MODEL LESSON

MATHEMATICS

CHAPTER NUMBER :~ 2
CHAPTER NAME :~ POLYNOMIALS
SUB TOPIC :~ REMAINDER THEOREM

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

PREVIOUS KNOWLEDGE TEST

Divide: $3x^4 - 4x^3 - 3x - 1$ by x − 1

LEARNING OUTCOME:~

Students will learn

a) Remainder theorem.

Exercise-2.3

Question 1.

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$

(iii) x

Solution:

Let $p(x) = x^3 + 3x^2 + 3x + 1$

(i) The zero of $x + 1$ is -1.

$\therefore p(-1) = (-1)3 + 3(-1)2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1 = 0$

Thus, the required remainder = 0

(iii) The zero of x is 0.

$\therefore p(0) = (0)^3 + 3(0)^2 + 3(0) + 1$

$= 0 + 0 + 0 + 1 = 1$

Thus, the required remainder = 1.

(iv) $x + \pi$

The zero of $x + \pi$ is $-\pi$.

$p(-\pi) = (-\pi)3 + 3(-\pi)22 + 3(-\pi) + 1$

$= -\pi3 + 3\pi2 + (-3\pi) + 1$

$= -\pi3 + 3\pi2 - 3\pi + 1$

Thus, the required remainder is $-\pi3 + 3\pi2 - 3\pi + 1$.

https://www.youtube.com/watch?v=F6onUHbWCus

"As great a genius as Archimedes could not invent analytical geometry, for the algebraic knowledge necessary for such as achievement was not available in his time..."

~ *Nathan A. Court...*

Remainder Theorem

In division, **Dividend = (Divisor X Quotient) + Remainder**
For example- When 15 is divided by 4 then we get,
15 = (4 X 3) + 3

In polynomials, Division is carried out in similar way.

Step by Step Guide for Division of Polynomials

Proceeding by way of an example.

Divide $x^2 + 2x - 7$ by $x - 2$

Here, Dividend: $x^2 + 2x - 7$ and Divisor: $x - 2$

Step 1- Arrange the terms of the polynomials (dividend and divisor) in descending order of their degrees.

Here, Dividend: $x^2 + 2x - 7$ and Divisor: $x - 2$

Step 2- Write down the problem in the standard form i.e. $\frac{Dividend}{Divisor}$

Here, $\frac{x^2+2x-7}{x-2}$

Step 3- Divide first term of the the dividend by first term of the divisor, to get the first term of the quotient.

Here, x^2 divided by x equals to x, which becomes the first term of the quotient

Step 4- Multiply this first term of the quotient obtained by the divisor and subtract it from the dividend which becomes the remainder

Here, $x \times (x - 2) = x^2 - 2x$ and then $(x^2 + 2x - 7) - (x^2 - 2x) = 4x - 7$

So the remainder is $4x - 7$

Step 5- Treat the remainder obtained as the new dividend and repeating the above steps using the divisor $x - 2$.

Proceeding like this till we get remainder 0 or the degree of the remainder polynomial is less than the degree of the divisor, we are done with the division.

What we have actually done is-

$$\begin{array}{r|l} & x + 4 \\ \hline x-2 & x^2 + 2x - 7 \\ & \underline{x^2 - 2x} \\ & \quad 4x - 7 \\ & \quad \underline{4x - 8} \\ & \qquad\quad 1 \end{array}$$

We can write-

$x^2 + 2x - 7 = [(x-2) \times (x+4)] + (1)$

In general, we can say that if p(x) and g(x) are polynomials such that degree of p(x) is greater than or equal to the degree of g(x) and g(x) is non-zero, then there exist polynomials q(x) and r(x) such that-

$$p(x) = [g(x) \times q(x)] + r(x)$$

where r(x) = 0 or degree of r(x) < degree of g(x)

We say that p(x) is divided by g(x) and q(x) → Quotient and r(x) → Remainder

Note that p(2) = 1 i.e. the value of the polynomial at the zero of the divisor (2) is equal to the remainder (1).

This is true for all polynomials when divided by a linear polynomial.

Formally this is the Remainder Theorem.

<u>Remainder Theorem</u>- Let p(x) be any polynomial of degree greater than or equal to one and let a be any real number. If p(x) is divided by the linear polynomial x – a , then the remainder is p(a).

Proof-

Let $p(x)$ be a polynomial of degree greater than or equal to one and let a be any real number.

Now, suppose $p(x)$ is divided by the linear polynomial $x - a$, then there exist polynomials $q(x)$ and $r(x)$ such that

$$p(x) = [(x - a) \times q(x)] + r(x)$$

where degree of $r(x)$ < degree of $(x - a)$

Since degree of $(x - a)$ is 1

So, degree of $r(x) = 0$

This implies that $r(x)$ is a constant, (say) r

i.e. $r(x) = r$

Now, $p(x) = [(x - a) \times q(x)] + r$

In particular for $x = a$

$p(a) = [(a - a) \times q(x)] + r$

$\quad = r$

Therefore, $p(a)$ is the remainder.

Hence Proved

Evaluation:-

a) Find the remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by x-1 .

HOMEWORK:-

EXERCISE - 2.3

QUESTION NUMBER-2,3

AHA:~

1.If the polynomials $ax^3 + 4x^2 + 3x - 4an$ d$x^3 - 4x + a$ leave the same remainder when divided by x~3, find ‘a’.

2.If the p(x)= $x^3 + 3x^2 + 3x + 1$ is divided by x+π, find the remainder.

THANKING YOU

ODM EDUCATIONAL GROUP