POLYNOMIALS

PPT-4

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02
CHAPTER NAME : POLYNOMIALS

CHANGING YOUR TOMORROW

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Learning outcome

- 1.Students will be able to know Division algorithm for polynomials
  2. Students will be able to establish relationship among dividend,divisior,quotient and the remainder.

PREVIOUS KNOWLEDGE TEST

Relationship between the zeros and the coefficients of a polynomial:

(i) If α, β are zeros of $p(x) = ax^2 + bx + c$, then

$$\text{Sum of zeros} = \alpha + \beta = \frac{-b}{a} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeros} = \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(ii) If α, β, γ are zeros of $p(x) = ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3}$$

(iii) If α, β are roots of a quadratic polynomial $p(x)$, then

$$p(x) = x^2 - (\text{sum of zeros})\, x + \text{product of zeros}$$

$$\Rightarrow p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

(iv) If α, β, γ are the roots of a cubic polynomial $p(x)$, then

$$p(x) = x^3 - (\text{sum of zeros})\, x^2 + (\text{sum of product of zeros taken two at a time})\, x - \text{product of zeros}$$

$$\Rightarrow p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

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- Division Algorithm for polynomials
- If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$
- https://youtu.be/vs2GYsMn9vw

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- Some more Division Algorithm for polynomials
- https://youtu.be/a9-ME46dX18(10.43)

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Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$ and verify the division algorithm.

[NCERT ; CBSE 2010]

Solution. Writing the given polynomials in standard form,

$$p(x) = -x^3 + 3x^2 - 3x + 5,\ g(x) = -x^2 + x - 1$$

Let us divide $p(x)$ by $g(x)$ by long division.

$$
\begin{array}{r|l}
 & x - 2 \\ \hline
-x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\
 & -x^3 + x^2 \quad - x \\
 & + \quad - \quad\quad + \\ \hline
 & \quad 2x^2 - 2x + 5 \\
 & \quad 2x^2 - 2x + 2 \\
 & \quad + \quad + \quad - \\ \hline
 & \quad\quad\quad\quad 3 \\ \hline
\end{array}
$$

$\therefore$ Quotient, $q(x) = x - 2$ and remainder, $r(x) = 3$

Now, Divisor × quotient + remainder $= (-x^2 + x - 1)(x - 2) + 3$

$$= x(-x^2 + x - 1) - 2(-x^2 + x - 1) + 3$$

$$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3 = -x^3 + 3x^2 - 3x + 5 = \text{Dividend}$$

Hence, the division algorithm is verified.

- Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in the following : $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

(*i*) Here $p(x) = x^3 - 3x^2 + 5x - 3$; $g(x) = x^2 - 2$

dividing $p(x)$ by $g(x)$

$$
\begin{array}{r|l}
 & x - 3 \\
\hline
x^2 - 2 & x^3 - 3x^3 + 5x - 3 \\
 & \underline{\underset{-}{x^3} \qquad \underset{+}{-}\, 2x} \\
 & -3x^2 + 7x - 3 \\
 & \underline{\underset{+}{-}3x^2 \qquad \underset{-}{+}\,6} \\
 & 7x - 9 \\
\end{array}
$$

Quotient $= x - 3$, Remainder $= 7x - 9$

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

- $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(*ii*) We have,

$$\begin{array}{r|l} & 2t^2 + 3t + 4 \\ \hline t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\ & \underline{\overset{-}{}2t^4 \qquad \mp 6t^2} \\ & 3t^3 + 4t^2 - 9t \\ & \underline{\overset{-}{}3t^3 \qquad \mp 9t} \\ & 4t^2 - 12 \\ & \underline{\overset{-}{}4t^2 \mp 12} \\ & 0 \end{array}$$

Clearly, remainder is zero, so $t^2 - 3$ is a factor of polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be $px + q$. Find the values of p and q

Solution.

$$
\begin{array}{r|l}
 & x^2 + 2x + 3 \\
\hline
x^2 + 5 & x^4 + 2x^3 + 8x^2 + 12x + 18 \\
 & x^4 \qquad + 5x^2 \\
 & - \qquad\quad - \\
\hline
 & 2x^3 + 3x^2 + 12x + 18 \\
 & 2x^3 \qquad + 10x \\
 & - \qquad\quad - \\
\hline
 & 3x^2 + 2x + 18 \\
 & 3x^2 \qquad + 15 \\
 & - \qquad\quad - \\
\hline
 & 2x + 3 \\
\hline
\end{array}
$$

As remainder is given to be $px + q$, so $px + q = 2x + 3 \Rightarrow p = 2$ and $q = 3$.

:HOME ASSIGNMENT Ex. 2.2 Q. No 1 to 2.

AHA

1. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be px + q. Find the values of p and q.

2. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by the polynomial $2x^2 - 5$, then find the values of a and b.

3. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be x + a, find the values of k and a.

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