ODM EDUCATIONAL GROUP

Changing your Tomorrow

REAL NUMBERS

PPT-7

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 01
CHAPTER NAME : REAL NUMBERS

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RECAPITULATION

- An algorithm is a series of well defined steps which gives a procedure for solving a type of problem
- A lemma is a proven statement used for proving another statement.
- A natural number which has exactly two factors, i.e. 1 and the number itself, is a **prime number**.
- Every non-prime number is a composite number. **Composite numbers** are those natural numbers which have more than two factors. Such numbers are divisible by other numbers as well.
- A number that cannot be expressed in the form p/q, where p and q are integers and q is not equal to 0,is called irrational number. (Example-$\sqrt{2}$).

Learning outcome

- Students will be able to **define** Lemma and Euclid's Division Lemma, Algorithm, Euclid's division algorithm, fundamental theorem of arithmetic, Irrational Numbers and rational Numbers
- Students will be able to **express** the relationship among dividend, divisor, quotient and remainder
- Students will be able to **formulate** numbers in algebraic form and **solve** analytical problems using Euclid's Division Lemma
- Students will be able to **FIND** the HCF of numbers using Euclid's division algorithm
- Students will be able to **find** HCF and LCM of numbers using prime factorization and the Establish relationship among them
- Students will be able to **prove** square root of some specific natural numbers are irrationals
- Students will be able to **find** the decimal expansions of Rational Numbers using different methods

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- Theorem 1.5 : Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form , p /q where p and q are coprime, and the prime factorization of q is of the form 2^m5^n. , where n, m are non-negative integers.
- Theorem 1.6 : Let x = p q be a rational number, such that the prime factorization of q is of the form 2^m5^n. , where n, m are non-negative integers. Then x has a decimal expansion which terminates.
- Let x = p q be a rational number, such that the prime factorization of q is not of the form 2^m5^n. , where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

- Amazing Tricks to Understand Real Numbers
- https://youtu.be/UUpa8t7H1Hg(10.34)

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Prove that $\sqrt{2}$ is irrational

- Let us assume, to the contrary, that $\sqrt{2}$ is rational.
- So we can find integers r and s ($\neq 0$) such that $\sqrt{2} = r/s$.
- Suppose r and s have a common factor other than 1. Then, we divide by the common factor to get $\sqrt{2} = a / b$, where a and b are coprime.
- So, $b\sqrt{2} = a$. Squaring on both sides and rearranging, we get
- 2b2 = a2 . Therefore,$\sqrt{2}$ divides a2 2 divides a. So, we can write a = 2c for some integer c.
- Substituting for a, we get 2b2 = 4c2 ,
- that is, b2 = 2c2 . This means that 2 divides b2 , and so 2 divides b .Therefore, a and b have at least 2 as a common factor. But this contradicts the fact that a and b have no common factors other than 1.
- This contradiction has arisen because of our incorrect assumption that 2 is rational.
- So, we conclude that $\sqrt{2}$ is irrational.

Show that square of any positive integer is of the form 3m or 3m +1 for some integer m.

Let a be any positive integer.

Then it is of the form 3q or 3q + 1 or 3q +2.

$x = 3q$

$$\Rightarrow (x)^2 = (3q)^2$$

$$\Rightarrow x^2 = 9q^2$$

$$\Rightarrow x^2 = 3 x 3q^2$$

$$= 3m \text{, where } m = 3q^2$$

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$x = 3q + 1$

$$\Rightarrow (x)^2 = (3q+1)^2$$

$$= (3q)^2 + 2x3qx1 + (1)^2$$

$$= 9q^2 + 6q + 1$$

$$= 3\left(3q^2 + 2q\right) + 1$$

$$= 3m + 1, \text{ where } m = 3q^2 + 2q$$

$x = 3q + 2$

$$\Rightarrow (x)^2 = (3q+2)^2$$

$$= (3q)^2 + 2 \times 3q \times 2 + (2)^2$$

$$= 9q^2 + 12q + 4$$

$$= 9q^2 + 12q + 3 + 1$$

$$= 3\left(3q^2 + 4q + 1\right) + 1$$

$$= 3m + 1$$

$\therefore$ Square of any positive integer is of the form 3m or 3m +1.

Prove that product of two consecutive positive integers is divisible by 2.

Let n and n + 1 be two consecutive positive integers n is in the form 2q and 2q + 1.

When $n=2q$

$$n(n+1)=2q(2q+1) \text{ divisible, by 2.}$$

When n = 2q + 1

$$n(n+1)=(2q+1)(2q+2)$$

$$=(2q+1)2(q+1)$$

$$=2(2q+1)(q+1) \qquad \text{Divisible by 2.}$$

$\therefore$ Product of two consecutive positive integer is divisible by 2.

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THANKING YOU

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