Chapter- 3

Matrices

Concept, notation, order, equality, types of matrices

A matrix is a rectangular arrangement of numbers or functions arranged into a fixed number of rows and columns. The element of a matrix is always enclosed in the bracket [ ] or (). Matrices are represented by capital letters like A, B, C, etc.

A matrix having m rows and n columns is called a matrix order m x n (read as m by n matrix). In general, a matrix of order m x n is written as.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{m\times n}$$

It can also be written in compact form as A^{-1} a_{ij} represent an element of ith row and jth column.

Question – 1

Write all possible order of matrices having 24 elements

Solution:- $1\times 2, 2\times 12, 3\times 8, 4\times 6, 6\times 4, 8\times 3, 12\times 2, 24\times 1$

Question – 2

Write a matrix of order 2×3 where $aij = 2i - j$

Solution:- $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$

Types of Matrices:-

Row Matrix:- A matrix is said to be a row matrix if it has only one row.

Example:- $A = \begin{bmatrix} 2 & 3 & 5 & 7 \end{bmatrix}_{1\times4}$

Column Matrix:- a matrix having any number of rows but only one column is called a column matrix.

Example:- $A = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}_{5\times1}$

Rectangular Matrix:-

A matrix having m rows and n columns where $m \neq n$ is called a rectangular matrix.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 6 \end{bmatrix}_{2\times3}$$

Square matrix:-

It a matrix in which number of rows = number of columns $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$

Diagonal Elements:-

The diagonal elements of a square matrix are the elements for which i = j. i.e the elements $a_{11}, a_{22}, a_{33}.....$ The line along which the diagonal elements lie is called the leading diagonal or principal diagonal.

Diagonal Matrix:-

It is a square matrix where diagonal elements are non-zero but the other elements are zero.

Example:- $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Scalar matrix:- It is a diagonal matrix where all the diagonal elements are equal

Example:- $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Equality of Matrices

Two matrices are said to be equal if their order is the same and their corresponding elements are equal.

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ then $a = 2, b = 3, c = 4, d = 5$

Question:-

If $\begin{bmatrix} x - y & 2x + z \\ 2x - y & 3z + w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ then find x, y, z and w

Solution:-

$$x - y = -1$$
$$2x + z = 5$$
$$2x - y = 0$$
$$3z + w = 13$$

Solving $x = 1, y = 2, z = 3, w = 4$

Addition, multiplication and scalar multiplication.

Addition of Matrices:-

Matrix addition is defined only when they are of the same order. The sum of matrices A and B is a matrix whose elements are obtained by adding the corresponding elements of A and B.

Example:-

Let $A=\begin{bmatrix}2 & 3\\ -4 & 5\end{bmatrix}, B=\begin{bmatrix}1 & -2\\ 0 & 4\end{bmatrix}$

$$A+B=\begin{bmatrix}3 & 1\\ -4 & 9\end{bmatrix}, A-B=\begin{bmatrix}1 & 5\\ -4 & 1\end{bmatrix}$$

Multiplication of a matrix by a scalar:-

If a scalar K is multiplied by a matrix A then all elements of matrix A are multiplied by constant K.

Example:-

If $A=\begin{bmatrix}3 & 4\\ -5 & 1\end{bmatrix}$ find 5A

Solution:-

$$5A=\begin{bmatrix}15 & 20\\ -25 & 5\end{bmatrix}$$

Example:-

If $A=\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & 2\\ 3 & 4 & 0\end{bmatrix}, B=\begin{bmatrix}2 & -1 & 0\\ 3 & 0 & 1\\ 0 & 0 & 4\end{bmatrix}$ find $3A+4B$

Solution:-

$$3A+4B=\begin{bmatrix}3&6&9\\0&-3&6\\9&12&0\end{bmatrix}+\begin{bmatrix}8&-4&0\\2&0&4\\0&0&16\end{bmatrix}=\begin{bmatrix}11&2&9\\12&-3&10\\9&12&16\end{bmatrix}$$

Simple properties of addition, multiplication, and scalar multiplication

Properties:-

Closure Law:- A matrix added with a matrix always gives a matrix. So Closure Law satisfies.

Commutative Law:- $A+B=B+A$

Associative Law:- $A+(B+C)=(A+B)+C$

Existence of Additive Identity:-

A null matrix of the same order with the given matrix is the additive identity of the matrix. $A+O=O+A=A$

Example:-

The additive identity of $A=\begin{bmatrix}2&3&4\\5&6&7\end{bmatrix}$ is $O_{2\times3}=\begin{bmatrix}0&0&0\\0&0&0\end{bmatrix}$

Existence of additive inverse:-

A is the additive inverse of A

Question – 1

Find x and y if $2\begin{bmatrix}1&3\\0&x\end{bmatrix}+\begin{bmatrix}y&0\\1&2\end{bmatrix}=\begin{bmatrix}5&6\\1&8\end{bmatrix}$

Solution:- $x=y=3$

Question – 2

If $A=\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}, B=\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}$ then find the matrix X, such that $2A+3X=5B$

Solution:-

$$X=\begin{bmatrix}-2 & -\frac{10}{3}\\ 4 & \frac{14}{3}\\ -\frac{31}{3} & -\frac{7}{3}\end{bmatrix}$$

Question – 3

Find the value of $x+y$ from matrix equation $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$

Solution:-

$x+y=2+9=11$

Multiplication of Matrices

If A and B be any two matrices, then their product AB will be defined only when the number of columns is A is equal to the number of rows in B. if $A=\left[a_{ij}\right]_{m\times n}$ and $B=\left[b_{ij}\right]_{n\times p}$ then their product $AB=C=\left[c_{ij}\right]$, will be a matrix of order $m\times x$, where, $(AB)_{ij}=c_{ij}=\sum_{r=1}^{n}a_{ir}b_{rj}$

Example:-

If $A=\begin{bmatrix}1 & 4 & 2\\ 2 & 3 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2\\ 2 & 2\\ 1 & 3\end{bmatrix}$

Then $AB = \begin{bmatrix} 1.1+4.2+2.1 & 1.2+4.2+2.3 \\ 2.1+3.2+1.1 & 2.2+3.2+1.3 \end{bmatrix}$

$AB = \begin{bmatrix} 11 & 16 \\ 9 & 13 \end{bmatrix}$

Properties of Matrix Multiplication:-

If A, B, and C are three matrices such that their product is defined, then

- $AB \neq BA$ (Generally not commutative)
- $(AB)C = A(BC)$ (Associative Law)
- $IA = A = AI$ (I is identity matrix for matrix multiplication)
- $A(B+C) = AB + AC$ (Distributive Law)
- If $AB = AC$ this not implies that B = C (Cancellation Law is not applicable)
- If $AB = 0$ It does not mean that A = 0 or B = 0 again product of two non-zero matrices may be zero matrix.

Note:-

- The multiplication of two diagonal matrices is again a diagonal matrix.
- The multiplication of two triangular matrices is again a triangular matrix
- The multiplication of two scalar matrices is also a scalar matrix.
- If A and B are two matrices of the same order, then
  - $(A+B)^2 = A^2 + B^2 + AB + BA$
  - $(A-B)^2 = A^2 + B^2 - AB - BA$
  - $(A-B)(A+B) = A^2 - B^2 + AB - BA$
  - $(A+B)(A-B) = A^2 - B^2 - AB + BA$
  - $A(-B) = (-A)B = -(AB)$

Positive Integral Powers of the matrix:-

The positive integral powers of matrix A are defined only when A is a square matrix. Also then

$A^2 = A.A \; A^3 = A.A.A = A^2A$

Also for any positive integers m, n

(a) $A^mA^n = A^{m+n}$ (b) $(A^m)^n = A^{mn} = (An)^m$ (c) $I^n = I, I^m = I$

(d) $A^o = I_n$ where A is a square matrix of order n.

Example – 1

If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $A^2 - 4A - nI = 0$, then find the value of n.

Solution:-

$$A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}, 4A = \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix}, nI = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$$

$$\Rightarrow A^2 - 4A - nI$$

$$= \begin{bmatrix} 5-8-n & -4+4-0 \\ -4+4-0 & 5-8-n \end{bmatrix} = \begin{bmatrix} -3-n & 0 \\ 0 & -3-n \end{bmatrix}$$

$$\therefore A^2 - 4A - nI = 0$$

$$\Rightarrow \begin{bmatrix} -3-n & 0 \\ 0 & -3-n \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow -3-n = 0 \Rightarrow n = -3$$

Example – 2

If $A=\begin{bmatrix}-1 & 2\\ 3 & -4\end{bmatrix}$ then find the element a_{21} of A^2

Solution:-

The element a_{21} is the product of the second row of A to the first column of A

$$\therefore a_{21}=\begin{bmatrix}3 & -4\end{bmatrix}\begin{bmatrix}-1\\ 3\end{bmatrix}=-3-12=-15$$

Transpose of a Matrix:-

The matrix obtained from a given matrix A by changing its rows into columns or columns into rows is called transpose of matrix A and is dented by A^T or A'. From the definition, it is obvious that if the order of A is $m\times n$.

Example:- Transpose of Matrix $\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{bmatrix}_{2\times 3}$ is $\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\\ a_3 & b_3\end{bmatrix}_{3\times 2}$

Properties of Transpose:-

(a) $\left(A^T\right)^T=A$ (b) $\left(A\pm B\right)^T=A^T\pm B^T$ (c) $\left(AB\right)^T=B^TA^T$

Non-commutative of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix

Non-Commutative of multiplication of matrices:-

Example:- If $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$ find AB, BA, show that $AB\neq BA$

Solution:- $AB=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}=\begin{bmatrix}0 & -4\\ 10 & 3\end{bmatrix}_{2\times 3}$, $BA=\begin{bmatrix}-10 & 2 & 21\\ -16 & 2 & 37\\ -2 & -2 & 11\end{bmatrix}_{3\times 3}$

Clearly $AB \neq BA$. So it is not commutative.

Zero Matrix as a product of two non zero matrices.

Example:- $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

If the product of two matrices is a zero matrix, one of the matrices doesn't need to be a zero matrix.

Application of Matrices:-

Example – 1

Use matrix multiplication to divide Rs. 30, 000 in two parts such that the total annual interest at 9% on the 1st part and 11% on the second part amount Rs. 3060/-

Solution:- $A = \begin{bmatrix} x & 30000 - x \end{bmatrix}$

$R = \begin{bmatrix} 0.09 \\ 0.11 \end{bmatrix}$ AR = 3060

$\Rightarrow \begin{bmatrix} x & 30000 - x \end{bmatrix} \begin{bmatrix} 0.09 \\ 0.11 \end{bmatrix} = 3060$

$\Rightarrow x = 1200$

Two parts are 1200, 1800

Example – 2 Let $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I be the identity matrix of order 2.Show that $I + A$

$= (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Solution:-

We have,

$$I+A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}=\begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{bmatrix}$$

And $I-A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 0\end{bmatrix}=\begin{bmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}$

$$\therefore (I-A)=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$\Rightarrow (I-A)=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}\begin{bmatrix}\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & -\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\\ \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\end{bmatrix}$$

$$\Rightarrow (I-A)=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & t\\ -t & 1\end{bmatrix}\begin{bmatrix}\frac{1-t^2}{1+t^2} & -\frac{2t}{1+t}\\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2}\end{bmatrix}$$ where $t=\tan\frac{\alpha}{2}$

$$\Rightarrow (I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}\frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2}\\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2}\end{bmatrix}$$

$$\Rightarrow (I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}\dfrac{1+t^2}{1+t^2} & \dfrac{-t(1+t^3)}{1+t^2}\\ \dfrac{t(1+t^2)}{1+t^2} & \dfrac{1+t^2}{1+t^2}\end{bmatrix}=\begin{bmatrix}1 & -t\\ t & 1\end{bmatrix}$$

$$\Rightarrow (I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & -\tan\dfrac{\alpha}{2}\\ \tan\dfrac{\alpha}{2} & 1\end{bmatrix}=I+A$$

Example – 3

If $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, prove that $(aI+bA)^n=a^nI+na^{n-1}bA$

Where I is a unit matrix of order 2 and n is a positive integer.

Solution:-

We shall prove the result by mathematical induction on n.

Step – 1, When n = 1, by the definition of integral powers of a matrix, we have

$$(aI+bA)^1=aI+bA=a^1I+1a^0bA=a^1I+1a^{1-1}bA$$

So, the result is true for n = 1

Step – 2, Let the result be true for n = m. Then

$(aI+bA)^m=a^mI+ma^{m-1}bA$. Now we shall show that the result is true for $n=m+1$

i.e $(aI+bA)^{m+1}=a^{m+1}I+(m+1)a^mbA$

By the definition of integral powers of a matrix, we have

$$(aI+bA)^{m+1}=(aI+bA)^m(aI+bA)$$

$$\Rightarrow (aI+bA)^{m+1} = (a^m I + ma^{m-1}bA)(aI+bA)$$

$$\Rightarrow (aI+bA)^{m+1} = (a^m I)(aI) + (a^m I)(bA) + (ma^{m-1}bA)(aI) + (ma^{m-1}bA)(bA)$$

$$\Rightarrow (aI+bA)^{m+1} = (a^m a)(I.I) + a^m b(IA) + ma^m b(AI) + ma^{m-1}b^2(AA)$$

$$\Rightarrow (aI+bA)^{m+1} = a^{m+1}I + a^m bA + ma^m bA + ma^{m-1}b^2A^2 \qquad [\because IA = AI = A, I.I = I]$$

$$\Rightarrow (aI+bA)^{m+1} = a^{m+1}I + (ma^m b + a^m b)A + ma^{m-1}b^2A^2$$

$$\Rightarrow (aI+bA)^{m+1} = a^{m+1}I + (m+1)a^m bA + ma^{m-1}b^2 o$$

$$\left[\because A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right]$$

$$\Rightarrow (aI+bA)^{m+1} = a^{m+1}I + (m+1)a^m bA$$

This shows that the result is true for $n = m+1$, whenever it is true for $n = m$. Hence, by the principle of mathematical induction, the result is valid for any positive integer n.

Example – 4 If A is a square matrix such that $A^2 = A$ show that $(I+A)^3 = 7A + I$

Solution:- Using matrix multiplication, we obtain

$$(I+A)^2 = (I+A)(I+A)$$

$$= I(I+A) + A(I+A)$$

$$= I^2 + IA + AI + A^2$$

$$= I + A + A + A^2$$

$$= I + 2A + A^2$$

$$= I + 2A + A$$

$= I + 3A$

$\therefore (I+A)^3 = (I+A)^2 (I+A)$

$\Rightarrow (I+A)^3 = (I+3A)(I+A)$

$= I(I+A) + 3A(I+A)$

$= I^2 + IA + 3(AI) + 3(AA)$

$= I + A + 3A + 3A^2$

$= I + A + 3A + 3A$.

$= I + 7A$

Symmetric and Skew symmetric matrices.

Symmetric Matrix:- A square matrix $A = [a_{ij}]$ is called symmetric matrix if $a_{ij} = a_{ji}$ for all I, j, or $A^T = A$.

Example:- $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

Note:-

- Every unit matrix and square zero matrix are symmetric matrices.
- Maximum number of different elements in a symmetric matrix is $\frac{n(n+1)}{2}$

Skew-Symmetric Matrix:- A square matrix $A = [a_{ij}]$ is called the skew-symmetric matrix. If $a_{ij} = -a_{ji}$ for all, I, j or $A^T = -A$

Example:- $\begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix}$

Note:-

- All principal diagonal elements of the skew-symmetric matrix are always zero because for any diagonal element $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$
- The diagonal elements of a skew-symmetric matrix are always 0.

  Proof:

Let $A = [a_{ij}]$ be a skew-symmetric matrix. Then,

$$
\begin{aligned}
& a_{ij} = -a_{ji} \quad \text{for all } i, j \\
\Rightarrow\ & a_{ii} = -a_{ii} \quad \text{for all values of } i \\
\Rightarrow\ & 2a_{ii} = 0 \\
\Rightarrow\ & a_{ii} = 0 \quad \text{for all values of } i \\
\Rightarrow\ & a_{11} = a_{22} = a_{33} = \dots = a_{nn} = 0.
\end{aligned}
$$

Example:-

1. If the matrix $A = \begin{bmatrix} 0 & a & 5 \\ 3 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric. Find the value of a, b, c

Answer:

For a skew-symmetric $A = [a_{ij}]$, we have

$a_{ij} = -a_{ji}$ for all $i \neq j$ and $a_{ii} = 0$ for all i

Thus, if $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric, then $A_{22} = 0$, $A_{12} = -A_{21}$ and $A_{31} = -A_{13}$.

$\Rightarrow \quad b = 0,\ a = -2$ and $c = -3$

2. Let A be a square matrix then prove that

- $A + A'$ is a symmetric matrix

  Prove:

  Let $P = A + A^T$. Then,

  $P^T = (A + A^T)^T = A^T + (A^T)^T$ [$\because (A+B)^T = A^T + B^T$]

  $\Rightarrow P^T = A^T + A$ [$\because (A^T)^T = A$]

  $\Rightarrow P^T = A + A^T = P$ [By commutativity of matrix addition]

  $\therefore$ P is a symmetric matrix.

- $A - A'$ is a skew-symmetric matrix

  Prove:

  Let $Q = A - A^T$. Then,

  $Q^T = (A - A^T)^T = A^T - (A^T)^T$ [$\because (A+B)^T = A^T + B^T$]

  $\Rightarrow Q^T = A^T - A$ [$\because (A^T)^T = A$]

  $\Rightarrow Q^T = -(A - A^T) = -Q$

  $\Rightarrow$ Q is skew-symmetric

- AA^T and A^TA are symmetric matrices

  Prove:

  $(AA^T)^T = (A^T)^T A^T$ [By reversal law]

  $\Rightarrow (AA^T)^T = AA^T$ [$\because (A^T)^T = A$]

  $\Rightarrow$ AA^T is symmetric

  Similarly, *it can be proved that* $A^T A$ is symmetric.

2. If A and B are symmetric matrices then show that AB is symmetric iff AB = BA

   Prove:

   AB is symmetric

   $\Leftrightarrow (AB)^T = AB$

   $\Leftrightarrow B^T A^T = AB$ [$\because (AB)^T = B^T A^T$]

   $\Leftrightarrow BA = AB$ [$\because$ A and B are symmetric matrices $\therefore A^T = A, B^T = B$]

4. Let A and B are symmetric matrices of same order then show that

- $A+B$ is a symmetric matrices

  Prove:

  We have,

  $(A+B)^T = A^T + B^T = A + B$ $[\because A^T = A, B^T = B]$

  $A+B$ is symmetric

- $AB-BA$ is a skew-symmetric matrix

  Prove:

  $(AB-BA)^T = (AB)^T - (BA)^T$

  $(AB-BA)^T = B^T A^T - A^T B^T$ [By reversal law]

  $(AB-BA)^T = BA - AB$ $[\because B^T = B, A^T = A]$

  $(AB-BA)^T = -(AB-BA)$

  $AB-BA$ is skew-symmetric.

- $AB+BA$ is a symmetric matrix

  Prove:

  $(AB+BA)^T = (AB)^T + (BA)^T$

  $= B^T A^T + A^T B^T$ [By reversal law]

  $= BA + AB$ $[\because A^T = A, B^T = B]$

  $= AB + BA$

  $AB+BA$ is symmetric matrix.

Theorem – 1

Every square matrix can be uniquely expressed as the sum of a symmetric and skew-symmetric matrix.

Proof

Let A be a square matrix then $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = P + Q$

Now $P^T = \frac{1}{2}(A + A^T)^T = \frac{1}{2}(A^T + A) = P$

$\Rightarrow P^T = P$, so T is a symmetric matrix.

$$Q^T = \left(\frac{1}{2}(A - A^T)\right)^T = \frac{1}{2}\left(A^T - (A^T)^T\right)$$

$$= \frac{1}{2}(A^T - A)$$

$$= -\frac{1}{2}(A - A^T) = -Q$$

$$\Rightarrow Q^T = -Q$$

So Q is a skew-symmetric matrix.

Question:-

Express the matrix A = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

Answer:

Here

$$B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

Let $P = \frac{1}{2}(B + B') = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix}$,

Now $P' = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} = P$

Thus $\quad P=\frac{1}{2}(B+B')$ is a symmetric matrix.

Also, let $\quad Q=\frac{1}{2}(B-B')=\frac{1}{2}\begin{bmatrix}0 & -1 & -5\\ 1 & 0 & 6\\ 5 & -6 & 0\end{bmatrix}=\begin{bmatrix}0 & \frac{-1}{2} & \frac{-5}{2}\\ \frac{1}{2} & 0 & 3\\ \frac{5}{2} & -3 & 0\end{bmatrix}$

Then $\quad Q'=\begin{bmatrix}0 & \frac{1}{2} & \frac{5}{3}\\ \frac{-1}{2} & 0 & -3\\ \frac{-5}{2} & 3 & 0\end{bmatrix}=-Q$

Thus $\quad Q=\frac{1}{2}(B-B')$ is a skew symmetric matrix.

Now $\quad P+Q=\begin{bmatrix}2 & \frac{-3}{2} & \frac{-3}{2}\\ \frac{-3}{2} & 3 & 1\\ \frac{-3}{2} & 1 & -3\end{bmatrix}+\begin{bmatrix}0 & \frac{-1}{2} & \frac{-5}{2}\\ \frac{1}{2} & 0 & 3\\ \frac{5}{2} & -3 & 0\end{bmatrix}=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}=B$

Thus, B is represented as the sum of a symmetric and a skew symmetric matrix.

Concept of elementary row and column operations.

Elementary Transformations or elementary operations of a matrix:-

The following three operations applied on the rows (columns) of a matrix are called elementary row (column) transformation.

- Interchange of any two rows (columns) denoted by $R_i \Leftrightarrow R_j$ or $C_i \Leftrightarrow C_j$
- Multiplying all elements of a row (column) of a matrix by a non-zero scalar denoted by $R_i \to kR_j$ or $C_i = kC_j$
- Adding to the elements of a row (column), the corresponding elements of any other row (column) multiplied by any scalar K, dented by $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Method of finding the inverse of a matrix by Elementary transformation:-

Let A be a non-singular matrix of order n. Then A can be reduced to the identity matrix I_n by a finite sequence of elementary transformation only. As we have discussed every elementary row transformation of a matrix is equivalent to pre-multiplication by the corresponding elementary matrix. Therefore there exist elementary matrices E_1, E_2 .....E_4 such that $\left(E_k E_{k-1} ..... E_2 E_1\right) A = I_n$

$\Rightarrow \left(E_k E_{k-1} ..... E_2 E_1\right) AA^{-1} = I_n A^{-1}$ (post multiplying by A^{-1})

$\Rightarrow \left(E_k E_{k-1} .... E_2 E_1\right) I_n = A^{-1} \quad \left(\therefore I_n A^{-1} = A^{-1} \text{ and } AA^{-1} = I_n\right)$

$\Rightarrow A^{-1} = \left(E_k E_{k-1} .... E_2 E_1\right) I_n$

Algorithm for finding the inverse of a non-singular matrix by elementary row transformations:-

Let A be a non-singular matrix of order n

Step – I:- Write $A = I_n A$

Step – II:- Perform a sequence of elementary row operations successively on the LHS and the prefactor I_n on the RHS till we obtain the result $I_n = BA$

Step – III:- Write $A^{-1} = B$.

The following steps will be helpful to find the inverse of a square matrix of order 3 by using elementary row transformations.

Step – I:- Introduce unity at the intersection of the first row and first column either by interchanging two rows or by adding a constant multiple of elements of some other row to the first row.

Step – II:- After introducing unity at (1, 1) place introduce zeros at all other places in the first column.

Step – III:- Introduce unity at the intersection 2^{nd} row and 2^{nd} column with the help of the 2^{nd} and 3^{rd} row.

Step – IV:- Introduce zeros at all other places in the second column except at the intersection of 2^{nd} and 2^{nd} column.

Step – V:- Introduce unity at the intersection of 3^{rd} row and third column.

Step – VI:- Finally introduce zeros at all other places in the third column except at the intersection of the third row and third column.

Example:- 1

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ using elementary row transformation.

Solution:-

$$A = IA$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \qquad \text{by } R_2 \to R_2 - 2R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \qquad \text{by } R_1 \to R_1 - 3R_2$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

Problem

Using elementary transformation find the inverse of following matrices.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}.$$

Answer:

In order to use elementary row operations we may write A = IA.

or $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$, then $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ (applying $R_2 \to R_2 - 2R_1$)

or $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} A$ (applying $R_2 \to -\frac{1}{5} R_2$)

or $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} A$ (applying $R_1 \to R_1 - 2R_2$)

Thus $A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix}$

Homework:-

Using elementary transformation find the inverse of following matrices.

(a) $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

Invertible matrices and proof of the uniqueness of inverse if it exists

The uniqueness of Inverse:-

The inverse of a square matrix, if it exists, is unique

Let $A = (a_{ij})_{n\times n}$ be any square matrix

If possible, A has two inverses B and C

$\Rightarrow AB = I = BA$ ....(1)

& $AC = I = CA$ ....(2)

Now, $B = BI = B(AC) = (BA)C = IC = C$

Example – 1

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$$

Solution:-

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad R_1 \Leftrightarrow R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \qquad R_3 \to R_3 - 3R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \qquad \begin{matrix} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 + 5R_2 \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} A \qquad \text{by } \begin{matrix} R_1 \to R_1 + R_3 \\ R_2 \to R_1 - 2R_3 \end{matrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$$

Problem – 1

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary row transformation.

Answer:

$A = IA$

or, $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ [Applying $R_2 \to R_2 + R_1$]

$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} A$ [Applying $R_2 \to R_2 + 2R_3$]

$\Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -4 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$ [Applying $R_1 \to R_1 + (-2) R_2$, $R_3 \to R_3 + 2R_2$]

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$ [Applying $R_1 \to R_1 + 2R_3$]

Hence, $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Problem – 2

$A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$ find A^{-1} using elementary row transformation.

Answer:

$A = IA$

or, $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & -1 & -1 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ [Applying $R_1 \to R_1 - R_2$]

$\Rightarrow \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ -3 & 3 & 1 \end{bmatrix} A$ [Applying $R_2 \to R_2 + (-2)\, R_1$ & $R_3 \to R_3 + (-3)\, R_1$]

$\Rightarrow \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 1/2 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3/2 & 0 \\ -3 & 3 & 1 \end{bmatrix} A$ [Applying $R_2 \to R_2(1/2)$]

$\Rightarrow \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 0 \\ -1 & 3/2 & 0 \\ -5 & 6 & 1 \end{bmatrix} A$ [Applying $R_1 \to R_1 + R_2$ and $R_3 \to R_3 + 2R_2$]

$\Rightarrow \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 0 \\ -1 & 3/2 & 0 \\ -5/4 & 3/2 & 1/4 \end{bmatrix} A$ [Applying $R_3 \to 1/4\, R_3$]

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5/8 & 5/4 & 1/8 \\ -3/8 & 3/4 & -1/8 \\ -5/4 & 3/2 & 1/4 \end{bmatrix} A$ [Applying $R_2 \to R_1 + (1/2)\, R_3$ & $R_2 \to R_2 + (1/2)\, R_3$]

Hence, $A^{-1} = \begin{bmatrix} -5/8 & 5/4 & 1/8 \\ -3/8 & 3/4 & -1/8 \\ -5/4 & 3/2 & 1/4 \end{bmatrix}$.

Home Work:-

Find A^{-1}, of the following matrices using elementary row transformation.

(a) $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix}$ Answer:- $-\frac{1}{30}\begin{bmatrix} -2 & 4 & -10 \\ 44 & -7 & -5 \\ -5 & -5 & 5 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$ Answer :- $\begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ Answer:- $\begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$

Word problems on matrices

EXAMPLE - 1

A factory produces three stickers P, Q and R printed with the messages 'SAVE TREES', 'SAVE ELECTRICITY' and 'SAVE FUEL' respectively. These stickers are sold in two markets, M_1 and M_2. Annual sale (in units) is indicated below:

	P	**Q**	**R**
M_1	2000	4000	10000
M_2	5000	7000	3000

Represent the above information in the form of a 2 × 3 matrix. What does the entry in the second row and first column represent?

Solution: We can represent the given information in the form of a 2 × 3 matrix given by

$$A = \begin{array}{c} \\ M_1 \\ M_2 \end{array} \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 2000 & 4000 & 10000 \\ 5000 & 7000 & 3000 \end{bmatrix} \end{array}.$$

Entry in second row and first column represents annual sale of sticker P in market M_2.

EXAMPLE - 2

Two dealers P and Q sell three items, viz., notebooks, pencils and erasers. In the month of April, the dealer P sells 250 notebooks, 400 pencils and 300 erasers, whereas the dealer Q sells 350 notebooks, 500 pencils and 470 erasers. In the month of May, the dealer P sells 380 notebooks, 440 pencils and 360 erasers, whereas the dealer Q sells 450 notebooks, 520 pencils and 420 erasers. Using matrices, represent

(i) the combined sale of two months for each dealer in each item?

(ii) the change in sales from April to May for each dealer in each item?

***Solution*:** We can represent the given sale information for the month of April as the following matrix:

$$A = \begin{matrix} & \text{Notebook} & \text{Pencil} & \text{Eraser} \\ \begin{matrix} P \\ Q \end{matrix} & \left[\begin{matrix} 250 \\ 350 \end{matrix}\right. & \begin{matrix} 400 \\ 500 \end{matrix} & \left.\begin{matrix} 300 \\ 470 \end{matrix}\right] \end{matrix}.$$

We can represent the given sale information for the month of May as the following matrix:

$$B = \begin{matrix} & \text{Notebook} & \text{Pencil} & \text{Eraser} \\ \begin{matrix} P \\ Q \end{matrix} & \left[\begin{matrix} 380 \\ 450 \end{matrix}\right. & \begin{matrix} 440 \\ 520 \end{matrix} & \left.\begin{matrix} 360 \\ 420 \end{matrix}\right] \end{matrix}.$$

(i) The matrix of combined sale of two months for each dealer in each item can be obtained by adding the two matrices, which is given by

$$A + B = \begin{bmatrix} 250 & 400 & 300 \\ 350 & 500 & 470 \end{bmatrix} + \begin{bmatrix} 380 & 440 & 360 \\ 450 & 520 & 420 \end{bmatrix}$$

$$= \begin{bmatrix} 250+380 & 400+440 & 300+360 \\ 350+450 & 500+520 & 470+420 \end{bmatrix} = \begin{bmatrix} 630 & 840 & 660 \\ 800 & 1020 & 890 \end{bmatrix}.$$

Hence, the combined sale of two months for each dealer in each item has the following form:

$$\begin{matrix} & \text{Notebook} & \text{Pencil} & \text{Eraser} \\ \begin{matrix} P \\ Q \end{matrix} & \left[\begin{matrix} 630 \\ 800 \end{matrix}\right. & \begin{matrix} 840 \\ 1020 \end{matrix} & \left.\begin{matrix} 660 \\ 890 \end{matrix}\right] \end{matrix}.$$

(ii) The matrix of change in sales from April to May for each dealer in each item can be obtained by taking the difference the two matrices, which is given by

$$B - A = \begin{bmatrix} 380 & 440 & 360 \\ 450 & 520 & 420 \end{bmatrix} - \begin{bmatrix} 250 & 400 & 300 \\ 350 & 500 & 470 \end{bmatrix}$$

$$= \begin{bmatrix} 380-250 & 440-400 & 360-300 \\ 450-350 & 520-500 & 420-470 \end{bmatrix} = \begin{bmatrix} 130 & 40 & 60 \\ 100 & 20 & -50 \end{bmatrix}.$$

Hence, the change in sales from April to May for each dealer in each item has the following form:

$$\begin{matrix} & \text{Notebook} & \text{Pencil} & \text{Eraser} \\ \begin{matrix} P \\ Q \end{matrix} & \left[\begin{matrix} 130 \\ 100 \end{matrix}\right. & \begin{matrix} 40 \\ 20 \end{matrix} & \left.\begin{matrix} 60 \\ -50 \end{matrix}\right] \end{matrix}.$$

■

EXAMPLE - 3

A builder is constructing a three-storey building. Each storey is to be installed with 20 fans, 30 tube lights and 25 LED bulbs. Represent the above information as a column matrix. Using scalar multiplication, find the total number of electrical items of each type in the building.

Solution**:** We can represent the given information of electrical items as the following matrix:

$$A = \begin{matrix} \text{Fan} \\ \text{Tube light} \\ \text{LED light} \end{matrix} \begin{bmatrix} 20 \\ 30 \\ 25 \end{bmatrix}.$$

The matrix of total electrical items of each type in the building can be obtained by multiplying the above matrix with the number of storeys, which is given by

$$3A = 3\begin{bmatrix} 20 \\ 30 \\ 25 \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 75 \end{bmatrix}.$$

Hence, the total number of fans, tube lights and LED lights in the building are 60, 90 and 75 respectively. ■

EXAMPLE - 4

A book store has 20 mathematics books, 15 physics books and 12 chemistry books. Their selling prices are ₹ 300, ₹ 320 and ₹ 340 each respectively. Find the total amount the store will receive from selling all the items.

Solution**:** We can represent the given price information as the following *Price Matrix*:

$$\begin{matrix} \\ \text{Mathematics} \\ \text{Physics} \\ \text{Chemistry} \end{matrix} \begin{matrix} \text{Price} \\ \begin{bmatrix} 300 \\ 320 \\ 340 \end{bmatrix} \end{matrix}.$$

We can represent the given sale information as the following *Sale Matrix*:

	Mathematics	Physics	Chemistry
Book store	20	15	12

Then, the amount received by the book store from selling all the items can be obtained by multiplying the sale matrix and price matrix, which is given by

$$\begin{bmatrix} 20 & 15 & 12 \end{bmatrix} \begin{bmatrix} 300 \\ 320 \\ 340 \end{bmatrix} = 6000 + 4800 + 4080 = 14880.$$

Hence, total amount is ₹ 14880. ■

EXAMPLE - 5

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand-made fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold are given below:

Article \ School	A	B	C
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also find the total funds collected for the purpose.

Solution: We can represent the given price information as the following *Price Matrix*:

$$\begin{matrix} \\ \text{Hand-fans} \\ \text{Mats} \\ \text{Plates} \end{matrix} \begin{matrix} \text{Price} \\ \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \end{matrix}.$$

We can represent the given sale information as the following *Sale Matrix*:

$$\begin{matrix} \\ \text{School } A \\ \text{School } B \\ \text{School } C \end{matrix} \begin{matrix} \begin{matrix} \text{Hand-fans} & \text{Mats} & \text{Plates} \end{matrix} \\ \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \end{matrix}.$$

Then, the matrix of funds collected by the schools can be obtained by multiplying the sale matrix and price matrix, which is given by

$$\begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 1000 + 5000 + 1000 \\ 625 + 4000 + 1500 \\ 875 + 5000 + 2000 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}.$$

Hence, amounts raised by school A, school B and school C are ₹ 7000, ₹ 6125 and ₹ 7875 respectively.

Also, the total funds collected $=$ ₹ $(7000 + 6125 + 7875) =$ ₹ 21000. ■