CHAPTER 1

INTEGERS

STUDY NOTES

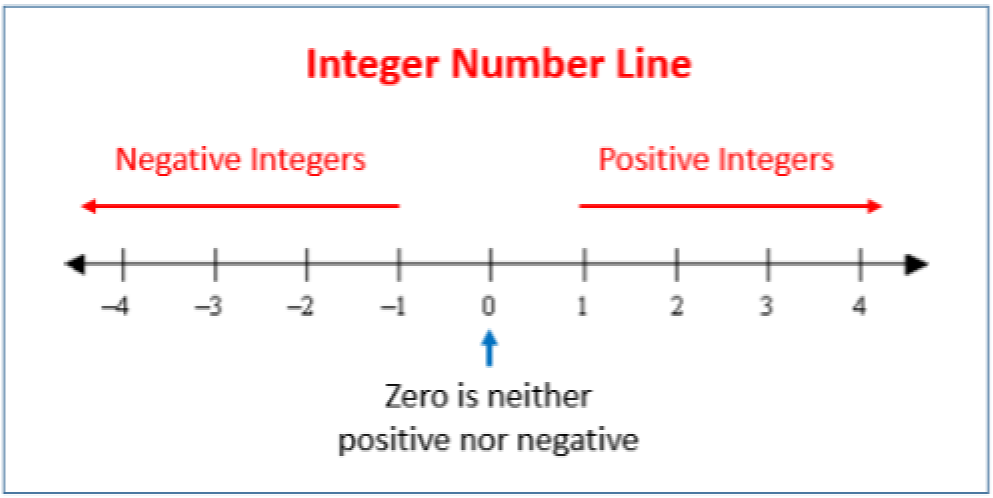
Integers

A whole number, from zero to positive or negative infinity is called **Integers**. I.e. it is a set of numbers which include zero, positive natural numbers and negative natural numbers. It is denoted by letter Z.

Z = {…,-2,-1, 0, 1, 2…}

Integers on Number Line

On the number line, for positive integers we move to the right from zero and for negative integers move to the left of zero.

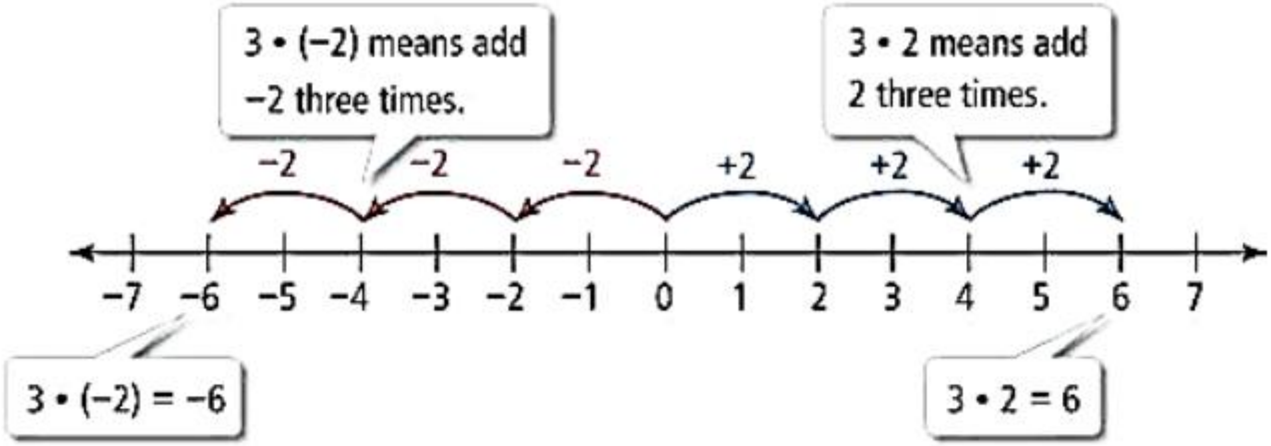


Multiplication of Integers

Multiplication of two integers is the repeated addition.

Example

- 3 × (-2) = three times (-2) = (-2) + (-2) + (-2) = – 6
- 3 × 2 = three times 2 = 2 + 2 + 2 = 6



Now let's see how to do the multiplication of integers without the number line.

1. Multiplication of a Positive Integer and a Negative Integer

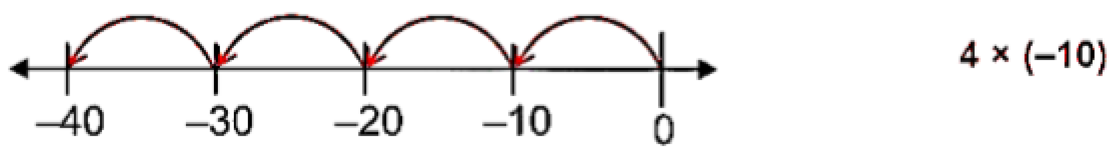
To multiply a positive integer with a negative integer, we can multiply them as a whole number and then put the negative sign before their product.

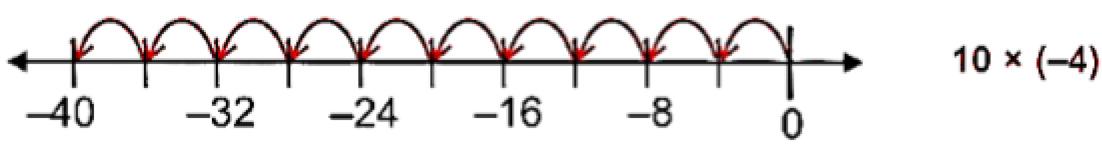
So the product of a negative and a positive integer will always be a negative integer.

For two integers p and q,
$p \times (-q) = (-p) \times q = -(p \times q) = -pq$

Example

$4 \times (-10) = (-4) \times 10 = -(4 \times 10) = -40$





2. Multiplication of Two Negative Integers

To multiply two negative integers, we can multiply them as a whole number and then put the positive sign before their product.

Hence, if we multiply two negative integers then the result will always be a positive integer.

For two integers p and q,
$(-p) \times (-q) = (-p) \times (-q) = p \times q$
Example

$(-10) \times (-3) = 30$

3. The Product of Three or More Negative Integers

It depends upon the number of negative integers.

a. If we multiply two negative integers then their product will be positive integer

$(-3) \times (-7) = 21$

b. If we multiply three negative integers then their product will be negative integer

$(-3) \times (-7) \times (-10) = -210$

If we multiply four negative integers then their product will be positive integer

$(-3) \times (-7) \times (-10) \times (-2) = 420$

Hence, if the number of negative integers is **even** then the result will be **a positive integer** and if the number of negative integers is **odd** then the result will be a **negative integer**.

Properties of Multiplication of Integers

1. Closure under Multiplication

In case of multiplication, the product of two integers is always integer so integers are closed under multiplication.

For all the integers p and q
$p \times q = r$, where r is an integer
Example

$(-10) \times (-3) = 30$

$(12) \times (-4) = -48$

2. Commutativity of Multiplication

If we change the order of the integers while multiplying then also the result will remain the same then it is said that **multiplication is commutative for integers.**
For any two integers p and q
$p \times q = q \times p$
Example

$20 \times (-30) = -600$

$(-30) \times 20 = -600$

There is no difference in answer after changing the order of the numbers.

3. Multiplication by Zero

If we multiply an integer with zero then the result will always be zero.

For any integer p,
p × 0 = 0 × p = 0
Example

9 × 0 = 0 × 9 = 0

0 × (-15) = 0

4. Multiplicative Identity

If we multiply an integer with 1 then the result will always the same as the integer.

For any integer q
q × 1 = 1 × q = q
Example

21 × 1 = 1 × 21 = 21

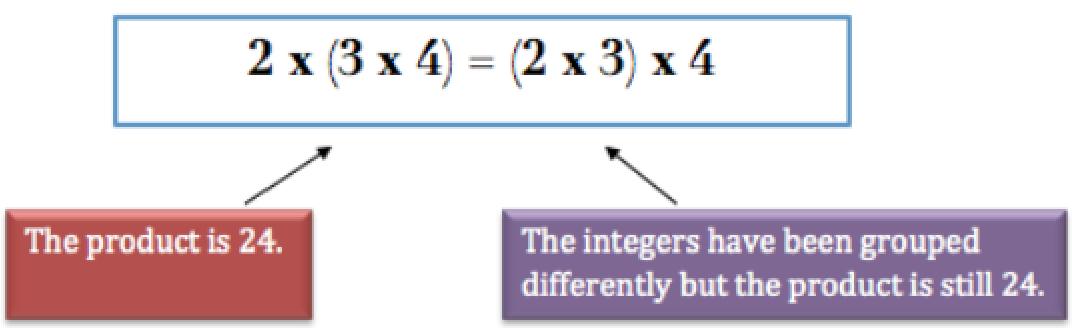
1 × (-15) = (-15)

5. Associative Property

If we change the grouping of the integers while multiplying in case of more than two integers and the result remains the same then it is said the associative property for multiplication of integers.

For any three integers, p, q and r
p × (q × r) = (p × q) × r
Example

If there are three integers 2, 3 and 4 and we change the grouping of numbers, then

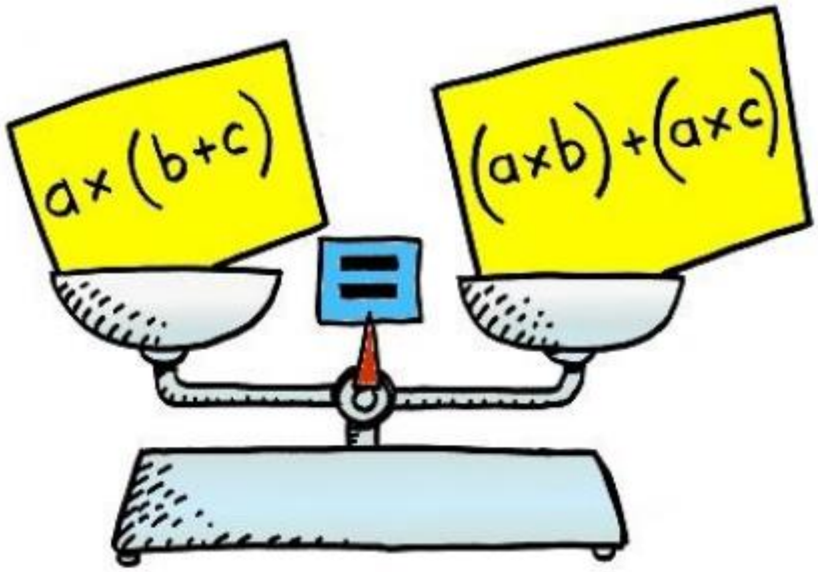


The result remains the same. Hence, multiplication **is associative for integers**.

6. Distributive Property

a. Distributivity of Multiplication over Addition.
For any integers a, b and c

a × (b + c) = (a × b) + (a × c)



Example

Solve the following by distributive property.

I. 35 × (10 + 2) = 35 × 10 + 35 × 2

= 350 + 70

= 420

II. (– 4) × [(–2) + 7] = (– 4) × 5 = – 20 And

= [(– 4) × (–2)] + [(– 4) × 7]

= 8 + (–28)

= –20

So, (– 4) × [(–2) + 7] = [(– 4) × (–2)] + [(– 4) × 7]

b. Distributivity of multiplication over subtraction

For any integers a, b and c

a × (b – c) = (a × b) – (a × c)

Example

5 × (3 – 8) = 5 × (- 5) = – 25

5 × 3 – 5 × 8 = 15 – 40 = – 25

So, 4 × (3 – 8) = 4 × 3 – 4 × 8.

Division of integers

1. Division of a Negative Integer by a Positive Integer

The division is the inverse of multiplication. So, like multiplication, we can divide them as a whole number and then place a negative sign prior to the result. Hence the answer will be in the form of a negative integer.

For any integers p and q,
$(-p) \div q = p \div (-q) = -(p \div q)$ where, $q \neq 0$

Example

$64 \div (-8) = -8$

2. Division of Two Negative Integers

To divide two negative integers, we can divide them as a whole number and then put the positive sign before the result.

The division of two negative integers will always be a positive integer.

For two integers p and q,
$(-p) \div (-q) = (-p) \div (-q) = p \div q$ where $q \neq 0$

Example

$(-10) \div (-2) = 5$

Properties of Division of Integers

For any integers p, q and r

Property	General form	Example	Conclusion
Closure Property	$p \div q$ is not always an integer	$10 \div 5 = 2$ $5 \div 10 = 1/2$ (not an integer)	The division is not closed under division.
Commutative Property	$p \div q \neq q \div p$	$10 \div 5 = 2$ $5 \div 10 = 1/2$	The division is not commutative for integer.
Division by Zero	$p \div 0$ = not defined $0 \div p = 0$	$0 \div 10 = 0$	No

Division Identity	p ÷ 1 = p	10 ÷ 1 = 10	Yes
Associative Property	(p ÷ q) ÷ r ≠ p ÷ (q ÷ r)	[(–16) ÷ 4] ÷ (–2) ≠ (–16) ÷ [4 ÷ (–2)] (-8) ÷ (-2) ≠ (-16) ÷ (-2) 4 ≠ 8	Division is not Associative for integers.