Chapter-6

Application of Derivatives

1. The radius of a circle is increasing at a uniform rate of 3cm/sec. At the instant when the radius of the circle is 2cm, its area increases at the rate of ____ cm^2/s.
2. The volume of a sphere is increasing at a rate of 3 cubic cm per second. Find the rate of increase of its surface area, when the radius is 2 cm.
3. Find the velocity and acceleration at the end of 2 seconds of the particle moving according to the rule $s = \sqrt{t} + 1$.
4. Find the rate of change of the area of a circle with respect to its radius r when $r = 3$ cm.
5. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?
6. Find the interval where the function $f(x) = \sin x$, $x \in [0, 2\pi]$ decreases.
7. Find the interval in which the function $f(x) = lnx$, $x > 0$ decreases.
8. Find the values of x for which the function $f(x) = 2 + 3x - x^3$ is decreasing.
9. Show that $y = \log(1 + x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x, throughout its domain.
10. Find the intervals in which the function is given by $f(x) = sinx + cosx$, $0 \leq x \leq 2\pi$ is
    (i)Increasing(ii) decreasing
11. Find the intervals in which the function $f(x) = (x - 1)^3(x - 2)^2$is increasing.
12. Find the intervals in which the function $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is decreasing.
13. Prove that $y = \frac{4\, sin\theta}{2+cos\theta} - \theta$ is an increasing function in $\left(0, \frac{\pi}{2}\right)$.
14. Prove that the function f defined by $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in $(-1, 1)$. Hence find the intervals in which $f(x)$ is strictly increasing.
15. Which of the following functions is decreasing on $\left(0, \frac{\pi}{2}\right)$?
    a) $sin2x$ b) $tanx$ c) $cosx$ d) $cos3x$
16. Find the slope of the tangent to the curve $y = 3x^2 - 4\text{x}$ at the point whose x-coordinate is 2.
17. Find the points on the curve $y = x^3 - 3\text{x}^2 - 4\text{x}$ at which the tangent lines are parallel to the line $4x + y - 3 = 0$.
18. Find the slope of the tangent to $x = t^2, y = 2t$at $t = 1$.

19. What is the slope of the normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 20$ at the point $(8, 64)$
20. Find the point at which the line $y = x + 1$ is a tangent to the curve $y^2 = 4x$.
21. Find the equation of tangents to the curve $y = x^3 + 2x - 4$ which are perpendicular to the line $x + 14y - 3 = 0$.
22. The equation of the tangent at $(2, 3)$ on the curve $y^2 = ax^3 + b$is $y = 4x - 5$. Find the values of $a$and b.
23. Find the equations of the tangent and normal to the curves $x = asin^3\theta$ and $y = a\ cos^3\theta$at $\theta = \frac{\pi}{4}$.
24. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which tangent is parallel to $x - axis$.
25. Find the equations of the tangent to the curve $y = x^2 - 2x + 7$ which is Perpendicular to the line $5y - 15x = 1$.
26. Using differentials, find the approximate value of $\sqrt{25.3}$ up to two places of decimals.
27. Using differentials, find the approximate value $(3.968)^{\frac{3}{2}}$.
28. Find the approximate value of $f(3.02)$ up to 2 places of decimal, where $f(x) = 3x^2 + 15x + 3$.
29. Using differentials, find the approximate value of $\sqrt{49.5}$
30. Using differentials, evaluate $\frac{1}{\sqrt{25.1}}$
31. If $y = a\ logx + bx^2 + x$ has extreme values at $x = -1$and $x = 2$. Find $a$and b.
32. Find the local maxima and local minima of the function $f(x) = sinx - cosx,\ 0 < x < 2\pi$. Also, find the local maximum and local minimum values.
33. If $(x) = \begin{cases} 3x + 2, & x \leq 0 \\ 2 - 3x, & x > 0 \end{cases}$, then which is the possible extreme point of $f(x)$ in $[-2, 2]$?
34. For what value of x, is the function $f(x) = 3 - 2x^2$ the maximum?
35. Give an example of a function that does not possess a relative maximum or relative minimum. A
36. What is the least value of $sin2x - x$in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$?
37. Find the critical points off$(x) = x + \frac{1}{x}$.
38. Find the point of local maximum for $f(x) = sinx + cosx,\ 0 < x < \pi$
39. State where $y = sinx$ attains a maximum value in the interval $[0, \pi]$
40. The sum of the perimeters of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of the square is double the radius of the circle.
41. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

42. Show that the surface area of a closed cuboid with a square base and given volume is minimum when it is a cube.
43. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.
44. A metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs ₹ 5 per cm^2 and the material for the sides costs ₹ 2.50 per cm^2. Find the least cost of the box.
45. Show that the semi-vertical angle of the cone of the maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
46. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}\,r$.
47. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius $r$is $\frac{4r}{3}$. Also, find the maximum volume in terms of the volume of the sphere.
48. Prove that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius $R$is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.
49. Prove that the semi-vertical angle of the right circular cone of a given volume and least curved surface area is $\cot^{-1}\sqrt{2}$
50. Of all the closed right circular cylindrical cans of volume$128\pi\ cm^3$, find the dimensions of the can which has a minimum surface area.
51. Find the equations of tangents to the curve $3x^2 - y^2 = 8$, which passes through the point $\left(\frac{4}{3}, 0\right)$.
52. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
53. Prove that all normal to the curves $x = a\,cost + at\,sint$ and $y = a\,sint - at\,cost$ are at a constant distance a from the origin.
54. Show that the right circular cone of the least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.
55. Show that of all rectangles of the given area, the square has the smallest perimeter.

56. Find the open interval in which the function $f(x) = x^{\frac{1}{x}}, x > 0$ is increasing.
57. If $f(x) = sinx - bx + c$ decreases for all real values of x show that $b > 1$.
58. Fill in the blank: The line $y = 2x + 1$ is a tangent to the curve $y^2 = 4x$ if the value of m is ________ .
59. Prove that $y = \frac{4\,sin\theta}{2+cos\theta} - \theta$ is an increasing function in $\left(0, \frac{\pi}{2}\right)$.