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PERIOD 6

MATHEMATICS

CHAPTER NUMBER :~ 8
CHAPTER NAME :~QUADRILATERALS

PREVIOUS KNOWLEDGE TEST

1. A diagonal of a parallelogram bisects one of its angles. Prove that it bisects the opposite angle also.

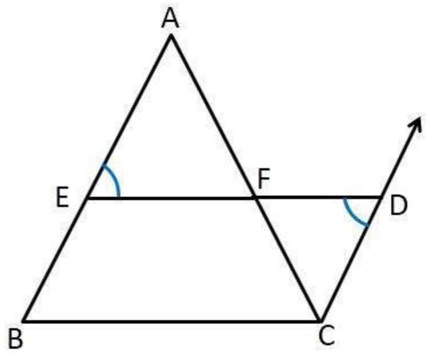
LEARNING OUTCOME:~

1.Students will be able to learn the midpoint theorem.

2.Students will be able to solve the sums related to midpoint theorem.

Theorem 8.9

The line segment joining the mid-points of two sides of a triangle is parallel to the third side.



Given : ABCD is a triangle where E and F are mid points of AB and AC respectively

To Prove : EF || BC

Construction : Through C draw a line segment parallel to AB & extend EF to meet this line at D.

Proof : Since AB || CD (By construction)

with transversal ED.

∠AEF = ∠CDF *(Alternate angles)* ...(1)

In $\triangle AEF$ and $\triangle CDF$

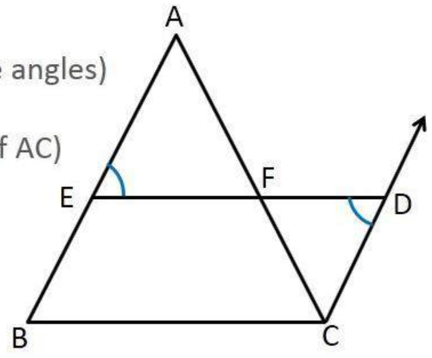
$\angle AEF = \angle CDF$ (From (1))

$\angle AFE = \angle CFD$ (Vertically opposite angles)

$AF = CF$ (As F is mid point of AC)

$\therefore \triangle AEF \cong \triangle CDF$ (AAS rule)

So, $EA = DC$ (CPCT)



But, $EA = EB$ (E is mid point of AB)

Hence, $EB = DC$

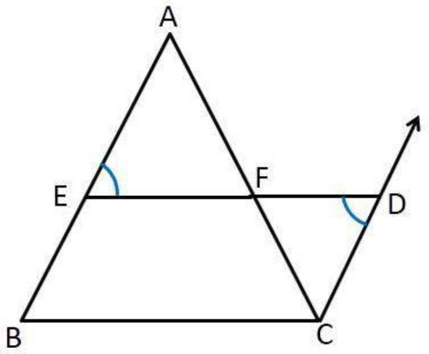
Now,

In EBCD,

$EB \parallel DC$ & $EB = DC$

Thus, one pair of opposite sides is equal and parallel.

Hence, EBCD is a parallelogram.



Since opposite sides of parallelogram are parallel.

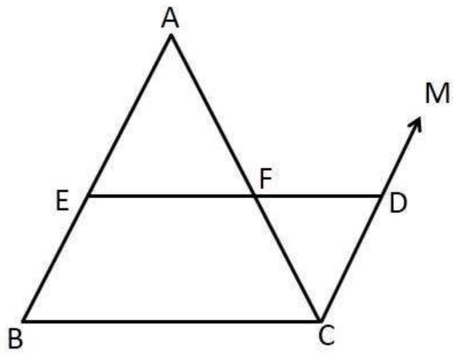
So, ED || BC

i.e. EF || BC

Hence, proved.

Theorem 8.10

The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.



Given : Δ ABC where

E is mid point of AB ,

F is some point on AC

& EF || BC

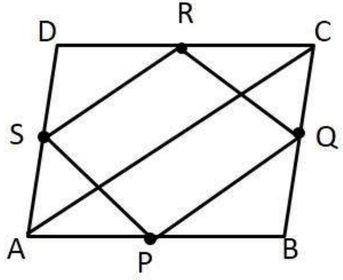
To Prove : F is a mid point of AC.

Construction : Through C draw CM || AB

Extend EF and let it cut CM at D.

Ex 8.2, 2

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.



Given: ABCD is rhombus where

P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively

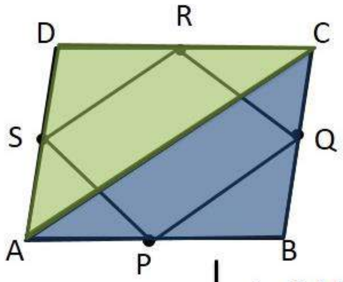
To prove: PQRS is a rectangle

Construction: Join A & C

Proof: A rectangle is a parallelogram with one angle 90°

First we will prove PQRS is a parallelogram,

and prove one angle 90 °



In **ΔABC**,

P is mid-point of AB,

Q is mid-point of BC

(Line segments joining the mid-points of two sides of a triangle is parallel to the third side and is half of it)

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(1)

In **ΔADC**,

R is mid-point of CD,

S is mid-point AD respectively.

(Line segments joining the mid-points of two sides of a triangle is parallel to the third side and is half of it)

$\therefore RS \parallel AC$ and $RS = \frac{1}{2}AC$ ...(2)

From (1) & (2)

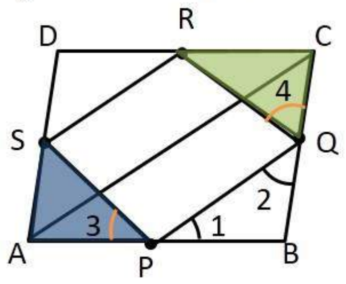
$PQ \parallel RS$ and $PQ = RS$

In PQRS,

one pair of opposite side is parallel and equal.

Hence, PQRS is a parallelogram.

Now we prove have to prove PQRS is a rectangle



Since AB = BC (Sides of rhombus are equal)

$\frac{1}{2}AB = \frac{1}{2}BC$

So, PB = BQ

P is mid-point of AB & Q is mid-point BC

Now, in Δ BPQ

PB = BQ

∴ ∠ 2 = ∠ 1 (Angles opposite to equal sides are equal) ...(3)

In Δ APS & Δ CQR

AP = CQ

AB = BC , $\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ BC , ⇒ AP = CQ

AS = CR

AD = CD , $\Rightarrow \frac{1}{2}$ AD = $\frac{1}{2}$ CD, ⇒ AS = CR

PS = QR (Opposite sides of parallelogram are equal)

∴ Δ APS ≅ Δ CQR (SSS congruence rule)

∠ 3 = ∠ 4 (CPCT) ...(4)

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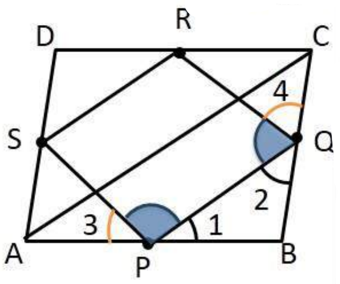
Now,

AB is a line

So, $\angle 3 + \angle SPQ + \angle 1 = 180°$ (Linear Pair) ...(5)



Similarly, for line BC

$\angle 2 + \angle PQR + \angle 4 = 180°$ (Linear Pair)

$\angle 1 + \angle PQR + \angle 3 = 180°$ (From (3)&(4)) ...(6)

From (5) & (6)

$\angle 1 + \angle SPQ + \angle 3 = \angle 1 + \angle PQR + \angle 3$

$\therefore \angle SPQ = \angle PQR$ ...(7)

Now,

PS || QR (Opposite sides of parallelogram are parallel)

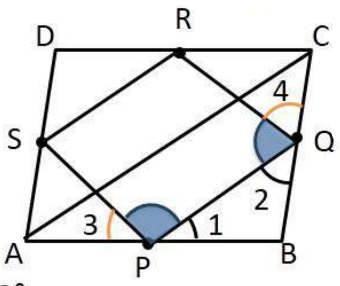
& PQ is a transversal

So, $\angle SPQ + \angle PQR = 180°$ (Interior angles on the same side of transversal are supplementary)

$\angle SPQ + \angle SPQ = 180°$ (From (7))

$2\angle SPQ = 180°$

$\angle SPQ = \frac{180°}{2} = 90°$



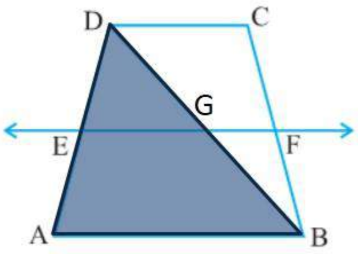
So, PQRS is a parallelogram with one angle 90°

∴ PQRS is a rectangle

Hence proved

Ex 8.2, 4

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F. Show that F is the mid-point of BC.



Given: ABCD is a trapezium where

AB || DC

E is the mid point of AD, i.e., AE = DE

& EF || AB

To prove: F is mid point of BC , i.e., BF = CF

Proof: Let EF intersect DB at G.

In Δ ABD

E is the mid-point of AD.

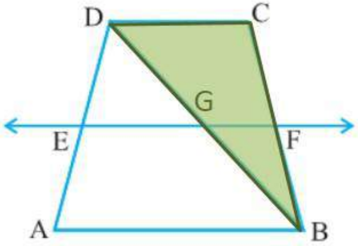
and EG || AB *(As EF || AB ,parts of parallel lines are parallel)*

∴ G will be the mid-point of DB.

(Line drawn through mid-point of one side of a triangle, parallel to another side, bisects the third side)

Given EF || AB and AB || CD,

∴ EF || CD



In ΔBCD,

G is the mid-point of side BD.

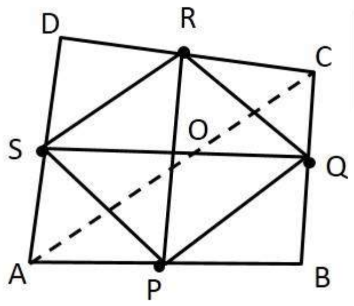
& GF || CD *(As EF || CD ,parts of parallel lines are parallel)*

∴ F is the mid-point of BC. (Line drawn through mid-point of one side of a triangle, parallel to another side, bisects the third side)

Hence proved

Ex 8.2, 6

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.



Given: Let ABCD is a quadrilateral

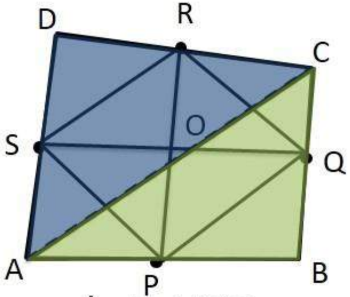
P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively

To prove: PR & SQ bisect each other

i.e. OP = OR & OQ = OS

Construction: Join A & C

Proof:



In **ΔADC**,

S is mid-point of AD

& R is mid-point of CD

Line segments joining the mid-points of two sides of a triangle is parallel to the third side and is half of it

$\therefore$ SR ∥ AC and SR = $\frac{1}{2}$ AC

In **ΔABC**,

P is mid-point of AB

& Q is mid-point of BC

Line segments joining the mid-points of two sides of a triangle is parallel to the third side and is half of it

$\therefore$ PQ ∥ AC and PQ = $\frac{1}{2}$ AC

From (1) & (2)

$\Rightarrow$ PQ = SR & PQ ∥ SR

So, In PQRS,

one pair of opposite sides is parallel and equal.

Hence, PQRS is a parallelogram.

PR & SQ are diagonals of parallelogram PQRS

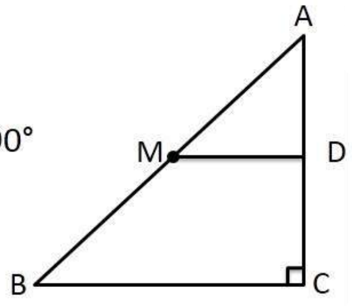
So, OP = OR & OQ = OS (Diagonals of a parallelogram bisect each other)

Hence proved

Ex 8.2, 7

ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC



Given: Δ ABC is right angled triangle, ∠ C = 90°

M is the mid-point of AB,

MD || BC

To prove: D is mid-point of AC, i.e., AD = CD

Proof:

In ΔABC,

M is the mid-point of AB

and MD || BC.

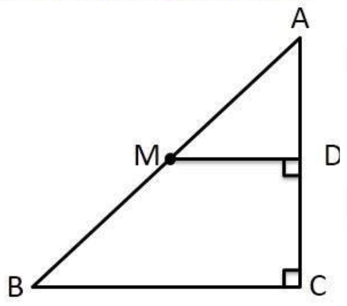
∴ D is the mid-point of AC.

(Line drawn through mid-point of one side of a triangle, parallel to another side, bisects the third side)

Ex 8.2, 7

ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(ii) MD ⊥ AC



As MD ∥ BC &

AC is transversal

∴ ∠MDC + ∠BCD = 180° *(Interior angles on the same side of transversal are supplementary)*

∠MDC + 90° = 180°

∠MDC = 90°

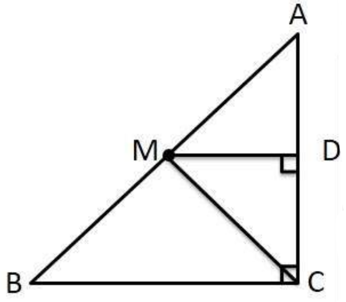
∴ MD ⊥ AC

Hence proved

Ex 8.2, 7

ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(iii) $CM = MA = \frac{1}{2} AB$



Join MC.

In ΔAMD and ΔCMD,

$AD = CD$ *(Proved in part(i) that D is the mid-point of AC)*

$\angle ADM = \angle CDM$ *(Both 90 ° as MD⊥ AC(proved in last part))*

$DM = DM$ *(Common)*

$\therefore \Delta AMD \cong \Delta CMD$ *(SAS congruence rule)*

$\therefore AM = CM$ *(CPCT) ...(1)*

However, $AM = \frac{1}{2} AB$ *(Given that M is mid-point of AB)* *...(2)*

From (1) & (2)

$\Rightarrow \quad CM = AM = \frac{1}{2} AB$

HOMEWORK ASSIGNMENT

Exercise 8.2

Question number 1,2,3,4

AHA

1.Prove that any two consecutive angles of a parallelogram are supplementary.

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