Chapter- 2

Inverse Trigonometric Functions

1. The principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is

   a) $\frac{\pi}{3}$ b) $-\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $-\frac{\pi}{6}$

2. The principal value of $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$ is

   a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{5\pi}{6}$ d) $\frac{2\pi}{3}$

3. The domain of $\sin^{-1} 2x$ is

   a) $[0, 1]$ b) $[-1, 1]$ c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ d) $[-2, 2]$

4. Write the principal value of the following $\left[\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(-\frac{1}{2}\right)\right]$

5. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

6. Find the principal value of $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

7. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is

   a) $\frac{\pi}{2}$ b) $\frac{3\pi}{2}$ c) $\frac{5\pi}{2}$ d) $\frac{7\pi}{2}$

8. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, then $\cot^{-1} x + \cot^{-1} y$ equals

   a) $\frac{\pi}{5}$ b) $\frac{2\pi}{5}$ c) $\frac{3\pi}{5}$ d) π

9. Write the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

10. Evaluate $\sin^{-1} x + \sin^{-1}(-x)$

11. What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?

12. Write the simplest form of $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

13. Write the simplest form of $\tan^{-1}\left[\frac{cosx-sinx}{cosx+sinx}\right], x < \pi$

14. Prove that $\cot^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right) = \frac{x}{2}, \ 0 < x < \frac{\pi}{2}$

15. If $\cot^{-1} x + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \frac{\pi}{4}$, then x is equal to

   a) 3 b) $\frac{1}{\sqrt{5}}$ c) 0 ` d) None of these

16. The value of $\sin(2\sin^{-1}(0.6))$ is

   a) 0.48 b) 0.96 c) 1.2 d) $\sin 1.2$

17. Prove that: $3 \sin^{-1} x = \sin^{-1}(3x - 4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right].$

18. Write into the simplest form: $\sin^{-1}\left(\sqrt{x}\sqrt{1-x^2} - x\sqrt{1-x}\right)$

19. Solve the equation: $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, x > 0.$

20. Prove that: $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

21. Prove that $\cos^{-1} x = 2 \sin^{-1}\sqrt{\frac{1-x}{2}}$

22. Prove that: $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

23. If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, then find the value of x.

24. Prove that $tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$

25. Prove the following

$\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right) = 0 \ (0 < xy, yz, zx < 1)$

26. Solve for x, $\tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$

27. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in (0, 1)$

28. Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$