

Chapter- 4

CUBES AND CUBE-ROOTS**Main Concepts and Results**

Cube is a 3-dimensional figure with all equal sides. If one cube has all the equal sides of 1 cm then how many such cubes are needed to make a new cube of side 2 cm?

8 such cubes are needed, and what if we need to make a cube of side 3 cm with the cubes of side 1 cm? The numbers 1, 8, 27 ...etc can be shown below in the

Cubes of Some Natural Numbers

Number	Cubes	Numbers	Cubes
1	$1^3 = 1$	11	$11^3 = 1331$
2	$2^3 = 8$	12	$12^3 = 1728$
3	$3^3 = 27$	13	$13^3 = 2197$
4	$4^3 = 64$	14	$14^3 = 2744$
5	$5^3 = 125$	15	$15^3 = 3375$
6	$6^3 = 216$	16	$16^3 = 4096$
7	$7^3 = 343$	17	$17^3 = 4913$
8	$8^3 = 512$	18	$18^3 = 5832$
9	$9^3 = 729$	19	$19^3 = 6859$
10	$10^3 = 1000$	20	$20^3 = 8000$

This table shows that

- There are only 10 perfect cubes between 1-1000.
- The cube of an **even** number is also even.
- The cube of an **odd** number is also an odd number.

One's digit of the Cubes

One's digit of the Cubes of a number having a particular number at the end will always remain same. Let's see in the following table:

Unit's digit of number	Last digit of its cube number	Example
1	1	$11^3 = 1331, 21^3 = 9261, \text{etc.}$
2	8	$2^3 = 8, 12^3 = 1728, 32^3 = 32768, \text{etc.}$
3	7	$13^3 = 2197, 53^3 = 148877, \text{etc.}$
4	4	$24^3 = 13824, 74^3 = 405224, \text{etc.}$
5	5	$15^3 = 3375, 25^3 = 15625, \text{etc.}$
6	6	$6^3 = 216, 26^3 = 17576, \text{etc.}$
7	3	$17^3 = 4913, 37^3 = 50653, \text{etc.}$
8	2	$8^3 = 512, 18^3 = 5832, \text{etc.}$
9	9	$19^3 = 6859, 39^3 = 59319, \text{etc.}$
10	0	$10^3 = 1000, 20^3 = 8000, \text{etc.}$

Some Interesting Patterns

1. Adding Consecutive Odd Numbers

$$\begin{aligned}1^3 &= 1 = 1 \\2^3 &= 8 = 3 + 5 \\3^3 &= 27 = 7 + 9 + 11 \\4^3 &= 64 = 13 + 15 + 17 + 19 \\5^3 &= 125 = 21 + 23 + 25 + 27 + 29 \\6^3 &= 216 = 31 + 33 + 35 + 37 + 39 + 41 \\7^3 &= 343 = 43 + 45 + 47 + 49 + 51 + 53 + 55 \\8^3 &= 512 = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71 \\9^3 &= 729 = 73 + 75 + 77 + 79 + 81 + 83 + 85 + 87 + 89 \\&\text{etc}\end{aligned}$$

This shows that if we add the consecutive odd numbers then we get the cube of the next number.

2. Cubes and their Prime Factors

Prime factorization of a number is done by finding the prime factors of the number and then pairing it in the group of three. If all the prime factors are in the pair of three then the number is a perfect cube.

Example

Calculate the cube root of 13824 by using prime factorization method.

Solution

First of all write the prime factors of the given number then pair them in the group of three.

$$13824 = \underbrace{2 \times 2 \times 2}_{2^3} \times \underbrace{2 \times 2 \times 2}_{2^3} \times \underbrace{2 \times 2 \times 2}_{2^3} \times \underbrace{3 \times 3 \times 3}_{3^3}$$

Since all the factors are in the pair of three the number 13824 is a perfect cube.

Smallest Multiple that is a Perfect Cube

As we have seen that the group of three prime factors makes a number perfect cube, so to make a number perfect cube we need to multiply it with the smallest multiple of that number.

Example

Check whether 1188 is a perfect cube or not. If not then which smallest natural number should be multiplied to 1188 to make it a perfect cube?

Solution

$$1188 = 2 \times 2 \times 3 \times 3 \times 3 \times 11$$

This shows that the prime numbers 2 and 11 are not in the groups of three. So, 1188 is not a perfect cube

To make it a perfect cube we need to multiply it with $2 \times 11 \times 11 = 242$, so, it will make the pair of 2, 3 and 11.

Hence the smallest natural number by which 1188 should be multiplied to make it a perfect cube is 242.

And the resulting perfect cube is $1188 \times 242 = 287496 (= 66^3)$.

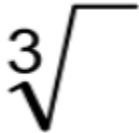
Cube Roots

Finding cube root is the inverse operation of finding the cube.

If $3^3 = 27$ then cube root of 27 is 3.

We write it as $\sqrt[3]{27} = 3$

Symbol of the Cube Root



Some of the cube roots are:

Statement	Inference	Statement	Inference
$1^3 = 1$	$\sqrt[3]{1} = 1$	$6^3 = 216$	$\sqrt[3]{216} = \sqrt[3]{6^3} = 6$
$2^3 = 8$	$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$	$7^3 = 343$	$\sqrt[3]{343} = \sqrt[3]{7^3} = 7$
$3^3 = 27$	$\sqrt[3]{27} = \sqrt[3]{3^3} = 3$	$8^3 = 512$	$\sqrt[3]{512} = \sqrt[3]{8^3} = 8$
$4^3 = 64$	$\sqrt[3]{64} = \sqrt[3]{4^3} = 4$	$9^3 = 729$	$\sqrt[3]{729} = \sqrt[3]{9^3} = 9$
$5^3 = 125$	$\sqrt[3]{125} = \sqrt[3]{5^3} = 5$	$10^3 = 1000$	$\sqrt[3]{1000} = \sqrt[3]{10^3} = 10$

Method of finding a Cube Root

There are two methods of finding a cube root

1. Prime Factorization Method

Step 1: Write the prime factors of the given number.

Step 2: Make the pair of three if possible.

Step 3: Then replace them with a single digit.

Step 4: Multiply these single digits to find the cube root.

Example

Find the cube root of 15625 by the prime factorization method.



$$\begin{array}{r|l}
 5 & 15625 \\
 \hline
 5 & 3125 \\
 \hline
 5 & 625 \\
 \hline
 5 & 125 \\
 \hline
 5 & 25 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

$$\begin{array}{l}
 15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\
 \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \\
 \therefore \sqrt[3]{15625} = 5 \qquad \times \qquad 5 \\
 = 25
 \end{array}$$

2. Estimation Method

This method is based on the estimation. Let's take the above example.

Step 1: If 15625 is the number then make the group of three digits starting from the right.
15 625

Step 2: Here 625 is the first group which tells us the unit's digit of the cube root. As the number is ending with 5 and we know that 5 comes at the unit's place of a number only when its cube root ends in 5.

So the unit place is 5.

Step 3: Now take the other group, i.e., 15. Cube of 2 is 8 and a cube of 3 is 27. 15 lie between 8 and 27. The number which is smaller among 2 and 3 is 2. The one's place of 2 is 2 itself. Take 2 as ten's place of the cube root of 15625. Thus,

$$3 \sqrt[3]{15625} = 25$$

