Preface

Mathematics — is a science of logical thinking and art of solving problems. This series '*Middle School Mathematics*' has been revised thoroughly, as per the latest curriculum prescribed by the Council for the ISC Examinations, New Delhi, keeping in mind exclusively the requirements of students of classes VI, VII and VIII.

While revising this series, great care has been taken to explain mathematical concepts and to provide adequate number of worked out examples. The endeavour has been directed towards giving an optimum amount of knowledge followed by problems which are compatible with that knowledge. Special stress has been laid on the simplicity of the language of the book. A lot of effort has been made to present the subject matter in clear, concise and laconic form for the benefit of both teachers and pupils.

Further, a sincere attempt has been made to make this series student friendly — the subject matter is well spaced out into sections and subsections with varied examples so as to solicit genuine interest in learning and understanding mathematical concepts and to enhance their skill in solving problems.

Suggestions for the improvement of this series will be gratefully acknowledged

— R.K. Bansal

Contents

MATHEMATICS CURRICULUM FOR CLASS VIII

TEACHING POINTS	KEY CONCEPTS
1. NUMBER SYSTEM	
Rational Numbers	• Properties of rational numbers. (including identities). Using general form of expression to describe properties • Representation of rational numbers on the number line • Understanding that between any two rational numbers there lies another rational number • Word problems
Exponents and Powers	• Laws of exponents with integral powers • Square and Square roots using factor method and division method for numbers containing (a) no more than total 4 digits and (b) no more than 2 decimal places • Cubes and cube roots (only factor method for numbers containing at most 3 digits)
Playing with numbers	• Writing and understanding a 2 and 3-digit number in generalized form (100a + 10b + c, where a, b, c can be only digit 0-9) and engaging with various puzzles Children to solve and create problems and puzzles. • Deducing the divisibility test rules of 2, 3, 5, 9, 10 for a two or three-digit number expressed in the general form.
Sets	• Union and intersection of sets • Disjoint set • Complement of a set
2. RATIO AND PROPORTION	
	• Slightly advanced problems involving applications on percentages, profit & loss, overhead expenses, discount, tax. • Difference between simple and compound interest (compounded yearly up to 3 years or half-yearly up to 3 steps only • Direct and inverse variations – Simple and direct word problems • Time and work problems– simple and direct word problems
3. ALGEBRA	
	• Algebraic Expressions • Multiplication and division of algebraic expression (Coefficient should be integers) • Identities $(a \pm b)^2 = a^2 \pm 2ab + b^2$, $a^2 - b^2 = (a - b)\ (a + b)$ • Factorisation (simple cases only) as examples the following types $a(x + y)$, $(x \pm y)^2$, $a^2 - b^2$, $(x + a)(x + b)$ • Solving linear equations in one variable in contextual problems involving multiplication and division (word problems) (avoid complex coefficient in the equations)

4. GEOMETRY

Understanding shapes

- Properties of quadrilaterals – Angle Sum property
- Properties of parallelogram (By verification) (i) Opposite sides of a parallelogram are equal, (ii) Opposite angles of a parallelogram are equal, (iii) Diagonals of a parallelogram bisect each other. (iv) Diagonals of a rectangle are equal and bisect each other. (v) Diagonals of a rhombus bisect each other at right angles. (vi) Diagonals of a square are equal and bisect each other at right angles.

Representing 3-D in 2-D

- Identify and match pictures with objects [more complicated e.g. nested, joint 2-D and 3-D shapes (not more than 2)].
- Drawing 2-D representation of 3-D objects (Continued and extended)
- Counting vertices, edges and faces and verifying Euler's relation for 3-D figures with flat faces (cubes, cuboids, tetrahedrons, prisms and pyramids)

Construction of Quadrilaterals

- Given four sides and one diagonal
- Three sides and two diagonals
- Three sides and two included angles
- Two adjacent sides and three angles

5. MENSURATION

- Area of a trapezium and a polygon.
- Surface area of a cube, cuboid, cylinder.
- Concept of volume, measurement of volume using a basic unit, volume of a cube, cuboid and cylinder
- Volume and capacity (measurement of capacity)

6. DATA HANDLING

- Arranging ungrouped data into groups, representation of grouped data through bargraphs, constructing and interpreting bar-graphs.
- Simple Pie charts with reasonable data numbers
- Consolidating and generalising the notion of chance in events like tossing coins, dice etc. Relating it to chance in life events.

Theme – 1
NUMBER
SYSTEM

RATIONAL NUMBERS 1

1.1 INTRODUCTION

We know that :

1. **Natural numbers** = Counting numbers
 = 1, 2, 3, 4, 5, ...
2. **Whole numbers** = 0 (zero) with natural numbers
 = 0, 1, 2, 3, 4, 5, ...
3. **Integers** = Negative of natural numbers together with whole numbers.
 = ..., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, ...
 <·························| |·························>
 Negative of natural numbers + Whole numbers

In class VII, rational numbers were introduced and we did addition, subtraction, multiplication and division of rational numbers. Now we shall be discussing rational numbers and operations on them in detail.

1.2 RATIONAL NUMBER

If p and q both are integers and q ≠ 0, then $\frac{p}{q}$ is called a rational number.

For example :

1. $\frac{-3}{7}$ is a rational number as –3 and 7 both are integers and 7 ≠ 0.
2. $\frac{15}{22}$ is a rational number as 15 and 22 both are integers and 22 ≠ 0.

Remember :

1. Zero (0) can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{5}, \frac{0}{-10}, \frac{0}{15}, \frac{0}{-22}$, etc. In each of these cases denominator is not equal to zero.
 So, zero can be expressed as a fraction with a non-zero denominator.
 ∴ **Zero (0) is a rational number**.
2. Every natural number, every whole number, every integer and every fraction is a rational number.
3. In the rational number $\frac{p}{q}$, where p and q are integers and $q \neq 0$, integer p is called numerator and integer q is called the denominator.
 For example :
 (i) In $\frac{-8}{15}$, numerator = –8 and denominator = 15.
 (ii) If numerator = 5 and denominator = –2, the rational number is $\frac{5}{-2}$.

4. A rational number is positive, if its numerator and denominator have same signs, whereas a rational number is negative, if its numerator and denominator have opposite signs.

Thus,

(i) each of $\frac{5}{8}, \frac{-5}{-8}, \frac{-12}{-17}, \frac{15}{19}$, etc. is positive.

(ii) each of $\frac{-5}{8}, \frac{5}{-8}, \frac{12}{-17}, \frac{-15}{19}$, etc. is negative.

5. If m is a non-zero integer and $\frac{p}{q}$ is a rational number, then

$\frac{p}{q} = \frac{p \times m}{q \times m}$ and $\frac{p}{q} = \frac{p \div m}{q \div m}$

Here $\frac{p \times m}{q \times m}$ and $\frac{p \div m}{q \div m}$ are rational numbers each equivalent to $\frac{p}{q}$.

6. Let $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that

$\frac{a}{b} = \frac{c}{d} \Rightarrow a \times d = b \times c$

Conversely, a × d = b × c $\Rightarrow \frac{a}{b} = \frac{c}{d}$ or, $\frac{a}{c} = \frac{b}{d}$, *etc*

7. A rational number $\frac{p}{q}$ is said to be in **standard form**, if :

(i) p and q have no common divisor (factor) other than one (1)

and (ii) q is positive.

For example :

(i) $\frac{3}{5}$ is a rational number in standard form.

(ii) The rational number $\frac{3}{-5}$ in standard form is $\frac{-3}{5}$.

(iii) $\frac{-21}{36}$ is not in standard form as 21 and 36 have 3 as a common divisor.

Since, $\frac{-21}{36} = \frac{-7 \times 3}{12 \times 3} = \frac{-7}{12}$

$\therefore \frac{-21}{36}$ in standard form is $\frac{-7}{12}$.

Similarly, $\frac{36}{-63} = \frac{4 \times 9}{-7 \times 9} = \frac{4}{-7} = \frac{-4}{7}$.

$\Rightarrow \frac{36}{-63}$ in standard form is $\frac{-4}{7}$.

1.3 PROPERTIES OF ADDITION OF RATIONAL NUMBERS

1. Closure property

If two rational numbers are added together, the result is always a rational number.

For example :

(i) Addition of rational numbers $\frac{3}{4}$ and $\frac{5}{6}$

$= \frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{9+10}{12} = \frac{19}{12}$, *which is a rational number.*

(ii) Addition of rational numbers $\frac{-3}{8}$ and $\frac{5}{12}$

$= \frac{-3}{8} + \frac{5}{12} = \frac{-9}{24} + \frac{10}{24} = \frac{-9+10}{24} = \frac{1}{24}$, *which is a rational number.*

Thus according to the closure property, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then their addition $\left(\frac{a}{b}+\frac{c}{d}\right)$ is also a rational number.

We say, set of rational numbers is closed for addition.

2. Commutativity

The addition of any two rational numbers is commutative.

According to commutative property of addition, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then : $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$.

Consider the rational numbers $\frac{-7}{12}$ and $\frac{5}{8}$.

$$\frac{-7}{12} + \frac{5}{8} = \frac{-14}{24} + \frac{15}{24} = \frac{-14+15}{24} = \frac{1}{24}$$

and, $\frac{5}{8} + \frac{-7}{12} = \frac{15}{24} + \frac{-14}{24} = \frac{15-14}{24} = \frac{1}{24}$

$\therefore \quad \frac{-7}{12} + \frac{5}{8} = \frac{5}{8} + \frac{-7}{12}$

The same can be verified with any pair of rational numbers.

3. Associativity

The addition of rational numbers is associative.

According to this property, if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

$$\frac{a}{b} + \left(\frac{c}{d}+\frac{e}{f}\right) = \left(\frac{a}{b}+\frac{c}{d}\right) + \frac{e}{f}$$

Consider the rational numbers $\frac{2}{3}$, $\frac{-5}{6}$ and $\frac{7}{12}$.

$$\therefore \quad \frac{2}{3} + \left(\frac{-5}{6} + \frac{7}{12}\right) = \frac{2}{3} + \left(\frac{-10}{12} + \frac{7}{12}\right)$$

$$= \frac{2}{3} + \frac{-3}{12} = \frac{8}{12} + \frac{-3}{12} = \frac{8-3}{12} = \frac{5}{12}$$

And, $$\left(\frac{2}{3} + \frac{-5}{6}\right) + \frac{7}{12} = \left(\frac{4}{6} + \frac{-5}{6}\right) + \frac{7}{12}$$

$$= \frac{-1}{6} + \frac{7}{12} = \frac{-2}{12} + \frac{7}{12} = \frac{-2+7}{12} = \frac{5}{12}$$

$$\therefore \quad \frac{2}{3} + \left(\frac{-5}{6} + \frac{7}{12}\right) = \left(\frac{2}{3} + \frac{-5}{6}\right) + \frac{7}{12}$$

In the same way,

(i) $-\frac{5}{8} + \left(\frac{3}{4} + \frac{-7}{16}\right) = \left(\frac{-5}{8} + \frac{3}{4}\right) + \frac{-7}{16}$

(ii) $\frac{15}{-22} + \left(\frac{-8}{11} + \frac{3}{2}\right) = \left(\frac{15}{-22} + \frac{-8}{11}\right) + \frac{3}{2}$ and so on.

4. Existence of additive identity of rational numbers

Additive identity for rational numbers is zero (0).

When the additive identity is added to any rational number or any rational number is added to the additive identity, the rational number remains the same.

$\therefore$ 0 + *a* rational number = The same rational number + 0

= The rational number itself

For example :

(i) $0 + \frac{-3}{5} = -\frac{3}{5} + 0 = -\frac{3}{5}$. (ii) $0 + \frac{7}{8} = \frac{7}{8} + 0 = \frac{7}{8}$ and so on.

Consider a rational number $\frac{a}{b}$, then $\frac{a}{b} + 0 = \frac{a}{b} = 0 + \frac{a}{b}$

The rational number 0 is called the identity element for addition of rational numbers.

5. Existence of additive inverse of a rational number

The negative of a rational number is called its additive inverse.

(i) The additive inverse of $\frac{3}{5} = -\frac{3}{5}$.

(ii) The additive inverse of $\frac{-5}{8} = \frac{5}{8}$ and so on.

The sum of a rational number and its additive inverse = Additive identity

i.e. Any rational number + its additive inverse = 0, the additive identity

$$\Rightarrow \quad \frac{3}{5} + \left(-\frac{3}{5}\right) = 0, \qquad \left(-\frac{5}{8}\right) + \frac{5}{8} = 0$$

$$\left(\frac{7}{-8}\right) + \frac{7}{8} = 0, \qquad \frac{8}{15} + \left(-\frac{8}{15}\right) = 0 \text{ and so on.}$$

Example 1 :

Add each pair of rational numbers, given below, and show that their addition (sum) is also a rational number :

(i) $\frac{7}{15}$ and $\frac{3}{5}$

(ii) $\frac{2}{5}$ and 2

(iii) $\frac{3}{8}$ and $\frac{-5}{12}$

(iv) $\frac{7}{-15}$ and $\frac{2}{-3}$

(v) $\frac{5}{-13}$ and $\frac{11}{26}$

Solution :

(i) $\frac{7}{15} + \frac{3}{5} = \frac{7}{15} + \frac{3\times3}{5\times3}$ [$\because$ L.C.M. of 15 and 5 = 15]

$= \frac{7}{15} + \frac{9}{15}$

$= \frac{7+9}{15} = \frac{16}{15}$, *which is a rational number.*

(ii) $\frac{2}{5} + 2 = \frac{2}{5} + \frac{2}{1}$ [$\because$ L.C.M. of 5 and 1 = 5]

$= \frac{2}{5} + \frac{2\times5}{1\times5}$

$= \frac{2}{5} + \frac{10}{5}$

$= \frac{2+10}{5} = \frac{12}{5}$, *which is a rational number.*

(iii) $\frac{3}{8} + \frac{-5}{12} = \frac{3\times3}{8\times3} + \frac{-5\times2}{12\times2}$ [$\because$ L.C.M. of 8 and 12 = 24]

$= \frac{9}{24} + \frac{-10}{24} = \frac{9-10}{24} = \frac{-1}{24}$, *which is a rational number.*

(iv) $\frac{7}{-15} + \frac{2}{-3} = \frac{-7}{15} + \frac{-2}{3}$

$= \frac{-7}{15} + \frac{-2\times5}{3\times5}$ [$\because$ L.C.M. of 15 and 3 = 15]

$= \frac{-7}{15} + \frac{-10}{15} = \frac{-7-10}{15} = \frac{-17}{15}$, *which is a rational number.*

(v) $\frac{5}{-13} + \frac{11}{26} = \frac{-5}{13} + \frac{11}{26}$

$= \frac{-5\times2}{13\times2} + \frac{11}{26}$ [∵ L.C.M. of 13 and 26 = 26]

$= \frac{-10}{26} + \frac{11}{26} = \frac{-10+11}{26} = \frac{1}{26}$, *which is a rational numbers.*

The examples, given above, show that the addition of two rational numbers is always a rational number. Thus, it verifies the closure property of addition of rational numbers.

Example 2 :

Evaluate :

(i) $\frac{3}{4} + \frac{5}{6} + \frac{-1}{4} + \frac{-7}{6}$ (ii) $\frac{9}{-10} + \frac{4}{15} + \frac{-3}{20} + \frac{-3}{10} + \frac{8}{15} + \frac{9}{-20}$

Solution :

Re-arrange the given rational numbers and group them in such a way that each group has rational numbers with same denominator.

(i) $\frac{3}{4} + \frac{5}{6} + \frac{-1}{4} + \frac{-7}{6} = \left(\frac{3}{4}+\frac{-1}{4}\right) + \left(\frac{5}{6}+\frac{-7}{6}\right)$

$= \frac{3-1}{4} + \frac{5-7}{6}$

$= \frac{2}{4} + \frac{-2}{6}$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1\times3}{2\times3} - \frac{1\times2}{3\times2}$ [∵ L.C.M. of 2 and 3 = 6]

$= \frac{3}{6} - \frac{2}{6} = \frac{3-2}{6} = \frac{1}{6}$ **(Ans.)**

(ii) $\frac{9}{-10} + \frac{4}{15} + \frac{-3}{20} + \frac{-3}{10} + \frac{8}{15} + \frac{9}{-20}$

$= \left(\frac{-9}{10}+\frac{-3}{10}\right) + \left(\frac{4}{15}+\frac{8}{15}\right) + \left(\frac{-3}{20}+\frac{-9}{20}\right)$

$= \frac{-9-3}{10} + \frac{4+8}{15} + \frac{-3-9}{20}$

$= \frac{-12}{10} + \frac{12}{15} + \frac{-12}{20}$

$= -\frac{6}{5} + \frac{4}{5} - \frac{3}{5} = \frac{-6+4-3}{5} = \frac{-5}{5} = -1$ **(Ans.)**

Example 3 :

Use rational numbers $\frac{4}{9}$ and $\frac{-7}{12}$ to verify the commutative proeprty for the addition of rational numbers.

Solution :

Show that : $\frac{4}{9} + \frac{-7}{12} = \frac{-7}{12} + \frac{4}{9}$.

$\because \quad \frac{4}{9} + \frac{-7}{12} = \frac{4\times4}{9\times4} + \frac{-7\times3}{12\times3}$ [$\because$ L.C.M. of 9 and 12 = 36]

$= \frac{16}{36} - \frac{21}{36} = \frac{16-21}{36} = \frac{-5}{36}$

And $\quad \frac{-7}{12} + \frac{4}{9} = \frac{-7\times3}{12\times3} + \frac{4\times4}{9\times4}$

$= \frac{-21}{36} + \frac{16}{36} = \frac{-21+16}{36} = \frac{-5}{36}$

$\therefore \quad \frac{4}{9} + \frac{-7}{12} = \frac{-7}{12} + \frac{4}{9}$

This verifies the commutative property for the addition of rational numbers.

Example 4 :

Use rational numbers $\frac{-4}{5}$, $\frac{7}{10}$ and $\frac{11}{-20}$ to verify the associative property of the addition of rational numbers.

Solution :

Show that : $\frac{-4}{5} + \left(\frac{7}{10} + \frac{11}{-20}\right) = \left(\frac{-4}{5} + \frac{7}{10}\right) + \frac{11}{-20}$.

$\because \quad \frac{-4}{5} + \left(\frac{7}{10} + \frac{11}{-20}\right) = \frac{-4}{5} + \left(\frac{7}{10} + \frac{-11}{20}\right)$

$= \frac{-4}{5} + \left(\frac{7\times2}{10\times2} + \frac{-11}{20}\right)$ [$\because$ L.C.M. of 10 and 20 = 20]

$= \frac{-4}{5} + \left(\frac{14}{20} + \frac{-11}{20}\right)$

$= \frac{-4}{5} + \left(\frac{14-11}{20}\right)$

$= \frac{-4}{5} + \frac{3}{20}$

$= \frac{-4\times4}{5\times4} + \frac{3}{20} = \frac{-16}{20} + \frac{3}{20} = \frac{-16+3}{20} = \frac{-13}{20}$

And, $\left(\frac{-4}{5}+\frac{7}{10}\right)+\frac{11}{-20}=\left(\frac{-4\times2}{5\times2}+\frac{7}{10}\right)+\frac{11}{20}$ [$\because$ L.C.M. of 5 and 10 = 10]

$$=\left(\frac{-8}{10}+\frac{7}{10}\right)+\frac{-11}{20}$$

$$=\frac{-8+7}{10}+\frac{-11}{20}$$

$$=\frac{-1}{10}+\frac{-11}{20}$$

$$=\frac{-1\times2}{10\times2}+\frac{-11}{20}$$

$$=\frac{-2}{20}+\frac{-11}{20}=\frac{-2-11}{20}=\frac{-13}{20}$$

$$\therefore \quad \frac{-4}{5}+\left(\frac{7}{10}+\frac{11}{-20}\right)=\left(\frac{-4}{5}+\frac{7}{10}\right)+\frac{11}{-20}$$

This verifies associative property of the addition of rational numbers.

Example 5 :

Write the additive inverse of :

(i) $\frac{3}{8}$ (ii) $\frac{-8}{15}$ (iii) $\frac{4}{-13}$ (iv) $\frac{-6}{-11}$

Solution :

The additive inverse of $\frac{a}{b}$ is $-\frac{a}{b}$ and the additive inverse of $-\frac{a}{b}$ is $\frac{a}{b}$.

(i) **The additive inverse of $\frac{3}{8}$ is $-\frac{3}{8}$.** **(Ans.)**

(ii) **The additive inverse of $\frac{-8}{15}=\frac{8}{15}$.** **(Ans.)**

(iii) **The additive inverse of $\frac{4}{-13}=\frac{4}{13}$.** **(Ans.)**

(iv) $\because \frac{-6}{-11}=\frac{6}{11}$

$\therefore$ **The additive inverse of $\frac{-6}{-11}$**

= The additive inverse of $\frac{6}{11}=\frac{-6}{11}$. **(Ans.)**

EXERCISE 1(A)

1. Add, each pair of rational numbers, given below, and show that their addition (sum) is also a rational number :

 (i) $\frac{-5}{8}$ and $\frac{3}{8}$ (ii) $\frac{-8}{13}$ and $\frac{-4}{13}$

 (iii) $\frac{6}{11}$ and $\frac{-9}{11}$ (iv) $\frac{5}{-26}$ and $\frac{8}{39}$

 (v) $\frac{5}{-6}$ and $\frac{2}{3}$ (vi) -2 and $\frac{2}{5}$

 (vii) $\frac{9}{-4}$ and $\frac{-3}{8}$ (viii) $\frac{7}{-18}$ and $\frac{8}{27}$

2. Evaluate :

 (i) $\frac{5}{9}+\frac{-7}{6}$ (ii) $4+\frac{3}{-5}$

 (iii) $\frac{1}{-15}+\frac{5}{-12}$ (iv) $\frac{5}{9}+\frac{3}{-4}$

 (v) $\frac{-8}{9}+\frac{-5}{12}$ (vi) $0+\frac{-2}{7}$

 (vii) $\frac{5}{-11}+0$ (viii) $2+\frac{-3}{5}$

 (ix) $\frac{4}{-9}+1$

3. Evaluate :

 (i) $\frac{3}{7}+\frac{-4}{9}+\frac{-11}{7}+\frac{7}{9}$

 (ii) $\frac{2}{3}+\frac{-4}{5}+\frac{1}{3}+\frac{2}{5}$

 (iii) $\frac{4}{7}+0+\frac{-8}{9}+\frac{-13}{7}+\frac{17}{9}$

 (iv) $\frac{3}{8}+\frac{-5}{12}+\frac{3}{7}+\frac{3}{12}+\frac{-5}{8}+\frac{-2}{7}$

4. For each pair of rational numbers, verify commutative property of additon of rational numbers :

 (i) $\frac{-8}{7}$ and $\frac{5}{14}$ (ii) $\frac{5}{9}$ and $\frac{5}{-12}$

 (iii) $\frac{-4}{5}$ and $\frac{-13}{-15}$ (iv) $\frac{2}{-5}$ and $\frac{11}{-15}$

 (v) 3 and $\frac{-2}{7}$ (vi) -2 and $\frac{3}{-5}$

5. For each set of rational numbers, given below, verify the associative property of addition of rational numbers :

 (i) $\frac{1}{2}$, $\frac{2}{3}$ and $-\frac{1}{6}$

 (ii) $\frac{-2}{5}$, $\frac{4}{15}$ and $\frac{-7}{10}$

 (iii) $\frac{-7}{9}$, $\frac{2}{-3}$ and $\frac{-5}{18}$

 (iv) -1, $\frac{5}{6}$ and $\frac{-2}{3}$

6. Write the additive inverse (negative) of :

 (i) $\frac{-3}{8}$ (ii) $\frac{4}{-9}$

 (iii) $\frac{-7}{5}$ (iv) $\frac{-4}{-13}$

 (v) 0 (vi) -2

 (vii) 1 (viii) $-\frac{1}{3}$

 (ix) $\frac{-3}{1}$

7. Fill in the blanks :

 (i) Additive inverse of $\frac{-5}{-12}$ = ...

 (ii) $\frac{-5}{-12}$ + its additive inverse = ...

 (iii) If $\frac{a}{b}$ is additive inverse of $\frac{-c}{d}$, then $\frac{-c}{d}$ is additive inverse of ....

 And so $\frac{a}{b}+\frac{(-c)}{d}=\frac{(-c)}{d}+\frac{a}{b}$ = ...

8. State, true or false :

 (i) $\frac{7}{9}=\frac{7+5}{9+5}$ (ii) $\frac{7}{9}=\frac{7-5}{9-5}$

 (iii) $\frac{7}{9}=\frac{7\times 5}{9\times 5}$ (iv) $\frac{7}{9}=\frac{7\div 5}{9\div 5}$

 (v) $\frac{-5}{-12}$ is a negative rational number

 (vi) $\frac{-13}{25}$ is smaller than $\frac{-25}{13}$.

1.4 SUBTRACTION OF RATIONAL NUMBERS

(i) Subtraction of $\frac{3}{4}$ from $\frac{5}{6}$

$$= \frac{5}{6} - \frac{3}{4}$$

$$= \frac{5\times2}{6\times2} - \frac{3\times3}{4\times3} \qquad [\because \text{ L.C.M. of 6 and 4} = 12]$$

$$= \frac{10}{12} - \frac{9}{12} = \frac{10-9}{12} = \frac{1}{12}$$

(ii) Subtraction of $\frac{-5}{8}$ from $\frac{-7}{12}$

$$= \frac{-7}{12} - \left(\frac{-5}{8}\right)$$

$$= \frac{-7}{12} + \frac{5}{8}$$

$$= \frac{-7\times2}{12\times2} + \frac{5\times3}{8\times3} \qquad [\because \text{ L.C.M. of 12 and 8} = 24]$$

$$= \frac{-14}{24} + \frac{15}{24} = \frac{-14+15}{24} = \frac{1}{24}$$

Example 6 :

The sum of two rational numbers is $\frac{-5}{8}$. If one of these numbers is $\frac{-7}{12}$, find the other.

Solution :

$\because$ The sum of two rational numbers = $\frac{-5}{8}$

and, one of the numbers = $\frac{-7}{12}$

$\therefore$ The other rational number

$$= \frac{-5}{8} - \left(\frac{-7}{12}\right)$$

$$= -\frac{5}{8} + \frac{7}{12}$$

$$= \frac{-5\times3}{8\times3} + \frac{7\times2}{12\times2} \qquad [\because \text{ L.C.M. of 8 and 12} = 24]$$

$$= \frac{-15}{24} + \frac{14}{24}$$

$$= \frac{-15+14}{24} = \frac{-1}{24}$$

(Ans.)

Example 7 :

What should be added to $-\frac{3}{8}$ to get $\frac{5}{6}$?

Solution :

Required rational number

$$= \frac{5}{6} - \left(-\frac{3}{8}\right)$$

$$= \frac{5}{6} + \frac{3}{8}$$

$$= \frac{5\times4}{6\times4} + \frac{3\times3}{8\times3}$$

$$= \frac{20}{24} + \frac{9}{24} = \frac{29}{24} = 1\frac{5}{24} \text{ (Ans.)}$$

Alternative method :

Let x be added

$$\therefore -\frac{3}{8} + x = \frac{5}{6}$$

$$\Rightarrow \quad x = \frac{5}{6} + \frac{3}{8}$$

$$= \frac{5\times4}{6\times4} + \frac{3\times3}{8\times3}$$

$$= \frac{20}{24} + \frac{9}{24} = \frac{29}{24} = 1\frac{5}{24} \quad \text{(Ans.)}$$

Example 8 :

What should be subtracted from $-\frac{3}{8}$ to get $\frac{5}{6}$?

Solution :

Required rational number

$$= -\frac{3}{8} - \frac{5}{6}$$

$$= \frac{-3\times3}{8\times3} - \frac{5\times4}{6\times4}$$

$$= \frac{-9}{24} - \frac{20}{24}$$

$$= \frac{-9-20}{24}$$

$$= \frac{-29}{24} \quad \text{(Ans.)}$$

Alternative method :

Let x be subtracted

$$\therefore \quad -\frac{3}{8} - x = \frac{5}{6}$$

$$\Rightarrow -\frac{3}{8} - \frac{5}{6} = x \text{ i.e. } x = -\frac{3}{8} - \frac{5}{6}$$

$$\Rightarrow \quad x = \frac{-3\times3}{8\times3} - \frac{5\times4}{6\times4}$$

$$= \frac{-9}{24} - \frac{20}{24}$$

$$= \frac{-9-20}{24} = \frac{-29}{24} \quad \text{(Ans.)}$$

1.5 PROPERTIES OF SUBTRACTION OF RATIONAL NUMBERS

1. Closure property

(i) $\frac{3}{5} - \frac{5}{10} = \frac{3\times2}{5\times2} - \frac{5}{10}$

$= \frac{6}{10} - \frac{5}{10} = \frac{6-5}{10} = \frac{1}{10}$, *which is a rational number.*

(ii) $\frac{7}{12} - \frac{5}{18} = \frac{7\times3}{12\times3} - \frac{5\times2}{18\times2}$

$= \frac{21}{36} - \frac{10}{36} = \frac{21-10}{36} = \frac{11}{36}$, *which is a rational number.*

Thus according to the closure property, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then $\frac{a}{b} - \frac{c}{d}$ is also a rational number. And so is $\frac{c}{d} - \frac{a}{b}$.

2. Commutativity

The subtraction of rational numbers is not commutative.

Consider the rational numbers $\frac{-7}{12}$ and $\frac{5}{8}$.

$$\frac{-7}{12} - \frac{5}{8} = \frac{-7\times 2}{12\times 2} - \frac{5\times 3}{8\times 3}$$
$$= \frac{-14}{24} - \frac{15}{24}$$
$$= \frac{-14-15}{24} = \frac{-29}{24}$$

and, $$\frac{5}{8} - \left(\frac{-7}{12}\right) = \frac{5}{8} + \frac{7}{12}$$
$$= \frac{5\times 3}{8\times 3} + \frac{7\times 2}{12\times 2}$$
$$= \frac{15}{24} + \frac{14}{24} = \frac{15+14}{24} = \frac{29}{24}$$

$\Rightarrow$ $$\frac{-7}{12} - \frac{5}{8} \neq \frac{5}{8} - \left(\frac{-7}{12}\right)$$

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then : $\frac{a}{b} - \frac{c}{d} \neq \frac{c}{d} - \frac{a}{b}$.
Hence, the subtraction of rational numbers is not commutative.

3. Associativity

The subtraction of rational numbers is not associative. i.e. if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

$$\frac{a}{b} - \left(\frac{c}{d} - \frac{e}{f}\right) \neq \left(\frac{a}{b} - \frac{c}{d}\right) - \frac{e}{f}$$

Consider the rational numbers $\frac{2}{3}$, $\frac{-5}{6}$ and $\frac{7}{12}$.

$\therefore$ $$\frac{2}{3} - \left(\frac{-5}{6} - \frac{7}{12}\right) = \frac{2}{3} - \left(\frac{-5\times 2}{6\times 2} - \frac{7}{12}\right)$$
$$= \frac{2}{3} - \left(\frac{-10}{12} - \frac{7}{12}\right)$$
$$= \frac{2}{3} - \left(\frac{-17}{12}\right)$$

$$= \frac{2}{3} + \frac{17}{12}$$

$$= \frac{2\times4}{12} + \frac{17}{12} = \frac{8+17}{12} = \frac{25}{12}$$

$$\left(\frac{2}{3} - \frac{-5}{6}\right) - \frac{7}{12} = \left(\frac{2\times2}{3\times2} - \frac{-5}{6}\right) - \frac{7}{12}$$

$$= \left(\frac{4}{6} + \frac{5}{6}\right) - \frac{7}{12}$$

$$= \frac{9}{6} - \frac{7}{12}$$

$$= \frac{9\times2}{6\times2} - \frac{7}{12}$$

$$= \frac{18}{12} - \frac{7}{12} = \frac{11}{12}$$

$$\Rightarrow \quad \frac{2}{3} - \left(\frac{-5}{6} - \frac{7}{12}\right) \neq \left(\frac{2}{3} - \frac{-5}{6}\right) - \frac{7}{12}$$

In the same way,

(i) $3 - \left(\frac{8}{9} - \frac{4}{7}\right) \neq \left(3 - \frac{8}{9}\right) - \frac{4}{7}$

(ii) $\frac{5}{8} - \left(\frac{7}{12} - \frac{9}{17}\right) \neq \left(\frac{5}{8} - \frac{7}{12}\right) - \frac{9}{17}$ and so on.

4. Existence of identity

For a rational number $\frac{a}{b}$,

$\frac{a}{b} - 0 = \frac{a}{b}$, but $0 - \frac{a}{b} \neq \frac{a}{b}$

$\therefore$ Subtraction has only right identity as $\frac{a}{b} - 0 = \frac{a}{b}$, $\frac{5}{8} - 0 = \frac{5}{8}$, $-4 - 0 = -4$ and so on.

And so we say subtraction has no identity.

5. Existence of inverse

Inverse for subtraction does not exist.

EXERCISE 1(B)

1. Evaluate :

(i) $\frac{2}{3} - \frac{4}{5}$ (ii) $\frac{-4}{9} - \frac{2}{-3}$

(iii) $-1 - \frac{4}{9}$ (iv) $\frac{-2}{7} - \frac{3}{-14}$

(v) $\frac{-5}{18} - \frac{-2}{9}$ (vi) $\frac{5}{21} - \frac{-13}{42}$

2. Subtract :

(i) $\frac{5}{8}$ from $\frac{-3}{8}$ (ii) $\frac{-8}{11}$ from $\frac{4}{11}$

(iii) $\frac{4}{9}$ from $\frac{-5}{9}$ (iv) $\frac{1}{4}$ from $\frac{-3}{8}$

(v) $\frac{-5}{8}$ from $\frac{-13}{16}$ (vi) $\frac{-9}{22}$ from $\frac{5}{33}$

3. The sum of two rational numbers is $\frac{9}{20}$. If one of them is $\frac{2}{5}$, find the other.

4. The sum of two rational numbers is $\frac{-2}{3}$. If one of them is $\frac{-8}{15}$, find the other.

5. The sum of the two rational numbers is –6. If one of them is $\frac{-8}{5}$, find the other.

6. Which rational number should be added to $\frac{-7}{8}$ to get $\frac{5}{9}$?

7. Which rational number should be added to $\frac{-5}{9}$ to get $\frac{-2}{3}$?

8. Which rational number should be subtracted from $\frac{-5}{6}$ to get $\frac{4}{9}$?

9. (i) What should be subtracted from –2 to get $\frac{3}{8}$?

   (ii) What should be added to –2 to get $\frac{3}{8}$?

10. Evaluate :

   (i) $\frac{3}{7} + \frac{-4}{9} - \frac{-11}{7} - \frac{7}{9}$

   (ii) $\frac{2}{3} + \frac{-4}{5} - \frac{1}{3} - \frac{2}{5}$

   (iii) $\frac{4}{7} - \frac{-8}{9} - \frac{-13}{7} + \frac{17}{9}$

1.6 MULTIPLICATION OF RATIONAL NUMBERS

Multiplication (product) **of two rational numbers**

$$= \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

(i) $\frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{1 \times 3}{5 \times 2} = \frac{3}{10}$

(ii) $\frac{-3}{5} \times \frac{4}{7} = \frac{-3 \times 4}{5 \times 7} = \frac{-12}{35}$

(iii) $\left(\frac{-15}{8}\right) \times \frac{-4}{5} = \frac{(-15) \times (-4)}{8 \times 5}$

$= \frac{15 \times 4}{8 \times 5} = \frac{3 \times 1}{2 \times 1} = \frac{3}{2}$

1.7 PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBERS

1. Closure property

If any two rational numbers are multiplied together, the result is always a rational number.

For example :

(i) Multiplication of $\frac{3}{4}$ and $\frac{5}{6}$

$$= \frac{3}{4} \times \frac{5}{6} = \frac{3\times5}{4\times6} = \frac{1\times5}{4\times2} = \frac{5}{8}$$, *which is a rational number.*

(ii) Multiplication of $\frac{-3}{8}$ and $\frac{5}{12}$

$$= \frac{-3}{8} \times \frac{5}{12} = \frac{-3\times5}{8\times12} = \frac{-1\times5}{8\times4} = \frac{-5}{32}$$, *which is a rational number.*

2. Commutativity

The multiplication of any two rational numbers is commutative.

Consider the rational numbers $\frac{-7}{12}$ and $\frac{5}{8}$.

$$\frac{-7}{12} \times \frac{5}{8} = \frac{-7\times5}{12\times8} = \frac{-35}{96}$$

and, $$\frac{5}{8} \times \frac{-7}{12} = \frac{5\times(-7)}{8\times12} = \frac{-35}{96}$$

$\therefore$ $$\frac{-7}{12} \times \frac{5}{8} = \frac{5}{8} \times \frac{-7}{12}$$

The same can be verified with any pair of rational numbers.

According to commutative property of multiplication, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then : $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$.

3. Associativity

According to this property, if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

$$\frac{a}{b} \times \left(\frac{c}{d}\times\frac{e}{f}\right) = \left(\frac{a}{b}\times\frac{c}{d}\right) \times \frac{e}{f}$$

Consider the rational numbers $\frac{2}{3}$, $\frac{-5}{6}$ and $\frac{7}{12}$.

$\therefore$ $$\frac{2}{3} \times \left(\frac{-5}{6}\times\frac{7}{12}\right) = \frac{2}{3} \times \left(\frac{-5\times7}{6\times12}\right)$$

$$= \frac{2}{3} \times \frac{-35}{72} = \frac{2\times-35}{3\times72} = \frac{1\times-35}{3\times36} = \frac{-35}{108}$$

And, $$\left(\frac{2}{3}\times\frac{-5}{6}\right) \times \frac{7}{12} = \left(\frac{2\times-5}{3\times6}\right) \times \frac{7}{12}$$

$$= \frac{-5}{9} \times \frac{7}{12} = \frac{-5\times7}{9\times12} = \frac{-35}{108}$$

$\Rightarrow$ $$\frac{2}{3} \times \left(\frac{-5}{6}\times\frac{7}{12}\right) = \left(\frac{2}{3}\times\frac{-5}{6}\right) \times \frac{7}{12}$$

In the same way,

(i) $-\frac{5}{8} \times \left(\frac{3}{4} \times \frac{-7}{16}\right) = \left(\frac{-5}{8} \times \frac{3}{4}\right) \times \frac{-7}{16}$

(ii) $\frac{15}{-22} \times \left(\frac{-8}{11} \times \frac{3}{2}\right) = \left(\frac{15}{-22} \times \frac{-8}{11}\right) \times \frac{3}{2}$ and so on.

4. Existence of multiplicative identity of rational numbers

When multiplicative identity is multiplied with any rational number or any rational number is multiplied with multiplicative identity, the rational number remains the same.

Multiplicative identity for rational numbers is one (1).

$\therefore$ 1 × any rational number = The same rational number × 1

= The number itself

That is, for national number $\frac{a}{b}$, $1 \times \frac{a}{b} = \frac{a}{b} \times 1 = \frac{a}{b}$.

For example :

(i) $\frac{5}{7} \times 1 = 1 \times \frac{5}{7} = \frac{5}{7}$.

(ii) $\frac{-20}{47} \times 1 = 1 \times \frac{-20}{47} = \frac{-20}{47}$ and so on.

5. Existence of multiplicative inverse of rational numbers

The reciprocal of a rational number is called its multiplicative inverse.

(i) The multiplicative inverse of $\frac{3}{5}$ = reciprocal of $\frac{3}{5} = \frac{5}{3}$.

(ii) The multiplicative inverse of $\frac{-5}{8}$ = reciprocal of $\frac{-5}{8} = \frac{8}{-5}$ and so on.

Rational number 0 (zero) does not have its multiplicative inverse.

The product of a rational number and its multiplicative inverse = multiplicative identity

i.e. A rational number × its multiplicative inverse = 1, *the multiplicative identity*

$\Rightarrow \quad \frac{3}{5} \times \frac{5}{3} = 1, \quad \left(\frac{-5}{8}\right) \times \left(\frac{8}{-5}\right) = 1$

$\left(\frac{7}{-8}\right) \times \left(\frac{-8}{7}\right) = 1, \quad \frac{-8}{-15} \times \frac{-15}{-8} = 1$ and so on.

For a rational number $\frac{a}{b}$

its multiplicative inverse is $\frac{b}{a}$ such that : $\frac{\mathbf{a}}{\mathbf{b}} \times \frac{\mathbf{b}}{\mathbf{a}} = \mathbf{1}$

1 is the multiplicative inverse of itself and so is –1.

6. Distributivity of multiplication over addition

The multiplication of rational numbers is distributive over their addition/subtraction.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

(i) $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$

(ii) $\frac{a}{b} \times \left(\frac{c}{d} - \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} - \frac{a}{b} \times \frac{e}{f}$

Consider any three numbers; $\frac{3}{4}, \frac{-4}{5}$ and $\frac{5}{6}$.

$$\therefore \quad \frac{3}{4} \times \left(\frac{-4}{5} + \frac{5}{6}\right) = \frac{3}{4} \times \left(\frac{-4 \times 6 + 5 \times 5}{30}\right)$$

$$= \frac{3}{4} \times \left(\frac{-24 + 25}{30}\right) = \frac{3}{4} \times \frac{1}{30} = \frac{3 \times 1}{4 \times 30} = \frac{1}{40}$$

And, $$\frac{3}{4} \times \frac{-4}{5} + \frac{3}{4} \times \frac{5}{6} = \frac{3 \times -4}{4 \times 5} + \frac{3 \times 5}{4 \times 6}$$

$$= \frac{-3}{5} + \frac{5}{8}$$

$$= \frac{-3 \times 8 + 5 \times 5}{40} = \frac{-24 + 25}{40} = \frac{1}{40}$$

$$\therefore \quad \frac{3}{4} \times \left(\frac{-4}{5} + \frac{5}{6}\right) = \frac{3}{4} \times \frac{-4}{5} + \frac{3}{4} \times \frac{5}{6}$$

In the same way,

$$\frac{3}{4} \times \left(\frac{-4}{5} - \frac{5}{6}\right) = \frac{3}{4} \times \left(\frac{-24 - 25}{30}\right)$$

$$= \frac{3}{4} \times \frac{-49}{30} = \frac{3 \times -49}{4 \times 30} = \frac{1 \times -49}{4 \times 10} = -\frac{49}{40}$$

And, $$\frac{3}{4} \times \frac{-4}{5} - \frac{3}{4} \times \frac{5}{6} = \frac{3 \times -4}{4 \times 5} - \frac{3 \times 5}{4 \times 6}$$

$$= \frac{-3}{5} - \frac{5}{8}$$

$$= \frac{-3 \times 8 - 5 \times 5}{40} = \frac{-24 - 25}{40} = -\frac{49}{40}$$

$$\therefore \quad \frac{3}{4} \times \left(\frac{-4}{5} - \frac{5}{6}\right) = \frac{3}{4} \times \frac{-4}{5} - \frac{3}{4} \times \frac{5}{6}$$

EXERCISE 1(C)

1. Evaluate :

(i) $\frac{-14}{5} \times \frac{-6}{7}$ (ii) $\frac{7}{6} \times \frac{-18}{91}$

(iii) $\frac{-125}{72} \times \frac{9}{-5}$ (iv) $\frac{-11}{9} \times \frac{-51}{-44}$

(v) $-\frac{16}{5} \times \frac{20}{8}$

2. Multiply :

(i) $\frac{5}{6}$ and $\frac{8}{9}$ (ii) $\frac{2}{7}$ and $\frac{-14}{9}$

(iii) $\frac{-7}{8}$ and 4 (iv) $\frac{36}{-7}$ and $\frac{-9}{28}$

(v) $\frac{-7}{10}$ and $\frac{-8}{15}$ (vi) $\frac{3}{-2}$ and $\frac{-7}{3}$

3. Evaluate :

(i) $\left(\frac{2}{-3}\times\frac{5}{4}\right) + \left(\frac{5}{9}\times\frac{3}{-10}\right)$

(ii) $\left(2\times\frac{1}{4}\right) - \left(\frac{-18}{7}\times\frac{-7}{15}\right)$

(iii) $\left(-5\times\frac{2}{15}\right) - \left(-6\times\frac{2}{9}\right)$

(iv) $\left(\frac{8}{5}\times\frac{-3}{2}\right) + \left(\frac{-3}{10}\times\frac{9}{16}\right)$

4. Multiply each rational number, given below, by one (1) :

(i) $\frac{7}{-5}$ (ii) $\frac{-3}{-4}$

(iii) 0 (iv) $\frac{-8}{13}$

(v) $\frac{-6}{-7}$

5. For each pair of rational numbers, given below, verify that the multiplication is commutative :

(i) $\frac{-1}{5}$ and $\frac{2}{9}$ (ii) $\frac{5}{-3}$ and $\frac{13}{-11}$

(iii) 3 and $\frac{-8}{9}$ (iv) 0 and $\frac{-12}{17}$

6. Write the reciprocal (multiplicative inverse) of each rational number, given below :

(i) 5 (ii) −3

(iii) $\frac{5}{11}$ (iv) $\frac{-7}{-8}$

(v) $\frac{-8}{-7}$ (vi) $\frac{15}{-17}$

7. Find the reciprocal (multiplicative inverse) of :

(i) $\frac{3}{5} \times \frac{2}{3}$ (ii) $\frac{-8}{3} \times \frac{13}{-7}$

(iii) $\frac{-3}{5} \times \frac{-1}{13}$

8. Verify that $(x + y) \times z = x \times z + y \times z$, if

(i) $x = \frac{4}{5}$, $y = -\frac{-2}{3}$ and $z = -4$

(ii) $x = 2$, $y = \frac{4}{5}$ and $z = \frac{3}{-10}$

9. Verify that $x \times (y - z) = x \times y - x \times z$, if

(i) $x = \frac{4}{5}$, $y = -\frac{7}{4}$ and $z = 3$

(ii) $x = \frac{3}{4}$, $y = \frac{8}{9}$ and $z = -5$

10. Name the multiplication property of rational numbers shown below :

(i) $\frac{3}{5} \times \frac{-8}{9} = \frac{-8}{9} \times \frac{3}{5}$

(ii) $\frac{-3}{4} \times \left(\frac{5}{7}\times\frac{-8}{15}\right) = \left(\frac{-3}{4}\times\frac{5}{7}\right) \times \frac{-8}{15}$

(iii) $\frac{4}{5} \times \left(\frac{3}{-8}+\frac{-4}{7}\right) = \frac{4}{5} \times \frac{3}{-8} + \frac{4}{5} \times \frac{-4}{7}$

(iv) $\frac{-7}{5} \times \frac{5}{-7} = 1$

(v) $\frac{8}{-9} \times 1 = 1 \times \frac{8}{-9} = \frac{8}{-9}$

11. Fill in the blanks :

(i) The product of two positive rational numbers is always ....

(ii) The product of two negative rational numbers is always ....

(iii) If two rational numbers have opposite signs then their product is always ....

(iv) The reciprocal of a positive rational number is .... and the reciprocal of a negative rational number is ....

(v) Rational number 0 has .... reciprocal.

(vi) The product of a rational number and its reciprocal is ....

(vii) The numbers .... and .... are their own reciprocals.

(viii) If m is reciprocal of n, then the reciprocal of n is ....

1.8 DIVISION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers such that $\frac{c}{d} \neq 0$, then :

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times (\text{reciprocal of } \frac{c}{d})$$

$$= \frac{a}{b} \times \frac{d}{c}$$

- If $\frac{c}{d} = 0$, the division $\frac{a}{b} \div \frac{c}{d}$ is *not defined.*

$\Rightarrow$ **division by 0 is not defined.**

- If m and n are two rational numbers such that $n \neq 0$, then m divided by n is the rational number obtained on multiplying m by the reciprocal of n. Thus $m \div n = m \times \frac{1}{n}$.
- Division is the inverse of multiplication.

For example :

(i) $\frac{3}{4}$ **divided by** $\frac{5}{12} = \frac{3}{4} \div \frac{5}{12} = \frac{3}{4} \times \frac{12}{5} = \frac{9}{5} = 1\frac{4}{5}$

(ii) $\frac{-6}{7}$ **divided by** $\frac{4}{21} = -\frac{6}{7} \div \frac{4}{21} = -\frac{6}{7} \times \frac{21}{4} = -\frac{9}{2}$

(iii) $-\frac{16}{27} \div \frac{-8}{9} = -\frac{16}{27} \times \frac{9}{-8} = \frac{2}{3}$

1.9 PROPERTIES OF DIVISION OF RATIONAL NUMBERS

1. Closure property

If a rational number is divided by some non-zero rational number, the result is always a rational number.

1. If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers and $\frac{c}{d} \neq 0$, then $\left(\frac{a}{b} \div \frac{c}{d}\right)$ is also a rational number.

2. For every rational number $\frac{a}{b}$

(i) $\frac{a}{b} \div 1 = \frac{a}{b}$ (ii) $\frac{a}{b} \div \frac{a}{b} = 1$

For example :

(i) $\frac{3}{4}$ and $\frac{5}{8}$ are two rational numbers. Since, $\frac{5}{8}$ is not equal to zero ($\frac{5}{8} \neq 0$), therefore $\frac{3}{4} \div \frac{5}{8}$ is a rational number.

$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$, a rational number.

(ii) 0 and $\frac{5}{2}$ are two rational numbers and $\frac{5}{2} \neq 0$,

then $0 \div \frac{5}{2} = 0 \times \frac{2}{5} = 0$, *a rational number.*

0 is a rational number.

(iii) 4 and $\frac{2}{3}$ are two rational numbers such that $\frac{2}{3} \neq 0$, then $4 \div \frac{2}{3} = 4 \times \frac{3}{2} = 6$, *a rational number.*

2. **Commutativity**

Division of two different rational numbers is not commutative.

i.e., If $\frac{a}{b}$ and $\frac{c}{d}$ are two non-zero rational numbers then : $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$.

The same can be verified with any pair of rational numbers.

3. **Associativity**

Division of rational numbers is not associative.

i.e. If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are rational numbers such that $\frac{c}{d} \neq 0$ and $\frac{e}{f} \neq 0$; then

$$\frac{a}{b} \div \left(\frac{c}{d} \div \frac{e}{f}\right) \neq \left(\frac{a}{b} \div \frac{c}{d}\right) \div \frac{e}{f}$$

4. Identity for division of rational numbers does not exist.

5. Inverse for division of rational numbers does not exist.

Example 9 :

The product of two rational numbers is $\frac{8}{9}$. If one of them is $-\frac{5}{6}$, find the other.

Solution :

$\because$ The product of two rational numbers is = $\frac{8}{9}$ and one of them is $-\frac{5}{6}$

$\therefore$ **The other number** = $\frac{8}{9} \div \left(-\frac{5}{6}\right)$

$$= \frac{8}{9} \times \left(-\frac{6}{5}\right) = -\frac{8\times 6}{9\times 5} = -\frac{8\times 2}{3\times 5} = -\frac{16}{15}$$

Example 10 :

By what number must $-\frac{5}{8}$ be multiplied, so that the product is $\frac{3}{4}$.

Solution :

$\because$ The product of two numbers is $\frac{3}{4}$ and one of them is $-\frac{5}{8}$

$\therefore$ **The other number** = $\frac{3}{4} \div \left(-\frac{5}{8}\right)$

$$= \frac{3}{4} \times \left(-\frac{8}{5}\right) = -\frac{24}{20} = -\frac{6}{5}$$

EXERCISE 1(D)

1. Evaluate :

   (i) $1 \div \frac{1}{3}$ (ii) $3 \div \frac{3}{5}$

   (iii) $-\frac{5}{12} \div \frac{1}{16}$ (iv) $-\frac{21}{16} \div \left(\frac{-7}{8}\right)$

   (v) $0 \div \left(-\frac{4}{7}\right)$ (vi) $\frac{8}{-5} \div \frac{24}{25}$

   (vii) $-\frac{3}{4} \div (-9)$ (viii) $\frac{3}{4} \div \left(-\frac{5}{12}\right)$

   (ix) $-5 \div \left(-\frac{10}{11}\right)$ (x) $\frac{-7}{11} \div \left(\frac{-3}{44}\right)$

2. Divide :

   (i) 3 by $\frac{1}{3}$ (ii) -2 by $\left(-\frac{1}{2}\right)$

   (iii) 0 by $\frac{7}{-9}$ (iv) $\frac{-5}{8}$ by $\frac{1}{4}$

   (v) $-\frac{3}{4}$ by $-\frac{9}{16}$

3. The product of two rational numbers is –2. If one of them is $\frac{4}{7}$, find the other.

4. The product of two numbers is $-\frac{4}{9}$. If one of them is $\frac{-2}{27}$, find the other.

5. m and n are two rational numbers such that $m \times n = -\frac{25}{9}$.

   (i) if $m = \frac{5}{3}$, find n, (ii) if $n = -\frac{10}{9}$, find m.

6. By what number must $-\frac{3}{4}$ be multiplied so that the product is $-\frac{9}{16}$?

7. By what number should $\frac{-8}{13}$ be multiplied to get 16 ?

8. If $3\frac{1}{2}$ litres of milk costs ₹ 49, find the cost of one litre of milk ?

9. Cost of $3\frac{2}{5}$ metre of cloth is ₹ $88\frac{1}{2}$. What is the cost of 1 metre of cloth ?

10. Divide the sum of $\frac{3}{7}$ and $\frac{-5}{14}$ by $-\frac{1}{2}$.

11. Find $(m + n) \div (m - n)$, if :

(i) $m = \frac{2}{3}$ and $n = \frac{3}{2}$

(ii) $m = \frac{3}{4}$ and $n = \frac{4}{3}$

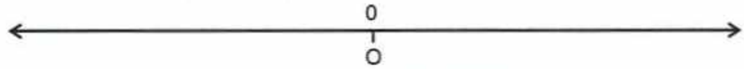
(iii) $m = \frac{4}{5}$ and $n = -\frac{3}{10}$

12. The product of two rational numbers is –5. If one of these numbers is $\frac{-7}{15}$, find the other.

13. Divide the sum of $\frac{5}{8}$ and $\frac{-11}{12}$ by the difference of $\frac{3}{7}$ and $\frac{5}{14}$.

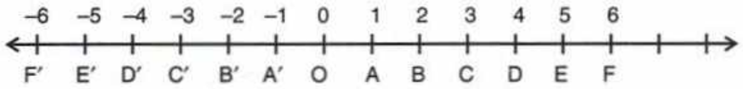
1.10 REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

Draw a line of suitable length. Nearly at the middle of this line, mark a point O that represents number zero (0).



Starting from O, mark points on this line at equal distances both on right as well as on left of O. Let A, B, C, D, etc. be the points on the right side of O and A′, B′, C′, D′, etc. be the points on the left side of O so that :

OA = AB = BC = CD = ... = OA′ = A′B′ = B′C′ = C′D′ = ...



If OA = 1 unit

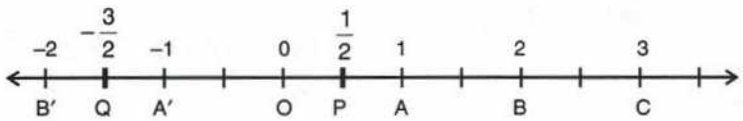
⇒ A, B, C, D, etc. represent integers 1, 2, 3, 4, ... respectively and A′, B′, C′, D′, etc. represent integers –1, –2, –3, –4, ... respectively.

Example 11 :

Represent $\frac{1}{2}$ and $-\frac{3}{2}$ on a number line.

Solution :

Draw a number line as shown below :



In this number line

OA = AB = ... = OA′ = A′B′ = ... = 1 unit

Since, denominator of each given rational number is 2; divide each of OA, AB, BC, OA′, A′B′, etc. into two equal parts.

To represent $\frac{1}{2}$, move one step towards right side of O to reach point P as shown.

$\because$ OA = 1 unit, therefore OP = $\frac{1}{2}$ unit and so **P represents $\frac{1}{2}$.** **(Ans.)**

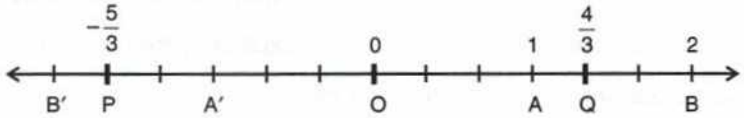
In the same way, to represent $-\frac{3}{2}$, move 3 steps towards the left side of O to reach point Q. Clearly Q, = $-\frac{3}{2}$. **(Ans.)**

Example 12 :

Represent $-\frac{5}{3}$ and $\frac{4}{3}$ on a number line.

Solution :

Draw a number line as shown below :



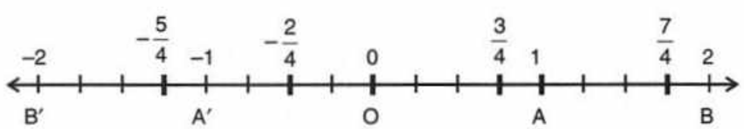
Since, the denominator of each given rational number is 3; divide each of OA, AB, OA′, A′B′, etc. into three equal parts.

To mark $-\frac{5}{3}$, move 5 parts to the left of O to reach point P, therefore P = $-\frac{5}{3}$.

To mark $\frac{4}{3}$, move 4 parts to the right of O to reach point Q, therefore Q = $\frac{4}{3}$.

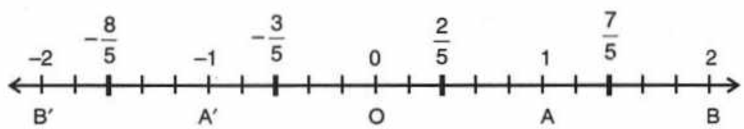
The following number lines will make the concept more clear

1.



- Since, each of the rational numbers $-\frac{5}{4}$, $-\frac{2}{4}$, $\frac{3}{4}$ and $\frac{7}{4}$ has 4 in its denominator, divide OA = AB = OA′ = A′B′, etc. into 4 equal parts.

2.



- Since, each of the rational numbers $-\frac{8}{5}$, $-\frac{3}{5}$, $\frac{2}{5}$ and $\frac{7}{5}$ has 5 in its denominator, divide OA = AB = OA′ = A′B′, etc. into 5 equal parts.

1.11 INSERTING RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS

First method :

If a and b are two rational numbers, then $\frac{a+b}{2}$ is also a rational number and its value lies between a and b.

(i) If $a < b \Rightarrow a < \frac{a+b}{2} < b$ *i.e.* $5 < 8 \Rightarrow 5 < \frac{5+8}{2} < 8$ *i.e.* $\mathbf{5 < 6{\cdot}5 < 8}$

(ii) If $a > b \Rightarrow a > \frac{a+b}{2} > b$ *i.e.* $8 > 5 \Rightarrow 8 > \frac{8+5}{2} > 5$ *i.e.* $\mathbf{8 > 6{\cdot}5 > 5}$

Example 13 :

Insert one rational number between 2 and 3.

Solution :

The rational number between 2 and 3 $= \frac{2+3}{2} = \frac{5}{2} = 2\frac{1}{2}$

It must be noted here that $2\frac{1}{2}$ is not the only rational number between 2 and 3. Infact, an infinite number of rational numbers exist between 2 and 3. Solution to such questions is not unique.

Example 14 :

Insert two rational numbers between 7 and 8.

Solution :

Given numbers = 7 and 8

$= 7, \frac{7+8}{2}, 8$ [Inserting one rational number between 7 and 8]

$= 7, 7{\cdot}5, 8$

$= 7, \frac{7+7{\cdot}5}{2}, 7{\cdot}5, 8 = 7, 7{\cdot}25, 7{\cdot}5, 8$

$\therefore$ **Required rational numbers between 7 and 8 are 7·25 and 7·5** **(Ans.)**

Alternative method :

Given numbers = 7 and 8

$= 7, \frac{7+8}{2}, 8$

$= 7, 7{\cdot}5, 8$

$= 7, 7{\cdot}5, \frac{7{\cdot}5+8}{2}, 8 = 7, 7{\cdot}5, 7{\cdot}75, 8$

$\therefore$ **Required rational numbers between 7 and 8 are 7·5 and 7·75** **(Ans.)**

Example 15 :

Insert three rational numbers between 3 and 4.

Solution :

Given numbers = 3 and 4

$$= 3, \frac{3+4}{2}, 4$$

$$= 3, 3{\cdot}5, 4$$

$$= 3, \frac{3+3{\cdot}5}{2}, 3{\cdot}5, \frac{3{\cdot}5+4}{2}, 4 = 3, 3{\cdot}25, 3{\cdot}5, 3{\cdot}75, 4$$

$\therefore$ **Required rational numbers between 3 and 4 are**

3·25, 3·5 and 3·75

Second method :

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a+c}{b+d}$ is also a rational number with its value lying between $\frac{a}{b}$ and $\frac{c}{d}$.

Example 16 :

Find three rational numbers between $\frac{3}{5}$ and $\frac{4}{7}$.

Solution :

Given numbers = $\frac{3}{5}$ and $\frac{4}{7}$

$$= \frac{3}{5}, \frac{3+4}{5+7}, \frac{4}{7}$$

$$= \frac{3}{5}, \frac{7}{12}, \frac{4}{7}$$

$$= \frac{3}{5}, \frac{3+7}{5+12}, \frac{7}{12}, \frac{7+4}{12+7}, \frac{4}{7}$$

$$= \frac{3}{5}, \frac{10}{17}, \frac{7}{12}, \frac{11}{19}, \frac{4}{7}$$

$\therefore$ **Required rational numbers between $\frac{3}{5}$ and $\frac{4}{7}$ are**

$\frac{10}{17}, \frac{7}{12}$ **and** $\frac{11}{17}$

1.12 METHOD OF FINDING LARGE NUMBER OF RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS.

Example 17 :

Insert five rational numbers between $\frac{3}{4}$ and $\frac{7}{8}$.

Solution :

Steps : 1. Find L.C.M. of denominators. L.C.M. of denominators 4 and 8 is 8.

2. Make denominator of each given rational number equal to 8 (the L.C.M.).

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8} \text{ and } \frac{7}{8} = \frac{7}{8}$$

3. Since, five rational numbers are required, multiply the numerator and the denominator of each rational number (obtained in step 2) by 5 + 1 = 6.

$$\therefore \frac{6}{8} = \frac{6 \times 6}{8 \times 6} = \frac{36}{48} \text{ and } \frac{7}{8} = \frac{7 \times 6}{8 \times 6} = \frac{42}{48}$$

Now every rational number with denominator 48 and numerator between 36 and 42 will have its value between the given rational numbers $\frac{3}{4}$ and $\frac{7}{8}$.

$\Rightarrow$ **Required rational numbers between $\frac{3}{4}$ and $\frac{7}{8}$ are**

$$= \frac{37}{48}, \frac{38}{48}, \frac{39}{48}, \frac{40}{48} \text{ and } \frac{41}{48}$$

$$= \frac{37}{48}, \frac{19}{24}, \frac{13}{16}, \frac{5}{6} \text{ and } \frac{41}{48}$$ **(Ans.)**

Example 18 :

Insert 7 rational numbers between $\frac{5}{6}$ and $\frac{7}{9}$.

Solution :

Step 1 : L.C.M. of denominators 6 and 9 = 18.

Step 2 : Make denominator of each given rational number equal to 18 (the L.C.M.).

$$\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18} \text{ and } \frac{7}{9} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18}$$

Step 3 : Since, seven rational numbers are required between $\frac{5}{6}$ and $\frac{7}{9}$; multiply the numerator and the denominator of each rational number (obtained in step 2) by 7 + 1 = 8

$$\frac{15}{18} = \frac{15 \times 8}{18 \times 8} = \frac{120}{144} \text{ and } \frac{14}{18} = \frac{14 \times 8}{18 \times 8} = \frac{112}{144}$$

⇒ **Required rational numbers between $\frac{5}{6}$ and $\frac{7}{9}$ are :**

$$\frac{119}{144}, \frac{118}{144}, \frac{117}{144}, \frac{116}{144}, \frac{115}{144}, \frac{114}{144} \text{ and } \frac{113}{144}$$

$$= \frac{119}{144}, \frac{59}{72}, \frac{13}{16}, \frac{29}{36}, \frac{115}{144}, \frac{19}{24} \text{ and } \frac{113}{144} \quad \textbf{(Ans.)}$$

EXERCISE 1(E)

1. Draw a number line and mark

   $\frac{3}{4}, \frac{7}{4}, \frac{-3}{4}$ and $\frac{-7}{4}$ on it.

2. On a number line mark the points

   $\frac{2}{3}, \frac{-8}{3}, \frac{7}{3}, \frac{-2}{3}$ and –2.

3. Insert one rational number between

   (i) 7 and 8 (ii) 3·5 and 5

   (iii) 2 and 3·2 (iv) 4·2 and 3·6

   (v) $\frac{1}{2}$ and 2

4. Insert two rational numbers between

   (i) 6 and 7 (ii) 4·8 and 6

   (iii) 2·7 and 6·3

5. Insert three rational numbers between

   (i) 3 and 4 (ii) 10 and 12

6. Insert five rational numbers between $\frac{3}{5}$ and $\frac{2}{3}$.

7. Insert six rational numbers between $\frac{5}{6}$ and $\frac{8}{9}$.

8. Insert seven rational numbers between 2 and 3.

EXPONENTS
(Powers)

2

2.1 REVIEW

Exponent	If x is a real number and n is an integer, we know : $x \times x \times x \times x \times$ .... n times = x^n where x^n is called an **exponential expression** with **base** x and **exponent** (or index, or power) **n.** x^n is read as 'x raised to the power n'.

2.2 LAWS OF EXPONENTS (FOR INTEGRAL POWERS)

1. **Product Law : $a^m \times a^n = a^{m+n}$**

$\Rightarrow$ $3^3 \times 3^5 = 3^{3+5} = 3^8$; $5^8 \times 5^5 = 5^{8+5} = 5^{13}$;

$7^2 \times 7^4 = 7^{2+4} = 7^6$; $2^{-5} \times 2^8 = 2^{-5+8} = 2^3$ and so on.

2. **Quotient Law : $\frac{a^m}{a^n} = a^{m-n}$, if m > n**

and $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$**, if n > m**

$\Rightarrow$ $\frac{2^{12}}{2^7} = 2^{12-7} = 2^5$; $\frac{2^6}{2^{13}} = \frac{1}{2^{13-6}} = \frac{1}{2^7}$;

$\frac{5^{12}}{5^{-3}} = 5^{12+3} = 5^{15}$; $\frac{3^{-6}}{3^3} = \frac{1}{3^{3+6}} = \frac{1}{3^9}$ and so on.

3. **Power Law : $(a^m)^n = a^{mn}$**

$\Rightarrow$ $(3^5)^2 = 3^{5 \times 2} = 3^{10}$; $(5^6)^{-3} = 5^{6 \times -3} = 5^{-18}$

$(7^{-2})^3 = 7^{-2 \times 3} = 7^{-6}$; $(5^{-3})^{-2} = 5^{-3 \times -2} = 5^6$ and so on.

$(-2)^3 = -2 \times -2 \times -2 = -8$,

$(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$,

$(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$,

$(-2)^6 = -2 \times -2 \times -2 \times -2 \times -2 \times -2 = 64$ and so on.

Thus,

(i) If n is **even**, $(-2)^n$ is **positive**.

(ii) if n is **odd**, $(-2)^n$ is **negative**.

In general, $(-a)^n = a^n$, if n is even

and, $(-a)^n = -a^n$, if n is odd

2.3 NEGATIVE INTEGRAL EXPONENT

For any non-zero rational number *a*

$a^{-n} = \frac{1}{a^n}$ and $a^n = \frac{1}{a^{-n}}$

i.e. a^{-n} and a^n are reciprocal of each other.

Thus, $5^{-3} = \frac{1}{5^3}$, $2^{-5} = \frac{1}{2^5}$,

$\left(\frac{2}{3}\right)^{-5} = \frac{1}{\left(\frac{2}{3}\right)^5} = \left(\frac{3}{2}\right)^5$, $\left(\frac{5}{8}\right)^{-6} = \frac{1}{\left(\frac{5}{8}\right)^6} = \left(\frac{8}{5}\right)^6$,

$7^3 = \frac{1}{7^{-3}}$, $11^5 = \frac{1}{11^{-5}}$ and so on.

Also

1. $(-2)^{-5} = \frac{1}{(-2)^5} = \frac{1}{-2^5} = \frac{1}{-2\times2\times2\times2\times2} = \frac{1}{-32} = -\frac{1}{32}$

2. $\left(\frac{4}{3}\right)^{-3} = \left(\frac{3}{4}\right)^3 = \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$

3. $\left(-\frac{2}{3}\right)^{-4} = \left(-\frac{3}{2}\right)^4 = \left(\frac{3}{2}\right)^4 = \frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2} = \frac{81}{16}$

4. $\frac{1}{5^{-3}} = 5^3 = 5\times5\times5 = 125$ and so on.

Example 1 :

Evaluate and express as a rational number of the form $\frac{m}{n}$:

(i) $\left(\frac{3}{5}\right)^{-2}\times\left(\frac{4}{5}\right)^{-3}$ (ii) $\left(-\frac{2}{3}\right)^{-4}\times\left(-\frac{3}{5}\right)^2$

Solution :

(i) $\left(\frac{3}{5}\right)^{-2}\times\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{3}\right)^2\times\left(\frac{5}{4}\right)^3$

$= \frac{5^2}{3^2}\times\frac{5^3}{4^3} = \frac{25\times125}{9\times64} = \frac{3125}{576}$ **(Ans.)**

(ii) $\left(-\frac{2}{3}\right)^{-4}\times\left(-\frac{3}{5}\right)^2 = \left(-\frac{3}{2}\right)^4\times\left(-\frac{3}{5}\right)^2$

$= \left(\frac{3}{2}\right)^4\times\left(\frac{3}{5}\right)^2 = \frac{81}{16}\times\frac{9}{25} = \frac{729}{400}$ **(Ans.)**

Example 2 :

Evaluate :

(i) $(2^{-1} \div 5^{-1})^2 \times \left(\frac{-5}{8}\right)^{-1}$ (ii) $(5^{-1} \times 3^{-1})^{-1} \div 6^{-1}$

(iii) $(4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1}$

Solution :

(i) $(2^{-1} \div 5^{-1})^2 \times \left(\frac{-5}{8}\right)^{-1} = \left(\frac{1}{2} \div \frac{1}{5}\right)^2 \times \left(\frac{8}{-5}\right)^1$

$= \left(\frac{1}{2} \times \frac{5}{1}\right)^2 \times \frac{8}{-5}$

$= \left(\frac{5}{2}\right)^2 \times \frac{8}{-5} = \frac{25}{4} \times \frac{8}{-5} = \frac{5 \times 2}{-1} = -10$ **(Ans.)**

(ii) $(5^{-1} \times 3^{-1})^{-1} \div 6^{-1} = \left(\frac{1}{5} \times \frac{1}{3}\right)^{-1} \div \frac{1}{6}$

$= \left(\frac{1}{15}\right)^{-1} \div \frac{1}{6} = \frac{15}{1} \times \frac{6}{1} = 90$ **(Ans.)**

(iii) $(4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1} = \left(\frac{1}{4} + \frac{1}{8}\right) \div \left(\frac{3}{2}\right)$

$= \left(\frac{2+1}{8}\right) \times \frac{2}{3} = \frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$ **(Ans.)**

Example 3 :

Evaluate : $\left\{\left(\frac{-3}{2}\right)^{-3}\right\}^2$

Solution :

$\left\{\left(\frac{-3}{2}\right)^{-3}\right\}^2 = \left(-\frac{3}{2}\right)^{-3 \times 2}$

$= \left(-\frac{3}{2}\right)^{-6}$

$= \left(-\frac{2}{3}\right)^6 = \left(\frac{2}{3}\right)^6 = \frac{2^6}{3^6} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{64}{729}$ **(Ans.)**

Example 4 :

Evaluate : $\left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{5}\right)^{-2}$.

Solution :

$$\left(\frac{1}{3}\right)^{-3}+\left(\frac{1}{4}\right)^{-3}-\left(\frac{1}{5}\right)^{-2}=\left(\frac{3}{1}\right)^{3}+\left(\frac{4}{1}\right)^{3}-\left(\frac{5}{1}\right)^{2}$$

$$= 3^3 + 4^3 - 5^2$$

$$= 27 + 64 - 25 = 66$$ **(Ans.)**

Example 5 :

If $3^{3x-1} \div 9 = 27$, find the value of x.

Solution :

$3^{3x-1} \div 9 = 27 \Rightarrow 3^{3x-1} \times \frac{1}{9} = 27$

$\Rightarrow 3^{3x-1} \times \frac{1}{3^2} = 3^3$

$\Rightarrow 3^{3x-1-2} = 3^3$

$\Rightarrow 3^{3x-3} = 3^3$

$\Rightarrow 3x - 3 = 3$

i.e. $3x = 6$ and $x = 2$ **(Ans.)**

Alternatively:

$3^{3x-1} \times \frac{1}{9} = 27$

$\Rightarrow 3^{3x-1} = 27 \times 9$

$= 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

$\Rightarrow 3x - 1 = 5$

i.e. $x = \frac{6}{3} = 2$ **(Ans.)**

EXERCISE 2(A)

1. Evaluate :

   (i) $(3^{-1} \times 9^{-1}) \div 3^{-2}$

   (ii) $(3^{-1} \times 4^{-1}) \div 6^{-1}$

   (iii) $(2^{-1} + 3^{-1})^3$

   (iv) $(3^{-1} \div 4^{-1})^2$

   (v) $(2^2 + 3^2) \times \left(\frac{1}{2}\right)^2$

   (vi) $(5^2 - 3^2) \times \left(\frac{2}{3}\right)^{-3}$

   (vii) $\left[\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right] \div \left(\frac{1}{6}\right)^{-3}$

   (viii) $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2$

   (ix) $\left\{\left(\frac{3}{5}\right)^{-2}\right\}^{-2}$

   (x) $(5^{-1} \times 3^{-1}) \div 6^{-1}$

2. If $1125 = 3^m \times 5^n$; find m and n.

3. Find *x*, if $9 \times 3^x = (27)^{2x-3}$

2.4 MORE ABOUT EXPONENTS

1. $(a \times b)^n = a^n \times b^n$

   e.g. $(a^5 \times b^{-3})^4 = (a^5)^4 \times (b^{-3})^4 = a^{20} \times b^{-12}$ and $(3^4 \times 5^{-3})^{-2} = 3^{-8} \times 5^6$

2. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

   e.g. $\left(\frac{a^{-3}}{b^4}\right)^6 = \frac{(a^{-3})^6}{(b^4)^6} = \frac{a^{-18}}{b^{24}}$ and $\left(\frac{5^7}{3^{-4}}\right)^{-3} = \frac{5^{-21}}{3^{12}}$

3. $a^0 = 1$; if $a \neq 0$

i.e. any non–zero number raised to the power zero is always equal to one (1).

e.g. $5^0 = 1, 7^0 = 1, (-8)^0 = 1, (2^{-5})^0 = 1$ and so on.

4. $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$; if $a \neq 0$

e.g. $2^{-3} = \frac{1}{2^3}, \frac{1}{5^{-7}} = 5^7, \frac{2^{-3}}{3^{-5}} = \frac{3^5}{2^3}$ and so on.

5. $\sqrt[n]{a} = a^{\frac{1}{n}}$ and $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

e.g. $\sqrt{5} = 5^{\frac{1}{2}}$ $\qquad \sqrt[6]{5^7} = 5^{\frac{7}{6}}$ $\qquad \sqrt[3]{a^2 \times b^4} = a^{\frac{2}{3}} \times b^{\frac{4}{3}}$, etc.

Also remember that :

(i) $(-a)^m = a^m$; *if m is even* and (ii) $(-a)^m = -a^m$; *if m is odd.*

e.g. $(-5)^4 = -5 \times -5 \times -5 \times -5 = 5^4$ and $(-5)^3 = -5 \times -5 \times -5 = -5^3$

Example 6 :

Evaluate :

(i) $4^{\frac{3}{2}} \times 125^{\frac{-2}{3}}$ (ii) $\left(\frac{8}{27}\right)^{\frac{2}{3}} \div (32)^{\frac{-2}{5}}$

(iii) $-2^4 - \left(\sqrt{3}\right)^0 \times (-2)^6 \div 4$

Solution :

(i) $4^{\frac{3}{2}} \times 125^{\frac{-2}{3}} = (2^2)^{\frac{3}{2}} \times (5^3)^{\frac{-2}{3}}$ $\qquad [4 = 2 \times 2 = 2^2, 125 = 5 \times 5 \times 5 = 5^3]$

$= 2^3 \times 5^{-2}$ $\qquad \left[2 \times \frac{3}{2} = 3 \text{ and } 3 \times \frac{-2}{3} = -2\right]$

$= \frac{8}{5^2}$ $\qquad \left[2^3 = 2 \times 2 \times 2 = 8 \text{ and } 5^{-2} = \frac{1}{5^2}\right]$

$= \frac{8}{25}$ **(Ans.)**

(ii) $\left(\frac{8}{27}\right)^{\frac{2}{3}} \div (32)^{\frac{-2}{5}} = \left(\frac{2}{3}\right)^{3 \times \frac{2}{3}} \div (2^5)^{\frac{-2}{5}}$ $\qquad \left[\frac{8}{27} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \left(\frac{2}{3}\right)^3 \text{ and } 32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5\right]$

$= \left(\frac{2}{3}\right)^2 \div 2^{-2}$ $\qquad \left[3 \times \frac{2}{3} = 2 \text{ and } 5 \times \frac{-2}{5} = -2\right]$

$= \frac{2^2}{3^2} \times \frac{1}{2^{-2}}$

$= \frac{4}{9} \times 2^2$ $\qquad \left[\frac{1}{2^{-2}} = 2^2\right]$

$= \frac{4 \times 4}{9} = \frac{16}{9} = 1\frac{7}{9}$ **(Ans.)**

(iii) **Given expression**

$= -2^4 - 1 \times 2^6 \div 2^2$ $\quad [(\sqrt{3})^0 = 1; (-2)^6 = 2^6 \text{ and } 4 = 2 \times 2 = 2^2]$

$= -2^4 - 2^4$ $\quad [2^6 \div 2^2 = 2^{6-2} = 2^4]$

$= -16 - 16 = -32$ **(Ans.)**

Example 7 :

Simplify : $\dfrac{x^{m+n} \times x^{n+l} \times x^{l+m}}{(x^m \times x^n \times x^l)^2}$

Solution :

Given expression $= \dfrac{x^{m+n+n+l+l+m}}{x^{2m} \times x^{2n} \times x^{2l}}$

$= \dfrac{x^{2m+2n+2l}}{x^{2m+2n+2l}} = 1$ **(Ans.)**

Example 8 :

Simplify : $\left(\dfrac{x^a}{x^b}\right)^{a+b} \times \left(\dfrac{x^b}{x^c}\right)^{b+c} \times \left(\dfrac{x^c}{x^a}\right)^{c+a}$

Solution :

Given expression $= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$

$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$

$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$ **(Ans.)**

EXERCISE 2(B)

1. Compute :
   (i) $1^8 \times 3^0 \times 5^3 \times 2^2$
   (ii) $(4^7)^2 \times (4^{-3})^4$
   (iii) $(2^{-9} \div 2^{-11})^3$
   (iv) $\left(\dfrac{2}{3}\right)^{-4} \times \left(\dfrac{27}{8}\right)^{-2}$
   (v) $\left(\dfrac{56}{28}\right)^0 \div \left(\dfrac{2}{5}\right)^3 \times \dfrac{16}{25}$
   (vi) $(12)^{-2} \times 3^3$
   (vii) $(-5)^4 \times (-5)^6 \div (-5)^9$
   (viii) $\left(-\dfrac{1}{3}\right)^4 \div \left(-\dfrac{1}{3}\right)^8 \times \left(-\dfrac{1}{3}\right)^5$
   (ix) $9^0 \times 4^{-1} \div 2^{-4}$ (x) $(625)^{-\frac{3}{4}}$
   (xi) $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$ (xii) $\left(\dfrac{1}{32}\right)^{-\frac{2}{5}}$
   (xiii) $(125)^{-\frac{2}{3}} \div (8)^{\frac{2}{3}}$
   (xiv) $(243)^{\frac{2}{5}} \div (32)^{-\frac{2}{5}}$
   (xv) $(-3)^4 - (\sqrt[4]{3})^0 \times (-2)^5 \div (64)^{\frac{2}{3}}$
   (xvi) $(27)^{\frac{2}{3}} \div (\frac{81}{16})^{-\frac{1}{4}}$
2. Simplify :
   (i) $8^{\frac{4}{3}} + 25^{\frac{3}{2}} - (\frac{1}{27})^{-\frac{2}{3}}$
   (ii) $[(64)^{-2}]^{-3} \div [\{(-8)^2\}^3]^2$
   (iii) $(2^{-3} - 2^{-4})(2^{-3} + 2^{-4})$
3. Evaluate :
   (i) $(-5)^0$ (ii) $8^0 + 4^0 + 2^0$
   (iii) $(8 + 4 + 2)^0$ (iv) $4x^0$
   (v) $(4x)^0$ (vi) $[(10^3)^0]^5$
   (vii) $(7x^0)^2$
   (viii) $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

4. Simplify :

(i) $\frac{a^5 b^2}{a^2 b^{-3}}$ (ii) $15y^8 \div 3y^3$

(iii) $x^{10}y^6 \div x^3y^{-2}$ (iv) $5z^{16} \div 15z^{-11}$

(v) $(36x^2)^{\frac{1}{2}}$ (vi) $(125x^{-3})^{\frac{1}{3}}$

(vii) $(2x^2y^{-3})^{-2}$ (viii) $(27x^{-3}y^6)^{\frac{2}{3}}$

(ix) $(-2x^{2/3}y^{-3/2})^6$

5. Simplify : $(x^{a+b})^{a-b} . (x^{b+c})^{b-c} . (x^{c+a})^{c-a}$

6. Simplify : (i) $\sqrt[5]{x^{20}y^{-10}z^5} \div \frac{x^3}{y^3}$

(ii) $\left(\frac{256a^{16}}{81b^4}\right)^{\frac{-3}{4}}$

7. Simplify and express as positive indices :

(i) $(a^{-2}b)^{-2}.(ab)^{-3}$ (ii) $(x^ny^{-m})^4 \times (x^3y^{-2})^{-n}$

(iii) $\left(\frac{125a^{-3}}{y^6}\right)^{\frac{-1}{3}}$ (iv) $\left(\frac{32x^{-5}}{243y^{-5}}\right)^{\frac{-1}{5}}$

(v) $(a^{-2}b)^{\frac{1}{2}} \times (ab^{-3})^{\frac{1}{3}}$

(vi) $(xy)^{m-n} \cdot (yz)^{n-l} \cdot (zx)^{l-m}$

8. Show that :

$$\left(\frac{x^a}{x^{-b}}\right)^{a-b} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b-c} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c-a} = 1$$

9. Evaluate : $\frac{x^{5+n} \times (x^2)^{3n+1}}{x^{7n-2}}$

10. Evaluate : $\frac{a^{2n+1} \times a^{(2n+1)(2n-1)}}{a^{n(4n-1)} \times (a^2)^{2n+3}}$

11. Prove that : $(m+n)^{-1}(m^{-1}+n^{-1}) = (mn)^{-1}$

12. Prove that :

(i) $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$

(ii) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$

13. Find the value of *n*, when :

(i) $12^{-5} \times 12^{2n+1} = 12^{13} \div 12^7$

(ii) $\frac{a^{2n-3} \times (a^2)^{n+1}}{(a^4)^{-3}} = (a^3)^3 \div (a^6)^{-3}$

14. Simplify :

(i) $\frac{a^{2n+3} \cdot a^{(2n+1)(n+2)}}{(a^3)^{2n+1} \cdot a^{n(2n+1)}}$

(ii) $\frac{x^{2n+7} \cdot (x^2)^{3n+2}}{x^{4(2n+3)}}$

SQUARES AND SQUARE ROOTS 3

3.1 REVIEW

1. Square	If a number is multiplied by itself, the product obtained is called the square of that number. *e.g.* (i) Since, $5 \times 5 = 25$; $\therefore$ 25 is square of 5 and we write $(5)^2 = 25$. (ii) 0·04 is square of 0·2 as $0{\cdot}2 \times 0{\cdot}2 = 0{\cdot}04$ and so on.
2. Square root	The **square root** of a given number **x** is the number whose square is **x**. *e.g.* square root of 36 is 6 as square of 6 is 36 *i.e.* $6^2 = 36$. The symbol of square root is radical sign $\sqrt{\ }$. Thus, square root of $64 = \sqrt{64} = 8$; square root of $1{\cdot}44 = \sqrt{1{\cdot}44} = 1{\cdot}2$ and so on. The sign $\sqrt{\ }$ is of the form of letter r, the first letter of the Latin word **radix** meaning **a root**.

1. $4^2 = 16$ is also read as; 4 **raised to the power** 2 is 16.
2. Squares of even numbers are always even.
   e.g. $2^2 = 2 \times 2 = 4$; $6^2 = 6 \times 6 = 36$; $14^2 = 14 \times 14 = 196$ and so on.
3. Squares of odd numbers are always odd.
   e.g. $3^2 = 3 \times 3 = 9$; $7^2 = 49$; $15^2 = 225$ and so on.
4. Whether the number is negative or positive, its square is always positive.
   e.g. $(3)^2 = 3 \times 3 = 9$, **which is a positive number.**
   $(-3)^2 = -3 \times -3 = 9$, **which is also a positive number.**
   Similarly, $(-5)^2 = 25$ and $(5)^2 = 25$, $(-8)^2 = 64$ and $8^2 = 64$.
5. Since, the square of every number is positive, the square root of a positive number can be obtained, but the square root of a negative number is not possible.

3.2 PERFECT SQUARE

A number, whose exact square root can be obtained, is called a **perfect square**.

e.g. 16, 49, 1·21, $\frac{9}{16}$, etc. are perfect squares as $\sqrt{16} = 4$, $\sqrt{49} = 7$, $\sqrt{1{\cdot}21} = 1{\cdot}1$ and so on.

> To find out whether a given number is a perfect square or not, express the number as a product of its prime factors. If the number is a perfect square, you would be able to group all the factors in pairs in such a way that both the factors in each pair are equal.

Example 1 :

Is 196 a perfect square ?

Solution :

$196 = 2 \times 2 \times 7 \times 7 = \overline{2 \times 2} \times \overline{7 \times 7}$

$\because$ The prime factors of 196 can be grouped in pairs; **196 is a perfect square. (Ans.)**

Example 2 :

Is 180 a perfect square ?

Solution :

$180 = 2 \times 2 \times 3 \times 3 \times 5 = \overline{2 \times 2} \times \overline{3 \times 3} \times 5$

Since, all the prime factors of 180 cannot be grouped in pairs. [One factor (*i.e.* 5) is left]

$\therefore$ **180 is not a perfect square.** **(Ans.)**

3.3 TO FIND THE SQUARE ROOT OF A PERFECT SQUARE NUMBER

(Using Prime Factor Method)

Example 3 :

Find the square root of 484.

Solution :

Square root of 484 $= \sqrt{484}$

Steps : 1. Resolve the number into prime factors : $= \sqrt{2 \times 2 \times 11 \times 11}$

2. Make pairs such that both the factors in each pair are equal : $= \sqrt{(2 \times 2) \times (11 \times 11)}$

3. Take one factor from each pair : $= 2 \times 11$

4. **The product is the square root of the given number = 22** **(Ans.)**

Example 4 :

Find the smallest number by which 980 be multiplied so that the product is a perfect square.

Solution :

$980 = \overline{2 \times 2} \times 5 \times \overline{7 \times 7}$

Since, the prime factor 5 is not in pair.

$\therefore$ **The given number should be multiplied by 5.** **(Ans.)**

$980 \times 5 = \overline{2 \times 2} \times \overline{5 \times 5} \times \overline{7 \times 7}, \quad \therefore \sqrt{980 \times 5} = 2 \times 5 \times 7 = 70$

Example 5 :

Find the smallest number by which 3150 be divided, so that the quotient is a perfect square.

Solution :

$3150 = 2 \times \overline{5 \times 5} \times \overline{3 \times 3} \times 7$

Since, the prime factors 2 and 7 cannot be paired.

$\therefore$ **The given number should be divided by 2 × 7 = 14** **(Ans.)**

$$\frac{3150}{14} = \frac{2 \times \overline{5 \times 5} \times \overline{3 \times 3} \times 7}{2 \times 7} = \overline{5 \times 5} \times \overline{3 \times 3}$$

Example 6 :

Find the square root of : (i) $2\frac{7}{9}$ (ii) 4·41

Solution :

(i) Square root of $2\frac{7}{9} = \sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3} = 1\frac{2}{3}$ **(Ans.)**

Square root of a fraction $= \dfrac{\textit{Square root of its numerator}}{\textit{Square root of its denominator}}$

(ii) $\sqrt{4{\cdot}41} = \sqrt{\frac{441}{100}} = \sqrt{\frac{3\times3\times7\times7}{2\times2\times5\times5}} = \frac{3\times7}{2\times5} = \frac{21}{10} = 2{\cdot}1$ **(Ans.)**

1. Instead of writing the prime factors of the given number in pairs, we can write them in index form and then in order to find the required square root, take half of each index value.

   e.g. $\sqrt{784} = \sqrt{2\times2\times2\times2\times7\times7} = \sqrt{2^4\times7^2} = 2^2\times7^1 = 28$

2. $\sqrt{9} = 3$, but $\sqrt{0{\cdot}9} \neq 0{\cdot}3$

   Reason : $(0{\cdot}3)^2 = 0{\cdot}3 \times 0{\cdot}3 = 0{\cdot}09 \quad \therefore \quad \sqrt{0{\cdot}09} = 0{\cdot}3$

   In the same way, $\sqrt{144} = 12$, but $\sqrt{14{\cdot}4} \neq 1{\cdot}2$

   Reason : $(1{\cdot}2)^2 = 1{\cdot}2 \times 1{\cdot}2 = 1{\cdot}44 \quad \therefore \quad \sqrt{1{\cdot}44} = 1{\cdot}2$

3. Square root of a perfect square even number is always an even number and square root of a perfect square odd number is always an odd number.

   e.g. (i) $\sqrt{4} = 2$, $\sqrt{16} = 4$, $\sqrt{36} = 6$, $\sqrt{64} = 8$, $\sqrt{100} = 10$ and so on.

   (ii) $\sqrt{9} = 3$, $\sqrt{25} = 5$, $\sqrt{49} = 7$, $\sqrt{81} = 9$, $\sqrt{121} = 11$ and so on.

Example 7 :

A man plants his orchard with 5625 trees and arranges them so that there are as many rows as there are trees in each row. How many rows are there ?

Solution :

Let the number of rows be x.

$\therefore$ Number of trees in each row = x

and, total number of trees planted = $x \times x = x^2$

Given : $x^2 = 5625 \Rightarrow x = \sqrt{5625} = \sqrt{\overline{5\times5}\times\overline{5\times5}\times\overline{3\times3}}$

$= 5 \times 5 \times 3 = 75$

$\therefore$ **The number of rows = 75** **(Ans.)**

Example 8 :

In a basket there are 50 flowers. A man goes to worship and puts as many flowers in each temple as there are temples in the city. Thus, he needs 8 baskets of flowers. Find the number of temples in the city.

Solution :

Let the number of temples in the city = x

$\therefore$ The number of flowers put in each temple = x

and, the total number of flowers used = $x \times x = x^2$

According to the given statement :

$x^2 = 50 \times 8 \Rightarrow x = \sqrt{50 \times 8} = \sqrt{\overline{5\times5}\times\overline{2\times2}\times\overline{2\times2}} = 5 \times 2 \times 2 = 20$

$\therefore$ **The number of temples in the city = 20** **(Ans.)**

Example 9 :

Find the smallest perfect square number, which is divisible by 8 and 12.

Solution :

The required smallest perfect square number divisible by 8 and 12 is divisible by L.C.M. of 8 and 12

Since, L.C.M. of 8 and 12 = 24 and $24 = \underline{2 \times 2} \times 2 \times 3$

To make it a perfect square, it must be multiplied by 2 and 3.

$\therefore$ **Required perfect square number** $= 24 \times 2 \times 3 =$ **144** **(Ans.)**

Example 10 :

Find the smallest perfect square number divisible by 24, 30 and 60.

Solution :

$\because$ L.C.M. of 24, 30 and 60 = 120

and $120 = \underline{2 \times 2} \times 2 \times 3 \times 5$ which will be the smallest perfect square on multiplying it with $2 \times 3 \times 5$.

$\therefore$ **Required perfect square number** $= 120 \times 2 \times 3 \times 5 =$ **3600** **(Ans.)**

EXERCISE 3(A)

1. Find the square of :
   (i) 59 (ii) 6·3 (iii) $15\frac{2}{3}$
2. By splitting into prime factors, find the square root of :
   (i) 11025 (ii) 396900 (iii) 194481
3. (i) Find the smallest number by which 2592 be multiplied so that the product is a perfect square.
   (ii) Find the smallest number by which 12748 be multiplied so that the product is a perfect square.
4. Find the smallest number by which 10368 be divided, so that the result is a perfect square. Also, find the square root of the resulting number.
5. Find the square root of :
   (i) 0·1764 (ii) $96\frac{1}{25}$ (iii) 0·0169
6. Evaluate :
   (i) $\sqrt{\frac{14.4}{22.5}}$ (ii) $\sqrt{\frac{0.225}{28.9}}$
   (iii) $\sqrt{\frac{25}{32} \times 2\frac{13}{18} \times 0.25}$
   (iv) $\sqrt{1\frac{4}{5} \times 14\frac{21}{44} \times 2\frac{7}{55}}$
7. Evaluate :
   (i) $\sqrt{3^2 \times 6^3 \times 24}$
   (ii) $\sqrt{(0.5)^3 \times 6 \times 3^5}$
   (iii) $\sqrt{\left(5+2\frac{21}{25}\right) \times \frac{0.169}{1.6}}$
   (iv) $\sqrt{5\left(2\frac{3}{4} - \frac{3}{10}\right)}$
   (v) $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$
8. A man, after a tour, finds that he had spent every day as many rupees as the number of days he had been on tour. How long did his tour last, if he had spent in all ₹ 1,296?
9. Out of 745 students, maximum are to be arranged in the school field for a P.T. display, such that the number of rows is equal to the number of columns. Find the number of rows if 16 students were left out after the arrangement.
10. 13 and 31 is a strange pair of numbers such that their squares 169 and 961 are also mirror images of each other. Find two more such pairs.
11. Find the smallest perfect square divisible by 3, 4, 5 and 6.

L.C.M. of 3, 4, 5 and 6 = 60.
Also, $60 = 2 \times 2 \times 5 \times 3$ in which 5 and 3 are not in pairs.
So, 60 should be multiplied by 5×3 to get a perfect square number.
$\therefore$ **The required least square number**
$= 60 \times 5 \times 3 =$ **900** **Ans.**

12. If $\sqrt{784} = 28$, find the value of :
    (i) $\sqrt{7{\cdot}84} + \sqrt{78400}$
    (ii) $\sqrt{0{\cdot}0784} + \sqrt{0{\cdot}000784}$

6.4 TO FIND THE SQUARE ROOT OF A PERFECT SQUARE NUMBER
(Using Division Method)

Example 11 :

Find the square root of 276676

Solution :

Steps :

1. Group the digits in pairs starting from right to left, thus $276676 = \overline{27}\ \overline{66}\ \overline{76}$. Take the first pair (*i.e.* 27) and find the largest whole number which when multiplied by itself gives 27 or just less than 27. Such a number is 5.
   Write 5 in the quotient and also in the divisor.
2. Subtract $5 \times 5 = 25$ from 27 (the first pair).
   The remainder is 2.
3. Bring down the second pair of digits (*i.e.* 66).
   Double the quotient (*i.e.* 5) and write the result ($5 \times 2 = 10$) on the left of 266.
   Put the largest possible digit on the right of 10, such that the product of this largest digit with the new number obtained does not exceed 266. Such a digit is 2 as $102 \times 2 = 204$.
   Write 2 in the quotient also.
4. Subtract $102 \times 2 = 204$ from 266. The remainder is 62.
   Bring down the next pair of digits (*i.e.* 76) and proceed as in (3).

	5 2 6	
5	$\overline{27}\ \overline{66}\ \overline{76}$	(Step 1)
	25	(Step 2)
102	2 66	(Step 3)
	2 04	
1046	62 76	(Step 4)
	62 76	
	×	

$\therefore$ $\sqrt{276676} = 526$ **(Ans.)**

Example 12 :

Using the division method find the square root of : (i) 4489 (ii) 46656

Solution :

(i)

	6 7
6	$\overline{44}\ \overline{89}$
	36
127	8 89
	8 89
	×

$\therefore$ $\sqrt{4489} = 67$ **(Ans.)**

(ii)

	2 1 6
2	$4\ \overline{66}\ \overline{56}$
	4
41	× 66
	41
426	25 56
	25 56
	×

$\therefore$ $\sqrt{46656} = 216$ **(Ans.)**

Example 13 :

Using the division method find the square root of : (i) 605·16 (ii) 0·000729

Solution :

In the mixed decimal numbers, starting from the decimal point, group the integral part from right to left and decimal part from left to right.

Thus, $605\cdot16 = 6\,\overline{05}\cdot\overline{16}$ and $0\cdot000729 = 0\cdot\overline{00}\,\overline{07}\,\overline{29}$

Now, proceed exactly in the same way as explained above. Just remember to put a decimal in the quotient as the decimal point in the dividend is crossed.

(i)

$$\begin{array}{r|l} & 2\;4\;\cdot6 \\ \hline 2 & 6\,\overline{05}\cdot\overline{16} \\ & 4 \\ \hline 44 & 2\,05 \\ & 1\,76 \\ \hline 486 & \quad 29\,16 \\ & \quad 29\,16 \\ \hline & \quad\;\times \end{array}$$

$\therefore \sqrt{605\cdot16} = 24\cdot6$ **(Ans.)**

(ii)

$$\begin{array}{r|r} & 0\cdot0\;\;2\;\;7 \\ \hline 2 & 0\cdot\overline{00}\,\overline{07}\,\overline{29} \\ & 4 \\ \hline 47 & 3\,29 \\ & 3\,29 \\ \hline & \times \end{array}$$

$\therefore \sqrt{0\cdot000729} = 0\cdot027$ **(Ans.)**

Examine the following results :

$\because\ (0\cdot7)^2 = 0\cdot49 \qquad \therefore\ \sqrt{0\cdot49} = 0\cdot7$

$\because\ (0\cdot03)^2 = 0\cdot0009 \qquad \therefore\ \sqrt{0\cdot0009} = 0\cdot03$

$\because\ (3\cdot2)^2 = 10\cdot24 \qquad \therefore\ \sqrt{10\cdot24} = 3\cdot2$

$\because\ (0\cdot015)^2 = 0\cdot000225 \qquad \therefore\ \sqrt{0\cdot000225} = 0\cdot015$

It is clear from these results that the square of any decimal number contains even number of decimal places and that the number of decimal places in the square is double the number of decimal places in the square root. Hence, a decimal number (or a mixed decimal number) can be a perfect square only when it has an even number of digits in its decimal part.

3.5 TO FIND THE SQUARE ROOT OF A NUMBER WHICH IS NOT A PERFECT SQUARE *(Using Division Method)*

Example 14 :

Find the square root of 24·729 correct to two places of decimal.

Solution :

When the square root is required correct to two places of decimal, we shall find the square root up to three places of decimal and then round it off upto two places of decimal.

Similarly, if the square root is required correct to three places of decimal, find the square root up to four places and then round it off upto three places and so on.

In order to find square root upto three places of decimal, we must have three pairs of digits after decimal.

For this purpose, $24\cdot729 = \overline{24}\cdot\overline{72}\,\overline{90}\,\overline{00}$

(Addition of any number of zeroes on the right of a decimal fraction does not change its value)

	4 ·9 7 2
4	$\overline{24}\cdot\overline{72}\,\overline{90}\,\overline{00}$
	16
89	8 72
	8 01
987	71 90
	69 09
9942	281 00
	198 84
	82 16

$\therefore \sqrt{24 \cdot 729}$ = 4·972 upto three places of decimal

= **4·97 correct to two places of decimal** **(Ans.)**

Example 15 :

Find the square root of :

(i) 3, correct to three places of decimal. (ii) 0·07688, correct to two places of decimal.

Solution :

(i) $3 = 3\cdot\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}$

	1 ·7 3 2 0
1	$3\cdot\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}$
	1
27	2 00
	1 89
343	11 00
	10 29
3462	71 00
	69 24
34640	17600

$\therefore \sqrt{3}$ = 1·7320

= **1·732** **(Ans.)**

(ii) $0{\cdot}07688 = 0\cdot\overline{07}\,\overline{68}\,\overline{80}$

	0 ·2 7 7
2	$0\cdot\overline{07}\,\overline{68}\,\overline{80}$
	4
47	3 68
	3 29
547	39 80
	38 29
	151

$\therefore \sqrt{0 \cdot 07688}$ = 0·277

= **0·28** **(Ans.)**

Example 16 :

Find the least number that must be subtracted from 2433 so that the remainder is a perfect square.

Solution :

	49
4	$\overline{24}\ \overline{33}$
	16
89	8 33
	8 01
	32

Clearly, if 32 is subtracted from 2433, the remainder will be a perfect square.

(Ans.)

Since, 2433 − 32 = 2401
and, $\sqrt{2401}$ = 49

Example 17 :

Find the least number which must be added to 18,265 to obtain a perfect square.

Solution :

	135
1	1 $\overline{82}$ $\overline{65}$
	1
23	82
	69
265	13 65
	13 25
	40

Clearly, 18265 is greater than 135^2.
∴ On adding the required number to 18265, we shall be getting 136^2 *i.e.* 18496

Hence, **the required number** = 18496 − 18265
= **231** **(Ans.)**

EXERCISE 3(B)

1. Find the square root of :
   (i) 4761 (ii) 7744 (iii) 15129
   (iv) 0·2916 (v) 0·001225
   (vi) 0·023104 (vii) 27·3529
2. Find the square root of :
   (i) 4·2025 (ii) 531·7636 (iii) 0·007225
3. Find the square root of :
   (i) 245 correct to two places of decimal.
   (ii) 496 correct to three places of decimal.
   (iii) 82·6 correct to two places of decimal.
   (iv) 0·065 correct to three places of decimal.
   (v) 5·2005 correct to two places of decimal.
   (vi) 0·602 correct to two places of decimal.
4. Find the square root of each of the following correct to two decimal places :
   (i) $3\frac{4}{5}$ (ii) $6\frac{7}{8}$

   (i) $3\frac{4}{5} = 3{\cdot}8$ (ii) $6\frac{7}{8} = 6{\cdot}875$
5. For each of the following, find the least number that must be subtracted so that the resulting number is a perfect square.
   (i) 796 (ii) 1886 (iii) 23497
6. For each of the following, find the least number that must be added so that the resulting number is a perfect square.
   (i) 511 (ii) 7172 (iii) 55078
7. Find the square root of 7 correct to two decimal places; then use it to find the value of $\sqrt{\frac{4+\sqrt{7}}{4-\sqrt{7}}}$ correct to three significant digits.

$$\sqrt{\frac{4+\sqrt{7}}{4-\sqrt{7}}} = \sqrt{\frac{(4+\sqrt{7})(4+\sqrt{7})}{(4-\sqrt{7})(4+\sqrt{7})}}$$
$$= \sqrt{\frac{(4+\sqrt{7})^2}{16-7}} = \frac{4+\sqrt{7}}{3}$$

8. Find the value of $\sqrt{5}$ correct to 2 decimal places; then use it to find the square root of $\frac{3-\sqrt{5}}{3+\sqrt{5}}$ correct to 2 significant digits.
9. Find the square root of :
   (i) $\frac{1764}{2809}$ (ii) $\frac{507}{4107}$
   (iii) $\sqrt{108 \times 2028}$ (iv) $0{\cdot}01 + \sqrt{0{\cdot}0064}$
10. Find the square root of 7·832 corret to :
    (i) 2 decimal places
    (ii) 2 significant digits.

11. Find the least number which must be subtracted from 1205 so that the resulting number is a perfect square.

By division method, find the square root of 1205

$\because \sqrt{1205} = 34{\cdot}713$ ......

$\therefore$ **Required number to be subtracted**

$= 1205 - 34^2 = \mathbf{49}$ **(Ans.)**

12. Find the least number which must be added to 1205 so that the resulting number is a perfect square

As done above, $\sqrt{1205} = 34{\cdot}713$ ....

$\therefore$ **Required number to be added**

$= 35^2 - 1205 = \mathbf{20}$ **(Ans.)**

13. Find the least number which must be subtracted from 2037 so that the resulting number is a perfect square.

14. Find the least number which must be added to 5483 so that the resulting number is a perfect square.

3.6 PROPERTIES OF SQUARE NUMBERS

1st Property :

The ending digit (*i.e.* the digit at unit's place) of the square of a number is 0, 1, 4, 5, 6 or 9.

For example :

(i) $11^2 = 121$ (ii) $22^2 = 484$ (iii) $53^2 = 2809$ (iv) $30^2 = 900$
(v) $4^2 = 16$ (vi) $25^2 = 625$ (vii) $46^2 = 2116$ (viii) $37^2 = 1369$
(ix) $68^2 = 4624$ (x) $19^2 = 361$

2nd Property :

A number having 2, 3, 7 or 8 at its unit's place is never a perfect square.

For example :

None of the following numbers is a perfect square.

(i) 12, 22, 32, 42, ... (ii) 13, 23, 33, 43, ...
(iii) 17, 27, 37, 47, ... (iv) 18, 28, 38, 48, ...

3rd Property :

If a number has 1 or 9 at its unit's place, then square of this number always has 1(one) at its unit place :

For example :

(i) Square of 1 = **1** (ii) $11^2 = 121$ (iii) $31^2 = 961$
(iv) $9^2 = 81$ (v) $29^2 = 841$ (vi) $49^2 = 2401$ and so on.

4th Property :

If the digit at the unit's place of a number is 4 or 6, then its square will always have 6 at its unit's place.

For example :

(i) $4^2 = 16$ (ii) $6^2 = 36$ (iii) $24^2 = 576$
(iv) $36^2 = 1296$ (v) $84^2 = 7056$ (vi) $96^2 = 9216$ and so on.

5^{th} Property :

If a number ends with *n* zeroes; its square ends with 2*n* zeroes.

For example :

(i) Square of 30 = 900 (ii) Square of 300 = 90000 and so on.

The number of zeroes at the end of a square number is always even. That is a number ending in an odd number of zeroes can never be a perfect square.

Thus, each of 40, 360, 49000, 2500000, etc. can not be a perfect square.

6^{th} Property :

A perfect square number leaves remainder 0 or 1 on dividing it by 3.

For example :

(i) 9 is a perfect square number and on dividing it by 3, the remainder is 0.

(ii) 16 is a perfect square number and on dividing it by 3, the remainder is 1.

When each of perfect square numbers 1, 4, 9, 16, 25, 36, ... is divided by 3, the remainder is either 0 or 1.

7^{th} Property :

For any natural number n,

$$(n + 1)^2 - n^2 = (n + 1) + n$$

For example :

(i) $8^2 - 7^2 = 8 + 7 = 15$ (ii) $15^2 - 14^2 = 15 + 14 = 29$

(iii) $35^5 - 34^2 = 35 + 34 = 69$ and so on.

1. The sum of first n odd natural numbers = n^2,

$\Rightarrow$ (i) **1 + 3** = sum of first 2 odd natural numbers = 2^2 = **4**

(ii) **1 + 3 + 5 + 7 + 9** = sum of first 5 odd natural numbers = 5^2 = **25**

(iii) **1 + 3 + 5 + 7 + 9 + ... + 19**

= sum of first 10 odd natural numbers = 10^2 = **100**

2. For any three natural numbers p, q and r,

if $p^2 + q^2 = r^2$ or $p^2 + r^2 = q^2$ or $q^2 + r^2 = p^2$; the numbers p, q and r are known as **Pythagorean triplets**.

For example :

(i) Natural numbers 3, 4 and 5 are Pythagorean triplets as $3^2 + 4^2 = 5^2$.

(ii) $5^2 + 12^2 = 13^2 \Rightarrow$ 5, 12 and 13 are Pythagorean triplets.

EXERCISE 3(C)

1. Seeing the value of the digit at unit's place, state which of the following can be square of a number ?

   (i) 3051 (ii) 2332 (iii) 5684
   (iv) 6908 (v) 50699

2. Squares of which of the following numbers will have 1(one) at their unit's place ?

   (i) 57 (ii) 81 (iii) 139
   (iv) 73 (v) 64

3. Which of the following numbers will not have 1(one) at their unit's place ?

   (i) 32^2 (ii) 57^2 (iii) 69^2
   (iv) 321^2 (v) 265^2

4. Squares of which of the following numbers will not have 6 at their unit's place ?

   (i) 35 (ii) 23 (iii) 64
   (iv) 76 (v) 98

5. Which of the following numbers will have 6 at their unit's place :

   (i) 26^2 (ii) 49^2 (iii) 34^2
   (iv) 43^2 (v) 244^2

6. If a number ends with 3 zeroes, how many zeroes will its square have ?

7. If the square of a number ends with 10 zeroes, how many zeroes will the number have ?

8. Is it possible for the square of a number to end with 5 zeroes ? Give reason.

9. Give reason to show that none of the numbers, given below, is a perfect square.

   (i) 2162 (ii) 6843
   (iii) 9637 (iv) 6598

10. State, whether the square of the following numbers is even or odd ?

    (i) 23 (ii) 54
    (iii) 76 (iv) 75

11. Give reason to show that none of the numbers 640, 81000 and 3600000 is a perfect square.

12. Evaluate :

    (i) $37^2 - 36^2$ (ii) $85^2 - 84^2$
    (iii) $101^2 - 100^2$

13. Without doing the actual addition, find the sum of :

    (i) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23
    (ii) 1 + 3 + 5 + 7 + 9 + ... + 39 + 41
    (iii) 1 + 3 + 5 + 7 + 9 + ... + 51 + 53

14. Write three sets of Pythagorean triplets such that each set has numbers less than 30

CUBES AND CUBE-ROOTS 4

4.1 INTRODUCTION

For any number m, $m \times m \times m$ is called **cube of m** or **m cube** and is written as m^3.

Thus, $\quad m \times m \times m = \text{cube of } m$

$= m^3 = m$ raised to the power 3, etc.

For example :

(i) **Cube of 5** $= 5^3$

$= 5 \times 5 \times 5 = 125$

(ii) **Cube of 8** $= 8^3$

$= 8 \times 8 \times 8 = 512$

(iii) **Cube of –4** $= -4 \times -4 \times -4$

$= -(4 \times 4 \times 4) = -64$

> Cube of a positive number is always positive and cube of a negative number is negative.

4.2 PERFECT CUBE

The cube of a number is called a **perfect cube**.

For example :

(i) $6^3 = 6 \times 6 \times 6 = 216 \Rightarrow$ **216 is a perfect cube.**

(ii) $15^3 = 15 \times 15 \times 15 = 3375 \Rightarrow$ **3375 is a perfect cube.**

Conversely, a given number is a perfect cube, if it can be expressed as the product of triplets of equal factors.

For example :

Consider the number 216, then

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= 6 \times 6 \times 6 = (6)^3$

$=$ Product of triplets of equal factors.

$\therefore$ **216 is a perfect cube.**

2	216
2	108
2	54
3	27
3	9
	3

Example 1 :

(i) Is 297 a perfect cube ? (ii) Is 2744 a perfect cube ?

Solution :

(i) $\therefore \quad 297 = 3 \times 3 \times 3 \times 11$

$= (3 \times 3 \times 3) \times 11$

Since, triplet of number 11 is not formed,

$\therefore$ **297 is not a perfect cube.** **Ans.**

3	297
3	99
3	33
	11

(ii) $\because$ 2744 $= 2 \times 2 \times 2 \times 7 \times 7 \times 7$
$= (2 \times 7) \times (2 \times 7) \times (2 \times 7)$
$= 14 \times 14 \times 14$
$= (14)^3$

$\therefore$ **2744 is a perfect cube** **Ans.**

2	2744
2	1372
2	686
7	343
7	49
	7

Example 2 :

What is the smallest number by which 3087 may be multiplied, so that the product is a perfect cube?

Solution :

3	3087
3	1029
7	343
7	49
	7

On finding the prime factors of 3087,

we get : 3087 $= 3 \times 3 \times 7 \times 7 \times 7$

Clearly, **3087 must be multiplied by 3** **Ans.**

3087 × 3 $= (3 \times 3 \times 7 \times 7 \times 7) \times 3$
$= 3 \times 3 \times 3 \times 7 \times 7 \times 7$
$= (3 \times 7) \times (3 \times 7) \times (3 \times 7) = 21 \times 21 \times 21 = (21)^3$

Example 3 :

What is the least number by which 6750 may be divided so that the quotient is a perfect cube ?

Solution :

2	6750
5	3375
5	675
5	135
3	27
3	9
	3

On finding the prime factors of 6750,

we get : 6750 $= 2 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3$
$= 2 \times (5 \times 5 \times 5) \times (3 \times 3 \times 3)$

Clearly, **6750 must be divided by 2** **Ans.**

$$\frac{6750}{2} = \frac{2 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3}{2}$$
$$= 5 \times 5 \times 5 \times 3 \times 3 \times 3$$
$$= (5 \times 3) \times (5 \times 3) \times (5 \times 3) = 15 \times 15 \times 15 = (15)^3$$

1. Cubes of odd natural numbers are odd, as : $1^3 = 1$, $3^3 = 27$, $5^3 = 125$, etc.
2. Cubes of even natural numbers are even, as : $2^3 = 8$, $4^3 = 64$, $6^3 = 216$, etc.

EXERCISE 4(A)

1. Find the cube of :
   (i) 7 (ii) 11 (iii) 16
   (iv) 23 (v) 31 (vi) 42
   (vii) 54

2. Find which of the following are perfect cubes ?
   (i) 243 (ii) 588 (iii) 1331
   (iv) 24000 (v) 1728 (vi) 1938

3. Find the cubes of :

(i) 2·1 (ii) 0·4 (iii) 1·6

(iv) 2·5 (v) 0·12 (vi) 0·02

(vii) 0·8

(v) Cube of 0·12 $= (0\cdot12)^3$

$= 0\cdot12 \times 0\cdot12 \times 0\cdot12$

$= \mathbf{0\cdot001728}$

4. Find the cubes of :

(i) $\frac{3}{7}$ (ii) $\frac{8}{9}$ (iii) $\frac{10}{13}$

(iv) $1\frac{2}{7}$ (v) $2\frac{1}{2}$

(v) Cube of $2\frac{1}{2} = \left(2\frac{1}{2}\right)^3$

$= \left(\frac{5}{2}\right)^3 = \frac{5\times5\times5}{2\times2\times2}$

$= \frac{125}{8} = \mathbf{15\frac{5}{8}}$

5. Find the cubes of :

(i) −3 (ii) −7 (iii) −12

(iv) −18 (v) −25 (vi) −30

(vii) −50

6. Which of the following are cubes of :

(i) an even number

(ii) an odd number.

216, 729, 3375, 8000, 125, 343, 4096 and 9261.

7. Find the least number by which 1323 must be multiplied so that the product is a perfect cube.

8. Find the smallest number by which 8768 must be divided so that the quotient is a perfect cube.

9. Find the smallest number by which 27783 be multiplied to get a perfect cube number.

10. With what least number must 8640 be divided so that the quotient is a perfect cube ?

11. Which is the smallest number that must be multiplied to 77175 to make it a perfect cube?

4.3 CUBE-ROOTS

The cube-root of a given number is the number whose cube is the given number.

⇒ if cube-root of number x is y, then cube of y is x.

⇒ if $\sqrt[3]{x} = y$, then $y^3 = x$.

For example :

(i) cube of 3 = 27 ⇒ cube-root of 27 = 3 *i.e.* $\sqrt[3]{27} = 3$

(ii) cube of 7 = 343 ⇒ cube-root of 343 = 7 *i.e.* $\sqrt[3]{343} = 7$ and so on.

In the same way :

(i) $125 = 5^3 \Rightarrow \sqrt[3]{125} = 5$

(ii) $512 = 8^3 \Rightarrow \sqrt[3]{512} = 8$ and so on.

4.4 CUBE-ROOT BY FACTORISATION

Steps :

1. Split the given number into its primes.
2. Form groups in triplets of the identical primes.
3. Take one prime number from each triplet.
4. Multiply all the prime numbers obtained in step 3 to get the required cube-root.

Example 4 :

Find the cube-root of 216.

2	216
2	108
2	54
3	27
3	9
	3

Solution :

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$= (2 \times 2 \times 2) \times (3 \times 3 \times 3)$$
$$\Rightarrow \quad \sqrt[3]{216} = 2 \times 3 = 6 \qquad \textbf{Ans.}$$

Example 5 :

Find the cube root 1728.

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

Solution :

$$1728 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3)$$
$$\Rightarrow \quad \text{Cube-root of } 1728 = 2 \times 2 \times 3 = 12 \qquad \textbf{Ans.}$$

Example 6 :

Find the smallest number by which 210125 must be multiplied so that the product is a perfect cube.

5	210125
5	42025
5	8405
41	1681
	41

Solution :

$$210125 = 5 \times 5 \times 5 \times 41 \times 41$$
$$= (5 \times 5 \times 5) \times 41 \times 41$$

Clearly, the required smallest number = 41 **Ans.**

1. Cube-root of a negative perfect cube :

If m is any positive integer, then $-m$ is a negative integer.

Since, $(-m)^3 = -m \times -m \times -m$
$= -m^3$

$\therefore \quad \sqrt[3]{-m^3} = -\sqrt[3]{m^3} = -m$

Cube-root of $(-m^3)$ = – (cube-root of m^3).

For example :

1. Cube-root of – 8 = – (cube-root of 8)
$= -2$

$\Rightarrow \quad \sqrt[3]{-8} = -2$

2. Cube-root of – 1000 = – (cube-root of 1000)
$= -10$

i.e. $\quad \sqrt[3]{-1000} = -10$

3. Cube-root of – 1 = – (cube root of 1)

$= -1$

i.e. $\sqrt[3]{-1} = -1$ and so on.

2. Cube-root of product of numbers :

(a) $\sqrt[3]{xy} = \sqrt[3]{x} \times \sqrt[3]{y}$

(b) $\sqrt[3]{xyz} = \sqrt[3]{x} \times \sqrt[3]{y} \times \sqrt[3]{z}$ and so on.

For example :

1. $\sqrt[3]{8\times125} = \sqrt[3]{8} \times \sqrt[3]{125}$
   $= 2 \times 5 = 10$

2. $\sqrt[3]{500\times54} = \sqrt[3]{500\times2\times27}$
   $= \sqrt[3]{1000\times27}$
   $= \sqrt[3]{1000} \times \sqrt[3]{27} = 10 \times 3 = 30$

3. $\sqrt[3]{-16\times32} = \sqrt[3]{-8\times2\times32}$
   $= \sqrt[3]{-8\times64}$
   $= \sqrt[3]{-8} \times \sqrt[3]{64} = -2 \times 4 = -8$

4. $\sqrt[3]{216\times-343} = \sqrt[3]{216} \times \sqrt[3]{-343}$
   $= 6 \times -7 = -42$

3. Cube-root of fractional numbers :

(a) $\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$

(b) $\sqrt[3]{\frac{xy}{z}} = \frac{\sqrt[3]{x}\times\sqrt[3]{y}}{\sqrt[3]{z}}$

(c) $\sqrt[3]{\frac{x}{yz}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}\times\sqrt[3]{z}}$ and so on.

For example :

1. $\sqrt[3]{\frac{125}{216}} = \frac{\sqrt[3]{125}}{\sqrt[3]{216}} = \frac{\sqrt[3]{5\times5\times5}}{\sqrt[3]{6\times6\times6}} = \frac{5}{6}$

2. $\sqrt[3]{\frac{-8}{27}} = \frac{\sqrt[3]{(-2)\times(-2)\times(-2)}}{\sqrt[3]{3\times3\times3}} = \frac{-2}{3}$

3. $\sqrt[3]{\frac{64}{-125}} = \frac{\sqrt[3]{4\times4\times4}}{\sqrt[3]{(-5)\times(-5)\times(-5)}} = \frac{4}{-5} = -\frac{4}{5}$

4. Cube-root of a decimal number :

Convert the given decimal number into a fractional number and then find its cube-root.

For example :

1. $\sqrt[3]{0\cdot027} = \sqrt[3]{\frac{0027}{1000}} = \sqrt[3]{\frac{27}{1000}} = \frac{3}{10} = 0\cdot3$

2. $\sqrt[3]{0\cdot008} = \sqrt[3]{\frac{0008}{1000}} = \sqrt[3]{\frac{8}{1000}} = \frac{2}{10} = 0\cdot2$

3. $\sqrt[3]{0\cdot125} = \sqrt[3]{\frac{125}{1000}} = \frac{5}{10} = 0\cdot5$

EXERCISE 4(B)

1. Find the cube-roots of :
   (i) 64 (ii) 343
   (iii) 729 (iv) 1728
   (v) 9261 (vi) 4096
   (vii) 8000 (viii) 3375

2. Find the cube-roots of :
   (i) $\frac{27}{64}$ (ii) $\frac{125}{216}$
   (iii) $\frac{343}{512}$ (iv) 64×729
   (v) 64×27 (vi) 729×8000
   (vii) 3375×512

3. Find the cube-roots of :
   (i) −216 (ii) −512 (iii) −1331
   (iv) $-\frac{27}{125}$ (v) $\frac{-64}{343}$ (vi) $-\frac{512}{343}$
   (vii) −2197 (viii) −5832 (ix) −2744000

4. Find the cube-roots of :
   (i) 2·744 (ii) 9·261
   (iii) 0·000027 (iv) −0·512
   (v) −15·625 (vi) -125×1000

5. Find the smallest number by which 26244 should be divided so that the quotient is a perfect cube.

6. What is the least number by which 30375 should be multiplied to get a perfect cube ?

7. Find the cube-roots of :
   (i) $700 \times 2 \times 49 \times 5$ (ii) -216×1728
   (iii) -64×-125 (iv) $-\frac{27}{343}$
   (v) $\frac{729}{-1331}$ (vi) 250·047
   (vii) −175616

PLAYING WITH NUMBERS 5

5.1 INTRODUCTION

In the previous classes, we have learnt about :

1. Natural numbers
2. Whole numbers
3. Integers
4. Rational numbers
5. Factors
6. Multiples
7. Relation between factors and multiples, etc.

5.2 GENERALIZED FORM OF NUMBERS

A **number** is said to be **in generalized** form, if it is expressed as the sum of the product of its digit with their place values.

For example :

(i) 68 = digit 6 × its place value + digit 8 × its place value
$= 6 \times 10 + 8 \times 1$

(ii) 257 = digit 2 × its place value + digit 5 × its place value + digit 7 × its place value
$= 2 \times 100 + 5 \times 10 + 7 \times 1$

(iii) $3479 = 3 \times 1000 + 4 \times 100 + 7 \times 10 + 9 \times 1$

5.3 TWO-DIGIT NUMBERS

Let a two-digit number has ***a*** at its ten's place and ***b*** at its unit's place then;

the number = $10a + b$

Remember : In a two digit number $\mathbf{10a + b}$; the ten's digit number ***a*** is any whole number from 1 to 9 and the unit digit number ***b*** is any whole number from 0 to 9.

Thus,

(i) $75 = 10 \times 7 + 5$ (ii) $68 = 10 \times 6 + 8$

(iii) $50 = 10 \times 5 + 0$ and so on.

In general, any two digit number *ab* made of digits a and b can be written as :

$ab = 10 \times a + b = 10a + b$ [Here, $ab \neq a \times b$]

and, $ba = 10 \times b + a = 10b + a$

5.4 THREE-DIGIT NUMBERS

Let a three-digit number has ***a*** at its hundred's place, ***b*** at its ten's place and ***c*** at unit's place.

The number = $100a + 10b + c$

Remember : In a three digit number $\mathbf{100a + 10b + c}$, the digit ***a*** at hundred's place is any whole number from 1 to 9, the digit ***b*** at ten's place is any whole number from 0 and 9 and the digit ***c*** at unit's place is any whole number from 0 to 9.

Thus,

(i) $428 = 100 \times 4 + 10 \times 2 + 8$ (ii) $300 = 100 \times 3 + 10 \times 0 + 0$

(iii) $579 = 100 \times 5 + 10 \times 7 + 9$

In general a three-digit number *abc* made up of digits *a*, *b* and *c* is written as :

$$\begin{aligned} abc &= 100 \times a + 10 \times b + c \\ &= 100a + 10b + c, \\ bca &= 100b + 10c + a, \\ cab &= 100c + 10a + b \quad \text{and so on.} \end{aligned}$$

5.5 SOME INTERESTING PROPERTIES

Property 1 :

Consider a two-digit number $ab = 10a + b$ and the number obtained on reversing its digits $ba = 10b + a$. Then.

$$\begin{aligned} ab + ba &= (10a + b) + (10b + a) \\ &= 11a + 11b = 11(a + b) \end{aligned}$$

$$\Rightarrow \quad a + b = \frac{ab + ba}{11} \text{ and } 11 = \frac{ab + ba}{a + b}$$

⇒ The sum of a two digit number *ab* and the number *ba* obtained on reversing its digits is completely divisible by (i) 11 and (ii) the sum of its digits *i.e.* $a + b$.

For example :

1. Consider the two digit number 35

The number obtained on reversing its digits = 53

Sum of these two numbers = 35 + 53 = 88

Now, $\frac{88}{11} = 8 \Rightarrow$ the sum of 35 and 53 is divisible by 11.

Also, $\frac{88}{3+5} = \frac{88}{8} = 11 \Rightarrow$ the sum of 35 and 53 is divisible by the sum of the digits 3 and 5.

2. Consider the two digit number 87

The number obtained on reversing its digits = 78

Sum of these two digit numbers = 87 + 78 = 165

Check whether 165 is divisible by 11 or not.

Also, check whether 165 is divisible by 8 + 7 = 15 or not.

Yes, 165 is divisible by 11 as $\frac{165}{11} = 15$

Also, 165 is divisible by sum of the digits 8 and 7

i.e. 165 is divisible by 8 + 7 = 15 as $\frac{165}{15} = 11$.

When $ab + ba$ is divided by 11, the quotient = $a + b$
and, when $ab + ba$ is divided by $a + b$, the quotient = 11

Example 1 :

Is the sum of two digit numbers 62 and 26 divisible by 8 and 11 ? Explain.

Solution :

$\because$ Out of 62 and 26, one number can be obtained by interchanging the digits of the other

Also, $6 + 2 = 8$; therefore when the sum of two given numbers (62 and 26) is divided by 8, the quotient will be 11 and when divided by 11, the quotient will be 8.

$\Rightarrow$ Sum of 62 and 26 is divisible by 8 and 11 both.

Property 2 :

Consider a two-digit number $ab = 10a + b$ and the number obtained on reversing its digits $ba = 10b + a$. Then

1.

$$ab - ba = (10a + b) - (10b + a) = 10a + b - 10b - a = 9a - 9b$$

$$ab - ba = 9(a - b)$$

$$\Rightarrow \frac{ab - ba}{9} = a - b \text{ and } \frac{ab - ba}{a - b} = 9$$

2.

$$ba - ab = (10b + a) - (10a + b) = 10b + a - 10a - b = 9b - 9a$$

$$\Rightarrow ba - ab = 9(b - a)$$

$$\Rightarrow \frac{ba - ab}{9} = b - a \text{ and } \frac{ba - ab}{b - a} = 9$$

$\Rightarrow$ The difference between a two digit number ab and the two digit number ba, obtained on reversing the digits, is completely divisible by :

(i) 9 and (ii) the difference between its digits

- If $a > b$; $ab > ba$ $\Rightarrow$ (i) on dividing $ab - ba$ by 9, quotient $= a - b$.
  (ii) on dividing $ab - ba$ by $a - b$, quotient = 9.
- If $b > a$; $ba > ab$ $\Rightarrow$ (i) on dividing $ba - ab$ by 9, quotient $= b - a$.
  (ii) on dividing $ba - ab$ by $b - a$, quotient = 9.

For example :

1. Consider the two digit number 73

The number obtained on reversing its digits = 37

The difference between these two numbers = 73 − 37 = 36

Now, $\frac{36}{9} = 4$ $\Rightarrow$ the difference between 73 and 37 is divisible by 9.

Also, $\frac{36}{7-3} = \frac{36}{4} = 9$ $\Rightarrow$ the difference between 73 and 37 is divisible by the difference between its digits 7 and 3.

2. Consider the two digit number 38

The number obtained on reversing its digits = 83

The difference between 38 and 83 = 83 − 38 = 45

Now, $\frac{45}{9} = 5$ $\Rightarrow$ the difference between 38 and 83 is divisible by 9.

Also, $\frac{45}{8-3} = \frac{45}{5} = 9$ $\Rightarrow$ the difference between 38 and 83 is divisible by the difference between its digits 8 and 3.

Example 2 :

Find the quotient when 83 – 38 is divided by (i) 9 (ii) 5.

Solution :

(i) $\because$ When $ab - ba$ is divided by 9, quotient is $a - b$

$\therefore$ When 83 – 38 is divided by 9, the **quotient is 8 – 3 = 5** **(Ans.)**

(ii) $\because$ When $ab - ba$ is divided by $a - b$, the quotient is 9

$\Rightarrow$ When 83 – 38 is divided by 8 – 3 = 5, the **quotient is 9** **(Ans.)**

Property 3 :

Consider a 3-digit number *abc*. On changing its digits in order as shown ahead *abc* ; *bca* ; *cab*, we get two more 3-digit numbers *bca* and *cab*. Clearly

$abc = 100a + 10b + c$, $bca = 100b + 10c + a$ and $cab = 100c + 10a + b$

Now, $abc + bca + cab = 111a + 111a + 111c$

$= 111\ (a + b + c)$

$\therefore \dfrac{abc + bca + cab}{111} = (a + b + c)$ and $\dfrac{abc + bca + cab}{a + b + c} = 111$

$\Rightarrow$ The sum of a three digit number and the two numbers obtained by changing its digits in order is completely divisible by (i) 111 and (ii) sum of its digits.

For example : Consider a three digit number 374. The two numbers obtained on changing its digits in order are 743 and 437.

Adding them, we get : 374 + 743 + 437 = 1554 ; sum of the digits *i.e.* 3 + 7 + 4 = 14.

We have $\dfrac{1554}{(3+7+4)} = 111$ *i.e.* $\dfrac{1554}{14} = 111$ and $\dfrac{1554}{111} = 14 = (3 + 7 + 4)$

$\Rightarrow$ The sum of 374, 743 and 437 is divisible by both 111 and the sum of digits *i.e.* 14.

Example 3 :

Find the quotient when 821 + 218 + 182 is divided by 111. Will the sum also be divisible by 11 ? Explain.

Solution :

$\because$ When $abc + bca + cab$ is divided by 111, the quotient is $a + b + c$.

$\therefore$ When 821 + 218 + 182 is divided by 111, the quotient will be 8 + 2 + 1 = **11**

$\because$ When $abc + bca + cab$ is divided by $a + b + c$, the quotient is 111.

We have 8 + 2 + 1 = 11. $\therefore$ 821 + 218 + 182 will be divisible by 11.

EXERCISE 5(A)

1. Write the quotient when the sum of 73 and 37 is divided by :
   (i) 11 (ii) 10
2. Write the quotient when the sum of 94 and 49 is divided by :
   (i) 11 (ii) 13
3. Find the quotient when 73 – 37 is divided by : (i) 9 (ii) 4
4. If $a = b$, show that $abc = bac$.
5. Find the quotient when 94 – 49 is divided by :
   (i) 9 (ii) 5
6. Show that 527 + 752 + 275 is exactly divisible by 14.
7. if $a > c$; show that $abc - cba = 99(a - c)$.
8. If $c > a$; show that $cba - abc = 99(c - a)$.
9. If $a = c$, show that $cba - abc = 0$.

5.6 LETTERS FOR DIGITS (CRYPTARITHMS)

Cryptarithm is a type of mathematical game consisting of numbers, whose digits are represented by letters and we have to identify the value of each letter.

Example 3 :

Solve the following cryptarithms :

(i) $\begin{array}{r} 3\,1\,A \\ +\,1\,A\,3 \\ \hline 5\,0\,1 \\ \hline \end{array}$ (ii) $\begin{array}{r} B\,9 \\ +\,4\,A \\ \hline 6\,5 \\ \hline \end{array}$ (iii) $\begin{array}{r} 8\,A\,5 \\ +\,9\,4\,A \\ \hline 1\,A\,3\,3 \\ \hline \end{array}$

Solution :

(i) $\begin{array}{r} 3\,1\,A \\ +\,1\,A\,3 \\ \hline 5\,0\,1 \\ \hline \end{array}$

- First, we have to find the value of letter A.
- Clearly, A + 3 is a number whose ones digit is 1.

$\Rightarrow$ A + 3 = 1, A + 3 = 11, A + 3 = 21 and so on
$\Rightarrow$ A = 1 − 3, A = 11 − 3, A = 21 − 3 and so on
$\Rightarrow$ A = −2, A = 8, A = 18 and so on

Since, A is a digit $\therefore$ A = 8

- **A = 8** satisfies the addition in tens and hundreds columns.

And so the puzzle can be solved as shown below :

$\begin{array}{r} 3\,1\,A \\ +\,1\,A\,3 \\ \hline 5\,0\,1 \\ \hline \end{array} = \begin{array}{r} 3\,1\,8 \\ +\,1\,8\,3 \\ \hline 5\,0\,1 \\ \hline \end{array}$ **(Ans.)**

(ii) $\begin{array}{r} B\,9 \\ +\,4\,A \\ \hline 6\,5 \\ \hline \end{array}$

- Clearly, 9 + A is a number whose ones digit is 5.

$\Rightarrow$ 9 + A = 5, 9 + A = 15, 9 + A = 25 and so on
$\Rightarrow$ A = 5 − 9, A = 15 − 9, A = 25 − 9 and so on
$\Rightarrow$ A = −4, A = 6, A = 16 and so on

$\begin{array}{r} {}^{1} \\ B\,9 \\ +\,4\,6 \\ \hline 6\,5 \\ \hline \end{array}$

Since, A is a digit $\therefore$ A = 6

$\because$ 9 + A = 9 + 6 = 15 ; 5 will come at ones place and digit 1 is carried over

Now, 1 + B + 4 = 6 $\Rightarrow$ B = 1

$\therefore$ The puzzle can be solved as shown below :

$\begin{array}{r} B\,9 \\ +\,4\,A \\ \hline 6\,5 \\ \hline \end{array} = \begin{array}{r} 1\,9 \\ +\,4\,6 \\ \hline 6\,5 \\ \hline \end{array}$ **(Ans.)**

(iii) $\begin{array}{r} 8\,A\,5 \\ +\,9\,4\,A \\ \hline 1\,A\,3\,3 \\ \hline \end{array}$

- 5 + A must give a number whose ones digit is 3

$\Rightarrow$ 5 + A = 3, 5 + A = 13, 5 + A = 23, ...
$\Rightarrow$ A = 3 − 5, A = 13 − 5, A = 23 − 5, ...
$\Rightarrow$ A = −2, A = 8, A = 18, ...
$\Rightarrow$ A = 8, as A is a digit

Writing 8 in place of each A in the given cryptarithm, we get :

$\begin{array}{r} 8\,A\,5 \\ +\,9\,4\,A \\ \hline 1\,A\,3\,3 \\ \hline \end{array} = \begin{array}{r} 8\,8\,5 \\ +\,9\,4\,8 \\ \hline 1\,8\,3\,3 \\ \hline \end{array}$ **(Ans.)**

Example 4 :

Find A and B in the addition :

$$\begin{array}{r} A \\ +A \\ +A \\ \hline BA \end{array}$$

Solution :

- At the ones column, the sum of three As is a number whose ones digit is A.
- This happens only when A = 0 or A = 5
- A = 0 $\Rightarrow$ A + A + A = 0 + 0 + 0 = 0, which makes B = 0.
- When A = 5, the puzzle is solved as shown alongside, where A = 5 and B = 1.

$$\begin{array}{r} 5 \\ +5 \\ +5 \\ \hline 15 \end{array}$$

Example 5 :

Find the values of A, B and C

$$\begin{array}{r} 4\,6\,A \\ -C\,B\,9 \\ \hline 2\,7\,5 \end{array}$$

Solution :

Given : 4 6 A − C B 9 = 2 7 5

$\Rightarrow$ 4 6 A = 2 7 5 + C B 9

$$\therefore \quad \begin{array}{r} 4\,6\,A \\ -C\,B\,9 \\ \hline 2\,7\,5 \end{array} \quad = \quad \begin{array}{r} 2\,7\,5 \\ +C\,B\,9 \\ \hline 4\,6\,A \end{array}$$

$$\begin{array}{r} {}^{1}\quad \\ 2\,7\,5 \\ +C\,B\,9 \\ \hline 4\,6\,4 \end{array}$$

- **In ones column :** 5 + 9 = 14

  $\Rightarrow$ A = 4 and 1 is carried over

- **In tens column :** 1 + 7 + B = 6

  $\Rightarrow$ 8 + B is a number with unit digit 6

  $\therefore$ 8 + B = 6 or 16 or 26 or ....

  $\Rightarrow$ B = −2 or 8 or 18 or ....

  $\Rightarrow$ B = 8 as B is a digit,

  $\because$ 1 + 7 + B = 1 + 7 + 8 = 16

  $\therefore$ In the result, 6 is at tens digit and 1 is carried over.

$$\begin{array}{r} 2\,7\,5 \\ +C\,8\,9 \\ \hline 4\,6\,4 \end{array}$$

$$\begin{array}{r} {}^{1}\quad\quad \\ 2\,7\,5 \\ +C\,8\,9 \\ \hline 4\,6\,4 \end{array}$$

- **At hundreds place :** 1 + 2 + C = 4 $\Rightarrow$ C = 1

$\therefore$ **A = 4, B = 8** and **C = 1** **(Ans.)**

$$\text{Clearly,} \quad \begin{array}{r} 4\,6\,A \\ -C\,B\,9 \\ \hline 2\,7\,5 \end{array} \quad = \quad \begin{array}{r} 2\,7\,5 \\ +C\,B\,9 \\ \hline 4\,6\,A \end{array} \quad = \quad \begin{array}{r} 2\,7\,5 \\ +1\,8\,9 \\ \hline 4\,6\,4 \end{array}$$

Example 6 :

Find the value of digits A, B and C from :

$$\begin{array}{r} B\,A \\ \times\ 6 \\ \hline C\,8\,8 \end{array}$$

Solution :

$$\begin{array}{r} B\,A \\ \times\ 6 \\ \hline C\,8\,8 \end{array}$$

$\Rightarrow$ A = 3 or A = 8 as 6 × 3 = 1**8** and 6 × 8 = 4**8**

For A = 3, we have

$$\begin{array}{r} {}^{1}\quad \\ B\,3 \\ \times\ 6 \\ \hline C\,8\,8 \end{array}$$

$\Rightarrow$ 6 × B + 1 = C8

i.e. $6 \times B + 1 = 10 \times C + 8$ [For a two digit number *ab*, $ab = 10a + b$]

$\Rightarrow$ $6 \times B = 10 \times C + 7$

Since no values of digits B and C satisfy this equation, $A \neq 3$.

$\therefore$ For A = 8, we have

$$\begin{array}{rrr} & \overset{4}{B} & 8 \\ & \times & 6 \\ \hline C & 8 & 8 \end{array} \quad \Rightarrow \quad 6 \times B + 4 = C8$$

i.e. $6 \times B + 4 = 10 \times C + 8 \Rightarrow 6 \times B = 10 \times C + 4$

The digits which satisfy this equation are :

B = 4 or 9 as $6 \times 4 + 4 = 2\mathbf{8}$ and $6 \times 9 + 4 = 5\mathbf{8}$ as and C = 2 or 5.

$\therefore$ Possible solutions are **A = 8, B = 4 and C = 2** or **A = 8, B = 9 and C = 5** **(Ans.)**

EXERCISE 5(B)

1. $$\begin{array}{rr} 3 & A \\ + 2 & 5 \\ \hline B & 2 \end{array}$$

2. $$\begin{array}{rrr} & 9 & 8 \\ + & 4 & A \\ \hline C & B & 3 \end{array}$$

3. $$\begin{array}{rr} A & 1 \\ + 1 & B \\ \hline B & 0 \end{array}$$

4. $$\begin{array}{rrr} 2 & A & B \\ + A & B & 1 \\ \hline B & 1 & 8 \end{array}$$

5. $$\begin{array}{rrr} 1 & 2 & A \\ + 6 & A & B \\ \hline A & 0 & 9 \end{array}$$

6. $$\begin{array}{rr} 1 & A \\ \times & A \\ \hline 9 & A \end{array}$$

7. $$\begin{array}{rrr} & A & B \\ & \times & 6 \\ \hline B & B & B \end{array}$$

8. $$\begin{array}{rrr} & A & B \\ & \times & 3 \\ \hline C & A & B \end{array}$$

9. $$\begin{array}{rrr} & A & B \\ & \times & 5 \\ \hline C & A & B \end{array}$$

10. $$\begin{array}{rrrr} & 8 & A & 5 \\ + & 9 & 4 & A \\ \hline 1 & A & 3 & 3 \end{array}$$

11. $$\begin{array}{rrrr} 6 & A & B & 5 \\ + D & 5 & 8 & C \\ \hline 9 & 3 & 5 & 1 \end{array}$$

5.7 TESTS OF DIVISIBILITY

1. Divisibility by 10 :

A number is divisible by 10, if its unit digit is zero.

For example :

Each of 10, 30, 70, 120, 500, etc. is divisible by 10.

2. Divisibility by 5 :

A number is divisible by 5, if its unit digit is 0 or 5.

For example :

Each of 15, 40, 65, 90, 115, 7620, etc. is divisible by 5.

3. Divisibility by 2 :

A number is divisible by 2, if its unit digit is zero or an even number.

For example :

Each of 20, 24, 36, 50, 78, 112, 230, etc. is divisible by 2.

4. Divisibility by 9 :

A number is divisible by 9, if sum of its digits is divisible by 9.

(i) **Consider the number 45387**

Sum of its digits = $4 + 5 + 3 + 8 + 7 = 27$

$\because$ 27 is divisible by 9 $\Rightarrow$ The number **45387 is divisible by 9**.

(ii) **Consider the number 3518**

Sum of its digits = 3 + 5 + 1 + 8 = 17

$\because$ 17 is not divisible by 9 $\Rightarrow$ The number **3518 is not divisible by 9.**

(iii) **Consider the number 43242876**

Sum of its digits = 4 + 3 + 2 + 4 + 2 + 8 + 7 + 6 = 36

$\because$ 36 is divisible by 9 $\Rightarrow$ The number **43242876 is divisible by 9.**

5. Divisibility by 3 :

A number is divisible by 3, if sum of its digits is divisible by 3.

(i) **Consider the number 645**

Sum of its digits = 6 + 4 + 5 = 15

$\because$ 15 is divisible by 3 $\Rightarrow$ The number **645 is divisible by 3.**

(ii) **Consider the number 364028**

Sum of its digits = 3 + 6 + 4 + 0 + 2 + 8 = 23

$\because$ 23 is not divisible by 3. $\Rightarrow$ The number **364028 is not divisible by 3.**

6. Divisibility by 6 :

A number is divisible by 6, if it is divisible by 2 and 3 both

Consider the number 540

$\therefore$ The unit digit of the number is 0, which is divisible by 2

$\therefore$ Sum of its digits = 5 + 4 + 0 = 9, which is divisible by 3

$\Rightarrow$ 540 is divisible by 3

$\because$ 540 is divisible by 2 and 3 both $\Rightarrow$ **540 is divisible by 6.**

7. Divisibility by 11 :

A number is divisible by 11, if the difference between the sum of its digits in even places and the sum of its digit in odd places is either 0 or divisible by 11.

(i) **Consider the number 352**

Counting from the right hand side, the sum of its digits in odd places = 2 + 3 = 5

And, the sum of its digits in even places = 5

The difference between these two sums = 5 – 5 = 0 $\therefore$ **352 is divisible by 11**

(ii) **Consider the number 61809**

Counting from the right hand side, the sum of its digits in odd places = 9 + 8 + 6 = 23

And, sum of the digits in even places = 0 + 1 = 1

The difference between these two sums = 23 – 1 = 22

$\because$ 22 is divisible by 11 $\Rightarrow$ **61809 is divisible by 11**

8. Divisibility by 4 :

A number is divisible by 4, if the two digit number formed by its ten's digit and unit digit is divisible by 4.

(i) **Consider the number 3516**

$\because$ 16 is divisble by 4 $\Rightarrow$ **3516 is divisible by 4**

(ii) **Consider the number 628093**

$\because$ 93 is not divisible by 4 $\Rightarrow$ **628093 is not divisible by 4**

EXERCISE 5(C)

1. Find which of the following numbers are divisible by 2 :
   (i) 192 (ii) 1660
   (iii) 1101 (iv) 2079
2. Find which of the following numbers are divsible by 3 :
   (i) 261 (ii) 777
   (iii) 6657 (iv) 2574
3. Find which of the following numbers are divisible by 4 :
   (i) 360 (ii) 3180
   (iii) 5348 (iv) 7756
4. Find which of the following numbers are divisible by 5 :
   (i) 3250 (ii) 5557
   (iii) 39255 (iv) 8204
5. Find which of the following numbers are divisible by 10 :
   (i) 5100 (ii) 4612
   (iii) 3400 (iv) 8399
6. Find which of the following numbers are divisible by 11 :
   (i) 2563 (ii) 8307
   (iii) 95635

Example 7 :

If 42x is divisible by 9, find the value of digit x.

Solution :

$\because$ 42x is divisible by 9

$\Rightarrow$ $4 + 2 + x$ is divisible by 9

$\Rightarrow$ $6 + x$ is a multiple of 9 [number divisible by 9 is a multiple of 9]

$\Rightarrow$ $6 + x = 0$, or 9, or 18, ...

$\Rightarrow$ $x = -6$, or $x = 3$ or $x = 12$, ...

Since, x is a digit $\Rightarrow$ $x = 3$ **(Ans.)**

Example 8 :

If 5x21 is divisible by 9; find the value of digit x.

Solution :

$\because$ 5x21 is divisible by 9

$\Rightarrow$ $5 + x + 2 + 1$ is a multiple of 9

$\Rightarrow$ $8 + x = 0$, or 9, or 18, or 27, ...

$\Rightarrow$ $8 + x = 0$ or $8 + x = 9$ or $8 + x = 18$, ...

$\Rightarrow$ $x = -8$ or $x = 1$ or $x = 10$, ...

Since, x is a digit $\Rightarrow$ $x = 1$ **(Ans.)**

Example 9 :

If 24a5 is a multiple of 3 *i.e.* divisible by 3; find the value of digit a.

Solution :

$\because$ 24a5 is a multiple of 3

$\Rightarrow$ $2 + 4 + a + 5$ is a multiple of 3

$\Rightarrow$ $11 + a = 0, 3, 6, 9, 12, 15, 18, 21$, ...

$\Rightarrow$ $a = -11, -8, -5, -2, 1, 4, 7, 10$, ...

Since, a is a digit $\therefore$ $a = 1, 4$ or 7 **(Ans.)**

Example 10 :

If $3x72$ is divisible by 3, find the value of x.

Solution :

$\because$ $3x72$ is divisible by 3

$\Rightarrow$ $3 + x + 7 + 2$ is a multiple of 3

$\Rightarrow$ $12 + x$ is a multiple of 3

$\Rightarrow$ $12 + x = 0, 3, 6, 9, 12, 15, 18, 21, 24, ...$

$\Rightarrow$ $x = -12, -9, -6, -3, 0, 3, 6, 9, 12, ...$

Since, x is a digit $\therefore$ **$x = 0, 3, 6$ or 9** **(Ans.)**

Example 11 :

$21y8$ is a multiple of 6, find the value of digit y.

Solution :

$\because$ $21y8$ is a multiple of 6

$\Rightarrow$ $21y8$ is a multiple of 2 and 3 both

Now $21y8$ is a multiple of 3

$\Rightarrow$ $2 + 1 + y + 8$ is a multiple of 3

$\Rightarrow$ $11 + y$ is a multiple of 3

$\Rightarrow$ $11 + y = 0, 3, 6, 9, 12, 15, 18, 21, ...$

$\Rightarrow$ $y = -11, -8, -5, -2, 1, 4, 7, 10, ...$ ....(i)

$21y8$ is a multiple of 2 as it has even number 8 at its units place.

Since, y is a digit, it can have values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ....(ii)

Values of digit y common to equations (i) and (ii) are 1, 4 and 7

$\therefore$ **$y = 1, 4$ or 7** **(Ans.)**

Example 12 :

$13z4$ is divisible by 6, find the value of digit z.

Solution :

$13z4$ is divisible by 6, if it is divisible by 2 and 3 both.

$13z4$ is divisible by 2 as it has even number 4 at its units place.

$13z4$ is divisible by 3

$\Rightarrow$ $8 + z = 0, 3, 6, 9, 12, 15, 18, 21, ...$ $[1 + 3 + 4 = 8]$

$\Rightarrow$ $z = -8, -5, -2, 1, 4, 7, 10, 13, ...$ .....(i)

Since, z is a digit, z can have values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ....(ii)

Values of digit z common to equations (i) and (ii) are 1, 4 and 7

$\therefore$ **$z = 1, 4$ or 7** **(Ans.)**

Example 13 :

$2y5$ is divisible by 11, find the value of digit y.

Solution :

Sum of the digit in even place = y

and, sum of the digits in odd places = $2 + 5 = 7$

$\therefore$ Difference of the sum of the digits in even places and in odd places = $y - 7$

$2y5$ is a multiple of 11

$\Rightarrow$ $y - 7$ must be multiple of 11

$\Rightarrow$ $y - 7 = 0, 11, 22, ...$

$\Rightarrow$ $y = 7, 18, 29, ...$

$\therefore$ **$y = 7$** **(Ans.)**

Example 14 :

$67x19$ is a multiple of 11. Find all possible values of digit x.

Solution :

Sum of the digits in even places = $1 + 7 = 8$

and, sum of the digits in odd places = $9 + x + 6 = 15 + x$

Difference between the sum of the digit in odd places and the sum of the digits in even places = $15 + x - 8 = 7 + x$

$67x19$ is a multiple of 11

$\Rightarrow$ $7 + x$ is a multiple of 11

$\Rightarrow$ $7 + x = 0, 11, 22, 33, 44, ...$

$\Rightarrow$ $x = -7, 4, 15, 26, ...$

$\therefore$ **$x = 4$** **(Ans.)**

Example 15 :

Find the value of digit z, if $12z4$ is divisible by 4.

Solution :

$12z4$ is divisible by 4

$\Rightarrow$ $z4$ is divisible by 4

$\Rightarrow$ $10z + 4$ is divisible by 4 with unit digit 4

$\Rightarrow$ $10z + 4 = 4, 24, 44, 64, 84, 104, ...$

$\Rightarrow$ $10z = 0, 20, 40, 60, 80, 100, ...$

$\Rightarrow$ $z = 0, 2, 4, 6, 8, 10, ...$

$\therefore$ **$z = 0, 2, 4, 6$ and 8** **(Ans.)**

EXERCISE 5(D)

For what value of digit x, is :

1. $1x5$ divisible by 3 ?
2. $31x5$ divisible by 3 ?
3. $28x6$ a multiple of 3 ?
4. $24x$ divisible by 6 ?
5. $3x26$ a multiple of 6 ?
6. $42x8$ divisible by 4 ?
7. $9142x$ a multiple of 4 ?
8. $7x34$ divisible by 9 ?
9. $5x555$ a multiple of 9 ?
10. $3x2$ divisible by 11 ?
11. $5x2$ a multiple of 11 ?

SETS 6

6.1 REVIEW

Set	A set is a collection of well-defined objects.
1. The collection of tall students of your class is not well-defined, so it **does not form a set**. 2. The collection of students of your class **with heights between 135 cm and 160 cm** is well-defined, so it forms a set.	
Elements	The objects (numbers, names, etc.) used to form a set are called elements or members of the set.
In general, a set is represented by capital letters of English alphabet. The elements of the set are written inside curly braces and separated by commas. (i) If A is the set of names : John, Geeta, Amit and Rohit; then : set A = {John, Geeta, Amit, Rohit}. (ii) If V is the set of vowels of English alphabet then, set V = {a, e, i, o, u}.	
Using '∈' and ∉	The symbol '∈' stands for 'belongs to' and the symbol '∉' stands for 'does not belong to'.
If an element x belongs to set A, we write : $x \in A$ and if 'x' does not belong to set A, we write $x \notin A$.	

6.2 REPRESENTATION OF A SET

There are mainly *two* ways of representing a set.

(i) Roster or Tabular Form (ii) Rule Method or Set–Builder Form.

1. *Roster* (or Tabular) *Form*

In this form, the elements of the set are enclosed in curly braces { } after separating them by commas.

For example, if a set A consists of numbers 2, 5, 7, 9 and 15, it is written as :

A = {2, 5, 7, 9, 15}.

More examples :

(i) The set of integers *i.e.* Z = {......., –2, –1, 0, 1, 2, 3, ...}

(ii) The set of whole numbers *i.e.* W = {0, 1, 2, 3, 4, ...}

(iii) The set of natural numbers *i.e.* N = {1, 2, 3, 4, ...}

> 1. The order in which the elements of a set are written is not important. *i.e.* {a, b, c}, {b, a, c} and {c, b, a} represent the same set.
> 2. An element of a set is written only once.
> *i.e.* (i) {2, 3, 3, 2, 4, 2} = {2, 3, 4}
> (ii) The set of letters in the word ALLAHABAD = {a, l, h, b, d}

2. *Set-Builder* Form (Rule Method)

In this form, the actual elements of the set are not written, but a statement or a formula or a rule is written in the briefest possible way to represent the elements of the set.

e.g. Let A be the set of natural numbers less than 7, then in set-builder form it is written as :

$A = \{ x : x \in N \text{ and } x < 7 \}$

and is read as "A is the set of x such that x is a natural number and x is less than 7."

> The symbol ':' is read as *such that.*

More examples :

1. $A = \{2, 3, 4, 5\}$ [Roster or Tubular Form]

   $= \{x : x \in N, 2 \leq x < 6\}$ [Set-builder Form]

> For x representing the nautral numbers 2, 3, 4 and 5 ; we can also write $1 < x < 6$ or $2 \leq x \leq 5$ or $1 < x \leq 5$.

2. $C = \{1, 3, 5, 7, 9, 11\}$ [Roster Form]

   $= \{x : x = 2n - 1, n \in N \text{ and } n \leq 6\}$

   or $\{x : x = 2n + 1, n \in W \text{ and } n \leq 5\}$ [Set-builder Form]

3. $D = \{x : x = 2n, n \in N \text{ and } n \leq 4\}$ [Set-builder Form]

   $= \{2 \times 1, 2 \times 2, 2 \times 3, 2 \times 4\}$

   $= \{2, 4, 6, 8\}$ [Roster Form]

Example 1 :

Write the following sets in tabular form :

(i) $\{x : x = \frac{2n}{n+2}, n \in W \text{ and } n < 3\}$ (ii) $\{x : x = 5y - 3, y \in Z \text{ and } -2 \leq y < 2\}$

(iii) $\{x : x \in W \text{ and } 8x + 5 < 23\}$

Solution :

(i) Since, $n \in W$ and $n < 3 \Rightarrow n = 0, 1$ and 2

$\therefore$ **Set in tabular form** $= \{\frac{2 \times 0}{0+2}, \frac{2 \times 1}{1+2}, \frac{2 \times 2}{2+2}\} = \{0, \frac{2}{3}, 1\}$ **(Ans.)**

(ii) $y \in Z$ and $-2 \leq y < 2 \Rightarrow y = -2, -1, 0$ and 1

$\therefore$ **Set in tabular form** $= \{5 \times -2 - 3, 5 \times -1 - 3, 5 \times 0 - 3, 5 \times 1 - 3\}$

$= \{-13, -8, -3, 2\}$ **(Ans.)**

(iii) $8x + 5 < 23 \Rightarrow 8x < 18$ and $x < 2{\cdot}25$

$\because x \in W$ and $x < 2{\cdot}25 \Rightarrow x = 0, 1$ and 2

$\therefore$ **Set in tabular form** $= \{0, 1, 2\}$ **(Ans.)**

Example 2 :

Express the following sets in set-builder form :

(i) $\{\frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{10}{11}, \frac{11}{12}\}$ (ii) $\{0, 3, 6, 9, 12, 15, 18\}$

(iii) $\{\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}\}$

(iv) {7, –1}

(v) {Natural numbers between –8 and 15}

(vi) {Natural numbers from –8 to 15}

Solution :

(i) $\{\frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{10}{11}, \frac{11}{12}\} = \{\frac{7}{7+1}, \frac{8}{8+1}, \frac{9}{9+1}, \frac{10}{10+1}, \frac{11}{11+1}\}$

$= \{x : x = \frac{n}{n+1}, n \in N \text{ and } 7 \le n \le 11\}$ **(Ans.)**

(ii) $\{0, 3, 6, 9, 12, 15, 18\} = \{3 \times 0, 3 \times 1, 3 \times 2, 3 \times 3, 3 \times 4, 3 \times 5, 3 \times 6\}$

$= \{x : x = 3n, n \in W \text{ and } x \le 6\}$ **(Ans.)**

(iii) $\{\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}\} = \{\frac{1}{3^1}, \frac{1}{3^2}, \frac{1}{3^3}, \frac{1}{3^4}, \frac{1}{3^5}\}$

$= \{x : \frac{1}{3^n}, n \in N \text{ and } n \le 5\}$ **(Ans.)**

(iv) $\{7, -1\} = \{x = 7 \text{ or } x = -1\}$

(v) $\{x : x \in N, -8 < x < 15\}$

(vi) $\{x : x \in N, -8 \le x \le 15\}$

EXERCISE 6(A)

1. Write the following sets in roster (Tabular) form :
   (i) $A_1 = \{x : 2x + 3 = 11\}$
   (ii) $A_2 = \{x : x^2 - 4x - 5 = 0\}$
   (iii) $A_3 = \{x : x \in Z, -3 \le x < 4\}$
   (iv) A_4 = {x : x is a two digit number and sum of the digits of x is 7}
   (v) $A_5 = \{x : x = 4n, n \in W \text{ and } n < 4\}$
   (vi) $A_6 = \{x : x = \frac{n}{n+2}; n \in N \text{ and } n > 5\}$
2. Write the following sets in set-builder (Rule Method) form :
   (i) $B_1 = \{6, 9, 12, 15, ...\}$
   (ii) $B_2 = \{11, 13, 17, 19\}$
   (iii) $B_3 = \left\{\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{9}{11}, ...\right\}$
   (iv) $B_4 = \{8, 27, 64, 125, 216\}$
   (v) $B_5 = \{-5, -4, -3, -2, -1\}$
   (vi) $B_6 = \{....., -6, -3, 0, 3, 6, ...\}$
3. (i) Is {1, 2, 4, 16, 64} = {x : x is a factor of 32} ? Give reason.
   (ii) Is {x : x is a factor of 27} ≠ {3, 9, 27, 54} ? Give reason.
   (iii) Write the set of even factors of 124.
   (iv) Write the set of odd factors of 72.
   (v) Write the set of prime factors of 3234.
   (vi) Is $\{x : x^2 - 7x + 12 = 0\} = \{3, 4\}$?
   (vii) Is $\{x : x^2 - 5x - 6 = 0\} = \{2, 3\}$?
4. Write the following sets in Roster form :
   (i) The set of letters in the word 'MEERUT'.
   (ii) The set of letters in the word 'UNIVERSAL'.
   (iii) $A = \{x : x = y + 3, y \in N \text{ and } y > 3\}$.
   (iv) $B = \{p : p \in W \text{ and } p^2 < 20\}$.
   (v) C = {x : x is a composite number and $5 \le x \le 21$}
5. List the elements of the following sets :
   (i) $\{x : x^2 - 2x - 3 = 0\}$
   (ii) $\{x : x = 2y + 5 ; y \in N \text{ and } 2 \le y < 6\}$
   (iii) {x : x is a factor of 24}
   (iv) $\{x : x \in Z \text{ and } x^2 \le 4\}$
   (v) $\{x : 3x - 2 \le 10 \text{ and } x \in N\}$
   (vi) $\{x : 4 - 2x > -6, x \in Z\}$

6.3 CARDINAL NUMBER OF A SET

The number of distinct elements in a finite set is called its **cardinal number.**

e.g. (i) If A = {c, d, f }, then the cardinal number of set A is 3 and we write n(A) = 3.

(ii) If B = {2, 4, 5, 6}, then n(B) = 4 and so on.

6.4 TYPES OF SETS

1. Finite Set :

A set with finite (limited) number of elements in it, is called a **finite set.**

e.g. (i) Set of boys in your class.

(ii) {x : x is a member of a particular family}.

(iii) {3, 4, 5, ..., 100} and so on.

2. Infinite Set :

A set, which is not finite, is called an **infinite set.** That is, a set with never ending number of elements is an infinite set.

e.g. (i) {x : x is a living thing}.

(ii) {x : x ∈ W and x > 1000} and so on.

3. Singleton or Unit Set :

A set, which has only one element in it, is called a **singleton** or **unit set.**

e.g. (i) {x : x is President of India}

(ii) Set of whole numbers between 6 and 8.

(iii) {x : 2x – 1 = 3} and so on.

4. Empty or Null Set :

The set, which has no element in it, is called the **empty** or the **null set.**

The empty set is denoted by the Danish letter '∅' and is pronounced as 'oe'.

∴ **Empty set** = { } = ∅.

Examples :

(i) The set of all odd numbers between 7 and 9.

(ii) {x : x ∈ N and x < 1} and so on.

1. The cardinal number of the empty set is 0, i.e. n(∅) = 0.
2. The set {0} is not an empty set, since it contains zero as its element.
3. The set {∅} is also not empty, since it contains ∅ as its element.

5. Joint or overlapping Sets :

Two sets are said to be **joint** or **overlapping sets,** if they have atleast one element in common.

e.g. Set A = {5, 7, 9, 11} and set B = {6, 9, 12, 15} are joint (overlapping) sets as number 9 is common to both the sets.

6. Disjoint Sets :

Two sets are said to be **disjoint,** if they have no element in common.

e.g. (i) Set A = {5, 7, 9, 11} and set B = {4, 6, 8, 10} are disjoint, as they have no element in common.

(ii) If A = {x : x is a student of Sophia Girls' School} and B = {x : x is a student of St. Mary's Academy}, then clearly sets A and B are disjoint.

7. Equivalent Sets :

Two sets are said to be **equivalent,** if they contain the same number of elements.

e.g. Set A = {a, b, c} and set B = {x, y, z} are equivalent as n(A) = n(B) and we write $A \leftrightarrow B$.

8. Equal Sets :

Two sets are **equal,** if both the sets have same (identical) elements.

e.g. If A = {1, 2, 3, 4, 5} and B = {x : x ∈ N and x < 6} ; then clearly A = B.

1. In equivalent sets, the number of elements are equal, whereas in equal sets, the elements are the same.
2. Equal sets are always equivalent, whereas the equivalent sets are not necessarily equal.

EXERCISE 6(B)

1. Find the cardinal number of the following sets :
   (i) $A_1 = \{-2, -1, 1, 3, 5\}$
   (ii) $A_2 = \{x : x \in N \text{ and } 3 \le x < 7\}$
   (iii) $A_3 = \{p : p \in W \text{ and } 2p - 3 < 8\}$
   (iv) $A_4 = \{b : b \in Z \text{ and } -7 < 3b - 1 \le 2\}$
2. If P = {p : p is a letter in the word 'PERMANENT'}, find n(P).
3. State, which of the following sets are finite and which are infinite :
   (i) A = {x : x ∈ Z and x < 10}
   (ii) B = {x : x ∈ W and 5x – 3 ≤ 20}
   (iii) P = {y : y = 3x – 2, x ∈ N and x > 5}
   (iv) $M = \{r : r = \frac{3}{n} ; n \in W \text{ and } 6 < n \le 15\}$
4. Find, which of the following sets are singleton sets :
   (i) The set of points of intersection of two non-parallel straight lines on the same plane.
   (ii) A = {x : 7x – 3 = 11}
   (iii) B = {y : 2y + 1 < 3 and y ∈ W}
5. Find, which of the following sets are empty :
   (i) The set of points of intersection of two parallel lines.
   (ii) A = {x : x ∈ N and 5 < x ≤ 6}.
   (iii) $B = \{x : x^2 + 4 = 0 \text{ and } x \in N\}$.
   (iv) C = {even numbers between 6 and 10}.
   (v) D = {prime numbers between 7 and 11}.
6. (i) Are the sets A = {4, 5, 6} and $B = \{x : x^2 - 5x - 6 = 0\}$ disjoint ?
   (ii) Are the sets A = {b, c, d, e} and B = {x : x is a letter in the word 'MASTER'} joint ?
7. State, whether the following pairs of sets are equivalent or not :
   (i) A = {x : x ∈ N and 11 ≥ 2x – 1} and B = {y : y ∈ W and 3 ≤ y ≤ 9}.
   (ii) Set of integers and set of natural numbers.
   (iii) Set of whole numbers and set of multiples of 3.
   (iv) P = {5, 6, 7, 8} and M = {x : x ∈ W and x ≤ 4}.
8. State, whether the following pairs of sets are equal or not :
   (i) A = {2, 4, 6, 8} and B = {2n : n ∈ N and n < 5}
   (ii) M = {x : x ∈ W and x + 3 < 8} and N = {y : y = 2n – 1, n ∈ N and n < 5}
   (iii) $E = \{x : x^2 + 8x - 9 = 0\}$ and F = {1, –9}
   (iv) A = {x : x ∈ N, x < 3} and $B = \{y : y^2 - 3y + 2 = 0\}$
9. State whether each of the following sets is a finite set or an infinite set :
   (i) The set of multiples of 8.
   (ii) The set of integers less that 10.
   (iii) The set of whole numbers less than 12.
   (iv) {x : x = 3n – 2, n ∈ W, n ≤ 8}

(v) $\{x : x = 3n - 2, n \in Z, n \leq 8\}$

(vi) $\{x : x = \frac{n-2}{n+1}, n \in W\}$

10. Answer, whether the following statements are **true** or **false**. Give reasons.

(i) The set of even natural numbers less than 21 and the set of odd natural numbers less than 21 are equivalent sets.

(ii) If E = {factors of 16} and F = {factors of 20}, then E = F.

(iii) The set A = {integers less than 20} is a finite set.

(iv) If A = {x : x is an even prime number}, then set A is empty.

(v) The set of odd prime numbers is the empty set.

(vi) The set of squares of integers and the set of whole numbers are equal sets.

(vii) If n(P) = n(M), then $P \leftrightarrow M$.

(viii) If set P = set M, then n(P) = n(M).

(ix) $n(A) = n(B) \Rightarrow A = B$.

6.5 SUBSET

If all the elements of the set A belong to the set B, the set A is said to be the subset of the set B.

And, if all the elements of the set B belong to the set A, the set B is said to be subset of the set A.

e.g. (i) If A = {5, 6, 7} and B = {2, 3, 4, 5, 6, 7}; clearly all the elements of the set A belong to the set B; therefore the set A is subset of the set B and we write : $A \subseteq B$.

(ii) If A = {x : x is a student of your school} and B = {x : x is a student of class VIII of your school}; then all the elements (students) of the set B belong to the set A; therefore B is subset of A and we write : $B \subseteq A$.

1. The notation '$A \subseteq B$' is read as : **A is subset of B** or **A is contained in B**.
2. '$B \subseteq A$' is read as : **B is subset of A** or **B is contained in A**.

More examples :

1. If $P = \{x : x \in N$ and x is divisible by 2$\}$ and $Q = \{x : x \in N$ and x is divisible by 4$\}$, then $Q \subseteq P$.

Reason : P = {2, 4, 6, 8, 10, 12, 14, ...} and Q = {4, 8, 12, 16, ...} Clearly, all the elements of the set Q belong to the set P; therefore $Q \subseteq P$.

2. $N \subseteq W$; since every natural number is also a whole number.
For the same reason : $N \subseteq Z$ (integers) and $W \subseteq Z$.

(i) Every set is a subset of itself, *i.e.* $A \subseteq A$, $B \subseteq B$, $E \subseteq E$ and so on.

(ii) Empty set $\varnothing$ is a subset of every set, *i.e.* $\varnothing \subseteq A$, $\varnothing \subseteq B$, $\varnothing \subseteq P$ and so on.

(iii) If $A \subseteq B$ and $B \subseteq A$, then A = B. Conversely, if A = B, then $A \subseteq B$ and $B \subseteq A$.

6.6 PROPER SUBSET

The set A is said to be a proper subset of set B if,

(i) all elements of set A are contained in set B and

(ii) there exists at least one element in B which is not in A.

Symbolically, we write it as $\mathbf{A \subset B}$ and read as '**A is proper subset of B**'.

e.g. (i) If A = {5, 6, 7} and B = {3, 5, 6, 7, 8}, then $A \subset B$.

(ii) The set of natural numbers (N) is a proper subset of the set of whole numbers (W) *i.e.* $N \subset W$.

6.7 NUMBER OF SUBSETS AND PROPER SUBSETS OF A GIVEN SET

If a set has n elements;

(i) the number of subsets of it = 2^n and

(ii) the number of proper subsets of it = $2^n - 1$.

	∅	{a}	{a, b}	{a, b, c}
1. Number of elements	0	1	2	3
2. No. of subsets	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$
3. No. of proper subsets :	$2^0 - 1 = 0$	$2^1 - 1 = 1$	$2^2 - 1 = 3$	$2^3 - 1 = 7$
4. Subsets :	∅	∅ and {a}	∅, {a}, {b} and {a, b}	∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c} and {a, b, c}
5. Proper subsets	Nil	∅	∅, {a} and {b}	∅, {a}, {b}, {c}, {a, b} {a, c} and {b, c}.

No set is proper subset of itself.

6.8 SUPER SET

If set A is a subset of set B, then B is called the **super set** of A and we write it as $\mathbf{B \supseteq A}$, which is read as **'B *is super set of* A'.**

6.9 UNIVERSAL SET

It is the set which contains all the sets under consideration as its subsets. A universal set is denoted by ξ (pxi) or U.

e.g. If A = {5, 6, 7, 8}, B = {1, 3, 5, 7} and C = {4, 6, 8, 10}, then the universal set for these sets may be taken as :

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

For the given sets, the choice of a universal set is not unique.

Sets under consideration	Universal Set
1. (i) A = {2, 4, 6, 7} (ii) B = {3, 5, 7, 9, 11} (iii) C = {0, 4, 8, 12, 16}	$\{x : x \in W \text{ and } x \le 16\}$ or, $\{x : x \in W\}$ or, $\{x : x \in Z\}$ or, $\{x : x \in Z, 0 \le x < 17\}$, etc.
2. (i) D = {Students of class 9 of your school} (ii) {Basketball players in your school} (iii) {Students of primary section of your school} (iv) {Students of secondary section of your school}	{Students of your school} or, {Students of your town} or, {x : x is a student}, etc.

Example 3 :

Given the universal set, $\xi = \{x : x \in N, 15 < x \leq 26\}$, list the elements of the following sets :

(i) $A = \{x : x > 20\}$ (ii) $B = \{x : x \leq 21\}$

Solution :

Since, the universal set, $\xi = \{16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26\}$,

$\therefore$ (i) $A = \{x : x > 20\}$ [Take the elements of ξ which are greater than 20]

$= \{21, 22, 23, 24, 25, 26\}$ **(Ans.)**

And, (ii) $B = \{x : x \leq 21\}$

$= \{16, 17, 18, 19, 20, 21\}$ **(Ans.)**

6.10 COMPLEMENT OF A SET

The complement of a set A is the set of all elements in the universal set which are not in set A. It is denoted by A′ and is read as 'complement of A' or 'A-dashed'.

e.g. (i) If universal set = {1, 2, 3, 4, 5, 6} and set A = {2, 3, 5}, then
Complement of set A = A′ = {1, 4, 6}

(ii) If universal set = $\{x : x \in N\}$ and A = {even natural numbers},
then A′ = {odd natural numbers}.

1. A set and its complement are disjoint, *i.e.* they do not have any common element. (In the example given above, the set A and its complement A′ are disjoint).
2. The complement of the empty set is universal set, i.e. $\varnothing' = \xi$.
3. The complement of a universal set is the empty set, i.e. $\xi' = \varnothing$.

EXERCISE 6(C)

1. Find all the subsets of each of the following sets :
   (i) A = {5, 7} (ii) B = {a, b, c}
   (iii) $C = \{x : x \in W, x \leq 2\}$
   (iv) {p : p is a letter in the word 'poor'}
2. If C is the set of letters in the word 'cooler', find:
   (i) set C (ii) n(C)
   (iii) number of its subsets
   (iv) number of its proper subsets
3. If T = {x : x is a letter in the word 'TEETH'}, find all its subsets.
4. Given the universal set = {–7, –3, –1, 0, 5, 6, 8, 9}, find :
   (i) $A = \{x : x < 2\}$ (ii) $B = \{x : -4 < x < 6\}$
5. Given the universal set = $\{x : x \in N$ and $x < 20\}$, find :
   (i) $A = \{x : x = 3p ; p \in N\}$
   (ii) $B = \{y : y = 2n + 3, n \in N\}$
   (iii) C = {x : x is divisible by 4}
6. Find the proper subsets of $\{x : x^2 - 9x - 10 = 0\}$.
7. Given, A = {Triangles}, B = {Isosceles triangles}, C = {Equilateral triangles}. State whether the following are **true** or **false**. Give reasons.
   (i) $A \subset B$ (ii) $B \subseteq A$
   (iii) $C \subseteq B$ (iv) $B \subset A$
   (v) $C \subset A$ (vi) $C \subseteq B \subseteq A$.
8. Given, A = {Quadrilaterals}, B = {Rectangles}, C = {Squares}, D = {Rhombuses}. State, giving reasons, whether the following are **true** or **false**.
   (i) $B \subset C$ (ii) $D \subset B$
   (iii) $C \subseteq B \subseteq A$ (iv) $D \subset A$
   (v) $B \supseteq C$ (vi) $A \supseteq B \supseteq D$
9. Given, universal set = $\{x : x \in N, 10 \leq x \leq 35\}$, $A = \{x \in N : x \leq 16\}$ and $B = \{x : x > 29\}$. Find:
   (i) A′ (ii) B′

10. Given, universal set = $\{x \in Z : -6 < x \leq 6\}$, N = {n : n is a non-negative number} and P = {x : x is a non-positive number}. Find :
    (i) N' (ii) P'
11. Let M = {letters of the word REAL} and N = {letters of the word LARE}. Write sets M and N in roster form and then state whether;
    (i) $M \subseteq N$ is true.
    (ii) $N \subseteq M$ is true.
    (iii) M = C is true.

6.11 SET OPERATIONS

1. Union of two sets

The union of two sets A and B consists of all the elements which belong either to set A or to set B or to both the sets. It is denoted by $A \cup B$ and is read as 'A *union* B' or 'A *cup* B'.

e.g. (i) If A = {5, 6, 7} and B = {6, 8}, then $A \cup B$ = {5, 6, 7, 8}.

(ii) If E = {numbers divisible by 2} and F = {numbers divisible by 3}, then $A \cup B$ = {numbers divisible by either 2 or by 3 or by both}.

1. Union of two sets is commutative *i.e.* $A \cup B = B \cup A$; $E \cup F = F \cup E$ and so on.
2. Union of sets is associative *i.e.* for any three sets A, B and C. $A \cup (B \cup C) = (A \cup B) \cup C$.
3. Since each element of set A is contained in $A \cup B$, therefore, $A \subset (A \cup B)$.
4. If $A \subset B$, then $A \cup B = B$.
5. The union of a set and its complement is the universal set, *i.e.* $A \cup A' = \xi$, $B \cup B' = \xi$ and so on.
6. The union of a set and the empty set is the set itself, *i.e.* $A \cup \varnothing = A$, $B \cup \varnothing = B$ and so on. Similarly, the union of a set and the universal set is the universal set, *i.e.* $A \cup \xi = \xi$, $B \cup \xi = \xi$, etc.

2. Intersection of two sets

The intersection of two sets A and B is the set of *elements which are common* to both the sets A and B. It is denoted by $A \cap B$ and is read as 'A *intersection* B' or 'A *cap* B'.

e.g. (i) If A = {5, 6, 7} and B = {6, 8], then $A \cap B$ = {6}.

(ii) If E = {numbers divisible by 2} and F = {numbers divisible by 3}. then $E \cap F$ = {numbers divisible by 2 and 3 both} = {numbers divisible by 6}.

1. Intersection of two sets is commutative, *i.e.* $A \cap B = B \cap A$; $M \cap N = N \cap M$, etc.
2. Intersection of sets is associative, *i.e.* $A \cap (B \cap C) = (A \cap B) \cap C$
3. Since all the elements of set $A \cap B$ are contained in set A, therefore $A \cap B$ is a subset of A *i.e.* $(A \cap B) \subset A$. Similarly, $(A \cap B) \subset B$.
4. If A and B are two disjoint sets, then $A \cap B = \varnothing$. Conversely, if $A \cap B = \varnothing$, the sets A and B are disjoint.
5. If $A \cap B \neq \varnothing$, the sets A and B are joint or overlapping sets.
6. $A \cap A' = \varnothing$.
7. The intersection of a set with the empty set is the empty set. *i.e.* $A \cap \varnothing = \varnothing$, $B \cap \varnothing = \varnothing$, etc.
8. The intersection of a set with the universal set is the set itself, *i.e.* $A \cap \xi = A$, $B \cap \xi = B$ and so on.

Example 4 :

If A = {factors of 24} and B = {factors of 36} ; find : (i) $A \cap B$, (ii) $A \cup B$.

Solution :

Given : A = {factors of 24} = {1, 2, 3. 4, 6, 8, 12, 24}

and, B = {factors of 36} = {1, 2, 3, 4, 6, 9, 12, 18, 36}

∴ (i) **$A \cap B$** = {elements common in both A and B}

= **{1, 2, 3, 4, 6, 12}** **(Ans.)**

and, (ii) **$A \cup B$** = {elements which belong either to A or B or both}

= **{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36}** **(Ans.)**

6.12 DIFFERENCE OF TWO SETS

If A and B are two given sets, then their difference A – B is the set of those elements which belong to set A but not to set B.

i.e. $A - B = \{x : x \in A \text{ and } x \notin B\}$

e.g. Let A = {b, c, d, e} and B = {a, b, c}, then A – B = {d, e} and B – A = {a}.

6.13 DISTRIBUTIVE LAWS

1. *The union is distributive over the intersection of two sets.*

   i.e. if A, B and C are three sets then, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   e.g. Let A = {2, 5, 8, 11}, B = {2, 4, 6, 8} and C = {5, 6, 7, 8} then $B \cap C$ = {6, 8}

   and $A \cup (B \cap C)$ = {2, 5, 8, 11} ∪ {6, 8} = {2, 5, 6, 8, 11} I

   Similarly, $(A \cup B) \cap (A \cup C)$ = {2, 4, 5, 6, 8, 11} ∩ {2, 5, 6, 7, 8, 11}

   = {2, 5, 6, 8, 11} II

   From I and II, it is verified that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2. *The intersection is distributive over the union of two sets.*

   i.e. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   It can be verified in the same way, as described above in law 1.

EXERCISE 6(D)

1. Given $A = \{x : x \in N \text{ and } 3 < x \leq 6\}$ and $B = \{x : x \in W \text{ and } x < 4\}$. Find :
   (i) sets A and B in roster form
   (ii) $A \cup B$ (iii) $A \cap B$
   (iv) A – B (v) B – A

2. If $P = \{x : x \in W \text{ and } 4 \leq x \leq 8\}$ and $Q = \{x : x \in N \text{ and } x < 6\}$. Find :
   (i) $P \cup Q$ and $P \cap Q$.
   (ii) Is $(P \cup Q) \supset (P \cap Q)$?

3. If A = {5, 6, 7, 8, 9}, $B = \{x : 3 < x < 8 \text{ and } x \in W\}$ and $C = \{x : x \leq 5 \text{ and } x \in N\}$. Find :
   (i) $A \cup B$ and $(A \cup B) \cup C$
   (ii) $B \cup C$ and $A \cup (B \cup C)$
   (iii) $A \cap B$ and $(A \cap B) \cap C$
   (iv) $B \cap C$ and $A \cap (B \cap C)$
   Is $(A \cup B) \cup C = A \cup (B \cup C)$?
   Is $(A \cap B) \cap C = A \cap (B \cap C)$?

4. Given A = {0, 1, 2, 4, 5}, B = {0, 2, 4, 6, 8} and C = {0, 3, 6, 9}. Show that :
   (i) $A \cup (B \cup C) = (A \cup B) \cup C$ *i.e.* the union of sets is associative.
   (ii) $A \cap (B \cap C) = (A \cap B) \cap C$ *i.e.* the intersection of sets is associative.

5. If $A = \{x \in W : 5 < x < 10\}$, $B = \{3, 4, 5, 6, 7\}$ and $C = \{x = 2n ; n \in N \text{ and } n \leq 4\}$. Find :

(i) $A \cap (B \cup C)$ (ii) $(B \cup A) \cap (B \cup C)$

(iii) $B \cup (A \cap C)$ (iv) $(A \cap B) \cup (A \cap C)$

Name the sets which are equal.

6. If P = {factors of 36} and Q = {factors of 48} ; find :

(i) $P \cup Q$ (ii) $P \cap Q$

(iii) $Q - P$ (iv) $P' \cap Q$

7. If $A = \{6, 7, 8, 9\}$, $B = \{4, 6, 8, 10\}$ and $C = \{x : x \in N : 2 < x \leq 7\}$; find :

(i) $A - B$ (ii) $B - C$

(iii) $B - (A - C)$ (iv) $A - (B \cup C)$

(v) $B - (A \cap C)$ (vi) $B - B$

8. If $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$; verify :

(i) $A - (B \cup C) = (A - B) \cap (A - C)$

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$.

9. Given $A = \{x \in N : x < 6\}$, $B = \{3, 6, 9\}$ and $C = \{x \in N : 2x - 5 \leq 8\}$. Show that :

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6.14 VENN-DIAGRAM

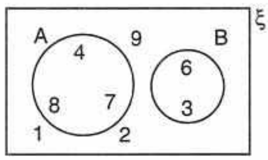
Venn-diagram is the most commonly used pictorial representation of sets. This idea was first developed by John Venn, an English mathematician, that is why the figures (geometrical figures) used in this type of representation are called *Venn-diagrams.*

In fact, in a Venn-diagram, a closed curve (figure) represents a set and the interior points within this closed curve represent the elements of the set.

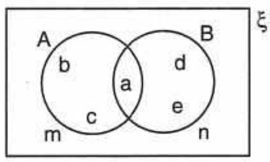
In a Venn-diagram, the universal set is represented by a rectangle and all other sets under consideration by circles or ovals within the rectangle.

6.15 USEING VENN-DIAGRAMS TO SHOW THE RELATIONSHIP BETWEEN THE SETS

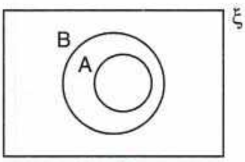
1. The diagram shows **two disjoint sets A and B.**



2. The diagram shows **A and B are joint or overlapping sets.**

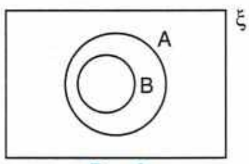


3. (i)



$A \subset B$

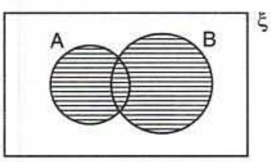
(ii)



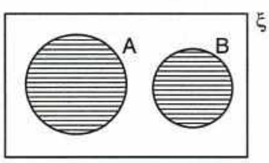
$B \subset A$

4. The **shaded portions** in the following figures show $A \cup B$.

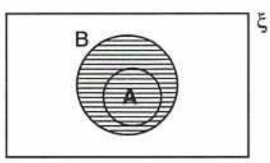
(i)



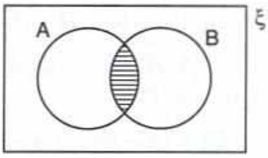
(ii)

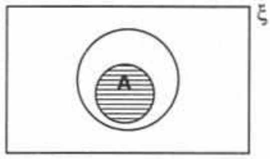


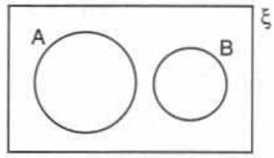
(iii)



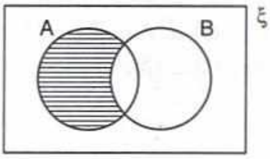
5. The **shaded portions** in the following figures show **A ∩ B**.

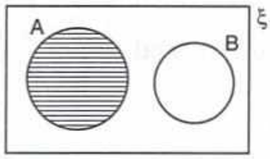
(i)


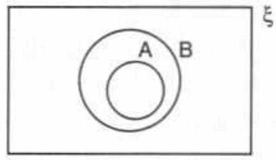
(ii)


(iii)


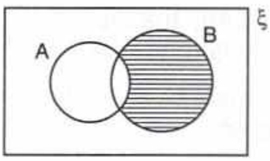
6. The **shaded portions** in the following figures show **A – B**.

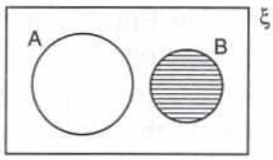
(i)


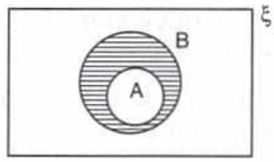
(ii)


(iii)


7. The **shaded portions** in the following figures show **B – A**.

(i)


(ii)


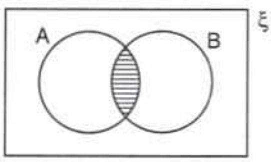
(iii)


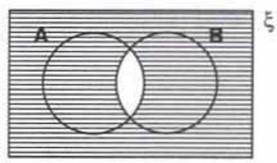
Example 5 :

For two overlapping sets A and B, draw Venn-diagrams to represent the following sets :

(i) (A ∩ B)′ (ii) (A ∪ B)′ (iii) A′ ∩ B

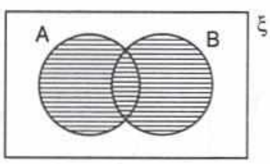
Solution :

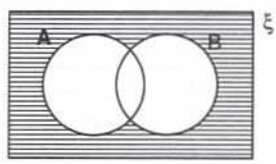
(i) Since, A ∩ B =


∴ **(A ∩ B)′** =


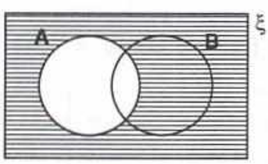
(Ans.)

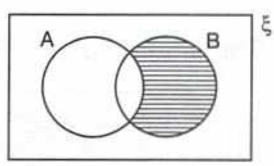
[The region of universal set, which is not in A ∩ B]

(ii) Since, A ∪ B =


∴ **(A ∪ B)′** =


(Ans.)

(iii) Since, A′ =


∴ **A′ ∩ B** =


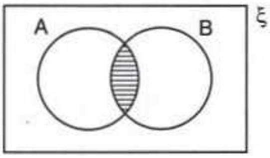
(Ans.)

Example 6 :

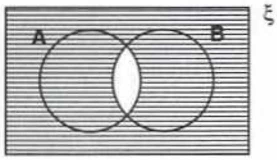
Use Venn-diagrams to prove that : $(A \cap B)' = A' \cup B'$.

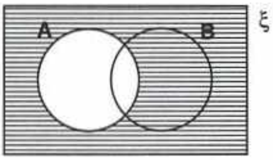
Solution :

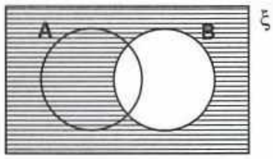
Consider A and B as two overlapping sets.

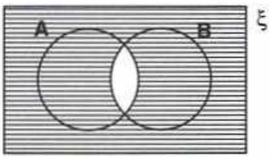
Since, $A \cap B =$ 

....I

$\therefore$ **$(A \cap B)' =$** 

Since, $A' =$ 

and $B' =$ 

$\therefore$ **$A' \cup B' =$** 

.... II

Since the shaded portions in diagrams I and II represent the same region,

$\therefore$ **$(A \cap B)' = A' \cup B'$** **Hence proved.**

Example 7 :

Given : $\xi = \{x : x \in N, 12 \leq x < 20\}$,

$A = \{x : x \text{ is divisible by } 3\}$ and $B = \{12, 14, 15, 16\}$.

Draw a Venn-diagram to show the relationship between the given sets.

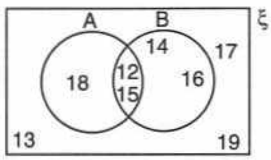
Solution :

Given : $\xi = \{12, 13, 14, 15, 16, 17, 18, 19\}$,

$A = \{12, 15, 18\}$ and $B = \{12, 14, 15, 16\}$.

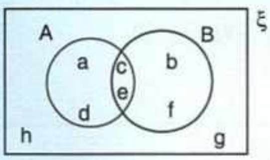
Since the sets A and B are overlapping sets,

therefore, the Venn-diagram should be as drawn alongside.

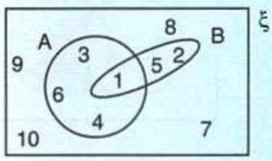


EXERCISE 6(E)

1. From the given diagram find :
   (i) $A \cup B$
   (ii) $A' \cap B$
   (iii) $A - B$
   (iv) $B - A$
   (v) $(A \cup B)'$



2. From the given diagram, find :
   (i) A'
   (ii) B'



   (iii) $A' \cup B'$
   (iv) $(A \cap B)'$

   Is $A' \cup B' = (A \cap B)'$?

3. Use the given diagram to find :

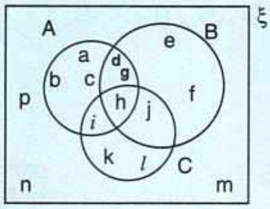
(i) A ∪ (B ∩ C)

(ii) B – (A – C)

(iii) A – B

(iv) A ∩ B′

Is A ∩ B′ = A – B ?



4. Use the given Venn-diagram to find :

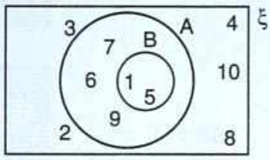
(i) B – A

(ii) A

(iii) B′

(iv) A ∩ B

(v) A ∪ B



5. Draw a Venn-diagram to show the relationship between two overlapping sets A and B. Now shade the region representing :

(i) A ∩ B (ii) A ∪ B (iii) B – A

6. Draw a Venn-diagram to show the relationship between sets A and B ; such that A ⊆ B. Now shade the region representing :

(i) A ∪ B (ii) B′ ∩ A (iii) A ∩ B

(iv) (A ∪ B)′

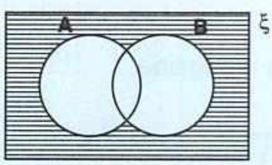
7. Two sets A and B are such that A ∩ B = ∅. Draw a Venn-diagram to show the relationship between A and B. Shade the region representing :

(i) A ∪ B (ii) (A ∪ B)′ (iii) B – A

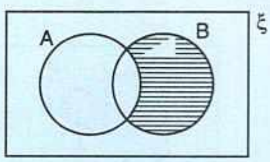
(iv) B ∩ A′

8. State the sets represented by the shaded portion of the following Venn-diagrams :

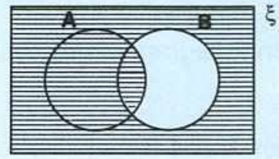
(i)



(ii)

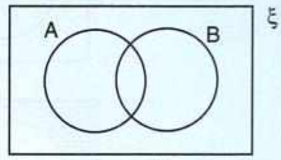


(iii)



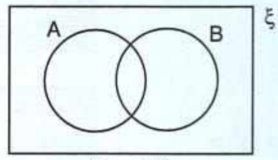
9. In each of the given diagrams, shade the region which represents the set given underneath the diagram :

(i)



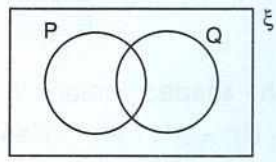
(B – A)′

(ii)



(A ∩ B)′

(iii)



(P ∪ Q)′

10. From the given diagram, find :

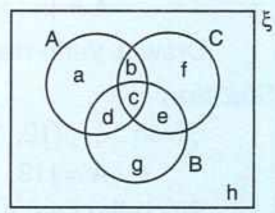
(i) (A ∪ B) – C

(ii) B – (A ∩ C)

(iii) (B ∩ C) ∪ A

Verify : A – (B ∩ C)

= (A – B) ∪ (A – C)



11. Using the given diagram, express the following sets in terms of set A and B.

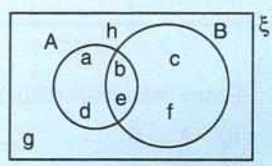
(i) {a, d}

(ii) {a, d, c, f}

(iii) {a, d, c, f, g, h}

(iv) {a, d, g, h}

(v) {g, h}



Theme – 2
RATIO AND PROPORTION

PERCENT AND PERCENTAGE 7

7.1 REVIEW

1.	**Percent**	*Percent* means; '**for every hundred**'. The word percent is abbreviated as p.c.; and is denoted by the symbol %.
2.	**Percentage**	A fraction, whose denominator is 100 is called a percentage and the numerator of such a fraction is called the rate percent. $\therefore \frac{8}{100} = 8$ percent *i.e.* 8 out of 100.
		Percent and percentage are used in the same sense.
3.	To express a given number as percent, multiply by 100 and in the same step attach the percentage sign (%). $\therefore \quad \frac{3}{5} = \frac{3}{5} \times 100\% = 60\%;\ 0{\cdot}45 = 0{\cdot}45 \times 100\% = 45\%$ and so on.	
4.	To express a given percent into a fraction, divide by 100 and in the same step remove the sign of percentage. $45\% = \frac{45}{100} = \frac{9}{20}$ (as a vulgar fraction) or $45\% = \frac{45}{100} = 0{\cdot}45$ (as a decimal fraction)	
5.	x as the percent of $y = \frac{x}{y} \times 100\%$ and $x\%$ of $y = \frac{x}{100} \times y$ (i) 5 as the percent of $20 = \frac{5}{20} \times 100\% = 25\%$ and (ii) 5% of $20 = \frac{5}{100} \times 20 = 1$	
6.	(i) Increase % = $\frac{\text{Increase in quantity}}{\text{Original quantity}} \times 100\%$ (ii) Decrease % = $\frac{\text{Decrease in quantity}}{\text{Original quantity}} \times 100\%$	

Example 1 :

(i) Find, 36 is what percent of 144.

(ii) 80 is 32% of a certain number, find the number.

(iii) Evaluate : 16% of 150 – 25% of 84 + 8% of 550.

Solution :

(i) ***Direct method :***

The required percent $= \frac{36}{144} \times 100\% = 25\%$ **(Ans.)**

Alternative method :

Let x% of 144 = 36 $\Rightarrow \frac{x}{100} \times 144 = 36$

$\Rightarrow x = \frac{36 \times 100}{144} = 25$

$\therefore$ **The required percent = 25%** **(Ans.)**

(ii) **Given :** 32% of a certain number = 80

$\Rightarrow \frac{32}{100} \times$ The number = 80

$\Rightarrow$ **The required number** $= 80 \times \frac{100}{32} =$ **250** **(Ans.)**

***Alternative method* :**

Let the required number be x

$\therefore$ 32% of $x = 80 \Rightarrow \frac{32}{100} \times x = 80$

$\Rightarrow x = 80 \times \frac{100}{32} = 250$

$\therefore$ **The required number = 250** **(Ans)**

(iii) 16% of 150 – 25% of 84 + 8% of 550

$= \frac{16}{100} \times 150 - \frac{25}{100} \times 84 + \frac{8}{100} \times 550$

$= 24 - 21 + 44 =$ **47** **(Ans.)**

Example 2 :

A man spends 65% of his salary and saves ₹ 525 per month. Find his monthly salary.

Solution :

Since, the man spends 65% of his salary

$\therefore$ He saves (100 – 65)% = 35% of his salary

Given : 35% of his monthly salary = ₹ 525

$\Rightarrow \frac{35}{100} \times$ his monthly salary = ₹ 525

$\therefore$ **His monthly salary** = ₹ 525 × $\frac{100}{35}$ = **₹ 1,500** **(Ans.)**

***Alternative method* :**

Let the man's monthly salary be ₹ 100

$\therefore$ He spends = 65% of ₹ 100 = $\frac{65}{100}$ × ₹ 100 = ₹ 65

and, saves = ₹ 100 – ₹ 65 = ₹ 35

Applying Unitary method :

When the man saves = ₹ 35, his monthly salary = ₹ 100

$\Rightarrow$ When the man saves = ₹ 1, his monthly salary = ₹ $\frac{100}{35}$

And, when the man saves = ₹ 525, his monthly salary = ₹ $\frac{100}{35} \times 525$

= ₹ 1,500

$\therefore$ **Man's monthly salary = ₹ 1,500** **(Ans.)**

Algebraic method :

Let the man's monthly salary = ₹ x

$\because$ He spends = 65% of ₹ x = ₹ $\frac{65x}{100}$

$\therefore$ He saves = ₹ x − ₹ $\frac{65x}{100}$ = ₹$(x - \frac{65x}{100})$

Given : $x - \frac{65x}{100} = 525 \Rightarrow \frac{100x - 65x}{100} = 525$

$\Rightarrow x = 525 \times \frac{100}{35} = 1500$

$\therefore$ **Man's monthly salary = ₹ 1,500** **(Ans.)**

Example 3 :

(i) A number 4·0 is wrongly read as 4·48; find the percentage error.

(ii) In a consignment of 500 articles, 70 articles are broken. Find the percentage of remaining articles.

Solution :

(i) $\because$ Error = 4·48 − 4·0 = 0·48

$\therefore$ **Percentage error** = $\frac{\text{Error}}{\text{Original number}} \times 100\%$

$= \frac{0{\cdot}48}{4{\cdot}0} \times 100\% =$ **12%** **(Ans.)**

(ii) Since, 70 articles are broken,

$\Rightarrow$ the number of remaining articles = 500 − 70 = 430

$\therefore$ **Percentage of remaining articles** = $\frac{\text{No. of remaining articles}}{\text{Original no. of articles}} \times 100\%$

$= \frac{430}{500} \times 100\% =$ **86%** **(Ans.)**

Example 4 :

In an election between two candidates, one candidate secured 43% of the total votes polled and lost the election by 4900 votes. Find the total number of votes polled.

Solution :

Since, losing candidate secured 43% of the votes polled

$\therefore$ Winning candidate secured (100 − 43)% = 57% of the votes polled

and, difference of their votes = 57% − 43%

= 14% of the votes polled

Given : 14% of votes polled = 4900

$\Rightarrow \frac{14}{100} \times$ votes polled = 4900

$\therefore$ **The total no. of votes polled** = $4900 \times \frac{100}{14}$ = **35,000** **(Ans.)**

Alternative method :

Let the total number of votes polled = 100

$\therefore$ The losing candidate gets 43% of 100 = 43 votes

and, the winning candidate gets = 100 − 43 = 57 votes

$\therefore$ The no. of votes by which the losing candidate has lost the election $= 57 - 43 = 14.$

Applying Unitary method :

When election is lost by 14 votes,

the total no. of votes polled $= 100$

$\Rightarrow$ when election is lost by 4900 votes,

the total no. of votes polled $= \dfrac{100}{14} \times 4900 =$ **35,000** **(Ans.)**

Algebraic method :

Let the total number of votes polled $= x$

$\therefore$ The votes secured by losing candidate $= 43\%$ of x

and, the votes secured by winning candidate $= (100 - 43)\%$ of x

$= 57\%$ of x.

Thus, 57% of $x - 43\%$ of $x = 4900$

$\Rightarrow$ 14% of $x = 4900$

i.e. $\dfrac{14}{100}x = 4900$ *i.e.,* $x = 4900 \times \dfrac{100}{14}$

$= 35000$

$\therefore$ **The total number of votes polled = 35,000** **(Ans.)**

Example 5 :

The cost of a machine depreciates every year by 10% of its cost at the beginning of the year. If the present cost of the machine is ₹ 10,000; find its cost :

(i) after one year (ii) after 2 years.

Solution :

(i) Since, present cost of the machine = ₹ 10,000

and, depreciation in its cost in 1st year = 10% of ₹ 10,000 = ₹ 1,000

$\therefore$ **The cost of the machine after one year** = ₹ (10,000 – 1,000)

= **₹ 9,000** **(Ans.)**

(ii) **For 2nd year :**

The cost of machine at the beginning = ₹ 9,000

and, depreciation = 10% of ₹ 9,000 = ₹ 900

$\therefore$ **The cost of the machine after two years** = ₹ (9,000 – 900)

= **₹ 8,100** **(Ans.)**

Alternative method **:**

When the cost is depreciating by $x\%$ every year;

(i) Value after one year = Present value $\left(1-\dfrac{x}{100}\right)$

and (ii) Value after 2 years = Present value $\left(1-\dfrac{x}{100}\right)\left(1-\dfrac{x}{100}\right)$ = Present value $\left(1-\dfrac{x}{100}\right)^2$

(i) **Cost after one year** = Present cost $\left(1-\frac{x}{100}\right)$

$$= ₹\ 10{,}000 \times \left(1-\frac{10}{100}\right)$$

$$= ₹\ 10{,}000 \times \frac{90}{100} = \mathbf{₹\ 9{,}000}$$ **(Ans.)**

(ii) **Cost after 2 years** = Present cost $\left(1-\frac{x}{100}\right)\left(1-\frac{x}{100}\right)$

$$= ₹\ 10{,}000 \times \left(1-\frac{10}{100}\right)\left(1-\frac{10}{100}\right)$$

$$= ₹\ 10{,}000 \times \frac{90}{100} \times \frac{90}{100} = \mathbf{₹\ 8{,}100}$$ **(Ans.)**

Example 6 :

The number 5,000 is first decreased by 10% and then increased by 20%, find the resulting number.

Solution :

The resulting number = The original number $\times \left(1-\frac{10}{100}\right) \times \left(1+\frac{20}{100}\right)$

$$= 5{,}000 \times \frac{90}{100} \times \frac{120}{100} = \mathbf{5{,}400}$$ **(Ans.)**

Example 7 :

The number 20,000 is first increased by 30% and then decreased by 20%, find the resulting number.

Solution :

The resulting number = The original number $\times \left(1+\frac{30}{100}\right) \times \left(1-\frac{20}{100}\right)$

$$= 20{,}000 \times \frac{130}{100} \times \frac{80}{100} = \mathbf{20{,}800}$$ **(Ans.)**

Example 8 :

The number 12,000 is first increased by 15% and then further increased by 25%. Find the resulting number.

Solution :

The resulting number = The original number $\times \left(1+\frac{15}{100}\right) \times \left(1+\frac{25}{100}\right)$

$$= 12{,}000 \times \frac{115}{100} \times \frac{125}{100} = \mathbf{17{,}250}$$ **(Ans.)**

Example 9 :

Find the percentage change in the cost of an article which first increases by 20% and then decreases by 8%.

Solution :

Let the original cost of the article be ₹ 100

After an increase of 20%, it becomes = ₹ 100 + 20% of ₹ 100

= ₹ 100 + ₹ 20 = ₹ 120

Since, now it is decreased by 8%

∴ Value of the article after a decrease of 8%

$$= ₹\ 120 - 8\% \text{ of } ₹\ 120$$

$$= ₹\ 120 - ₹\ 9.60 = ₹\ 110.40$$

∴ Change (increase) on the whole

$$= \text{Final value} - \text{Initial value}$$

$$= ₹\ 110.40 - ₹\ 100 = ₹\ 10.40$$

∴ Percentage change (increase) = **10·4%** **(Ans.)**

> Change (increase or decrease) on 100 is called percentage change.

Alternative method :

Since, the cost of the article was first increased by 20% and then decreased by 8%

⇒ If initial value of the article is ₹ 100,

$$\text{its final value} = ₹\ 100 \times \left(1+\frac{20}{100}\right)\left(1-\frac{8}{100}\right)$$

$$= ₹\ 100 \times \frac{120}{100} \times \frac{92}{100} = ₹\ 110.40$$

∴ **Percentage change** (increase) = (₹ 110.40 – ₹ 100)% = **10.40%** **(Ans.)**

EXERCISE 7(A)

1. Evaluate :
   (i) 55% of 160 + 24% of 50 – 36% of 150
   (ii) 9·3% of 500 – 4·8% of 250 – 2·5% of 240
2. (i) A number is increased from 125 to 150; find the percentage increase.
   (ii) A number is decreased from 125 to 100; find the percentage decrease.
3. Find :
   (i) 45 is what percent of 54 ?
   (ii) 2·7 is what percent of 18 ?
4. (i) 252 is 35% of a certain number, find the number.
   (ii) If 14% of a number is 315; find the number.
5. Find the percentage change, when a number is changed from :
   (i) 80 to 100 (ii) 100 to 80
   (iii) 6·25 to 7·50
6. An auctioneer charges 8% for selling a house. If the house is sold for ₹ 2,30,500. Find the charges of the auctioneer.
7. Out of 800 oranges, 50 are found rotten. Find the percentage of good oranges.
8. A cistern contains 5 thousand litres of water. If 6% water is leaked, find how many litres of water would be left in the cistern.
9. A man spends 87% of his salary. If he saves ₹ 325; find his salary.
10. (i) A number 3·625 is wrongly read as 3·265; find the percentage error.
    (ii) A number $5{\cdot}78 \times 10^3$ is wrongly written as $5{\cdot}87 \times 10^3$; find the percentage error.
11. In an election between two candidates, one, candidate secured 58% of the votes polled and won the election by 18,336 votes. Find the total number of votes polled and the votes secured by each candidate.
12. In an election between two candidates, one, candidate secured 47% of votes polled and lost the election by 12,366 votes. Find the total votes polled and the votes secured by the winning candidate.
13. The cost of a scooter depreciates every year by 15% of its value at the beginning of the year. If the present cost of the scooter is ₹ 8,000; find its cost :
    (i) after one year (ii) after 2 years.
14. In an examination, the pass mark is 40%. If a candidate gets 65 marks and fails by 3 marks; find the maximum marks.

    > 40% of the max. marks = 65 + 3

15. In an examination, a candidate secured 125 marks and failed by 15 marks. If the pass percentage was 35%; find the maximum marks.
16. In an objective type paper of 150 questions; John got 80% correct answers and Mohan got 64% correct answers.
    (i) How many correct answers did each get ?
    (ii) What percent is Mohan's correct answers to John's correct answers ?
17. The number 8,000 is first increased by 20% and then decreased by 20%. Find the resulting number.
18. The number 12,000 is first decreased by 25% and then increased by 25%. Find the resulting number.
19. The cost of an article is first increased by 20% and then decreased by 30%, find the percentage change in the cost of the article.
20. The cost of an article is first decreased by 25% and then further decreased by 40%. Find the percentage change in the cost of the article.

Example 10 :

5% of the population of a certain town was killed in a bombardment and 7% of the remaining died in panic. If the population of the town is now 44175; find the population of the town at the beginning before the bombardment.

Solution :

Let the population of the town at the beginning be 100.

$\therefore$ Population killed in bombardment = 5% of 100 = 5

Remaining population = 100 – 5 = 95

Population died in panic = 7% of 95 $= \frac{7}{100} \times 95 = \frac{133}{20}$

Remaining population $= 95 - \frac{133}{20} = \frac{1767}{20}$

Applying unitary method :

If the remaining population is $\frac{1767}{20}$, population at the beginning = 100

If the remaining population is 1, population at the beginning $= 100 \times \frac{20}{1767}$

and, if the remaining population is 44175, **population in the beginning**

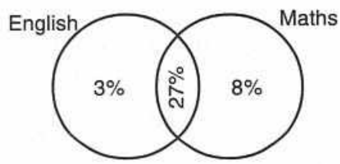
$$= 100 \times \frac{20}{1767} \times 44175 = \mathbf{50,000}$$ **(Ans.)**

Example 11 :

In an examination, 30 percent candidates failed in English, 35 percent failed in Mathematics and 27 percent failed in both the subjects. Find :

(i) percentage of total failed.
(ii) percentage of total passed.
(iii) the total number of candidates; if 248 passed in both.

Solution :



(i) Since, failed only in English = 30% – 27% = 3%

failed only in Mathematics = 35% – 27% = 8%

and failed in both = 27%

$\therefore$ **Total failed** = 3% + 8% + 27% = **38%** **(Ans.)**

(ii) **Total passed** = (100 – 38)% = **62%** **(Ans.)**

(iii) Since, 62% of the candidates = 248

$\Rightarrow \frac{62}{100} \times$ No. of candidates = 248

$\Rightarrow$ **No. of candidates** $= 248 \times \frac{100}{62} =$ **400** **(Ans.)**

Example 12 :

A's income is 10 percent more than B's; how much percent is B's income less than A's ?

Solution :

Let B's income = ₹ 100

then, A's income = ₹ 100 + 10% of ₹ 100

$= ₹\ 100 + \frac{10}{100} \times ₹\ 100 = ₹\ 110$

If A's income is ₹ 110, B's income = ₹ 10 less than A [₹ (110 – 100)]

If A's income = ₹ 1, B's = ₹ $\frac{10}{110}$ less than A

and, if A's income = ₹ 100, B's is ₹ $\frac{10}{110} \times 100$ less than A

$\Rightarrow$ **B's income is** $\frac{100}{11}$% less *i.e.* **$9\frac{1}{11}$% less than A's.** **(Ans.)**

Example 13 :

If the price of wheat is increased by 20% today, at what percent should it be decreased tomorrow, so as to bring down the price back to the original ?

Solution :

Let the original price of wheat be ₹ 100

$\therefore$ Today's price = ₹ 100 + ₹ 20 = ₹ 120.

In order to bring down the price to original *i.e.* to ₹ 100; its price should be decreased by ₹ 120 – ₹ 100 = ₹ 20 on ₹ 120.

i.e. On ₹ 120, the price should be decreased by ₹ 20.

$\Rightarrow$ On ₹ 1, the price should be decreased by ₹ $\frac{20}{120}$

and, on ₹ 100, the price should be decreased by ₹ $\frac{20}{120} \times 100 = ₹\ \frac{50}{3} = ₹\ 16\frac{2}{3}$

$\therefore$ **The price should be decreased by $16\frac{2}{3}$%** **(Ans.)**

Example 14 :

A number decreased by 18% becomes 410. Find the number.

Solution :

Let the number be 100.

Since, decrease in number = 18% of 100 = 18

$\therefore$ After decrease, the number becomes = 100 – 18 = 82.

Applying unitary method :

When the decreased number = 82, the original number = 100

$\Rightarrow$ When the decreased number = 1, the original number = $\frac{100}{82}$

and, when the decreased number = 410,

the original number = $\frac{100}{82} \times 410 = 500$ **(Ans.)**

***Alternative method** (Algebraic method) :*

Let the original number be x.

$\therefore \quad x - 18\% \text{ of } x = 410 \quad \Rightarrow x - \frac{18x}{100} = 410$

i.e. $\frac{100x - 18x}{100} = 410 \quad \Rightarrow \quad \frac{82x}{100} = 410$ *i.e.* $x = 410 \times \frac{100}{82} = 500$

$\therefore$ **The original number = 500** **(Ans.)**

Direct method :

If a number is decreased by $x\%$,

the new number = $\left(\frac{100 - x}{100}\right) \times$ the original number.

and, if a number is increased by $x\%$,

the new number = $\left(\frac{100 + x}{100}\right) \times$ the original number.

Here, the decrease in number = 18% and the new (decreased) number is 410.

$\therefore$ The new number = $\frac{100 - 8}{100} \times$ the original number

$\Rightarrow$ $410 = \frac{82}{100} \times$ the original number

$\Rightarrow$ **The original no.** = $\frac{410 \times 100}{82} = 500$ **(Ans.)**

Example 15 :

Two numbers are respectively 10% and 25% more than a third number. What percent is the first of the second ?

Solution :

Let the third number be 100.

$\therefore$ The first number = 100 + 10% of 100 = 110

and, the second number = 100 + 25% of 100 = 125.

$\therefore$ **The first no. as the percent of the second** = $\frac{110}{125} \times 100\% = 88\%$ **(Ans.)**

EXERCISE 7(B)

1. A man bought a certain number of oranges; out of which 13 percent were found rotten. He gave 75% of the remaining in charity and still has 522 oranges left. Find how many had he bought ?

2. 5% pupil in a town died due to some diseases and 3% of the remaining left the town. If 2,76,450 pupil are still in the town; find the original number of pupil in the town.

3. In a combined test in English and Physics; 36% candidates failed in English; 28% failed in Physics and 12% in both; find :
   (i) the percentage of passed candidates.
   (ii) the total number of candidates appeared, if 208 candidates have failed.
4. In a combined test in Maths and Chemistry, 84% candidates passed in Maths, 76% in Chemistry and 8% failed in both. Find :
   (i) the percentage of failed candidates.
   (ii) if 340 candidates passed in the test, then, how many candidates had appeared in the test ?
5. A's income is 25% more than B's. Find, how much percent B's income is less than A's ?
6. Mona is 20% younger than Neetu. By how much percent is Neetu older than Mona ?
7. If the price of sugar is increased by 25% today, by what percent should it be decreased tomorrow to bring the price back to the original ?
8. A number increased by 15% becomes 391. Find the number.
9. A number decreased by 23% becomes 539. Find the number.
10. Two numbers are respectively 20 percent and 50 percent more than a third number. What percent is the second of the first ?
11. Two numbers are respectively 20 percent and 50 percent of a third number. What percent is the second of the first ?
12. Two numbers are respectively 30 percent and 40 percent less than a third number. What percent is the second of the first ?

EXERCISE 7(C)

1. A bag contains 8 red balls, 11 blue balls and 6 green balls. Find the percentage of blue balls in the bag.
2. Mohan gets ₹ 1,350 from Geeta and ₹ 650 from Rohit. Out of the total money that Mohan gets from Geeta and Rohit, what percent does he get from Rohit ?
3. The monthly income of a man is ₹ 16,000. 15 percent of it is paid as income-tax and 75% of the remainder is spent on rent, food, clothing, etc. How much money is still left with the man ?
4. A number is first increased by 20% and the resulting number is then decreased by 10%. Find the overall change in the number as percent.
5. A number is increased by 10% and the resulting number is again increased by 20%. What is the overall percentage increase in the number ?
6. During 2003, the production of a factory decreased by 25%. But, during 2004, it (production) increased by 40% of what it was at the beginning of 2004. Calculate the resulting change (increase or decrease) in production during these two years.
7. Last year, oranges were available at ₹ 24 per dozen; but this year, they are available at ₹ 50 per score. Find the percentage change in the price of oranges [1 score = 20.]
8. In an examination, Kavita scored 120 out of 150 in Maths, 136 out of 200 in English and 108 out of 150 in Science. Find her percentage score in each subject and also on the whole (aggregate).
9. A is 25% older than B. By what percent is B younger than A ?
10. (i) Increase 180 by 25%.
    (ii) Decrease 140 by 18%.
11. In an election, three candidates contested and secured 29200, 58800 and 72000 votes. Find the percentage of votes scored by the winning candidate.
12. (i) A number when increased by 23% becomes 861; find the number.
    (ii) A number when decreased by 16% becomes 798; find the number.
13. The price of sugar is increased by 20%. By what percent must the consumption of sugar be decreased so that the expenditure on sugar may remain the same ?

PROFIT, LOSS AND DISCOUNT

(Including Overhead Expenses and Tax)

8

8.1 REVIEW

1. **Profit**	When the selling price (**S.P.**) of an article is **more than** its cost price (**C.P.**); the article is said to be sold at a **profit** (gain) And, Profit = Selling Price – Cost Price *i.e.* Profit = S.P. – C.P.
2. **Loss**	When the selling price (**S.P.**) of an article is **less than** its cost price (**C.P.**); the article is said to be sold at a **loss**. And, Loss = Cost Price – Selling Price *i.e.* Loss = C.P. – S.P.
3. **Profit = S.P. – C.P.**	⇒ (i) S.P. = C.P. + Profit and, (ii) C.P. = S.P. – Profit
4. **Loss = C.P. – S.P.**	⇒ (i) S.P. = C.P. – Loss and, (ii) C.P. = S.P. + Loss
5. Profit (gain)% = $\frac{\text{Profit}}{\text{C.P.}} \times 100\%$ and Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100\%$	
6. Profit % and loss % are always calculated on cost price.	

Example 1 :

Articles, bought at 10 for ₹ 8, are sold at 8 for ₹ 10. Find the gain percent.
Also, find the number of articles bought and sold in order to gain ₹ 144.

Solution :

Whenever the cost price and the selling price are given for different number of identical articles; first of all, find the C.P. and the S.P. of equal number of articles and then calculate the profit percent or the loss percent, as the case may be.

Also, in order to have a certain gain,

$$\text{the number of articles bought and sold} = \frac{\text{Total profit}}{\text{Profit on one article}}$$

Given : C.P. of 10 articles = ₹ 8

∴ C.P. of 1 article = ₹ $\frac{8}{10}$ = ₹ 0·80

Also, given : S.P. of 8 articles = ₹ 10

∴ S.P. of 1 article = ₹ $\frac{10}{8}$ = ₹ 1·25

Profit on 1 article = S.P. – C.P. = ₹ 1·25 – ₹ 0·80 = ₹ 0·45 and, profit % $= \frac{\text{Profit}}{\text{C.P.}} \times 100\%$

$= \frac{₹\,0{\cdot}45}{₹\,0{\cdot}80} \times 100\% = 56{\cdot}25\%$ **(Ans.)**

Also, **the number of articles bought and sold**

$= \frac{\text{Total profit}}{\text{Profit on one article}}$

$= \frac{₹\,144}{₹\,0{\cdot}45} = 320$ **(Ans.)**

8.2 OVERHEAD EXPENSES

When an article is purchased at one place and is taken to some other place; an additional money for transportation, labour, packing, etc. is to be spent. This additional money spent is termed as **overheads** or **overhead expenses**.

The overheads (if any) incurred is added to the actual cost price to get the total cost price of the article and then the profit or loss is calculated on this total cost price.

Example 2 :

Raju goes from Agra to Delhi to buy an article, which costs ₹ 6,500 in Delhi. He sells this article in Agra for ₹ 8,000. Find his gain or loss per cent. Consider that he spends ₹ 700 on his transportation, food, etc.

Solution :

Given : Actual price paid for the article = ₹ 6,500

and, overhead expenses = ₹ 700

∴ Total cost price = ₹ 6,500 + ₹ 700 = ₹ 7,200

Since, selling price = ₹ 8,000

∴ Gain = ₹ 8,000 – ₹ 7,200 [Gain = S.P. – C.P.]

= ₹ 800

and, gain % $= \frac{₹\,800}{₹\,7{,}200} \times 100\% = 11\frac{1}{9}\%$ **(Ans.)**

Example 3 :

A man sold his bicycle for ₹ 810; losing one-ninth of its selling price, Find :

(i) the loss (ii) the cost price of the bicycle (iii) the loss as percent.

Solution :

(i) Since, S. P. = ₹ 810 ; loss $= \frac{1}{9} \times ₹\,810 = ₹\,90$ **(Ans.)**

(ii) C.P. = S.P. + loss = ₹ 810 + ₹ 90 = ₹ 900 **(Ans.)**

(iii) Loss % $= \frac{\text{Loss}}{\text{C.P.}} \times 100\%$

$= \frac{₹\,90}{₹\,900} \times 100\% = 10\%$ **(Ans.)**

Example 4 :

The selling price of a table is $\frac{27}{25}$ times its cost price. Find the loss or the profit as percent.

Solution :

Let the cost price of the table = ₹ 100

$\therefore$ Its selling price = $\frac{27}{25} \times$ ₹ 100 = ₹ 108

Profit = S.P. – C.P. = ₹ 108 – ₹ 100 = ₹ 8

$\therefore$ **Profit %** = $\frac{\text{Profit}}{\text{C.P.}} \times 100\%$ = $\frac{₹ 8}{₹ 100} \times 100\%$ = **8%** **(Ans.)**

Algebraic method :

Let the C.P. = ₹ x

$\Rightarrow$ S.P. = ₹ $\frac{27}{25}x$

$\therefore$ Profit = S.P. – C.P. = ₹ $\frac{27}{25}x$ – ₹ x = ₹ $\frac{27-25}{25}x$ = ₹ $\frac{2}{25}x$

And, **profit %** = $\frac{₹ \frac{2}{25}x}{₹ x} \times 100\%$

$= \frac{2}{25} \times 100\%$ = **8%** **(Ans.)**

Example 5 :

The cost price of an article is $\frac{5}{4}$ times its selling price. Find the loss or the profit as percent.

Solution :

Let the selling price = ₹ 100

$\therefore$ The cost price = $\frac{5}{4} \times$ ₹ 100 = ₹ 125

Loss = C.P. – S.P. = ₹ 125 – ₹ 100 = ₹ 25

And, **loss %** = $\frac{\text{Loss}}{\text{C.P.}} \times 100\%$ = $\frac{₹ 25}{₹ 125} \times 100\%$

= **20%** **(Ans.)**

Algebraic method :

Let the selling price = ₹ x

$\therefore$ The cost pice = ₹ $\frac{5}{4}x$

Loss = C.P. – S.P. = ₹ $\frac{5}{4}x$ – ₹ x = ₹ $\left(\frac{5-4}{4}\right)x$ = ₹ $\frac{1}{4}x$

$\therefore$ **Loss %** = $\frac{₹ \frac{1}{4}x}{₹ \frac{5}{4}x} \times 100\%$ = $\frac{1}{4} \times \frac{4}{5} \times 100\%$ = **20%** **(Ans.)**

EXERCISE 8(A)

1. Megha bought 10 note-books for ₹ 40 and sold them at ₹ 4·75 per note-book. Find her gain percent.
2. A fruit-seller buys oranges at 4 for ₹ 3 and sells them at 3 for ₹ 4. Find his profit percent.
3. A man buys a certain number of articles at 15 for ₹ 112·50 and sells them at 12 for ₹ 108. Find :
   (i) his gain as percent;
   (ii) the number of articles sold to make a profit of ₹ 75.
4. A boy buys an old bicycle for ₹ 162 and spends ₹ 18 on its repairs before selling the bicycle for ₹ 207. Find his gain or loss as percent.
5. An article is bought from Jaipur for ₹ 4,800 and is sold in Delhi for ₹ 5,820. If ₹ 1,200 is spent on its transportations, etc.; find the loss or the gain as percent.
6. Mohit sold a T.V. for ₹ 3,600; gaining one-sixth of its selling price. Find :
   (i) the gain. (ii) the cost price of the T.V.
   (iii) the gain percent.
7. By selling a certain number of goods for ₹ 5,500; a shopkeeper loses equal to one-tenth of their selling price. Find :
   (i) the loss incured
   (ii) the cost price of the goods
   (iii) the loss as percent.
8. The selling price of a sofa-set is $\frac{4}{5}$ times of its cost price. Find the gain or the loss as percent.
9. The cost price of an article is $\frac{4}{5}$ times of its selling price. Find the loss or the gain as percent.
10. A shopkeeper sells his goods at 80% of their cost price. Find his percent gain or loses?
11. The cost price of an article is 90% of its selling price. What is the profit or the loss as percent ?
12. The cost price of an article is 30 percent less than its selling price. Find, the profit or the loss as percent.
13. A shopkeeper bought 300 eggs at 80 paisa each. 30 eggs were broken in transit and then he sold the remaining eggs at one rupee each. Find, his gain or loss as percent.
14. A man sold his bicycle for ₹ 405 losing one-tenth of its cost price. Find :
   (i) its cost price; (ii) the loss percent.
15. A man sold a radio-set for ₹ 250 and gained one-ninth of its cost price. Find :
   (i) its cost price; (ii) the profit percent.

8.3 TO FIND S.P., WHEN C.P. AND GAIN (OR LOSS) PERCENT ARE GIVEN

Example 6 :

Bhanu bought a fountain pen for ₹ 12. For how much should she sell it to gain 15% ?

Solution :

Since, C.P. of the pen = ₹ 12

∴ Gain = 15% of the C.P.

$$= 15\% \text{ of ₹ } 12 = \frac{15}{100} \times ₹\,12 = ₹\,1{\cdot}80$$

S.P. = C.P. + Gain ⇒ S.P. = ₹ 12 + ₹ 1·80 = **₹ 13·80** **(Ans.)**

Alternative method **:**

$$\text{S.P.} = \frac{(100 + \text{gain}\%)}{100} \times \text{C.P.}$$

$$\Rightarrow \quad \text{S.P.} = \frac{100+15}{100} \times ₹\,12 = \frac{115}{100} \times ₹\,12 = ₹\,13{\cdot}80$$ **(Ans.)**

Example 7 :

An article bought for ₹ 450 is sold at a loss of 20%. Find its selling price.

Solution :

Since, C.P. = ₹ 450

∴ Loss = 20% of ₹ 450 = $\frac{20}{100}$ × ₹ 450 = ₹ 90

S.P. = C.P. – Loss ⇒ **S.P.** = ₹ 450 – ₹ 90 = **₹ 360** **(Ans.)**

***Alternative method* :**

$$\text{S.P.} = \frac{(100 - \text{loss\%})}{100} \times \text{C.P.}$$

⇒ **S.P.** = $\frac{(100-20)}{100}$ × ₹ 450 = $\frac{80}{100}$ × ₹ 450 = **₹ 360** **(Ans.)**

8.4 TO FIND C.P., WHEN S.P. AND GAIN (OR LOSS) PERCENT ARE GIVEN

Example 8 :

Ram sells an article for ₹ 360 at a gain of 20%. Find its cost price.

Solution :

Let C.P. of the article = ₹ 100

∴ Gain = 20% of ₹ 100 = ₹ 20

and, S.P. = ₹ 100 + ₹ 20 = ₹ 120

When S.P. is ₹ 120 ; C.P. = ₹ 100

When S.P. is ₹ 1 ; C.P. = ₹ $\frac{100}{120}$

When S.P. is ₹ 360 ; **C.P.** = ₹ $\frac{100}{120}$ × 360 = **₹ 300** **(Ans.)**

***Alternative method* :**

$$\text{C.P.} = \frac{100}{(100 + \text{gain\%})} \times \text{S.P.}$$

⇒ **C.P.** = $\frac{100}{100+20}$ × ₹ 360 = $\frac{100}{120}$ × ₹ 360 = **₹ 300** **(Ans.)**

Example 9 :

By selling an article for ₹ 382·50 a man loses 15%. Find its cost price.

Solution :

Let C.P. = ₹ 100

∴ Loss = 15% of ₹ 100 = ₹ 15

and, S.P. = ₹ 100 – ₹ 15 = ₹ 85

When S.P. is ₹ 85 ; C.P. = ₹ 100

When S.P. is ₹ 1 ; C.P. = ₹ $\frac{100}{85}$

When S.P. is ₹ 382·20 ; C.P. = ₹ $\frac{100}{85}$ × 382·50 = ₹ 450 **(Ans.)**

Alternative method :

$$\text{C.P.} = \frac{100}{(100 - \text{loss\%})} \times \text{S.P.}$$

⇒ C.P. = $\frac{100}{100-15}$ × ₹ 382·50 = $\frac{100}{85}$ × ₹ 382·50 = ₹ 450 **(Ans.)**

Example 10 :

By selling an article for ₹ 810; a man loses 10%. At what price should he sell it in order to gain 8% ?

Solution :

Given : S.P. = ₹ 810 and loss = 10%

∴ C.P. = $\left(\frac{100}{100-10}\right)$ × ₹ 810 $\left[\because \text{C.P.} = \left(\frac{100}{100-\text{loss\%}}\right) \times \text{S.P.}\right]$

= $\frac{100}{90}$ × ₹ 810 = ₹ 900

Now, C.P. = ₹ 900, gain = 8% and required to find S.P.

S.P. = $\left(\frac{100+8}{100}\right)$ × ₹ 900 $\left[\because \text{S.P.} = \left(\frac{100+\text{gain\%}}{100}\right) \times \text{S.P.}\right]$

= $\frac{108}{100}$ × ₹ 900 = ₹ 972 **(Ans.)**

Example 11 :

Peter sells two watches for ₹ 198 each; gaining 20% on one and losing 20% on the other. Find his gain % or loss % on the whole.

Solution :

For one watch : S.P. = ₹ 198 and gain = 20%

⇒ C.P. = $\frac{100}{(100+20)}$ × ₹ 198 $\left[\text{C.P.} = \frac{100}{(100+\text{gain\%})} \times \text{S.P.}\right]$

= ₹ 165

For the other watch : S.P. = ₹ 198 and loss = 20%

⇒ C.P. = $\frac{100}{(100-20)}$ × ₹ 198 $\left[\text{C.P.} = \frac{100}{(100-\text{loss\%})} \times \text{S.P.}\right]$

= ₹ 247·50

Total C.P. of both the watches= ₹ 165 + ₹ 247·50 = ₹ 412·50

Total S.P. of both the watches= ₹ 198 + ₹ 198 = ₹ 396

∴ Loss on the whole = ₹ 412·50 – ₹ 396 = ₹ 16·50

and, loss % on the whole = $\frac{16 \cdot 50}{412 \cdot 50}$ × 100% = 4% **(Ans.)**

EXERCISE 8(B)

1. Find the selling price, if :
   (i) C.P. = ₹ 950 and profit = 8%
   (ii) C.P. = ₹ 1,300 and loss = 13%
2. Find the cost price, if :
   (i) S.P. = ₹ 1,680 and profit = 12%
   (ii) S.P. = ₹ 1,128 and loss = 6%
3. By selling an article for ₹ 900; a man gains 20%. Find his cost price and the gain.
4. By selling an article for ₹ 704; a person loses 12%. Find his cost price and the loss.
5. Find the selling price, if :
   (i) C.P. = ₹ 352; overheads = ₹ 28 and profit = 20%.
   (ii) C.P. = ₹ 576; overheads = ₹ 44 and loss = 16%.
6. If John sells his bicycle for ₹ 637, he will suffer a loss of 9%. For how much should it be sold, if he desires a profit of 5% ?
7. A man sells a radio-set for ₹ 605 and gains 10%. At what price should he sell another radio of the same kind, in order to gain 16% ?
8. By selling a sofa-set for ₹ 2,500; the shopkeeper loses 20%. Find his loss percent or profit percent; if he sells the same sofa-set for ₹ 3,150.
9. Mr. Sinha sold two tape-recorders for ₹ 990 each; gaining 10% on one and losing 10% on the other. Find his total loss or gain, as percent, on the whole transaction.
10. A tape-recorder is sold for ₹ 2,760 at a gain of 15% and a C.D. player is sold for ₹ 3,240 at a loss of 10%. Find :
    (i) the C.P. of the tape-recorder.
    (ii) the C.P. of the C.D. player.
    (iii) the total C.P. of both.
    (iv) the total S.P. of both.
    (v) the gain % or the loss% on the whole.
11. Rajesh sold his scooter to Rahim at 8% loss and Rahim, in turn, sold the same scooter to Prem at 5% gain. If Prem paid ₹ 14,490 for the scooter; find :
    (i) the S.P. and the C.P. of the scooter for Rahim.
    (ii) the S.P. and the C.P. of the scooter for Rajesh.
12. John sold an article to Peter at 20% profit and Peter sold it to Mohan at 5% loss. If Mohan paid ₹ 912 for the article; find how much did John pay for it ?

Example 12:

A fruit-seller buys oranges at ₹ 20 per dozen and sells them at a profit of 20%. Find the price paid by the customer for buying :

(i) 4 oranges (ii) 3 dozen oranges

Solution :

For one dozen (12) ***oranges*** : C.P. = ₹ 20 and profit = 20%

$$\therefore \quad \text{S.P.} = \frac{100+20}{100} \times ₹\,20 \qquad \left[\because \text{S.P.} = \left(\frac{100+\text{profit}\,\%}{100}\right) \times \text{C.P.}\right]$$

$$= \frac{120}{100} \times ₹\,20 = ₹\,24$$

(i) Since, S.P. of 12 oranges = ₹ 24 ⇒ S.P. of 1 orange = ₹ $\frac{24}{12}$ = ₹ 2

∴ **For buying 4 oranges, the customer paid** = 4 × ₹ 2 = **₹ 8** **(Ans.)**

(ii) We know, 3 dozen = 3 × 12 = 36

∴ **For 3 dozen oranges, the customer paid** = 36 × ₹ 2 = **₹ 72** **(Ans.)**

Example 13 :

A fruit-seller sells 8 bananas for ₹ 6 gaining 25%. How many bananas did he buy for ₹ 6 ?

Solution :

For 8 bananas : S.P. = ₹ 6 and gain = 25%

$\therefore$ C.P. = $\left(\frac{100}{100+25}\right) \times ₹\ 6$ $\left[\text{C.P.} = \left(\frac{100}{100+\text{gain\%}}\right) \times \text{S.P.}\right]$

$= \frac{100}{125} \times ₹\ 6 = ₹\ 4{\cdot}80$

$\Rightarrow$ For ₹ 4·80, the fruit-seller buys 8 bananas

$\Rightarrow$ **For ₹ 6, he buys** $= \frac{8}{4{\cdot}80} \times 6$ = **10 bananas** **(Ans.)**

Example 14 :

The cost price of 10 articles is equal to the selling price of 9 articles. Find the profit percent.

Solution :

Let the C.P. of 1 article be ₹ 1

$\therefore$ C.P. of 10 articles = ₹ 10

According to the question,

S.P. of 9 articles = ₹ 10

$\therefore$ S.P. of 1 article = ₹ $\frac{10}{9}$

Profit = ₹ $\left(\frac{10}{9} - 1\right)$ = ₹ $\frac{1}{9}$

and, **profit percent** = $\frac{\frac{1}{9}}{1} \times 100\% = 11\frac{1}{9}\%$ **(Ans.)**

Example 15 :

A man bought a piece of land for ₹ 15,000. He sold $\frac{1}{3}$ of this land at a loss of 5 percent. At what gain percent should he sell the remaining land in order to gain 8% on the whole ?

Solution :

Since, C.P. of the whole land = ₹ 15,000

and, the gain desired on the whole = 8%

$\therefore$ Total S.P. of the whole land = $\frac{100+8}{100} \times ₹\ 15{,}000 = ₹\ 16{,}200$

C.P. of $\frac{1}{3}$ of the land = $\frac{1}{3}$ of ₹ 15,000 = ₹ 5,000

Since, loss on it = 5%

$\therefore$ S.P. of it = $\frac{100-5}{100} \times ₹\ 5{,}000 = ₹\ 4{,}750$

Now, C.P. of remaining land = ₹ 15,000 – ₹ 5,000 = ₹ 10,000

and, S.P. of the remaining land = ₹ 16,200 – ₹ 4,750 = ₹ 11,450

$\therefore$ Gain on the remaining land = ₹ 11,450 – ₹ 10,000 = ₹ 1,450

and, **gain percent on the remaining land** = $\frac{1,450}{10,000} \times 100\%$ = **14·5%** **(Ans.)**

Example 16 :

A shopkeeper sells an article at 15% gain. Had he sold it for ₹ 18 more, he would have gained 18%. Find the cost price of the article.

Solution :

Let the C.P. of the article be ₹ 100

when gain = 15% ; S.P. = ₹ (100 + 15) = ₹ 115

and, when gain = 18% ; S.P. = ₹ (100 + 18) = ₹ 118

Difference of the two selling prices = ₹ 118 – ₹ 115 = ₹ 3

Applying unitary method :

When sold for ₹ 3 more, the C.P. of the article = ₹ 100

When sold for ₹ 18 more, **the C.P. of the article** = ₹ $\frac{100}{3} \times 18$ = **₹ 600** **(Ans.)**

EXERCISE 8(C)

1. A stationer buys pens at 5 for ₹ 28 and sells them at a profit of 25%. How much should a customer pay; if he buys
   (i) only one pen ? (ii) three pens ?
2. A fruit-seller sells 4 oranges for ₹ 3, gaining 50%. Find :
   (i) C.P. of 4 oranges.
   (ii) C.P. of one orange.
   (iii) S.P. of one orange
   (iv) profit made by selling one orange
   (v) number of oranges need to be bought and sold in order to gain ₹ 24.
3. A man sells 12 articles for ₹ 80 gaining $33\frac{1}{3}$%. Find the number of articles bought by the man for ₹ 90.
4. The cost price of 20 articles is same as the selling price of 16 articles. Find the gain percent.
5. The selling price of 15 articles is equal to the cost price of 12 articles. Find the gain or loss as percent.
6. By selling 8 pens, Shyam loses equal to the cost price of 2 pens. Find his loss percent.
7. A shopkeeper bought rice worth ₹ 4,500. He sold one-third of it at 10% profit. If he desires a profit of 12% on the whole; find :
   (i) the selling price of the rest of the rice.
   (ii) the percentage profit on the rest of the rice.
8. Mohan bought a certain number of note-books for ₹ 600. He sold $\frac{1}{4}$ of them at 5 percent loss. At what price should he sell the remaining note-books so as to gain 10% on the whole ?
9. Raju sells a watch at 5% profit. Had he sold it for ₹ 24 more; he would have gained 11%. Find the cost price of the watch.
10. A man sold a bicycle at 5% profit. If the cost had been 30% less and the selling price ₹ 63 less, he would have made a profit of 30%. What was the cost price of the bicycle ?
11. Renu sold an article at a loss of 8 percent. Had she bought it at 10% less and sold for ₹ 36 more; she would have gained 20%. Find the cost price of the article.

8.5 DISCOUNT

In order to dispose of the old or the damaged goods; some shopkeepers offer a reduction on the marked price of their goods. This reduction is called **discount**.

1. Discount is always given on the marked price (M.P.).
2. Selling price = Marked Price – Discount

   i.e. S.P. = M.P. – Discount

   Also, discount = M.P. – S.P.
3. If discount = d%, $S.P. = \left(\frac{100-d}{100}\right) \times M.P. \Rightarrow M.P. = \left(\frac{100}{100-d}\right) \times S.P.$
4. Marked price is also called **list price, printed price**, etc.

Example 17 :

A tradesman marks his goods at 35 percent above the cost price and then allows purchasers a discount of 15 percent. What profit percent does he save ?

Solution :

Let the C.P. = ₹ 100

∴ Marked Price = ₹ (100 + 35) = ₹ 135

Discount = 15% of ₹ 135 = ₹ 20·25

∴ Selling Price = ₹ 135 – ₹ 20·25 = ₹ 114·75

∴ Profit = S.P. – C.P. = ₹ 114·75 – ₹ 100 = ₹ 14·75

and, **profit percent** $= \frac{14 \cdot 75}{100} \times 100\% =$ **14·75%** **(Ans.)**

Example 18 :

A dealer allows his customers a discount of 25% and still gains 25%. If an article costs ₹ 1,440 to the dealer; find :

(i) its selling price (ii) its marked price.

Solution :

(i) Since, C.P. = ₹ 1,440 and profit = 25%

∴ **S.P.** $= \left(\frac{100+25}{100}\right) \times$ ₹ 1,440 $\left[S.P. = \left(\frac{100 + \text{profit}\,\%}{100}\right) \times C.P.\right]$

$= \frac{125}{100} \times$ ₹ 1,440 = **₹ 1,800** **(Ans.)**

(ii) Since, S.P. = ₹ 1,800 and discount = 25%

∴ **M.P.** $= \left(\frac{100}{100-25}\right) \times$ ₹ 1,800 $\left[M.P. = \left(\frac{100}{100-d}\right) \times S.P.\right]$

$= \frac{100}{75} \times$ ₹ 1,800 = **₹ 2,400** **(Ans.)**

Example 19 :

Find a single discount (as percent) equivalent to successive discounts of 10% and 20%.

Solution :

Let M.P. = ₹ 100

1st discount = 10% of ₹ 100 = ₹ 10

Since, ₹ 100 – ₹ 10 = ₹ 90

∴ 2nd discount = 20% of ₹ 90 = ₹ 18

∴ S.P. = ₹ 90 – ₹ 18 = ₹ 72

Single equivalent discount = M.P. – S.P. = ₹ 100 – ₹ 72 = ₹ 28

Since, the discount of ₹ 28 is on ₹ 100

∴ **Required single equivalent discount as percent = 28%** **(Ans.)**

Single equivalent discount = Sum of all the discounts

= ₹ 10 + ₹ 18 = ₹ 28

Since, this ₹ 28 is discount on ₹ 100 (M.P.)

∴ **Single equivalent discount as percent = 28%** **(Ans.)**

Alternative method :

∵ $$\text{S.P.} = \text{M.P.} \times \left(\frac{100 - d_1}{100}\right) \times \left(\frac{100 - d_2}{100}\right)$$

$$= ₹\ 100 \times \frac{100 - 10}{100} \times \frac{100 - 20}{100} = ₹\ 72$$

∴ **Single equivalent discount as percent** = (100 – 72)% = **28%** **(Ans.)**

Example 20 :

An article is sold at two successive discounts of 50% each. Find the single equivatent discount as percent.

Solution :

Let M.P. of the article = ₹ 100

∴ Price of the article, after 1st discount

= ₹ 100 – 50% of ₹ 100

$$= ₹\ 100 - \frac{50}{100} \times ₹\ 100 = ₹\ 100 - ₹\ 50 = ₹\ 50$$

Now, the second discount will be on this ₹ 50

∴ Price of the article, after 2nd discount

= ₹ 50 – 50% of ₹ 50

$$= ₹\ 50 - \frac{50}{100} \times ₹\ 50 = ₹\ 50 - ₹\ 25 = ₹\ 25$$

∴ **Single equivalent discount as percent** = (100 – 25)% = **75%** **(Ans.)**

EXERCISE 8(D)

1. An article is marked for ₹ 1,300 and is sold for ₹ 1,144. Find the discount percent.
2. The marked price of a dinning table is ₹ 23,600 and is available at a discount of 8%. Find its selling price.
3. A wrist-watch is available at a discount of 9%. If the list-price of the watch is ₹ 1,400. Find the discount given and the selling price of the watch.
4. A shopkeeper sells an article for ₹ 248.50 after allowing a discount of 10%. Find the list price of the article.
5. A shopkeeper buys an article for ₹ 450. He marks it at 20% above the cost price. Find :
   (i) the marked price of the article.
   (ii) the selling price, if he sells the article at 10 percent discount.
   (iii) the percentage discount given by him, if he sells the article for ₹ 496.80.
6. The list price of an article is ₹ 800 and is available at a discount of 15 percent. Find :
   (i) the selling price of the article;
   (ii) the cost price of the article, if a profit of $13\frac{1}{3}$% is made on selling it.
7. An article is marked at ₹ 2,250. By selling it at a discount of 12%, the dealer makes a profit of 10%. Find :
   (i) the selling price of the article.
   (ii) the cost price of the article for the dealer.
8. By selling an article at 20% discount, a shopkeeper gains 25%. If the selling price of the article is ₹ 1,440; find :
   (i) the marked price of the article.
   (ii) the cost price of the article.
9. A shopkeeper marks his goods at 30 percent above the cost price and then gives a discount of 10 percent. Find his gain percent.
10. A ready-made garments shop in Delhi allows 20 percent discount on its garments and still makes a profit of 20 percent. Find the marked price of a dress which is bought by the shopkeeper for ₹ 400.
11. At 12% discount, the selling price of a pen is ₹ 13.20. Find its marked price. Also, find the new selling price of the pen, if it is sold at 5% discount.
12. The cost price of an article is ₹ 2,400 and it is marked at 25% above the cost price. Find the profit and the profit percent, if the article is sold at 15% discount.
13. Thirty articles are bought at ₹ 450 each. If one-third of these articles be sold at 6% loss; at what price must each of the remaining articles be sold in order to make a profit of 10% on the whole ?
14. The cost price of an article is 25% below the marked price. If the article is available at 15% discount and its cost price is ₹ 2,400; find :
    (i) its marked price (ii) its selling price
    (iii) the profit percent.
15. Find a single discount (as percent) equivalent to following successive discounts :
    (i) 20% and 12% (ii) 10%, 20% and 20%
    (iii) 20%, 10% and 5%
16. Find the single discount (as percent) equivalent to successive discounts of :
    (i) 80% and 80% (ii) 60% and 60%
    (iii) 60% and 80%

8.6 TAX

The central government as well as every state government need money :

(i) to provide various types of facilities.
(ii) for construction and maintenance of roads, hospitals, etc.
(iii) to meet the administrative expenses.
(iv) to execute the welfare and developement schemes.
(v) to meet the expenses on salaries of employees, etc.

In order to collect money for the expenses made by or to be made by different governments; the state governments as well as government at centre, levy taxes on the sale of goods.

8.7 COMPUTATION OF TAX

The calculation of tax is very easy as it involves only very simple concepts of percentage.

The rates of tax depend upon the nature of goods purchased and are different for different goods (items). Some items of necessity and/or of daily use for common persons are completely/partially exempted from Tax.

1. Tax is calculated on the sale price
2. $\text{Tax} = \dfrac{\text{Rate of tax} \times \text{Sale price}}{100}$
3. $\text{Rate of Tax} = \dfrac{\text{Tax}}{\text{sale price}} \times 100\%$

If the rate of tax is $x\%$, then price paid for the item $= \text{Its sale-price} \times \left(\dfrac{100 + x}{100}\right)$

The amount of money paid by a customer for an article

= The sale price of the article + Tax on it, if any.

Example 21 :

Rohit purchased a pair of shoes costing ₹ 850. Calculate the total amount to be paid by him, if the rate of Tax is 6%.

Solution :

Sale price of shoes = ₹ 850

and, Tax = 6% of ₹ 850 = ₹ 51

⇒ Total amount to be paid by Rohit

= ₹ 850 + ₹ 51 = **₹ 901 Ans.**

Direct method :

Amount paid by Rohit

$= ₹\ 850 \times \left(\dfrac{100 + 6}{100}\right)$

= **₹ 901** **Ans.**

Example 22 :

Mr. Gupta purchased an article for ₹ 702 including tax. If the rate of tax is 8%, find the sale price of the article.

Solution :

Let the sale price of the article be ₹ x

⇒ x + 8% of x = ₹ 702

⇒ $x = ₹\ 702 \times \dfrac{100}{108}$

= ₹ 650

⇒ Sale price of the article= **₹ 650 Ans.**

Direct method :

$₹\ 702 = \text{Sale-price} \left(\dfrac{100 + 8}{100}\right)$

⇒ $₹\ 702 \times \dfrac{100}{108} = \text{Sale-price}$

⇒ **Sale-price = ₹ 650** **Ans.**

Example 23 :

Geeta purchased a face-cream for ₹ 79.10 including tax. If the printed price of the face-cream is ₹ 70; find the rate of tax.

Solution :

Total price (including tax) = ₹ 79.10 and printed price = ₹ 70

⇒ Tax paid = ₹ 79.10 − ₹ 70 = ₹ 9.10

and, **the rate of sales tax** $= \dfrac{9.10}{70} \times 100\%$ = **13%** **Ans.**

Example 24 :

Mrs. Sharma purchased confectionery goods costing ₹ 165 on which the rate of tax is 6% and some tooth-paste, shaving-cream, soap, etc., costing ₹ 230 on which the rate of Tax is 10%. If she gives a five-hundred rupee note to the shopkeeper, what money will he return to Mrs. Sharma ?

Solution :

Price of confectionery goods including tax

= ₹ 165 + 6% of ₹ 165 = ₹ 174.90

Price of tooth-paste, shaving-cream, soap, etc. including tax

= ₹ 230 + 10% of ₹ 230 = ₹ 253

∴ Total amount to be paid by Mrs. Sharma

= ₹ 174.90 + ₹ 253 = ₹ 427.90

Since Mrs. Sharma gave a five-hundred rupee note to the shopkeeper, the money that the shopkeeper will return to Mrs. Sharma

= ₹ 500 – ₹ 427.90 = **₹ 72.10** **Ans.**

Example 25 :

Smith buys an article marked at ₹ 2,200. The rate of tax is 12%. He asks the shopkeeper to reduce the price of the article to such an extent that he does not have to pay anything more than ₹ 2,240 including tax. Calculate the reduction, as percent, needed in the marked price of the article.

Solution :

Let the cost of the article be reduced to ₹ x.

∴ x + 12% of x = 2,240

On solving, we get : x = 2,000

⇒ Reduced price of the article = ₹ 2,000

Reduction needed = ₹ 2,200 – ₹ 2,000 = ₹ 200

Hence, reduction as percent of marked-price

$$= \frac{\text{Reduction}}{\text{Marked-price}} \times 100\%$$

$$= \frac{200}{2{,}200} \times 100\% = 9\frac{1}{11}\%$$ **Ans.**

Example 26 :

The price of an article inclusive of 12% tax is ₹ 2,016. Find its marked price. If the tax is reduced to 7%, how much less will the customer pay for the article ?

Solution :

Let marked price be ₹ x.

∴ x + 12% of x = 2,016

On solving, we get : x = 1,800

∴ Marked price of the article = **₹ 1,800** **Ans.**

Since, new tax = 7%

∴ Now, the customer will pay = ₹ 1,800 + 7% of ₹ 1,800

$$= ₹\ 1,800 + \frac{107}{100} \times ₹\ 1,800 = ₹\ 1,926$$

∴ **Customer will pay for the article** ₹ (2,016 − 1,926) or **₹ 90 less** **Ans.**

EXERCISE 8(E)

1. Rajat purchases a wrist-watch costing ₹ 540. The rate of tax is 8%. Find the total amount paid by Rajat for the watch.
2. Ramesh paid ₹ 345.60 as tax on a purchase of ₹ 3,840. Find the rate of tax.
3. The price of a washing machine, inclusive of Tax, is ₹ 13,530/-. If the tax is 10%, find its basic (cost) price.
4. Sarita purchases biscuits costing ₹ 158 on which the rate of tax is 6%. She also purchases some cosmetic goods costing ₹ 354 on which the rate of tax is 9%. Find the total amount to be paid by Sarita.
5. The price of a T.V. set inclusive of tax of 9% is ₹ 13,407. Find its marked price. If tax is increased to 13%, how much more does the customer has to pay for the T.V. set ?
6. The price of an article is ₹ 8,250 which includes tax at 10%. Find how much more or less does a customer pay for the article, if the tax on the article:
   (i) increases to 15%
   (ii) decreases to 6%
   (iii) increases by 2%
   (iv) decreases by 3%.
7. A bicycle is available for ₹ 1,664 including tax. If the list price of the bicycle is ₹ 1,600, find :
   (i) the rate of Tax.
   (ii) the price a customer will pay for the bicycle if the tax is increased by 6%.
8. When the rate of Tax is decreased from 9% to 6% for a coloured T.V.; Mrs Geeta will save ₹ 780 in buying this T.V. Find the list price of the T.V.
9. A shopkeeper sells an article for ₹ 21,384 including 10% tax. However, the actual rate of Tax is 8%. Find the extra profit made by the dealer.

8.8 GOODS AND SERVICES TAX (GST)

Goods and services tax is a single indirect tax for the whole nation. This tax has been recently introduced by the government of India with the principle — **one nation and one tax**. With the introduction of GST all other indirect taxes such as excise, value added tax (VAT), service tax, etc. have been abolished.

- India has moved into GST from 01-07-2017.
- GST means a tax on supply of goods or services or both.
- By the implementation of GST, it is assumed that the movement of goods will now become cheaper and much simpler across the whole country.

- The tax paid by a customer, at the time of purchase of goods or services or both, is called **indirect tax**.
- All the indirect taxes, that used to exist prior to GST are merged into GST.

It is presumed that the GST will :

1. bring about uniformity in taxation throughout the country.
2. remove disparity of taxes in different states.
3. reduce tax evasion.

4. reduce the tax payer's difficulties by decreasing the physical interface between the tax-payer and the tax-authorities.

8.9 GST COMPRISES OF

- **Central-GST** (CGST), which is levied by central government for the ***intrastate transaction*** or ***movement*** of goods and services.

Intra-state means : supply within the same state.

- **State-GST** (SGST), which is levied by the state government for the ***intrastate transaction*** or ***movement*** of goods and services.
- **Integrated-GST** (IGST), which is levied by the central government for ***interstate transaction*** or ***movement*** (including imports) of goods and services.

Inter-state means : supply from one state to another state.

Note :

1. In case of intra-state transaction, the seller collects both CGST and SGST from the buyer and deposits CGST with central government whereas SGST remains with the state government.
2. In case of inter-state transaction, the seller collects IGST from the buyer and deposits it with the central government. The central government distributes IGST between central and state governments as per the law.

- The tax collected on the ***intra-sate movements*** of goods and services is shared equally by the central government and the state government.

Therefore, if the rate of GST = 12%

then the rate of central GST (CGST) = 6%

and the rate of state GST (SGST) = 6%

- In case of ***inter-state movements*** of goods and services, whole of the tax (GST) is levied by the central government

Therefore, if the rate of GST = 12%

rate of intergrated GST (IGST) = 12%

At present, the rates of GST are 0%, 5%, 12%, 18% and 28% as applicable.

Example 27 :

In an intra-state transaction, goods worth ₹ 20,000, are bought . If GST rate is 28%, find the amount of bill.

Solution :

In intra-state transaction, IGST = 00

Since, it is the case of intra-state transaction, the rate of CGST = $\frac{28}{2}\%$ = 14% and rate of SGST is also 14%

$\because$ Cost of goods = ₹ 20,000

CGST = 14% of ₹ 20,000 = ₹ 2,800

and, SGST = 14% of ₹ 20,000 = ₹ 2,800

$\therefore$ **Amount of bill = ₹ 25,600** **Ans.**

Example 28 :

In an intra-state transaction, goods worth ₹ 20,000 are bought at 40% discount. If GST rate is 28%, find the amount of bill.

Solution :

$\because$ Cost of goods bought = ₹ 20,000

Discount = 40% of ₹ 20,000 = ₹ 8,000

$\therefore$ Taxable cost of goods = ₹ 20,000 − ₹ 8,000 = ₹ 12,000

CGST = 14% of ₹ 12,000 = ₹ 1,680

SGST = 14% of ₹ 12,000 = ₹ 1,680

$\therefore$ **Amount of bill** = ₹ 12,000 + ₹ 1,680 + ₹ 1,680

= **₹ 15,360** **Ans.**

Example 29 :

In an inter-state transaction, goods worth ₹ 15,000 are bought at 20% discount. If the rate of GST is 5%, find the amount of bill.

Solution :

In case of inter-state transaction, CGST = 00 and SGST = 00

$\because$ Cost of goods bought = ₹ 15,000

Discount = 20% of ₹ 15,000 = ₹ 3,000

$\therefore$ Taxable cost of goods = ₹ 15,000 − ₹ 3,000 = ₹ 12,000

IGST = 5% of ₹ 12,000 = ₹ 600

$\therefore$ **Amount of bill** = ₹ 12,000 + ₹ 600

= **₹ 12,600** **Ans.**

Example 30 :

Some financial related services are provided by a bank in Delhi. How much will Manoj, a resident of Delhi, pay for the services costing ₹ 8,000, ₹ 6,500 and ₹ 9,500, if the rate of GST is 18% ?

Solution :

Cost of all the services = ₹ 8,000 + ₹ 6,500 + ₹ 9,500

= ₹ 24,000

Since, it is a case of intra-state transaction :

CGST = $\frac{18}{2}$ % of ₹ 24,000 = ₹ 2,160

SGST = ₹ 2,160

$\therefore$ **Manoj will pay** = ₹ 24,000 + ₹ 2,160 + ₹ 2,160

= **₹ 28,320** **Ans.**

Example 31 :

A dealer in Mumbai sells two different goods/services to Amar in the same city. One of these goods/services is marked at ₹ 4,000 (available at 40% discount) and the other is marked at ₹ 6,000 (available at 30% discount). If the rate of GST is 12%, find how much will Amar pay in all ?

Solution :

$\because$ Marked price of the 1st goods/services = ₹ 4,000 at 40% discount

$\therefore$ Their sale (discounted) price = ₹ 4,000 – 40% of ₹ 4,000

= ₹ 2,400

Marked price of the second goods/services = ₹ 6,000 at 30% discount

$\therefore$ Their sale (discounted) price = ₹ 6,000 – 30% of ₹ 6,000

= ₹ 4,200

Total sale-price = ₹ 2,400 + ₹ 4,200

= ₹ 6,600

Since it is a case of intra-state transaction and $\frac{12}{2}\% = 6\%$

$\therefore$ CGST = 6% of ₹ 6,600 = ₹ 396

SGST = 6% of ₹ 6,600 = ₹ 396

$\therefore$ **Amar will pay** = ₹ 6,600 + ₹ 396 + ₹ 396

= **₹ 7,392** **Ans.**

EXERCISE 8(F)

1. Some goods/services cost ₹ 16,000 and rate of GST on them is 12%. Find the amount of bill, in case of :
   (i) intra-state transaction.
   (ii) inter-state transaction.
2. John belongs to Delhi. He buys goods, worth ₹ 25,000 from a shop in Delhi. If the rate of GST is 5%, find how much money in all, will John pay for these goods ?
3. Find the amount of bill for the following inter-state transaction of goods/services :
   Cost of transaction = ₹ 30,000; discount = 30% and GST = 28%.
4. For both the following inter-state transaction of services, find the total amount of bill.
   (i) Cost of services = ₹ 5,000, discount = 20% and GST = 12%
   (ii) Cost of services = ₹ 12,500, discount = 40% and GST = 18%
5. A shopkeeper in Indore, sells 20 identical articles for ₹ 450 each. Find the amount of bill if he gives 20% discount and then charges GST = 28%
6. A dealer in Bihar supplied goods to a dealer in Mumbai. The dealer in Mumbai buys :
   (i) 40 articles for ₹ 800 each at 30% discount
   (ii) 75 articles for ₹ 1,000 each at 20% disocunt.

   If the rate of GST on the whole is 12%, find how much will the dealer at Mumbai pay to dealer in Bihar.

INTEREST 9

(Simple and Compound)

9.1 REVIEW

1. Principal (P)	It is the money (sum) borrowed or the sum lent.
2. Interest (I)	(i) It is the money paid by the borrower to the money lender, for the use of money borrowed. (ii) **The simple interest** (S.I.) and **the interest** mean the same.
3. Rate (R)	(i) It is the interest on every ₹ 100. (ii) If rate is 12% per year; it means, ₹ 12 is the interest of one year on ₹ 100. (iii) If rate is 2% per month; it means, ₹ 2 is the interest of one month on ₹ 100.
4. Time (T)	(i) It is the time for which the money is lent or is borrowed. (ii) If the rate of interest is per year, say 8% per year; the time (T) must be taken in years. (iii) If the rate of interest is per month, say 1·5% per month; the time (T) must be taken in months.
5. Amount (A)	It is the total of the sum borrowed and the interest on it. *i.e.* Amount = Sum borrowed + Interest ⇒ Amount = Principal + Interest *i.e.* **A = P + I**

The interest (I) depends on :

1. Principal (P) 2. Rate or rate percent (R) 3. The time (T).

The formula for calculating interest $\mathbf{I} = \frac{\mathbf{P \times R \times T}}{\mathbf{100}}$

Since, Amount = Principal + Interest

⇒ $\mathbf{A = P + I}$

⇒ $\mathbf{A = P + \frac{P \times R \times T}{100}}$ *i.e.* $\mathbf{A = P\left(1 + \frac{RT}{100}\right)}$

***Example* 1 :**

Find the simple interest on ₹ 1,300 from December 23, 2002 to May 18, 2003 at $7\frac{1}{2}$% per annum.

***Solution* :**

(i) **Given :** P = ₹ 1,300 and R = $\frac{15}{2}$ %

Also, T = 146 days

$= \frac{146}{365}$ years $= \frac{2}{5}$ years

∴ S.I. = ₹ $\frac{1,300 \times 15 \times 2}{100 \times 2 \times 5}$

= ₹ 39 **(Ans.)**

To calculate time (T) :

Dec.	=	8 days	(31 – 23)
Jan.	=	31 days	
Feb.	=	28 days	
March	=	31 days	
April	=	30 days	
May	=	18 days	
Total	=	**146 days**	

1. The day on which the money is borrowed is not included in the time.
2. The day on which the money is paid back to the money lender is included in the time.

9.2 TO FIND THE PRINCIPAL (P) ; The Rate Per cent (R) and The Time (T)

The formula for interest, $I = \frac{P \times R \times T}{100}$ can be re-written as :

(i) $P = \frac{100 \times I}{R \times T}$ *i.e.* $\text{Principal} = \frac{100 \times \text{Interest}}{\text{Rate} \times \text{Time}}$

(ii) $R = \frac{100 \times I}{P \times T}$ *i.e.* $\text{Rate} = \frac{100 \times \text{Interest}}{\text{Principal} \times \text{Time}}$

and, (iii) $T = \frac{100 \times I}{P \times R}$ *i.e.* $\text{Time} = \frac{100 \times \text{Interest}}{\text{Principal} \times \text{Rate}}$

EXERCISE 9(A)

1. Find the interest and the amount on :
   (i) ₹ 750 in 3 years 4 months at 10% per annum.
   (ii) ₹ 5,000 at 8% per year from 23rd December 2011 to 29th July 2012.
   (iii) ₹ 2,600 in 2 years 3 months at 1% per month.
   (iv) ₹ 4,000 in $1\frac{1}{3}$ years at 2 paise per rupee per month.
2. Rohit borrowed ₹ 24,000 at 7·5 percent per year. How much money will he pay at the end of 4 years to clear his debt ?
3. The interest on a certain sum of money is ₹ 1,480 in 2 years and at 10 percent per year. Find the sum of money.
4. On what principal will the simple interest be ₹ 7,008 in 6 years 3 months at 5% per year ?
5. Find the principal which will amount to ₹ 4,000 in 4 years at 6.25% per annum.
6. (i) At what rate per cent per annum will ₹ 630 produce an interest of ₹ 126 in 4 years ?
   (ii) At what rate percent per year will a sum double itself in $6\frac{1}{4}$ years ?
7. (i) In how many years will ₹ 950 produce ₹ 399 as simple interest at 7% ?
   (ii) Find the time in which ₹ 1,200 will amount to ₹ 1,536 at 3·5% per year.
8. The simple interest on a certain sum of money is $\frac{3}{8}$ of the sum in $6\frac{1}{4}$ years. Find the rate percent charged.
9. What sum of money borrowed on 24th May will amount to ₹ 10,210·20 on 17th October of the same year at 5 percent per annum simple interest.
10. In what time will the interest on a certain sum of money at 6% be $\frac{5}{8}$ of itself ?
11. Ashok lent out ₹ 7,000 at 6% and ₹ 9,500 at 5%. Find his total income from the interest in 3 years.
12. Raj borrows ₹ 8,000; out of which ₹ 4,500 at 5% and remaining at 6%. Find the total interest paid by him in 4 years.
13. Mohan lends ₹ 4,800 to John for $4\frac{1}{2}$ years and ₹ 2,500 to Shyam for 6 years and receives a total sum of ₹ 2,196 as interest. Find the rate per cent per annum, it being the same in both the cases.
14. John lent ₹ 2,550 to Mohan at 7·5 per cent per annum. If Mohan discharges the debt after 8 months by giving an old black and white television and ₹ 1,422·50. Find the price of the television.

Example 2 :

Find the rate of interest per year, if the interest charged for 8 months be 0·06 times of the money borrowed.

Solution :

Let the money borrowed be ₹ 100 *i.e.* P = ₹ 100

Given : Interest (I) charged = 0·06 × ₹ 100 = ₹ 6 and $T = \frac{8}{12}$ years = $\frac{2}{3}$ years

$\therefore$ **Rate** $= \frac{I \times 100}{P \times T}\% = \frac{6 \times 100}{100 \times \frac{2}{3}}\% =$ **9%** **(Ans.)**

Example 3 :

A sum of money lent out at 9 percent for 5 years produces twice as much interest as ₹ 4,800 in $4\frac{1}{2}$ years at 10 per cent. Find the sum.

Solution :

Let the required sum be ₹ *x*. According to the given statement :

$$₹\ \frac{x \times 9 \times 5}{100} = 2 \times ₹\ \frac{4,800 \times 10 \times 9}{100 \times 2}$$

On solving, we get : $x =$ ₹ 9,600

$\therefore$ **The required sum = ₹ 9,600** **(Ans.)**

Example 4 :

A certain sum amounts to ₹ 9,440 in 3 years and to ₹ 10,400 in 5 years. Find the sum and the rate percent.

Solution :

Amount in 3 years = ₹ 9,440 ⇒ P + I of 3 years = ₹ 9,440 I

Amount in 5 years = ₹ 10,400 ⇒ P + I of 5 years = ₹ 10,400 II

$\therefore$ Eq. II – Eq. I ⇒ Interest of 2 years = ₹ 10,400 – ₹ 9,440 = ₹ 960

⇒ Interest of 1 year = ₹ $\frac{960}{2}$ = ₹ 480

And, Interest of 3 years = ₹ 480 × 3 = ₹ 1,440

From equation I, we get :

P + ₹ 1,440 = ₹ 9,440 ⇒ P = ₹ 8,000

Taking P = ₹ 8,000, I = ₹ 480 and T = 1 year

We get, rate $= \frac{I \times 100}{P \times T}\% = \frac{480 \times 100}{8000 \times 1}\% = 6\%$

$\therefore$ **The sum = ₹ 8,000 and rate percent = 6%** **(Ans.)**

EXERCISE 9(B)

1. The interest on a certain sum of money is 0·24 times of itself in 3 years. Find the rate of interest.
2. If ₹ 3,750 amount to ₹ 4,620 in 3 years at simple interest. Find :
   (i) the rate of interest.
   (ii) the amount of ₹ 7,500 in $5\frac{1}{2}$ years at the same rate of interest.
3. A sum of money, lent out at simple interest, doubles itself in 8 years. Find :
   (i) the rate of interest.
   (ii) in how many years will the sum become triple (three times) of itself at the same rate percent ?
4. Rupees 4,000 amount to ₹ 5,000 in 8 years; in what time will ₹ 2,100 amount to ₹ 2,800 at the same rate ?
5. What sum of money lent at 6·5% per annum will produce the same interest in 4 years as ₹ 7,500 produce in 6 years at 5% per annum ?
6. A certain sum amounts to ₹ 3,825 in 4 years and to ₹ 4,050 in 6 years. Find the rate percent and the sum.
7. At what rate per cent of simple interest will the interest on ₹ 3,750 be one-fifth of itself in 4 years? To what will it amount in 15 years?
8. On what date will ₹ 1,950 lent on 5th January, 2011 amount to ₹ 2,125·50 at 5 per cent per annum simple interest ?
9. If the interest on ₹ 2,400 be more than the interest on ₹ 2,000 by ₹ 60 in 3 years at the same rate per cent; find the rate.
10. Divide ₹ 15,600 into two parts such that the interest on one at 5 percent for 5 years may be equal to that on the other at $4\frac{1}{2}$ per cent for 6 years.

9.3 COMPOUND INTEREST (By simple interest method)

Money is said to be lent at **compound Interest,** when at the end of a year (or, some other fixed period) the interest is not paid to the money lender, but is added to the sum lent, and the amount thus obtained becomes the principal for the next period. This process is repeated until the amount for the last period has been found. The difference between the original sum and the final amount is the compound interest.

i.e. Compound Interest = Final amount – Original principal

= Amount – Principal

⇒ **C.I. = A – P**

Example 5 :

Calculate the compound interest on ₹ 6,000 for 2 years at 10% per year.

Solution :

For 1st year : Principal (P) = ₹ 6,000, Rate (R) = 10% and Time (T) = 1 year

∴ Interest $= \frac{P \times R \times T}{100} = \frac{₹\,6{,}000 \times 10 \times 1}{100} = ₹\,600$

And, amount = principal + interest accrued

= ₹ 6,000 + ₹ 600 = ₹ 6,600

For 2nd year : P = ₹ 6,600, R = 10% and T = 1 year

∴ Interest $= \frac{P \times R \times T}{100} = \frac{₹\,6{,}600 \times 10 \times 1}{100} = ₹\,660$

And, final amount = principal + interest

= ₹ 6,600 + ₹ 660 = ₹ 7,260

∴ **Compound interest** = Final amount – Original principal

= ₹ 7,260 – ₹ 6,000 = **₹ 1,260** **(Ans.)**

Example 6 :

Calculate the amount and the compound interest on ₹ 8,000 for 3 years at 5% per annum.

Solution :

For 1st year : Principal (P) = ₹ 8,000, Rate (R) = 5% and Time (T) = 1 year

∴ Interest = $\frac{₹\,8{,}000 \times 5 \times 1}{100}$ = ₹ 400

And, amount = P + I = ₹ 8,000 + ₹ 400 = ₹ 8,400

For 2nd year : P = ₹ 8,400; R = 5% and T = 1 year

∴ Interest = $\frac{₹\,8{,}400 \times 5 \times 1}{100}$ = ₹ 420

And, amount = ₹ 8,400 + ₹ 420 = ₹ 8,820

For 3rd year : P = ₹ 8,820; R = 5% and T = 1 year

∴ Interest = $\frac{₹\,8{,}820 \times 5 \times 1}{100}$ = ₹ 441

⇒ **Amount** = ₹ 8,820 + ₹ 441 = **₹ 9,261** **(Ans.)**

And, **C.I.** = ₹ 9,261 – ₹ 8,000 = **₹ 1,261** **(Ans.)**

In order to understand the difference between simple interest and compound interest, study the following table prepared with a principal of ₹ 8,000 and rate of interest 5% per annum:

		Under Simple Interest	Under Compound Interest
First year	Principal Interest at 5% Amount	₹ 8,000 ₹ 400 **₹ 8,400**	₹ 8,000 ₹ 400 **₹ 8,400**
Second year	Principal Interest at 5% Amount	₹ 8,000 ₹ 400 **₹ (8,400 + 400) = ₹ 8,800**	₹ 8,400 ₹ 420 **₹ 8,820**
Third year	Principal Interest at 5% Amount	₹ 8,000 ₹ 400 **₹ (8,800 + 400) = ₹ 9,200**	₹ 8,820 ₹ 441 **₹ 9,261**

∴ Interest earned by simple Interest

= ₹ (9,200 – 8,000) = ₹ 1,200

And, interest earned by compound Interest

= ₹(9,261 – 8,000) = ₹ 1,261

1. In the case of simple interest, the principal remains the same every year for the whole period, whereas in case of compound interest, the principal keeps on increasing every year.
2. Since, in the case of the compound interest, the principal keeps on increasing every year, the interest of every year also keeps on increasing.

3. The compound interest for the certain period can also be obtained by adding the interest of different years.

For example, in example 5, the compound interest of two years

= Interest of the first year + Interest of the second year

= ₹ 600 + ₹ 660 = **₹ 1,260.**

And, in example 6,

C.I. in 3 years = ₹ 400 + ₹ 420 + ₹ 441 = **₹ 1,261**

Example 7 :

Calculate the amount and the compound interest on ₹ 5,000 in 2 years, if the rates of interest for the successive years be 8% and 10% respectively.

Solution :

For 1st year : Principal (P) = ₹ 5,000, Rate (R) = 8% and Time (T) = 1 year

$\therefore$ Interest $= \frac{₹ 5,000 \times 8 \times 1}{100} = ₹ 400$

And, amount = ₹ 5,000 + ₹ 400 = ₹ 5,400

For 2nd year : P = ₹ 5,400; R = 10% and T = 1 year

$\therefore$ Interest $= \frac{₹ 5,400 \times 10 \times 1}{100} = ₹ 540$

$\Rightarrow$ **Amount** = ₹ 5,400 + ₹ 540 = **₹ 5,940** **(Ans.)**

And, **C.I.** = ₹ 5,940 − ₹ 5,000 = **₹ 940** **(Ans.)**

Example 8 :

Calculate the compound interest for the second year on ₹ 4,000 invested for 3 years at 10% per annum.

Solution :

For 1st year : Principal (P) = ₹ 4,000, Rate (R) = 10% and Time (T) = 1 year

$\therefore$ Interest $= \frac{₹ 4,000 \times 10 \times 1}{100} = ₹ 400$

And, amount = ₹ 4,000 + ₹ 400 = ₹ 4,400 [$\because$ A = P + I]

For 2nd year : P = ₹ 4,400; R = 10% and T = 1 year

$\therefore$ Interest $= \frac{₹ 4,400 \times 10 \times 1}{100} = ₹ 440$

$\Rightarrow$ **Compound interest for second year = ₹ 440** **(Ans.)**

Example 9 :

Calculate the difference between the compound interest and the simple interest on ₹ 10,000 in two years and at 5% per year.

Solution :

For the Compound interest :

Principal for 1st year = ₹ 10,000

$$\text{Interest of 1st year} = \frac{₹\,10{,}000 \times 5 \times 1}{100} = ₹\ 500$$

$$\text{Amount of 1st year} = ₹\ 10{,}500$$

$$\text{Principal for 2nd year} = ₹\ 10{,}500$$

$$\text{Interest of 2nd year} = \frac{₹\,10{,}500 \times 5 \times 1}{100} = ₹\ 525$$

$$\text{Amount of 2nd year} = ₹\ 11{,}025$$

$\therefore$ The compound interest in 2 years = ₹ 11,025 − ₹ 10,000 = ₹ 1,025

For the simple interest :

P = ₹ 10,000; R = 5% and T = 2 years

$\Rightarrow$ $\text{Interest} = \dfrac{₹\,10{,}000 \times 5 \times 2}{100} = ₹\,1{,}000$

$\Rightarrow$ **The required difference between C.I. and S.I.** = C.I. − S.I.

= ₹ 1,025 − ₹ 1,000 = **₹ 25 (Ans.)**

9.4 INTEREST COMPOUNDED HALF-YEARLY

Example 10 :

Calculate the amount and the compound interest on ₹ 8,000 for 1 year at 10% per annum compounded half-yearly.

Solution :

For 1st half-year : P = ₹ 8,000, R = 10% and Time $T = \frac{1}{2}$ year

$\therefore$ $\text{Interest} = \dfrac{P \times R \times T}{100} = \dfrac{₹\,8{,}000 \times 10 \times 1}{100 \times 2} = ₹\,400$

and, amount = P + I

= ₹ 8,000 + ₹ 400 = ₹ 8,400

For 2nd half-year : P = ₹ 8,400, R = 10% and $T = \frac{1}{2}$ year

$\therefore$ $\text{Interest} = \dfrac{P \times R \times T}{100} = \dfrac{₹\,8{,}400 \times 10 \times 1}{100 \times 2} = ₹\,420$

and, amount = P + I

= ₹ 8,400 + ₹ 420 = ₹ 8,820

$\therefore$ **Required amount** = **₹ 8,820** **(Ans.)**

And, **required C.I.** = ₹ 8,820 − ₹ 8,000 = **₹ 820** **(Ans.)**

Example 11 :

Calculate the amount and the compound interest on ₹ 5,000 in $1\frac{1}{2}$ years when interest is compound half-yearly at the rate of 20% per year.

Solution :

For 1st half-year : $P = ₹ 5,000$, $R = 20\%$ and Time $T = \frac{1}{2}$ year

$\therefore$ Interest $= \frac{P \times R \times T}{100} = \frac{₹ 5,000 \times 20 \times 1}{100 \times 2} = ₹ 500$

and, amount $= P + I$

$= ₹ 5,000 + ₹ 500 = ₹ 5,500$

For 2nd half-year : $P = ₹ 5,500$, $R = 20\%$ and $T = \frac{1}{2}$ year

$\therefore$ Interest $= \frac{P \times R \times T}{100} = \frac{₹ 5,500 \times 20 \times 1}{100 \times 2} = ₹ 550$

and, amount $= P + I$

$= ₹ 5,500 + ₹ 550 = ₹ 6,050$

For 3rd half-year : $P = ₹ 6,050$, $R = 20\%$ and $T = \frac{1}{2}$ year

$\therefore$ Interest $= \frac{P \times R \times T}{100} = \frac{₹ 6,050 \times 20 \times 1}{100 \times 2} = ₹ 605$

and, amount $= P + I$

$= ₹ 6,050 + ₹ 605 = ₹ 6,655$

$\therefore$ **Amount $= ₹ 6,655$** **(Ans.)**

And, C.I. $= ₹ 6,655 - ₹ 5,000 =$ **₹ 1,655** **(Ans.)**

9.5 USING FORMULAE

1. Let A be the amount of ₹ P in n years and at r% per annum.

   (i) If compounded yearly :

   $$A = P\left(1 + \frac{r}{100}\right)^n$$

   (ii) If compounded half-yearly :

   $$A = P\left(1 + \frac{r}{2 \times 100}\right)^{n \times 2}$$

2. Let R_1% be the rate of interest for first year, R_2% be the rate of interest for second year, R_3% be the rate of interest for third year and so on.

   $$A = \left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right) \ldots\ldots$$

Example 12 :

Find the amount and the compound interest on ₹ 16,000 in 3 years at 10% per annum.

Solution :

Given : $P = ₹ 16,000$; $n = 3$ years and $r = 10\%$

$\therefore$ Amount (A) $= P\left(1+\frac{r}{100}\right)^n$

$$= ₹\ 16,000\left(1+\frac{10}{100}\right)^3$$

$$= ₹\ 16,000\left(\frac{11}{10}\right)^3$$

$$= ₹\ 16,000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} = ₹\ 21,296$$

$\therefore$ **Amount = ₹ 21,296** **(Ans)**

and, **C.I.** = ₹ 21,296 – ₹ 16,000 = **₹ 5,296** **(Ans.)**

***Example* 13 :**

Find the amount and the compound interest on ₹ 8,000 in $1\frac{1}{2}$ years at 10% per year compounded half-yearly.

***Solution* :**

Given : P = ₹ 8,000; $n = \frac{3}{2}$ years and $r = 10\%$

$\therefore$ Amount (A) $= P\left(1+\frac{r}{2\times100}\right)^{n\times2}$ [Since compounded half-yearly]

$$= ₹\ 8,000\left(1+\frac{10}{2\times100}\right)^{\frac{3}{2}\times2}$$

$$= ₹\ 8,000\left(\frac{21}{20}\right)^3$$

$$= ₹\ 8,000 \times \frac{9261}{8000} = ₹\ 9,261$$

$\therefore$ **Amount = ₹ 9,261** **(Ans.)**

and, **C.I.** = ₹ 9,261 – ₹ 8,000 = **₹ 1,261** **(Ans.)**

***Example* 14 :**

Find the amount and the compound interest on ₹ 12,000 in 3 years, when the rates of interest for successive years are 10%, 12% and 15% respectively.

***Solution* :**

Given : P = ₹ 12,000; $n = 3$ years; $R_1 = 10\%$, $R_2 = 12\%$ and $R_3 = 15\%$

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

$$= ₹\ 12,000\left(1+\frac{10}{100}\right)\left(1+\frac{12}{100}\right)\left(1+\frac{15}{100}\right)$$

$$= ₹\ 12,000 \times \frac{11}{10} \times \frac{28}{25} \times \frac{23}{20}$$

$= ₹\ 17,001.60$ **(Ans.)**

and, C.I. $= ₹\ 17001.60 - ₹\ 12,000 = ₹\ 5,001.60$ **(Ans.)**

EXERCISE 9(C)

1. A sum of ₹ 8,000 is invested for 2 years at 10% per annum compound interest. Calculate :
   (i) interest for the first year.
   (ii) principal for the second year.
   (iii) interest for the second year.
   (iv) final amount at the end of the second year.
   (v) compound interest earned in 2 years.
2. A man borrowed ₹ 20,000 for 2 years at 8% per year compound interest. Calculate :
   (i) the interest of the first year.
   (ii) the interest of the second year.
   (iii) the final amount at the end of the second year.
   (iv) the compound interest of two years.
3. Calculate the amount and the compound interest on ₹ 12,000 in 2 years at 10% per year.
4. Calculate the amount and the compound interest on ₹ 10,000 in 3 years at 8% per annum.
5. Calculate the compound interest on ₹ 5,000 in 2 years; if the rates of interest for successive years be 10% and 12% respectively.
6. Calculate the compound interest on ₹ 15,000 in 3 years; if the rates of interest for successive years be 6%, 8% and 10% respectively.
7. Mohan borrowed ₹ 16,000 for 3 years at 5% per annum compound interest. Calculate the amount that Mohan would have to pay at the end of 3 years.
8. Rekha borrowed ₹ 40,000 for 3 years at 10% per annum compound interest. Calculate the interest paid by her for the second year.
9. Calculate the compound interest for the second year on ₹ 15,000 invested for 5 years at 6% per annum.
10. A man invests ₹ 9,600 at 10% per annum compound interest for 3 years. Calculate :
    (i) the interest for the first year.
    (ii) the amount at the end of the first year.
    (iii) the interest for the second year.
    (iv) the interest for the third year.
11. A person invests ₹ 5,000 for two years at a certain rate of interest compounded annually. At the end of one year, this sum amounts to ₹ 5,600. Calculate.
    (i) the rate of interest per annum.
    (ii) the amount at the end of the second year.
12. Calculate the difference between the compound interest and the simple interest on ₹ 7,500 in two years and at 8% per annum.
13. Calculate the difference between the compound interest and the simple interest on ₹ 8,000 in three years at 10% per annum.
14. Rohit borrowed ₹ 40,000 for 2 years at 10% per annum C.I. and Manish borrowed the same sum for the same time at 10·5% per annum simple interest. Which of these two gives less interest and by how much ?
15. Mr. Sharma lends ₹ 24,000 at 13% p.a. simple interest and an equal sum at 12% p.a. compound interest. Find the total interest earned by Mr. Sharma in 2 years.
16. Peter borrows ₹ 12,000 for 2 years at 10% p.a. compound interest. He repays ₹ 8,000 at the end of first year. Find :
    (i) the amount at the end of first year, before making the repayment.
    (ii) the amount at the end of first year, after making the repayment.
    (iii) the principal for the second year.
    (iv) the amount to be paid at the end of second year, to clear the account.
17. Gautam takes a loan of ₹ 16,000 for 2 years at 15% p.a. compound interest. He repays ₹ 9,000 at the end of first year. How much must he pay at the end of second year to clear the debt ?

18. A certain sum of money, invested for 5 years at 8% p.a. simple interest, earns an interest of ₹ 12,000. Find :
    (i) the sum of money.
    (ii) the compound interest earned by this money in two years at 10% p.a. compound interest.
19. Find the amount and the C.I. on ₹ 12,000 in one year at 10% per annum compounded half-yearly.
20. Find the amount and the C.I. on ₹ 8,000 in $1\frac{1}{2}$ years at 20% per year compounded half-yearly.
21. Find the amount and the compound interest on ₹ 24,000 for 2 years at 10% per annum compounded yearly.
22. Find the amount and the compound interest on ₹ 16,000 for 3 years at 5% per annum compounded annually.
23. Find the amount and the compound interest on ₹ 20,000 for $1\frac{1}{2}$ years at 10% per annum compounded half-yearly.
24. Find the amount and the compound interest on ₹ 32,000 for 1 year at 20% per annum compounded half-yearly.
25. Find the amount and the compound interest on ₹ 4,000 in 2 years, if the rate of interest for first year is 10% and for the second year is 15%.
26. Find the amount and the compound interest on ₹ 10,000 in 3 years, if the rates of interest for the successive years are 10%, 15% and 20% respectively.

DIRECT AND INVERSE VARIATIONS 10

(Including Time and Work)

10.1 VARIATIONS

Two quantities are said to be in variation if change in one quantity causes corresponding change in the other quantity.

We come across many such situations in our day-to-day life where we see variation in one quantity bringing variation in another quantity.

For example :

1. **More** is the number of articles bought, **more** is their cost.
2. **More** is the money deposited in a bank **more** is the interest earned.
3. **More** is the speed of a car, **less** is the time taken to cover a certain distance.

10.2 TYPES OF VARIATIONS

There are two types of variations :

1. Direct variation 2. Inverse variation

1. **Direct Variation :** Two quantities are said to have direct variation, if the increase in one quantity causes an increase in the other and, the decrease in one quantity causes a decrease in the other.
2. **Inverse Variation :** Two quantities are said to vary inversely, if any increase in one quantity, decreases the other quantity. And, decrease in one quantity, increases the second quantity.

1. If 7 bags weigh 560 kg $\Rightarrow$ 1 bag weighs $\frac{560}{7}$ kg = 80 kg

   And, 14 bags weigh 14×80 kg = 1120 kg

   Clearly, it is a **case of direct variation.**

2. If 7 men do a certain piece of work in 560 days

   $\Rightarrow$ 1 man will do the same work in 7×560 days = 3920 days

   And, 14 men will do the same work in $\frac{3920}{14}$ days = 280 days.

   It is the **case of inverse variation.**

10.3 DIRECT VARIATION

Consider the following table which shows the cost of different number of identical articles.

Number of articles	8	10	15	25	30	40	60
Cost (in ₹)	40	50	75	125	150	200	300

Now, in each case, if x and y are in direct variation (proportion), $\frac{x}{y}$ is a constant. On finding the ratio of the number of articles and their costs, we get :

$\frac{8}{40} = \frac{1}{5}$, $\frac{10}{50} = \frac{1}{5}$, $\frac{15}{75} = \frac{1}{5}$ and so on.

In this case, the cost of articles (in ₹) varies with the number of articles in such a way that the ratio between the number of articles and their cost is always the same.

i.e. $\frac{\text{number of articles}}{\text{their cost}}$ is always the same.

Such a case is an example of direct variation and for this we say :

If two quantities x and y are in direct variation, then the ratio $\frac{x}{y}$ is always constant.

If two quantities x and y are in direct variation, the ratio of any two or more values of x is in proportion to the corresponding values of y.

i.e. if x_1, x_2, x_3, ... are some different values of x and y_1, y_2, y_3, ... are corresponding values of y, then $\frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3}$ = ... = a constant.

In the table, given above,

$\frac{x_1}{y_1} = \frac{8}{40} = \frac{1}{5}$, $\frac{x_2}{y_2} = \frac{10}{50} = \frac{1}{5}$, $\frac{x_3}{y_3} = \frac{15}{75} = \frac{1}{5}$ and so on.

Example 1 :

Observe the table, given below, and find whether x and y are in direct variation or not :

x	6	10	14	18	24
y	18	30	42	54	72

Solution :

$\because \quad \frac{x_1}{y_1} = \frac{6}{18} = \frac{1}{3}, \quad \frac{x_2}{y_2} = \frac{10}{30} = \frac{1}{3}$

$\frac{x_3}{y_3} = \frac{14}{42} = \frac{1}{3}, \quad \frac{x_4}{y_4} = \frac{18}{54} = \frac{1}{3}$ and $\frac{x_5}{y_5} = \frac{24}{72} = \frac{1}{3}$

$\Rightarrow \quad \frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} = \frac{x_4}{y_4} = \frac{x_5}{y_5}$ = a constant

$\therefore$ **x and y are in direct variation** **(Ans.)**

Example 2 :

From the table, given below, find whether x and y are in direct proportion or not.

x	30	20	24	16
y	36	30	30	20

Solution :

$\because \quad \frac{x_1}{y_1} = \frac{30}{36} = \frac{5}{6}, \frac{x_2}{y_2} = \frac{20}{30} = \frac{2}{3},$

$\frac{x_3}{y_3} = \frac{24}{30} = \frac{4}{5}$ and $\frac{x_4}{y_4} = \frac{16}{20} = \frac{4}{5}$

$\Rightarrow \quad \frac{x_1}{y_1} \neq \frac{x_2}{y_2} \neq \frac{x_3}{y_3} = \frac{x_4}{y_4}$

$\therefore$ **x and y are not in direct variation.** **(Ans.)**

Example 3 :

If p and q are in direct variation, find the values of x, y and z in the table, given below :

p	6	x	y	30
q	72	60	96	z

Solution :

p and q are in direct variation

$\Rightarrow \quad \frac{6}{72} = \frac{x}{60} = \frac{y}{96} = \frac{30}{z}$

$\Rightarrow \quad \frac{6}{72} = \frac{x}{60}, \frac{6}{72} = \frac{y}{96}$ and $\frac{6}{72} = \frac{30}{z}$

$\Rightarrow \quad x = \frac{6}{72} \times 60, y = \frac{6}{72} \times 96$ and $z = 30 \times \frac{72}{6}$

$\Rightarrow$ **x = 5, y = 8 and z = 360** **(Ans.)**

Example 4 :

If x and y vary directly, find the missing entries :

x	5	a	b	42
y	10	16	48	c

Solution :

Since, x and y vary directly

$\frac{5}{10} = \frac{a}{16} = \frac{b}{48} = \frac{42}{c}$

$\Rightarrow \quad \frac{5}{10} = \frac{a}{16}, \frac{5}{10} = \frac{b}{48}$ and $\frac{5}{10} = \frac{42}{c}$

$\Rightarrow$ **a = 8, b = 24 and c = 84** **(Ans.)**

Example 5 :

A car covers a distance of 216 km consuming 12 litres of petrol. How much distance will it cover consuming 18 litres of petrol ?

Solution :

Let the car cover x km consuming 18 litres of petrol.

Petrol (in litres)	12	18
Distance (in km)	216	x

It is a case of direct variation

$\Rightarrow \quad \frac{x_1}{y_1} = \frac{x_2}{y_2} \Rightarrow \frac{12}{216} = \frac{18}{x}$

i.e. $12x = 18 \times 216 \quad \Rightarrow x = \frac{18 \times \overset{18}{\cancel{216}}}{\cancel{12}} = 324$ km

∴ **The distance covered by the car consuming 18 litres of petrol = 324 km (Ans.)**

Example 6 :

15 m of cloth costs ₹ 1,940. Find, how many metres of cloth of the same kind can be bought for ₹ 4,656 ?

Solution :

Let x m of cloth can be bought for ₹ 4,656

Cloth (in m)	15	x
Cost (in ₹)	1940	4656

Since, it is a case of direct variation

$\therefore \quad \frac{15}{1940} = \frac{x}{4656}$

$\Rightarrow \quad x = \frac{15}{1940} \times 4656 = 36$

$\Rightarrow$ **36 m cloth can be bought for ₹ 4,656 (Ans.)**

Example 7 :

A journey of 240 km costs ₹ 4,080. How much distance will cost ₹ 7,344 ?

Solution :

Let the journey of x km distance costs ₹ 7,344

Distance (in km)	240	x
Cost (in ₹)	4080	7344

Clearly $\frac{240}{4080} = \frac{x}{7344}$

$\Rightarrow \quad x = \frac{240}{4080} \times 7344 = 432$

$\therefore$ **A journey of 432 km will cost ₹ 7,344** **(Ans.)**

Example 8 :

A vertical pole, 2 m 80 cm high, casts a shadow 1 m 60 cm long. Find, at the same time,

(i) the length of shadow casted by a 5 m 25 cm high pole

(ii) the height of the pole which casts a shadow of length 2 m 50 cm.

Solution :

Let the required length of shadow be x cm and the height of the pole is y cm.

Height of the pole (in cm)	280	525	y
Length of the shadow (in cm)	160	x	250

Since, it is a case of direct variation :

$$\frac{280}{160} = \frac{525}{x} = \frac{y}{250}$$

(i) $\frac{280}{160} = \frac{525}{x} \Rightarrow 280x = 160 \times 525$

$\Rightarrow x = \frac{160 \times 525}{280} = 300$

$\therefore$ **Required length** = 300 cm = **3 m** **(Ans.)**

(ii) $\frac{280}{160} = \frac{y}{250} \Rightarrow y = \frac{280}{160} \times 250 = 437.5$

$\therefore$ **Required height of the pole** = 437.5 cm = **4.375 m** **(Ans.)**

EXERCISE 10(A)

1. In which of the following tables, x and y vary directly :

(i)

x	3	5	8	11
y	4.5	7.5	12	16.5

(ii)

x	16	30	40	56
y	32	60	80	84

(iii)

x	27	45	54	75
y	81	180	216	225

2. If x and y vary directly, find the vlaues of x, y and z :

x	3	x	y	10
y	36	60	96	z

3. A truck consumes 28 litres of diesel for moving through a distance of 448 km. How much distance will it cover in 64 litres of diesel ?
4. For 100 km, a taxi charges ₹ 1,800. How much will it charge for a journey of 120 km ?
5. If 27 identical articles cost ₹ 1,890, how many articles can be bought for ₹ 1,750 ?
6. 7 kg of rice costs ₹ 1,120. How much rice can be bought for ₹ 3,680 ?
7. 6 note-books cost ₹ 156, find the cost of 54 such note-books.
8. 22 men can dig a 27 m long trench in one day. How many men should be employed for digging 135 m long trench of the same type in one day ?
9. If the total weight of 11 identical articles is 77 kg, how many articles of the same type would weigh 224 kg ?
10. A train is moving with uniform speed of 120 km per hour.
    (i) How far will it travel in 36 minutes ?
    (ii) In how much time will it cover 210 km ?

10.4 INVERSE VARIATION

Two quantities x and y are said to be in inverse variation, if an increase in x causes a corresponding decrease in y and a decrease in x causes a corresponding increase in y.

1. x and y are in inverse variation (proportion)
   ⇒ product of x and y is constant
   ⇒ xy is a constant
2. If x_1, x_2, x_3, ... are some different values of x
   and y_1, y_2, y_3, ... are the corresponding values of y,
   then, $x_1 y_1 = x_2y_2 = x_3y_3 = ... =$ a constant

Example 9 :

In which of the following tables x and y vary inversely.

(i)

x	2	4	8
y	32	16	8

(ii)

x	9	18	2	12
y	2	1	7·5	3

Solution :

(i) ∵ $x_1y_1 = 2 \times 32 = 64$; $x_2y_2 = 4 \times 16 = 64$

and $x_3y_3 = 8 \times 8 = 64$

∴ $x_1y_1 = x_2y_2 = x_3y_3$

⇒ **x and y vary inversely** **(Ans.)**

(ii) $\because$ $x_1y_1 = 9 \times 2 = 18$; $x_2y_2 = 18 \times 1 = 18$

$x_3y_3 = 2 \times 7.5 = 15$ and $x_4y_4 = 12 \times 3 = 36$

$\therefore$ $x_1y_1 = x_2y_2 \neq x_3y_3 \neq x_4y_4$

$\Rightarrow$ **x and y do not vary inversely** **(Ans.)**

Example 10 :

If p and q vary inversely, find the values of x, y and z.

(i)

p	8	2	y	10
q	2.5	x	5	z

(ii)

p	16	32	8	z
q	16	x	y	4

Solution :

(i) $\because$ p and q vary inversely

$x_1y_1 = x_2y_2 = x_3y_3 = x_4y_4$

$\Rightarrow$ $8 \times 2.5 = 2 \times x = y \times 5 = 10 \times z$

$\Rightarrow$ $20 = 2x = 5y = 10z$

$\Rightarrow$ $2x = 20$, $5y = 20$ and $10z = 20$

$\Rightarrow$ **$x = 10$, $y = 4$ and $z = 2$** **(Ans.)**

(ii) $\because$ p and q vary inversely

$x_1y_1 = x_2y_2 = x_3y_3 = x_4y_4$

$\Rightarrow$ $16 \times 16 = 32 \times x = 8 \times y = z \times 4$

$\Rightarrow$ $32x = 256$, $8y = 256$ and $4z = 256$

$\Rightarrow$ **$x = 8$, $y = 32$ and $z = 64$** **(Ans.)**

Example 11 :

If 78 men can do a work in 140 days, in how many days will 42 men do the same work ?

Solution :

Let 42 men will do the same work in d days

Number of men	78	42
Number of days	140	d

$\because$ It is a case of inverse variation

$\therefore$ $78 \times 140 = 42 \times d$ [In inverse variation, $x_1y_1 = x_2y_2$]

$\Rightarrow$ $d = \dfrac{78 \times 140}{42} = 260$

$\therefore$ **42 men will do the same work in 260 days** **(Ans.)**

EXERCISE 10(B)

1. Check whether x and y vary inversely or not.

(i)

x	4	3	12	1
y	6	8	2	24

(ii)

x	30	120	60	24
y	60	30	30	75

(iii)

x	10	30	60	10
y	90	30	20	90

2. If x and y vary inversely, find the values of l, m and n :

(i)

x	4	8	2	32
y	4	l	m	n

(ii)

x	24	32	m	16
y	l	12	8	n

3. 36 men can do a piece of work in 7 days. How many men will do the same work in 42 days ?
4. 12 pipes, all of the same size, fill a tank in 42 minutes. How long will it take to fill the same tank, if 21 pipes of the same size are used ?
5. In a fort 150 men had provisions for 45 days. After 10 days, 25 men left the fort. How long would the food last at the same rate ?
6. 72 men do a piece of work in 25 days. In how many days will 30 men do the same work ?
7. If 56 workers can build a wall in 180 hours, how many workers will be required to do the same work in 70 hours ?
8. A car takes 6 hours to reach a distination by travelling at the speed of 50 km per hour. How long will it take when the car travels at the speed of 75 km per hour ?

10.5 USING UNITARY METHOD

Example 12 :

A fort had provisions for 300 men for 90 days. After 20 days, 50 men left the fort. How long would the food last at the same rate ?

Solution :

After 20 days :

For 300 men, provisions will last (90 – 20) days = 70 days

$\Rightarrow$ For 1 man, the provisions will last 300 × 70 days

And, for (300 – 50) = 250 men, **the provisions will last** for $\frac{300 \times 70}{250}$ days

= **84 days** **(Ans.)**

Example 13 :

A hostel had provisions for 75 students for 30 days. After 6 days, 15 more students joined the hostel. How long would the remaining provisions last at the same rate for all the students ?

Solution :

After 6 days :

For 75 students, provisions are sufficient for (30 – 6) days = 24 days

$\Rightarrow$ For 1 student, the provisions are sufficient for (75 × 24) days

And, for 90 students, **the provisions are sufficient for** $\left(\frac{75 \times 24}{90}\right)$ days

= **20 days** **(Ans.)**

Example 14 :

8 men or 6 women earn ₹ 960 in one day.

Find : (i) one day's earning of a man.

(ii) one day's earning of a woman.

(iii) one day's earning of 4 men and 5 women.

Solution :

(i) Since, one day's earning of 8 men = ₹ 960

$\Rightarrow$ **One day's earning of a man** = ₹ $\frac{960}{8}$ = **₹ 120** **(Ans.)**

(ii) Since, one day's earning of 6 women = ₹ 960

$\Rightarrow$ **One day's earning of a woman** = ₹ $\frac{960}{6}$ = **₹ 160** **(Ans.)**

(iii) **One day's earning of 4 men and 5 women**

= 4 × ₹ 120 + 5 × ₹ 160 = **₹ 1,280** **(Ans.)**

> While applying unitary method, arrange a statement in such a way that, whatever is asked to find in the question, is written at the end of the statement.

Example 15 :

2 men or 3 women can do a piece of work in 45 days. Find, in how many days will 6 men and 1 woman be able to complete the same work ?

Solution :

According to the amount of work done, in the same time, 2 men are equivalent to 3 women

i.e. 2 men $\equiv$ 3 women

$$1 \text{ man} \equiv \frac{3}{2} \text{ women}$$

$$\text{and, } 6 \text{ men} \equiv \frac{3}{2} \times 6 \text{ women} \equiv 9 \text{ women.}$$

$\therefore$ 6 men + 1 woman $\equiv$ 9 women + 1 woman $\equiv$ 10 women

Since, 3 women can do the work in 45 days

$\Rightarrow$ 1 woman will do the work in 45 × 3 days = 135 days

$\Rightarrow$ 10 women will do the work in $\frac{135}{10}$ days = $13\frac{1}{2}$ days.

$\therefore$ **6 men and 1 woman will complete the work in $13\frac{1}{2}$ days** **(Ans.)**

Example 16 :

3 men and 4 boys can complete a certain amount of work in 28 days, whereas 4 men and 6 boys can complete the same work in 20 days.

Find : (i) according to the amount of work done, one man is equivalent to how many boys.

(ii) the number of days required by 7 men and 6 boys to complete the same work.

Solution :

(i) In 28 days, the work can be completed by 3 men and 4 boys

$\therefore$ In 1 day, the work can be completed by 28(3 men + 4 boys)

i.e. by 84 men + 112 boys.

$\because$ In 20 days, the same work can be completed by 4 men + 6 boys

$\therefore$ In 1 day, the same work can be completed by 20(4 men + 6 boys)

i.e. by 80 men + 120 boys.

$\therefore$ According to the amount of work done,

84 men + 112 boys $\equiv$ 80 men + 120 boys

$\Rightarrow$ 4 men $\equiv$ 8 boys and **1 man $\equiv$ 2 boys** **(Ans.)**

(ii) Since, 3 men + 4 boys $\equiv$ 3 $\times$ 2 boys + 4 boys [1 man $\equiv$ 2 boys]

$\equiv$ 10 boys

And, 7 men + 6 boys $\equiv$ 7 $\times$ 2 boys + 6 boys

$\equiv$ 20 boys

Given, 3 men + 4 boys can complete the work in 28 days

i.e., 10 boys can complete the work in 28 days

$\Rightarrow$ 1 boy will complete the same work in 28 $\times$ 10 days = 280 days

$\Rightarrow$ 20 boys (7 men + 6 boys) will complete the same work in $\frac{280}{20}$ days = 14 days

$\Rightarrow$ **7 men and 6 boys require 14 days to complete the same work** **(Ans.)**

EXERCISE 10(C)

1. Cost of 24 identical articles is ₹ 108. Find the cost of 40 similar articles.
2. If 15 men can complete a piece of work in 30 days, in how many days will 18 men complete it ?
3. In order to complete a work in 28 days, 60 men are required. How many men will be required if the same work is to be completed in 40 days ?
4. A fort had provisions for 450 soldiers for 40 days. After 10 days, 90 more soldiers come to the fort. Find for how many days will the remaining provisions last at the same rate?
5. A garrison has sufficient provisions for 480 men for 12 days. If the number of men is reduced by 160; find how long will the provisions last ?
6. $\frac{3}{5}$ quintal of wheat costs ₹ 210. Find the cost of :

   (i) 1 quintal of wheat (ii) 0·4 quintal of wheat.
7. If $\frac{2}{9}$ of a property costs ₹ 2,52,000; find the cost of $\frac{4}{7}$ of it.
8. 4 men or 6 women earn ₹ 360 in one day. Find, how much will :

   (i) a man earn in one day ?

   (ii) a woman earn in one day ?

   (iii) 6 men and 4 women earn in one day ?
9. 16 boys went to a canteen to have tea and snacks together. The bill amounted to ₹ 114.40. What will be the contribution of a boy who pays for himself and 5 others ?

10. 50 labourers can dig a pond in 16 days. How many labourers will be required to dig another pond, double in size, in 20 days ?

11. If 12 men or 18 women can complete a piece of work in 7 days, in how many days can 4 men and 8 women complete the same work ?

12. If 3 men or 6 boys can finish a work in 20 days, how long will 4 men and 12 boys take to finish the same work ?

13. A particular work can be completed by 6 men and 6 women in 24 days; whereas the same work can be completed by 8 men and 12 women in 15 days. Find :
    (i) according to the amount of work done, one man is equivalent to how many women.
    (ii) the time taken by 4 men and 6 women to complete the same work.

14. If 12 men and 16 boys can do a piece of work in 5 days and, 13 men and 24 boys can do it in 4 days, how long will 7 men and 10 boys take to do it ?

10.6 MORE EXAMPLES

Example 17 :

30 men can build a wall in 50 days. How many more men are required to build another wall, double in size, in 75 days ?

Solution :

In order to build the wall in 50 days, no. of men required = 30

$\Rightarrow$ To build the same wall in 1 day, no. of men required = 30×50

$\Rightarrow$ To build the same wall in 75 days, no. of men required = $\frac{30 \times 50}{75} = 20$

$\therefore$ To build the wall double in size, the no. of the men required = 2×20 men = 40 men

Since, 30 men were already at work

$\therefore$ (40 – 30) men = **10 more men are required.** **(Ans.)**

Example 18 :

A camp had sufficient food for 400 soldiers for 20 days. However, some soldiers left on the first day only and then the food lasted for 32 days. Find, how many soldiers had left ?

Solution :

For 20 days, the food is sufficient for 400 soldiers

$\Rightarrow$ For 1 day, the food will be sufficient for (400×20) soldiers

And, for 32 days, the food will be sufficient for $\frac{400 \times 20}{32}$ soldiers = 250 soldiers

$\therefore$ **The no. of soldiers left** = (400 – 250) soldiers = **150 soldiers** **(Ans.)**

Example 19 :

Eight men can dig a field in 14 days, working 6 hours a day. In how many days can 7 men dig the same field, working 8 hours a day ?

Solution :

Since, 8 men working 6 hours a day, dig the field in 14 days.

$\Rightarrow$ 8 men working 1 hour a day, dig the field in 14×6 days

⇒ 1 man working 1 hour a day, digs the field in 14 × 6 × 8 days

⇒ 7 men working 1 hour a day, dig the field in $= \frac{14 \times 6 \times 8}{7}$ days

⇒ **7 men working 8 hours a day, dig the field in $= \frac{14 \times 6 \times 8}{7 \times 8}$ = 12 days** **(Ans.)**

Example 20 :

108 kg of ration is sufficient for 18 students for 15 days. Find, for how many students will 70 kg of ration be sufficient for 25 days.

Solution :

Given, 108 kg ration is sufficient for 15 days for 18 students

⇒ 108 kg ration is sufficient for 1 day for (18 × 15) students

⇒ 1 kg ration is sufficient for 1 day for $\left(\frac{18 \times 15}{108}\right)$ students *i.e.* for $\frac{5}{2}$ students

⇒ 70 kg ration is sufficient for 1 day for $\left(\frac{5}{2} \times 70 = 175\right)$ students

⇒ **70 kg ration is sufficient for 25 days for** $\frac{175}{25}$ students = **7 students** **(Ans.)**

Example 21 :

A contractor undertook to build a road in 200 days. He employed 140 men. After 60 days, he found that only one-fourth of the road could be built. How many additional men should he employed to complete the work in time ?

Solution :

Here, incomplete work $= 1 - \frac{1}{4} = \frac{3}{4}$ and remaining days = 200 – 60 = 140

According to the given statement :

$\frac{1}{4}$ of the road is built in 60 days by 140 men

⇒ $\frac{1}{4}$ of the road can be built in 1 day by 140 × 60 = 8400 men

⇒ Complete road can be built in 1 day by 8400 × 4 = 33600 men

⇒ Complete road can be built in 140 days by $\frac{33600}{140} = 240$ men

⇒ $\frac{3}{4}$ of the road can be built in 140 days by $240 \times \frac{3}{4}$ = 180 men

∴ **Additional men required = 180 – 140 = 40 men** **(Ans.)**

10.7 ARROW METHOD

In general this method is used to solve the problems based on unitary method.

Example 22 : (*Based on inverse proportion*) :

A certain sum is divided equally among 50 boys and each boy gets ₹ 75. If the same sum is divided equally among 60 boys; how much will each get ?

Solution :

Let each boy gets ₹ x

1. Form two columns, as shown alongside, one heading **no. of boys** and other heading **each gets**.

   Since we are to find the share of each boy ; column heading **each gets** must be on the extreme right. Now write the different quantities as shown :

No. of boys	Each gets
50 ↑	₹ 75 ↓
60	₹ x

2. Mark an arrow in the downward direction for the column on the extreme right.
3. Since it is case of inverse proportion, mark an arrow in the first column in the upward direction (*i.e.* direction opposite to the arrow on the extreme right column).
4. Now according to the arrow taken :

$$\frac{\text{Value on head}}{\text{Value on tail}} \text{ for one arrow} = \frac{\text{Value on head}}{\text{Value on tail}} \text{ for another arrow}$$

$$\Rightarrow \quad \frac{x}{75} = \frac{50}{60} \quad \textit{i.e. } x = \frac{50 \times 75}{60} = 62{\cdot}5$$

$\therefore$ **Each boy gets ₹ 62·50** **(Ans.)**

Example 23 : (*Based on direct proportion*) :

The cost of 15 pens is ₹ 375. Find, how many pens can be bought for ₹ 800.

Solution :

Let x pens can be bought for ₹ 800.

Arranging given data and number of pens (x) according to the previous example, we get the columns as shown alongside :

Rupee	No. of pens
375 ↓	15 ↓
800	x

Since, it is the case of direct proportion, arrows must be in the same direction and so :

$$\frac{\text{Value at the head}}{\text{Value at the tail}} \text{ for one arrow} = \frac{\text{Value at the head}}{\text{Value at the tail}} \text{ for another arrow}$$

$$\Rightarrow \quad \frac{x}{15} = \frac{800}{375} \quad \textit{i.e. } x = 32$$

$\therefore$ **32 pens can be bought for ₹ 800** **(Ans.)**

When three or more than three types of quantities are taken, then :

for the column on extreme right

$$= \frac{\text{Value on head}}{\text{Value on tail}} \text{ for 1st column} \times \frac{\text{Value on head}}{\text{Value on tail}} \text{ for 2nd column} \quad \text{and so on.}$$

For example 19, given above :

Let 7 men will dig the field in x days working 8 hours a day.

$\therefore$

No. of men (1st column)	Hrs. per day (2nd column)	No. of days (Last column)
8 ↑	6 ↑	14 ↓
7	8	x

Mark arrow for extreme right column in the downward direction, then compare each column with this last column and each time mark arrows **upwards**, if it is the case of **inverse proportion** and **downwards**, if it is the case of **direct proprotion**.

Now use the equation in the box, given above, to get :

$$\frac{x}{14} = \frac{8}{7} \times \frac{6}{8} \Rightarrow x = 12$$

∴ **The field can be dug in 12 days.** **(Ans.)**

The 1st and the 2nd column can be taken in any order. For example, we can take as :

Hrs. per day	No. of men	No. of days
6 ↑	8 ↑	14 ↓
8	7	x

$$\Rightarrow \frac{x}{14} = \frac{6}{8} \times \frac{8}{7} \quad \textit{i.e. } x = 12.$$

For example 20, given above :

Ration	Days	Students
108 kg ↓	15 ↑	18 ↓
70 kg	25	x

$$\Rightarrow \frac{x}{18} = \frac{15}{25} \times \frac{70}{108} \quad \textit{i.e. } x = 7.$$

∴ **Required number of students**

= **7** **(Ans.)**

For example 21, given above :

Days	Work	Men
60 ↑	$\frac{1}{4}$ ↓	140 ↓
140	$\frac{3}{4}$	x

$$\Rightarrow \frac{x}{140} = \frac{\frac{3}{4}}{\frac{1}{4}} \times \frac{60}{140} \quad \textit{i.e. } x = 180$$

⇒ Total no. of men required = 180

i.e. **Additional no. of men required**

= 180 − 140 = **40** **(Ans.)**

EXERCISE 10(D)

1. Eight oranges can be bought for ₹ 10.40. How many more can be bought for ₹ 16.90 ?
2. Fifteen men can build a wall in 60 days. How many more men are required to build another wall of same size in 45 days ?
3. Six taps can fill an empty cistern in 8 hours. How much more time will be taken, if two taps go out of order ? Assume, all the taps supply water at the same rate.
4. A contractor undertakes to dig a canal, 6 kilometre long, in 35 days and employed 90 men. He finds that after 20 days only 2 km of canal have been dug. How many more men must he employ to finish the work in time ?
5. If 10 horses consume 18 bushels in 36 days, how long will 24 bushels last for 30 horses ?
6. A family of 5 persons can be maintained for 20 days with ₹ 2,480. Find, how long will ₹ 6,944 maintain a family of 8 persons ?
7. 90 men can complete a work in 24 days working 8 hours a day. How many men are required to complete the same work in 18 days working $7\frac{1}{2}$ hours a day?
8. Twelve typists, all working with same speed, type a certain number of pages in 18 days working 8 hours a day. Find, how many hours per day must sixteen typists work in order to type the same number of pages in 9 days ?
9. If 25 horses consume 18 quintal in 36 days, how long will 28 quintal last for 30 horses ?
10. If 70 men dig 15,000 sq. m of a field in 5 days. Find how many men will dig 22,500 sq. m field in 25 days ?

11. A contractor undertakes to build a wall 1000 m long in 50 days. He employs 56 men, but at the end of 27 days, he finds that only 448 m of wall is built. How many extra men must the contractor employ so that the wall is completed in time ?
12. A group of labourers promises to do a piece of work in 10 days, but five of them become absent. If the remaining labourers complete the work in 12 days, find their original number in the group.
13. Ten men, working for 6 days of 10 hours each, finish $\frac{5}{21}$ of a piece of work. How many men working at the same rate and for the same number of hours each day, will be required to complete the remaining work in 8 days?

10.8 TIME AND WORK

1. If a man can do a piece of work in 12 days; his one day work = $\frac{1}{12}$

   i.e. one day's work = $\frac{1}{\text{no. of days required to complete the work}}$
2. If a man's 1 day work = $\frac{1}{15}$ then, he can complete the work in 15 days.

   i.e. no. of days required to do a certain work = $\frac{1}{\text{one day's work}}$
3. No. of days required to complete a certain work = $\frac{\text{work to be completed}}{\text{one day's work}}$

Example 24 :

A can do a piece of work in 80 days and B in 100 days. They work together for first 20 days before B goes away. In how many days will A manage to finish the remaining work ?

Solution :

A's 1 day work = $\frac{1}{80}$ and B's 1 day work = $\frac{1}{100}$

$\therefore$ (A + B)'s 1 day work = $\frac{1}{80} + \frac{1}{100} = \frac{5+4}{400} = \frac{9}{400}$

$\therefore$ (A + B)'s 20 days work = $\frac{9}{400} \times 20 = \frac{9}{20}$

Remaining work = $1 - \frac{9}{20} = \frac{20-9}{20} = \frac{11}{20}$

$\therefore$ **No. of days taken by A to finish the remaining work**

$= \frac{\text{Remaining work}}{\text{A's 1 day work}} = \frac{\frac{11}{20}}{\frac{1}{80}} = \frac{11}{20} \times \frac{80}{1} =$ **44 days** **(Ans.)**

Example 25 :

A and B can finish a piece of work in 15 days, B and C in 20 days while C and A in 30 days. How long will they take to finish it together ? How long will each take to finish the work alone ?

Solution :

(A + B)'s 1 day work = $\frac{1}{15}$ I

(B + C)'s 1 day work = $\frac{1}{20}$ .... II and, $(C + A)$'s 1 day work $= \frac{1}{30}$ III

$\therefore$ $2(A + B + C)$'s 1 day work $= \frac{1}{15} + \frac{1}{20} + \frac{1}{30}$ [On adding I, II and III]

$$= \frac{4+3+2}{60} = \frac{9}{60} = \frac{3}{20}$$

$\Rightarrow$ $(A + B + C)$'s 1 day work $= \frac{3}{20 \times 2} = \frac{3}{40}$ IV

$\therefore$ **They together will complete the work in $\frac{40}{3}$ days $= 13\frac{1}{3}$ days.** **(Ans.)**

Now, A's 1 day work $= \frac{3}{40} - \frac{1}{20}$ [Subtracting II from IV]

$$= \frac{3-2}{40} = \frac{1}{40}$$

$\therefore$ **A alone will complete the work in 40 days.** **(Ans.)**

B's 1 day work $= \frac{3}{40} - \frac{1}{30}$ [Subtracting III from IV]

$$= \frac{9-4}{120} = \frac{5}{120} = \frac{1}{24}$$

$\therefore$ **B alone will complete the work in 24 days.** **(Ans.)**

C's 1 day work $= \frac{3}{40} - \frac{1}{15}$ [Subtracting I from IV]

$$= \frac{9-8}{120} = \frac{1}{120}$$

$\therefore$ **C alone will complete the work in 120 days.** **(Ans.)**

Example 26 :

A can do $\frac{2}{3}$ of a certain work in 12 days and B can do $\frac{1}{6}$ of the same work in 4 days. Find, in how many days will they together complete the work ?

Solution :

Since, A's 12 days work $= \frac{2}{3}$, $\therefore$ A's 1 day work $= \frac{2}{3 \times 12} = \frac{1}{18}$

Since, B's 4 days work $= \frac{1}{6}$ $\therefore$ B's 1 day work $= \frac{1}{6 \times 4} = \frac{1}{24}$

$\therefore$ $(A + B)$'s 1 day work $= \frac{1}{18} + \frac{1}{24} = \frac{4+3}{72} = \frac{7}{72}$

$\Rightarrow$ **A and B together can complete the work in $\frac{72}{7}$ days $= 10\frac{2}{7}$ days.** **(Ans.)**

Example 27 :

A and B can do a piece of work in 45 days and 40 days respectively. They began the work together but A leaves after some days and B finishes the remaining work in 23 days. After how many days did A leave the work ?

Solution :

Since, B's 1 day work $= \frac{1}{40}$

$\Rightarrow$ Work done by B alone in 23 days $= \frac{1}{40} \times 23 = \frac{23}{40}$

$\therefore$ Work done by A and B together $= 1 - \frac{23}{40} = \frac{17}{40}$

$\because$ (A + B)'s 1 day work $= \frac{1}{45} + \frac{1}{40} = \frac{8+9}{360} = \frac{17}{360}$

$\therefore$ No. of days A and B worked together $= \frac{\frac{17}{40}}{\frac{17}{360}} = \frac{17}{40} \times \frac{360}{17} = 9$

Hence, A left the work after 9 days. **(Ans.)**

Alternative method :

Let A leave after x days

$\Rightarrow$ A and B worked together for x days

$\Rightarrow$ x days work of A and B + 23 days work of B = 1

$$\Rightarrow \quad x\left(\frac{1}{45} + \frac{1}{40}\right) + 23 \times \frac{1}{40} = 1$$

$$\Rightarrow \quad x\left(\frac{8+9}{360}\right) = 1 - \frac{23}{40}$$

$$\Rightarrow \quad x \times \frac{17}{360} = \frac{17}{40}$$

$$\Rightarrow \quad x = \frac{17}{40} \times \frac{360}{17} = 9$$

$\therefore$ **A left the work after 9 days.** **(Ans.)**

Third method :

Let A leave after x days

$\Rightarrow$ A and B worked together for x days

$\Rightarrow$ A worked for x days and B worked for $(x + 23)$ days

$\Rightarrow$ x days work of A + $(x + 23)$ days work of B = 1

$$\Rightarrow \quad x \times \frac{1}{45} + (x + 23) \times \frac{1}{40} = 1$$

$$\Rightarrow \quad \frac{8x + 9x + 207}{360} = 1 \text{ i.e. } 17x + 207 = 360$$

$$\Rightarrow \quad 17x = 360 - 207 = 153 \text{ and } x = \frac{153}{17} = 9$$

$\therefore$ **A left the work after 9 days.** **(Ans.)**

Example 28 :

An empty cistern can be filled by two pipes A and B in 12 minutes and 16 minutes respectively and the full cistern can be emptied by a third pipe C in 8 minutes. If all the pipes be turned on at the same time, in how much time will the empty cistern be full ?

Solution :

Since, A fills in 1 minute $= \frac{1}{12}$ of the cistern, B fills in 1 minute $= \frac{1}{16}$ of the cistern and C empties in 1 minute $= \frac{1}{8}$ of the cistern,

$\therefore$ A, B and C together fill in 1 minute $= \left(\frac{1}{12} + \frac{1}{16} - \frac{1}{8}\right)$ of the cistern

$= \frac{4+3-6}{48}$ of the cistern

$= \frac{1}{48}$ of the cistern

$\therefore$ **A, B and C together will take 48 minutes to fill the cistern.** **(Ans.)**

EXERCISE 10(E)

1. A can do a piece of work in 10 days and B in 15 days. How long will they take to finish it working together?
2. A and B together can do a piece of work in $6\frac{2}{3}$ days, but B alone can do it in 10 days. How long will A take to do it alone ?
3. A can do a work in 15 days and B in 20 days. If they work together on it for 4 days, what fraction of the work will be left ?
4. A, B and C can do a piece of work in 6 days, 12 days and 24 days respectively. In what time will they altogether do it ?
5. A and B working together can mow a field in 56 days and with the help of C, they could have mowed it in 42 days. How long would C take to mow the field by himself ?
6. A can do a piece of work in 24 days, A and B can do it in 16 days and A, B and C in $10\frac{2}{3}$ days. In how many days can A and C do it working together ?
7. A can do a piece of work in 20 days and B in 15 days. They worked together on it for 6 days and then A left. How long will B take to finish the remaining work ?
8. A can finish a piece of work in 15 days and B can do it in 10 days. They worked together for 2 days and then B goes away. In how many days will A finish the remaining work ?
9. A can do a piece of work in 10 days, B in 18 days, and A, B and C together in 4 days. In what time would C do it alone ?
10. A can do $\frac{1}{4}$ of a work in 5 days and B can do $\frac{1}{3}$ of the same work in 10 days. Find the number of days in which both working together will complete the work.
11. One tap can fill a cistern in 3 hours and the waste pipe can empty the full cistern in 5 hours. In what time will the empty cistern be full, if the tap and the waste pipe are kept open together?
12. A and B can do a work in 8 days, B and C in 12 days, and A and C in 16 days. In what time can they do it, all working together ?
13. A and B complete a piece of work in 24 days. B and C do the same work in 36 days; and A, B and C together finish it in 18 days. In how many days will :
    (i) A alone,
    (ii) C alone,
    (iii) A and C together, complete the work ?
14. A and B can do a piece of work in 40 days, B and C in 30 days, and C and A in 24 days.
    (i) How long will it take them to do the work, working together ?
    (ii) In what time can each finish it working alone ?
15. A can do a piece of work in 10 days, B in 12 days and C in 15 days. All begin together but A leaves the work after 2 days and B leaves 3 days before the work is finished. How long did the work last ?

    Suppose the work last for x days.
    $\therefore$ A worked for 2 days, B for $(x-3)$ days and C for x days.
    $\Rightarrow$ A's 2 days work + B's$(x-3)$ days work + C's x days work = 1
    $\Rightarrow 2 \times \frac{1}{10} + (x-3) \times \frac{1}{12} + x \times \frac{1}{15} = 1$

16. Two pipes P and Q would fill an empty cistern in 24 minutes and 32 minutes respectively. Both the pipes being opened together, find when the first pipe must be turned off so that the empty cistern may be just filled in 16 minutes.

Theme – 3
ALGEBRA

ALGEBRAIC EXPRESSIONS 11

(Including Operations on Algebraic Expressions)

11.1 REVIEW

1. Constant	A symbol, which has a fixed value, is called a **constant.** *e.g.* $8, 23, 7\frac{3}{4}, -15, \sqrt{3}, 2+\sqrt{5}$, etc.
2. Variable	A symbol, which does not have any fixed value, but may be assigned value (values) according to the requirement, is called a **variable** or a **literal**. *e.g.* x, y, z, p, q, etc.

Remember :

1. Combination of a constant and a variable is a variable.

   e.g. $5x, 5 + x, 5 \div x, x \div 5, x - 5, 5 - x, 8x$, etc.

2. Combination of two or more variables is also a variable.

   e.g. $xy, \frac{x}{y}, \frac{y}{x}, x - y, y - x, x + y, xyz, \frac{xy}{z}, \frac{x}{yz}$, etc.

   In the same way, each of the following combinations is a variable.

   $15 + x - y, 15x - y, 15 - xy, \frac{15x}{y}, \frac{15}{xy}, 15xy$, etc.

3. Term :	A term is a number (constant), a variable, a combination (product or quotient) of numbers and variables. *e.g.* $7, x, 5x, 3xy, -6, \frac{18}{xy}, \frac{yz}{x}$ etc.

Remember :

$7x = 7 \times x, xy = x \times y, 3xy = 3 \times x \times y, 5xyz = 5 \times x \times y \times z, \frac{4xy}{pq} = \frac{4 \times x \times y}{p \times q}$, etc.

11.2 ALGEBRAIC EXPRESSIONS

A single term or a combination of two or more terms connected by *plus* (+) or *minus* (–) signs; forms an *algebraic expression.*

e.g. $5 - y, 3x^2 - 5x, 6xy + z, 8 + x^2 - 3x, 8x + 5y - 7z, xy - 6z + 4$, etc.

Note :

1. Various parts of an algebraic expression which are separated by the sign of addition (+) or subtraction (–) are called the **terms** of the expression.
   (i) $5x^3 - 8xy + 7y^2$ is an algebraic expression consisting of three terms $5x^3$, $-8xy$ and $7y^2$.
   (ii) $4xy^2 + 8xy - 7y^2 + 5$ is an algebraic expression consisting of four terms $4xy^2$, $8xy$, $-7y^2$ and $+5$.

2. The signs of *multiplication* (×) and *division* (÷) do not separate the terms.

e.g. $3x^2 \times 8y$ is a single term; similarly, $3x^2 \div 8y$ is also a single term.

1. Monomial	An algebraic expression, which contains ***only one term,*** is called ***monomial.*** *e.g.* $3, \frac{6}{11}, x, 7x, 8xy, \frac{9x^2}{y}, \frac{yz}{x}, \frac{5x^2y}{z}, \frac{6y}{xz^2}$, etc.
2. Binomial	An algebraic expression, which contains only ***two different terms,*** is called a ***binomial.*** *e.g.* $8 + x, xy - 4, \frac{4x}{y} - 8y, 4x^2 + 6xy, yz^2 + zx$, etc.
3. Trinomial	An algebraic expression, which contains ***three different terms only,*** is called a ***trinomial.*** *e.g.* $3 + x + y, 7x + 11 - y^2, ax^2 + bx + c, xy + yz - zx$, etc.
4. Polynomial (Multinomial)	An algebraic expression, which contains ***more than one term,*** is called a ***polynomial (multinomial).*** *e.g.* $x^2 - 5x, x - 3xy + y^2, 1 - 5y + xy + x^2y$, etc. Thus, every binomial, every trinomial, etc. is a **multinomial** (polynomial).

Remember :

An algebraic expression of the form :

$\frac{x^2+xy+y^2}{x-3y}, \frac{x^2+a^2}{x+a}, \frac{x}{2y}, \frac{3x-y}{8ax}$, etc. does not form a polynomial.

11.3 DEGREE OF A POLYNOMIAL

(a) When the polynomial contains only one variable (literal)

The highest power of the variable is the degree of the polynomial.

e.g. In polynomial $3x^2 - 8x + 4$, the term containing greatest power of the variable x is $3x^2$ and its power is 2.

∴ The degree of polynomial $3x^2 - 8x + 4$ is 2.

Similarly,

(i) The degree of polynomial $4x - 7x^5 + 8$ is 5.

(ii) The degree of $\frac{3}{7}y^2 + \frac{4}{9}y^5 - \frac{1}{5}y^8 + 15y$ is 8.

(iii) The degree of $8 - 5x$ is 1 and so on.

Remember :

An algebraic expression is a polynomial if degree of each term, used in it, is a non-negative integer.

(b) When the polynomial contains two or more variables

Steps :

1. Find the sum of the powers of the variables in each term.
2. The highest sum of the powers is taken to be the degree of the polynomial.

For example :

Consider the polynomial : $7x^3y^4 - 8x^2y^3z^4 + 5x^4y^3z$

The sum of the powers of the term $7x^3y^4 = 3 + 4 = 7$,

the sum of the powers of the term $- 8x^2y^3z^4 = 2 + 3 + 4 = 9$ and

the sum of the powers of $5x^4y^3z = 4 + 3 + 1 = 8$

Clearly, the degree of the polynomial $7x^3y^4 - 8x^2y^3z^4 + 5x^4y^3z$ is 9.

Similarly,

(i) The degree of polynomial $5x^2y - 4x^3y^5 + 6 = 3 + 5 = 8$

(ii) The degree of $xy^2z^3 + 4x^3y^3z^4 - 5x^2yz^3 = 3 + 3 + 4 = 10$

Important :

1. $5x + \frac{7}{x}$ is an algebraic expression but not a polynomial.

   Reason : $5x + \frac{7}{x}$ contains a term namely $\frac{7}{x}$ $(= 7x^{-1})$ in which the power of x is -1, which is not a non-negative integer.

   In the same way, each of $2x^2 - x^{5/2} + x + 7$, $2\sqrt{x} - 5x + 8$, $3 - \frac{1}{x^2} + \frac{1}{x} + x$, etc. is an expression but not a polynomial.

2. Each of $5xy^2 - 6x\sqrt{y} + 3y^2$, $8xy^2 - 5x^2y + \frac{2x}{y} + 6$, $\frac{1}{x^3} - y^5$ is an expression but not a polynomial.

A polynomial is said to be :

(i) *Linear polynomial,* if its degree is 1 (one). (ii) *Quadratic polynomial,* if its degree is 2 (two).

(iii) *Cubic polynomial,* if its degree is 3 (three) (iv) *Constant polynomial,* if its degree is 0.

11.4 PRODUCT, FACTOR AND COEFFICIENT

1. **Product :** When *two or more* numbers (constants or variables or both) are multiplied together, the result is called the **product**.

   e.g. (i) 3abc is the product of 3, a, b and c.

   (ii) $5x^2yz$ is the product of 5, x^2, y and z; or we can say, it is the product of 5, x, x, y and z.

2. **Factor :** Each of the quantities (constant or variable or both), which forms a product, is called a **factor of the product**.

   e.g. 5 and x are the factors of 5x

Note : In any given quantity (term) the **constant factor** is called **numeral factor** and the factor containing only **literals** is called a **literal factor.**

Thus in 5x; 5 is the numeral factor and x is the literal factor.

Each factor of a quantity completely divides the quantity.

e.g. (i) Factors of 5x are 5 and x as 5x is completely divisible by 5 and also by x.

(ii) For $5x^2y$, its numeral factor is 5 and the literal factors are x, y, x^2, xy and x^2y.

3. Coefficient : Any factor of an algebraic quantity is called the coefficient of the remaining quantity.

For example :

In the algebraic term (quantity) 7xyz :

7 is coefficient of xyz, 7x is coefficient of yz,

7xy is coefficient of z, xz is coefficient of 7y and so on.

11.5 LIKE AND UNLIKE TERMS

The terms having the **same literal coefficients** are called **like terms** and those having **different literal coefficients** are called **unlike terms.**

For example :

(i) $6xy^2, -8xy^2, 15xy^2, 23xy^2$ and $-\frac{5}{7}xy^2$ are like terms as the literal coefficient of all the terms is same *i.e.* xy^2.

(ii) 5xyz, 8xyz, – 6xyz and $\frac{2}{3}$xyz are like terms.

(iii) x^2y and xy^2 are unlike terms.

(iv) $7xy^2z$, $8x^2yz$ and $-15xyz^2$ are unlike terms.

EXERCISE 11(A)

1. Separate the constants and variables from the following :

$-7, 7 + x, 7x + yz, \sqrt{5}, \sqrt{xy}, \frac{3yz}{8}, 4{\cdot}5y - 3x,$

$8 - 5, 8 - 5x, 8x - 5y \times p$ and $3y^2z \div 4x$

2. Write the number of terms in each of the following polynomials :

(i) $5x^2 + 3 \times ax$

(ii) $ax \div 4 - 7$

(iii) $ax - by + y \times z$

(iv) $23 + a \times b \div 2$

3. Separate monomials, binomials, trinomials and polynomials from the following algebraic expressions :

$8 - 3x, xy^2, 3y^2 - 5y + 8, 9x - 3x^2 + 15x^3 - 7,$ $3x \times 5y, 3x \div 5y, 2y \div 7 + 3x - 7$ and $4 - ax^2 + bx + y$

4. Write the degree of each polynomial given below :

(i) $xy + 7z$ (ii) $x^2 - 6x^3 + 8$

(iii) $y - 6y^2 + 5y^8$ (iv) $xyz - 3$

(v) $xy + yz^2 - zx^3$

(vi) $x^5y^7 - 8x^3y^8 + 10x^4y^4z^4$

5. Write the coefficient of :

(i) ab in 7abx (ii) 7a in 7abx

(iii) $5x^2$ in $5x^2 - 5x$ (iv) 8 in $a^2 - 8ax + a$

(v) 4xy in $x^2 - 4xy + y^2$

6. In $\frac{5}{7}xy^2z^3$, write the coefficient of :

(i) 5 (ii) $\frac{5}{7}$ (iii) 5x

(iv) xy^2 (v) z^3 (vi) xz^3

(vii) $5xy^2$ (viii) $\frac{1}{7}yz$ (ix) z

(x) yz^2 (xi) 5xyz.

7. In each polynomial, given below, separate the like terms :

(i) $3xy, -4yx^2, 2xy^2, 2{\cdot}5x^2y, -8yx, -3{\cdot}2y^2x$ and x^2y

(ii) $y^2z^3, xy^2z^3, -5x^2yz, -4y^2z^3, -8xz^3y^2,$ $3x^2yz$ and $2z^3y^2$

11.6 COMBINING LIKE TERMS

Adding or subtracting the like terms means combining of these terms.

1. In combining like terms; combine their numerical coefficients and place the result before their common literal factor.

   (i) $3x + 2x = (3 + 2)x = 5x$; $4x^2 + 15x^2 = (4 + 15)x^2 = 19x^2$,

   (ii) $8xy - 5xy = (8 - 5)xy = 3xy$,

   (iii) $9xy^2 - 15xy^2 + 11xy^2 = (9 - 15 + 11)xy^2 = 5xy^2$ and so on.

2. Two unlike terms cannot be added or subtracted to give a single term.
3. For combining polynomials; the like terms of the given polynomials are combined together.

Example 1 :

Add : 6 – 3a + b; a – 7 – 6b and 3b + 2 – a.

Solution :

$$\begin{array}{r} 6 - 3a + b \\ -7 + a - 6b \\ +2 - a + 3b \\ \hline 1 - 3a - 2b \\ \hline \end{array}$$

Arrange the polynomials with like terms one below the other, then combine the like terms.

(Ans.)

Alternative method :

Addition of 6 – 3a + b; a – 7 – 6b and 3b + 2 – a

$= (6 - 3a + b) + (a - 7 - 6b) + (3b + 2 - a)$

$= 6 - 3a + b + a - 7 - 6b + 3b + 2 - a$

$= -3a + a - a + b - 6b + 3b + 6 - 7 + 2$

$= -3a - 2b + 1$ **(Ans.)**

Example 2 :

Subtract : $5x - 3x^2 + 8xy$ from $7x^2 + 3xy - 4x$.

Solution :

$$\begin{array}{r} 7x^2 + 3xy - 4x \\ -3x^2 + 8xy + 5x \\ + \quad - \quad - \\ \hline 10x^2 - 5xy - 9x \\ \hline \end{array}$$

Arrange the polynomials with like terms one below the other. Change the signs of each term to be subtracted and then combine the like terms.

(Ans.)

Alternative method :

Subtraction of $5x - 3x^2 + 8xy$ from $7x^2 + 3xy - 4x$

$= (7x^2 + 3xy - 4x) - (5x - 3x^2 + 8xy)$

$= 7x^2 + 3xy - 4x - 5x + 3x^2 - 8xy$

$= 7x^2 + 3x^2 + 3xy - 8xy - 4x - 5x$

$= 10x^2 - 5xy - 9x$ **(Ans.)**

Example 3 :

Subtract $12x + 3y - z$ from the sum of $7x + 4y - 5z + 5$ and $6x - 7z - 8$.

Solution :

$$\begin{array}{l} 7x + 4y - 5z + 5 \\ 6x \quad\quad - 7z - 8 \\ \hline \mathbf{13x + 4y - 12z - 3} \\ \hline \end{array} \qquad \Bigg| \qquad \begin{array}{l} 13x + 4y - 12z - 3 \\ 12x + 3y - z \\ - \quad - \quad + \\ \hline \mathbf{x + y - 11z - 3} \\ \hline \end{array}$$

(Ans.)

Alternative method :

The required result

$= (7x + 4y - 5z + 5) + (6x - 7z - 8) - (12x + 3y - z)$
$= 7x + 4y - 5z + 5 + 6x - 7z - 8 - 12x - 3y + z$
$= 7x + 6x - 12x + 4y - 3y - 5z - 7z + z + 5 - 8$
$= 13x - 12x + y - 12z + z - 3 = \mathbf{x + y - 11z - 3}$ **(Ans.)**

Example 4 :

Take away $8x - 7y + 8p + 10q$ from $10x + 12y - 7p + 9q$.

Solution :

The required result

$= (10x + 12y - 7p + 9q) - (8x - 7y + 8p + 10q)$
$= 10x + 12y - 7p + 9q - 8x + 7y - 8p - 10q$
$= 10x - 8x + 12y + 7y - 7p - 8p + 9q - 10q = \mathbf{2x + 19y - 15p - q}$ **(Ans.)**

Example 5 :

If $a = 3x - 5y$, $b = 6x + 3y$ and $c = 2y - 4x$.
Find : (i) $a + b - c$, (ii) $2a - 3b + 4c$

Solution :

(i) $\mathbf{a + b - c} = (3x - 5y) + (6x + 3y) - (2y - 4x)$
$= 3x - 5y + 6x + 3y - 2y + 4x$
$= 3x + 6x + 4x - 5y + 3y - 2y = \mathbf{13x - 4y}$ **(Ans.)**

(ii) $\mathbf{2a - 3b + 4c} = 2(3x - 5y) - 3(6x + 3y) + 4(2y - 4x)$
$= 6x - 10y - 18x - 9y + 8y - 16x$
$= 6x - 18x - 16x - 10y - 9y + 8y = \mathbf{-28x - 11y}$ **(Ans.)**

Example 6 :

The perimeter of a triangle is $15x^2 - 23x + 9$ and two of its sides are $5x^2 + 8x - 1$ and $6x^2 - 9x + 4$. Find the third side.

Solution :

Since, the perimeter of a triangle = sum of the lengths of its three sides.

$\Rightarrow \quad 15x^2 - 23x + 9 = (5x^2 + 8x - 1) + (6x^2 - 9x + 4) + \text{third side}$
$\Rightarrow \quad 15x^2 - 23x + 9 = 11x^2 - x + 3 + \text{third side}$
$\therefore$ **The third side** $= 15x^2 - 23x + 9 - 11x^2 + x - 3$
$= \mathbf{4x^2 - 22x + 6}$ **(Ans.)**

EXERCISE 11(B)

1. Evaluate :
   (i) $-7x^2 + 18x^2 + 3x^2 - 5x^2$
   (ii) $b^2y - 9b^2y + 2b^2y - 5b^2y$
   (iii) $abx - 15abx - 10abx + 32abx$
   (iv) $7x - 9y + 3 - 3x - 5y + 8$
   (v) $3x^2 + 5xy - 4y^2 + x^2 - 8xy - 5y^2$
2. Add :
   (i) $5a + 3b,\ a - 2b,\ 3a + 5b$
   (ii) $8x - 3y + 7z,\ -4x + 5y - 4z,\ -x - y - 2z$
   (iii) $3b - 7c + 10,\ 5c - 2b - 15,\ 15 + 12c + b$
   (iv) $a - 3b + 3,\ 2a + 5 - 3c,\ 6c - 15 + 6b$
   (v) $13ab - 9cd - xy,\ 5xy,\ 15cd - 7ab,\ 6xy - 3cd$
   (vi) $x^3 - x^2y + 5xy^2 + y^3,\ -x^3 - 9xy^2 + y^3,\ 3x^2y + 9xy^2$
3. Find the total savings of a boy who saves ₹ $(4x - 6y)$, ₹ $(6x + 2y)$, ₹ $(4y - x)$ and ₹ $(y - 2x)$ in four consecutive weeks.
4. Subtract :
   (i) $4xy^2$ from $3xy^2$
   (ii) $-2x^2y + 3xy^2$ from $8x^2y$
   (iii) $3a - 5b + c + 2d$ from $7a - 3b + c - 2d$
   (iv) $x^3 - 4x - 1$ from $3x^3 - x^2 + 6$
   (v) $6a + 3$ from $a^3 - 3a^2 + 4a + 1$
   (vi) $cab - 4cad - cbd$ from $3abc + 5bcd - cda$
   (vii) $a^2 + ab + b^2$ from $4a^2 - 3ab + 2b^2$
5. (i) Take away $-3x^3 + 4x^2 - 5x + 6$ from $3x^3 - 4x^2 + 5x - 6$.
   (ii) Take $m^2 + m + 4$ from $-m^2 + 3m + 6$ and the result from $m^2 + m + 1$.
6. Subtract the sum of $5y^2 + y - 3$ and $y^2 - 3y + 7$ from $6y^2 + y - 2$.
7. What must be added to $x^4 - x^3 + x^2 + x + 3$ to obtain $x^4 + x^2 - 1$?
8. (i) How much more than $2x^2 + 4xy + 2y^2$ is $5x^2 + 10xy - y^2$?
   (ii) How much less $2a^2 + 1$ is than $3a^2 - 6$?
9. If $x = 6a + 8b + 9c$; $y = 2b - 3a - 6c$ and $z = c - b + 3a$; find :
   (i) $x + y + z$ (ii) $x - y + z$
   (iii) $2x - y - 3z$ (iv) $3y - 2z - 5x$
10. The sides of a triangle are $x^2 - 3xy + 8$, $4x^2 + 5xy - 3$ and $6 - 3x^2 + 4xy$. Find its perimeter.
11. The perimeter of a triangle is $8y^2 - 9y + 4$ and its two sides are $3y^2 - 5y$ and $4y^2 + 12$. Find its third side.
12. The two adjacent sides of a rectangle are $2x^2 - 5xy + 3z^2$ and $4xy - x^2 - z^2$. Find its perimeter.
13. What must be subtracted from $19x^4 + 2x^3 + 30x - 37$ to get $8x^4 + 22x^3 - 7x - 60$?
14. How much smaller is $15x - 18y + 19z$ than $22x - 20y - 13z + 26$?
15. How much bigger is $5x^2y^2 - 18xy^2 - 10x^2y$ than $-5x^2 + 6x^2y - 7xy$?

11.7 MULTIPLICATION

For the literals x, y, z, etc.

(i) $x^m \times x^n = x^{m+n}$ *i.e.* $x^3 \times x^5 = x^{3+5} = x^8$

(ii) $x^2y^3 \times x^3y^5 = x^{2+3} \cdot y^{3+5} = x^5y^8$

(iii) $x^4y^2z^3 \times x^3yz^2 \times x^2y^5z = x^{4+3+2} \cdot y^{2+1+5} \cdot z^{3+2+1} = x^9y^8z^6$ and so on.

1. Multiplying a monomial by a monomial

Example 7 :

Multiply $6a^2b^3$ and $-4a^3b^4$.

Multiplication of monomials = (Product of their numeral coefficients) × (Product of their literal coefficients)

Solution :

Steps : 1. Multiply the numeral coefficients *i.e.* $6 \times -4 = -24$

2. Multiply the literal coefficients *i.e.* $a^2b^3 \times a^3b^4 = a^5b^7$

3. Multiply the results of steps 1 and 2 *i.e.* $-24 \times a^5b^7 = -24a^5b^7$ **(Ans.)**

Directly, $6a^2b^3 \times -4a^3b^4 = (6 \times -4)(a^2b^3 \times a^3b^4) = -24a^5b^7$ **(Ans.)**

In the same way :

$$4x^2y^5z \times -5xy^3z^2 \times 2x^3yz^4 = (4 \times -5 \times 2)(x^2y^5z \times xy^3z^2 \times x^3yz^4)$$
$$= -40x^6y^9z^7 \quad \textbf{(Ans.)}$$

2. Multiplying a polynomial by a monomial

Example 8 :

Multiply $8a^2b - 3ab + 5b^2$ by $6ab$.

Solution :

Multiply each term of the polynomial by the monomial.

$$\begin{array}{r} 8a^2b - 3ab + 5b^2 \\ \times\ 6ab \\ \hline 48a^3b^2 - 18a^2b^2 + 30ab^3 \\ \hline \end{array} \quad \textbf{(Ans.)}$$

$[8a^2b \times 6ab = 48a^3b^2$
$-3ab \times 6ab = -18a^2b^2$
$5b^2 \times 6ab = 30ab^3]$

Alternative method :

$$(8a^2b - 3ab + 5b^2) \times 6ab = 8a^2b \times 6ab - 3ab \times 6ab + 5b^2 \times 6ab$$
$$= 48a^3b^2 - 18a^2b^2 + 30ab^3 \quad \textbf{(Ans.)}$$

3. Multiplying a polynomial by a polynomial

Example 9 :

Multiply $x^2 - 4x + 7$ by $x - 2$.

Solution :

Multiply each term of one polynomial by each term of the other polynomial and then combine the terms.

$$\therefore \begin{array}{r} x^2 - 4x + 7 \\ x - 2 \\ \hline x^3 - 4x^2 + 7x \qquad \\ -2x^2 + 8x - 14 \\ \hline x^3 - 6x^2 + 15x - 14 \\ \hline \end{array}$$

[Multiplying by x]
[Multiplying by – 2]
[Combining the terms] **(Ans.)**

Alternative method :

$$(x - 2)(x^2 - 4x + 7) = x(x^2 - 4x + 7) - 2(x^2 - 4x + 7)$$
$$= x^3 - 4x^2 + 7x - 2x^2 + 8x - 14$$
$$= x^3 - 6x^2 + 15x - 14 \quad \textbf{(Ans.)}$$

or, $$(x^2 - 4x + 7)(x - 2) = x^2(x - 2) - 4x(x - 2) + 7(x - 2)$$
$$= x^3 - 2x^2 - 4x^2 + 8x + 7x - 14$$
$$= x^3 - 6x^2 + 15x - 14 \quad \textbf{(Ans.)}$$

Example 10 :

Multiply $2x^3 - 7x + 8$ by $3x^2 + 2x$.

Solution :

$$\begin{array}{r} 2x^3 - 7x + 8 \\ 3x^2 + 2x \\ \hline 6x^5 - 21x^3 + 24x^2 \\ - 14x^2 + 4x^4 + 16x \\ \hline 6x^5 - 21x^3 + 10x^2 + 4x^4 + 16x \\ \hline \end{array}$$

(Ans.)

OR, **The required multiplication**

$= (2x^3 - 7x + 8)(3x^2 + 2x)$

$= 2x^3(3x^2 + 2x) - 7x(3x^2 + 2x) + 8(3x^2 + 2x)$

$= 6x^5 + 4x^4 - 21x^3 - 14x^2 + 24x^2 + 16x$

$= 6x^5 + 4x^4 - 21x^3 + 10x^2 + 16x$ **(Ans.)**

Example 11 :

The adjacent sides of a rectangle are $3x^2 - 2xy + 5y^2$ and $2x^2 + 5xy - 3y^2$. Find the area of the rectangle.

Solution :

One of the adjacent sides of a rectangle can be taken as its length and the other side as its breadth.

$\therefore$ Area of a rectangle = length × breadth

Let length = $3x^2 - 2xy + 5y^2$

$\therefore$ Breadth = $2x^2 + 5xy - 3y^2$

Area of the given rectangle

= length × breadth

$= (3x^2 - 2xy + 5y^2) \times (2x^2 + 5xy - 3y^2)$

$= 3x^2(2x^2 + 5xy - 3y^2) - 2xy(2x^2 + 5xy - 3y^2) + 5y^2(2x^2 + 5xy - 3y^2)$

$= 6x^4 + 15x^3y - 9x^2y^2 - 4x^3y - 10x^2y^2 + 6xy^3 + 10x^2y^2 + 25xy^3 - 15y^4$

$= 6x^4 + 11x^3y - 9x^2y^2 + 31xy^3 - 15y^4$ **(Ans.)**

Example 12 :

Multiply $3x^2y$ and $-2xy^2$. Verify the product for x = 1 and y = 2.

Solution :

$(3x^2y) \times (-2xy^2) = (3 \times -2)(x^2 \times x)(y \times y^2)$

$= -6x^3y^3$

For x = 1 and y = 2

$(3x^2y) \times (-2xy^2) = (3 \times 1^2 \times 2)(-2 \times 1 \times 2^2)$

$= 6 \times (-8) = -48$

And, $-6x^3y^3 = -6 \times 1^3 \times 2^3$

$= -6 \times 1 \times 8 = -48$

$\therefore$ For x = 1 and y = 2, it is verified that

$(3x^2y) \times (-2xy^2) = -6x^3y^3$ **(Ans.)**

Example 13 :

Multiply $-5xy^2$, $-3x^2yz$ and $8yz^2$. Verify the result for x = 1, y = 2 and z = 3.

Solution :

$$(-5xy^2)(-3x^2yz)(8yz^2) = (-5 \times -3 \times 8)(x \times x^2)(y^2 \times y \times y)(z \times z^2)$$
$$= 120x^3y^4z^3$$

For x = 1, y = 2 and z= 3, we have

$$(-5xy^2)(-3x^2yz)(8yz^2) = (-5 \times 1 \times 2^2)(-3 \times 1^2 \times 2 \times 3)(8 \times 2 \times 3^2)$$
$$= (-20)(-18)(144) = 51840$$
$$120x^3y^4z^3 = 120 \times 1^3 \times 2^4 \times 3^3$$
$$= 120 \times 1 \times 16 \times 27 = 51840$$

∴ For x = 1, y = 2 and z = 3, it is verified that

$(-5xy^2)(-3x^2yz)(8yz^2) = 120x^3y^4z^3$ **(Ans.)**

EXERCISE 11(C)

1. Multiply :
   (i) $8ab^2$ by $-4a^3b^4$
   (ii) $\frac{2}{3}ab$ by $-\frac{1}{4}a^2b$
   (iii) $-5cd^2$ by $-5cd^2$
   (iv) $4a$ and $6a + 7$
   (v) $-8x$ and $4 - 2x - x^2$
   (vi) $2a^2 - 5a - 4$ and $-3a$
   (vii) $x + 4$ by $x - 5$
   (viii) $5a - 1$ by $7a - 3$
   (ix) $12a + 5b$ by $7a - b$
   (x) $x^2 + x + 1$ by $1 - x$
   (xi) $2m^2 - 3m - 1$ and $4m^2 - m - 1$
   (xii) a^2, ab and b^2
   (xiii) abx, $-3a^2x$ and $7b^2x^3$
   (xiv) $-3bx$, $-5xy$ and $-7b^3y^2$
   (xv) $-\frac{3}{2}x^5y^3$ and $\frac{4}{9}a^2x^3y$
   (xvi) $-\frac{2}{3}a^7b^2$ and $-\frac{9}{4}ab^5$
   (xvii) $2a^3 - 3a^2b$ and $-\frac{1}{2}ab^2$
   (xviii) $2x + \frac{1}{2}y$ and $2x - \frac{1}{2}y$
2. Multiply :
   (i) $5x^2 - 8xy + 6y^2 - 3$ by $-3xy$
   (ii) $3 - \frac{2}{3}xy + \frac{5}{7}xy^2 - \frac{16}{21}x^2y$ by $-21x^2y^2$
   (iii) $6x^3 - 5x + 10$ by $4 - 3x^2$
   (iv) $2y - 4y^3 + 6y^5$ by $y^2 + y - 3$
   (v) $5p^2 + 25pq + 4q^2$ by $2p^2 - 2pq + 3q^2$
3. Simplify :
   (i) $(7x - 8)(3x + 2)$
   (ii) $(px - q)(px + q)$
   (iii) $(5a + 5b - c)(2b - 3c)$
   (iv) $(4x - 5y)(5x - 4y)$
   (v) $(3y + 4z)(3y - 4z) + (2y + 7z)(y + z)$
4. The adjacent sides of a rectangle are $x^2 - 4xy + 7y^2$ and $x^3 - 5xy^2$. Find its area.
5. The base and the altitude of a triangle are $(3x - 4y)$ and $(6x + 5y)$ respectively. Find its area.
6. Multiply $-4xy^3$ and $6x^2y$ and verify your result for x = 2 and y = 1.
7. Find the value of $(3x^3) \times (-5xy^2) \times (2x^2yz^3)$ for x = 1, y = 2 and z = 3.
8. Evaluate $(3x^4y^2)(2x^2y^3)$ for x = 1 and y = 2.
9. Evaluate $(x^5) \times (3x^2) \times (-2x)$ for x = 1.
10. If x= 2 and y = 1;
    find the value of $(-4x^2y^3) \times (-5x^2y^5)$.
11. Evaluate :
    (i) $(3x - 2)(x + 5)$ for x = 2.
    (ii) $(2x - 5y)(2x + 3y)$ for x = 2 and y = 3.
    (iii) $xz(x^2 + y^2)$ for x = 2, y = 1 and z = 1.
12. Evaluate :
    (i) $x(x - 5) + 2$ for x = 1.
    (ii) $xy^2(x - 5y) + 1$ for x = 2 and y = 1.
    (iii) $2x(3x - 5) - 5(x - 2) - 18$ for x = 2.
13. Multiply and then verify :
    $-3x^2y^2$ and $(x - 2y)$ for x = 1 and y = 2.
14. Multiply :
    (i) $2x^2 - 4x + 5$ by $x^2 + 3x - 7$
    (ii) $(ab - 1)(3 - 2ab)$
15. Simplify : $(5 - x)(6 - 5x)(2 - x)$.

11.8 DIVISION

For the literals x, y, z, etc.

(i) $x^m \div x^n = \dfrac{x^m}{x^n} = x^{m-n}$; if m > n and $x^m \div x^n = \dfrac{x^m}{x^n} = \dfrac{1}{x^{n-m}}$; if n > m.

(ii) $\dfrac{x^7}{x^5} = x^{7-5} = x^2$ and $\dfrac{x^5}{x^7} = \dfrac{1}{x^{7-5}} = \dfrac{1}{x^2}$

(iii) $\dfrac{x^8 \cdot y^4}{x^3y^2} = x^{8-3} \cdot y^{4-2} = x^5y^2$ or, directly : $\dfrac{x^8y^4}{x^3y^2} = x^5y^2$

(iv) $\dfrac{x^5y^2z^7}{x^2y^6z^3} = \dfrac{x^3z^4}{y^4}, \dfrac{18y^8z^3}{3y^4z^5} = \dfrac{6y^4}{z^2}$ and so on.

1. Dividing a monomial by a monomial :

Example 14 :

Divide : (i) $36a^7$ by $-12a^3$ (ii) $-50a^2b^3$ by $-15a^4b^2$

Solution :

Division of a monomial by another monomial = (Division of their numeral coefficients) × (Division of their literal coefficients)

(i) $\dfrac{36a^7}{-12a^3} = \left(\dfrac{36}{-12}\right)\left(\dfrac{a^7}{a^3}\right) = -3a^4$ (ii) $\dfrac{-50a^2b^3}{-15a^4b^2} = \left(\dfrac{-50}{-15}\right)\left(\dfrac{a^2b^3}{a^4b^2}\right) = \dfrac{10b}{3a^2}$ **(Ans.)**

2. Dividing a polynomial by a monomial :

Example 15 :

Divide : $9a^5 - 6a^2$ by $3a^2$

Solution :

Divide each term of the polynomial by the monomial and simplify.

$\therefore \dfrac{9a^5 - 6a^2}{3a^2} = \dfrac{9a^5}{3a^2} - \dfrac{6a^2}{3a^2} = 3a^3 - 2$ **(Ans.)**

In the same way :

$$(8x^2y - 6xy + 5xy^2) \div 2xy = \frac{8x^2y}{2xy} - \frac{6xy}{2xy} + \frac{5xy^2}{2xy}$$
$$= 4x - 3 + \frac{5}{2}y$$

3. Dividing a polynomial by a polynomial :

Example 16 :

Divide : $8x^2 - 45y^2 + 18xy$ by $2x - 3y$.

Solution :

Dividend = $8x^2 + 18xy - 45y^2$ and divisor = $2x - 3y$

Steps : 1. Arrange the terms of divisor and the dividend in ascending or descending powers of their literal coefficients.

2. Divide the first term in the dividend by the first term in the divisor to obtain the first term of the quotient.

i.e. $\frac{8x^2}{2x} = 4x$ (quotient).

3. Multiply the complete divisor by the quotient and subract.

4. Use the remainder as the new dividend.

5. Repeat the steps 2, 3 and 4 as long as required.

$$\begin{array}{r|l} & 4x + 15y \\ 2x - 3y & 8x^2 + 18xy - 45y^2 \\ & 8x^2 - 12xy \\ & - \quad + \\ \hline & 30xy - 45y^2 \\ & 30xy - 45y^2 \\ & - \quad + \\ \hline & \times \quad \times \end{array}$$

∴ **Answer = $4x + 15y$**

Example 17 :

Find the quotient and the remainder (if any), when, $2x^3 - 8x^2 + 5x - 8$ is divided by $x - 2$.

Solution :

$$\begin{array}{r|l} & 2x^2 - 4x - 3 \\ x - 2 & 2x^3 - 8x^2 + 5x - 8 \\ & 2x^3 - 4x^2 \\ & - \quad + \\ \hline & -4x^2 + 5x - 8 \\ & -4x^2 + 8x \\ & + \quad - \\ \hline & -3x - 8 \\ & -3x + 6 \\ & + \quad - \\ \hline & -14 \end{array}$$

Verification :

Quotient × Divisor + Remainder

$= (2x^2 - 4x - 3)(x - 2) + (-14)$

$= 2x^3 - 4x^2 - 4x^2 + 8x - 3x + 6 - 14$

$= 2x^3 - 8x^2 + 5x - 8$

= **Dividend**

∴ **Quotient = $2x^2 - 4x - 3$** and **remainder = -14** **(Ans.)**

Remember : Dividend = Quotient × Divisor + remainder

EXERCISE 11(D)

1. Divide :
   (i) $-70a^3$ by $14a^2$
   (ii) $24x^3y^3$ by $-8y^2$
   (iii) $15a^4b$ by $-5a^3b$
   (iv) $-24x^4d^3$ by $-2x^2d^5$
   (v) $63a^4b^5c^6$ by $-9a^2b^4c^3$
   (vi) $8x - 10y + 6c$ by 2
   (vii) $15a^3b^4 - 10a^4b^3 - 25a^3b^6$ by $-5a^3b^2$
   (viii) $-14x^6y^3 - 21x^4y^5 + 7x^5y^4$ by $7x^2y^2$
   (ix) $a^2 + 7a + 12$ by $a + 4$
   (x) $x^2 + 3x - 54$ by $x - 6$
   (xi) $12x^2 + 7xy - 12y^2$ by $3x + 4y$
   (xii) $x^6 - 8$ by $x^2 - 2$
   (xiii) $6x^3 - 13x^2 - 13x + 30$ by $2x^2 - x - 6$
   (xiv) $4a^2 + 12ab + 9b^2 - 25c^2$ by $2a + 3b + 5c$
   (xv) $16 + 8x + x^6 - 8x^3 - 2x^4 + x^2$ by $x + 4 - x^3$

2. Find the quotient and the remainder (if any); when :
   (i) $a^3 - 5a^2 + 8a + 15$ is divided by $a + 1$.
   (ii) $3x^4 + 6x^3 - 6x^2 + 2x - 7$ is divided by $x - 3$.
   (iii) $6x^2 + x - 15$ is divided by $3x + 5$.
   In each case, verify your answer.

3. The area of a rectangle is $x^3 - 8x^2 + 7$ and one of its sides is $x - 1$. Find the length of the adjacent side.

4. The product of two numbers is $16x^4 - 1$. If one number is $2x - 1$, find the other.

5. Divide $x^6 - y^6$ by the product of $x^2 + xy + y^2$ and $x - y$.

11.9 SIMPLIFICATION

(Using removal of brackets and principle of BODMAS)

1. Brackets :

The signs for different types of brackets are :

(i) ——— ; *Vinculum* or bar brackets,

(ii) (   ) ; *Parenthesis* or small brackets,

(iii) {   } ; *Curly brackets* or middle brackets,

(iv) [   ] ; *Square brackets* or big brackets.

In a combined operation, the brackets must be removed in the same order as written above.

2. Principle of BODMAS :

The principle of BODMAS helps in remembering the order in which the combined operation must be done. According to this principle, the order of operations must be as given below :

1. **B** stands for **bracket** *i.e.* [{ (¯) }]
2. **O** stands for **of** (means multiply)
3. **D** stands for **division** *i.e.* ÷
4. **M** stands for **multiplication** *i.e.* ×
5. **A** stands for **addition** *i.e.* +
6. **S** stands for **subtraction** *i.e.* –

Example 18 :

Simplify : $84 - 7[-11x - 4\{-17x + 3(8 - \overline{9 - 5x})\}]$

Solution :

The given expression $= 84 - 7[-11x - 4\{-17x + 3(8 - 9 + 5x)\}]$

$= 84 - 7[-11x - 4\{-17x + 3(5x - 1)\}]$

$= 84 - 7[-11x - 4\{-17x + 15x - 3\}]$

$= 84 - 7[-11x - 4\{-2x - 3\}]$

$= 84 - 7[-11x + 8x + 12]$

$= 84 - 7[-3x + 12]$

$= 84 + 21x - 84 =$ **$21x$** **(Ans.)**

Example 19 :

Simplify : (i) $x^5 \div x^7 \times x^4$ (ii) $x^5 \times x^7 \div x^4$ (iii) $5y \times [6y \div 3 + \{4 - (3y - \overline{2y - 4})\}]$

Solution :

(i) $x^5 \div x^7 \times x^4$

$= \frac{x^5}{x^7} \times x^4$

$= \frac{x^9}{x^7} = x^2$ **(Ans.)**

(ii) $x^5 \times x^7 \div x^4$

$= \frac{x^5 \times x^7}{x^4}$

$= \frac{x^{12}}{x^4} = x^8$ **(Ans.)**

(iii) **The given expression** $= 5y \times [6y \div 3 + \{4 - (3y - 2y + 4)\}]$

$= 5y \times [6y \div 3 + \{4 - (y + 4)\}]$

$= 5y \times [6y \div 3 + \{4 - y - 4\}]$

$= 5y \times [6y \div 3 + \{-y\}]$

$= 5y \times [6y \div 3 - y]$

$= 5y \times \left[\frac{6y}{3} - y\right] = 5y \times [2y - y]$

$= 5y \times y = 5y^2$ **(Ans.)**

EXERCISE 11(E)

Simplify :

1. $a^2 - 2a + \{5a^2 - (3a - 4a^2)\}$
2. $x - y - \{x - y - (x + y) - \overline{x - y}\}$
3. $-3(1 - x^2) - 2\{x^2 - (3 - 2x^2)\}$
4. $2\{m - 3(n + \overline{m - 2n})\}$
5. $3x - [3x - \{3x - (3x - \overline{3x - y})\}]$
6. $p^2x - 2\{px - 3x(x^2 - \overline{3a - x^2})\}$
7. $2[6 + 4\{m - 6(7 - \overline{n + p}) + q\}]$
8. $a - [a - \overline{b + a} - \{a - (a - \overline{b - a})\}]$
9. $3x - [4x - \overline{3x - 5y} - 3\{2x - (3x - \overline{2x - 3y})\}]$
10. $a^5 \div a^3 + 3a \times 2a$
11. $x^5 \div (x^2 \times y^2) \times y^3$
12. $(x^5 \div x^2) \times y^2 \times y^3$
13. $(y^3 - 5y^2) \div y \times (y - 1)$
14. $3a \times [8b \div 4 - 6\{a - (5a - \overline{3b - 2a})\}]$
15. $7x + 4\{x^2 \div (5x \div 10)\} - 3\{2 - x^3 \div (3x^2 \div x)\}$

IDENTITIES 12

12.1 REVIEW

1. Special Products	The multiplications of certain types of expressions can be obtained by direct or short cut method. Such multiplications are known as ***special products***. For example (The product of two binomials) : **1. $(x + a)(x + b)$** $= x(x + b) + a(x + b)$ $= x^2 + bx + ax + ab$ $= x^2 + ax + bx + ab =$ $\mathbf{x^2 + (a + b)x + ab}$ **2. $(x + a)(x - b)$** $= x(x - b) + a(x - b)$ $= x^2 - bx + ax - ab$ $= x^2 + ax - bx - ab =$ $\mathbf{x^2 + (a - b)x - ab}$ **3. $(x - a)(x + b)$** $= x(x + b) - a(x + b)$ $= x^2 + bx - ax - ab$ $= x^2 - ax + bx - ab =$ $\mathbf{x^2 - (a - b)x - ab}$ **4. $(x - a)(x - b)$** $= x(x - b) - a(x - b)$ $= x^2 - bx - ax + ab$ $= x^2 - ax - bx + ab =$ $\mathbf{x^2 - (a + b)x + ab}$

Examples (Using direct method) :

1. $(x + 5)(x + 3) = x^2 + (5 + 3)x + 5 \times 3 = \mathbf{x^2 + 8x + 15}$
2. $(x + 5)(x - 3) = x^2 + (5 - 3)x - 5 \times 3 = \mathbf{x^2 + 2x - 15}$
3. $(x - 5)(x + 3) = x^2 - (5 - 3)x - 5 \times 3 = \mathbf{x^2 - 2x - 15}$
4. $(x - 5)(x - 3) = x^2 - (5 + 3)x + 5 \times 3 = \mathbf{x^2 - 8x + 15}$

12.2 IMPORTANT

While using direct method, the product of two binomials gives three terms :

(i) The first term = Product of the first terms of the two binomials

(ii) The middle term = (First term of first binomial × second term of second binomial) + (second term of first binomial × first term of second binomial)
= Product of outer terms + Product of inner terms

(iii) The third term = Product of the second terms of the two binomials.

Example 1 :

Evaluate :

(i) $(2x + 3y)(3x + 4y)$ (ii) $(2a + 3)(5a - 7)$

(iii) $(4a - 3b)(2a + 5b)$ (iv) $(7x - 3)(2x - 9)$.

Solution :

(i) $(2x + 3y)(3x + 4y) = (2x \times 3x) + (2x \times 4y + 3y \times 3x) + (3y \times 4y)$

$= 6x^2 + (8xy + 9xy) + (12y^2)$

$= 6x^2 + 17xy + 12y^2$ **(Ans.)**

(ii) $(2a + 3)(5a - 7) = (2a \times 5a) + (2a \times -7 + 3 \times 5a) + (3 \times -7)$
$= 10a^2 + (-14a + 15a) + (-21)$
$= 10a^2 + a - 21$ **(Ans.)**

(iii) $(4a - 3b)(2a + 5b) = (4a \times 2a) + (4a \times 5b + -3b \times 2a) + (-3b \times 5b)$
$= 8a^2 + (20ab - 6ab) + (-15b^2)$
$= 8a^2 + 14ab - 15b^2$ **(Ans.)**

(iv) $(7x - 3)(2x - 9) = (7x \times 2x) + (7x \times -9 + -3 \times 2x) + (-3 \times -9)$
$= 14x^2 + (-63x - 6x) + (27)$
$= 14x^2 - 69x + 27$ **(Ans.)**

12.3 PRODUCT OF SUM AND DIFFERENCE OF TWO TERMS

Consider the two terms 5x and 4y.

the sum of these two terms = 5x + 4y and

the difference of these terms = 5x − 4y.

And, **the product of their sum and their difference**

$= (5x + 4y)(5x - 4y)$
$= 5x(5x - 4y) + 4y(5x - 4y)$
$= 25x^2 - 20xy + 20xy - 16y^2$
$= 25x^2 - 16y^2$
$= (5x)^2 - (4y)^2 =$ **(First Term)^2 − (Second Term)^2**

Thus : $(x + y)(x - y) = x^2 - y^2$ and $x^2 - y^2 = (x + y)(x - y)$.

Example 2 :

Evaluate :

(i) $(x - 2)(x + 2)(x^2 + 4)$ (ii) $(2a - 5b)(2a + 5b)(4a^2 + 25b^2)$

Solution :

(i) $(x - 2)(x + 2)(x^2 + 4) = [(x - 2)(x + 2)](x^2 + 4)$
$= (x^2 - 2^2)(x^2 + 4)$
$= (x^2 - 4)(x^2 + 4)$
$= (x^2)^2 - (4)^2 = x^4 - 16$ **(Ans.)**

(ii) $(2a - 5b)(2a + 5b)(4a^2 + 25b^2) = [(2a - 5b)(2a + 5b)](4a^2 + 25b^2)$
$= [(2a)^2 - (5b)^2](4a^2 + 25b^2)$
$= (4a^2 - 25b^2)(4a^2 + 25b^2)$
$= (4a^2)^2 - (25b^2)^2 = 16a^4 - 625b^4$ **(Ans.)**

Example 3 :

Use of the formula $(a + b)(a - b) = a^2 - b^2$ to find the value of :

(i) 107×93 (ii) 30.8×29.2

Solution :

(i) $107 \times 93 = (100 + 7)(100 - 7)$
$= (100)^2 - (7)^2 = 10000 - 49 = 9951$ **(Ans.)**

(ii) $30.8 \times 29.2 = (30 + 0.8)(30 - 0.8)$
$= (30)^2 - (0.8)^2 = 900 - 0.64 = 899.36$ **(Ans.)**

EXERCISE 12(A)

1. Use direct method to evaluate the following products :
   (i) $(x + 8)(x + 3)$ (ii) $(y + 5)(y - 3)$
   (iii) $(a - 8)(a + 2)$ (iv) $(b - 3)(b - 5)$
   (v) $(3x - 2y)(2x + y)$ (vi) $(5a + 16)(3a - 7)$
   (vii) $(8 - b)(3 + b)$
2. Use direct method to evaluate :
   (i) $(x + 1)(x - 1)$ (ii) $(2 + a)(2 - a)$
   (iii) $(3b - 1)(3b + 1)$ (iv) $(4 + 5x)(4 - 5x)$
   (v) $(2a + 3)(2a - 3)$ (vi) $(xy + 4)(xy - 4)$
   (vii) $(ab + x^2)(ab - x^2)$
   (viii) $(3x^2 + 5y^2)(3x^2 - 5y^2)$
   (ix) $\left(z-\frac{2}{3}\right)\left(z+\frac{2}{3}\right)$
   (x) $\left(\frac{3}{5}a+\frac{1}{2}\right)\left(\frac{3}{5}a-\frac{1}{2}\right)$
   (xi) $(0{\cdot}5 - 2a)(0{\cdot}5 + 2a)$
   (xii) $\left(\frac{a}{2}-\frac{b}{3}\right)\left(\frac{a}{2}+\frac{b}{3}\right)$
3. Evaluate :
   (i) $(a + 1)(a - 1)(a^2 + 1)$
   (ii) $(a + b)(a - b)(a^2 + b^2)$
   (iii) $(2a - b)(2a + b)(4a^2 + b^2)$
   (iv) $(3 - 2x)(3 + 2x)(9 + 4x^2)$
   (v) $(3x - 4y)(3x + 4y)(9x^2 + 16y^2)$
4. Use the formula :
   $(a + b)(a - b) = a^2 - b^2$ to evaluate :
   (i) 21×19 (ii) 33×27
   (iii) 103×97 (iv) $9{\cdot}8 \times 10{\cdot}2$
   (v) $7{\cdot}7 \times 8{\cdot}3$ (vi) $4{\cdot}6 \times 5{\cdot}4$
5. Evaluate :
   (i) $(6 - xy)(6 + xy)$
   (ii) $\left(7x+\frac{2}{3}y\right)\left(7x-\frac{2}{3}y\right)$
   (iii) $\left(\frac{a}{2b}+\frac{2b}{a}\right)\left(\frac{a}{2b}-\frac{2b}{a}\right)$
   (iv) $\left(3x-\frac{1}{2y}\right)\left(3x+\frac{1}{2y}\right)$
   (v) $(2a + 3)(2a - 3)(4a^2 + 9)$
   (vi) $(a + bc)(a - bc)(a^2 + b^2c^2)$
   (vii) $(5x + 8y)(3x + 5y)$
   (viii) $(7x + 15y)(5x - 4y)$
   (ix) $(2a - 3b)(3a + 4b)$
   (x) $(9a - 7b)(3a - b)$

12.4 EXPANSIONS

In expansion, we study the multiplication of an expression by itself to obtain its second, third or higher power.

1. $(a + b)^2 = (a + b)(a + b)$
   $= a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

(Sum of two terms)^2 = (Ist term)^2 + 2 × 1st term × 2nd term + (2nd term)^2

2. $(a - b)^2 = (a - b)(a - b)$
   $= a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$

(Difference of two terms)^2 = (Ist term)^2 − 2 × Ist term × 2nd term + (2nd term)^2

Examples :

1. $(3x + 4y)^2$ = (Ist term)^2 + 2 × Ist term × 2nd term + (2nd term)^2
   $= (3x)^2 + 2 \times 3x \times 4y + (4y)^2$
   $= 9x^2 + 24xy + 16y^2$ **(Ans.)**

2. $\left(\frac{3x}{2y}-\frac{2y}{3x}\right)^2$ = (Ist term)^2 − 2 × Ist term × 2nd term + (2nd term)^2

$$= \left(\frac{3x}{2y}\right)^2 - 2 \times \frac{3x}{2y} \times \frac{2y}{3x} + \left(\frac{2y}{3x}\right)^2 = \frac{9x^2}{4y^2} - 2 + \frac{4y^2}{9x^2}$$ **(Ans.)**

3. $(208)^2 = (200 + 8)^2$

$= (200)^2 + 2 \times 200 \times 8 + (8)^2$

$= 40000 + 3200 + 64 = 43264$ **(Ans.)**

4. $(9{\cdot}7)^2 = (10 - 0{\cdot}3)^2$

$= (10)^2 - 2 \times 10 \times 0{\cdot}3 + (0.3)^2 = 100 - 6 + 0{\cdot}09 = 94{\cdot}09$ **(Ans.)**

12.5 IMPORTANT FORMULAE TO BE MEMORISED

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$
4. $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$
5. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
   $= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
6. $(a + b - c)^2 = a^2 + b^2 + (-c)^2 + 2(a \times b) + 2(b \times -c) + 2(-c \times a)$
   $= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

Example 4 :

Expand : (i) $\left(2x + \frac{1}{2x}\right)^2$ (ii) $\left(3a - \frac{1}{a}\right)^2$

(iii) $(a + 2b - 5c)^2$ (iv) $(a - 2b - 5c)^2$

Solution :

(i) $\left(2x + \frac{1}{2x}\right)^2 = (2x)^2 + \left(\frac{1}{2x}\right)^2 + 2 \times 2x \times \frac{1}{2x}$

$= 4x^2 + \frac{1}{4x^2} + 2$ **(Ans.)**

(ii) $\left(3a - \frac{1}{a}\right)^2 = (3a)^2 + \left(\frac{1}{a}\right)^2 - 2 \times 3a \times \frac{1}{a}$

$= 9a^2 + \frac{1}{a^2} - 6$ **(Ans.)**

(iii) $(a + 2b - 5c)^2 = (a)^2 + (2b)^2 + (-5c)^2 + 2(a \times 2b) + 2(2b \times -5c) + 2(-5c \times a)$

$= a^2 + 4b^2 + 25c^2 + 4ab - 20bc - 10ca$ **(Ans.)**

(iv) $(a - 2b - 5c)^2 = (a)^2 + (-2b)^2 + (-5c)^2 + 2(a \times -2b) + 2(-2b \times -5c) + 2(-5c \times a)$

$= a^2 + 4b^2 + 25c^2 - 4ab + 20bc - 10ca$ **(Ans.)**

12.6 CUBES OF BINOMIALS

1. $(a + b)^3 = (a + b)(a + b)^2$
$= (a + b)(a^2 + 2ab + b^2)$
$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$
$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$

2. $(a - b)^3 = (a - b)(a - b)^2$
$= (a - b)(a^2 - 2ab + b^2)$
$= a(a^2 - 2ab + b^2) - b(a^2 - 2ab + b^2)$
$= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3 = a^3 - 3a^2b + 3ab^2 - b^3$

> 1. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
> $= a^3 + b^3 + 3ab(a + b)$
> 2. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
> $= a^3 - b^3 - 3ab(a - b)$

Example 5 :

Expand : (i) $(3x + 2y)^3$ (ii) $(5y - 3x)^3$

Solution :

(i) Since, $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\therefore$ $(3x + 2y)^3 = (3x)^3 + 3 \times (3x)^2 \times 2y + 3 \times 3x \times (2y)^2 + (2y)^3$

$= 27x^3 + 54x^2y + 36xy^2 + 8y^3$ **(Ans.)**

(ii) Since, $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$\therefore$ $(5y - 3x)^3 = (5y)^3 - 3 \times (5y)^2 \times 3x + 3 \times 5y \times (3x)^2 - (3x)^3$

$= 125y^3 - 225y^2x + 135yx^2 - 27x^3$ **(Ans.)**

EXERCISE 12(B)

1. Expand :
(i) $(2a + b)^2$ (ii) $(a - 2b)^2$
(iii) $\left(a + \frac{1}{2a}\right)^2$ (iv) $\left(2a - \frac{1}{a}\right)^2$
(v) $(a + b - c)^2$ (vi) $(a - b + c)^2$
(vii) $\left(3x + \frac{1}{3x}\right)^2$ (viii) $\left(2x - \frac{1}{2x}\right)^2$

2. Find the square of :
(i) $x + 3y$ (ii) $2x - 5y$
(iii) $a + \frac{1}{5a}$ (iv) $2a - \frac{1}{a}$
(v) $x - 2y + 1$ (vi) $3a - 2b - 5c$
(vii) $2x + \frac{1}{x} + 1$ (viii) $5 - x + \frac{2}{x}$
(ix) $2x - 3y + z$ (x) $x + \frac{1}{x} - 1$

3. Evaluate using expansion of $(a + b)^2$ or $(a - b)^2$:
(i) $(208)^2$ (ii) $(92)^2$
(iii) $(415)^2$ (iv) $(188)^2$
(v) $(9.4)^2$ (vi) $(20.7)^2$

4. Expand :
(i) $(2a + b)^3$ (ii) $(a - 2b)^3$
(iii) $(3x - 2y)^3$ (iv) $(x + 5y)^3$
(v) $\left(a + \frac{1}{a}\right)^3$ (vi) $\left(2a - \frac{1}{2a}\right)^3$

5. Find the cube of :
(i) $a + 2$ (ii) $2a - 1$
(iii) $2a + 3b$ (iv) $3b - 2a$
(v) $2x + \frac{1}{x}$ (vi) $x - \frac{1}{2}$

12.7 APPLICATION OF FORMULAE

Example 6 :

(i) If $a + b = 8$ and $ab = 15$, find $a^2 + b^2$.

(ii) If $a - b = 3$ and $a^2 + b^2 = 29$, find ab.

Solution :

(i) $(a + b)^2 = a^2 + b^2 + 2ab = a^2 + b^2 + 2ab$

$\Rightarrow \quad (8)^2 = a^2 + b^2 + 2 \times 15$

$\Rightarrow \quad 64 - 30 = a^2 + b^2 \qquad \therefore\ a^2 + b^2 = 34$ **(Ans.)**

(ii) $(a - b)^2 = a^2 + b^2 - 2ab = a^2 + b^2 - 2ab$

$\Rightarrow \quad (3)^2 = 29 - 2ab$

$\Rightarrow \quad 2ab = 29 - 9 = 20 \qquad \therefore\ ab = \frac{20}{2} = 10$ **(Ans.)**

Example 7 :

If $a^2 + b^2 = 73$ and $ab = 24$; find : (i) $a + b$ (ii) $a - b$

Solution :

(i) $(a + b)^2 = a^2 + b^2 + 2ab \Rightarrow (a + b)^2 = 73 + 2 \times 24 = 73 + 48 = 121$

$\therefore \quad a + b = \pm\sqrt{121} = \pm 11$ **(Ans.)**

(ii) $(a - b)^2 = a^2 + b^2 - 2ab \Rightarrow (a - b)^2 = 73 - 2 \times 24 = 25$

$\therefore \quad a - b = \pm\sqrt{25} = \pm 5$ **(Ans.)**

Example 8 :

If $a^2 + \frac{1}{a^2} = 2$; find : (i) $a + \frac{1}{a}$ (ii) $a - \frac{1}{a}$

Solution :

(i) $\left(a+\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2 \Rightarrow \left(a+\frac{1}{a}\right)^2 = 2 + 2 = 4$

$\therefore \quad a + \frac{1}{a} = \pm\sqrt{4} = \pm 2$ **(Ans.)**

(ii) $\left(a-\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2 \Rightarrow \left(a-\frac{1}{a}\right)^2 = 2 - 2 = 0$

$\therefore \quad a - \frac{1}{a} = \sqrt{0} = 0$ **(Ans.)**

Example 9 :

If $a + b + c = 9$ and $a^2 + b^2 + c^2 = 29$, find $ab + bc + ca$.

Solution :

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow \quad 9^2 = 29 + 2(ab + bc + ca)$ [By substituting the given values]

$\Rightarrow \quad 81 - 29 = 2(ab + bc + ca)$

$\Rightarrow \quad 52 = 2(ab + bc + ca)$

$\Rightarrow \quad ab + bc + ca = \frac{52}{2} = 26$ **(Ans.)**

Example 10 :

If $a + b = 5$ and $ab = 6$, find $a^3 + b^3$.

Solution :

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$\Rightarrow \quad (5)^3 = a^3 + b^3 + 3 \times 6 \times 5$$

$$\Rightarrow \quad 125 - 90 = a^3 + b^3$$

$$\therefore \quad a^3 + b^3 = 35$$

(Ans.)

Example 11 :

If $a - \frac{1}{a} = 3$; find $a^3 - \frac{1}{a^3}$

Solution :

Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$$\Rightarrow \quad \left(a - \frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3a \times \frac{1}{a}\left(a - \frac{1}{a}\right)$$

$$\Rightarrow \quad (3)^3 = a^3 - \frac{1}{a^3} - 3 \times 3$$ [Given $a - \frac{1}{a} = 3$]

$$\Rightarrow \quad 27 + 9 = a^3 - \frac{1}{a^3}$$

$$\Rightarrow \quad a^3 - \frac{1}{a^3} = 36$$

(Ans.)

Example 12 :

The sum of two numbers is 4 and their product is 3. Find :

(i) the sum of their squares. (ii) the sum of their cubes.

Solution :

Let the numbers be x and y.

$\therefore$ $x + y = 4$ and $xy = 3$

To find (i) $x^2 + y^2$ (ii) $x^3 + y^3$.

(i) $(x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow$ $4^2 = x^2 + y^2 + 2 \times 3 \Rightarrow x^2 + y^2 = 10$ **(Ans.)**

(ii) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow$ $4^3 = x^3 + y^3 + 3 \times 3 \times 4 \Rightarrow x^3 + y^3 = 64 - 36 = 28$ **(Ans.)**

EXERCISE 12(C)

1. If $a + b = 5$ and $ab = 6$, find $a^2 + b^2$
2. If $a - b = 6$ and $ab = 16$, find $a^2 + b^2$
3. If $a^2 + b^2 = 29$ and $ab = 10$, find :
   (i) $a + b$ (ii) $a - b$
4. If $a^2 + b^2 = 10$ and $ab = 3$; find :
   (i) $a - b$ (ii) $a + b$
5. If $a + \frac{1}{a} = 3$, find : $a^2 + \frac{1}{a^2}$
6. If $a - \frac{1}{a} = 4$, find : $a^2 + \frac{1}{a^2}$
7. If $a^2 + \frac{1}{a^2} = 23$, find : $a + \frac{1}{a}$
8. If $a^2 + \frac{1}{a^2} = 11$, find : $a - \frac{1}{a}$

9. If $a + b + c = 10$ and $a^2 + b^2 + c^2 = 38$, find : $ab + bc + ca$

10. Find : $a^2 + b^2 + c^2$, if $a + b + c = 9$ and $ab + bc + ca = 24$.

11. Find : $a + b + c$, if $a^2 + b^2 + c^2 = 83$ and $ab + bc + ca = 71$.

12. If $a + b = 6$ and $ab = 8$, find : $a^3 + b^3$.

13. If $a - b = 3$ and $ab = 10$, find : $a^3 - b^3$.

14. Find : $a^3 + \frac{1}{a^3}$, if $a + \frac{1}{a} = 5$.

15. Find : $a^3 - \frac{1}{a^3}$, if $a - \frac{1}{a} = 4$.

16. If $2x - \frac{1}{2x} = 4$, find :
    (i) $4x^2 + \frac{1}{4x^2}$ (ii) $8x^3 - \frac{1}{8x^3}$

17. If $3x + \frac{1}{3x} = 3$, find :
    (i) $9x^2 + \frac{1}{9x^2}$ (ii) $27x^3 + \frac{1}{27x^3}$

18. The sum of the squares of two numbers is 13 and their product is 6. Find :
    (i) the sum of the two numbers.
    (ii) the difference between them.

EXERCISE 12(D)

1. Evaluate :
   (i) $\left(3x+\frac{1}{2}\right)\left(2x+\frac{1}{3}\right)$
   (ii) $(2a + 0.5)(7a - 0.3)$
   (iii) $(9 - y)(7 + y)$ (iv) $(2 - z)(15 - z)$
   (v) $(a^2 + 5)(a^2 - 3)$ (vi) $(4 - ab)(8 + ab)$
   (vii) $(5xy - 7)(7xy + 9)$
   (viii) $(3a^2 - 4b^2)(8a^2 - 3b^2)$

2. Evaluate :
   (i) $\left(2x-\frac{3}{5}\right)\left(2x+\frac{3}{5}\right)$
   (ii) $\left(\frac{4}{7}a+\frac{3}{4}b\right)\left(\frac{4}{7}a-\frac{3}{4}b\right)$
   (iii) $(6 - 5xy)(6 + 5xy)$
   (iv) $\left(2a+\frac{1}{2a}\right)\left(2a-\frac{1}{2a}\right)$
   (v) $(4x^2 - 5y^2)(4x^2 + 5y^2)$
   (vi) $(1.6x + 0.7y)(1.6x - 0.7y)$
   (vii) $(m + 3)(m - 3)(m^2 + 9)$
   (viii) $(3x + 4y)(3x - 4y)(9x^2 + 16y^2)$
   (ix) $(a + bc)(a - bc)(a^2 + b^2c^2)$
   (x) 203×197
   (xi) 20.8×19.2

3. Find the square of :
   (i) $3x + \frac{2}{y}$ (ii) $\frac{5a}{6b} - \frac{6b}{5a}$
   (iii) $2m^2 - \frac{2}{3}n^2$ (iv) $5x + \frac{1}{5x}$
   (v) $8x + \frac{3}{2}y$ (vi) 607
   (vii) 391 (viii) 9.7

4. If $a + \frac{1}{a} = 2$, find :
   (i) $a^2 + \frac{1}{a^2}$ (ii) $a^4 + \frac{1}{a^4}$

5. If $m - \frac{1}{m} = 5$, find :
   (i) $m^2 + \frac{1}{m^2}$ (ii) $m^4 + \frac{1}{m^4}$
   (iii) $m^2 - \frac{1}{m^2}$

6. If $a^2 + b^2 = 41$ and $ab = 4$, find :
   (i) $a - b$ (ii) $a + b$

7. If $2a + \frac{1}{2a} = 8$, find :
   (i) $4a^2 + \frac{1}{4a^2}$ (ii) $16a^4 + \frac{1}{16a^4}$

8. If $3x - \frac{1}{3x} = 5$, find :
   (i) $9x^2 + \frac{1}{9x^2}$ (ii) $81x^4 + \frac{1}{81x^4}$

9. Expand :
   (i) $(3x - 4y + 5z)^2$ (ii) $(2a - 5b - 4c)^2$
   (iii) $(5x + 3y)^3$ (iv) $(6a - 7b)^3$

10. If $a + b + c = 9$ and $ab + bc + ca = 15$, find : $a^2 + b^2 + c^2$.

11. If $a + b + c = 11$ and $a^2 + b^2 + c^2 = 81$, find : $ab + bc + ca$.

12. If $3x - 4y = 5$ and $xy = 3$, find : $27x^3 - 64y^3$.

13. If $a + b = 8$ and $ab = 15$, find : $a^3 + b^3$.

14. If $3x + 2y = 9$ and $xy = 3$, find : $27x^3 + 8y^3$.

15. If $5x - 4y = 7$ and $xy = 8$, find : $125x^3 - 64y^3$.

16. The difference between two numbers is 5 and their product is 14. Find the difference between their cubes.

FACTORISATION 13

13.1 REVIEW

Factors	Each of the numbers (constant or variable), which form a product is called a factor of the product. (i) 5 and x are factors of the product 5x. (ii) $(2x - 5)$ and $(3x + 2)$ are the factors of $(2x - 5)(3x + 2)$. Since, $(2x - 5)(3x + 2) = 2x(3x + 2) - 5(3x + 2)$ $= 6x^2 + 4x - 15x - 10$ $= 6x^2 - 11x - 10$ $\therefore$ $2x - 5$ and $3x + 2$ are the factors of $6x^2 - 11x - 10$.

13.2 FACTORISATION

Factorisation means to find two or more expressions whose product is equal to the given expression.

13.3 FACTORISATION BY TAKING OUT COMMON FACTORS

Steps : 1. Find by inspection, the largest monomial that will divide each term of the given polynomial completely.

2. Divide each term of the given polynomial by this monomial (factor) and enclose the quotient within brackets keeping this common monomial outside the bracket.

Example 1 :

Factorise : (i) $5x^2 - 10x$ (ii) $3x^2y - 6xy^2 + 9xy$

Solution :

(i) By inspection, we find that the largest monomial which divides each term of the given polynomial $5x^2 - 10x$ is $5x$. [Step 1]

$$\therefore \quad 5x^2 - 10x = 5x\left(\frac{5x^2}{5x} - \frac{10x}{5x}\right) \quad \text{[Step 2]}$$

$$= 5x(x - 2) \quad \textbf{(Ans.)}$$

(ii)
$$3x^2y - 6xy^2 + 9xy = 3xy\left(\frac{3x^2y}{3xy} - \frac{6xy^2}{3xy} + \frac{9xy}{3xy}\right)$$

$$= 3xy(x - 2y + 3) \quad \textbf{(Ans.)}$$

Example 2 :

Factorise :

(i) $-10a^4x^2 - 15a^6x^4 + 20a^7x^5$ (ii) $2x(a + b) - 3y(a + b)$

Solution :

(i)
$$-10a^4x^2 - 15a^6x^4 + 20a^7x^5 = -5a^4x^2\left(\frac{-10a^4x^2}{-5a^4x^2} - \frac{15a^6x^4}{-5a^4x^2} + \frac{20a^7x^5}{-5a^4x^2}\right)$$

$$= -5a^4x^2(2 + 3a^2x^2 - 4a^3x^3) \quad \textbf{(Ans.)}$$

(ii) $2x(a+b) - 3y(a+b) = (a+b)\left[\frac{2x(a+b)}{a+b} - \frac{3y(a+b)}{a+b}\right]$

$= (a+b)(2x-3y)$ **(Ans.)**

EXERCISE 13(A)

Factorise :

1. $15x + 5$
2. $a^3 - a^2 + a$
3. $3x^2 + 6x^3$
4. $4a^2 - 8ab$
5. $2x^3b^2 - 4x^5b^4$
6. $15x^4y^3 - 20x^3y$
7. $a^3b - a^2b^2 - b^3$
8. $6x^2y + 9xy^2 + 4y^3$
9. $17a^6b^8 - 34a^4b^6 + 51a^2b^4$
10. $3x^5y - 27x^4y^2 + 12x^3y^3$
11. $x^2(a-b) - y^2(a-b) + z^2(a-b)$
12. $(x+y)(a+b) + (x-y)(a+b)$
13. $2b(2a+b) - 3c(2a+b)$
14. $12abc - 6a^2b^2c^2 + 3a^3b^3c^3$
15. $4x(3x-2y) - 2y(3x-2y)$
16. $(a+2b)(3a+b) - (a+b)(a+2b) + (a+2b)^2$
17. $6xy(a^2+b^2) + 8yz(a^2+b^2) - 10xz(a^2+b^2)$

13.4 FACTORISATION BY GROUPING

A given algebraic expression, containing an even number of terms may be resolved into factors, if its terms can be arranged in groups such that each group has a common factor.

Steps :
1. Arrange the terms of the given expression in suitable groups such that each group has a common factor.
2. Factorise each group.
3. Take out the factor which is common to each group.

Example 3 :

Factorise : $ax - bx + ay - by$

Solution :

$ax - bx + ay - by$

$= (ax - bx) + (ay - by)$ [Step 1]

$= x(a-b) + y(a-b)$ [Step 2]

$= (a-b)(x+y)$ **(Ans.)** [Step 3]

Or $ax - bx + ay - by$

$= ax + ay - bx - by$ [Step 1]

$= a(x+y) - b(x+y)$ [Step 2]

$= (x+y)(a-b)$ **(Ans.)** [Step 3]

Example 4 :

Factorise :

(i) $y^3 - 3y^2 + 2y - 6 - xy + 3x$ (ii) $a^2 - (b+5)a + 5b$

Solution :

(i) $y^3 - 3y^2 + 2y - 6 - xy + 3x = (y^3 - 3y^2) + (2y - 6) - (xy - 3x)$ [Step 1]

$= y^2(y-3) + 2(y-3) - x(y-3)$ [Step 2]

$= (y-3)(y^2 + 2 - x)$ **(Ans.)** [Step 3]

(ii) $a^2 - (b + 5)a + 5b = a^2 - ab - 5a + 5b$ [Removing the bracket]

$= (a^2 - ab) - (5a - 5b)$ [Step 1]

$= a(a - b) - 5(a - b)$ [Step 2]

$= (a - b)(a - 5)$ **(Ans.)** [Step 3]

EXERCISE 13(B)

Factorise :

1. $a^2 + ax + ab + bx$
2. $a^2 - ab - ca + bc$
3. $ab - 2b + a^2 - 2a$
4. $a^3 - a^2 + a - 1$
5. $2a - 4b - xa + 2bx$
6. $xy - ay - ax + a^2 + bx - ab$
7. $3x^5 - 6x^4 - 2x^3 + 4x^2 + x - 2$
8. $-x^2y - x + 3xy + 3$
9. $6a^2 - 3a^2b - bc^2 + 2c^2$
10. $3a^2b - 12a^2 - 9b + 36$
11. $x^2 - (a - 3)x - 3a$
12. $x^2 - (b - 2)x - 2b$
13. $a(b - c) - d(c - b)$
14. $ab^2 - (a - c)b - c$
15. $(a^2 - b^2)c + (b^2 - c^2)a$
16. $a^3 - a^2 - ab + a + b - 1$
17. $ab(c^2 + d^2) - a^2cd - b^2cd$
18. $2ab^2 - aby + 2cby - cy^2$
19. $ax + 2bx + 3cx - 3a - 6b - 9c$
20. $2ab^2c - 2a + 3b^3c - 3b - 4b^2c^2 + 4c$

13.5 FACTORISATION OF DIFFERENCE OF TWO SQUARES

Since, the product of $(x + y)$ and $(x - y)$ is $x^2 - y^2$

$\therefore$ Factors of $x^2 - y^2 = (x + y)(x - y)$

Difference of squares of two terms = Sum of the two terms × their difference.

Example 5 :

Factorise : $25a^2 - 36b^2$

Solution :

$$25a^2 - 36b^2 = (5a)^2 - (6b)^2 = (5a + 6b)(5a - 6b) \quad \textbf{(Ans.)}$$

Example 6 :

Factorise :

(i) $1 - 4(a - 2b)^2$ (ii) $9(x + y)^2 - 16(x - 3y)^2$

Solution :

(i) $1 - 4(a - 2b)^2 = 1 - 2^2(a - 2b)^2$

$= 1 - [2(a - 2b)]^2$

$= 1^2 - (2a - 4b)^2$

$= (1 + \overline{2a - 4b})(1 - \overline{2a - 4b})$

$= (1 + 2a - 4b)(1 - 2a + 4b)$ **(Ans.)**

(ii) $9(x + y)^2 - 16(x - 3y)^2 = [3(x + y)]^2 - [4(x - 3y)]^2$

$= (3x + 3y)^2 - (4x - 12y)^2$

$= (\overline{3x + 3y} + \overline{4x - 12y})(\overline{3x + 3y} - \overline{4x - 12y})$

$= (3x + 3y + 4x - 12y)(3x + 3y - 4x + 12y)$

$= (7x - 9y)(15y - x)$ **(Ans.)**

EXERCISE 13(C)

Factorise :

1. $16 - 9x^2$
2. $1 - 100a^2$
3. $4x^2 - 81y^2$
4. $\frac{4}{25} - 25b^2$
5. $(a + 2b)^2 - a^2$
6. $(5a - 3b)^2 - 16b^2$
7. $a^4 - (a^2 - 3b^2)^2$
8. $(5a - 2b)^2 - (2a - b)^2$
9. $1 - 25(a + b)^2$
10. $4(2a + b)^2 - (a - b)^2$
11. $25(2x + y)^2 - 16(x - y)^2$
12. $49(x - y)^2 - 9(2x + y)^2$
13. $\left(6\frac{2}{3}\right)^2 - \left(2\frac{1}{3}\right)^2$
14. $\left(7\frac{3}{10}\right)^2 - \left(2\frac{1}{10}\right)^2$
15. $(0{\cdot}7)^2 - (0{\cdot}3)^2$
16. $(4{\cdot}5)^2 - (1{\cdot}5)^2$
17. $75(x + y)^2 - 48(x - y)^2$
18. $a^2 + 4a + 4 - b^2$
19. $a^2 - b^2 - 2b - 1$
20. $x^2 + 6x + 9 - 4y^2$

13.6 FACTORISATION OF TRINOMIALS

Since, the product of two binomials $(2a + b)$ and $(3a - 5b)$

$= (2a + b)(3a - 5b)$

$= 6a^2 - 7ab - 5b^2$, **which is a trinomial.**

∴ The factors of a trinomial $6a^2 - 7ab - 5b^2$ are the binomials $(2a + b)$ and $(3a - 5b)$.

Before learning the factorisation of a trinomial, it is essential to know how to find out the two numbers whose product and sum are given.

Example 7 :

Find the numbers whose :

(i) product = 6 and sum = 5
(ii) product = 6 and sum = – 5
(iii) product = – 6 and sum = 5
(iv) product = – 6 and sum = – 5

Solution :

(i) Since, product = 6 and sum = 5. The *product* and the *sum* of two numbers are *positive* only when *both the numbers are positive.*

By trial, we find that the required two numbers are 3 and 2. **(Ans.)**

Product of 3 and 2 = $3 \times 2 = 6$ and their sum = $3 + 2 = 5$

(ii) Since, product = 6 and sum = – 5

The *product* of two numbers is *positive* and their *sum* is *negative* only when *both the numbers are negative.*

∴ Required numbers are – 3 and – 2. **(Ans.)**

(iii) Since, product = – 6 and sum = 5.

The *product* of two numbers is *negative* and their *sum* is *positive* only when the *larger* of the two numbers is *positive* and the *smaller* is *negative.*

By trial, we find that the required two numbers are 6 and – 1. **(Ans.)**

(iv) Since, product = – 6 and sum = – 5. The *product* of two numbers is *negative* and their *sum* is also *negative* only when the *larger* is *negative* and the *smaller* is *positive.*

∴ By trial, we find that the required two numbers are – 6 and 1. **(Ans.)**

Note : The standard forms of a trinomial are :

(i) $6x^2 + 11x + 3$ *i.e.* descending order of the powers of its literal coefficients.

(ii) $3 + 11x + 6x^2$ *i.e.* ascending order of the powers of its literal coefficients.

To factorise a given trinomial, the following steps should be adopted :

1. Find the product of the first and the last terms of the trinomial with their signs. In case of trinomial $6x^2 + 11x + 3$, the product of its first and last terms $= 6x^2 \times 3 = 18x^2$.
2. Split the middle term of the given trinomial (*i.e.* $+ 11x$) such that the sum of these two terms is equal to the middle term and their product is equal to the product obtained in step 1 (*i.e.* $18x^2$)

   By trial, we find that the two such terms are $+ 9x$ and $+ 2x$.
3. Now by forming the suitable groups, factorise the given trinomial.

   i.e. $6x^2 + 11x + 3 = 6x^2 + 9x + 2x + 3$

   $= 3x(2x + 3) + 1(2x + 3)$

   $= (2x + 3)(3x + 1)$ **(Ans.)**

Example 8 :

Factorise :

(i) $x^2 - 9x + 20$ (ii) $y^2 + 5y - 24$ (iii) $1 - 3a - 28a^2$

Solution :

(i) Given trinomial $= x^2 - 9x + 20$

The product of its first and the last terms $= x^2 \times 20 = 20x^2$

Splitting the middle term (*i.e.* $-9x$) into two terms so that their product is $20x^2$ and sum is $-9x$; we get : $-5x$ and $-4x$.

$\therefore$ $x^2 - 9x + 20 = x^2 - 5x - 4x + 20$

$= x(x - 5) - 4(x - 5)$

$= (x - 5)(x - 4)$ **(Ans.)**

(ii) Given trinomial is $y^2 + 5y - 24$

The product of its first and the last terms $= y^2 \times -24 = -24y^2$

and, the middle term $= +5y$.

Now find two terms whose product should be $-24y^2$ and sum should be $+ 5y$. By trial, we find that the required two terms are $+ 8y$ and $- 3y$.

$\therefore$ $y^2 + 5y - 24 = y^2 + 8y - 3y - 24$

$= y(y + 8) - 3(y + 8) = (y + 8)(y - 3)$ **(Ans.)**

(iii) Given trinomial is $1 - 3a - 28a^2$

Product of the first and the last terms $= 1 \times -28a^2 = -28a^2$

and, the middle term $= -3a$

By trial, we find that two terms whose product is $-28a^2$ and sum is $-3a$ are $-7a$ and $+4a$.

$\therefore$ $1 - 3a - 28a^2 = 1 - 7a + 4a - 28a^2$

$= 1(1 - 7a) + 4a(1 - 7a) = (1 - 7a)(1 + 4a)$ **(Ans.)**

Example 9 :

Factorise : (i) $(a+b)^2 - 11(a+b) - 42$ (ii) $7 + 10(x-y) - 8(x-y)^2$

Solution :

(i) $(a+b)^2 - 11(a+b) - 42 = x^2 - 11x - 42$ [Taking a + b = x]

$= x^2 - 14x + 3x - 42$ [Splitting the middle term]

$= x(x-14) + 3(x-14)$

$= (x-14)(x+3)$

$= (a+b-14)(a+b+3)$ **(Ans.)** [Substituting x = a + b]

(ii) $7 + 10(x-y) - 8(x-y)^2 = 7 + 10a - 8a^2$ [Taking x – y = a]

$= 7 + 14a - 4a - 8a^2$ [Splitting the middle term]

$= 7(1+2a) - 4a(1+2a)$

$= (1+2a)(7-4a)$

$= [1+2(x-y)][7-4(x-y)]$ [Substituting a = x – y]

$= (1+2x-2y)(7-4x+4y)$ **(Ans.)**

EXERCISE 13(D)

Factorise :

1. $x^2 + 6x + 8$
2. $x^2 + 4x + 3$
3. $a^2 + 5a + 6$
4. $a^2 - 5a + 6$
5. $a^2 + 5a - 6$
6. $x^2 + 5xy + 4y^2$
7. $a^2 - 3a - 40$
8. $x^2 - x - 72$
9. $x^2 - 10xy + 24y^2$
10. $2a^2 + 7a + 6$
11. $3a^2 - 5a + 2$
12. $7b^2 - 8b + 1$
13. $2a^2 - 17ab + 26b^2$
14. $2x^2 + xy - 6y^2$
15. $4c^2 + 3c - 10$
16. $14x^2 + x - 3$
17. $6 + 7b - 3b^2$
18. $5 + 7x - 6x^2$
19. $4 + y - 14y^2$
20. $5 + 3a - 14a^2$
21. $(2a+b)^2 + 5(2a+b) + 6$
22. $1 - (2x+3y) - 6(2x+3y)^2$
23. $(x-2y)^2 - 12(x-2y) + 32$
24. $8 + 6(a+b) - 5(a+b)^2$
25. $2(x+2y)^2 - 5(x+2y) + 2$

13.7 FACTORISING A PERFECT SQUARE TRINOMIAL

Square of a binomial is called a perfect square trinomial.

Since, $(a+b)^2 = a^2 + 2ab + b^2$

and, $(a-b)^2 = a^2 - 2ab + b^2$

∴ $a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$ are perfect square trinomials.

Any trinomial which can be expressed as $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$ is a perfect square trinomial.

Example 10 :

(i) Is $4x^2 + 12xy + 9y^2$ a perfect square trinomial ?

(ii) Is $x^2 - 6xy + 36y^2$ a perfect square trinomial ?

Solution :

(i) $4x^2 + 12xy + 9y^2 = (2x)^2 + 2 \times 2x \times 3y + (3y)^2$

$= a^2 + 2ab + b^2$ [Taking 2x = a and 3y = b]

$= (a+b)^2$

$= (2x+3y)^2$ **(Ans.)**

∴ **The given trinomial $4x^2 + 12xy + 9y^2$ is a perfect square trinomial.**

(ii) $x^2 - 6xy + 36y^2 = (x)^2 - x \times 6y + (6y)^2$

$= a^2 - ab + b^2$ [Taking x = a and 6y = b]

Since, the given trinomial cannot be expressed as $a^2 - 2ab + b^2$; **it is not a perfect square trinomial.** **(Ans.)**

13.8 FACTORISING COMPLETELY

Example 11 :

Factorise completely : (i) $8x^3 - 18xy^2$ (ii) $3x^2 + 12x - 36$

Solution :

(i) $8x^3 - 18xy^2 = 2x(4x^2 - 9y^2)$ [Taking out the common]

$= 2x[(2x)^2 - (3y)^2]$ [Converting in the form $a^2 - b^2$]

$= 2x(2x + 3y)(2x - 3y)$ **(Ans.)**

(ii) $3x^2 + 12x - 36 = 3(x^2 + 4x - 12)$ [Taking out the common factor]

$= 3(x^2 + 6x - 2x - 12)$ [Factorising the trinomial]

$= 3[x(x + 6) - 2(x + 6)]$

$= 3(x + 6)(x - 2)$ **(Ans.)**

Example 12 :

Factorise completely : (i) $x^2 + 4xy + 4y^2 - 9z^2$ (ii) $16x^4 - y^4$

Solution :

(i) In the given expression $x^2 + 4xy + 4y^2$ is a perfect square trinomial as :

$x^2 + 4xy + 4y^2 = x^2 + 2 \times x \times 2y + (2y)^2$

$= a^2 + 2ab + b^2$ [Taking x = a and 2y = b]

$= (a + b)^2$

$= (x + 2y)^2$ [Substituting]

$\therefore x^2 + 4xy + 4y^2 - 9z^2 = (x + 2y)^2 - (3z)^2$

$= (x + 2y + 3z)(x + 2y - 3z)$ **(Ans.)**

(ii) $16x^4 - y^4 = (4x^2)^2 - (y^2)^2$

$= (4x^2 + y^2)(4x^2 - y^2)$

$= (4x^2 + y^2)[(2x)^2 - (y)^2]$

$= (4x^2 + y^2)(2x + y)(2x - y)$ **(Ans.)**

EXERCISE 13(E)

1. In each case, find whether the trinomial is a perfect square or not :
   (i) $x^2 + 14x + 49$ (ii) $a^2 - 10a + 25$
   (iii) $4x^2 + 4x + 1$ (iv) $9b^2 + 12b + 16$
   (v) $16x^2 - 16xy + y^2$ (vi) $x^2 - 4x + 16$

Factorise completely :

2. $2 - 8x^2$
3. $8x^2y - 18y^3$
4. $ax^2 - ay^2$
5. $25x^3 - x$
6. $a^4 - b^4$
7. $16x^4 - 81y^4$
8. $625 - x^4$
9. $x^2 - y^2 - 3x - 3y$
10. $x^2 - y^2 - 2x + 2y$
11. $3x^2 + 15x - 72$
12. $2a^2 - 8a - 64$
13. $5b^2 + 45b + 90$
14. $3x^2y + 11xy + 6y$
15. $5ap^2 + 11ap + 2a$
16. $a^2 + 2ab + b^2 - c^2$
17. $x^2 + 6xy + 9y^2 + x + 3y$
18. $4a^2 - 12ab + 9b^2 + 4a - 6b$
19. $2a^2b^2 - 98b^4$
20. $a^2 - 16b^2 - 2a - 8b$

EXERCISE 13(F)

1. Factorise :
   (i) $6x^3 - 8x^2$
   (ii) $35a^3b^2c + 42ab^2c^2$
   (iii) $36x^2y^2 - 30x^3y^3 + 48x^3y^2$
   (iv) $8(2a + 3b)^3 - 12(2a + 3b)^2$
   (v) $9a(x - 2y)^4 - 12a(x - 2y)^3$
2. Factorise :
   (i) $a^2 - ab - 3a + 3b$
   (ii) $x^2y - xy^2 + 5x - 5y$
   (iii) $a^2 - ab(1 - b) - b^3$
   (iv) $xy^2 + (x - 1)y - 1$
   (v) $(ax + by)^2 + (bx - ay)^2$
   (vi) $ab(x^2 + y^2) - xy(a^2 + b^2)$
   (vii) $m - 1 - (m - 1)^2 + am - a$
3. Factorise :
   (i) $a^2 - (b - c)^2$
   (ii) $25(2x - y)^2 - 16(x - 2y)^2$
   (iii) $16(5x + 4)^2 - 9(3x - 2)^2$
   (iv) $9x^2 - \frac{1}{16}$
   (v) $25(x - 2y)^2 - 4$
4. Factorise :
   (i) $a^2 - 23a + 42.$ (ii) $a^2 - 23a - 108$
   (iii) $1 - 18x - 63x^2$ (iv) $5x^2 - 4xy - 12y^2$
   (v) $x(3x + 14) + 8$ (vi) $5 - 4x(1 + 3x)$
   (vii) $x^2y^2 - 3xy - 40$
   (viii) $(3x - 2y)^2 - 5(3x - 2y) - 24$
   (ix) $12(a + b)^2 - (a + b) - 35$
5. Factorise :
   (i) $15(5x - 4)^2 - 10(5x - 4)$
   (ii) $3a^2x - bx + 3a^2 - b$
   (iii) $b(c - d)^2 + a(d - c) + 3(c - d)$
   (iv) $ax^2 + b^2y - ab^2 - x^2y$
   (v) $1 - 3x - 3y - 4(x + y)^2$
6. Factorise :
   (i) $2a^3 - 50a$ (ii) $54a^2b^2 - 6$
   (iii) $64a^2b - 144b^3$ (iv) $(2x - y)^3 - (2x - y)$
   (v) $x^2 - 2xy + y^2 - z^2$
   (vi) $x^2 - y^2 - 2yz - z^2$
   (vii) $7a^5 - 567a$ (viii) $5x^2 - \frac{20x^4}{9}$
7. Factorise $xy^2 - xz^2$, Hence, find the value of :
   (i) $9 \times 8^2 - 9 \times 2^2$
   (ii) $40 \times 5.5^2 - 40 \times 4.5^2$
8. Factorise :
   (i) $(a - 3b)^2 - 36b^2$
   (ii) $25(a - 5b)^2 - 4(a - 3b)^2$
   (iii) $a^2 - 0{\cdot}36b^2$
   (iv) $a^4 - 625$
   (v) $x^4 - 5x^2 - 36$
   (vi) $15(2x - y)^2 - 16(2x - y) - 15$
9. Factorise $a^2b - b^3$. Using this result, find the value of $101^2 \times 100 - 100^3$
10. Evaluate (using factors) : $301^2 \times 300 - 300^3$.

LINEAR EQUATIONS IN ONE VARIABLE 14

(With Problems Based on Linear Equations)

14.1 REVIEW

1. **Equation**	A statement which states that the two algebraic *expressions are equal* is called an *equation.* *e.g.* Each of the following algebraic statement is an equation. (i) $7x^2 + 8 = x - 3$, (ii) $3x - 4y = 8$, (iii) $x - 3 = 3x + 8$, etc.
2. **Linear Equation**	The equation involving only one variable (unknown) in first order *(i.e. with highest power equal to one)* is called a linear equation. *e.g.* $3x - 5 = 0$, $8 - y = 15$, $7 + 3z = 10$, etc.

1. To solve an equation means to find the value of its variable (*i.e.* x, y or z, etc).
2. A **linear equation** has **only one solution,** which is called its **root.**

An equation remains unaltered (unchanged) on :

(i) *adding the same number to both sides of it;*
(ii) *subtracting the same number from both sides of it;*
(iii) *multiplying both sides of it by the same number;* and
(iv) *dividing both sides of it by the same number.*

Example 1 :

Solve : $21 - 3(a - 7) = a + 20$

Solution :

$$21 - 3a + 21 = a + 20$$

$$\Rightarrow \quad 42 - 20 = a + 3a \Rightarrow 22 = 4a \quad \therefore a = \frac{22}{4} = 5\frac{1}{2}$$ **(Ans.)**

Example 2 :

Solve : $\frac{y+2}{4} - \frac{y-3}{3} = \frac{1}{2}$

Solution :

Since, L.C.M. of denominators 4, 3 and 2 = 12

$$\therefore \quad 12 \times \frac{y+2}{4} - 12 \times \frac{y-3}{3} = 12 \times \frac{1}{2}$$ [Multiplying each term by 12]

$$\Rightarrow \quad 3(y + 2) - 4(y - 3) = 6$$

$$\Rightarrow \quad 3y + 6 - 4y + 12 = 6$$

$$\Rightarrow \quad -y = -12 \Rightarrow y = 12$$ **(Ans.)**

Example 3 :

Solve : (i) $\frac{5}{x} = \frac{7}{x-4}$ (ii) $\frac{a-2}{a+4} = \frac{a-3}{a+1}$

Solution :

On cross-multiplying; we get :

(i)

$$7x = 5(x - 4)$$
$$\Rightarrow 7x = 5x - 20$$
$$\Rightarrow 7x - 5x = -20$$
$$\Rightarrow 2x = -20$$
$$\Rightarrow x = -10$$ **(Ans.)**

(ii)

$$(a-2)(a+1) = (a-3)(a+4)$$
$$\Rightarrow a^2 - 2a + a - 2 = a^2 - 3a + 4a - 12$$
$$\Rightarrow a^2 - a - 2 = a^2 + a - 12$$
$$\Rightarrow a^2 - a - a^2 - a = -12 + 2$$
$$\Rightarrow -2a = -10$$
$$\Rightarrow a = 5$$ **(Ans.)**

Example 4 :

Solve : $\frac{2x+1}{10} - \frac{3-2x}{15} = \frac{x-2}{6}$. Hence, find the value of y, if $\frac{1}{x} + \frac{1}{y} = 3$.

Solution :

Since, L.C.M. of denominators 10, 15 and 6 = 30, multiply each fraction by 30 to get :

$$30 \times \frac{2x+1}{10} - 30 \times \frac{3-2x}{15} = 30 \times \frac{x-2}{6}$$
$$\Rightarrow 3(2x+1) - 2(3-2x) = 5(x-2)$$
$$\Rightarrow 6x + 3 - 6 + 4x = 5x - 10$$
$$\Rightarrow 10x - 5x = -10 + 3$$
$$\Rightarrow 5x = -7 \text{ and } x = -\frac{7}{5}$$ **(Ans.)**

Now, $\frac{1}{x} + \frac{1}{y} = 3 \Rightarrow -\frac{5}{7} + \frac{1}{y} = 3$ $\quad [x = -\frac{7}{5} \Rightarrow \frac{1}{x} = -\frac{5}{7}]$

$$\Rightarrow \frac{1}{y} = 3 + \frac{5}{7} = \frac{26}{7}$$
$$\therefore y = \frac{7}{26}$$ **(Ans.)**

EXERCISE 14(A)

Solve the following equations :

1. $20 = 6 + 2x$
2. $15 + x = 5x + 3$
3. $\frac{3x+2}{x-6} = -7$
4. $3a - 4 = 2(4 - a)$
5. $3(b - 4) = 2(4 - b)$
6. $\frac{x+2}{9} = \frac{x+4}{11}$
7. $\frac{x-8}{5} = \frac{x-12}{9}$
8. $5(8x + 3) = 9(4x + 7)$
9. $3(x + 1) = 12 + 4(x - 1)$
10. $\frac{3x}{4} - \frac{1}{4}(x - 20) = \frac{x}{4} + 32$
11. $3a - \frac{1}{5} = \frac{a}{5} + 5\frac{2}{5}$
12. $\frac{x}{3} - 2\frac{1}{2} = \frac{4x}{9} - \frac{2x}{3}$
13. $\frac{4(y+2)}{5} = 7 + \frac{5y}{13}$
14. $\frac{a+5}{6} - \frac{a+1}{9} = \frac{a+3}{4}$
15. $\frac{2x-13}{5} - \frac{x-3}{11} = \frac{x-9}{5} + 1$
16. $6(6x - 5) - 5(7x - 8) = 12(4 - x) + 1$
17. $(x - 5)(x + 3) = (x - 7)(x + 4)$
18. $(x - 5)^2 - (x + 2)^2 = -2$
19. $(x - 1)(x + 6) - (x - 2)(x - 3) = 3$

20. $\dfrac{3x}{x+6} - \dfrac{x}{x+5} = 2$

21. $\dfrac{1}{x-1} + \dfrac{2}{x-2} = \dfrac{3}{x-3}$

22. $\dfrac{x-1}{7x-14} = \dfrac{x-3}{7x-26}$

23. $\dfrac{1}{x-1} - \dfrac{1}{x} = \dfrac{1}{x+3} - \dfrac{1}{x+4}$

24. Solve : $\dfrac{2x}{3} - \dfrac{x-1}{6} + \dfrac{7x-1}{4} = 2\dfrac{1}{6}$.

Hence, find the value of 'a', if $\dfrac{1}{a} + 5x = 8$.

25. Solve : $\dfrac{4-3x}{5} + \dfrac{7-x}{3} + 4\dfrac{1}{3} = 0$.

Hence, find the value of 'p', if $3p - 2x + 1 = 0$.

26. Solve : $0{\cdot}25 + \dfrac{1{\cdot}95}{x} = 0{\cdot}9$.

27. Solve : $5x - (4x + \dfrac{5x-4}{7}) = \dfrac{4x-14}{3}$.

14.2 TO SOLVE PROBLEMS BASED ON LINEAR EQUATIONS

Steps :

1. Read the problem carefully to know what is given and what is to be found.
2. Represent the unknown quantity as x or some other letter such as a, b, y, z, etc.
3. According to the conditions, given in the problem, write the relation between the given quantity and quantity to be found.
4. Solve the equation to obtain the value of the unknown.

Example 5 :

Find a number such that one-fifth of it is less than its one-fourth by 3.

Solution :

Let the required number be x.

Since, one-fifth of x = $\dfrac{x}{5}$ and one-fourth of it = $\dfrac{x}{4}$; then according to the given statement :

$$\frac{x}{4} - \frac{x}{5} = 3 \Rightarrow \frac{5x-4x}{20} = 3 \qquad \text{[L.C.M. of 4 and 5 = 20]}$$

$$\Rightarrow \quad x = 3 \times 20 = 60 \qquad \textbf{(Ans.)}$$

Example 6 :

The difference of the squares of two consecutive even natural numbers is 92. Taking x as the smaller of the two numbers, form an equation in x and hence find the larger of the two.

Solution :

Since, the consecutive even natural numbers differ by 2 and it is given that the smaller of the two numbers is x; therefore, the next (larger) even number is x + 2.

According to the given statement :

$$(x+2)^2 - x^2 = 92 \qquad \text{[Difference of the squares]}$$

$$\Rightarrow \quad x^2 + 4x + 4 - x^2 = 92$$

$$\Rightarrow \quad 4x = 92 - 4 = 88$$

$$\Rightarrow \quad x = 22$$

$$\therefore \quad \text{Larger even number} = x + 2 = 22 + 2 = 24 \qquad \textbf{(Ans.)}$$

In case of integers, natural numbers and whole numbers :

1. Consecutive numbers are taken as : x + 1, x + 2, ...
2. Consecutive even numbers are taken as : x, x + 2, x + 4, ...; where x is an even number.
3. Consecutive odd numbers are also taken as : x, x + 2, x + 4, ...; where x is an odd number.
4. Consecutive multiples of 3 are taken as : x, x + 3, x + 6, ...; ...., where x is a multiiple of 3.

Example 7 :

A rectangle is 8 cm long and 5 cm wide. Its perimeter is doubled when each of its sides is increased by x cm. Form an equation in x and find the new length of the rectangle.

Solution :

Since, length of the rectangle = 8 cm and its width = 5 cm

$\therefore$ Its perimeter = 2(length + width)

= 2(8 + 5)cm = 26 cm

On increasing each of its sides by x cm,

its new length = (8 + x) cm

and, new width = (5 + x) cm

$\therefore$ Its new perimeter = 2(8 + x + 5 + x) cm

= (26 + 4x) cm

Given : new perimeter = 2 times the original perimeter

$\Rightarrow \quad 26 + 4x = 2 \times 26$

$\Rightarrow \quad 4x = 52 - 26 = 26$

$\Rightarrow \quad x = \frac{26}{4} = 6.5$ cm

And, **the new length of the rectangle** = (8 + x) cm

= (8 + 6.5) cm = **14.5 cm.** **(Ans.)**

Example 8 :

A man is 24 years older than his son. In 2 years, his age will be twice the age of his son. Find their present ages.

Solution :

Let the present age of the son be x years

$\therefore$ Present age of the father = (x + 24) years

In 2 years :

The man's age will be (x + 24) + 2 = (x + 26) years

and son's age will be x + 2 years

According to the problem : $x + 26 = 2(x + 2)$

On solving we get : $x = 22$

$\therefore$ **Present age of the man** = x + 24 = 22 + 24 = **46 years**

and, **Present age of the son** = x = **22 years.** **(Ans.)**

Example 9 :

One day a boy walked from his house to his school at the speed of 4 km/hr and he reached ten minutes late to the school. Next day, he ran at the speed of 8 km/hr and was 5 minutes early to the school. Find the distance between his house and school.

Solution :

Let the distance between his house and school be x km.

Since, time = $\frac{\text{distance}}{\text{speed}}$

$\therefore$ To reach the school, first day he takes $\frac{x}{4}$ hrs and next day he takes $\frac{x}{8}$ hrs .

Since, the difference of two timings = 10 minutes + 5 minutes = 15 minutes = $\frac{1}{4}$ hrs

$$\therefore \quad \frac{x}{4} - \frac{x}{8} = \frac{1}{4}.$$

On solving, we get : **x = 2 km.** **(Ans.)**

Example 10 :

Two consecutive even numbers are such that half of the larger exceeds one-fourth of the smaller by 5. Find the numbers.

Solution :

Let the required even numbers be x and x + 2.

Given : $\frac{1}{2}(x+2) - \frac{1}{4}x = 5$

$\Rightarrow \quad \frac{2x+4-x}{4} = 5$

$\Rightarrow \quad x + 4 = 20 \quad$ *i.e.* $\quad x = 20 - 4 = 16$

$\therefore$ **Required numbers** = x and x + 2

= 16 and 16 + 2 = **16 and 18** **(Ans.)**

Example 11 :

A person is paid ₹ 150 for each day he works and is fined ₹ 30 for each day he remains absent. If in 40 days, he earned ₹ 3,300; find for how many days did he work ?

Solution :

Let the man works for x days

$\therefore$ He remains absent for (40 – x) days.

Since, the man gets ₹ 150 for each day he worked and is fined ₹ 30 for each day he remains absent.

$\therefore \quad 150x - 30\,(40 - x) = 3{,}300$

$\Rightarrow \quad 150x - 1200 + 30x = 3{,}300$

$\Rightarrow \quad 180x = 3{,}300 + 1{,}200 = 4{,}500$

and, $\quad x = \frac{4{,}500}{180} = 25$

$\therefore$ **The man worked for 25 days** **(Ans.)**

EXERCISE 14(B)

1. Fifteen less than 4 times a number is 9. Find the number.
2. If Megha's age is increased by three times her age, the result is 60 years. Find her age.
3. 28 is 12 less than 4 times a number. Find the number.
4. Five less than 3 times a number is – 20. Find the number.
5. Fifteen more than 3 times Neetu's age is the same as 4 times her age. How old is she ?
6. A number decreased by 30 is the same as 14 decreased by 3 times the number. Find the number.
7. A's salary is same as 4 times B's salary. If together they earn ₹ 3,750 a month, find the salary of each.
8. Separate 178 into two parts so that the first part is 8 less than twice the second part.
9. Six more than one-fourth of a number is two-fifth of the number. Find the number.
10. The length of a rectangle is twice its width. If its perimeter is 54 cm, find its length.
11. A rectangle's length is 5 cm less than twice its width. If the length is decreased by 5 cm and width is increased by 2 cm; the perimeter of the resulting rectangle will be 74 cm. Find the length and the width of the original rectangle.
12. The sum of three consecutive odd numbers is 57. Find the numbers.
13. A man's age is three times that of his son and in twelve years he will be twice as old as his son would be. What are their present ages ?
14. A man is 42 years old and his son is 12 years old. In how many years will the age of the son be half the age of the man at that time ?
15. A man completed a trip of 136 km in 8 hours. Some parts of the trip was covered at 15 km/hr and the remaining at 18 km/hr. Find the part of the trip covered at 18 km/hr.
16. The difference of two numbers is 3 and the difference of their square is 69. Find the numbers.
17. Two consecutive natural numbers are such that one-fourth of the smaller exceeds one-fifth of the greater by 1. Find the numbers.
18. Three consecutive whole numbers are such that if they be divided by 5, 3 and 4 respectively; the sum of the quotients is 40. Find the numbers.
19. If the same number be added to the numbers 5, 11, 15 and 31, the resulting numbers are in proportion. Find the number.

> a, b, c and d are in proportion $\Rightarrow \frac{a}{b} = \frac{c}{d}$.

20. The present age of a man is twice that of his son. Eight years hence, their ages will be in the ratio 7 : 4. Find their present ages.

EXERCISE 14(C)

1. Solve :

(i) $\frac{1}{3}x - 6 = \frac{5}{2}$

(ii) $\frac{2x}{3} - \frac{3x}{8} = \frac{7}{12}$

(iii) $(x+2)(x+3)+(x-3)(x-2)-2x(x+1)=0$

(iv) $\frac{1}{10} - \frac{7}{x} = 35$

(v) $13(x-4)-3(x-9)-5(x+4)=0$

(vi) $x + 7 - \frac{8x}{3} = \frac{17x}{6} - \frac{5x}{8}$

(vii) $\frac{3x-2}{4} - \frac{2x+3}{3} = \frac{2}{3} - x$

(viii) $\frac{x+2}{6} - \left(\frac{11-x}{3} - \frac{1}{4}\right) = \frac{3x-4}{12}$

(ix) $\frac{2}{5x} - \frac{5}{3x} = \frac{1}{15}$

(x) $\frac{x+2}{3} - \frac{x+1}{5} = \frac{x-3}{4} - 1$

(xi) $\frac{3x-2}{3} + \frac{2x+3}{2} = x + \frac{7}{6}$

(xii) $x - \frac{x-1}{2} = 1 - \frac{x-2}{3}$

(xiii) $\frac{9x+7}{2} - (x - \frac{x-2}{7}) = 36$

(xiv) $\frac{6x+1}{2} + 1 = \frac{7x-3}{3}$

2. After 12 years, I shall be 3 times as old as I was 4 years ago. Find my present age.
3. A man sold an article for ₹ 396 and gained 10% on it. Find the cost price of the article.
4. The sum of two numbers is 4500. If 10% of one number is 12.5% of the other, find the numbers.
5. The sum of two numbers is 405 and their ratio is 8 : 7. Find the numbers.
6. The ages of A and B are in the ratio 7 : 5. Ten years hence, the ratio of their ages will be 9 : 7. Find their present ages.
7. Find the number whose double is 45 greater than its half.
8. The difference between the squares of two consecutive numbers is 31. Find the numbers.
9. Find a number such that when 5 is subtracted from 5 times the number, the result is 4 more than twice the number.
10. The numerator of a fraction is 5 less than its denominator. If 3 is added to the numerator and denominator both, the fraction becomes $\frac{4}{5}$. Find the original fraction.

LINEAR INEQUATIONS 15
(Including Number Lines)

15.1 INTRODUCTION

1. Equation	A *statement*, which says that *one thing is equal to another*, is called an *equation*. *e.g.* (i) $x = 5$ (ii) $3x = 7$ (iii) $2x - 5 = 10$, etc.
2. Inequation	A *statement*, which says that *one thing is not equal to another* (*i.e.*, either it is greater or lesser), is called an *inequation*. *e.g.* (i) $x < 7$ (read as x *is less than* 7) (ii) $x > 5$ (read as x *is greater than* 5)
3. Connecting-verbs	The symbols =, ≠, <, >, etc. are called *connecting verbs*. (i) '<' means; 'is less than', (ii) '>' means; 'is greater than', (iii) '≤' means; 'is less than or equal to', (iv) '≥' means; 'is greater than or equal to'.
4. Linear inequation	If a and b are real numbers and $a \neq 0$, then each of the following statements is called a linear inequation. (i) $ax + b > 0$ (ii) $ax + b < 0$ (iii) $ax + b \geq 0$ (iv) $ax + b \leq 0$

15.2 REPLACEMENT SET AND SOLUTION SET

For any linear inequation in x, the set from which the value(s) of variable x is chosen, is called the **replacement set** or the **universal set.**

The set of elements of the replacement set (universal set), which satisfy the given inequation, is called **the solution set** or **the truth set.**

e.g. Consider the inequation (statement) $x > 6$;

(i) if replacement set = {2, 4, 6, 8, 10}

then, the solution set = {8, 10}

(ii) if replacement set = {1, 3, 5, 7, 9, 11}

then, the solution set = {7, 9, 11}

The following table will make the concept more clear.

Replacement set	Linear inequation	Solution set
1. N, set of natural numbers	(i) $x < 4$ (ii) $5 < x \leq 10$	$\{x : x \in N \text{ and } x < 4\} = \{1, 2, 3\}$ $\{x : x \in N \text{ and } 5 < x \leq 10\}$ $= \{6, 7, 8, 9, 10\}$
2. {–4, –3, –2, –1, 0, 1, 2}	$x > -2$	{–1, 0, 1, 2}
3. {–5, –4, –3, –2, –1, 0}	$x \leq -2$	{–5, –4, –3, –2}

15.3 PROPERTIES

1. Adding the same number to each side of an inequation, does not change the sign of inequality.

   i.e. if $a > b$, then $a + c > b + c$

   and, if $a < b$, then $a + c < b + c$.

   For example :

   (i) $8 > 5 \Rightarrow 8 + 13 > 5 + 13$ (ii) $5 < 8 \Rightarrow 5 + 13 < 8 + 13$

   (iii) $8 > 5 \Rightarrow 8 + (-13) > 5 + (-13)$ (iv) $5 < 8 \Rightarrow 5 + (-13) < 8 + (-13)$

2. Subtracting the same number from each side of an inequation, does not change the sign of inequality.

   i.e. if $a > b$, then $a - c > b - c$

   and, if $a < b$, then $a - c < b - c$.

   For example :

   (i) $13 > 8 \Rightarrow 13 - 5 > 8 - 5$ (ii) $8 < 13 \Rightarrow 8 - 5 < 13 - 5$

3. Multiplying each side of an inequation by a positive number, does not change the sign of inequality.

   i.e. if $a > b$ and c is positive (*i.e.* $c > 0$) then, $a.c > b.c$

   also, if $a < b$ and $c > 0$; then $a \cdot c < b \cdot c$.

   For example :

   (i) $12 > 7 \Rightarrow 12 \times 6 > 7 \times 6$ (ii) $7 < 12 \Rightarrow 7 \times 8 < 12 \times 8$

4. Multiplying each side of an inequation by a negative number, reverses the sign of inequality.

   i.e. if $a > b$ and c is negative (*i.e.* $c < 0$), then $a \cdot c < b \cdot c$;

   also, if $a < b$ and $c < 0$; then $a \cdot c > b \cdot c$.

   For example :

   (i) $15 > 9 \Rightarrow 15 \times -4 < 9 \times -4$ (ii) $6 < 11 \Rightarrow 6 \times -5 > 11 \times -5$

5. Dividing each side of an inequation by a positive number, does not change the sign of inequality.

   i.e. if $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$

   also, if $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

   For example :

   (i) $14 > 9 \Rightarrow \frac{14}{5} > \frac{9}{5}$ (ii) $9 < 14 \Rightarrow \frac{9}{5} < \frac{14}{5}$

6. Dividing each side of an inequation by a negative number, reverses the sign of inequality.

   i.e. if $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$

   also, if $a < b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.

   For example :

   (i) $25 > 16 \Rightarrow \frac{25}{-9} < \frac{16}{-9}$ (ii) $8 < 15 \Rightarrow \frac{8}{-4} > \frac{15}{-4}$

Example 1 :

Find the solution set of the inequation :

(i) $12 + 6x > 0$; where x is a negative integer.

(ii) $30 - 4(2x - 1) < 30$; where x is a positive integer.

Solution :

(i) $12 + 6x > 0 \Rightarrow 6x > -12$

$\Rightarrow x > -2$ [Dividing by 6]

$\because$ x is a negative integer $\therefore$ **Solution set = {– 1}** **(Ans.)**

(ii) $30 - 4(2x - 1) < 30 \Rightarrow 30 - 8x + 4 < 30$

$\Rightarrow 34 - 8x < 30$

$\Rightarrow -8x < 30 - 34$

$\Rightarrow -8x < -4$

$\Rightarrow \frac{-8x}{-8} > \frac{-4}{-8}$ [Dividing by –8]

$\Rightarrow x > \frac{1}{2}$

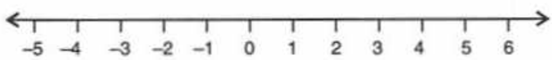
$\because$ x is a positive integer $\therefore$ **Solution set = {1, 2, 3, 4, 5, ...}** **(Ans.)**

EXERCISE 15(A)

1. If the replacement set is the set of natural numbers, solve :
   (i) $x - 5 < 0$ (ii) $x + 1 \le 7$
   (iii) $3x - 4 > 6$ (iv) $4x + 1 \ge 17$
2. If the replacement set = {– 6, – 3, 0, 3, 6, 9}; find the truth set of the following :
   (i) $2x - 1 > 9$ (ii) $3x + 7 \le 1$
3. Solve : $7 > 3x - 8$; $x \in N$.
4. Solve : $-17 < 9y - 8$; $y \in Z$.
5. Solve : $9x - 7 \le 28 + 4x$; $x \in W$.
6. Solve : $\frac{2}{3}x + 8 < 12$; $x \in W$.
7. Solve : $-5(x + 4) > 30$; $x \in Z$.
8. Solve the inequation $8 - 2x \ge x - 5$; $x \in N$.
9. Solve the inequality $18 - 3(2x - 5) > 12$; $x \in W$.
10. Solve : $\frac{2x+1}{3} + 15 \le 17$; $x \in W$.
11. Solve : $-3 + x < 2$, $x \in N$.
12. Solve : $4x - 5 > 10 - x$, $x \in \{0, 1, 2, 3, 4, 5, 6, 7\}$.
13. Solve : $15 - 2(2x - 1) < 15$, $x \in Z$.
14. Solve : $\frac{2x+3}{5} > \frac{4x-1}{2}$, $x \in W$.

15.4 NUMBER LINE

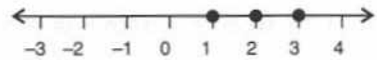
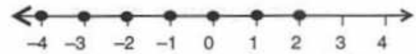
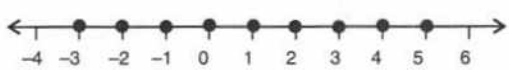
A number line is a graph (straight line) on which real numbers are marked as shown below :



(A number line)

The solution of every inequation can be represented on a number line.

For example :

	Inequation	Solution set	Corresponding number line
1.	x < 4 and x ∈ N	{1, 2, 3}	
2.	x < 5; x ∈ W	{0, 1, 2, 3, 4}	
3.	x < 3; x ∈ Z	{......, –3, –2, –1, 0, 1, 2}	
4.	– 3 ≤ x < 6; x ∈ W	{0, 1, 2, 3, 4, 5}	
5.	– 3 ≤ x < 6; x ∈ Z	{–3, –2, –1, 0, 1, 2, 3, 4, 5}	

> Thick dots on the number line represent the solution.

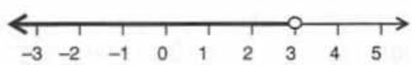
> The dark arrow on the left side shows that the solution set continues towards left side.

IMPORTANT

1. For x ≤ 3 where x is a real number; the number line will be as shown below :

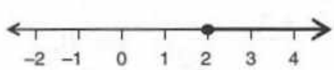
The dark circle around 3, shows 3 is included in the solution and the dark line with dark arrow on the left of number 3 shows that every number less than 3 is also included in the solution.

2. For x < 3 where x ∈ R; the number line will be as shown below :

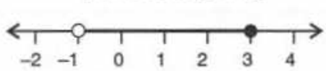


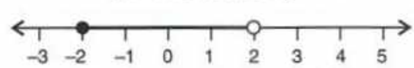
The hollow circle around 3, shows 3 is not included in the solution and the dark line with dark arrow on the left of number 3 shows that every number less than 3 is included in the solution.

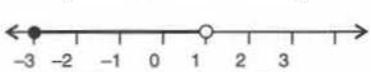
Similarly consider the following number lines :

1. 
[x ≥ 2 and x ∈ R]

2. 
[x > 2 and x ∈ R]

3. 
[–1 < x ≤ 3 and x ∈ R]

4. 
[– 2 ≤ x < 2 and x ∈ R]

5. 
[– 3 ≤ x < 1 and x ∈ R]

Example 2 :

Graph the solution set on a number line if – 2x + 14 < 6; where x is a real number.

Solution :

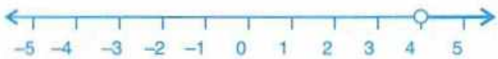
$-2x + 14 < 6 \Rightarrow -2x < 6 - 14$

$\Rightarrow -2x < -8$

$\Rightarrow \frac{-2x}{-2} > \frac{-8}{-2}$ [Division by a negative number, reverses the sign of inequality]

$\Rightarrow x > 4$ **(Ans.)**

$\therefore$ The required graph is :



EXERCISE 15(B)

Solve and graph the solution set on a number line :

1. $x - 5 < -2; x \in N$
2. $3x - 1 > 5; x \in W$
3. $-3x + 12 < -15; x \in R$
4. $7 \geq 3x - 8; x \in W$
5. $8x - 8 \leq -24; x \in Z$
6. $8x - 9 \geq 35 - 3x; x \in N$
7. $5x + 4 > 8x - 11; x \in Z$
8. $\frac{2x}{5} + 1 < -3; x \in R$
9. $\frac{x}{2} > -1 + \frac{3x}{4}; x \in N$
10. $\frac{2}{3}x + 5 \leq \frac{1}{2}x + 6; x \in W$
11. Solve the inequation $5(x - 2) > 4(x + 3) - 24$ and represent its solution on a number line. Given the replacement set is $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$.
12. Solve $\frac{2}{3}(x - 1) + 4 < 10$ and represent its solution on a number line. Given replacement set is $\{-8, -6, -4, 3, 6, 8, 12\}$.
13. For each inequation, given below, represent the solution on a number line :

    (i) $\frac{5}{2} - 2x \geq \frac{1}{2}, x \in W$

    (ii) $3(2x - 1) \geq 2(2x + 3), x \in Z$

    (iii) $2(4 - 3x) \leq 4(x - 5), x \in W$

    (iv) $4(3x + 1) > 2(4x - 1)$, x is a negative integer

    (v) $\frac{4 - x}{2} < 3, x \in R$

    (vi) $-2(x + 8) \leq 8, x \in R$

Theme – 4
GEOMETRY

UNDERSTANDING SHAPES 16

(Including Polygons)

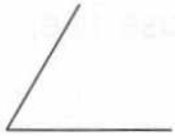
16.1 INTRODUCTION

On a plane sheet of paper, draw different types of figures.

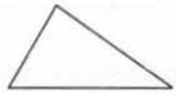
Some of the figures are shown below :

(i)

(ii)

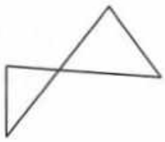


(iii)

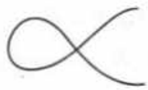


(iv)

(v)

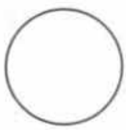


(vi)



(vii)

(viii)



All these figures are called **curves**.

In our day-to-day life, the word **curve** means not **straight**. However, in mathematics, a **straight line** is also called a **curve**.

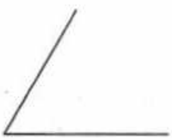
16.2 DIFFERENT TYPES OF CURVES

1. **Open curve :**

A curve which does not cut itself is called an **open curve**. e.g.

(i)

(ii)



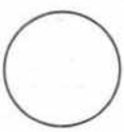
(iii)

, etc.

2. **Closed curve :**

A curve which cuts itself is called a **closed curve**. e.g.

(i)



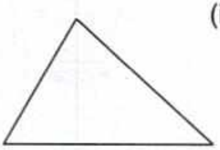
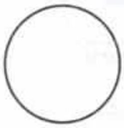
(ii)

(iii)

, etc.

3. **Simple closed curve :**

A closed curve is called a **simple closed curve**, if it does not pass through any point of it, more than once. e.g.

(i)  (ii)  (iii)  etc.

A closed curve, drawn on a plane paper, divides the plane into three parts.

(i) **the interior of the curve :**

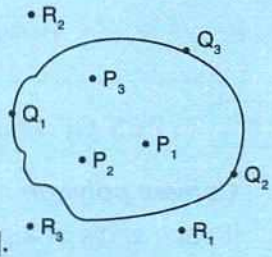
[The points P_1, P_2, P_3, ... lie in the interior (inside) of the curve.]

(ii) **the boundary of the curve :**

[The points Q_1, Q_2, Q_3,........ lie on the boundary of the curve].

(iii) **the exterior of the curve :**

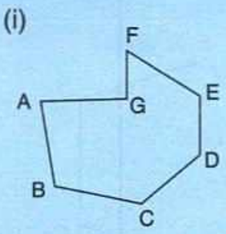
[The points R_1, R_2, R_3, ... lie outside (exterior) of the curve].

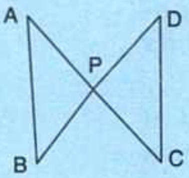


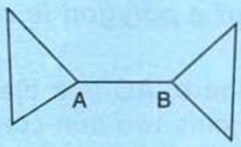
The interior of the curve together with its boundary is called its **region**.

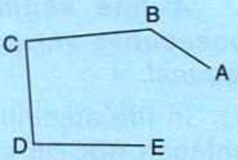
16.3 POLYGON

Polygon	It is a closed plane figure, bounded by straight-line segments. The line segments forming a polygon intersect only at end-points and each end-point is shared by only two line segments.

(i) 

(ii) 

(iii) 

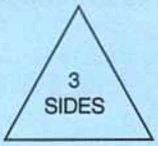
(iv) 

1. The figure (i), given above, **represents a polygon.**
2. In figure (ii), given above, the line segments AC and BD intersect at point P which is not an end-point, therefore, the figure **does not represent a polygon.**
3. The figure (iii), given above, **does not represent a polygon** as points A and B are the end-points of three line segments.
4. The figure (iv), given above, **does not represent** a **polygon** as it is not a closed figure. Also, the end-points A and E are not shared by two line segments.

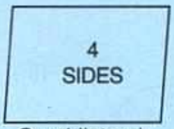
The segments which make up a polygon are called the *sides* of the polygon and the end-points of the segments are called the vertices of the polygon.

Polygons are named according to the number of sides they contain.

e.g.

(i) 

Triangle

(ii) 

Quadrilateral

(iii) 

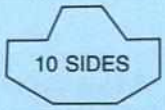
Pentagon

(iv) 

Hexagon

(v) 

Octagon

(vi) 

Decagon, etc.

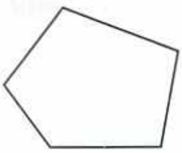
16.4 TYPES OF POLYGONS

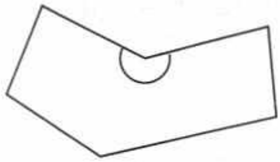
1. ***Convex polygon :***

If *each angle* of a polygon *is less than 180°*, it is called a *convex polygon.*

2. ***Concave polygon :***

If *at least one angle* of a polygon *is more than 180°*, it is called a *concave* or re-entrant polygon.


Convex polygon

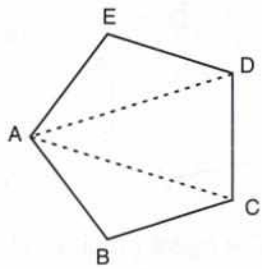

Concave polygon

Unless it is stated, a polygon means a convex polygon.

Remember :

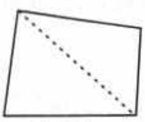
A line segment joining any two non-consecutive vertices of a polygon is called its diagonal.

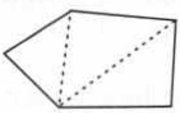
In the adjoining figure, AC is a diagonal of pentagon ABCDE as it joins two non-consecutive vertices A and C of the pentagon. Similarly, AD is also a diagonal. More diagonals can be drawn through the vertices B, C, D and E of the pentagon ABCDE.

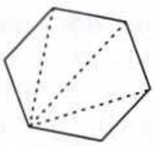


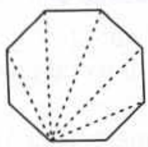
16.5 SUM OF ANGLES OF A POLYGON

Draw all possible diagonals through a single vertex of a polygon to form as many triangles as possible.


4 sides
2 triangles


5 sides
3 triangles


6 sides
4 triangles


8 sides
6 triangles

It is observed that the number of triangles formed is two less than the number of sides in the polygon.

So, if a polygon has n sides, the number of triangles formed will be n – 2.

Since, the sum of angles of a triangle = 180°

∴ The sum of angles of (n – 2) triangles = (n – 2) × 180°

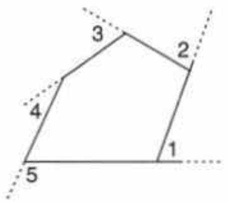
⇒ **Sum of angles** (interior angles) **of a polygon with n sides** = (n – 2) × 180°

= (2n – 4) × 90°

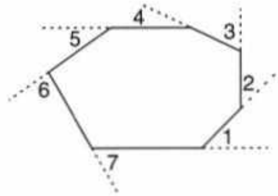
= **(2n – 4) right angles**

16.6 SUM OF EXTERIOR ANGLES OF A POLYGON

If the sides of a polygon are produced in order, the sum of exterior angles so formed is always 4 right angles *i.e.* 360°.



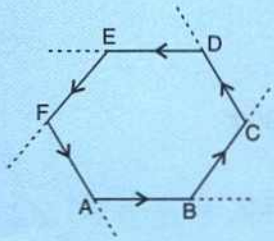
[∠1 + ∠2 + ∠3 + ∠4 + ∠5 = 360°]

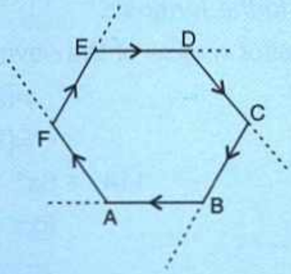


[∠1 + ∠2 + ... + ∠7 = 360°]

If a man walks along the sides of a polygon and each side of the polygon is produced in the direction of motion of the man, the sides of the polygon are said to be produced in order.

e.g.





In each diagram, the direction of motion of the man is represented by arrows.

Example 1 :

Is it possible to have a polygon, the sum of whose interior angles is 9 right angles.

Solution :

Let the number of sides be n.

∴ The sum of its interior angles = (2n – 4) × 90°

According to the given statement :

(2n – 4) × 90° = 9 × 90° [∵ 9 right angles = 9 × 90°]

⇒ 2n – 4 = 9

⇒ n = 6·5; **which is not possible** **(Ans.)**

1. The number of sides in a polygon is always a natural number and is never in fraction or decimals.
2. The smallest number of sides in a polygon is 3, which is in case of a triangle.

Example 2 :

The sides of a pentagon are produced in order. If the measures of exterior angles so obtained are x°, $(2x)^\circ$, $(3x)^\circ$, $(4x)^\circ$ and $(5x)^\circ$, find all the exterior angles.

Solution :

Since, the sum of exterior angles obtained in the above case = 360°

$\Rightarrow \quad x^\circ + (2x)^\circ + (3x)^\circ + (4x)^\circ + (5x)^\circ = 360^\circ$

$\Rightarrow \quad (15x)^\circ = 360^\circ$ *i.e.* $x = \frac{360}{15} = 24$

$\therefore$ **Exterior angles** $= 24^\circ, (2 \times 24)^\circ, (3 \times 24)^\circ, (4 \times 24)^\circ$ and $(5 \times 24)^\circ$

$=$ **24°, 48°, 72°, 96° and 120°** **(Ans.)**

Example 3 :

One angle of a seven-sided polygon is 114° and each of the other six angles is x°. Find the magnitude of x°.

Solution :

Since, each of the other six angles is x°

$\Rightarrow$ Sum of these six angles $= 6x^\circ$

$\Rightarrow$ Sum of all the seven angles $= 114^\circ + 6x^\circ$ I

According to the formula :

Sum of interior angles of the seven-side polygon

$= (2n - 4) \times 90^\circ$

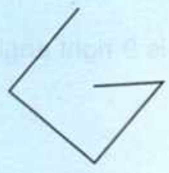
$= (2 \times 7 - 4) \times 90^\circ = 900^\circ$ II

$\therefore \quad 114^\circ + 6x^\circ = 900^\circ$ [From I and II]

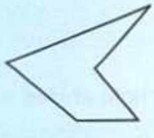
$\Rightarrow \quad 6x^\circ = 900^\circ - 114^\circ = 786^\circ \Rightarrow$ **$x^\circ = 131^\circ$** **(Ans.)**

EXERCISE 16(A)

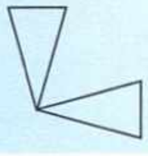
1. State which of the following are polygons :

(i)


(ii)


(iii)


(iv)


(v)


If the given figure is a polygon, name it as **convex** or **concave.**

2. Calculate the sum of angles of a polygon with :
   (i) 10 sides (ii) 12 sides
   (iii) 20 sides
3. Find the number of sides in a polygon if the sum of its interior angles is :
   (i) 900° (ii) 1620°
   (iii) 16 right angles
4. Is it possible to have a polygon, whose sum of interior angles is :
   (i) 870° (ii) 2340°
   (iii) 7 right angles ?
5. (i) If all the angles of a hexagon are equal, find the measure of each angle.

(ii) If all the angles of a 14-sided figure are equal, find the measure of each angle.

6. Find the sum of exterior angles obtained on producing, in order, the sides of a polygon with:

(i) 7 sides (ii) 10 sides (iii) 250 sides

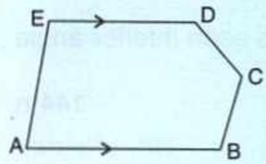
7. The sides of a hexagon are produced in order. If the measures of exterior angles so obtained are $(6x - 1)°$, $(10x + 2)°$, $(8x + 2)°$, $(9x - 3)°$, $(5x + 4)°$ and $(12x + 6)°$; find each exterior angle.

8. The interior angles of a pentagon are in the ratio 4 : 5 : 6 : 7 : 5. Find each angle of the pentagon.

9. Two angles of a hexagon are 120° and 160°. If the remaining four angles are equal, find each equal angle.

10. The figure, given below, shows a pentagon ABCDE with sides AB and ED parallel to each other, and $\angle B : \angle C : \angle D = 5 : 6 : 7$.

(i) Using formula, find the sum of interior angles of the pentagon.



(ii) Write the value of $\angle A + \angle E$.

(iii) Find angles B, C and D.

11. Two angles of a polygon are right angles and the remaining are 120° each. Find the number of sides in it.

$$2 \times 90° + (n - 2) \times 120° = (2n - 4) \times 90°.$$

12. In a hexagon ABCDEF, side AB is parallel to side FE and $\angle B : \angle C : \angle D : \angle E = 6 : 4 : 2 : 3$. Find $\angle B$ and $\angle D$.

13. The angles of a hexagon are $x + 10°$, $2x + 20°$, $2x - 20°$, $3x - 50°$, $x + 40°$ and $x + 20°$. Find x.

14. In a pentagon, two angles are 40° and 60°, and the rest are in the ratio 1 : 3 : 7. Find the biggest angle of the pentagon.

16.7 REGULAR POLYGON

A polygon is said to be a regular polygon, if all its

(i) interior angles are equal, (ii) sides are equal and

(iii) exterior angles are equal.

(a) If a regular polygon has n sides :

1. The sum of its interior angles = $(2n - 4) \times 90°$

And, each interior angle = $\dfrac{(2n-4) \times 90°}{n}$

2. The sum of its exterior angles = 360°

And, each exterior angle = $\dfrac{360°}{n}$

3. No. of sides (n) of the regular polygon = $\dfrac{360°}{\text{Exterior angle}}$

(b) Whether the given polygon is regular or not, at each vertex of the polygon :

exterior angle + interior angle = 180°.

Since, both the angles together form a straight line angle.

Example 4 :

If each interior angle of a regular polygon is 144°, find the number of sides in it.

Solution :

Let the number of sides of the regular polygon be n.

$\therefore$ Its each interior angle = $\frac{(2n-4)\times 90^\circ}{n}$ *i.e.* $144^\circ = \frac{(2n-4)\times 90^\circ}{n}$

$\Rightarrow$ $144\,n = 180\,n - 360$ *i.e.* $n = 10$

$\therefore$ **No. of sides = 10** **(Ans.)**

***Alternative method* :**

Given: Each interior angle = 144°

and we know, interior angle + exterior angle = 180°

$\Rightarrow$ 144° + exterior angle = 180°

i.e. exterior angle = 36°

Since, no. of sides of a regular polygon = $\frac{360^\circ}{\text{exterior angle}}$

$\therefore$ **No. of sides in the given polygon** = $\frac{360^\circ}{36^\circ} = \mathbf{10}$ **(Ans.)**

Example 5 :

Is it possible to have a regular polygon with each interior angle equal to 105° ?

Solution :

The number of sides in a polygon is always a whole number which is greater than or equal to 3.

Let the number of sides in the regular polygon be n.

$\therefore$ $\frac{(2n-4)\times 90^\circ}{n} = 105^\circ$ $\Rightarrow$ $180\,n - 360 = 105\,n$

$\Rightarrow$ $180\,n - 105\,n = 360$

$\Rightarrow$ $75\,n = 360$

and, $n = \frac{360}{75} = 4\frac{4}{5}$

Since, $n = 4\frac{4}{5}$ is not a whole number.

$\therefore$ **No regular polygon is possible with each interior angle equal to 105°. (Ans.)**

Example 6 :

The sum of the interior angles of a regular polygon is equal to six times the sum of exterior angles. Find the number of sides of the polygon.

Solution :

Let the number of sides of the regular polygon be n.

$\therefore$ Sum of interior angles = $(2n - 4) \times 90^\circ$

$\therefore$ Sum of exterior angles = 360°

$\Rightarrow$ $(2n - 4) \times 90^\circ = 6 \times 360^\circ$

$$\Rightarrow \quad 2n - 4 = \frac{6 \times 360}{90} = 24$$

$$\Rightarrow \quad 2n = 28 \text{ and } n = 14$$

$\therefore$ **The number of sides of the regular polygon = 14** **(Ans.)**

Example 7 :

An exterior angle and an interior angle of a regular polygon are in the ratio 2 : 7. Find the number of sides in the polygon.

Solution :

Given : Exterior angle : Interior angle = 2 : 7

$\Rightarrow$ If exterior angle = 2x, the interior angle = 7x

Since, an exterior angle + interior angle = 180°

$\Rightarrow$ $2x + 7x = 180°$ *i.e.* $9x = 180°$ and $x = 20°$

$\therefore$ Exterior angle of the given regular polygon = $2x = 2 \times 20° = 40°$

And, **the no. of sides in the polygon** = $\dfrac{360°}{\text{exterior angle}}$

$$= \frac{360°}{40°} = 9$$ **(Ans.)**

Example 8 :

The ratio of the number of sides of two regular polygons is 1 : 2, and the ratio of the sum of their interior angles is 3 : 8. Find the number of sides in each polygon.

Solution :

Since, the ratio between the number of sides of the two polygons is 1 : 2.

Let the number of sides be x and 2x.

Since, the sum of interior angles of a polygon = $(2n - 4) \times 90°$

$\therefore$ The sum of interior angles of the 1st polygon = $(2x - 4) \times 90°$

and, the sum of interior angles of the 2nd polygon = $(2 \times 2x - 4) \times 90°$

$= (4x - 4) \times 90°$

Given, the ratio of the sum of interior angles of the two regular polygons is 3 : 8.

$$\Rightarrow \quad \frac{(2x-4)\times 90°}{(4x-4)\times 90°} = \frac{3}{8} \quad \textit{i.e.} \quad \frac{2x-4}{4x-4} = \frac{3}{8}$$

$$\Rightarrow \quad 16x - 32 = 12x - 12 \quad \textit{i.e.} \quad 4x = 20$$

$$\Rightarrow \quad x = \frac{20}{4} = 5$$

$\therefore$ **The number of sides in the two polygons** are x and 2x = **5 and 10** **(Ans.)**

EXERCISE 16(B)

1. Fill in the blanks :

In case of regular polygon, with :

no. of sides	each exterior angle	each interior angle
(i) ..8.....	....	....
(ii) ..12..	....	....
(iii)	.......72°.....	....
(iv)	......45°.....	....
(v)	....	.......150°....
(vi)	....	......140°......

2. Find the number of sides in a regular polygon, if its each interior angle is :
   (i) 160° (ii) 135°
   (iii) $1\frac{1}{5}$ of a right angle.
3. Find the number of sides in a regular polygon, if its each exterior angle is :
   (i) $\frac{1}{3}$ of a right angle
   (ii) two-fifths of a right angle
4. Is it possible to have a regular polygon whose each interior angle is :
   (i) 170° (ii) 138°
5. Is it possible to have a regular polygon whose each exterior angle is :
   (i) 80° (ii) 40% of a right angle
6. Find the number of sides in a regular polygon, if its interior angle is equal to its exterior angle.
7. The exterior angle of a regular polygon is one-third of its interior angle. Find the number of sides in the polygon.
8. The measure of each interior angle of a regular polygon is five times the measure of its exterior angle. Find :
   (i) measure of each interior angle,
   (ii) measure of each exterior angle and
   (iii) number of sides in the polygon
9. The ratio between the interior angle and the exterior angle of a regular polygon is 2 : 1. Find :
   (i) each exterior angle of the polygon,
   (ii) number of sides in the polygon.
10. The ratio between the exterior angle and the interior angle of a regular polygon is 1 : 4. Find the number of sides in the polygon.
11. The sum of interior angles of a regular polygon is twice the sum of its exterior angles. Find the number of sides of the polygon.
12. AB, BC and CD are three consecutive sides of a regular polygon. If angle BAC = 20°, find:
    (i) its each interior angle
    (ii) its each exterior angle
    (iii) the number of sides in the polygon.
13. Two alternate sides of a regular polygon, when produced, meet at right angle. Calculate the number of sides in the polygon.
14. In a regular pentagon ABCDE, draw a diagonal BE and then find the measure of :
    (i) $\angle BAE$ (ii) $\angle ABE$ (iii) $\angle BED$
15. The difference between the exterior angles of two regular polygons, having the sides equal to (n – 1) and (n + 1) is 9°. Find the value of n.

> When number of sides of a regular polygon = n –1,
> the value of its each exterior angle = $\frac{360°}{n-1}$
> And, when number of sides of a regular polygon = n + 1,
> the value of its each exterior angle = $\frac{360°}{n+1}$
> Given : $\frac{360°}{n-1} - \frac{360°}{n+1} = 9°$
> On solving, we get **n = 9**

16. If the difference between the exterior angle of a n sided regular polygon and an (n + 1) sided regular polygon is 12°, find the value of n.
17. The ratio between the number of sides of two regular polygons is 3 : 4 and the ratio between the sum of their interior angles is 2 : 3. Find the number of sides in each polygon.
18. Three of the exterior angles of a hexagon are 40°, 51° and 86°. If each of the remaining exterior angles is x°, find the value of x.
19. Calculate the number of sides of a regular polygon, if :
    (i) its interior angle is five times its exterior angle.

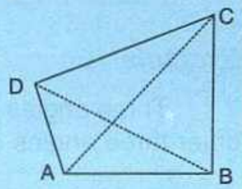
(ii) the ratio between its exterior angle and interior angle is 2 : 7.

(iii) its exterior angle exceeds its interior angle by 60°.

20. The sum of interior angles of a regular polygon is thrice the sum of its exterior angles. Find the number of sides in the polygon.

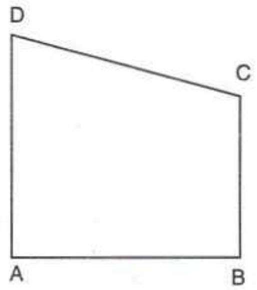
16.8 QUADRILATERAL

Quadrilateral	**A quadrilateral is a closed polygon with four sides.** The adjoining figure shows a quadrilateral ABCD which has : (i) **four sides :** AB, BC, CD and DA (ii) **four vertices :** A, B, C and D (iii) **four angles :** ∠ABC, ∠BCD, ∠CDA and ∠DAB (iv) **two diagonals :** AC and BD The sum of angles of a quadrilateral = 4 right angles = 360°.



Theorem 1 :

The sum of the angles of a quadrilateral is 360° (4 right angles). Prove it.



Given : A quadrilateral ABCD.

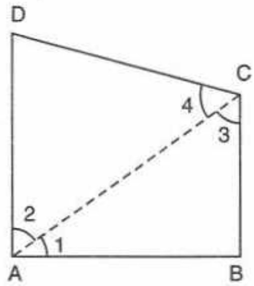
To prove : ∠A + ∠B + ∠C + ∠D = 360°

Construction :

Draw diagonal AC which divides ∠A into two parts, ∠1 and ∠2. Also, AC divides ∠C into two parts, ∠3 and ∠4.

Clearly, ∠A = ∠1 + ∠2

and, ∠C = ∠3 + ∠4



Proof : Since, the sum of the angles of a triangle is 180°.

∴ In ΔABC, ∠1 + ∠B + ∠3 = 180°

and, in ΔADC, ∠2 + ∠D + ∠4 = 180°

On adding, we get :

(∠1 + ∠2) + ∠B + ∠D + (∠3 + ∠4) = 180° + 180°

⇒ ∠A + ∠B + ∠D + ∠C = 360°

i.e. **∠A + ∠B + ∠C + ∠D = 360°** **Hence Proved.**

Example 9 :

The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Find all its angles.

Solution :

Since, 3 + 4 + 5 + 6 = 18 and sum of the angles of a quadrilateral is 360°.

∴ **First angle** = $\frac{3}{18} \times 360°$ = **60°**, **second angle** = $\frac{4}{18} \times 360°$ = **80°**, third angle $= \frac{5}{18} \times 360° = 100°$ and, fourth angle $= \frac{6}{18} \times 360° = 120°$ **(Ans.)**

Alternative method :

Let the angles of the quadrilateral be 3x, 4x, 5x and 6x.

$\therefore$ $3x + 4x + 5x + 6x = 360°$ $\Rightarrow$ $18x = 360°$ and $x = 20°$

$\therefore$ First angle $= 3x = 3 \times 20° = 60°$, second angle $= 4x = 4 \times 20° = 80°$,

third angle $= 5x = 5 \times 20° = 100°$ and fourth angle $= 6x = 6 \times 20° = 120°$ **(Ans.)**

Example 10 :

Three angles of a quadrilateral are in the ratio 4 : 6 : 3. If the fourth angle is 100°; find the other three angles of the quadrilateral.

Solution :

Let the three angles be 4x, 6x and 3x

$\therefore$ $4x + 6x + 3x + 100° = 360°$

$\Rightarrow$ $13x = 360° - 100° = 260°$ and, $x = \frac{260°}{13} = 20°$

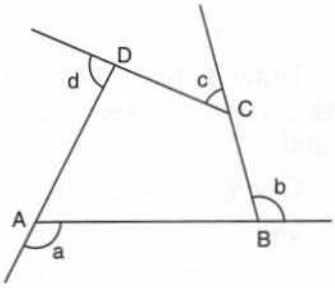
$\therefore$ The other three angles are : 4x, 6x and 3x

$= 4 \times 20°, 6 \times 20°$ and $3 \times 20°$

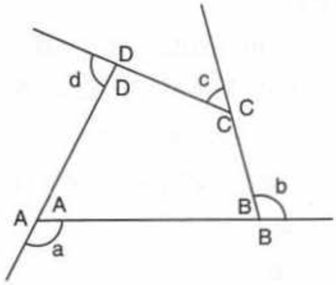
$= 80°, 120°$ and $60°$ **(Ans.)**

Example 11 :

Using the adjoining figure, find $a + b + c + d$.



Solution :



At vertex A, $\angle A + a = 180°$,

at vertex B, $\angle B + b = 180°$,

at vertex C, $\angle C + c = 180°$

and, at vertex D, $\angle D + d = 180°$

On adding, we get :

$(\angle A + \angle B + \angle C + \angle D) + (a + b + c + d) = 720°$

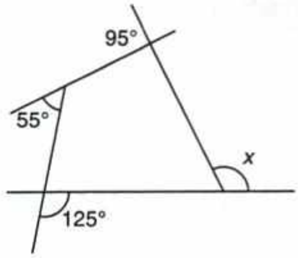
$\Rightarrow$ $360° + (a + b + c + d) = 720°$

$\Rightarrow$ $a + b + c + d = 720° - 360° = 360°$ **(Ans.)**

$\therefore$ Sum of exterior angles of a quadrilateral is 360°.

Example 12 :

From the adjoining figure, find the value of x.



Solution :

$\because$ The sum of exterior angles of a qadrilateral is 360°

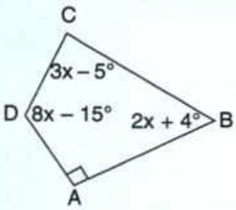
$\Rightarrow \quad x + 95° + 55° + 125° = 360°$

$\Rightarrow \quad x + 275° = 360°$

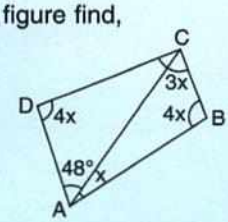
$\Rightarrow \quad x = 360° - 275° = 85°$ **(Ans.)**

EXERCISE 16(C)

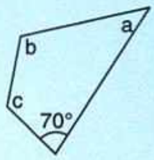
1. Two angles of a quadrilateral are 89° and 113°. If the other two angles are equal, find the equal angles.
2. Two angles of a quadrilateral are 68° and 76°. If the other two angles are in the ratio 5 : 7, find the measure of each of them.
3. Angles of a quadrilateral are $(4x)°$, $5(x + 2)°$, $(7x - 20)°$ and $6(x + 3)°$. Find :
   (i) the value of x.
   (ii) each angle of the quadrilateral.
4. Use the information given in the following figure to find :
   (i) x.
   (ii) ∠B and ∠C.



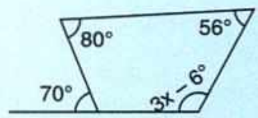
5. In quadrilateral ABCD, side AB is parallel to side DC. If ∠A : ∠D = 1 : 2 and ∠C : ∠B = 4 : 5.
   (i) Calculate each angle of the quadrilateral.
   (ii) Assign a special name to quadrilateral ABCD.
6. From the following figure find,
   (i) x
   (ii) ∠ABC
   (iii) ∠ACD



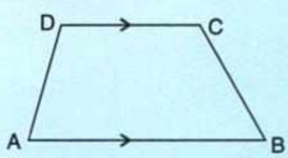
7. Given : In quadrilateral ABCD, ∠C = 64°, ∠D = ∠C − 8°; ∠A = $5(a + 2)°$ and ∠B = $2(2a + 7)°$. Calculate ∠A.
8. In the given figure :
   ∠b = 2a + 15° and
   ∠c = 3a + 5°,
   find the values of b and c.



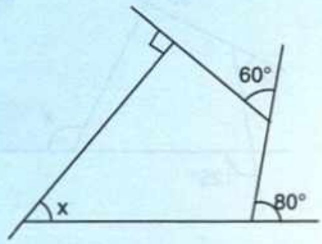
9. Three angles of a quadrilateral are equal. If the fourth angle is 69°, find the measure of equal angles.
10. In quadrilateral PQRS, ∠P : ∠Q : ∠R : ∠S = 3 : 4 : 6 : 7. Calculate each angle of the quadrilateral and then prove that PQ and SR are parallel to each other.
    (i) Is PS also parallel to QR ?
    (ii) Assign a special name to quadrilateral PQRS.
11. Use the information given in the following figure to find the value of x.



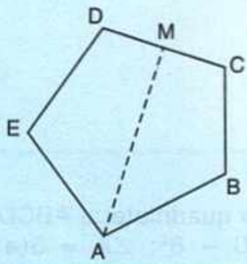
12. The following figure shows a quadrilateral in which sides AB and DC are parallel.
    If ∠A : ∠D = 4 : 5, ∠B = $(3x - 15)°$
    and ∠C = $(4x + 20)°$, find each angle of the quadrilateral ABCD.



13. Use the following figure to find the value of *x*.

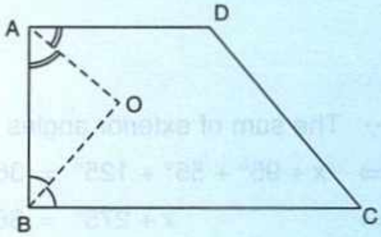


14. ABCDE is a regular pentagon. The bisector of angle A of the pentagon meets the side CD in point M. Show that $\angle AMC = 90°$.



15. In a quadrilateral ABCD, AO and BO are bisectors of angle A and angle B respectively. Show that :

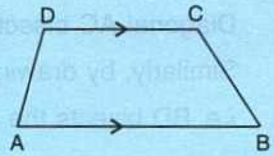
$$\angle AOB = \frac{1}{2}(\angle C + \angle D)$$



SPECIAL TYPES OF QUADRILATERALS 17

17.1 TRAPEZIUM

A trapezium is a quadrilateral in which one pair of opposite sides are parallel but other two sides of it are non-parallel.

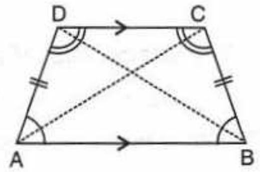


The given figure shows a trapezium ABCD in which side AB is parallel to side DC and side AD is not parallel to side BC.

Since, AB is parallel to DC, we have :

$\angle A + \angle D = 180°$ and $\angle B + \angle C = 180°$

If the non-parallel sides of a trapezium are equal, it is called an **isosceles trapezium**.



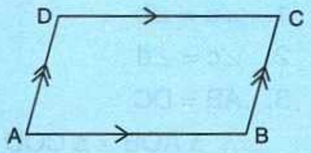
In an isosceles trapezium ABCD, side AB // side DC and non-parallel sides AD and BC are equal. Also,

(i) $\angle A = \angle B$ and $\angle C = \angle D$

and, (ii) diagonal AC = diagonal BD.

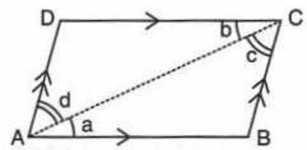
17.2 PARALLELOGRAM

A parallelogram is a quadrilateral in which both the pairs of opposite sides are parallel.



The given figure shows a parallelogram ABCD as AB is parallel to DC and AD is parallel to BC.

Theorem 2



In a parallelogram :

(i) opposite sides are equal,

(ii) opposite angles are equal and

(iii) each diagonal bisects the parallelogram.

Given : A parallelogram ABCD.

To prove : (i) AB = DC and AD = BC

(ii) $\angle A = \angle C$ and $\angle B = \angle D$

(iii) AC bisects the parallelogram *i.e.* Δ ABC = Δ ADC and also, BD bisects the parallelogram *i.e.* Δ ABD = Δ BCD.

Construction : Draw diagonal AC

Proof :

Statement	Reason
In Δ ABC and Δ ADC,	
1. $\angle a = \angle b$	Alternate angles, as AC cuts parallel sides AB and DC
2. $\angle c = \angle d$	Alternate angles, as AC cuts parallel sides AD and BC.
3. AC = AC	Common.
$\therefore$ Δ ABC $\cong$ Δ ADC	A.S.A.

(i) Since, Δ ABC $\cong$ Δ ADC
AB = DC and AD = BC — Corresponding parts of congruent triangles are congruent.

(ii) Since, Δ ABC $\cong$ Δ ADC
$\therefore$ $\angle$B = $\angle$D — Corresponding parts of congruent triangles are congruent.
and $\angle$A = $\angle$C — From 1 and 2; $\angle a + \angle d = \angle b + \angle c$ *i.e.* $\angle A = \angle C$

(iii) Since, Δ ABC $\cong$ Δ ADC
$\therefore$ **Δ ABC = Δ ADC** — Congruent triangles are equal.

Diagonal AC bisects parallelogram ABCD.

Similarly, by drawing the diagonal BD, it can be proved that **Δ ABD = Δ BCD**.

i.e. BD bisects the parallelgoram ABCD.

Hence Proved.

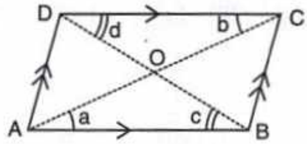
Theorem 3

The diagonals of a parallelogram bisect each other.

Given : A parallelogram ABCD whose diagonals intersect each other at O.

To prove : Diagonals bisect each other. *i.e.* OA = OC and OB = OD.

Proof :



Statement	*Reason*
In Δ AOB and Δ COD,	
1. $\angle a = \angle b$	Alternate angles.
2. $\angle c = \angle d$	Alternate angles.
3. AB = DC	Opposite sides of a parallelogram are equal.
$\therefore$ Δ AOB $\cong$ Δ COD	A.S.A.
$\therefore$ **OA = OC** and **OB = OD**	Corresponding parts of congruent triangles are congruent.

Hence Proved.

Theorem 4

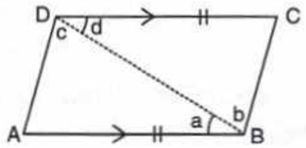
If a pair of opposite sides of a quadrilateral are equal and parallel, it is a parallelogram.

Given : A quadrilateral ABCD in which AB = DC and AB is parallel to DC.

To prove : ABCD is a parallelogram.

Construction : Join B and D.

Proof :



Statement	*Reason*
In Δ ABD and Δ CDB;	
1. AB = DC	Given
2. BD = BD	Common (identity).

3. $\angle a = \angle d$ — Alternate angles, since BD cuts parallel sides AB and DC

$\therefore \Delta ABD \cong \Delta CDB$ — S.A.S.

$\Rightarrow \angle c = \angle b$ — Corresponding parts of congruent triangles are congruent.

But these are alternate angles,

$\therefore$ AD is parallel to BC. — When alternate angles are equal, the lines are parallel.

Since, both the pairs of opposite sides of the quadrilateral ABCD are parallel, it is a parallelogram.

Hence Proved.

In order to prove that given quadrilateral is a parallelogram; show that :

(i) opposite sides are parallel or, (ii) opposite sides are equal or,
(iii) opposite angles are equal or, (iv) diagonals bisect each other or,
(v) a pair of opposite sides is equal and parallel.

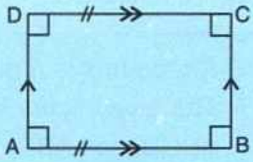
17.3 RECTANGLE

A rectangle is a quadrilateral in which :

(i) opposite sides are equal *i.e.* AB = DC and AD = BC

(ii) opposite sides are parallel *i.e.* AB // DC and AD // BC and

(iii) each angle is 90° *i.e.* $\angle A = \angle B = \angle C = \angle D = 90°$.



1. A quadrilateral, with each angle 90° is called a **rectangle.** See Fig. (a)
2. A parallelogram, with any angle 90° is called a **rectangle.** See Fig. (b)
3. A parallelogram, with equal diagonals is called a **rectangle.** See Fig. (c)


Fig. (a)

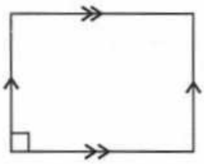


Fig. (b)

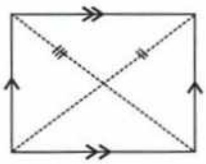
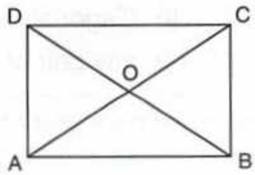


Fig. (c)

1. In order to prove that a given quadrilateral is a rectangle, show that each of its angles is 90°
2. In order to prove that the given parallelogram is a rectangle; show that :
   (i) any angle of it is 90° or (ii) its diagonals are equal.

Theorem 5

Diagonals of a rectangle are equal and bisect each other.

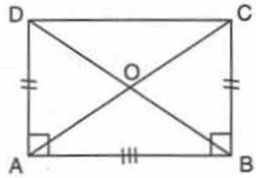


Given : A rectangle ABCD with diagonals AC and BD intersecting each other at point O.

To prove : (i) AC = BD (ii) OA = OC and OB = OD.

Proof :

(i) In Δ ABC and Δ BAD :



$AB = AB$ [Common]

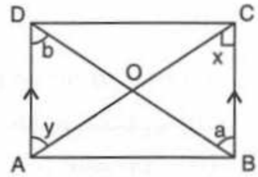
$BC = AD$ [Opposite sides of a rectangle are equal]

$\angle BAD = \angle ABC$ [Each equal to 90°]

$\therefore \Delta ABC \cong \Delta BAD$ [By SAS]

$\Rightarrow$ **AC = BD** **Hence Proved.**

(ii) Since, rectangle is a parallelogram, so its opposite sides are equal and parallel



$\Rightarrow$ AD is parallel to BC

$\therefore$ In Δ AOD and Δ COB,

$AD = BC$

$\angle a = \angle b$ [Alternate angles]

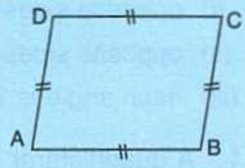
and $\angle x = \angle y$ [Alternate angles]

$\Rightarrow \Delta AOD \cong \Delta COB$ [By ASA]

$\Rightarrow$ **OA = OC** and **OB = OD** **Hence Proved.**

17.4 RHOMBUS

A rhombus is a quadrilateral in which all the sides are equal.
The given figure shows a rhombus
ABCD, so : AB = BC = CD = DA



1. A quadrilateral, with all the sides equal is called a **rhombus.** See. Fig. (a)
2. A parallelogram, with any pair of adjacent sides equal, is called a rhombus. See Fig. (b)
3. A parallelogran whose diagonals intersect at 90°, is a rhombus. See Fig.(c).

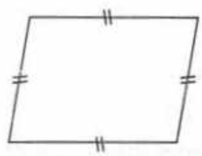


Fig. (a)

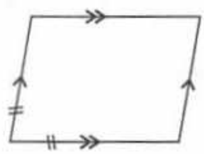


Fig. (b)

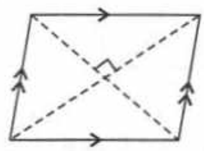


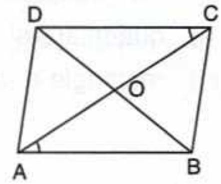
Fig. (c)

1. In order to prove that given quadrilateral is a rhombus, show that :
   (i) its all sides are equal or
   (ii) its diagonals bisect each other at 90°.
2. In order to prove that given parallelogram is a rhombus, show that :
   (i) diagonals intersect each other at 90°
   (ii) any pair of adjacent sides is equal.

Since, in parallelogram, opposite sides are already equal, so if the adjacent sides are also equal, it is a rhombus.

Theorem 6

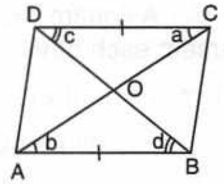
Diagonals of a rhombus bisect each other at right angle.



Given : A rhombus ABCD whose diagonals AC and BD intersect each other at point O.

To prove : (i) OA = OC, OB = OD (ii) ∠AOB = ∠BOC = 90°.

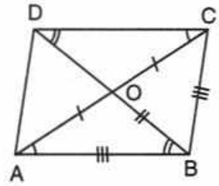
Proof :



(i) In Δ AOB and Δ COD,

AB = DC	[Sides of a rhombus are equal]
∠a = ∠b	[Alternate angles]
∠c = ∠d	[Alternate angles]
⇒ Δ AOB ≅ Δ COD	[By ASA]

⇒ **OA = OC** and **OB = OD** **Hence Proved.**



(ii) In Δ AOB and Δ COB,

OA = OC	[Proved above]
OB = OB	[Common]
AB = BC	[Sides of a rhombus are equal]
⇒ Δ AOB ≡ Δ COB	[By SSS]

⇒ ∠AOB = ∠COB

But, ∠AOB + ∠COB = 180° [AC is a straightl line]

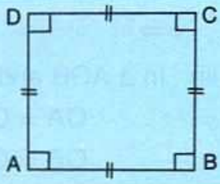
$$\Rightarrow \angle AOB = \angle COB = \frac{180°}{2} = 90°$$ **Hence Proved.**

17.5 SQUARE

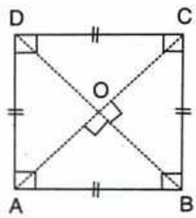
A square is a quadrilateral in which all the sides are equal and each angle is 90°.

The given figure shows a square ABCD, so,

(i) AB = BC = CD = DA and (ii) ∠A = ∠B = ∠C = ∠D = 90°.



1. A quadrilateral is a square when :
   (i) all its sides are equal. (ii) each angle is 90°.
   (iii) diagonals bisect each other at right angle. (iv) diagonals are equal.
2. A parallelogram is a square when :
   (i) all its sides are equal,
   (ii) each angle is a right-angle,
   (iii) diagonals are equal,
   (iv) diagonals intersect at right-angle.



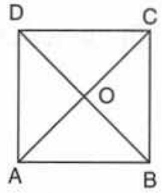
3. A rectangle is a square when :
   (i) its adjacent sides are equal, (ii) diagonals intersect at right-angle.
4. A rhombus is a square when :
   (i) each angle is 90° (ii) diagonals are equal.

A square satisfies all the properties of a :

(i) quadrilateral, (ii) trapezium, (iii) parallelogram,
(iv) rectangle and (v) rhombus.

Theorem 7

Diagonals of a square are equal and bisect each other at 90°.



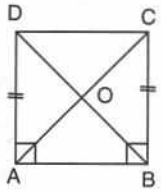
Given : A square ABCD whose diagonals AC and BD intersect each other at point O.

To prove : (i) AC = BD (ii) OA = OC and OB = OD and
(iii) ∠AOB = ∠BOC = 90°

Proof :

(i) In Δ ABC and Δ BAD,

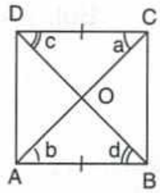
	AB = AB	[Common]
	AD = BC	[Sides of a square are equal]
	∠ABC = ∠BAD	[Each is 90°]
⇒	Δ ABC ≅ Δ BAD	[By SAS]
⇒	**AC = BD**	



Hence Proved.

(ii) In Δ AOB and Δ COD,

	AB = DC	[Sides of a square are equal]
	∠a = ∠b	[Alternate angles]
and	∠c = ∠d	[Alternate angles]
⇒	Δ AOB ≅ Δ COD	[By ASA]
⇒	**OA = OC** and **OB = OD**	



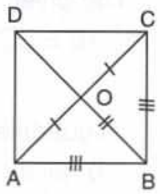
Hence Proved.

(iii) In Δ AOB and Δ BOC,

	OA = OC	[Proved above]
	OB = OB	[Common]
and	AB = BC	[Sides of a square are equal]
⇒	Δ AOB ≅ Δ BOC	[By SSS]
⇒	∠AOB = ∠BOC	

But ∠AOB + ∠BOC = 180° [AC is a straight line]

⇒ **∠AOB = ∠BOC = 90°**



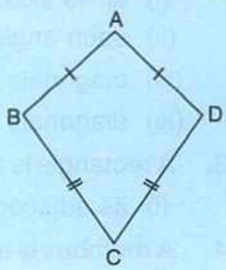
Hence Proved.

Kite

A quadrilateral, in which two pairs of adjacent side are equal, is called a **kite**.

The adjoining figure shows a quadrilateral ABCD in which AB = AD and BC = DC.

⇒ Quadrilateral ABCD is a **kite**.



In a kite shaped figure, ABCD,

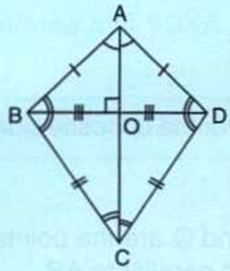
(i) $\angle ABC = \angle ADC$.

(ii) $AC \perp BD$

i.e. $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$.

(iii) $OB = OD$

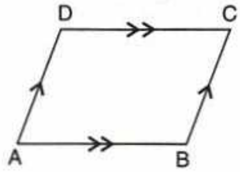
(iv) $\angle OAB = \angle OAD$ and $\angle OCB = \angle OCD$.



Example 1 :

Prove that consecutive angles of a parallelogram are supplementary.

Solution :



Consider a parallelogram ABCD in which AB//DC and AD//BC.

To prove : $\angle A + \angle B = 180°$

Proof :

We know the opposite angles of a parallelogram are equal, therefore $\angle A = \angle C$ and $\angle B = \angle D$.

Also, as the sum of interior angles of a quadrilateral = 360°

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \angle A + \angle B + \angle A + \angle B = 360°$ [$\because \angle C = \angle A$ and $\angle D = \angle B$]

$\Rightarrow 2\angle A + 2\angle B = 360°$

i.e. $\angle A + \angle B = \frac{360°}{2} = 180°$ **Hence proved.**

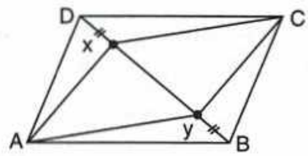
Similary, we can prove that :

$\angle B + \angle C = 180°$, $\angle C + \angle D = 180°$ and $\angle D + \angle A = 180°$.

Example 2 :

In a parallelogram ABCD, X and Y are points on diagonal BD such that DX = BY. Prove that AXCY is a parallelogram.

Solution :



According to the given statement, the figure will be as shown alongside.

In Δ ADX and Δ CBY

	$AD = BC$	[Opposite sides of a parallelgram are equal]
	$DX = BY$	[Given]
	$\angle ADX = \angle CBY$	[Alternate angles as AD//BC and BD is transversal]
$\therefore$	$\Delta ADX \cong \Delta CBY$	[By S.A.S.]
$\Rightarrow$	$AX = CY$	[Corresponding parts of congruent triangles]
Similarly,	$\Delta CDX \cong \Delta ABY$	[By S.A.S.]
$\Rightarrow$	$CX = AY$	

$AX = CY$ and $CX = AY$

$\Rightarrow$ **AXCY is a parallelogram.**

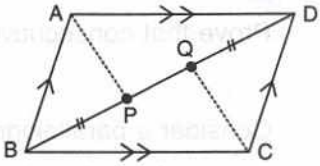
Hence proved.

Whenever the opposite sides of a quadrilateral are equal, the quadrilateral is a parallelogram.

Example 3 :

P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP.

Proof :



According to the given statement, the figure will be as shown alongside.

P and Q are the points of trisection of the diagonal BD.

$\Rightarrow \quad BP = PQ = QD = \frac{1}{3}BD$

In Δ APD and Δ BQC : $BQ = \frac{2}{3}BD$ and $PD = \frac{2}{3}BD \Rightarrow BQ = PD$

$\therefore \quad BQ = PD$ [Proved above]

$AD = BC$ [Opposite sides of the parallelogram ABCD]

and, $\angle ADB = \angle CBD$ [Alternate angles]

$\therefore \quad \Delta APD \cong \Delta BQC$ [By S.A.S.]

$\Rightarrow \quad \angle APD = \angle CQB$ [Corresponding parts of congruent triangles]

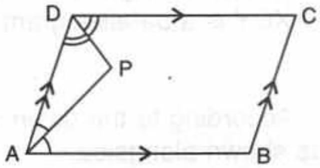
But these are alternate angles and whenever the alternate angles are equal, the lines are parallel.

$\therefore$ **CQ is parallel to AP.** **Hence proved.**

Example 4 :

In parallelogram ABCD, the bisectors of adjacent angles A and D intersect each other at point P. Prove that $\angle APD = 90°$.

Solution :



According to the given statement, the figure will be as shown alongside :

Since, AB is parallel to DC and AD is transversal,

$\therefore \quad \angle BAD + \angle ADC = 180°$ I

Since, AP bisects angle BAD

$\therefore \quad \angle PAD = \frac{1}{2}\angle BAD$

And, DP bisects angle ADC

$\Rightarrow \quad \angle PDA = \frac{1}{2}\angle ADC$

$\therefore \quad \angle PAD + \angle PDA = \frac{1}{2}\angle BAD + \frac{1}{2}\angle ADC$

$$= \frac{1}{2}(\angle BAD + \angle ADC)$$

$$= \frac{1}{2} \times 180° = 90°$$ [From equation I]

In triangle APD,

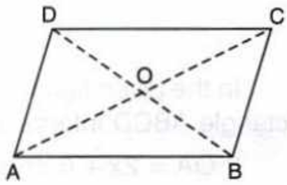
$$\angle PAD + \angle PDA + \angle APD = 180°$$ [Sum of angles of a Δ = 180°]

$$\Rightarrow \quad 90° + \angle APD = 180°$$

$$\Rightarrow \quad \angle APD = 180° - 90° = 90°$$ **Hence proved.**

Example 5 :

The given figure shows a parallelogram ABCD whose diagonals intersect each other at point O. If OB = 6 cm and AC is 6 cm more than BD, find OC.



Solution :

$\because$ The diagonals of a parallelogram bisect each other.

$$\therefore \quad OB = \frac{1}{2}BD \quad \Rightarrow \quad 6 \text{ cm} = \frac{1}{2}BD$$

$$\Rightarrow \quad BD = 12 \text{ cm}$$

And, $AC = BD + 6$ cm

$= 12$ cm $+ 6$ cm $= 18$ cm

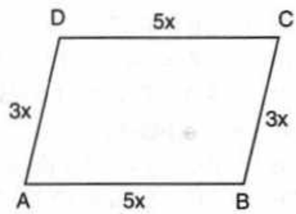
and $OC = \frac{1}{2}AC$

$= \frac{1}{2} \times 18$ cm $=$ **9 cm** **(Ans.)**

Example 6 :

The adjacent sides of a parallelogram are in the ratio 5 : 3. If its perimeter is 96 cm, find the sides of the parallelogram.

Solution :



Let the adjacent sides be $5x$ cm and $3x$ cm

Perimeter = 96 cm

$\Rightarrow \quad 2(5x + 3x) = 96$

$\Rightarrow \quad 16x = 96$ and $x = 6$

$\therefore$ **Required sides** $= 5x$ and $3x$

$= 5 \times 6$ cm and 3×6 cm

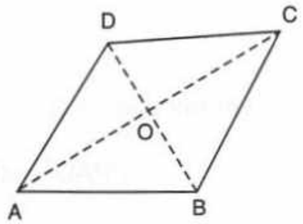
$=$ **30 cm and 18 cm** **(Ans.)**

Example 7 :

One of the diagonals of a rhombus is equal to its sides. Find the angles of the rhombus.

Solution :

Let ABCD be a rhombus such that its diagonal BD is equal to its sides.



i.e. AB = BC = CD = AD = BD

$\Rightarrow$ ΔABD and ΔBCD are equilateral

$\Rightarrow \quad \angle A = \angle C = 60°$

$\because \quad \angle A + \angle B = 180° \quad \Rightarrow 60° + \angle B = 180°$

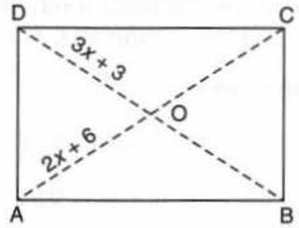
$\Rightarrow \angle B = 120° = \angle D$

$\therefore \quad \angle A = 60° = \angle C$ and $\angle B = \angle D = 120°$ **(Ans.)**

Example 8 :

In the given figure, the diagonals AC and BD of a rectangle ABCD intersect each other at point O.

If OA = 2*x* + 6 and OD = 3*x* + 3; find the value of *x*.



Solution :

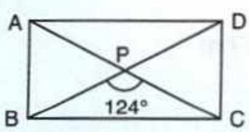
The diagonals of a rectangle are equal and bisect each other

$\Rightarrow \quad OD = OA \quad \Rightarrow \quad 3x + 3 = 2x + 6$

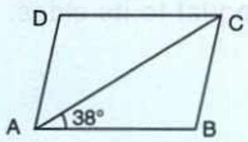
$\Rightarrow \quad x = 3$ **(Ans.)**

EXERCISE 17

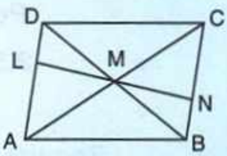
1. In parallelogram ABCD, $\angle A$ = 3 times $\angle B$. Find all the angles of the parallelogram. In the same parallelogram, if AB = 5x – 7 and CD = 3x + 1; find the length of CD.
2. In parallelogram PQRS, $\angle Q = (4x - 5)°$ and $\angle S = (3x + 10)°$. Calculate : $\angle Q$ and $\angle R$.
3. In rhombus ABCD :
   (i) if $\angle A = 74°$; find $\angle B$ and $\angle C$.
   (ii) if AD = 7·5 cm; find BC and CD.
4. In square PQRS :
   (i) if PQ = 3x – 7 and QR = x + 3; find PS.
   (ii) if PR = 5x and QS = 9x – 8. Find QS.
5. ABCD is a rectangle. If $\angle BPC = 124°$. Calculate :
   (i) $\angle BAP$ (ii) $\angle ADP$.



6. ABCD is a rhombus. If $\angle BAC = 38°$, find :
   (i) $\angle ACB$
   (ii) $\angle DAC$
   (iii) $\angle ADC$.

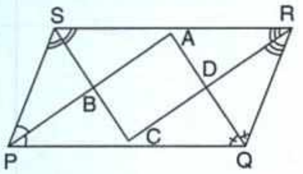


7. ABCD is a rhombus. If $\angle BCA = 35°$, find $\angle ADC$.
8. PQRS is a parallelogram whose diagonals intersect at M.
   If $\angle PMS = 54°$, $\angle QSR = 25°$ and $\angle SQR = 30°$; find :
   (i) $\angle RPS$ (ii) $\angle PRS$ (iii) $\angle PSR$.
9. Given : Parallelogram ABCD in which diagonals AC and BD intersect at M.
   Prove : M is mid-point of LN.



10. In an isosceles-trapezium, show that the opposite angles are supplementary.
11. ABCD is a parallelogram. What kind of quadrilateral is it if :
    (i) AC = BD and AC is perpendicular to BD ?
    (ii) AC is perpendicular to BD but is not equal to it ?
    (iii) AC = BD but AC is not perpendicular to BD ?

12. Prove that the diagonals of a parallelogram bisect each other.
13. If the diagonals of a parallelogram are of equal lengths, the parallelogram is a rectangle. Prove it.
14. In parallelogram ABCD, E is the mid-point of AD and F is the mid-point of BC. Prove that BFDE is a parallelogram.
15. In parallelogram ABCD, E is the mid-point of side AB and CE bisects angle BCD. Prove that :
    (i) AE = AD
    (ii) DE bisects $\angle ADC$ and
    (iii) Angle DEC is a right angle.
16. In the alongside diagram, the bisectors of interior angles of the parallelogram PQRS enclose a quadrilateral ABCD.

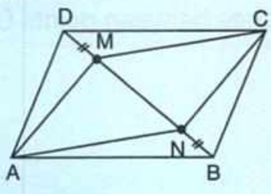


Show that :

(i) $\angle PSB + \angle SPB = 90°$ (ii) $\angle PBS = 90°$
(iii) $\angle ABC = 90°$ (iv) $\angle ADC = 90°$
(v) $\angle A = 90°$ (vi) ABCD is a rectangle

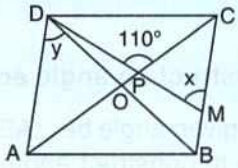
Thus, the bisectors of the angles of a parallelogram enclose a rectangle.

17. In parallelogram ABCD, X and Y are mid-points of opposite sides AB and DC respectively. Prove that :
    (i) AX = YC. (ii) AX is parallel to YC
    (iii) AXCY is a parallelogram.
18. The given figure shows parallelogram ABCD. Points M and N lie in diagonal BD such that DM = BN.

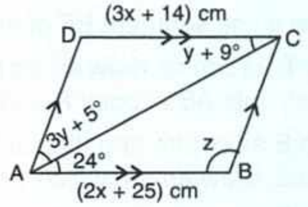


Prove that :
(i) $\Delta DMC \cong \Delta BNA$ and so CM = AN.
(ii) $\Delta AMD \cong \Delta CNB$ and so AM = CN.
(iii) ANCM is a parallelogram.

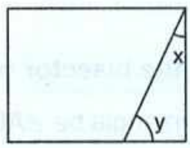
19. The given figure shows a rhombus ABCD in which angle BCD = 80°. Find angles x and y.



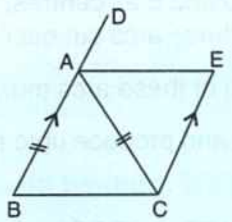
20. Use the information given in the following diagram to find the values of x, y and z.



21. The following figure is a rectangle in which x : y = 3 : 7; find the values of x and y.



22. In the given figure, AB // EC, AB = AC and AE bisects $\angle DAC$. Prove that :
    (i) $\angle EAC = \angle ACB$
    (ii) ABCE is a parallelogram.

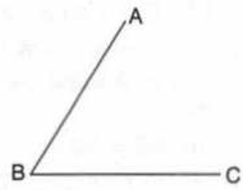


18 CONSTRUCTIONS

(Using ruler and compasses only)

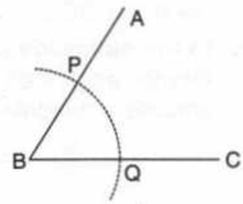
18.1 CONSTRUCTION OF AN ANGLE

1. To construct an angle equal to given angle.



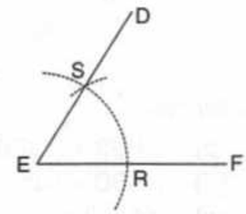
Let the given angle be ∠ABC as shown alongside and we have to construct another angle (say, ∠DEF) equal to ∠ABC.

Steps :

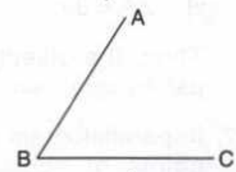


1. Draw a line segment EF of any suitable size.
2. With B as centre, draw an arc of any suitable radius which cuts AB at point P and BC at point Q.
3. With E as centre and the same radius as taken in step 2, draw an arc which cuts EF at point R.
4. With R as centre and radius equal to PQ, draw an arc which cuts the previous arc at point S.
5. Join E and S, and produce upto point D.

   ∠DEF so obtained is equal to ∠ABC.

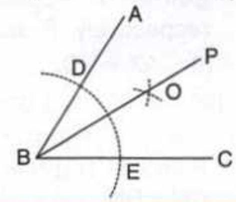


2. To draw the bisector of a given angle.



Let the given angle be ∠ABC whose bisector is to be drawn.

Steps :



1. With B as centre, draw an arc of any suitable radius which cuts AB at point D and BC at point E.
2. Taking D and E as centres, draw arcs of equal radii and let these arcs cut each other at point O.

> The radii of these arcs must be more than half the distance between points D and E.

3. Join BO and produce upto point P.

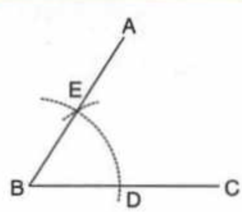
   ∴ **BP is the required bisector of ∠ABC.**

   Thus, $\angle ABP = \angle PBC = \frac{1}{2}\angle ABC$.

3. Construction of angles of 60°, 30°, 90° and 45°.

1. Construction of angle of 60° :

Steps :



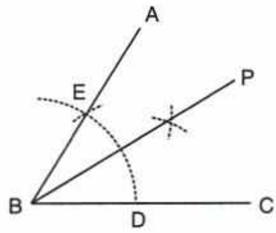
1. Draw a line segment BC of any suitable length.
2. With B as centre, draw an arc of any suitable radius which cuts BC at point D.

3. With D as centre and the same radius as taken in step 2, draw one more arc which cuts the previous arc at point E.
4. Join BE and produce upto any point A.

   $\therefore$ **$\angle ABC$ so obtained is of $60°$ *i.e.* $\angle ABC = 60°$.**

2. Construction of angle of 30° :

Steps :

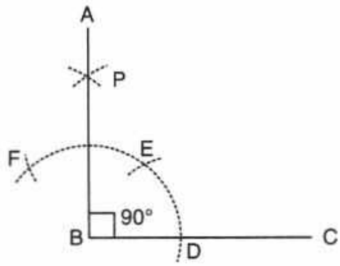


1. Draw angle ABC = 60°.
2. Draw BP, the bisector of $\angle ABC$.

   $\therefore \angle PBC = \frac{1}{2}\angle ABC = \frac{1}{2} \times 60° = 30°$

3. Construction of angle of 90° :

Steps :



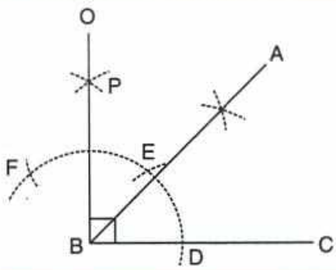
1. Draw a line segment BC of any suitable length.
2. Taking B as centre, draw an arc of any suitable radius, which cuts BC at point D.
3. With D as centre and the same radius, as taken in step 2, draw an arc which cuts previous arc at point E.
4. With E as centre and the same radius, draw one more arc which cuts the first arc at point F.
5. With E and F as centres and radii equal to more than half the distance between E and F, draw arcs which cut each other at point P.
6. Join BP and produce upto any point A.

   $\therefore$ **$\angle ABC$ so obtained is of $90°$ *i.e.* $\angle ABC = 90°$.**

Since, $\angle ABC = 90° \Rightarrow$ AB and BC are perpendicular to each other.

4. Construction of angle of 45° :

Steps :



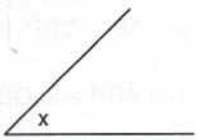
1. Draw a line segment BC of any suitable length.
2. Construct angle OBC = 90°.
3. Draw BA, the bisector of angle OBC.

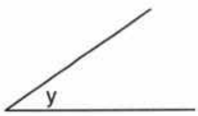
   $\therefore$ **$\angle ABC$ so obtained is the angle of $45°$.**

Since, BA is bisector of angle OBC, $\angle ABC = \angle ABO = \frac{90°}{2} = 45°$.

Example 1 :

Given below are the two angles *x* and *y*.



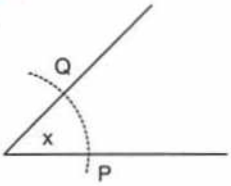


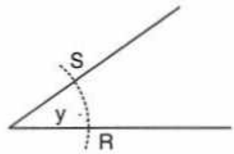
Construct an angle ABC such that :

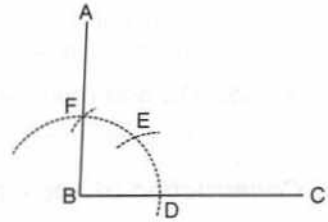
(i) $\angle ABC = x + y$ (ii) $\angle ABC = 2x + y$

Solution :

(i)







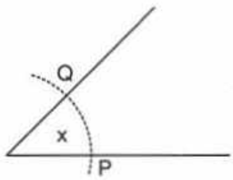
Steps :

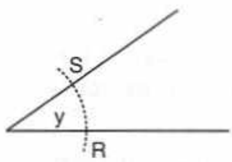
As shown above :

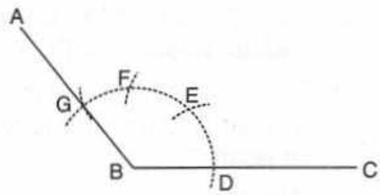
1. Draw line segment BC of any suitable length.
2. With B as centre, draw an arc of any suitable radius. With the same radius, draw arcs with the vertices of given angles as centres. Let these arcs cut arms of the angle *x* at points P and Q, and arms of the angle *y* at points R and S.
3. From the arc, with centre B, cut DE = PQ = x and EF = SR = y.
4. Join BF and produce upto point A.

   Thus, **∠ABC = x + y**

(ii)





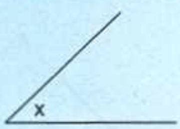


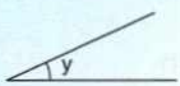
Proceed in exactly the same way as in part (i) taking DE = PQ = x, EF = PQ = x and FG = RS = y.

Thus, **∠ABC** = x + x + y = **2x + y**

EXERCISE 18(A)

1. Given below are the angles *x* and *y*.





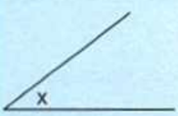
Without measuring these angles, construct :

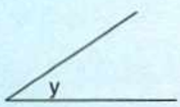
(i) ∠ABC = x + y

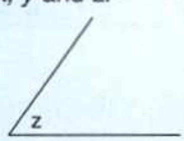
(ii) ∠ABC = 2x + y

(iii) ∠ABC = x + 2y

2. Given below are the angles *x*, *y* and *z*.







Without measuring these angles construct :

(i) ∠ABC = x + y + z

(ii) ∠ABC = 2x + y + z

(iii) ∠ABC = x + 2y + z

3. Draw a line segment BC = 4 cm. Construct angle ABC = 60°.
4. Construct angle ABC = 45° in which BC = 5 cm and AB = 4·6 cm.
5. Construct angle ABC = 90°. Draw BP, the bisector of angle ABC. State, the measure of angle PBC.
6. Draw angle ABC of any suitable measure.

   (i) Draw BP, the bisector of angle ABC.

   (ii) Draw BR, the bisector of angle PBC and draw BQ, the bisector of angle ABP.

   (iii) Are the angles ABQ, QBP, PBR and RBC equal ?

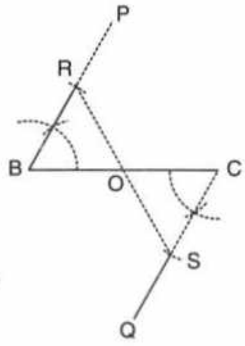
   (iv) Are the angles ABR and QBC equal ?

4. Construction of bisector of a line segment.

Steps :

1. Draw the given line segment and represent it by BC.
2. At B, construct angle PBC of any suitable measure and at C, construct angle QCB equal to angle PBC.

   i.e. $\angle PBC = \angle QCB$.
3. From BP, cut BR of any suitable length and from CQ, cut CS = BR.
4. Join R and S.
5. Let RS cut the given line segment BC at point O.

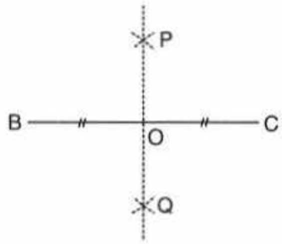


Thus, **RS is a bisector of BC** such that $OB = OC = \frac{1}{2} BC$.

Alternative method :

Steps :

1. Draw BC.
2. With B as centre, and radius equal to more than half of BC, draw arcs on both the sides of BC.
3. With C as centre and with the same radius as taken in step 2, draw arcs on both the sides of BC.
4. Let the arcs intersect each other at points P and Q.
5. Join P and Q.
6. The line PQ intersects the given line segment BC at mid-point O.



Thus, **PQ is a bisector of BC** such that $OB = OC = \frac{1}{2} BC$.

5. Construction of perpendicular bisector of a line segment.

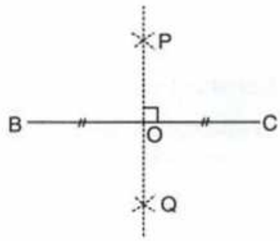
Steps :

1. Draw the given line segment and represent it by BC.
2. Now proceed in exactly the same way as in alternative method of construction 4, given above.

   In this construction, the line PQ bisects the given line segment BC and is perpendicular to it.

   i.e. $OB = OC$ and $\angle POC = 90°$.

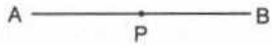
   Hence, **PQ is perpendicular bisector of BC.**



6. Construction of perpendicular to a line.

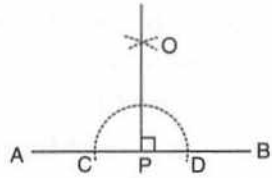
1. To construct the perpendicular to a line at a given point in it :

Let P be the given point in the given line AB.



Steps :

1. With P as centre, draw an arc with a suitable radius which cuts AB at points C and D.
2. Taking C and D as centres, draw arcs of equal radii which cut each other at point O.



The radius must be more than half the distance between C and D.

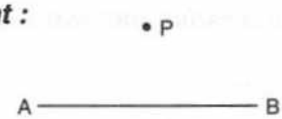
3. Join P and O

   Then, **OP is the required perpendicular.**

So, $\angle OPA = \angle OPB = 90°$

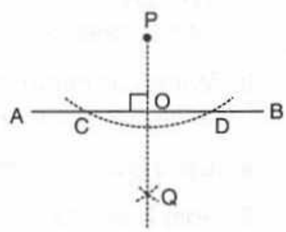
2. To construct the perpendicular to a line from an external point :

Let P be the given external point of line AB.



Steps :

1. With P as centre, draw an arc of a suitable radius which cuts AB as points C and D.
2. With C and D as centres, draw arcs of equal radii and let these arcs intersect each other at point Q.



The radius of these arcs must be more than half of CD and both the arcs must be drawn on the other side.

3. Join P and Q.
4. Let PQ cut AB at point O.

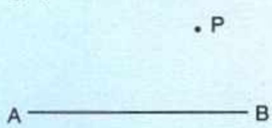
   Thus, **OP is the required perpendicular.**

Clearly, $\angle AOP = \angle BOP = 90°$

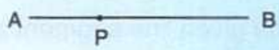
EXERCISE 18(B)

1. Draw a line segment AB of length 5·3 cm. Using two different methods bisect AB.
2. Draw a line segment PQ = 4·8 cm. Construct the perpendicular bisector of PQ.
3. In each of the following, draw a perpendicular through point P to the line segment AB :

   (i)



   (ii)



   (iii)

4. Draw a line segment AB = 5·5 cm. Mark a point P, such that PA = 6 cm and PB = 4·8 cm. From the point P, draw a perpendicular to AB.
5. Draw a line segment AB = 6·2 cm. Mark a point P in AB such that BP = 4 cm. Through point P draw a perpendicular to AB.

7. Constructions of Parallel Lines.

1. To construct a line parallel to a given line and passing through a given point :

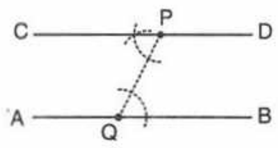
Let the given line be AB and the given point be P.

First Method : (*By drawing alternate angles*)

Steps :

1. Take any point Q in line AB and join it with the given point P.
2. At point P, construct ∠CPQ = ∠PQB.
3. Produce CP upto any point D.

   Thus, **CPD is the required parallel line.**

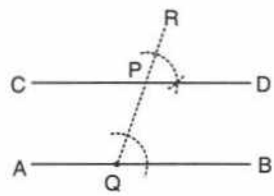


Alternative method : (By drawing corresponding angles)

Steps :

1. Join QP and produce it to any point R.
2. At P, construct ∠RPD = ∠PQB.
3. Produce DP upto any point C.

   Thus, **CPD is the required parallel line.**

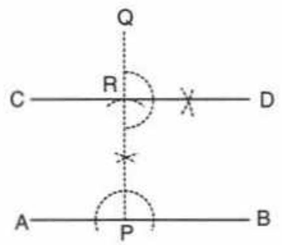


2. To construct a line parallel to a given line at a given distance from it :

Steps :

1. At any point P in line AB, draw PQ perpendicular to AB.
2. With P as centre and radius equal to 3·6 cm, draw an arc which cuts PQ at point R.
3. At point R, draw RD perpendicular to PQ.
4. Produce DR upto any point C.

   Then, **CD is the required parallel line.**



EXERCISE 18(C)

1. Draw a line AB = 6 cm. Mark a point P any where outside the line AB. Through the point P, construct a line parallel to AB.
2. Draw a line MN = 5·8 cm. Locate a point A which is 4·5 cm from M and 5 cm from N. Through A draw a line parallel to line MN.
3. Draw a straight line AB = 6·5 cm. Draw another line which is parallel to AB at a distance of 2·8 cm from it.
4. Construct an angle PQR = 80°. Draw a line parallel to PQ at a distane of 3 cm from it and another line parallel to QR at a distance of 3·5 cm from it. Mark the point of intersection of these parallel lines as A.
5. Draw an angle ABC = 60°. Draw the bisector of it. Also draw a line parallel to BC a distance of 2·5 cm from it.

   Let this parallel line meet AB at point P and angle bisectors at point Q. Measure the lengths of BP and PQ. Is BP = PQ ?
6. Construct an angle ABC = 90°. Locate a point P which is 2·5 cm from AB and 3·2 cm from BC.

8. Construction of quadrilateral ABCD.

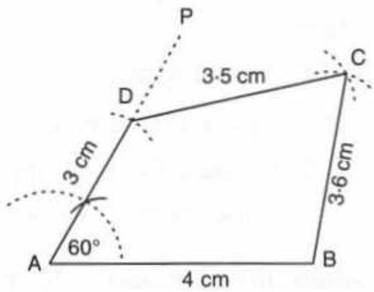
1. When four sides and one angle are given :

Let AB = 4 cm, BC = 3·6 cm, CD = 3·5 cm, AD = 3 cm and ÐA = 60°.

Steps :

1. Draw AB = 4 cm.
2. At A, construct angle PAB = 60° and from AP cut AD = 3 cm.
3. Taking D as centre, draw an arc of radius 3·5 cm (= CD) and taking B as centre draw one more arc of radius 3·6 cm (= BC) which cuts the previous arc at point C.
4. Join CD and CB.

   Then, **ABCD is the required quadrilateral.**



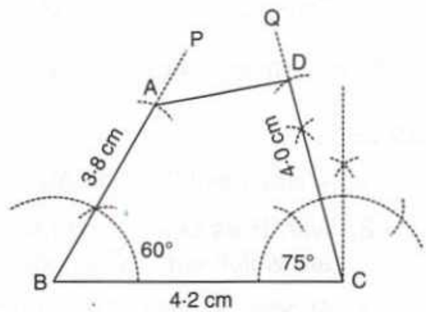
2. When three consecutive sides and two included angles are given :

Let AB = 3·8 cm, BC = 4·2 cm, CD = 4·0 cm, ∠B = 60° and ∠C = 75°.

Steps :

1. Draw BC = 4·2 cm.
2. At B, construct angle PBC = 60° and at C, construct angle QCB = 75°.
3. From BP cut AB = 3·8 cm and from CQ cut CD = 4·0 cm.
4. Join A and D.

   Then, **ABCD is the required quadrilateral.**



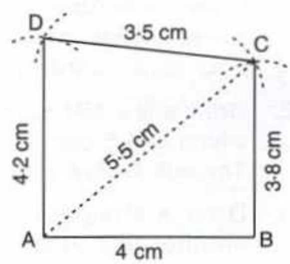
3. When four sides and one diagonal are given :

Let AB = 4 cm, BC = 3·8 cm, CD = 3·5 cm, AD = 4·2 cm and diagonal AC = 5·5 cm.

Steps :

1. Draw AB = 4 cm.
2. Taking B as centre, draw an arc of radius 3·8 cm (= BC) and taking A as centre, draw one more arc of radius 5·5 cm (= diagonal AC). Let the two arcs intersect at point C.
3. Taking C as centre, draw an arc of radius 3·5 cm (= CD) and taking A as centre, draw one more arc of radius 4·2 cm (= AD). Let the two arcs intersect at point D.

   Then, **ABCD is the required quadrilateral.**



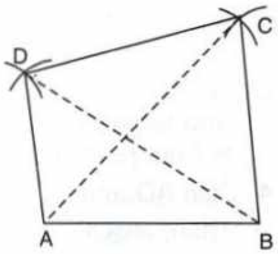
4. *When three sides and two diagonals are given :*

Let side AB = 5 cm, side AD = 4·6 cm, side BC = 5·3 cm, diagonal AC = 5 cm and diagonal BD = 6 cm.

Steps :

1. Draw AB = 5 cm.
2. Taking A as centre and radius = 5 cm (= AC), draw an arc.
3. Taking B as centre and radius = 5·3 cm (= BC), draw one more arc which meets the first arc at point C.
4. With A as centre, draw an arc with radius = 4·6 cm (= AD) and with B as centre draw one more are with radius = 6 cm (= BD)which meets AD at point D.

Then, **ABCD is the required quadrilateral.**



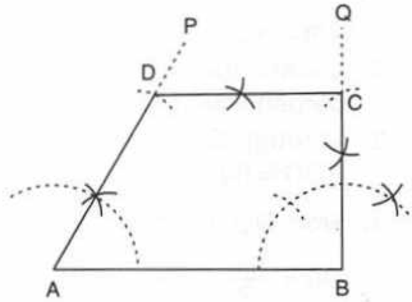
5. *When two adjacent sides and three angles are given.*

Let AB = 6 cm, BC = 5 cm, ∠A = 60°, ∠B = 90° and ∠C = 90°.

Steps :

1. Draw AB = 6 cm.
2. At A, draw AP such that ∠PAB = 60°.
3. At B, draw BQ such that ∠QBA = 90°.
4. From BQ, cut BC = 5 cm.
5. At C, draw a line CD which meets AP at point D such that ∠BCD = 90°.

Then, **ABCD is the required quadrilateral.**



9. Construction of parallelogram ABCD.

1. *When two consecutive sides and the included angles are given :*

Let BC = 4·2 cm, CD = 3·6 cm and ∠C = 60°.

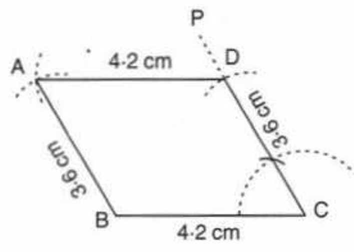
> You know that the opposite sides of a parallelogram are always equal.

∴ BC = 4·2 cm = AD and CD = 3·6 cm = AB.

Steps :

1. Draw BC = 4·2 cm.
2. At C, construct angle PCB = 60° and from CP cut CD = 3·6 cm.
3. Taking D as centre, draw an arc of radius 4·2 cm (= AD) and taking B as centre draw one more arc of radius 3·6 cm (= AB). Let the two arcs intersect at point A.
4. Join AB and AD.

Then, **ABCD is the required parallelogram.**



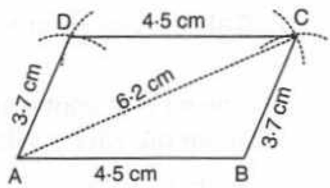
2. When two consecutive sides and one diagonal are given :

Let AB = 4·5 cm, BC = 3·7 cm and diagonal AC = 6·2.

Steps :

1. Draw AB = 4·5 cm.
2. Taking B as centre, draw an arc of radius 3·7 cm (= BC) and taking A as centre draw one more arc of radius 6·2 cm (= diagonal AC). Let the two arcs intersect at point C. Join B and C.
3. Taking C as centre, draw an arc of radius 4·5 cm (= AB) and taking A as centre draw one more arc of radius 3·7 cm (= BC). Let the two arcs intersect at point D.
4. Join AD and CD.

   Then, **ABCD is the required parallelogram.**



3. When both the diagonals and the angle between them are given :

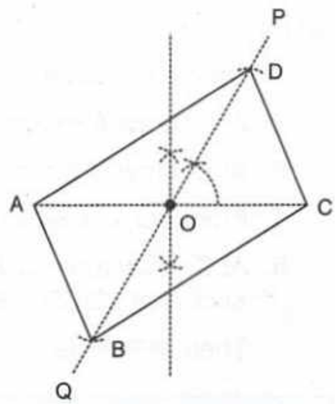
Let the diagonal AC = 5·5 cm, diagonal BD = 6·0 cm and the angle between them = 60°.

You know that the diagonals of a parallelogram bisect each other.

Steps :

1. Draw AC = 5·5 cm.
2. Locate the mid-point O of AC by drawing its perpendicular bisector.
3. Through O, construct a line POQ so that angle POC = 60°.
4. From PQ, cut OD = $\frac{BD}{2} = \frac{6 \cdot 0}{2}$ = 3·0 cm and also OB = $\frac{6 \cdot 0}{2}$ = 3·0 cm.
5. Join AB, BC, CD and DA.

   Then, **ABCD is the required parallelogram.**



10. Construction of rectangle ABCD.

1. When two adjacent sides are given :

Let AB = 5·0 cm and BC = 3·5 cm.

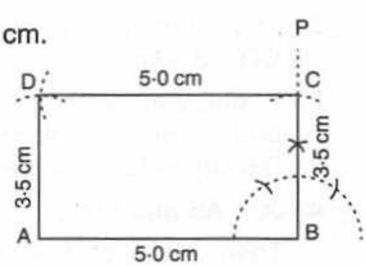
The opposite sides of a rectangle are equal and each angle of it is 90°.

∴ AB = 5·0 cm = DC, BC = 3·5 cm = AD and ∠A = ∠B = ∠C = ∠D = 90°.

Steps :

1. Draw AB = 5·0 cm.
2. At B, construct angle PBA = 90°. From BP cut BC = 3·5 cm.
3. Taking C as centre, draw an arc of radius 5·0 cm (= AB) and taking A as centre, draw one more arc of radius 3·5 cm (= BC). Let these two arcs intersect at point D.
4. Join AD and CD.

   Then, **ABCD is the required rectangle.**



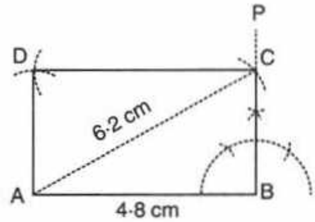
2. *When one side and one diagonal are given :*

Let AB = 4·8 cm and diagonal AC = 6·2 cm.

Steps :

1. Draw AB = 4·8 cm.
2. At B, construct angle PBA = 90°.
3. Taking A as centre, draw an arc of radius 6·2 cm (= AC) which cuts BP at point C.
4. Taking C as centre, draw an arc of radius 4·8 cm (= AB) and taking A as centre draw another arc of radius equal to BC. Let these two arcs intersect at point D.
5. Join AD and CD.

   Then, **ABCD is the required rectangle.**



3. *When one diagonal and the angle between the two diagonals are given :*

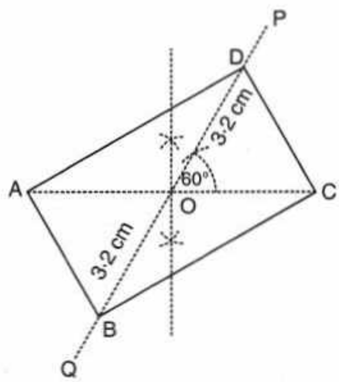
Let diagonal AC = 6·4 cm and the angle between the two diagonals be 60°.

The diagonals of a rectangle are equal *i.e.* AC = BD = 6·4 cm.

Steps :

1. Draw AC = 6·4 cm.
2. Draw the perpendicular bisector of AC to locate the mid-point of AC. Let the perpendicular bisector intersect AC at point O.
   Therefore, O is the mid-point of AC.
3. Through O, construct a line POQ so that angle POC = 60°.
4. From OP cut OD equal to OC (*i.e.* 3·2 cm) and from OQ cut OB equal to OA (*i.e.* 3·2 cm).
5. Join AB, BC, CD and DA.

   Then, **ABCD is the required rectangle.**



11. Construction of rhombus ABCD.

1. *When one side and one angle are given :*

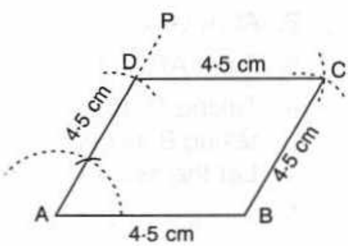
Let the side AB = 4·5 cm and angle A = 60°.

You know that the sides of a rhombus are equal *i.e.* AB = BC = CD = AD = 4·5 cm

Steps :

1. Draw AB = 4·5 cm.
2. At A, construct angle PAB = 60°
3. From AP, cut AD = 4·5 m.
4. Taking D as centre, draw an arc of radius 4·5 cm (= AB) and taking B as centre draw one more arc of radius 4·5 cm (= AB). Let the two arcs intersect at point C.
5. Join BC and DC.

   Then, **ABCD is the required rhombus.**



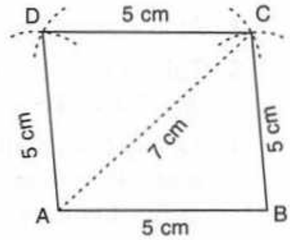
2. *When one side and one diagonal are given :*

Let the side AB = 5 cm and the diagonal AC = 7 cm.

Steps :

1. Draw AB = 5 cm.
2. Taking A as centre, draw an arc of radius 7 cm (= AC) and taking B as centre, draw one more arc of radius 5 cm (= AB). Let the two arcs intersect at point C.
3. Taking C as centre, draw an arc of radius 5 cm (= AB) and taking A as centre, draw one more arc of radius 5 cm (= AB). Let the two arcs intersect at point D.
4. Join BC, CD and DA.

   Then, **ABCD is the required rhombus.**



3. *When both the diagonals are given :*

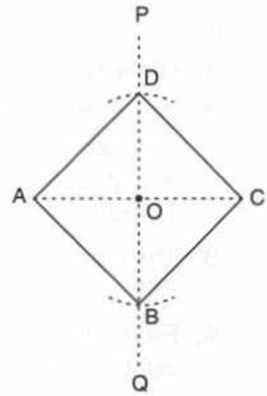
Let the diagonal AC = 4·8 cm and the diagonal BD = 5·4 cm.

The diagonals of a rhombus bisect each other at 90°.

Steps :

1. Draw AC = 4·8 cm.
2. Draw the perpendicular bisector of AC. Let PQ be the perpendicular bisector of AC which bisects AC at point O.
3. From OP, cut OD = $\frac{BD}{2} = \frac{5 \cdot 4 \text{ cm}}{2}$ = 2·7 cm and from OQ, cut OB = $\frac{BD}{2}$ = 2·7 cm.
4. Join AB, BC, CD and DA.

   Then, **ABCD is the required rhombus.**



12. Construction of square ABCD.

1. *When one side is given :*

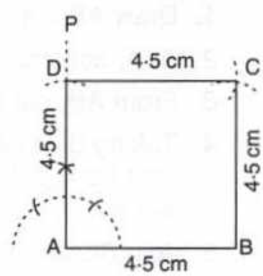
Let side AB = 4·5 cm.

You know that sides of a square are equal *i.e.* AB = BC = CD = AD = 4·5 cm and each angle of the square is 90°.

Steps :

1. Draw AB = 4·5 cm.
2. At A, construct angle PAB = 90°
3. From AP, cut AD = 4·5 cm.
4. Taking D as centre, draw an arc of radius 4·5 cm and taking B as centre, draw one more arc of radius 4·5 cm. Let the two arcs intersect at point C.
5. Join BC and DC.

   Then, **ABCD is the required square.**



2. *When a diagonal is given :*

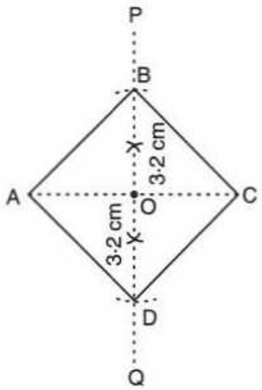
Let diagonal AC = 6·4 cm.

In a square, the diagonals bisect each other at 90°. Also, the diagonals of a square are equal *i.e.* diagonal AC = diagonal BD = 6·4 cm.

Steps :

1. Draw AC = 6·4 cm.
2. Construct POQ, the perpendicular bisector of AC which intersects AC at point O.
3. From OP, cut OB = $\frac{6 \cdot 4}{2}$ cm = 3·2 cm and from OQ cut OD = 3·2 cm.
4. Join AB, BC, CD and DA.

   Then, **ABCD is the required square.**



EXERCISE 18(D)

Students are advised to draw a rough free-hand sketch in each case, before starting the actual construction.

1. Construct a quadrilateral ABCD; if :
   (i) AB = 4·3 cm, BC = 5·4 cm, CD = 5 cm, DA = 4·8 cm and angle ABC = 75°.
   (ii) AB = 6 cm, CD = 4·5 cm, BC = AD = 5 cm and ∠BCD = 60°.
   (iii) AB = 8 cm, BC = 5·4 cm, AD = 6 cm, ∠A = 60° and ∠B = 75°.
   (iv) AB = 5 cm, BC = 6·5 cm, CD = 4·8 cm, ∠B = 75° and ∠C = 120°.
   (v) AB = 6 cm = AC, BC = 4 cm, CD = 5 cm and AD = 4·5 cm.
   (vi) AB = AD = 5 cm, BD = 7 cm and BC = DC = 5·5 cm.
2. Construct a parallelogram ABCD, if :
   (i) AB = 3·6 cm, BC = 4·5 cm and ∠ABC = 120°.
   (ii) BC = 4·5 cm, CD = 5·2 cm and ∠ADC = 75°.
   (iii) AD = 4 cm, DC = 5 cm and diagonal BD = 7 cm.
   (iv) AB = 5·8 cm, AD = 4·6 cm and diagonal AC = 7·5 cm.
   (v) diagonal AC = 6·4 cm, diagonal BD = 5·6 cm and angle between the diagonals is 75°.
   (vi) lengths of diagonals AC and BD are 6·3 cm and 7·0 cm respectively, and the angle between them is 45°.
   (vii) lengths of diagonals AC and BD are 5·4 cm and 6·7 cm respectively, and the angle between them is 60°.
3. Construct a rectangle ABCD; if :
   (i) AB = 4·5 cm and BC = 5·5 cm.
   (ii) BC = 6·1 cm and CD = 6·8 cm.
   (iii) AB = 5·0 cm and diagonal AC = 6·7 cm.
   (iv) AD = 4·8 cm and diagonal AC = 6·4 cm.
   (v) each diagonal is 6 cm and the angle between them is 45°.
   (vi) each diagonal is 5·5 cm and the angle between them is 60°.
4. Construct a rhombus ABCD, if :
   (i) AB = 4 cm and ∠B = 120°.
   (ii) BC = 4·7 cm and ∠B = 75°.
   (iii) CD = 5 cm and diagonal BD = 8·5 cm.
   (iv) BC = 4·8 cm and diagonal AC = 7 cm.
   (v) diagonal AC = 6 cm and diagonal BD = 5·8 cm
   (vi) diagonal AC = 4·9 cm and diagonal BD = 6 cm.
   (vii) diagonal AC = 6·6 cm and diagonal BD = 5·3 cm.
5. Construct a square, if :
   (i) its one side is 3·8 cm.
   (ii) its each side is 4·3 cm.
   (iii) one diagonal is 6·2 cm.
   (iv) each diagonal is 5·7 cm.

6. Construct a quadrilateral ABCD in which; $\angle A = 120°$, $\angle B = 60°$, AB = 4 cm, BC = 4·5 cm and CD = 5 cm.
7. Construct a quadrilateral ABCD, such that AB = BC = CD = 4·4 cm, $\angle B = 90°$ and $\angle C = 120°$.
8. Using ruler and compasses only, construct a parellelogram ABCD, in which : AB = 6 cm, AD = 3 cm and $\angle DAB = 60°$.

   In the same figure draw the bisector of angle DAB and let it meet DC at point P. Measure angle APB.
9. Draw a parallelogram ABCD, with AB = 6 cm, AD = 4·8 cm and $\angle DAB = 45°$.

   Draw the perpendicular bisector of side AD and let it meet AD at point P. Also, draw the diagonals AC and BD, and let them intersect at point O. Join O and P. Measure OP.
10. Using ruler and compasses only, construct a rhombus whose diagonals are 8 cm and 6 cm. Measure the length of its one side.

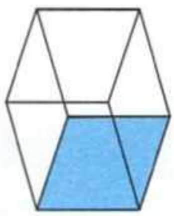
REPRESENTING 3-D IN 2-D 19

19.1 INTRODUCTION

In this chapter, students will visualize two dimensional cross-sections of representations of three dimensional objects. In particular, this lesson will help the students who have difficulties recognizing and drawing 2-dimensional cross-sections along a plane of representation of a 3-dimensional object.

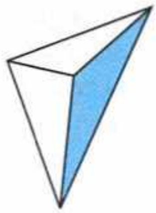
The following diagram shows 2-dimensional representations of 3-dimensional objects.

(i)



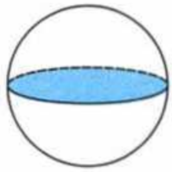
(Cube)

(ii)



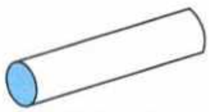
(Tetrahedron)

(iii)



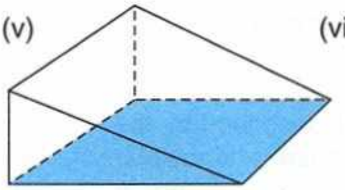
(Sphere)

(iv)



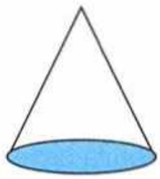
(Cylinder)

(v)



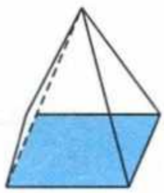
(Triangular prism)

(vi)



(Cone)

(vii)



(Rectangular pyramid)

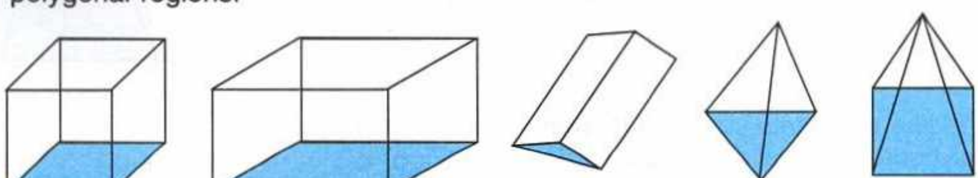
The figures given here show the 2-D representation of 3-D objects.

19.2 POLYHEDRON

A three-dimensional figure bounded by polygonal regions is called **polyhedron**.

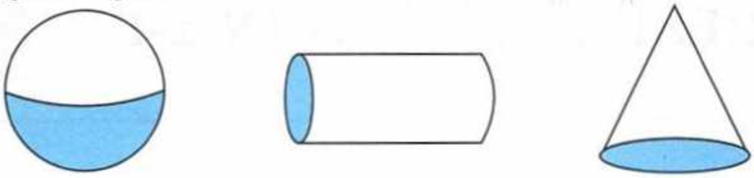
For example :

1. Cubes, cuboids, prisms and pyramids are polyhedrons as all these are bounded by polygonal regions.



Polygonal region means each face of the figure is a polygon.

2. Spheres, cylinders and cones are not polyhedrons as all these are not bounded by polygonal region.

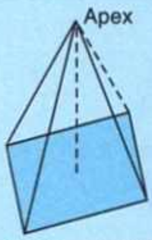


Polyhedra is the plural of polyhedron.

Pyramids

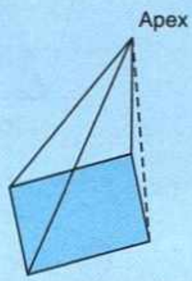
A pyramid is a polyhedron for which the base is a polygon and all lateral surfaces are triangles.

If the lateral surfaces of a pyramid are congruent triangles, it is called a right pyramid, indicating that the apex (the vertex at which the lateral surfaces meet) is directly above the centre of the base.



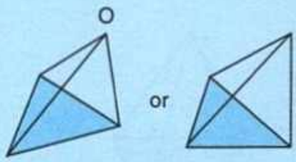
(Right pyramid)

In this chapter, when we use the term pyramid, we mean a right pyramid.



(Non-right pyramid)

A pyramid is typically described by the shape of its base. For instance, a trianglular pyramid has a base that is a triangle. A pyramid with triangular base is also called a **tetrahedron** which has four sides. A regular tetrahedron has its faces as equilateral triangles where all sides are equal and the angle between them is 60°.



(Tetrahedran) (Tetrahedron)

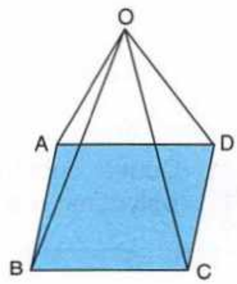
19.3 FACES, EDGES AND VERTICES

1. **Faces (F) :**

   The polygons forming a polyhedron are known as **faces**.

   The given figure shows a pyramid with five polygonal regions : ABCD, OAD, OBC, OAB and OCD.

   $\therefore$ Number of faces in it = 5.



2. **Edges (E) :**

   The line segments common to intersecting faces of a polyhedron are called **edges**.

   The given figure has 8 edges.

   Namely; OA, OB, OC, OD, AB, BC, CD and DA.

3. **Vertices (V) :**

Points of intersection of the edges of a polyhedron are called its vertices

The given figure has 5 vertices.

Namely; O, A, B, C and D.

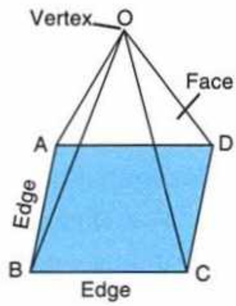
19.4 EULER'S FORMULA

Euler's formula deals with shapes called polyhedra. A polyhedron is a closed solid shape which has flat faces and straight edges.

According to the Euler's formula, if a polyhedron has :

1. Number of faces = F
2. Number of edges = E and
3. Number of vertices = V, then

$$F + V - E = 2$$



In the adjoining figure, the pyramid shows

(i) 5 faces *i.e.* F = 5,

(ii) 8 edges *i.e.* E = 8, and

(iii) 5 vertices *i.e.* V = 5

$\therefore$ **F + V − E** = 5 + 5 − 8

= 10 − 8 = **2**

Hence, the Euler's formula.

19.5 MORE POLYHEDRONS (Polyhedra)

1. **Cube :**

F = Number of faces = 6

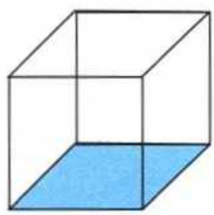
V = Number of vertices = 8

and E = Number of edges = 12

$\therefore$ **F + V − E** = 6 + 8 − 12

= **2**

Hence, the Euler's formula.



2. **Cuboid :**

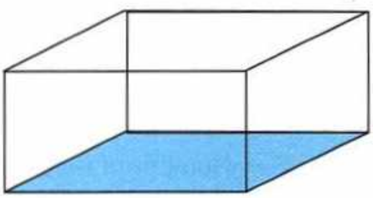
F = Number of faces = 6

V = Number of vertices = 8

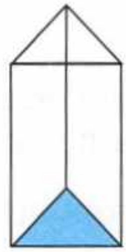
and E = Number of edges = 12

$\therefore$ **F + V − E** = 6 + 8 − 12

= **2**



3. Triangular prism :



F = Number of faces = 5

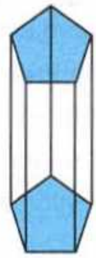
V = Number of vertices = 6

and E = Number of edges = 9

∴ **F + V − E** = 5 + 6 − 9

= **2**

4. Pentagonal prism :



F = Number of faces = 7

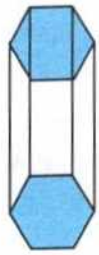
V = Number of vertices = 10

and E = Number of edges = 15

∴ **F + V − E** = 7 + 10 − 15

= **2**

5. Hexagonal prism :



F = Number of faces = 8

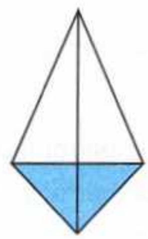
V = Number of vertices = 12

and E = Number of edges = 18

∴ **F + V − E** = 8 + 12 − 18

= **2**

6. Tetrahedron (triangular pyramid) **:**



F = Number of faces = 4

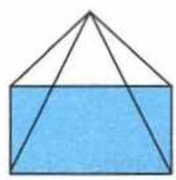
V = Number of vertices = 4

and E = Number of edges = 6

∴ **F + V − E** = 4 + 4 − 6

= **2**

7. Rectangular pyramid :



F = Number of faces = 5

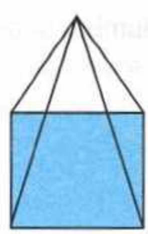
V = Number of vertices = 5

and E = Number of edges = 8

∴ **F + V − E** = 5+ 5 − 8

= **2**

8. Square pyramid :



F = Number of faces = 5

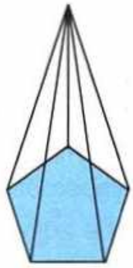
V = Number of vertices = 5

and E = Number of edges = 8

∴ **F + V − E** = 5 + 5 − 8

= **2**

9. Pentagonal pyramid :



F = Number of faces = 6

V = Number of vertices = 6

and E = Number of edges = 10

$\therefore$ **F + V − E** = 6 + 6 − 10

= **2**

10. Hexagonal pyramid :

F = Number of faces = 7

V = Number of vertices = 7

and E = Number of edges = 12

$\therefore$ **F + V − E** = 7 + 7 − 12

= **2**

Example 1 :

Can a polyhedron have 14 faces, 24 edges and 33 vertices ?

Solution :

Clearly, F = 14, E = 24 and V = 33.

$\therefore$ **F + V − E** = 14 + 33 − 24

= 47 − 24 = **23**

$\Rightarrow$ F + V − E is not equal to 2

$\therefore$ **A polyhedron can not have 14 faces, 24 edges and 33 vertices** **(Ans.)**

Example 2 :

Use Euler's formula to find the values of *a, b* and *c*.

	Faces	Vertices	Edges
(i)	8	*a*	12
(ii)	*b*	6	9
(iii)	20	12	*c*

Solution :

(i) F + V − E = 2

$\Rightarrow$ 8 + *a* − 12 = 2

$\Rightarrow$ ***a*** = 2 + 12 − 8 = **6** **(Ans.)**

(ii) F + V − E = 2

$\Rightarrow$ *b* + 6 − 9 = 2

$\Rightarrow$ ***b*** = 2 + 9 − 6 = **5** **(Ans.)**

(iii) F + V − E = 2

$\Rightarrow$ 20 + 12 − *c* = 2

$\Rightarrow$ 20 + 12 − 2 = C

C = 30 **(Ans.)**

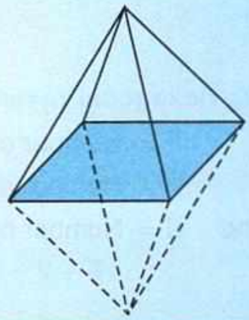
Do you know :

1. A prism is a solid, whose side faces are parallelograms and whose ends (bases) are congruent and parallel rectilinear figures.
2. A pyramid is a polyhedron whose base is a polygon of any number of sides and whose other faces are triangles with a common vertex.
3. The adjoining figure is called an octahedron which has :

   (i) 8 triangular faces *i.e.* F = 8

   (ii) 6 vertices *i.e.* V = 6

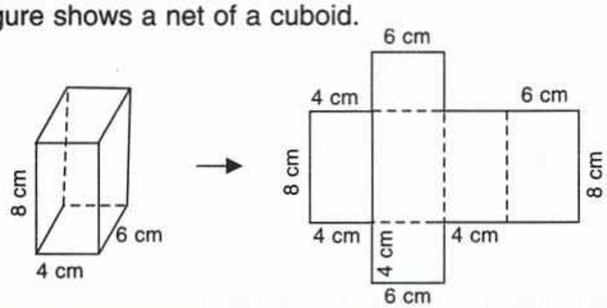
and, (iii) 12 edges *i.e.* E = 12



19.6 NETS OF A SOLID

A net is a pattern made when the surface of a 3-dimensional figure is laid out flat showing each face of the figure.

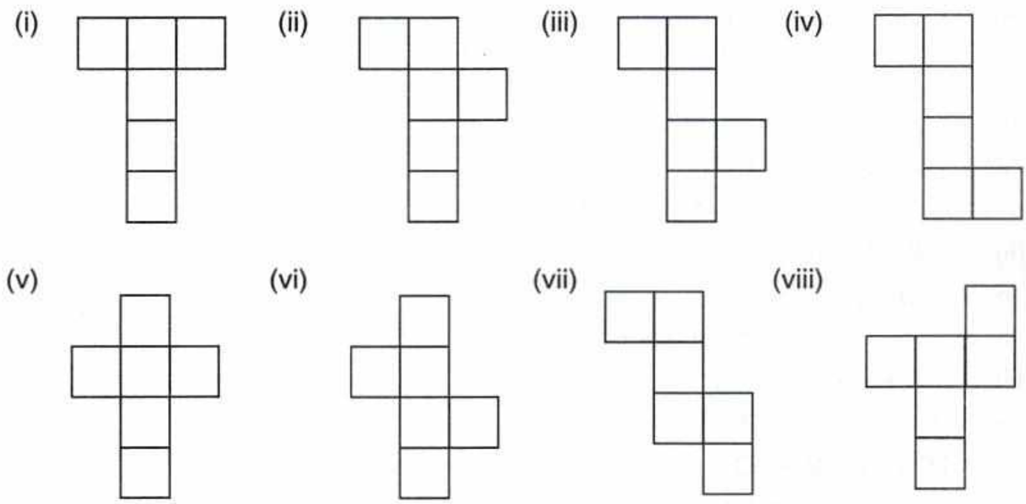
The following figure shows a net of a cuboid.

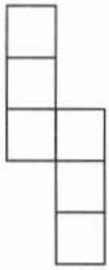
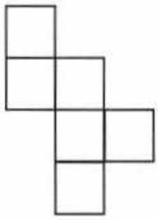
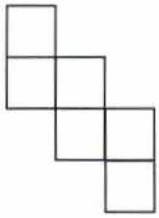


A 3-dimension figure (solid) may have several nets.

1. Nets of a cube

A cube has 11 nets as shown below :

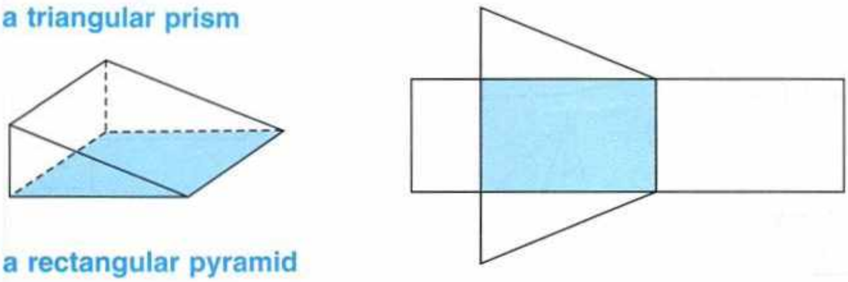


(ix)  (x)  (xi) 

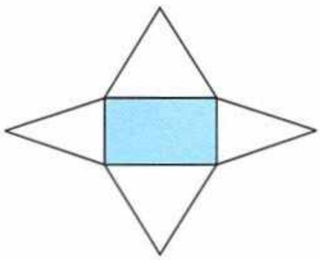
1. Make sure that the solid and the net have the same number of faces and that the shapes of the faces of the solid match the shapes of the corresponding faces in the net.
2. Visualize how the net is to be folded to form the solid and make sure that all the sides fit together properly.

Nets are helpful when we need to find the surface area of the solids.

2. Net of a triangular prism

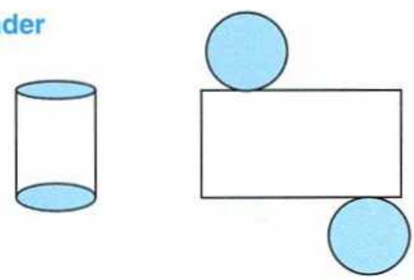


3. Net of a rectangular pyramid



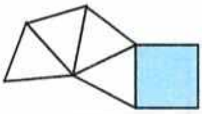
4. Net of a cone

5. Net of a cylinder

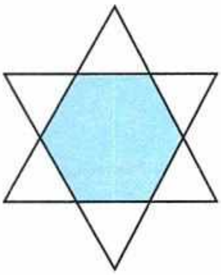


6. More nets

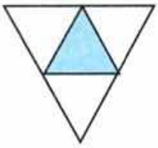
(i)



(ii)



(iii)

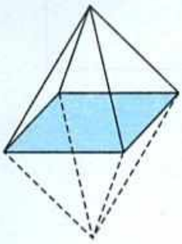


(A square pyramid) (A hexagonal pyramid) (A tetrahedron)

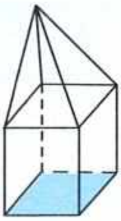
EXERCISE 19

1. If a polyhedron has 10 faces and 8 vertices, find the number of edges in it.
2. If a polyhedron has 10 vertices and 7 faces, find the number of edges in it.
3. State, the number of faces, number of vertices and number of edges of :
   (i) a pentagonal pyramid
   (ii) a hexagonal prism
4. Verify Euler's formula for the following three dimensional figures :

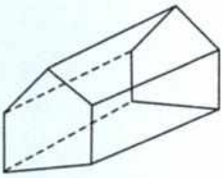
(i)



(ii)



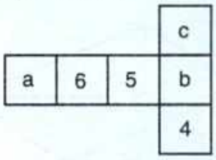
(iii)



5. Can a polyhedron have 8 faces, 26 edges and 16 vertices ?
6. Can a polyhedron have :
   (i) 3 triangles only ?
   (ii) 4 triangles only ?
   (iii) a square and four triangles?
7. Using Euler's formula, find the values of x, y and z

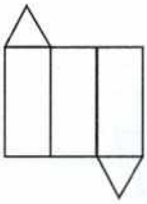
	Faces	Vertices	Edges
(i)	x	15	20
(ii)	6	y	8
(iii)	14	26	z

8. What is the least number of planes that can enclose a solid ? What is the name of the solid.
9. Is a square prism same as a cube ?
10. The dimensions of a cuboid are 6 cm × 4 cm × 2 cm. Draw two different nets of it.
11. Dice are cubes where the sum of the numbers on the opposite faces is 7. Find the missing numbers *a*, *b* and *c*.

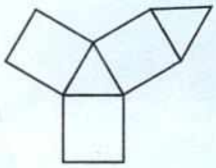


12. Name the polyhedron that can be made by folding each of the following nets :

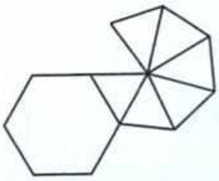
(i)



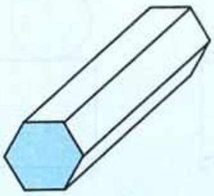
(ii)

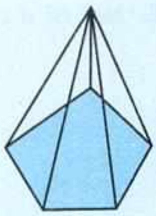


(iii)



13. Draw two nets for each of the following polyhedrons :





Theme – 5
MENSURATION

AREA OF A TRAPEZIUM AND A POLYGON 20

20.1 REVIEW

Perimeter	The perimeter of a plane figure is the length of its boundary.
Area	The area of a plane figure is the amount of surface enclosed by its sides (boundary).

Common units of perimeter are metre (m), centimetre (cm), decimetre (dm), etc.

1. 1 m = 100 cm and 1 cm = $\frac{1}{100}$ m = 0·01 m
2. 1 m = 10 dm and 1 dm = $\frac{1}{10}$ m = 0·1 m
3. 1 dm = 10 cm and 1 cm = $\frac{1}{10}$ dm = 0·1 dm
4. 1 m = 1000 mm and 1 mm = $\frac{1}{1000}$ m = 0·001 m and so on.

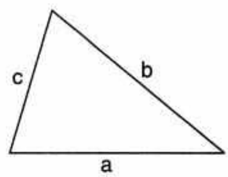
Common units of area are square metre (sq. m or m^2), square centimetre (sq. cm or cm^2), square millimetre (sq. mm or mm^2), etc.

1. $1\ m^2 = 100 \times 100\ cm^2 = 10{,}000\ cm^2$ and $1\ cm^2 = \frac{1}{100 \times 100}\ m^2 = 0{\cdot}0001\ m^2$
2. $1\ m^2 = 10 \times 10\ dm^2 = 100\ dm^2$ and $1\ dm^2 = \frac{1}{10 \times 10}\ m^2 = 0{\cdot}01\ m^2$
3. $1\ dm^2 = 10 \times 10\ cm^2 = 100\ cm^2$ and $1\ cm^2 = \frac{1}{100}\ dm^2 = 0{\cdot}01\ dm^2$
4. $1\ cm^2 = 10 \times 10\ mm^2 = 100\ mm^2$ and $1\ mm^2 = \frac{1}{100}\ cm^2 = 0{\cdot}01\ cm^2$ and so on.

20.2 PERIMETER AND AREA OF TRIANGLES

1. If a, b and c are the three sides of a triangle; then its
   (i) perimeter = a + b + c
   (ii) area = $\sqrt{s(s-a)(s-b)(s-c)}$

   Where, s = semi-perimeter of the triangle = $\frac{a+b+c}{2}$

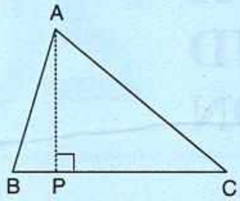


2. If one side (base) and the corresponding height (altitude) of the triangle are known, its

$$\text{area} = \frac{1}{2}\ \text{base} \times \text{height}$$

Any side of the triangle can be taken as its base and the corresponding height means : the length of perpendicular to this side from the opposite vertex.

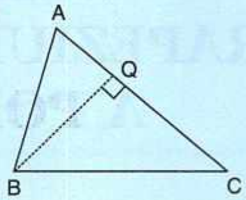
(i)



If BC is taken as base

area = $\frac{1}{2}$ × BC × AP

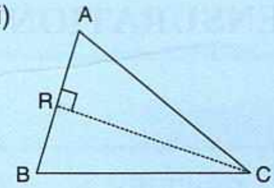
(ii)



If AC is taken as base

area = $\frac{1}{2}$ × AC × BQ

(iii)



If AB is taken as base

area = $\frac{1}{2}$ × AB × CR

Also, area = $\frac{1}{2}$ × base × height ⇒ (i) base = $\frac{2 \times \text{area}}{\text{height}}$

(ii) height = $\frac{2 \times \text{area}}{\text{base}}$

Example 1 :

Find the area of a triangle whose sides are 9 cm, 12 cm and 15 cm. Also, find the length of altitude corresponding to the largest side of the triangle.

Solution :

Let a = 9 cm, b = 12 cm and c = 15 cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{9+12+15}{2} \text{ cm} = \frac{36}{2} \text{ cm} = 18 \text{ cm}$$

$$\therefore \quad \textbf{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-9)(18-12)(18-15)}$$

$$= \sqrt{18 \times 9 \times 6 \times 3} = \sqrt{2916} = \mathbf{54 \text{ cm}^2} \qquad \textbf{(Ans.)}$$

Also, area of triangle = $\frac{1}{2}$ base × corresponding altitude

$$\therefore \quad 54 = \frac{1}{2} \times 15 \times h \qquad \text{[Taking largest side as the base]}$$

$$\Rightarrow \quad h = \frac{54 \times 2}{15} \text{ cm} = \mathbf{7{\cdot}2 \text{ cm}} \qquad \textbf{(Ans.)}$$

Example 2 :

Find the area of an equilateral triangle, whose one side is a cm.

Solution :

$$s = \frac{a+b+c}{2} = \frac{a+a+a}{2} = \frac{3a}{2} \qquad \text{[Sides of an equilateral triangle are equal]}$$

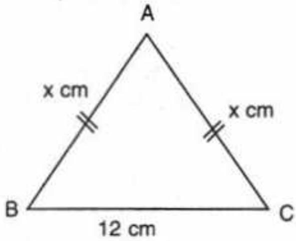
$$\therefore \quad \textbf{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \frac{a \times a}{2 \times 2}\sqrt{3} = \mathbf{\frac{\sqrt{3}}{4} a^2 \text{cm}^2} \qquad \textbf{(Ans.)}$$

Example 3 :

The base of an isosceles triangle is 12 cm and its perimeter is 32 cm. Find the area of the triangle.



Solution :

Let each of the two equal sides of the given isosceles triangle be x cm.

Since, perimeter of the triangle is 32 cm

$\Rightarrow \quad x + x + 12 = 32 \Rightarrow x = 10$

Hence, the sides of the given isosceles triangle are 10 cm, 10 cm and 12 cm

Let $\quad a = 10$ cm, $b = 10$ cm and $c = 12$ cm

$\therefore \quad s = \frac{a+b+c}{2} = \frac{10+10+12}{2}$ cm = 16 cm

Area of the Δ $= \sqrt{s(s-a)(s-b)(s-c)}$

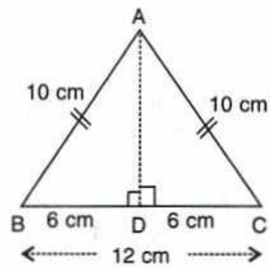
$= \sqrt{16(16-10)(16-10)(16-12)}\ cm^2$

$= \sqrt{16 \times 6 \times 6 \times 4}\ cm^2 = 4 \times 6 \times 2\ cm^2 =$ **48 cm²** **(Ans.)**

Alternative method :

Draw AD perpendicular to base BC of the given triangle.

Since, the perpendicular from the vertex of an isosceles triangle to its base bisects the base, therefore



$BD = CD = \frac{BC}{2} = \frac{12}{2}$ cm = 6 cm.

In right-angled triangle ABD,

$AD^2 + BD^2 = AB^2$

$\Rightarrow \quad AD^2 + 6^2 = 10^2$

$\Rightarrow \quad AD^2 = 100 - 36 = 64$ and $AD = \sqrt{64}$ cm = 8 cm

Now the base BC of the given triangle is 12 cm and its height AD is 8 cm

$\therefore$ **Area of the Δ** $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 12\ cm \times 8\ cm =$ **48 cm²** **(Ans.)**

EXERCISE 20(A)

1. Find the area of of a triangle; whose sides are :
   (i) 10 cm, 24 cm and 26 cm
   (ii) 18 mm, 24 mm and 30 mm
   (iii) 21 m, 28 m and 35 m
2. Two sides of a triangle are 6 cm and 8 cm. If height of the triangle corresponding to 6 cm side is 4 cm; find :
   (i) area of the triangle
   (ii) height of the triangle corresponding to 8 cm side.
3. The sides of a triangle are 16 cm, 12 cm and 20 cm. Find :
   (i) area of the triangle
   (ii) height of the triangle, corresponding to the largest side

(iii) height of the triangle, corresponding to the smallest side.

4. Two sides of a triangle are 6·4 m and 4·8 m. If height of the triangle corresponding to 4·8 m side is 6 m; find :
   (i) area of the triangle;
   (ii) height of the triangle corresponding to 6·4 m side

5. The base and the height of a triangle are in the ratio 4 : 5. If the area of the triangle is 40 m^2; find its base and height.

> Let base be 4x m and height be 5x m.
> $\therefore \ \frac{1}{2} \times 4x \times 5x = 40.$

6. The base and the height of a triangle are in the ratio 5 : 3. If the area of the triangle is 67·5 m^2; find its base and height.

7. The area of an equilateral triangle is $144\sqrt{3}$ cm^2; find its perimeter.

> Area of an equilateral triangle
> $= \frac{\sqrt{3}}{4} \times (\text{side})^2$ and its perimeter = 3 × side

8. The area of an equilateral triangle is numerically equal to its perimeter. Find its perimeter correct to 2 decimal places.

9. A field is in the shape of a quadrilateral ABCD in which side AB = 18 m, side AD = 24 m, side BC = 40 m, DC = 50 m and angle A = 90°. Find the area of the field.

10. The lengths of the sides of a triangle are in the ratio 4 : 5 : 3 and its perimeter is 96 cm. Find its area.

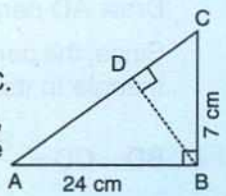
11. One of the equal sides of an isosceles triangle is 13 cm and its perimeter is 50 cm. Find the area of the triangle.

12. The altitude and the base of a triangular field are in the ratio 6 : 5. If its cost is ₹ 49,57,200 at the rate of ₹ 36,720 per hectare and 1 hectare = 10,000 sq. m, find (in metre) dimensions of the field.

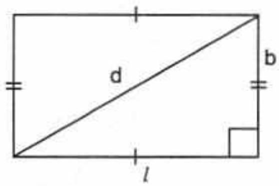
> **Area of the given triangular field**
> $= \frac{\text{Cost of the field}}{\text{Rate}} = \frac{49,57,200}{36,720}$ hectare
> = 135 hectare
> = 135 × 10,000 sq. m = 1350000 m^2.

13. Find the area of the right-angled triangle with hypotenuse 40 cm and one of the other two sides 24 cm.

14. Use the information given in the adjoining figure to find :
   (i) the length of AC.
   (ii) the are a of Δ ABC.
   (iii) the length of BD, correct to one decimal place.



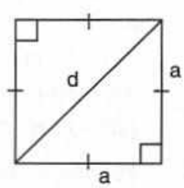
20.3 PERIMETER AND AREA OF RECTANGLES



1. Perimeter = length of boundary
   $= 2l + 2b = 2(l + b)$
2. Area = length × breadth
   $= l \times b$
3. Since, $d^2 = l^2 + b^2$ [Applying Pythagoras Theorem]

   $\therefore$ Diagonal (d) $= \sqrt{l^2 + b^2}$

20.4 PERIMETER AND AREA OF SQUARES



1. Perimeter = 4a = 4 × side
2. Area $= a \times a = a^2 = (\text{side})^2$
3. Diagonal (d) $= \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2} = \text{side}\sqrt{2}$

Example 4 :

The perimeter of a rectangle is 28 cm and its length is 8 cm. Find its :

(i) breadth (ii) area (iii) diagonal

Solution :

(i) Since, perimeter $= 2(l + b)$

$\Rightarrow$ $28 = 2(8 + b) \Rightarrow$ **b = 6 cm** **(Ans.)**

(ii) **Area** $= l \times b = 8 \times 6 \text{ cm}^2 =$ **48 cm²** **(Ans.)**

(iii) **Diagonal (d)** $= \sqrt{l^2 + b^2} = \sqrt{8^2 + 6^2} =$ **10 cm** **(Ans.)**

Example 5 :

The area of a rectangle is $5{\cdot}4 \text{ m}^2$. If its breadth is 1·5 m; find its :

(i) length (ii) perimeter

Solution :

(i) **Area** $= l \times b$

$\Rightarrow$ $5{\cdot}4 = l \times 1{\cdot}5$ or $l = \frac{5{\cdot}4}{1{\cdot}5}$ m = **3·6 m** **(Ans.)**

(ii) **Perimeter** $= 2(l + b) = 2\,(3{\cdot}6 + 1{\cdot}5)$ m = **10·2 cm** **(Ans.)**

Example 6 :

The perimeter of a square is 28 cm. Find its :

(i) one side (ii) area (iii) diagonal

Solution :

(i) **Perimeter** $= 4 \times \text{side}$ $\Rightarrow$ $28 = 4 \times \text{side}$

$\Rightarrow$ **side** $= \frac{28}{4} =$ **7 cm** **(Ans.)**

(ii) **Area** $= (\text{side})^2 = 7^2 \text{ cm}^2 =$ **49 cm²** **(Ans.)**

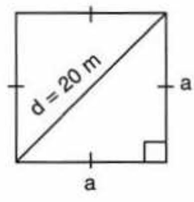
(iii) **Diagonal (d)** $= \text{side}\sqrt{2} = 7 \times 1{\cdot}414$ $[\sqrt{2} = 1{\cdot}414]$

$=$ **9·898 cm** **(Ans.)**

Example 7 :

The diagonal of a square is 20 m. Find its :

(i) area (ii) length of one side (iii) perimeter



Solution :

(i) If each side of the square is a m;

Then, $d^2 = a^2 + a^2$ [Pythagoras Theorem]

$\Rightarrow$ $(20)^2 = 2a^2$

$\Rightarrow$ $a^2 = \frac{400}{2} = 200$

$\therefore$ (i) **Area** $= a^2 =$ **200 m²** **(Ans.)**

(ii) Since, $a^2 = 200 \Rightarrow$ **a** $= \sqrt{200}$ m = **14·1 m** **(Ans.)**

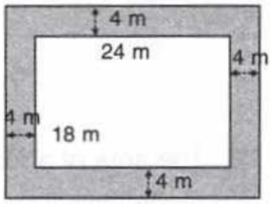
(iii) **Perimeter** $= 4a = 4 \times 14{\cdot}1$ m = **56·4 m** **(Ans.)**

Example 8 :

A path of uniform width 4 m runs around the outside of a rectangular field 24 m by 18 m. Find the area of the path.

Solution :

According to the given information, the figure will be as shown alongside in which the shaded portion is the area of the path.



Clearly :

Length of the field excluding path = 24 m

and, width of the field excluding path = 18 m

∴ **Area of the field excluding path** = 24 m × 18 m = **432 m²**

Now, length of the field including path = (24 + 2 × 4) m = 32 m

and, width of the field including path = (18 + 2 × 4) m = 26 m

∴ **Area of the field including path** = 32 m × 26 m = **832 m²**

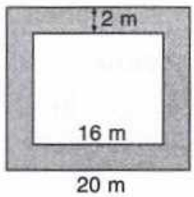
∴ **Area of the path** = $832 \text{ m}^2 - 432 \text{ m}^2$ = **400 m²** **(Ans.)**

Example 9 :

A path of uniform width 2 m runs around the inside of a square field of side 20 m. Find the area of the path.

Solution :

According to the given information, the figure will be as shown alongside :



Clearly :

area of the field including path = 20 m × 20 m = 400 m²

Each side of the square field excluding path = (20 − 2 × 2) m = 16 m

∴ Area of the field excluding path = 16 m × 16 m = 256 m²

∴ **Area of the path** = $400 \text{ m}^2 - 256 \text{ m}^2$ = **144 m²** **(Ans.)**

Example 10 :

A rectangular hall is 5·25 m long and 3·78 m wide. Its floor is to be covered with square tiles, each of side 21 cm. Find the cost of tiles required at the rate of ₹ 5 per tile.

Solution :

Since, floor area of hall = 5·25 m × 3·78 m

= 525 × 378 cm²

And, area of each tile = 21 × 21 cm²

$$\text{Number of tiles required} = \frac{\text{Floor area of hall}}{\text{Area of each tile}}$$

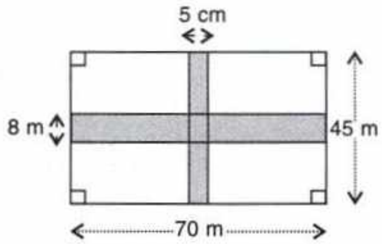
$$= \frac{525 \times 378}{21 \times 21} = 450$$

Since, cost of each tile = ₹ 5

∴ **The cost of all the tiles required** = 450 × ₹ 5 = **₹ 2,250** **(Ans.)**

Example 11 :

The adjoining figure shows a rectangular field 70 m long and 45 m wide. The shaded portion shows two mutually perpendicular roads, one of width 5 m and the other with width 8 cm. Find the cost of levelling the roads at the rate of ₹ 180 per sq. m.



Solution :

Area of road along the length = 70 m × 8 m = 560 m^2,

area of road along the width = 45 m × 5 m = 225 m^2

and, area common to both the roads = 5 m × 8 m = 40 m^2

∴ Actual area of the two roads = 560 m^2 + 225 m^2 – 40 m^2

= 745 m^2

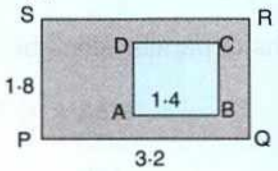
⇒ **The cost of levelling the roads** = Area of roads × Rate of levelling

= 745 × ₹ 180 = **₹ 1,34,100** **(Ans.)**

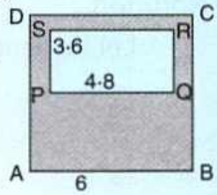
EXERCISE 20(B)

1. Find the length and perimeter of a rectangle, whose area = 120 cm^2 and breadth = 8 cm.
2. The perimeter of a rectangle is 46 m and its length is 15 m. Find its :
   (i) breadth (ii) area (iii) diagonal
3. The diagonal of a rectangle is 34 cm. If its breadth is 16 cm; find its :
   (i) length (ii) area
4. The area of a small rectangular plot is 84 m^2. If the difference between its length and the breadth is 5 m; find its perimeter.
5. The perimeter of a square is 36 cm; find its area.
6. Find the perimeter of a square whose area is 1·69 m^2.
7. The diagonal of a square is 12 cm long; find its area and length of one side.
8. The diagonal of a square is 15 m; find the length of its one side and perimeter.
9. The area of a square is 169 cm^2. Find its :
   (i) one side (ii) perimeter
10. The length of a rectangle is 16 cm and its perimeter is equal to the perimeter of a square with side 12·5 cm. Find the area of the rectangle.
11. The perimeter of a square is numerically equal to its area. Find its area.
12. Each side of a rectangle is doubled. Find the ratio between :
    (i) perimeters of the original rectangle and the resulting rectangle
    (ii) areas of the original rectangle and the resulting rectangle
13. In each of the following cases ABCD is a square and PQRS is a rectangle. Find, in each case, the area of the shaded portion. (All measurements are in metre).

(i)



(ii)



14. A path of uniform width, 3 m, runs around the outside of a square field of side 21 m. Find the area of the path.
15. A path of uniform width, 2·5 m, runs around the inside of a rectangular field 30 m by 27 m. Find the area of the path.
16. The length of a hall is 18 m and its width is 13·5 m. Find the least number of square tiles, each of side 25 cm, required to cover the floor of the hall,
    (i) without leaving any margin.
    (ii) leaving a margin of width 1·5 m all around.

    In each case, find the cost of the tiles required at the rate of ₹ 6 per tile.
17. A rectangular field is 30 m in length and 22 m in width. Two mutually perpendicular roads, each 2·5 m wide, are drawn inside the field so that one road is parallel to the length of the

field and the other road is parallel to its width. Calculate the area of the crossroads.

18. The length and the breadth of a rectangular field are in the ratio 5 : 4 and its area is 3380 m^2. Find the cost of fencing it at the rate of ₹ 75 per m.

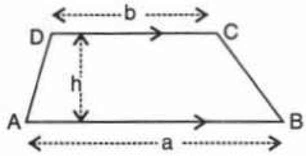
19. The length and the breadth of a conference hall are in the ratio 7 : 4 and its perimeter is 110 m. Find :
    (i) area of the floor of the hall.
    (ii) number of tiles, each a rectangle of size 25 cm × 20 cm, required for flooring of the hall.
    (iii) the cost of the tiles at the rate of ₹ 1,400 per hundred tiles.

20.5 TRAPEZIUM

Area of a trapezium

$$= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (a + b) \times h$$



1. Here, a and b are the parallel sides of the trapezium and h is the height.
2. Height of the trapezium means the distance between its parallel sides.

Example 12 :

The lengths of parallel sides of a trapezium are in the ratio 3 : 5 and the distance between them is 10 cm. If the area of the trapezium is 120 cm^2; find the lengths of its parallel sides.

Solution :

Let the lengths of parallel sides be 3x cm and 5x cm

Since, $\text{area} = \frac{1}{2}$ (the sum of parallel sides) × height

$\Rightarrow \quad 120 = \frac{1}{2}(3x + 5x) \times 10$ *i.e.* $x = \frac{120}{40} = 3$

∴ **Lengths of parallel sides** = 3x cm and 5x cm

= 3 × 3 cm and 5 × 3 cm = **9 cm and 15 cm** **(Ans.)**

Example 13 :

The area of a trapezium is 880 cm^2. The lengths of the parallel sides are 60 cm and 28 cm respectively. Find the distance between the parallel sides.

Solution :

Let the distance between the parallel sides = h cm

i.e. Height of the trapezium = h cm

$\because \frac{1}{2} \times$ (sum of the parallel sides) × height = Area of the trapezium

$\Rightarrow \quad \frac{1}{2} \times (60 + 28) \times h = 880$

$\Rightarrow \quad h = 880 \times \frac{2}{88} = 20$

∴ **The distance between the parallel sides is 20 cm** **(Ans.)**

Example 14 :

The area of a trapezium shaped field is 1920 m^2, the height is 30 m and one of the parallel sides is 40 m. Find the other parallel side.

Solution :

Let the other parallel side = x m

$\because \frac{1}{2} \times$ (sum of the parallel sides) $\times$ height = 1920

$\Rightarrow \quad \frac{1}{2} \times (40 + x) \times 30 = 1920$

$\Rightarrow \quad (40 + x) \times 15 = 1920$

$\Rightarrow \quad 40 + x = \frac{1920}{15} = 128$

$\therefore \quad x = 128 - 40 = 88$

$\therefore$ **The other parallel side of the trapezium = 88 m** **(Ans.)**

Example 15 :

The area of the trapezium is 210 cm^2 and its height is 14 cm. If one of the parallel sides is longer than the other by 6 cm, find the two parallel sides.

Solution :

Let the length of the smaller parallel side = x cm

$\therefore$ The length of bigger parallel side = (x + 6) cm

Given, height of the trapezium = 14 cm and its area = 210 cm^2

$\because \frac{1}{2} \times$ (sum of the parallel sides) $\times$ height = area

$\Rightarrow \quad \frac{1}{2} \times (x + 6 + x) \times 14 = 210$

$\Rightarrow \quad 2x + 6 = \frac{210 \times 2}{14} = 30$

$\therefore \quad 2x = 30 - 6 = 24$ and $x = \frac{24}{2} = 12$

$\therefore$ The two parallel sides are (x + 6) cm and x cm

= (12 + 6) cm and 12 cm = **18 cm and 12 cm** **(Ans.)**

Example 16 :

The perimeter of a trapezium is 52 cm and its non-parallel sides are 9 cm and 11 cm respectively. If its height is 8 cm, find the area of the trapezium.

Solution :

Given : perimeter of the trapezium = 52 cm and its height = 8 cm

$\Rightarrow$ Sum of the parallel sides + sum of non-parallel sides = 52 cm

$\Rightarrow$ Sum of the parallel sides + (9 cm + 11 cm) = 52 cm

$\Rightarrow$ Sum of the parallel sides = 52 cm – 20 cm = 32 cm

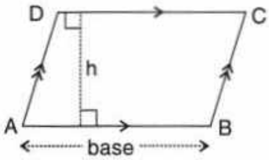
$\therefore$ **Area of the trapezium** $= \frac{1}{2} \times$ (sum of the parallel sides) $\times$ height

$= \frac{1}{2} \times 32 \times 8$ cm^2 = **128 cm^2** **(Ans.)**

20.6 PARALLELOGRAM

Area = Base × corresponding height

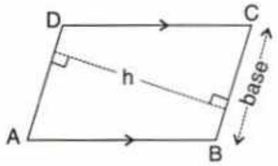
e.g. (i)



For parallelogram ABCD, if side AB is taken as base, the corresponding height, is the distance between parallel sides AB and DC.

∴ **Area = AB × h**

(ii)



And, if the side BC is taken as base, the corresponding height is the distance between parallel sides BC and AD.

∴ **Area = BC × h**

Example 17 :

A parallelogram has sides of 12 cm and 8 cm. If the distance between the 12 cm sides is 5 cm; find the distance between 8 cm sides.

Solution :

According to the question; if base = 12 cm; height = 5 cm

∴ Area = base × height = 12×5 cm^2 = 60 cm^2.

Now, base = 8 cm and area = 60 cm^2

∴ Area = base × height

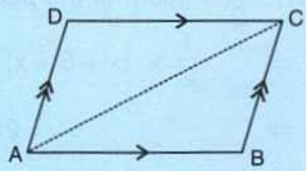
$\Rightarrow \quad 60 = 8 \times h \Rightarrow h = \frac{60}{8} = 7{\cdot}5$ cm **(Ans.)**

Each diagonal bisects a parallelogram;

i.e. Δ ABC = Δ ADC = $\frac{1}{2}$ (// gm ABCD)

Similarly, if diagonal BD is drawn :

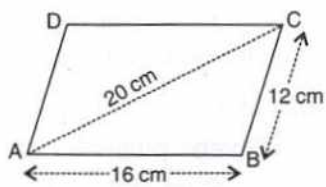
Δ ABD = Δ BCD = $\frac{1}{2}$ (// gm ABCD).



Example 18 :

In parallelogram ABCD; AB = 16 cm, BC = 12 cm and diagonal AC = 20 cm. Find the area of the parallelogram.

Solution :



For triangle ABC,

AB = 16 cm, AC = 20 cm and BC = 12 cm

$\therefore \quad s = \frac{a+b+c}{2} = \frac{16+20+12}{2} = 24$ cm

$\therefore \quad$ Area of Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24\ (24-16)(24-20)(24-12)}$

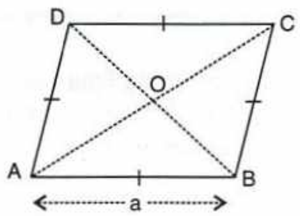
$= \sqrt{24 \times 8 \times 4 \times 12} = \sqrt{9216} = 96$ cm^2

∴ **Area of parallelogram ABCD** = 2×96 cm^2 = **192 cm^2** **(Ans.)**

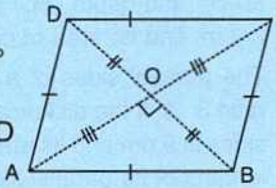
20.7 RHOMBUS

A rhombus is a parallelogram whose all the sides are equal.

The adjoining figure shows a rhombus ABCD in which AB = BC = CD = DA = a (say).



1. Perimeter of a rhombus = 4 × side = 4a
2. $$\text{Area of a rhombus} = \frac{1}{2} \times \text{product of its diagonals}$$
$$= \frac{1}{2} \times AC \times BD$$



1. Diagonals of a rhombus bisect each other at right angle.
   $\therefore$ $OA = OC = \frac{1}{2}AC$, $OB = OD = \frac{1}{2}BD$ and $\angle AOB = 90°$
2. $\Delta AOB = \Delta BOC = \Delta COD = \Delta DOA = \frac{1}{4} \times$ rhombus ABCD
3. Since, a rhombus is a parallelogram :
   (i) each diagonal bisects it
   i.e. $\Delta ABC = \Delta ADC = \frac{1}{2} \times$ rhombus ABCD
   and, $\Delta ABD = \Delta BCD = \frac{1}{2} \times$ rhombus ABCD
   (ii) area of rhombus = base × height

Example 19 :

The diagonals of a rhombus are 16 cm and 12 cm; find :

(i) its area (ii) length of its side (iii) its perimeter.

Solution :

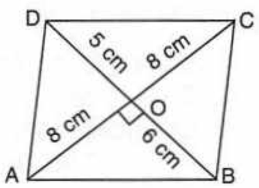
(i) **Area of rhombus** $= \frac{1}{2} \times$ product of its diagonals

$= \frac{1}{2} \times 16 \times 12\ cm^2 =$ **$96\ cm^2$** **(Ans.)**

(ii) Given diagonal AC = 16 cm; then $OA = OC = \frac{16}{2}$ cm = 8 cm

Given diagonal BD = 12 cm, then $OB = OD = \frac{12}{2}$ cm = 6 cm

Since, the diagonals of a rhombus bisect at 90°,



$\therefore$ Applying Pythagoras Theorem in Δ AOB; we get :

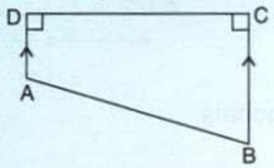
$$(AB^2) = OA^2 + OB^2$$
$$= 8^2 + 6^2 = 100$$
$$\therefore \quad AB = \sqrt{100} = 10\ cm$$

$\therefore$ **Length of its side = 10 cm** **(Ans.)**

(iii) **Perimeter of rhombus** = 4 × side = 4 × 10 = **40 cm** **(Ans.)**

EXERCISE 20(C)

1. The following figure shows the cross-section ABCD of a swimming pool which is a trapezium in shape.



If the width DC, of the swimming pool is 6·4 m, depth (AD) at the shallow end is 80 cm and depth (BC) at the deepest end is 2·4 m, find its area of cross-section.

2. The parallel sides of a trapezium are in the ratio 3 : 4. If the distance between the parallel sides is 9 dm and its area is 126 dm^2, find the lengths of its parallel sides.
3. The two parallel sides and the distance between them are in the ratio 3 : 4 : 2. If the area of the trapezium is 175 cm^2; find its height.
4. A parallelogram has sides of 15 cm and 12 cm. If the distance between the 15 cm sides is 6 cm, find the distance between 12 cm sides.
5. A parallelogram has sides of 20 cm and 30 cm. If the distance between its shorter sides is 15 cm; find the distance between the longer sides.
6. The adjacent sides of a parallelogram are 21 cm and 28 cm. If its one diagonal is 35 cm; find the area of the parallelogram.
7. The diagonals of a rhombus are 18 cm and 24 cm. Find ;

   (i) its area (ii) length of its sides

   (iii) its perimeter
8. The perimeter of a rhombus is 40 cm. If one diagnonal is 16 cm, find :

   (i) its other diagonal (ii) area
9. Each side of a rhombus is 18 cm. If the distance between two parallel sides is 12 cm, find its area.
10. The length of the diagonals of a rhombus is in the ratio 4 : 3. If its area is 384 cm^2, find its side.
11. A thin metal iron-sheet is a rhombus in shape, with each side 10 m. If one of its diaginals is 16 m, find the cost of painting its both sides at the rate of ₹ 6 per m^2.

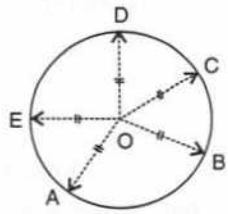
    Also, find the distance between the opposite sides of this rhombus.
12. The area of a trapezium is 279 sq. cm and the distance between its two parallel sides is 18 cm. If one of its parallel sides is longer than the other side by 5 cm, find the lengths of its parallel sides.
13. The area of a rhombus is equal to the area of a triangle. If base of triangle is 24 cm, its corresponding altitude is 16 cm and one of the diagonals of the rhombus is 19·2 cm, find its other diagonal.
14. Find the area of the trapezium ABCD in which AB//DC, AB = 18 cm, $\angle B = \angle C = 90°$, CD = 12 cm and AD = 10 cm.

20.8 CIRCLE

A ***circle*** is a closed curve, obtained on joining all points that are at the same distance from a fixed point and lie in the same plane.

Infact, the circle is obtained only when the points, discussed above are joined together by a free-hand closed curve.

The adjoining figure shows a fixed point O and points A, B, C, D, ..., etc., such that, points A, B, C, D, etc. are at equal distances from the fixed point O, *i.e.* OA = OB = OC = OD = ... The free-hand closed curve passing through the points A, B, C, D, ..., etc., is the ***circle***.



The ***fixed point*** is called the ***centre*** and the ***constant distance*** is called the ***radius*** of the circle. Thus, point O is the centre and radius = OA = OB = OC = OD and so on.

The radius of a circle is in general represented by *r*.

1. **Diameter :** A straight line, joining any two points on the circumference of the circle and passing through the centre, is called **diameter** of the circle. Thus, BOD is a diameter.

Diameter = 2 × radius *i.e.* $d = 2r$.

2. **Circumference :** The *perimeter* of the circle is called its *circumference.*

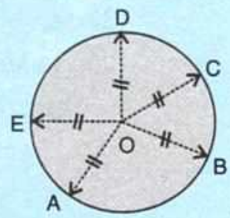
The Greek letter π (pronounced as Pie) represents the ratio between the circumference and diameter of a circle.

i.e. $\dfrac{\text{circumference of the circle}}{\text{its diameter (d)}} = \pi$

$\Rightarrow$ **circumference of the circle** $= \pi \times d = \mathbf{2\pi r}$ [As, d = 2r]

3. **Area of a circle** is the region bounded by the circumference of the circle.

Area of a circle $= \pi r^2$; where $\pi = 3\frac{1}{7} = \frac{22}{7}$ and r = radius of the circle.



Example 20 :

Find radius and area of a circle whose circumference is 132 cm.

Solution :

Given circumference = 132

$\Rightarrow$ $2\pi r = 132$

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 132 \Rightarrow r = \frac{132 \times 7}{2 \times 22} = 21$ cm **(Ans.)**

$\therefore$ **Area** $= \pi r^2 = \frac{22}{7} \times (21)^2 = 1386$ cm^2 **(Ans.)**

Example 21 :

Find circumference of the circle, whose area is 24·64 m^2.

Solution :

Given : $\pi r^2 = 24{\cdot}64 \Rightarrow \frac{22}{7} r^2 = 24{\cdot}64$

$\Rightarrow r^2 = 24{\cdot}64 \times \frac{7}{22} = 7{\cdot}84 \Rightarrow r = 2{\cdot}8$ m

$\therefore$ **Circumference** $= 2\pi r = 2 \times \frac{22}{7} \times 2{\cdot}8$ m $= 17{\cdot}6$ m **(Ans.)**

Example 22 :

The perimeter of a square, whose each side is 22 cm, is the same as circumference of a circle. Find the area of the circle.

Solution :

The perimeter of the square = 4 × its side

= 4 × 22 cm = 88 cm

$\Rightarrow$ The circumference of the circle = 88 cm

i.e., $2\pi r = 88$ cm [r = radius of the circle]

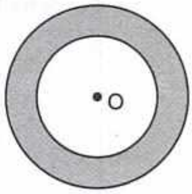
$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 88 \text{ cm} \quad i.e. \quad r = 14 \text{ cm}$$

$\therefore$ **Area of the circle** $= \pi r^2$

$$= \frac{22}{7} \times 14 \text{ cm} \times 14 \text{ cm} = \mathbf{616 \text{ cm}^2} \qquad \textbf{(Ans.)}$$

Example 23 :

The shaded portion in the adjoining figure shows a circular path enclosed by two concentric circles. If the inner circumference of the path is 176 m and the uniform width of the circular path is 3·5 m; find the area of the path.



Solution :

Let the radius of the inner circle be *r* m

$$\therefore \quad 2\pi r = 176 \Rightarrow 2 \times \frac{22}{7} \times r = 176 \text{ m}$$

$$\Rightarrow \quad r = 176 \times \frac{7}{2 \times 22} \text{ m} = 28 \text{ m}$$

Since, the width of the path = 3·5 m

the radius of the outer circle (R) = r + 3·5 m = 28 m + 3·5 m = 31·5 m

The area of the circular path = Area of the outer circle – Area of the inner circle

$$= \pi R^2 - \pi r^2$$

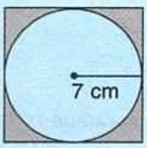
$$= \left(\frac{22}{7} \times 31 \cdot 5 \times 31 \cdot 5 - \frac{22}{7} \times 28 \times 28\right) \text{m}^2$$

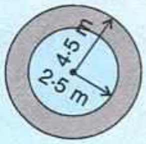
$$= (3118 \cdot 5 - 2464) \text{m}^2 = \mathbf{654 \cdot 5 \text{ m}^2} \qquad \textbf{(Ans.)}$$

> Whenever the value of π is not given, take : $\pi = \frac{22}{7}$.

EXERCISE 20(D)

1. Find the radius and area of a circle, whose circumference is :
   (i) 132 cm (ii) 22 m
2. Find the radius and circumference of a circle, whose area is :
   (i) 154 cm^2 (ii) 6·16 m^2
3. The circumference of a circular table is 88 m. Find its area.
4. The area of a circle is 1386 sq. cm, find its circumference.
5. Find the area of a flat circular ring formed by two concentric circles (circles with same centre) whose radii are 9 cm and 5 cm.
6. Find the area of the shaded portion in each of the following diagrams :

   (i) 

   (ii) 

7. The radii of the inner and outer circumferences of a circular-running-track are 63 m and 70 m respectively. Find :
   (i) the area of the track
   (ii) the difference between the lengths of the two circumferences of the track
8. A circular field of radius 105 m has a circular path of uniform width of 5 m along and inside its boundary. Find the area of the path.
9. There is a path of uniform width 7 m round and outside a circular garden of diameter 210 m. Find the area of the path.
10. A wire, when bent in the form of a square, encloses an area of 484 cm^2. Find :
    (i) one side of the square
    (ii) length of the wire
    (iii) the largest area enclosed, if the same wire is bent to form a circle.
11. A wire, when bent in the form of a square, encloses an area of 196 cm^2. If the same wire

is bent to form a circle, find the area of the circle.

12. The radius of a circular wheel is 42 cm. Find the distance travelled by it in :

 (i) 1 revolution (ii) 50 revolutions

 (iii) 200 revolutions

> The distance travelled by a wheel in one revolution is equal to its circumference.

13. The diameter of the wheel of a car is 0·70 m. Find the distance covered by it in 500 revolutions.

 If the wheel takes 5 minutes to make 500 revolutions; find its speed in :

 (i) m/s (ii) km/hr

14. A bicycle wheel, diameter 56 cm, is making 45 revolutions in every 10 seconds. Calculate the speed, in kilometre per hour, of the bicycle.

15. A roller has a diameter of 1·4 m. Find :

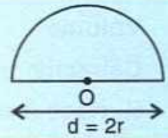
 (i) its circumference

 (ii) the number of revolutions it makes while travelling 61·6 m.

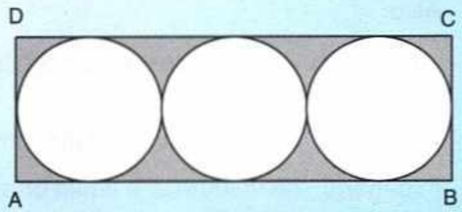
16. Find the area of the circle, length of whose circumference is equal to the sum of the lengths of the circumferences of circles with radii 15 cm and 13 cm.

17. A piece of wire of length 108 cm is bent to form a semicircular arc bounded by its diameter. Find its radius and enclosed area.

> $\frac{2\pi r}{2} + d = 108$
>
> Enclosed area $= \frac{\pi r^2}{2}$



18. In the following figure, a rectangle ABCD encloses three circles. If BC = 14 cm, find the area of the shaded portion. (Take $\pi = 3\frac{1}{7}$)



SURFACE AREA, VOLUME AND CAPACITY 21

(Cuboid, Cube and Cylinder)

21.1 INTRODUCTION

Volume	The space occupied by a body (solid) is called its volume.
Capacity	The capacity of a container is its internal volume.
Surface area	The sum of areas of all the faces of a body is called its surface area.

Units of length	Unit of volume	Unit of surface-area
m (metre) cm mm	m^3 (cubic metre) cm^3 mm^3	m^2 (square metre) cm^2 mm^2

Also,

$1\ m^3 = 100 \times 100 \times 100\ cm^3 = 1000000\ cm^3$ and $1\ cm^3 = \frac{1}{100 \times 100 \times 100}\ m^3$

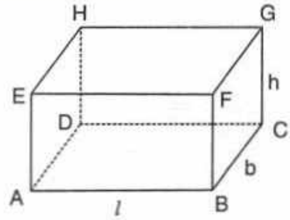
$1\ cm^3 = 10 \times 10 \times 10\ mm^3 = 1000\ mm^3$ and $1\ mm^3 = \frac{1}{1000}\ cm^3$

In general, the volume of a liquid or a gas is measured in litres, such that

$1\ m^3 = 1000$ litre and 1 litre $= 1000\ cm^3$ (c.c. or millilitre)

21.2 CUBOID (a rectangular solid)

A **cuboid** is a solid bounded by six rectangular faces.



1. **Volume of a cuboid**

$= $ length $\times$ breadth $\times$ height

$= l \times b \times h$

2. **Total surface area of a cuboid = Area of six rectangular faces**

Since, Area of ABCD + Area of EFGH = $2(l \times b)$ [Opposite faces are equal]

Area of BCGF + Area of ADHE = $2(b \times h)$ [Opposite faces are equal]

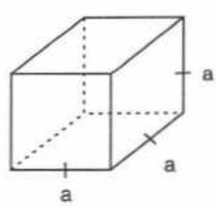
and Area of ABFE + Area of DCGH = $2(h \times l)$ [Opposite faces are equal]

$\therefore$ **Total surface area of cuboid = $2(l \times b + b \times h + h \times l)$**

21.3 CUBE

A **cube** is a rectangular solid whose *each face* is a *square*.

In other words, a cube is a cuboid whose, length = breadth = height = a (say)



1. Since volume of a cuboid $= l \times b \times h$

$\therefore$ **Volume of a cube** $= a \times a \times a$

$= \mathbf{a^3} = (\text{its edge})^3$

2. **Total surface area of a cube** $= 2(a \times a + a \times a + a \times a)$

$= \mathbf{6a^2} = 6\ (\text{edge})^2$

Example 1 :

The length, breadth and height of a cuboid are in the ratio 6 : 5 : 4. If its volume is 15,000 cm^3; find : (i) its dimensions (ii) its surface area.

Solution :

Dimension means : Its length, breadth and height.

(i) Given : Length : breadth : height = 6 : 5 : 4

$\Rightarrow$ If length = 6x cm, breadth = 5x cm and height = 4x cm

$\therefore$ Length $\times$ breadth $\times$ height = volume

$\Rightarrow$ $6x \times 5x \times 4x = 15{,}000$

$\Rightarrow$ $x^3 = \dfrac{15{,}000}{6 \times 5 \times 4} = 125 = 5 \times 5 \times 5 = 5^3$

$\therefore$ $x = 5$

i.e. **length** = 6x cm = 6 × 5 cm = **30 cm**

breadth = 5x cm = 5 × 5 cm = **25 cm**

and, **height** = 4x cm = 4 × 5 cm = **20 cm** **(Ans.)**

(ii) **Surface area of the cuboid** = $2(l \times b + b \times h + h \times l)$

$= 2(30 \times 25 + 25 \times 20 + 20 \times 30)\ cm^2$

$= 2(750 + 500 + 600)\ cm^2 =$ **3700 cm²** **(Ans.)**

Example 2 :

The total surface area of a cube is 294 cm², find its volume.

Solution :

Since the total surface area of a cube = $6 \times (side)^2$

$\Rightarrow$ $6 \times (side)^2 = 294$

$\Rightarrow$ side = 7 cm

$\therefore$ **volume** = $(side)^3 = (7\ cm)^3 =$ **343 cm³** **(Ans.)**

Example 3 :

A rectangular solid of metal has dimensions 50 cm, 64 cm and 72 cm. It is melted and recasted into identical cubes each with edge 4 cm, find the number of cubes formed.

Solution :

$\therefore$ Volume of rectangular solid melted = length × breadth × height

$= 50 \times 64 \times 72\ cm^3$

And, volume of each cube formed = $(edge)^3$

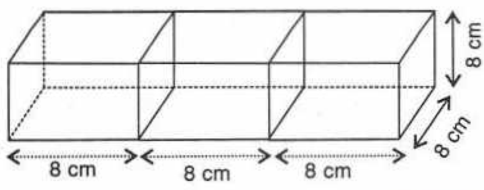
$= (4)^3\ cm^3 = 4 \times 4 \times 4\ cm^3$

$\therefore$ **Number of cubes formed** = $\dfrac{\text{Volume of solid melted}}{\text{Volume of each cube}}$

$= \dfrac{50 \times 64 \times 72}{4 \times 4 \times 4} =$ **3600** **(Ans.)**

Example 4 :

Three cubes, each of edge 8 cm, are joined as shown alongside. Find the total surface area and the volume of the cuboid.



Solution :

Since, length (l) of the resulting cuboid $= 3 \times 8$ cm $= 24$ cm,

its breadth (b) $= 8$ cm and its height (h) $= 8$ cm

Total surface area $= 2(l \times b + b \times h + h \times l)$

$= 2(24 \times 8 + 8 \times 8 + 8 \times 24)$ cm^2 $=$ **896 cm^2** **(Ans.)**

Volume $= l \times b \times h$

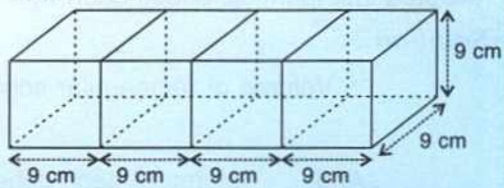
$= 24 \times 8 \times 8$ cm^3 $=$ **1536 cm^3** **(Ans.)**

EXERCISE 21(A)

1. Find the volume and the total surface area of a cuboid, whose :
   (i) length = 15 cm, breadth = 10 cm and height = 8 cm
   (ii) $l = 3{\cdot}5$ m, b = 2·6 m and h = 90 cm.
2. (i) The volume of a cuboid is 3456 cm^3. If its length = 24 cm and breadth = 18 cm, find its height.
   (ii) The volume of a cuboid is 7·68 m^3. If its length = 3·2 m and height = 1·0 m; find its breadth.
   (iii) The breadth and height of a rectangular solid are 1·20 m and 80 cm respectively. If the volume of the cuboid is 1·92 m^3, find its length.
3. The length, the breadth and the height of a cuboid are in the ratio 5 : 3 : 2. If its volume is 240 cm^3; find its dimensions. Also, find the total surface area of the cuboid.
4. The length, breadth and height of a cuboid are in the ratio 6 : 5 : 3. If its total surface area is 504 cm^2, find its dimensions. Also, find the volume of the cuboid.
5. Find the volume and total surface area of a cube whose each edge is :
   (i) 8 cm (ii) 2 m 40 cm.
6. Find the length of each edge of a cube, if its volume is :
   (i) 216 cm^3 (ii) 1·728 m^3
7. The total surface area of a cube is 216 cm^2. Find its volume.
8. A solid cuboid of metal has dimensions 24 cm, 18 cm and 4 cm. Find its volume.
9. A wall 9 m long, 6 m high and 20 cm thick, is to be constructed, using bricks of dimensions 30 cm, 15 cm and 10 cm. How many bricks will be required ?

$$\text{No. of bricks} = \frac{\text{Volume of wall}}{\text{Volume of one brick}}$$

10. A solid cube of edge 14 cm is melted down and recast into smaller and equal cubes each of edge 2 cm. Find the number of smaller cubes obtained.
11. A closed box is a cuboid in shape with length = 40 cm, breadth = 30 cm and height = 50 cm. It is made of thin metal sheet. Find the cost of metal sheets required to make 20 such boxes, if 1 m^2 of metal sheet costs ₹ 45.
12. Four cubes, each of edge 9 cm, are joined as shown below :



Write the dimensions of the cuboid obtained. Also, find total surface area and volume.

21.4 APPLICATIONS

1. For a room :

Every room has four walls; two walls along its length and two walls along its width.

∴ (i) Area of each wall along the length = $l \times h$

and, (ii) Area of each wall along the width = $b \times h$

∴ **Area of 4 walls of the room** = $2 \times l \times h + 2 \times b \times h$

$= \mathbf{2(l + b) \times h}$

> This area includes the area of doors and windows.

Also, (iii) **The area of roof = the area of floor =** $\mathbf{l \times b}$

Example 5 :

The internal length, breadth and height of a rectangular room are 6 m, 5·2 m and 4·5 m respectively. It has two doors each of 1·2 m by 2 m and three windows each of 1 m by 80 cm. Find the total internal area of the room to be whitewashed.

Also, find the cost of whitewashing the room (excluding the doors and windows) at the rate of ₹ 6 per m^2.

Solution :

For the room, its $l =$ 6 m, $b =$ 5·2 m and $h =$ 4·5 m

∴ Area of its four walls = $2(l + b)h$

$= 2(6 + 5{\cdot}2) \times 4{\cdot}5\ m^2 = 100{\cdot}8\ m^2$

Area of its roof = $l \times b = 6 \times 5{\cdot}2\ m^2 = 31{\cdot}2\ m^2$

Since, area of one door = $1{\cdot}2 \times 2\ m^2 = 2{\cdot}4\ m^2$

∴ area of two doors = $2 \times 2{\cdot}4\ m^2 = 4{\cdot}8\ m^2$

Also, area of each window = $1 \times 0{\cdot}80\ m^2 = 0{\cdot}80\ m^2$ [80 cm = 0·80 m]

∴ Area of three windows = $3 \times 0{\cdot}80\ m^2 = 2{\cdot}40\ m^2$

∴ **Total internal area of the room to be whitewashed**

= (Area of four walls + Area of roof) – (Area of two doors + Area of three windows)

$= (100{\cdot}8 + 31{\cdot}2) - (4{\cdot}8 + 2{\cdot}4)\ m^2$

$= \mathbf{124{\cdot}8\ m^2}$ **(Ans.)**

Cost of whitewashing = ₹ 6 × 124·8 = **₹ 748·80** **(Ans.)**

2. For a box :

(i) Space occupied by it = its external volume

(ii) Its capacity = its internal volume

(iii) Volume of material in it = its external volume – its internal volume.

3. For a closed box :

If its external length, breadth and height are l, b and h respectively, and its walls are x unit thick throughout, then :

(i) Its **internal length** = External length – twice the thickness of walls

$= l - 2x$

(ii) Its **internal breadth** = b – 2x and

(iii) Its **internal height** = h – 2x

Conversely, if the internal dimensions of a box are l, b and h respectively and its sides (walls) are x unit thick everywhere, then its external dimensions are $l + 2x$, $b + 2x$ and $h + 2x$ respectively.

Example 6 :

The external length, breadth and height of a closed wooden box are 30 cm, 18 cm and 20 cm respectively. If the walls of the box are 1·5 cm thick, find :

(i) capacity of the box;

(ii) volume of the wood used in making the box;

and (iii) weight of the box; if 1 cm^3 of the wood weighs 0·80 g.

Solution :

Given, external length of the box = 30 cm

external breadth of the box = 18 cm

and, external height of the box = 20 cm

∴ External volume of the box = $30 \times 18 \times 20$ cm^3

= 10,800 cm^3

Since, the walls of the box are 1·5 cm thick throughout;

∴ Internal length of the box = $(30 - 2 \times 1{\cdot}5)$ cm = 27 cm

internal breadth of the box = $(18 - 2 \times 1{\cdot}5)$ cm = 15 cm

and, internal height of the box = $(20 - 2 \times 1{\cdot}5)$ cm = 17 cm

∴ Internal volume of the box = $27 \times 15 \times 17$ cm^3;

= 6,885 cm^3

(i) **Capacity of the box** = its internal volume

= **6,885 cm^3** **(Ans.)**

(ii) **Volume of the wood used** = External volume – Internal volume

= 10,800 cm^3 – 6,885 cm^3

= **3,915 cm^3** **(Ans.)**

(iii) Since, 1 cm^3 of wood weighs 0·80 g

∴ **Weight of the box** = $3{,}915 \times 0{\cdot}80$ g

= 3132 g = **3·132 kg** **(Ans.)**

EXERCISE 21(B)

1. How many persons can be accommodated in a big-hall of dimensions 40 m, 25 m and 15 m, assuming that each person requires 5 m^3 of air ?

   No. of persons $= \dfrac{\text{Volume of hall}}{\text{Volume of air required for each person}}$

2. The dimensions of a class-room are, length = 15 m, breadth = 12 m and height = 7·5 m. Find, how many children can be accommodated in this class-room, assuming 3·6 m^3 of air is needed for each child.

3. The length, breadth and height of a room are 6 m, 5·4 m and 4 m respectively. Find the area of : (i) its four-walls (ii) its roof

4. A room 5 m long, 4·5 m wide and 3·6 m high has one door 1·5 m by 2·4 m and two windows, each 1 m by 0·75 m. Find :

(i) the area of its walls, excluding doors and windows.
(ii) the cost of distempering its walls at the rate of ₹ 4·50 per m^2.
(iii) the cost of painting its roof at the rate of ₹ 9 per m^2.

5. The dining-hall of a hotel is 75 m long, 60 m broad and 16 m high. It has five-doors 4 m by 3 m each and four windows 3 m by 1·6 m each. Find the cost of :
   (i) papering its walls at the rate of ₹ 12 per m^2;
   (ii) carpeting its floor at the rate of ₹ 25 per m^2.
6. Find the volume of wood required to make a closed box of external dimensions 80 cm, 75 cm and 60 cm, the thickness of walls of the box being 2 cm throughout.
7. A closed box measures 66 cm, 36 cm and 21 cm from outside. If its walls are made of metal-sheet, 0·5 cm thick; find :
   (i) the capacity of the box;
   (ii) volume of metal-sheet and
   (iii) weight of the box, if 1 cm^3 of metal weighs 3·6 g.
8. The internal length, breadth and height of a closed box are 1 m, 80 cm and 25 cm respectively. If its sides are made of 2·5 cm thick wood; find :
   (i) the capacity of the box
   (ii) the volume of wood used to make the box.
9. Find the area of metal-sheet required to make an open tank of length = 10 m, breadth = 7·5 m and depth = 3·8 m.

> The area of metal sheet = Area of 4 walls of the tank + area of its base = $2(l + b)\,h + l \times b$

10. A tank 30 m long, 24 m wide and 4·5 m deep is to be made. It is open from the top. Find the cost of iron-sheet required, at the rate of ₹ 65 per m^2, to make the tank.

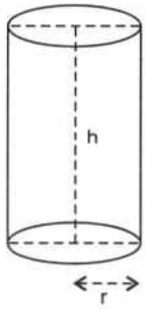
EXERCISE 21(C)

1. The edges of three solid cubes are 6 cm, 8 cm and 10 cm. These cubes are melted and recasted into a single cube. Find the edge of the resulting cube.
2. Three solid cubes of edges 6 cm, 10 cm and x cm are melted to form a single cube of edge 12 cm, find the value of x.
3. The length of the diagonal of a cube is $8\sqrt{3}$ cm. Find its :
   (i) edge (ii) total surface area
   (iii) volume

> Diagonal of a cube = edge $\times\sqrt{3}$

4. A cube of edge 6 cm and a cuboid with dimensions 4 cm × x cm × 15 cm are equal in volume. Find :
   (i) the value of x.
   (ii) total surface area of the cuboid.
   (iii) total surface area of the cube.
   (iv) which of these two has greater surface and by how much ?
5. The capacity of a rectangular tank is 5·2 m^3 and the area of its base is 2.6×10^4 cm^2; find its height (depth).
6. The height of a rectangular solid is 5 times its width and its length is 8 times its height. If the volume of the wall is 102·4 cm^3, find its length.
7. The ratio between the lengths of the edges of two cubes are in the ratio 3 : 2. Find the ratio between their :
   (i) total surface area
   (ii) volume.
8. The length, breadth and height of a cuboid (rectangular solid) are 4 : 3 : 2.
   (i) If its surface area is 2548 cm^2, find its volume.
   (ii) If its volume is 3000 m^3, find its surface area.

21.5 CYLINDER



A solid (a pipe shaped figure), which has uniform **circular cross-section**, is called a **cylinder**.

Let the radius of the circular cross-section = r cm and the height of the cylinder = h cm.

∴ 1. Area of cross-section = πr^2

2. Perimeter of cross-section = $2\pi r$

3. Curved surface area = Perimeter of cross-section × height

$= 2\pi r \times h = 2\pi rh$

4. Total surface area = Curved surface area + 2(area of cross-section)

$= 2\pi rh + 2(\pi r^2)$

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r)$

5. Volume = Area of cross-section × height (or length)

$= \pi r^2 \times h = \pi r^2 h$

> When the value of π is not given, take $\pi = \frac{22}{7}$

Example 7 :

The curved surface area of a cylinder is 17,600 cm^2 and the circumference of its base is 220 cm. Find :

(i) the height of the cylinder.

(ii) the volume of the cylinder.

Solution :

(i) Given : $2\pi rh = 17600\ cm^2$ and $2\pi r = 220$ cm

$\therefore \quad \frac{2\pi rh}{2\pi r} = \frac{17600}{220} \Rightarrow$ **h = 80 cm** **(Ans.)**

(ii) Since, $2\pi r = 220$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 220$

$\Rightarrow \quad r = 220 \times \frac{7}{2 \times 22} = 35$ cm

and **volume** $= \pi r^2 h$

$= \frac{22}{7} \times 35 \times 35 \times 80\ cm^3$

$=$ **3,08,000 cm^3** **(Ans.)**

EXERCISE 21(D)

1. The height of a circular cylinder is 20 cm and the diameter of its base is 14 cm. Find :
   (i) its volume
   (ii) its total surface area
2. Find the curved surface area and the toal surface area of a right circular cylinder whose height is 15 cm and the diameter of the cross-section is 14 cm.
3. Find the height of the cylinder whose radius is 7 cm and the total surface are is 1100 cm^2.
4. The curved surface area of a cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.
5. The ratio between the curved surface area and the total surface area of a cylinder is 1 : 2. Find the ratio between the height and the radius of the cylinder

6. Find the capacity of a cylindrical container with internal diameter 28 cm and height 20 cm.
7. The total surface area of a cylinder is 6512 cm^2 and the circumference of its base is 88 cm. Find :
   (i) its radius (ii) its volume
8. The sum of the radius and the height of a cylinder is 37 cm and the total surface area of the cylinder is 1628 cm^2. Find the height and the volume of the cylinder.
9. A cylindrical pillar has radius 21 cm and height 4 m. Find :
   (i) the curved surface area of the pillar
   (ii) cost of polishing 36 such cylindrical pillars at the rate of ₹ 12 per m^2.
10. If radii of two cylinders are in the ratio 4 : 3 and their heights are in the ratio 5 : 6, find the ratio of their curved surfaces.

21.6 MIXED PRACTICE

Example 8 :

Find the length of the longest rod that can be placed in a small box with length 20 cm, breadth 20 cm and height = 10 cm.

Solution :

Since, l = 20 cm, b = 20 cm and h = 10 cm

Required length of the rod = Length of the diagonal of the box

$$= \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{(20)^2 + (20)^2 + (10)^2} \text{ cm}$$

$$= \sqrt{400 + 400 + 100} \text{ cm} = \sqrt{900} \text{ cm} = 30 \text{ cm} \quad \textbf{(Ans.)}$$

Example 9 :

The total surface area of a cylinder of diameter 10 cm is 330 cm^2. Find the height of the cylinder.

Solution :

Given, diameter of the cylinder = 10 cm

$$\Rightarrow \quad \text{Its radius} = \frac{10}{2} \text{ cm} = 5 \text{ cm}$$

$$\text{Total surface area of the cylinder} = 2\pi r\,(h + r)$$

$$\Rightarrow \quad 2\pi r\,(h + r) = 330$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times 5 \times (h + 5) = 330$$

$$\Rightarrow \quad h + 5 = 330 \times \frac{7}{2 \times 22 \times 5} = 10{\cdot}5$$

$$\Rightarrow \quad h = 10{\cdot}5 \text{ cm} - 5 \text{ cm} = 5{\cdot}5 \text{ cm}$$

∴ **The height of the cylinder = 5·5 cm** **(Ans.)**

Example 10 :

How many 3 cm cubes can be cut from a cuboid measuring 18 cm × 12 cm × 9 cm ?

Solution :

$$\text{Number of cubes cut} = \frac{\text{Volume of the cuboid}}{\text{Volume of each cube}}$$

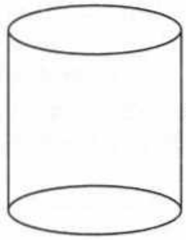
$$= \frac{18 \times 12 \times 9}{3 \times 3 \times 3} = 72 \quad \textbf{(Ans.)}$$

Example 11 :

A rectangular sheet of paper 44 cm × 18 cm is rolled along its length and a cylinder is formed. Find the radius and the volume of the cylinder formed.

Solution :

⇒



Clearly, length of the rectanguler sheet = Circumference of the base of the cylinder formed

$\Rightarrow \quad 44 \text{ cm} = 2\pi r$

$\Rightarrow \quad 44 = 2 \times \frac{22}{7} \times r$

$\Rightarrow \quad r = 44 \times \frac{7}{44} \text{ cm} = 7 \text{ cm}$ **(Ans.)**

Also, height of the cylinder formed = Breadth of the sheet

$\Rightarrow \quad h = 18 \text{ cm}$

Volume of the cylinder formed $= \pi r^2 h$

$= \frac{22}{7} \times 7^2 \times 18 \text{ cm}^3 = 2772 \text{ cm}^3$ **(Ans.)**

EXERCISE 21(E)

1. A cuboid is 8 m long, 12 m broad and 3·5 m high. Find its
   (i) total surface area
   (ii) lateral surface area
2. How many bricks will be required for constructing a wall which is 16 m long, 3 m high and 22·5 cm thick, if each brick measures 25 cm × 11·25 cm × 6 cm ?
3. The length, breadth and height of cuboid are in the ratio 6 : 5 : 3. If its total surface area is 504 cm^2, find its volume.
4. The external dimensions of an open wooden box are 65 cm, 34 cm and 25 cm. If the box is made up of wood 2 cm thick, find the capacity of the box and the volume of wood used to make it.
5. The curved surface area and the volume of a toy, cylindrical in shape, are 132 cm^2 and 462 cm^3 respectively. Find, its diameter and its length.
6. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of ₹ 10 per m^2 is ₹ 15,000, find the height of the hall.
7. The length of a hall is double its breadth. Its height is 3 m. The area of its four walls (including doors and windows) is 108 m^2, find its volume.
8. A solid cube of side 12 cm is cut into 8 identical cubes. What will be the side of the new cube ? Also, find the ratio between the surface area of the original cube and the total surface area of all the small cubes formed.
9. The diameter of a garden roller is 1·4 m and it is 2 m long. Find the maximum area covered by it in 50 revolutions ?
10. In a building, there are 24 cylindrical pillars. For each pillar, radius is 28 cm and height is 4 m. Find the total cost of painting the curved surface area of the pillars at the rate of ₹ 8 per m^2.

Theme – 6
DATA HANDLING
(Statistics)

DATA HANDLING 22

22.1 INTRODUCTION

The word *statistics* is used in two different senses :

1. Singular   2. Plural

1. In the singular sense, it implies the whole subject as a branch of knowledge to be used in collecting, analysing, presenting and interpreting some numerical informations (data).
2. In the plural sense, it implies the collection of numerical data in a systematic manner with some definite object in view.

   e.g. Statistics of :

   (i) population   (ii) taxes   (iii) number of failures in your school, etc.

22.2 DATA

A set of numerical facts, collected with a definite object in view, is called ***data***.

e.g. consider the heights (in centimetre) of 9 children in your class :

Naresh	153	Ashok	150	Ritu	154
Manisha	147	Peter	152	Monu	153

Here, the set of numbers (numerical facts representing the heights, in cm, of different children), 153, 147, 150, 152, 154 and 153 is called a ***set of data***.

> The data can be obtained by individuals (*e.g.* one may go from person to person to know his income, age, savings, etc.) and by government sources (*e.g.* the birth rate in a particular period, the rise in prices in a particular period, etc.)

Whatever be the method of collecting the data, once the data is collected, it must be arranged in a systematic form to get a fair idea of the essential points.

The arrangment of data in a systematic form, generally in a table form, is called **tabulation**. *e.g.* consider the following tabulation :

Class	First division	Second division	Detained	Total
IX	42	37	10	89
X	30	25	nil	55
XI	22	27	8	57

This table represents the results of a particular examination in a certain school.

22.3 FREQUENCY

Consider the set of data : 5, 7, 3, 8, 7, 5, 5, 3, 5, 8, 7.

In this set of data, the number 5 occurs four times and we say, the frequency of number 5 is 4.

Similarly, the frequency of number 3 is 2, as it occurs two times in the given set of data, the frequency of number 7 is 3 (why?) and the frequency of number 8 is 2.

Thus *frequency is a number which tells how many times does a particular data* (number), *appears in the given set of data.*

22.4 RAW DATA, ARRAYED DATA AND FREQUENCY DISTRIBUTION

Let 30 students of a class score the following marks (out of 10) in a class test :

9, 8, 6, 10, 5, 6, 8, 7, 10, 5, 6, 4, 5, 7, 8, 10, 8, 9, 6, 5, 4, 8, 8, 9, 10, 6, 6, 6, 5, 4.

The data recorded in the original form, as above, is called ***raw data.***

If the given data is arranged in **ascending** or **descending** order of their magnitudes, it is called **arrayed data** or simply, an **array.**

Thus 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9, 9, 10, 10, 10, 10 is an array in which, the given data is arranged in ascending order. Similarly, we can arrange it in descending order also.

The above arrayed data can also be represented in the form of a table as given below :

Marks	4	5	6	7	8	9	10
No. of students (Frequency)	3	5	7	2	6	3	4

Such a tabular arrangement of data, showing the frequency of each number (observation), is called a **frequency distribution** and the table so formed is called a **frequency distribution table** or simply, a frequency table.

22.5 CONSTRUCTING A FREQUENCY TABLE

Example 1 :

The minimum temperature (in °C) during the day at Delhi has been recorded for 30 days, as given below :

11·8	11·6	11·6	11·4	11·3	11·4
11·7	11·7	11·3	11·4	11·7	11·7
11·6	11·3	11·7	11·5	11·8	11·3
11·3	11·4	11·5	11·3	11·5	11·5
11·8	11·5	11·4	11·3	11·6	11·5

Construct a frequency table for the above data.

Solution :

Temperature (in °C)	Tally Marks	Frequency
11·3	𝍸 II	7
11·4	𝍸	5
11·5	𝍸 I	6
11·6	IIII	4
11·7	𝍸	5
11·8	III	3

Steps :

1. In the first column, write the temperatures in ascending (or descending) order.
2. The temperatures of different days are recorded in the second column (headed by Tally Marks) by making a short vertical line called a stroke. The marking of strokes is done as follows :
   (i) The first temperature in the given data is 11·8, so make a stroke in the Tally marks column against temperature 11·8.
   (ii) The next temperature is 11·6, so make a stroke against temperature 11·6 and so on.

(iii) When four strokes are made against any particular temperature, don't make the fifth stroke in the same way but make a stroke across the first four (like 卌). This gives a bundle of five strokes. The next stroke starts a new bundle.

(iv) When the marking of the strokes is completed, count the strokes against each and write the numbers in the column headed by ***Frequency***.

The table obtained in Example 1 (above) represents ungrouped frequency distribution.

22.6 GROUPED FREQUENCY DISTRIBUTION

Example 2 :

Given below are the marks obtained by 40 students in an examination :

40	56	24	41	51	22	59	12	26	46
51	14	23	59	58	29	41	35	36	40
42	43	36	33	38	52	23	24	13	55
18	54	20	57	30	24	43	50	26	15

Taking class intervals 10-20, 20-30, ..., 50-60; construct a frequency table for the above distribution.

Solution :

The frequency table for the given distribution is :

Marks	Tally Marks	Frequency
10-20	卌	5
20-30	卌 卌	10
30-40	卌 I	6
40-50	卌 III	8
50-60	卌 卌 I	11

(Ans.)

In this frequency distribution, the mark 20 is included in the class interval 20-30 and not in 10-20. Similarly, mark 30 is included in the class interval 30-40 and not in 20-30, and so on.

22.7 CLASS-INTERVALS AND CLASS-LIMITS

In example 2, given above, 10-20 is called a *class-interval* which is bounded by two numbers 10 and 20. These numbers (10 and 20) are called class-limits, the smaller number 10 is called the *lower class limit* and the larger number is called the *upper class limit*. Similarly, 20-30 is the second class-interval in the same example 2, 20 being the *lower class limit* and 30, the *upper class limit* and so on.

22.8 CLASS MARK

The *class-mark* of a class-interval is the value midway between its lower class limit and upper class limit.

Thus, class-mark of a class-interval $= \dfrac{\text{its lower class limit} + \text{upper class limit}}{2}$

e.g. class mark of 10-20 $= \dfrac{10+20}{2} = 15;$

class mark of 55-60 $= \dfrac{55+60}{2} = 57{\cdot}5$ and so on.

EXERCISE 22(A)

1. Arrange the following data as an array (in ascending order) :
   (i) 7, 5, 15, 12, 10, 11, 16
   (ii) 6·3, 5·9, 9·8, 12·3, 5·6, 4·7
2. Arrange the following data as an array (descending order) :
   (i) 2, 0, 3, 4, 1, 2, 3, 5
   (ii) 9·1, 3·7, 5·6, 8·3, 11·5, 10·6
3. Construct a frequency table for the following data :
   (i) 6, 7, 5, 6, 8, 9, 5, 5, 6, 7, 8, 9, 8, 10, 10, 9, 8, 10, 5, 7, 6, 8
   (ii) 3, 2, 1, 5, 4, 3, 2, 5, 5, 4, 2, 2, 2, 1, 4, 1, 5, 4
4. Following are the marks obtained by 30 students in an examination :

15	20	8	9	10
16	17	20	24	30
44	47	38	36	40
27	25	28	30	19
7	11	21	31	41
37	47	23	20	17

   Taking class intervals 0-10, 10-20, ..., 40-50; construct a frequency table.
5. Construct a frequency distribution table for the following data, taking class-intervals 4-6, 6-8, ..., 14-16.

11·5	6·3	7·8	9·2	10·5
4·5	6	8·3	12·5	15·8
7·4	5·3	8·4	15·2	8·9
9·8	8·25	6·5	5·8	10·5
4·6	6·4	8·9	10·8	12·7
14·2	15·3	11·7	9·9	8·8
6·6	4·3	4·7	9·4	10·1
15·5	14·4	12·2	7·7	5·5

6. Fill in the blanks :
   (i) Lower class limit of 15 - 18 is ....
   (ii) Upper class limit of 24 - 30 is ....
   (iii) Upper limit of 5 - 12·5 is ....
   (iv) If the upper and the lower limits of a class interval are 16 and 10, the class-interval is ....
   (v) If the lower and the upper limits of a class interval are 7·5 and 12·5, the class-interval is ....

22.9 GRAPHICAL REPRESENTATION

As compared to written statement, the graphical representation of statistical data has a more lasting effect on the mind. Of course, a graphical representation should be properly titled and labelled so as to convey maximum information on what it is about.

Out of various types of graphical representations, the followings are discussed in this chapter :

1. Bar graph or Bar chart
2. Pie graph or Pie chart
3. Histogram

1. Bar graph (Bar chart)

A **bar graph** is the simplest and the most widely used graphical representation of the numerical data with heights of rectangular bars of equal width.

In a bar graph
1. All the bars are of the same width. Infact width of the bars does not represent any thing.
2. The heights of different bars are directly proportional to the number (data) they represent.
3. Same space is left between consecutive bars.

Example 3 :

The approximate speeds of some objects are given below. Draw a bar graph to represent them.

Name of objects	Bicycle	Scooter	Car	Bus	Train
Speed (in km/hr)	10	40	60	50	80

Solution :

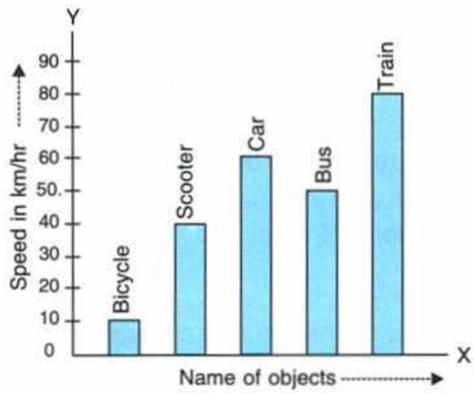
Steps :

1. On a graph paper, draw two mutually perpendicular lines OX and OY which intersect each other at point O.

   The line OX is taken horizontal and is called the x-axis, whereas the line OY is taken vertical and is called the y-axis.
2. On the x-axis, starting from O, mark points at equal distances. At these points write the names of items (objects) given.

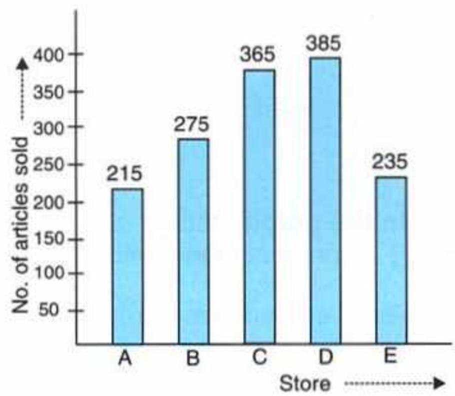
   In this example, the names of the items are Bicycle, Scooter, Car, Bus and Train.
3. Along y-axis, mark the heights of the bars (rectangles) in proportion of the given data (speed).
4. Remember :
   (i) The heights of different bars (rectangles) are parallel to y-axis and the mid-points of their bases are on the points marked on the x-axis (step 2).
   (ii) The bars may be taken of any suitable width, but widths of all the bars must be same.
   (iii) The space (distance) between consecutive bars may be of any suitable value, but the spaces between all the consecutive bars must be the same.

The required bar graph is as given alongside :



Example 4 :

The given bar graph shows the number of identical articles sold by 5 different stores A, B, C, D and E during a particular month.



Study the bar graph and answer the following questions :

(i) Which store sold the largest number of articles ?

(ii) By what per cent is the sale of store D more than that of store B ?

(iii) Find the ratio of the total sale of the two highest selling stores to the total sale of the two least selling stores.

(iv) By how much must store E increase its sale to catch up with its nearest rival store ?

Solution :

The following table shows the monthly sale of identical articles as given in the above graph.

Store	A	B	C	D	E
No. of articles sold	215	275	365	385	235

(i) Clearly, **store D sold the largest number of articles.** **(Ans.)**

(ii) $\because$ Sale of store D = 385

and, sale of store B = 275

$\therefore$ Difference of two sales = 385 − 275 = 110

And, **required percentage** = $\frac{110}{275} \times 100\% = 40\%$ **(Ans.)**

(iii) Total sale of the two highest selling stores = 385 + 365 = 750

and, total sale of the two lowest selling stores = 215 + 235 = 450

$\therefore$ **Required ratio** = $750 : 450 = \frac{750}{450} = \frac{5}{3} = 5 : 3$ **(Ans.)**

(iv) Present sale of store E = 235

and, present sale of nearest rival store (store B) = 275

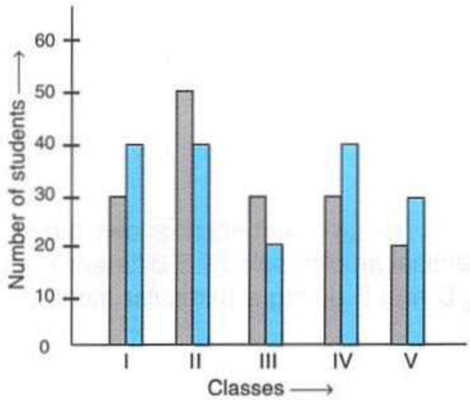
$\therefore$ **Store E must increase its sale by** = 275 − 235 = 40 **(Ans.)**

2. Double bar graph

In a double bar graph, two bar graphs are drawn side by side which help to compare two collections of data.

Example 5 :

Examine the given graph carefully.



In the graph, shown above, ▆ represents the number of girls in a particular class and ▆ represents the number of boys in the same class.

Read and answer the following questions.

(i) Which class has the least number of students ?

(ii) Which class has maximum number of students ?

(iii) What is the ratio between the number of girls and the number of boys of class V.

Solution :

(i) **Classes III and V have the least number of students** **(Ans.)**

(ii) **Class II has the maximum number of students** **(Ans.)**

(iii) **Required ratio** $= \dfrac{\text{Number of girls in class V}}{\text{Number of boys in class V}}$

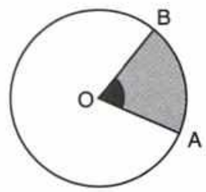
$$= \frac{20}{30} = \frac{2}{3} = 2 : 3$$ **(Ans.)**

3. Pie graph (or Pie chart)

When the numerical data is represented by the sectors of a circle, the graph obtained is called a ***pie graph*** or ***pie chart***.

An angle, whose vertex is the centre of a circle, is called ***central angle*** and the region of the circle enclosed by the arms of the angle is called ***sector***.

In the adjoining figure, ∠AOB is the central angle and the shaded portion of the circle is the sector.



Example 6 :

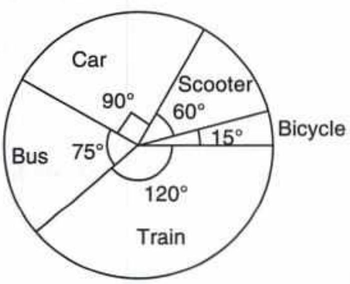
Use the information given in example 3, to draw a pie-graph.

Solution :

Steps :

1. Since the angle about the centre of a circle is 360°, divide 360° in proportion of given data. In this example, divide 360° in proportion 10 : 40 : 60 : 50 : 80 to obtain the corresponding central angle.
2. Make a table as given below.
3. Draw a circle of any suitable radius.
4. For each central-angle, draw a sector.

Name of object	Speed in km/hr	Central angle
Bicycle	10	$\frac{10}{240} \times 360° = 15°$
Scooter	40	$\frac{40}{240} \times 360° = 60°$
Car	60	$\frac{60}{240} \times 360° = 90°$
Bus	50	$\frac{50}{240} \times 360° = 75°$
Train	80	$\frac{80}{240} \times 360° = 120°$
	240	360°



Example 7 :

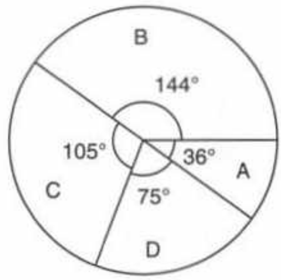
The adjoining diagram shows a pie-chart with central angles for number of players playing four games A, B, C and D. If each player takes part in only one game and there are 480 players in all, find :

(i) how many players play the game D ?

(ii) how many players do not play game B ?

(iii) what is the ratio of players playing game A to the players playing game C ?

(iv) what percentage of players are playing game B ?



Solution :

(i) **Number of players playing game** D = $\frac{75}{360} \times 480 = 100$ **(Ans.)**

(ii) $\because$ Number of players playing game B = $\frac{144}{360} \times 480 = 192$

$\Rightarrow$ **Number of players not playing game B** = 480 − 192 = **288** **(Ans.)**

(iii) $\because$ Number of players playing game A = $\frac{36}{360} \times 480 = 48$

and, number of players playing game C = $\frac{105}{360} \times 480 = 140$

$\Rightarrow$ **Required ratio** = $\frac{48}{140} = \frac{12}{35}$ = **12 : 35** **(Ans.)**

Alternative method :

Required ratio = $\frac{\text{Central angle for A}}{\text{Central angle for C}}$

$= \frac{36}{105} = \frac{12}{35}$ = **12 : 35** **(Ans.)**

(iv) **Percentage of players playing game B**

$= \frac{144}{360} \times 100\% = 40\%$ **(Ans.)**

EXERCISE 22(B)

1. Hundred students from a certain locality use different modes of travelling to school as given below. Draw a bar graph.

Bus	Car	Rickshaw	Bicycle	Walk
32	16	24	20	8

2. Mr. Mirza's monthly income is ₹ 7,200. He spends ₹ 1,800 on rent, ₹ 2,700 on food, ₹ 900 on education of his children, ₹ 1,200 on other things and saves the rest.

Draw a pie-chart to represent it.

3. The percentage of marks obtained, in different subjects by Ashok Sharma (in an examination) are given below. Draw a bar graph to represent it.

English	Hindi	Maths	Science	Social Studies
85	60	35	50	70

4. The following table shows the market position of different brands of tea-leaves.

Brand	A	B	C	D	others
% Buyers	35	20	20	15	10

Draw a pie-chart to represent the above information.

5. Students of a small school use different modes of travel to school as shown below :

Mode	Bus	Car	Bicyle	Auto	On foot
No. of students	142	98	50	34	16

Draw a suitable bar graph

6. For the following table, draw a bar-graph

A	B	C	D	E	F
230	400	350	200	380	160

7. Manoj appeared for ICSE examination 2018 and secured percentage of marks as shown in the following table :

Subject	Hindi	English	Maths	Science	Social Study
Marks as percent	60	45	42	48	75

Represent the above data by drawing a suitable bar graph.

8. For the data given above in question number 7, draw a suitable pie-graph.

9. Mr. Kapoor compares the prices (in ₹) of different items at two different shops A and B. Examine the following table carefully and represent the data by a double bar graph.

Items	Price (in ₹) at shop A	Price (in ₹) at Shop B
Tea-set	900	950
Mixie	700	800
Coffee-maker	600	700
Dinner set	600	500

10. The folliwng table shows the mode of transport used by boys and girls for going to the same school.

	Bus	Bicycle	Walking	Other sources
Number of boys	80	60	20	85
Number of girls	90	75	35	60

Draw a double bar graph representing the above data.

PROBABILITY 23

23.1 INTRODUCTION

The word **'probability'** is commonly used even without knowing its actual meaning. Terms like possible, probable, **chance**, likely, etc. convey the same sense that there is some uncertainty about the occurrence of the event in consideration.

The probability of an event is based on what has actually happened. In the theory of probability we deal with events which are outcomes of an experiment.

1. An **experiment** is an action which results in some well defined outcomes.
2. A **random experiment** is an experiment with more than one possible outcomes and it is not possible to predict any outcome in advance.
3. In this chapter wherever the word experiment will be used, it will mean random experiment.
4. Some examples of experiment (random experiment) are :
   (a) Tossing a coin (outcomes are head H and tail T).
   (b) Throwing a die (outcomes are 1, 2, 3, 4, 5 and 6).
   (c) Drawing a card from a pack of 52 playing cards, etc.

Remember :

Random experiment means all the outcomes of the experiment are known in advance, but any specific outcome of experiment is not known in advance.

Throwing a die is a random experiment because we know well in advance that there are only six possible outcomes of the experiment *i.e.* 1, 2, 3, 4, 5 and 6. But it is not possible to know which of these six numbers will finally come up.

Example 1 :

Write all possible outcomes, when :

(i) a coin is tossed once.

(ii) a coin is tossed two times or two coins are tossed together once.

(iii) a coin is tossed three times.

Solution :

(i) **Outcomes are :** Head (H) and tail (T). **(Ans.)**

(ii) **Outcomes are :** HH, HT, TH and TT. **(Ans.)**

(iii) **Outcomes are :** HHH, HHT, HTT
HTH, THT and TTT
THH, TTH **(Ans.)**

Example 2 :

Write all possible outcomes, when :

(i) a die is thrown once.

(ii) a die is thrown two times

Solution :

(i) **Outcomes are :** 1, 2, 3, 4, 5 and 6. **(Ans.)**

(ii) **Outcomes are :** (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5) and (6, 6) **(Ans.)**

23.2 SOME BASIC TERMS

1. A trial

(i) When a coin is tossed one time, two times, three times, etc. each toss of the coin is called **a trial**.

(ii) When a die is thrown any number of times, each throw of the die is called **a trial**.

(iii) When two coins are tossed simultaneously, any number of times, each simultaneous toss of both the coins is called **a trial**.

2. Equally likely outcomes

If all the outcomes of an experiment have equal chances to take place, the outcomes are said to be equally likely outcomes.

The outcomes of tossing of a coin, the outcomes of throwing of a die, the outcomes of drawing a card from a pack of 52 playing cards, etc. are all examples of equally likely outcomes.

Each outcome of an experiment is called an event.

An important discussion

Let a coin be tossed 100 times.

If head comes up 53 times, clearly tail comes up 100 – 53 = 47 times.

Now, if the coin is tossed once more, its chance to come up with head is 53 out of 100 and its chance to come up with tail is 47 out of 100.

We say :

for getting head, favourable outcomes = 53

and, for getting tail, favourable outcomes = 47

23.3 PROBABILITY

If in a random experiment, the total number of outcomes (events) are n out of which m outcomes (events) are favourable to a particular event E; then the probability of happening of event E is denoted by P(E) and is equal to $\frac{m}{n}$.

i.e. P(E) = Probability of the happening of event E

$$= \frac{m}{n}$$

$$= \frac{\text{Number of favourable outcomes (events)}}{\text{Total number of all possible outcomes}}$$

1. **Sure event :**

   An event, which will surely (certainly) happen, is called a **sure event.**

   For example :

   When a die is thrown, then the event getting a natural number upto 6 is a **sure event.**

   If E is a sure event, its probability is always one (1), *i.e.* for a sure event E, P(E) = 1.

2. **Impossible event**

   An event, which will never happen, is called an **impossible event.**

   For example :

   When a die is thrown, then the event getting a natural number greater than 6 is an **impossible event.**

   If E is an impossible event, its probability is always zero (0), *i.e.* for an impossible event E, P(E) = 0.

3. Probability of an event can never be less than 0 or more than 1, *i.e.* for an event E

$$0 \leq P(E) \leq 1$$

Example 3 :

Can the probability of an event be 1·25 ? Give reason.

Solution :

No, the probability of an even can not be 1·25. **(Ans.)**

Reason : The probability of an event is never more than 1 (one).

Example 4 :

Can the probability of an event be –2 ? Give reason.

Solution :

No, the probability of an even can not be –2. **(Ans.)**

Reason : The probability of an event is never less than 0 (zero).

Example 5 :

A die is thrown once. Find the probability of getting :

(i) an even number.

(ii) a multiple of 3.

(iii) not a multiple of 3.

Solution :

When a die is thrown once, the total number of possible outcomes = 6 (1, 2, 3, 4, 5 and 6)

(i) **For an even number :**

Number of favourable outcomes = 3 (2, 4 and 6)

$\therefore$ **P (getting an even number)** $= \dfrac{\text{Number of favourable outcomes (events)}}{\text{Total number of outcomes}}$

$= \dfrac{3}{6} = \dfrac{1}{2}$ **(Ans.)**

(ii) **For a multiple of 3 :**

Number of favourable outcomes = 2 (3 and 6)

$\therefore$ **P (getting a multiple of 3)** $= \dfrac{2}{6} = \dfrac{1}{3}$ **(Ans.)**

(iii) **For not a multiple of 3 :**

Number of favourable outcomes = 4 (1, 2, 4 and 5)

$\therefore$ **P (not a multiple of 3)** $= \dfrac{\text{Number of favourable outcomes (events)}}{\text{Total number of possible outcomes}}$

$= \dfrac{4}{6} = \dfrac{2}{3}$ **(Ans.)**

Example 6 :

Two coins are tossed together. Find the probability of getting :

(i) no tail (ii) two tails (iii) exactly one tail

Solution :

When two coins are tossed together, the total number of possible outcomes

= 4 (*i.e.* HH, HT, TH and TT)

(i) **For no tail :**

Possible number of favourable outcomes = 1 (*i.e.* HH)

$\therefore$ **P(no tail)** $= \dfrac{1}{4}$ **(Ans.)**

(ii) **For two tails :**

Possible number of favourable outcomes = 1 (*i.e.* TT)

$\therefore$ **P(two tails)** $= \dfrac{1}{4}$ **(Ans.)**

(iii) **For exactly one tail :**

Possible number of favourable outcomes = 2 (*i.e.* TH and HT)

$\therefore$ **P(exactly one tail)** $= \dfrac{2}{4} = \dfrac{1}{2}$ **(Ans.)**

Example 7 :

A letter is chosen from the word 'TRIANGLE'. What is the probability that it is a vowel ?

Solution :

Total number of possible outcomes = 8(t, r, i, a, n, g, l, e)

Total number of favourable outcomes = 3(*a*, *e* or *i*)

$\therefore$ **Required probability** = $\frac{3}{8}$ **(Ans.)**

Example 8 :

A bag contains 4 red, 6 black and 5 white balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is :

(i) white (ii) not black (iii) red or black.

Solution :

Total number of possible outcomes = Total number of balls

= 4 red balls + 6 black balls + 5 white balls

= 15 balls = 15

(i) **For a white ball**

Number of favourable outcomes = Number of white balls = 5

$\therefore$ **Required probability** = $\frac{5}{15} = \frac{1}{3}$ **(Ans.)**

(ii) **For not a black ball**

Number of favourable outcomes = Number of red balls + Number of white balls

= 4 + 5 = 9

$\therefore$ **Required probability** = $\frac{9}{15} = \frac{3}{5}$ **(Ans.)**

(iii) **For red or black ball**

Number of favourable outcomes = Number of red balls + Number of black balls

= 4 + 6 = 10

$\therefore$ **Required probability** = $\frac{10}{15} = \frac{2}{5}$ **(Ans.)**

EXERCISE 23

1. A die is thrown, find the probability of getting :
   (i) a prime number
   (ii) a number greater than 4
   (iii) a number not greater than 4.
2. A coin is tossed. What is the probability of getting :
   (i) a tail ? (ii) a head ?
3. A coin is tossed twice. Find the probability of getting :
   (i) exactly one head
   (ii) exactly one tail
   (iii) two tails
   (iv) two heads
4. A letter is chosen from the word 'PENCIL', what is the probability that the letter chosen is a consonant ?
5. A bag contains a black ball, a red ball and a green ball, all the balls are identical in shape and size. A ball is drawn from the bag without looking into it. What is the probability that the ball drawn is :
   (i) a red ball (ii) not a red ball
   (iii) a white ball.

6. In a single throw of a die, find the probability of getting a number
   (i) greater than 2
   (ii) less than or equal to 2
   (iii) not greater than 2.
7. A bag contains 3 white, 5 black and 2 red balls, all of the same size. A ball is drawn from the bag without looking into it, find the probability that the ball drawn is :
   (i) a black ball (ii) a red ball
   (iii) a white ball (iv) not a red ball
   (v) not a black ball
8. In a single throw of a die, find the probability that the number :
   (i) will be an even number
   (ii) will be an odd number
   (iii) will not be an even number.
9. In a single throw of a die, find the probability of getting :
   (i) 8
   (ii) a number greater than 8
   (iii) a number less than 8
10. Which of the following can not be the probability of an event ?
   (i) $\frac{2}{7}$ (ii) 3·8
   (iii) 37% (iv) −0·8
   (v) 0·8 (vi) $\frac{-2}{5}$
   (viii) $\frac{7}{8}$
11. A bag contains six identical black balls. A boy withdraws one ball from the bag without looking into it. What is the probability that he takes out :
   (i) a white ball ?
   (ii) a black ball ?
12. Three identical coins are tossed together. What is the probability of obtaining :
   (i) all heads ?
   (ii) exactly two heads ?
   (iii) exactly one head ?
   (iv) no head ?
13. A book contains 92 pages. A page is chosen at random. What is the probability that the sum of the digits in the page number is 9 ?
14. Two coins are tossed together. What is the probability of getting :
   (i) at least one head ?
   (ii) both heads or both tails ?
15. From 10 identical cards, numbered 1, 2, 3, ..., 10, one card is drawn at random. Find the probability that the number on the card drawn is a multiple of :
   (i) 2 (ii) 3
   (iii) 2 and 3 (iv) 2 or 3
16. Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is :
   (i) 0 (ii) 12
   (iii) less than 12
   (iv) less than or equal to 12
17. A die is thrown once. Find the probability of getting :
   (i) a prime number
   (ii) a number greater than 3
   (iii) a number other than 3 and 5
   (iv) a number less than 6
   (v) a number greater than 6.
18. Two coins are tossed together. Find the probability of getting :
   (i) exactly one tail
   (ii) at least one head
   (iii) no head
   (iv) at most one head

ANSWERS

Exercise 1 (A)

1. (i) $-\frac{1}{4}$ (ii) $-\frac{12}{13}$ (iii) $-\frac{3}{11}$ (iv) $\frac{1}{78}$ (v) $-\frac{1}{6}$ (vi) $-\frac{8}{5}$ (vii) $-\frac{21}{8}$ (viii) $\frac{-5}{54}$ **2.** (i) $-\frac{11}{18}$ (ii) $\frac{17}{5} = 3\frac{2}{5}$ (iii) $-\frac{29}{60}$ (iv) $-\frac{7}{36}$ (v) $\frac{-47}{36}$ (vi) $-\frac{2}{7}$ (vii) $-\frac{5}{11}$ (viii) $\frac{7}{5} = 1\frac{2}{5}$ (ix) $\frac{5}{9}$ **3.** (i) $-\frac{17}{21}$ (ii) $\frac{3}{5}$ (iii) $-\frac{2}{7}$ (iv) $-\frac{23}{84}$ **6.** (i) $\frac{3}{8}$ (ii) $\frac{4}{9}$ (iii) $\frac{7}{5} = 1\frac{2}{5}$ (iv) $-\frac{4}{13}$ (v) 0 (vi) 2 (vii) –1 (viii) $\frac{1}{3}$ (ix) $\frac{3}{1} = 3$
7. (i) $-\frac{5}{12}$ (ii) 0 (iii) $\frac{a}{b}$ (iv) 0 **8.** (i) false (ii) false (iii) true (iv) true (v) false (vi) false

Exercise 1 (B)

1. (i) $-\frac{2}{15}$ (ii) $\frac{2}{9}$ (iii) $-\frac{13}{9}$ (iv) $-\frac{1}{14}$ (v) $-\frac{1}{18}$ (vi) $\frac{23}{42}$ **2.** (i) –1 (ii) $\frac{12}{11} = 1\frac{1}{11}$ (iii) –1 (iv) $-\frac{5}{8}$ (v) $-\frac{3}{16}$ (vi) $\frac{37}{66}$ **3.** (i) $\frac{1}{20}$ **4.** $-\frac{2}{15}$ **5.** $-\frac{22}{5}$ **6.** $\frac{103}{72} = 1\frac{31}{72}$ **7.** $-\frac{1}{9}$ **8.** $-\frac{23}{18}$ **9.** (a) $-\frac{19}{8}$ (b) $\frac{19}{8} = 2\frac{3}{8}$ **10.** (i) $\frac{7}{9}$ (ii) $-\frac{13}{15}$ (iii) $5\frac{13}{63}$

Exercise 1 (C)

1. (i) $\frac{12}{5} = 2\frac{2}{5}$ (ii) $-\frac{3}{13}$ (iii) $\frac{25}{8} = 3\frac{1}{8}$ (iv) $-\frac{17}{12}$ (v) –8 **2.** (i) $\frac{20}{27}$ (ii) $-\frac{4}{9}$ (iii) $-\frac{7}{2}$ (iv) $\frac{81}{49} = 1\frac{32}{49}$ (v) $\frac{28}{75}$ (vi) $\frac{7}{2} = 3\frac{1}{2}$ **3.** (i) –1 (ii) $\frac{-7}{10}$ (iii) $\frac{2}{3}$ (iv) $-\frac{411}{160}$ **4.** (i) $\frac{7}{-5}$ (ii) $\frac{3}{4}$ (iii) 0 (iv) $\frac{-8}{13}$ (v) $\frac{6}{7}$
6. (i) $\frac{1}{5}$ (ii) $-\frac{1}{3}$ (iii) $\frac{11}{5} = 2\frac{1}{5}$ (iv) $\frac{8}{7} = 1\frac{1}{7}$ (v) $\frac{7}{8}$ (vi) $-\frac{17}{15}$ **7.** (i) $\frac{5}{2} = 2\frac{1}{2}$ (ii) $\frac{21}{104}$ (iii) $\frac{65}{3} = 21\frac{2}{3}$
10. (i) Commutativity (ii) Associativity (iii) Distributivity (iv) Existence of inverse (v) Existence of identity
11. (i) positive (ii) positive (iii) negative (iv) positive, negative (v) no (vi) 1 (vii) 1 and –1 (viii) m

Exercise 1 (D)

1. (i) 3 (ii) 5 (iii) $-\frac{20}{3}$ (iv) $\frac{3}{2} = 1\frac{1}{2}$ (v) 0 (vi) $-\frac{5}{3}$ (vii) $\frac{1}{12}$ (viii) $-\frac{9}{5}$ (ix) $\frac{11}{2} = 5\frac{1}{2}$ (x) $9\frac{1}{3}$
2. (i) 9 (ii) 4 (iii) 0 (iv) $-\frac{5}{2}$ (v) $\frac{4}{3} = 1\frac{1}{3}$ **3.** $-\frac{7}{2}$ **4.** 6 **5.** (i) $-\frac{5}{3}$ (ii) $\frac{5}{2} = 2\frac{1}{2}$ **6.** $\frac{3}{4}$ **7.** –26
8. ₹ 14 **9.** ₹ $26\frac{1}{34}$ **10.** $-\frac{1}{7}$ **11.** (i) $-\frac{13}{5}$ (ii) $-\frac{25}{7}$ (iii) $\frac{5}{11}$ **12.** $\frac{75}{7} = 10\frac{5}{7}$ 13. $\frac{-49}{12}$ or $\frac{49}{12} = 4\frac{1}{12}$

Exercise 1 (E)

3. (i) $\frac{15}{2} = 7{\cdot}5$ (ii) 4·25 (iii) 2·6 (iv) 3·9 (v) 1·25 **4.** (i) 6·25 and 6·50 or, 6·50 and 6·75 (ii) 5·1 and 5·4 or, 5·4 and 5·7 (iii) 3·6 and 4·5 or, 4·5 and 5·4 **5.** (i) 3·25, 3·5 and 3·75 (ii) 10·5, 11 and 11·5 **6.** $\frac{11}{18}$, $\frac{28}{45}$, $\frac{19}{30}$, $\frac{29}{45}$ and $\frac{59}{90}$ **7.** $\frac{53}{63}$, $\frac{107}{126}$, $\frac{6}{7}$, $\frac{109}{126}$, $\frac{55}{63}$ and $\frac{37}{42}$
8. $\frac{17}{8}$, $\frac{9}{4}$, $\frac{19}{8}$, $\frac{5}{2}$, $\frac{21}{8}$, $\frac{11}{4}$ and $\frac{23}{8} = 2\frac{1}{8}$, $2\frac{1}{4}$, $2\frac{3}{8}$, $2\frac{1}{2}$, $2\frac{5}{8}$, $2\frac{3}{4}$ and $2\frac{7}{8}$

Exercise 2 (A)

1. (i) $\frac{1}{3}$ (ii) $\frac{1}{2}$ (iii) $\frac{125}{216}$ (iv) $\frac{16}{9} = 1\frac{7}{9}$ (v) $\frac{13}{4} = 3\frac{1}{4}$ (vi) 54 (vi) $\frac{37}{216}$ (viii) $\frac{256}{81} = 3\frac{13}{81}$ (ix) $\frac{81}{625}$ (x) $\frac{2}{5}$ **2.** m = 2 and n = 3 **3.** $x = 2\frac{1}{5}$

Exercise 2 (B)

1. (i) 500 (ii) 16 (iii) 64 (iv) $\frac{4}{9}$ (v) 10 (vi) $\frac{3}{16}$ (vii) –5 (viii) $-\frac{1}{3}$ (ix) 4 (x) $\frac{1}{125}$ (xi) $1\frac{7}{9}$ (xii) 4 (xiii) $\frac{1}{100}$ (xiv) 36 (xv) 83 (xvi) $13\frac{1}{2}$ **2.** (i) 132 (ii) 1 (iii) $\frac{3}{256}$ **3.** (i) 1 (ii) 3 (iii) 1 (iv) 4 (v) 1 (vi) 1(vii) 49 (viii) $3\frac{62}{81}$ **4.** (i) a^3b^5 (ii) $5y^5$ (iii) x^7y^8 (iv) $\frac{1}{3}z^{27}$ (v) 6x (vi) $\frac{5}{x} = 5x^{-1}$ (vii) $\frac{y^6}{4x^4} = \frac{1}{4}.y^6.x^{-4}$ (viii) $\frac{9y^4}{x^2} = 9x^{-2}y^4$ (ix) $\frac{64x^4}{y^9} = 64x^4.y^{-9}$ **5.** 1

6. (i) xyz (ii) $\frac{27b^3}{64a^{12}} = \frac{27}{64}.a^{-12}b^3$ **7.** (i) $\frac{a}{b^5}$ (ii) $\frac{x^ny^{2n}}{y^{4m}} = x^n.y^{2n-4m}$ (iii) $\frac{ay^2}{5}$ (iv) $\frac{3x}{2y}$ (v) $\frac{1}{a^{2/3}.b^{1/2}}$ (vi) $\frac{x^ly^mz^n}{x^ny^lz^m} = x^{l-n}.y^{m-l}.z^{n-m}$ **9.** x^9 **10.** $\frac{1}{a^{n+6}}$ **13.** (i) 5 (ii) 4 **14.** (i) a^2 (ii) $\frac{1}{x}$

Exercise 3 (A)

1. (i) 3481 (ii) 39·69 (iii) $245\frac{4}{9}$ **2.** (i) 105 (ii) 630 (iii) 441 **3.** (i) 2 (ii) 3187 **4.** 2 ; 72 **5.** (i) 0·42 (ii) $9\frac{4}{5}$ (iii) 0·13 **6.** (i) 0·8 (ii) $\frac{3}{34}$ (iii) $\frac{35}{48}$ (iv) $7\frac{49}{110}$ **7.** (i) 216 (ii) 13·5 (iii) 0·91 (iv) $3\frac{1}{2}$ (v) 16 **8.** 36 days **9.** 27 **10.** 12 and 21; 102 and 201 **12.** (i) 282·8 (ii) 0·308

Exercise 3 (B)

1. (i) 69 (ii) 88 (iii) 123 (iv) 0·54 (v) 0·035 (vi) 0·152 (vii) 5·23 **2.** (i) 2·05 (ii) 23·06 (iii) 0·085 **3.** (i) 15·65 (ii) 22·271 (iii) 9·09 (iv) 0·255 (v) 2·28 (vi) 0·78 **4.** (i) 1·95 (ii) 2·62 **5.** (i) 12 (ii) 37 (iii) 88 **6.** (i) 18 (ii) 53 (iii) 147 **7.** 2·65; 2·22 **8.** 2·24, 0·38 **9.** (i) $\frac{42}{53}$ (ii) $\frac{13}{37}$ (iii) 468 (iv) 0·09 **10.** (i) 2·80 (ii) 2·8 **13.** 12 **14.** 142

Exercise 3 (C)

1. (i), (iii) and (v) **2.** (ii) and (iii) **3.** (i), (ii) and (v) **4.** (i), (ii) and (v) **5.** (i), (iii) and (v) **6.** six zeroes **7.** five zeroes **8.** no; it will always have an even number of zeroes **9.** A number having 2,3, 7 or 8 at the unit place is never a perfect square **10.** (i) odd (ii) even (iii) even (iv) odd **11.** No number has an even number of zeroes **12.** (i) 73 (ii) 169 (iii) 201 **13.** (i) $12^2 = 144$ (ii) $21^2 = 441$ (iii) $27^2 = 729$ **14.** 6, 8 and 10; 15, 20 and 25; 10, 24 and 26

Exercise 4 (A)

1. (i) 343 (ii) 1331 (iii) 4096 (iv) 12167 (v) 29791 (vi) 74088 (vii) 157464 **2.** (iii) and (v) **3.** (i) 9·261 (ii) 0·064 (iii) 4·096 (iv) 15·625 (vi) 0·000008 (vii) 0·512 **4.** (i) $\frac{27}{343}$ (ii) $\frac{512}{729}$ (iii) $\frac{1000}{2197}$ (iv) $2\frac{43}{343}$ **5.** (i) –27 (ii) –343 (iii) –1728 (iv) –5832 (v) –15625 (vi) –27000 (vii) –125000 **6.** (i) 216, 8000, 4096 (ii) 729, 3375, 125, 343, 9261 **7.** 7 **8.** 137 **9.** 9 **10.** 5 **11.** 15

Exercise 4 (B)

1. (i) 4 (ii) 7 (iii) 9 (iv) 12 (v) 21 (vi) 16 (vii) 20 (viii) 15 **2.** (i) $\frac{3}{4}$ (ii) $\frac{5}{6}$ (iii) $\frac{7}{8}$ (iv) 36 (v) 12 (vi) 180 (vii) 120 **3.** (i) −6 (ii) −8 (iii) −11 (iv) $-\frac{3}{5}$ (v) $-\frac{4}{7}$ (vi) $-\frac{8}{7}$ (vii) −13 (viii) −18 (ix) −140 **4.** (i) 1·4 (ii) 2·1 (iii) 0·03 (iv) −0·8 (v) −2·5 (vi) 50 **5.** 36 **6.** 3 **7.** (i) 70 (ii) −72 (iii) 20 (iv) $-\frac{3}{7}$ (v) $-\frac{9}{11}$ (vi) 6·3 (vii) −56

Exercise 5 (A)

1. (i) 10 (ii) 11 **2.** (i) 13 (ii) 11 **3.** (i) 4 (ii) 9 **5.** (i) 5 (ii) 9

Exercise 5 (B)

1. A = 7 and B = 6 **2.** A = 5, B = 4 and C = 1 **3.** A = 7 and B = 9 **4.** A = 4 and B = 7
5. A = 8 and B = 1 **6.** A = 6 **7.** A = 7 and B= 4 **8.** A = 5, B = 0 and C = 1
9. A = 5, B = 0 and C = 2 **10.** A = 8 **11.** A = 7, B = 6, C = 6 and D = 2

Exercise 5 (C)

1. (i) and (ii) **2.** All **3.** All **4.** (i) and (iii) **5.** (i) and (iii) **6.** only (i)

Exercise 5 (D)

1. 0, 3, 6 or 9 **2.** 0, 3, 6 or 9 **3.** 2, 5 or 8 **4.** 0 or 6
5. 1, 4 or 7 [Since, 3 x 26 has 6 at its unit place and 6 is an even number, therefore given number 3 × 26 is a multiple of 2] **6.** 0, 2, 4, 6 or 8 **7.** 0, 4 or 8 **8.** 4 **9.** 7 **10.** 5 **11.** 7

Exercise 6 (A)

1. (i) {4} (ii) {5, −1} (iii) {−3, −2, −1, 0, 1, 2, 3} (iv) {16, 25, 34, 43, 52, 61, 70} (v) {0, 4, 8, 12} (vi) $\left\{\frac{3}{4}, \frac{7}{9}, \frac{4}{5}, \frac{9}{11}, ...\right\}$ **2.** (i) {x : x = 3n + 3; x ∈ N} (ii) {x : x is a prime number between 10 and 20} (iii) {x : x = $\frac{n}{n+2}$, where n is an odd natural number} (iv) {x : x = n^3; n ∈ N and 2 ≤ n ≤ 6} (v) {x : x ∈ Z, − 5 ≤ x ≤ − 1} (vi) {x : x = 3n, n ∈ Z} **3.** (i) No, 64 is not a factor of 32 (ii) Yes, 54 is not a factor of 27 (iii) {2, 4, 62, 124} (iv) {1, 3, 9} (v) {2, 3, 7, 11} (vi) Yes (vii) No **4.** (i) {m, e, r, u, t} (ii) {u, n, i, v, e, r, s, a, l} (iii) {7, 8, 9, 10, ...} (iv) {0, 1, 2, 3, 4} (v) {6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21} **5.** (i) 3 and − 1 (ii) 9, 11, 13 and 15 (iii) 1, 2, 3, 4, 6, 8, 12 and 24 (iv) −2, −1, 0, 1 and 2 (v) 1, 2, 3 and 4 (vi) 4, 3, 2, 1, 0, −1, ...

Exercise 6 (B)

1. (i) 5 (ii) 4 (iii) 6 (iv) 3 **2.** 7 **3.** (i) Infinite (ii) Finite (iii) Infinite (iv) Finite **4.** All **5.** (i), (iii) and (v) **6.** (i) No (ii) Yes **7.** (i) Not equivalent; (ii) Not equivalent, as the two sets are not finite (iii) Not equivalent, as the two sets are not finite (iv) Not equivalent **8.** (i) Equal (ii) Not equal (iii) Equal (iv) Equal **9.** (i) Infinite (ii) Infinite (iii) Finite (iv) Finite (v) Infinite (vi) Finite **10.** (i) True, since both the sets have 10 elements (ii) False, since E = {1, 2, 4, 8, 16} and F = {1, 2, 4, 5, 10, 20} (iii) False; since A = {19, 18, ..., 0, −1, −2,.. .....} (iv) False, since A = {2} (v) False, since the given set has 3, 5, 7, 11, etc (vi) False (vii) True (viii) True (ix) False, the sets are equivalent

Exercise 6 (C)

1. (i) ϕ, {5}, {7}, {5, 7} (ii) ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c} (iii) ϕ, {0}, {1}, {2}, {0, 1}, {0, 2}, {1, 2}, {0, 1, 2} (iv) ϕ, {p}, {o}, {r}, {p, o}, {p, r}, {o, r}, {p, o, r} **2.** (i) C = {c, o, l, e, r} (ii) 5 (iii) 32 (iv) 31 **3.** ϕ, {t}, {e}, {h}, {t, e}, {t, h}, {e, h}, {t, e, h} **4.** (i) {−7, −3, −1, 0} (ii) {−3, −1, 0, 5} **5.** (i) {3, 6, 9, 12, 15, 18}

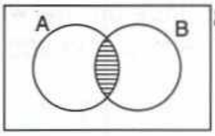
(ii) {5, 7, 9, 11, 13, 15, 17, 19} (iii) {4, 8, 12, 16} **6.** ϕ, {10}, {–1} **7.** (i) False, since each triangle is not isosceles (ii) True (iii) True, since each equilateral triangle is isosceles also (iv) True (v) True (vi) True **8.** (i) False (ii) False (iii) True (iv) True (v) True (vi) False **9.** (i) $\{x \in N : 17 \le x \le 35\}$ (ii) $\{x : 10 \le x \le 29\}$ **10.** (i) {–5, –4, –3, –2, –1} (ii) {1, 2, 3, 4, 5, 6} **11.** M = {r, e, a, *l*} and N = {*l*, a, r, e}; (i) Yes (ii) Yes (iii) Yes

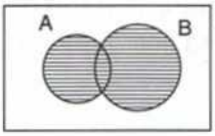
Exercise 6 (D)

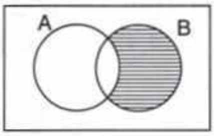
1. (i) A = {4, 5, 6}, B = {0, 1, 2, 3} (ii) {0, 1, 2, 3, 4, 5, 6} (iii) ϕ (iv) {4, 5, 6} (v) {0, 1, 2, 3} **2.** (i) $P \cup Q$ = {1, 2, 3, 4, 5, 6, 7, 8} and $P \cap Q$ = {4, 5} (ii) Yes **3.** (i) {4, 5, 6, 7, 8, 9} and {1, 2, 3, 4, 5, 6, 7, 8, 9} (ii) {1, 2, 3, 4, 5, 6, 7} and {1, 2, 3, 4, 5, 6, 7, 8, 9} (iii) {5, 6, 7} and {5} (iv) {4, 5} and {5}, yes ; yes **5.** (i) {6, 7, 8} (ii) {3, 4, 5, 6, 7, 8} (iii) {3, 4, 5, 6, 7, 8} (iv) {6, 7, 8}. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $(B \cup A) \cap (B \cup C) = B \cup (A \cap C)$ **6.** (i) {1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48} (ii) {1, 2, 3, 4, 6 12} (iii) {8, 16, 24, 48} (iv) {8, 16, 24, 48} **7.** (i) {7, 9} (ii) {8, 10} (iii) {4, 6, 10} (iv) {9} (v) {4, 8, 10} (vi) ϕ

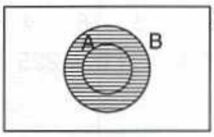
EXERCISE 6 (E)

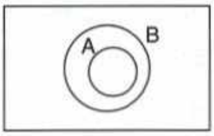
1. (i) $A \cup B$ = {a, b, c, d, e, f} (ii) $A' \cap B$ = {b, f} (iii) A – B = {a, d} (iv) B – A = {b, f} (v) $(A \cup B)'$ = {h, g} **2.** (i) A′ = {2, 5, 7, 8, 9, 10} (ii) B′ = {3, 4, 6, 7, 8, 9, 10} (iii) $A' \cup B'$ = {2, 3, 4, 5, 6, 7, 8, 9, 10} (iv) {2, 3, 4, 5, 6, 7, 8, 9, 10}; Yes **3.** (i) {a, b, c, d, g, h, i, j} (ii) {e, f, h, j} (iii) {a, b, c, i} (iv) {a, b, c, i}; yes **4.** (i) ϕ (ii) {1, 5, 6, 7, 9} (iii) {2, 3, 4, 6, 7, 8, 9, 10} (iv) {1, 5} (v) {1, 5, 6, 7, 9}

5. (i)


(ii)


(iii)


6. (i)


(ii)


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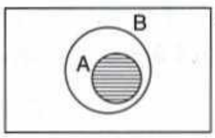

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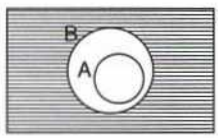
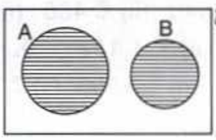
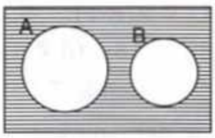
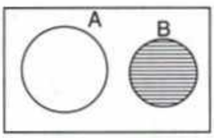
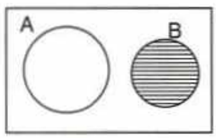


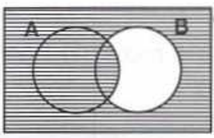
7. (i)


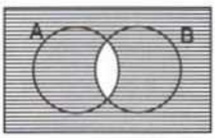
(ii)


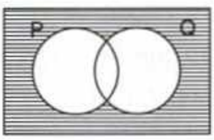
(iii)


(iv)


8. (i) $(A \cup B)'$ (ii) B – A or $A' \cap B$ (iii) $(B - A)'$

9. (i)


(ii)


(iii)


10. (i) {a, d, g} (ii) {d, e, g} (iii) {a, b, c, d, e} **11.** (i) A – B (ii) $(A \cup B) - (A \cap B)$ or $(A - B) \cup (B - A)$ (iii) $(A \cap B)'$ (iv) B′ (v) $(A \cup B)'$

Exercise 7 (A)

1. (i) 46 (ii) 28·5 **2.** (i) 20% (ii) 20% **3.** (i) $83\frac{1}{3}$% (ii) 15% **4.** (i) 720 (ii) 2250 **5.** (i) 25% increase (ii) 20% decrease (iii) 20% increase **6.** ₹ 18,440 **7.** $93\frac{3}{4}$% **8.** 4,700 litres **9.** ₹ 2,500 **10.** (i) 9·93% (ii) 1·56% **11.** 1,14,600; 66,468 and 48,132 **12.** 2,06,100 and 1,09,233 **13.** (i) ₹ 6,800 (ii) ₹ 5,780 **14.** 170 **15.** 400 **16.** (i) John = 120; Mohan = 96 (ii) 80%. **17.** 7,680 **18.** 11,250 **19.** 16% decrease **20.** Decreased by 55%

Exercise 7 (B)

1. 2,400 **2.** 3,00,000 **3.** (i) 48% (ii) 400 **4.** (i) 32% (ii) 500 **5.** 20% **6.** 25% **7.** 20% **8.** 340 **9.** 700 **10.** 125% **11.** 250% **12.** $85\frac{5}{7}$%

Exercise 7 (C)

1. 44% **2.** 32·5% **3.** ₹ 3,400 **4.** 8% increase **5.** 32% **6.** 5% increase **7.** 25% increase **8.** Maths = 80%, English = 68% and Science = 72%. 72·8% **9.** 20% **10.** (i) 225 (ii) 114·8 **11.** 45% **12.** (i) 700 (ii) 950 **13.** $16\frac{2}{3}$%.

Exercise 8 (A)

1. $18\frac{3}{4}$% **2.** $77\frac{7}{9}$% **3.** (i) 20% (ii) 50 **4.** Gain = 15% **5.** Loss = 3% **6.** (i) ₹ 600 (ii) ₹ 3,000 (iii) 20% **7.** (i) ₹ 550 (ii) ₹ 6,050 (iii) $9\frac{1}{11}$% **8.** Loss = 20% **9.** gain = 25% **10.** 20% loss **11.** $11\frac{1}{9}$% profit **12.** $42\frac{6}{7}$% profit **13.** 12·5% gain **14.** (i) ₹ 450 (ii) 10% **15.** (i) ₹ 225 (ii) $11\frac{1}{9}$%

Exercise 8 (B)

1. (i) ₹ 1,026 (ii) ₹ 1,131 **2.** (i) ₹ 1,500 (ii) ₹ 1,200 **3.** ₹ 750 and ₹ 150 **4.** ₹ 800 and ₹ 96 **5.** (i) ₹ 456 (ii) ₹ 520·80 **6.** ₹ 735 **7.** ₹ 638 **8.** 0·8% profit **9.** 1% loss **10.** (i) ₹ 2,400 (ii) ₹ 3,600 (iii) ₹ 6,000 (iv) ₹ 6,000 (v) there is no gain and no loss on the whole. **11.** (i) For Rahim : S.P. = ₹ 14,490 and C.P. = ₹ 13,800 (ii) For Rajesh : S.P. = ₹ 13,800 and C.P. = ₹ 15,000 **12.** ₹ 800.

Exercise 8 (C)

1. (i) ₹ 7 (ii) ₹ 21 **2.** (i) ₹ 2 (ii) ₹ 0·50 (iii) ₹ 0·75 (iv) ₹ 0·25 (v) 96 **3.** 18 **4.** 25% **5.** 20% loss **6.** 25% **7.** (i) ₹ 3,390 (ii) 13% **8.** ₹ 517·50 **9.** ₹ 400 **10.** ₹ 450 **11.** ₹ 225

Exercise 8 (D)

1. 12% **2.** ₹ 21,712 **3.** ₹ 126 and ₹ 1,274 **4.** ₹ 276·11 **5.** (i) ₹ 540 (ii) ₹ 486 (iii) 8% **6.** (i) ₹ 680 (ii) ₹ 600 **7.** (i) ₹ 1,980 (ii) ₹ 1,800 **8.** (i) ₹ 1,800 (ii) ₹ 1,152 **9.** 17% **10.** ₹ 600 **11.** (i) ₹ 15 (ii) ₹ 14·25 **12.** ₹ 150 and 6·25% **13.** ₹ 531 **14.** (i) ₹ 3,200 (ii) ₹ 2,720 (iii) $13\frac{1}{3}$% **15.** (i) 29·6% (ii) 42·4% (iii) 31·6% **16.** (i) 96% (ii) 84% (iii) 92%

Exercise 8 (E)

1. ₹ 583.20 **2.** 9% **3.** ₹ 12,300 **4.** ₹ 553.34 **5.** ₹ 12,300; ₹ 492 **6.** (i) ₹ 375 more (ii) ₹ 300 less (iii) ₹ 150 more (iv) ₹ 225 less **7.** (i) 4% (ii) ₹ 1,760 **8.** ₹ 26,000 **9.** ₹ 388.80

Exercise 8 (F)

1. (i) ₹ 17,920 (ii) ₹ 17,920 **2.** ₹ 26,250 **3.** ₹ 26,880 **4.** ₹ 13,330 **5.** ₹ 9,216 **6.** ₹ 92,288

Exercise 9 (A)

1. (i) ₹ 250 and ₹ 1,000 (ii) ₹ 240 and ₹ 5,240 (iii) ₹ 702 and ₹ 3,302 (iv) ₹ 1,280 and ₹ 5,280 **2.** ₹ 31,200 **3.** ₹ 7,400 **4.** ₹ 22,425.60 **5.** ₹ 3,200 **6.** (i) 5% (ii) 16% **7.** (i) 6 years (ii) 8 years **8.** 6% **9.** ₹ 10,010 **10.** 10 years and 5 months **11.** ₹ 2,685 **12.** ₹ 1,740 **13.** 6% **14.** ₹ 1,255

Exercise 9 (B)

1. 8% **2.** (i) $7\frac{11}{15}$% (ii) ₹ 10,690 **3.** (i) 12·5% (ii) 16 years **4.** 10 years and 8 months **5.** ₹ 8,653.85 **6.** $3\frac{1}{3}$% and ₹ 3,375 **7.** 5% and ₹ 6,562.50 **8.** 23rd Oct. 1992 **9.** 5% **10.** ₹ 8,100 and ₹ 7,500

Exercise 9 (C)

1. (i) ₹ 800 (ii) ₹ 8,800 (iii) ₹ 880 (iv) ₹ 9,680 (v) ₹ 1,680 **2.** (i) ₹ 1,600 (ii) ₹ 1,728 (iii) ₹ 23,328 (iv) ₹ 3,328 **3.** ₹ 14,520 and ₹ 2,520 **4.** ₹ 12,597.12 and ₹ 2,597.12 **5.** ₹ 1,160 **6.** ₹ 3,889.20 **7.** ₹ 18,522 **8.** ₹ 4,400 **9.** ₹ 954 **10.** (i) ₹ 960 (ii) ₹ 10,560 (iii) ₹ 1,056 (iv) ₹ 1,161.60 **11.** (i) 12% (ii) ₹ 6,272 **12.** ₹ 48 **13.** ₹ 248 **14.** Both get equal interests **15.** ₹ 12,345.60 **16.** (i) ₹ 13,200 (ii) ₹ 5,230 (iii) ₹ 5,200 (iv) ₹ 5,720 **17.** ₹ 10,810 **18.** (i) ₹ 30,000 (ii) ₹ 6,300 **19.** ₹ 13,230 and ₹ 1,230 **20.** ₹ 10,648 and ₹ 2,648 **21.** ₹ 29,040 and ₹ 5,040 **22.** ₹ 18,522 and ₹ 2,522 **23.** ₹ 23,152.50 and ₹ 3,152.50 **24.** ₹ 38,720 and ₹ 6,720 **25.** ₹ 5,060 and ₹ 1,060 **26.** ₹ 15,180 and ₹ 5,180

Exercise 10 (A)

1. only (i) **2.** x = 5, y = 8 and z = 120 **3.** 1024 km **4.** ₹ 2,160 **5.** 25 **6.** 23 kg **7.** ₹ 1,404 **8.** 110 men **9.** 32 **10.** (i) 72 km (ii) 1 hour 45 minutes

Exercise 10 (B)

1. only (i) **2.** (i) $l = 2$, m = 8 and n = 0·5 (ii) $l = 16$, m = 48 and n = 24 **3.** 6 men **4.** 24 minutes **5.** 42 days **6.** 60 days **7.** 144 **8.** 4 hours

Exercise 10 (C)

1. ₹ 180 **2.** 25 days **3.** 42 men **4.** 25 days **5.** 18 days **6.** (i) ₹ 350 (ii) ₹ 140 **7.** ₹ 6,48,000 **8.** (i) ₹ 90 (ii) ₹ 60 (iii) ₹ 780 **9.** ₹ 42·90 **10.** 80 **11.** 9 days **12.** 6 days **13.** (i) $1\frac{1}{2}$ women (ii) 30 days **14.** $8\frac{1}{3}$ days

Exercise 10 (D)

1. 5 oranges **2.** 5 men **3.** 4 hours **4.** 150 men **5.** 16 days **6.** 35 days **7.** 128 men **8.** 12 hours/days **9.** $46\frac{2}{3}$ days **10.** 21 men **11.** 25 **12.** 30 **13.** 24

Exercise 10 (E)

1. 6 days **2.** 20 days **3.** $\frac{8}{15}$ **4.** $3\frac{3}{7}$ days **5.** 168 days **6.** $13\frac{5}{7}$ days **7.** $4\frac{1}{2}$ days **8.** 10 days **9.** $10\frac{10}{17}$ days **10.** 12 days **11.** $7\frac{1}{2}$ hours **12.** $7\frac{5}{13}$ days **13.** (i) 36 days (ii) 72 days (iii) 24 days **14.** (i) 20 days (ii) A in 60 days; B in 120 days and C in 40 days **15.** 7 days **16.** After 12 minutes

Exercise 11 (A)

1. Constants are : -7, $\sqrt{5}$ and $8 - 5$, variables are : $7 + x$, $7x + yz$, $\sqrt{xy}$, $\frac{3yz}{8}$, $4{\cdot}5y - 3x$, $8 - 5x$, $8x - 5y \times p$ and $3y^2z \div 4x$ **2.** (i) 2 (ii) 2 (iii) 3 (iv) 2 **3.** Monomials : xy^2, $3x \times 5y$, $3x \div 5y$; Binomials : $8 - 3x$; Trinomials : $3y^2 - 5y + 8$, $2y \div 7 + 3x - 7$, Polynomials : $8 - 3x$, $3y^2 - 5y + 8$, $9x - 3x^2 + 15x^3 - 7$, $2y \div 7 + 3x - 7$, $4 - ax^2 + bx + y$ **4.** (i) 2 (ii) 3 (iii) 8 (iv) 3 (v) 4 (vi) 12 **5.** (i) 7x (ii) bx (iii) 1 (iv) $-ax$ (v) -1 **6.** (i) $\frac{1}{7}xy^2z^3$ (ii) xy^2z^3 (iii) $\frac{1}{7}y^2z^3$ (iv) $\frac{5}{7}z^3$ (v) $\frac{5}{7}xy^2$ (vi) $\frac{5}{7}y^2$ (vii) $\frac{1}{7}z^3$ (viii) $5xyz^2$ (ix) $\frac{5}{7}xy^2z^2$ (x) $\frac{5}{7}xyz$ (xi) $\frac{1}{7}yz^2$ **7.** (i) $3xy$ and $-8yx$; $-4yx^2$, $2{\cdot}5x^2y$ and x^2y; $2xy^2$ and $-3{\cdot}2y^2x$ (ii) y^2z^3, $-4y^2z^3$ and $2z^3y^2$; xy^2z^3 and $-8xz^3y^2$; $-5x^2yz$ and $3x^2yz$

Exercise 11 (B)

1. (i) $9x^2$ (ii) $-11b^2y$ (iii) 8abx (iv) $4x - 14y + 11$ (v) $4x^2 - 3xy - 9y^2$ **2.** (i) $9a + 6b$ (ii) $3x + y + z$ (iii) $2b + 10c + 10$ (iv) $3a + 3b + 3c - 7$ (v) $6ab + 3cd + 10xy$ (vi) $2x^2y + 5xy^2 + 2y^3$ **3.** ₹ (7x+ y) **4.** (i) $-xy^2$ (ii) $10x^2y - 3xy^2$ (iii) $4a + 2b - 4d$ (iv) $2x^3 - x^2 + 4x + 7$ (v) $a^3 - 3a^2 - 2a - 2$ (vi) $2abc + 3cad + 6bcd$ (vii) $3a^2 - 4ab + b^2$ **5.** (i) $6x^3 - 8x^2 + 10x - 12$ (ii) $3m^2 - m - 1$ **6.** $3y - 6$ **7.** $x^3 - x - 4$ **8.** (i) $3x^2 + 6xy - 3y^2$ (ii) $a^2 - 7$ **9.** (i) $6a + 9b + 4c$ (ii) $12a + 5b + 16c$ (iii) $6a + 17b + 21c$ (iv) $-45a - 32b - 65c$ **10.** $2x^2 + 6xy + 11$ **11.** $y^2 - 4y - 8$ **12.** $2x^2 - 2xy + 4z^2$ **13.** $11x^4 - 20x^3 + 37x + 23$ **14.** $7x - 2y - 32z + 26$ **15.** $5x^2y^2 - 18xy^2 - 16x^2y + 5x^2 + 7xy$

Exercise 11 (C)

1. (i) $-32a^4b^6$ (ii) $-\frac{1}{6}a^3b^2$ (iii) $25c^2d^4$ (iv) $24a^2 + 28a$ (v) $-32x + 16x^2 + 8x^3$ (vi) $-6a^3 + 15a^2 + 12a$ (vii) $x^2 - x - 20$ (viii) $35a^2 - 22a + 3$ (ix) $84a^2 + 23ab - 5b^2$ (x) $1 - x^3$ (xi) $8m^4 - 14m^3 - 3m^2 + 4m + 1$ (xii) a^3b^3 (xiii) $-21a^3b^3x^5$ (xiv) $-105b^4x^2y^3$ (xv) $-\frac{2}{3}a^2x^8y^4$ (xvi) $\frac{3}{2}a^8b^7$ (xvii) $-a^4b^2 + \frac{3}{2}a^3b^3$ (xviii) $4x^2 - \frac{1}{4}y^2$ **2.** (i) $-15x^3y + 24x^2y^2 - 18xy^3 + 9xy$ (ii) $-63x^2y^2 + 14x^3y^3 - 15x^3y^4 + 16x^4y^3$ (iii) $-18x^5 + 39x^3 - 30x^2 - 20x + 40$ (iv) $6y^7 + 6y^6 - 22y^5 - 4y^4 + 14y^3 + 2y^2 - 6y$ (v) $10p^4 + 40p^3q - 27p^2q^2 + 67pq^3 + 12q^4$ **3.** (i) $21x^2 - 10x - 16$ (ii) $p^2x^2 - q^2$ (iii) $10ab + 10b^2 - 17bc - 15ac + 3c^2$ (iv) $20x^2 - 41xy + 20y^2$ (v) $11y^2 + 9yz - 9z^2$ **4.** $(x^5 - 4x^4y + 2x^3y^2 + 20x^2y^3 - 35xy^4)$ sq. unit **5.** $\frac{1}{2}(18x^2 - 9xy - 20y^2)$ sq. unit **6.** $-24x^3y^4$ **7.** -6480 **8.** 192 **9.** -6 **10.** 320 **11.** (i) 28 (ii) -143 (iii) 10 **12.** (i) -2 (ii) -5 (iii) -14 **13.** $6x^2y^3 - 3x^3y^2$ **14.** (i) $2x^4 + 2x^3 - 21x^2 + 43x - 35$ (ii) $5ab - 2a^2b^2 - 3$ **15.** $60 - 92x + 41x^2 - 5x^3$

Exercise 11 (D)

1. (i) $-5a$ (ii) $-3x^3y$ (iii) $-3a$ (iv) $\frac{12x^2}{d^2}$ (v) $-7a^2bc^3$ (vi) $4x - 5y + 3c$ (vii) $-3b^2 + 2ab + 5b^4$ (viii) $-2x^4y - 3x^2y^3 + x^3y^2$ (ix) $a + 3$ (x) $x + 9$ (xi) $4x - 3y$ (xii) $x^4 + 2x^2 + 4$ (xiii) $3x - 5$ (xiv) $2a + 3b - 5c$ (xv) $-x^3 + x + 4$ **2.** (i) Quotient = $a^2 - 6a + 14$ and remainder = 1 (ii) Quotient = $3x^3 + 15x^2 + 39x + 119$ and remainder = 350 (iii) Quotient = $2x - 3$ and remainder = 0 **3.** $x^2 - 7x - 7$ **4.** $8x^3 + 4x^2 + 2x + 1$ **5.** $x^3 + y^3$

Exercise 11 (E)

1. $10a^2 - 5a$ **2.** 2x **3.** $3 - 3x^2$ **4.** $6n - 4m$ **5.** $3x - y$ **6.** $p^2x - 2px + 12x^3 - 18ax$ **7.** $8m + 48n + 48p + 8q - 324$ **8.** 2b **9.** $5x - 14y$ **10.** $7a^2$ **11.** x^3y **12.** x^3y^5 **13.** $y^3 - 6y^2 + 5y$ **14.** $108a^2 - 48ab$ **15.** $x^2 + 15x - 6$

Exercise 12 (A)

1. (i) $x^2 + 11x + 24$ (ii) $y^2 + 2y - 15$ (iii) $a^2 - 6a - 16$ (iv) $b^2 - 8b + 15$ (v) $6x^2 - xy - 2y^2$ (vi) $15a^2 + 13a - 112$ (vii) $24 + 5b - b^2$ **2.** (i) $x^2 - 1$ (ii) $4 - a^2$ (iii) $9b^2 - 1$ (iv) $16 - 25x^2$ (v) $4a^2 - 9$ (vi) $x^2y^2 - 16$ (vii) $a^2b^2 - x^4$ (viii) $9x^4 - 25y^4$ (ix) $z^2 - \frac{4}{9}$ (x) $\frac{9}{25}a^2 - \frac{1}{4}$ (xi) $0{\cdot}25 - 4a^2$ (xii) $\frac{a^2}{4} - \frac{b^2}{9}$ **3.** (i) $a^4 - 1$ (ii) $a^4 - b^4$ (iii) $16a^4 - b^4$ (iv) $81 - 16x^4$ (v) $81x^4 - 256y^4$ **4.** (i) 399 (ii) 891 (iii) 9991 (iv) 99·96 (v) 63·91 (vi) 24·84 **5.** (i) $36 - x^2y^2$ (ii) $49x^2 - \frac{4}{9}y^2$ (iii) $\frac{a^2}{4b^2} - \frac{4b^2}{a^2}$ (iv) $9x^2 - \frac{1}{4y^2}$ (v) $16a^4 - 81$ (vi) $a^4 - b^4c^4$ (vii) $15x^2 + 49xy + 40y^2$ (viii) $35x^2 + 47xy - 60y^2$ (ix) $6a^2 - ab - 12b^2$ (x) $27a^2 - 30ab + 7b^2$

Exercise 12 (B)

1. (i) $4a^2+4ab+b^2$ (ii) $a^2-4ab+4b^2$ (iii) $a^2+1+\frac{1}{4a^2}$ (iv) $4a^2-4+\frac{1}{a^2}$ (v) $a^2+b^2+c^2+2ab-2bc-2ca$ (vi) $a^2+b^2+c^2-2ab-2bc+2ca$ (vii) $9x^2+2+\frac{1}{9x^2}$ (viii) $4x^2-2+\frac{1}{4x^2}$
2. (i) $x^2+6xy+9y^2$ (ii) $4x^2-20xy+25y^2$ (iii) $a^2+\frac{2}{5}+\frac{1}{25a^2}$ (iv) $4a^2-4+\frac{1}{a^2}$ (v) $x^2+4y^2+1-4xy-4y+2x$ (vi) $9a^2+4b^2+25c^2-12ab+20bc-30ca$ (vii) $4x^2+\frac{1}{x^2}+5+\frac{2}{x}+4x$ (viii) $21+x^2+\frac{4}{x^2}-10x+\frac{20}{x}$ (ix) $4x^2+9y^2+z^2-12xy-6yz+4zx$ (x) $x^2+\frac{1}{x^2}+3-\frac{2}{x}-2x$
3. (i) 43264 (ii) 8464 (iii) 172225 (iv) 35344 (v) 88·36 (vi) 428·49 **4.** (i) $8a^3+12a^2b+6ab^2+b^3$ (ii) $a^3-6a^2b+12ab^2-8b^3$ (iii) $27x^3-54x^2y+36xy^2-8y^3$ (iv) $x^3+15x^2y+75xy^2+125y^3$ (v) $a^3+3a+\frac{3}{a}+\frac{1}{a^3}$ (vi) $8a^3-6a+\frac{3}{2a}-\frac{1}{8a^3}$ **5.** (i) $a^3+6a^2+12a+8$ (ii) $8a^3-12a^2+6a-1$ (iii) $8a^3+36a^2b+54ab^2+27b^3$ (iv) $27b^3-54b^2a+36ba^2-8a^3$ (v) $8x^3+12x+\frac{6}{x}+\frac{1}{x^3}$ (vi) $x^3-\frac{3x^2}{2}+\frac{3x}{4}-\frac{1}{8}$

Exercise 12 (C)

1. 13 **2.** 68 **3.** (i) ±7 (ii) ±3 **4.** (i) ±2 (ii) ±4 **5.** 7 **6.** 18 **7.** ±5 **8.** ±3 **9.** 31 **10.** 33 **11.** ±15 **12.** 72 **13.** 117 **14.** 110 **15.** 76 **16.** (i) 18 (ii) 76 **17.** (i) 7 (ii) 18 **18.** (i) ±5 (ii) ±1

Exercise 12 (D)

1. (i) $6x^2+2x+\frac{1}{6}$ (ii) $14a^2+2{\cdot}9a-0{\cdot}15$ (iii) $63+2y-y^2$ (iv) $30-17z+z^2$ (v) a^4+2a^2-15 (vi) $32-4ab-a^2b^2$ (vii) $35x^2y^2-4xy-63$ (viii) $24a^4-41a^2b^2+12b^4$ **2.** (i) $4x^2-\frac{9}{25}$ (ii) $\frac{16}{49}a^2-\frac{9}{16}b^2$ (iii) $36-25x^2y^2$ (iv) $4a^2-\frac{1}{4a^2}$ (v) $16x^4-25y^4$ (vi) $2{\cdot}56x^2-0{\cdot}49y^2$ (vii) m^4-81 (viii) $81x^4-256y^4$ (ix) $a^4-b^4c^4$ (x) 39991 (xi) 399·36 **3.** (i) $9x^2+\frac{12x}{y}+\frac{4}{y^2}$ (ii) $\frac{25a^2}{36b^2}-2+\frac{36b^2}{25a^2}$ (iii) $4m^4-\frac{8}{3}m^2n^2+\frac{4}{9}n^4$ (iv) $25x^2+2+\frac{1}{25x^2}$ (v) $64x^2+24xy+\frac{9}{4}y^2$ (vi) 368449 (vii) 152881 (viii) 94·09 **4.** (i) 2 (ii) 2 **5.** (i) 27 (ii) 727 (iii) $5\sqrt{29}$ **6.** (i) $\sqrt{33}$ (ii) 7 **7.** (i) 62 (ii) 3842 **8.** (i) 27 (ii) 727 **9.** (i) $9x^2+16y^2+25z^2-24xy-40yz+30zx$ (ii) $4a^2+25b^2+16c^2-20ab+40bc-16ca$ (iii) $125x^3+225x^2y+135xy^2+27y^3$ (iv) $216a^3-756a^2b+882ab^2-343b^3$ **10.** 51 **11.** 20 **12.** 665 **13.** 152 **14.** 243 **15.** 3703 **16.** 335

Exercise 13 (A)

1. $5(3x+1)$ **2.** $a(a^2-a+1)$ **3.** $3x^2(1+2x)$ **4.** $4a(a-2b)$ **5.** $2x^3b^2(1-2x^2b^2)$ **6.** $5x^3y(3xy^2-4)$ **7.** $b(a^3-a^2b-b^2)$ **8.** $y(6x^2+9xy+4y^2)$ **9.** $17a^2b^4(a^4b^4-2a^2b^2+3)$ **10.** $3x^3y(x^2-9xy+4y^2)$ **11.** $(a-b)(x^2-y^2+z^2)$ **12.** $2x(a+b)$ **13.** $(2a+b)(2b-3c)$ **14.** $3abc(4-2abc+a^2b^2c^2)$ **15.** $2(3x-2y)(2x-y)$ **16.** $(a+2b)(3a+2b)$ **17.** $2(a^2+b^2)(3xy+4yz-5xz)$

Exercise 13 (B)

1. $(a+x)(a+b)$ **2.** $(a-b)(a-c)$ **3.** $(a-2)(b+a)$ **4.** $(a-1)(a^2+1)$ **5.** $(a-2b)(2-x)$ **6.** $(x-a)(y-a+b)$ **7.** $(x-2)(3x^4-2x^2+1)$ **8.** $(xy+1)(3-x)$ **9.** $(2-b)(3a^2+c^2)$ **10.** $3(b-4)(a^2-3)$ **11.** $(x-a)(x+3)$ **12.** $(x-b)(x+2)$ **13.** $(b-c)(a+d)$ **14.** $(b-1)(ab+c)$ **15.** $(a-c)(b^2+ac)$ **16.** $(a-1)(a^2-b+1)$ **17.** $(ac-bd)(bc-ad)$ **18.** $(2b-y)(ab+cy)$ **19.** $(a+2b+3c)(x-3)$ **20.** $(b^2c-1)(2a+3b-4c)$

Exercise 13 (C)

1. $(4 + 3x)(4 - 3x)$ **2.** $(1 + 10a)(1 - 10a)$ **3.** $(2x + 9y)(2x - 9y)$ **4.** $\left(\frac{2}{5}+5b\right)\left(\frac{2}{5}-5b\right)$ **5.** $4b(a + b)$ **6.** $(5a + b)(5a - 7b)$ **7.** $3b^2(2a^2 - 3b^2)$ **8.** $(7a - 3b)(3a - b)$ **9.** $(1 + 5a + 5b)(1 - 5a - 5b)$ **10.** $3(5a + b)(a + b)$ **11.** $3(14x + y)(2x + 3y)$ **12.** $(13x - 4y)(x - 10y)$ **13.** 39 **14.** $48\frac{22}{25}$ **15.** 0·4 **16.** 18 **17.** $3(9x + y)(x + 9y)$ **18.** $(a + 2 + b)(a + 2 - b)$ **19.** $(a + b + 1)(a - b - 1)$ **20.** $(x + 3 + 2y)(x + 3 - 2y)$

Exercise 13 (D)

1. $(x + 4)(x + 2)$ **2.** $(x + 3)(x + 1)$ **3.** $(a + 3)(a + 2)$ **4.** $(a - 3)(a - 2)$ **5.** $(a + 6)(a - 1)$ **6.** $(x + 4y)(x + y)$ **7.** $(a - 8)(a + 5)$ **8.** $(x - 9)(x + 8)$ **9.** $(x - 6y)(x - 4y)$ **10.** $(2a + 3)(a + 2)$ **11.** $(a - 1)(3a - 2)$ **12.** $(b - 1)(7b - 1)$ **13.** $(2a - 13b)(a - 2b)$ **14.** $(x + 2y)(2x - 3y)$ **15.** $(c + 2)(4c - 5)$ **16.** $(2x + 1)(7x - 3)$ **17.** $(2 + 3b)(3 - b)$ **18.** $(1 + 2x)(5 - 3x)$ **19.** $(1 + 2y)(4 - 7y)$ **20.** $(1 + 2a)(5 - 7a)$ **21.** $(2a + b + 3)(2a + b + 2)$ **22.** $(1 - 6x - 9y)(1 + 4x + 6y)$ **23.** $(x - 2y - 8)(x - 2y - 4)$ **24.** $(4 + 5a + 5b)(2 - a - b)$ **25.** $(x + 2y - 2)(2x + 4y - 1)$

Exercise 13 (E)

1. (i) Yes (ii) Yes (iii) Yes (iv) No (v) No (vi) No **2.** $2(1 + 2x)(1 - 2x)$ **3.** $2y(2x + 3y)(2x - 3y)$ **4.** $a(x + y)(x - y)$ **5.** $x(5x + 1)(5x - 1)$ **6.** $(a^2 + b^2)(a + b)(a - b)$ **7.** $(4x^2 + 9y^2)(2x + 3y)(2x - 3y)$ **8.** $(25 + x^2)(5 + x)(5 - x)$ **9.** $(x + y)(x - y - 3)$ **10.** $(x - y)(x + y - 2)$ **11.** $3(x + 8)(x - 3)$ **12.** $2(a - 8)(a + 4)$ **13.** $5(b + 6)(b + 3)$ **14.** $y(x + 3)(3x + 2)$ **15.** $a(p + 2)(5p + 1)$ **16.** $(a + b + c)(a + b - c)$ **17.** $(x + 3y)(x + 3y + 1)$ **18.** $(2a - 3b)(2a - 3b + 2)$ **19.** $2b^2(a + 7b)(a - 7b)$ **20.** $(a + 4b)(a - 4b - 2)$

Exercise 13 (F)

1. (i) $2x^2(3x - 4)$ (ii) $7ab^2c(5a^2 + 6c)$ (iii) $6x^2y^2(6 - 5xy + 8x)$ (iv) $4(2a + 3b)^2(4a + 6b - 3)$ (v) $3a(x - 2y)^3(3x - 6y - 4)$ **2.** (i) $(a - b)(a - 3)$ (ii) $(x - y)(xy + 5)$ (iii) $(a - b)(a + b^2)$ (iv) $(y + 1)(xy - 1)$ (v) $(a^2 + b^2)(x^2 + y^2)$ (vi) $(bx - ay)(ax - by)$ (vii) $(m - 1)(2 - m + a)$ **3.** (i) $(a + b - c)(a - b + c)$ (ii) $3(14x - 13y)(2x + y)$ (iii) $11(29x + 10)(x + 2)$ (iv) $(3x + \frac{1}{4})(3x - \frac{1}{4})$ (v) $(5x - 10y + 2)(5x - 10y - 2)$ **4.** (i) $(a - 21)(a - 2)$ (ii) $(a - 27)(a + 4)$ (iii) $(1 - 21x)(1 + 3x)$ (iv) $(x - 2y)(5x + 6y)$ (v) $(x + 4)(3x + 2)$ (vi) $(1 - 2x)(5 + 6x)$ (vii) $(xy - 8)(xy + 5)$ (viii) $(3x - 2y - 8)(3x - 2y + 3)$ (ix) $(4a + 4b - 7)(3a + 3b + 5)$ **5.** (i) $5(5x - 4)(5x - 14)$ (ii) $(3a^2 - b)(x + a)$ (iii) $(c - d)(bc - bd - a + 3)$ (iv) $(a - y)(x + b)(x - b)$ (v) $(1 - 4x - 4y)(1 + x + y)$ **6.** (i) $2a(a + 5)(a - 5)$ (ii) $6(3ab + 1)(3ab - 1)$ (iii) $16b(2a + 3b)(2a - 3b)$ (iv) $(2x - y)(2x - y + 1)(2x - y - 1)$ (v) $(x - y + z)(x - y - z)$ (vi) $(x + y + z)(x - y - z)$ (vii) $7a(a^2 + 9)(a + 3)(a - 3)$ (viii) $5x^2\left(1+\frac{2x}{3}\right)\left(1-\frac{2x}{3}\right)$ **7.** $x(y + z)(y - z)$ (i) 540 (ii) 400 **8.** (i) $(a + 3b)(a - 9b)$ (ii) $(7a - 31b)(3a - 19b)$ (iii) $(a + 0{\cdot}6\,b)(a - 0{\cdot}6\,b)$ (iv) $(a^2 + 25)(a + 5)(a - 5)$ (v) $(x^2 + 4)(x + 3)(x - 3)$ (vi) $(6x - 3y - 5)(10x - 5y + 3)$ **9.** $b(a + b)(a - b)$ and 20100 **10.** 180300

Exercise 14 (A)

1. 7 **2.** 3 **3.** 4 **4.** 2·4 **5.** 4 **6.** 7 **7.** 3 **8.** 12 **9.** – 5 **10.** 108 **11.** 2 **12.** 4·5 **13.** 13 **14.** $-\frac{1}{7}$ **15.** 14 **16.** 3 **17.** – 13 **18.** $x = 1\frac{9}{14}$ **19.** $1\frac{1}{2}$ **20.** $-4\frac{8}{13}$ **21.** $1\frac{1}{2}$ **22.** 8 **23.** $-1\frac{1}{2}$ **24.** $x = 1$ and $a = \frac{1}{3}$ **25.** $x = 8$ and $p = 5$ **26.** 3 **27.** 5

Exercise 14 (B)

1. 6 **2.** 15 years **3.** 10 **4.** –5 **5.** 15 years **6.** 11 **7.** A's = ₹ 3000 ; B's = ₹ 750 **8.** 116 and 62 **9.** 40 **10.** 18 cm **11.** 25 cm and 15 cm **12.** 17, 19 and 21 **13.** 36 years and 12 years **14.** 18 years **15.** 96 km **16.** 13 and 10 **17.** 24 and 25 **18.** 50, 51 and 52 **19.** 1 **20.** 48 years and 24 years

Exercise 14 (C)

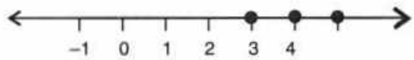
1. (i) $25\frac{1}{2}$ (ii) 2 (iii) 6 (iv) $-\frac{70}{349}$ (v) 9 (vi) $1\frac{25}{31}$ (vii) 2 (viii) 11 (ix) −19 (x) 19 (xi) $\frac{1}{3}$ (xii) $\frac{7}{5}$ (xiii) 9 (xiv) $-\frac{15}{4}$ **2.** 12 years **3.** ₹ 360 **4.** 2500 and 2000 **5.** 216 and 189 **6.** 35 years and 25 years **7.** 30 **8.** 15 and 16 **9.** 3 **10.** $\frac{17}{22}$

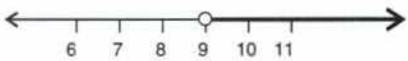
Exercise 15 (A)

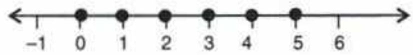
1. (i) {1, 2, 3, 4} (ii) {1, 2, 3, 4, 5, 6} (iii) {4, 5, 6, ...} (iv) {4, 5, 6, ...} **2.** (i) {6, 9} (ii) {−6, −3} **3.** {1, 2, 3, 4} **4.** {0, 1, 2, 3, ...} **5.** {0, 1, 2, 3, 4, 5, 6, 7} **6.** {0, 1, 2, 3, 4, 5} **7.** {−11, −12, −13, ...} **8.** {1, 2, 3, 4} **9.** {0, 1, 2, 3} **10.** {0, 1, 2} **11.** {1, 2, 3, 4} **12.** {4, 5, 6, 7} **13.** {1, 2, 3, 4, 5, ...} **14.** {0}

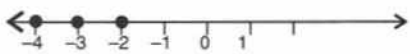
Exercise 15 (B)

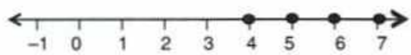
1. $x < 3$; 

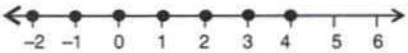
2. $x > 2$; 

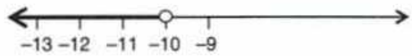
3. $x > 9$; 

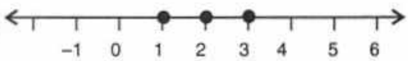
4. $x \le 5$; 

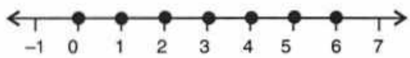
5. $x \le -2$; 

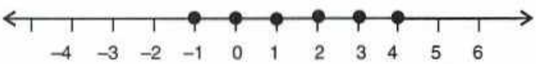
6. $x \ge 4$; 

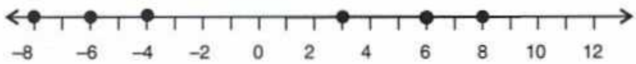
7. $x < 5$; 

8. $x < -10$; 

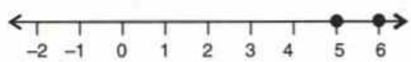
9. $x < 4$; 

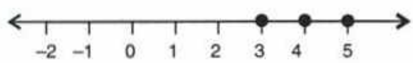
10. $x \le 6$; 

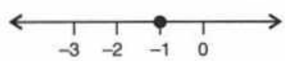
11. $x > -2$; 

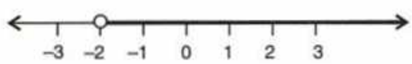
12. $x < 10$ 

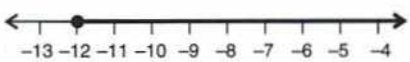
13. (i) $x \le 1, x \in W \Rightarrow$ Solution = {0, 1} ⇒ 

(ii) $x \ge 4\frac{1}{2}, x \in Z \Rightarrow$ Solution = {5, 6, ...} ⇒ 

(iii) $x \ge 2{\cdot}8, x \in W \Rightarrow$ Solution = {3, 4, 5, ...} ⇒ 

(iv) $x > -1{\cdot}5$, $x \in$ {negative integers} ⇒ Solution = {−1} ⇒ 

(v) $x > -2, x \in R \Rightarrow$ Solution = $\{x > -2, x \in R\}$ ⇒ 

(vi) Solution = $\{x \ge -12, x \in R\}$ ⇒ 

Exercise 16 (A)

1. (ii), (iii) and (v). (ii) concave (iii) concave **2.** (i) 1440° (ii) 1800° (iii) 3240° **3.** (i) 7 (ii) 11 (iii) 10 **4.** (i) No (ii) Yes (iii) No **5.** (i) 120° (ii) $\left(154\frac{2}{7}\right)^\circ$ **6.** (i) 360° (ii) 360° (iii) 360° **7.** 41°, 72°, 58°, 60°, 39° and 90° **8.** 80°, 100°, 120°, 140°, and 100° **9.** 110° **10.** (i) 540° (ii) ∠A + ∠E = 180° (iii) ∠B = 100°, ∠C = 120° and ∠D = 140° **11.** 5 **12.** ∠B = 216°, ∠C = 144°, ∠D = 72° and ∠E = 108° **13.** x = 70° **14.** 280°

Exercise 16 (B)

1. (i) 45° and 135° (ii) 30° and 150° (iii) 5 and 108° (iv) 8 and 135° (v) 12 and 30° (vi) 9 and 40° **2.** (i) 18 (ii) 8 (iii) 5 **3.** (i) 12 (ii) 10 **4.** (i) Yes (ii) No **5.** (i) No (ii) Yes **6.** 4 **7.** 8 **8.** (i) 150° (ii) 30° (iii) 12 **9.** (i) 60° (ii) 6 **10.** 10 **11.** 6 **12.** (i) 140° (ii) 40° (iii) 9 **13.** 8 **14.** (i) 108° (ii) 36° (iii) 72° **15.** n = 9 **16.** n = 5 **17.** 6 and 8 **18.** 61 **19.** (i) 12 (ii) 9 (iii) 3 **20.** 8

Exercise 16 (C)

1. 79° each **2.** 90° and 126° **3.** (i) x = 16° (ii) 64°, 90°, 92° and 114° **4.** (i) x = 22° (ii) ∠B = 48° and ∠C = 61° **5.** (i) ∠A = 60°, ∠B = 100°, ∠C = 80° and ∠D = 120° (ii) Trapezium **6.** (i) x = 26° (ii) ∠ABC = 104° (iii) ∠ACD = 28° **7.** 130° **8.** ∠b = 105° and ∠c = 140° **9.** 97° each **10.** ∠P = 54°, ∠Q = 72°, ∠R = 108° and ∠S = 126° (i) No (ii) Trapezium **11.** 40° **12.** ∠A = 80°, ∠B = 60° ∠C = 120° and ∠D = 100° **13.** 50°

Exercise 17

1. 135°, 45°, 135° and 45°, CD = 13 units **2.** 55° and 125° **3.** (i) 106° and 74° (ii) BC = CD = 7·5 cm **4.** (i) 8 (ii) 10 **5.** (i) 62° (ii) 28° **6.** (i) 38° (ii) 38° (iii) 104° **7.** 110° **8.** (i) 96° (ii) 29° (iii) 55° **11.** (i) Square (ii) Rhombus (iii) Rectangle **19.** x = 70°, y = 50° **20.** x = 11, y = 15° and z = 106° **21.** x = 27° and y = 63°

Exercise 18 (A)

5. ∠PBC = 45° **6.** (iii) Yes (iv) Yes

Exercise 18 (C)

5. Yes

Exercise 18 (D)

8. 90° **9.** 3 cm **10.** 5 cm

Exercise 19

1. 16 **2.** 15 **3.** (i) 6, 6, 10 (ii) 8, 12, 18 **5.** no **6.** (i) no (ii) yes (iii) yes **7.** (i) x = 7 (ii) y = 4 (iii) 38 **8.** 4, Tetrahedron **9.** yes **11.** a = 2, b = 1 and c = 3
12. (i) Triangular prism (ii) Triangular prism (iii) Hexagonal pyramid

Exercise 20 (A)

1. (i) 120 cm^2 (ii) 216 mm^2 (iii) 294 m^2 **2.** (i) 12 cm^2 (ii) 3 cm **3.** (i) 96 cm^2 (ii) 9·6 cm (iii) 16 cm **4.** (i) 14·4 m^2 (ii) 4·5 m **5.** 8 m and 10 m **6.** 15 m and 9 m **7.** 72 cm **8.** $12\sqrt{3}$ unit = 20·78 unit **9.** 816 m^2 **10.** 384 cm^2 **11.** 60 cm^2 **12.** 1800 m and 1500 m **13.** 384 cm^2 **14.** (i) 25 cm (ii) 84 cm^2 (iii) 6·7 cm

Exercise 20 (B)

1. 15 cm and 46 cm **2.** (i) 8 m (ii) 120 m^2 (iii) 17 m **3.** (i) 30 cm (ii) 480 cm^2 **4.** 38 m **5.** 81 cm^2 **6.** 5·2 m **7.** 72 cm^2 and 8·49 cm **8.** 10·6 m and 42·4 m **9.** (i) 13 cm (ii) 52 cm **10.** 144 cm^2 **11.** 16 sq. units **12.** (i) 1 : 2 (ii) 1 : 4 **13.** (i) 3·8 m^2 (ii) 18·72 sq. m **14.** 288 m^2 **15.** 260 m^2 **16.** (i) 3888 and ₹ 23,328 (ii) 2520 and ₹ 15,120 **17.** 123·75 m^2 **18.** ₹ 17,550 **19.** (i) 700 m^2 (ii) 14,000 (iii) ₹ 1,96,000

Exercise 20 (C)

1. 10·24 sq m **2.** 12 dm and 16 dm **3.** 10 cm **4.** 7·5 cm **5.** 10 cm **6.** 588 cm^2 **7.** (i) 216 cm^2 (ii) 15 cm (iii) 60 cm **8.** (i) 12 cm (ii) 96 cm^2 **9.** 216 cm^2 **10.** 20 cm **11.** ₹ 1,152; 9·6 m **12.** 13 cm and 18 cm **13.** 20 cm **14.** 120 cm^2

Exercise 20 (D)

1. (i) 21 cm and 1386 cm^2 (ii) 3·5 m and 38·5 m^2 **2.** (i) 7 cm and 44 cm (ii) 1·4 m and 8·8 m **3.** 616 m^2 **4.** 132 cm **5.** 176 cm^2 **6.** (i) 42 cm^2 (ii) 44 m^2 **7.** (i) 2926 m^2 (ii) 44 m **8.** $3221\frac{3}{7}$ m^2 **9.** 4774 m^2 **10.** (i) 22 cm (ii) 88 cm (iii) 616 cm^2 **11.** 249·45 cm^2 **12.** (i) 2·64 m (ii) 132 m (iii) 528 m

13. 1100 m (i) $3\frac{2}{3}$ m/s (ii) 13·2 kmh^{-1} **14.** 28·512 kmh^{-1} **15.** (i) 4·4 m (ii) 14 **16.** 2464 cm^2 **17.** 21 cm and 693 cm^2 **18.** 126 cm^2

Exercise 21 (A)

1. (i) 1200 cm^3 ; 700 cm^2 (ii) 8·19 m^3 ; 29·18 m^2 **2.** (i) 8 cm (ii) 2·4 m (iii) 2 m **3.** 10 cm, 6 cm and 4 cm ; 248 cm^2 **4.** 12 cm, 10 cm and 6 cm ; 720 cm^3 **5.** (i) 512 cm^3 ; 384 cm^2 (ii) 13·824 m^3 ; 34·56 m^2 **6.** (i) 6 cm (ii) 1·2 m **7.** 216 cm^3 **8.** 1728 cm^3 **9.** 2400 **10.** 343 **11.** ₹ 846 **12.** length = 36 cm, breadth = 9 cm and height = 9 cm. Total surface area = 1458 cm^2; Volume = 2916 cm^3.

Exercise 21 (B)

1. 3000 **2.** 375 **3.** (i) 91·2 m^2 (ii) 32·4 m^2 **4.** (i) 63·3 m^2 (ii) ₹ 284.85 (iii) ₹ 202.50 **5.** (i) ₹ 50889.60 (ii) ₹ 1,12,500 **6.** 57824 cm^3 **7.** (i) 45500 cm^3 (ii) 4396 cm^3 (iii) 15825·6 g **8.** (i) 0·2 m^3 (ii) 0·06775 m^3 **9.** 208 m^2 **10.** ₹ 78390

Exercise 21 (C)

1. 12 cm **2.** $x = 8$ **3.** (i) 8 cm (ii) 384 cm^2 (iii) 512 cm^3 **4.** (i) $x = 3·6$ (ii) 256·8 cm^2 (iii) 216 cm^3 (iv) cuboid, by 40·8 cm^3 **5.** 2 m **6.** 32 cm **7.** (i) 9 : 4 (ii) 27 : 8 **8.** (i) 8232 cm^3 (ii) 1300 m^2

Exercise 21 (D)

1. (i) 3080 cm^3 (ii) 1188 cm^2 **2.** 660 cm^2 and 968 cm^2 **3.** 18 cm **4.** 2 cm **5.** 1 : 1 **6.** 12320 cm^3

7. (i) 14 cm (ii) 36960 cm^3 **8.** 30 cm and 4620 cm^3 **9.** (i) 5·28 m^2 (ii) ₹ 2280.96 **10.** 10 : 9

Exercise 21 (E)

1. (i) 332 m^2 (ii) 140 m^2 **2.** 6400 **3.** 720 cm^3 **4.** 42090 cm^3 and 13160 cm^3 **5.** 14 cm and 3 cm

6. 6 m **7.** 216 m^3 **8.** 6 cm, 1 : 2 **9.** 440 m^2 **10.** ₹ 1,351·68

Exercise 22 (A)

1. (i) 5, 7, 10, 11, 12, 15, 16 (ii) 4·7, 5·6, 5·9, 6·3, 9·8, 12·3

2. (i) 5, 4, 3, 3, 2, 2, 1, 0 (ii) 11·5, 10·6, 9·1, 8·3, 5·6, 3·7

3. (i)

Date	Tally Marks	Frequency
5	IIII	4
6	IIII	4
7	III	3
8	卌	5
9	III	3
10	III	3

(ii)

Date	Tally Marks	Frequency
1	III	3
2	卌	5
3	II	2
4	IIII	4
5	IIII	4

4.

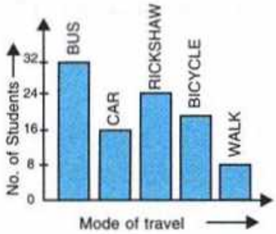
C.I.	Tally Marks	Frequency
0–10	III	3
10–20	卌 II	7
20–30	卌 IIII	9
30–40	卌 I	6
40–50	卌	5

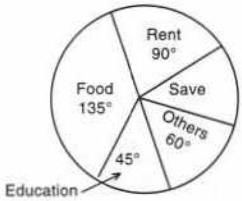
5.

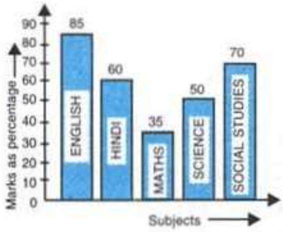
C.I.	Tally Marks	Frequency
4–6	卌 II	7
6–8	卌 III	8
8–10	卌 卌	10
10–12	卌 I	6
12–14	III	3
14–16	卌 I	6

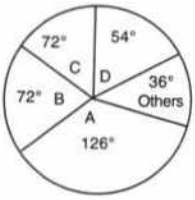
6. (i) 15 (ii) 30 (iii) 12·5 (iv) 10–16 (v) 7·5 – 12·5

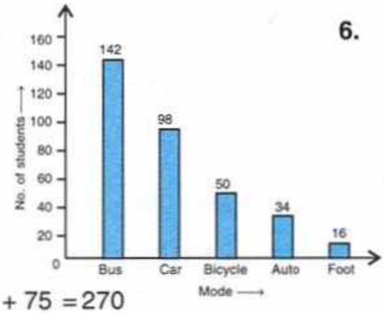
Exercise 22 (B)

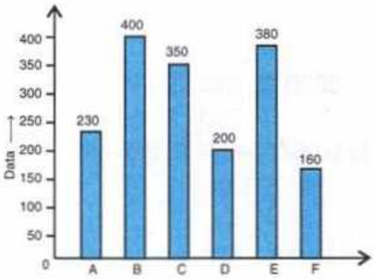
1. 

2. 

3. 

4. 

5. 

6. 

8. $\because$ $60 + 45 + 42 + 48 + 75 = 270$

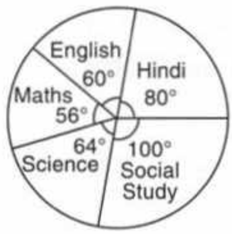
$\therefore$ Central angle for Hindi $= \dfrac{60}{270} \times 360° = 80°$,

Central angle for English $= \dfrac{45}{270} \times 360° = 60°$,

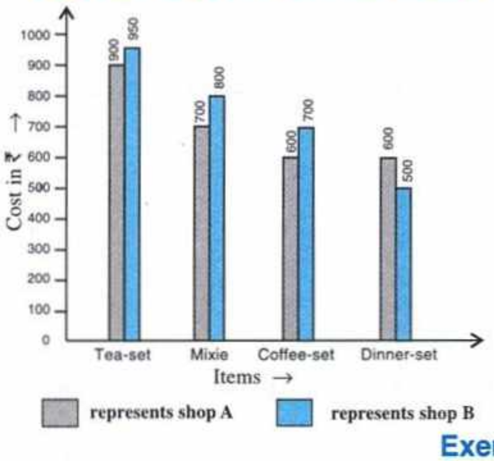
Central angle for Maths $= \dfrac{42}{270} \times 360° = 56°$,

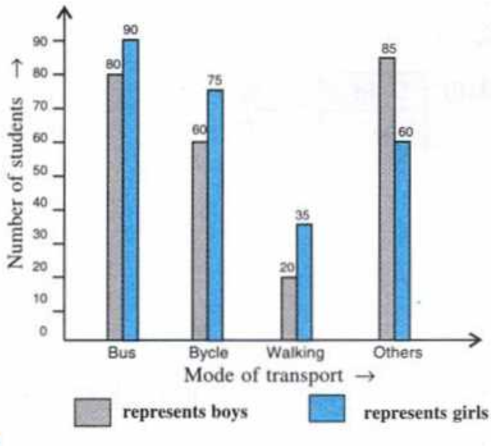
Central angle for Science $= \dfrac{48}{270} \times 360° = 64°$,

and, central angle for Social study $= \dfrac{75}{270} \times 360° = 100°$,



$\therefore$ **Required Pie-graph is as shown alongside :**

9. 

10. 

Exercise 23

1. (i) $\frac{1}{2}$ (ii) $\frac{1}{3}$ (iii) $\frac{2}{3}$ **2.** (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ **3.** (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{4}$ (iv) $\frac{1}{4}$ **4.** $\frac{2}{3}$ **5.** (i) $\frac{1}{3}$ (ii) $\frac{2}{3}$ (iii) 0 **6.** (i) $\frac{2}{3}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{3}$ **7.** (i) $\frac{1}{2}$ (ii) $\frac{1}{5}$ (iii) $\frac{3}{10}$ (iv) $\frac{4}{5}$ (v) $\frac{1}{2}$ **8.** (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{2}$ **9.** (i) 0 (ii) 0 (iii) 1 **10.** (ii), (iv) and (vi) **11.** (i) 0 (ii) 1 **12.** (i) $\frac{1}{8}$ (iii) $\frac{3}{8}$ (iii) $\frac{3}{8}$ (iv) $\frac{1}{8}$ **13.** $\frac{5}{46}$ **14.** (i) $\frac{3}{4}$ (ii) $\frac{1}{2}$ **15.** (i) $\frac{1}{2}$ (ii) $\frac{3}{10}$ (iii) $\frac{1}{10}$ (iv) $\frac{7}{10}$ **16.** (i) 0 (ii) $\frac{1}{36}$ (iii) $\frac{35}{36}$ (iv) 1 **17.** (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{2}{3}$ (iv) $\frac{5}{6}$ (v) 0 **18.** (i) $\frac{1}{2}$ (ii) $\frac{3}{4}$ (iii) $\frac{1}{4}$ (iv) $\frac{3}{4}$