CHAPTER 2

RATIONAL NUMBERS

A rational number **is defined as a number that can be expressed in the form p / q, where p and q are integers and q≠0.**

- In our daily lives, we use some quantities which are not whole numbers but can be expressed in the form of p / q. Hence we need rational numbers.

Equivalent Rational Numbers

- By **multiplying or dividing the numerator and denominator** of a rational number by a **same non zero integer**, we obtain another rational number equivalent to the given rational number. These are called **equivalent fractions**.

Rational Numbers in Standard Form

- A rational number is said to be in the **standard form** if its denominator is a positive integer and the **numerator and denominator have no common factor other than 1.**
- **Positive and Negative Rational Numbers**
  A rational number whose numerator and denominator both are positive integers is called a positive rational number.
  A rational number, whose numerator is a negative integer and denominator is a positive integer, is called a negative rational number. Similarly, if the numerator is positive integer and denominator is a negative integer; is also a negative rational number.
- **Reduction of a Rational Number to its Lowest Form**
  To reduce a rational number to its standard form (or lowest form), we divide its numerator and denominator by their HCF ignoring the negative sign, if any.
  However, if there is a negative sign in the denominator, we divide by -HCF'.
- **Comparison of Rational Numbers**
  Two positive rational numbers can be compared exactly as we compare two fractions.
  Two negative rational numbers can be compared by ignoring their negative signs and then reversing the order.
  Comparison of a negative and a positive rational number is obvious as a negative rational number is always less than a positive rational number.

Rational Numbers between Two Rational Numbers

- There are unlimited number (infinite number) of rational numbers between any two rational numbers.

Properties of Rational Numbers

Closure property

Rational numbers are closed under addition, subtraction and multiplication; this implies that the product after these operations will also be rational number.

Commutative property

Rational numbers are commutative under addition and multiplication i.e. changing the order of rational number during these operations will not affect the product and addition but not subtraction and division

Subtraction and division of rational numbers do not hold the commutative property.

Associative Property

When rational numbers are rearranged among two or more same operations and still their result does not change that is called associative property.

Addition and subtraction of rational numbers hold the associative property and multiplication and division does not hold the associative property.

Role of Zero in Numbers (Additive Identity)

Zero is the additive identity for whole numbers, integers as well as rational numbers.

Role of one in Numbers (Multiplicative Identity)

One is the multiplicative identity for whole numbers, integers as well as rational numbers.

Product of Reciprocals
The product of a rational number with its reciprocal is always 1.

Rational Numbers on a Number Line
Positive rational numbers are marked on the right of 0 on the number line whereas negative rational numbers are marked on the left of 0 on the number line.
The method of representation is the same as the method of representation of fractions on the number line.

While adding rational numbers with same denominators, we add the numerators keeping the denominators same.

$$e.g.\quad \frac{-2}{5}+\frac{3}{5}=\frac{-2+3}{5}=\frac{1}{5}.$$

While subtracting two rational numbers, we add the additive inverse of the rational, number that is being Subtracted, to the other rational number.

$$e.g.\quad \frac{5}{7}-\frac{3}{8}=\frac{5}{7}+\left(\frac{-3}{8}\right)=\frac{19}{56}.$$

While multiplying a rational number by a positive integer, we multiply the numerator by that integer, keeping the denominator unchanged.

$$e.g.\quad \frac{-2}{9}\times(-5)=\frac{-2\times(-5)}{9}=\frac{10}{9}.$$

Product of reciprocals is always equal to 1.

$$e.g.\quad \frac{-4}{9}\times\frac{-9}{4}=1$$

To divide one rational number by the other non – zero rational number, we multiply the rational number by the reciprocal of the other.

$$e.g.\quad \frac{-6}{5}\div\frac{-2}{3}=\frac{-6}{5}\times\frac{3}{-2}=\frac{-6\times 3}{5\times -2}=\frac{18}{10}.$$