

PERIOD~8

MATHEMATICS

CHAPTER NUMBER :~ 2

CHAPTER NAME :~ POLYNOMIALS

SUB TOPIC :~ APPLICATIONS OF ALGEBRAIC IDENTITIES

CHANGING YOUR TOMORROW

PREVIOUS KNOWLEDGE TEST

Algebraic Identities

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$

LEARNING OUTCOME:~

Students will learn

a) Applications of Algebraic identities.

EXERCISE-2.5

Question 1.

Use suitable identities to find the following products

(i) $(x + 4)(x + 10)$

(ii) $(x+8) (x -10)$

(iii) $(3x + 4) (3x - 5)$

Solution:

(i) We have, $(x+ 4) (x + 10)$

Using identity,

$$(x+ a) (x+ b) = x^2 + (a + b) x+ ab.$$

$$\begin{aligned} \text{We have, } (x + 4) (x + 10) &= x^2+(4 + 10) x + (4 \times 10) \\ &= x^2 + 14x+40 \end{aligned}$$

(ii) We have, $(x + 8) (x - 10)$

Using identity,

$$(x + a) (x + b) = x^2 + (a + b) x + ab$$

$$\begin{aligned} \text{We have, } (x + 8) (x - 10) &= x^2 + [8 + (-10)] x + (8) (-10) \\ &= x^2 - 2x - 80 \end{aligned}$$

(iii) We have, $(3x + 4) (3x - 5)$

Using identity,

$$(x + a) (x + b) = x^2 + (a + b) x + ab$$

$$\begin{aligned} \text{We have, } (3x + 4) (3x - 5) &= (3x)^2 + (4 - 5) x + (4) (-5) \\ &= 9x^2 - x - 20 \end{aligned}$$

Question 2.

Evaluate the following products without multiplying directly

(i) 103×107

(ii) 95×96

(iii) 104×96

Solution:

(i) We have, $103 \times 107 = (100 + 3) (100 + 7)$

$$= (100)^2 + (3 + 7) (100) + (3 \times 7)$$

[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$$= 10000 + (10) \times 100 + 21$$

$$= 10000 + 1000 + 21 = 11021$$

(ii) We have, $95 \times 96 = (100 - 5) (100 - 4)$
 $= (100)^2 + [(-5) + (-4)] 100 + (-5 \times -4)$
[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
 $= 10000 + (-9) + 20 = 9120$
 $= 10000 + (-900) + 20 = 9120$

(iii) We have $104 \times 96 = (100 + 4) (100 - 4)$
 $= (100)^2 - 4^2$
[Using $(a + b)(a - b) = a^2 - b^2$]
 $= 10000 - 16 = 9984$

Question 3.

Factorise the following using appropriate identities

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - y^2$

Solution:

(i) We have, $9x^2 + 6xy + y^2$

$$= (3x)^2 + 2(3x)(y) + (y)^2$$

$$= (3x + y)^2$$

[Using $a^2 + 2ab + b^2 = (a + b)^2$]

$$= (3x + y)(3x + y)$$

(ii) We have, $4y^2 - 4y + 1$

$$= (2y)^2 + 2(2y)(-1) + (-1)^2$$

$$= (2y - 1)^2$$

[Using $a^2 - 2ab + b^2 = (a - b)^2$]

$$= (2y - 1)(2y - 1)$$

Question 4.

Expand each of the following, using suitable identity

(i) $(x+2y+ 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(- 2x + 3y + 2z)^2$

(iv) $(3a -7b - c)^2$

(v) $(- 2x + 5y - 3z)^2$

Solution:

We know that

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

(i) $(x + 2y + 4z)^2$

$$= x^2 + (2y)^2 + (4z)^2 + 2 (x) (2y) + 2 (2y) (4z) + 2(4z) (x)$$

$$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8 zx$$

$$\begin{aligned} \text{(ii)} \quad (2x - y + z)^2 &= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x) \\ &= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (-2x + 3y + 2z)^2 &= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x) \\ &= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (3a - 7b - c)^2 &= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a) \\ &= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad (-2x + 5y - 3z)^2 &= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x) \\ &= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx \end{aligned}$$

Question 5.

Factorise

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Solution:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$$

$$= (2x + 3y - 4z)^2 = (2x + 3y + 4z)(2x + 3y - 4z)$$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(-\sqrt{2}x)$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$$

Question 6.

Write the following cubes in expanded form
Solution:

$$\text{We have, } (x + y)^3 = x^3 + y^3 + 3xy(x + y) \dots(1)$$

$$\text{and } (x - y)^3 = x^3 - y^3 - 3xy(x - y) \dots(2)$$

$$\begin{aligned} \text{(i) } (2x + 1)^3 &= (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1) \text{ [By (1)]} \\ &= 8x^3 + 1 + 6x(2x + 1) \\ &= 8x^3 + 12x^2 + 6x + 1 \end{aligned}$$

$$\begin{aligned} \text{(ii) } (2a - 3b)^3 &= (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b) \text{ [By (2)]} \\ &= 8a^3 - 27b^3 - 18ab(2a - 3b) \\ &= 8a^3 - 27b^3 - 36a^2b + 54ab^2 \end{aligned}$$

Question 7.

Evaluate the following using suitable identities

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Solution:

(i) We have, $99 = (100 - 1)$

$$\therefore 99^3 = (100 - 1)^3$$

$$= (100)^3 - 1^3 - 3(100)(1)(100 - 1)$$

[Using $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]

$$= 1000000 - 1 - 300(100 - 1)$$

$$= 1000000 - 1 - 30000 + 300$$

$$= 1000300 - 30001 = 970299$$

(ii) We have, $102 = 100 + 2$

$$\therefore 102^3 = (100 + 2)^3$$

$$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$$

$$[\text{Using } (a + b)^3 = a^3 + b^3 + 3ab(a + b)]$$

$$= 1000000 + 8 + 600(100 + 2)$$

$$= 1000000 + 8 + 60000 + 1200 = 1061208$$

(iii) We have, $998 = 1000 - 2$

$$\therefore (998)^3 = (1000 - 2)^3$$

$$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$$

$$[\text{Using } (a - b)^3 = a^3 - b^3 - 3ab(a - b)]$$

$$= 1000000000 - 8 - 6000(1000 - 2)$$

$$= 1000000000 - 8 - 6000000 + 12000$$

$$= 994011992$$

<https://www.youtube.com/watch?v=C46fQQeHgQE>

“As great a genius as Archimedes could not invent analytical geometry, for the algebraic knowledge necessary for such an achievement was not available in his time...”

~ Nathan A. Court...

Application of Algebraic Identities

- To find- 99×101 without actual multiplication

Solution: We can write, $99 \times 101 = (100 - 1)(100 + 1)$

$$\begin{aligned} &= (100)^2 - (1)^2 \\ &= 10000 - 1 \\ &= 9999 \end{aligned}$$

- Evaluate: $(999)^3$

Solution: We can write, $(999)^3 = (1000 - 1)^3$

$$\begin{aligned} &= (1000)^3 - (1)^3 - 3(1000)(1)(1000 - 1) \\ &= 997002999 \end{aligned}$$

Evaluation:~

1. Factorize: $8x^3 + 27y^3 + 36x^2y + 54xy^2$.

HOMEWORK:-
EXERCISE - 2.5
QUESTION NUMBER-8 TO 16

AHA:~

1. $a+b+c=6$, $ab+bc+ca=11$,

Find $a^3+b^3+c^3-3abc$.

2. $x+1/x=3$, Find x^2+1/x^2 .

THANKING YOU
ODM EDUCATIONAL GROUP