Chapter- 15

Linear Inequations

Inequality: Two real numbers or two algebraic expressions related by the symbols, '<', '>', '≤' or '≥,' form an inequality.
Linear inequality in one variable: ax + b > c
Here, the term — one variable— represents the variable x in the inequality.
There exist different types of inequalities, such as numerical inequalities, literal inequalities, strict inequalities, slack inequalities, linear inequality in one variable, linear inequality in two variables, etc.

Type	Examples
Numerical inequalities	10 > 3; 12 < 19
literal inequalities	a > 4; b < 5; x ≥ 11, y ≤ 3
Strict inequalities	ax+b < 0; ax+b > 0; ax+by < c; ax+by > c
Slack inequalities	ax+by ≤ c; ax+by ≥ c
Linear inequalities in one variable	ax+b < 0; ax+b > 0; ax+by ≤ c; ax+by ≥ c
Linear inequalities in two variable	ax+b < c; ax+b > c; ax+by ≤ c; ax+by ≥ c

Algebraic solutions of inequalities in one variable

Consider the inequality:
2x + 3 < 12, where x ∈ Z
x = 0 ⇒ LHS: 2 x 0 + 3 = 0 + 3 = 3 < 12
x = 1 ⇒ LHS: 2 x 1 + 3 = 2 + 3 = 5 < 12
x = 2 ⇒ LHS: 2 x 2 + 3 = 4 + 3 = 7 < 12
x = 3 ⇒ LHS: 2 x 3 + 3 = 6 + 3 = 9 < 12
x = 4 ⇒ LHS: 2 x 4 + 3 = 8 + 3 = 11 < 12
x = 5 ⇒ LHS: 2 x 5 + 3 = 10 + 3 = 13 ≮ 12

In the inequality, when the value of x is substituted form zero through four, we got the value on the left hand side less than the value on the right hand side.

For all these values of x, the statement of inequality holds true.
The statement does not hold true for x is equal to five.

Further, it can be observed that the statement holds true even for the values of x less than zero.

x < 0

The solution set for the inequality (S) = {..... -2, -1, 0,1, 2,3,4}
Solution set consists of integers.

For the same inequality, the solution sets of natural numbers and real numbers:
(S) = {1, 2,3,4} (Solution set of natural numbers)
S = {x: x ∈ R where x < 4.5} (Solution set of real numbers)
The values of x for which the statement holds true are called the solutions of the inequality.

Any solution of an inequality in one variable is a value of the variable that makes it a true statement.

The solutions were found for this inequality by trial and error.

Rules that can be used in finding the solutions of an inequality:

Rule 1: Equal numbers may be added to (or subtracted from) both the sides of an inequality without affecting the sign of inequality.

Ex: $x + 3 > 8$
$x + 3 - 3 > 8 - 3$
$x > 5$
The nature of the inequality does not change.

This applies for addition also.

$x + 3 + 5 > 8 + 5$

$x + 8 > 13$
Rule 2: Both the sides of an inequality can be multiplied (or divided) by the same positive number without affecting the sign of inequality.
$3x > 40$
$5 \times 3 \times x > 40 \times 5$
$15x > 200$
$3x/2 > 40/2$

$3x/2 > 20$

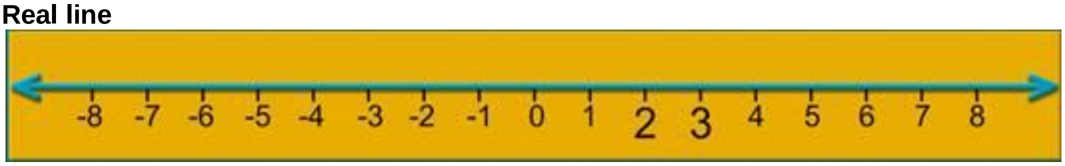
The nature of the inequality does not change by dividing the inequality on both the sides.

Rule 3: When the inequality is divided or multiplied by a negative number, the sign of inequality is reversed.

$3x > 40$
$3x \times (-1) > 40 \times (-1)$
$-3x < -40$
$-3x/-1 < -40/-1$

The sign of the inequality changes when divided by a negative number on both the sides.

Real line

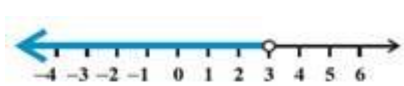


$2 < 3$
$2 \times -1 < 3 \times -1$
$-3 < -2$

The numbers on the real line, -3 is less than -2.

We change signs when we multiply with a negative number.

Finding solutions of inequalities using rules:
Ex: Solve the inequality, $3x + 4 < 22$ where $x \in R$ and $x \in N$.
To solve an inequality, the coefficient of x is made as one. For this, all the constants are taken to the other side of the inequality sign.
$3x + 4 < 22$
$3x + 4 - 4 < 22 - 4$
$3x < 18$
$3x/3 < 18/3$
$x < 6$
$S = (-\infty, 6) = \{x: x \in R, \text{ where } x < 6\}$
Graphical representation of inequality
The solution set contains numbers less than six.



S = {1,2,3,4,5} (Solution set of natural numbers)

Ex: Solve $6x + 7 \geq 3x - 8$, where $x \in R$

$6x + 7 \geq 3x - 8$

$6x + 7 - 7 \geq 3x - 8 - 7$

$\Rightarrow 6x \geq 3x - 15$

$\Rightarrow 6x - 3x \geq 3x - 15 - 3x$

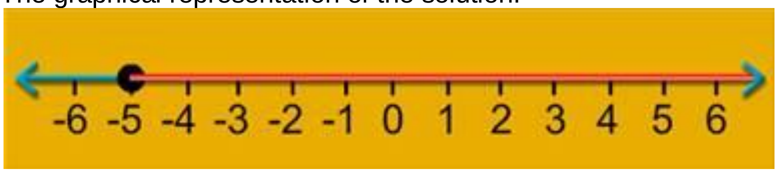
$\Rightarrow 3x \geq - 15$

$\Rightarrow 3x/3 \geq - 15/3$

$\Rightarrow x \geq -5$

$S = [-5, \infty) = \{x : x \in R, \text{Where } x \geq -5\}$

The graphical representation of the solution:



Here, the solution includes the number -5.