Chapter- 2

POLYNOMIALS

Introduction:-

1. Polynomial: An algebraic expression in the form of $a_0 + a_1x^1 + a_2x^2 + ... + a_n.x^n$ where $a_0, a_1, a_2, ...a_n$ are real numbers, n is a non-negative integer and $a_n \neq 0$ is called a polynomial of degree n.
2. Degree of Polynomials: The highest power of (variable) x in a polynomial p(x) is called the degree of polynomial.
3. Types of Polynomial:
   (a) Constant polynomial: a polynomial of degree zero is called a constant polynomial and it is of the form $P(x) = k$.
   (b) Linear Polynomial: A polynomial of degree one is called linear polynomial and it is of the form $P(x) = ax + b$, where a, b are real numbers and $a \neq 0$.
   (c) Cubic Polynomial: A polynomial of degree three is called cubic polynomial and it is of the form $P(x) = ax^3 + bx^2 + cx + d$, where a, b, c are real numbers and $a \neq 0$.
   (d) Cubic polynomial: a polynomial of degree three is called cubic polynomial and it is of the form $P(x) = ax^3 + bx^2 + cx + d$ where a, b, c, d are real numbers and $a \neq 0$.
   (e) Bi-quadratic polynomial: A polynomial of degree four is called Bi-quadratic polynomial or quadratic polynomial and it is of the form $P(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d and e are real numbers and $a \neq 0$.
   (f) Value of a polynomial: If $p(x)$ is a polynomial in the form of $a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_nx^n$ with $a_n \neq 0$, then value of the polynomial p(x) for a real value at $x = 0$ will be given by $P(a) = a_0 + a_1 \times (a) + a_2 \times (a)^2 + a_3 \times (a)^3 + ... + a_n \times (a)^n$.
   (g) Zero of a polynomial: A number 'a' is said to be the zero of a polynomial p(x), if on replacing each x in the polynomial by 'a' the value of polynomial comes out to be zero.

Geometrical Representation of the zeros of a polynomial:-

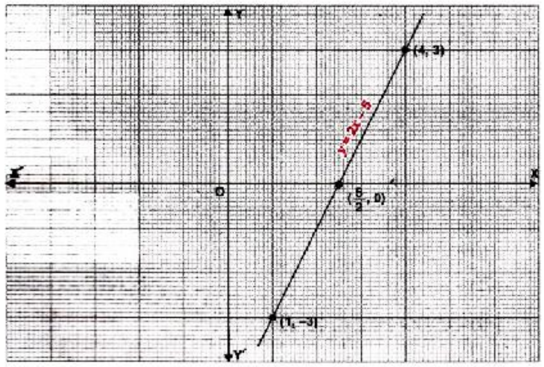
Graph of a polynomial p(x) can be drawn in the coordinate plane with the help of table of values obtained from the equation y = p(x)

Geometrical Representation of the zero of a linear polynomial:-

Consider a linear polynomial 2x – 5. It has one and only one zero 5/2. Let us drawn the graph of the equation y = 2x – 5. We known that it will be a straight line.

Table values

x	1	2
$y = 2x - 5$	-3	3



From the figure we observe that the graph of the equation y = 2x – 5 intersects the x – axis at the point $A\left(\frac{5}{2}, 0\right)$ whose x – coordinate is 5/2 and we also know that 5/2 is the only zero of the linear polynomial $2x - 5$. By taking a few more illustration, we find the same situation. Therefore, we conclude that the linear polynomial $ax + b$ has one and only one zero which is the x-coordinate of the point where the graph of $y = ax + b$ intersects the x-axis.

Geometrical Representation of the zeros of a Quadratic Polynomial:-

$ax^2 + bx + c$ is a quadratic polynomial when $a \neq 0$. Here, either a > or a < 0.

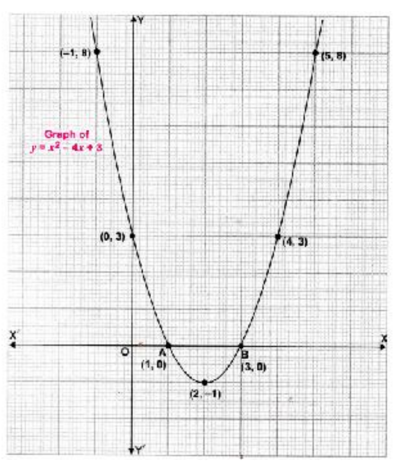
Case – 1, when a > 0, i.e coefficient of leading term is positive.

We consider a quadratic polynomial $x^2 - 4x + 3$. It has two zero, 1 and 3. Let us draw the graph of the equation $y = x^2 - 4x + 3$.

Table of Values

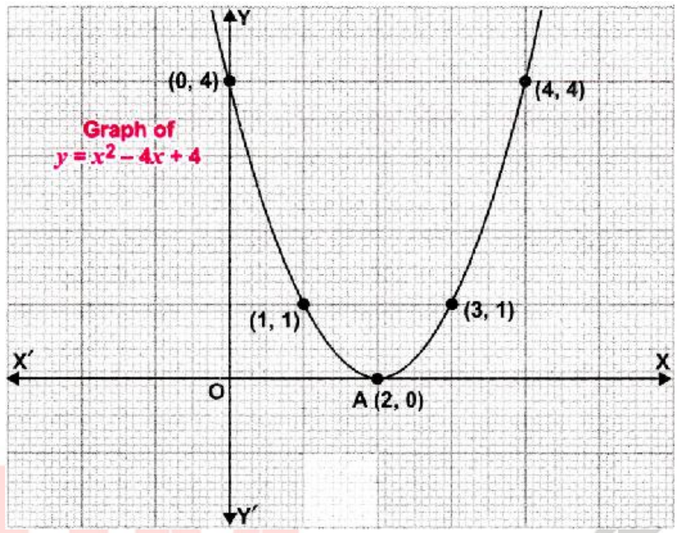
x	-1	0	1	2	3	4	5
$y = x^2 - 4x + 3$	8	3	0	-1	0	3	8

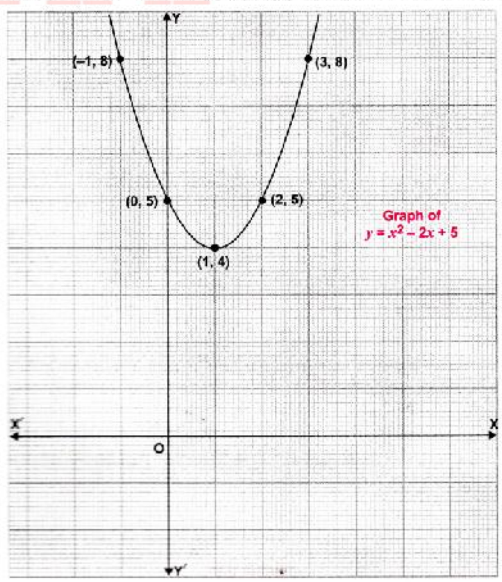
The graph of $y = x^2 - 4x + 3$ in figure is a parabola which is open the upward side, i.e an upward parabola. It is this like U.



Meets the x-axis in two points A (1, 0) and B (3, 0)

The x-coordinates of the points A (1, 0) and B (3,0) are 1 and 3 respectively, which are the zeros of the corresponding quadratic polynomial $x^2 - 4x + 3$. Similarly, we can draw the graphs of the equations $y = x^2 - 4x + 4$ and $y = x^2 - 2x + 5$ as shown below in figure.





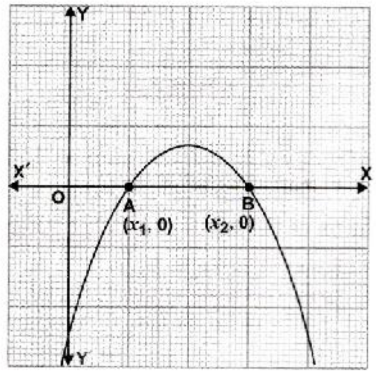
In figure the graph of $y = x^2 - 4x + 4$, touches the x-axis at a single point A (2,3) whose x-coordinate is 2 and also 2 is a zero of the quadratic polynomial $x^2 - 4x + 4$. We observe that $x^2 - 4x + 4 = (x-2)(x-2)$ which gives two zeroes, each equal 2. Thus, a quadratic polynomial $ax^2 + bx + c$ having both zeros equal to same number.

In figure the graph of $y = x^2 - 2x + 5$, neither intersects nor touches the x-axis. It is an upward parabola above the x-axis. It means that the polynomial $x^2 - 2x + 5$ does not have any real zero.

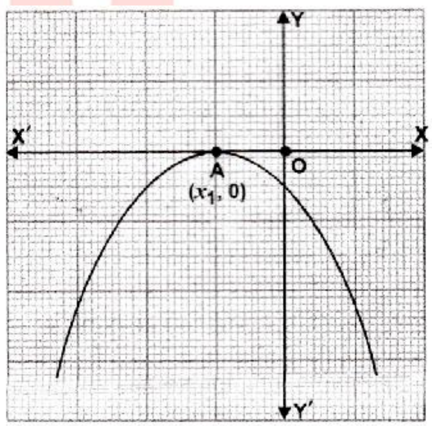
Case – II, When the coefficient of the leading term of the quadratic polynomial $ax^2 + bx + c$ is negative, i.e a < 0.

As in case – I, we have three types of situation about the graph of the equation $y = ax^2 + bx + c$. In each situation, the graph is a downward parabola.

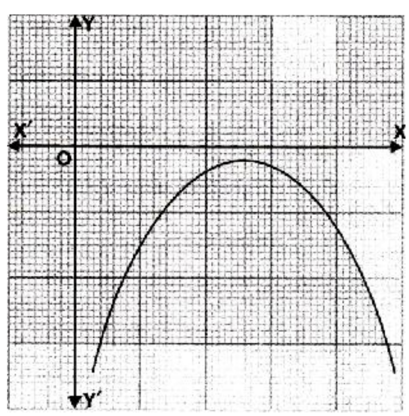
In figure graph of the equation $y = ax^2 + bx + c$ is a downward parabola which intersects the x-axis at two points $A(x_1, 0)$ and $B(x_2, 0)$. Here, x_1, x_2 are two distinct real zeros of the quadratic polynomial $ax^2 + bx + c$



In figure graph of the equation $y = ax^2 + bx + c$ is a downward parabola which touches the x-axis at the point $A(x_1, 0)$. Here x_1 is a zero of the quadratic polynomial $ax^2 + bx + c$. Moreover, the two zeros of $ax^2 + bx + c$ are x_1 and x_1, i.e, of same value.



In figure, graph of the equation $y = ax^2 + bx + c$ is a downward parabola which is below the x-axis. i.e neither intersects nor touches the x-axis. In this case the quadratic polynomial $ax^2 + bx + c$ does not have any real zero.



Remark:- Graph of $y = ax^2 + bx + c$ intersects the x-axis at most in two points and hence the quadratic polynomial can have at the most two distinct real zeros.

Geometric Representation of the zeros of a cubic polynomial

For a cubic polynomial ax^3+bx^2+cx+d, the equation $y=ax^3+bx^2+cx+d$ gives a graph which meets the x-axis at the most in three distinct points but it will certainly meet the x-axis atleast in one point. Thus, we conclude that a cubic polynomial ax^3+bx^2+cx+d can have at most three dinstinct real zeros but atleast one real zero is certain. Now, we will learn these facts through some illustrative examples.

Concept – 2 Relationship between zeroes and coefficients of a polynomial:-

Sl. No.	Types of Polynomial	General Form	No. of Zeroes	Relationship between Zeroes
1	Linear	$ax+b, a\neq 0$	1	$k=-\frac{b}{a}$, i.e, $k=-\frac{(\text{Constant term})}{\text{Coefficient of } x}$
2	Quadratic	$ax^2+bx+c,\ a\neq 0$	2	Sum of zeroes $(\alpha+\beta)=-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=-\frac{b}{a}$ Product of zeroes $(\alpha\beta)=\frac{\text{Constant}}{\text{Coefficient of } x^2}=\frac{c}{a}$
3	Cubic	$ax^3+bx^2+cx+d,\ a\neq 0$	3	Sum of zeroes $(\alpha+\beta+\gamma)=-\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}=-\frac{b}{a}$ Product of zeros $(\alpha\beta\gamma)=-\frac{\text{Constant term}}{\text{Coefficient of } x^3}=-\frac{d}{a}$ Sum of the product of zeroes taken 2 at a time $(\alpha\beta+\beta\gamma+\gamma\alpha)=\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}=\frac{c}{a}$

Formation of Polynomial when their zeroes are given:

Sl. No.	Type of polynomial	Zeroes of polynomial	Sum and products of polynomial	Expression so obtained in the form of polynomial
1	Constant Polynomial	There is no zeroes for such polynomial	Does not exist	$p(x)=k$ or $p(x)=c$ or any other constant number
2	Linear Polynomial	Say its zero is 'α' because linear polynomial has only one zero	Sum and product in this case is same i.e only α	$p(x)=k[x-\alpha]$
3	Quadratic polynomial	Let α & β are its zeroes	Sum is $'S'=\alpha+\beta$ Product $P=\alpha\times\beta$	$k[x^2-(\alpha+\beta)x+\alpha.\beta]$ Or K [x^2 – (sum of zeroes) x + product of zeroes].
4	Cubic Polynomial	Let three zeroes are α, β, γ	(a) Sum of the zeroes $S=\alpha+\beta+r$ (b) Sum of zeroes taken 2 at a time say $S'=\alpha\beta+\beta r+r\alpha$ (c) Product of all three zeroes = $\alpha.\beta.\gamma$	$p(x)=k\begin{bmatrix} x^3-(\alpha+\beta+\gamma)x^2+ \\ (\alpha\beta+\beta\gamma+\gamma\alpha)x+\alpha\beta.\gamma \end{bmatrix}$ Or $p(x)=k[x^3-sx^2+s'x-p]$