REAL NUMBERS

PPT-5

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 01
CHAPTER NAME : REAL NUMBERS

PREVIOUS KNOWLEDGE TEST :

- An algorithm is a series of well defined step which gives a procedure for solving a type of problem
- A lemma is a proven statement used for proving another statement.
- A natural number which has exactly two factors, i.e. 1 and the number itself, is a **prime number**.
- Every non-prime number is a composite number. **Composite numbers** are those natural numbers which have more than two factors. Such numbers are divisible by other numbers as well.
- A number that cannot be expressed in the form p/q, where p and q are integers and q is not equal to 0,is called irrational number. Example - $\sqrt{2}$.

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Learning outcome

1. Students will be able to **define** Irrational Numbers.
2. Students will be able to **explain** Irrational Numbers.

3.Students will be able to **prove** square root of some specific natural numbers which are irrationals.

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Irrational numbers with illustration.

- https://www.youtube.com/watch?v=CtRtXoT_2Ps

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Theorem 1.3 Let P be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

Let P be a prime number. If P divides a^2 then P divides a where **a** is a positive integer.

From FTA Integer a can be factorized as the product of primes.

Let a = P_1, P_2, P_3 …………… P_n be the prime factorization of a where P_1, P_2 ….. P_n are primes not necessarily distinct.

Now,

$a = P_1 P_2 P_3 \ldots\ldots P_n$

$\Rightarrow a^2 = P_1^2 P_2^2 P_3^2 \ldots\ldots P_n^2$

Now it is given that P is prime and it divides a^2….. P is a prime factor of a^2 from the uniqueness part of FTA it follows that the only prime factors of a2 are P_1, P_2, P_3, ….. P_n. So P is one of P_1, P_2, P_3, …. P_n.

P/ P_1, P_2, P_3….. P_n

P/a

Prove that $\sqrt{2}$ is irrational by contradiction method.

Let $\sqrt{2}$ be rational. Then there exist integers **a** and **b** such that

$\sqrt{2} = \frac{a}{b}$ where a and b co-prime and $b \neq 0$

$\Rightarrow (\sqrt{2})^2 = \frac{a^2}{b^2}$ $\Rightarrow 2 = \frac{a^2}{b^2}$

$\Rightarrow 2b^2 = a^2$ $\Rightarrow$ 2 divides a^2

$\Rightarrow$ 2 divides a

Now, we can write a = 2k for some integer K placing a = 2K we get

$2b^2 = (2K)^2$ $\Rightarrow 2b^2 = 4K^2$

$\Rightarrow b^2 = 2K^2$ $\Rightarrow$ a divides b^2

$\Rightarrow$ 2 divides b

$\therefore$ a and b have at least 2 as a common factor. But this contradicts the So our supposition is wrong.

$\sqrt{2}$ is irrational

Prove that $3+2\sqrt{5}$ is irrational.

Let $3+2\sqrt{5}$ be rational. Then there exist integers a and b $(b \neq 0)$ such that

$3+2\sqrt{5}=\frac{a}{b}$, where a and b are co-prime

$\Rightarrow 2\sqrt{5}=\frac{a}{b}-3$ $\Rightarrow 2\sqrt{5}=\frac{a-3b}{b}$

$\Rightarrow \sqrt{5}=\frac{a-3b}{2b}$ $\Rightarrow$ a, b, 2, 3 are integers

$\Rightarrow \frac{a-3b}{2b}$ is rational

$\Rightarrow$ $\sqrt{5}$ is also rational

But we know that $\sqrt{5}$ is irrational. So our supposition is wrong.

$3+2\sqrt{5}$ is irrational

Prove that $3\sqrt{5}$ is irrational

Let $3\sqrt{5}$ be rational. Then fare exist integers a and b $(b \neq 0)$ such that

$3\sqrt{5} = \frac{a}{b}$ a and b are co-prime

$\Rightarrow \sqrt{5} = \frac{a}{3b}$

$\Rightarrow$ a, b, 3 are integers

$\Rightarrow \frac{a}{3b}$ is rational

$\Rightarrow \sqrt{5}$ is also rational

But we know that, $\sqrt{5}$ is irrational.

So, our supposition is wrong.

$3\sqrt{5}$ is irrational

Home assignment

- HW- Ex. 1.3 Q. No 1 to 3. .

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THANKING YOU

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