SURFACE AREAS AND VOLUMES

SUBJECT : MATHEMATICS

CHAPTER NO: 13

CHAPTER NAME: SURFACE AREAS AND VOLUMES

ODM EDUCATIONAL GROUP
Changing your Tomorrow

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Sol. Given, radius of metallic sphere = 4.2 cm

$$\therefore \quad \text{Volume} = \frac{4}{3}\pi(4.2)^3 \qquad ...(i)$$

$\because$ Sphere is melted and recast into a cylinder of radius 6 cm and height h.

$$\therefore \text{ Volume of cylinder} = \pi r^2 h = \pi(6)^2 \times h \qquad ...(ii)$$

According to question,

Volume of cylinder = Volume of sphere

$$\Rightarrow \pi(6)^2 h = \frac{4}{3}\pi(4.2)^3 \Rightarrow 36h = \frac{4}{3} \times \frac{42 \times 42 \times 42}{1000}$$

$$h = \frac{4 \times 42 \times 42 \times 42}{36 \times 3 \times 1000} \text{ cm}$$

$$h = \frac{4 \times 7 \times 7 \times 14}{1000} \text{ cm}$$

$$h = \frac{2744}{1000} \text{ cm} = 2.74 \text{ cm}$$

2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol. Radius of 1st metallic sphere = 6 cm

$\therefore$ Volume of 1st metallic sphere $= \frac{4}{3}\pi(6)^3$ cm^3

Radius of 2nd metallic sphere = 8 cm

$\therefore$ Volume of 2nd metallic sphere $= \frac{4}{3}\pi(8)^3$ cm^3

Radius of 3rd metallic sphere = 10 cm

$\therefore$ Volume of 3rd metallic sphere $= \frac{4}{3}\pi(10)^3$ cm^3

Volume of all three metallic spheres

$$= \frac{4}{3}\pi(6^3+8^3+10^3) \text{ cm}^3$$

$\because$ 3 spheres are melted and recast into a new metallic sphere of radius r.

$\therefore$ Volume of new metallic sphere $= \frac{4}{3}\pi r^3$

According to the question,

$$\frac{4}{3}\pi(6^3+8^3+10^3) = \frac{4}{3}\pi r^3 \Rightarrow 6^3+8^3+10^3 = r^3$$

EXAMPLE 22 *A hemispherical bowl of internal diameter 36 cm contains a liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. How many bottles are required to empty the bowl?*

SOLUTION We have,

Radius of hemispherical bowl = 18 cm

$\therefore$ Volume of hemispherical bowl $= \frac{2}{3}\pi \times (18)^3 \text{ cm}^3$ $\left[\because V = \frac{2}{3}\pi r^3\right]$

and, Radius of a cylindrical bottle = 3 cm

Height of a cylindrical bottle = 6 cm

$\therefore$ Volume of a cylindrical bottle $= (\pi \times 3^2 \times 6) \text{ cm}^3$ $\left[\because V = \pi r^2 h\right]$

Suppose x bottles are required to empty the bowl.

Volume of x cylindrical bottles $= (\pi \times 9 \times 6 \times x) \text{ cm}^3$

Clearly, Volume of liquid in x bottles = Volume of bowl

$$\Rightarrow \quad \pi \times 9 \times 6 \times x = \frac{2\pi}{3} \times (18)^3$$

$$\Rightarrow \quad x = \frac{2\pi \times 18^3}{3 \times \pi \times 9 \times 6} = 72$$

Hence, 72 bottles are required to empty the bowl.

3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

=

5. A

h

OLUTION We have,

Volume of the earth taken out of the well

$= \text{Volume of a cylinder of radius } \frac{7}{2} \text{ m and height } 20 \text{ m}$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 20 \text{ m}^3 = 770 \text{ m}^3$$

Let the height raised of 22 m × 14 m platform be equal to h metres. Then,

Volume of the earth in platform = Volume of the earth taken out of the well

$$\Rightarrow \quad 22 \times 14 \times h = 770$$

$$\Rightarrow \quad h = \frac{770}{22 \times 14} \text{ m} \Rightarrow h = \frac{5}{2} \text{ m} = 2.5 \text{ m}.$$

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Sol. Given, diameter of the well = 3 m $\Rightarrow$ Radius = $\frac{3}{2}$ m

Depth of the well = 14 m

Volume of the earth taken out from the well = $\pi r^2 h$

$= \pi\left(\frac{3}{2}\right)^2 \times 14 = \frac{\pi \times 9 \times 14}{4} = \frac{63}{2}\pi \text{ m}^3$

$\because$ Earth taken out from the well evenly spread to form an embankment having height h and width of embankment around the well is 4 m.

$\therefore$ External radius (R) = radius of well + width of the embankment = $\frac{3}{2}$ m + 4 m = $\frac{11}{2}$ m

Internal radius = $\frac{3}{2}$ m = radius of well

Volume of the earth used for embankment

$= \pi\ (R^2 - r^2)\ h$

$= \pi\left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]h = \pi\left(\frac{121}{4} - \frac{9}{4}\right)h \text{ m}^3$

$= \pi\left(\frac{112}{4}\right)h \text{ m}^3 = \pi\ (28)h \text{ m}^3$

According to the question,

$$\frac{63}{2}\pi = \pi \times 28\ h$$

$$\Rightarrow \quad h = \frac{63}{2 \times 28} = \frac{9}{8} = 1.125 \text{ m}$$

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