ODM EDUCATIONAL GROUP
Changing your Tomorrow

MODEL LESSON

MATHEMATICS

CHAPTER NUMBER :~ 2
CHAPTER NAME :~ POLYNOMIALS
SUB TOPIC :~ FACTOR THEOREM

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PREVIOUS KNOWLEDGE TEST

a) Find the remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by x-1

LEARNING OUTCOME:~

Students will learn

a) factor theorem.

Question 2.

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Solution:

We have, $p(x) = x^3 - ax^2 + 6x - a$ and zero of $x - a$ is a.

$\therefore p(a) = (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a = 5a$

Thus, the required remainder is 5a.

Question 3.

Check whether 7 + 3x is a factor of 3x3+7x.

Solution:

We have, p(x) = 3x3+7x. and zero of 7 + 3x is −73.

$$\therefore \ p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right)$$

$$= 3\left(\frac{-343}{27}\right) + \left(\frac{-49}{3}\right) = -\frac{343}{9} - \frac{49}{3} = -\frac{490}{9}$$

Since,(−4909) ≠ 0

i.e. the remainder is not 0.

∴ 3x3 + 7x is not divisib1e by 7 + 3x.

Thus, 7 + 3x is not a factor of 3x3 + 7x.

https://www.youtube.com/watch?v=4YXE7HDyInM

"As great a genius as Archimedes could not invent analytical geometry, for the algebraic knowledge necessary for such as achievement was not available in his time…"

~ *Nathan A. Court…*

Factorization of Polynomials

Factor Theorem- If p(x) is a polynomial of degree n where n is greater than or equal to one and a is any real number then-

1) x – a is a factor of p(x) if p(a) = 0
2) p(a) = 0 if x – a is a factor of p(x)

Proof-

Using Remainder theorem, we can write-

$$p(x) = [(x - a) \times q(x)] + p(a) \qquad \text{Here, } r(x) = r = p(a)$$

(1) Suppose, p(a) = 0 . Then p(x) = (x – a) X q(x)
Clearly, x – a is a factor of p(x)

(2)Suppose x – a is a factor of p(x). Then we can write, p(x) = (x – a) X g(x) for some polynomial g(x). So, p(a) = 0

Applications of Factor Theorem

Example 1. Find the value of k, if x – 1 is a factor of $4x^3 + 3x^2 - 4x + k$

Solution: By Factor theorem, p(1) = 0

p(1) = 4 (1) + 3 (1) – 4 (1) + k = 0

K = - 3

Factorization of Quadratic polynomials using Splitting the middle term method

Consider the quadratic polynomial $ax^2 + bx + c$
Suppose (p x + q) and (r x + s) be its factors.
So we can write, $ax^2 + bx + c = (px + q) X (rx + s)$
On comparing the coefficients of the like terms in L.H.S and R.H.S we get,
a = p r , b = p s + q r , c = q s
And clearly, (p s) X (q r) = (p r) X (q s) = (a c) .

So, to factorize the quadratic polynomial, we need to write b as the sum of two numbers whose product is (a c)

For example- Factorize $6x^2 + 17x + 5$ by-
1) Splitting the middle term method
2) Factor theorem

Solution-
1) We can write $6x^2 + 17x + 5 = 6x^2 + (15 + 2)x + 5$
$= 6x^2 + 15x + 2x + 5$
$= 3x(2x + 5) + 1(2x + 5)$
$= (3x + 1)(2x + 5)$

2) We can write $6x^2 + 17x + 5 = 6(x^2 + (17/6)x + (5/6))$
Taking, $p(x) = x^2 + (17/6)x + (5/6)$
So, $6x^2 + 17x + 5 = 6\,p(x)$
Suppose a and b are zeroes of p(x). Then x – a and x – b are factors of p(x) and we have
$p(x) = (x - a)(x - b)$
So, $6x^2 + 17x + 5 = 6(x - a)(x - b)$ ⟶ **(*)**
On comparing the coefficients of like terms, we get, $ab = 5/6$
Possibilities for the values of a and b satisfying $ab = 5/6$ are- $\frac{\pm 1}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{5}{2}, \pm 1$

Also, these values of a and b must satisfy $p(a) = 0$ and $p(b) = 0$
So, by checking all the values found above, we get that $a = -1/3$ and $b = -5/2$ satisfies the above conditions. So, **(*)**, becomes- $6x^2 + 17x + 5 = (3x + 1)(2x + 5)$

To factorize a cubic polynomial p(x) , we first find a zero of the polynomial by hit and trial method (by putting different values of x in the polynomial equation p(x) = 0)

Then the obtained root of the polynomial equation say x = a (or simply, the zero of the polynomial p(x)) implies that x – a is a factor of p(x).

So we can write, $p(x) = (x - a)\, g(x) \longrightarrow$ **(**)**

where g(x) is a polynomial such that the degree of g(x) < degree of p(x)

Here, as degree of p(x) = 3 So, degree of g(x) = 2 i.e. g(x) is a quadratic polynomial and using factor theorem or splitting the middle term method we can factorize g(x) and thus we get the desired factorization by putting the resultant g(x) in **(**)**

Evaluation:-

1. Find the value of 'k' if x-1is a factor of $4x^3 + 3x^2 - 4x - k$.

2. Factorize $6x^2 + 17x + 5$ by splitting the middle term and by using factor theorem.

HOMEWORK:-EXERCISE – 2.4
QUESTION NUMBER-1,2,3

AHA:~

1. For what value of ‘a’ is $2x^3 + ax^2 - 11x + a + 3$ is exactly divisible by (2x − 1)?

2. Find the value of ‘a’ and ‘b’ so that the polynomial $x^3 + 10x^2 + ax + a$ is exactly divisible by x − 1 and x −2.

THANKING YOU

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