CHAPTER 5

EXPONENTS

We can write large numbers in a short form using exponents.
For example: $10{,}000 = 10 \times 10 \times 10 \times 10 = 10^4$
Here, '10' is called the base and '4' the exponent. The number 10^4 is read as 10 raised to the power of 4 or simply as the fourth power of 10.
10^4 is called the exponential form of 10,000.

$(1)^{\text{any natural number}} = 1$

$(-1)^{\text{an odd natural number}} = -1$

$(-1)^{\text{an even natural number}} = +1$

$a^m \times a^n = a^{m+n}$, where m and n are whole numbers and a ($\neq 0$) is an integer.
This formula can be used to write answers to above questions.

For any non-zero integer a,
$a^m \div a^n = a^{m-n}$ where m and n are whole numbers and $m > n$.

For any non-zero integer a,
$(a^m)^n = a^{mn}$ (where m and n are whole numbers)

For any non-zero integer a
$a^m \times b^m = (ab)^m$ (where m is any whole number)

$$a^m \div b^m = \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

(where m is a whole number; a and b are any non-zero integers)
$a^0 = 1$ (for any non-zero integer a)
Any number (except 0) raised to the power (or exponent) 0 is 1.

Decimal Number System

$10{,}000 = 10^4$
$1000 = 10^3$
$100 = 10^2$
$10 = 10^1$

$1 = 10^0$

We can write the expansion of a number using powers of 10 in the exponent form.

Expressing Large Numbers in the Standard Form

Large numbers can be expressed conveniently using exponents. Such a number is said to be in standard form if it can be expressed as $k \times 10^m$, where $1 \leq, k < 10$ and m is a natural number.

Note that, one less than the digit count (number of digits) to the left of the decimal point in a given number, is the exponent of 10 in the standard form.

For any rational number a and positive integer n, we define a^n as $a \times a \times a \times \ldots\ldots \times a$ (n times). a^n is known as the nth power of a and is read as 'a raised to the power n'. The rational a is called the base and n is called the exponent or power.

e.g. $10{,}000 = 10 \times 10 \times 10 \times 10 = 10^4$.

10 is the base and 4 is the exponent.

Reciprocal of $\left(\frac{a}{b}\right)^m = \frac{b^m}{a^m} = \left(\frac{b}{a}\right)^m$, so the reciprocal of $\left(\frac{a}{b}\right)^m$ is $\left(\frac{b}{a}\right)^m$.

Multiplying Powers with the Same Base: If a is any non-zero integer and whole numbers are m and n, then $a^m \times a^n = a^{m+n}$

e.g. $2^4 \times 2^2$

$a = 2, m = 4, n = 2$

$2^4 \times 2^2 = 2^{4+2} = 2^6$

Dividing Powers with the Same Base: If a is any non-zero integer and m, n are the whole number, then $a^m \div a^n = a^{m-n}$

e.g. $2^4 \div 2^2$

$a = 2, m = 4, n = 2$

$2^4 \div 2^2 = 2^{4-2} = 2^2$

Taking Power of a Power: If a is any non-zero integer and m, n are whole numbers, $(a^m)^n = a^{mn}$

e.g. $(6^2)^4$

$a = 6, m = 2, n = 4$

$(6^2)^4 = (6)^{2\times4} = 6^8$.

Multiplying Powers with the Same Exponents: If a, b are two non-zero integers and m is any whole number, then

$a^m \times b^n = (a \times b)^m$

e.g. $2^3 \times 3^3$

a = 2, b = 3, m = 3
$2^3 \times 3^3 = (2 \times 3)^3 = 6^3$.

Dividing Powers with the Same Exponents: If a, b are two non-zero integers and m is a whole number, then

$$a^m \div b^m = \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

$$\begin{aligned} e.g.\quad & 2^3 \div 3^3 \\ & a = 2, b = 3, m = 3 \\ & 2^3 \div 3^3 = \frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3 \end{aligned}$$

Numbers with Exponent Zero: If a be any non-zero integer, then, $a^0 = 1$

$$e.g.\quad \frac{2^5}{2^5} = 2^{5-5} = 2^0 = 1$$

Numbers with Negative Exponent: If a is any non-zero integer, then a^{-1} = 1a
e.g. 2^{-5} = 125