[CLASS-X] | MATHEMATICS | STUDY NOTES

Chapter-1

NUMBER SYSTEMS

Main Concepts and Results

- Algorithm: An algorithm means a series of well defined steps which gives a procedure for solving a type of problem.
- > Lemma: It is a proven statement used for proving another statement.
- > Euclid's Division Lemma: If a and b are two positive integers then there exist unique integers q and r satisfying a = bq + r, for all $q \ge 0$, $0 \le r < b$
- Euclid's Division Algorithm: To obtain the HCF of two positive integers say c and d where c>d then follow the steps below:

Step-1: Applying **Euclid's** division Lemma to c and d we find whole numbers q and r such that

c = dq + r, for all $q \ge 0$, $0 \le r < d$

Step-2: If r = 0, d is the HCF of c and d, if $r \neq 0$ apply **Euclid's** divisions lemma to d and r.

Step-3: Continue the process till the remainder is zero. The divisor at this stage will be required HCF.

- Fundamental Theorem of Arithmetic: Every composite number can be expressed as a product of primes any this factorization is unique, apart from the order, in which the prime factors occur.
- Prime Numbers: A natural number different from 1 is said to be prime, if it has no factors other than one and itself.

e.g.: 2, 3, 5, 7, 11, 13, 17...are the first few primes.

- Composite Numbers: A natural number different from 1 is said to be composite, if it is not prime. e.g., 4, 6, 8, 9 ... are first few composite numbers.
- To find the HCF and LCM of two positive integers using the fundamental theorem of arithmetic (Prime Factorization Method) following steps are used.
- For Two positive Integers : Let a and b are two positive integers then,
 - First factorize both the numbers.
 - HCF (a, b) = Product of the smallest power of each common prime factor in the Factorization of the numbers.
 - LCM (a, b) = Product of the greatest power of each prime factor, in the Factorization of the numbers.
 - Further HCF (a, b) \times LCM (a, b) = a \times b
 - i.e., HCF×LCM = Product of two numbers

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- For Three positive Integers : Consider three positive integers a, b and c
 - First factorize all the 3 numbers.
 - HCF (a, b, c) = Product of the smallest power of each common prime factors in the Factorization of the numbers.
 - LCM (a, b, c) = Product of greatest power of each prime factor, in the Factorization of the numbers.
 - HCF (a, b, c) \times LCM(a,b, c) \neq a \times b \times c

• HCF
$$(a, b, c) = \frac{a \times b \times c \times LCM(a, b, c)}{LCM(a, b) \times LCM(b, c) \times LCM(c, a)}$$

• LCM
$$(a, b, c) = \frac{a \times b \times c \times HCF(a, b, c)}{HCF(a, b) \times HCF(b, c) \times HCF(c, a)}$$

Rational Numbers: The numbers of the form $\frac{p}{q}$, where, p and q are co-primes and q $\neq 0$. e.g.,

5, -6, 2/3, -1/7.

> Irrational Numbers: The number cannot be expressed in the form $\frac{p}{q}$, where p and q are

integers and $q \neq 0$ e.g., $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational numbers.

- \blacktriangleright Let 'p' be a prime number, if p divide a^2 , then 'p' divides a, where 'a' + positive integer.
- A Rational number $\frac{p}{q}$ in standard form will have a terminating decimal representation, if the prime factorization of q is of the form 2^m.5ⁿ where m and n are non negative integers.
- Rational number $\frac{p}{q}$ in standard form will have a non-terminating repeating decimal representation, if the prime factorization of q is **not** of the form 2^m.5ⁿ where m and n are non negative integers.
- > The sum and difference of rational and irrational number is irrational.
- The product and quotient of a non-zero rational numbers are an irrational number is irrational.

[CLASS-X] | MATHEMATICS | STUDY NOTES CHAPTER : I REAL NUMBERS ***************** Division Lemma Given positive integers a nd b, there exist unique integers q and r satisfying. a = bq + r; 0 < r < b..... 1. Let y be a prime number. If p divides a2, then p divides a, where a is a positive integer 2. $\sqrt{2}\sqrt{3}$ are irrational Euclid's Theorems Division Algorithm 3. Let x be a rational number whose Steps to obtain the HCF of two decimal expansion terminates. Then x can be positive integers, say c and d, expressed in the form, $\frac{p}{d}$ where p & q are with c>d coprime, the prime factorisation of g is of the form 2n 5w where n, n are non-Step 1: Apply Euclid's Division negative integers Lemma, to c & d. c = dq + r..... **Real Numbers** 4. Let $x = \frac{p}{q}$ be a rational number such Step 2: If r = zero, d is the HCF of c and d If r + 0, apply Euclid's that the prime factorisation of q is of the form 2" 5" where n, m are non-negative Division to d and r integers. Then, x has a decimal expansion which terminates. ••••••• Fundamental Theorem of Arithmetic Step 3: Continue the process Prime Factorization Method till the remainder is zero 5. Let $x = \frac{p}{a}$ be a rational number, such that the prime factorisation of q is not of the form of 205w where n, m are non-negative integers. Then, x has a decimal expansion Every composite number which is non-terminating repeating ********** can be expressed as a product of ×..... For any two positive integers, Composite Number primes, and this factorisation is unique, a and b $\mathbf{x} = \mathbf{P}_1 \times \mathbf{P}_2 \times \mathbf{P}_3 \dots \times \mathbf{P}_m$ apart from the order in which the HCF $(a, b) \times LCM (a, b) = a \times b$ where P1P2 ... Pn are prime factors occur For Example prime numbers $f(x) = 3x^2y$ ••••••••••••• $g(x) = 6xy^2$ HCF = 3xy $LCM = 6x^2y^2$ ***********************************