Chapter- 5

Continuity

SECTION-A

Very Short Type[1 mark Questions]

1. If $f(x) = 2x$ and$g(x) = \frac{x^2}{2} + 1$, then which of the following may not be a continuous function?

a) $f(x) + g(x)$ b) $f(x) - g(x)$ c)$f(x).g(x)$ d) $\frac{g(x)}{f(x)}$

2. The function $f(x) = cotx$ is discontinuous on the set

a) $\{x = n\pi : n \in Z\}$ b) $\{x = 2n\pi : n \in Z\}$

c) $\left\{x = (2n+1)\frac{\pi}{2} : n \in Z\right\}$ d) $\left\{x = \frac{n\pi}{2} : n \in Z\right\}$

3. The function $f(x) = \frac{4-x^2}{4x-x^3}$

a) Discontinuous at only one point b) discontinuous at exactly two points

c) Discontinuous at exactly three points d) none of these

4. Determine the value of $'k'$ for which the following function is continuous at $x = 3$.

$$f(x) = \begin{cases} \frac{(x+3)^2-36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

5. Determine the value of the constant $'k'$ so that the function $f(x) = \begin{cases} \frac{kx}{|x|} & if\ x < 0 \\ 3 & if\ x \geq 0 \end{cases}$ is continuous at $x = 0$.

SECTION-B

Long Type – I [4 Marks Questions]

6. For what value of k is the function continuous at x=2? *f(x)*= $\begin{cases} 2x+1; & x<2 \\ k; & x=2 \\ 3x-1; & x>2 \end{cases}$

7. Discuss the continuity of the function x=0: *f(x)*= $\begin{cases} \frac{x^4+2x^3+x^2}{\tan^{-1} x}, & x \neq 0 \\ 0, & x=0 \end{cases}$

8. If the function *f(x)* given by *f(x)*= $\begin{cases} 3ax+b; x>1 \\ 11\ ; x=1 \\ 5ax-2b; x<1 \end{cases}$ is continuous at x=1, find the value of a & b.

9. Find the relationship between a and b so that the function *f* defined by : *f(x)*= $\begin{cases} ax+1, x\le 3 \\ bx+3, x>3 \end{cases}$ is continuous at x=3.

10. Find the value of k,for which f(x)= $\begin{cases} \dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, -1\le x<0 \\ \dfrac{2x+1}{x-1}, 0\le x<1 \end{cases}$ is continuous at x=0.

11. Find the value of k so that the function f ,defined by *f(x)* = $\begin{cases} kx+1, x\le \pi \\ \cos x, x>\pi \end{cases}$ is continuous at x= π

12. Find the value of a for which the function f defined as *f(x)*= $\begin{cases} a\sin\dfrac{\pi}{2}(x+1), x\le 0 \\ \dfrac{\tan x-\sin x}{x^3}, x>0 \end{cases}$ is continuous at x=0.

13. Find all points of discontinuity of f, where f is defined as follows f(x)= $\begin{cases} |x|+3; x\le -3 \\ -2x; -3<x<3 \\ 6x+2; x\ge 3 \end{cases}$

14.For what value of k, the following function is continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{1-cos4x}{8x^2} & if\ x\ \neq 0 \\ k\ if\ x=0 \end{cases}$$

15.Find the value of a such that the function f defined by $f(x) = \begin{cases} \frac{x^2-3x+2}{x^2-1} & when\ x \neq 1 \\ 4a & when\ x=1 \end{cases}$ is continuous at $x = 1$.

16.The function $f(x)$ is defined as $f(x) = \begin{cases} x^2+ax+b,\ 0\le x<2 \\ 3x+2,\ 2\le x\le 4 \\ 2ax+5b,\ 4<x\le 8 \end{cases}$. If $f(x)$ is continuous in $[0,8]$, find the values of a and b.

Differentiability

SECTION-A

Very Short Type[1 mark Questions]

17. The function $f(x) = e^{|x|}$is

a) Continuous everywhere but not differentiable at $x = 0$
b) Continuous and differentiable everywhere
c) Not continuous at $x = 0$
d) None of these

18. The set of points where the function f given by $f(x) = |2x - 1|\, sinx$ is differentiable is

a) R b) $R - \left\{\frac{1}{2}\right\}$ c)$(0, \infty)$ d) none of these

19. Fill in the blank: The greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at $x =$ ________ .

SECTION-B

Long Type – I [4 Marks Questions]

20. If the function $f(x)$ is differentiable at $x = 2$, then find the value of a and b. $f(x) = \begin{cases} x^2 \; if \; x \leq 2 \\ ax + b \; if \; x > 2 \end{cases}$

21. Find the values of a and b so that the function $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$ is differentiable for $x \in R$.

22. Show that the function f(x) = $|x-3|$,$x \in$ R is continuous but not differentiable at x=3.

23. Show that the function $f(x) = |2x + 1|$ is not differentiable at $x = -\frac{1}{2}$.

24. If function $f(x) = |x - 3| + |x - 4|$, show that f is not differentiable at $x = 3$ and $x = 4$.

25. Discuss the differentiability of the function $f(x) = x|x|$at $x = 0$.

SECTION-C

Long Type – II [6 Marks Questions]

26. Show that the function defined as follows is continuous at $x = 1, x = 2$ but not differentiable at $x = 2$. $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$

DIFFERENTIATION

27. If $y = \log(\cos e^x)$, then find $\frac{dy}{dx}$.

28. If $y = cosec(cot\sqrt{x})$, then find $\frac{dy}{dx}$.

29. If f(x)=$\sqrt{\frac{\sec x - 1}{\sec x + 1}}$ then find f′(x).

30. Differentiate with respect to x: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

31. If $y = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, find $\frac{dy}{dx}$.

32. If $y = \cos^{-1}\left(\frac{3x+4\sqrt{1-x^2}}{5}\right)$,find $\frac{dy}{dx}$

33. Differentiate the function w.r.t x: $f(x) = \tan^{-1}\left(\frac{1-x}{1+x}\right) - \tan^{-1}\left(\frac{x+2}{1-2x}\right)$

34. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}-1}{x}\right)$ w.r.t x.

35. Differentiate $x^{\cos x} + \frac{x^2+1}{x^2-1}$ w.r.t x

36. Find $\frac{dy}{dx}$,if y= $(\cos x)^x + (\sin x)^{1/x}$

37. Differentiate the function w.r.t x: $(x)^{\cos x} + (\sin x)^{\tan x}$

38. If $(\cos x)^y = (\sin y)^x$,find $\frac{dy}{dx}$.

39. Find $\frac{dy}{dx}$,if $(x^2+y^2)^2 = xy$

40. If siny=x sin(a+y),Prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

41. If $\log(x^2+y^2)=2\tan^{-1}(y/x)$,then show that $\frac{dy}{dx}=\frac{x+y}{x-y}$

42. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$,then show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

43.If $x^y=e^{x-y}$ then show that $\frac{dy}{dx}=\frac{\log x}{[\log(xe)]^2}$

44. If $y=\log\tan(\frac{\pi}{4}+\frac{x}{2})$ then show that $\frac{dy}{dx}=\sec x$.

45. If $x^m y^m=(x+y)^{m+n}$ find $\frac{dy}{dx}$

46. If $x^{16}y^9=(x^2+y)^{17}$ then find $\frac{dy}{dx}$

47. If $x\sin(a+y)+\sin a.\cos(a+y)=0$ prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

48. If y=$\frac{\sin^{-1}x}{\sqrt{1-x^2}}$,then prove that $(1-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0$

49. If $y=e^x(\sin x+\cos x)$,then show that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

50. If x=a(cost+tsint) and y=a(sint-tcost),then find $\frac{d^2y}{dx^2}$.

51. If $y=e^{a\sin^{-1}x}, -1\le x\le 1$ then show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$

52. If $y=\mathrm{cosec}^{-1}x$,x>1 then show that $x(x^2-1)\frac{d^2y}{dx^2}+(2x^2-1)\frac{dy}{dx}=0$

53. If y=a cos(logx)+b sin(logx) then show that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

54. Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]=\sqrt{a^2-x^2}$

55. If x=a(t-sint),y=a(1+cost),then find $\frac{d^2y}{dx^2}$

56. If $y=\log\left[x+\sqrt{x^2+1}\right]$ then show that $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$

57. If y=$\sin^{-1}$x,then show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$

58. If x=a$\cos^3$t and y=a$\sin^3$t then find the value of $\frac{d^2y}{dx^2}$ at t=$\frac{\pi}{6}$

59. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$

60. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

61. Verify Rolle's theorem for $f(x)=sin2x$ in $\left[0,\frac{\pi}{2}\right]$ and find the value of $c\ \in\ \left[0,\frac{\pi}{2}\right]$.

62. Using Rolle's theorem find a point on the curve $y=sinx+cosx-1, x\in\left[0,\frac{\pi}{2}\right]$, where tangent is parallel to $x-axis$.

63. Verify Mean value Theorem for $f(x)=(x-1)(x-2)(x-3)$in $[1,4]$.

64. Verify Lagrange's mean value theorem for the function f(x) =x^2+2x+3, for [4,6].

65. For what value of c, Mean value theorem is applicable for the function $f(x)=x+\frac{1}{x}$ on $[1,3]$?

66. Find $\frac{dy}{dx}$ if $x=a\sec^3\theta\ and\ y=a\tan^3\theta$

67. Find $\frac{dy}{dx}$ if $x=\frac{e^t+e^{-t}}{2}$ and $y=\frac{e^t-e^{-t}}{2}$