Chapter- 6

SETS

Main Concepts and Results

What is a Set?

- A set is a well-defined collection of distinct objects.
  - Example: A = {1, 2, 3, 4, 5}

What is an element of a Set?

- The objects in a set are called its elements.
  - So in case of the above Set A, the elements would be 1, 2, 3, 4, and 5. We can say, $1 \in A$, $2 \in A$ etc.
- Usually we denote Sets by CAPITAL LETTERs like A, B, C, etc. while their elementsare denoted in small letters like *x, y, z* etc.
- If *x* is an element of A, then we say *x* belongs to A and we represent it as $x \in A$
- If *x* is not an element of A, then we say that *x* does not belong to A and we represent it as
  $x \notin A$

How to describe a Set?

- Roaster Method or Tabular Form
  - In this form, we just list the elements
  - Example A = {1, 2, 3, 4} or B = {*a, b, c, d, e*}
- Set- Builder Form or Rule Method or Description Method
  - In this method, we list the properties satisfied by all elements of the set
  - Example $A = \{x : x \in N, x < 5\}$

Some examples of Roster Form vs Set-builder Form

	Roster Form	Set-builder Form
1	{1, 2, 3, 4, 5}	$\{x \mid x \in N, x < 6\}$
2	{2, 4, 6, 8, 10}	$\{x \mid x = 2n, n \in N, 1 \le n \le 5\}$
3	{1, 4, 9, 16, 25, 36}	$\{x \mid x = n^2, n \in N, 1 \le n \le 6\}$

Sets of Numbers

1. Natural Numbers (N)
   - N = {1, 2, 3,4 ,5 6, 7, …}
2. Integers (Z)
   - Z = {…, -3, -2, -1, 0, 1, 2, 3, 4, …}
3. Whole Numbers (W)
   - W = {0, 1, 2, 3, 4, 5, 6…}
4. Rational Numbers (Q)
   - $Q=\{\frac{p}{q} : p \in Z, q \in Z, q \neq 0\}$

Finite Sets & Infinite Sets

- Finite Set: A set where the process of counting the elements of the set wouldsurely come to an end is called finite set
  - Example: All natural numbers less than 50
  - All factors of the number 36
- Infinite Set: A set that consists of uncountable number of distinct elements iscalled infinite set.
  - Example: Set containing all natural numbers $\{x \mid x \in N, x > 100\}$

Cardinal number of Finite Set

- The number of distinct elements contained in a finite set A is called the cardinalnumber of A and is denoted by $n(A)$
  - Example A = {1, 2, 3, 4} then n(A) = 4

- A = {x | x is a letter in the word 'APPLE'}. Therefore A = {A, P, L, E} and
  $n(A) = 4$

- A = {x | x is the factor of 36}, Therefore A = { 1, 2, 3, 4, 6, 9, 12, 18, 36}and n(A) = 9

Empty Set

- A set containing no elements at all is called an empty set or a null set or a void set.
- It is denoted by φ (phai)
- In roster form you write φ = { }
- Also n (φ) = 0
    - Examples: $\{x \mid x \in N, 3 < x < 4\} = \varphi$
    - $\{x \mid x$ is an even prime number, $x > 5\} = \varphi$

Non Empty Set

- A set which has at least one element is called a non-empty set
    - Example: A = {1, 2, 3} or B = {1}

Singleton Set

- A set containing exactly one element is called a singleton set
    - Example: A = {*a*} or B = {1}

Equal Sets

- Two set A and B are said to be equal sets and written as A = B if every element ofA is in B and every element of B is in A
    - Example A = {1, 2, 3, 4} and B = {4, 2, 3, 1}
- It is not about the number of elements. It is the elements themselves.
- If the sets are not equal, then we write as A ≠ B

Equivalent Sets

- Two finite sets A and B are said to be equivalent, written as A ⟷ B,if n(A) = n(B), that is they have the same number of elements.
    - Example: A = {a, e, i, o, u} and B = {1, 2, 3, 4, 5}, Therefore n(A) = 5 andn(B) = 5 therefore A ⟷ B
- Note: Two equal sets are always equivalent but two equivalent sets need not beequal.

Subsets

- If A and B are two sets given in such a way that every element of A is in B, thenwe say A is a subset of B and we write it as $A \subseteq B$
- Therefore is $A \subseteq B$ and *x* ⍰ A then *x* ⍰ B
- If A is a subset of B, we say B is a superset of A and is written as B ⍰ A
- Every set is a subset of itself.
    - i.e. $A \subseteq A$, $B \subseteq B$ etc.
- Empty set is a subset of every set
    - i.e. $\varphi \subseteq A$, $\varphi \subseteq B$
- If $A \subseteq B$ and $B \subseteq A$, then A = B
- Similarly, if A = B, then $A \subseteq B$ and $B \subseteq A$
- If set A contains *n* elements, then there are 2^n subsets of A

Power Set

- The set of all possible subsets of a set A is called the power set of A, denoted by P(A). IfA contains *n* elements, then $P(A) = 2^n$ sets.
    - i.e. if A = {1, 2}, then $P(A) = 2^2 = 4$
    - Empty set is a subset of every set
    - So in this case the subsets are {1}, {2}, {2, 3} & φ

Proper Subset

- Let A be any set and let B be any non-empty subset. Then A is called a proper subset ofB, and is written as A □ B, if and only if every element of A is in B, and there exists at least one element in B which is not there in A.
  - i.e. if A ⊆ B and A □ B, then A □ B
  - Please note that φ has no proper subset
  - A set containing n elements has (2n – 1) proper subsets.
    - i.e. if A = {1, 2, 3, 4}, then the number of proper subsets is $(2^4 – 1) = 15$

Universal Set

- If there are some sets in consideration, then there happens to be a set which is a supersetof each one of the given sets. Such a set is known as universal set, to be denoted by U or

□□

- i.e. if A = {1, 2}, B = {3, 4}, and C = {1, 5}, then U or □ = {1, 2, 3, 4, 5}

Operations on Sets

- *Union of Sets*
  - The union of sets A and B, denoted by A □ B, is the set of all those elements, eachone of which is either in A or in B or in both A and B
  - If there is a set A = {2, 3} and B = {*a*, *b*}, then A □ B = {2, 3, *a*, *b*}
  - So if A □ B = {*x* | *x* □ A or *x* □ B}
  - , then *x* □ A □ B which means *x* □ A or *x* □ B
  - And if *x* □ A □ B which means *x* □ A or *x* □ B

- *Interaction of Sets*
  - The intersection of sets A and B is denoted by A □ B, and is a set of all

elementsthat are common in sets A and B.

- i.e. if $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$, then $A \cap B = \{2\}$ as 2 is the only commonelement.
- Thus $A \cap B = \{x: x \in A \text{ and } x \in B\}$ then $x \in A \cap B$ i.e. $x \in A$ and $x \in B$
- And if $x \in A \cap B$ i.e. $x \in A$ and $x \in B$
- Disjointed Sets
  - Two sets A and B are called disjointed, if they have no element in common.Therefore:
  - $A \cap B = \varphi$ i.e. if $A = \{2, 3\}$ and $B = \{4, 5\}$, then $A \cap B = \varphi$
- Intersecting sets
  - Two sets are said to be intersecting or overlapping or joint sets, if they haveat least one element in common.
  - Therefore two sets A and B are overlapping if and only if $A \cap B \neq \varphi$
  - Intersection of sets is Commutative
    - i.e. $A \cap B = B \cap A$ for any sets A and B

- Intersection of sets is Associative
  - i.e. for any sets, A, B, C,
    - $(A \cap B) \cap C = A \cap (B \cap C)$
    - If $A \subseteq B$, then $A \cap B = A$
  - Since $A \subseteq U$, so $A \cap U = A$
    - For any sets A and B, we have
      - $A \cap B \subseteq A$ and $A \cap B \subseteq B$
    - $A \cap \varphi = \varphi$ for every set A

- <u>*Difference of Sets*</u>
  - For any two sets A and B, the difference A – B is a set of all those elements of Awhich are not in B.

i.e. if A = {1, 2, 3, 4, 5} and B = {4, 5, 6}, Then A – B = {1, 2, 3} and B – A = {6}

Therefore A – B = {x | x □ A and x □ B}, then x □ A – B then x □ A but x □ B

- If A □ B then A – B = □

- *Complement of a Set*

  - Let x be the universal set and let A □ x. Then the complement of A, denoted by A'is the set of all those elements of x which are not in A.

    - i.e. let □ = {1, 2, 3, 4, 5,6 ,7 ,8} and A = {2, 3,4 }, then A' = {1, 5, 6, 7, 8}
    - Thus A' = {x | x □ □ and x □ A} clearly x □ A' and x □ A

  - Please note

    - □' = □ and □'= □
    - A □ A' = □ and A □ A' = φ

Disruptive laws for Union and Intersection of Sets

- For any three sets A, B, C, we have the following

  - A □ (B □ C) = (A □ B) □ (A □ C)

    Say A = {1, 2}, B = {2, 3} and C = {3, 4}

    Therefore A □ (B □ C) = {1, 2, 3} and

    And (A □ B) □ (A □ C) = {1, 2, 3} and hence equal

  - A □ (B □ C) = (A □ B) □ (A □ C)

    Say A = {1, 2}, B = {2, 3} and C = {3, 4}

    Then A □ (B □ C) = {2} and (A □ B) □ (A □ C) = {2} and hence equal

Disruptive laws for Union and Intersection of Sets

- De-Morgan's Laws

  – Let A and B be two subsets of a universal set □, then

- (A □ B)’ = A’ □ B’
- (A □ B)’ = A’ □ B’

  Let □ = {1, 2, 3, 4, 5, 6} and A = {1, 2, 3} and B = {3, 4, 5}

  Then A □ B = {1, 2, 3, 4, 5}, therefore (A □ B)’ = {6}

  A’ = {4, 5, 6} and B’ = {1, 2, 6}

  Therefore A’ □ B’ = {6}. Hence proven