

POLYNOMIALS

PPT-6

SUBJECT : MATHEMATICS

CHAPTER NUMBER: 02

CHAPTER NAME : POLYNOMIALS

CHANGING YOUR TOMORROW

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RECAPITULATION

1. Division algorithm for polynomials.

$p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$. This result is known as division algorithm for polynomials.

2. Every **linear polynomial** in one variable has a **unique zero**, a non-zero constant polynomial has no zero, and every real number is a zero of the zero polynomial

3 A **quadratic polynomial** can have at most **2 zeroes** and a **cubic polynomial** can have at most **3 zeroes**

4. General form of linear polynomials $ax + b$ where $a \neq 0$

5. General form of quadratic polynomials $ax^2 + bx + c$ where $a \neq 0$

1. sum of zeroes = - (Coefficient of x) / Coefficient of x^2 = $-b/a$

2. product of zeroes = Constant term / Coefficient of x^2 = c/a

Learning outcome

1. Students will be able to define polynomials, linear, quadratic & cubic polynomials,
2. Students will be able to express polynomials in general form.
3. Students will be able to know the zero of a polynomial.
4. Students will be able to know the geometrical meaning of the zeroes of a polynomial.
5. Students will be able to know the relationship between zeroes and coefficients of a quadratic polynomial

6. Students will be able to know Division algorithm for polynomials
7. Students will be able to establish relationship among dividend, divisor, quotient and the remainder.
8. Students will be able to find the remaining zeroes of a polynomial when some of its zeroes are given.

- Division algorithm for polynomials

<https://youtu.be/vs2GYsMn9vw> (3.22)

Quick revision of polynomial

<https://youtu.be/YmDnGcol-gs> (10.06)

Previous years questions

https://youtu.be/F140P_dJbmo (12.02)

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in the following : $P(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(i) Here $p(x) = x^3 - 3x^2 + 5x - 3$; $g(x) = x^2 - 2$
dividing $p(x)$ by $g(x)$

$$\begin{array}{r}
 x - 3 \\
 x^2 - 2 \overline{) x^3 - 3x^2 + 5x - 3} \\
 \underline{-x^3 \qquad \quad + 2x} \\
 -3x^2 + 7x - 3 \\
 \underline{+3x^2 \qquad \quad + 6} \\
 7x - 9
 \end{array}$$

Quotient = $x - 3$, Remainder = $7x - 9$

_____) If α, β are the two zeroes of the polynomial $3x^2 - 4x + 1$, find a quadratic polynomial whose zeroes are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

Solution. Given $f(x) = 3x^2 - 4x + 1$ $\therefore \alpha + \beta = \frac{4}{3}$ and $\alpha\beta = \frac{1}{3}$

For the required quadratic polynomial,

$$\text{Sum of the zeroes} = \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta} = \frac{\frac{64}{27} - 3 \times \frac{1}{3} \times \frac{4}{3}}{\frac{1}{3}} = \frac{28}{9} = -\frac{b}{a}$$

$$\text{Product of the zeroes} = \frac{\alpha^2}{\beta} \cdot \frac{\beta^2}{\alpha} = \alpha\beta = \frac{1}{3} = \frac{c}{a}$$

$$\Rightarrow a = 9, b = -28, c = 3$$

$$\therefore \text{Required polynomial} = 9x^2 - 28x + 3.$$

If one root of the quadratic polynomial $2x^2 - 3x + p$ is 3, find the other root. Also, find the value of p .

Sol. \because 3 is a root (zero) of $p(x)$

$$\Rightarrow 2(3)^2 - 3 \times 3 + p = 0$$

$$\Rightarrow 18 - 9 + p = 0 \Rightarrow p = -9$$

$$\begin{aligned}\text{Now } p(x) &= 2x^2 - 3x - 9 = 2x^2 - 6x + 3x - 9 \\ &= 2x(x - 3) + 3(x - 3) \\ &= (x - 3)(2x + 3)\end{aligned}$$

For roots of polynomial, $p(x) = 0$

$$\Rightarrow (x - 3)(2x + 3) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -\frac{3}{2}, \text{ Other root} = -\frac{3}{2}$$

If the sum of the zeroes of the quadratic polynomial $ky^2 + 2y - 3k$ is equal to twice their product, find the value of k . [CBSE 2014]

Sol. $p(y) = ky^2 + 2y - 3k$

$$a = k, b = 2, c = -3k$$

A.T.Q., Sum of zeroes = $2 \times$ product of zeroes

$$\Rightarrow \frac{-b}{a} = 2 \times \frac{c}{a} \Rightarrow \frac{-2}{k} = 2 \times \frac{-3k}{k}$$

$$\Rightarrow \frac{2}{k} = 6 \Rightarrow k = \frac{1}{3}$$

On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Sol. $p(x) = x^3 - 3x^2 + x + 2$

$g(x) = ?$

$$\begin{array}{r}
 x^2 - x + 1 \\
 x - 2 \overline{) x^3 - 3x^2 + 3x - 2} \\
 \underline{x^3 - 2x^2} \\
 -x^2 + 3x - 2 \\
 \underline{-x^2 + 2x} \\
 +x - 2 \\
 \underline{x - 2} \\
 0 \\
 \underline{0} \\
 0
 \end{array}$$

Quotient = $x - 2$;

Remainder = $-2x + 4$

On dividing $p(x)$ by $g(x)$, we have

$$p(x) = g(x) \times \text{quotient} + \text{remainder}$$

$$\Rightarrow x^3 - 3x^2 + x + 2$$

$$= g(x)(x - 2) + (-2x + 4)$$

$$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \times (x - 2)$$

$$\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x) (x - 2)$$

$$\Rightarrow g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$\therefore g(x) = x^2 - x + 1.$$

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