Chapter- 13

Factorisation

The factors of an algebraic expression could be anything like numbers, variables and expressions.

As we have seen above that the factors of algebraic expression can be seen easily but in some case like 2y + 4, x^2 + 5x etc. the factors are not visible, so we need to decompose the expression to find its factors.

Methods of Factorisation

1. Method of Common Factors

- In this method, we have to write the irreducible factors of all the terms
- Then find the common factors amongst all the irreducible factors.
- The required factor form is the product of the common term we had chosen and the leftover terms.

Example

$$
\begin{aligned}
4x^2 + 6x &= \boxed{2} \times 2 \times \boxed{x} \times x + \boxed{2} \times 3 \times \boxed{x} \\
&= \boxed{2 \times x} \times 2 \times x + \boxed{2 \times x} \times 3 \\
&= \boxed{2x} \times 2x + \boxed{2x} \times 3 \\
&= 2x(2x+3)
\end{aligned}
$$

2. Factorisation by Regrouping Terms

Sometimes it happens that there is no common term in the expressions then

- We have to make the groups of the terms.
- Then choose the common factor among these groups.
- Find the common binomial factor and it will give the required factors.

Example

Factorise $3x^2 + 2x + 12x + 8$ by regrouping the terms.

Solution:

First, we have to make the groups then find the common factor from both the groups.

$$(3x^2 + 2x) + (12x + 8)$$

$$\downarrow x \qquad \downarrow 4$$

$$x(3x + 2) + 4(3x + 2)$$

$$(x + 4)(3x + 2)$$

Now the common binomial factor i.e. $(3x + 2)$ has to be taken out to get the two factors of the expression.

3. Factorisation Using Identities

Remember some identities to factorise the expression

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a – b)^2 = a^2 – 2ab + b^2$
- $(a + b)(a – b) = a^2 – b^2$

We can see the different identities from the same expression.

$(2x + 3)^2 = (2x)^2 + 2(2x)(3) + (3)^2$
$= \underline{4x^2 + 12x + 9}$
$(2x - 3)^2 = (2x)^2 – 2(2x)(3) + (3)^2$
$= \underline{4x^2 -12x + 9}$
$(2x + 3)(2x - 3) = (2x)^2 – (3)^2$
$= \underline{4x^2 - 9}$

Example 1

Factorise $x– (2x – 1)^2$ using identity.

Solution:

This is using the identity $(a + b)(a – b) = a^2 – b^2$
$x^2 - (2x - 1)^2 = [(x + (2x - 1))] [x – (2x-1))]$
$= (x + 2x - 1)(x – 2x + 1)$
$= (3x – 1)(- x + 1)$

Example 2

Factorize $9x^2 - 24xy + 16y^2$ using identity.

Solution:

1. First, write the first and last terms as squares.

$9x^2 - 24xy + 16y^2$

$= (3x)^2 – 24xy + (4y)^2$

2. Now split the middle term.

$= (3x)^2 – 2(3x)(4y) + (4y)^2$

3. Now check it with the identities

$$= \underbrace{(3x)^2}_{a^2} - \underbrace{2(3x)(4y)}_{2ab} + \underbrace{(4y)^2}_{b^2}$$

$$\boxed{a^2 - 2ab + b^2 = (a - b)^2}$$

4. This is $(3x – 4y)^2$

5. Hence the factors are $(3x – 4y)(3x – 4y)$.

Example 3

Factorise $x^2 + 10x + 25$ using identity.

Solution:

$x^2 + 10x + 25$

$= (x)^2 + 2(5)(x) + (5)^2$

We will use the identity $(a + b)^2 = a^2 + 2ab + b^2$ here.

Therefore,

$x^2 + 10x + 25 = (x + 5)^2$

4. Factors of the form (x + a) (x + b)

$(x + a)(x + b) = x^2 + (a + b)x + ab$.

Example:

Factorise $x^2 + 3x + 2$.

Solution:

If we compare it with the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

We get to know that $(a + b) = 3$ and $ab = 2$.

This is possible when $a = 1$ and $b = 2$.

Substitute these values into the identity,

$x^2 + (1 + 2)x + 1 \times 2$

$(x + 1)(x + 2)$

Division of Algebraic Expressions

Division is the inverse operation of multiplication.

1. Process to divide a monomial by another monomial

- Write the irreducible factors of both the monomials
- Cancel out the common factors.
- The balance is the answer to the division.

Example

Solve $54y^3 \div 9y$

Solution:

Write the irreducible factors of the monomials

$54y^3 = 3 \times 3 \times 3 \times 2 \times y \times y \times y$

$9y = 3 \times 3 \times y$

$$\frac{54y^3}{9y} = \frac{3 \times 3 \times 3 \times 2 \times y \times y \times y}{3 \times 3 \times y} = 2 \times 3 \times y \times y = 6y^2$$

2. Process to divide a polynomial by a monomial

- Write the irreducible form of the polynomial and monomial both.
- Take out the common factor from the polynomial.
- Cancel out the common factor if possible.
- The balance will be the required answer.

Example

Solve $4x^3 + 2x^2 + 2x \div 2x$.

Solution:

Write the irreducible form of all the terms of polynomial

$4x^3 + 2x^2 + 2x$

$= 4(x)\,(x)\,(x) + 2(x)\,(x) + 2x$

Take out the common factor i.e.2x

$= 2x\,(2x^2 + x + 1)$

$$\frac{4x^3 + 2x^2 + 2x}{2x} = \frac{2x(2x^2 + x + 1)}{2x} = (2x^2 + x + 1)$$

3. Process to divide a polynomial by a polynomial

In the case of polynomials we need to reduce them and find their factors by using identities or by finding common terms or any other form of factorization. Then cancel out the common factors and the remainder will be the required answer.

Example

Solve $z\,(5z^2 - 80) \div 5z\,(z + 4)$

Solution:

Find the factors of the polynomial

$= z\,(5z^2 - 80)$

$= z\,[(5 \times z^2) - (5 \times 16)]$

$= z \times 5 \times (z^2 - 16)$

$= 5z \times (z + 4)\,(z - 4)$ [using the identity $a^2 - b^2 = (a + b)\,(a - b)$]

$$\frac{\{z(5z^2 - 80)\}}{5z(z + 4)} = \frac{5z(z + 4)(z - 4)}{5z(z + 4)} = z - 4$$

Some Common Errors

- While adding the terms with same variable students left the term with no coefficient but the variable with no coefficient means 1.

  $2x + x + 3 = 3x + 3$ not $2x + 3$

  We will consider x as 1x while adding the like terms.

- If we multiply the expressions enclosed in the bracket then remember to multiply all the terms.

  $2(3y + 9) = 6y + 18$ not $6y + 9$

We have to multiply both the terms with the constant.

- If we are substituting any negative value for the variables then remember to use the brackets otherwise it will change the operation and the answer too.

  If $x = -5$

  Then $2x = 2(-5) = -10$

  Not, $2 - 5 = -3$

- While squaring of the monomial we have to square both the number and the variable.

  $(4x)^2 = 16x^2$ not $4x^2$

  We have to square both the numerical coefficient and the variable.

- While squaring a binomial always use the correct formulas.

  $(2x + 3)^2 \neq 4x^2 + 9$ But $(2x + 3)^2 = 4x^2 + 12x + 9$

- While dividing a polynomial by a monomial remember to divide each term of the polynomial in the numerator by the monomial in the denominator.

$$\frac{x + 5}{5} \neq x + 1 \text{ but } \frac{x + 5}{5} = \frac{x}{5} + 1$$