Chapter- 8

Factors and Multiples

STUDY NOTES

This lesson will help you to:

- Find the divisibility of a number by 2, 3, 4, 5, 6, 8, 9, 10, 11, 12 and 15.
- Recognize and find the factors and multiples of any given number.
- Know about the properties and facts of factors and multiples.
- Know about the prime and composite numbers.
- Find the HCF and LCM of two or more numbers using different methods.
- Relationship between H.C.F. and L.C.M. and the numbers.
- Understand the real life applications of factors and multiples.

ODD AND EVEN NUMBER :

- **A number which is a multiple of 2 is called an even number.**
  Example : 2, 4, 6, 8..... are even numbers.
- **A number which is not a multiple of 2 is called an odd number.**
  Example : 1, 3, 5, 7,..... are odd numbers.

TESTS OF DIVISIBILITY OF NUMBERS :

Rohit and Rakhi have 17 pencils.

They want to divide these 17 pencils in two equal parts. But it is not possible, because 17 is not divisible by 2.

A number is divisible by another number, if it leaves no remainder.

- **Divisibility by 2** : A number is divisible by 2, if the end digit of the number is 0, 2, 4, 6 or 8. Example : 12, 216, 5028, 56790
- **Divisibility by 3 :** A number is divisible by 3, if the sum of all of its digits is divisible by 3. Example : 216, 5028

  2 + 1 + 6 = 9, is divisible by 3.

  5 + 0 + 2 + 8 = 15, is divisible by 3.

- **Divisibility by 4 :** A number is divisible by 4, if the last two digits of the number are divisible by 4.

  Example : 512, 6024

  In 512 , 12 is divisible by 4. So, 512 is number by 4.

  In 6024, 24 is divisible by 4. So, 6024 is also number by 4.

- **Divisibility by 5 :** A number is divisible by 5, if the number ends with 0 or 5. Example : 10, 25, 30, 415, 610 are divisible by 5.

- **Divisibility by 6 :** A number is divisible by 6, if it is divisible by 2 and 3. Example : Is 630 divisible by 6 ?

  Sol. In 630, last digit is 0; therefore, it is divisible by 2 and

  Sum of their digits 6 + 3 + 0=9. So it is divisible by 3. So, the number 630 is divisible by 6.

- **Divisibility by 8 :** A number is divisible by 8, if the number by the last digits of the given number is divisible by 8.

  Example : 96432

  432 is divisible by 8.

  So, 96432 is divisible by 8.

- **Divisibility by 9 :** A number is divisible by 9, if and only if the sum of its digits is divisible by 9. Example : 918

  Sum of their digits 9 + 1 + 8 = 18. So it is divisible by 9. 18 is divisible by 9, so 918 is divisible by 9.

- **Divisibility by 10 :** A number is divisible by 10, if the number ends with 0.

  Example : 10, 20, 50 , 490 are divisible by 10.

- **Divisibility by 12 :** If the number is divisible by both 3 and 4 , then it is divisible by 12.

  Example: 24, 60, 2700, 56100… etc.

- **Divisibility by 15 :** If the number is divisible by both 3 and 5 , then it is divisible by 15.

  Example: 45, 90,450, 2700… etc.

- **Divisibility by 11 :** If the number difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number. If the difference is either 0 or divisible by 11, then the number is divisible by 11.

Example: 308, 1331, 61809 etc.

Number	Sum of the digits (at odd places) From the right	Sum of the digits (at even places) From the right	Difference
308	8 + 3 = 11	0	11 - 0 = 11
1331	1 + 3 = 4	3 + 1 = 4	4 - 4 = 0
61809	9 + 8 + 6 = 23	0 + 1 = 1	23 - 1 = 22

PRIME AND COMPOSITE NUMBERS :

Now, we are familiar with the factors of a number. Observe the number of factors of a few numbers arranged in the given table:

Numbers	Factors	Number of Factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6

We find that :

a. The number 1 has only one factor. (i.e. itself)

b. There are numbers, having exactly two factors 1 and the number itself. Such numbers are 2, 3, 5, 7, 11 etc. These numbers are prime numbers.

The numbers other than 1 whose only factors are 1 and the number itself are called Prime numbers.

c. There are numbers having more than two factors like 4, 6, 8, 9, 10 and so on. These are composite numbers.

The numbers having more than two factors are called Composite numbers.

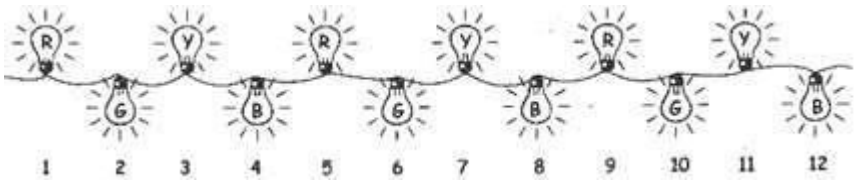
d. Two prime numbers which differ by 2 are called **twin primes**. Some examples of pairs of twin primes are : 3,5 ; 5,7 ; 11,13 .

QUICK CONCEPT REVIEW

MULTIPLES

It is Diwali night, Pinki & her family are celebrating & enjoying together. Pinki saw the lightning all around

She saw many colorful lights. She started counting the number of bulbs.



She saw that blue bulbs are placed at 4th place, 8th place, and 12th place and so on.....

Let us observe these numbers. 4, 8, 12.....

This is similar to the table of 4

or we can say that these numbers are the multiples of 4.

Definition of multiples :

- A multiple is a number that is the product of a given number and some other number.

For example: If A x B = C, C is multiple of both A and B.

- A multiple is basically a time table.
- We can find the multiple of a number by multiplying it by 1,2,3,4 and so on.

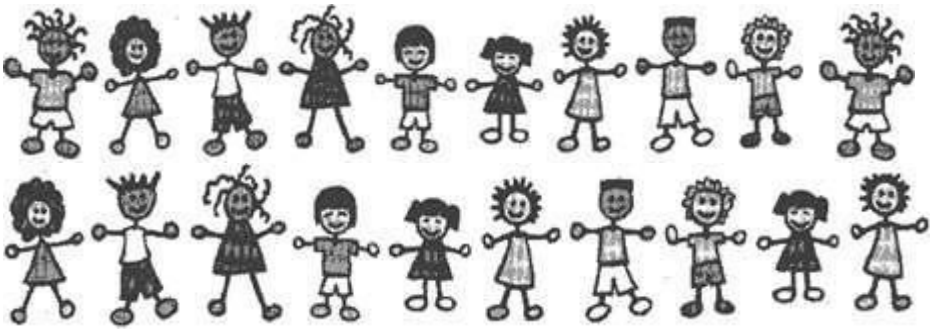
Properties of multiples :

1. Every number is a multiple of itself.
2. Every number is a multiple of 1.
3. A multiple of a number is always greater than or equal to the number itself.
4. A number has an uncountable number of multiples.
5. There is no largest multiple of a number.

FACTORS

It was picture day in Ria's school. Her teacher made all the students stand in a single line. But all of them could not come in the frame.

So, she decided to make 2 lines of 10 students each.



This way also all the students were not fitting in the frame.

Then she made 4 lines of 5 each. Now all the students could fit in the frame.

So here we saw three different ways to make 20 students stand in lines. The first way is 1×20
The second way is 2×10
& the third way is 4×5

Therefore, we can say that 1, 2, 4, 5, 10 & 20 are the factors of 20.

Definition of factors: The factors of a number are those which divide the number without leaving any remainder.

Thus, factors of a number divide the number completely.

Properties of factors :

2. 1 is a factor of every number.
3. Every number is a factor of itself.

4. A number has a limited number of factors.
5. A factor of a number is neither equal to or smaller than the number.

COMMON FACTORS

The same factors of two or more than two different numbers are called common factors.

Let us find out the factors of 15 & 21.

Factors of 15 are 1, 3, 5 & 15.

Factors of 21 are 1, 3, 7 & 21.

Therefore, Common factors of 15 & 21 are 1 and 3.

HIGHEST COMMON FACTOR (H.C.F) OR GREATEST COMMON FACTOR (G.C.F.)

The common factor which is highest among the common factors of two or more than two numbers is called H.C.F. of those number.

Methods to find H.C.F. :

(i) By listing factors:

Factors of 12 = 1. 2. 3. 4. 6, 12

Factors of 32 = 1. 2.4. 8,16, 32

Common factors of 12 and 32 = 1.2,4

H.C.F. of 12 and 32 = 4

(ii) By prime factorization method:

H. C.F. of 12 and 24

2	12
2	6
3	3
	1

2	24
2	12
2	6
3	3
	1

12=2×2×3

24=2×2×2×3

Common prime factors

=2×2×3=12.

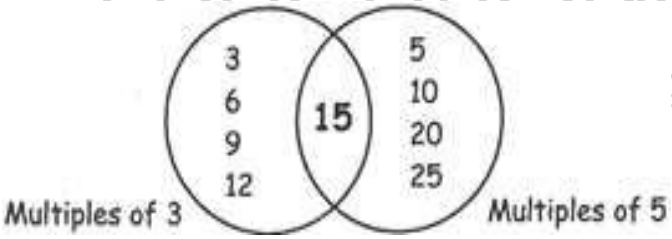
So, H.C.F. of 12 and 24 = 12

COMMON MULTIPLES

For Example, let us find out the multiples of 3 & 5.

Multiples of 3: 3, 6, 9,12,15,18, 21, 24....

Multiples of 5: 5,10,15, 20, 25



Therefore, the common multiple of 3 & 5 is 15.

LOWEST COMMON MULTIPLES (L.C.M.)

The lowest common multiple among the common multiples of two or more than two numbers is called their L.C.M.

Properties of L.C.M.:

1. The L.C.M. of two prime numbers is their product.

Example: L.C.M. of 3 and 2 = 6

L.C.M. of 7 and 2 = 14

2. I f a number is a factor of another number, then the L.C.M. is the greater number itself.

Example : L.C.M. of 3 and 9 = 9

L.C.M. of 8 and 24 = 24

3. The L.C.M. of two or more numbers cannot be less than either of them.

4. The L.C.M. of two consecutive numbers is the product of the numbers.

Methods to find L.C.M.

(i) Listing multiples and then finding out the least common multiple.

Example : Find the LCM of 5 and 7.

Multiples of 5 = 5,10,15, 20, 25, 30, 35, 40,……

Multiples of 7 = 7, 14, 21, 28, 35,42, ……

Common multiples of 5 and 7 = 35, 70, ……

Least common multiple of 5 and 7 = 35.

(ii) Prime factorization method

Example : Find the LCM of 8, 24 and 36.

Step -1 : Find the prime factors of each number.

Prime factors of 8 : 2 × 2 × 2

Prime factors of 24 : 2 × 2 × 2 × 3

Prime factors of 36 : 2 × 2 × 3 × 3

Step-2: Ring all common factors. Taking ony one factor out of a set of common factors, multiply all prime factors. This gives the LCM.

LCM = 2 × 2 × 2 × 3 × 3 =72

Answer : 72

(iii) Division method

2	36, 72
2	18, 36
3	9, 18
3	3, 6
2	1, 2
	1, 1

Now, 2×2×3×3×2=72

Hence, L.C.M. of 36 and 72 = 72.

Relationship between L.C.M. , H.C.F. and the numbers :

Product of the two numbers = L.C.M × H.C.F.

Consider 6 and 8.

L.C.M. of 6 and 8 =24

H.C.F. of 6 and 8 =2

L.C.M × H.C.F. 24 × 2 = 48

The product of 8 and 6 =48

Therefore, **L.C.M × H.C.F. = Product of the two numbers**

I) L.C.M. of two numbers = $\frac{\textbf{Their product}}{\textbf{Their H.C.F.}}$

II) H.C.F. of two numbers = $\frac{\text{Their product}}{\text{Their L.C.M.}}$

III) $\frac{\text{H.C.F.} \times \text{L.C.M.}}{\text{One number}}$ **= The other number**

Story sums :

Find the largest number that divide 12 and 20 without a remainder.

Ans : The largest number that divides 12 and 20 without a remainder is the HCF of 12 and 20.

12 = 2 × 2 × 3

20 = 2 × 2 × 5

HCF = 2 × 2 = 4

So, 4 is the largest number that divides 12 and 20 without a remainder

IN A NUT SHELL

A number is Divisible by	If the last digit is
2	0, 2, 4, 6, 8
5	0, 5
10	0

A number is Divisible by	If the sum of its digit is divisible by
3	3
9	9

A number is Divisible by	If it is divisible by
6	2 and 3
12	3 and 4
15	3 and 5

SUMMARY :

- Whenever we multiply a number with 1,2,3,4,5,6,7, …… we get multiples of that number.
- Every number is a multiple of 1.
- Every multiple of a number is either greater than or equal to the number.
- A number which is a multiple of 2 is called an even number.
- A number which is not a multiple of 2 called an odd number.
- When we multiply any two or more numbers, we get a product. Then each number is a factor of the product.
- Every number is a factor of itself.
- Every factor of a number is either less than or equal to the number.
- A number is divisible by another number, if dividing the remainder is 0.
- If the first number is a factor of the second number, then the second number is divisible by the first number.
- A number that has exactly two factors is called a prime number.
- A number which has more than two factors is called a composite number.
- The number 1 has only one factor. It is neither a prime nor a composite number.
- Product of the two numbers = L.C.M × H.C.F.

MIND MAP