CHAPTER 11

FUNDAMENTAL CONCEPTS
(INCLUDING FUNDAMENTAL OPERATIONS)

Constant

Constant is a quantity which has a fixed value.

Terms of Expression

Parts of an expression which are formed separately first and then added are known as **terms**. They are added to form expressions.

Example: Terms 4x and 5 are added to form the expression (4x +5).

Coefficient of a term

The numerical factor of a term is called **coefficient** of the term.
Example: 10 is the coefficient of the term 10xy in the expression 10xy+4y

Definition of Variables

- Any algebraic expression can have any number of variables and constants.
  - Variable
    - A variable is a quantity that is prone to change with the context of the situation.
    - a,x,p,… are used to denote variables.

    To know more about Variables, visit here.

  - Constant
- It is a quantity which has a fixed value.
- In the expression 5x+4, the variable here is x and the constant is 4.
- The value 5x and 4 are also called terms of expression.
- In the term 5x, 5 is called the coefficient of x. Coefficients are any numerical factor of a term.

Factors of a term

Factors of a term are quantities which can not be further factorised. A term is a product of its factors.
Example: The term –3xy is a product of the factors –3, x and y.

Formation of Algebraic Expressions

- Variables and numbers are used to construct terms.
- These terms along with a combination of operators constitute an algebraic expression.
- The algebraic expression has a value that depends on the values of the variables.
- For example, let $6p^2-3p+5$ be an algebraic expression with variable p
  The value of the expression when p=2 is,
  $6(2)^2 - 3(2) + 5$
  $\Rightarrow 6(4) - 6 + 5 = 23$
  The value of the expression when p=1 is,
  $6(1)^2 - 3(1) + 5$
  $\Rightarrow 6 - 3 + 5 = 8$

To know more about Algebraic Expressions, visit here.

Like and Unlike Terms

Like terms

- Terms having same algebraic factors are like terms.
  Example: 8xy and 3xy are like terms.

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Unlike terms

- Terms having different algebraic factors are unlike terms.
  Example: 7xy and −3x are unlike terms.

Monomial, Binomial, Trinomial and Polynomial Terms

Types of expressions based on the number of terms

Based on the number of terms present, algebraic expressions are classified as:

- **Monomial:** An expression with only one term.
  Example: 7xy, −5m, etc.
- **Binomial:** An expression which contains two, unlike terms.
  Example: 5mn+4, x+y, etc

- **Trinomial:** An expression which contains three terms.
  Example: x+y+5, a+b+ab, etc.

Polynomials

- An expression with one or more terms.
  Example: x+y, 3xy+6+y, etc.

To know more about Polynomial, visit here.

Addition and Subtraction of Algebraic Equations

- Mathematical operations like addition and subtraction can be applied to algebraic terms.
- For adding or subtracting two or more algebraic expression, like terms of both the expressions are grouped together and unlike terms are retained as it is.
- Sum of two or more like terms is a like term with a numerical coefficient equal to the sum of the numerical coefficients of all like terms.
- Difference between two like terms is a like term with a numerical coefficient equal to the difference between the numerical coefficients of the two like terms.
- For example, $2y + 3x - 2x + 4y$
  $\Rightarrow x(3-2) + y(2+4)$
  $\Rightarrow x+6y$
- Summation of algebraic expressions can be done in two ways:
  Consider the summation of the algebraic expressions $5a^2+7a+2ab$ and $7a^2+9a+11b$
- **Horizontal method**
  $5a^2+7a+2ab+7a^2+9a+11b$
  $= (5+7)a^2+(7+9)a+2ab+11b$
  $= 12a^2+16a+2ab+11b$

Vertical method

$\mathbf{5a^2+7a+2ab}$

$\mathbf{7a^2+9a+11b}$

$\mathbf{12a^2+16a+2ab+11b}$

Multiplication of Algebraic Expressions Introduction

There exist a number of situations when we need to multiply algebraic expressions. For example, in finding area of a rectangle whose sides are given as expressions.

Multiplying a Monomial by a Monomial

A monomial multiplied by a monomial always gives a monomial.

Multiplying Two Monomials

In the product of two monomials
Coefficient = coefficient of the first monomial × coefficient of the second monomial
Algebraic factor = algebraic factor of a first monomial × algebraic factor of the second monomial

Multiplying Three or More Monomials

We first multiply the first two monomials and then multiply the resulting monomial by the third monomial. This method can be extended to the product of any number of monomials.

Rules of Signs

The product of two factors is positive or negative accordingly as the two factors have like signs or unlike signs. Note that

(i) (+) × (+) = +
(ii) (+) × (-) = –
(iii) (-) × (+) = –
(iv) (-) × (-) = +
If x is a variable and p, q are positive integers, then $x^p \times x^q = x^{p+q}$

Multiplying a Monomial by a Polynomial

While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.

Multiplying a Monomial by a Binomial

By using the distributive law, we carry out the multiplication term by term.
It states that if P, Q and R are three monomials, then

- P × (Q + R) = (P × Q) + (P × R)
- (Q + R) × P = (Q × P) + (R × P)

Multiplying a Monomial by a Trinomial

By using the distributive law, we carry out the multiplication term by term.

Multiplying A Polynomial by a Polynomial

We multiply each term of one polynomial by each term of the other polynomial. Also, we combine the like terms in the product.

Multiplying a Binomial by a Binomial

We use distributive law and multiply each of the two terms of one binomial by each of the two terms of the other binomial and combine like terms in the product.

Thus, if P, Q, R and S are four monomials, then

$(P + Q) \times (R + S) = P \times (R + S) + Q \times (R + S)$

$= (P \times R + P \times S) + (Q \times R + Q \times S)$

$= PR + PS + QR + QS.$

Multiplying a Binomial by a Trinomial

We use distributive law and multiply each of the three terms in the trinomial by each of the two terms in the binomial and combine like terms in the product.

Division of a monomial by another monomial

To divide a monomial by a monomial, first express the numerator and the denominator in their irreducible form, and then cancel the common factors.

Division of a polynomial by a monomial

To divide a polynomial by a monomial, either divide each term of the numerator by the denominator or factorise the numerator by the common factor method.

Division of a polynomial by a polynomial

To divide a polynomial by a polynomial, first factorise the numerator and the denominator by using the appropriate method and then cancel the common factors.

Dividend = Divisor × Quotient + Remainder.

Hint: Division is the inverse operation of multiplication.

$2x \times (2x+3) = 4x^2 + 6x$, then

$(4x^2 + 6x) \div 2x = (2x+3)$