

$$\text{Increase \%} = \frac{\text{Increase in value}}{\text{Original value}} \times 100\%$$

$$\text{and decrease \%} = \frac{\text{Decrease in value}}{\text{Original value}} \times 100\%$$

For example :

(i) If the price of milk increases from ₹ 24 per litre to ₹ 32·40 per litre,

Increase in price = ₹ 32·40 – ₹ 24 = ₹ 8·40

and **increase %** = $\frac{\text{Increase in price}}{\text{Original price}} \times 100\% = \frac{₹ 8 \cdot 40}{₹ 24} \times 100\% =$ **35%.**

(ii) If the price of sugar decreases from ₹ 40 per kg to ₹ 32 per kg,

Decrease in price = ₹ 40 – ₹ 32 = ₹ 8

and **decrease%** = $\frac{\text{Decrease in price}}{\text{Original price}} \times 100\% = \frac{8}{40} \times 100\% =$ **20%.**

Example 5 :

(i) 70 is increased by 40%. Find the increased number.

(ii) The cost of an article is decreased by 15%. If the original cost is ₹ 80, find the decreased cost.

Solution :

(i) Since original number = 70

And increase in it = 40% of 70 = $\frac{40}{100} \times 70 = 28$

∴ **Increased number** = 70 + 28 = **98** **(Ans.)**

(ii) Since the original cost = ₹ 80

And decrease in it = 15% of ₹ 80 = $\frac{15}{100}$ × ₹ 80 = ₹ 12

∴ **Decreased cost** = ₹ 80 – ₹ 12 = **₹ 68** **(Ans.)**

Example 6 :

Out of ₹ 36,000, two-fifth were kept in a bank. Of the remaining money, 40% is spent on food and 15% on rent. Find how much money is spent on food and how much on rent ?

Solution :

∵ Money kept in bank = $\frac{2}{5}$ × ₹ 36,000 = ₹ 14,400

∴ Remaining money = ₹ 36,000 – ₹ 14,400 = ₹ 21,600

Now, **money spent on food** = 40% of ₹ 21,600

= $\frac{40}{100}$ × ₹ 21,600 = **₹ 8,640** **(Ans.)**

And **money spent on rent** = 15% of ₹ 21,600

= $\frac{15}{100}$ × ₹ 21,600 = **₹ 3,240** **(Ans.)**

16.7 PROBLEMS RELATED TO ENVIRONMENT

Example 7 :

Most of the water on the earth is salty, *i.e.* unfit for drinking. Only 2·7% of the available water (by volume) is fresh. Find :

(i) the percentage of water on the earth that is unfit for drinking.

(ii) out of 5,00,000 m^3 of water (taken from different parts of the earth and mixed together), what amount of water is fit for drinking.

Solution :

(i) Since, 2·7% of the available water is fit for drinking

∴ **The percentage of water unfit for drinking**

$= (100 - 2{\cdot}7)\% = \mathbf{97{\cdot}3\%}$ **(Ans.)**

(ii) **Quantity of water fit for drinking**

$= 2{\cdot}7\%$ of the water taken

$= \frac{2{\cdot}7}{100} \times 5{,}00{,}000\ m^3 = \mathbf{13{,}500\ m^3}$ **(Ans.)**

EXERCISE 16(C)

1. The price of rice rises from ₹ 30 per kg to ₹ 36 per kg. Find the percentage rise in the price of rice.
2. The population of a small locality was 4000 in 1979 and 4500 in 1981. By what percent had the population increased ?
3. The price of a scooter was ₹ 8,000 in 1975. It came down to ₹ 6,000 in 1980. By what percent had the price of the scooter came down ?
4. Find the resulting quantity when :
   (i) ₹ 400 is decreased by 8%.
   (ii) 25 km is increased by 5%.
   (iii) a speed of 600 km h^{-1} is increased by $12\frac{1}{2}$ %.
   (iv) there is 2·5% increase in a salary of ₹ 62,500.
5. The population of a village decreased by 12%. If the original population was 25,000, find the population after the decrease ?
6. Out of a salary of ₹ 13,500, I keep one-third as savings. Of the remaining money, I spend 50% on food and 20% on house rent. How much do I spend on food and house rent ?
7. A tank can hold 50 litres of water. At present it is only 30% full. How many litres of water shall I put into the tank so that it becomes 50% full ?
8. In an election, there are a total of 80,000 voters and two candidates, A and B. 80% of the voters go to the polls, out of which 60% vote for A. How many votes does B get ?
9. 70% of our body weight is made up of water. Find the weight of water in the body of a person whose body weight is 56 kg.

10. Only one-fifth of water is available in liquid form. This limited amount of water is replenished and used by man recurrently. Express this information as percent, showing :
   (i) water available in liquid form.
   (ii) water available in frozen form.

11. By weight, 90% of tomato and 78% of potato is water. Find :
   (i) the weight of water in 25 kg of tomato.
   (ii) the total quantity, by weight, of water in 90 kg of potato and 30 kg of tomato.

Revision Exercise (Chapter 16)

1. Rohit's age is 12 years and Geeta's age is 15 years. Express :
   (i) Rohit's age as a percent of Geeta's age.
   (ii) Geeta's age as a percent of Rohit's age.

2. A class has 30 boys and 20 girls. Find :
   (i) the percentage of girls in the class. (ii) the percentage of boys in the class.
   (iii) percentage of number of boys as compared with number of girls.

3. Mrs. Sharma went to the market with ₹ 800 in her purse. When she returned to her home, ₹ 240 were still left in her purse. What percent of her money did she spend in the market ?

4. In a mixture of two liquids *A* and *B* , 35% is liquid *B*. If the total quantity of the mixture is 20 kg, find the quantity of *A* by weight.

5. A girl got 375 marks out of 500 in the first term examination, 560 marks out of 800 in the second term examination and 840 marks out of 1200 in the third term examination. Find :
   (i) her percentage score in the first term examination.
   (ii) her percentage score in the second term examination.
   (iii) her percentage score in the third term examination.
   (iv) the total marks secured in all the three examinations.
   (v) the total marks scored in all the three examinations.
   (vi) her percentage score on the whole in all the three examinations.

6. Out of his monthly income of ₹ 2,500, a man spends ₹ 1,750. What percent of his income does he save every month ?

7. Mr. Singh's monthly salary is ₹ 15,000. This month he was promoted with an increment of ₹ 3,000 in his salary. Express his increment as a percent of his original salary.

8. (i) The price of an article increased from ₹ 16 to ₹ 20; find the percentage increase.
   (ii) The price of an article decreased from ₹ 20 to ₹ 16; find the percentage decrease.

9. (i) The salary of a man is ₹ 7,200 per month, which is now increased by 8%. Find his new salary per month.
   (ii) The salary of Mr. Sahni is ₹ 8,400 per month, which is now decreased by 5%. Find his new salary per month.

10. Find the percentage change from the first quantity to the second :
   (i) ₹ 80, ₹ 120 (ii) 75 kg, 60 kg (iii) 50 cm, 45 cm

11. The original price of an article is ₹ 640. Find its new price when its price is :
   (i) increased by 30% (ii) decreased by 20%

12. Find the number that is :

(i) 50% more than 48 (ii) 30% less than 70.

13. Evaluate : (i) 8% of 900 – 12% of 750 + 20% of 165.

(ii) 70% of 70 + 90% of 90 – 120% of 120.

14. Approximately 97·3% water on the earth is not fit for drinking. Find :

(i) the percentage of water on the earth that is fit for drinking.

(ii) the total volume of water available in a certain part of the earth where there is 21,600 m^3 of drinking water

15. Air is an important inexhaustible natural resource. It is essential for the survival of human beings, microbes, plants and animals. The following table shows the percentage of various gases in air.

Contents of air	**Percentage** (by volume)
Nitrogen	78
Oxygen	21
Others (carbon dioxide, inert gases, water vapour, etc.)	1

In 800 m^3 of air, calculate the approximate quantities of nitrogen, oxygen and other gases.

IDEA OF SPEED, DISTANCE AND TIME 17

17.1 SPEED

Speed of a body is the distance covered by the body in unit time.

$\therefore$ Speed = $\frac{\text{Distance}}{\text{Time}}$ $\Rightarrow$ (i) Distance = Speed × Time

and, (ii) Time = $\frac{\text{Distance}}{\text{Speed}}$

1. In order ***to find speed,*** if :
   (i) ***distance*** *is in metre* ***(m)*** *and* ***time*** *in second* ***(s)****; then the* ***speed*** *is in metre per second* ***(m s^{-1})****.*
   (ii) ***distance*** *is in kilometre* ***(km)*** *and* ***time*** *in hour* ***(h)****; then the* ***speed*** *is in kilometre per hour* ***(km h^{-1})****.*
2. *In order* ***to find distance****, if :*
   (i) ***speed*** *is in* ***m s^{-1}****,* ***time*** *must be in* ***second.***
   (ii) ***speed*** *is in* ***km h^{-1}****,* ***time*** *must be in* ***hour.***
3. *In order* ***to find time****, if :*
   (i) ***speed*** *is in* ***km h^{-1}****,* ***distance*** *must be in* ***kilometre****.*
   (ii) ***speed*** *is in* ***m s^{-1}****,* ***distance*** *must be in* ***metre****.*

Example 1 :

A boy covers a distance of 1·2 km in 40 minutes. Find his speed in :

(i) km per hour (km h^{-1}) (ii) metre per second (m s^{-1})

Solution :

(i) In order to get **speed in km per hour; the distance** covered must be **in km** and **the time taken** must be **in hour.**

Given : distance = 1·2 km and time = 40 min = $\frac{40}{60}$ h = $\frac{2}{3}$ h

$\therefore$ **Speed** = $\frac{\text{Distance}}{\text{Time}}$

$$= \frac{1{\cdot}2\text{ km}}{\frac{2}{3}\text{h}} = 1{\cdot}2 \times \frac{3}{2}\text{ km h}^{-1} = \mathbf{1{\cdot}8\text{ km h}^{-1}}$$ **(Ans.)**

(ii) In order **to get speed in metre per second; the distance** covered must be **in metre** and **the time** taken must be **in second.**

Given : distance = 1·2 km = 1·2 × 1000 m = 1,200 m

And, time = 40 min = 40 × 60 sec = 2400 sec

$$\therefore \quad \textbf{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{1200\text{ m}}{2400\text{ sec}} = \frac{1}{2}\text{ m s}^{-1} = \mathbf{0{\cdot}5\ m\ s^{-1}} \qquad \textbf{(Ans.)}$$

17.2 UNIFORM SPEED AND VARIABLE SPEED

If a body *covers equal distances in equal intervals of time,* its *speed is said* to be *uniform,* otherwise its speed is *variable.*

For example :

(i) If a car covers 60 km in first hour, 60 km in second hour, 60 km in third hour and so on, its *speed is uniform.*

(ii) If a car covers 60 km in first hour, 67 km in second hour, 58 km in third hour and so on, its *speed is variable.*

(iii) If a car covers first 60 km in one hour, second 60 km in 1 hour 20 minutes, third 60 km in 1 hour 30 minutes and so on, then also its **speed is variable.**

Example 2 :

A man runs 200 metre in 25 seconds. Find :

(i) his speed (ii) the distance run by him in 5 seconds

(iii) the time taken by him to cover $\frac{2}{5}$ km.

Solution :

(i) $\textbf{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{200\text{ m}}{25\text{ sec}} = \mathbf{8\ m\ s^{-1}}$ **(Ans.)**

(ii) **Distance run in 5 sec** = Speed × Time

$= 8\text{ m s}^{-1} \times 5\text{ sec} = \mathbf{40\ m}$ **(Ans.)**

(iii) **Time taken to cover $\frac{2}{5}$ km** $= \frac{\text{Distance}}{\text{Speed}}$

$= \frac{400\text{m}}{8\text{ms}^{-1}}$ $\left[\frac{2}{5}\text{ km} = \frac{2}{5} \times 1000\text{ m} = 400\text{ m}\right]$

$=$ **50 seconds** **(Ans.)**

Example 3 :

A train covers first 120 km in 2 hours, next 160 km in 3 hours and last 140 km again in 2 hours. Find the average speed of the train.

Solution :

$$\text{Average speed of an object} = \frac{\text{Total distance covered by it}}{\text{Total time taken}}$$

Since, total distance covered = 120 km + 160 km + 140 km

= 420 km

And, total time taken = 2 hr + 3 hr + 2 hr = 7 hr.

$\therefore$ **Average speed** = $\frac{420 \text{ km}}{7 \text{ hr}}$ = **60 km h^{-1}** **(Ans.)**

Example 4 :

A man covers first 60 km of his journey at 30 km h^{-1} and remaining 50 km at 20 km h^{-1}. Find :

(i) the total time taken,

(ii) his average speed during the whole journey.

Solution :

(i) Time taken to cover 1st 60 km = $\frac{60}{30}$ h $\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

= 2 h

And, time taken to cover remaining 50 km = $\frac{50}{20}$ h = $\frac{5}{2}$ h

$\therefore$ **Total time taken** = 2 h + $\frac{5}{2}$ h = $\frac{9}{2}$ h = **$4\frac{1}{2}$ h** **(Ans.)**

(ii) Since, total distance covered = 60 km + 50 km = 110 km

and total time taken = $\frac{9}{2}$ h

$\therefore$ **Average speed** = $\frac{110}{\frac{9}{2}}$ km h^{-1} $\left[\because \text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}\right]$

= $\frac{110 \times 2}{9}$ km h^{-1} = **$24\frac{4}{9}$ km h^{-1}** **(Ans.)**

EXERCISE 17(A)

1. A train covers 51 km in 3 hours. Calculate its speed. How far does the train go in 30 minutes ?
2. A motorist travelled the distance between two towns, which is 65 km, in 2 hours and 10 minutes. Find his speed in metre per minute.
3. A train travels 700 metres in 35 seconds. What is its speed in km h^{-1} ?
4. A racing car covered 600 km in 3 hours 20 minutes. Find its speed in metre per second. How much distance will the car cover in 50 sec ?
5. Rohit goes 350 km in 5 hours. Find :

   (i) his speed (ii) the distance covered by Rohit in 6·2 hours

   (iii) the time taken by him to cover 210 km,

   [Assume that throughout the journey, the speed of Rohit remains uniform].

6. A boy drives his scooter with a uniform speed of 45 km h^{-1}. Find :
   (i) the distance covered by him in 1 hour 20 min.
   (ii) the time taken by him to cover 108 km.
   (iii) the time taken to cover 900 m.
7. I travel a distance of 10 km and come back in $2\frac{1}{2}$ hours. What is my speed ?
8. A man walks a distance of 5 km in 2 hours. Then he goes in a bus to a nearby town, which is 40 km, in further 2 hours. From there, he goes to his office in an autorickshaw, a distance of 5 km, in $\frac{1}{2}$ hour. What was his average speed during the whole journey?
9. Jagan went to another town such that he covered 240 km by a car going at 60 km h^{-1}. Then he covered 80 km by a train, going at 100 km h^{-1} and the rest 200 km, he covered by a bus, going at 50 km h^{-1}. What was his average speed during the whole journey ?
10. The speed of sound in air is about 330 m s^{-1}. Express this speed in km h^{-1}. How long will the sound take to travel 99 km ?

17.3 CONVERTING SPEED FROM ONE UNIT TO OTHER UNIT

To convert speed in kilometre per hour (km h^{-1}) into metre per second (m s^{-1}), multiply by $\frac{5}{18}$. And, to convert ms^{-1} into km h^{-1}, multiply by $\frac{18}{5}$.

Reason : $1 \text{ km h}^{-1} = \frac{1 \text{ kilometre}}{1 \text{ hour}} = \frac{1000 \text{ metre}}{60 \times 60 \text{ second}} = \frac{5}{18} \text{ m s}^{-1}$.

Example 5 :

Convert : (i) 90 km h^{-1} into m s^{-1} (ii) 15 ms^{-1} into km h^{-1}
(iii) 75 cm s^{-1} into km h^{-1} (iv) 45 km h^{-1} into m min^{-1}

Solution :

(i) $90 \text{ km h}^{-1} = 90 \times \frac{5}{18} \text{ m s}^{-1} = 25 \text{ m s}^{-1}$ **(Ans.)**

(ii) $15 \text{ m s}^{-1} = 15 \times \frac{18}{5} \text{ km h}^{-1} = 54 \text{ km h}^{-1}$ **(Ans.)**

(iii) $75 \text{ cm s}^{-1} = 0{\cdot}75 \text{ ms}^{-1}$ $\left[\because 75 \text{ cm} = \frac{75}{100} \text{ m} = 0{\cdot}75 \text{ m}\right]$

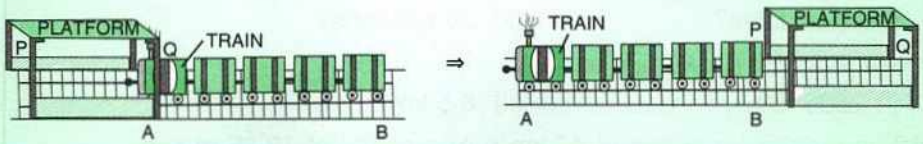
$= 0{\cdot}75 \times \frac{18}{5} \text{ km h}^{-1} = 2{\cdot}7 \text{ km h}^{-1}$ **(Ans.)**

(iv) $45 \text{ km h}^{-1} = \frac{45 \text{ km}}{1 \text{ h}} = \frac{45 \times 1000 \text{ m}}{60 \text{ min}} = 750 \text{ m min}^{-1}$ **(Ans.)**

When a train passes a :

(i) **pole** or any other stationary object, etc.,
the minimum distance covered by the train = length of the train

(ii) **platform**, the minimum distance covered by the train
= length of the train + length of the platform.



Example 6 :

A 160 m long train is travelling at a speed of 72 km h^{-1}, find the time taken by the train to pass :

(i) a telegraph post (ii) a 200 m long platform.

Solution :

(i) Distance to be covered = length of the train = 160 m

And, speed = $72 \text{ km h}^{-1} = 72 \times \frac{5}{18} \text{ m s}^{-1} = 20 \text{ m s}^{-1}$

$\therefore$ **Time taken** = $\frac{\text{Distance}}{\text{Speed}} = \frac{160}{20}$ sec = **8 sec** **(Ans.)**

(ii) Distance to be covered = length of the train + length of the platform

= 160 m + 200 m = 360 m

$\therefore$ **Time taken** = $\frac{\text{Distance}}{\text{Speed}} = \frac{360}{20}$ sec = **18 sec** **(Ans.)**

Example 7 :

P and Q run with speeds 8 km h^{-1} and 11 km h^{-1}. They start running from the same point, find the distance between them after 2 hours, if they run in the

(i) same direction. (ii) opposite directions (moving away from each other).

Solution :

(i) Required distance = Difference between the distances covered by P and Q

(ii) Required distance = Sum of the distances covered by P and Q.

Distance run by P in 2 hours = speed × time

= 8 km h^{-1} × 2 hours = 16 km

Distance run by Q in 2 hours = 11 km h^{-1} × 2 hours = 22 km

(i) **Required distance** = 22 km − 16 km = **6 km** **(Ans.)**

(ii) **Required distance** = 22 km + 16 km = **38 km** **(Ans.)**

EXERCISE 17(B)

1. A train 180 m long is running at a speed of 90 km/h. How long will it take to pass a railway signal ?
2. A train whose length is 150 m, passes a telegraph pole in 10 sec. Find the speed of the train in km/h.
3. A train, 120 m long, passes a railway platform 160 m long in 14 sec. How long will it take to pass another platform which is 100 m long ?
4. Mr. Amit can walk 8 km in 1 hour 20 minutes.

   (a) How far does he go in :

   (i) 10 minutes? (ii) 30 seconds?

   (b) How long will it take him to walk :

   (i) 2500 m? (ii) 6·5 km?
5. Which is greater : a speed of 45 km/h or a speed of 12·25 m/sec ?

   How much is the distance travelled by each in 2 seconds ?
6. A and B start from the same point and at the same time with speeds 15 km/h and 12 km/h respectively. Find the distance between A and B after 6 hours if both move in :

   (i) same direction (ii) the opposite directions.
7. A and B start from the same place, in the same direction and at the same time with speeds 6 km/h and 2m/sec respectively. After 5 hours who will be ahead and by how much ?
8. Mohit covers a certain distance in 6 hrs by his scooter at a speed of 40 km h^{-1}.

   (i) Find the time taken by Manjoor to cover the same distance by his car at the speed of 60 km h^{-1}.

   (ii) Find the speed of Joseph, if he takes 8 hrs to complete the same distance.
9. A boy swims 200 m in still water and then returns back to the point of start in total 10 minutes. Find the speed of his swim in (i) m s^{-1} (ii) km h^{-1}.
10. A distance of 14·4 km is covered in 2 hours 40 minutes. Find the speed in m s^{-1}.

    With this speed Sakshi goes to her school, 240 m away from her house and then returns back. How much time, in all, will Sakshi take ?

Theme – 3
ALGEBRA

FUNDAMENTAL CONCEPTS 18

18.1 ALGEBRA

Algebra is a *generalized form of arithmetic.* In arithmetic, we use numbers like 5, –8, 0·64, etc., each with a definite value, whereas in algebra, we use letters (a, b, c, ..., x, y, z, etc.) along with numbers.

For example :

$7x,\ 3x - 2,\ 5a + b,\ 2y - 5x,\ x + 2y - 7z$ and so on

The letters used in algebra are called ***variables*** or ***literal numbers*** or simply ***literals.*** They do not have a fixed value.

18.2 SIGNS AND SYMBOLS

In algebra, the signs +, –, × and ÷ are used in the same sense as they are used in arithmetic.

Also, the following *signs and symbols* are *frequently used in algebra,* each with the same meaning in every branch of mathematics.

$=$	means	"*is equal to*"	$\neq$	means	"*is not equal to*"
$<$	means	"*is less than*"	$>$	means	"*is greater than*"
$\nless$	means	"*is not less than*"	$\ngtr$	means	"*is not greater than*"
$\therefore$	means	"*therefore*"	$\because$	means	"*because*" or "*since*"
$\sim$	means	"*difference between*"	$\Rightarrow$	means	"*implies that*".

18.3 WRITING A GIVEN STATEMENT IN ALGEBRAIC FORM

	Statement	Algebraic Form
(i)	x subtracted from 8 is less than y	$8 - x < y$
(ii)	y divided by 5 equals 2	$\frac{y}{5} = 2$
(iii)	z increased by 2x is 23	$z + 2x = 23$

Conversely,

	Algebraic Form		Statement
(i)	$x + y = 3$		x plus y is equal to 3
		or	sum of x and y is equal to 3.
(ii)	$p - 5 = x$		*p* minus 5 is equal to x
		or	*p* decreased by 5 is equal to x.
		or	*p* exceeds 5 by x
(iii)	$5x > 7$		5 multiplied by x is greater than 7
		or	product of 5 and x is greater than 7
(iv)	$\frac{8}{y} < 3$		8 divided by y is less than 3.

EXERCISE 18(A)

1. Express each of the following statements in **algebraic form** :

(i)	The sum of 8 and x is equal to y.	....
(ii)	x decreased by 5 is equal to y.	....
(iii)	The sum of 2 and x is greater than y.	....
(iv)	The sum of x and y is less than 24.	....
(v)	15 multiplied by m gives 3n.	....
(vi)	Product of 8 and y is equal to 3x.	....
(vii)	30 divided by b is equal to p.	....
(viii)	z decreased by 3x is equal to y.	....
(ix)	12 times of x is equal to 5z.	....
(x)	12 times of x is greater than 5z.	....
(xi)	12 times of x is less than 5z.	....
(xii)	3z subtracted from 45 is equal to y.	....
(xiii)	8x divided by y is equal to 2z.	....
(xiv)	7y subtracted from 5x gives 8z.	....
(xv)	7y decreased by 5x gives 8z.	....

2. For each of the following algebraic expressions, write a suitable statement in words :

(i)	$3x + 8 = 15$	....
(ii)	$7 - y > x$	....
(iii)	$2y - x < 12$	....
(iv)	$5 \div z = 5$	....
(v)	$a + 2b > 18$	....
(vi)	$2x - 3y = 16$	....
(vii)	$3a - 4b > 14$	....
(viii)	$b + 7a < 21$	....
(ix)	$(16 + 2a) - x > 25$	....
(x)	$(3x + 12) - y < 3a$	....

18.4 CONSTANTS AND VARIABLES

In algebra, we come across *two types of symbols,* namely, *constants* and *variables.*

A symbol with a *fixed numerical value* in all situations is called a **constant**, e.g. 5, 30, 256, –7, $\frac{5}{3}$, $\frac{7}{9}$, etc., whereas a symbol whose *value changes with situation* is called a **variable,** such as; x, y, p, q, 5x, etc.

In 3x, 3 is a constant and x a variable but, together, 3x is a variable.
Reason : *As the value of x will change, the value of 3x will also change accordingly.*
Similarly 3 is constant and x is variable but, together, each of 3 + x, x – 3 and x ÷ 3 is a variable.
So, we conclude that every combination of a constant and a variable is always a variable.

18.5 TERM

*A **term** is a constant or a variable or a product or a quotient of constants and variables.*

For example :

(i) 4 is a term, which is a *constant*

(ii) x is a term, which is a *variable*

(iii) 4x is a term, which is the *product of a constant and a variable.*

(iv) $\frac{3}{y}$ is a term, which is the *quotient of a constant and a variable.*

A term is called a **constant term** if it does not contain any literal (variable).

Thus, each of 3, –20, $\frac{5}{7}$, $-\frac{4}{9}$, etc. is a constant term.

Constants (fixed numbers) and variables (literal numbers) may be combined to form several types of terms.

For example :

The constants 2, 5, –8, 4, $\frac{3}{2}$, etc., and the variables x, y, z, etc., may be combined to form terms such as 2x, 5y, 5xy, 5xyz, 4xz, $\frac{3}{2}yz$, ...

(i) Like Terms :

The terms having *the same literal coefficients* are called ***like terms.*** They may differ only in their numeral coefficients.

For example :

Each having the same literal coefficient : xy

(i) xy, 5xy, – 4xy, etc. are like terms

(ii) $-8x^2y$, $7x^2y$, $1{\cdot}5x^2y$, etc. are like terms and so on.

(ii) Unlike Terms :

The terms that *do not have the same literal coefficients* are called ***unlike terms.***

For example :

(i) 6a, 6ab and 6ac are unlike terms.

(ii) 2xy, $2x^2y$ and $2xy^2$ are unlike terms and so on.

18.6 ALGEBRAIC EXPRESSIONS

An algebraic expression is a collection of one or more terms, which are separated from each other by the signs + (plus) and/or – (minus).

For example :

Algebraic expressions	Number of terms used	Terms
(i) $5x$	1	$5x$
(ii) $8xy^2$	1	$8xy^2$
(iii) $3x + 8z$	2	$3x$ and $8z$
(iv) $4x - y + 7$	3	$4x$, y and 7
(v) $7xy + \frac{2a}{y} - 3z + 8$	4	$7xy$, $\frac{2a}{y}$, $3z$ and 8

and so on.

In the algebraic expression $4x - y + 7$, 7 is the constant term as it does not contain a literal. Similarly, in the algebraic expression $7xy + \frac{2a}{y} - 3z + 8$; 8 is the constant term.

18.7 TYPES OF ALGEBRAIC EXPRESSIONS

According to the number of terms used to form an algebraic expression, it is called monomial, binomial, trinomial, and so on as explained below.

(i) Monomial :

An *algebraic expression* with *only one term* is called a ***monomial.***

For example : -8, z, xy, $2x$, $5y$, $\frac{2x}{5y}$, etc. are all monomials.

(ii) Binomial :

An algebraic expression of *two unlike terms* is called a ***binomial.***

A binomial is a polynomial of two terms

For example :

(i) $5x + 2y$, $7 - x$, $4x + y$, $y + zy$, etc. (ii) $2a + \frac{b}{2}$, $\frac{a}{3} - \frac{b}{3}$, $\frac{ab}{2} + \frac{26}{3}$, etc.

(iii) Trinomial :

An algebraic expression containing *three unlike terms* is called a ***trinomial.***

A trinomial is a polynomial of three terms

For example :

$ax^2 + bx + c$, $2x^2 - 7x + 4$, $xy - x + y^2$, etc.

(iv) Multinomial :

An algebraic expression with two or *more than two* terms is called a ***multinomial.***

For example :

(i) Each of $3x + 2$, $5 - x$, $a^2 - 7x$ is a *multinomial of two terms.*

(ii) $7 + x - xy + y^2$ is a *multinomial of four terms.*

(iii) $a + ab - b^2 + 7x - z$ is a *multinomial of five terms* and so on.

(v) Polynomial :

An *algebraic expression* with one *or more (unlike) terms,* is called a ***polynomial.***

For example :

(i) Each of –20, 8, x, 5x, $3xy^2$, etc., is a polynomial.

(ii) 3x + 2y is a *polynomial of two terms.*

(iii) x + 4yz – 7z + 8 is a *polynomial of four terms.*

(iv) Every monomial, every binomial, every trinomial and every multinomial is a polynomial.

> For each literal used in a polynomial, its power must always be a whole number.

(v) A polynomial can not be of the form : $\frac{1}{x}$, $\frac{3}{x+5}$, $\frac{2x}{x-5}$, $\frac{5}{x^2}$, $\frac{7x}{x^2+8}$, $x^{2/3}$, $x^{1/2}$, etc.

> *Terms are separated by plus (+) and minus (–) signs only.*
> *The signs of multiplication (×) and division (÷) do not separate terms.*
> *Thus, 3p + 5z – 7y has three terms, whereas 3p × 5z – 7y has two terms only.*
> *In the same way, 8 – 4x + 7y + 2z has four terms, whereas 8 × 4x × 7y ÷ 2z has only one term.*

18.8 PRODUCTS AND FACTORS

A **product** is the result of the multiplication of two or more constants or literals or of both.

For example :

5xy is the product of 5, x and y.

Each constant and each literal multiplied together to form a product is called a **factor** of that product.

18.9 COEFFICIENT

Any factor or group of factors of a product is known as the **coefficient** of the remaining factors.

For example :

In the product 5axy,

5 is the coefficient of axy, 5x is the coefficient of ay, xy is the coefficient of 5a, axy is the coefficient of 5 and so on.

If a factor is a *numerical quantity,* it is called a *numeral coefficient* of the remaining factors, and if a factor involves *letters,* it is called a *literal coefficient* of the remaining factors.

For example :

In a product 3xy,

3 is a *numeral coefficient* of xy, x is a *literal coefficient* of 3y, xy is a *literal coefficient* of 3, y is *literal coefficient* of 3x, 3y is *literal coefficient* of x and so on.

> *When the coefficient is unity, i.e. 1 (one), it is usually omitted, i.e. 1b is written as b.*

18.10 POWER OF LITERAL QUANTITIES

When *a quantity is multiplied by itself* any number of times, the product is called *a power of that quantity.* This product is expressed by writing the number of like factors in it to the right of the quantity slightly raised.

For example :

$a \times a$ has 2 like factors, so we express it as : $a \times a = a^2$

Similarly, (i) $a \times a \times a$ has 3 like factors, so we write : $a \times a \times a = a^3$.

(ii) $a \times a \times a \times a \times a$ has 5 like factors, so we write : $a \times a \times a \times a \times a = a^5$.

The following table will make the concept more clear :

	Product	Write as :	Read as :
(i)	$a \times a$	a^2	*a* squared or *a* raised to the power 2.
(ii)	$a \times a \times a$	a^3	*a* cubed or *a* raised to the power 3.
(iii)	$m \times m \times m \times m \times m$	m^5	*m* raised to the power 5 or fifth power of *m*.

In a^8, ***a*** is called the **base** and **8** is called the **exponent** or the **index** or the **power.**

Similarly, in x^5, **x** is the **base** and **5** is the **exponent** or the **index** or the **power** and so on.

1. For all values of x, $x^1 = x$ *i.e.* $5^1 = 5$, $8^1 = 8$, $35^1 = 35$ and so on
2. For all values of x, $x^0 = 1$ *i.e.* $5^0 = 1$, $8^0 = 1$, $35^0 = 1$ and so on

Example 1 :

Write each of the following products in **index form :**

(i) $m \times m \times n \times n \times n \times n$ (ii) $3 \times b \times b \times b \times b \times p \times p \times p$

Solution :

(i) $m \times m \times n \times n \times n \times n = \mathbf{m^2n^4}$ **(Ans.)**

(ii) $3 \times b \times b \times b \times b \times p \times p \times p = \mathbf{3b^4p^3}$ **(Ans.)**

Example 2 :

Write each of the following in **product form :**

(i) $3p^4$ (ii) $7b^2q^3$ (iii) $a^3m^4n^2$

Solution :

(i) $3p^4 = \mathbf{3 \times p \times p \times p \times p}$ **(Ans.)**

(ii) $7b^2q^3 = \mathbf{7 \times b \times b \times q \times q \times q}$ **(Ans.)**

(iii) $a^3m^4n^2 = \mathbf{a \times a \times a \times m \times m \times m \times m \times n \times n}$ **(Ans.)**

18.11 POLYNOMIAL IN ONE VARIABLE AND ITS DEGREE

When an **algeraic expression** is **made of one variable** only, it is called a **polynomial in one variable.**

For example :

(i) $3 + 5x - 7x^2$ is a polynomial in variable x.

(ii) $9y^3 - 5y^2 + 8$ is a polynomial in variable y.

The **degree of a polynomial** in one variable **is the greatest** of the **exponents** (powers) **of its various terms.**

For example :

1. For polynomial $4x^2 - 3x^5 + 8x^6$

   (i) the exponent of the term $4x^2 = 2$,

   (ii) the exponent of the term $3x^5 = 5$ and

   (iii) the exponent of the term $8x^6 = 6$.

   Since the greatest exponent is 6

   ∴ The **degree** of the polynomial $4x^2 - 3x^5 + 8x^6 =$ **6**

The polynomial $3x^4 - x^3 + 5x - 7$ is in one variable only, the variable being x.
The polynomial $8y^5 - 3y^2 + 8$ is also in one variable only, the variable being y.

2. The **degree** of the polynomial $25 - x^4$ is **4.**
3. The **degree** of the polynomial $5x - 3$ is **1.**

$\because x = x^1$

4. The **degree** of the polynomial $4x^3 - 15x^5 - 7x^8$ is **8** and so on.

Polynomials of two or more variables and their degree

For example :

(i) $3x + xy^2 - 8yz$ is a polynomial made of three variables, x, y and z.

(ii) $5y^3 - 3y^2x + 8x^2y^2 - 3x^5$ is a polynomial of two variables, x and y.

In order to find the degrees of such polynomials, find :

(a) The sum of the powers of all the variables used in each term of a given polynomial.

(b) The greatest of these sums is the degree of the given polynomial.

For example :

For polynomial $3x + xy^2 - 8yz$

The terms used are $3x$, xy^2 and $8yz$

Since the sum of the powers of the variables in $3x$ used = 1, [$3x = 3x^1$]

the sum of the powers of the variables in $xy^2 = 1 + 2 = 3$

and the sum of the powers of the variables used in $8yz = 1 + 1 = 2$

Clearly, **degree of the given polynomial = 3**

EXERCISE 18(B)

1. Separate the **constants** and the **variables** from each of the following :

$6,\ 4y,\ -3x,\ \frac{5}{4},\ \frac{4}{5}xy,\ az,\ 7p,\ 0,\ \frac{9x}{y},\ \frac{3}{4x},\ -\frac{xz}{3y}$

2. Group the like terms together :

(i) $4x, -3y, -x, \frac{2}{3}x, \frac{4}{5}y$ and y. (ii) $\frac{2}{3}xy, -4yx, 2yz, \frac{-2}{3}yz, \frac{zy}{3}$ and yx.

(iii) $-ab^2, b^2a^2, 7b^2a, -3a^2b^2$ and $2ab^2$ (iv) $5ax, -5by, \frac{by}{7}, 7xa$ and $\frac{2ax}{3}$.

3. State whether **true** or **false** :

(i) 16 is a constant and y is a variable, but 16y is variable.

(ii) 5x has two terms 5 and x. (iii) The expression 5 + x has two terms 5 and x.

(iv) The expression $2x^2 + x$ is a trinomial. (v) $ax^2 + bx + c$ is a trinomial.

(vi) $8 \times ab$ is a binomial. (vii) 8 + ab is a binomial.

(viii) $x^3 - 5xy + 6x + 7$ is a polynomial. (ix) $x^3 - 5xy + 6x + 7$ is a multinomial.

(x) The coefficient of x in 5x is 5x. (xi) The coefficient of ab in – ab is –1.

(xii) The coefficient of y in –3xy is –3.

4. State the number of terms in each of the following expressions :

(i) $2a - b$ (ii) $3 \times x + \frac{a}{2}$ (iii) $3x - \frac{x}{p}$

(iv) $a \div x \times b + c$ (v) $3x \div 2 + y + 4$ (vi) $xy \div 2$

(vii) $x + y \div a$ (viii) $2x + y + 8 \div y$ (ix) $2 \times a + 3 \div b + 4$

5. State whether **true** or **false** :

(i) xy and –yx are like terms. (ii) x^2y and $-y^2x$ are like terms.

(iii) a and –a are like terms. (iv) –ba and 2ab are unlike terms.

(v) 5 and 5x are like terms. (vi) 3xy and 4xyz are unlike terms.

6. For each expression given below, state whether it is a *monomial, or a binomial* or *a trinomial.*

(i) xy (ii) $xy + x$ (iii) $2x \div y$ (iv) $-a$ (v) $ax^2 - x + 5$

(vi) $-3bc + d$ (vii) $1 + x + y$ (viii) $1 + x \div y$ (ix) $x + xy - y^2$

7. Write down the coefficient of x in the following monomials :

(i) x (ii) $-x$ (iii) $-3x$ (iv) $-5ax$ (v) $\frac{3}{2}xy$ (vi) $\frac{ax}{y}$

8. Write the coefficients of :

(i) x in $-3xy^2$ (ii) x in $-ax$ (iii) y in $-y$ (iv) y in $\frac{2}{a}y$

(v) xy in $-2xyz$ (vi) ax in $-axy^2$ (vii) x^2y in $-3ax^2y$ (viii) xy^2 in $5axy^2$

9. State the numeral coefficients of the following monomials :

(i) $5xy$ (ii) abc (iii) $5pqr$ (iv) $\frac{-2x}{y}$

(v) $\frac{2}{3}xy^2$ (vi) $\frac{-15xy}{2z}$ (vii) $-7x \div y$ (viii) $-3x \div (2y)$

10. Write the degree of each of the following polynomials :

(i) $x + x^2$ (ii) $5x^2 - 7x + 2$ (iii) $x^3 - x^8 + x^{10}$

(iv) $1 - 100x^{20}$ (v) $4 + 4x - 4x^3$ (vi) $8x^2y - 3y^2 + x^2y^5$

(vii) $8z^3 - 8y^2z^3 + 7yz^5$ (viii) $4y^2 - 3x^3 + y^2x^7$

FUNDAMENTAL OPERATIONS 19
(Related to Algebraic Expressions)

19.1 BASIC CONCEPT

In Mathematics, the operations **addition** (+), **subtraction** (–), **multiplication** (×) and **division** (÷) are the **four fundamental operations**.

Students are familiar with these operations as they have already studied about these operations in Arithmetic in their lower classes.

(i) ADDITION OF LIKE TERMS :

The addition of like terms is a single term (like to the given terms) whose coefficient is equal to the sum of the coefficients of the given (like) terms.

Thus :

(i) Addition of 3x and 8x $= 3x + 8x = (3 + 8)x =$ **11x**

(ii) Addition of $8x^2y$ and $-5x^2y = 8x^2y + (-5x^2y)$

$= 8x^2y - 5x^2y = (8 - 5)x^2y =$ **$3x^2y$**

(iii) $2xy + 3xy + 5xy = (2 + 3 + 5)xy =$ **10xy**

(iv) $7y^2 - 4y^2 + 3y^2 = (7 - 4 + 3)y^2$

$= (10 - 4)y^2 =$ **$6y^2$**

For addition, the terms are taken with their given signs, e.g.

(i) addition of 7xy and $-3xy = 7xy - 3xy = (7 - 3)xy =$ **4xy**

(ii) addition of $-7xy$ and $3xy = -7xy + 3xy = (-7 + 3)xy =$ **$-4xy$**

and (iii) $-7x - 3x = (-7 - 3)x =$ **$-10x$**

In the same way :

(i) addition of $-3xy^2$, $-5xy^2$ and $-xy^2$

$= (-3xy^2) + (-5xy^2) + (-xy^2)$

$= -3xy^2 - 5xy^2 - xy^2$

$= (-3 - 5 - 1)xy^2 =$ **$-9xy^2$**

(ii) addition of 7ab, –2ab, –5ab, 6ab and –ab

$= 7ab - 2ab - 5ab + 6ab - ab$

$= (7 - 2 - 5 + 6 - 1)ab$

$= (13 - 8)ab =$ **5ab**

(ii) ADDITION OF UNLIKE TERMS :

As shown above, the sum of two or more like terms is a single like term, but two unlike terms cannot be added together to get a single term.

For example : the unlike terms 2ab and 4bc cannot be added together to form a single term. All that can be done is to connect them by the sign of addition and leave the result in the form 2ab + 4bc.

In the same way,

(i) addition of $5x^2$ and $8xy$

$= 5x^2 + 8xy$

(ii) addition of $2y^3$, $-5xy$ and $3x^3$

$= 2y^3 - 5xy + 3x^3$ and so on

(iii) SUBTRACTION OF LIKE TERMS :

For subtraction of like terms, the rules are the same as those for subtraction of integers.

For example :

Since $-4 + 2 = -2$,

$\therefore$ $-4x + 2x = -2x$

$-4 + 2 = -2$

Since $3 - 7 = -4$,

$\therefore$ $3x - 7x = -4x$

$3 - 7 = -4$

Example 1 :

Subtract : (i) 4x from $-8x$ (ii) $-3x$ from $-7x$

Solution :

In each subtraction, change the sign of the term to be subtracted.

(i) $-8x - (4x) = -8x - 4x = -12x$ **(Ans.)**

(ii) $-7x - (-3x) = -7x + 3x = -4x$ **(Ans.)**

The result of subtraction of two like terms is also a like term.

(iv) SUBTRACTION OF UNLIKE TERMS :

Just as it is with addition of unlike terms, we cannot get a single term by the subtraction of unlike terms. For example, 2ab and 4bc are two unlike terms, the subtraction of 2ab from 4bc is 4bc − 2ab, which cannot be simplified further to get a single term.

Similarly, the subtraction of 4bc from 2ab is 2ab − 4bc, which cannot be simplified further to get a single term.

Example 2 :

Evaluate : (i) $3x - 4x + 7x$ (ii) $6ab + 3ab - 4ab$

(iii) $5ax + ax - 8ax$ (iv) $8a + 3a - 5a - 2a$

Solution :

1. *Add the positive terms together and separately add the negative terms together as well.*
2. *Find the result of the two terms obtained.*

(i) $3x - 4x + 7x = 10x - 4x$

$= 6x$ **(Ans.)**

$3x + 7x = 10x$

(ii) $6ab + 3ab - 4ab = 9ab - 4ab = 5ab$ **(Ans.)**

(iii) $5ax + ax - 8ax = 6ax - 8ax = -2ax$ **(Ans.)**

(iv) $8a + 3a - 5a - 2a = 11a - 7a = 4a$ **(Ans.)**

Example 3 :

Evaluate :

(i) $3x + 1\frac{2}{5}x$ (ii) $5a - 2\frac{1}{2}a + 1\frac{1}{2}a$

Solution :

(i) $3x + 1\frac{2}{5}x = \frac{3x}{1} + \frac{7x}{5}$

$1\frac{2}{5} = \frac{1\times5+2}{5} = \frac{7}{5}$

$= \frac{15x + 7x}{5} = \frac{22x}{5} = 4\frac{2}{5}x$ **(Ans.)**

(ii) $5a - 2\frac{1}{2}a + 1\frac{1}{2}a = \frac{5a}{1} - \frac{5a}{2} + \frac{3a}{2}$

$= \frac{10a - 5a + 3a}{2} = \frac{13a - 5a}{2} = \frac{8a}{2} = 4a$ **(Ans.)**

EXERCISE 19(A)

1. Fill in the blanks :

(i) $5 + 4$ = ... and $5x + 4x$ = ...

(ii) $12 + 18$ = ... and $12x^2y + 18x^2y$ = ...

(iii) $7 + 16$ = ... and $7a + 16b$ = ...

(iv) $1 + 3$ = ... and $x^2y + 3xy^2$ = ...

(v) $7 - 4$ = ... and $7ab - 4ab$ = ...

(vi) $12 - 5$ = ... and $12x - 5y$ = ...

(vii) $35 - 16$ = ... and $35ab - 16ba$ = ...

(viii) $28 - 13$ = ... and $28ax^2 - 13a^2x$ = ...

2. Fill in the blanks :

(i) The sum of -2 and -5 = ... and the sum of $-2x$ and $-5x$ = ...

(ii) The sum of 8 and -3 = ... and the sum of $8ab$ and $-3ab$ = ...

(iii) The sum of -15 and -4 = ... and the sum of $-15x$ and $-4y$ = ...

(iv) $15 + 8 + 3$ = ... and $15x + 8y + 3x$ = ...

(v) $12 - 9 + 15$ = ... and $12ab - 9ab + 15ba$ = ...

(vi) $25 - 7 - 9$ = ... and $25xy - 7xy - 9yx$ = ...

(vii) $-4 - 6 - 5$ = ... and $-4ax - 6ax - 5ay$ = ...

3. Add :

(i) $8xy$ and $3xy$
(ii) $2xyz$, xyz and $6xyz$
(iii) $2a$, $3a$ and $4b$
(iv) $3x$ and $2y$
(v) $5m$, $3n$ and $4p$
(vi) $6a$, $3a$ and $9ab$
(vii) $3p$, $4q$ and $9q$
(viii) $5ab$, $4ba$ and $6b$
(ix) $50pq$, $30pq$ and $10pr$
(x) $-2y$, $-y$ and $-3y$
(xi) $-3b$ and $-b$
(xii) $5b$, $-4b$ and $-10b$
(xiii) $-2c$, $-c$ and $-5c$

4. Evaluate :

(i) $6a - a - 5a - 2a$
(ii) $2b - 3b - b + 4b$
(iii) $3x - 2x - 4x + 7x$
(iv) $5ab + 2ab - 6ab + ab$
(v) $8x - 5y - 3x + 10y$

5. Evaluate :

(i) $-7x + 9x + 2x - 2x$
(ii) $5ab - 2ab - 8ab + 6ab$
(iii) $-8a - 3a + 12a + 13a - 6a$
(iv) $19abc - 11abc - 12abc + 14abc$

6. Subtract the first term from the second :

(i) $4ab$, $6ba$
(ii) $4{\cdot}8b$, $6{\cdot}8b$
(iii) $3{\cdot}5abc$, $10{\cdot}5abc$
(iv) $3\frac{1}{2}mn$, $8\frac{1}{2}nm$

7. Simplify :

(i) $2a^2b^2 + 5ab^2 + 8a^2b^2 - 3ab^2$
(ii) $4a + 3b - 2a - b$
(iii) $2xy + 4yz + 5xy + 3yz - 6xy$
(iv) $ab + 15ab - 11ab - 2ab$
(v) $6a^2 - 3b^2 + 2a^2 + 5b^2 - 4a^2$
(vi) $8abc + 2ab - 4abc + ab$
(vii) $9xyz + 15yxz - 10zyx - 2zxy$
(viii) $13pqr + 2p + 4q - 6pqr + 5pqr$
(ix) $4ab + 0 - 2ba$
(x) $6x^2y - 2xy^2 + 5x^2y - xy^2$

19.2 MORE ABOUT ADDITION AND SUBTRACTION

(i) Addition of Polynomials :

Example 4 :

Add : $4a + 2b$, $3a - 3b + c$ and $-2a + 4b + 2c$.

Solution :

First method (Row method) :

Steps :

1. Write all the given polynomials in a row.
2. Group the like terms.
3. Add the like terms.

The required addition

$= (4a + 2b) + (3a - 3b + c) + (-2a + 4b + 2c)$ [Step 1]

$= 4a + 2b + 3a - 3b + c - 2a + 4b + 2c$ [Step 2]

$= 4a + 3a - 2a + 2b - 3b + 4b + c + 2c$ [Step 3]

$= \mathbf{5a + 3b + 3c}$ **(Ans.)**

Second method ***(Column method) :***

Arrange the given polynominals so that the like terms of the polynominals are one below the other in a vertical column, then add.

In general, the column method is preferred.

$$\therefore \begin{array}{r} 4a + 2b \\ 3a - 3b + c \\ -2a + 4b + 2c \\ \hline 5a + 3b + 3c \\ \hline \end{array} \quad \textbf{(Ans.)}$$

Example 5 :

(i) Add : $3x^3 - 5x^2 + 8x + 10$, $15x^3 - 6x - 23$ and $9x^2 - 4x + 15$.

(ii) Add : $3ab^2 - 2b^2 + a^2$, $5a^2b - 2ab^2 - 3a^2$ and $8a^2 - 5b^2$.

Solution :

Using the row method :

(i) $(3x^3 - 5x^2 + 8x + 10) + (15x^3 - 6x - 23) + (9x^2 - 4x + 15)$

$= 3x^3 - 5x^2 + 8x + 10 + 15x^3 - 6x - 23 + 9x^2 - 4x + 15$

$= 3x^3 + 15x^3 - 5x^2 + 9x^2 + 8x - 6x - 4x + 10 - 23 + 15$

$= 18x^3 + 4x^2 + 8x - 10x + 25 - 23$

$= 18x^3 + 4x^2 - 2x + 2$ **(Ans.)**

(ii) $(3ab^2 - 2b^2 + a^2) + (5a^2b - 2ab^2 - 3a^2) + (8a^2 - 5b^2)$

$= 3ab^2 - 2b^2 + a^2 + 5a^2b - 2ab^2 - 3a^2 + 8a^2 - 5b^2$

$= 5a^2b + 3ab^2 - 2ab^2 + a^2 - 3a^2 + 8a^2 - 2b^2 - 5b^2$

$= 5a^2b + ab^2 + 6a^2 - 7b^2$ **(Ans.)**

Using the column method :

(i)
$$\begin{array}{rrrr} 3x^3 & - 5x^2 & + 8x & + 10 \\ 15x^3 & & - 6x & - 23 \\ & + 9x^2 & - 4x & + 15 \\ \hline 18x^3 & + 4x^2 & - 2x & + 2 \\ \hline \end{array} \quad \textbf{(Ans.)}$$

(ii)
$$\begin{array}{rrrr} & 3ab^2 & -2b^2 & + a^2 \\ 5a^2b & - 2ab^2 & & - 3a^2 \\ & & -5b^2 & + 8a^2 \\ \hline 5a^2b & + ab^2 & -7b^2 & + 6a^2 \\ \hline \end{array} \quad \textbf{(Ans.)}$$

(ii) Subtraction in Polynomials :

Steps (for the row method) :

1. Enclose the expression to be subtracted in brackets with a minus sign prefixed.
2. Remove the bracket by changing the sign of each term kept in the bracket.

Examples :

(i) $(2x - y) - (x + 5y) = 2x - y - x - 5y$

(ii) $(3a + b - c) - (2a - 3b + c) = 3a + b - c - 2a + 3b - c$, etc.

3. Combine the like terms and add.

Example 6 :

Subtract : $3a - 4b + 5c$ from $4a - b + 6c$.

Solution :

$4a - b + 6c - (3a - 4b + 5c)$ [Step 1]

Row method is used

$= 4a - b + 6c - 3a + 4b - 5c$ [Step 2]

$= 4a - 3a - b + 4b + 6c - 5c$ [Step 3]

$= a + 3b + c$ **(Ans.)**

> Whenever there is a negative sign before a bracket, open (remove) the bracket and at the same time, change the sign of each term inside the bracket.
> *e.g.* $(x + y) - (x - y + z) = x + y - x + y - z = 2y - z$

***Alternative method* (column method) :**

Steps (for the column method) **:**

1. Rewrite the given expressions in two lines (rows) such that the lower line is the expression to be subtracted and like terms of both the expressions are one below the other.
2. Change the sign of each term in the lower line, *i.e.* change the sign of each term of the expression to be subtracted.
3. Add column-wise.

Thus, for Example 6 given above, we have :

Step 1 :	$4a$	$-$	b	$+$	$6c$
	$3a$	$-$	$4b$	$+$	$5c$
Step 2 :	$-$		$+$		$-$
Step 3 :	a	$+$	$3b$	$+$	c

(Ans.)

Example 7 :

From the sum of $5x^2 - 7x + 4$ and $-3x^2 + 5x + 2$ subtract $x^2 + x + 1$.

Solution :

Row method :

$(5x^2 - 7x + 4) + (-3x^2 + 5x + 2) - (x^2 + x + 1)$

$= 5x^2 - 7x + 4 - 3x^2 + 5x + 2 - x^2 - x - 1$

$= 5x^2 - 3x^2 - x^2 - 7x + 5x - x + 4 + 2 - 1$

$= 5x^2 - 4x^2 - 8x + 5x + 6 - 1$

$= x^2 - 3x + 5$

Column method :

$5x^2$	$-7x$	$+4$	
$-3x^2$	$+5x$	$+2$	Add
$2x^2$	$-2x$	$+6$	
x^2	$+x$	$+1$	
$-$	$-$	$-$	Subtract
x^2	$-3x$	$+5$	**(Ans.)**

EXERCISE 19(B)

1. Find the sum of :
   (i) $3a + 4b + 7c$, $-5a + 3b - 6c$ and $4a - 2b - 4c$.
   (ii) $2x^2 + xy - y^2$, $-x^2 + 2xy + 3y^2$ and $3x^2 - 10xy + 4y^2$.
   (iii) $x^2 - x + 1$, $-5x^2 + 2x - 2$ and $3x^2 - 3x + 1$.
   (iv) $a^2 - ab + bc$, $2ab + bc - 2a^2$ and $-3bc + 3a^2 + ab$.
   (v) $4x^2 + 7 - 3x$, $4x - x^2 + 8$ and $-10 + 5x - 2x^2$.
   (vi) $3x + 4xy - y^2$, $xy - 4x + 2y^2$ and $3y^2 - xy + 6x$.
2. Add the following expressions :
   (i) $-17x^2 - 2xy + 23y^2$, $-9y^2 + 15x^2 + 7xy$ and $13x^2 + 3y^2 - 4xy$.
   (ii) $-x^2 - 3xy + 3y^2 + 8$, $3x^2 - 5y^2 - 3 + 4xy$ and $-6xy + 2x^2 - 2 + y^2$.
   (iii) $a^3 - 2b^3 + a$, $b^3 - 2a^3 + b$ and $-2b + 2b^3 - 5a + 4a^3$.
3. Evaluate :
   (i) $3a - (a + 2b)$ (ii) $(5x - 3y) - (x + y)$ (iii) $(8a + 15b) - (3b - 7a)$
   (iv) $(8x + 7y) - (4y - 3x)$ (v) $7 - (4a - 5)$ (vi) $(6y - 13) - (4 - 7y)$
4. Subtract :
   (i) $5a - 3b + 2c$ from $a - 4b - 2c$. (ii) $4x - 6y + 3z$ from $12x + 7y - 21z$.
   (iii) $5 - a - 4b + 4c$ from $5a - 7b + 2c$. (iv) $-8x - 12y + 17z$ from $x - y - z$.
   (v) $2ab + cd - ac - 2bd$ from $ab - 2cd + 2ac + bd$.
5. (i) Take $-ab + bc - ca$ from $bc - ca + ab$.
   (ii) Take $5x + 6y - 3z$ from $3x + 5y - 4z$.
   (iii) Take $\frac{-3}{2}p + q - r$ from $\frac{1}{2}p - \frac{1}{3}q - \frac{3}{2}r$.
   (iv) Take $1 - a + a^2$ from $a^2 + a + 1$.
6. From the sum of $x + y - 2z$ and $2x - y + z$ subtract $x + y + z$.
7. From the sum of $3a - 2b + 4c$ and $3b - 2c$ subtract $a - b - c$.
8. Subtract $x - 2y - z$ from the sum of $3x - y + z$ and $x + y - 3z$.
9. Subtract the sum of $x + y$ and $x - z$ from the sum of $x - 2z$ and $x + y + z$.

19.3 MULTIPLICATION

(i) Multiplication of monomials :

In multiplication, the powers of like factors are added

We know, $a^2 = a \times a$ and $a^3 = a \times a \times a$

$\therefore$ Multiplication of a^2 and $a^3 = a^2 \times a^3$

$= (a \times a) \times (a \times a \times a)$

$= a \times a \times a \times a \times a = a^5$

or, simply : $a^2 \times a^3 = a^{2+3} = a^5$

1. **Product Law used in exponents :**

Students already know : $a^m \times a^n = a^{m+n}$

(i) $x^5 \times x^7 = x^{5+7} = x^{12}$.

(ii) $x^2y^3 \times x^6y^4$

$= x^{2+6} \times y^{3+4} = x^8y^7$.

(iii) $xy^2z^5 \times x^4y^3z^2 \times x^7y^5z^4$

$= x^{1+4+7} \times y^{2+3+5} \times z^{5+2+4}$

$= x^{12}y^{10}z^{11}$ and so on

2. **The multiplication** (product) **of given monomials**

= (Multiplication of their coefficients) × (multiplication of their literals).

Example 8 :

Multiply : (i) $8x^2y^3$, $6y^2z^5$ and $3xz^2$

(ii) $-5a^2xy^3$, $\frac{7}{3}ax^3y^2$ and $-9xy$

Solution :

(i) $8x^2y^3 \times 6y^2z^5 \times 3xz^2$

= (Multiplication of their coefficients) × (multiplication of their literals)

$= (8 \times 6 \times 3) \times (x^2 \times x) \times (y^3 \times y^2) \times (z^5 \times z^2)$

$= 144 \times x^3 \times y^5 \times z^7 = 144x^3y^5z^7$ **(Ans.)**

(ii) $-5a^2xy^3 \times \frac{7}{3}ax^3y^2 \times -9xy$

$= \left(-5 \times \frac{7}{3} \times -9\right) \times (a^2 \times a) \times (x \times x^3 \times x) \times (y^3 \times y^2 \times y)$

$= 105a^3x^5y^6$ **(Ans.)**

(ii) Multiplication of a polynomial and a monomial :

Multiply each term of the polynomial by the monomial

Multiplication of $4x^2y - 3xy^2 + 4xy$ and $2xy$

$= 2xy \times (4x^2y - 3xy^2 + 4xy)$

$= (2xy \times 4x^2y) - (2xy \times 3xy^2) + (2xy \times 4xy)$

$= 8x^3y^2 - 6x^2y^3 + 8x^2y^2$

In the same way :

(a) $5x^2(3x^3y^2 - 2xy^3 + y^5)$

$= (5x^2 \times 3x^3y^2) - (5x^2 \times 2xy^3) + (5x^2 \times y^5)$

$= 15x^5y^2 - 10x^3y^3 + 5x^2y^5$

(b) $-3a(4a^2 - 8a^3 + 7a^4)$

$= -12a^3 + 24a^4 - 21a^5$

(iii) Multiplication of two binomials :

Steps :

1. Multiply each term of the first binomial by each term of the other one.
2. In the product obtained, combine the like terms.

(i) **Multiplication of x + 3 and x + 5** $= (x + 3) . (x + 5)$

$= x . (x + 5) + 3 . (x + 5)$

$= x . x + x . 5 + 3 . x + 15$

$= x^2 + 5x + 3x + 15$

$= \mathbf{x^2 + 8x + 15}$

(ii) $(3x - 2y) (5x + 3y) = 3x(5x + 3y) - 2y(5x + 3y)$

$= 15x^2 + 9xy - 10xy - 6y^2$

$= 15x^2 - xy - 6y^2$

Alternative method :

Steps :

1. Write the binomials one below the other.
2. Multiply the first term of the binomial in the lower line with each term of the binomial in the upper line.
3. Multiply the second term of the binomial in the lower line with each term of the binomial in the upper line, then place the like terms one below the other.
4. Add the like terms column-wise.

∴ **Multiplication of x + 3 and x + 5 is :**

$$\begin{array}{rrrl} & x & + 3 & \\ \times & x & + 5 & \\ \hline & x^2 & + 3x & [\because x . (x + 3) = x^2 + 3x] \\ & & + 5x + 15 & [\because 5 . (x + 3) = 5x + 15] \\ \hline \text{On adding :} & x^2 & + 8x + 15 & \text{(Ans.)} \\ \hline \end{array}$$

Example 9 :

Multiply : 3x − 4y and 4x + 5x

Solution :

$(3x - 4y) . (4x + 5y) = 3x . (4x + 5y) - 4y . (4x + 5y)$

$= 12x^2 + 15xy - 16xy - 20y^2 = 12x^2 - xy - 20y^2$ **(Ans.)**

Alternative method :

$$\begin{array}{rrrrl} & 3x & - 4y & & \\ \times & 4x & + 5y & & \\ \hline & 12x^2 & - 16xy & & [\because 4x . (3x - 4y) = 12x^2 - 16xy] \\ & & + 15xy & - 20y^2 & [\because 5y . (3x - 4y) = 15xy - 20y^2] \\ \hline \text{On adding :} & 12x^2 & - xy & - 20y^2 & \text{(Ans.)} \\ \hline \end{array}$$

EXERCISE 19(C)

1. Fill in the blanks :

(i) 6×3 = ... and $6x \times 3x$ = ...

(ii) 6×3 = ... and $6x^2 \times 3x^3$ = ...

(iii) 5×4 = ... and $5x \times 4y$ = ...

(iv) 4×7 = ... and $4ax \times 7x$ = ...

(v) 6×2 = ... and $6xy \times 2xy$ = ...

(vi) 12×4 = ... and $12ax^2 \times 4ax$ = ...

(vii) 1×8 = ... and $a^2xy^2 \times 8a^3x^2y$ = ...

(viii) 15×3 = ... and $15x \times 3x^5y^2$ = ...

2. Fill in the blanks :

(i) $4x \times 6x \times 2$ = ... (ii) $3ab \times 6ax$ = ...

(iii) $x \times 2x^2 \times 3x^3$ = ... (iv) $5 \times 5a^3$ = ...

(v) $6 \times 6x^2 \times 6x^2y^2$ = ... (vi) $-8x \times -3x$ = ...

(vii) $-5 \times -3x \times 5x^2$ = ... (viii) $8 \times -4xy^2 \times 3x^3y^2$ = ...

(ix) $-4x \times 5xy \times 3z$ = ...

(x) $5x \times 2x^2y \times -7y^3 \times 2x^3y^2$ = ...

3. Find the value of :

(i) $3x^3 \times 5x^4$ (ii) $5a^2 \times 7a^7$ (iii) $3abc \times 6ac^3$

(iv) $a^2b^2 \times 5a^3b^4$ (v) $2x^2y^3 \times 5x^3y^4$ (vi) $abc \times bcd$

4. Multiply :

(i) $a + b$ by ab (ii) $3ab - 4b$ by $3ab$ (iii) $2xy - 5by$ by $4bx$

(iv) $4x + 2y$ by $3xy$ (v) $x^2 - x$ by $2x$ (vi) $1 + 4x$ by x

(vii) $9xy^2 + 3x^2y$ by $5xy$ (viii) $6x - 5y$ by $3axy$

5. Multiply :

(i) $-x + y - z$ and $-2x$ (ii) $xy - yz$ and x^2yz^2 (iii) $2xyz + 3xy$ and $-2y^2z$

(iv) $-3xy^2 + 4x^2y$ and $-xy$ (v) $4xy$ and $-x^2y - 3x^2y^2$

6. Multiply :

(i) $3a + 4b - 5c$ and $3a$ (ii) $-5xy$ and $-xy^2 - 6x^2y$

7. Multiply :

(i) $x + 2$ and $x + 10$ (ii) $x + 5$ and $x - 3$ (iii) $x - 5$ and $x + 3$

(iv) $x - 5$ and $x - 3$ (v) $2x + y$ and $x + 3y$ (vi) $3x - 5y$ and $x + 6y$

(vii) $x + 9y$ and $x - 5y$ (viii) $2x + 5y$ and $2x + 5y$

8. Multiply :

(i) $3abc$ and $-5a^2b^2c$ (ii) $x - y + z$ and $-2x$

(iii) $2x - 3y - 5z$ and $-2y$ (iv) $-8xyz + 10x^2yz^3$ and xyz

(v) xyz and $-13xy^2z + 15x^2yz - 6xyz^2$ (vi) $4abc - 5a^2bc - 6ab^2c$ and $-2abc^2$

9. Find the product of :

(i) $xy - ab$ and $xy + ab$ (ii) $2abc - 3xy$ and $2abc + 3xy$

(iii) $a + b - c$ and $2a - 3b$ (iv) $5x - 6y - 7z$ and $2x + 3y$

(v) $5x - 6y - 7z$ and $2x + 3y + z$ (vi) $2a + 3b - 4c$ and $a - b - c$

17.4 DIVISION

(i) Division of a monomial by a monomial :

(a) 10ab divided by 5a $= \frac{10ab}{5a} = \frac{2 \times \cancel{5} \times \cancel{a} \times b}{\cancel{5} \times \cancel{a}} = 2b$

i.e. when 10ab is divided by 5a, the quotient is 2b.

(b) 21ayz divided by 7az

$= 21ayz \div 7az = \frac{21ayz}{7az} = \frac{3 \times \cancel{7} \times \cancel{a} \times y \times \cancel{z}}{\cancel{7} \times \cancel{a} \times \cancel{z}} = 3y$

(c) Divison of $12m^5$ by $4m^3$

$= 12m^5 \div 4m^3$

$= \frac{3 \times \cancel{4} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times m \times m}{\cancel{4} \times \cancel{m} \times \cancel{m} \times \cancel{m}}$

$= 3 \times m \times m = 3m^2$

Write each term in its expanded form and then cancel the terms that are common to the numerator and the denominator.

Using exponents (quotient law) :

(i) $\frac{a^m}{a^n} = a^{m-n}$, if m > n and

(ii) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, if n > m

$\therefore \frac{x^7}{x^3} = x^{7-3} = x^4, \quad \frac{x^3}{x^7} = \frac{1}{x^{7-3}} = \frac{1}{x^4},$

$\frac{x^5y^3}{x^2y^8} = \frac{x^{5-2}}{y^{8-3}} = \frac{x^3}{y^5}$ and so on

$\therefore \quad 12m^5 \div 4m^3 = \frac{12m^5}{4m^3}$

$= 3 \times m^{5-3} = 3 \times m^2 = 3m^2$

(d) Divison of $35a^3b^5$ by $5a^6b^2$.

$= 35a^3b^5 \div 5a^6b^2 = \frac{35a^3b^5}{5a^6b^2} = \frac{5 \times 7 \times b^{5-2}}{5 \times a^{6-3}}$ [Using exponents]

$= \frac{7b^3}{a^3}$

More examples :

(i) $\frac{24\,a^4b^6}{8\,a^9b^5} = \frac{3 \times 8 \times b^{6-5}}{8 \times a^{9-4}} = \frac{3b}{a^5}$ (ii) $-45m^3n^5 \div 9mn^2 = \frac{-45m^3n^5}{9mn^2} = -5m^2n^3$

(iii) $-15a^5b^7x^2 \div -5a^2b^3x^8 = \frac{-15a^5b^7x^2}{-5a^2b^3x^8} = \frac{3a^3b^4}{x^6}$ and so on.

(ii) Division of a polynomial by a monomial :

Divide each term of the polynomial by the monomial :

(i) Division of $12x^5 - 9x^3$ by $3x^2$

$= \frac{12x^5}{3x^2} - \frac{9x^3}{3x^2} = \mathbf{4x^3 - 3x}$

(ii) Division of $15x^2y^3 - 21x^3y^4 + 18x^4y^2$ by $3x^2y^2$

$= \frac{15x^2y^3}{3x^2y^2} - \frac{21x^3y^4}{3x^2y^2} + \frac{18x^4y^2}{3x^2y^2} = \mathbf{5y - 7xy^2 + 6x^2}$

Example 10 : Divide : $24x^3y^3 + 30x^4y^5 - 12x^5y^4$ by $-6x^2y^3$

Solution :

$$\frac{24x^3y^3 + 30x^4y^5 - 12x^5y^4}{-6x^2y^3} = \left(\frac{24x^3y^3}{-6x^2y^3}\right) + \left(\frac{30x^4y^5}{-6x^2y^3}\right) - \left(\frac{12x^5y^4}{-6x^2y^3}\right)$$

$$= (-4x) + (-5x^2y^2) - (-2x^3y)$$

$$= -4x - 5x^2y^2 + 2x^3y$$ **(Ans.)**

EXERCISE 19(D)

1. Divide :

   (i) $3a$ by a (ii) $15x$ by $3x$ (iii) $16m$ by 4
   (iv) $20x^2$ by $5x$ (v) $30p^2$ by $10p^2$ (vi) $14a^3b^3$ by $2a^2$
   (vii) $18pqr^2$ by $3pq$ (viii) 100 by $50b$

2. Simplify :

   (i) $2x^5 \div x^2$ (ii) $6a^8 \div 3a^3$ (iii) $20xy \div -5xy$
   (iv) $-24a^2b^2c^2 \div 6ab$ (v) $-5x^2y \div xy^2$ (vi) $40p^3q^4r^5 \div 10p^3q$
   (vii) $-64x^4y^3z \div 4x^3y^2z$ (viii) $35xy^5 \div 7x^2y^4$

3. Divide :

   (i) $-\frac{3m}{4}$ by $2m$ (ii) $-15p^6q^8$ by $-5p^6q^7$ (iii) $-21m^5n^7$ by $14m^2n^2$
   (iv) $36a^4x^5y^6$ by $4x^2a^3y^2$ (v) $20x^3a^6$ by $5xy$ (vi) $\frac{28a^2b^3}{c^2}$ by $4abc$
   (vii) $\frac{2a^2}{9b^2}$ by $\frac{3b}{2a}$ (viii) $\frac{-5 \cdot 5x^2}{y}$ by $\frac{11x}{y}$ (ix) $\frac{64x^2y^2}{z^2}$ by $\frac{8xy}{z}$

4. Simplify :

   (i) $\frac{-15m^5n^2}{-3m^5}$ (ii) $\frac{35x^4y^2}{-15x^2y^2}$ (iii) $\frac{-24x^6y^2}{6x^6y}$

5. Divide :

   (i) $9x^3 - 6x^2$ by $3x$ (ii) $6m^2 - 16m^3 + 10m^4$ by $-2m$
   (iii) $15x^3y^2 + 25x^2y^3 - 36x^4y^4$ by $5x^2y^2$
   (iv) $36a^3x^5 - 24a^4x^4 + 18a^5x^3$ by $-6a^3x^3$.

SUBSTITUTION
(Including Use of Brackets as Grouping Symbols) 20

20.1 BASIC CONCEPT

The value of an expression depends on the value(s) of its variable(s).

Consider the algebraic expression : $3x + 2$

In the expression $3x + 2$, the variable used is x, and so the value of the expression $3x + 2$ depends on the value of its variable x. That is :

(i) if $x = 2$, the value of the expression $3x + 2 = 3 \times 2 + 2 = 6 + 2 = 8$

(ii) if $x = 0$, the value of the expression $3x + 2 = 3 \times 0 + 2 = 0 + 2 = 2$

(iii) if $x = -2$, the value of the expression $3x + 2 = 3 \times -2 + 2 = -6 + 2 = -4$ and so on.

Now consider the algebraic expression $5x - 2y$.

This expression consists of variables x and y.

Now, if :

(i) $x = 3$ and $y = 2$, $5x - 2y = 5 \times 3 - 2 \times 2 = 15 - 4 = 11$

(ii) $x = 8$ and $y = 5$, $5x - 2y = 5 \times 8 - 2 \times 5 = 40 - 10 = 30$.

The process of finding the value of an algebraic expression by substituting the given value (values) of its variable (variables) is called **substitution**.

Example 1 :

If $x = 6$ and $y = 3$, find the value of :

(i) $4x + y$ (ii) $3x - 4y$ (iii) $3xy$ (iv) $\frac{5x}{4y}$

Solution :

(i) $4x + y = 4 \times 6 + 3 = 24 + 3 = 27$ **(Ans.)**

(ii) $3x - 4y = 3 \times 6 - 4 \times 3 = 18 - 12 = 6$ **(Ans.)**

(iii) $3xy = 3 \times 6 \times 3 = 54$ **(Ans.)**

(iv) $\frac{5x}{4y} = \frac{5 \times 6}{4 \times 3} = \frac{30}{12} = \frac{5}{2} = 2\frac{1}{2}$ **(Ans.)**

Example 2 :

If $a = 2$, $b = 5$ and $c = 8$, find the value of : $3ab + 10bc - 2abc$

Solution :

$$\begin{aligned} 3ab + 10bc - 2abc &= 3 \times 2 \times 5 + 10 \times 5 \times 8 - 2 \times 2 \times 5 \times 8 \\ &= 30 + 400 - 160 \\ &= 430 - 160 = 270 \end{aligned}$$ **(Ans.)**

Example 3 :

If $p = 8$, $q = 1$ and $r = 2$, find the value of : $\dfrac{10pq - 3qr}{4pqr - 2p}$

Solution :

$$\frac{10pq - 3qr}{4pqr - 2p} = \frac{10 \times 8 \times 1 - 3 \times 1 \times 2}{4 \times 8 \times 1 \times 2 - 2 \times 8} = \frac{80 - 6}{64 - 16} = \frac{74}{48} = \frac{37}{24} = 1\frac{13}{24} \quad \textbf{(Ans.)}$$

Example 4 :

If $x = 2$, find the value of $3x^2 + x$.

Solution :

$$3x^2 + x = 3(2)^2 + 2$$
$$= 3 \times 2 \times 2 + 2 = 12 + 2 = 14 \quad \textbf{(Ans.)}$$

Example 5 :

If $x = 5$, $y = 6$ and $z = 10$, find the value of :

(i) $\dfrac{3x^2}{x}$ (ii) $\dfrac{xy}{xz}$ (iii) $\dfrac{x^2y}{z}$

Solution :

(i) $\dfrac{3x^2}{x} = \dfrac{3 \times 5^2}{5} = \dfrac{3 \times 5 \times 5}{5} = 3 \times 5 = 15$ **(Ans.)**

Alternative method :

$\dfrac{3x^2}{x} = \dfrac{3 \times x \times x}{x} = 3 \times x = 3 \times 5 = 15$ **(Ans.)**

(ii) $\dfrac{xy}{xz} = \dfrac{5 \times 6}{5 \times 10} = \dfrac{30}{50} = \dfrac{3}{5}$ **(Ans.)**

Alternative method :

$\dfrac{xy}{xz} = \dfrac{y}{z} = \dfrac{6}{10} = \dfrac{3}{5}$ **(Ans.)**

(iii) $\dfrac{x^2y}{z} = \dfrac{5^2 \times 6}{10} = \dfrac{5 \times 5 \times 6}{10} = \dfrac{150}{10} = 15$ **(Ans.)**

Example 6 :

If $a = 2$, $b = 3$ and $c = 4$, find the value of :

(i) a^b (ii) b^a (iii) c^b (iv) $a^2 - b^2 + c^2$

Solution :

(i) $a^b = 2^3 = 2 \times 2 \times 2 = 8$ **(Ans.)**

(ii) $b^a = 3^2 = 3 \times 3 = 9$ **(Ans.)**

(iii) $c^b = 4^3 = 4 \times 4 \times 4 = 64$ **(Ans.)**

(iv) $a^2 - b^2 + c^2 = (2)^2 - (3)^2 + (4)^2$
$= 4 - 9 + 16 = 20 - 9 = 11$ **(Ans.)**

EXERCISE 20(A)

1. Fill in the following blanks, when : $x = 3$, $y = 6$, $z = 18$, $a = 2$, $b = 8$, $c = 32$ and $d = 0$.

(i)	$x + y$ = **3 + 6 = 9**	(ix)	$a + b + x$ = ...
(ii)	$y - x$ = ...	(x)	$b + z - d$ = ...
(iii)	$\frac{y}{x}$ = ...	(xi)	$a - b + y$ = ...
(iv)	$c \div b$ = ...	(xii)	$z - a - b$ = ...
(v)	$z \div x$ = ...	(xiii)	$d - a + x$ = ...
(vi)	$y \times d$ = ...	(xiv)	$xy - bd$ = ...
(vii)	$d \div x$ = ...	(xv)	$xz + cd$ = ...
(viii)	$ab + y$ = ...		

2. Find the value of :
   (i) $p + 2q + 3r$, when $p = 1$, $q = 5$ and $r = 2$
   (ii) $2a + 4b + 5c$, when $a = 5$, $b = 10$ and $c = 20$
   (iii) $3a - 2b$, when $a = 8$ and $b = 10$
   (iv) $5x + 3y - 6z$, when $x = 3$, $y = 5$ and $z = 4$
   (v) $2p - 3q + 4r - 8s$, when $p = 10$, $q = 8$, $r = 6$, and $s = 2$
   (vi) $6m - 2n - 5p - 3q$, when $m = 20$, $n = 10$, $p = 2$ and $q = 9$
3. Find the value of :
   (i) $4pq \times 2r$, when $p = 5$, $q = 3$ and $r = 1/2$ (ii) $\frac{yx}{z}$, when $x = 8$, $y = 4$ and $z = 16$
   (iii) $\frac{a+b-c}{2a}$, when $a = 5$, $b = 7$ and $c = 2$
4. If $a = 3$, $b = 0$, $c = 2$ and $d = 1$, find the value of :
   (i) $3a + 2b - 6c + 4d$ (ii) $6a - 3b - 4c - 2d$ (iii) $ab - bc + cd - da$
   (iv) $abc - bcd + cda$ (v) $a^2 + 2b^2 - 3c^2$ (vi) $a^2 + b^2 - c^2 + d^2$
   (vii) $2a^2 - 3b^2 + 4c^2 - 5d^2$
5. Find the value of : $5x^2 - 3x + 2$, when $x = 2$.
6. Find the value of : $3x^3 - 4x^2 + 5x - 6$, when $x = -1$.
7. Show that the value of : $x^3 - 8x^2 + 12x - 5$ is zero, when $x = 1$.
8. State **true** and **false** :
   (i) The value of $x + 5 = 6$, when $x = 1$ (ii) The value of $2x - 3 = 1$, when $x = 0$
   (iii) $\frac{2x - 4}{x + 1} = -1$, when $x = 1$.
9. If $x = 2$, $y = 5$ and $z = 4$, find the value of each of the following :
   (i) $\frac{x}{2x^2}$ (ii) $\frac{xz}{yz}$ (iii) z^x (iv) y^x
   (v) $\frac{x^2 y^2 z^2}{xz}$ (vi) $\frac{5x^4 y^2 z^2}{2x^2}$ (vii) $xy \div y^2z$ (viii) $\frac{x^2 y^x}{x}$
10. If $a = 3$, find the values of a^2 and 2^a.
11. If $m = 2$, find the difference between the values of $4m^3$ and $3m^4$.

20.2 BRACKETS

In general, the symbols ( ), { }, [ ], etc., are called **brackets**.

1. ( ) *is called a pair of **small brackets** or **parenthesis**.*
2. { } *is called a pair of **middle brackets** or **curly brackets**.*
3. [ ] *is called a pair of **big brackets** or **square brackets**.*

Note : *If one more bracket is needed, we use the bar bracket, i.e. a line ——— is drawn over a group of terms.*

Thus, in $2x + \overline{3y - 4z}$*, the line over 3y – 4z serves as the **bar bracket** and is **called vinculum**.*

If an expression is enclosed within a bracket, it is considered a single quantity even if it is made up of many terms.

For example :

Each of (x – y) and (2a + 3b – 2) will be treated as a single quantity.

While simplifying an expression containing a bracket, first of all, the terms inside brackets are operated (combined).

Example 7 :

Evaluate : (i) 15 – (8 – 6) (ii) (15 – 8) – 6

Solution :

(i) 15 – (8 – 6) = 15 – 2 (Simplifying the terms in the bracket first)

= 13 **(Ans.)**

(ii) (15 – 8) – 6 = 7 – 6 = 1 **(Ans.)**

Similarly,

15 – (8 + 6) = 15 – 14 = 1; (15 – 8) + 6 = 7 + 6 = 13 and so on.

20.3 OPENING OR REMOVING BRACKETS

Removing brackets

(i) ***When there is + ve** (positive) **sign before a pair of brackets :***

The brackets are removed without changing the signs of the terms in the brackets.

For example :

(i) 10 + (7 – 3)
= 10 + 7 – 3 = **14**

(ii) a + (b – c + d)
= **a + b – c + d**

(ii) ***When there is – ve** (negative) **sign before a pair of brackets :***

The brackets are removed and, at the same time, the sign of each term inside the brackets is also changed,

i.e. + sign will change to – sign, and – sign will change to + sign.

For example :

(i) $12 - (8 - 5)$ $= 12 - 8 + 5 = 17 - 8 = \mathbf{9}$

(ii) $a - (b - c + d)$ $= \mathbf{a - b + c - d}$

20.4 REMOVAL OF BRACKETS

In a combined operation, the brackets should be removed in the following order :

(i) $\overline{\quad\quad}$ (ii) ( ) (iii) { } (iv) [ ]

For example :

(i) $3x - (2y - \overline{x - y})$

$= 3x - (2y - x + y)$ [On removing the bar bracket]

$= 3x - (3y - x)$

$= 3x - 3y + x$ [On removing the small brackets]

$= \mathbf{4x - 3y}$

(ii) $6a - \{a + (2a - \overline{4 - a})\}$

$= 6a - \{a + (2a - 4 + a)\}$

$= 6a - \{a + (3a - 4)\}$

$= 6a - \{a + 3a - 4\}$

$= 6a - \{4a - 4\} = 6a - 4a + 4 = \mathbf{2a + 4}$

First of all remove the bar bracket and simplify, then remove the small brackets ( ) and simplify. Finally remove the curly brackets { } and simplify.

Example 8 :

Simplify :

(i) $a - [b - \{c - (a - \overline{b - c})\}]$ (ii) $3a - [2a - \{a + (a - \overline{b - c} + c)\}]$

Solution :

(i) $\mathbf{a - [b - \{c - (a - \overline{b - c})\}]}$

$= a - [b - \{c - (a - b + c)\}]$ [On removing the bar brackets]

$= a - [b - \{c - a + b - c\}]$ [On removing the small brackets]

$= a - [b - c + a - b + c]$ [On removing the middle brackets]

$= a - b + c - a + b - c$ [On removing the square brackets]

$= \mathbf{0}$ **(Ans.)**

(ii) $\mathbf{3a - [2a - \{a + (a - \overline{b - c} + c)\}]}$

$= 3a - [2a - \{a + (a - b + c + c)\}]$

$= 3a - [2a - \{a + a - b + c + c\}]$

$= 3a - [2a - a - a + b - c - c]$

$= 3a - 2a + a + a - b + c + c = \mathbf{3a - b + 2c}$ **(Ans.)**

Order of removing the brackets is : $\overline{\quad}$, ( ), { } and finally [ ].

> *A number placed before a pair of brackets indicates that each term inside the brackets is to be multiplied by that number.*
>
> Thus : (i) $5(x + y) = 5x + 5y$ (ii) $2(2a - b) = 4a - 2b$
>
> (iii) $3(m - 2n + 4a) = 3m - 6n + 12a$ *and so on.*

Example 9 :

Simplify : (i) $2x + 3(x - y)$ (ii) $5a - 2\{2b - 3(a - b)\}$

Solution :

(i) $2x + 3(x - y) = 2x + 3x - 3y$

$= 5x - 3y$ **(Ans.)**

(ii) $5a - 2\{2b - 3(a - b)\} = 5a - 2\{2b - 3a + 3b\}$

$= 5a - 4b + 6a - 6b$

$= 5a + 6a - 4b - 6b = 11a - 10b$ **(Ans.)**

EXERCISE 20(B)

1. Evaluate :
   (i) $(23 - 15) + 4$ (ii) $5x + (3x + 7x)$ (iii) $6m - (4m - m)$
   (iv) $(9a - 3a) + 4a$ (v) $35b - (16b + 9b)$ (vi) $(3y + 8y) - 5y$

2. Simplify :
   (i) $12x - (5x + 2x)$ (ii) $10m + (4n - 3n) - 5n$
   (iii) $(15b - 6b) - (8b + 4b)$ (iv) $-(-4a - 8a)$
   (v) $x - (x - y) - (-x + y)$ (vi) $p + (-q - r - s) - (p - q - r)$
   (vii) $(a + b) - (c + d) - (e - f)$ (viii) $3x + (8x - 5x) - (7x - x)$
   (ix) $a - (a - b - c)$ (x) $6a^2 + (2a^2 - a^2) - (a^2 - b^2)$
   (xi) $2m - (3m + 2n - 6n)$ (xii) $-m - n - (-m) - m$
   (xiii) $x + y - (x + \overline{y - x})$ (xiv) $25y - (5x - 10y + 6x - 3y)$
   (xv) $3x + (2x - \overline{x + 2})$ (xvi) $a - (2a - \overline{4a + 3a})$
   (xvii) $5x^2 - (3x - \overline{x^2 - 4})$ (xviii) $-(y - x) - (x + y - \overline{2x + y})$

3. Simplify :
   (i) $x - (y - z) + x + (y - z) + y - (z + x)$ (ii) $x - [y + \{x - (y + x)\}]$
   (iii) $4x + 3(2x - 5y)$ (iv) $2(3a - b) - 5(a - 3b)$
   (v) $p + 2(q - \overline{r + p})$ (vi) $a - [-\{-(a - \overline{b - c})\}]$
   (vii) $3x - [5y - \{6y + 2(10y - x)\}]$ (viii) $5\{a^2 - a(a - \overline{a - 2})\}$

20.5 INSERTING BRACKETS

1. When any part of an expression is inserted within a pair of brackets with a *positive sign* before it, *the sign of each term kept inside the brackets remains unchanged.*

For example :

The expression $a - b + c - d$ may be written as :

$$a + (-b + c - d) \quad \text{or} \quad a - b + (c - d)$$

2. When any part of an expression is to be inserted within a pair of brackets with a *negative sign* before it, *the sign of each term kept inside the brackets gets changed.*

For example :

The expression a − b + c − d may be written as :

$$a - (b - c + d) \quad \text{or} \quad a - b - (-c + d)$$

EXERCISE 20(C)

1. Fill in the blanks :
   (i) $2a + b - c = 2a +$ (................)
   (ii) $3x - z + y = 3x -$ (................)
   (iii) $6p - 5x + q = 6p -$ (................)
   (iv) $a + b - c + d = a +$ (................)
   (v) $5a + 4b + 4x - 2c$ $= 4x -$ (....... ....)
   (vi) $7x + 2z + 4y - 3 = -3 + 4y +$ (................)
   (vii) $3m - 2n + 6 = 6 -$ (................)
   (viii) $2t + r - p - q + s = 2t + r -$ (................)
2. Insert the bracket as indicated :
   (i) $x - 2y = -$ (................)
   (ii) $m + n - p = -$ (................)
   (iii) $a + 4b - 4c = a +$ (................)
   (iv) $a - 3b + 5c = a -$ (................)
   (v) $x^2 - y^2 + z^2 = x^2 -$ (................)
   (vi) $m^2 + x^2 - p^2 = -$ (................)
   (vii) $2x - y + 2z = 2z -$ (................)
   (viii) $ab + 2bc - 3ac = 2bc -$ (................)

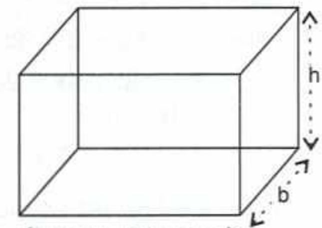
FRAMING ALGEBRAIC EXPRESSIONS 21

(Including Evaluation)

21.1 FRAMING ALGEBRAIC EXPRESSIONS

To express a given statement in terms of some variables in the form of an algebraic expression is called framing of an algebraic expression.

For example :



1. Area of a rectangle = its length × its breadth

   $= l \times b$

2. Speed of a car that travels distance d in time t

   $= \frac{d}{t}$

3. Total area of the adjoining cuboid

   = 2 × (length × breadth + breadth × height + height × length)

   $= 2 \times (l \times b + b \times h + h \times l)$

21.2 FRAMING A FORMULA

To express a given statement in the form of an algebraic equation is called **framing of formula**.

A formula is a statement expressed in symbols (letters) showing the relationship of related quantities.

Statement	Corresponding formula
1. The sum of two numbers x and y is 75.	$x + y = 75$
2. The velocity (V m s^{-1}) of a car which travels d metres in t seconds.	$V = \frac{d}{t}$
3. The balance ₹ B in my bank account, if I started with ₹ A and withdrew ₹ W.	$B = A - W$
4. An article is bought for ₹ x and is sold for ₹ y. If x is greater than y, profit P is :	$P = x - y$

***Example* 1 :**

Express each of the following as an algebraic expression :

(i) The sum of m and n.

(ii) The product of m and n.

(iii) The subtraction of y from $6x$.

(iv) The product of 8, x and y.

(v) The product of x and $b - c$.

Solution :

(i) $m + n$ **(Ans.)**

(ii) mn **(Ans.)**

> The notations in algebra are the same as in arithmatic except that the multiplication sign in algebra is usually omitted e.g. $m \times n$ is written as mn, $3 \times n$ as $3n$ and so on.

(iii) $6x - y$ **(Ans.)**

(iv) $8xy$ **(Ans.)**

(v) $x(b - c)$ **(Ans.)**

Example 2 :

For each statement, given below, write a formula :

(i) time (t hours) taken by a hiker who walks D kilometres at S kilometre per hour.

(ii) the total wage ₹ W of a man whose basic wage is ₹ B for first t hours and ₹ R an hour for the next m hours.

Solution :

(i) $\because$ Time $= \frac{\text{Distance}}{\text{Speed}} \Rightarrow t = \frac{D}{S}$ **(Ans.)**

(ii) Total wages = Wages for first t hours + wages for next m hours

$\Rightarrow \quad W = B + Rm$ **(Ans.)**

21.3 EVALUATION OF ALGEBRAIC EXPRESSIONS

Evaluation is the process used to find out the value of the given algebraic expression for the given value (values) of variable (variables) used in it.

Example 3 :

Evaluate :

(i) $4m - 9$ for $m = -3$. (ii) $4x + 21$ for $x = 4$.

(iii) $5p + 24$ for $p = -3$.

Solution :

(i) $4m - 9$ for $m = -3$

$= 4 \times (-3) - 9$

$= -12 - 9 = -21$ **(Ans.)**

(ii) $4x + 21$ for $x = 4$

$= 4 \times 4 + 21$

$= 16 + 21 = 37$ **(Ans.)**

(iii) $5p + 24$ for $p = -3$

$= 5 \times (-3) + 24$

$= -15 + 24 = 9$ **(Ans.)**

Example 4 :

If $x = 5$, evaluate :

(i) $3x$ (ii) x^2 (iii) $2x^3$

Solution :

(i) $3x = 3 \times 5 = 15$ **(Ans.)**

(ii) $x^2 = 5^2 = 5 \times 5 = 25$ **(Ans.)**

(iii) $2x^3 = 2 \times 5^3 = 2 \times 5 \times 5 \times 5 = 250$ **(Ans.)**

Example 5 :

If $x = 2$ and $y = 3$, evaluate :

(i) xy (ii) $5x^2y$ (iii) $5y^2$ (d) $(5y)^2$

Solution :

(i) $xy = x \times y = 2 \times 3 = 6$ **(Ans.)**

(ii) $5x^2y = 5 \times x \times x \times y = 5 \times 2 \times 2 \times 3 = 60$ **(Ans.)**

(iii) $5y^2 = 5 \times y \times y = 5 \times 3 \times 3 = 45$ **(Ans.)**

(iv) $(5y)^2 = 5y \times 5y = 5 \times 5 \times y \times y = 25 \times 3 \times 3 = 225$ **(Ans.)**

Example 6 :

If $y = 6$, evaluate :

(i) $5y - 6$ (ii) $3y^2 + 2y$ (iii) $\frac{y^3}{4}$

Solution :

(i) $5y - 6 = (5 \times y) - 6 = (5 \times 6) - 6 = 30 - 6 = 24$ **(Ans.)**

(ii) $3y^2 + 2y = (3 \times y \times y) + (2 \times y) = (3 \times 6 \times 6) + 2 \times 6 = 108 + 12 = 120$ **(Ans.)**

(iii) $\frac{y^3}{4} = \frac{y \times y \times y}{4} = \frac{6 \times 6 \times 6}{4} = 54$ **(Ans.)**

Example 7 :

If $y = -6$, evaluate :

(i) $5y - 6$ (ii) $3y^2 + 2y$ (iii) $\frac{y^3}{4}$

Solution :

(i) $5y - 6 = (5 \times y) - 6 = (5 \times -6) - 6 = -30 - 6 = -36$ **(Ans.)**

(ii) $3y^2 + 2y = (3 \times y \times y) + (2 \times y) = (3 \times -6 \times -6) + 2 \times -6$

$= 108 - 12 = 96$ **(Ans.)**

(iii) $\frac{y^3}{4} = \frac{y \times y \times y}{4} = \frac{-6 \times -6 \times -6}{4} = -54$ **(Ans.)**

Example 8 :

If $x = -5$, evaluate :

(i) $4x$ (ii) $2x^2$ (iii) $(3x)^3$

Solution :

(i) $4x = 4 \times x = 4 \times (-5) = -20$ **(Ans.)**

(ii) $2x^2 = 2 \times x \times x = 2 \times (-5) \times (-5) = 2 \times 25 = 50$ **(Ans.)**

(iii) $(3x)^3 = 3x \times 3x \times 3x = (3 \times -5) \times (3 \times -5) \times (3 \times -5)$

$= -15 \times -15 \times -15 = -3375$ **(Ans.)**

Example 9 :

If $x = 2$ and $y = 3$, evaluate :

(i) $4xy$ (ii) $3x^2y^2$ (iii) $5xy^3$

Solution :

(i) $4xy = 4 \times x \times y = 4 \times 2 \times 3 = 24$ **(Ans.)**

(ii) $3x^2y^2 = 3 \times x \times x \times y \times y = 3 \times 2 \times 2 \times 3 \times 3 = 108$ **(Ans.)**

(iii) $5xy^3 = 5 \times x \times y \times y \times y = 5 \times 2 \times 3 \times 3 \times 3 = 270$ **(Ans.)**

Example 10 :

If $a = 3$ and $b = -4$, evaluate : $2a^2 - b^2$.

Solution :

$$2a^2 - b^2 = (2 \times a \times a) - (b \times b)$$
$$= (2 \times 3 \times 3) - (-4 \times -4)$$
$$= 18 - (16) = 18 - 16 = 2$$ **(Ans.)**

EXERCISE 21

1. Write in the form of an algebraic expression :
   (i) Perimeter (P) of a rectangle is two times the sum of its length (l) and its breadth (b).
   (ii) Perimeter (P) of a square is four times its side.
   (iii) Area of a square is square of its side.
   (iv) Surface area of a cube is six times the square of its edge.
2. Express each of the following as an algebraic expression :
   (i) The sum of x and y minus m.
   (ii) The product of x and y divided by m.
   (iii) The subtraction of $5m$ from $3n$ and then adding $9p$ to it.
   (iv) The product of 12, x, y and z minus the product of 5, m and n.
   (v) Sum of p and $2r - s$ minus sum of a and $3n + 4x$.
3. Construct a formula for the following :

   Total wages (₹ W) of a man whose basic wage is (₹ B) for t hours a week plus overtime at (₹ R) per hour, if he works a total of T hours.

Wages for t hours = ₹ B

$\because$ Overtime = $(T - t)$ hour

$\therefore$ Wages for overtime = ₹ R $(T - t)$

$\Rightarrow$ Total wages = Wages for t hours + wages for overtime of $(T - t)$ hours

$\Rightarrow$ **₹ W = ₹ B + ₹ R (T − t)**

4. If $x = 4$, evaluate :

(i) $3x + 8$ (ii) $x^2 - 2x$ (iii) $\frac{x^2}{2}$

5. If $m = 6$, evaluate :

(i) $5m - 6$ (ii) $2m^2 + 3m$ (iii) $(2m)^2$

6. If $x = 4$, evaluate :

(i) $12x + 7$ (ii) $5x^2 + 4x$ (iii) $\frac{x^2}{8}$

7. If $m = 2$, evaluate :

(i) $16m - 7$ (ii) $15m^2 - 10m$ (iii) $\frac{1}{4}m^3$

8. If $x = 10$, evaluate :

(i) $100x + 225$ (ii) $6x^2 - 25x$ (iii) $\frac{1}{50}x^3$

9. If $a = -10$, evaluate :

(i) $5a$ (ii) a^2 (iii) a^3

10. If $x = -6$, evaluate :

(i) $11x$ (ii) $4x^2$ (iii) $2x^3$

11. If $m = -7$, evaluate :

(i) $12m$ (ii) $2m^2$ (iii) $2m^3$

12. Find the average (A) of four quantities p, q, r and s.
If A = 6, $p = 3$, $q = 5$ and $r = 7$; find the value of s.

13. If $a = 5$ and $b = 6$, evaluate :

(i) $3ab$ (ii) $6a^2b$ (iii) $2b^2$

14. If $x = 8$ and $y = 2$, evaluate :

(i) $9xy$ (ii) $5x^2y$ (iii) $(4y)^2$

15. If $x = 5$ and $y = 4$, evaluate :

(i) $8xy$ (ii) $3x^2y$ (iii) $3y^2$

16. If $y = 5$ and $z = 2$, evaluate :

(i) $100yz$ (ii) $9y^2z$ (iii) $5y^2$ (iv) $(5z)^3$

17. If $x = 2$ and $y = 10$, evaluate :

(i) $30xy$ (ii) $50xy^2$ (iii) $(10x)^2$ (iv) $5y^2$

18. If $m = 3$ and $n = 7$, evaluate :

(i) $12mn$ (ii) $5mn^2$ (iii) $(10m)^2$ (iv) $4n^2$

19. If $a = -10$, evaluate :

(i) $3a - 2$ (ii) $a^2 + 8a$ (iii) $\frac{1}{5} \times a^2$

20. If $x = -6$, evaluate :

(i) $4x - 9$ (ii) $3x^2 + 8x$ (iii) $\frac{x^2}{2}$

21. If $m = -8$, evaluate :

(i) $2m + 21$ (ii) $m^2 + 9m$ (iii) $\frac{m^2}{4}$

22. If $p = -10$, evaluate :

(i) $6p + 50$ (ii) $3p^2 - 20p$ (iii) $\frac{p^2}{50}$

23. If $y = -8$, evaluate :

(i) $6y + 53$ (ii) $y^2 + 12y$ (iii) $\frac{y^3}{4}$

24. If $x = 2$ and $y = -4$, evaluate :

(i) $11xy$ (ii) $5x^2y$ (iii) $(5y)^2$ (iv) $8x^2$

25. If $m = 9$ and $n = -2$, evaluate :

(i) $4mn$ (ii) $2m^2n$ (iii) $(2n)^3$

26. If $m = -8$ and $n = -2$, evaluate :

(i) $12mn$ (ii) $3m^2n$ (iii) $(4n)^2$

27. If $x = -5$ and $y = -8$, evaluate :

(i) $4xy$ (ii) $2xy^2$ (iii) $4x^2$ (iv) $3y^2$

28. Find T, if $T = 2a - b$, $a = 7$ and $b = 3$.

29. From the formula $B = 2a^2 - b^2$, calculate the value of B when $a = 3$ and $b = -1$.

30. The wages ₹ W of a man earning ₹ x per hour for t hours are given by the formula $W = xt$. Find his wages for working 40 hours at a rate of ₹ 39.45 per hour.

31. The temperature in Fahrenhiet scale is represented by F and the temperature in Celsius scale is represented by C. If $F = \frac{9}{5} \times C + 32°$, find F when $C = 40°$.

SIMPLE (Linear) EQUATIONS 22
(Including Word Problems)

22.1 BASIC CONCEPT :

A mathematical statement which shows that two algebraic expressions are equal is called an equation.

For example :

If the expressions $3x - 5$ and $x + 8$ are equal, we write $3x - 5 = x + 8$, which is an equation.

Similarly :

(i) If the expressions $x - 3$ and $7 - x$ are equal, it is written as $x - 3 = 7 - x$ [Which is an equation]

(ii) Seven subtracted from a number (x) equals 4 [A statement]
$\Rightarrow$ $x - 7 = 4$ [An equation]

(iii) A certain number (x) multiplied by 4 equals 20 [A statement]
$\Rightarrow$ $4x = 20$ [An equation]

(iv) A number (x) divided by 7 equals 2 [A statement]
$\Rightarrow$ $\frac{x}{7} = 2$ [An equation]

An equation is said to be a **linear equation** if it contains only **one variable** (literal) with **highest power 1** (one).

Since each of the equations discussed above has only one variable (which is x) with highest power 1, each of these equations is a linear equation.

22.2 SOLVING A LINEAR EQUATION

Solving a linear equation means finding the value of an unknown algebraic quantity (variable) used in the equation.

For example :

(i) To solve the equation $x + 5 = 7$ means to find the value of x.

(ii) To solve the equation $3y + 2 = 9$ means to find the value of y.

(iii) To solve the equation $\frac{2a}{3} + 4a = 10$ means to find the value of a and so on.

22.3 RULES FOR SOLVING A LINEAR EQUATION

Rule 1 : The given equation does not change if the same quantity is added on both sides.

e.g. $x + 5 = 2 \Rightarrow x + 5 + \mathbf{7} = 2 + \mathbf{7}$,

$3x - 2 = 8 \Rightarrow 3x - 2 + \mathbf{4} = 8 + \mathbf{4}$, etc.

Rule 2 : The given equation does not change if the same quantity is subtracted from both the sides of it.

e.g. $x + 5 = 2 \Rightarrow x + 5 - \mathbf{7} = 2 - \mathbf{7}$,

$3x - 2 = 8 \Rightarrow 3x - 2 - \mathbf{4} = 8 - \mathbf{4}$, etc.

Rule 3 : The given equation does not change if each of its terms is multiplied by the same quantity.

e.g. $5x = 2 \Rightarrow 5x \times \mathbf{3} = 2 \times \mathbf{3}$,

$\frac{3x}{2} = 7 \Rightarrow \frac{3x}{2} \times 2 = 7 \times 2$, etc.

Rule 4 : The given equation does not change if each of its terms is divided by the same non-zero quantity.

e.g. $3x = 5 \Rightarrow \frac{3x}{3} = \frac{5}{3}$,

$7x = 8 \Rightarrow \frac{7x}{4} = \frac{8}{4}$, etc.

22.4 SOLVING AN EQUATION OF THE FORM $x + a = b$.

Example 1 : Solve : $x + 3 = 10$

Solution :

$x + 3 = 10 \Rightarrow x + 3 - 3 = 10 - 3$ [**Rule 2 :** Subtracting 3 from both the sides]

$\Rightarrow x = 7$ **(Ans.)**

22.5 SOLVING AN EQUATION OF THE FORM $x - a = b$

Example 2 : Solve : $x - 5 = 2$

Solution :

$x - 5 = 2 \Rightarrow x - 5 + 5 = 2 + 5$ [**Rule 1 :** Adding 5 on both the sides]

$\Rightarrow x = 7$ **(Ans.)**

22.6 SOLVING AN EQUATION OF THE FORM $ax = b$

Example 3 : Solve : $2x = 6$

Solution :

$2x = 6 \Rightarrow \frac{2x}{2} = \frac{6}{2}$ [**Rule 4 :** Dividing each term by 2]

$\Rightarrow x = 3$ **(Ans.)**

22.7 SOLVING AN EQUATION OF THE FORM $\frac{x}{a} = b$

Example 4 : Solve : $\frac{y}{2} = 5$

Solution :

$$\frac{y}{2} = 5 \Rightarrow \frac{y}{2} \times 2 = 5 \times 2$$ [**Rule 3 :** Multiplying each term by 2]

$$\Rightarrow y = 10$$ **(Ans.)**

Example 5 : Solve : (i) $p - 2\cdot5 = 7\cdot3$ (ii) $x + 3\frac{1}{3} = 6$

Solution :

(i) $p - 2\cdot5 = 7\cdot3 \Rightarrow p - 2\cdot5 + 2\cdot5 = 7\cdot3 + 2\cdot5$

$\Rightarrow p = 9\cdot8$ **(Ans.)**

(ii) $x + 3\frac{1}{3} = 6 \Rightarrow x + \frac{10}{3} - \frac{10}{3} = 6 - \frac{10}{3}$

$\Rightarrow x = \frac{18 - 10}{3} = \frac{8}{3} = 2\frac{2}{3}$ **(Ans.)**

EXERCISE 22(A)

1. Solve :

(i) $x + 2 = 6$ (ii) $x + 6 = 2$ (iii) $y + 8 = 5$

(iv) $x + 4 = -3$ (v) $y + 2 = -8$ (vi) $b + 2\cdot5 = 4\cdot2$

(vii) $p + 4\cdot6 = 8\cdot5$ (viii) $y + 3\cdot2 = -6\cdot5$ (ix) $a + 8\cdot9 = -12\cdot6$

(x) $x + 2\frac{1}{3} = 5$ (xi) $z + 2 = 4\frac{1}{5}$ (xii) $m + 3\frac{1}{2} = 4\frac{1}{4}$

(xiii) $x + 2 = 1\frac{1}{4}$ (xiv) $y + 5\frac{1}{3} = 4$ (xv) $a + 3\frac{1}{5} = 1\frac{1}{2}$

2. Solve :

(i) $x - 3 = 2$ (ii) $m - 2 = -5$ (iii) $b - 5 = 7$

(iv) $a - 2\cdot5 = -4$ (v) $y - 3\frac{1}{2} = 6$ (vi) $z - 2\frac{1}{3} = -6$

(vii) $p - 5\cdot4 = 2\cdot7$ (viii) $x - 1\cdot5 = -4\cdot9$ (ix) $n - 4 = -4\frac{1}{5}$

3. Solve :

(i) $3x = 12$ (ii) $2y = 9$ (iii) $5z = 8\cdot5$

(iv) $2\cdot5m = 7\cdot5$ (v) $3\cdot2p = 16$ (vi) $2a = 4\cdot6$

4. Solve :

(i) $\frac{x}{2} = 5$ (ii) $\frac{y}{3} = -2$ (iii) $\frac{a}{5} = -15$

(iv) $\frac{z}{4} = 3\frac{1}{4}$ (v) $\frac{m}{6} = 2\frac{1}{2}$ (vi) $\frac{n}{7} = -2\cdot8$

5. Solve :

(i) $-2x = 8$ (ii) $-3\cdot5y = 14$ (iii) $-5z = 4$

(iv) $-5 = a + 3$ (v) $2 = p + 5$ (vi) $4\cdot5 = m - 2\cdot7$

(vii) $3\frac{2}{5} = x - 2\frac{1}{3}$ (viii) $5 = m + 3\frac{4}{7}$ (ix) $-2\frac{1}{5} = y - 4$

22.8 SOLVING EQUATIONS USING MORE THAN ONE PROPERTY

Example 6 :

Solve : (i) $3x + 8 = 14$ (ii) $\frac{m}{3} + 7 = 11$ (iii) $2 + \frac{5x}{3} = x + 6$

Solution :

(i) $3x + 8 = 14 \Rightarrow 3x + 8 - 8 = 14 - 8$ [Subtracting 8 from both the sides]

$\Rightarrow 3x = 6$

$\Rightarrow \frac{3x}{3} = \frac{6}{3}$ [Dividing each side by 3]

$\Rightarrow x = 2$ **(Ans.)**

(ii) $\frac{m}{3} + 7 = 11 \Rightarrow \frac{m}{3} + 7 - 7 = 11 - 7$ [Subtracting 7 from each side]

$\Rightarrow \frac{m}{3} = 4$

$\Rightarrow \frac{m}{3} \times 3 = 4 \times 3$ [Multiplying each side by 3]

$\Rightarrow m = 12$ **(Ans.)**

(iii) $2 + \frac{5x}{3} = x + 6 \Rightarrow 2 + \frac{5x}{3} - 2 = x + 6 - 2$ [Subtracting 2]

$\Rightarrow \frac{5x}{3} = x + 4$

$\Rightarrow \frac{5x}{3} - x = x + 4 - x$ [Subtracting x]

$\Rightarrow \frac{5x}{3} - x = 4$

$\Rightarrow \frac{5x}{3} \times 3 - x \times 3 = 4 \times 3$ [Multiplying each term by 3]

$\Rightarrow 5x - 3x = 12$

$\Rightarrow 2x = 12 \Rightarrow x = 6$ **(Ans.)**

Multiplying each term of an equation by the same number does not change the equation

22.9 SOLVING A LINEAR EQUATION USING TRANSPOSITION

Transposition of a positive or a negative term.

In transposition (shifting), a positive term becomes negative and a negative term becomes positive :

positive term becomes negative

e.g. $x + 3 = 7 \Rightarrow x = 7 - 3$

and $2x - 5 = 8 \Rightarrow 2x = 8 + 5$

negative term becomes positive

Example 7 : Solve : (i) $x + 5 = 32$ (ii) $y - 4 = 3$

(iii) $3z - 1 = 8$ (iv) $\frac{x}{3} + 4 = 12$

Solution :

(i) $x + 5 = 32 \Rightarrow x = 32 - 5$ [Transposing + 5]

$\Rightarrow x = 27$ **(Ans.)**

(ii) $y - 4 = 3 \Rightarrow y = 3 + 4$ [Transposing − 4]

$\Rightarrow y = 7$ **(Ans.)**

(iii) $3z - 1 = 8 \Rightarrow 3z = 8 + 1 = 9$ [Transposing − 1]

$\Rightarrow \frac{3z}{3} = \frac{9}{3}$ [Dividing by 3]

$\Rightarrow z = 3$ **(Ans.)**

(iv) $\frac{x}{5} + 4 = 2 \Rightarrow \frac{x}{5} = 2 - 4$ [Transposing + 4]

$\Rightarrow \frac{x}{5} = -2$

$\Rightarrow \frac{x}{5} \times 5 = -2 \times 5$ [Multiplying by 5]

$\Rightarrow x = -10$ **(Ans.)**

EXERCISE 22(B)

1. Solve :

(i) $2x + 5 = 17$ (ii) $3y - 2 = 1$ (iii) $5p + 4 = 29$

(iv) $4a - 3 = -27$ (v) $2z + 3 = -19$ (vi) $7m - 1 = 20$

(vii) $2.4x - 3 = 4.2$ (viii) $4m + 9.4 = 5$ (ix) $6y + 4 = -4.4$

2. Solve :

(i) $\frac{x}{3} - 5 = 2$ (ii) $\frac{y}{2} - 3 = 8$ (iii) $\frac{z}{7} + 1 = 2\frac{1}{2}$

(iv) $\frac{a}{2.4} - 5 = 2.4$ (v) $\frac{b}{1.6} + 3 = -2.5$ (vi) $\frac{m}{4} - 4.6 = -3.1$

3. Solve :

(i) $-8m - 2 = -10$ (ii) $4x + 2x = 3 + 5$

(iii) $4x - x + 5 = 8$ (iv) $6x + 2 = 2x + 10$

(v) $18 - (2a - 12) = 8a$ (vi) $3x + 5 + 2x + 6 + x = 4x + 21$

(vii) $3.5x - 9 - 3 = x + 1$ (viii) $8x + 6 + 2x - 4 = 4x + 8$

(ix) $-m + (3m - 6m) = -8 - 14$ (x) $5x - 14 = x - (24 + 4x)$

EXERCISE 22(C)

Solve :

1. $5 - x = 3$
2. $2 - y = 8$
3. $8{\cdot}4 - x = -2$
4. $x + 2\frac{1}{5} = 3$
5. $y - 3\frac{1}{2} = 2\frac{1}{3}$
6. $5\frac{2}{3} - z = 2\frac{1}{2}$
7. $1{\cdot}6z = 8$
8. $3a = -2{\cdot}1$
9. $\frac{z}{4} = -1{\cdot}5$
10. $\frac{z}{6} = -1\frac{2}{3}$
11. $-5x = 10$
12. $2{\cdot}4z = -4{\cdot}8$
13. $2y - 5 = -11$
14. $2x + 4{\cdot}6 = 8$
15. $5y - 3{\cdot}5 = 10$
16. $3x + 2 = -2{\cdot}2$
17. $\frac{y}{2} - 5 = 1$
18. $\frac{z}{3} - 1 = -5$
19. $\frac{x}{4} + 3{\cdot}6 = -1{\cdot}1$
20. $-3y - 2 = 10$
21. $4z - 5 = 3 - z$
22. $7x - 3x + 2 = 22$
23. $6y + 3 = 2y + 11$
24. $3(x + 5) = 18$
25. $5(x - 2) - 2(x + 1) = 3$
26. $(5x - 3) \div 4 = 3$
27. $3(2x + 1) - 2(x - 5) - 5(5 - 2x) = 16$

22.10 SOLVING WORD PROBLEMS

Steps :

1. Read the given statement carefully to know what is given and what is required to be found.
2. Take the unknown quantity required to be found as x or y or z, etc.
3. Form an equation according to the given relationship between the knowns and unknowns.
4. Solve the equation obtained in step 3 to get the required unknown quantity.

Example 8 :

A number increased by 13 is equal to 31. Find the number.

Solution :

Step 1 : On reading the given statement carefully, we conclude that we have to find a number that satisfies the given condition.

Step 2 : Let the required number be x.

Step 3 : The given relationship is :

The required number increased by 13 = 31.

$\Rightarrow \quad x + 13 = 31$

Step 4 : $x = 31 - 13 = 18$

$\therefore$ **The required number = 18** **(Ans.)**

Example 9 :

One-third of a number added to one-fifth of it gives 32. Find the number.

Solution :

Let the required number be x.

Given : $\frac{1}{3}x + \frac{1}{5}x = 32$

$\Rightarrow \frac{5x + 3x}{15} = 32$

$\Rightarrow \frac{8x}{15} = 32$ and $x = 32 \times \frac{15}{8} = 60$

$\therefore$ **The required number = 60** **(Ans.)**

Example 10 :

A number is decreased by 15 and the new number so obtained is multiplied by 3; the result is 81. Find the number.

Solution :

Let the number be x.

The number decreased by 15 = x – 15

The new number (x – 15) multiplied by 3 = 3(x – 15)

Given : $3(x - 15) = 81 \Rightarrow 3x - 45 = 81$

$\Rightarrow 3x = 81 + 45 = 126$

$\Rightarrow x = \frac{126}{3} = 42$

$\therefore$ **The required number = 42** **(Ans.)**

Example 11 :

The age of a man is 38 years more than the age of his son. If the sum of their ages is 82 years, find the age of the son and his father.

Solution :

Let the age of the son = x years

$\because$ The age of the man is 38 years more than the age of his son

$\therefore$ The age of the man = (x + 38) years

Given : The sum of the ages of the man and his son = 82 years

$\therefore (x + 38) + x = 82$

$\Rightarrow x + 38 + x = 82$ and $2x + 38 = 82$

$\Rightarrow 2x = 82 - 38 = 44$ and $x = \frac{44}{2} = 22$

$\therefore$ **The age of the son** = x years = **22 years**

And **the age of his father** = (x + 38) years = (22 + 38) years

= **60 years** **(Ans.)**

EXERCISE 22(D)

1. A number increased by 17 becomes 54. Find the number.
2. A number decreased by 8 equals 26; find the number.
3. One-fourth of a number added to two-seventh of it gives 135; find the number.
4. Two-fifth of a number subtracted from three-fourth of it gives 56; find the number.
5. A number is increased by 12 and the new number obtained is multiplied by 5. If the resulting number is 95, find the original number.
6. A number is increased by 26 and the new number obtained is divided by 3. If the resulting number is 18, find the original number.
7. The age of a man is 27 years more than the age of his son. If the sum of their ages is 47 years, find the age of the son and his father.
8. The difference between the ages of Gopal and his father is 26 years. If the sum of their ages is 56 years, find the ages of Gopal and his father.
9. When two consecutive natural numbers are added, the sum is 31; find the numbers.

> Let the two consecutive natural numbers be x and $x + 1$.
>
> $$\therefore \quad x + (x + 1) = 31$$
>
> Similarly, three consecutive natural numbers can be taken as : x, $x + 1$ and $x + 2$

10. When three consecutive natural numbers are added, the sum is 66, find the numbers.
11. A natural number decreased by 7 is 12. Find the number.
12. One-fourth of a number added to one-sixth of it is 15. Find the number.
13. A whole number is increased by 7 and the number so obtained is multiplied by 5; the result is 45. Find the whole number.
14. The age of a man and the age of his daughter differ by 23 years, and the sum of their ages is 41 years. Find the age of the man.
15. The difference between the ages of a woman and her son is 19 years, and the sum of their ages is 37 years, find the age of the son.
16. Two natural numbers differ by 6 and their sum is 36. Find the larger number.
17. The difference between two numbers is 15. Taking the smaller number as x, find :
    (i) the expression for the larger number.
    (ii) the larger number if the sum of these numbers is 71.
18. The difference between two numbers is 23. Taking the larger number as x, find :
    (i) the expression for smaller number.
    (ii) the smaller number, if the sum of these two numbers is 91.
19. Find three consecutive integers whose sum is 78.
20. The sum of three consecutive numbers is 54. Taking the middle number as x, find :
    (i) the expressions for the smallest number and the largest number.
    (ii) the three numbers.

Theme – 4
GEOMETRY

FUNDAMENTAL CONCEPTS 23

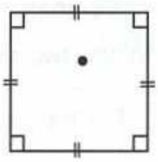
23.1 BASIC CONCEPT

Geometry *is the study of the position, shape, size and other properties of different figures.* Geometrical terms such as *point, line, plane, etc.,* carry the *basic ideas* for the *development of geometry.*

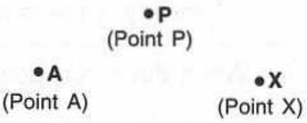
1. POINT :

A point is a mark of position. It has neither length nor width nor thickness; and occupies no space.

A small dot made by a sharp pencil on a plane sheet of paper represents a point.



A point is represented by a dot. In general, it is denoted by a capital letter, such as A, P, X, etc. as shown alongside, and is read as '*point P*', '*point X*' and so on.



2. LINE :

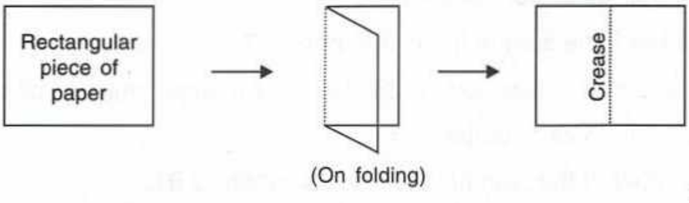
A line has only length. It has neither width nor thickness.

A line, as shown alongside, is represented by a straight mark with two arrow heads and is denoted by two capital letters.

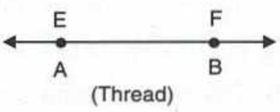
The word 'line' usually refers to a straight line.

More about a line

1. Take a rectangular piece of paper and fold it exactly at its middle and press the two parts together. On unfolding it, we will see that a straight crease is formed. The crease so obtained is the physical example of a part of the line. See below :

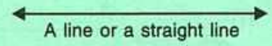


2. Another physical example of a part of a line is a taut straight thread.



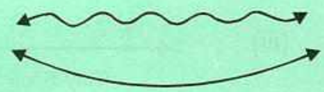
A line through points A and B is denoted as $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ and read as 'line AB' or 'line BA.'

1. *The two arrows of a line drawn in opposite directions indicate that the line has unlimited length, i.e. it can be extended up to any distance on either side.*


A line or a straight line

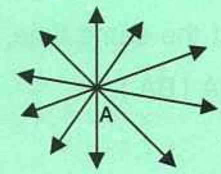
2. *A line may be straight or curved, but when we say 'a line', it means a straight line only.*

   The basic idea of a line is its straightness and that it extends infinitely in both directions.


[Curved lines]

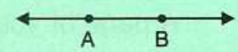
3. *An unlimited number of lines can be drawn through a given fixed point.*

   The adjoining figure shows a fixed point A. It is clear from the figure that an unlimited number of lines can be drawn through A.



4. *One and only one line can be drawn through any two fixed points.*

   In the adjoining figure, A and B are two fixed points. It is clear from the figure that only one line can be drawn through the fixed points A and B.

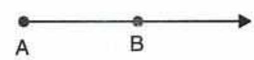


5. *Every line has an infinite (uncountable) number of points in it.*

3. RAY :

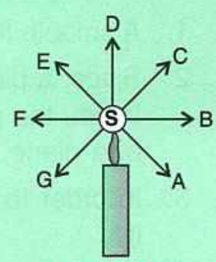
It is a line, i.e. a straight line, *that starts from a given fixed point and moves in the same direction.*

The adjoining figure shows a straight line that starts from a given fixed point A and moves through point B in the same direction. Therefore, it is a ray and is written as $\overrightarrow{AB}$.



1. *A ray has only one end* (fixed) *point, which is also known as its initial point.*
2. *A ray extends indefinitely in only one direction.*
3. *An unlimited number of rays can be drawn with the same initial point.*

   The adjoining figure shows a point source of light ,S. SA, SB, SC, SD, etc., are rays of light starting from the source S. Clearly, there are a very large number of such rays each having the same initial point S.
4. *A ray is a part of a line.*



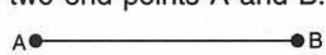
Lighted Candle

4. LINE SEGMENT :

A line segment is a part of a straight line.

The adjoining figure shows a line segment AB, which has two end points A and B.

AB and BA both represent the same line segment.



A line segment is a part of a line as well as of a ray.

Make the following facts clear :

(i) 

[Line AB, *i.e.* $\overleftrightarrow{AB}$ or line BA, *i.e.* $\overleftrightarrow{BA}$]

(ii) 

[Ray AB = $\overrightarrow{AB}$]

(iii) 

[Ray BA = $\overrightarrow{BA}$]

(iv) 

[Line segment AB (or BA)]

Clearly, line segment AB (or BA) is a part of both the rays AB ($\overrightarrow{AB}$) and BA ($\overrightarrow{BA}$). At the same time, the line segment AB (or BA) is a part of both the lines AB ($\overleftrightarrow{AB}$) and BA ($\overleftrightarrow{BA}$).

5. SURFACE :

A surface has length and width, but no thickness.

A page of your book, the outside of a box, a black-board, etc., represent surfaces.

A surface may be *flat* or *curved.*

The *surface of a wall* is *flat,* while the *surface of a cricket ball* is *curved.*

6. PLANE :

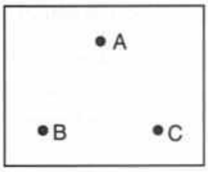
It is a flat surface. A plane has length and width, but no thickness.

A page of your book, the surface of a wall, the top of a table, etc., are some examples of planes.

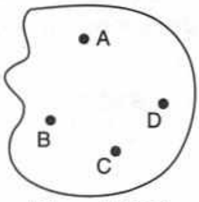
(i) *When a straight line is drawn through any two points on a plane, the line lies entirely within the plane.*
(ii) *A plane is a surface that extends indefinitely in all directions.*

More about a plane

1. A smooth flat surface is known as a plane surface, otherwise it is a curved surface.
2. Since, a plane extends indefinitely in all directions, it can not be drawn on paper. Infact, the piece of paper, page of your note-book, etc., represent only a portion of a plane.
3. In order to represent a plane, three or more non-collinear points are marked on it.

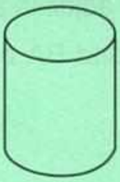


(Plane ABC)



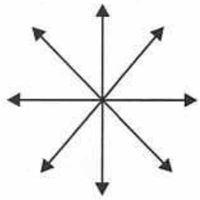
(Plane ABCD)

Each of the figures, given alongside, is a surface but not a plane and so a curved surface.


and


Properties of a plane

1. An unlimited number of lines can be drawn through each point of the plane.

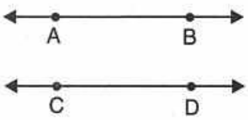


2. Through any two points on a plane, one and only one line can be drawn.

7. PARALLEL LINES :

Two straight lines are said to be ***parallel*** *to each other if they lie in the same plane and do not meet even when produced upto any extent on either side.*

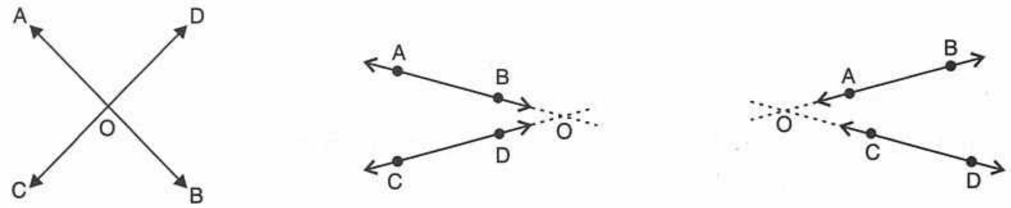
The given figure shows two lines AB and CD that are parallel to each other.



The line AB is parallel to the line CD is symbolically represented by writing AB // CD, read as *line AB is parallel to line CD.*

8. INTERSECTING LINES :

If two lines lie in the same plane and are not parallel to each other, they are called ***intersecting lines****.*



The figure given above shows two lines AB and CD that are not parallel. Such lines either intersect at point O or will intersect at point O if extended.

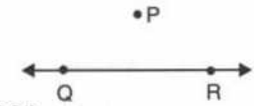
1. *The distance between two parallel lines is always the same whereas the distance between two non-parallel lines (lines that are not parallel) keeps changing.*
2. *Two different lines in a plane are either parallel or they intersect at only one point.*
3. *The definitions of parallel lines and intersecting lines are applicable only when the lines are in the same plane, i.e. when the lines are co-planar.*

9. COLLINEAR POINTS :

If three or more points lie on the same straight line, then the points are called ***collinear points.***

The given figure shows the collinear points A, B, C and D; as these points lie on the same line.

The following figure shows three non-collinear points : P, Q and R. These points do not lie on the same straight line.



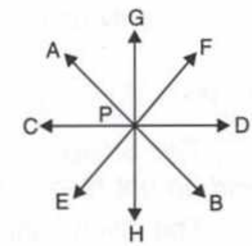
Consider any three points A, B and C,

(i) if point B lies on line AC; the points A, B and C are collinear.

(ii) if point B does not lie on the line AC; the points A, B and C are non-collinear.

10. CONCURRENT LINES :

If three or more straight lines (in the same plane) pass through the same point, the lines are called ***concurrent lines and the point is called the point of concurrence.***



The given figure shows the concurrent lines AB, CD, EF and GH; all these lines are in the same plane and pass through the same point P. Clearly, *point P is the point of concurrence.*

Example 1 :

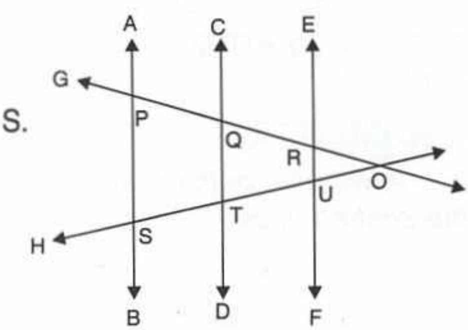
Use the adjoining figure to name :

(i) all pairs of parallel lines.

(ii) lines which intersect at point Q.

(iii) lines whose point of intersection is point S.

(iv) collinear points.



Solution :

(i) AB // CD, CD // EF and AB // EF.

(ii) OG and CD.

(iii) OH and AB.

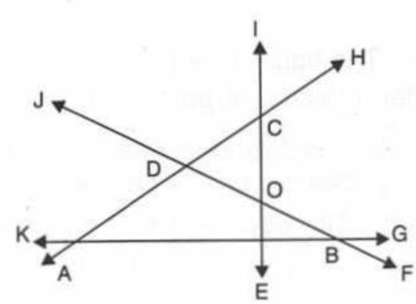
(iv) G, P, Q, R and O; H, S, T, U and O; A, P, S and B; C, Q, T and D; E, R, U and F.

Example 2 :

Use the adjoining figure to write :

(i) lines intersecting at point D.

(ii) lines concurrent at point O.



Solution :

(i) JF and AH.

(ii) JF and IE

EXERCISE 23(A)

1. State, **true** or **false**, if **false**, correct the statement :

(i) A dot has width but no length.

(ii) A ray has an infinite length only on one side of it.

(iii) A line segment PQ is written as $\overleftrightarrow{PQ}$.

(iv) $\overleftrightarrow{PQ}$ represents a straight line.

(v) Three points are said to be collinear, if they lie in the same plane.

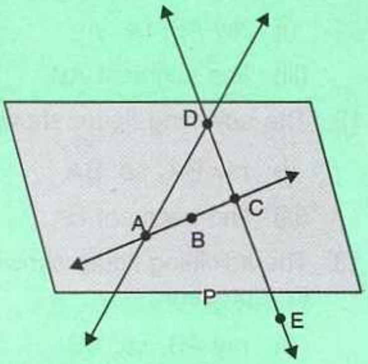
(vi) Three or more points, all lying in the same line, are called collinear points.

2. Write how many lines can be drawn, passing through :

(i) a given point ? (ii) two given fixed points ?

(iii) three collinear points ? (iv) three non-collinear points ?

3. The shaded region of the given figure shows a plane :



(a) Name :

(i) three collinear points.

(ii) three non-collinear points.

(iii) a pair of intersecting lines.

(b) State whether **true** or **false** :

(i) Line DE is contained in the given plane P.

(ii) Lines AB and DE intersect at point C.

(iii) Points D, B and C are collinear.

(iv) Points D, B and E are collinear.

4. Correct the statement, if it is wrong :

(i) A ray can be extended infinitely on either side.

(ii) A ray has a definite length. (iv) A line has two end points.

(iii) A line segment has a definite length. (v) A ray has only one end point.

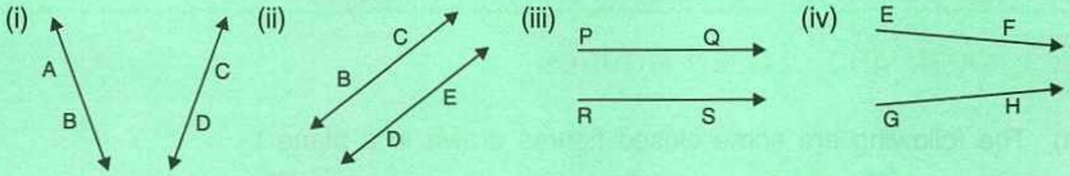
5. State **true** or **false**, if **false**, give the correct statement :

(i) A line has a countable number of points in it.

(ii) Only one line can pass through a given point.

(iii) The intersection of two planes is a straight line.

6. State whether the following *pairs of lines* or *rays* appear to be *parallel* or *intersecting.*



If, on extending the two lines on either side, they intersect or appear to intersect, they are intersecting lines.

7. Give two examples, from your surroundings, for each of the following :

(i) points (ii) line segments

(iii) plane surfaces (iv) curved surfaces.

8. Under what condition will two straight lines in the same plane have :

(i) no point in common. (ii) only one point in common.

(iii) an infinite number of points in common.

If possible, draw diagrams in support of your answer.

9. Mark two points A and B on a page of your exercise book. Mark a third point P such that :

(i) P lies between A and B and the three points A, P and B are collinear.

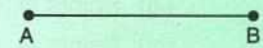
(ii) P does not lie between A and B yet the three points are collinear.
(iii) the three points do not lie in a line.

10. Mark two points P and Q on a piece of paper. How many lines can you draw :
    (i) passing through both the points P and Q ? (ii) passing through the point P ?
    (iii) passing through the point Q ?

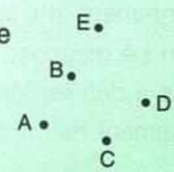
11. The adjoining figure shows a line AB. Draw figures to represent :
    (i) ray AB, *i.e.* $\overrightarrow{AB}$ (ii) ray BA, *i.e.* $\overrightarrow{BA}$
    (iii) line segment AB.

12. The adjoining figure shows a ray AB. Draw figures to show :
    (i) ray BA, *i.e.* $\overrightarrow{BA}$ (ii) line AB
    (iii) line segment BA.

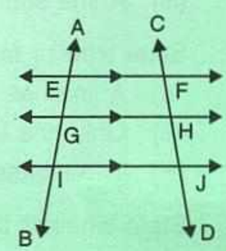
13. The adjoining figure shows a line segment AB. Draw figures to represent :
    (i) ray AB, *i.e.* $\overrightarrow{AB}$ (ii) line AB, *i.e.* $\overleftrightarrow{AB}$
    (iii) ray BA.



14. Use a ruler and find whether the points in the given figure are collinear or not :
    (i) D, A and C (ii) A, B and C
    (iii) A, B and E (iv) B, C and E



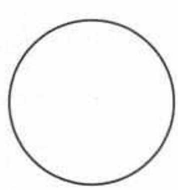
15. From the adjoining figure, write :
    (i) all pairs of parallel lines.
    (ii) all the lines which intersect EF.
    (iii) lines whose point of intersection is G.



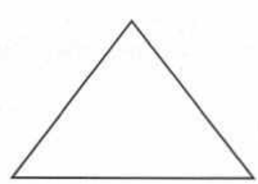
23.2 OPEN AND CLOSED FIGURES

(a) The following are some closed figures drawn in a plane :

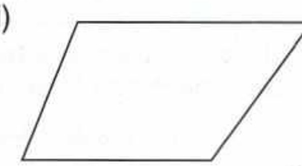
(i)



(ii)



(iii)

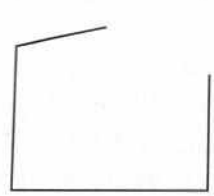


, etc.

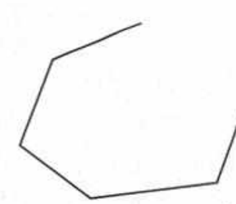
(b) The following are some open figures drawn in a plane :

(i)

(ii)



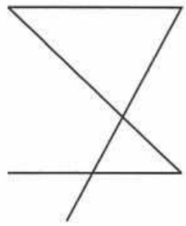
(iii)



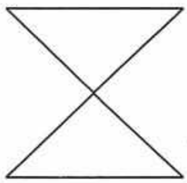
, etc.

(c) The following figures are in the same plane having intersecting line segments.

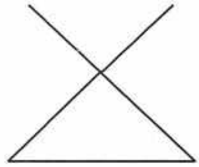
(i)



(ii)



(iii)



, etc.

What is the difference among the different sets of figures drawn above : for (a), for (b) and for (c) ?

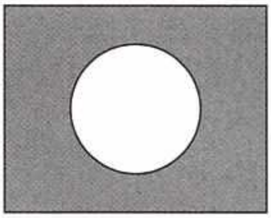
The figures in (a) drawn above, are *plane closed figures* because :

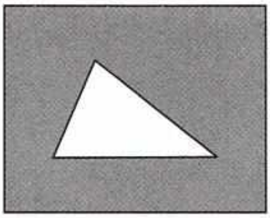
(i) they are bounded (enclosed) by continuous curves/lines and

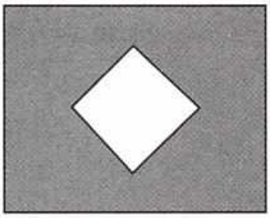
(ii) the curves/lines in the figure do not intersect each other.

Interior and exterior of a closed figure

In each of the following closed figures, the unshaded part shows interior of the figure whereas the shaded portion shows the exterior of the figure.





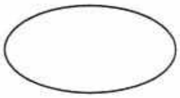


Curvilinear and linear boundries

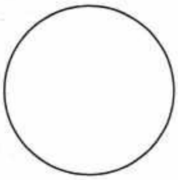
If a closed figure is not bounded by line segments, the figure is said to have **curvilinear boundries**.

For example :

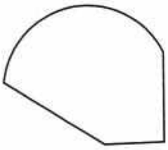
(i)



(ii)



(iii)

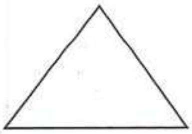


, etc.

If a closed figure is bounded by line segments only, the figure is said to have **linear boundries**.

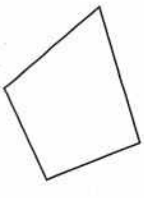
For example :

(i)



(ii)

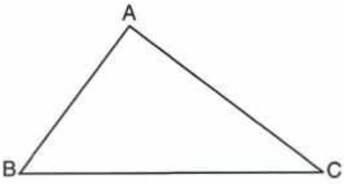
(iii)



, etc.

1. TRIANGLE :

*A **triangle** is a plane closed figure bounded by three line segments.*



The adjoining figure shows a triangle ABC bounded by the three line segments AB, BC and CA.

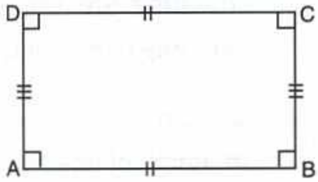
The *line segments* AB, BC and CA that form triangle ABC are called the sides of triangle ABC.

2. RECTANGLE :

*A **rectangle** is a plane closed figure bounded by four line segments such that :*

(i) *the opposite line segments (sides) are equal*

(ii) *each angle of the figure is 90°.*



The adjoining figure shows a rectangle ABCD bounded by the line segments AB, BC, CD and DA such that :

(i) the opposite line segments (sides) are equal.

i.e. AB = CD and AD = BC

(ii) each angle of it is 90°,

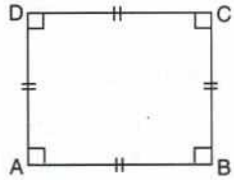
i.e. angle A = angle B = angle C = angle D = 90°.

3. SQUARE :

*A **square** is a plane closed figure bounded by four line segments* (sides) *such that :*

(i) *the four line-segments are equal to each other*

(ii) *each angle of the figure is 90°.*



The adjoining figure shows a square ABCD bounded by the line segments AB, BC, CD and DA such that :

(i) the four line segments (sides) are equal,

i.e. AB = BC = CD = DA

(ii) each angle in it is 90°

i.e. angle A = angle B = angle C = angle D = 90°.

4. CIRCLE :

*A **circle** is yet another type of plane closed figure, but it is not bounded by line segments.*

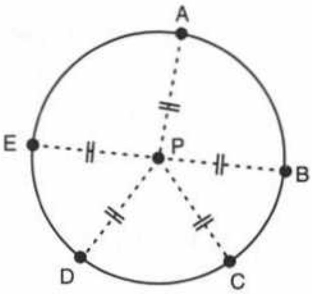
For example :

1. A wheel of : a cycle, a car, a bus, etc.
2. The disc of the full moon, etc.

A circle is a closed smooth curve of which each point is equidistant from a fixed point inside it.

The adjoining figure shows a circle. P is a fixed point inside it and A, B, C, D, E, ..., etc. are some points on the circle.

Clearly, PA = PB = PC = PD = ...



The *fixed point* inside the circle is called the ***centre*** of the circle, whereas the *distance* of each *point* on the circle *from the fixed point* (centre) is called the ***radius*** of the circle.

∴ P = centre of the circle

and PA = PB = PC = ... = radius of the circle.

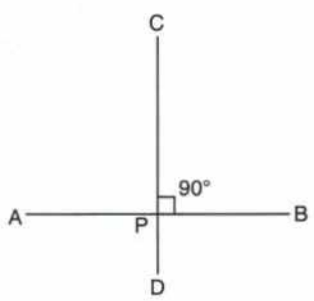
23.3 PERPENDICULAR TO A LINE SEGMENT

Your geometry box has a pair of set-squares. Each set-square has three edges (sides). The two sides (edges) of each set square contain an angle of 90°, hence these two sides are *perpendicular* to each other.

Similarly, when you look at a page of your textbook carefully, what do you observe ? You observe that every pair of adjacent sides contains an angle of 90°. Therefore, each pair of adjacent sides of a page of your textbook is perpendicular to each other.

Thus, two lines are said to be ***perpendicular*** *to each other, if they contain an angle of 90°* (right angle) *between them.*

The adjoining figure shows two line segments AB and CD so that the angle between these line segments is 90°. Therefore, the two line segments are perpendicular to each other, *i.e.* AB is perpendicular to CD and CD is perpendicular to AB.

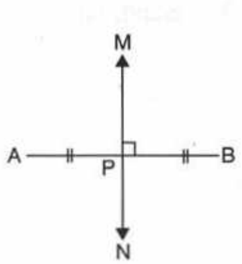


In your class-room, each wall is perpendicular to its floor. In the same way, the floor of your class-room is perpendicular to each wall of the class.

23.4 PERPENDICULAR BISECTOR OF A LINE SEGMENT :

For a line segment, the line that passes through its mid-point and is also perpendicular to it is called its ***perpendicular bisector***.

The given figure shows a line segment AB whose mid-point is P. The line MN is perpendicular to AB at point P, *i.e.* the line MN passes through the mid-point P of the line segment AB and is also perpendicular to AB. Therefore line **MN is the perpendicular bisector of the line segment AB**.

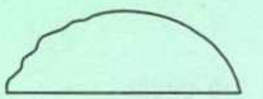
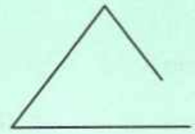
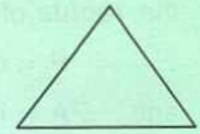


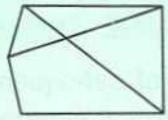
1. A perpendicular bisector of a line segment always :
   (i) passes through the mid-point of the line segment *i.e.* bisects the given line segment and
   (ii) is perpendicular to the given line segment.
2. One and only one perpendicular bisector can be drawn to a given line segment.

   In other words; *every line segment has a unique perpendicular bisector of it.*

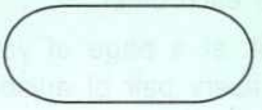
EXERCISE 23(B)

1. State, which of the following is a *plane closed figure* :

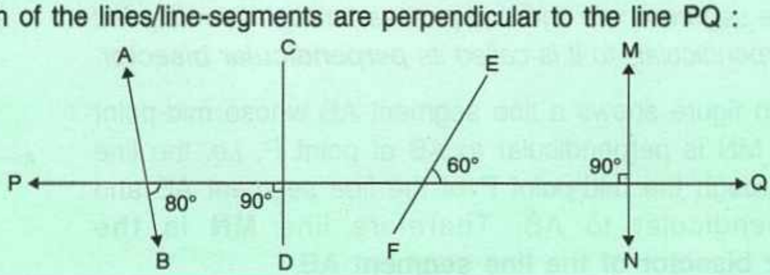
(i)

(ii)

(iii)

(iv)

(v)

(vi)

(vii)

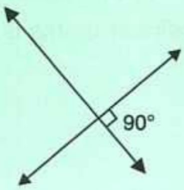
(viii)


2. Fill in the blanks :
   (i) .... is a three sided plane closed figure.
   (ii) A square is a plane closed figure which is not bounded by ....
   (iii) A rectangle is a .... sided plane ....
   (iv) A rectangle has opposite sides .... and adjacent sides .... to each other.
   (v) The sides of a square are .... to each other and each angle is ....
   (vi) For a line segment, a line making angle of .... with it, is called perpendicular to it.
   (vii) For a line segment, a line .... it and making angle of .... with it, is called .... bisector of the line segment.
   (viii) How many perpendiculars can be drawn to a line segment of length 6 cm ?
   (ix) How many perpendicular bisectors can be drawn to a line segment of length 6 cm ?
   (x) A perpendicular to a line segment will be its perpendicular bisector if it passes through the .... of the given line segment.
3. State, which of the lines/line-segments are perpendicular to the line PQ :

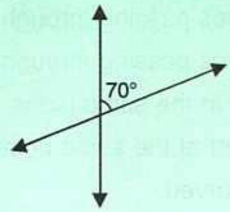


4. Which of the following figures shows two mutually perpendicular lines :

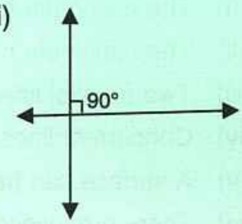
(i)



(ii)

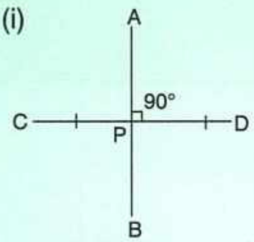


(iii)

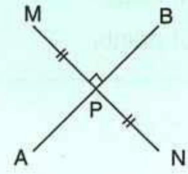


5. For each figure given below name the line segment that is perpendicular bisector of the other :

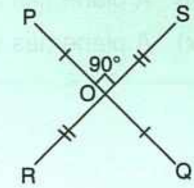
(i)



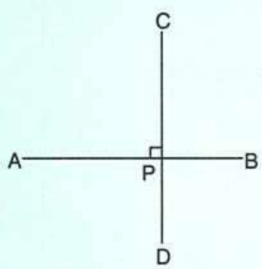
(ii)



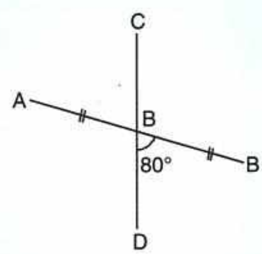
(iii)



(iv)

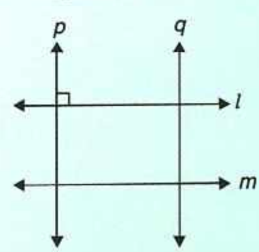


(v)



6. Name three objects from your surroundings that contain perpendicular edges.

7. Using the given figure, answer the following :



   (i) Name the pairs of parallel lines.
   (ii) Name the pairs of mutually perpendicular lines.
   (iii) Is the line p parallel to the line l ?
   (iv) Is the line q perpendicular to the line m ?

8. Place a scale (ruler) on a sheet of paper and hold it firmly with one hand. Now draw two line segments AB and CD along the longer edges of the scale. State whether segment AB is parallel to or perpendicular to segment CD.

9. Check your textbook :
   (i) How many pairs of its edges are parallel to each other ?
   (ii) How many pairs of its edges are perpendicular to each other ?

10. Give two examples from your surroundings for each of the following :
   (i) intersecting lines
   (ii) parallel lines
   (iii) perpendicular lines.

11. State **true** or **false**, if **false**, give the correct statement :

(i) The maximum number of lines passing through three collinear points is three.

(ii) The maximum number of lines passing through three non-collinear points is three.

(iii) Two parallel lines always lie in the same plane.

(iv) Concurrent lines always meet at the same point.

(v) A surface can be plane or curved.

(vi) There are infinite number of points in a line segment of length 10 cm.

(vii) There are infinite number of points in a line.

(viii) A plane has infinite number of lines.

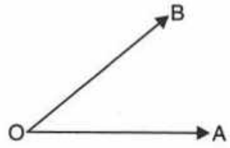
(ix) A plane has infinite number of points.

ANGLES
(With Their Types)
24

24.1 CONCEPT OF AN ANGLE

Two different rays starting from the same fixed point form an angle.

In the adjoining figure, two different rays OA and OB start from the same fixed point O to form angle AOB.



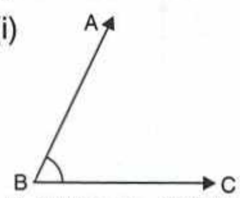
The point O, which is common to both the rays, is called the *vertex* of the angle AOB, whereas the rays OA and OB are called *the sides* or *arms* of the angle AOB.

The symbol '∠' is used to represent an angle.

Thus, angle AOB can be written as ∠AOB., *i.e.* angle AOB = ∠AOB.

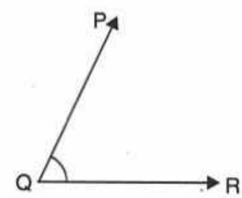
An angle is represented by three capital letters taken in such a way that the letter in the middle is always the vertex of the angle.

For example : (i)



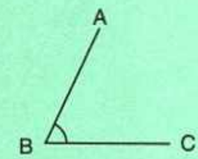
[∠ABC or ∠CBA]

(ii)



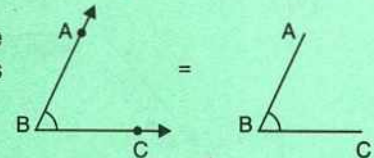
[∠PQR or ∠RQP]

1. An angle can be represented by the name of its vertex also.
   Thus, ∠ABC = ∠CBA = ∠B; ∠PQR = ∠RQP = ∠Q and so on.
2. Sometimes, two line segments with a common end point also form an angle at that point.

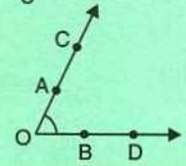
   In the adjoining figure, line segments BA and BC form angle ABC at their common end point.

   It should be noted here that, whether or not BA and BC are rays or line segments, the measure of angle ABC remains the same.

   IMPORTANT : If the arms of an angle have several points on them, the same angle can be written in various ways.

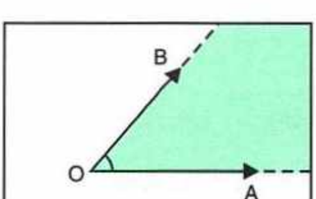
   In the given figure : ∠O = ∠AOB = ∠BOA = ∠COB = ∠COD
   = ∠BOC = ∠DOA = ∠DOC = ∠AOD

24.2 INTERIOR OF AN ANGLE :

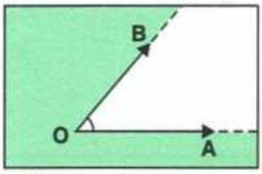
The interior of an angle is the region that lies within an angle. In other words, it is the region bounded by the arms of an angle.

The shaded portion of the given figure shows the interior of the angle AOB.



[Interior of an angle]

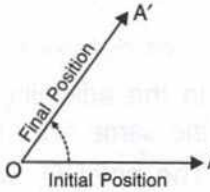
24.3 EXTERIOR OF AN ANGLE :



[Exterior of an angle]

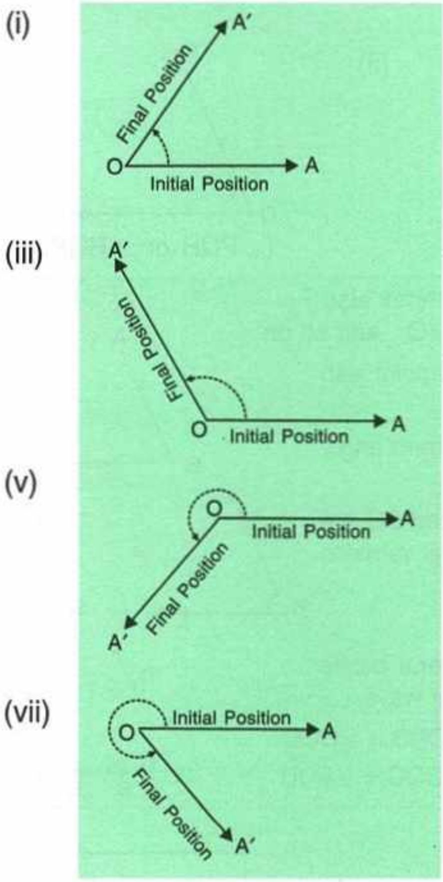
The exterior of an angle is the region that lies outside the angle. The shaded portion of the adjoining figure shows the exterior of the angle AOB.

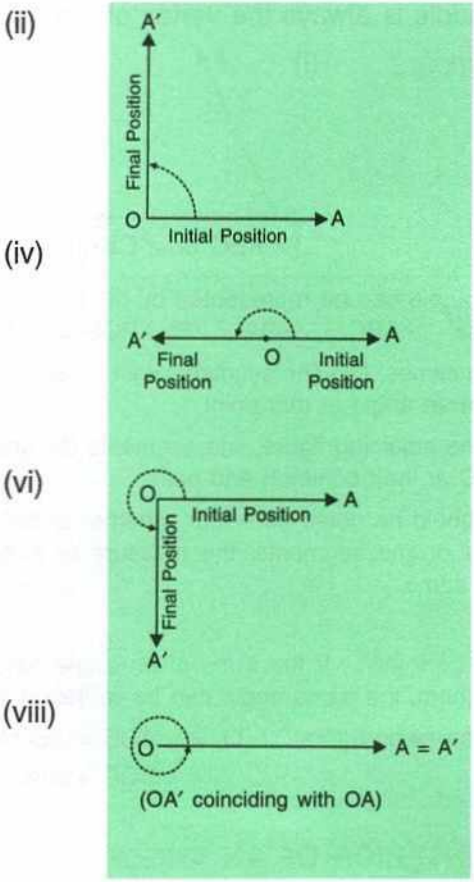
24.4 ANGLE FORMED BY ROTATION :



Consider a ray OA rotating about its end point O (OA is the ray's original or initial position). As OA rotates about the fixed point O, it attains several positions before it returns to its original position.

At any particular moment, the position of ray OA, shown above as OA′, is termed its final position. It can be observed easily that the initial position OA and the final position OA′ form an angle ∠AOA′. The angle becomes bigger and bigger as OA′ rotates till it coincides with its initial position OA (see the following figures carefully).



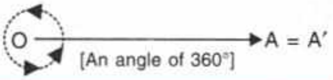


24.5 MEASURING AN ANGLE :

The unit of measurement of an angle is ***degree***. The symbol for degree is °.

Thus, if measure of an angle is **60 degrees**, we write **60°**.

The angle measure of one complete rotation is **360 degrees** *i.e.* **360°** [see alongside]



> Angle due to one complete rotation is also known as a complete angle.

When the angle formed by one complete rotation of a ray is divided into 360 equal parts, each part is called one degree, *i.e.* 1°.

If one degree is further divided into 60 equal parts, each part is called a minute, which is denoted by **1′ (one prime)**.

Thus, 1 minute = 1′, 5 minutes = 5′, 40 minutes = 40′ and so on.

If we further divide one minute into 60 equal parts, each part is called **a second**, which is denoted by **1″ (two primes)**.

Thus, 1 second = 1″, 45 seconds = 45″, 40 seconds = 40″ and so on.

Hence, 1 complete rotation = 360° [Three sixty degrees]

1° = 60′ [Sixty minutes]

1′ = 60″ [Sixty seconds]

5 minutes 30 seconds = 5′ 30″

Similarly, 25 degrees 30 minutes 15 seconds = 25° 30′ 15″ and so on.

***Example* 1 :**

Add : (i) 32° 23′ 15″ and 49° 17′ 32″ (ii) 74° 35′ 18″ and 9° 20′ 53″

Solution :

(i)

32° 23′ 15″
\+ 49° 17′ 32″
____
81° 40′ 47″ **(Ans.)**

(ii)

74° 35′ 18″
\+ 9° 20′ 53″
____
83° 55′ 71″

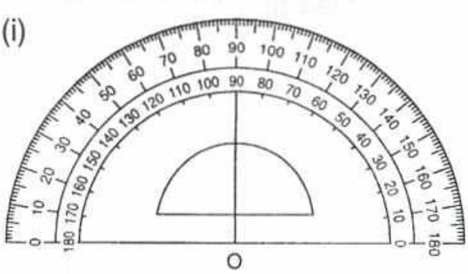
Since 60″ = 1′ ∴ 71″ = 1′ 11″

and so 83° 55′ 71″ = **83° 56′ 11″** **(Ans.)**

24.6 USING A PROTRACTOR FOR MEASURING AN ANGLE :

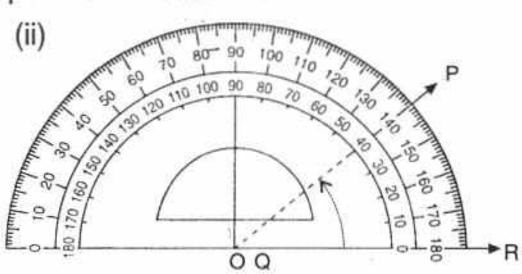
A protractor, as shown below, is a semi-circular plastic (or metallic) disc marked in degrees from 0° to 180° on its semi-circular part. The centre of this semi-circular piece is marked as O, which is also the mid-point of its base-line.

(i)



[A protractor]

(ii)



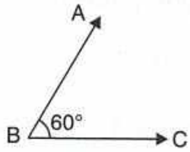
[Measuring angle PQR]

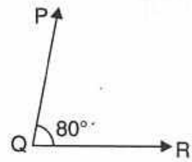
In order to measure an angle, say angle PQR, as shown above, the base line of the protractor is kept on arm QR of the angle, such that its centre O coincides with the vertex of the angle PQR. Now the position of the other arm, *i.e.* arm PQ, of the angle PQR is read from the markings on the protractor.

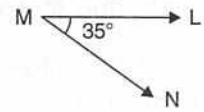
In the figure given above ∠PQR = 40°

24.7 TYPES OF ANGLES :

1. **Acute Angle :** An **angle** of **less than 90°** is known as an **acute angle.**

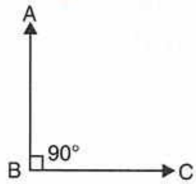
e.g. (i) 

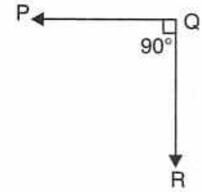
(ii) 

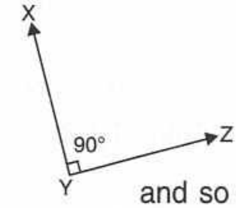
(iii) 

and so on.

2. **Right Angle :** An **angle** of **90°** is known as a **right angle**.

e.g. (i) 

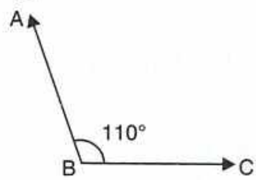
(ii) 

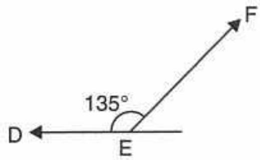
(iii) 

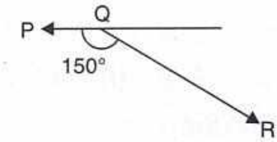
and so on.

Two lines (rays, line segments) that form an angle of 90° are said to be perpendicular to each other.

3. **Obtuse Angle :** An **angle** of **more than 90° but less than 180°** is known as an **obtuse angle.**

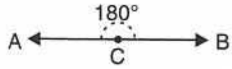
e.g. (i) 

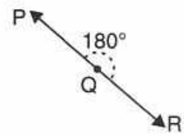
(ii) 

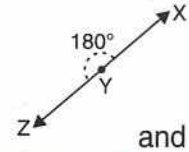
(iii) 

and so on.

4. **Straight Line Angle :** An **angle** of **180°** is known as a **straight angle** or **straight line angle**.

e.g. (i) 

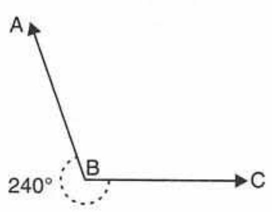
(ii) 

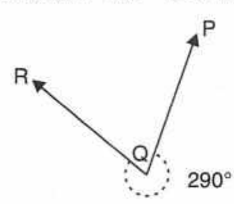
(iii) 

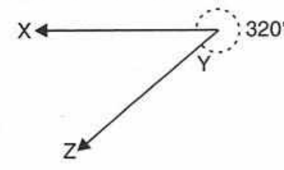
and so on.

A straight line angle = 180° = 2 × 90° = **Two right angles**

5. **Reflex Angle :** An **angle** of **more than 180° but less than 360°** is called a **reflex angle**.

e.g. (i) 

(ii) 

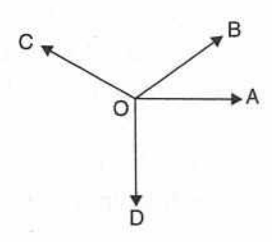
(iii) 

and so on.

24.8 AN IMPORTANT RESULT :

The sum of the angles around a point is always 360°.

For example :



In the adjoining figure,

$\angle AOB + \angle BOC + \angle COD + \angle DOA = 360°$.

24.9 ADJACENT ANGLES :

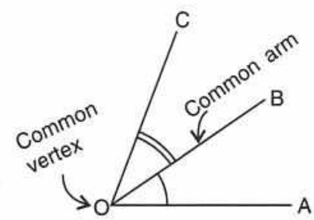
Two angles are said to be adjacent angles if :

(i) they have their vertices at the same point, *i.e.* they have a common vertex,

(ii) they have one common arm

and, (iii) the other arms of the angles are on the opposite sides of the common arm.

The figure given alongside shows two angles AOB and BOC having arm OB in common, vertex O in common, and also with their other arms OA and OC lying on opposite sides of the common arm OB.

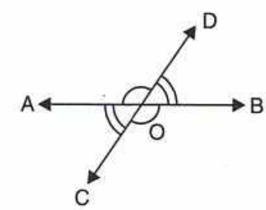


∴ ∠ AOB and ∠ BOC are adjacent angles.

24.10 VERTICALLY OPPOSITE ANGLES :

When two straight lines intersect, the angles on the opposite sides of their point of intersection are called vertically opposite angles.

The adjoining figure shows two lines AB and CD intersecting at point O.



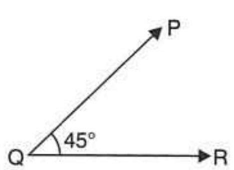
Clearly, ∠ AOC and ∠ BOD are vertically opposite angles and ∠ BOC and ∠ AOD are also vertically opposite angles.

> Vertically opposite angles are always equal.
> *i.e.* ∠ AOC = ∠ BOD and ∠ BOC = ∠ AOD

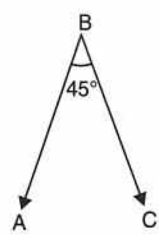
24.11 CONGRUENT (EQUAL) ANGLES :

Angles having the same measure (value), are said to be congruent angles.

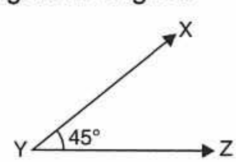
e.g. (i)



(ii)



(iii)

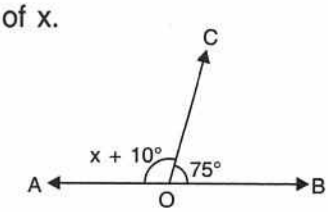


and so on.

As shown above, the angles PQR, ABC and XYZ are congruent, since these angles have the same measure, *i.e.* 45°.

Example 2 :

In the given figure, AOB is a straight line. Find the value of x.



Solution :

Since AOB is a straight line

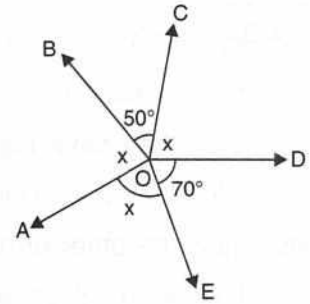
∴ ∠AOB = 180° ⇒ x + 10° + 75° = 180°

⇒ x + 85° = 180°

⇒ x = 180° – 85° = 95° **(Ans.)**

Example 3 :

Use the given figure to find x.



Solution :

Since the sum of all the angles around a point is 360°.

$\therefore \quad x + x + 70° + x + 50° = 360°$

$\Rightarrow \quad 3x + 120° = 360°$

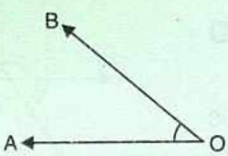
$\Rightarrow \quad 3x = 360° - 120° = 240°$

$\Rightarrow \quad x = \frac{240°}{3} = 80°$ **(Ans.)**

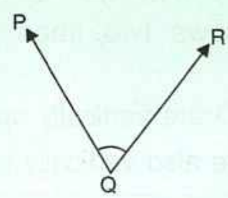
EXERCISE 24(A)

1. For each angle given below, write the name of the vertex, the names of the arms and the name of the angle.

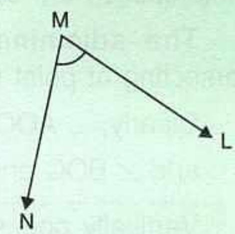
(i)



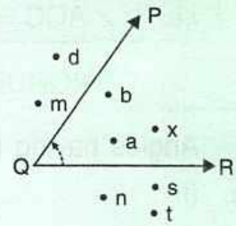
(ii)



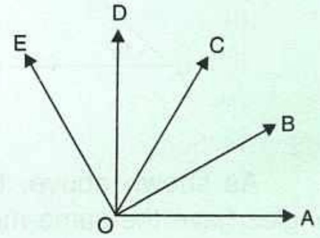
(iii)



2. Name the points :
   (i) in the interior of the angle PQR.
   (ii) in the exterior of the angle PQR.

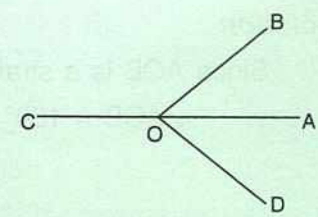


3. In the adjoining figure, figure out the number of angles formed within the arms OA and OE.

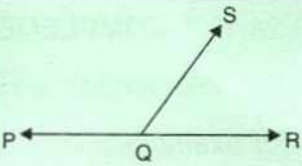


4. Add :
   (i) 29° 16′ 23″ and 8° 27′ 12″
   (ii) 9° 45′ 56″ and 73° 8′ 15″
   (iii) 56° 38′ and 27° 42′ 30″
   (iv) 47° and 61° 17′ 4″

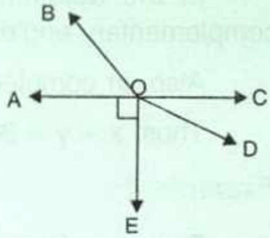
5. In the figure given alongside, name :
   (i) three pairs of adjacent angles.
   (ii) two acute angles.
   (iii) two obtuse angles.
   (iv) two reflex angles.



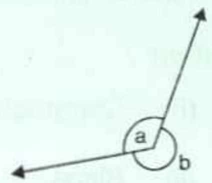
6. In the given figure :
PQR is a straight line. If :
(i) $\angle SQR = 75°$; find $\angle PQS$.
(ii) $\angle PQS = 110°$; find $\angle RQS$.



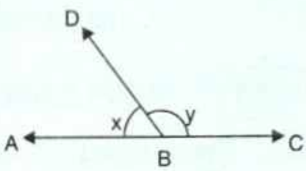
7. In the given figure, AOC is a straight line. If angle AOB = 50°, angle AOE = 90° and angle COD = 25°, find the measure of :
(i) angle BOC (ii) angle EOD
(iii) obtuse angle BOD (iv) reflex angle BOD
(v) reflex angle COE.



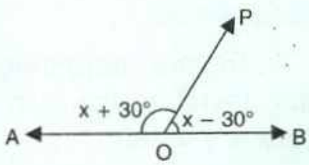
8. In the given figure, if :
(i) a = 130°, find b.
(ii) b = 200°, find a.
(iii) a = 5/3 right angle, find b.



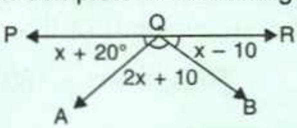
9. In the figure given alongside, ABC is a straight line.
(i) If x = 53°, find y.
(ii) If $y = 1\frac{1}{2}$ right angles, find x.



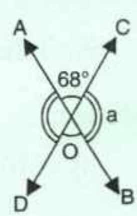
10. In the figure given alongside, AOB is a straight line. Find the value of x and also answer each of the following :
(i) $\angle AOP$ = ...
(ii) $\angle BOP$ = ...
(iii) which angle is obtuse ?
(iv) which angle is acute ?



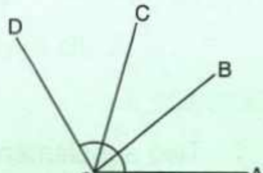
11. In the figure given alongside, PQR is a straight line. Find x. Then complete the following:
(i) $\angle AQB$ = ...
(ii) $\angle BQP$ = ...
(iii) $\angle AQR$ = ...



12. In the figure given alongside, lines AB and CD intersect at point O.
(i) Find the value of $\angle a$.
(ii) Name all the pairs of vertically opposite angles.
(iii) Name all the pairs of adjacent angles.
(iv) Name all the reflex angles formed and write the measure of each.



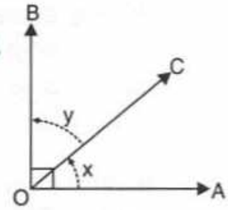
13. In the figure given alongside :
(i) if $\angle AOB = 45°$, $\angle BOC = 30°$ and $\angle AOD = 110°$; find angles COD and BOD.
(ii) if $\angle BOC = \angle DOC = 34°$ and $\angle AOD = 120°$; find angle AOB and angle AOC.
(iii) if $\angle AOB = \angle BOC = \angle COD = 38°$; find reflex angle AOC and reflex angle AOD.



24.12 COMPLEMENTARY ANGLES :

Two angles are said to be complementary if their sum is 90°.

For example :



In the adjoining figure, angle x and angle y are complementary angles as their sum is 90°, *i.e.* x + y = 90°.

Also, in complementary angles; each angle is called the complement of the other.

Thus, x + y = 90° $\Rightarrow$ (i) x is complement of y and (ii) y is complement of x.

Example 4 :

Find complement of each given angle : (i) 35° (ii) $\frac{2}{3}$ of 90°

Solution :

(i) **Complement of 35°** = 90° − 35° = **55°** **(Ans.)**

(ii) Since $\frac{2}{3}$ of 90° = $\frac{2}{3}$ × 90° = **60°**

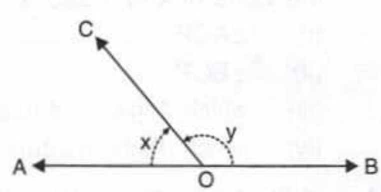
$\therefore$ Its **complement** = 90° − 60° = **30°** **(Ans.)**

24.13 SUPPLEMENTARY ANGLES :

Two angles are said to be supplementary if their sum is 180°.

For example :

In the adjoining figure, AOB is a straight line; therefore the sum of the angles x and y is 180°, *i.e.* x + y = 180°.



Thus, angles x and y are supplementary angles.

Also, in supplementary angles, each angle is called the supplement of the other.

Thus, x + y = 180° $\Rightarrow$ (i) x is the supplement of y and, (ii) y is the supplement of x.

Example 5 :

Find the supplement of each given angle : (i) 48° (ii) $\frac{4}{5}$ of 90°

Solution :

(i) **Supplement of 48°** = 180° − 48° = **132°** **(Ans.)**

(ii) Since $\frac{4}{5}$ of 90° = $\frac{4}{5}$ × 90° = **72°**

$\therefore$ Its **supplement** = 180° − 72° = **108°** **(Ans.)**

Example 6 :

Two supplementary angles are in the ratio 5 : 4. Find the angles.

Solution :

The ratio of the supplementary angles is 5 : 4, and 5 + 4 = 9

$\therefore$ The angles are $\frac{5}{9} \times 180°$ and $\frac{4}{9} \times 180°$

$= 100°$ and $80°$, respectively **(Ans.)**

Alternative method :

Let the angles be 5x and 4x. [As ratio of the angles is 5 : 4]

Since sum of supplementary angles = 180°

$\Rightarrow \quad 5x + 4x = 180°$

$\Rightarrow \quad 9x = 180°$ and $x = 20°$

$\therefore$ **Required angles** $= 5x$ and $4x$

$= 5 \times 20°$ and $4 \times 20°$

$= 100°$ and $80°$ **(Ans.)**

EXERCISE 24(B)

1. Write the complement angle of :
   (i) 45° (ii) x° (iii) (x – 10)° (iv) 20° + y°
2. Write the supplement angle of :
   (i) 49° (ii) 111° (iii) (x – 30)° (iv) 20° + y°
3. Find the complement angle of :
   (i) 1/2 of 60° (ii) 1/5 of 160° (iii) 2/5 of 70° (iv) 1/6 of 90°
4. Find the supplement angle of :
   (i) 50% of 120° (ii) 1/3 of 150° (iii) 60% of 100° (iv) 3/4 of 160°
5. Find the angle : (i) that is equal to its complement ?
   (ii) that is equal to its supplement ?
6. Two complementary angles are in the ratio 7 : 8. Find the angles.
7. Two supplementary angles are in the ratio 7 : 11. Find the angles.
8. The measures of two complementary angles are $(2x - 7)°$ and $(x + 4)°$. Find x.
9. The measures of two supplementary angles are $(3x + 15)°$ and $(2x + 5)°$. Find x.
10. For an angle x°, find :
    (i) the complementary angle
    (ii) the supplementary angle.
    (iii) the value of x° if its supplementary angle is three times its complementary angle.

PROPERTIES OF ANGLES AND LINES
(Including Parallel Lines)
25

25.1 PROPERTIES OF ADJACENT AND VERTICALLY OPPOSITE ANGLES

***Property* 1 :**

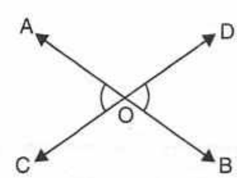
When two straight lines intersect :

(i) *the sum of each pair of adjacent angles is always* 180°.

(ii) *the vertically opposite angles are always equal.*

For example :

In the adjoining figure, the straight lines AB and CD intersect each other at point O, and so, we have :



(i) the sum of adjacent angles = 180°,

i.e. $\angle AOD + \angle DOB = 180°$,

$\angle BOD + \angle BOC = 180°$,

$\angle BOC + \angle COA = 180°$,

and $\angle COA + \angle AOD = 180°$.

(ii) the vertically opposite angles are equal, *i.e.* $\angle AOC = \angle BOD$ and $\angle BOC = \angle AOD$

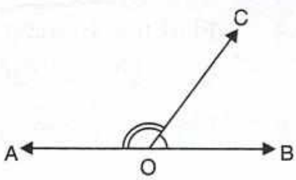
***Property* 2 :**

If the sum of two adjacent angles is 180°, their exterior arms are always in the same straight line.

Conversely, **if the exterior arms of two adjacent angles are in the same straight line, the sum of the angles is always 180°.**

For example :

Considering the two adjacent angles given alongside :

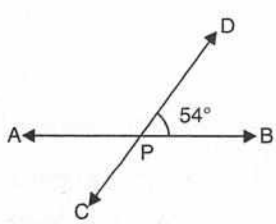


(i) if $\angle AOC + \angle BOC = 180°$, the exterior arms OA and OB are in the same straight line, *i.e.* AOB is a straight line.

(ii) if exterior arms OA and OB are in the same straight line, *i.e.* if AOB is a straight line, $\angle AOC + \angle BOC = 180°$.

***Example* 1 :**

Two straight lines AB and CD intersect at point P. If angle BPD = 54°, find, giving reason :



(i) $\angle APD$ (ii) $\angle APC$.

***Solution* :**

(i) When two straight lines intersect each other, the adjacent angles are supplementary.

$\Rightarrow \angle APD + \angle BPD = 180° \Rightarrow \angle APD + 54° = 180°$

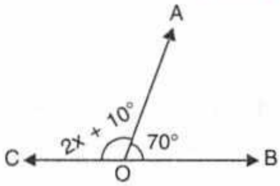
$\Rightarrow$ **$\angle APD$** $= 180° - 54° =$ **126°** **(Ans.)**

(ii) When two straight lines intersect each other, the vertically opposite angles are equal.

$\Rightarrow \quad \angle APC = \angle BPD = 54°$ **(Ans.)**

Example 2 :

The figure given alongside, shows two adjacent angles AOB and AOC whose exterior arms OB and OC are along the same straight line. Find the value of *x*.



Solution :

Since the exterior arms of the adjacent angles are in a straight line, the adjacent angles are supplementary.

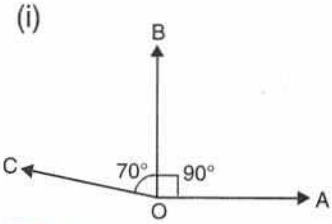
$\therefore \angle AOC + \angle AOB = 180° \Rightarrow 2x + 10° + 70° = 180°$

$\Rightarrow 2x = 180° - 80° \Rightarrow x = \dfrac{100°}{2} = 50°$ **(Ans.)**

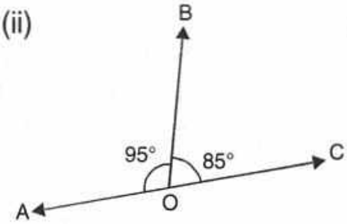
Example 3 :

Each figure given below shows a pair of adjacent angles AOB and BOC. Find whether or not the exterior arms OA and OC are in the same straight line.

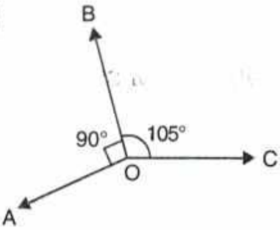
(i)



(ii)



(iii)



Solution :

(i) $\angle AOB + \angle BOC = 90° + 70° = 160°$

Since the sum of adjacent angles AOB and BOC is not 180°, the **exterior arms OA and OB are not in the same straight line.** **(Ans.)**

(ii) $\angle AOB + \angle BOC = 95° + 85° = 180°$

$\Rightarrow$ The sum of adjacent angles AOB and BOC is 180°.

$\therefore$ ***The exterior arms OA and OC are in the same straight line.*** **(Ans.)**

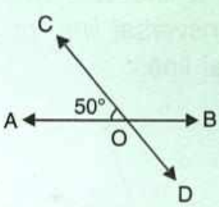
(iii) $\angle AOB + \angle BOC = 90° + 105° = 195°$; *which is not 180°.*

$\Rightarrow$ **The exterior arms OA and OC are *not in the same straight line.*** **(Ans.)**

EXERCISE 25(A)

1. Two straight lines AB and CD intersect each other at a point O and angle AOC = 50°; find :

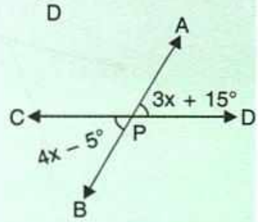
   (i) angle BOD (ii) ∠AOD (iii) ∠BOC



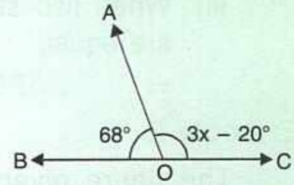
2. The adjoining figure shows two straight lines AB and CD intersecting at point P. If ∠BPC = 4x − 5° and ∠APD = 3x + 15°, find :

   (i) the value of *x*. (ii) ∠APD

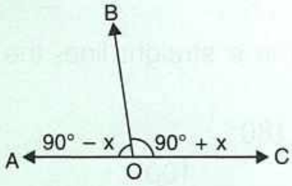
   (iii) ∠BPD (iv) ∠BPC.

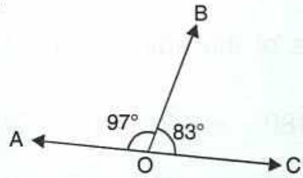


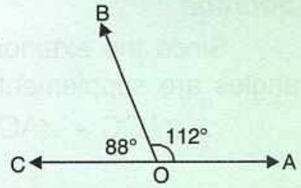
3. The figure, given alongside, shows two adjacent angles AOB and AOC whose exterior sides are along the same straight line. Find the value of x.



4. Each figure, given below, shows a pair of adjacent angles AOB and BOC. Find whether or not the exterior arms OA and OC are in the same straight line.

(i)


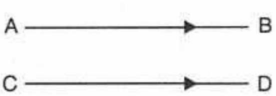
(ii)


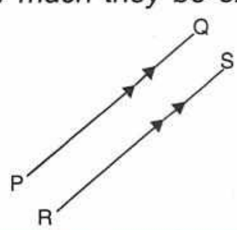
(iii)


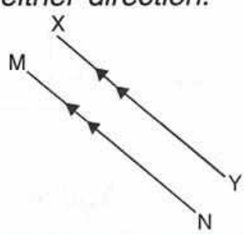
5. A line segment AP stands at point P of a straight line BC such that $\angle APB = 5x - 40°$ and $\angle APC = x + 10°$; find the values of x and angle APB.

25.2 PARALLEL LINES

Two straight lines, lying in the same plane, are said to be parallel if they do not meet (intersect), no matter how much they be extended in either direction.

e.g. (i)


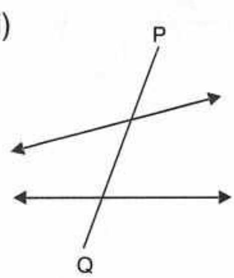
(ii)


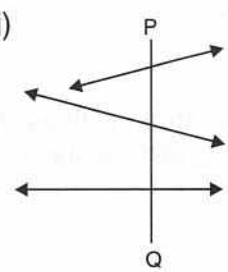
(iii)


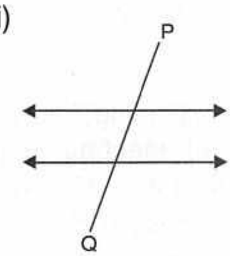
1. *Two parallel lines are represented by drawing arrows on both lines in the same direction. See the figures drawn above.*
2. *The distance between two parallel lines does not change, i.e. neither does it increase nor does it decrease. That is why parallel lines never meet (intersect).*
   Example; railway lines.

25.3 CONCEPT OF TRANSVERSAL LINES

When a line cuts two or more lines, whether (parallel or non-parallel), *the line is called a transversal line, or simply, a transversal.* In each of the following figures, PQ is a transversal line.

(i)


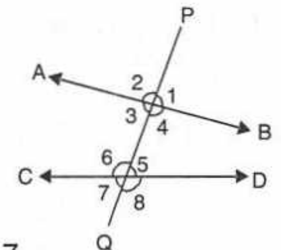
(ii)


(iii)


25.4 ANGLES FORMED BY TWO LINES AND THEIR TRANSVERSAL LINE

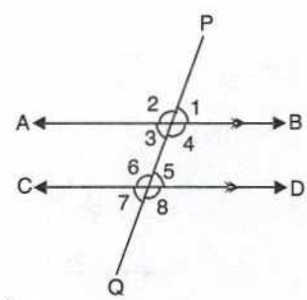
When a transversal PQ cuts two parallel or non-parallel lines AB and CD (as shown), *eight* (8) *angles are formed :* they are marked 1 to 8 in the following figure :

These angles can be distinguished as stated below :



1. ***Exterior Angles :***
   The angles marked 1, 2, 7 and 8 are *exterior angles.*
2. ***Interior Angles :***
   The angles marked 3, 4, 5 and 6 are interior angles.
3. ***Exterior Alternate Angles :***
   The two pairs of exterior alternate angles are 2 and 8; 1 and 7.
4. ***Interior Alternate Angles :***
   The two pairs of interior alternate angles are 3 and 5; 4 and 6. In general, ***interior alternate angles*** are simply called as ***alternate angles***.
5. ***Corresponding Angles :***
   The four pairs of corresponding angles are 1 and 5; 2 and 6; 3 and 7; 4 and 8.
6. ***Co-interior or Conjoined or Allied Angles :***
   The two pairs of co-interior or allied angles are 3 and 6; 4 and 5.
7. ***Exterior Allied Angles :***
   The two pairs of exterior allied angles are 2 and 7; 1 and 8.

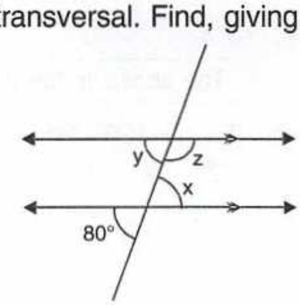
25.5 WHEN TWO PARALLEL LINES ARE CUT BY A TRANSVERSAL



1. *Exterior alternate angles are equal,*
   i.e. $\angle 1 = \angle 7$ and $\angle 2 = \angle 8$
2. *Interior alternate angles are equal,*
   i.e. $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$
3. *Corresponding angles are equal,*
   i.e. $\angle 1 = \angle 5$ and $\angle 2 = \angle 6$
   $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$
4. *Co-interior* (conjoined or allied) *angles are supplementary,*
   i.e. $\angle 4 + \angle 5 = 180^\circ$ and $\angle 3 + \angle 6 = 180^\circ$
5. *Exterior allied angles are supplementary,*
   i.e. $\angle 2 + \angle 7 = 180^\circ$ and $\angle 1 + \angle 8 = 180^\circ$

Example 4 :

In the figure given alongside, two parallel lines are cut by a transversal. Find, giving reasons, the values of the angles x, y and z.



Solution :

$\angle x = 80^\circ$ [Vertically opposite angles]

$\angle y = \angle x$ [Alternate angles]

$= 80^\circ$

$\angle x + \angle z = 180^\circ$ [Co-interior angles are supplementary]

$80^\circ + \angle z = 180^\circ$

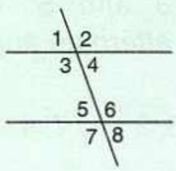
$\Rightarrow \quad \angle z = 180^\circ - 80^\circ = 100^\circ$

$\therefore \quad$ **$\angle x = 80^\circ$, $\angle y = 80^\circ$ and $\angle z = 100^\circ$** **(Ans.)**

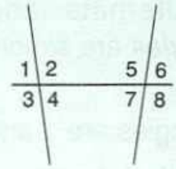
EXERCISE 25(B)

1. Identify the pair of angles in each of the figures given below : adjacent angles, vertically opposite angles, interior alternate angles, corresponding angles or exterior alternate angles.

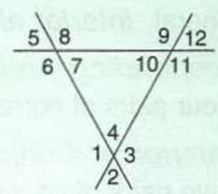
(a) (i) $\angle 2$ and $\angle 4$
(ii) $\angle 1$ and $\angle 8$
(iii) $\angle 4$ and $\angle 5$
(iv) $\angle 1$ and $\angle 5$
(v) $\angle 3$ and $\angle 5$



(b) (i) $\angle 2$ and $\angle 7$
(ii) $\angle 4$ and $\angle 8$
(iii) $\angle 1$ and $\angle 8$
(iv) $\angle 1$ and $\angle 5$
(v) $\angle 4$ and $\angle 7$

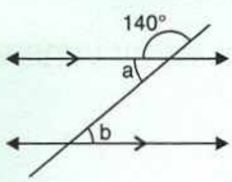


(c) (i) $\angle 1$ and $\angle 10$
(ii) $\angle 6$ and $\angle 12$
(iii) $\angle 8$ and $\angle 10$
(iv) $\angle 4$ and $\angle 11$
(v) $\angle 2$ and $\angle 8$
(vi) $\angle 5$ and $\angle 7$

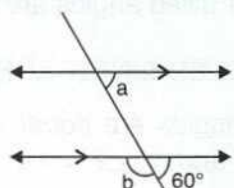


2. Each figure given below shows a pair of parallel lines cut by a transversal. For each case, find *a* and *b*, giving reasons.

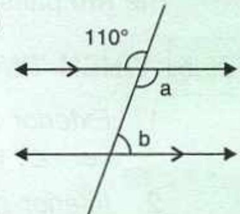
(i)



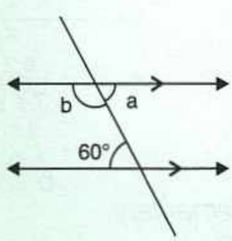
(ii)



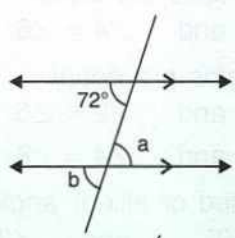
(iii)



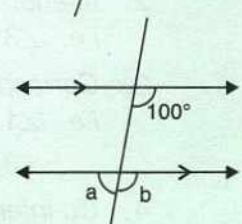
(iv)



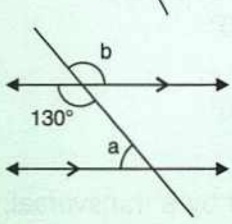
(v)



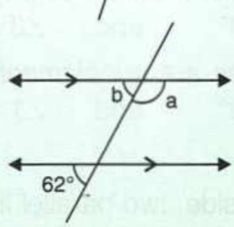
(vi)



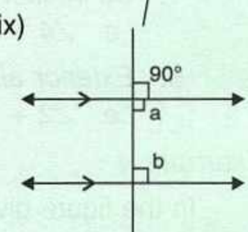
(vii)



(viii)

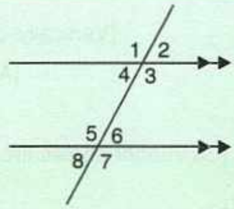


(ix)

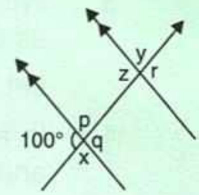


The arrows in the diagrams given above, indicate that the lines are parallel.

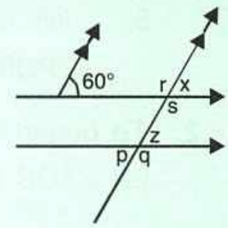
3. If $\angle 1 = 120^\circ$, find the measures of : $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$, $\angle 7$ and $\angle 8$. Give reasons.



4. In the figure given alongside, find the measure of the angles denoted by x, y, z, p, q and r.



5. Using the figure given alongside, fill in the blanks :

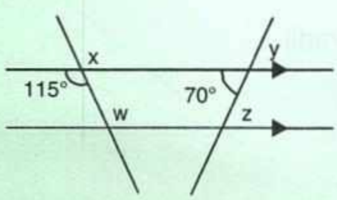


∠x = ...; ∠z = ...;

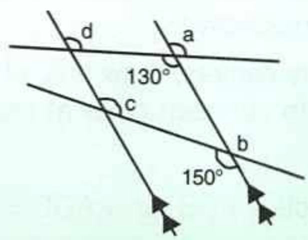
∠p = ...; ∠q = ...;

∠r = ...; ∠s = ...;

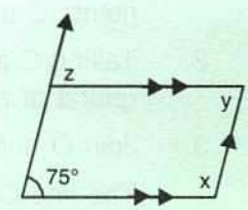
6. In the figure given alongside, find the angles shown by x, y, z and w. give reasons.



7. Find a, b, c and d in the figure given below :



8. Find x, y and z in the figure given below :



CONSTRUCTION OF ANGLES (Using ruler and compass)

25.6 RULER AND COMPASS CAN BE USED :

(i) To copy a given angle.

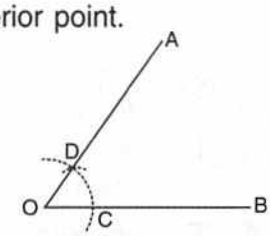
(ii) To bisect a given angle.

(iii) To construct certain angles from a given point.

(iv) To bisect a given line segment by drawing its perpendicular bisector.

(v) To drop a perpendicular on to a line from a given exterior point.

(vi) To draw a perpendicular at a point on a given line.



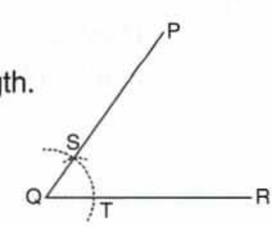
1. *Copying a given angle*

[To draw an angle equal to the given angle].

Let ∠AOB be the given angle of certain size, that we have to copy at a given point Q.

Steps :

1. At point Q, draw line segment QR of any suitable length.
2. With O as centre, draw an arc of any suitable radius, to cut the arms of the angle at C and D.



3. With Q as centre, draw an arc of the same radius as drawn for C and D.

   Let this arc cuts the line, segment QR at point T.

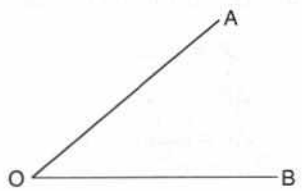
4. In a pair of compasses, take the distance equal to the distance between C and D, and then, with T as centre, draw an arc which cuts the first arc at point S.

5. Join QS and extend up to a suitable point P.

   ∠PQR so obtained is equal to the given ∠AOB.

2. *To bisect a given angle.*

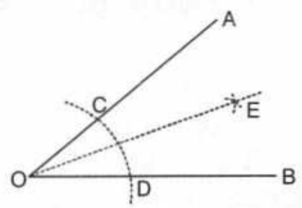
Let ∠AOB be the angle to be bisected.



Steps :

1. With O as centre, draw an arc of any suitable measure that cuts the two arms AO and BO at points C and D, respectively.
2. Taking C and D as centres, draw arcs of equal radii (plural of radius) to cut each other at point E.
3. Join O and E.

   The line OE bisects ∠AOB *i.e.* ∠AOE = ∠BOE



> The radius of each arc in step 2 must be more than half the distance between C and D.

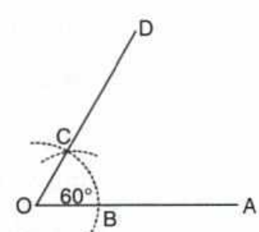
25.7 CONSTRUCTION OF PARTICULAR ANGLES :

Such as : 60°, 30°, 90°, 45°, 120°, 135°, 75°, 105°, 15°, 165°, etc.

3. *To construct an angle of 60°.*

Steps :

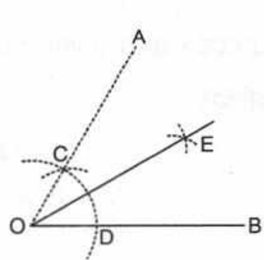
1. Draw a line segment OA of any suitable length.
2. With O as centre, draw an arc of any suitable radius that cuts OA at point B.
3. With B as centre, draw an arc of same size to cut the first arc at point C.
4. Join OC and extend upto a suitable point D.
   Then, **∠DOA = 60°**.



4. *To construct an angle of 30°.*

Steps :

1. Draw an angle AOB of 60°, as explained above.
2. Bisect this angle to get two angles of 30° each.
   Thus, **∠EOB = 30°**.

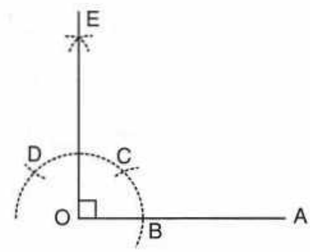


5. *To construct an angle of 90°.*

1st method :

Let OA be the line segment on which an angle of 90° is to be constructed at point O.

Steps :

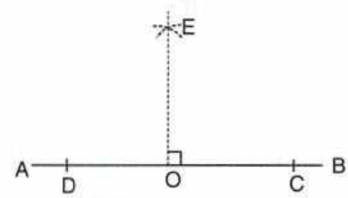


1. With O as centre, draw an arc of a suitable radius that cuts OA at point B.
2. With B as centre, draw an arc (with the same radius, as taken in step 1) that cuts the first arc at point C.
3. Again, with C as centre and with the same radius, draw one more arc so that it cuts the first arc at point D.
4. With C and D as centres, draw two arcs of equal radii so that they intersect at point E.
5. Join O and E. Then, **∠AOE = 90°.**

2nd method :

Let AB be the line segment and O the point where an angle of 90° is to be drawn.

Steps :



1. With O as centre, draw two arcs (both of the same radii) to cut AB at points C and D.
2. With C and D as centres, draw two more arcs of equal radii so that they intersect at point E.
3. Join points O and E. Then, **∠AOE = 90°** and **∠BOE = 90°**.

> In both the constructions discussed above, **OE** is said to be **perpendicular** to line segment OA at point O.
>
> Sign of perpendicular is ⊥. For example AB is perpendicular to CD is expressed by writing AB ⊥ CD.

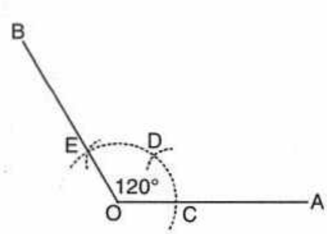
6. *To construct an angle of 45°.*

Draw an angle of 90° and bisect it. Each angle so obtained will be 45°.

7. *To construct an angle of 120°.*

Steps :

1. With centre O on the line segment OA, draw an arc to cut OA at point C.
2. With C as centre, draw one more arc with the same radius so that it cuts the first arc at point D.
3. With D as centre, draw one more arc of the same radius so that it cuts the first arc at E.
4. Join OE and extend it up to a suitable point B.
   Then, **∠AOB = 120°.**



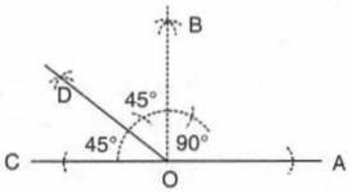
8. ***To construct an angle of 135°.***

Steps :

1. Draw an angle BOA = 90° at point O of the given line segment AC.
2. Bisect the angle BOC (clearly, angle BOC is also 90°).

   Thus, ∠BOD = ∠COD = 45°

   And, **∠AOD** = 90° + 45° = **135°.**



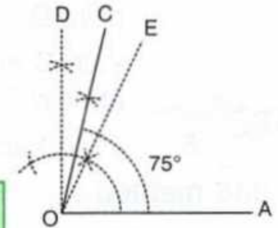
9. ***To construct an angle of 75°.***

Steps :

1. Draw an angle AOD = 90° at point O of the line segment OA.
2. At the same point O, draw angle AOE = 60°.
3. Bisect ∠DOE so that

   ∠EOC = ∠DOC = 15°

   Thus, **∠AOC** = ∠AOE + ∠EOC = 60° + 15° = **75°.**



Many more angles can be drawn with such combinations.
e.g. : (i) 105° = 90° + 15°
(ii) 150° = 90° + 60° or 150 ° = 120° + 30° and so on.

25.8 PERPENDICULARS :

10. ***To draw the perpendicular bisector of a given line segment.***

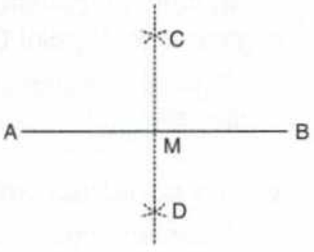
Let AB be the given line segment.

Steps :

1. With A and B as centres, draw arcs of equal radii on both the sides of AB. *The radii of these arcs must be more than half the length of AB.*
2. Let these arcs cut each other at points C and D.
3. Join CD, which cuts AB at M.

   Then, AM = BM. And ∠AMC = 90°

   Thus, the line segment CD is the perpendicular bisector of AB as it bisects AB at M and is also perpendicular to AB.



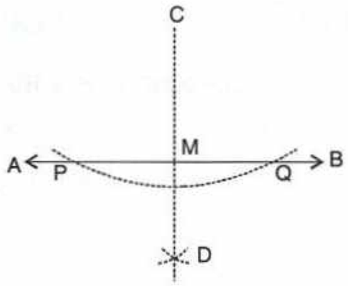
11. ***To draw a perpendicular on to a given line from a given point outside the line.***

Let AB be the given line and C the given point lying outside the line AB.

Steps :

1. Taking C as centre, draw an arc of a suitable radius, it cuts AB at the two points P and Q.

2. With P and Q as centres, draw two arcs of equal radii intersecting at point D on the other side of AB.
3. Join C and D. Let CD cuts line AB at point M. **CM is the required perpendicular on to the given line AB from the exterior point C.**



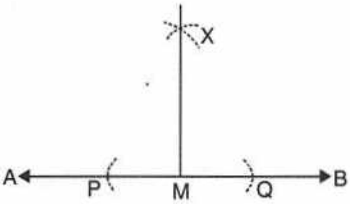
12. *To draw a perpendicular on to a line through a given point on the given line.*

Let AB be the given line and let M be a point on the line AB.

Steps :

1. Taking M as centre, draw two arcs of the same radii. Let these arcs cut AB at points P and Q.
2. Now taking P and Q as centres, draw arcs of equal radii intersecting at point X.
3. Join M and X.

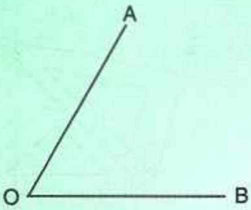
MX is the required perpendicular on to the line AB through point M on it.



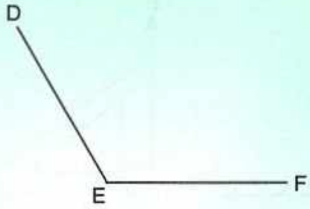
EXERCISE 25(C)

1. Draw in your note-book the following angles using a ruler and a compass only.

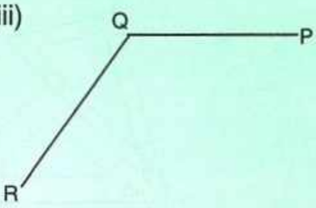
(i)



(ii)



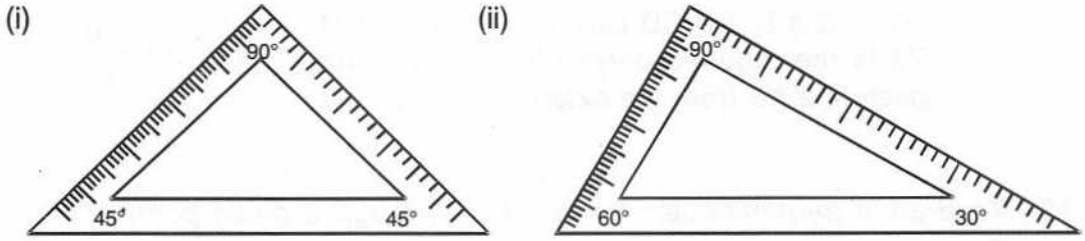
(iii)



2. Construct the following angles, using a ruler and a compass only.
   (i) 60° (ii) 90° (iii) 45° (iv) 30° (v) 120° (vi) 135°
3. Draw line AB = 6 cm. Construct angle ABC = 60°. Then draw the bisector of angle ABC.
4. Draw a line segment PQ = 8 cm. Construct the perpendicular bisector of the line segment PQ. Let the perpendicular bisector drawn meets PQ at point R. Measure the lengths of PR and QR. Is PR = QR ?
5. Draw a line segment AB = 7 cm. Mark a point P on AB such that AP = 3 cm. Draw perpendicular on to AB at point P.
6. Draw a line segment AB = 6·5 cm. Locate a point P that is 5 cm from A and 4·6 cm from B. Through the point P, draw a perpendicular on to the line segment AB.

25.9 USING SET-SQUARES :

A set-square is a triangular piece of plastic or metal.



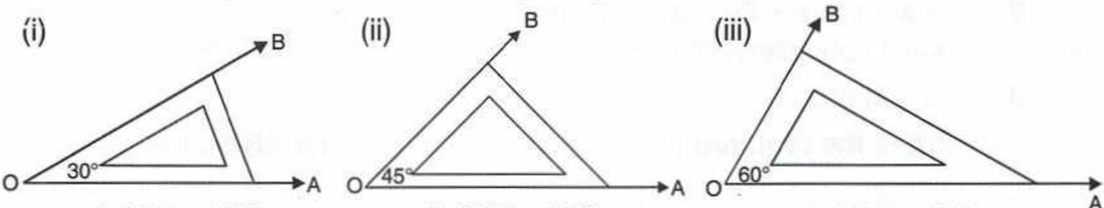
In general, set-squares are of two types :

(i) a set-square with angles 45°, 90° and 45°.

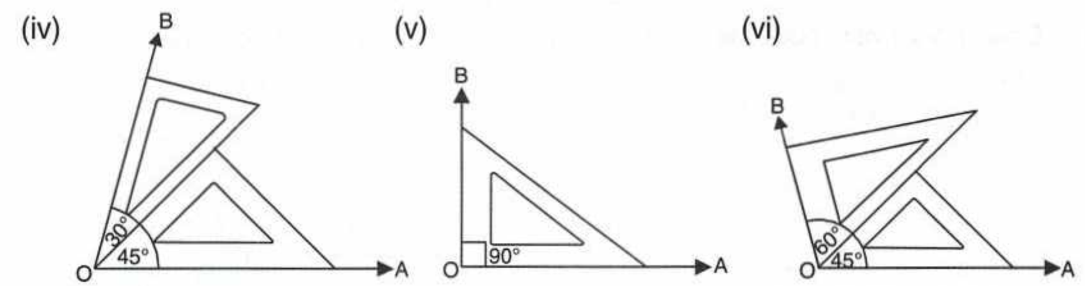
(ii) a set-square with angles 60°, 90° and 30°.

1. *To draw the angles of 30°, 45°, 60°, 75°, 90° and 105° using a set-square.*

The lines drawn along the sides of a set-square give the angles mentioned above.

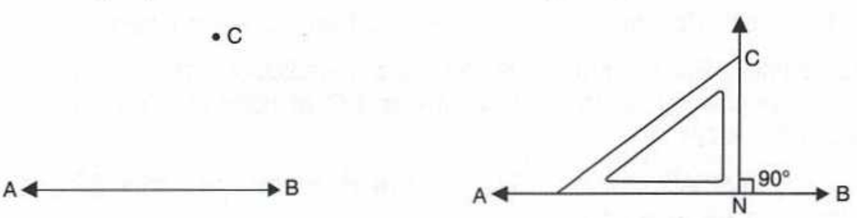


[∠AOB = 30°] [∠AOB = 45°] [∠AOB = 60°]



[∠AOB = 75°] [∠AOB = 90°] [∠AOB = 105°]

2. *To draw a perpendicular on to a line through a point outside the line.*

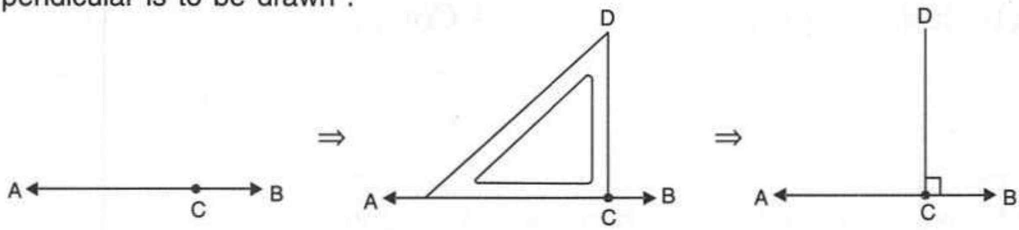


Let AB be the given line and C the given point outside the line AB. Place the suitable set-square in such a way that one of its edges, *i.e.* the one containing angle 90°, coincides with line AB and the other edge with point C (see the figure given above). Then, through point C, draw a line along the edge of the set-square such that it meets the given line AB at point N.

∴ CN is the required perpendicular on to AB through the external point C.

3. *To construct a perpendicular on to a line at a point on the line.*

Let the given line be AB; the given point on AB is C, through which the perpendicular is to be drawn :



Place the suitable set-square at point C in such a way that one edge of it, *i.e.* the one containing 90°, coincides with line AB. Now, through C, draw a line segment CD along the other edge of the set-square containing 90°.

∴ **CD is the required perpendicular on to AB through point C on the line AB.**

EXERCISE 25(D)

1. Draw a line segment OA = 5 cm. Use set-squares to construct angle AOB = 60° such that OB = 3 cm. Join A and B; then measure the length of AB.
2. Draw a line segment OP = 8 cm. Use set-squares to construct $\angle POQ = 90°$ such that OQ = 6 cm. Join P and Q; then measure the length of PQ.
3. Draw $\angle ABC = 120°$. Bisect the angle using ruler and compass. Measure each angle so obtained and check whether or not the new angles obtained on bisecting $\angle ABC$ are equal.
4. Draw $\angle PQR = 75°$ by using set-squares. On PQ mark a point M such that MQ = 3 cm. On QR mark a point N such that QN = 4 cm. Join M and N. Measure the length of MN.

TRIANGLES

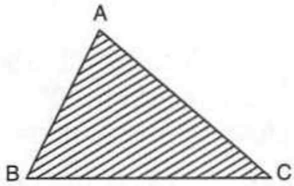
(Including Types, Properties and Constructions)

26

26.1 TRIANGLE :

A triangle is a plane closed figure bounded by three line segments.

In the adjoining figure, the line segments AB, BC and CA form the triangle ABC.



The three line segments *AB, BC and CA are the sides of the triangle ABC.*

A triangle is denoted by the Greek letter Δ (delta).

Thus, triangle ABC can be written as Δ ABC.

26.2 VERTEX :

Vertex of a triangle is a point where any two of its sides meet.

In the figure given above, the sides AB and AC meet at point A.

∴ A is a vertex of Δ ABC.

Similarly, vertex B = the point where the sides BC and AB meet.

And, vertex C = the point where the sides AC and BC meet.

The plural of vertex is vertices.

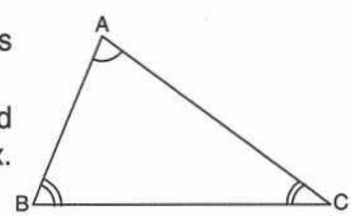
Thus, A, B and C are the three vertices of the triangle ABC.

BC is the side opposite to vertex A and A is the vertex opposite to side BC. The same is true for vertex B and side AC, and for vertex C and side AB.

26.3 ANGLES (INTERIOR ANGLES) OF A TRIANGLE :

Every triangle has three angles.

In the triangle ABC drawn alongside, the three angles (interior angles) are : ∠BAC, ∠ABC and ∠ACB.



(i) An interior angle of a triangle can also be denoted by the letter representing the corresponding vertex. Consider ∠ABC, since it is formed at vertex B, it can be written as ∠B.

Thus, ∠ABC = ∠B, ∠BCA = ∠C and ∠BAC = ∠A, all are interior angles of the triangle ABC.

(ii) The sum of the interior angles of a triangle is always 180°, *i.e.* two right angles.

∴ In Δ ABC, $\angle A + \angle B + \angle C = 180°$,

and in Δ PQR, $\angle P + \angle Q + \angle R = 180°$ and so on.

1. Each triangle has three sides, three vertices and three angles (interior angles).
2. Δ ABC can also be written as Δ BAC or Δ BCA or Δ CAB, or Δ ACB or Δ CBA *i.e.* the three letters representing a triangle can be written in any order.

26.4 EXTERIOR ANGLE OF A TRIANGLE :

When any side of a triangle is extended, the angle formed outside the triangle is called an ***exterior angle****.*

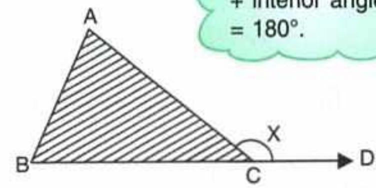
26.5 SOME IMPORTANT RESULTS :

1. *An exterior angle of a triangle is an adjacent and supplementary angle to the corresponding interior angle of the triangle.*

At each vertex, exterior angle + interior angle = 180°.

For example :

In the figure given alongside, the side BC of Δ ABC is extended up to point D, thus forming an exterior angle ACD. The exterior angle ACD is adjacent and supplementary to the corresponding interior ∠ACB of the Δ ABC *i.e.* ∠ACD + ∠ACB = 180°.



2. *An exterior angle of a triangle is always equal to the sum of its two opposite interior angles.*

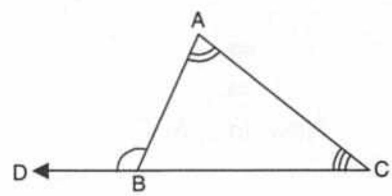
For example :

In the given figure, exterior angle ABD is formed by extending the side CB of the triangle ABC.

∴ Exterior angle ABD

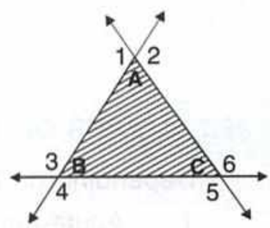
= Sum of interior opposite angles A and C,

i.e. ∠ABD = ∠A + ∠C.



3. *On extending the sides of a triangle, six exterior angles are formed, two at each vertex.*

The adjoining figure shows the six exterior angles formed by extending the sides of the triangle ABC.



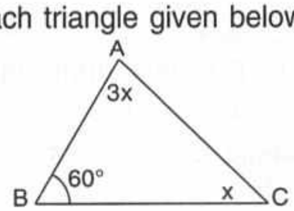
26.6 CONSIDER THE FOLLOWING TABLE :

Exterior Angle	Adjacent Interior Angle	Interior opposite Angles	Relation between an exterior angle and its adjacent interior angle	Relation between an exterior angle and the interior opposite angles
∠1	∠A	∠B and ∠C	∠1 + ∠A = 180°	∠1 = ∠B + ∠C
∠2	∠A	∠B and ∠C	∠2 + ∠A = 180°	∠2 = ∠B + ∠C
∠3	∠B	∠A and ∠C	∠3 + ∠B = 180°	∠3 = ∠A + ∠C
∠4	∠B	∠A and ∠C	∠4 + ∠B = 180°	∠4 = ∠A + ∠C
∠5	∠C	∠A and ∠B	∠5 + ∠C = 180°	∠5 = ∠A + ∠B
∠6	∠C	∠A and ∠B	∠6 + ∠C = 180°	∠6 = ∠A + ∠B

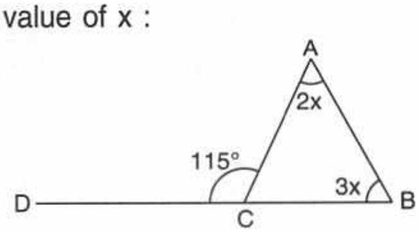
Example 1 :

For each triangle given below, find the value of x :

(i)



(ii)



Solution :

(i) $\angle A + \angle B + \angle C = 180°$

$\Rightarrow 3x + 60° + x = 180°$

$4x = 180° - 60°$

$4x = 120°$

$\therefore \quad \mathbf{x} = \dfrac{120°}{4} = \mathbf{30°}$ **(Ans.)**

Sum of the angles of a Δ is 180°

(ii) $\angle ACD = \angle A + \angle B$

$\Rightarrow 115° = 2x + 3x$

$5x = 115°$

$\therefore \quad \mathbf{x} = \dfrac{115°}{5} = \mathbf{23°}$ **(Ans.)**

Exterior angle of a Δ = sum of interior opposite angles.

Alternative method :

$\angle ACD + \angle ACB = 180°$

$\Rightarrow 115° + \angle ACB = 180°$

$\Rightarrow \angle ACB = 180° - 115° = 65°$

At each vertex, exterior angle + interior angle = 180°

Now, in Δ ABC

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 2x + 3x + 65° = 180°$

$5x = 180° - 65° = 115° \quad \therefore \mathbf{x} = \dfrac{115°}{5} = \mathbf{23°}$ **(Ans.)**

26.7 TYPES OF TRIANGLES ACCORDING TO ANGLES :

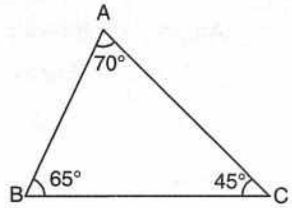
Depending on the sizes of its angles, a triangle can be classified as :

1. Acute-angled triangle
2. Right-angled triangle
3. Obtuse-angled triangle.

1. Acute-angled triangle :

*If each angle of a triangle is acute (less than 90°), it is called an **acute-angled triangle**.*

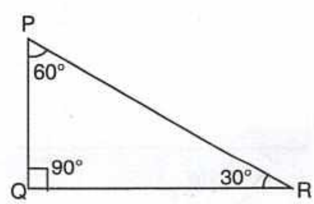
The adjoining figure shows an acute-angled triangle, each of its angles is less than 90°.



2. Right-angled triangle :

*If one of the angles of a triangle is a right angle i.e. 90°, it is called a **right-angled triangle**.*

The figure given alongside shows a right angled triangle PQR, as $\angle PQR = 90°$.

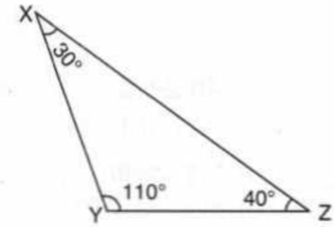


Sum of the two acute angles of a right angled triangle is always 90°, *i.e.* $\angle P + \angle R = 90°$.

In a right-angled triangle, the side opposite to the right angle is called the ***hypotenuse.*** Hypotenuse is the largest side of a right angled triangle.

In the given Δ PQR, side PR is opposite to angle Q, which is a right angle. Therefore, PR is the hypotenuse. Also, PR > PQ and PR > QR.

3. Obtuse-angled triangle :



*If an angle of a triangle is obtuse (more than 90°), the triangle is called an **obtuse-angled triangle**.*

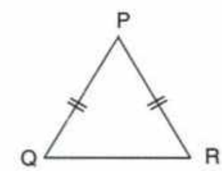
In the adjoining figure, Δ XYZ is an obtuse-angled triangle, as ∠XYZ = 110°, *i.e.* ∠XYZ is an obtuse angle.

26.8 TYPES OF TRIANGLES ACCORDING TO SIDES :

On the basis of sides, a triangle can be classified as :

1. Isosceles triangle 2. Equilateral triangle 3. Scalene triangle.

1. Isosceles triangle :

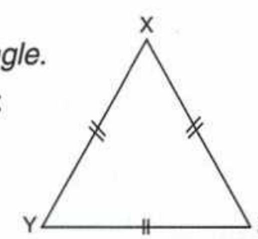


A triangle, with at least two sides equal is called an isosceles triangle.

In the given figure, PQR is an isosceles triangle, as PQ = PR.

> *In an isosceles triangle, the angles opposite to the equal sides are equal.*
> *Thus, PQ = PR ⇒ Angle opposite to PQ = Angle opposite to PR,* i.e. ∠R = ∠Q.

2. Equilateral triangle :



A triangle, with all its sides equal is called an equilateral triangle.

In the figure given alongside, XYZ is an equilateral triangle as :

side XY = side YZ = side XZ.

In an equilateral triangle, all angles are equal, *i.e.* ∠XYZ = ∠YXZ = ∠XZY.

Since, the sum of all the three interior angles of every triangle is 180°

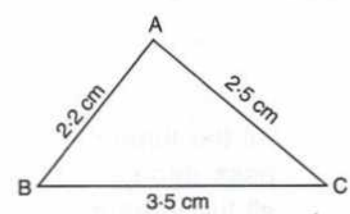
therefore each interior angle of an equilateral triangle = $\frac{180°}{3}$ = 60°.

Thus, in an equilateral Δ XYZ :

(i) XY = YZ = ZX and (ii) ∠XYZ = ∠YXZ = ∠XZY = 60°.

> Every equilateral triangle is isosceles, but the converse is not always true.

3. Scalene triangle :

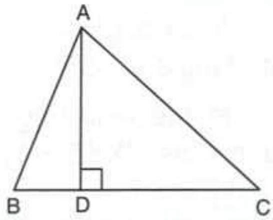


If the three sides of a triangle are unequal, *i.e.* if the sides are of different lengths, the triangle is called a *scalene triangle.*

In a scalene triangle all the angles are of different sizes. That is, in Δ ABC, ∠D ≠ ∠B ≠ ∠C.

26.9 ALTITUDE AND MEDIAN OF A TRIANGLE

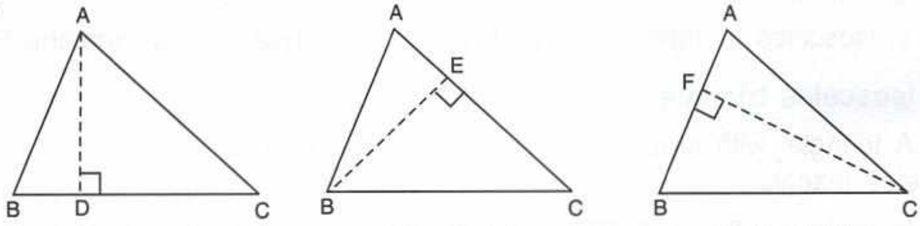
1. *An altitude of a triangle is the perpendicular from a vertex to the opposite side.*



In the given figure, AD is perpendicular to side BC of triangle ABC.

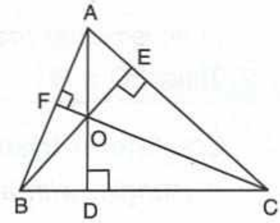
⇒ AD is an altitude of triangle ABC

In fact, every triangle has three vertices and three sides, so three altitudes. Ses below :



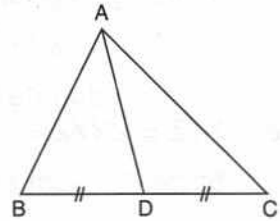
(AD is altitude) (BE is altitude) (CF is altitude)

All the three altitudes of a triangle are concurrent *i.e.* they pass through the same point. The point of intersection of all the three altitudes is denoted by letter O, where the altitudes AD, BE and CF meet.



The point O, where all the three altitudes of the triangle ABC meet, is called **orthocentre** of the triangle ABC.

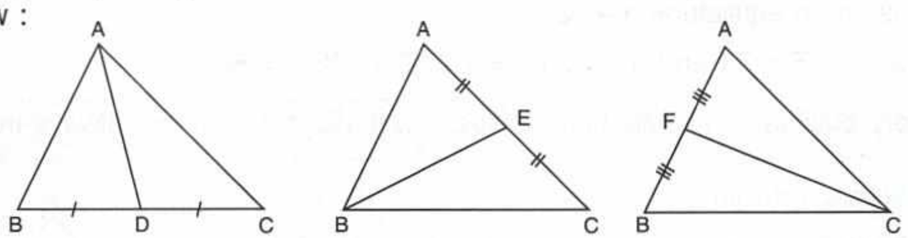
2. *Median : A median of a triangle is the line joining a vertex of the triangle with the mid-point of the opposite side.*



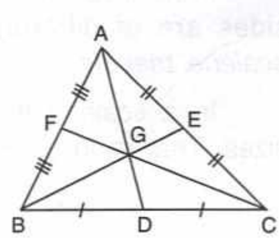
In the given figure, line AD joins vertex A of triangle ABC and the mid-point D of side BC.

⇒ AD is a median of triangle ABC.

In fact, every triangle has three vertices and three sides, so three medians. See below :



All the three medians of a triangle are concurrent *i.e.* they pass through the same point. The point of intersection of all the three medians is denoted by the letter G, where the medians AD, BE and CF meet.

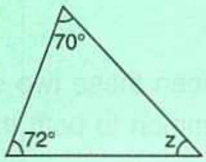


The point G, where all the three medians of the triangle ABC meet, is called the **centroid** of the triangle ABC.

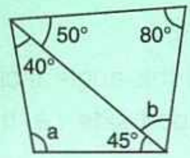
EXERCISE 26(A)

1. In each of the following, find the marked unknown angles :

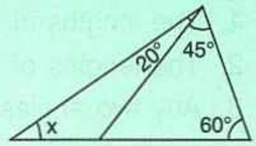
(i)



(ii)



(iii)



2. Can a triangle together have the following angles ?

(i) 55°, 55° and 80° (ii) 33°, 74° and 73° (iii) 85°, 95° and 22°

> A triangle can together have the given angles if the sum of these angles is 180°.

3. Find x, if the angles of a triangle are :

(i) x°, x°, x° (ii) x°, 2x°, 2x° (iii) 2x°, 4x°, 6x°

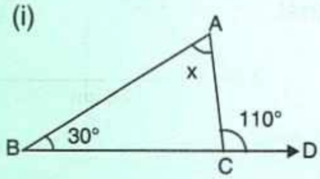
4. One angle of a right-angled triangle is 70°. Find the other acute angle.

5. In Δ ABC, $\angle A = \angle B = 62°$; find $\angle C$.

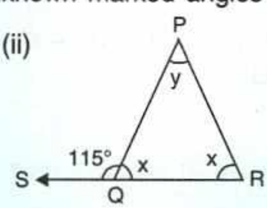
6. In Δ ABC, $\angle B = \angle C$ and $\angle A = 100°$; find $\angle B$.

7. Find, giving reasons, the unknown marked angles in each triangle drawn below :

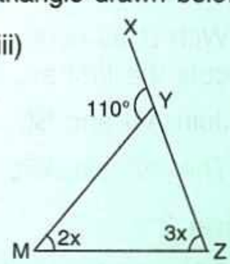
(i)



(ii)

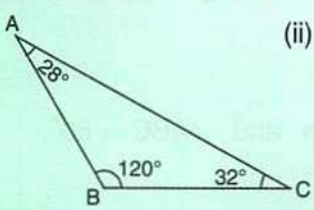


(iii)

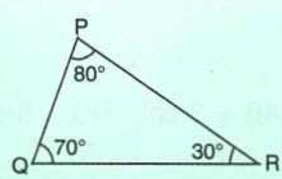


8. Classify the following triangles according to angle :

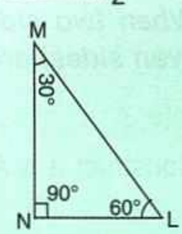
(i)



(ii)

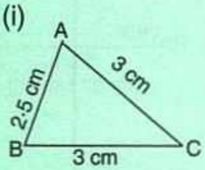


(iii)

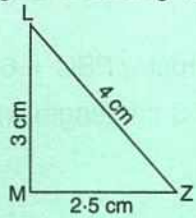


9. Classify the following triangles according to side :

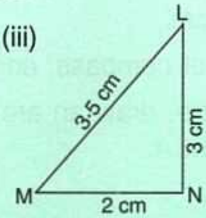
(i)



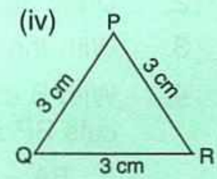
(ii)



(iii)



(iv)



26.9 CONSTRUCTION OF TRIANGLES :

Here, we shall be constructing a triangle when any one of the following three conditions is given :

1. The lengths of the three sides.
2. The lengths of two sides and the angle included between these two sides.
3. Any two angles and the included side *i.e.* the side common to both the angles.

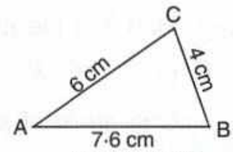
Construction 1 :

When the lengths of three sides are given.

Example 2 :

Construct a Δ ABC such that CB = 4 cm, AC = 6 cm and AB = 7·6 cm

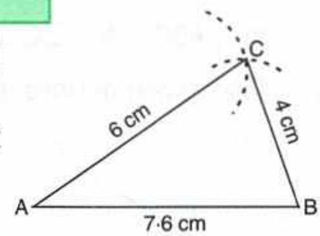
Steps :



[Rough sketch]

1. Draw a rough sketch of the triangle, as shown alongside.
2. Draw one of the sides, say AB = 7·6 cm.

> *We can draw any side first, but usually we start with the longest side.*



3. Using compass and taking A as centre, draw an arc of radius 6 cm.
4. With B as centre, draw an arc of radius 4 cm, that cuts the first arc at point C.
5. Join AC and BC.

The triangle ABC so obtained is the required triangle.

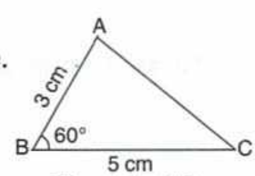
Construction 2 :

When two sides and the included angle (i.e. the angle formed between the two given sides) are given.

Example 3 :

Construct a Δ ABC given AB = 3 cm, BC = 5 cm and $\angle$ABC = 60°.

Steps :

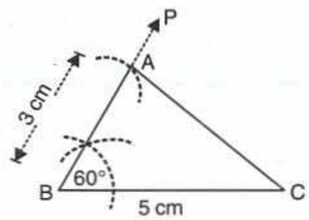


[Rough sketch]

1. Draw a rough sketch of the triangle as shown alongside.
2. Draw BC = 5 cm.
3. With the help of compass, construct $\angle$PBC = 60°.
4. With B as centre, draw an arc of 3 cm length which cuts BP at point A.

   $\therefore$ BA = 3 cm
5. Join A and C.

   Clearly, the triangle ABC so obtained is the required triangle.

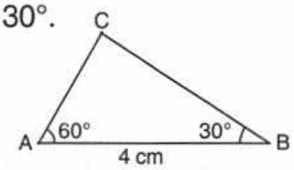


Construction 3 :

When two angles and included side are given.

Example 4 :

Construct Δ ABC when AB = 4 cm, $\angle A = 60°$ and $\angle B = 30°$.

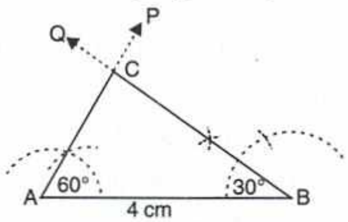


[Rough sketch]

Steps :

1. Draw AB = 4 cm.
2. At A, draw AP making an angle of 60° with AB, *i.e.* draw angle PAB = 60°.
3. At B, draw BQ making an angle of 30° with AB, *i.e.* $\angle QBA = 30°$.
4. AP and BQ intersect each other at point C.

The triangle ABC so obtained is the required triangle.



EXERCISE 26(B)

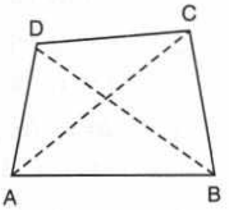
Construct triangle ABC when :

1. AB = 6 cm, BC = 8 cm and AC = 4 cm.
2. AB = 3·5 cm, AC = 4·8 cm and BC = 5·2 cm.
3. AB = BC = 5 cm and AC = 3 cm. Measure angles A and C. Is $\angle A = \angle C$?
4. AB = BC = CA = 4·5 cm. Measure all the angles of the triangle. Are they equal ?
5. AB = 3 cm, BC = 7 cm and $\angle B = 90°$.
6. AC = 4·5 cm, BC = 6 cm and $\angle C = 60°$.
7. AC = 6 cm, $\angle A = 60°$ and $\angle C = 45°$. Measure AB and BC.
8. AB = 5·4 cm, $\angle A = 30°$ and $\angle B = 90°$. Measure $\angle C$ and side BC.
9. AB = 7 cm, $\angle B = 120°$ and $\angle A = 30°$. Measure AC and BC.
10. BC = 3 cm, AC = 4 cm and AB = 5 cm. Measure angle ACB. Give a special name to this triangle.

QUADRILATERAL 27

27.1 INTRODUCTION

A quadrilateral is a four sided closed figure.



The adjoining figure shows a quadrilaterial ABCD. Clearly, the quadrilaterial ABCD is a plane and closed figure, which has :

(i) **four sides :** AB, BC, CD and DA

(ii) **four vertices :** vertex A, vertex B, vertex C and vertex D

(iii) **four angles :** $\angle$ABC, $\angle$BCD, $\angle$CDA and $\angle$DAB

(iv) **two diagonals :** AC and BD

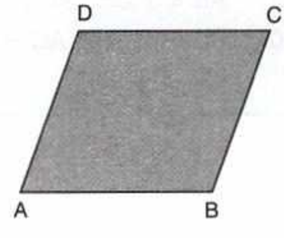
(v) **four pairs of adjacent sides :** AB and BC; BC and CD; CD and DA; DA and AB

(vi) **two pairs of opposite sides :** AB and DC; BC and AD.

If **each angle** of a quadrilateral **is less than 180°**; it is called a **convex quadrilateral**. And, **if** at **least one angle** of a quadrilateral **is more than 180°**, it is called a **concave quadrilateral**.

In this chapter, we will be studying about convex quadrilaterals only.

27.2 INTERIOR AND EXTERIOR OF A QUADRILATERAL



The adjoining figure shows a quadrilateral ABCD whose inside is shaded.

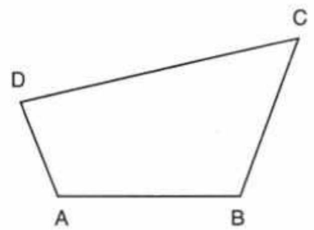
The shaded portion of the figure is called inside of the quadrilateral ABCD.

And, the part of the plane which is outside the quadrilateral ABCD is called exterior of quadrilateral ABCD.

An important property

The sum of the interior angles of a quadrilateral is 360° (4-right angles).

Thus, for quadrilateral ABCD,



$\angle ABC + \angle BCD + \angle CDA + \angle DAB = 360°$

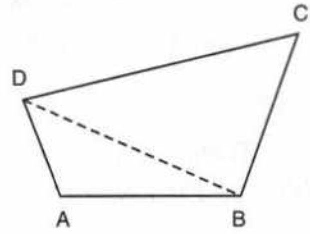
i.e. $\angle B + \angle C + \angle D + \angle A = 360°$

$\Rightarrow$ $\angle A + \angle B + \angle C + \angle D = 360°$

Explanation

Draw any of the two diagonals of the quadrilateral ABCD.

In the adjoining figure, diagonal BD is drawn which divides quadrilateral ABCD into two triangles.



Since, the sum of angles of a triangle = $180°$

$\Rightarrow$ The sum of angles of 2-triangles = $2 \times 180° = 360°$

$\Rightarrow$ **Sum of the angles of the quadrilateral = $360°$**

Example 1 :

If three angles of a quadrilateral are $84°$, $100°$ and $93°$. Find its fourth angle.

Solution :

Let the fourth angle of the quadrilateral be x

$\because$ Sum of the angles of the quadrilateral = $360°$

$\Rightarrow 84° + 100° + 93° + x = 360°$

$\Rightarrow x = 360° - 277° = 83°$

$\therefore$ **The fourth angle = $83°$** **(Ans.)**

Example 2 :

Angles of a quadrilateral are $85°$, $95°$, $x°$ and $(x + 10)°$. Find the value of x.

Solution :

$\because$ Sum of the angles of the quadrilateral = $360°$

$\Rightarrow 85° + 95° + x° + (x + 10)° = 360°$

$\Rightarrow 180° + x° + x° + 10° = 360°$

$\Rightarrow 2x° + 190° = 360°$

i.e. $2x° = 360° - 190° = 170°$

$\therefore x° = \dfrac{170°}{2} = 85°$

$\therefore$ **$x = 85$** **(Ans.)**

Example 3 :

The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Find all its angles.

Solution :

Since, $3 + 4 + 5 + 6 = 18$ and sum of the angles of a quadrilateral is $360°$.

$\therefore$ **First angle** = $\frac{3}{18} \times 360° =$ **$60°$**, **second angle** = $\frac{4}{18} \times 360° =$ **$80°$**,

third angle = $\frac{5}{18} \times 360° =$ **$100°$** and, **fourth angle** = $\frac{6}{18} \times 360° =$ **$120°$** **(Ans.)**

Alternative method :

Let the angles of the quadrilateral be $3x$, $4x$, $5x$ and $6x$.

$\therefore\ 3x + 4x + 5x + 6x = 360^\circ \quad \Rightarrow 18x = 360^\circ$ and $x = 20^\circ$

$\therefore$ **First angle** $= 3x = 3 \times 20^\circ = $ **60°**, **second angle** $= 4x = 4 \times 20^\circ =$ **80°**,

third angle $= 5x = 5 \times 20^\circ =$ **100°** and **fourth angle** $= 6x = 6 \times 20^\circ =$ **120°** **(Ans.)**

Example 4 :

Three angles of a quadrilateral are in the ratio 4 : 6 : 3. If the fourth angle is 100°; find the other three angles of the quadrilateral.

Solution :

Let the three angles be 4x, 6x and 3x

$\therefore\ 4x + 6x + 3x + 100^\circ = 360^\circ$

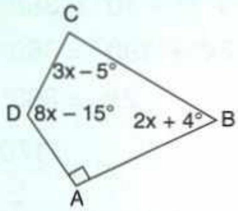
$\Rightarrow \quad 13x = 360^\circ - 100^\circ = 260^\circ$ and, $x = \frac{260^\circ}{13} = 20^\circ$

$\therefore$ **The other three angles** are 4x, 6x and 3x

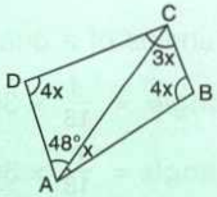
$= 4 \times 20^\circ, 6 \times 20^\circ$ and $3 \times 20^\circ =$ **80°, 120°** and **60°** **(Ans.)**

EXERCISE 27(A)

1. Two angles of a quadrilateral are 89° and 113°. If the other two angles are equal, find the equal angles.
2. Two angles of a quadrilateral are 68° and 76°. If the other two angles are in the ratio 5 : 7, find the measure of each of them.
3. Angles of a quadrilateral are $(4x)^\circ$, $5(x + 2)^\circ$, $(7x - 20)^\circ$ and $6(x + 3)^\circ$. Find :
   (i) the value of x.
   (ii) each angle of the quadrilateral.
4. Use the information given in the following figure to find :
   (i) x.
   (ii) ∠B and ∠C.



5. In quadrilateral ABCD, side AB is parallel to side DC. If ∠A : ∠D = 1 : 2 and ∠C : ∠B = 4 : 5.
   Calculate each angle of the quadrilateral.
6. From the following figure find,
   (i) x
   (ii) ∠ABC
   (iii) ∠ACD



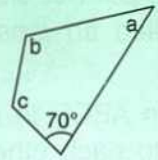
7. In quadrilateral ABCD, ∠C = 64°, ∠D = ∠C – 8°; ∠A = $5(a + 2)^\circ$ and ∠B = $2(2a + 7)^\circ$. Calculate ∠A.

8. In the given figure :

   b = 2a + 15° and

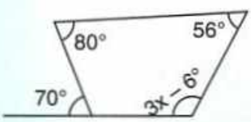
   c = 3a + 5°,

   find the values of b and c.



9. Three angles of a quadrilateral are equal. If the fourth angle is 69°, find the measure of equal angles.
10. In quadrilateral PQRS, ∠P : ∠Q : ∠R : ∠S = 3 : 4 : 6 : 7. Calculate each angle of the quadrilateral and then prove that PQ and SR are parallel to each other.

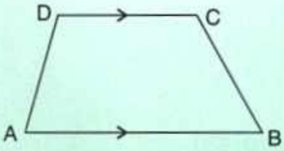
    Is PS also parallel to QR ?
11. Use the information given in the following figure to find the value of x.



12. The following figure shows a quadrilateral in which sides AB and DC are parallel.

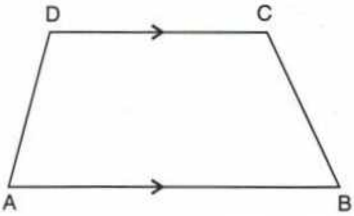
    If ∠A : ∠D = 4 : 5, ∠B = (3x – 15)°

    and ∠C = (4x + 20)°, find each angle of the quadrilateral ABCD.



27.3 TYPES OF QUADRILATERALS

1. **Trapezium :** *A trapezium is a quadrilateral in which one pair of opposite sides are parallel whereas the other pair of opposite sides are non-parallel.*



The given figure shows a quadrilateral ABCD in which opposite sides AB and DC are parallel to each other whereas, the opposite sides AD and BC are not parallel to each other, therefore quadrilateral ABCD is a **trapezium**.

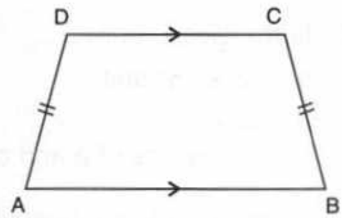
1. As the trapezium ABCD is a quadrilateral, sum of its interior angles is 360°

   i.e. ∠A + ∠B + ∠C + ∠D = 360°.
2. Since, the sides AB and CD of trapezium, given above, are parallel (AB//DC), we get two pairs of co-interior angles, one pair is ∠A and ∠D whereas the other pair is ∠B and ∠C.

   We know that the sum of each pair of co-interior angles is 180°

   ∴ ∠A + ∠D = 180° and ∠B + ∠C = 180°.

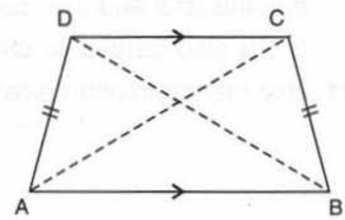
2. **Isosceles trapezium :** If the *non-parallel sides* of a trapezium *are equal*, it is called an *isosceles trapezium*.



The given figure shows a trapezium ABCD in which opposite sides AB and DC are parallel to each other but opposite sides AD and BC are not parallel to each other.

If the non-parallel sides AD and BC are equal in length, the given trapezium is an **isosceles trapezium**.

In case of an isosceles trapezium ABCD (as shown alongside) :



(i) $\angle A = \angle B$ and $\angle C = \angle D$

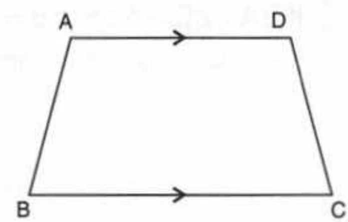
(ii) $\angle A + \angle D = 180°$ and $\angle B + \angle C = 180°$

(iii) $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

(iv) diagonal AC = diagonal BD

Example 5 :

In a trapezium ABCD, side BC is parallel to side AD. If $\angle D = 100°$, find $\angle C$.



Solution :

$\because$ BC//AD and DC is transversal

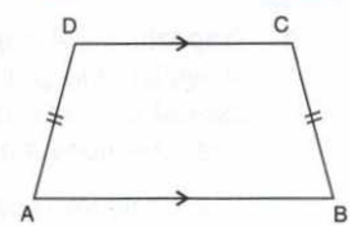
$\therefore$ $\angle D$ and $\angle C$ are co-interior angles with their sum = 180°

i.e. $\angle D + \angle C = 180°$

$\Rightarrow$ $100° + \angle C = 180°$ *i.e.* $\angle C = 180° - 100° = 80°$ **(Ans.)**

Example 6 :

Angle B of isosceles trapezium ABCD is 82°, find angles A, C and D.



Solution :

$\angle A = \angle B = 82°$

$\angle C + \angle B = 180° \Rightarrow \angle C + 82° = 180°$

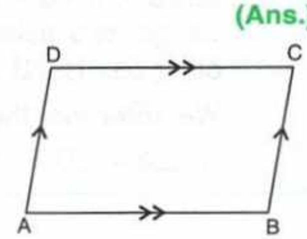
i.e. $\angle C = 180° - 82° = 98°$

Also, $\angle D = \angle C = 98°$

$\therefore$ **$\angle A = 82°$, $\angle C = 98°$ and $\angle D = 98°$** **(Ans.)**

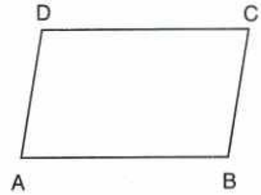
3. ***Parallelogram*** *A parallelogram is a quadrilateral in which both the pairs of opposite sides are parallel.*

In the given quadrilateral ABCD, side AB is parallel to its opposite side DC and side AD is parallel to its opposite side BC.



Therefore, ABCD is a parallelogram.

In case of a parallelogram ABCD (as shown alongside).



(i) *opposite sides are equal :*

i.e. AB = DC and AD = BC

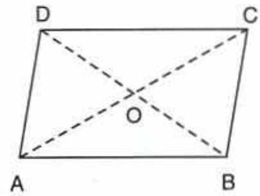
(ii) *opposite angles are euqal :*

i.e. $\angle A = \angle C$ and $\angle B = \angle D$

(iii) *adjacent angles are supplementary*

i.e. $\angle A + \angle B = 180°$, $\angle B + \angle C = 180°$,

$\angle C + \angle D = 180°$ and $\angle D + \angle A = 180°$

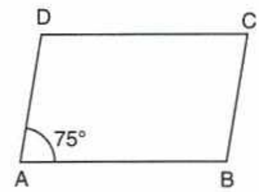


(iv) *diagonals bisect each other.*

i.e. if diagonals AC and BD intersect each other at point O; $OA = OC = \frac{1}{2}AC$ and $OB = OD = \frac{1}{2}BD$

Example 7 :

In parallelogram ABCD, $\angle A = 75°$, find angles B, C and D.



Solution :

$\because$ Adjacent angles of a parallelogram are supplementary, therefore

$$\angle A + \angle B = 180° \quad \Rightarrow \quad 75° + \angle B = 180°$$

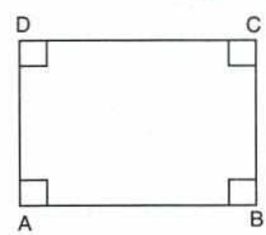
i.e. $\angle B = 180° - 75° = \mathbf{105°}$

$\because$ Opposite angles of a parallelogram are equal, therefore,

$$\angle C = \angle A = 75° \quad \text{and} \quad \angle D = \angle B = 105°$$

$\therefore$ **$\angle B = 105°$, $\angle C = 75°$ and $\angle D = 105°$** **(Ans.)**

4. **Rectangle :** *A rectangle is a quadrilateral in which every angle is 90° (a right-angle).*

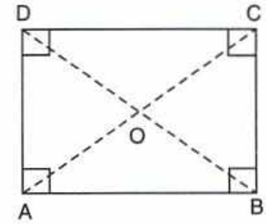


In the given quadrilateral ABCD,

$\angle A = \angle B = \angle C = \angle D = 90°$

$\Rightarrow$ **Quadrilateral ABCD is a rectangle**

In case of rectangle ABCD (as shown alongside).



(i) *Each angle is* 90°

i.e. $\angle A = \angle B = \angle C = \angle D = 90°$

(ii) *Opposite sides are equal*

i.e. AB = DC and AD = BC.

(iii) *Opposite sides are parallel.*

i.e. AB // DC and AD // BC

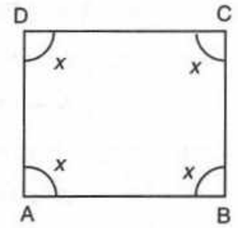
(iv) *Diagonals are equal i.e.* $AC = BD$.

(v) *Diagonals bisect each other*

i.e. $OA = OC = \frac{1}{2}AC$ and $OB = OD = \frac{1}{2}BD$

Example 8 :

The adjoining figure shows a quadrilateral with each angle equal to x. Show that the quadrilateral is a rectangle.



Solution :

The given quadrilateral will be a rectangle, if its each angle is 90°
i.e. if $x = 90°$

Since, the sum of interior angles of a quadrilateral is 360°

$\Rightarrow \quad x + x + x + x = 360°$ *i.e.* $4x = 360°$

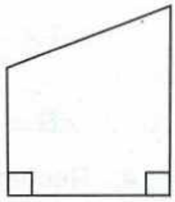
$\Rightarrow \quad x = \frac{360°}{4} = 90°$

$\Rightarrow$ Each angle of the quadrilateral is 90°

$\Rightarrow$ **The given quadrilateral is a rectangle.**

Example 9 :

Two angles of a quadrilateral are 90° each. Is this quadrilateral a rectangle ? Give reason.



Solution :

No, it is not a rectangle.

Reason : Each angle of the quadrilateral is not 90°.

Example 10 :

What are the additional properties which a parallelogram must have to be a rectangle.

Solution :

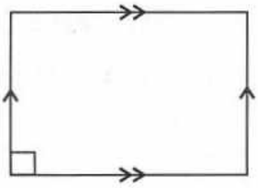
First property : Diagonals must be equal.

Second property : Any angle is 90°.

If any angle of a parallelogram is 90°, its each angle is 90° and so, it is a rectangle.

Example 11 :

Opposite sides of a quadrilateral are parallel and one of its angles is 90°, is it a reactangle ? Give reason.

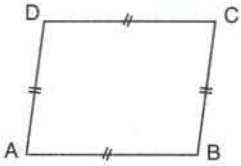


Solution :

Yes, it is a rectangle.

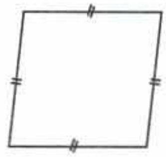
Reason : Opposite sides are parallel means the quadrilateral is a parallelogram and one of the angles of the parallelgram is 90° means the resulting figure is a rectangle.

5. **Rhombus :** *A rhombus is a quadrilateral in which all sides are equal.*

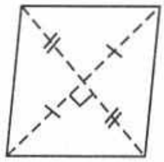


In the adjoining figure, sides AB, BC, CD and DA are equal *i.e.* AB = BC = CD = DA; therefore ABCD is a rhombus.

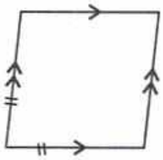
(i) A quadrilateral, with all the sides equal, is called a rhombus.



(ii) A quadrilateral is a rhombus, if its diagonals bisect each other at 90°.



(iii) A parallelogram, with any pair of adjacent sides equal, is a rhombus.



Example 12 :

State two properties each of which makes a given quadrilateral a rhombus.

Solution :

(i) All the sides are equal.

(ii) Diagonals bisect each other at 90°.

Example 13 :

State, two properties each of which makes a given parallelogram a rhombus.

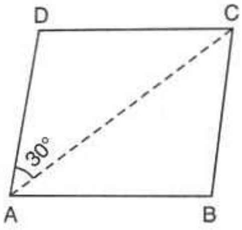
Solution :

(i) Diagonals bisect each other at 90°.

(ii) Any pair of adjacent sides is equal.

Example 14 :

The adjoining figure shows a rhombus ABCD, with a diagonal = AC and angle DAC = 30°. Find all the angles of the rhombus.



Solution :

In a rhombus, diagonals bisect the angles at vertex.

$\therefore \angle CAB = \angle DAC = 30° \Rightarrow \angle DAB = 30° + 30° = 60°$

In a rhombus, opposite angles are equal and adjacent angles are supplementary.

$\therefore \quad \angle BCD = \angle DAB = 60^\circ$

$\angle DAB + \angle ABC = 180^\circ \Rightarrow 60^\circ + \angle ABC = 180^\circ$

i.e. $\angle ABC = 180^\circ - 60 = 120^\circ$

Also, $\angle ADC = \angle ABC = 120^\circ$

$\therefore$ **Required angles are 60°, 120°, 60° and 120°** **(Ans.)**

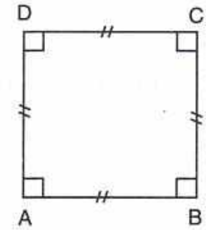
Example 15 :

Is rhombus a rectangle ? Give reason.

Solution :

No, rhombus is not a rectangle. **(Ans.)**

Reason : Since, no angle of a rhombus is 90° and its diagonals are also not equal, the rhombus is not a rectangle.



6. **Square :** *A square is a quadrilateral in which :*
   (i) all the sides are equal and
   (ii) each angle is 90°.

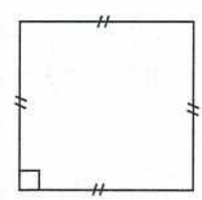
The figure, given alongside, is a square ABCD as :

(i) AB = BC = CD = DA

(ii) $\angle A = \angle B = \angle C = \angle D = 90^\circ$

Also, the diagonals are equal and bisect each other at 90°.

Example 16 :



The perimeter (sum of all the sides) of a square is 24 cm; find its each side. Also, state the value of each interior angle.

Solution :

$\because$ Sum of the four equal sides = 24 cm

$\therefore$ **Each side** $= \dfrac{24 \text{ cm}}{4} =$ **6 cm** **(Ans.)**

Each interior angle is 90°. **(Ans.)**

Example 17 :

If all the angles of a quadrilateral are 90° each, is this quadrilateral a square ? Give reason.

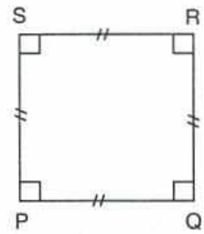
Solution :

The given quadrilateral is not necessarily a square. **(Ans.)**

Reason : When all the angles of a quadrilateral are 90° each, it is always a rectangle. And, if any pair of adjacent sides of this rectangle are equal, it is a square.

∴ For a quadrilateral to be a square, its each angle must be equal to 90° and any pair of adjacent sides must be equal.

Example 18 :



In a square PQRS, if PQ = $3x - 7$ and QR = $x + 3$, find PS.

Solution :

∵ All the sides of a square are equal

∴ PQ = QR

$\Rightarrow 3x - 7 = x + 3$ *i.e.* $3x - x = 7 + 3$

$\Rightarrow 2x = 10$ and $x = \frac{10}{2} = 5$

$x = 5 \Rightarrow$ PQ = $3x - 7$

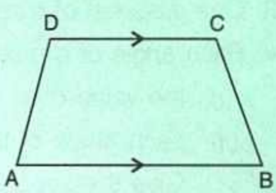
$= 3 \times 5 - 7 = 8$

∵ All the sides of a square are equal.

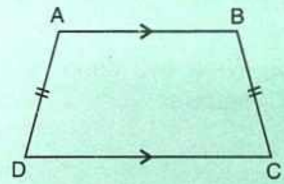
$\Rightarrow$ PS = PQ $\Rightarrow$ **PS = 8** **(Ans.)**

EXERCISE 27(B)

1. In a trapezium ABCD, side AB is parallel to side DC. If $\angle A = 78°$ and $\angle C = 120°$, find angles B and D.

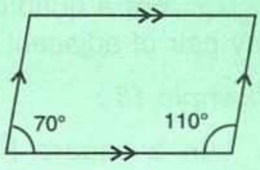


2. In a trapezium ABCD, side AB is parallel to side DC. If $\angle A = x°$ and $\angle D = (3x - 20)°$; find the value of x.
3. The angles A, B, C and D of a trapezium ABCD are in the ratio 3 : 4 : 5 : 6.
   i.e. $\angle A : \angle B : \angle C : \angle D = 3 : 4 : 5 : 6$. Find all the angles of the trapezium. Also, name the two sides of this trapezium which are parallel to each other. Give reason for your answer.
4. In an isosceles trapezium one pair of opposite sides are .... to each other and the other pair of opposite sides are .... to each other.
5. Two diagonals of an isosceles trapezium are x cm and $(3x - 8)$ cm. Find the value of x.
6. Angle A of an isosceles trapezium ABCD is 115°; find the angles B, C and D.



7. Two opposite angles of a parallelogram are 100° each. Find each of the other two opposite angles.

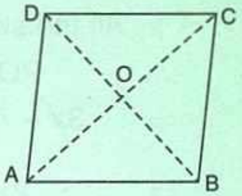
8. Two adjacent angles of a parallelogram are 70° and 110° respectively. Find the other two angles of it.



9. The angles A, B, C and D of a quadrilateral are in the ratio 2 : 3 : 2 : 3. Show that this quadrilateral is a parallelogram.

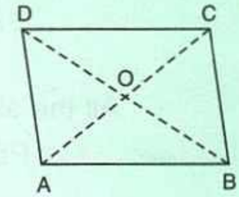
10. In a parallelogram ABCD, its diagonals AC and BD intersect each other at point O.

If AC = 12 cm and BD = 9 cm; find : lengths of OA and OD.



11. In parallelogram ABCD, its diagonals intersect at point O.

If OA = 6 cm and OB = 7·5 cm, find the lengths of AC and BD.



12. In a parallelogram ABCD, $\angle A = 90°$

(i) what is the measure of angle B ?

(ii) write the special name of the parallelogram.

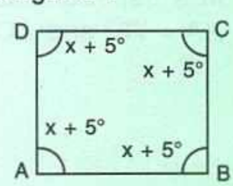
13. One diagonal of a rectangle is 18 cm. What is the length of its other diagonal ?

14. Each angle of a quadrilateral is $x + 5°$. Find :

(i) the value of x.

(ii) each angle of the quadrilateral.

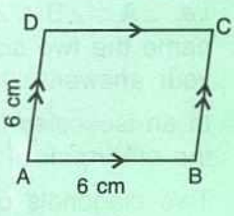
Give the special name of the quadrilateral taken.



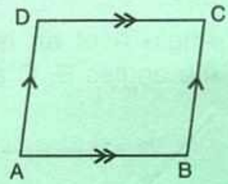
15. If three angles of a quadrilateral are 90° each, show that the given quadrilateral is a rectangle.

16. The diagonals of a rhombus are 6 cm and 8 cm. State the type of angles these diagonals make when they intersect.

17. Write, giving reason, the name of the figure drawn alongside. Under what condition will this figure be a square ?



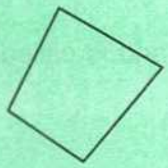
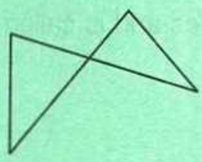
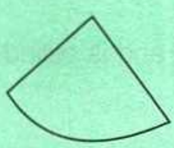
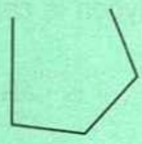
18. Write two conditions that will make the adjoining figure a square



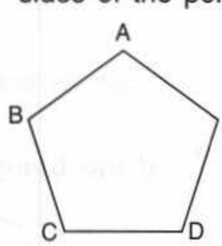
POLYGONS 28

28.1 INTRODUCTION

A **polygon is a closed plane figure**, bounded by straight line segments.

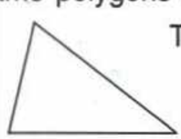
1.		It is a polygon as it is (i) a closed plane figure. (ii) bounded by straight line segments.
2.		It is not a polygon as the given figure is not bounded by straight line segments. The straight line segments, forming a polygon, intersect only at vertex.
3.		It is not a polygon as the given figure has one curved side *i.e.* it is not bounded by straight line segments.
4.		It is not a polygon as the given figure is not closed.

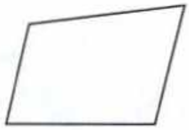
1. The straight line segments which make up a polygon are called the sides of the polygon and end-points of the line segments (the points where the adjacent sides of the polygon intersect each other) are called vertices of the polygon.

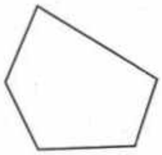


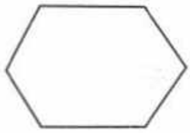
Sides of the adjoining polygon are AB, BC, CD, DE and EA, whereas its vertices are A, B, C, D and E.

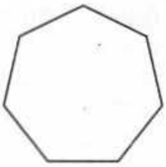
We name polygons according to the number of sides they contain. e.g.

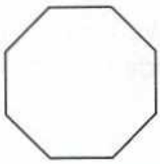
(i)  The adjoining polygon has three sides and is called a ***triangle***.

(ii)  The adjoining polygon has four sides and is called a ***quadrilateral***.

(iii)  The adjoining polygon has five sides and is called a ***pentagon***.

(iv)  The adjoining polygon has six sides and is called a ***hexagon***.

(v)  The adjoining polygon has seven sides and is called a ***septagon***.

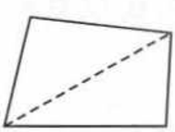
(vi)  The adjoining polygon has eight sides and is called an ***octagon***.

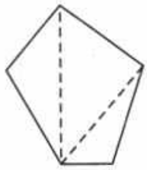
1. If each angle of a polygon is less than 180°, the polygon is called a convex polygon.
2. If at least one angle of a polygon is more than 180°, it is called a concave polygon.
3. Unless it is stated, a polygon means a convex polygon.

In a polygon : Number of sides in it = Number of angles in it.

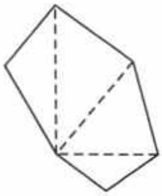
28.2 SUM OF ANGLES OF A POLYGON

1. Through any vertex of the polygon, draw as many diagonals as possible to form the maximum number of triangles.

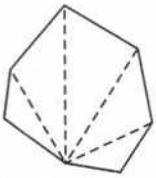
   (i)  A **quadrilateral** has **four sides** and two **triangles** are formed by its diagonal.

(ii) 

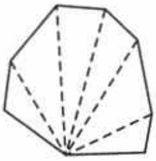
A **pentagon** has **five sides** and **three triangles** are formed by its diagonals.

(iii) 

A **hexagon** has **six sides** and **four triangles** are formed by its diagonals.

(iv) 

A **septagon** has **seven sides** and **five triangles** are formed by its diagonals.

(v) 

An **octagon** has **eight sides** and **six triangles** are formed by its diagonals.

It can easily be seen, from above, that if a polygon has n sides and the diagonals are drawn through only one vertex of it, we get (n – 2) triangles.

∴ **Sum of the angles of an n-sided polygon**

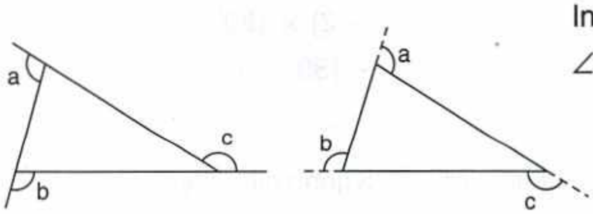
= Sum of the angles of (n – 2) triangles

= **$(n - 2) \times 180°$**

Sum of the angles of a polgyon means, sum of its interior angles.

28.3 SUM OF EXTERIOR ANGLES OF A POLYGON

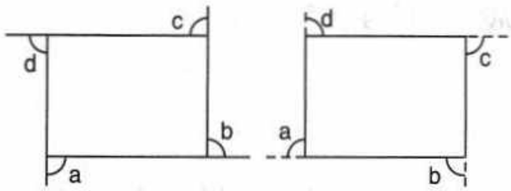
If the sides of a polygon are produced in order, the sum of exterior angles so formed is 4-right angles i.e. 360°.

1. 

In each case :

$\angle a + \angle b + \angle c = 360°$

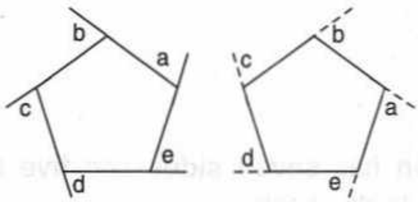
2.



In each case :

$\angle a + \angle b + \angle c + \angle d = 360°$

3.



In each case :

$\angle a + \angle b + \angle c + \angle d + \angle e = 360°$ and so on.

1. Whatever be the number of sides in a polygon, the sum of its exterior angles is 360° only.
2. The number of sides of a polygon is always a whole number which is greater than or equal to 3. *i.e.* the smallest number of sides in a polygon is 3 (three).

Example 1 :

Find the sum of the interior angles of a polygon with number of sides :

(i) 6 (ii) 8 (iii) 15

Solution :

(i) Given n = 6

$\therefore$ **Sum of interior angles** $= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 4 \times 180° =$ **720°** **(Ans.)**

(ii) Given n = 8

$\therefore$ **Sum of interior angles** $= (n - 2) \times 180°$

$= (8 - 2) \times 180°$

$= 6 \times 180° =$ **1080°** **(Ans.)**

(ii) Given n = 15

$\therefore$ **Sum of interior angles** $= (n - 2) \times 180°$

$= (15 - 2) \times 180°$

$= 13 \times 180° =$ **2340°** **(Ans.)**

Example 2 :

Find the sum of exterior angles of a polygon with sides :

(i) 5 (ii) 12

Solution :

Sum of the exterior angles of a polygon does not depend on its number of sides.

$\therefore$ In both the given cases, the sum of exterior angles of the polygons will be 360°.

Example 3 :

Can the sum of angles of a polygon be 7 right angles ?

Solution :

Let the number of sides in the polygon be n.

$\therefore$ Sum of its angles $= (n - 2) \times 180°$

According to the given statement :

$$(n - 2) \times 180° = 7 \times 90° \qquad [\because 7 \text{ right angles} = 7 \times 90°]$$

$$\Rightarrow \quad 180°n - 360° = 630°$$

$$\Rightarrow \quad 180°n = 630° + 360° = 990°$$

i.e. $$n = \frac{990}{180} = 5{\cdot}5$$

But $n = 5{\cdot}5$ is not a whole number.

$\therefore$ No polygon will have the sum of its interior angles equal to seven right angles. **(Ans.)**

Example 4 :

The sides of a triangle are produced in order. If the exterior angles formed are 3x°, 5x° and 4x°; find the value of x°.

Solution :

$\because$ Sum of exterior angles of each polygon = 360°

$\Rightarrow$ Sum of exterior angles of the triangle = 360°

$$\Rightarrow \quad 3x° + 5x° + 4x° = 360°$$

$$\Rightarrow \quad 12x° = 360°$$

and, $$x° = \frac{360°}{12} = 30°$$

$\therefore$ $x = 30$ **(Ans.)**

Example 5 :

Find the number of sides in a polygon, if the sum of its interior angles is 1080°.

Solution :

Let number of sides be n.

$\therefore$ The sum of its interior angles $= (n - 2) \times 180°$

Given : $$(n - 2) \times 180° = 1080°$$

$$\Rightarrow \quad 180°n - 360° = 1080°$$

$\Rightarrow \quad 180°n = 1080° + 360° = 1440°$

and, $\quad n = \frac{1440°}{180°} = 8$

$\therefore$ **Required number of sides = 8** **(Ans.)**

Example 6 :

If all the angles of a 12-sided polygon are equal, find the measure of each angle.

Solution :

Since, $\quad n = 12$

$\therefore$ Sum of interior angles of the polygon

$= (n - 2) \times 180°$

$= (12 - 2) \times 180°$

$= 10 \times 180° = 1800°$

Since, all the interior angles are equal and there are 12 interior angles in the 12 sided polygon; therefore

Measure of each angle $= \frac{1800°}{12} =$ **150°** **(Ans.)**

Example 7 :

The interior angles of a pentagon are in the ratio 5 : 4 : 7 : 5 : 6. Find each angle of the pentagon.

Solution :

Let the angles be 5x°, 4x°, 7x°, 5x° and 6x°.

Since, $\quad n = 5$

$\therefore$ Sum of interior angles of the polygon $= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 3 \times 180° = 540°$

Given : $\quad 5x° + 4x° + 7x° + 5x° + 6x° = 540°$

$\Rightarrow \quad 27x = 540$ and $x = \frac{540}{27} = 20$

$\therefore$ **Required angles** = 5x°, 4x°, 7x°, 5x° and 6x°

$= 5 \times 20°, 4 \times 20°, 7 \times 20°, 5 \times 20°$ and $6 \times 20°$

= **100°, 80°, 140°, 100° and 120°** **(Ans.)**

Example 8 :

Two angles of a hexagon are 100° and 120°. If the remaining four angles are equal, find each equal angle.

Solution :

Let each equal angle be x°

Since, a hexagon has six angles, therefore sum of all its six interior angles

$= 100° + 120° + x° + x° + x° + x°$

$= 220° + 4x°$

Also, sum of the interior angles of the hexagon

$= (n - 2) \times 180°$ [where n = 6]

$= (6 - 2) \times 180°$

$= 4 \times 180° = 720°$

Given : $220° + 4x° = 720°$

$\Rightarrow \quad 4x° = 720° - 220° = 500°$

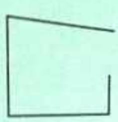
$\Rightarrow \quad x° = \dfrac{500°}{4} = 125°$

$\therefore$ **Each equal angle = 125°** **(Ans.)**

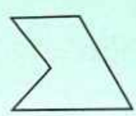
EXERCISE 28(A)

1. State, which of the following are polygons :

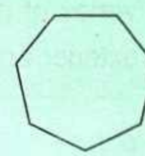
(i)



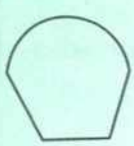
(ii)



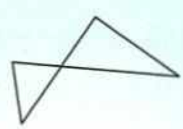
(iii)



(iv)



(v)



2. Find the sum of interior angles of a polygon with :
   (i) 9 sides (ii) 13 sides (iii) 16 sides
3. Find the number of sides of a polygon, if the sum of its interior angles is :
   (i) 1440° (ii) 1620°
4. Is it possible to have a polygon, whose sum of interior angles is 1030°.
5. (i) If all the angles of a hexagon are equal, find the measure of each angle.
   (ii) If all the angles of an octagon are equal, find the measure of each angle.
6. One angle of a quadrilateral is 90° and all other angles are equal; find each equal angle.
7. If angles of a quadrilateral are in the ratio 4 : 5 : 3 : 6; find each angle of the quadrilateral.
8. If one angle of a pentagon is 120° and each of the remaining four angles is x°, find the magnitude of x.
9. The angles of a pentagon are in the ratio 5 : 4 : 5 : 7 : 6; find each angle of the pentagon.
10. Two angles of a hexagon are 90° and 110°. If the remaining four angles are equal, find each equal angle.

28.4 REGULAR POLYGON

A polygon is said to be a regular polygon, if all its

(i) sides are equal,

(ii) interior angles are equal and

(iii) exterior angles are equal.

If a regular polygon has n sides :

1. The sum of its interior angles = $(n - 2) \times 180°$

   And, each interior angle = $\frac{(n-2)\times 180°}{n}$

2. The sum of its exterior angles = $360°$

   And, each exterior angle = $\frac{360°}{n}$

3. Number of sides (n) of a regular polygon = $\frac{360°}{\text{exterior angle}}$

4. Whether the given polygon is regular or non-regular, at each vertex of the polygon :

   exterior angle + interior angle = $180°$

Example 9 :

Find each interior angle of the regular polygon with number of sides.

(i) 20 (ii) 15

Solution :

(i) Given number of sides $n = 20$

Each interior angle of the 20 sided regular polygon

$$= \frac{(n-2)\times 180°}{n}$$

$$= \frac{(20-2)\times 180°}{20} \qquad [\because n = 20]$$

$$= \frac{18\times 180°}{20} = 162° \qquad \textbf{(Ans.)}$$

Alternative method :

$n = 20$

$\Rightarrow$ Sum of interior angles of the 20-sided regular polygon

$= (n - 2) \times 180°$

$= (20 - 2) \times 180° = 18 \times 180° = 3240°$

$\therefore$ Each interior angle of the 20 sided regular polygon

$$= \frac{3240°}{20} = 162° \quad \textbf{(Ans.)}$$

(ii) Given number of sides, n = 15

Each interior angle of 15-sided regular polygon

$$= \frac{(n-2)\times 180°}{n}$$

$$= \frac{(15-2)\times 180°}{15} \quad [\because n = 15]$$

$$= \frac{13\times 180°}{15} = 13 \times 12° = 156° \quad \textbf{(Ans.)}$$

Alternative method :

$$n = 15$$

$\Rightarrow$ Sum of interior angles of the 15-sided regular polygon

$$= (n - 2) \times 180°$$

$$= (15 - 2) \times 180°$$

$$= 13 \times 180° = 2340°$$

$\therefore$ Each interior angle of the 15-sided regular polygon

$$= \frac{2340°}{15} = 156° \quad \textbf{(Ans.)}$$

Example 10 :

Find each exterior angle of a regular polygon with sides :

(i) 12 (ii) 18

Solution :

If the number of sides of a regular polygon = n,

its each exterior angle = $\frac{360°}{n}$

(i) $\because$ Number of sides, n = 12

$\therefore$ **Each exterior angle** = $\frac{360°}{n}$

$$= \frac{360°}{12} = 30° \quad \textbf{(Ans.)}$$

(ii) $\because$ Number of sides, n = 18

$\therefore$ **Each exterior angle** = $\frac{360°}{n}$

$$= \frac{360°}{18} = 20° \quad \textbf{(Ans.)}$$

Example 11 :

Find the number of sides of a regular polygon with each exterior angle :

(i) 18° (ii) 72°

Solution :

When exterior angle of a regular polygon is given,

$$\text{the number of its sides} = \frac{360^\circ}{\text{exterior angle}}$$

(i) Exterior angle of the regular polygon = 18°

$\Rightarrow$ **Number of sides in it** $= \dfrac{360^\circ}{\text{exterior angle}}$

$= \dfrac{360^\circ}{18^\circ} =$ **20** **(Ans.)**

(ii) Exterior angle of the regular polygon = 72°

$\Rightarrow$ **Number of sides in it** $= \dfrac{360^\circ}{\text{exterior angle}}$

$= \dfrac{360^\circ}{72^\circ} =$ **5** **(Ans.)**

Example 12 :

Is it possible to have a regular polygon with each exterior angle 75° ?

Solution :

Since, each exterior angle = 75°

$\therefore$ Number of sides of the regular polgyon $= \dfrac{360^\circ}{\text{exterior angle}}$

$= \dfrac{360^\circ}{75^\circ} = \dfrac{24}{5} = 4\dfrac{4}{5}$, which is not a whole number

$\therefore$ **No regular polygon is possible with each exterior angle 75°.** **(Ans.)**

Example 13 :

Is it possible to have a regular polygon with each interior angle 135° ?

Solution :

If n be the number of sides of the regular polygon

$\therefore \dfrac{(n-2)\times 180^\circ}{n} = 135^\circ \Rightarrow 180n - 360 = 135n$

$\Rightarrow 180n - 135n = 360$

$\Rightarrow 45n = 360$

and, $n = \dfrac{360}{45} = 8$

Since, n = 8 is a whole number, therefore **regular polygon with each interior angle 135° is possible.** **(Ans.)**

Alternative method :

Given each interior angle = 135°

and we know, at each vertex of a polygon,

interior angle + exterior angle = 180°

$\Rightarrow$ 135° + exterior angle = 180°

$\Rightarrow$ Exterior angle = 180° − 135°

= 45°

Now, **number of sides** $= \dfrac{360°}{\text{exterior angle}}$

$= \dfrac{360°}{45°} = \mathbf{8}$ **(Ans.)**

EXERCISE 28(B)

1. Fill in the blanks :

In case of regular polygon, with :		
Number of sides	**Each exterior angle**	**Each interior angle**
(i) 6	....	....
(ii) 8	....	....
(iii)	36°	....
(iv)	20°	....
(v)	....	135°
(vi)	....	165°

2. Find the number of sides in a regular polygon, if its each interior angle is :

(i) 160° (ii) 150°

3. Find the number of sides in a regular polygon, if its each exterior angle is :

(i) 30° (ii) 36°

4. Is it possible to have a regular polygon whose each interior angle is :

(i) 135° ? (ii) 155° ?

5. Is it possible to have a regular polygon whose each exterior angle is :

(i) 100° ? (ii) 36° ?

6. The ratio between the interior angle and the exterior angle of a regular polygon is 2 : 1. Find :

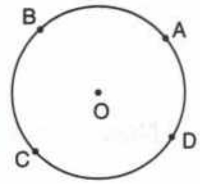
(i) each exterior angle of this polygon.

(ii) number of sides in the polygon.

THE CIRCLE 29

29.1 CIRCLE :

A circle is a closed curve such that all the points on its circumference (boundary) are equidistant from a fixed point inside it.



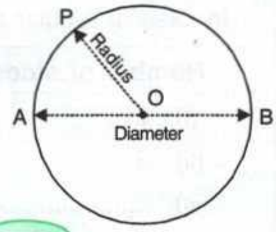
The given figure shows a circle and a fixed point O inside the circle. The points A, B, C and D, which are marked on the circumference (boundary) of the circle, are all at the same distance from the fixed point O, *i.e.* OA = OB = OC = OD and so on.

In fact, a **circle** is defined as the ***path traced* by a *moving point* that always remains at a *fixed distance* from a given *fixed point.***

The path so traced by a moving point is called the ***circumference*** of a circle. The fixed point is called the ***centre*** of the circle and *the fixed distance* from this fixed point to any point on the circumference is called ***radius.***

29.2 RADIUS :

The radius of a circle is the length of the line segment joining the centre of the circle with any point on its circumference.



In the adjacent figure, OP is the line that joins the centre O of the circle with a point P on its circumference; therefore OP = radius of the circle.

Each of OA and OB is a radius.

29.3 DIAMETER :

A ***straight line*** *passing* ***through the centre*** *of a circle and with both its* ***ends on the circumference*** *of the circle is called the* ***diameter*** *of the circle.*

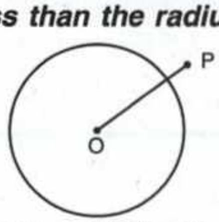
In the figure given above, AB is the diameter.

1. In a circle, the length of the diameter is always double the length of the radius,
   i.e. Diameter = 2 × Radius and Radius = $\frac{\text{Diameter}}{2}$
2. The diameter of a circle always divides the circle into two equal parts, each of these two equal parts is called a **semi-circle**.

IMPORTANT : Let a circle with centre O and a point P lie in the same plane such that the distance between O and P is :

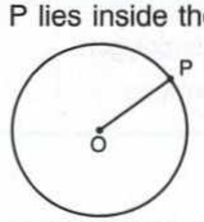
(i) ***greater than the radius***, the point P lies outside the circle.
(ii) ***equal to the radius***, the point P lies on the circumference of the circle.
(iii) ***less than the radius***, the point P lies inside the circle.

Thus, (i)



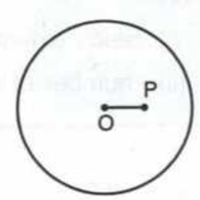
[P lies outside the circle ⇒ OP > radius]

(ii)



[P lies on the circumference ⇒ OP = radius]

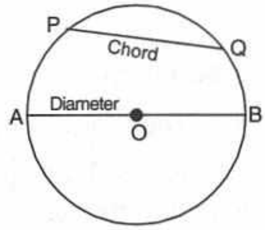
(iii)



[P lies inside the circle ⇒ OP < radius]

29.4 CHORD :

A straight line joining any two points on the circumference of a circle, is called a ***chord****.*



In the figure given alongside, the straight line PQ is obtained by joining the points P and Q lying on the circumference of the circle, therefore, PQ is a chord of the circle.

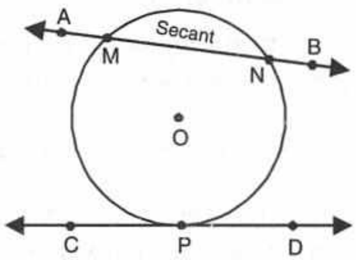
In the same way, diameter AB too is a chord of the circle.

1. A chord passing through the centre of a circle is also its diameter; viz. AB.
2. The diameter of a circle is the longest chord of that circle.

29.5 SECANT AND TANGENT :

A straight line intersecting a circle at two points is called a ***secant.***

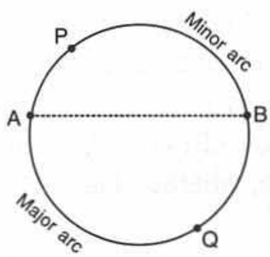
A straight line touching a circle at one point only is called a ***tangent.***



In the figure given alongside, line AB intersects the given circle at points M and N. Therefore, AB is a *secant.* And line CD, which touches the circle at point P only is a *tangent* to the circle. The point P, at which the tangent touches the circle, is the **point of contact**.

29.6 ARC :

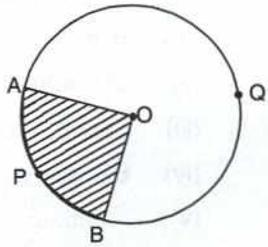
An arc is a part of the circumference of a circle.



Let A and B be two points on the circumference of a circle, as shown alongside. On joining AB, the circumference of the circle is divided into two parts, namely APB and AQB. These two parts are known as the arcs of a circle.

It is clear from the figure that *the length of the arc APB is smaller* than the *length of the arc AQB*. Therefore, **arc APB** is the ***minor arc*** and **arc AQB** is the ***major arc***.

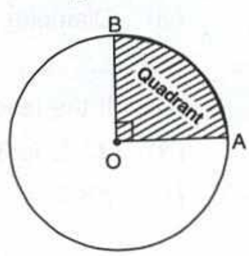
29.7 SECTOR :



The part of a circle enclosed by any two radii (plural of radius) and an arc *is called a* ***sector****.*

In the figure given alongside, the arc APB and the radii OA and OB enclose the shaded portion OAPB, which is thus a sector of the circle.

It is clear from the figure that the radii OA and OB divide the circle into two unequal sectors; the *minor sector is the shaded portion* OAPB and *the major sector is the unshaded portion* OAQB.



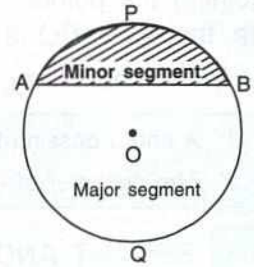
Note : The part of a circle enclosed by two mutually perpendicular radii, as shown alongside, is called a ***quadrant***.

29.8 SEGMENT :

Every chord divides a circle into two parts, these two parts of a circle are called its ***segments.***

Therefore, *a segment is a part of a circle enclosed by a chord and an arc.*

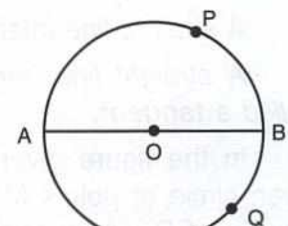
In the figure given alongside, the chord AB divides the circle into two unequal parts.



The ***smaller,*** *i.e.* the *shaded part of the circle, is called the* ***minor segment,*** and the ***larger,*** *i.e.* the *unshaded part of the circle, is called the* ***major segment.***

1. The major segment contains the centre of the circle.
2. A chord through the centre, (*i.e.* a diameter) divides a circle into two equal parts, each equal part is called a **semi-circle.**

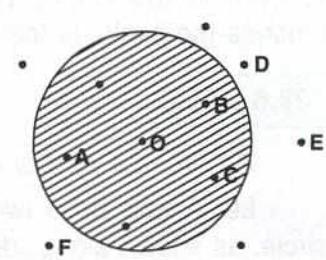
In the figure given alongside, AB is a diameter so APB and AQB are semi-circles.



29.9 INTERIOR AND EXTERIOR OF A CIRCLE :

The *part of the plane,* or the set of points in a plane *that lie inside a circle is called the interior of the circle.*

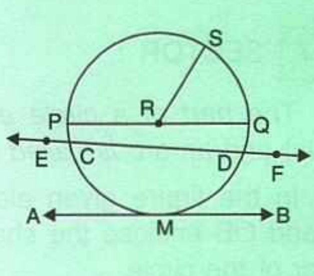
The part of the plane, or the set of points in a plane, *that lie outside a circle is called the exterior of the circle.*



The shaded part of the adjoining figure shows the interior of the circle. The points O, A, B, C, etc., lie inside the circle, whereas the points D, E, F, etc., lie outside the circle.

EXERCISE 29(A)

1. Use the figure given alongside to fill in the blanks :



(i) R is the .... of the circle.

(ii) Diameter of a circle is ....

(iii) Tangent to a circle is ....

(iv) EF is a .... of the circle.

(v) .... is a chord of the circle.

(vi) Diameter = 2 × ....

(vii) .... is a radius of the circle.

(viii) If the length of RS is 5 cm, the length of PQ = ...

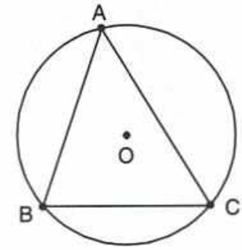
(ix) If PQ is 8 cm long, the length of RS = ...

(x) AB is a .... of the circle.

2. Draw a circle of radius 4·2 cm. Mark its centre as O. Take a point A on the circumference of the circle. Join AO and extend it till it meets point B on the circumference of the circle.
   (i) Measure the length of AB. (ii) Assign a special name to AB.
3. Draw circles with diameter :
   (i) 6 cm (ii) 8·4 cm
   In each case, measure the length of the radius of the circle drawn.
4. Draw a circle of radius 6 cm. In the circle, draw a chord AB = 6 cm.
   (i) If O is the centre of the circle, join OA and OB.
   (ii) Assign a special name to Δ AOB.
   (iii) Write the measure of angle AOB.
5. Draw a circle of radius 4·8 cm and mark its centre as P.
   (i) Draw radii PA and PB such that ∠APB = 45°.
   (ii) Shade the major sector of the circle.
6. Draw a circle of radius 3·6 cm. In the circle, draw a chord AB = 5 cm. Now shade the minor segment of the circle.
7. Mark two points A and B, 4 cm apart.
   Draw a circle passing through B and with A as centre.
8. Draw a line AB = 8·4 cm. Now draw a circle with AB as diameter.
   Mark a point C on the circumference of the circle. Measure angle ACB.

29.10 CIRCUMCIRCLE OF A TRIANGLE :

*A circle that passes through all the three vertices of a triangle is called the **circumcircle of the triangle**.*



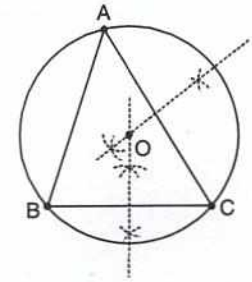
To construct the circumcircle of a given triangle ABC.

Steps :

1. Draw the perpendicular bisectors of any two sides of the triangle.

   Let the perpendicular bisectors of the sides BC and AC be drawn such that they intersect at point O.

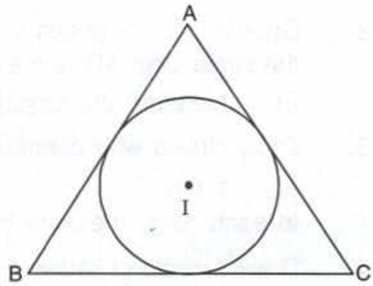
2. Draw a circle taking O as centre and OA or OB or OC as radius. This circle will pass through all the three vertices, A, B and C.



1. *The point where the perpendicular bisectors of the sides of a triangle meet, shown here as O, is called **circumcentre.***
2. *OA = OB = OC = radius of the circle = **circumradius.***

29.11 IN-CIRCLE OF A TRIANGLE :

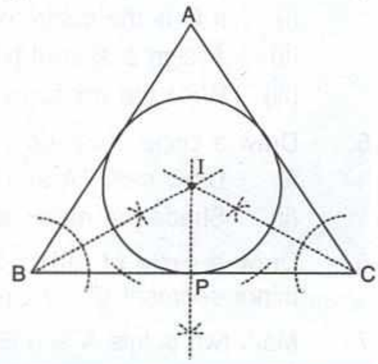
A circle drawn inside a triangle *such that it touches all the three sides of the triangle is called the* ***in-circle*** *of the triangle.*



To construct the in-circle of a given triangle ABC.

Steps :

1. Draw the bisectors of any two angles of the triangle. Let these bisectors meet at point I.
2. From the point I, drop perpendicular IP on to BC.
3. With I as centre and IP as radius, draw a circle; this circle will touch all the three sides of the triangle.



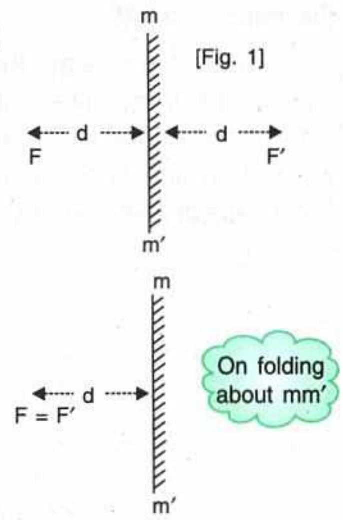
1. *The point where the bisectors of the angles of a triangle meet, shown here as I, is called* ***incentre.***
2. *The length of the perpendicular, here IP, is called* ***inradius.***

EXERCISE 29(B)

1. Construct a triangle ABC with AB = 4·2 cm, BC = 6 cm and AC = 5 cm. Construct the circumcircle of the triangle drawn.
2. Construct a triangle PQR with QR = 5·5 cm, $\angle$Q = 60° and angle R = 45°. Construct the circumcircle of the triangle PQR.
3. Construct a triangle ABC with AB = 5 cm, $\angle$B = 60° and BC = 6·4 cm. Draw the incircle of the triangle ABC.
4. Construct a triangle XYZ in which XY = YZ = 4·5 cm and ZX = 5·4 cm. Draw the circumcircle of the triangle and measure its circumradius.
5. Construct a triangle PQR in which PQ = QR = RP = 5·7 cm. Draw the incircle of the triangle and measure its radius.

SYMMETRY 30
(Including Constructions On Symmetry)

30.1 CONCEPT OF SYMMETRY (LINEAR SYMMETRY)

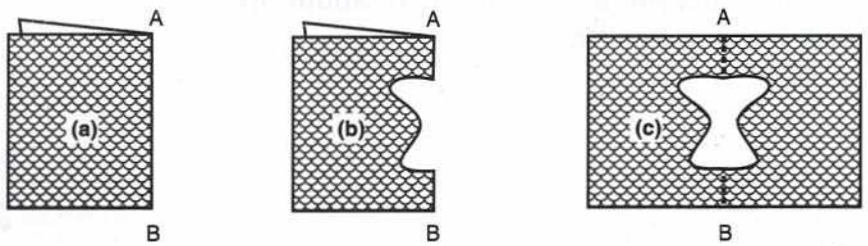


Consider a plane mirror *mm′*. If an object *F* is kept at a distance 'd' in front of the mirror; the image of the object, *F′* as seen in the mirror, is formed at distance 'd' behind the mirror.

Now, if Figure 1, given alongside, is folded about the mirror *mm′*, the object *F* and its image *F′* coincide. Since the two parts of the figure coincide when the figure is folded along the mirror line *mm′*, we say that the figure is symmetrical about the mirror line *mm′*. For this reason, the mirror line *mm′* is called the **line of symmetry** of the whole figure, *i.e.* the object *F*, its image *F′* and the mirror *mm′* taken as a whole.

Making the concept more clear :

Fold a rectangular piece of paper as shown in Figure (a) below. Then cut a piece of any design from the folded side of the paper as shown in Figure (b) below.



When we unfold the paper, there emerges a design, as shown in Figure (c) above. It is clear that the pattern of the design is identical on both the sides of the folding line (crease) of the paper, as shown here by the dotted line AB. So, AB is the line of symmetry here.

A figure that is identical on both the sides of a line in it is said to be *symmetrical about that line*, and *the line* about which the figure is *symmetrical* is called the *line of symmetry* or the *axis of symmetry.*

In order to find whether or not a given figure is symmetrical about a line in it, fold the figure about that line. If the part of the figure that lies on one side of the line coincides with the part of the figure on the other side of the line, the figure is symmetrical about that line.

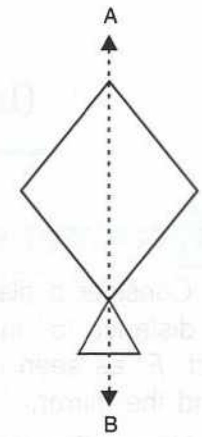
30.2 2-D SYMMETRICAL OBJECTS

An object that can be drawn on a plane surface, is called a 2-D object.

In this chapter, we shall be studying on 2-D symmetrical objects for reflection symmetry.

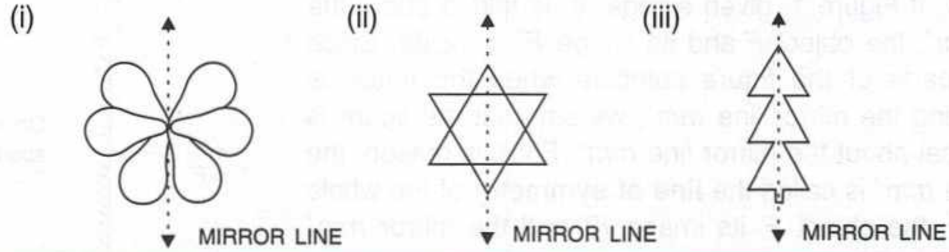
Meaning of reflection symmetry

The adjoining figure is said to have reflection symmetry about dotted line AB.



Reason : If the dotted line AB is considered to be a plane mirror, the image of the figure on the left side of mirror line AB concides with the figure on the right side of the mirror line AB.

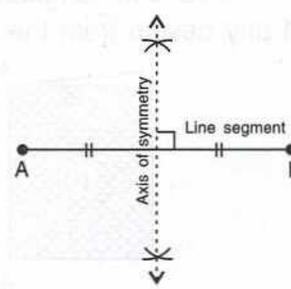
In the same way, the image of the figure on the right side of the mirror line concides with the figure on the left side of the mirror line AB. In each of the following figures, the dotted line is the mirror line and the figure is said to have reflection symmetry about this mirror line.



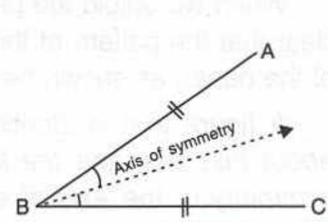
Each of the figures given above, has reflection symmetry about the dotted line.

Examples :

1. A *line segment* is *symmetrical* about its *perpendicular bisector.*

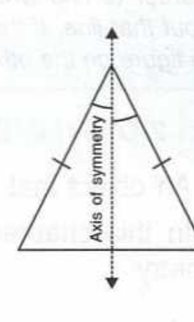


2. An *angle* (with equal arms) is *symmetrical* about its *bisector.*

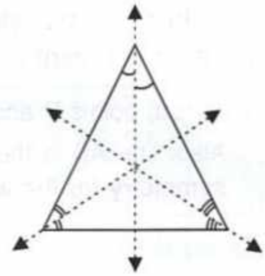


In each figure, the line of symmetry is represented by the dotted line

3. An *isosceles triangle* has *one line of symmetry.* The line of symmetry is the bisector of the angle contained by the two equal sides.



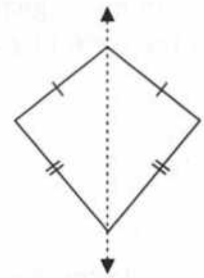
4. An equilateral triangle has *three lines of symmetry*. Each of the three *bisectors* of *angles* is a line of symmetry.



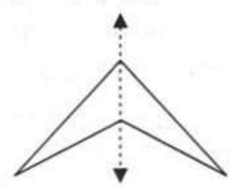
A scalene triangle has no line of symmetry

> A figure may have many lines of symmetry, e.g. a circle.

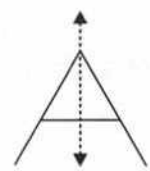
5. A *kite-shaped* figure has *one line* of symmetry.



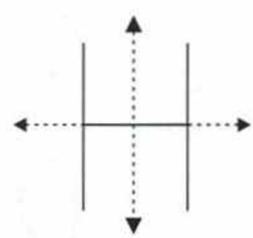
6. A figure of the shape of an *arrow-head* has *one line of symmetry.*



7. The letter 'A' has *one line of symmetry.*

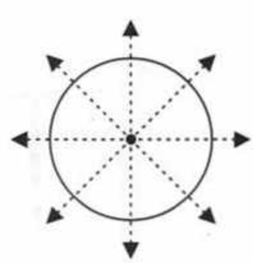


8. The letter 'H' has *two* lines of symmetry.



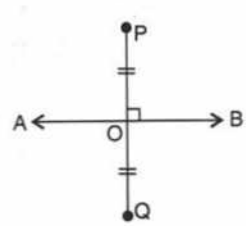
9. A circle has an *infinite* number of *lines* of *symmetry.*

Every line that passes *through the centre of a circle* is a *line of symmetry.*



30.3 SYMMETRIC POINT

Consider a point P and a line AB. From point P, draw PO perpendicular to AB and then extend PO up to point Q such that OP = OQ. Now fold the figure obtained about the line AB. What do you observe ?



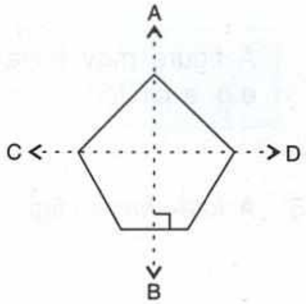
Points P and Q coincide.

In this case, point Q is said to be the **symmetric point** of the given point P with respect to the given line AB.

In fact, points P and Q are symmetric to each other with respect to the line AB.
Also, line AB is the perpendicular bisector of the line segment PQ. **Line AB is the line of symmetry** for the whole figure obtained.

***Example* 1 :**

In the adjoining figure, which one is the mirror line (*i.e.*, line of symmetry) AB or CD.

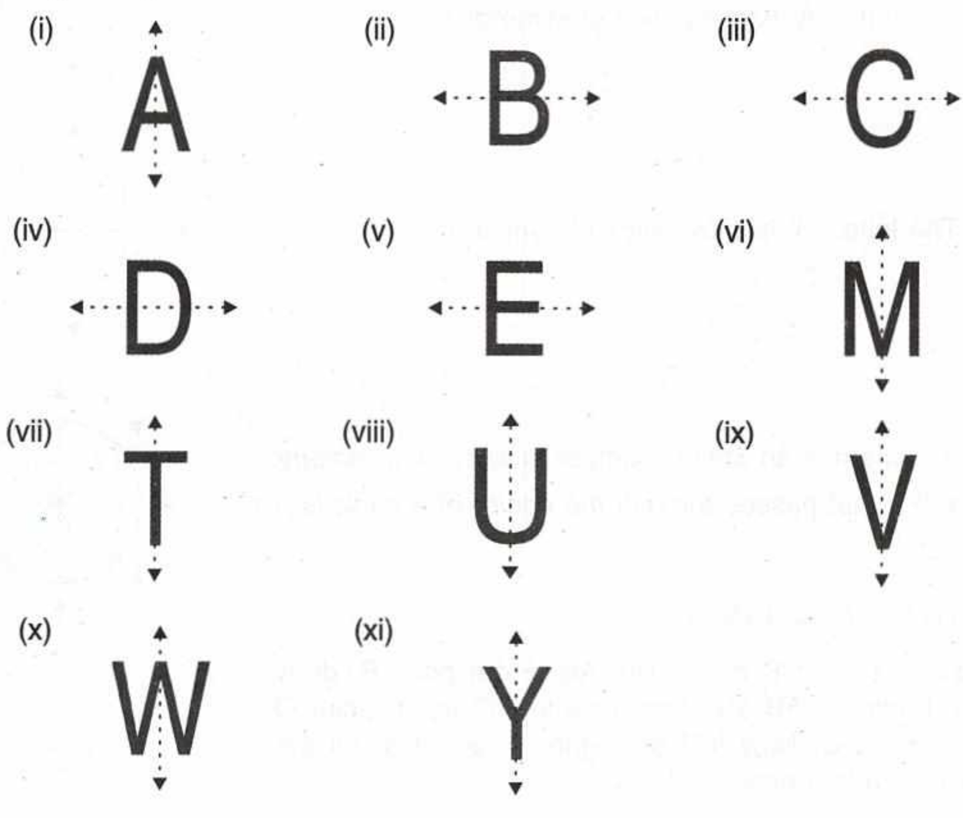


Solution :

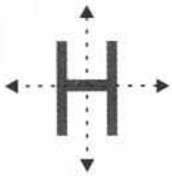
Fold the figure about the dotted line AB. The two parts of the figure (one on the left side of AB and the other on the right side of AB) coincide. This implies that the given figure is symmetric about the mirror line AB.

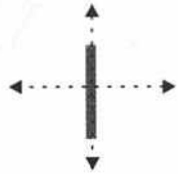
Now, fold the figure about the dotted line CD. The two parts of the figure (one below CD and the other above CD) do not coincide. This implies that the given figure is not symmetric about the line CD *i.e.* CD is not the mirror line for the given figure.

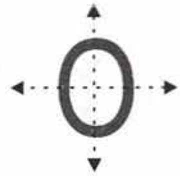
Note 1 : Given below are the letters of English alphabet which have exactly one line of symmetry :

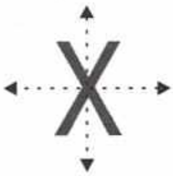


Note 2 : Given below are the letters of Egnlsih alphabet which have exactly two lines of symmetry :

(i)


(ii)


(iii)


(iv)


Note 3 : Given below are the letters of English alphabet which have no line of symmetry :

(i) F (ii) G (iii) J

(iv) K (v) L (vi) N

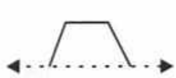
(vii) P (viii) Q (ix) R

(x) S (xi) Z

***Example* 2 :**

Complete each of the following figures in such a way that the completed figure is symmetric about the given dotted line (line of symmetry) :

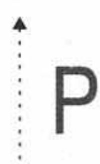
(i)


(ii)


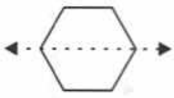
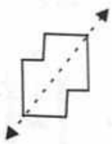
(iii)

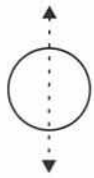

(iv)


(v)


(vi)


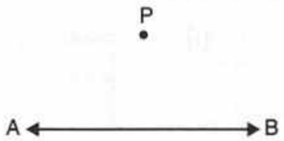
Solution :

(i)  (ii)  (iii) 

(iv)  (v)  (vi) 

30.4 TO LOCATE A POINT THAT IS SYMMETRIC TO A GIVEN POINT WITH RESPECT TO A GIVEN LINE

Given a point P and a line AB, we want to find the point that is symmetric to the given point P with respect to the given line AB.

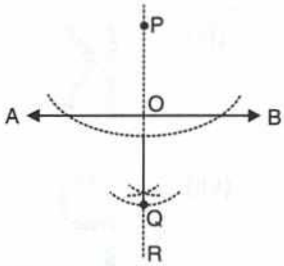


For this,

1. Draw PO perpendicular to AB, and extend it upto a point R.
2. From OR cut OQ = PO.

Clearly, point Q is symmetric to the given point P with respect to the given line AB.

Thus, AB is the line of symmetry.



To check your construction, fold the figure about the line of symmetry AB; you will find that P and Q coincide, i.e. they occur at the same point.

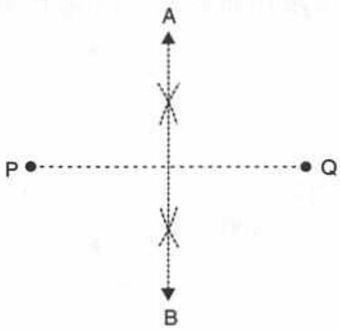
30.5 CONSTRUCTING THE LINE OF SYMMETRY WHEN TWO FIXED POINTS ARE SYMMETRIC WITH RESPECT TO THE REQUIRED LINE

Given two fixed points P and Q, we want to construct a line so that P and Q are symmetric with respect to this line.

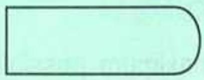
For this :

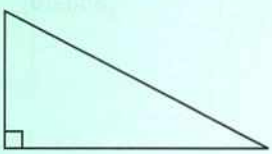
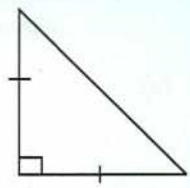
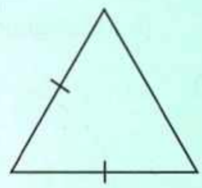
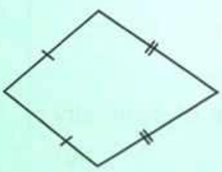
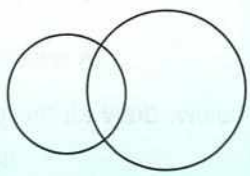
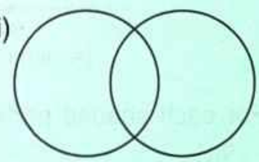
1. Join P and Q.
2. Draw perpendicular bisector of the line segment PQ.

The obtained perpendicular bisector AB is the required line of symmetry, *i.e.* with respect to AB, the two points P and Q are symmetric.

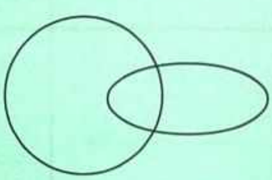
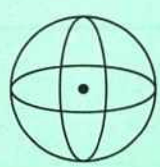
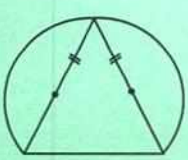


EXERCISE 30

1. State **true** or **false** :
   (i) The letter B has one line of symmetry.
   (ii) The letter F has no line of symmetry.
   (iii) The letter O has only two lines of symmetry.
   (iv) The figure  has no line of symmetry.
   (v) The letter N has one line of symmetry.
   (vi) The figure  has one line of symmetry.
   (vii) The letter D has only one line of symmetry.
   (viii) A scalene triangle has three lines of symmetry.
2. Construct a triangle ABC in which AB = AC = 5 cm and BC = 6 cm. Draw all its lines of symmetry.
3. Examine each of the following figures carefully, draw line(s) of symmetry in which ever figure possible :

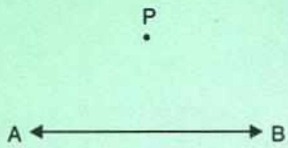
(i)

(ii)

(iii)

(iv)

(v)

(vi)

[Equal circles]

4. Construct a triangle XYZ in which XY = YZ = ZX = 4·5 cm. Draw all its lines of symmetry.
5. Construct a triangle ABC in which AB = BC = 4 cm and $\angle ABC = 60°$. Draw all its lines of symmetry.
6. Draw the line(s) of symmetry for each figure drawn below :

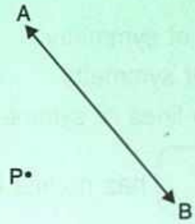
(i)

(ii)

(iii)

(iv)


7. In each of the following case, construct a point that is symmetric to the given point P with respect to the given line AB.

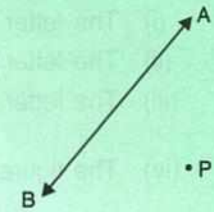
(i)



(ii)



(iii)

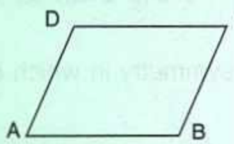


8. Mark two points A and B 5·5 cm apart. Draw a line PQ so that A and B are symmetric with respect to the line PQ. Give a special name to line PQ.

9. For each letter of the English alphabet, draw the maximum possible number of lines of symmetry.

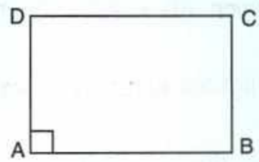
10. Draw all the possible lines of symmetry for each figure given below :

(i)



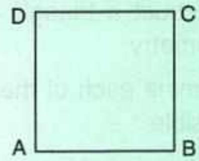
[Parallelogram]

(ii)



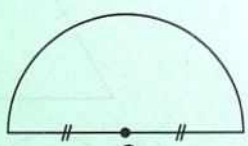
[Rectangle]

(iii)



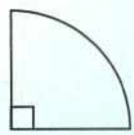
[Square]

(iv)



[A semi-circle]

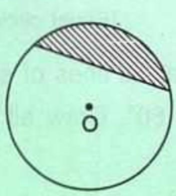
(v)



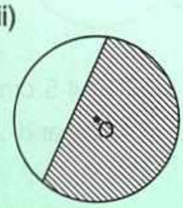
[A quadrant]

11. For each shaded portion given below, draw all the possible lines of symmetry :

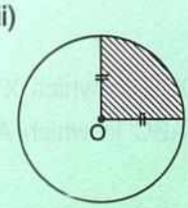
(i)



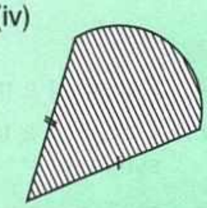
(ii)



(iii)



(iv)



RECOGNITION OF SOLIDS 31

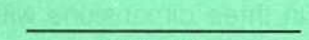
31.1 IDENTIFICATION OF 3D SHAPES

[Cube, cuboid, cylinder, sphere, cone, prism and pyramid]

Solid : An object that occupies space and has a fixed shape is called a **solid**. A book, a brick, a ball, etc., are some examples of a solid.

1. A thin straight line drawn on paper, *i.e.* a line drawn on a plane, has only length.

   Thus, we say that **straight line** has **only one dimension**, namely, its **length**.

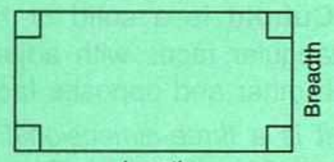


Straight line

2. A rectangle, drawn on paper, has length and breadth.

   Thus, we say that a **rectangle** has **two dimensions**, namely, **length** and **breadth**.

   In fact, every rectilinear figure (such as : square, parallelogram, trapezium, etc.) is a **two-dimensional** figure.

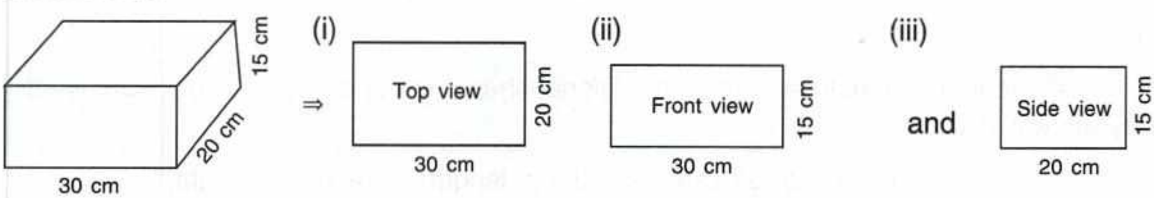


3. **Solids** have **length**, **breadth** and **height**. For this reason, every solid is a **three-dimensional** figure.

Every solid object has mainly three different views :

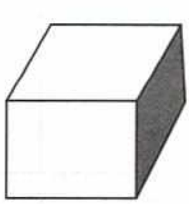
1. Top view 2. Side view 3. Front view

If you ask a carpenter to make a box with wood, it will be easy for him to make the box, if we draw and give him the top view, the side view and the front view with dimensions.



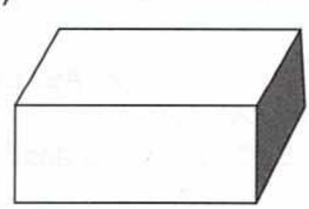
Some three dimensional shapes are given below :

(i)



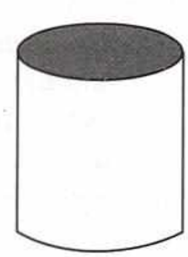
(Cube)

(ii)



(Cuboid)

(iii)

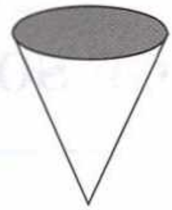


(Cylinder)

(iv)

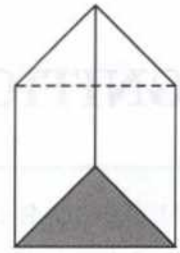
(Sphere)

(v)



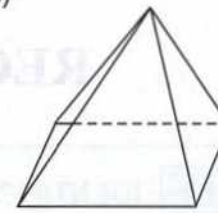
(Cone)

(vi)



(Prism)

(vii)

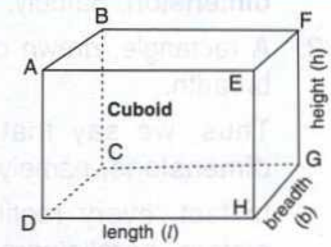


(Pyramid)

31.2 RECOGNIZING FACES, EDGES AND VERTICES (CORNERS) OF A POLYHEDRON AND COUNTING THEM

In elementary geometry, a **polyhedron** (plural : polyhedra or polyhedrons) is a solid in three dimensions with flat polygonal faces, straight edges and sharp corners (or, vertices.)

(a) Cuboid (a rectangular solid) :



Cuboid is a solid or hollow body which has six rectangular faces with adjacent faces at right angle to each other and opposite faces parallel to each other.

It is a three-dimensional solid all of whose sides are not necessarily equal. That is, in general, a cuboid has length, breadth and height of different values (sizes).

The given figure shows a cuboid. It is clear from the figure that a cuboid has :

(i) ***six faces,*** namely, ABCD, ABFE, AEHD, CGHD, CGFB and EFGH.

Each face of a cuboid is a rectangle.

(ii) ***twelve edges,*** namely, AB, BC, CD, DA, AE, EH, HD, EF, FG, GH, BF and CG.

(iii) ***eight vertices*** (corners), namely, A, B, C, D, E, F, G and H.

Also, (i) length (l) of the cuboid = AE = DH = CG = BF

(ii) breadth (b) of the cuboid = AB = DC = HG = EF

(iii) height (h) of the cuboid = AD = BC = EH = FG

(b) Cube :

Cube is a symmetrical three-dimensional shape, either solid or hollow, contained by six equal squares.

A cube is a cuboid with all sides equal, i.e. length = breadth = height.

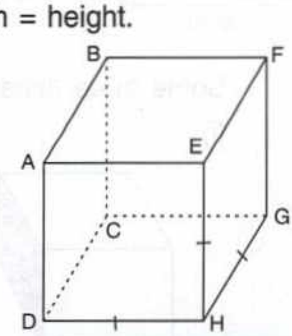
The adjoining figure shows a cube.

Since, a cube is a cuboid, it also has :

(i) ***six faces :*** ABCD, ABFE, AEHD, CGHD, CGFB and EFGH.

(ii) ***twelve edges :*** AB, BC, CD, DA, AE, EH, HD, EF, FG, GH, BF and CG.

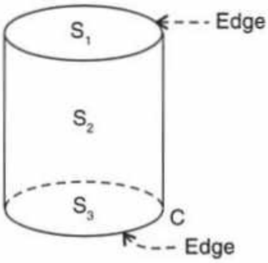
(iii) ***eight corners :*** A, B, C, D, E, F, G and H.



Each face of a cube is a square in shape and all the six faces of a cube are congruent (equal).

(c) Cylinder :

Cylinder is a solid or hollow geometrical figure with a curved side and two identical circular flat ends.

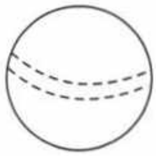


Cylinder is a 3D figure with no vertex, **two edges** (shown in the adjoining figure) and **three faces** shown by S_1, S_2 and S_3, where S_1 and S_3 are circular in shape and S_2 is a curved surface.

(d) Sphere :

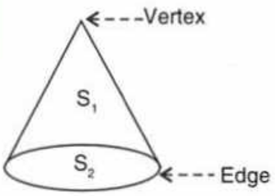
Sphere is a round solid or hollow figure, with every point on its surface equidistant from its centre.

Sphere is a 3D figure with **no vertex**, **no edge** and **only one surface** which is a curved surface.



(e) Cone :

Cone is a solid or hollow object which tapers from a circular base to a point.

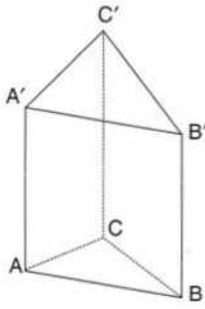


Cone is a 3D figure with **one vertex**, **one edge** and **two surface** represented by S_1 and S_2, where S_1 is a curved surface and S_2 is a circle.

(f) Prism :

Prism is a solid geometric figure whose two ends are similar, equal and parallel rectilinear figures and whose side-faces are parallelograms or rectangles.

The adjoining figure shows a prism with :



(i) ***three side faces,*** namely, AA′C′C, ABB′A′ and BB′C′C ; each of these three faces is a parallelogram (or rectangle).

This prism also has two congruent end-faces, *i.e.* bases, namely, triangles ABC and A′B′C′.

The two end-faces (bases) are always parallel to each other.

(ii) ***nine edges,*** namely, AB, AC, BC, A′B′, A′C′, B′C′, AA′, BB′ and CC′.

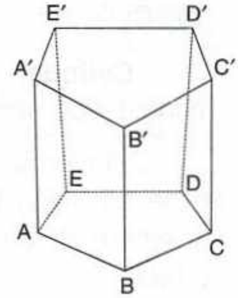
Of these nine edges, AA′, BB′ and CC′ are parallel to one another, AB is parallel to A′B′, BC is parallel to B′C′ and AC is parallel to A′C′.

(iii) ***six vertices,*** namely, A, B, C, A′, B′ and C′.

Thus, a prism is a solid, whose side-faces are parallelograms (or, rectangles) and whose end-faces, i.e. bases, are two parallel and congruent polygons.

The following figure shows a prism with :

(i) ***five side-faces,*** namely, ABB′A′, BCC′B′, CDD′C′, DEE′D′ and AEE′A′, each of which is a rectangle.



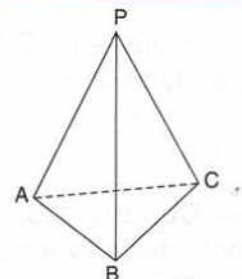
The prism also has two end-faces, ABCDE and A′B′C′D′E′, which are congruent and parallel to each other.

(ii) ***fifteen edges,*** AA′ // BB′ // CC′ // DD′ // EE′,
AB // A′ B′, BC // B′C′, CD // C′D′, DE // D′E′ and AE // A′E′.

(iii) ***ten vertices,*** A, B, C, D, E and A′, B′, C′, D′, E′.

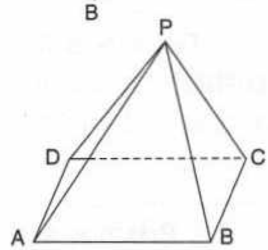
(g) Pyramid :

A pyramid is a solid whose base is a plane rectilinear figure, such as a triangle, a quadrilateral, and whose side-faces are triangles with a common vertex. This common vertex must lie outside the plane of the base.



The adjoining figure shows a pyramid with triangular base ABC and side-faces (each of which is also a triangle) PAB, PBC and PAC. Point P is the common vertex of the side-faces.

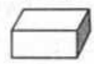
Since, the base of this pyramid is a triangle, it is called a ***triangular pyramid*** or ***tetrahedron***.



In the same way, the adjoining figure shows a pyramid whose base is a quadrilateral ABCD and side-faces are ΔPAB, ΔPBC, ΔPCD and ΔPDA. Clearly, P is the common vertex of the side-faces and it does not lie on the plane of the base.

Since, the base of this pyramid is a quadrilateral, it is called a ***quadrilateral pyramid.***

The number of vertices (V), no. of edges (E) and the no. of faces (F) of different 3D figures are as given below :

Figure		V	E	F
1. *Cube*		8	12	6
2. *Cuboid*		8	12	6
3. *Cylinder*		0	2	3
4. *Sphere*		0	0	1
5. *Cone*		1	1	2
6. *Prism*		6	9	5
7. *Triangular pyramid (or tetrahedron)*		4	6	4
8. *Square (or, rectangular pyramid)*		5	8	5

31.3 NETS OF 3D FIGURES

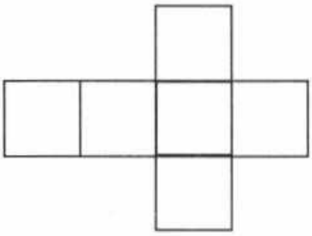
[For cube, cuboid, cylinder, cone, prism, pyramid, etc.]

A pattern, that can be cut and folded to make a model of a solid shape, is called a **net**.

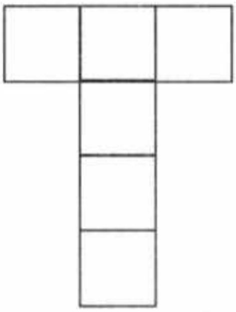
1. Nets of cube :

Some of the nets of a cube are shown below :

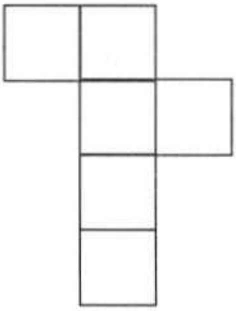
(i)



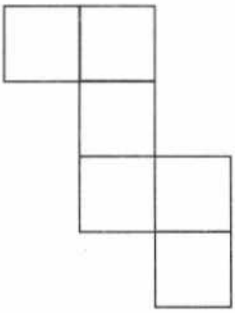
(ii)



(iii)



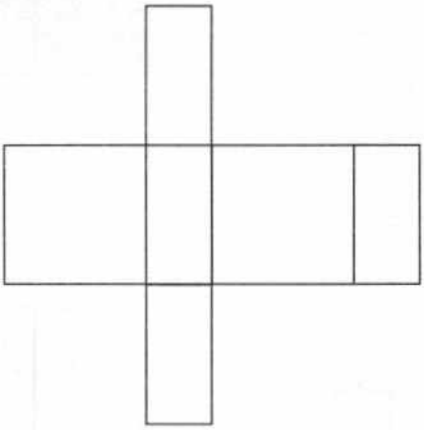
(iv)



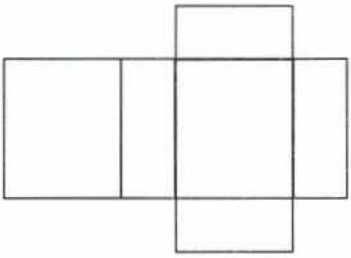
2. Nets of cuboid :

Some of the nets of a cuboid are shown below :

(i)

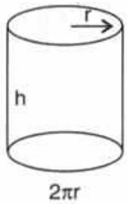


(ii)



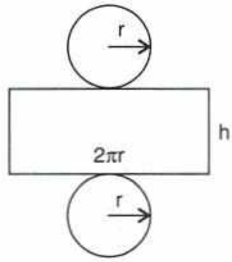
3. Nets of cylinder :

Let radius of the base (cross-section) be *r* unit and height be *h* cm.

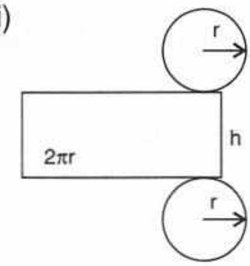


⇒

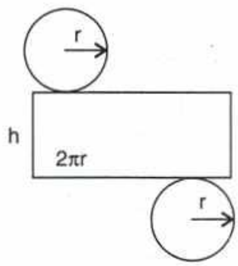
(i)



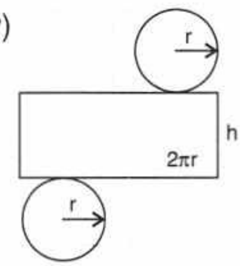
(ii)



(iii)



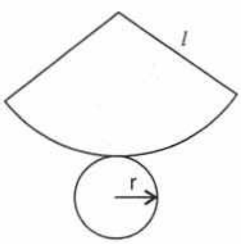
(iv)



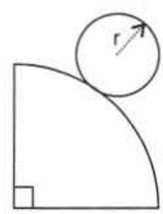
, etc.

4. Nets of cone :

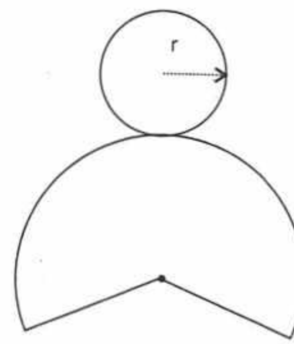
(i)



(ii)

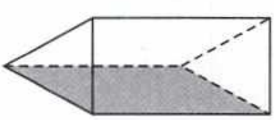


(iii)

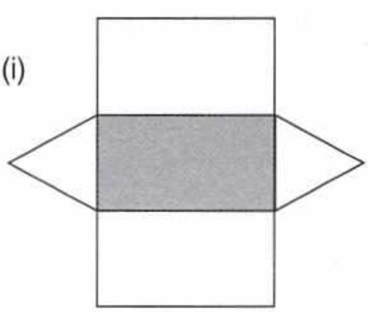


5. Nets of prism :

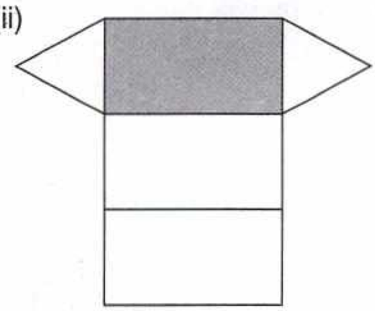
(a) **Triangular prism**



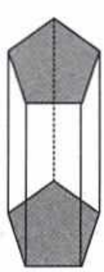
(i)



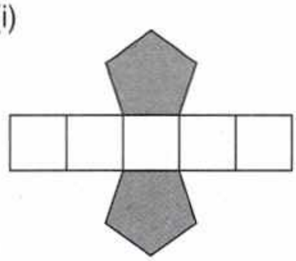
(ii)



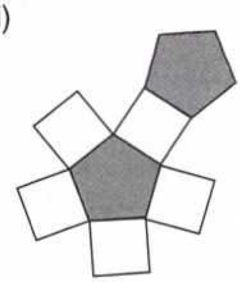
(b) **Pentagonal prism**



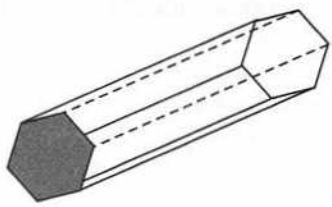
(i)

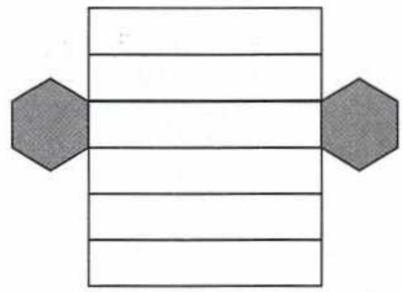


(ii)



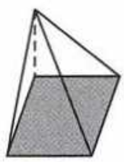
(c) **Hexagonal prism**



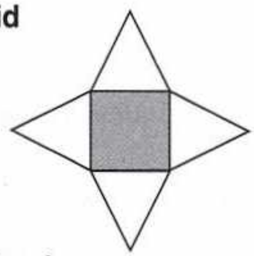


6. Nets of pyramid :

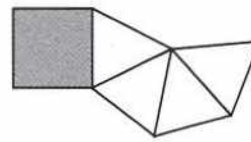
(a) **Square or rectangular pyramid**



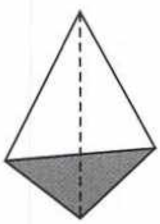
(i)



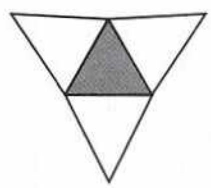
(ii)



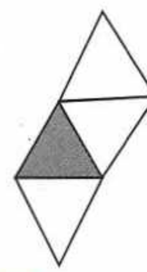
(b) **Triangular pyramid (Tetrahedron)**



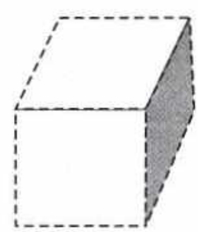
(i)



(ii)



31.4 DRAWING OBLIQUE SKETCHES OF CUBE AND CUBOID



The figure, given alongside, shows a cube and gives a clear idea of how does the cube look ?

In the figure, all the sides drawn do not have equal lengths, still one can easily recognise it as a cube.

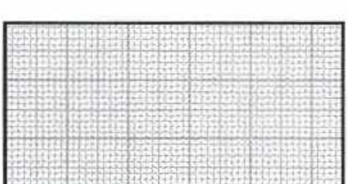
Such a sketch of a solid is called an **oblique sketch**.

(a) *To draw an oblique sketch of a cube* :

Let we are to draw an oblique sketch of a cube with sides 4 cm × 4 cm × 4 cm *i.e.* each edge of the cube is 4 cm.

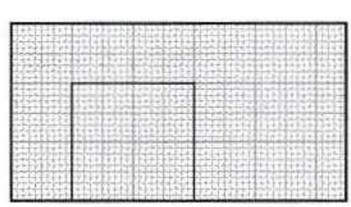
Step 1 :

Take a squared (graph) paper.



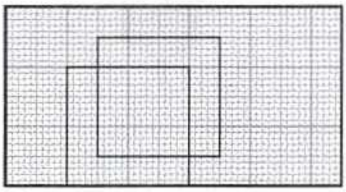
Step 2 :

Draw the *front face*.



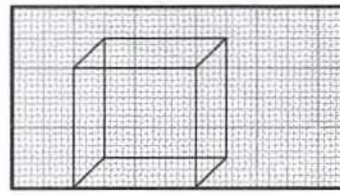
Step 3 :

Draw the *opposite face* of the same size as that of front face.



Step 4 :

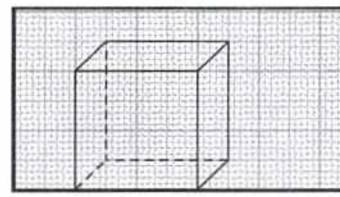
Join the corresponding corners.



Step 5 :

Draw the figure again with hidden edges dotted.

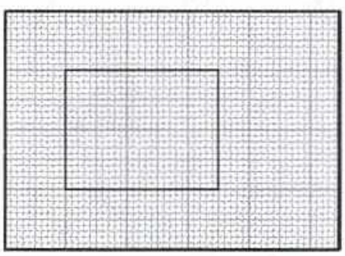
[It is a convention that we show hidden edges by dotted lines].



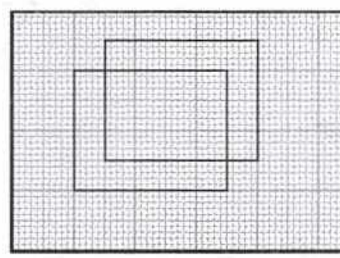
(b) *To draw oblique sketch of a cuboid* :

Let the sides of the cuboid be 5 cm × 4 cm × 3 cm.

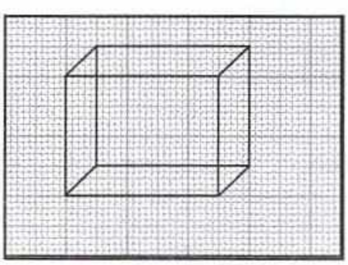
Step 1 :



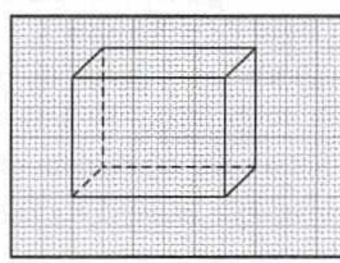
Step 2 :



Step 3 :



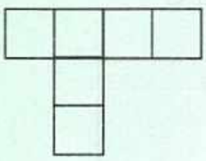
Step 4 :



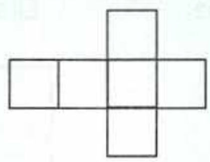
EXERCISE 31

1. Identify the nets which can be used to form cubes :

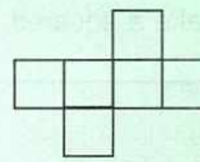
(i)



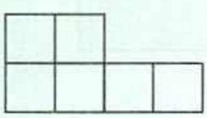
(ii)



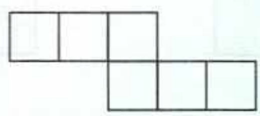
(iii)



(iv)



(v)



2. Draw at least three different nets for making cube.
3. The dimensions of a cuboid are 6 cm, 4 cm and 3 cm. Draw two different types of oblique sketches for this cuboid.
4. Two cubes, each with 3 cm edge, are placed side by side to form a cuboid. For this cuboid, draw an oblique sketch.
5. The figures, given below, show shadows of some 3D objects, when seen under the lamp of an overhead projector :

(i) 
A square

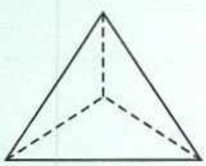
(ii) 
A rectangle

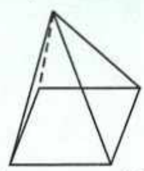
(iii) 
A circle

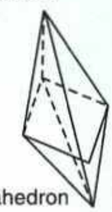
(iv) 
a triangle

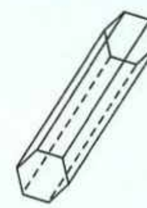
In each case, name the object

6. Look at the solids, drawn below, and fill the given chart.

(i) 
Triangular pyramid

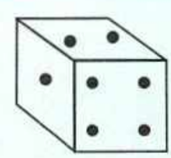
(ii) 
Square pyramid

(iii) 
Hexahedron

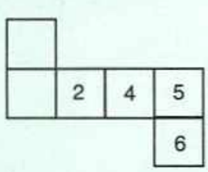
(iv) 
Hexagonal

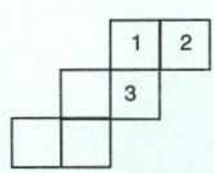
Polyhedon	Faces (F)	Vertices (V)	Edges (E)	F – E + V
(a) *Triangular pyramid*				
(b) *Square pyramid*				
(c) *Hexahedron*				
(d) *Hexagonal prism*				

7. Dice are cubes with dot or dots on each face. Opposite faces of a die always have a total of seven on them.



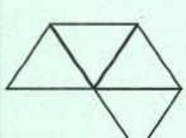
Below are given two nets to make dice (cube), the numbers inserted in each square indicate the number of dots in it.

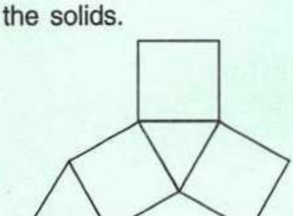
(i) 

(ii) 

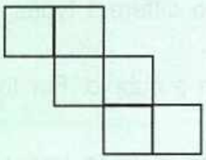
Insert suitable numbers in each blank so that numbers in opposite faces of the die have a total of seven dots.

8. The following figures represent nets of some solids. Name the solids.

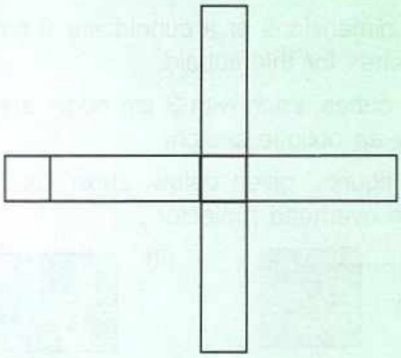
(i) 

(ii) 

(iii)



(iv)



Theme – 5
MENSURATION

PERIMETER AND AREA OF PLANE FIGURES 32

32.1 INTRODUCTION

In mensuration, we deal with measurements of length, area, volume, surface area, etc. Knowledge of mensuration is of great use in our day-to-day life, specially, for instance, when we buy :

(i) **cloth** for shirts by **length,**

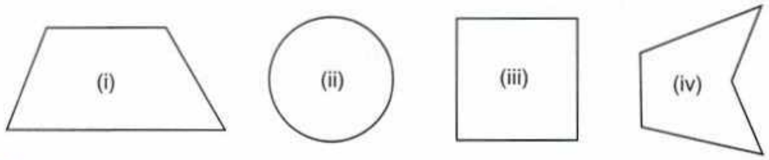
(ii) **a plot of land** by **area**

(iii) **milk, petrol,** etc., by **volume** and so on.

32.2 CLOSED FIGURE

Any geometrical plane figure bounded by lines (straight or curved) is called a **plane closed figure**.

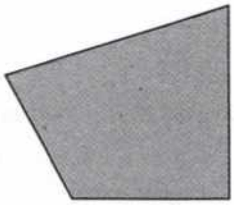
Each of the following figures is a plane closed figure.



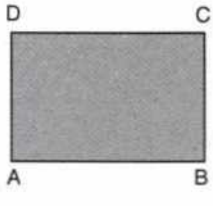
, etc.

32.3 REGION

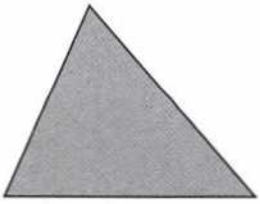
Consider the figure as shown alongside. The part of the plane enclosed by this figure is known as the interior of the figure. The interior of the figure alongwith its boundary is called **region of the figure**.



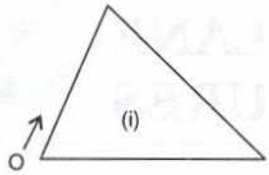
Consider the rectangle ABCD, shown alongside. This rectangle encloses a certain part of the plane which is called the interior of the rectangle. The interior part of the rectangle alongwith its boundary is called region of the rectangle or rectangular region.

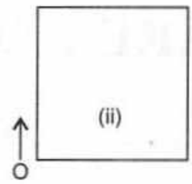


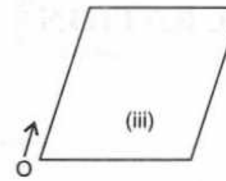
In the same way, the shaded portion (interior of the triangle) alongwith its boundary is known as triangular region.



32.4 CONCEPT OF PERIMETER





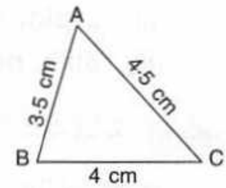


Consider each of the figures, given above. In each case, start moving from O along the line segments to reach again at the same point O by making a complete round, then the distance covered in making one full round is the perimeter of the figure.

Perimeter :

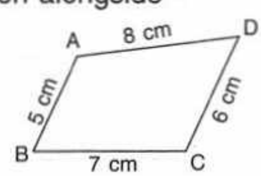
The perimeter of a closed figure is the length of its boundary.

For example :



1. **Perimeter of Δ ABC** given alongside
   = Length of the boundary of Δ ABC
   = Length of AB + length of BC + length of CA
   = 3·5 cm + 4 cm + 4·5 cm = **12 cm**

2. **Perimeter of the plane figure (quadrilateral) ABCD**, given alongside
   = AB + BC + CD + DA
   = 5 cm + 7 cm + 6 cm + 8 cm = **26 cm**



(ii) **Unit of Perimeter :**

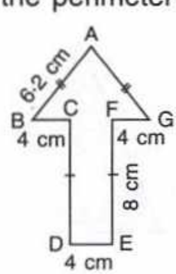
The unit of perimeter is the same as the unit of length, i.e. centimetre (cm), metre (m), etc.

1. 1 cm = $\frac{1}{100}$ m *and* 1 m = 100 cm
2. *For finding the perimeter of any plane-figure, convert each length into the same unit,*
   e.g. if the lengths of the sides of a triangular figure are 80 cm, 1·2 m and 95 cm,
   its ***perimeter*** = 80 cm + 1·2 m + 95 cm
   = 80 cm + 120 cm + 95 cm (1·2 m = 1·2 × 100 cm = 120 cm)
   = **295 cm**
   OR, ***perimeter*** *of the given triangle* = 80 cm + 1·2 m + 95 cm
   = 0·8 m + 1·2 m + 0·95 m = **2·95 m**

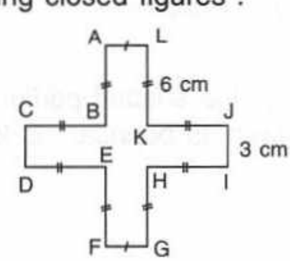
Example 1 :

Find the perimeter of each of the following closed figures :

(i)



(ii)



Solution :

(i) **Required perimeter**

$= AB + BC + CD + DE + EF + FG + GA$

$= 6{\cdot}2\text{ cm} + 4\text{ cm} + 8\text{ cm} + 4\text{ cm} + 8\text{ cm} + 4\text{ cm} + 6{\cdot}2\text{ cm}$

$=$ **40·4 cm**

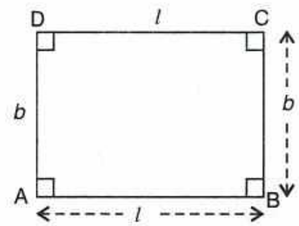
(ii) **Required perimeter**

$= AB + BC + CD + DE + EF + FG + GH + HI + IJ + JK + KL + LA$

$= 6\text{ cm} + 6\text{ cm} + 3\text{ cm} + 6\text{ cm} + 6\text{ cm} + 3\text{ cm} + 6\text{ cm} + 6\text{ cm} + 3\text{ cm} + 6\text{ cm} + 6\text{ cm} + 3\text{ cm}$

$=$ **60 cm**

32.5 PERIMETER OF A RECTANGLE



The sum of all the sides of a rectangle is called its perimeter.

The figure, given alongside, shows a rectangle ABCD in which

side AB = side DC = l (length of the rectangle)

and, side BC = side AD = b (breadth of the rectangle)

∴ **Perimeter (P) of this rectangle**

= Sum of the lengths of its sides

$= AB + BC + CD + DA$

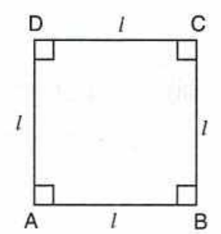
$= l + b + l + b$

$= 2l + 2b = \mathbf{2(l + b)}$

$P = 2(l + b) \Rightarrow$ (i) $l = \frac{P}{2} - b$ *i.e.* length $= \frac{\text{Perimeter}}{2}$ − breadth

(ii) $b = \frac{P}{2} - l$ *i.e.* breadth $= \frac{\text{Perimeter}}{2}$ − length

32.6 PERIMETER OF A SQUARE



A square has all the four sides equal to each other.

∴ **Perimeter of the square ABCD**

$= AB + BC + CD + DA$

$= l + l + l + l = 4l =$ **4 × side**

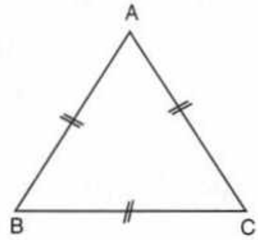
Perimeter of a square = 4 × side

$\Rightarrow$ its side $= \frac{\text{Perimeter}}{4}$.

32.7 PERIMETER OF AN EQUILATERAL TRIANGLE

In an equilateral triangle ABC, all its sides are equal

i.e. AB = BC = CA



∴ **Perimeter of equilateral triangle ABC**

= AB + BC + CA

= AB + AB + AB [∵ AB = BC = CA]

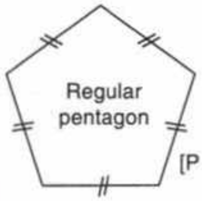
= 3 × AB

= **3 × side of the equilateral triangle**

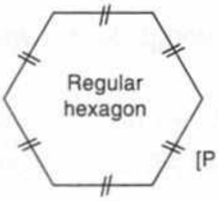
> In case of an equilateral triangle
>
> Perimeter = 3 × side and side = $\frac{\text{Perimeter}}{3}$.

In the same way, consider the following figures :

(i)



(ii)



Example 2 :

Find the perimeter of a rectangle whose :

(i) length = 50 cm and breadth = 40 cm

(ii) length = 8 m and breadth = 5 m

(iii) length = 6 m and breadth = 80 cm

(iv) length = 4·8 m and breadth = 3·6 m

Solution :

(i) ∵ Length = 50 cm and breadth = 40 cm

∴ **Perimeter** = 2(length + breadth)

= 2(50 cm + 40 cm)

= 2 × 90 cm = 180 cm = $\frac{180}{100}$ m = **1·8 m** **(Ans.)**

(ii) ∵ Length = 8 m and breadth = 5 m

∴ **Perimeter** = 2(length + breadth)

= 2(8 m + 5 m)

= 2 × 13 m = **26 m** **(Ans.)**

(iii) ∵ Length = 6 m and breadth = 80 cm = $\frac{80}{100}$ m = 0·8 m

∴ **Perimeter** = 2(length + breadth)

= 2(6 m + 0·8 m)

= 2 × 6·8 m = **13·6 m** **(Ans.)**

(iv) $\because$ Length = 4·8 m and breadth = 3·6 m

$\therefore$ **Perimeter** = 2(length + breadth)

= 2(4·8 m + 3·6 m)

= 2 × 8·4 m = **16·8 m** **(Ans.)**

***Example* 3 :**

If P denotes perimeter of a rectangle, l denotes its length and b denotes its breadth, find :

(i) l, if P = 48 cm and b = 10 cm

(ii) b, if P = 3·2 m and l = 60 cm

***Solution* :**

(i) **Length, l** $= \frac{P}{2} - b$

$= \frac{48}{2}$ cm − 10 cm = 24 cm − 10 cm = **14 cm** **(Ans.)**

(ii) **Breadth, b** $= \frac{P}{2} - l$

$= \frac{3 \cdot 2}{2}$ m − 0·6 m [$\because$ 60 cm = $\frac{60}{100}$ m = 0·6 m]

= 1·6 m − 0·6 m = **1·0 m** **(Ans.)**

***Example* 4 :**

Find :

(i) the perimeter of a square whose each side is 2·8 m.

(ii) the side of the square whose perimeter is 88 cm.

***Solution* :**

(i) $\because$ Side of the square = 2·8 m

$\therefore$ Its **perimeter** = 4 × side

= 4 × 2·8 m = **11·2 m** **(Ans.)**

(ii) $\because$ Perimeter of the square = 88 cm

$\therefore$ Its **side** $= \frac{\text{Perimeter}}{4}$

$= \frac{88 \text{ cm}}{4}$ = **22 cm** **(Ans.)**

***Example* 5 :**

A square field has each side 80 m whereas a rectangular field has length = 120 m and breadth 60 m. Which of these two fields has greater perimeter and by how much ?

Solution :

Perimeter of the square field = 4 × side

= 4 × 80 m = 320 m

Perimeter of the rectangular field= 2 × (length + breadth)

= 2 × (120 m + 60 m)

= 2 × 180 m = 360 m

∴ **Rectangular field has greater perimeter by 360 m – 320 m = 40 m (Ans.)**

Example 6 :

A rectangular field has length = 200 m and breadth = 160 m. Find :

(i) the perimeter of the field.

(ii) the length of fence of this field.

(iii) the cost of fencing the field at the rate of ₹ 50 per metre.

Solution :

(i) **The perimeter of the field** = 2 × (length + breadth)

= 2 × (200 m + 160 m)

= 2 × 360 m = **720 m** **(Ans.)**

(ii) **The length of fence** = The perimeter of the rectangular field

= **720 m** **(Ans.)**

(ii) **The cost of fence** = Length of fence × Rate of fence

= 720 m × ₹ 50 per metre

= **₹ 36000** **(Ans.)**

Example 7 :

Each side of a square field is 60 m. Find the cost of fencing this square field at the rate of ₹ 150 per metre.

Solution :

∵ Perimeter of the square field = 4 × its side

= 4 × 60 m = 240 m

∴ Length of required fence = 240 m

The cost of fence = its length × its rate

= 240 m × ₹ 150 per metre

= **₹ 36000** **(Ans.)**

Example 8 :

For a rectangular playground, its length is 80 m and its breadth = 60 m. Find :

(i) the distance moved by a boy to make one complete round of the field.

(ii) the distance moved by the same boy to make ten complete rounds of this field.

Solution :

(i) **Distance covered by the boy to make one complete round of the field**

= Perimeter of the field

= 2 × (length + breadth)

= 2 × (80 m + 60 m) = 2 × 140 m = **280 m** **(Ans.)**

(ii) **Distance covered by the boy to make ten complete rounds of this field**

= 280 m × 10 = **2800 m** **(Ans.)**

Example 9 :

A makes four full rounds of a square field of side 100 m. B also makes four full rounds of a rectangular field 150 m long and 55 m wide. Find, who covers larger distance and by how much ?

Solution :

∵ Distance covered by A in one full round

= Perimeter of the square field

= 4 × side = 4 × 100 m = 400 m

⇒ Distance covered by A in four full rounds

= 4 × 400 m = 1600 m

∵ Distance covered by B in one full round

= Perimeter of the rectangular field

= 2 × (length + breadth)

= 2 × (150 m + 55 m)

= 2 × 205 m = 410 m

⇒ Distance covered by B in four full rounds

= 4 × 410 m = **1640 m**

Clearly, **B covers larger distance** and by 1640 m – 1600 m = **40 m** **(Ans.)**

Example 10 :

The length of a rectangular field is thrice of its breadth. A boy ran along the boundary of this field. In making 6 rounds of the field, he covered a distance of 6 km. Find the length of the field.

Solution :

Let the breadth of the field = x m

∴ its length = $3x$ m

And, its perimeter = 2 × (length + breadth)

= $2 \times (3x + x) = 2 \times 4x = 8x$ m

∴ Distance covered in 1 round = $8x$ m

⇒ Distance covered in 6 rounds = $6 \times 8x$ m = $48x$ m

$\therefore$ $48x = 6000$ [$\because$ 6 km = 6 × 1000 m = 6000 m]

$\Rightarrow$ $x = \dfrac{6000}{48} = 125$

$\therefore$ The length of the rectangular field

$= 3x$ m = 3 × 125 m = **375 m** **(Ans.)**

***Example* 11 :**

Find the perimeter of :

(i) an equilateral triangle of side 13 cm.

(ii) an isosceles triangle with each equal side = 10 cm and the third side = 15 cm.

(iii) a regular pentagon with side = 12 cm.

(iv) a regular hexagon with side = 9 cm.

***Solution* :**

(i) **The perimeter of equilateral triangle** = 3 × side

= 3 × 13 cm = **39 cm** **(Ans.)**

(ii) **Required perimeter** = 10 cm + 10 cm + 15 cm

= **35 cm** **(Ans.)**

(iii) **Perimeter of given pentagon** = 5 × side

= 5 × 12 cm = **60 cm** **(Ans.)**

(iv) **Perimeter of given hexagon** = 6 × 9 cm

= **54 cm** **(Ans.)**

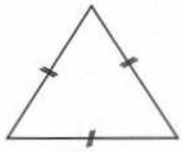
32.8 SHAPES OF DIFFERENT KINDS WITH THE SAME PERIMETER

Consider an equilateral triangle with each side = 12 cm

The perimeter of the triangle

= 3 × side

= 3 × 12 cm = 36 cm

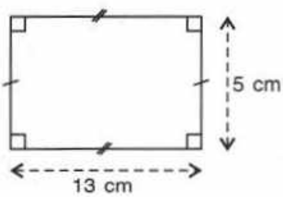


Now, consider a rectangle with length = 13 cm and breadth = 5 cm

Its perimeter = 2 × (length + breadth)

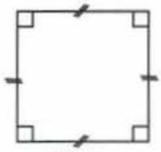
= 2 × (13 cm + 5 cm)

= 2 × 18 cm = 36 cm



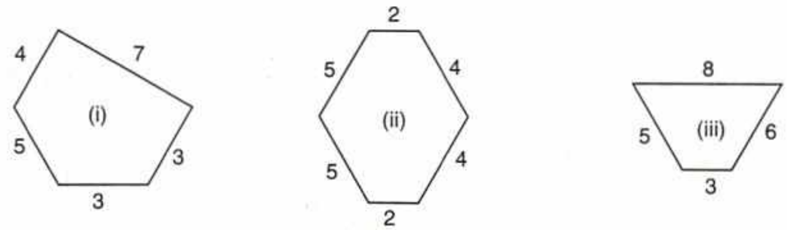
Further, consider a square of side 9 cm,

Its perimeter = 4 × 9 cm = 36 cm



In the example, given above, we find that even if the dimensions (length, breadth, etc.) of different figures are not equal, their perimeters are same.

Now consider the following shapes which have sides of different lengths but same perimeter.



P = 22 units   P = 22 units   P = 22 units

***Example* 12 :**

Each side of an equilateral triangle is 20 cm. Find the perimeter of the triangle. A square has its perimeter same as that of the above triangle. Find the side of the square.

***Solution* :**

$\because$ Each side of the given equilateral triangle = 20 cm

$\therefore$ Perimeter of the triangle = 3 × side

= 3 × 20 cm = 60 cm

$\Rightarrow$ Perimeter of the square = 60 cm

$\Rightarrow$ 4 × the side of square = 60 cm

$\Rightarrow$ **The side of the square** = $\frac{60 \text{ cm}}{4}$ = **15 cm** **(Ans.)**

***Example* 13 :**

A wire is bent in the form of a square of side 25 cm. Find the length of the wire. If the same wire is bent in the form of a rectangle of length 30 cm; find the width of the rectangle.

***Solution* :**

Length of the wire = 4 × side of the square

= 4 × 25 cm = **100 cm** **(Ans.)**

Now, length of the wire = perimeter of the rectangle

= 2 × (length + breadth)

$\Rightarrow$ 100 cm = 2 × (30 cm + breadth)

$\Rightarrow$ 100 cm = 60 cm + 2 × breadth

$\Rightarrow$ 100 cm − 60 cm = 2 × breadth

$\Rightarrow$ 40 cm = 2 × breadth

$\Rightarrow$ **breadth** = $\frac{40}{2}$ cm = **20 cm** **(Ans.)**

***Example* 14 :**

Perimeter of a regular pentagon is same as the perimeter of a regular hexagon. If each side of the given hexagon is 15 cm, find the side of the given pentagon.

***Solution* :**

Perimeter of regular hexagon = 6 × length of the side of given hexagon

= 6 × 15 cm = 90 cm

Clearly, perimeter of the given pentagon

= 90 cm

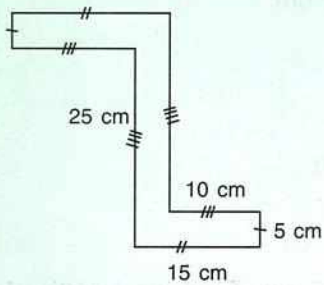
⇒ 5 × side of the given pentagon = 90 cm

⇒ **side of the given pentagon** = $\frac{90}{5}$ cm = **18 cm** **(Ans.)**

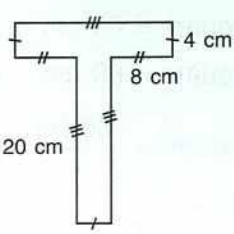
EXERCISE 32(A)

1. What do you understand by a plane closed figure ?
2. The interior of a figure is called region of the figure. Is this statement true ?
3. Find the perimeter of each of the following closed figures :

(i)



(ii)



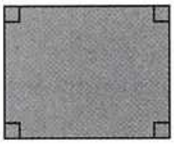
4. Find the perimeter of a rectangle whose :
   (i) length = 40 cm and breadth = 35 cm
   (ii) length = 10 m and breadth = 8 m
   (iii) length = 8 m and breadth = 80 cm
   (iv) length = 3·6 m and breadth = 2·4 m
5. If P denotes perimeter of a rectangle, l denotes its length and b denotes its breadth, find :
   (i) l, if P = 38 cm and b = 7 cm
   (ii) b, if P = 3·2 m and l = 100 cm
   (iii) P, if l = 2 m and b = 75 cm
6. Find the perimeter of a square whose each side is 1·6 m.
7. Find the side of the square whose perimeter is 5 m.
8. A square field has each side 70 m whereas a rectangular field has length = 50 m and breadth = 40 m. Which of the two fields has greater perimeter and by how much ?
9. A rectangular field has length = 160 m and breadth = 120 m. Find :
   (i) the perimeter of the field.
   (ii) the length of fence required to enclose the field.
   (iii) the cost of fencing the field at the rate of ₹ 80 per metre.

10. Each side of a square plot of land is 55 m. Find the cost of fencing the plot at the rate of ₹ 32 per metre.
11. Each side of a square field is 70 m. How much distance will a boy walk in order to make :
    (i) one complete round of this field ?
    (ii) 8 complete rounds of this field ?
12. A school playground is rectangular in shape with length = 120 m and breadth = 90 m. Some school boys run along the boundary of the playground and make 15 complete rounds in 45 minutes. How much distance did they run during this period.
13. Mohit makes 8 full rounds of a rectangular field with length = 120 m and breadth = 75 m. John makes 10 full rounds of a square field with each side 100 m. Find who covers larger distance and by how much ?
14. The length of a rectangle is twice of its breadth. If its perimeter is 60 cm, find its length.
15. Find the perimeter of :
    (i) an equilateral triangle of side 9·8 cm.
    (ii) an isosceles triangle with each equal side = 13 cm and the third side = 10 cm.
    (iii) a regular pentagon of side 8·2 cm.
    (iv) a regular hexagon of side 6·5 cm.
16. An equilateral triangle and a square have equal perimeters. If side of the triangle is 9·6 cm; what is the length of the side of the square ?
17. A rectangle with length = 18 cm and breadth = 12 cm has same perimeter as that of a regular pentagon. Find the side of the pentagon.
18. A regular pentagon of each side 12 cm has same perimeter as that of a regular hexagon. Find the length of each side of the hexagon.
19. Each side of a square is 45 cm and a rectangle has length 50 cm. If the perimeters of both (square and rectangle) are same, find the breadth of the rectangle.
20. A wire is bent in the form of an equilateral triangle of each side 20 cm. If the same wire is bent in the form of a square, find the side of the square.

32.9 CONCEPT OF AREA

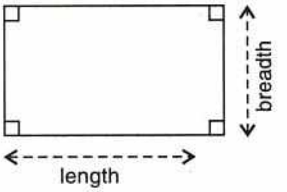
Area of any plane surface is the length of its boundary plus its interior.

For example :



The perimeter of a rectangle together with its interior is called area of the rectangle.

32.10 AREA OF A RECTANGLE



In general, the **larger side** of a rectangle is called its **length** and the shorter side is called its **breadth**.

Area of the rectangle = its length × its breadth
i.e. $A = l \times b$

32.11 AREA OF A SQUARE

A square is a rectangle whose length is equal to its breadth.

$\therefore$ Area of a square = Length × breadth

= length × length

= $(\text{length})^2 = (\text{side})^2$

1. Area of a rectangle = its length × its breadth

i.e., $A = l \times b$

$\Rightarrow \quad l = \frac{A}{b} \quad$ *i.e.* $\quad$ length = $\frac{\text{Area}}{\text{Breadth}}$

and, $\quad b = \frac{A}{l} \quad$ *i.e.* $\quad$ breadth= $\frac{\text{Area}}{\text{Length}}$

Also, in case of a rectangle :

(i) if unit of length is cm and the unit of breadth is also cm, then unit of area is cm^2 (square centimetre)

(ii) if unit of length is m and the unit of breadth is also m, then unit of area is m^2 (square metre) and so on.

2. Area of square = $(\text{side})^2$

$\Rightarrow$ its side = $\sqrt{\text{Area of the square}}$

Also, in case of a square :

(i) if unit of length of its side is cm, then unit of its area is cm^2 (square cm)

(ii) if unit of length of its side = m, then unit of its area is m^2 (square metre).

***Example* 15 :**

Find the area of a rectangle whose :

(i) length = 12 cm and breadth = 6 cm

(ii) length = 1·4 m and breadth = 0·5 m

(iii) length = 3 m and breadth = 80 cm

(iv) length = 15 cm and breadth = 130 mm

***Solution* :**

(i) Length = 12 cm and breadth = 6 cm

$\Rightarrow$ **Area of the rectangle** = length × breadth

= 12 cm × 6 cm = **72 cm^2** **(Ans.)**

(ii) $\because$ Length = 1·4 m and breadth = 0·5 m

$\therefore$ **Area of the rectangle** = length × breadth

$= 1{\cdot}4 \text{ m} \times 0{\cdot}5 \text{ m} = \mathbf{0{\cdot}70 \text{ m}^2}$ **(Ans.)**

(iii) $\because$ Length = 3 m and breadth = 80 cm

$\therefore$ **Area of the rectangle** = length × breadth

$= 3 \text{ m} \times 0{\cdot}8 \text{ m}$ $\quad [\because 80 \text{ cm} = \frac{80}{100} \text{ m} = 0{\cdot}8 \text{ m}]$

$= \mathbf{2{\cdot}4 \text{ m}}$ **(Ans.)**

(iv) $\because$ Length = 15 cm and

breadth $= 130 \text{ mm} = \frac{130}{10} \text{ cm} = 13 \text{ cm}$

$\therefore$ **Area of the rectangle** = length × breadth

$= 15 \text{ cm} \times 13 \text{ cm} = \mathbf{195 \text{ cm}^2}$ **(Ans.)**

Example 16 :

Find the area of a square whose each side is :

(i) 3 m (ii) 2·4 cm (iii) 1·8 m

Solution :

(i) **Area of the square** = (its side)^2

$= (3 \text{ m})^2 = 3 \text{ m} \times 3 \text{ m} = \mathbf{9 \text{ m}^2}$ **(Ans.)**

(ii) **Area of the square** = (its side)^2

$= (2{\cdot}4 \text{ cm})^2 = 2{\cdot}4 \text{ cm} \times 2{\cdot}4 \text{ cm} = \mathbf{5{\cdot}76 \text{ cm}^2}$ **(Ans.)**

(iii) **Area of the square** = (its side)^2

$= (1{\cdot}8 \text{ m})^2 = 1{\cdot}8 \text{ m} \times 1{\cdot}8 \text{ m} = \mathbf{3{\cdot}24 \text{ m}^2}$ **(Ans.)**

Example 17 :

If A denotes area of a rectangle, l represents its length and b represents its breadth, find :

(i) l, if A = 45 cm^2 and b = 10 cm

(ii) b, if A = 77 cm^2 and l = 22 cm

Solution :

(i) $l = \frac{A}{b}$ $\quad [\because A = l \times b \Rightarrow l = \frac{A}{b}]$

$\Rightarrow l = \frac{45 \text{ cm}^2}{10 \text{ cm}} = \mathbf{4{\cdot}5 \text{ cm}}$ **(Ans.)**

(ii) $b = \frac{A}{l}$ $\quad [\because A = l \times b \Rightarrow b = \frac{A}{l}]$

$\Rightarrow b = \frac{77 \text{ cm}^2}{22 \text{ cm}} = \frac{7}{2} \text{ cm} = \mathbf{3{\cdot}5 \text{ cm}}$ **(Ans.)**

Example 18 :

Each side of a square is 9 cm. Find its :

(i) perimeter (ii) area.

Solution :

(i) **Perimeter** = $4 \times$ side

$= 4 \times 9$ cm = **36 cm** **(Ans.)**

(ii) **Area** = $(\text{side})^2$

$= (9 \text{ cm})^2 = 9 \text{ cm} \times 9 \text{ cm} =$ **81 cm²** **(Ans.)**

Example 19 :

The perimeter of a square field is 112 m, find its area.

Solution :

Given perimeter of the square field = 112 m

and, we know that the perimeter of the square = $4 \times$ its side

$\therefore\ 4 \times s = 112 \text{ m}$ *i.e.* $s = \frac{112 \text{ m}}{4} = 28 \text{ m}$

$\Rightarrow$ **Area of the square field** = $(\text{its side})^2$

$= (28 \text{ m})^2$

$= 28 \text{ m} \times 28 \text{ m} =$ **784 m²** **(Ans.)**

Example 20 :

The area of a square field is 1600 m². Find its perimeter.

Solution :

$\because$ Area of the square field = 1600 m²

$\therefore$ Its each side = $\sqrt{1600}$ m = 40 m

$\Rightarrow$ **Perimeter of the square field** = $4 \times$ side

$= 4 \times 40$ m = **160 m** **(Ans.)**

Example 21 :

For a rectangular piece of paper, area = 96 cm² and length = 12 cm, find its perimeter.

Solution :

$\because$ Area of a rectangle = its length $\times$ its breadth

$\Rightarrow$ 96 cm² = 12 cm $\times$ breadth

$\Rightarrow$ breadth = $\frac{96 \text{ cm}^2}{12 \text{ cm}} = 8$ cm

And, **perimeter** = $2 \times (l + b)$

$= 2 \times (12 \text{ cm} + 8 \text{ cm}) = 2 \times 20 \text{ cm} =$ **40 cm** **(Ans.)**

Example 22 :

Find the perimeter of a rectangle whose area is 650 cm^2 and its breadth is 13 cm.

Solution :

$\because$ Area of a rectangle= 650 cm^2 and its breadth = 13 cm

$$\therefore \quad \textbf{Its length} = \frac{\text{Area}}{\text{Breadth}}$$

$$= \frac{650 \text{ cm}^2}{13 \text{ cm}} = 50 \text{ cm}$$

$$\Rightarrow \quad \textbf{Perimeter of the rectangle} = 2 \times (\text{length} + \text{breadth})$$

$$= 2 \times (50 \text{ cm} + 13 \text{ cm})$$

$$= 2 \times 63 \text{ cm} = \textbf{126 cm}$$ **(Ans.)**

Example 23 :

What will happen to the area of rectangle, if its length and breadth both are doubled ?

Solution :

Let the original length of the rectangle = l and its original breadth = b

$\therefore$ Its orginal area = length × breadth

i.e. $A = l \times b$

Increased length = $2l$

and, increased breadth = $2b$

$\therefore$ New area $= 2l \times 2b$

$= 4 \times l \times b$

= 4 × area of original rectangle

$\Rightarrow$ **Area of the new rectangle = 4 times the area of original rectangle. (Ans.)**

Example 24 :

Find the change in area of a rectangle, if its length is trebled and breadth is doubled.

Solution :

Let original length of the rectangle = l and its original breadth = b

$\therefore$ Its original area = length × breadth

i.e. $A = l \times b$

Since, increased length = $3l$

and, increased breadth = $2b$

$\therefore$ New area $= 3l \times 2b = 6 \times l \times b = 6A$ $[\because A = l \times b]$

$\therefore$ **The change** (increase) **in area**

= New area − original area

$= 6A - A$

$= 5A$ = **5 times the original area** **(Ans.)**

Example 25 :

What will happen to the area of a square field when its each side is :

(i) doubled (ii) trebled (iii) halved?

Solution :

⇒ **Original area of the square** = $(\text{side})^2 = s^2$ *i.e.* $A = s^2$

(i) ∵ New length = $2s$

∴ New area $= (2s)^2$

$= 2s \times 2s = 4s^2 = 4A$

⇒ **The area of the new square**

= 4 times the area of the given square **(Ans.)**

(ii) ∵ New length = $3s$

∴ New area $= (3s)^2$

$= 3s \times 3s = 9s^2 = 9A$

⇒ **The area of the new square**

= 9 times the area of the given square **(Ans.)**

(ii) ∵ New length = $\frac{s}{2}$

∴ New area $= \left(\frac{s}{2}\right)^2$

$= \frac{s}{2} \times \frac{s}{2} = \frac{1}{4} \times s^2 = \frac{1}{4} \times A$

⇒ **The area of the new square**

$= \frac{1}{4}$ **times the area of the given square** **(Ans.)**

Example 26 :

A room is 6 m long and 4 m wide. How many rectangular tiles will be required to cover the floor of the room, if each tile is 25 cm long and 20 cm wide.

Solution :

Area to be covered by the tiles = Area of the floor of the room

= its length × its breadth

= 6 m × 4 m

= 600 cm × 400 cm

= 240000 cm^2

∵ Area of each rectangular tile = 25 cm × 20 cm = 500 cm^2

∴ **Required number of tiles** = $\frac{\text{Area to be covered by tiles}}{\text{Area of each tile}}$

$= \frac{240000 \text{ cm}^2}{500 \text{ cm}^2} = \mathbf{480}$ **(Ans.)**

Example 27 :

The floor of a big hall is in the shape of a square of side 8 m. The floor is to be covered by square tiles each of side 40 cm. Find the number of tiles required.

Solution :

$\because$ Area of the floor of the square hall $= (\text{side})^2$

$= (8 \text{ m})^2 = 8 \text{ m} \times 8 \text{ m} = 64 \text{ m}^2$

And, area of each square tile $= (\text{side})^2$

$= (40 \text{ cm})^2$

$= 40 \text{ cm} \times 40 \text{ cm}$

$= \frac{40}{100} \text{ m} \times \frac{40}{100} \text{ m}$

$= 0{\cdot}4 \text{ m} \times 0{\cdot}4 \text{ m} = 0{\cdot}16 \text{ m}^2$

$\therefore$ **Required number of tiles** $= \frac{\text{Area of the floor of the hall}}{\text{Area of each tile}}$

$= \frac{64 \text{ m}^2}{0{\cdot}16 \text{ m}^2} = \frac{64 \times 100}{16} =$ **400** **(Ans.)**

Alternative method :

$\because$ Area of the floor of the hall $= (\text{side})^2$

$= (8 \text{ m})^2$

$= 8 \text{ m} \times 8 \text{ m} = 800 \text{ cm} \times 800 \text{ cm}$

And, area of each square tile $= (\text{side})^2$

$= 40 \text{ cm} \times 40 \text{ cm}$

$\therefore$ **Required number of tiles** $= \frac{\text{Area of the floor of the hall}}{\text{Area of each tile}}$

$= \frac{800 \text{ cm} \times 800 \text{ cm}}{40 \text{ cm} \times 40 \text{ cm}}$

$= 20 \times 20 =$ **400** **(Ans.)**

Example 28 :

The perimeter of a square plot is 200 m. Find :

(i) the area of the plot.

(ii) cost of levelling the plot at the rate of ₹ 48 per square metre.

Solution :

$\because$ Perimeter of the square $= 4 \times$ its side

$\therefore$ $4 \times$ side of the square $= 200$ m

$\Rightarrow$ side of the square $= \frac{200}{4} \text{ m} = 50 \text{ m}$

(i) **The area of the square plot** $= (\text{side})^2$

$= (50 \text{ m})^2$

$= 50 \text{ m} \times 50 \text{ m} =$ **2500 m^2** **(Ans.)**

(ii) **Cost of levelling the plot** = Area of the plot × Rate of levelling

$= 2500 \text{ m}^2 \times ₹\ 48 \text{ per m}^2$

= **₹ 1,20,000** **(Ans.)**

***Example* 29 :**

The perimeter of a rectangular field is 300 m and its length is 90 m. Find :

(i) its breadth

(ii) its area

(iii) cost of ploughing the field at the rate of ₹ 80 per square metre.

***Solution* :**

(i) ∵ Perimeter of a rectangle = 2 × (length + breadth)

⇒ 300 m = 2 × (90 m + breadth)

⇒ 150 m = 90 m + breadth $\quad [\frac{300 \text{ m}}{2} = 150 \text{ m}]$

⇒ 150 m − 90 m = breadth

i.e. **breadth = 60 m** **(Ans.)**

(ii) **Area of the rectangular field** = length × breadth

$= 90 \text{ m} \times 60 \text{ m} =$ **5400 m^2** **(Ans.)**

(iii) **Cost of ploughing the field** = area of the field × rate of ploughing

$= 5400 \text{ m}^2 \times ₹\ 80$ per square metre

= 5400 × ₹ 80

= **₹ 4,32,000** **(Ans.)**

***Example* 30 :**

The cost of flooring a room at ₹ 68 per square metre is ₹ 2040. If the length of the room is 7·5 m, find :

(i) its breadth (ii) its perimeter.

***Solution* :**

∵ Total cost of flooring the room = ₹ 2040

and cost of flooring per square metre = ₹ 68

∴ Area of the room $= \frac{\text{Total cost of flooring}}{\text{Cost of flooring per square metre}}$

$= \frac{2040}{68} \text{ m}^2 = 30 \text{ m}^2$

(i) $\because$ length × breadth = area

$\Rightarrow$ 7·5 m × breadth = 30 m^2

$\Rightarrow$ **breadth** $= \dfrac{30\ m^2}{7{\cdot}5\ m}$ = **4 m** **(Ans.)**

(ii) **Perimeter** = 2 × (length + breadth)

= 2 × (7·5 m + 4 m)

= 2 × 11·5 m = **23 m** **(Ans.)**

EXERCISE 32(B)

1. Find the area of a rectangle whose :
   (i) length = 15 cm and breadth = 6·4 cm
   (ii) length = 8·5 m and breadth = 5 m
   (iii) length = 3·6 m and breadth = 90 cm
   (iv) length = 24 cm and breadth = 180 mm
2. Find the area of a square, whose each side is :
   (i) 7·2 cm (ii) 4·5 m (iii) 4·1 cm
3. If A denotes area of a rectangle, l represents its length and b represents its breadth, find :
   (i) l, if A = 48 cm^2 and b = 6 cm
   (ii) b, if A = 88 m^2 and l = 11 m
4. Each side of a square is 3·6 m; find its
   (i) perimeter (ii) area.
5. The perimeter of a square is 60 m, find :
   (i) its each side
   (ii) its area
   (iii) its new area obtained on increasing each of its sides by 2 m.
6. Each side of a square is 7 m. If its each side be increased by 3 m, what will be the increase in its area.
7. The perimeter of a square field is numerically equal to its area. Find each side of the square.
8. A rectangular piece of paper has area = 24 cm^2 and length = 5 cm. Find its perimeter.
9. Find the perimeter of a rectangle whose area = 2600 m^2 and breadth = 50 m.
10. What will happen to the area of a rectangle, if its length and breadth both are trebled ?
11. Length of a rectangle is 30 m and its breadth is 20 m. Find the increase in its area if its length is increased by 10 m and its breadth is doubled.
12. The side of a square field is 16 m. What will be the increase in its area, if :
    (i) each of its sides is increased by 4 m
    (ii) each of its sides is doubled.
13. Each rectangular tile is 40 cm long and 30 cm wide. How many tiles will be required to cover the floor of a room with length = 4·8 m and breadth = 2·4 m.
14. Each side of a square tile is 60 cm. How many tiles will be required to cover the floor of a hall with length = 50 m and breadth = 36 m.

15. The perimeter of a square plot = 360 m. Find :
    (i) its area.
    (ii) cost of fencing its boundary at the rate of ₹ 40 per metre.
    (iii) cost of levelling the plot at ₹ 60 per square metre.
16. The perimeter of a rectangular field is 500 m and its length = 150 m. Find :
    (i) its breadth.
    (ii) its area.
    (iii) cost of ploughing the field at the rate of ₹ 1.20 per square metre.
17. The cost of flooring a rectangular hall at ₹ 64 per square metre is ₹ 2,048. If the breadth of the hall is 5 m, find :
    (i) its length.
    (ii) its perimeter.
    (iii) cost of fixing a border of very small width along its boundary at the rate of ₹ 60 per metre.
18. The length of a rectangle is three times its breadth. If the area of the rectangle is 1875 sq. cm, find its perimeter.

Theme – 6
STATISTICS

33 DATA HANDLING
(Including Pictograph and Bar Graph)

33.1 INTRODUCTION

Statistics is the science that deals with the collection, classification, tabulation, representation and interpretation of data.

In statistics, numerical facts are collected in the form of numbers.

If we have collected information about the heights of Class 6 children from ten different schools of Delhi, then this information in the form of numbers is called **statistics**.

33.2 DATA

Each number collected for giving required information is called **data**.

Suppose information is required about the number of members in different families of a certain locality. In order to do so, a certain number of families (say, 50 families) of that locality are visited and the information, so collected, is summarized in the form of a table as given below :

No. of members in a family	No. of families
1	3
2	5
3	12
4	22
5 and above	8

Whatever be the method adopted, once the data is collected, it should be put in a suitable form, such that it easily gives a fair idea of the necessary information contained in the data.

In statistics data is collected and used in many ways, by government departments, educational institutions, various companies, etc.

In particular, data on population, food, education, taxes, finances, post and telegraph, agriculture, etc. is of great use. In fact, there is hardly any field left where statistical data is not used.

33.3 RAW DATA

Consider the following list of numbers :

72, 77, 67, 74, 82, 80, 66, 90, 80, 78, 57, 56, 54, 74, 72, 92, 87, 77, 67 and 82.

Each entry in the above list is a numerical fact which is called an observation. Such collection of observations collected initially is called **raw data**.

33.4 REPRESENTATION OF DATA

The raw data can be arranged in two ways :

(i) Ascending order.

(ii) Descending order.

The raw data when put in ascending or descending order of magnitude is called an **array**.

Consider the numbers 15, 23, 16, 28, 29, 45, 52, 47, 32, 30 and 20.

On writing these numbers in ascending order, we get. :

15, 16, 20, 23, 28, 29, 30, 32, 45, 47 and 52.

And on writing these numbers in descending order, we get :

52, 47, 45, 32, 30, 29, 28, 23, 20, 16 and 15.

Let the marks obtained by 30 students of class VI in a class test, out of 100 marks, according to their roll numbers be :

70, 50, 30, 60, 30, 40, 20, 80, 20, 40, 30, 00, 20, 10, 20, 30, 30, 30, 30, 80, 80, 20, 80, 60, 30, 40, 60, 10, 00 and 40.

The data (known as raw data or ungrouped data) as given above can be arranged as :

Roll no.	Marks	Roll no.	Marks	Roll no.	Marks
1	70	11	30	21	80
2	50	12	00	22	20
3	30	13	20	23	80
4	60	14	10	24	60
5	30	15	20	25	30
6	40	16	30	26	40
7	20	17	30	27	60
8	80	18	30	28	10
9	20	19	30	29	00
10	40	20	80	30	40

When the number of data is long, we write them in the form of a table as given below :

Data written in the form of a table are called tabulated data.

Steps of working

1. Form a table with three columns.

   The **first column** is headed by **marks**, the **second column** is headed by **Tally-marks** and the **third column** is headed by **number of students** which is also called **frequency**. See below :

Marks	Tally-marks	No. of students (Frequency)
00		
10		
20		
30		
40		
50		
60		
70		
80		

2. In the first column, write all the marks from lowest to highest.
3. Look at the first value in the given raw data (which is 70) and put a bar (a vertical line or a slightly slant line) in the second column opposite to it.

Now see the second value in the given raw data (which is 50) and mark a bar in the table opposite to the raw data taken *i.e.* against 50.

The process is repeated till all the observations in the given raw data are exhausted. The bars drawn in the second column are known as tally marks and to facilitate, we record these tally marks in bunches of five. The fifth tally mark is drawn diagonally across the first four.

Thus, 5 tally marks = ~~||||~~, 6 tally marks = ~~||||~~ |, 7 tally marks = ~~||||~~ || and so on.

The resulting tabulated data, for the raw data, given above, will be :

Marks	Tally-marks	No. of students
00	||	2
10	||	2
20	~~||||~~	5
30	~~||||~~ |||	8
40	||||	4
50	|	1
60	|||	3
70	|	1
80	||||	4

The way of representing the given raw data in the form of a table, as done above, is known as **frequency distribution**.

For any data given in the first column, the corresponding number in the third column is called its **frequency**.

In the above distribution table, the frequency of 20 is 5, frequency of 60 is 3 and so on.

Now consider the data :

36, 36, 37, 38, 39, 38, 37, 36, 36, 39, 40, 37, 39, 37, 37, 36, 38, 38, 37 and 40.

When the above data are put in the form of a frequency distribution table we get :

Marks	Tally-marks	Frequency
36	~~IIII~~	5
37	~~IIII~~ I	6
38	IIII	4
39	III	3
40	II	2

Example 1 :

For the following data, construct a frequency distribution table :

55, 56, 56, 54, 57, 57, 56, 55, 55, 56, 56, 57, 55, 56, 56, 54, 56, 55, 54, 57, 57, 56, 55, 54 and 55.

Solution :

The required frequency table will be as shown below :

Marks	Tally-marks	Frequency
54	IIII	4
55	~~IIII~~ II	7
56	~~IIII~~ IIII	9
57	~~IIII~~	5
Total		**25**

The frequency distribution obtained here is also known as discrete frequency distribution.

EXERCISE 33(A)

1. Marks scored by 30 students of class VI are as given below :

   38, 46, 33, 45, 63, 53, 40, 85, 52, 75, 60, 73, 62, 22, 69, 43, 45, 33, 47, 41, 29, 43, 37, 49, 83, 44, 55, 22, 35 and 45. State :

   (i) the highest marks scored. (ii) the lowest marks scored.

   (iii) the range of marks.

Range of marks = Highest mark scored − Lowest mark scored.

2. For the following raw data, form a discrete frequency distribution :

30, 32, 32, 28, 34, 34, 32, 30, 30, 32, 32, 34, 30, 32, 32, 28, 32, 30, 28, 30, 32, 32, 30, 28 and 30.

3. Define :

(i) data (ii) frequency of an observation.

4. Rearrange the following raw data in descending order :

5·3, 5·2, 5·1, 5·7, 5·6, 6·0, 5·5, 5·9, 5·8, 6·1, 5·5, 5·8, 5·7, 5·9 and 5·4. Then write the :

(i) highest value (ii) lowest value

(iii) range of values

5. Represent the following data in the form of a frequency distribtuion :

52, 56, 72, 68, 52, 68, 52, 68, 52, 60, 56, 72, 56, 60, 64, 56, 48, 48, 64 and 64.

6. In a study about the number of accidents per day in Delhi, the data collected for 30 days are as follows :

6	3	5	6	4	3	2	5	4	2
4	0	5	3	6	1	5	5	2	6
2	1	2	2	0	5	4	6	1	6

Construct a suitable frequency distribution table.

7. The following data represents the weekly wages (in ₹) of 15 workers in a factory :

900, 850, 800, 850, 800, 750, 950, 900, 950, 800, 750, 900, 750, 800 and 850.

Prepare a frequency distribution table. Then, find,

(i) how many workers are getting less than ₹ 850 per week ?

(ii) how many workers are getting more than ₹ 850 per week ?

8. Using the data, given below, construct a frequency distribution table :

9, 17, 12, 20, 9, 18, 25, 17, 19, 9, 12, 9, 12, 18, 17, 19, 20, 25, 9 and 12.

Now answer the following :

(i) How many numbers are less than 19 ?

(ii) How many numbers are more than 20 ?

(iii) Which of the numbers, given above, is occuring most frequently ?

9. Using the following data, construct a frequency distribution table :

46. 44, 42, 54, 52, 60, 50, 58, 56, 62, 50, 56, 54, 58 and 48.

Now answer the following :

(i) What is the range of the numbers ?

(ii) How many numbers are greater than 50 ?

(iii) How many numbers are between 40 and 50 ?

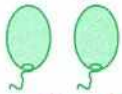
33.5 PICTOGRAPH

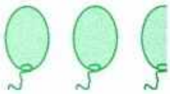
In order to extract some useful information, data are usually depicted by pictures. *The study of numerical data through pictures is known as the pictorial representation of data.* The graph drawn using symbolic pictures is called a pictograph.

The representation of an information through pictures is called **pictograph.**

Let us assume that the picture of a balloon represents 100 balloons *i.e.*

 = 100 balloons, then

 = 2 × 100 balloons = 200 balloons

 $= 2\frac{1}{2} \times 100$ balloons $= \frac{5}{2} \times 100$ balloons = 250 balloons

and so on.

Here, we have taken scale : one balloon = 100 balloons.

Example 2 :

The number of cricket bats sold by a shop during a certain week are given below:

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of bats	24	12	20	32	16	12

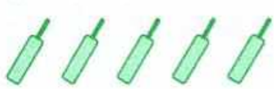
Draw a suitable pictograph.

Solution :

First of all assume a scale.

Since 4 is the largest number by which each of the given data 24, 12, 20, 32, 16 and 12 is completely divisible, we take the scale : one bat in picture = 4 bats.

∴ The required pictograph is as shown below :

Day	Number of bats sold	
Monday		$\because \frac{24}{4} = 6$
Tuesday		$\because \frac{12}{4} = 3$
Wednesday		$\because \frac{20}{4} = 5$
Thursday		$\because \frac{32}{4} = 8$
Friday		$\because \frac{16}{4} = 4$
Saturday		$\because \frac{12}{4} = 3$

Example 3 :

The modes of travelling to school by 160 students are given below :

Mode	By walking	On bicycle	By car	By bus
No. of students	30	50	20	60

Draw a suitable pictograph.

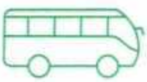
Solution :

Since, every given number data is completely divisible by 10, therefore the scale will be :

 = 10 students (walking)

 = 10 students (using bicycle)

 = 10 students (using car)

 = 10 students (using bus)

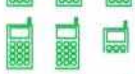
The required pictograph will be as shown below :

Mode	Number of students
By walking	
On bicycles	
By car	
By bus	

Example 4 :

The following pictograph shows the number of mobile sets sold by a store during a week.

The scale used :  = 6 mobile sets.

Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

Using the pictograph shown above, answer the following questions :

(i) How many mobile sets were sold on Tuesday ?

(ii) How many mobile sets were sold on Monday ?

(iii) How many mobile sets were sold during the whole week ?

Solution :

(i) $4 \times 6 = 24$ **(Ans.)**

(ii) $6 \times 6 = 36$ **(Ans.)**

(iii) $(6 + 4 + 2\frac{1}{2} + 7 + 5 + 3\frac{1}{2}) \times 6 = 28 \times 6 = 168$ **(Ans.)**

EXERCISE 33(B)

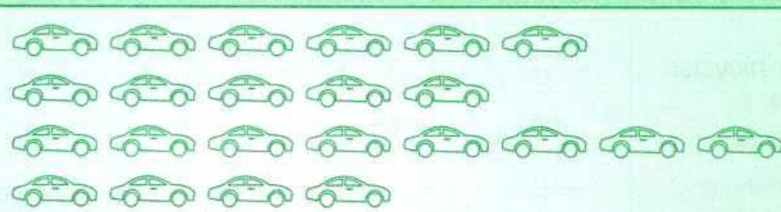
1. The sale of vehicles, in a particular city, during the first six months of the year 2016 is shown below :

Month	Jan	Feb	March	April	May	June
Number of vehicles sold	3000	2500	4000	1000	1500	3500

Draw a pictograph to represent the above data.

2. The following pictograph shows the number of cars sold by four dealers A, B, C and D in a city. Scale :  = 50 cars.

Dealer	Number of cars
A	
B	
C	
D	



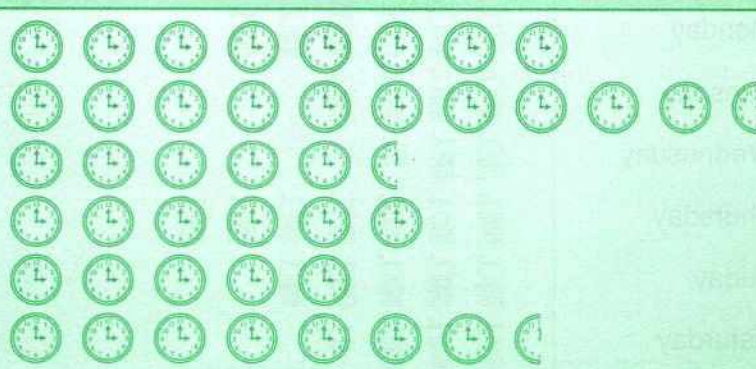
Using the pictograph, drawn above, answer the following questions :

(i) How many more cars has dealer A sold as compared to dealer D ?

(ii) What is the total number of cars sold by all the dealers ?

3. The following pictograph shows the number of watches manufactured by a factory, in a particular week.

Day	Number of watches
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	



Scale :  = 100 watches

Find :

(i) on which day were the least number of watches manufactured ?

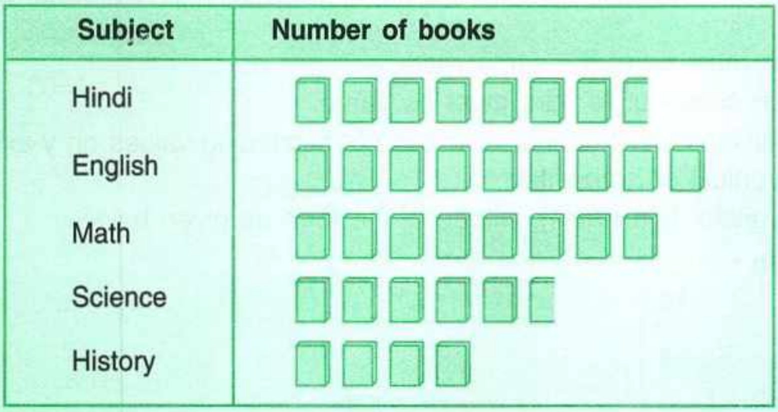
(ii) total number of watches manufactured in the whole week ?

4. The number of animals in five villages are as follows :

Village	A	B	C	D	E
Number of animals	160	240	180	80	120

Prepare a pictograph of these animals using one symbol to represent 20 animals.

5. The following pictograph shows different subject books which are kept in a school library.



Taking symbol of one book = 50 books, find :

(i) how many History books are there in the library ?

(ii) how many Science books are there in the library ?

(iii) which books are maximum in number ?

33.6 BAR (COLUMN) GRAPH

A bar graph is the representation of numerical data by rectangles (or bars) of equal width and varying heights.

Example 5 :

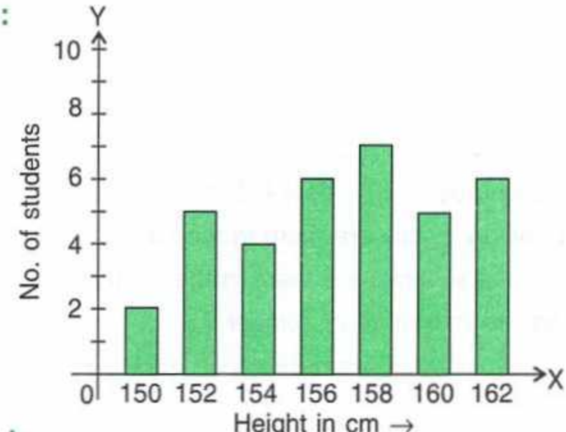
The following table shows the heights of 35 students of a particular class. Draw a bar graph on the basis of it and show the number of students of different heights in the graph.

No. of students	2	5	4	6	7	5	6
Height (in cm)	150	152	154	156	158	160	162

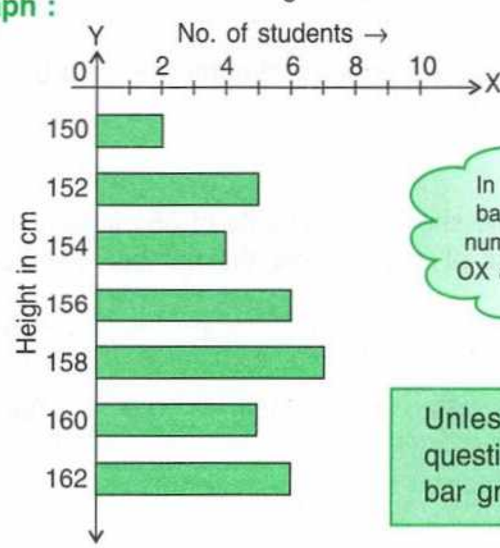
Steps of working :

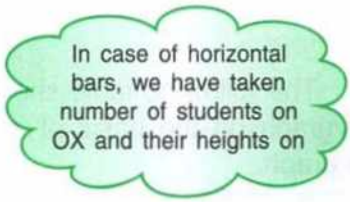
1. On a graph paper or on a plane sheet of paper, draw two mutually perpendicular lines OX and OY.
2. In general, OX is drawn horizontal and is called x-axis, whereas, OY is drawn vertical and is called y-axis.
3. Out of the two given sets of values, take one along x-axis and the other along y-axis. In this example, we are taking height (in cm) along horizontal axis OX and number of students along vertical axis OY.
4. Along the x-axis (*i.e.* along line OX), mark points at equal distances and below these points write the heights of the students.
5. For the given number of students choose a suitable scale. Then for the given data (number of students), mark on OY heights for number of students.
6. At each point marked on x-axis, draw bars, each perpendicular to x-axis and parallel to y-axis.
7. (i) All the bars drawn must be of the same width.
   (ii) Distances between consecutive bars must be same.
   (iii) The height of a bar must be proportional to the corresponding values on y-axis.
8. Bars may be drawn vertical or horizontal.
   The bar graph of the given data (table), will be of the form as given below.

(i) **Vertical bar graph :**



(ii) **Horizontal bar graph :**

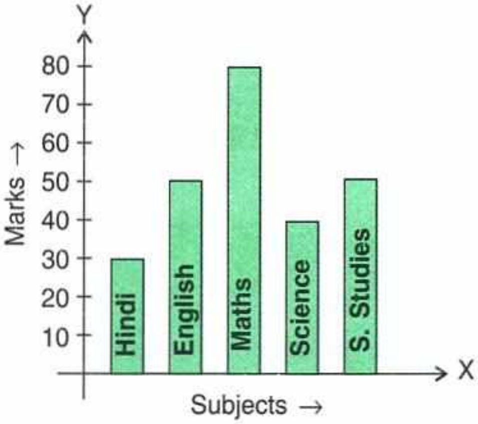




Unless mentioned in the question, we draw vertical bar graph only.

Example 6 :

Given below is a bar graph showing the marks obtained by Rohit in five subjects in his annual examination :



Read the graph carefully and answer the questions given below :

(i) In which subject does Rohit get the lowest marks ?

(ii) In which subject does Rohit get the highest marks ?

(iii) How many marks does he get in English ?

(iv) In which subject/subjects does Rohit get less than 50 marks ?

Solution :

(i) Rohit gets the lowest marks in **Hindi**. **(Ans.)**

(ii) Rohit getss highest marks in **Maths**. **(Ans.)**

(iii) He gets 50 marks in **English**. **(Ans.)**

(iv) Rohit gets less than 50 marks in **Hindi and Science**. **(Ans.)**

EXERCISE 33(C)

1. The following table gives the number of students in class VI in a school during academic years 2011-2012 to 2015-2016.

Academic years	2011-12	2012-13	2013-14	2014-15	2015-16
No. of students	80	120	130	150	180

Represent the above data by a bar graph.

2. The attendence of a particular class for the six days of a week are as given below :

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Attendence	48	44	40	36	39	43

Draw a suitable bar graph.

3. The total number of students present in class VI B, for the six days in a particular week were as given below. Draw a suitable bar graph.

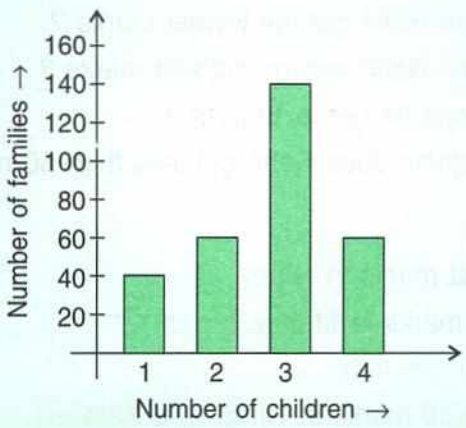
Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of students present	40	30	35	25	10	20

4. The following table shows the population of a particular city at different years :

Year	1996	2001	2006	2011	2016
Population in Lakh	45	57	70	90	110

Represent the above information with the help of a suitable bar graph.

5. In a survey of 300 families of a colony, the number of children in each family was recorded and the data has been represented by the bar graph, given below :



Read the graph carefully and answer the following questions :

(i) How many families have 2 children each ?

(ii) How many families have no child ?

(iii) What percentage of families have 4 children ?

6. Use the data, given in the following table, to draw a bar graph :

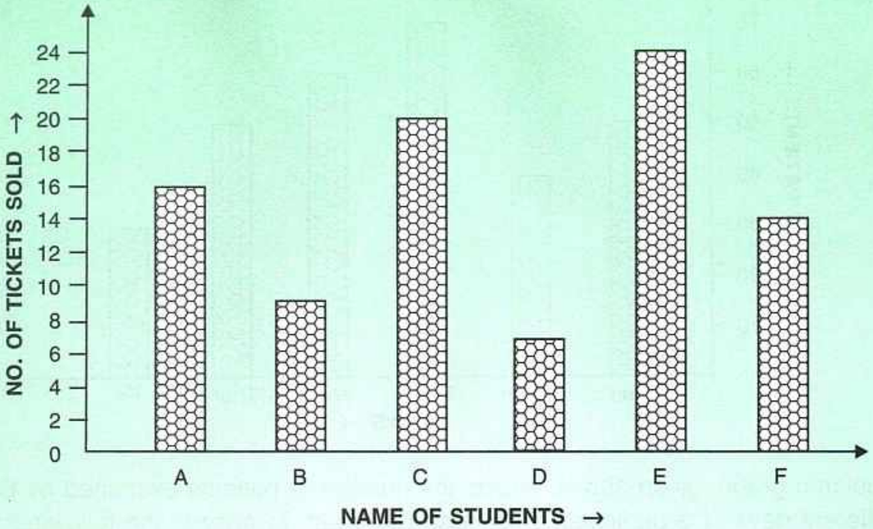
A	B	C	D	E	F
250	300	225	350	275	325

Out of A, B, C, D, E and F

(i) Which has the maximum value.

(ii) Which is greater A + D or B + E.

7. The bar graph drawn below shows the number of tickets sold during a fair by 6 students, A, B, C, D, E and F.



Using the bar graph, answer the following questions :

(i) How many tickets were sold by each of A, B, C, D, E and F ?

(ii) Who sold the least number of tickets ?

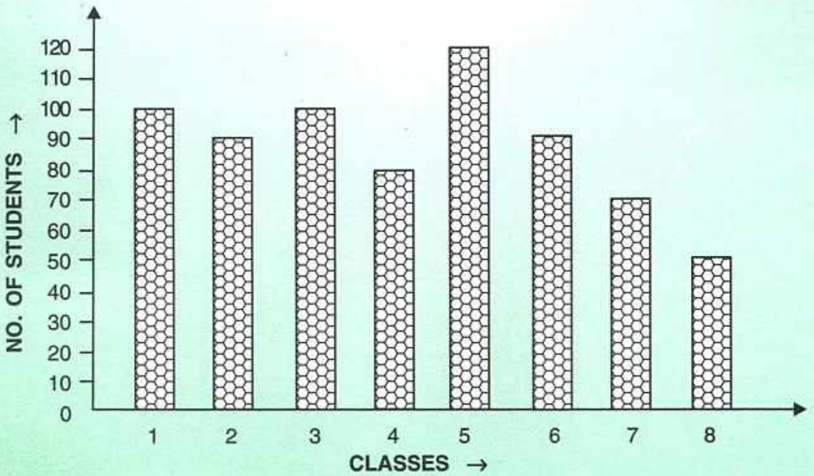
(iii) Who sold the maximum number of tickets ?

(iv) How many tickets were sold by A, B and C taken together ?

(v) How many tickets were sold by D, E and F taken together ?

(vi) What is the average number of tickets sold per student ?

8. The following bar graph shows the number of students in various classes, in a school in Delhi.



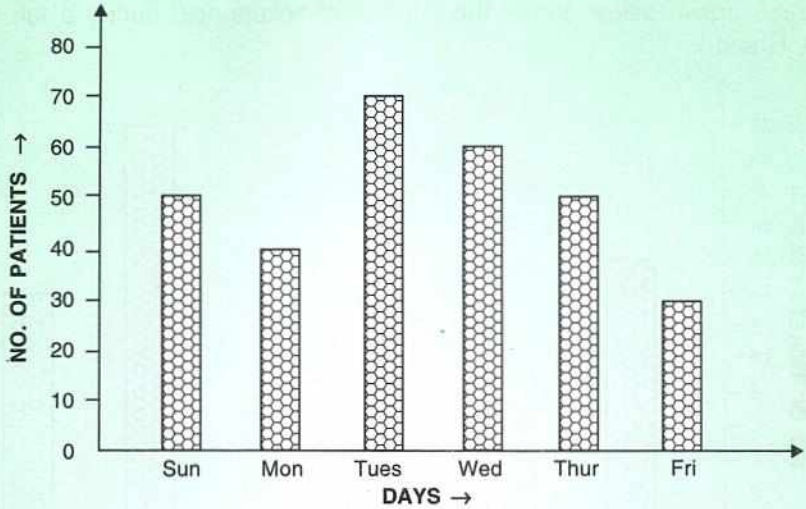
Using the given bar graph, find :

(i) the number of students in each class.

(ii) the total number of students from class 6 to class 8.

(iii) how many more students are there in class 5 compared to class 6 ?

(iv) the total number of students from class 1 to class 8.

9. 

The column graph, given above, shows the number of patients examined by Dr. V.K. Bansal on different days of a particular week. Use the graph to answer the following :

(i) On which day were the maximum number of patients examined ?

(ii) On which day were the least number of patients examined ?

(iii) On which days were an equal number of patients examined ?

(iv) What is the total number of patients examined in the week ?

10. A student spends his pocket money on various items, as given below :
Books : ₹ 380, Postage : ₹ 30, Cosmetics : ₹ 240, Stationary : ₹ 220 and Entertainment : ₹ 120. Draw a bar graph to represent his expenses.

MEAN AND MEDIAN 34

34.1 INTRODUCTION

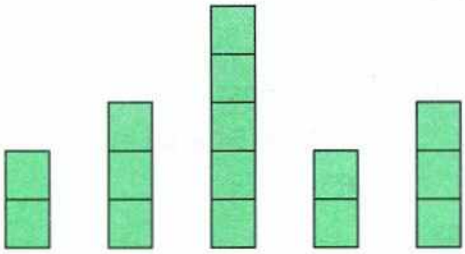
We are familiar with the term 'average'. What do you understand when we hear that the average score of Virat Kohli is 86 in one day Internationals ? It simply means that if we add all the runs scored by Virat Kohli in one day Internationals and divide it with the number of (one day international) matches he has played, we get the number 86. Same is the meaning when we speak of average income, average speed, average marks, etc.

Mathematicians have realised that there are three useful ways of looking at the average of a set of data. They are called the **mean**, the **median** and the **mode** of the data.

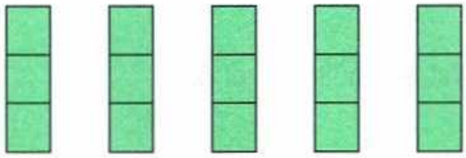
In this chapter, we shall be studying **mean** and **median** only.

34.2 MEAN

Look at the following diagram showing the strips of equal squares.



Now re-arrange the strips of squares so that there are the same number of squares in each strip. [See the following diagram].



In the first diagram, the total number of squares is 15 and the number of strips is 5.

∴ The number of squares in each strip of the second diagram

$$= \frac{\text{Total number of squares}}{\text{Total number of strips}}$$

$= \frac{15}{5} = 3$; which gives **mean** squares in the five

$$\therefore \text{ Mean} = \frac{\text{sum of all the values (observations)}}{\text{number of values (observations)}}$$

The mean gives us a single number which indicates an average of the data. It is not necessary that the mean is a member of the data.

Example 1 :

Find the mean of the data : 5, 13, 10, 13, 15, 9, 17 and 14.

Solution :

$$\textbf{Required mean} = \frac{\text{Sum of data values (observations)}}{\text{Number of data values (observations)}}$$

$$= \frac{5+13+10+13+15+9+17+14}{8}$$

$$= \frac{96}{8} = \mathbf{12}$$ **(Ans.)**

Example 2 :

A batsman secored the following number of runs in six innings :
40, 39, 54, 50, 64 and 59.
Find the mean runs scored by him.

Solution :

Sum of runs scored in all the six innings

$= 40 + 39 + 54 + 50 + 64 + 59$

$= 306$

Number of innings $= 6$

$$\therefore \textbf{ Required mean score} = \frac{306}{6} = \mathbf{51}$$ **(Ans.)**

Example 3 :

Find the mean of the first six multiples of 5.

Solution :

The first six multiples of 5 are : 5, 10, 15, 20, 25 and 30

$\therefore$ Sum of these numbers $= 5 + 10 + 15 + 20 + 25 + 30$

$= 105$

and, number of their numbers $= 6$

$$\therefore \textbf{ Required mean} = \frac{105}{6} = \mathbf{17{\cdot}5}$$ **(Ans.)**

Example 4 :

Find the value of x, if the mean of 10, 12, 13, x and 17 is 14.

Solution :

$$\because \quad \text{Mean} = \frac{\text{Sum of data}}{\text{Number of data}}$$

$$\Rightarrow \quad 14 = \frac{10+12+13+x+17}{5}$$

$$\Rightarrow \quad 14 = \frac{52+x}{5}$$

$\Rightarrow \quad 70 = 52 + x$ and $x = 70 - 52 = 18$ **(Ans.)**

Example 5 :

The numbers of games won by a football team over the last 9 seasons have been: 5, 7, 3, 6, 5, 9, 8, 7 and 5.

Find the mean.

Solution :

Sum of the values = Sum of games won

= 5 + 7 + 3 + 6 + 5 + 9 + 8 + 7 + 5

= 55

and, number of values = number of seasons

= 9

$$\therefore \quad \text{The mean} = \frac{\text{sum of games won}}{\text{number of games seasons}}$$

$= \frac{55}{9} = 6{\cdot}1$ (approximately) **(Ans.)**

Suppose the team mentioned above, wins 7 games in the next season. The mean will increase because the new data (7 games won) is greater than the mean obtained above.

In this case, the sum of games won = 55 + 7 = 62

And, the number of games seasons = 9 + 1 = 10

$$\therefore \quad \text{Now, the mean} = \frac{62}{10}$$

$= 6{\cdot}2$ **(Ans.)**

EXERCISE 34(A)

1. Find the mean of :
   (i) 7, 10, 4 and 17
   (ii) 12, 9, 6, 11 and 17
   (iii) 3, 1, 5, 4, 4 and 7
   (iv) 7, 5, 0, 3, 0, 6, 0, 9, 1 and 4
   (v) 2·1, 4·5, 5·2, 7·1 and 9·3
   (vi) 5, 2·4, 6·2, 8·9, 4·1 and 3·4
2. Find the mean of :
   (i) first eight natural numbers
   (ii) first six even natural numbers
   (iii) first five odd natural numbers
   (iv) all prime numbers upto 30
   (v) all prime numbers between 20 and 40.
3. Heights (in cm) of 7 boys of a locality are 144 cm, 155 cm, 168 cm, 163 cm, 167 cm, 151 cm and 158 cm. Find their mean height.
4. Find the mean of 35, 44, 31, 57, 38, 29, 26, 36, 41 and 43.
5. The mean of 18, 28, x, 32, 14 and 36 is 23. Find the value of x.
6. If the mean of x, $x + 2$, $x + 4$, $x + 6$ and $x + 8$ is 13, find the value of x.

34.3 MEDIAN

When the given data is arranged from smallest to largest, the value of middle data is the median.

Example :

To find the median of the data 5, 7, 3, 6, 5, 9, 8, 7 and 5.

We arrange the given data from smallest to largest (*i.e.* in ascending order) as shown below :

3 5 5 5 **6** 7 7 8 9

Clearly, the middle term is 6 and so the required **median** = 6

When the given data is in ascending (smallest to largest) form, its median splits the data in two halves.

In 3 5 5 5 **6** 7 7 8 9, the median is 6 and it splits the data in two halves *i.e.* 3 5 5 5 and 7 7 8 9.

- If there is an odd number of data, there will be one middle value. This middle value is the median. Median in this case is one of the data.
- If there is an even number of data, there will be two middle values. The median is the average of these values.

Example 6 :

Find the median of :

(i) 9, 7, 6, 14, 10, 4 and 11
(ii) 2, 5, 9, 4, 12, 3, 7, 4, 10 and 7.

Solution :

(i) On writing the given numbers from smallest to largest (*i.e.* in ascending order) we get :

4 6 7 **9** 10 11 14

Clearly, middle term is 9; therefore **median = 9** **(Ans.)**

(ii) On writing the given numbers in ascending order, we get :

2 3 4 4 5 7 7 9 10 12

Number of data = 10, *which is even*

2 3 4 4 **5** **7** 7 9 10 12

$\Rightarrow$ The two middle data = 5 and 7

$\therefore$ Median = Average of 5 and 7 = $\frac{5+7}{2}$ = **6** **(Ans.)**

Example 7 :

Find the median :

9, 11, 15, 12, 14, 12, 14, 12, 13, 12, 13 and 16.

Solution :

On writing the given data in ascending (smallest to largest) order, we get :

9 11 12 12 12 **12** **13** 13 14 14 15 16

Number of data = 12

$\Rightarrow$ The two middle data = 12 and 13

Clearly, **median** = Average of 12 and 13

$= \frac{12+13}{2} = $ **12·5** **(Ans.)**

EXERCISE 34(B)

1. Find the median of
   (i) 21, 21, 22, 23, 23, 24, 24, 24, 24, 25 and 25
   (ii) 3·2, 4·8, 5·6, 5·6, 7·3, 8·9 and 9·1
   (iii) 17, 23, 36, 12, 18, 23, 40 and 20
   (iv) 26, 33, 41, 18, 30, 22, 36, 45 and 24
   (v) 80, 48, 66, 61, 75, 52, 45 and 70
2. Find the mean and the median of :
   (i) 1, 3, 4, 5, 9, 9 and 11
   (ii) 10, 12, 12, 15, 15, 17, 18, 18, 18 and 19
   (iii) 2, 4, 5, 8, 10, 13 and 14
   (iv) 5, 8, 10, 11, 13, 16, 19 and 20
   (v) 1·2, 1·9, 2·2, 2·6 and 2·9
   (vi) 0·5, 5·6, 3·8, 4·9, 2·7 and 4·4.

ANSWERS

Exercise 1 (A)

1. (i) 537 (ii) 2428 (iii) 10,35,729 **2.** (i) 428 (ii) 2497 (iii) 3297

3. (i) 45427 (ii) 381007 (iii) 63520 **4.** (i) 540276 > 369998 (ii) 6983245 > 6893254

5. 5223791 < 5432972 < 23106293 < 23182634 < 54344782

6. 54,82,900 > 27,32,940 > 5,43,287 > 78,396 > 43,877 > 4,999

7. (i) 84 and 5942 (ii) 5499 and 54909 **8.** (i) 7532 and 2357 (ii) 9641 and 1469

(iii) 7420 and 2047 (iv) 8531 and 1358 (v) 9760 and 6079 **10.** 9857 and 1052

11. (i) 99999 and 100000 (ii) 1000 – 999 = 1 (iii) 100 + 99 = 199 (iv) 100000, six (v) 999, largest

12. 965320 **13.** 1023 and 9876 **14.** 97865 and 10823 **15.** 10999 **16.** 899999

17. (i) 2000 (ii) 2000 **18.** 2000 **19.** 368, 386, 638, 683, 863 and 836

20. (i) 9547 and 4597 (ii) 7594 and 4597 (iii) 9475 and 5479

Exercise 1 (B)

1. 4,23,414 **2.** B > A by 57,100 **3.** 2,98,256 **4.** 3000 **5.** 51,11,079 **6.** 2151

7. A wins by 32332 votes **8.** 52965 **9.** 1,73,460 **10.** ₹ 1,47,184 **11.** Greater by 72,315

12. 13 shirts; cloth left = 10 cm **13.** 720 kg 14. 60,160 m = 60·160 km **15.** 202

16. (i)75,88,870 (ii) 1,59,62,410 **17.** 11,60,469

Exercise 2 (A)

1. (i) 60 (ii) 270 (iii) 540 (iv) 8260 (v) 6290 (vi) 3010 (vi) 72330

2. (i) 700 (ii) 800 (iii) 2700 (iv) 5400 (v) 6400 (vi) 59200

3. (i) 6000 (ii) 7000 (iii) 25000 (iv) 33000 (v) 9000 (vi) 83000

4. (i) 580 (ii) 600 (iii) 4000 (iv) 30000 (v) 30000

5. (i) 860, 900 and 1000 (ii) 1250, 1200 and 1000 (iii) 54550, 54500 and 55000

(iv) 68080, 68100 and 68000 (v) 56290, 56300 and 56000 (vi) 7290, 7300 and 7000

(vii) 8924380, 8924400 and 8924000 **6.** (i) ₹ 560 (ii) 840 m (iii) 550 cm (iv) ₹30

7. 25, 26, 27, 28, 29, 30, 31, 32, 33 and 34 **8.** 45, 46, 47, 48, 49, 50, 51, 52, 53 and 54

9. 85 and 94 **10.** 125 and 134

Exercise 2 (B)

1. (i) 70 + 40 = 110 (ii) 30 + 90 = 120 (iii) 90 (iv) 100 (v) 160 (vi) 140 (vii) 640 (viii) 640 (ix) 800

2. (i) 1100 (ii) 800 (iii) 800 (iv) 9600 (v) 6600 (vi) 69300

3. (i) 90,000 (ii) 79000 **4.** (i) 50 (ii) 60 (iii) 260 **5.** (i) 500 (ii) 200 (iii) 4300

6. (i) 7000 (ii) 28000 **7.** (i) 2500 (ii) 2400 (iii) 1500 (iv) 4500 (v) 4900 (vi) 900

8. (i) 100,000 (ii) 120,000 (iii) 60,000 (iv) 150000 (v) 40000 (vi) 150000

9. (i) 9000 (ii) 43000 (iii) 10000 (iv) 21000 (v) 56000 (vi) 12000 **10.** (i) 3 (ii) 4 (iii) 4

(iv) $198 \div 24 = \frac{200}{20} = 10$ (v) $355 \div 26 = \frac{360}{30} = 12$ (vi) $444 \div 44 = \frac{440}{40} = 11$ (vii) 28

Exercise 3

1. (i) four lakh thirty five thousand three hundred forty two
   (ii) thirty six lakh seventy one thousand four hundred thirty
   (iii) four crore twenty eight lakh thirty thousand four
   (iv) seventy five million one hundred thirty two thousand six hundred eighty four
   (v) eight hundred fifteen thousand nine hundred six
   (vi) five million four hundred twenty thousand seven hundred

2. (i) 8,35,629 (ii) 3,56,40,254 (iii) 28,26,040 **3.** (i) 6,509,820 (ii) 428,140,584 (iii) 63,560,981

4. (i) 4,67,306 = 467,306 = four hundred sixty seven thousand three hundred six
   (ii) 13,00,045 = 1,300,045 = one million three hundred thousand forty five.

5. (i) 604,847 = 6,04,847 = six lakh four thousand eight hundred forty seven
   (ii) 2,310,104 = 23,10,104 = twenty three lakh ten thousand one hundred four = 2,310, 104

Exercise 4 (A)

1. (i) kg, 20 (ii) m, 80 (iii) 5 cm (iv) 24 km (v) (a) 53, fifty three (b) 9, nine (c) 240, two hundred forty

2. (i) 600 (ii) 8000 (iii) 5000 (iv) 6000 **3.** 2995 **4.** (i) 500 (ii) 60 (iii) 560

5. (ii) seventy six thousand = $7 \times 10000 + 6 \times 1000$
   (iii) six lakh twenty three thousand = $6 \times 100000 + 2 \times 10000 + 3 \times 1000$
   (iv) forty thousand seventy five = $4 \times 10000 + 7 \times 10 + 5 \times 1$
   (v) fifty thousand four = $5 \times 10000 + 4 \times 1$ **6.** 69930

Exercise 4 (B)

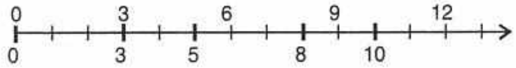
1. (i) 1000 (ii) 9999 (iii) 9 coins (iv) ₹ 100 (v) 9999 boys (vi) 999 toys

2. a four digit number **3.** 10,00,000 **4.** 99999, 99998, 99997, 99996, 99995 and 99994

5. 1000000, 1000001, 1000002, 1000003, 1000004 **6.** none **7.** 90,000

Exercise 5 (A)

1. (i) 1 (ii) 0 (iii) not possible (iv) not possible (v) whole numbers (vi) natural numbers
   (vii) 4100 (viii) 4229 **2.**



3. (i) true (ii) false (iii) true (iv) true (v) true (vi) true

4. (i) 54 (ii) 497 (iii) 286 (iv) 286 (v) b **5.** (i) 708 (ii) 800 (iii) 500 (iv) 2400 **6.** yes

7. (i) closure property for addition of whole numbers
   (ii) associativity for addition of whole numbers
   (iii) distributivity of multiplication over addition of whole numbers
   (iv) distributivity (v) distributivity (vi) 0 is additive identity
   (vii) sum of a number and its additive inverse is zero

8. (i) false (ii) true (iii) true (iv) false (v) true (vi) false (vii) true (viii) true (ix) true (x) false
   (xi) true (xii) false

Exercise 5 (B)

1. (i) yes, yes (ii) no, yes **2.** (i) 8, –8, commutative (ii) –5, whole number, whole numbers (iii) –11, –16, 13, –6, No, associative **3.** not possible **4.** inverse does not exist

5. $12 \times 3 = 36$; $108 - 72 = 36$; yes, yes, distributivity

6. $8 \times 24 = 192$; $384 - 192 = 192$; yes, distributivity

Exercise 5 (C)

1. (i) 0 (ii) 592 (iii) 573 (iv) 578 (v) 2 (vi) 3 (vii) 54 + 16; 70; 5810 (viii) 98 – 75; 23

2. (i) 48700 (ii) 44400 (iii) 900000 **3.** (i) 100368 (ii) 386540 (iii) 4460892

4. (i) 53704 (ii) 912912 (iii) 2185629 **5.** (i) 6790 (ii) 2840 (iii) 5587300 (iv) 79840 (v) 8324000 (vi) 3333000 **6.** (i) 9990000 (ii) 9989001

7. (i) 440 (ii) 6 + 4 + 2; $11 \times 12 = 132$ **8.** (i) 6390 (ii) 74568 (iii) 740580 (iv) 79992

Exercise 5 (D)

1. (i) $5 \div 8$ is not a whole number, $5 \div 0$ is not possible, etc.

(ii) $5 \div 1 = 5$, $16 \div 1 = 16$, etc (iii) $8 \div 8 = 1$, $12 \div 12 = 1$, etc.

(iv) $0 \div 6 = 0$, $0 \div a = 0$, if $a \neq 0$, etc. (v) $7 \div 0$ is not defined, $16 \div 0$ is not defined, etc.

2. $x = 1$ **3.** (i) 987 (ii) 0 (iii) 335 (iv) 0 **4.** (iii) and (iv)

Exercise 5 (E)

1. 90001 **2.** 9900001 **3.** 24 and 22 **4.** yes, $a = 1$ **5.** Quotient = 138 and remainder = 2

Exercise 5 (F)

1. (i)
$1234 \times 9 + 4 = 11110$
$12345 \times 9 + 5 = 111110$
$123456 \times 9 + 6 = 1111110$

(ii)
$9876 \times 9 + 4 = 88888$
$98765 \times 9 + 3 = 888888$
$987654 \times 9 + 2 = 8888888$

(iii)
$1234 \times 8 + 4 = 9876$
$12345 \times 8 + 5 = 98765$
$123456 \times 8 + 6 = 987654$

(iv)
$444 \div 12 = 37$
$555 \div 15 = 37$
$666 \div 18 = 37$

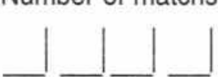
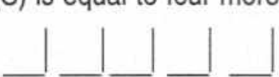
2. (i) 2, 1 and 3 (row-wise) (ii) 9, 11, 3, 6 and 5 (row-wise) (iii) 12, 6, 14, 8 and 18 (row-wise)

3.

n	1	2	3	4
S	7	10	13	16

(i) $S = 3n + 4$ (ii) (1) 49 (2) 124

(iii) Number of matchsticks (S) is equal to four more than three times the number of the figure

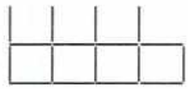
4. (i)  and 

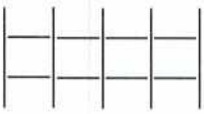
(ii)

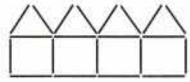
n	1	2	3	4	5
L	2	4	6	8	10

(iii) $L = 2n$ (iv) (1) 24 (2) 40

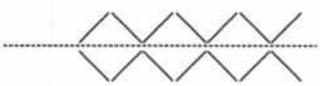
5. (a)  (i) F = $3n + 2$ (ii) 50 and 92

(b)  (i) F = $4n + 1$ (ii) 65 and 121

(c)  (i) F = $5n + 3$ (ii) 83 and 153

(d)  (i) F = $5n + 1$ (ii) 81 and 151

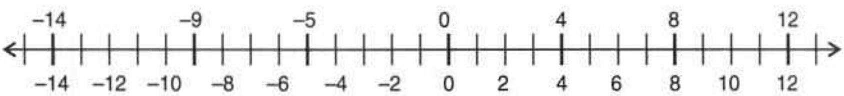
(e)  (i) F = $4n + 1$ (ii) 65 and 121

(f)  (i) F = $4n - 2$ (ii) 62 and 118

Exercise 6

1. (i) 20 (ii) 0 (iii) –8 (iv) loss of ₹ 10 (v) positive (vi) > (vii) right side (viii) left side

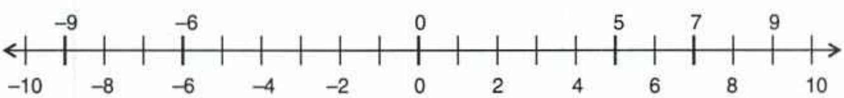
2. (i)



∴ Given integers in ascending order are :

$-14 < -9 < -5 < 0 < 4 < 8 < 12$

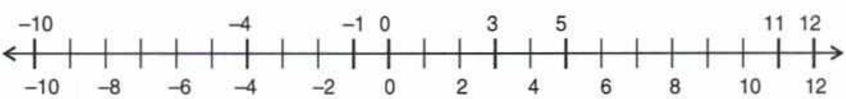
(ii)



∴ Given integers in ascending order are :

$-9 < -6 < 0 < 5 < 7 < 9$

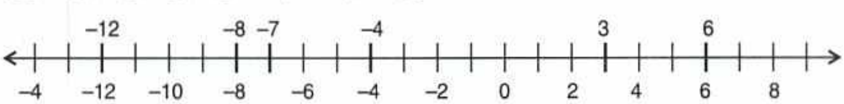
3. (i)



∴ Given integers in descending order are :

$12 > 11 > 5 > 3 > 0 > -1 > -4 > -10$

(ii)



∴ Given integers in descending order are :

$6 > 3 > -4 > -7 > -8 > -12$

4. (i) 28 (ii) 2 (iii) –2 (iv) –28 **5.** (i) 473 (ii) –771 (iii) –297 (iv) –215 (v) –876 (vi) 3

6. (i) 3 (ii) 13 (iii) –11 (iv) 6 (v) 15 (vi) 3 **7.** (i) 576 (ii) 66 (iii) –453 (iv) 222

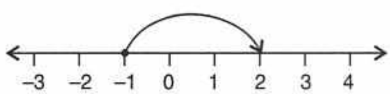
Exercise 7(A)

1. (i) greater (ii) left (iii) right (iv) less, greater (v) greater, less (vi) greater, less (vii) less, greater (viii) greater, less **2.** (i) –15 (ii) 15 (iii) 8 (iv) 0 **3.** (i) –6 (ii) –3 (iii) –51 (iv) 0 **4.** (i) 3 > 0 (ii) 0 > –8 (iii) – 9 < –3 (iv) –3 < 3 (v) 5 > –1 (vi) –13 < 0 (vii) –8 > –18 **5.** (i) –8, –5, –1, 0, 4, 5 (ii) –7, –6, –3, 0, 2, 3, 4 **6.** (i) 15, 8, 0, –2, –3, –5 (ii) 23, 12, 7, 6, 0, –11 **7.** (i) False (ii) True (iii) True (iv) True (v) False (vi) False

Exercise 7(B)

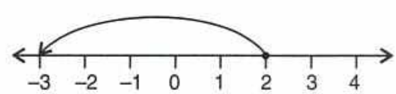
1. (i) +11 (ii) +6 (iii) +5 **2.** (i) +1 (ii) –2 (ii) +3 **3.** (i) +2 (ii) –3 (iii) –4 **4.** (i) –3 (ii) –7 (iii) –7 **5.** (i) +8 (ii) +13 (iii) –8 (iv) –12 (v) +6 (vi) –4

6. (i)



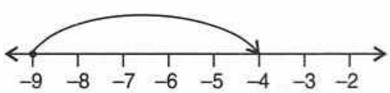
[–1 + 3 = 2]

(ii)



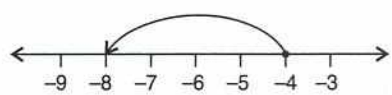
[2 – 5 = –3]

(iii)



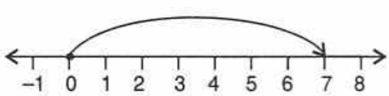
[–9 + 5 = –4]

(iv)



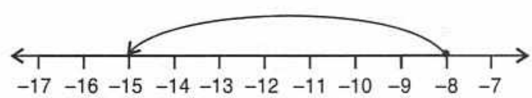
[–4 – 4 = –8]

(v)



[0 + 7 = 7]

(vi)



[–8 – 7 = –15]

Exercise 8(A)

1. (i) 1, 3, 5 and 15 (ii) 1, 5, 11 and 55 (iii) 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48 (iv) 1, 2, 3, 4, 6, 9, 12, 18 and 36 (v) 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42 and 84 **2.** (i) 2, 3, 5, 7, 11, 13, 17, 19 and 23 (ii) 17, 19, 23, 29 and 31 (iii) 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71 and 73 **3.** (i) 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 and 43 (ii) 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31 (iii) 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47 (iv) 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53 and 59 **4.** (i) 2 (ii) 3 (iii) 5 and 7 (iv) 7 **5.** (i) 2 and 3 (ii) 2 and 3 (iii) 2 and 5 (iv) 2, 3 and 7

Exercise 8(B)

1. (i) 1 (ii) 5 (iii) 3 (iv) 2 (v) 3 **2.** (i) 1 (ii) 1 (iii) 20 (iv) 4 (v) 4 **3.** (i) 8 (ii) 6 (iii) 1 (iv) 10 (v) 2 **4.** (i) 15 (ii) 12 (iii) 33 (iv) 12 (v) 15 **5.** 45 **7.** 15 and 16; 15 and 28; 16 and 21 **8.** 18

Exercise 8(C)

1. (i) 24 (ii) 60 (iii) 36 **2.** (i) 288 (ii) 600 (iii) 294 (iv) 726 (v) 510 **3.** 100 **4.** 6

5. 480 **6.** 360 **7.** 1439 **8.** 867

Revision Exercise (Chapter 8)

1. (i) 12 (ii) 6 **2.** (i) 5040 (ii) 2640 **3.** (i) True (5 and 11 are two prime numbers and their H.C.F. is 1) (ii) True (4 and 9 are two co-prime numbers and their H.C.F. is 1) (iii) True (5 and 11 are prime numbers and their L.C.M. = 5 × 11 = 55 (iv) True (4 and 9 are two co-prime numbers and their L.C.M. = 4 × 9 = 36) **4.** 336 **5.** 24 **6.** (i) 84, which is the L.C.M. of given numbers (ii) 14, which is the H.C.F. of given numbers. **7.** 840 and 28 **8.** 18 and 2700

Exercise 9 (A)

1. 10 **2.** 60 **3.** 24 **4.** –2 **5.** 21 **6.** 14 **7.** –56 **8.** 17 **9.** 38 **10.** 59 **11.** 27 **12.** 14

Exercise 9 (B)

1. (i) 1, 2 (ii) 3, 0 (iii) exact divisor, the number (iv) itself (v) itself (vi) one (vii) finite, infinite (viii) multiple

2. (i) 1, 2, 4, 8 and 16 (ii) 1, 3, 7 and 21 (iii) 1, 3, 13 and 39 (iv) 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48 (v) 1, 2, 4, 8, 16, 32 and 64 (vi) 1, 2, 7, 14, 49 and 98

3. (i) 4, 8, 12, 16, 20 and 24 (ii) 9, 18, 27, 36, 45 and 54 (iii) 11, 22, 33, 44, 55, and 66 (iv) 15, 30, 45, 60, 75 and 90 (v) 18, 36, 54, 72, 90 and 108 (vi) 16, 32, 48, 64, 80 and 96 **5.** 4 and 12 **6.** 6 and 9

10. (i) 1608 = 1600 + 8 = 8(200 +1) = 8 × 201 (ii) 56008 = 56000 + 8 = 8(7000 + 1) = 8 × 7001 (iii) 240008 = 240000 + 8 = 8(30000 + 1) = 8 × 30001

Exercise 9 (C)

1. (i) 352 (iii) 496 **2.** (ii) 532 (iv) 9232 **3.** (ii) 2536 (iii) 92760 (iv) 444320

4. (ii) 543 (iv) 92349 **5.** (i) 1332 (ii) 4968 **6.** (i) 324 (ii) 2010 **7.** (i) 5080 (ii) 755

8. (i) 9990 (ii) 0 **9.** (i) 5918 (ii) 68717 (iv) 10857 **10.** (i) 960 (ii) 8295

11. (i) M = 2 (ii) M = 1 (iii) M = 1 **12.** (i) M = 4 (ii) M = 5 (iii) M = 3

13. (i) M = 8 (ii) M = 7 (iii) M = 3 (iv) M = 5 **14.** (i) false (ii) true (iii) true (iv) true

Exercise 10(A)

1. (i) No; some problems may be easy for one person but may not be easy to some other persons. Objects are not well-defined. (ii) Yes (iii) Yes. (iv) No; it is not mentioned that the boys must be taller than which boy. If we consider three boys A, B and C; boy A can be taller than B but not necessarily taller than C. (v) Yes (vi) No; it may be difficult for one student to draw a given triangle but to some other student it may not be difficult to draw the same triangle. (vii) Yes (viii) No; a fruit may be tasty for one person and may not be tasty to other person/persons. (ix) No; clever in what respect ? (x) No; all the people cannot find the same schools as good schools. So, the objects are not well-defined. (xi) Yes (xii) Yes (xiii) Yes.

Exercise 10(B)

1. (i) True (ii) True (iii) False (iv) False **2.** (i) wrong; $5 \notin B$ (ii) correct (iii) correct (iv) wrong; $9 \notin B$ (v) correct (vi) correct **3.** (i) False (ii) True (iii) False (iv) True (v) True (vi) True (vii) True **4.** (i) { 1, 2, 3, 4, 5 } (ii) {acute-angle, right-angle, obtuse-angle} (iii) {scalene-triangle, isosceles-triangle, equilateral-triangle} (iv) { Write the name of each member of your family } (v) { b, c, d, f, g, h }

(vi) { a, e, i, o }   (vii) { Indira Gandhi, Atal Bihari Vajpayee, Dr. Manmohan Singh }
5. (a) (i) 3, 8, 5, 15, 12 and 7   (ii) c, m, n, o and s   (b) (i) { 2, 4, 8, 16, 64, 128 }
(ii) { 3, 5, 15, 45, 75, 90 }   **6.** (i) { b, h, o, p, a, l }   (ii) { e, a }   (iii) { h, n, g, k }

Exercise 10(C)

1. (i) {3, 6, 9, 12, 15} (ii) {–3, –2, –1, 0, 1, 2, 3} (iii) {s, c, h, o, l} (iv) {11, 13, 15, 17, 19} (v) {a, e, i} (vi) {m, d, r, s} **2.** (i) {2, 3, 5, 7, 11, 13, 17, 19} (ii) $\{1^2, 2^2, 3^2, 4^2\} = \{1, 4, 9, 16\}$ (iii) {2, 4, 6, 8} (iv) {a, b, c, d, e, f, g, h} (v) {b, a, s, k, e, t} (vi) {Jaipur, Jodhpur, Jalandhar, Jhansi} (vii) {△, ○, ▱, ⬠} (viii) {o, a} (ix) {0, 1, 4, 9} as $0 = 0^2$, $1 = 1^2$, $4 = 2^2$ and $9 = 3^2$
3. (i) {x : x is an even natural number less than 12} (ii) {x : x is a prime number less than 12} (iii) {x : x is a month whose name starts with letter J} (iv) {x : x is a vowel in English alphabet} (v) {x : x is a day of the week whose name starts with letter T } (vi) {x : x is a perfect square natural number upto 25} (vii) {x : x is a natural number upto 30 and divisible by 5}.
4. (i) {1, 2, 3, 4, 6, 8, 12, 24}; {x : x is a natural number which divides 24 completely} (ii) {21, 23, 25, 27, 29, 31, 33}; {x : x is an odd number between 20 and 35} (iii) { c, a, l, u, t }; { x : x is a letter used in the word 'CALCUTTA' } (iv) {January, February, March, April, May}; {x : x is name of first five months of a year} (v) {16, 25, 36, 49, 64, 81}; {x : x is a perfect square two digit number}. **5.** (i) {5, 15, 25, 35} (ii) {18, 24, 30} (iii) {Friday, Saturday, Sunday} (iv) {September, October, November, December}

Exercise 10(D)

1. (i) Infinite (ii) Finite (iii) Infinite (iv) Finite **2.** (i), (iii) and (iv) **3.** (i) Equal (ii) Equivalent (iii) Equal (iv) Equivalent **4.** (i) Infinite (ii) Infinite (iii) Infinite (iv) Finite (v) Infinite (vi) Finite (vii) Finite (viii) Infinite (ix) Infinite (x) Infinite **5.** (i) Not empty (ii) Empty (iii) Empty (iv) Not empty (v) Not empty (vi) Empty (vii) Not empty **6.** (i) Equivalent (ii) Equal (iii) Equal (iv) Equal (v) Equal (vi) Equivalent **7.** (i) Finite (ii) Infinite (iii) Finite (iv) Finite (v) Infinite (vi) Finite **8.** (i) False (ii) False (iii) True (iv) True (each set is the empty set) (v) True (vi) False (6 is an even natural number which is divisible by 3) (vii) True (no positive number can be less than 0) (viii) False
9. (i) Disjoint sets; as no girl can be of age below 15 years and also above 15 years (ii) Overlapping sets; as boys above 27 years are also above 20 years. (iii) Overlapping sets; as natural numbers from 51 to 59 are common to both the sets. (iv) Overlapping sets; as students of Class IX studying in I.C.S.E. Board are common. (v) Overlapping sets; as natural number 24 is common to both the sets. (vi) Disjoint sets; as no letter is common to both the sets.

Exercise 10(E)

1. (i) 4 (ii) 6 (iii) 0 (iv) 3 (v) 4 (vi) 7 **2.** (i) 9 (ii) 4 (iii) 4 (iv) 0
3. (i) False; n(A) = 1 (ii) False; n(∅) = 0 (iii) True (iv) False; n(B) = 4.

Exercise 11(A)

1. (a) (i) 2 : 3 (ii) 8 : 9 (iii) 4 : 5 (b) (i) 25 : 4 (ii) 3 : 20 (iii) 10 : 3 (iv) 8 : 3 (v) 4 : 3 (vi) 16 : 3 (c) (i) 3 : 5 (ii) 1 : 2 (iii) 28 : 42 : 15 (iv) x : 4 (v) 5 : 3 (vi) 1 : 2 **2.** 3 : 4 **3.** (i) false (ii) true (iii) true **4.** yes **5.** yes **6.** 70 : 95 : 56 **7.** 3 : 2

Exercise 11(B)

1. (i) 24 : 17 (ii) 17 : 7 (iii) 7 : 24 **2.** (i) 6 : 7 (ii) 7 : 13 (iii) 13 : 6 **3.** (i) 1 : 2 (ii) 2 : 3 (iii) 3 : 1 **4.** (i) 2 : 3 (ii) 4 : 7 (iii) 8 : 13 (iv) 5 : 3 (v) 2 : 1 **5.** 1440 **6.** 3 : 2 **7.** 1 : 10 **8.** 2 : 3 **9.** (i) 5 : 18 (ii) 3 : 2 (iii) 6 : 5 (iv) 2 : 7 **10.** 9 : 4 **11.** (i) B (ii) 2, 18, 4, 16, 3 **12.** 180

Exercise 11(C)

1. Hari = ₹ 75; Gopi = ₹ 45 **2.** 45 and 27 **3.** 18, 27 and 36
4. ₹ 4,800, ₹ 3,200 and ₹ 2,400 **5.** ₹ 600, ₹ 900 and ₹ 1,000 **6.** 80° and 30°
7. 9 cm, 6 cm and 12 cm **8.** 10 kg **9.** A = ₹ 18,000, B = ₹ 9,000 and C = ₹ 4,500
10. Ashok = ₹ 60,000; Mohit = ₹ 15,000 and Geeta = ₹ 6,000

Exercise 11(D)

1. (i) $\frac{5}{9}$ (ii) $\frac{6}{13}$ **2.** (i) $\frac{9}{17}$ (ii) $\frac{7}{15}$ **3.** 152 **4.** 175 **5.** 300 and then 540 **6.** 300 and 225
7. 1800 and 540 **8.** 2800 and 4900 **9.** 13 : 10 **10.** 5 : 6

Exercise 12(A)

1. (i) No (ii) Yes (iii) Yes (iv) Yes (v) Yes (vi) Yes **2.** (i) 3 (ii) 18 (iii) 12 (iv) 20 (v) 12 (vi) 9 **3.** (i) 4 (ii) 2 (iii) 5 (iv) 12 (v) 6 (vi) 90 **4.** 25 **5.** 27 **6.** 48 **7.** 100 m **8.** ₹ 80 **9.** 10·8 kg **10.** (i) 90 (ii) 315 **11.** 45

Exercise 12(B)

1. (iii) **2.** (iii) and (iv) **3.** (i) 9 (ii) 0.24 **4.** (i) 9 (ii) $9\frac{1}{3}$ (iii) 0.1 **5.** 28.7 **6.** (ii) 9 (iii) 30
7. (i) 11 : 35 (ii) 22 (iii) 105

Exercise 12(C)

1. (i) Yes (ii) No **2.** (i) 25 (ii) 270 **3.** ₹ 1,600 **4.** 28 cm **5.** 15 years **6.** A = ₹ 3,500; B = ₹ 4,200 and C = ₹ 2,800 **7.** No **8.** No **10.** (i) 16.07 m (ii) 13.72 m **11.** 6 **12.** 49

Exercise 13(A)

1. (i) ₹ 70 (ii) ₹ 910 **2.** (i) 66 km (ii) 165 km **3.** (i) ₹ 38.40 (ii) ₹ 576.00 (iii) ₹ 1382.40
4. 30 **5.** ₹ 210 **6.** 12 **7.** 272 km **8.** ₹ 1008 **9.** (i) 108 km (ii) 1.5 hrs **10.** 644 cm
11. (i) ₹ 324 (ii) 24 kg **12.** Manoj [*Reason* : For Rohit, the cost of each pen $= \frac{₹\,150}{10}$ = ₹ 15 and for Manoj the cost of each pen = $\frac{₹\,168}{14}$ = ₹ 12] **13.** 9.6 m **14.** 216 km

Exercise 13(B)

1. 18 kg **2.** B **3.** 540 words **4.** 13.5°C **5.** ₹ 48,000 **6.** 300 litre **7.** ₹ 9,000 **8.** 280 steps **9.** (ii) ₹ 810 (ii) 40 m **10.** ₹ 3,780

Exercise 14(A)

1. (i) $\frac{2}{7}$ (ii) $\frac{5}{17}$ (iii) $\frac{3}{5}$ **2.** (i) proper (ii) improper (iii) improper (iv) 1 (v) 1 (vi) mixed (vii) like (viii) unlike (ix) equal (x) like (xi) 2 ; $\frac{41}{13}$ (xii) $\frac{4\times5+3}{5} = \frac{23}{5}$

3. (i) $\frac{2}{9}, \frac{7}{15}, \frac{11}{20}, \frac{18}{23}$ and $\frac{27}{35}$ (ii) $\frac{4}{3}$ and $\frac{20}{11}$. **4.** (i) $\frac{11}{5}$ (ii) $\frac{13}{4}$ (iii) $\frac{57}{8}$ (iv) $\frac{23}{11}$

5. (i) $5\frac{15}{17}$ (ii) $7\frac{4}{11}$ (iii) $29\frac{6}{7}$ (iv) $7\frac{8}{15}$ **6.** (i) $\frac{20}{60}, \frac{24}{60}, \frac{45}{60}, \frac{10}{60}$ (ii) $\frac{100}{120}, \frac{105}{120}, \frac{110}{120}, \frac{36}{120}$ (iii) $\frac{32}{112}, \frac{98}{112}, \frac{40}{112}, \frac{63}{112}$

Exercise 14(B)

1. (i) $\frac{4}{5}$ (ii) $\frac{2}{3}$ (iii) $\frac{2}{9}$ (iv) $\frac{1}{3}$ (v) $\frac{3}{2}$ **2.** (i) False (ii) True (iii) False (iv) False (v) False

3. (i) $\frac{2}{3}$ (ii) $\frac{3}{4}$ (iii) $\frac{11}{14}$ **4.** (i) $\frac{3}{8}$ (ii) $\frac{8}{15}$ (iii) $\frac{10}{39}$ **5.** (i) $\frac{7}{8}, \frac{13}{24}, \frac{5}{16}$ (ii) $\frac{4}{5}, \frac{3}{4}, \frac{11}{20}, \frac{7}{15}$ (iii) $\frac{9}{11}, \frac{5}{7}, \frac{3}{8}$ **6.** (i) $\frac{1}{4}, \frac{9}{16}, \frac{7}{12}$ (ii) $\frac{2}{7}, \frac{1}{3}, \frac{5}{6}, \frac{8}{9}$ (iii) $\frac{3}{8}, \frac{5}{9}, \frac{2}{3}, \frac{5}{6}$ **7.** $\frac{7}{12}$ **8.** (i) < (ii) < (iii) = (iv) > **9.** (i) $\frac{25}{18}$ (ii) $\frac{7}{25}$

Exercise 14(C)

1. (i) $2\frac{1}{8}$ (ii) $3\frac{3}{10}$ (iii) $5\frac{1}{8}$ (iv) $7\frac{13}{24}$ (v) $7\frac{1}{3}$ (vi) $8\frac{41}{48}$ **2.** (i) $1\frac{5}{48}$ (ii) $\frac{11}{12}$ (iii) $2\frac{13}{42}$ (iv) $2\frac{7}{12}$ (v) $4\frac{23}{30}$ (vi) $3\frac{1}{4}$ (vii) $3\frac{23}{24}$ (viii) $\frac{3}{10}$ (ix) $3\frac{5}{24}$ (x) $2\frac{11}{12}$ (xi) $\frac{29}{45}$

Exercise 14(D)

1. (i) $\frac{6}{35}$ (ii) $\frac{4}{15}$ (iii) $3\frac{1}{3}$ (iv) $\frac{1}{4}$ (v) $13\frac{1}{56}$ (vi) $\frac{1}{8}$ (vii) 1 (viii) $2\frac{1}{7}$ **2.** (i) $\frac{5}{9}$ (ii) $10\frac{1}{8}$ (iii) $\frac{5}{2} = 2\frac{1}{2}$ (iv) 1 (v) $1\frac{1}{3}$ (vi) $3\frac{9}{11}$ **3.** (i) $\frac{20}{21}$ (ii) $\frac{15}{32}$ (iii) 0 (iv) $\frac{8}{9}$ (v) $3\frac{15}{16}$ (vi) $\frac{48}{49}$ (vii) $6\frac{17}{18}$ (viii) $\frac{1}{3}$ **4.** (i) $7\frac{6}{11}$ (ii) $1\frac{9}{11}$ (iii) $\frac{28}{111}$ (iv) $\frac{11}{20}$ (v) $\frac{30}{49}$ (vi) $2\frac{1}{4}$ (vii) $\frac{1}{1152}$

5. (i) 10 (ii) $1\frac{1}{60}$ (iii) $3\frac{3}{32}$ (iv) $\frac{4}{5}$ (v) $-1\frac{1}{8}$ (vi) $16\frac{1}{14}$ (vii) $\frac{1}{30}$ (viii) $-\frac{7}{204}$ (ix) $\frac{173}{330}$ (x) $2\frac{1}{5}$ (xi) $23\frac{8}{9}$

Exercise 14(E)

1. $5\frac{7}{8}$ m **2.** $4\frac{4}{5}$ m **3.** ₹ $51\frac{1}{2}$ **4.** 340 **5.** 88 g **6.** $\frac{1}{6}$ **7.** ₹ 1,152 **8.** 400 **9.** 4 km **10.** 20 km **11.** ₹ 1,944

Exercise 15(A)

1. (i) 2 (ii) 3 (iii) 1 (iv) 4 (v) 5 **2.** (i) 1·360, 239·800 and 47·008 (ii) 507·07520, 8·52073 and 0·80800 (iii) 459·220000, 7·030930 and 0·200037 **3.** (i) 0·7 (ii) 4·7 (iii) 3·43 (iv) 0·003 (v) 0·07295 (vi) 0·000289 (vii) 0·95 **4.** (i) 0·75 (ii) 0·075 (iii) 0·008 (iv) 0·28 **5.** (i) $\frac{1}{20}$ (ii) $3\frac{19}{20}$ (iii) $4\frac{1}{200}$ (iv) $\frac{219}{250}$ (v) $50\frac{3}{50}$ (vi) $\frac{43}{4000}$ (vii) $4\frac{4403}{5000}$

Exercise 15(B)

1. (i) 5·722 (ii) 1·5938 (iii) 258·21 (iv) 232·37 (v) 0·65046 **2.** (i) 4·16 (ii) 0·9696 (iii) 0·02 (iv) 0·488 **3.** (i) 0·54 (ii) 20·0978 (iii) 0·42 (iv) 15·998 (v) 9·05 **4.** (i) 1·145 (ii) 4·124 (iii) 1·197 (iv) 25·234 (v) 1·1098 **5.** 52·185 **6.** 11·66 **7.** 27·467 **8.** 2·829 **9.** 20·4627 **10.** 910·624

Exercise 15(C)

1. (i) 44·8 (ii) 346·14 (iii) 58·466 (iv) 1·5855 (v) 18·75 (vi) 3·384 (vii) 5·8478 (viii) 1·16388 **2.** (i) 0·0208 (ii) 3·61 (iii) 10·368 (iv) 0·4374 (v) 3·375 (vi) 0·000625 (vii) 0·0000004 **3.** (i) 39, 390 and 3900 (ii) 28·9, 289 and 2890 (iii) 0·829, 8·29 and 82·9 (iv) 403, 4030 and 40300 (v) 3·725, 37·25 and 372·5 **4.** (i) 1·08 (ii) 0·0012 (iii) 1·29 (iv) 0·1068 (v) 3·27 (vi) 0·356 (vii) 0·252 (viii) 1·03 (ix) 0·6 **5.** (i) 4·979, 0·4979 and 0·04979 (ii) 0·0923, 0·00923 and 0·000923 (iii) 0·00704, 0·000704 and 0·0000704 **6.** (i) 20 (ii) 70 (iii) 2·4 (iv) 5·4 (v) 10·33 (vi) 110 (vii) 640 (viii) 2025 **7.** (i) 10 (ii) 1000 (iii) 100 (iv) 10000 (v) 1000 (vi) 1000 (vii) 100 (viii) 10 (ix) 1000 (x) 100 (xi) 10000 **8.** (i) 6·466 (ii) 65·14 (iii) 297·106 (iv) 248·45 (v) 1·1346

Exercise 15(D)

1. (i) 840 p (ii) 97 p (iii) 9 p (iv) 6235 p **2.** (i) ₹ 0·55 (ii) ₹ 0·08 (iii) ₹ 6·95 (iv) ₹ 32·79 **3.** (i) 600 cm (ii) 854 cm (iii) 308 cm (iv) 87 cm (v) 3 cm (vi) 2504 cm **4.** (i) 2·50 m (ii) 23·28 m (iii) 0·86 m (iv) 0·04 m (v) 1·07 m **5.** (i) 6000 gm (ii) 5543 gm (iii) 78 gm (iv) 3620 gm (v) 4500 gm **6.** (i) 7 kg (ii) 6·839 kg (iii) 0·445 kg (iv) 0·093 kg (v) 0·008 kg (vi) 13·545 kg **7.** (i) ₹ 17·37 (ii) ₹ 29·35 (iii) ₹ 2·81 (iv) ₹ 15·44 **8.** (i) ₹ 27·48 (ii) ₹ 4·16 (iii) ₹ 0·66 = 66 paise **9.** (i) 4·18 m (ii) 11·38 m (iii) 1·57 m **10.** (i) 14·57 m (ii) 1028·72 m (iii) 0·38 m **11.** (i) 59·924 kg (ii) 2·717 kg (iii) 4·088 kg **12.** (i) 6·138 kg (ii) 18·683 kg (iii) 3·371 kg

Exercise 15(E)

1. ₹ 106 **2.** ₹ 8·73 **3.** 2·58 m **4.** 10·75 m **5.** 3·225 km **6.** ₹ 1,265·75 **7.** 35·7 m **8.** 5·65 m = 5 m 65 cm **9.** (i) 0·467 kg = 467 gm (ii) 2·335 kg **10.** 240·36 m; 20·03 m

Exercise 16(A)

1. (i) 65% (ii) 70% are good **2.** (i) 1·5% (ii) $83\frac{1}{3}$ % (iii) $81\frac{1}{4}$ % = 81·25% (iv) $66\frac{2}{3}$ % **3.** (i) 10% (ii) 2% (iii) 70% (iv) 15% (v) 3·2% **4.** (i) $\frac{2}{25}$ (ii) $\frac{1}{5}$ (iii) $\frac{17}{20}$ (iv) $\frac{5}{2} = 2\frac{1}{2}$ (v) $\frac{1}{8}$ **5.** (i) 0·25 (ii) 1·08 (iii) 0·95 (iv) 0·045 (v) 0·292 **6.** (i) 700% (ii) 200% (iii) 1950% (iv) 537%

Exercise 16(B)

1. (i) 20% (ii) 20% (iii) 25% (iv) 20% **2.** (i) 20% (ii) $8\frac{1}{3}$ % (iii) $11\frac{2}{3}$ %. **3.** (i) ₹ 30 (ii) 117 (iii) 0·3 min = 18 sec (iv) 37·5 kg **4.** 30% **5.** $81\frac{1}{4}$ % = 81·25% **6.** 76% **7.** 85% **8.** 11 kg **9.** Arithmetic (92·86%); $92\frac{6}{7}$ %; 80% **10.** 12% **11.** 25% **12.** (i) 80% (ii) 20% **13.** ₹ 26,880 **14.** (i) ₹ 540 (ii) ₹ 1,260 **15.** (i) 0 (ii) ₹ 25

Exercise 16(C)

1. 20% **2.** 12·5% **3.** 25% **4.** (i) ₹ 368 (ii) 26·25 km (iii) 675 km/h (iv) ₹ 64,062·50 **5.** 22,000 **6.** Food = ₹ 4,500; House rent = ₹ 1,800 and on both = ₹ 6,300 **7.** 10 litres **8.** 25,600 **9.** 39·2 kg **10.** (i) 20% (ii) 80% **11.** (i) 22·5 kg (ii) 97·2 kg

Revision Exercise (Chapter 16)

1. (i) 80% (ii) 125% **2.** (i) 40% (ii) 60% (iii) 150% **3.** 70% **4.** 13 kg **5.** (i) 75% (ii) 70% (iii) 70% (iv) 2500 (v) 1775 (vi) 71% **6.** 30% **7.** 20% **8.** (i) 25% (ii) 20% **9.** (i) ₹ 7,776 (ii) ₹ 7,980 **10.** (i) 50% increase (ii) 20% decrease (iii) 10% decrease **11.** (i) ₹ 832 (ii) ₹ 512 **12.** (i) 72 (ii) 49 **13.** (i) 15 (ii) −14 **14.** (i) 2·7% (ii) 8,00,000 m^3 **15.** Nitrogen = 624 m^3, Oxygen = 168 m^3 and others = 8 m^3

Exercise 17(A)

1. 17 km/h ; 8·5 km **2.** 500 m/minute **3.** 72 km/h **4.** 50 m/sec ; 2500 m = 2·5 km **5.** (i) 70 km/h (ii) 434 km (iii) 3 hours **6.** (i) 60 km (ii) 2·4 hours = 2 hours 24 min (iii) 1·2 min = 1 min 12 seconds **7.** 8 km/h **8.** $11\frac{1}{9}$ km/h **9.** $59\frac{1}{11}$ km/h **10.** 1188 km/h, 300 sec = 5 min

Exercise 17(B)

1. 7·2 sec **2.** 54 km/h **3.** 11 sec **4.** (a) (i) 1 km (ii) 0·05 km = 50 m (b) (i) 25 min (ii) 65 min = 1 hour 5 min **5.** 45 km/h is greater. 25 m and 24·5 m **6.** (i) 18 km (ii) 162 km

7. B will be 6 km ahead of A **8.** (i) 4 hrs (ii) 30 km h^{-1} **9.** (i) $\frac{2}{3}$ m s^{-1} (ii) 2·4 km h^{-1}
10. 1·5 m s^{-1} ; 5 min. 20 sec.

Exercise 18(A)

1. (i) $8 + x = y$ (ii) $x - 5 = y$ (iii) $2 + x > y$ (iv) $x + y < 24$ (v) $15 \times m = 3n$ (vi) $8 \times y = 3x$ (vii) $30 \div b = p$ (viii) $z - 3x = y$ (ix) $12 \times x = 5z$ (x) $12 \times x > 5z$ (xi) $12 \times x < 5z$ (xii) $45 - 3z = y$ (xiii) $8x \div y = 2z$ (xiv) $5x - 7y = 8z$ (xv) $7y - 5x = 8z$
2. (i) 3x plus 8 is equal to 15 (ii) 7 decreased by y is greater than x
(iii) 2y decreased by x is less than 12 (iv) 5 divided by z is equal to 5
(v) a increased by 2b is greater than 18 (vi) 2x decreased by 3y is equal to 16 (vii) 3a decreased by 4b is greater than 14 (viii) b increased by 7a is less than 21
(ix) The sum of 16 and 2a decreased by x is greater than 25
(x) The sum of 3x and 12 decreased by y is less than 3a.

Exercise 18(B)

1. 6, $\frac{5}{4}$ and 0 are constants. 4y, – 3x, $\frac{4}{5}xy$, az, 7p, $\frac{9x}{y}$, $\frac{3}{4x}$ and $-\frac{xz}{3y}$ are variables
2. (i) 4x, – x, $\frac{2}{3}x$ and – 3y, $\frac{4}{5}$ y, y (ii) $\frac{2}{3}$ xy, –4yx, yx and 2yz, $-\frac{2}{3}yz$ and $\frac{zy}{3}$ (iii) $-ab^2$, $7b^2a$, $2ab^2$ and b^2a^2, $-3a^2b^2$ (iv) 5ax, 7xa, $\frac{2ax}{3}$ and – 5by, $\frac{by}{7}$ **3.** (i) True (ii) False (iii) True (iv) False (v) True (vi) False (vii) True (viii) True (ix) True (x) False (xi) True (xii) False **4.** (i) 2 (ii) 2 (iii) 2 (iv) 2 (v) 3 (vi) 1 (vii) 2 (viii) 3 (ix) 3
5. (i) True (ii) False (iii) True (iv) False (v) False (vi) True **6.** (i) Monomial (ii) Binomial (iii) Monomial (iv) Monomial (v) Trinomial (vi) Binomial (vii) Trinomial (viii) Binomial (ix) Trinomial **7.** (i) 1 (ii) –1 (iii) –3 (iv) –5a (v) $\frac{3}{2}y$ (vi) $\frac{a}{y}$ **8.** (i) $-3y^2$ (ii) – a (iii) – 1 (iv) $\frac{2}{a}$ (v) – 2z (vi) $-y^2$ (vii) – 3a (viii) 5a **9.** (i) 5 (ii) 1 (iii) 5 (iv) –2 (v) $\frac{2}{3}$ (vi) $-\frac{15}{2}$ (vii) – 7 (viii) $-\frac{3}{2}$ **10.** (i) 2 (ii) 2 (iii) 10 (iv) 20 (v) 3 (vi) 7 (vii) 6 (viii) 9

Exercise 19(A)

1. (i) 9 and 9x (ii) 30 and $30x^2y$ (iii) 23 and 7a + 16b (iv) 4 and $x^2y + 3xy^2$ (v) 3 and 3ab (vi) 7 and 12x – 5y (vii) 19 and 19 ab (viii) 15 and $28ax^2 - 13a^2x$
2. (i) –7 and –7x (ii) 5 and 5ab (iii) –19 and –15x –4y (iv) 26 and 18x + 8y (v) 18 and 18ab (vi) 9 and 9xy (vii) –15 and –10ax –5ay **3.** (i) 11xy (ii) 9xyz (iii) 5a + 4b (iv) 3x + 2y (v) 5m + 3n + 4p (vi) 9a + 9ab (vii) 3p + 13q (viii) 9ab + 6b (ix) 80pq + 10pr (x) – 6y (xi) – 4b (xii) – 9b (xiii) – 8c **4.** (i) – 2a (ii) 2b (iii) 4x (iv) 2ab (v) 5x + 5y **5.** (i) 2x (ii) ab (iii) 8a (iv) 10abc **6.** (i) 2ab (ii) 2b (iii) 7abc (iv) 5 mn **7.** (i) $10a^2b^2 + 2ab^2$ (ii) 2a + 2b (iii) xy + 7yz (iv) 3ab (v) $4a^2 + 2b^2$ (vi) 4abc + 3ab (vii) 12xyz (viii) 12pqr + 2p + 4q (ix) 2ab (x) $11x^2y - 3xy^2$

Exercise 19(B)

1. (i) $2a + 5b - 3c$ (ii) $4x^2 - 7xy + 6y^2$ (iii) $-x^2 - 2x$ (iv) $2a^2 + 2ab - bc$ (v) $x^2 + 5 + 6x$ (vi) $5x + 4xy + 4y^2$ **2.** (i) $11x^2 + xy + 17y^2$ (ii) $4x^2 - 5xy - y^2 + 3$ (iii) $3a^3 + b^3 - 4a - b$ **3.** (i) $2a - 2b$ (ii) $4x - 4y$ (iii) $15a + 12b$ (iv) $11x + 3y$ (v) $12 - 4a$ (vi) $13y - 17$ **4.** (i) $-4a - b - 4c$ (ii) $8x + 13y - 24z$ (iii) $6a - 3b - 2c - 5$ (iv) $9x + 11y - 18z$ (v) $-ab - 3cd + 3ac + 3bd$ **5.** (i) $2ab$ (ii) $-2x - y - z$ (iii) $2p - \frac{4}{3}q - \frac{1}{2}r$ (iv) $2a$ **6.** $2x - y - 2z$ **7.** $2a + 2b + 3c$ **8.** $3x + 2y - z$ **9.** 0

Exercise 19(C)

1. (i) 18 and $18x^2$ (ii) 18 and $18x^5$ (iii) 20 and $20xy$ (iv) 28 and $28ax^2$ (v) 12 and $12x^2y^2$ (vi) 48 and $48a^2x^3$ (vii) 8 and $8a^5x^3y^3$ (viii) 45 and $45x^6y^2$ **2.** (i) $48x^2$ (ii) $18a^2bx$ (iii) $6x^6$ (iv) $25a^3$ (v) $216x^4y^2$ (vi) $24x^2$ (vii) $75x^3$ (viii) $-96x^4y^4$ (ix) $-60x^2yz$ (x) $-140x^6y^6$ **3.** (i) $15x^7$ (ii) $35a^9$ (iii) $18a^2bc^4$ (iv) $5a^5b^6$ (v) $10x^5y^7$ (vi) ab^2c^2d **4.** (i) $a^2b + ab^2$ (ii) $9a^2b^2 - 12ab^2$ (iii) $8bx^2y - 20b^2xy$ (iv) $12x^2y + 6xy^2$ (v) $2x^3 - 2x^2$ (vi) $x + 4x^2$ (vii) $45x^2y^3 + 15x^3y^2$ (viii) $18ax^2y - 15axy^2$ **5.** (i) $2x^2 - 2xy + 2xz$ (ii) $x^3y^2z^2 - x^2y^2z^3$ (iii) $-4xy^3z^2 - 6xy^3z$ (iv) $3x^2y^3 - 4x^3y^2$ (v) $-4x^3y^2 - 12x^3y^3$ **6.** (i) $9a^2 + 12ab - 15ac$ (ii) $5x^2y^3 + 30x^3y^2$ **7.** (i) $x^2 + 12x + 20$ (ii) $x^2 + 2x - 15$ (iii) $x^2 - 2x - 15$ (iv) $x^2 - 8x + 15$ (v) $2x^2 + 7xy + 3y^2$ (vi) $3x^2 + 13xy - 30y^2$ (vii) $x^2 + 4xy - 45y^2$ (viii) $4x^2 + 20xy + 25y^2$ **8.** (i) $-15a^3b^3c^2$ (ii) $-2x^2 + 2xy - 2xz$ (iii) $-4xy + 6y^2 + 10yz$ (iv) $-8x^2y^2z^2 + 10x^3y^2z^4$ (v) $-13x^2y^3z^2 + 15x^3y^2z^2 - 6x^2y^2z^3$ (vi) $-8a^2b^2c^3 + 10a^3b^2c^3 + 12a^2b^3c^3$ **9.** (i) $x^2y^2 - a^2b^2$ (ii) $4a^2b^2c^2 - 9x^2y^2$ (iii) $2a^2 - ab - 2ac - 3b^2 + 3bc$ (iv) $10x^2 + 3xy - 14xz - 18y^2 - 21yz$ (v) $10x^2 + 3xy - 9zx - 18y^2 - 27yz - 7z^2$ (vi) $2a^2 + ab - 6ac - 3b^2 + bc + 4c^2$.

Exercise 19(D)

1. (i) 3 (ii) 5 (iii) $4m$ (iv) $4x$ (v) 3 (vi) $7ab^3$ (vii) $6r^2$ (viii) $\frac{2}{b}$ **2.** (i) $2x^3$ (ii) $2a^5$ (iii) -4 (iv) $-4abc^2$ (v) $-\frac{5x}{y}$ (vi) $4q^3r^5$ (vii) $-16xy$ (viii) $\frac{5y}{x}$ **3.** (i) $-\frac{3}{8}$ (ii) $3q$ (iii) $-\frac{3}{2}m^3n^5$ (iv) $9ax^3y^4$ (v) $\frac{4x^2a^6}{y}$ (vi) $\frac{7ab^2}{c^3}$ (vii) $\frac{4a^3}{27b^3}$ (viii) $-\frac{x}{2} = -0{\cdot}5x$ (ix) $\frac{8xy}{z}$ **4.** (i) $5n^2$ (ii) $-\frac{7x^2}{3}$ (iii) $-4y$ **5.** (i) $3x^2 - 2x$ (ii) $-3m + 8m^2 - 5m^3$ (iii) $3x + 5y - \frac{36}{5}x^2y^2$ (iv) $-6x^2 + 4ax - 3a^2$

Exercise 20(A)

1. (ii) 3 (iii) 2 (iv) 4 (v) 6 (vi) 0 (vii) 0 (viii) 22 (ix) 13 (x) 26 (xi) 0 (xii) 8 (xiii) 1 (xiv) 18 (xv) 54 **2.** (i) 17 (ii) 150 (iii) 4 (iv) 6 (v) 4 (vi) 63 **3.** (i) 60 (ii) 2 (iii) 1 **4.** (i) 1 (ii) 8 (iii) –1 (iv) 6 (v) – 3 (vi) 6 (vii) 29 **5.** 16 **6.** –18 **8.** (i) True (ii) False (iii) True **9.** (i) $\frac{1}{4}$ (ii) $\frac{2}{5}$ (iii) 16 (iv) 25 (v) 200 (vi) 4000 (vii) $\frac{1}{10}$ (viii) 50 **10.** (i) $a^2 = 9$ and $2^a = 8$ **11.** 16

Exercise 20(B)

1. (i) 12 (ii) 15x (iii) 3m (iv) 10a (v) 10b (vi) 6y **2.** (i) 5x (ii) 10m – 4n (iii) – 3b (iv) 12a (v) x (vi) – s (vii) a + b – c – d – e + f (viii) 0 (ix) b + c (x) $6a^2 + b^2$ (xi) 4n – m (xii) – m – n (xiii) x (xiv) 38y – 11x (xv) 4x – 2 (xvi) 6a (xvii) $6x^2 - 3x - 4$ (xviii) 2x – y **3.** (i) x + y – z (ii) x (iii) 10x – 15y (iv) a + 13b (v) 2q – 2r – p (vi) b – c (vii) x + 21y (viii) $5a^2 - 10a$

Exercise 20(C)

1. (i) 2a + (b – c) (ii) 3x – (z – y) (iii) 6p – (5x – q) (iv) a + (b – c + d) (v) 4x – (2c – 5a – 4b) (vi) – 3 + 4y + (7x + 2z) (vii) 6 – (2n – 3m) (viii) 2t + r – (p + q – s) **2.** (i) – (– x + 2y) = – (2y – x) (ii) – (– m – n + p) = – (p – m – n) (iii) a + (4b – 4c) (iv) a – (3b – 5c) (v) $x^2 - (y^2 - z^2)$ (vi) $-(-m^2 - x^2 + p^2) = -(p^2 - m^2 - x^2)$ (vii) 2z – (– 2x + y) = 2z – (y – 2x) (viii) 2bc – (– ab + 3ac) = 2bc – (3ac – ab)

Exercise 21

1. (i) $P = 2(l + b)$ (ii) P = 4 × side (iii) $A = (\text{side})^2$ (iv) $S = 6\,(\text{edge})^2$ **2.** (i) x + y – m (ii) $\frac{xy}{m}$ (iii) 3n – 5m + 9p (iv) 12xyz – 5mn (v) p + 2r – s – (a + 3n + 4x) **3.** W = B + R(T – t)

4. (i) 20 (ii) 8 (iii) 8 **5.** (i) 24 (ii) 90 (iii) 144 **6.** (i) 55 (ii) 96 (iii) 2

7. (i) 25 (ii) 40 (iii) 2 **8.** (i) 1225 (ii) 350 (iii) 20 **9.** (i) –50 (ii) 100 (iii) –1000

10. (i) –66 (ii) 144 (iii) –432 **11.** (i) –84 (ii) 98 (iii) –686 **12.** $A = \frac{p+q+r+s}{4}$ and s = 9

13. (i) 90 (ii) 900 (iii) 72 **14.** (i) 144 (ii) 640 (iii) 64 **15.** (i) 160 (ii) 300 (iii) 48

16. (i) 1000 (ii) 450 (iii) 125 (iv) 1000 **17.** (i) 600 (ii) 1000 (iii) 400 (iv) 500

18. (i) 252 (ii) 735 (iii) 900 (iv) 196 **19.** (i) –32 (ii) 20 (iii) 20 **20.** (i) –33 (ii) 60 (iii) 18

21. (i) 5 (ii) –8 (iii) 16 **22.** (i) –10 (ii) 500 (iii) 2 **23.** (i) 5 (ii) –32 (iii) –128

24. (i) –88 (ii) –80 (iii) 400 (iv) 32 **25.** (i) –72 (ii) –324 (iii) –64 **26.** (i) 192 (ii) –384 (iii) 64

27. (i) 160 (ii) –640 (iii) 100 (iv) 192 **28.** 11 **29.** 17 **30.** ₹ 1578 **31.** 104°

Exercise 22(A)

1. (i) $x = 4$ (ii) $x = -4$ (iii) $y = -3$ (iv) $x = -7$ (v) $y = -10$ (vi) $b = 1{\cdot}7$ (vii) $p = 3{\cdot}9$ (viii) $y = -9{\cdot}7$ (ix) $a = -21{\cdot}5$ (x) $x = 2\frac{2}{3}$ (xi) $z = 2\frac{1}{5}$ (xii) $m = \frac{3}{4}$ (xiii) $x = -\frac{3}{4}$ (xiv) $y = -1\frac{1}{3}$ (xv) $a = -1\frac{7}{10}$ **2.** (i) $x = 5$ (ii) $m = -3$ (iii) $b = 12$ (iv) $a = -1{\cdot}5$ (v) $y = 9\frac{1}{2}$ (vi) $z = -3\frac{2}{3}$ (vii) $p = 8{\cdot}1$ (viii) $x = -3{\cdot}4$ (ix) $n = -\frac{1}{5} = -0{\cdot}2$ **3.** (i) $x = 4$ (ii) $y = 4\frac{1}{2} = 4{\cdot}5$ (iii) $z = 1{\cdot}7$ (iv) $m = 3$ (v) $p = 5$ (vi) $a = 2{\cdot}3$ **4.** (i) $x = 10$ (ii) $y = -6$ (iii) $a = -75$ (iv) $z = 13$ (v) $m = 15$ (vi) $n = -19{\cdot}6$ **5.** (i) $x = -4$ (ii) $y = -4$ (iii) $z = -\frac{4}{5} = -0{\cdot}8$ (iv) $a = -8$ (v) $p = -3$ (vi) $m = 7{\cdot}2$ (vii) $x = 5\frac{11}{15}$ (viii) $m = 1\frac{3}{7}$ (ix) $y = 1\frac{4}{5}$

Exercise 22(B)

1. (i) $x = 6$ (ii) $y = 1$ (iii) $p = 5$ (iv) $a = -6$ (v) $z = -11$ (vi) $m = 3$ (vii) $x = 3$ (viii) $m = -1{\cdot}1$ (ix) $y = -1{\cdot}4$ **2.** (i) $x = 21$ (ii) $y = 22$ (iii) $z = 10\frac{1}{2}$ (iv) $a = 17{\cdot}76$ (v) $b = -8{\cdot}8$ (vi) $m = 6$ **3.** (i) $m = 1$ (ii) $x = 1\frac{1}{3}$ (iii) $x = 1$ (iv) $x = 2$ (v) $a = 3$ (vi) $x = 5$ (vii) $x = 5\frac{1}{5}$ (viii) $x = 1$ (ix) $m = 5\frac{1}{2}$ (x) $x = -1\frac{1}{4}$

Exercise 22(C)

1. $x = 2$ **2.** $y = -6$ **3.** $x = 10{\cdot}4$ **4.** $x = \frac{4}{5}$ **5.** $y = 5\frac{5}{6}$ **6.** $z = 3\frac{1}{6}$ **7.** $z = 5$ **8.** $a = -0{\cdot}7$ **9.** $z = -6$ **10.** $z = -10$ **11.** $x = -2$ **12.** $z = -2$ **13.** $y = -3$ **14.** $x = 1{\cdot}7$ **15.** $y = 2{\cdot}7$ **16.** $x = -1{\cdot}4$ **17.** $y = 12$ **18.** $z = -12$ **19.** $x = -18{\cdot}8$ **20.** $y = -4$ **21.** $z = 1{\cdot}6$ **22.** $x = 5$ **23.** $y = 2$ **24.** 1 **25.** 5 **26.** 3 **27.** 2

Exercise 22(D)

1. 37 **2.** 34 **3.** 252 **4.** 160 **5.** 7 **6.** 28 **7.** Son = 10 years and father = 37 years **8.** Gopal = 15 years, father = 41 years **9.** 15 and 16 **10.** 21,22 and 23 **11.** 19 **12.** 36 **13.** 2 **14.** 32 years **15.** 9 years **16.** 21 **17.** (i) $x + 15$ (ii) 43 **18.** (i) $x - 23$ (ii) 34 **19.** 25, 26 and 27 **20.** (i) $x - 1$, x and $x + 1$ (ii) 17, 18 and 19.

Exercise 23(A)

1. (i) False; a dot has no length and no width (ii) True (iii) False; a line segment PQ is just written as PQ (iv) True (v) False; three points are said to be collinear if they lie in the same straight line (vi) True

2. (i) Infinite (ii) Only one (iii) Only one (iv) None **3.** (a) (i) A, B and C (ii) A, D and C (iii) AC and DC (b) (i) true (ii) true (iii) false (iv) false

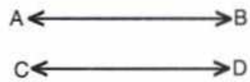
4. (i) A ray can be extended infinitely on only one side of it (ii) A ray has indefinite length (iii) Correct (iv) A line has no end-point (v) Correct

5. (i) False ; a line has an infinite number of points on it (ii) False; an infinite number of lines can pass through a given point (iii) True

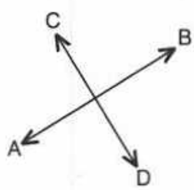
6. (i) Intersecting (ii) Parallel (iii) Parallel (iv) Intersecting

7. (i) Tip of your pencil and tip of a nail (ii) Lines printed on your note-book and edge of your table-top (iii) Top of your study table and a wall of your class-room (iv) Surface of a cricket-ball and surface of your pen.

8. (i) When lines are parallel to each other

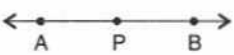


(ii) The lines intersect



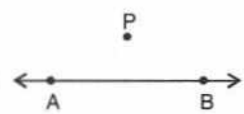
(iii) Lines coincide

9. (i)



(ii)

(iii)



10. (i) only one (ii) Infinite (iii) Infinite

11. (i)

(ii)

(iii)

12. (i)

(ii)

(iii)

13. (i)

(ii)

(iii)

14. (i) no (ii) no (iii) yes (iv) no

15. (i) EF // GH, EF //IJ and GH // IJ (ii) AB and CD (iii) AB and GH

Exercise 23(B)

1. (i), (iii), (iv), (vi) and (viii) **2.** (i) Triangle (ii) any curved line (iii) four; closed figure (iv) parallel (also equal); perpendicular (v) equal ; 90° (vi) 90° (vii) bisecting ; 90° ; perpendicular (viii) Infinite (ix) Only one (x) mid-point **3.** CD and MN **4.** (i) and (iii) **5.** (i) AB of CD (ii) AB of MN (iii) PQ of RS and RS of PQ *i.e.,* PQ and RS are perpendicular bisectors of each other (iv) None (v) None **6.** Your class-room, your text-book and your note-book **7.** (i) $l \parallel m$ and $p \parallel q$ (ii) p and l, p and m, q and l and q and m (iii) No (iv) Yes **8.** Parallel **9.** (i) 18 (ii) 24 (three at each corner) **10.** (i) Edges of your book through any corner of it (ii) Opposite edges of your book (iii) Adjacent edges at each end of your book **11.** (i) False; only one line passes through three collinear points (ii) False; no line passes through all the three non-collinear points (iii) True (iv) True (v) True (vi) True (vii) True (viii) True (ix) True

Exercise 24(A)

1. (i) Vertex = 0; arms are OA and OB; angle AOB or ∠AOB or ∠O (ii) Vertex = Q; arms are QP and QR; angle PQR or ∠PQR or ∠Q (iii) Vertex = M; arms are MN and ML; angle NML = ∠LMN = ∠M **2.** (i) a, b and x (ii) d, m, n, s and t. **3.** ∠AOB, ∠AOC, ∠AOD, ∠AOE, ∠BOC, ∠BOD, ∠BOE, ∠COD, ∠COE and ∠DOE. **4.** (i) 37°43′35″ (ii) 82°54′11″ (iii) 84°20′30″ (iv) 108°17′4″ **5.** (i) ∠AOB and ∠BOC; ∠BOC and ∠COD; ∠COD and ∠DOA; (ii) ∠AOB and ∠AOD (iii) ∠BOC and ∠COD. (iv) reflex ∠AOB and reflex ∠COD **6.** (i) 105° (ii) 70° **7.** (i) 130° (ii) 65° (iii) 155° (iv) 205° (v) 270° **8.** (i) 230° (ii) 160° (iii) 210° **9.** (i) 127° (ii) 45° **10.** x = 90° (i) 120° (ii) 60° (iii) ∠AOP (iv) ∠BOP **11.** x = 40° (i) 90° (ii) 150° (iii) 120° **12.** (i) 112° (ii) ∠AOC and ∠BOD; ∠AOD and ∠BOC (iii) ∠AOC and ∠BOC; ∠BOC and ∠BOD; ∠BOD and ∠DOA; ∠DOA and ∠AOC (iv) Reflex ∠BOC = 248°; Reflex ∠AOC = 292°; Reflex ∠AOD = 248°; Reflex ∠BOD = 292° **13.** (i) 35° and 65° (ii) 52° and 86° (iii) 284° and 246°.

Exercise 24(B)

1. (i)45° (ii) (90 – x)° (iii) (100 – x)° (iv) (70 – y)° **2.** (i) 131° (ii) 69° (iii) (210 – x)° (iv) (160 – y)° **3.** (i) 60° (ii) 58° (iii) 62° (iv) 75° **4.** (i) 120° (ii) 130° (iii) 120° (iv) 60° **5.** (i) 45° (ii) 90° **6.** 42° and 48° **7.** 70° and 110° **8.** 31 **9.** 32 **10.** (i) (90 – x)° (ii) (180 – x)° (iii) 45°

Exercise 25(A)

1. (i) 50° (ii) 130° (iii) 130° **2.** (i) 20° (ii) 75° (iii) 105° (iv) 75° **3.** x = 44° **4.** (i) Yes (ii) Yes (iii) No **5.** x = 35° and ∠APB = 135°

Exercise 25(B)

1. (a) (i) adjacent angles (ii) alternate exterior angles (iii) interior alternate angles (iv) corresponding angles (v) allied angles (b) (i) alternate interior angles (ii) corresponding angles (iii) alternate exterior angles (iv) corresponding angles (v) allied angles (c) (i) corresponding (ii) alternate exterior (iii) alternate interior (iv) alternate interior (v) alternate exterior (vi) vertically opposite **2.** (i) a = 40°; b = 40° (ii) a = 60°; b = 120° (iii) a = 110°; b = 70° (iv) a = 60°; b = 120° (v) a = 72°; b = 72° (vi) a = 80°; b = 100° (vii) a = 50°; b = 130° (viii) a = 118°; b = 62° (ix) a = 90°; b = 90° **3.** ∠2 = 60°; ∠3 = 120°; ∠4 = 60°; ∠5 = 120°; ∠6 = 60°; ∠7 = 120° and ∠8 = 60° **4.** x = 80°; y = 80°, z = 100°, p = 80°; q = 100°, r = 100° **5.** x = 60°; z = 60°, p = 60°, q = 120°, r = 120°, s = 120° **6.** x = 115°, y = 70°, z = 70°, w = 115° **7.** a = 130°; b = 150°; c = 150°; d = 130° **8.** x = 105°, y = 75°, z = 75°.

Exercise 25(C)

4. PR = 4 cm and QR = 4 cm. Yes, PR = QR.

Exercise 25(D)

1. 5·8 cm (approximately) **2.** 10 cm **3.** Each angle = 60°; yes, angles obtained on bisecting are equal **4.** MN = 4·3 cm (approximately)

Exercise 26(A)

1. (i) z = 38° (ii) a = 95°; b = 50° (iii) x = 55° **2.** (i) No (ii) Yes (iii) No

3. (i) 60 (ii) 36 (iii) 15 **4.** 20° **5.** 56° **6.** 40° **7.** (i) x = 80° (ii) x = 65°; y = 50°

(iii) x = 22°; 2x = 44°; 3x = 66° **8.** (i) Obtuse-angled triangle (ii) Acute angled (iii) Right angled **9.** (i) Isosceles triangle (ii) Scalene (iii) Scalene (iv) Equilateral

Exercise 26(B)

3. Yes; ∠A = ∠C = 72·5° (approximately) **4.** Yes; each angle is 60°

7. AB = 4·4 cm and BC = 5·4 cm **8.** ∠C = 60° and side BC = 3·1 cm

9. BC = 7 cm and AC = 12 cm **10.** ∠ACB = 90°; right-angled triangle.

Exercise 27(A)

1. 79° each **2.** 90°, 126° **3.** (i) x = 16 (ii) 64°, 90°, 92° and 114°

4. (i) x = 22° (ii) ∠B = 48° and ∠C = 61° **5.** ∠A = 60°, ∠B = 100°, ∠C = 80° and ∠D = 120°

6. (i) x = 26° (ii) 104° (iii) 28° **7.** ∠A = 130° **8.** b = 105° c = 140° **9.** 97°

10. ∠P = 54°, ∠Q = 72°, ∠R = 108° and ∠S = 126°

∠P + ∠S = 54° + 126° = 180° ⇒ PQ and SR are parallel to each other

∠P + ∠Q = 54° + 72° = 126° ≠ 180° ⇒ PS is not parallel to QR

11. x = 40° **12.** ∠A = 80°, ∠D = 100°, ∠B = 60° and ∠C = 120°

Exercise 27(B)

1. ∠B = 60°, ∠D = 102° **2.** x = 50 **3.** ∠A = 60°, ∠B = 80°, ∠C = 100° and ∠D = 120°

∵ ∠A + ∠D = 60° + 120° = 180° ⇒ AB is parallel to DC (since the sum of co-interior angles is 180°, the lines are parallel) **4.** Parallel, equal **5.** x = 4 **6.** ∠B = 115°, ∠C = 65° and ∠D = 65°

7. 80° each **8.** 70°, 110° **9.** ∠A = 72°, ∠B = 108°, ∠C = 72° and ∠D = 108°. Since opposite angles are equal, it is a parallelogram **10.** OA = 6 cm, OD = 4.5 cm **11.** AC = 12 cm, BD = 15 cm

12. (i) ∠B = 90° (ii) rectangle **13.** 18 cm **14.** (i) x = 85° (ii) 90°, rectangle

15. each angle is 90°, so quadrilateral is a rectangle **16.** 90°

17. Rhombus, as all its four sides are equal. The given figure will be a square when any angle of it is 90° **18.** All the sides must be equal and any angle is 90°

Exercise 28(A)

1. (ii) and (iii) **2.** (i) 1260° (ii) 1980° (iii) 2520° **3.** (i) 10 (ii) 11 **4.** No **5.** (i) 120° (ii) 135°
6. 90° **7.** 80°, 100°, 60° and 120° **8.** 105° **9.** 100°, 80°, 100°, 140° and 120° **10.** 130°

Exercise 28(B)

1. (i) 60°, 120° (ii) 45°, 135° (iii) 10, 144° (iv) 18, 160° (v) 8, 45° (vi) 24, 15° **2.** (i) 18 (ii) 12
3. (i) 12 (ii) 10 **4.** (i) yes, 8 sides (ii) No **5.** (i) No (ii) Yes, 10 sides **6.** (i) 60° (ii) 6

Exercise 29(A)

1. (i) centre (ii) PQ (iii) AB (iv) secant (v) CD (vi) radius (vii) RS (viii) 10 cm
(ix) 4 cm (x) tangent **2.** (i) 8·4 cm (ii) diameter **3.** (i) 3 cm (ii) 4·2 cm
4. (ii) Equilateral triangle (iii) 60° **8.** ∠ACB = 90°

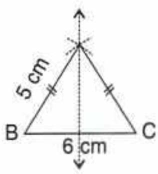
Exercise 29(B)

4. 2·8 cm **5.** 3·3 cm (approx.)

Exercise 30

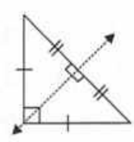
1. (i) True, if both the semi-circles of B are equal
(ii) True (iii) False (iv) False (v) False (vi) True (vii) True (viii) False

2.

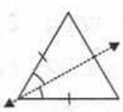


3. (i) Not possible

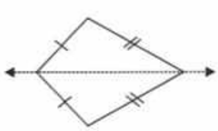
(ii)



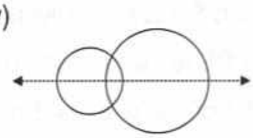
(iii)



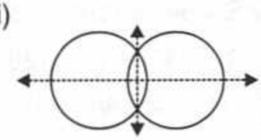
(iv)



(v)



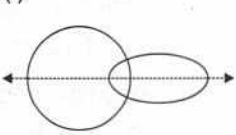
(vi)



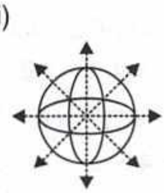
4. Three lines of symmetry; bisector of each angle.

5. Three lines of symmetry, as ΔABC is equilateral.

6. (i)



(ii)



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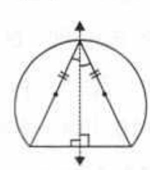
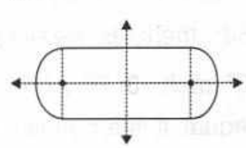


(iv)

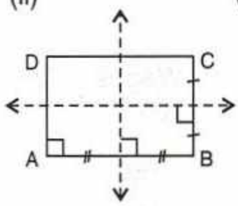


7. In each case draw perpendicular from point P on the line AB, which meets AB at point Q. From PQ produced cut QR = PQ. Point R, so obtained, is the required point.

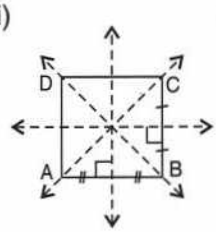
8. PQ is perpendicular bisector of the line segment joining A and B.

10. (i) Parallelogram has no line of symmetry.

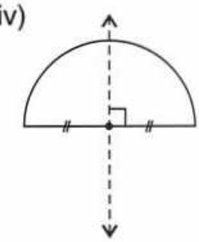
(ii)



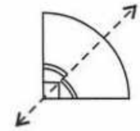
(iii)



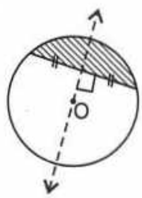
(iv)



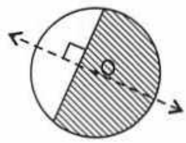
(v)



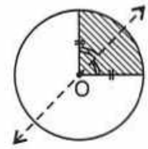
11. (i)



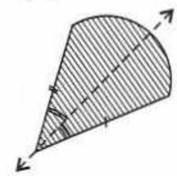
(ii)



(iii)



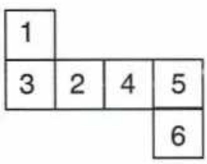
(iv)



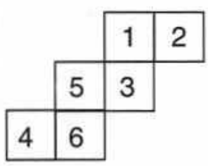
Exercise 31

1. (ii), (iii) and (v) **5.** (i) cube (ii) cuboid (iii) sphere (iv) prism **6.** (a) 4, 4, 6, 2 (b) 5, 5, 8, 2 (c) 8, 6, 12, 2 (d) 8, 12, 18, 2

7. (i)



(ii)



8. (i) Tetrahedron (ii) Triangular prism (iii) Cube (iv) Cuboid

Exercise 32(A)

1. A figure that lies in a plane and is bounded by straight or curved lines.

2. No. The interior of the figure together with its boundary is region of the figure.

3. (i) 110 cm (ii) 88 cm **4.** (i) 150 cm (ii) 36 m (iii) 17.60 m = 1760 cm (iv) 12 m

5. (i) 12 cm (ii) 0.6 m (iii) 550 cm = 5.50 m **6.** 6.4 m **7.** 1.25 m **8.** square by 100 m

9. (i) 560 m (ii) 560 m (iii) ₹ 44,800 **10.** ₹ 7,040 **11.** (i) 280 m (ii) 2240 m

12. 6300 m **13.** John by 880 m **14.** 20 cm **15.** (i) 29.4 cm (ii) 36 cm (iii) 41 cm (iv) 39 cm

16. 7.2 cm **17.** 12 cm **18.** 10 cm **19.** 40 cm **20.** 15 cm

Exercise 32(B)

1. (i) 96 cm^2 (ii) 42.5 m^2 (iii) 3.24 m^2 (iv) 432 cm^2

2. (i) 51.84 cm^2 (ii) 20.25 m^2 (iii) 16.81 cm^2

3. (i) 8 cm (ii) 8 m **4.** (i) 14.4 m (ii) 12.96 m^2 **5.** (i) 15 m (ii) 225 m^2 (iii) 289 m^2

6. 51 m^2 **7.** 4 unit **8.** 19.6 cm **9.** 204 m

10. Area will become 9 times *i.e.* it will increase 8 times **11.** 1000 m^2

12. (i) 144 m^2 (ii) 768 m^2 **13.** 96 **14.** 5000

15. (i) 8100 m^2 (ii) ₹ 14,400 (iii) ₹ 4,86,000 **16.** (i) 100 m (iii) 15000 m^2 (iii) ₹ 18,000

17. (i) 6.4 m (ii) 22.8 m (iii) ₹ 1,368 **18.** 200 cm

Exercise 33(A)

1. (i) 85 (ii) 22 (iii) 63

2.

Marks	Tally-marks	Frequency
28	IIII	4
30	𝍸 III	8
32	𝍸 𝍸	10
34	III	3
	Total	**25**

4. 6.1, 6, 5.9, 5.9, 5.8, 5.8, 5.7, 5.7, 5.6, 5.5, 5.5, 5.4, 5.3, 5.2, 5.1

(i) 6.1 (ii) 5.1 (iii) 1

5.

Marks	Tally-marks	Frequency
48	II	2
52	IIII	4
56	IIII	4
60	II	2
64	III	3
68	III	3
72	II	2
	Total	**20**

6.

Number of accidents per day	Tally-marks	Frequency
0	II	2
1	III	3
2	𝍸 I	6
3	III	3
4	IIII	4
5	𝍸 I	6
6	𝍸 I	6
	Total	**30**

7.

Wages (in ₹)	Tally-marks	Frequency
750	III	3
800	IIII	4
850	III	3
900	III	3
950	II	2
	Total	**15**

(i) 7 (ii) 8

8.

Marks	Tally-marks	Frequency
9	𝍸	5
12	IIII	4
17	III	3
18	II	2
19	II	2
20	II	2
25	II	2
	Total	**20**

(i) 14 (ii) 2 (iii) 9

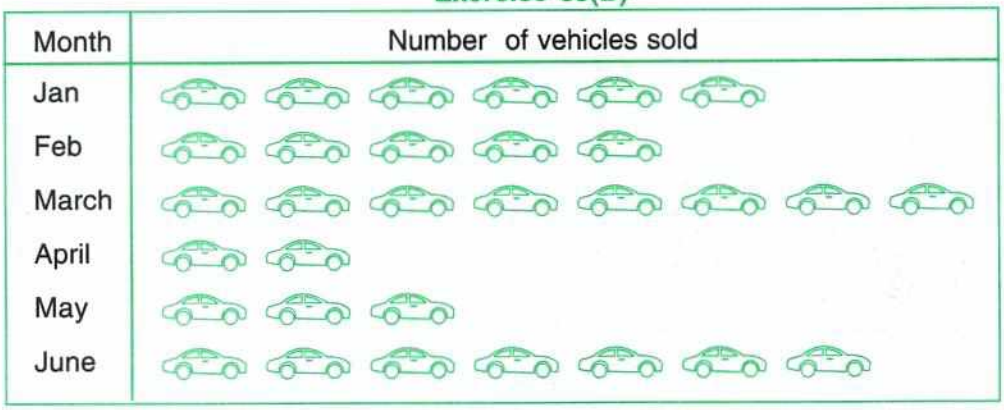
9.

Marks	Tally-marks	Frequency
42	I	1
44	I	1
46	I	1
48	I	1
50	II	2
52	I	1
54	II	2
56	II	2
58	II	2
60	I	1
62	I	1
	Total	**15**

(i) 20 (ii) 9 (iii) 4

Exercise 33(B)

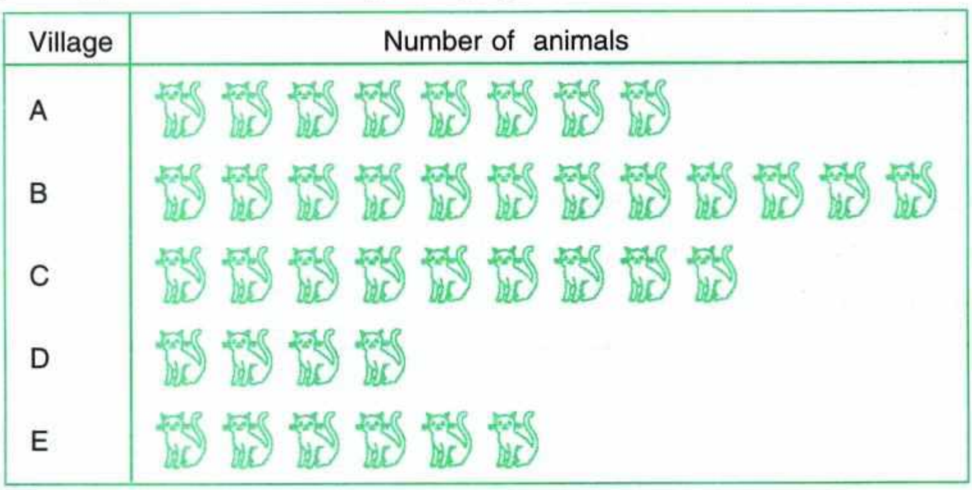
1.



Scale : = 500 vehicles

2. (i) 100 (ii) $23 \times 50 = 1150$ **3.** (i) Friday (ii) $42.5 \times 100 = 4250$

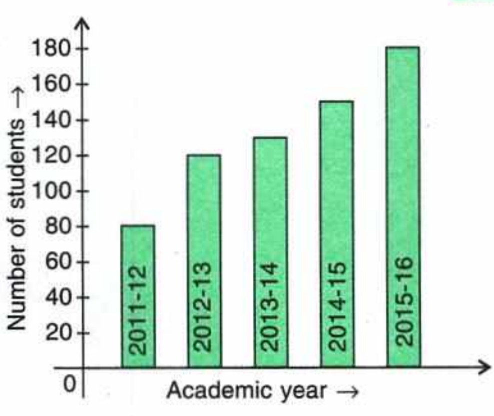
4.



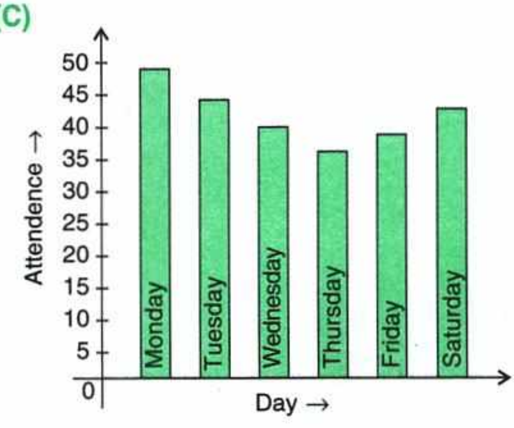
5. (i) 200 (ii) 275 (iii) English

Exercise 33(C)

1.

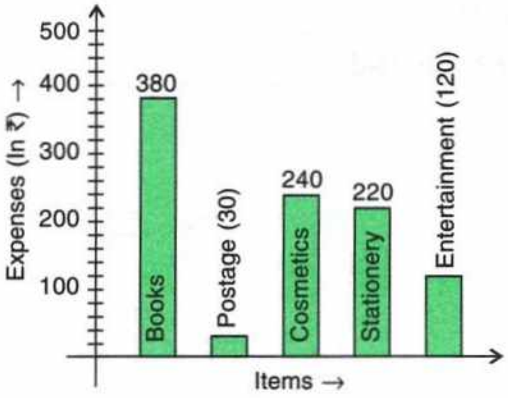


2.



5. (i) 60 (ii) 0 (iii) 20% **6.** (i) D (ii) A + D **7.** (i) A = 16, B = 9, C = 21, D = 7, E = 24 and F = 15 (ii) D (iii) E (iv) 46 (v) 46 (vi) 15·3 **8.** (i) 1—100, 2—90, 3—100, 4—80, 5—120, 6—90, 7—70, 8—50 (ii) 210 (iii) 30 (iv) 700 **9.** (i) Tuesday (ii) Friday (iii) Sunday, Thursday (iv) 300

10.



Exercise 34(A)

1. (i) 9·5 (ii) 11 (ii) 4 (iv) 3·5 (v) 5·64 (vi) 5

2. (i) 4·5 (ii) 7 (iii) 5 (iv) 12·9 (v) 30

3. 158 cm **4.** 38 **5.** 10 **6.** 9

Exercise 34(B)

1. (i) 24 (ii) 5·6 (iii) 21·5 (iv) 30 (v) 63·5

2. (i) 6 and 5 (ii) 15·4 and 16 (iii) 8 and 8 (iv) 12·75 and 12

(v) 2·16 and 2·2 (vi) 3·65 and 4·1