PERIOD-9

MATHEMATICS

CHAPTER NUMBER :~ 2
CHAPTER NAME :~ POLYNOMIALS
SUB TOPIC :~ RECAPITULATION OF POLYNOMIAL

PREVIOUS KNOWLEDGE TEST

FACTORISE

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

LEARNING OUTCOME:-

1.Students will learn

Polynomials in one variable ,Term,Coefficient,General form and types of polynomials

2. Students will learn about Zeroes of the Polynomials

3.Students will learn about Division of Polynomials Remainder theorem ,

Factorization of Polynomials (Factor Theorem)

Factorization of Polynomials using Factor Theorem and splitting the middle term,

Algebraic identities

Application of Algebraic identities.

Question 8.

Factorise each of the following

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

Solution:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 6ab(2a + b)$

$= (2a)^3 + (b)^3 + 3(2a)(b)(2a + b)$

$= (2 a + b)^3$

[Using $a^3 + b^3 + 3\ ab(a + b) = (a + b)^3$]

$= (2a + b)(2a + b)(2a + b)$

(ii) $8a^3 - b^3 - 12o^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 3(2a)(b)(2a - b)$

$= (2a - b)^3$

[Using $a^3 + b^3 + 3\ ab(a + b) = (a + b)^3$]

$= (2a - b)\ (2a - b)\ (2a - b)$

$(iii)\ 27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3(3)(5a)(3 - 5a)$

$= (3 - 5a)^3$

[Using $a^3 + b^3 + 3\ ab(a + b) = (a + b)^3$]

$= (3 - 5a)\ (3 - 5a)\ (3 - 5a)$

$(iv)\ 64a^3 \sim 27b^3 \sim 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$

$= (4a - 3b)^3$

[Using $a^3 - b^3 - 3\ ab(a - b) = (a - b)^3$]

$= (4a - 3b)(4a - 3b)(4a - 3b)$

Question 9.
Verify

(i) $x^3 + y^3 = (x + y)\text{-}(x^2 – xy + y^2)$
(ii) $x^3 – y^3 = (x – y)\ (x^2 + xy + y^2)$

Solution:

(i) $\because (x + y)^3 = x^3 + y^3 + 3xy(x + y)$
$\Rightarrow (x + y)^3 – 3(x + y)(xy) = x^3 + y^3$
$\Rightarrow (x + y)[(x + y)2\text{-}3xy] = x^3 + y^3$
$\Rightarrow (x + y)(x^2 + y^2 – xy) = x^3 + y^3$
Hence, verified.

(ii) $\because (x – y)^3 = x^3 – y^3 – 3xy(x – y)$
$\Rightarrow (x – y)^3 + 3xy(x – y) = x^3 – y^3$
$\Rightarrow (x – y)[(x – y)^2 + 3xy)] = x^3 – y^3$
$\Rightarrow (x – y)(x^2 + y^2 + xy) = x^3 – y^3$
Hence, verified.

Question 10.
Factorise each of the following

(i) $27y^3 + 125z^3$
(ii) $64m^3 - 343n^3$

Solution:

(i) We know that
$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
We have, $27y^3 + 125z^3 = (3y)^3 + (5z)^3$
$= (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$
$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) We know that
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
We have, $64m^3 - 343n^3 = (4m)^3 - (7n)^3$
$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$
$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

Question 11.

Factorise $27x^3 + y^3 + z^3 - 9xyz$.

Solution:

We have,

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

Using the identity,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

We have, $(3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)[(3x)^3 + y^3 + z^3 - (3x \times y) - (y \times 2) - (z \times 3x)]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

Question 12.

Verify that

$x^3 + y^3 + z^3 - 3xyz = 12\ (x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Solution:

R.H.S

$= 12(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= 12\ (x + y + 2)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (z^2 + x^2 - 2zx)]$

$= 12\ (x + y + 2)(x^2 + y^2 + y^2 + z^2 + z^2 + x^2 - 2xy - 2yz - 2zx)$

$= 12\ (x + y + z)[2(x^2 + y^2 + z^2 - xy - yz - zx)]$

$= 2\ x\ 12\ x\ (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= x^3 + y^3 + z^3 - 3xyz =$ L.H.S.

Hence, verified.

Question 13.

If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3\ xyz$.

Solution:

Since, $x + y + z = 0$

$\Rightarrow x + y = -z\ (x + y)^3 = (-z)^3$

$\Rightarrow x^3 + y^3 + 3xy(x + y) = -z^3$

$\Rightarrow x^3 + y^3 + 3xy(-z) = -z^3$ [$\because x + y = -z$]

$\Rightarrow x^3 + y^3 - 3xyz = -z^3$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$

Hence, if $x + y + z = 0$, then

$x^3 + y^3 + z^3 = 3xyz$

Question 14.

Without actually calculating the cubes, find the value of each of the following

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Solution:

(i) We have, $(-12)^3 + (7)^3 + (5)^3$

Let x = -12, y = 7 and z = 5.

Then, x + y + z = -12 + 7 + 5 = 0

We know that if x + y + z = 0, then, $x^3 + y^3 + z^3 = 3xyz$

$\therefore (-12)^3 + (7)^3 + (5)^3 = 3[(-12)(7)(5)]$

= 3[-420] = -1260

(ii) We have, $(28)^3 + (-15)^3 + (-13)^3$

Let x = 28, y = -15 and z = -13.

Then, x + y + z = 28 – 15 – 13 = 0

We know that if x + y + z = 0, then $x^3 + y^3 + z^3 = 3xyz$

$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

= 3(5460) = 16380

Question 15.

Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given

(i) Area $25a^2 – 35a + 12$

(ii) Area $35y^2 + 13y – 12$

Solution:

Area of a rectangle = (Length) x (Breadth)

(i) $25a^2 – 35a + 12 = 25a^2 – 20a – 15a + 12 = 5a(5a – 4) – 3(5a – 4) = (5a – 4)(5a – 3)$

Thus, the possible length and breadth are $(5a – 3)$ and $(5a – 4)$.

(ii) $35y^2 + 13y - 12 = 35y^2 + 28y – 15y - 12$

$= 7y(5y + 4) – 3(5y + 4) = (5y + 4)(7y – 3)$

Thus, the possible length and breadth are $(7y – 3)$ and $(5y + 4)$.

Question 16.

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume $3x^2 - 12x$

(ii) Volume $12ky^2 + 8ky - 20k$

Solution:

Volume of a cuboid = (Length) x (Breadth) x (Height)

(i) We have, $3x^2 - 12x = 3(x^2 - 4x)$

$= 3 \times x \times (x - 4)$

∴ The possible dimensions of the cuboid are 3, x and $(x - 4)$.

Solution:

Volume of a cuboid = (Length) x (Breadth) x (Height)

(ii) We have, $12ky^2 + 8ky - 20k$

$= 4[3ky^2 + 2ky - 5k] = 4[k(3y^2 + 2y - 5)]$

$= 4 \times k \times (3y^2 + 2y - 5)$

$= 4k[3y^2 - 3y + 5y - 5]$

$= 4k[3y(y - 1) + 5(y - 1)]$

$= 4k[(3y + 5) \times (y - 1)]$

$= 4k \times (3y + 5) \times (y - 1)$

Thus, the possible dimensions of the cuboid are 4k, (3y + 5) and (y - 1).

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"As great a genius as Archimedes could not invent analytical geometry, for the algebraic knowledge necessary for such as achievement was not available in his time…"

~ *Nathan A. Court…*

Evaluation:-

a) Without calculating cubes find $(-12)^3+(5)^3+(7)^3$.

b) Find 'k' if $x+3$ is a factor of $3x^2+kx+6$.

c) If $a+b+c=0$, prove that $a^3+b^3+c^3=3abc$.

HOMEWORK:-

REVISE THE CHAPTER -2

AHA:~

1. $a-b=5$, $ab=12$ Find a^2+b^2.
2. If $x^3+ax^2-bx+10$ is divisible by x^3-3x+2 find 'a' and 'b'.

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THANKING YOU

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