Chapter- 1

RATIONAL NUMBERS

If p and q both are integers and $q \neq 0$, then $\frac{p}{q}$ is called a rational numbers.

For example : $\frac{-5}{7}, \frac{5}{7}$

Remember :

1. Zero (O) can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{5}$ in each of these cases denominator is not equal to zero.

   So, zero can be expressed as a fraction with a non-zero denominator.

   $\therefore$ Zero (0) is a rational number.

2. Every natural number, every whole number, every integer and every fraction is a rational number.

3. In the rational number $\frac{p}{q}$, where p and q are integers and $q \neq 0$, integer p is called numerator and integer q is called the denominator.

Properties of Addition of Rational Numbers

1. Closure Property

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then their addition $\left(\frac{a}{b}+\frac{c}{d}\right)$ is also a rational number.

Ex - $\frac{-3}{8}+\frac{5}{4}=\frac{1}{24}$, which is a rational number.

2. Commutativity

The addition of any two rational numbers is commutative.

According to commutative property of addition, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then : $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$

Ex- $\frac{5}{8}+\frac{-7}{12}=\frac{15}{24}+\frac{-14}{24}=\frac{15-14}{24}=\frac{1}{24}$

$\frac{-7}{12}+\frac{5}{8}=\frac{-14}{24}+\frac{15}{24}=\frac{-14+15}{24}=\frac{1}{24}$

$\therefore \frac{-7}{12}+\frac{5}{8}=\frac{5}{8}+\frac{-7}{12}$

3. Associativity

If $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}$.

4. Existence of additive identity of rational numbers.

Additive identity for rational numbers is zero (0).

5. Existence of additive inverse of a rational number.

The negative of a rational number is called its additive inverse.

Ex - The additive inverse of $\frac{3}{5}=-\frac{3}{5}$.

Properties of Subtraction of Rational Numbers

(i) Rational Numbers are closed under subtraction.

(ii) Subtraction of rational numbers is not commutative.

(iii) The subtraction of rational numbers is not associative.

(iv) Subtraction has no identity.

(v) Inverse for substraction does not exist.

Properties Multification of Rational Numbers

(i) **Closer properties** : If any two rational numbers are multified together, the result is always a rational numbers.

(ii) Commutativity : If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then : $\frac{a}{b}x\frac{c}{d}=\frac{c}{d}x\frac{a}{b}$.

(iii) Associativity : If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

$$\frac{a}{b}x\left(\frac{c}{d}x\frac{e}{f}\right)=\left(\frac{a}{b}x\frac{c}{d}\right)x\frac{e}{f}$$

(iv) Existence of multiplicative identity of rational numbers : Multiplicative identity is multiplied with any rational number or any rational number is multiplied with multiplicative identity, the rational number remains the same.

Ex - $1 x \frac{a}{b} = \frac{a}{b} x 1 = 1$

(v) Existence of multiplicative inverse of rational numbers : The reciprocal of a rational number is called its multiplicative inverse.

Note : Rational number 0 (zero) does not have its multiplicative inverse.

(vi) Distributivity of multiplication over addition : The multiplication of rational numbers is distributive over their addition/subtraction.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

(i) $\frac{a}{b} x \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} x \frac{c}{d} + \frac{a}{b} x \frac{e}{f}$

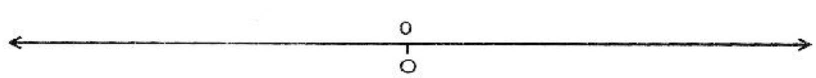
(ii) $\frac{a}{b} x \left(\frac{c}{d} - \frac{e}{f}\right) = \frac{a}{b} x \frac{c}{d} - \frac{a}{b} x \frac{e}{f}$

Properties of Division Of Rational Numbers

1. Closure Property : If a rational number is divided by some non-zero rational number, the result is always a rational number.
2. Commutativity : Division of two different rational numbers is not commutative.
3. Associativity : Division of rational numbers is not associative.
4. Identity for division of rational numbers does not exist.
5. Inverse for division of rational numbers does not exist.

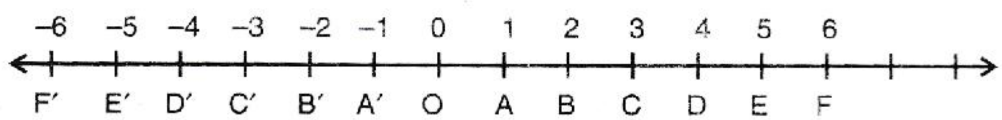
Representation of Rational Numbers on the Number Line.

Draw a line of suitable length. Nearly at the middle of this line, mark a point O that represents number zero (0).



Starting from O, mark points on this line at equal distances both on right as well as on left of O. Let A, B, C, D etc. be the points on the right side of O and A', B', C', D', etc. be the point on the left side of O so that :

OA = AB = BC = CD = ………….. OA' = A'B' = B'C' = C'D' = …………………



If OA = 1 unit

= A,B,C,D etc, represent integers 1,2,3,4,…… respectively and A' , B', C', D'. etc, represent integers -1,-2,-3,-4 ………… respectively.

<u>Insetrting Rational Numbers Between Two given rational Numbers.</u>

If a and b are two rational numbers, then $\frac{a+b}{2}$ is also a rational numbers and its value lies between a and b.

If $a<b \Rightarrow a<\frac{a+b}{2}<b$ i.e., $5<8 \Rightarrow 5<\frac{5+8}{2}<8$ i.e., 5 < 6.5 < 8

Note : For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a+c}{b+d}$ is also a rational number with its value lying between $\frac{a}{b}$ and $\frac{c}{d}$