Chapter- 1

NUMBER SYSTEMS

Main Concepts and Results

- **Algorithm:** An algorithm means a series of well defined steps which gives a procedure for solving a type of problem.
- **Lemma:** It is a proven statement used for proving another statement.
- **Euclid's Division Lemma:** If a and b are two positive integers then there exist unique integers q and r satisfying a = bq + r, for all q≥0, $0 \le r < b$
- **Euclid's Division Algorithm:** To obtain the HCF of two positive integers say c and d where c>d then follow the steps below:

  Step-1: Applying **Euclid's** division Lemma to c and d we find whole numbers q and r such that c = dq + r, for all q≥0, 0 ≤ r < d

  Step-2: If r = 0, d is the HCF of c and d, if $r \neq 0$ apply **Euclid's** divisions lemma to d and r.

  Step-3: Continue the process till the remainder is zero. The divisor at this stage will be required HCF.
- **Fundamental Theorem of Arithmetic:** Every composite number can be expressed as a product of primes any this factorization is unique, apart from the order, in which the prime factors occur.
- **Prime Numbers:** A natural number different from 1 is said to be prime, if it has no factors other than one and itself.

  e.g.: 2, 3, 5, 7, 11, 13, 17…are the first few primes.
- **Composite Numbers:** A natural number different from 1 is said to be composite, if it is not prime. e.g., 4, 6, 8, 9 … are first few composite numbers.
- To find the HCF and LCM of two positive integers using the fundamental theorem of arithmetic (Prime Factorization Method) following steps are used.
- **For Two positive Integers :** Let a and b are two positive integers then,
  - First factorize both the numbers.
  - HCF (a, b) = Product of the smallest power of each common prime factor in the Factorization of the numbers.
  - LCM (a, b) = Product of the greatest power of each prime factor, in the Factorization of the numbers.
  - Further HCF (a, b) × LCM (a, b) = $a \times b$
  - i.e., $HCF \times LCM$ = Product of two numbers

- **For Three positive Integers :** Consider three positive integers a, b and c
  - First factorize all the 3 numbers.
  - HCF (a, b, c) = Product of the smallest power of each common prime factors in the Factorization of the numbers.
  - LCM (a, b, c) = Product of greatest power of each prime factor, in the Factorization of the numbers.
  - HCF (a, b, c) $\times LCM(a,b,c) \neq a \times b \times c$
  - $HCF\ (a, b, c) = \frac{a \times b \times c \times LCM(a, b, c)}{LCM(a, b) \times LCM\ (b, c) \times LCM(c,a)}$
  - $LCM\ (a, b, c) = \frac{a \times b \times c \times HCF(a, b, c)}{HCF(a, b) \times HCF\ (b, c) \times HCF(c,a)}$
- Rational Numbers: The numbers of the form $\frac{p}{q}$, where, p and q are co-primes and $q \neq 0$. e.g., 5, -6, 2/3, -1/7.
- Irrational Numbers: The number cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ e.g., $\sqrt{2}, \sqrt{3}, \sqrt{5}$ are irrational numbers.
- Let 'p' be a prime number, if p divide a^2, then 'p' divides a, where 'a' + positive integer.
- A Rational number $\frac{p}{q}$ in standard form will have a terminating decimal representation, if the prime factorization of q is of the form $2^m.5^n$ where m and n are non negative integers.
- Rational number $\frac{p}{q}$ in standard form will have a non-terminating repeating decimal representation, if the prime factorization of q is **not** of the form $2^m.5^n$ where m and n are non negative integers.
- The sum and difference of rational and irrational number is irrational.
- The product and quotient of a non-zero rational numbers are an irrational number is irrational.

CHAPTER : 1 REAL NUMBERS