SURFACE AREAS AND VOLUMES

SUBJECT : MATHEMATICS
CHAPTER NO: 13
CHAPTER NAME: SURFACE AREAS AND VOLUMES

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14.2 SOME USEFUL FORMULAE

CUBOID *Let l, b and h denote respectively the length, breadth and height of a cuboid. Then,*

(i) *Total surface area of the cuboid* $= 2(lb + bh + lh)$ *square units*

(ii) *Volume of the cuboid* = *Area of the base* × *Height* = *Length* × *Breadth* × *Height*

$= lbh$ *cubic units*

(iii) *Diagonal of the cuboid* $= \sqrt{l^2 + b^2 + h^2}$ *units.*

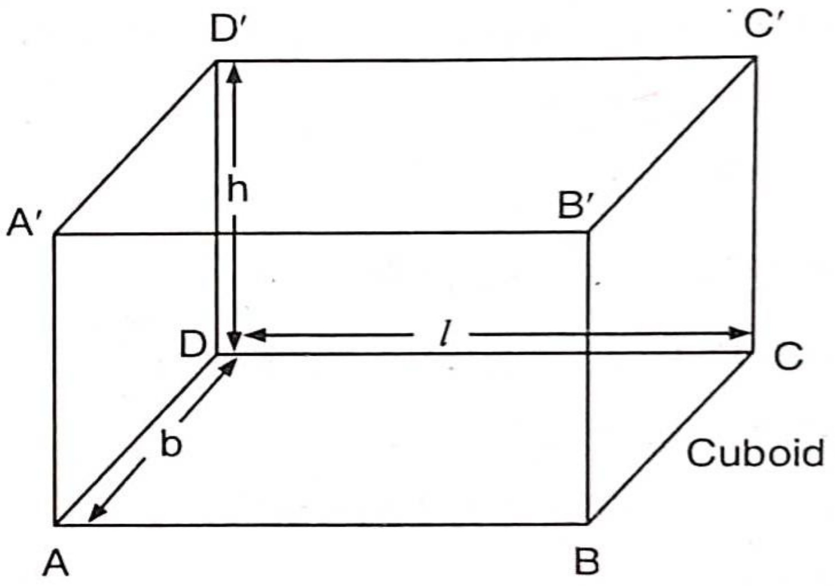


Fig. 14.1

(iv) *Area of four walls of a room* $= lh + lh + bh + bh = 2(l + b)h$ *square units.*

CUBE *If the length of each edge of a cube is 'a' units, then*

(i) *Total surface area of the cube* $= 6a^2$ *square units*

(ii) *Volume of the cube* $= a^3$ *cubic units*

(iii) *Diagonal of the cube* $= \sqrt{3}\,a$ *units*

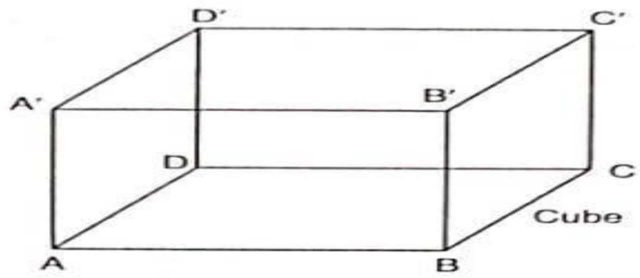


Fig. 14.2

RIGHT CIRCULAR CYLINDER *For a right circular cylinder of base radius r and height (or length) h, we have*

(i) *Area of each end* = *Area of base* $= \pi r^2$

(ii) *Curved surface area* $= 2\pi r h$

$= 2\pi r \times h$

= *Perimeter of the base* × *Height*

(iii) *Total surface area* = *Curved surface area* + *Area of circular ends*

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

(iv) *Volume* $= \pi r^2 h$

= *Area of the base* × *Height*

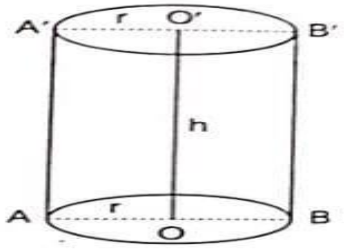


Fig. 14.3

RIGHT CIRCULAR HOLLOW CYLINDER *Let R and r be the external and internal radii of a hollow cylinder of height h. Then,*

(i) *Area of each end* $= \pi(R^2 - r^2)$

(ii) *Curved surface area of hollow cylinder*

= *External surface area* + *Internal surface area*

$= 2\pi Rh + 2\pi rh$

$= 2\pi h(R + r)$

(iii) *Total surface area* $= 2\pi Rh + 2\pi rh + 2(\pi R^2 - \pi r^2)$

$= 2\pi h(R + r) + 2\pi(R + r)(R - r)$

$= 2\pi(R + r)(R + h - r)$

(iv) *Volume of material* = *External volume* − *Internal volume*

$= \pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - r^2)$

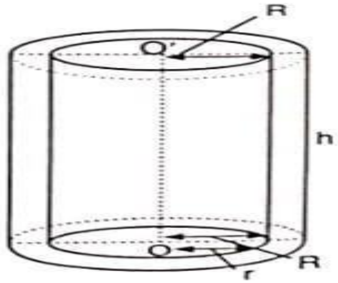


Fig. 14.4

RIGHT CIRCULAR CONE *For a right circular cone of height h, slant height l and radius of base r, we have*

(i) $l^2 = r^2 + h^2$

(ii) *Curved surface area* $= \pi r l$ *sq. units*

(iii) *Total surface area* $=$ *Curved surface area* $+$ *Area of the base*

$$= \pi r l + \pi r^2$$

$$= \pi r (l + r) \text{ sq. units}$$

(iv) *Volume* $= \dfrac{1}{3}\pi r^2 h$

$$= \frac{1}{3}(\text{Area of the base}) \times \text{Height}$$

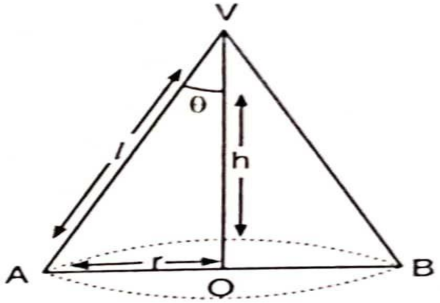


Fig. 14.5

SPHERE *For a sphere of radius r, we have*

(i) *Surface area* $= 4\pi r^2$

(ii) *Volume* $= \dfrac{4}{3}\pi r^3$

For a hemisphere of radius r, we have

(i) *Surface area* $= 2\pi r^2$

(ii) *Total surface area* $= 2\pi r^2 + \pi r^2 = 3\pi r^2$

(iii) *Volume* $= \dfrac{2}{3}\pi r^3$

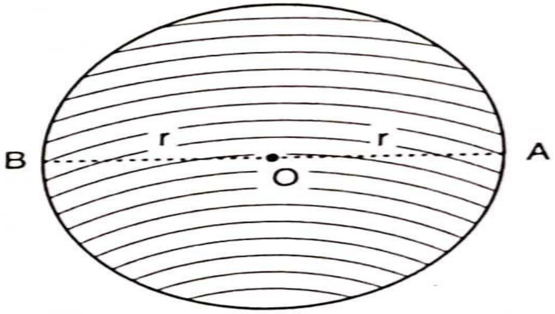


Fig. 14.6

SPHERICAL SHELL *If R and r are respectively the outer and inner radii of a spherical shell, then*

(i) *Outer surface area* $= 4\pi R^2$

(ii) *Volume of material* $= \dfrac{4}{3}\pi (R^3 - r^3)$

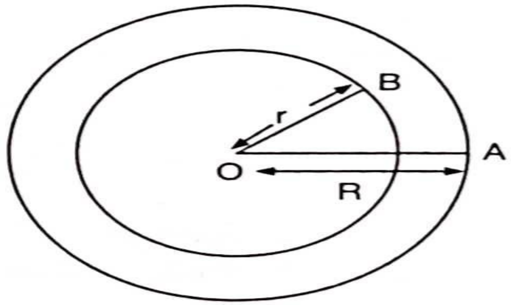
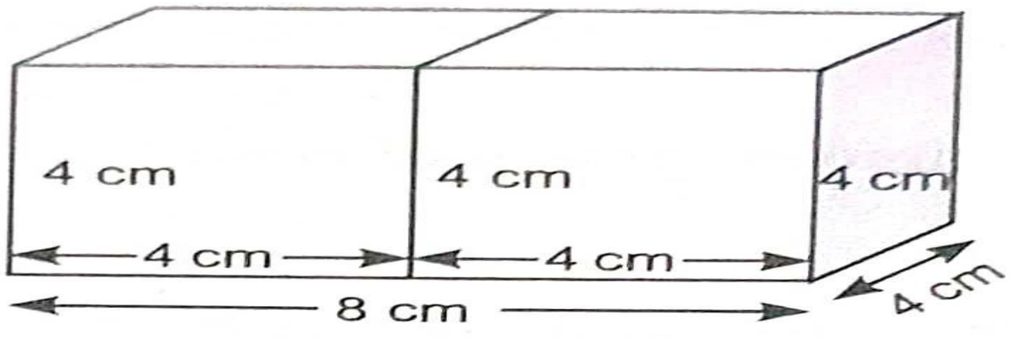


Fig. 14.7

1. **2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.**

Sol. Volume of one cube = 64 cm^3

Let, edge of one cube be a



Volume of the cube = $(\text{edge})^3$

$\therefore \quad a^3 = 64 \quad \Rightarrow \quad a = 4$ cm

Similarly, edge of the another cube = 4 cm.

Now, both cubes are joined together and a cuboid is formed as shown in the figure.

Now, length of the cuboid (l) = 8 cm

breadth of the cuboid (b) = 4 cm

height of the cuboid (h) = 4 cm

Surface area of the cuboid so formed

$= 2(lb + bh + hl)$

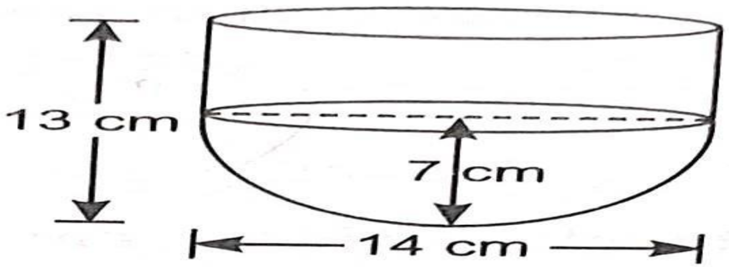
$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)$

$= 2(32 + 16 + 32) = 160$ cm^2

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Sol. Given, diameter of hemisphere = 14 cm

$\therefore$ Radius $= \frac{14}{2} = 7$ cm



Curved surface area of hemisphere $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$

$= 14 \times 22 \text{ cm}^2 = 308 \text{ cm}^2$

Here, total height of the vessel = 13 cm

$\therefore$ Height of the cylinder = total height − radius of the hemisphere

$= 13 \text{ cm} - 7 \text{ cm} = 6 \text{ cm}$

andradius of the cylinder=radius of the hemisphere

$= 7$ cm

Inner surface area of the cylinder $= 2\pi rh$

$= \frac{2 \times 22 \times 7 \times 6}{7}$

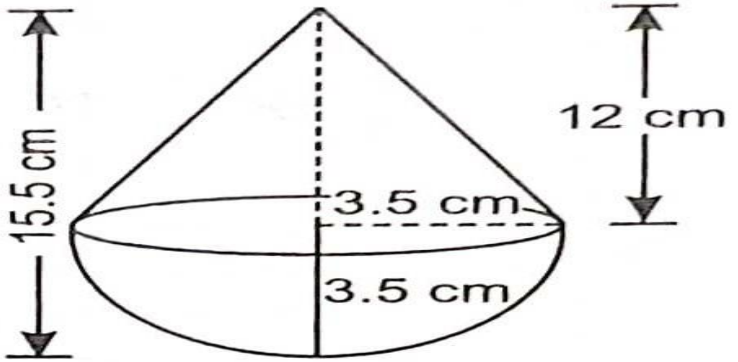
$= 2 \times 22 \times 6 = 264 \text{ cm}^2$

$\therefore$ Inner surface area of the vessel = inner surface area of the cylinder + curved surface area of the hemisphere

$= 264 \text{ cm}^2 + 308 \text{ cm}^2 = 572 \text{ cm}^2$

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Sol. Given, radius of hemisphere = 3.5 cm



Surface area of hemisphere $= 2\pi r^2 = 2 \times \frac{22}{7} \times 3.5 \times 3.5 \text{ cm}^2$

$$= \frac{2 \times 22 \times 35 \times 35}{7 \times 10 \times 10} = 77 \text{ cm}^2$$

Height of conical portion $= 15.5 \text{ cm} - 3.5 \text{ cm} = 12 \text{ cm}$

Radius of conical portion = 3.5 cm

Slant height of conical portion $= \sqrt{(12)^2 + (3.5)^2}$ cm

$= \sqrt{144 + 12.25}$ cm $= \sqrt{156.25} = 12.5$ cm

Curved surface area of conical portion $= \pi r l$

$= \frac{22}{7} \times \frac{35}{10} \times \frac{125}{10} \text{ cm}^2 = \frac{11 \times 25}{2} \text{ cm}^2 = \frac{275}{2} \text{ cm}^2$

Total surface area of the toy = surface area of hemisphere + surface area of conical portion

$= 77 \text{ cm}^2 + \frac{275}{2} \text{ cm}^2 = \frac{154 + 275}{2} \text{ cm}^2$

$= \frac{429}{2} \text{ cm}^2 = 214.5 \text{ cm}^2$

Example 2 : The decorative block shown in Fig. 13.7 is made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block. (Take $\pi = \frac{22}{7}$)

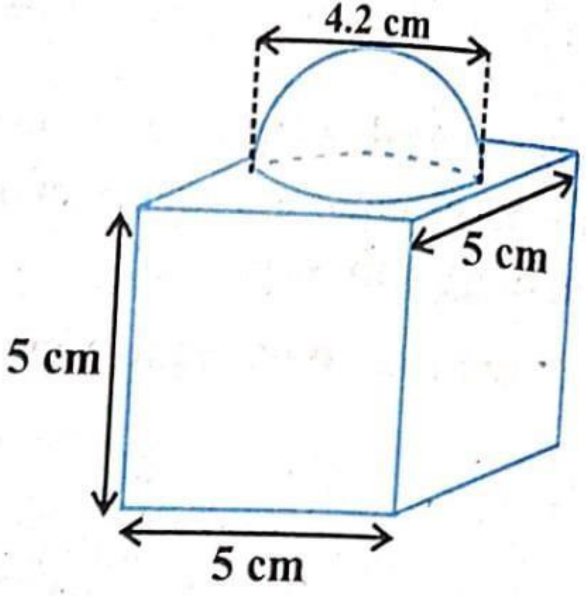


Fig. 13.7

Solution : The total surface area of the cube = $6 \times (\text{edge})^2 = 6 \times 5 \times 5 \text{ cm}^2 = 150 \text{ cm}^2$

Note that the part of the cube where the hemisphere is attached is not included in the surface area.

So, the surface area of the block = TSA of cube – base area of hemisphere + CSA of hemisphere

$$= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2) \text{ cm}^2$$

$$= 150 \text{ cm}^2 + \left(\frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2}\right) \text{cm}^2$$

$$= (150 + 13.86) \text{ cm}^2 = 163.86 \text{ cm}^2$$

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