POLYNOMIALS

PPT-3

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02
CHAPTER NAME : POLYNOMIALS

CHANGING YOUR TOMORROW

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Learning outcome

1. Students will be able to **know and find** the relationship between zeroes and coefficients of a cubic polynomial

2.. Students will be able to solve questions involving relationship between zeroes and coefficients of a cubic polynomial

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PREVIOUS KNOWLEDGE TEST

1. sum of zeroes =- (Coefficient of x)/ Coefficient of x^2 =-b/a
2. product of zeroes = = Constant term/ Coefficient of x^2= c/a
3. A **quadratic polynomial** can have at most **2 zeroes** and **a cubic polynomia**l can have atmost **3 zeroes**
4. General form of linear polynomials $ax + b\ where\ a \neq 0$
5. General from of quadratic polynomials $ax^2 + bx + c$ where a≠O
6. General form of cubic polynomial$ax^3 + bx^2 + cx + d, where\ a$ ≠0

Verify that $3, -1, -\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$ and then verify the relationship between the zeroes and its coefficients.

Solution. Given polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$

Comparing the given polynomial to the general cubic polynomial $ax^3 + bx^2 + cx + d$, we get $a = 3,\ b = -5,\ c = -11,\ d = -3$

$$p(3) = 3.3^3 - 5.3^2 - 11.3 - 3 = 81 - 45 - 33 - 3 = 0$$

$$p(-1) = 3(-1)^3 - 5.(-1)^2 - 11.(-1) - 3 = -3 - 5 + 11 - 3 = 0$$

$$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^3 - 5.\left(-\frac{1}{3}\right)^2 - 11\left(-\frac{1}{3}\right) - 3$$

$$= -3 \times \frac{1}{27} - \frac{5}{9} + \frac{11}{3} - 3 = \frac{-1 - 5 + 33 - 27}{9} = \frac{0}{9} = 0$$

$\therefore$ 3, -1 and $\left(-\frac{1}{3}\right)$ are zeroes of the polynomial $p(x)$.

We take $\alpha = 3,\ \beta = -1$ and $\gamma = -\frac{1}{3}$. Then,

(i) $\alpha + \beta + \gamma = 3 + (-1) + \left(-\frac{1}{3}\right) = 3 - \frac{4}{3} = \frac{9-4}{3} = \frac{5}{3} = -\frac{-5}{3} = \frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{b}{a}$

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha = 3(-1) + (-1).\left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)(3)$

$$= -3 + \frac{1}{3} - 1 = -4 + \frac{1}{3} = \frac{-11}{3} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$$

(iii) $\alpha\beta\gamma = 3(-1)\left(-\frac{1}{3}\right) = 1 = -\frac{-3}{3} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{d}{a}$.

If α, β, γ are the zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.

Solution. Given $p(x) = 6x^3 + 3x^2 - 5x + 1$

On comparing with $p(x) = ax^3 + bx^2 + cx + d$, we get : $a = 6, \quad b = 3, \quad c = -5, \quad d = 1$

As α, β, γ are the zeroes of $p(x)$, so

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{3}{6} = -\frac{1}{2}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = -\frac{5}{6}$$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{6}$$

$$\therefore \quad \alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} = \frac{-5/6}{-1/6} = 5.$$

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If α, β are the two zeroes of the polynomial $2x^2 - 5x + 7$, find the quadratic polynomial whose zeroes are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

Solution. Given $f(x) = 2x^2 - 5x + 7$

$$\therefore \quad \alpha + \beta = \frac{5}{2} \quad \text{and} \quad \alpha\beta = \frac{7}{2}$$

For the required quadratic polynomial,

Sum of the zeroes $= (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = \frac{25}{2} = -\frac{b}{a}$

Product of the zeroes $= (2\alpha + 3\beta)(3\alpha + 2\beta) = 6(\alpha^2 + \beta^2) + 13\alpha\beta = 6[(\alpha + \beta)^2 - 2\alpha\beta] + 13\alpha\beta$

$$= 6\left[\frac{25}{4} - 2 \times \frac{7}{2}\right] + 13 \times \frac{7}{2} = \frac{82}{2} = \frac{c}{a}$$

$\Rightarrow \quad a = 2, \quad b = -25, \quad c = 82$

$\therefore$ Required polynomial $= 2x^2 - 25x + 82$.

Some Useful Relations Involving α and β -

1. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta.$
2. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
3. $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$
4. $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
5. $\alpha^3 - \beta^3 = (\alpha - \beta)^3 - 3\alpha\beta(\alpha - \beta)$
6. $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$
7. $\alpha^4 - \beta^4 = (\alpha - \beta)(\alpha + \beta)(\alpha^2 + \beta^2) = (\alpha + \beta)[(\alpha + \beta)^2 - 2\alpha\beta]\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$.

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If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$, satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k.

[CBSE OD 17]

Solution. Given $p(x) = 2x^2 + 5x + k \Rightarrow a = 2,\ b = 5,\ c = k$

$$\therefore \quad \alpha + \beta = -\frac{b}{a} = -\frac{5}{2} \text{ and } \alpha\beta = \frac{c}{a} = \frac{k}{2}$$

Given $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4} \Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$

$$\Rightarrow \quad \left(-\frac{5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4} \Rightarrow -\frac{k}{2} = \frac{21}{4} - \frac{25}{4} = -\frac{4}{4} = -1$$

$$\therefore \quad k = 2.$$

If α and $\frac{1}{\alpha}$ are the zeroes of the polynomial $4x^2 - 2x + (k - 4)$, find the value of k.
[CBSE 10]

Solution. Given $f(x) = 4x^2 - 2x + (k - 4)$

Product of the zeroes $= \alpha \cdot \frac{1}{\alpha} = \frac{c}{a} = \frac{k-4}{4}$

$\therefore \quad \frac{k-4}{4} = 1 \quad \Rightarrow \quad k = 8.$

If α, β are zeroes of quadratic polynomial $x^2 - (k+6)x + 2(2k-1)$, find k if $\alpha + \beta = \frac{1}{2}\alpha\beta$.

[CBSE 10]

Solution. Given $f(x) = x^2 - (k+6)x + 2(2k-1)$

$\therefore$ $\alpha + \beta = -\frac{b}{a} = k + 6$ and $\alpha\beta = \frac{c}{a} = 2(2k-1)$

But $\alpha + \beta = \frac{1}{2}\alpha\beta \Rightarrow k + 6 = \frac{1}{2} \times 2(2k-1) \Rightarrow k = 7.$

HOME ASSIGNMENT ; Example 5.& AHA
Ex 2.4 Q1 &2

THANKING YOU

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