Chapter- 10

Factors and multiples

STUDY NOTES

LEARN ABOUT:

- **FACTORS AND PROPERTIES OF FACTORS**
- **PRIME AND COMPOSITE NUMBERS**
- **PRIME FACTORS**
- **MULTIPLES AND PROPERTIES OF MULTIPLES**
- **COMMON FACTORS AND HCF BY DIFFERENT METHODS**
- **COMMON MULTIPLES AND LCM BY DIFFERENT METHODS**

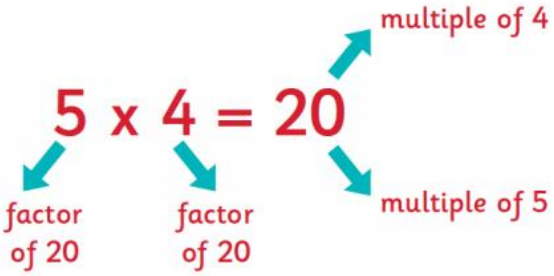
- **FACTORS-**

  When we multiply 2 numbers, each of the numbers being multiplied is called a factor of the product.

  5 x 4 = 20, here 5 and 4 are the factors of 20.

  6 x 5 = 30, here 6 and 5 are the factors of 30.

  8 x 9 = 72, here 8 and 9 are the factors of 72.



(Note: 1. For a number to be a factor of any number, it must completely divide that number without leaving any remainder.

2. Every number will have at least 2 factors, 1 and the number itself.)

- **PROPERTIES OF FACTORS-**

  1. 1 is a factor of every number. Each number can be written as a product of 1 and the number itself.

EXAMPLE- 1 x 17 = 17; 1 x 36 = 36

2. 1 is the only number which has only one factor.

3. A factor of a number (other than zero) is either less than or equal to the number itself.
   EXAMPLE- 12 x 1 = 12; 2 x 6 = 12; 3 x 4 = 12
   So, factors of 12 are 1, 2, 3, 4, 6 and 12.
   1 is the smallest factor and the number itself is the greater factor.

Therefore, the factor of a number is either less than or equal to itself.

EXAMPLE- Find all the factors of 30.

SOLUTION-
1 x 30 = 30
2 x 15 = 30
3 x 10 = 30
5 x 6 = 30

- **PRIME AND COMPOSITE NUMBERS-**

Composite numbers are the numbers having more than two factors i.e other than 1 and the number itself.
EXAMPLE- 4, 6, 8, 9, 10 and so on……

A prime number is a number which has only two factors, namely 1 and the number itself. These numbers are not completely divisible by any other number, except 1 and the number itself.
EXAMPLE- 2, 3, 5, 7, 11 and so on……

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(Note: 1. 1 is not a prime number since it has only one factor, that is itself.
2. 2 is the first even prime number.)

- **PRIME FACTORS-**

A prime factor of a given number is a prime number that completely divides the given number.
Prime factors can be obtained by using 2 methods.

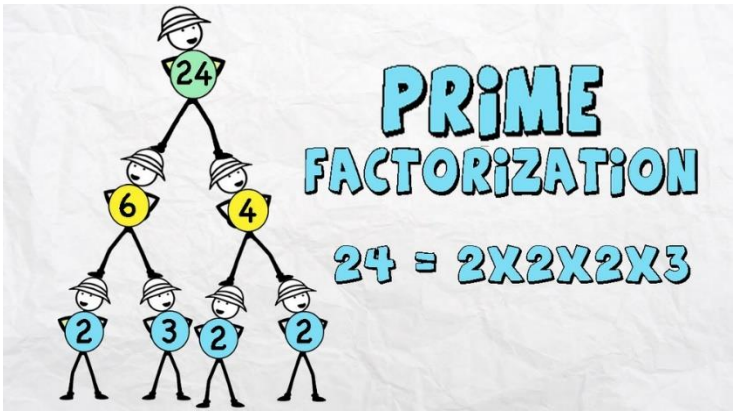
1. Factor tree method
2. Prime factorisation method

FACTOR TREE METHOD-
In factor tree method, we keep on breaking a number into factors until we get all prime factors. These prime factors are circled and written in the end to represent the number as their product.

EXAMPLE-

PRIME FACTORIZATION METHOD-
In this method we start dividing the number with its smallest prime factor and keep on dividing till we get1 as a quotient. As it is difficult to make factor trees for larger numbers, this method is more useful and compact.

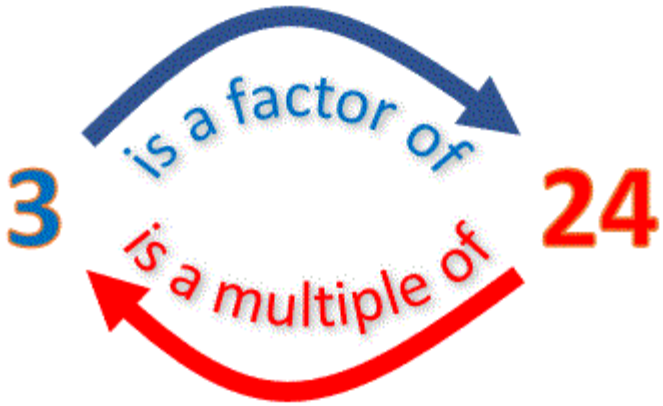
EXAMPLE-



(Note: Every composite number can be expressed as a product of all its prime factors.)

- **MULTIPLES-**

When we multiply two given numbers, their product is a multiple of each of the numbers.
5 x 6 = 30, here 30 is a multiple of both 5 and 6.
4 x 7 = 28, here 28 is a multiple of both 4 and 7.



- **PROPERTIES OF MULTIPLES-**

1. The smallest multiple of a number is the number itself.
2. Every number is a multiple of 1. Every number is a multiple of itself.

   EXAMPLE-
   1 x 15 = 15, so 15 is a multiple of 1 and 15.
3. 0 is a multiple of every number.

   EXAMPLE-
   25 x 0 = 0. Thus 0 is a multiple of every number.
4. Every (non- zero) multiple of a whole number is either greater than or equal to the number.

   EXAMPLE-
   Multiple of 6 are 6,12, 18, 24……
   The smallest multiple of 6 is 6 and the other multiples of 6 are greater than 6.
5. Multiples of a number are infinite, they carry on and on.

   EXAMPLE-
   Multiples of 20 are 20, 40, 60, 80…….

- ## COMMON FACTORS AND MULTIPLES-

A. COMMON FACTORS-

Factors common to 2 or more numbers are said to be common factors for those numbers.

EXAMPLE-
30 and 48 can both be divided by 2. So, 2 is a common factor of 30 and 48.

(Note: The largest factor that is common to all the given numbers is called the highest common factor (HCF) or greatest common factor (GCF) of the given numbers.)

HCF of 2 or more numbers can be found out by 2 methods.

1. **LISTING METHOD:**
   In this method, we list the factors of the given numbers separately. Then we circle the common factors of the numbers and find the greatest factor amongst the common factors.

   EXAMPLE-
   Find the HCF of 36 and 54 by listing method.

   SOLUTION-
   STEP-1 List all the factors of 36 and 54.
   Factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18, 36.
   Factors of 54 = 1, 2, 3, 6, 9, 18, 27, 54.

   STEP-2 Compare and circle the common factors of 36 and 54.

   Factors of 36 = 1, (2), (3), 4, (6), (9), 12, (18), 36

   Factors of 54 = 1, (2), (3), (6), (9), (18), 27, 54

   STEP-3 Identify the highest common factor among the circled numbers.
   Here, 18 is the last common factor of 36 and 54.
   So, the HCF of 36 and 54 is 18.

2. **COMMON DIVISION METHOD:**
   In this method, we find the HCF of the given numbers by dividing them together by their least common factor. Then we multiply the common factors to get the HCF of the given numbers.

EXAMPLE-
Find the HCF of 48 and 92 using common division method.

SOLUTION-

2	48, 92
2	24, 46
	12, 23

HCF = 2 × 2 = 4

B. COMMON MULTIPLES:

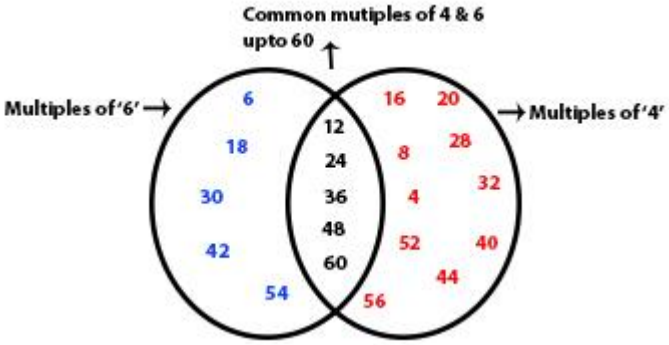
When a particular number is a multiple of 2 or more numbers, it is called a common multiple.

EXAMPLE-
Find the common multiples of 2 and 4.

SOLUTION-
Multiples of 2 = 2, (4), 6, (8), 10, (12),.....

Multiples of 4 = (4), (8), (12), 16, 20, 24,.....

Multiples that are common to both the numbers are 4, 8 and 12.



Least common multiple (LCM) is the smallest common multiple of the given numbers.
LCM of any given numbers can be found out by 2 methods.
1. Listing method
2. Common division method

LISTING METHOD -

In this method, we list first few multiples of the given numbers. Then we circle the common multiples and identify the least common multiple of the given numbers among the circled ones.

EXAMPLE-
Find the LCM of 12, 15 and 20.

SOLUTION-
STEP-1 List the multiples of each number.

Multiples of 12 = 12, 24, 36, 48, 60, 72, 84, 96, 108, 120….
Multiples of 15 = 15, 30, 45, 60, 75, 90, 105, 120, 135……
Multiples of 20 = 20, 40, 60, 80, 100, 120, 140, 160……

STEP-2 Circle the common multiples of 12, 15 and 20.

Multiples of 12 = 12, 24, 36, 48, (60), 72, 84, 96, 108, (120)…

Multiples of 15 = 15, 30, 45, (60), 75, 90, 105, (120), 135…..

Multiples of 20 = 20, 40, 60, 80, 100, 120, 140, 160…..

STEP-3 Identify the least common multiples among the circled numbers.
Here, 60 is the first common multiple of 12, 15 and 20.
So, LCM of 12, 15 and 20 is 60.

COMMON DIVISION METHOD-

In this method, we start by dividing at least one of the given numbers by the smallest prime number. Bring down the numbers that are indivisible as it is. Keep on repeating the method till all the quotients are 1 in the last ro. Then, multiply all the prime numbers to get the LCM of the given numbers.

EXAMPLE-
Find the LCM of 16, 24 and 30.

SOLUTION-

STEP-1: Write all the numbers in a row, separated by commas.

	16, 24, 30

STEP-2: Choose the smallest prime number that divides any one of the given numbers.

2	16, 24, 30
	8, 12, 15

STEP-3: Keep on dividing the numbers by the smallest prime numbers and bring the indivisible numbers down as it is.

2	16, 24, 30
2	8, 12, 15
2	4, 6, 15

STEP-4: Repeat till you get all ones (1) in the last row.

2	16, 24, 30
2	8, 12, 15
2	4, 6, 15
	2, 3, 15

STEP-5: Multiply all the prime numbers on the left to get the LCM of 16, 24 and 30.
So, LCM of 16, 24 and 30 is 2 x 2 x 2 x 2 x 3 x 5 = 240

2	16, 24, 30
2	8, 12, 15
2	4, 6, 15
3	2, 3, 15
	2, 1, 5

----------X---X---X----------