Chapter- 1

Relations and Functions

1. The relation $R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$ on set $A = \{1,2,3\}$ is
   a) Reflexive but not symmetric b) Reflexive but not transitive
   c) Symmetric and transitive d) Neither symmetric nor transitive
2. Fill in the blank: Let the relation R be defined in N by $a\ R\ b$ if $2a + 3b = 30$. Then $R =$ ________ .
3. State the reason for the relation R in the set $\{1,2,3\}$ given by $R = \{(1,2), (2,1)\}$ not to be transitive.
4. Let R be a relation in the set of natural numbers N defined by $R = \{(a,b) \in N \times N : a < b\}$. Is relation R reflexive. Give a reason.
5. Given set $A = \{3,4,5,6\}$. Let R be the relation in set A and defined as $R = \{(a,b): a + b \leq 12,\ a,b \in A\}$. Is this relation symmetric?
6. Let $A = \{ a, b, c\}$, find the total number of distinct relations in set A.
7. Let $A = \{ a, b. c\}$. Write the smallest and largest equivalence relations on A.
8. Write $X = \{x, y\}$ and an equivalence relation on X defined as $R = \{(x,x), (x,y), (y,y), (y,x)\}$. How many equivalence classes are there?
9. Let A be any non-empty set and $P(A)$ be the power set of A. A relation R defined on $P(A)$ by $X\ R\ Y \Leftrightarrow X \cap Y = X,\ X, Y \in P(A)$. Examine whether R is symmetric.
10. If R_1 and R_2 are symmetric, then prove that $R_1 \cup R_2$ is symmetric.
11. If Z is the set of all integers and R is the relation defined on Z defined as $R = \{ (a,b): a,b \in Z \text{ and } a - b \text{ is divisible by } 5 \}$. Prove that R is an equivalence relation.
12. Show that the relation R in the set of integers given by $R = \left\{(a,b): \frac{a}{b} \text{ is a power of } 3\right\}$ is an equivalence relation.

13. Let R be a relation on the set of all lines in a plane defined by ' is parallel to', $i.e. (l_1, l_2) \in R \Leftrightarrow l_1 || l_2$. Show that R is an equivalence relation.

14. If N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)R\ (c, d)$if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

15. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is divisible by 2} is an equivalence relation. Write all the equivalence classes of R.

16. Relation R defined in a set of real numbers as $R = \{(x, y): x + y\ is\ divisible\ by\ 5\ \forall\ x, y\ \in R\}$. Show that R is neither reflexive nor transitive but symmetric.

17. Set A has three elements and set B has four elements. The number of injections that can be defined from A to B is

a) 144　　b) 12　　c) 24　　d) 64

18. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mapping from A to B is

a) 720　　b) 120　　c) 0　　d) None of these

19. Let the function $f: R \rightarrow R$ be defined by $f(x) = cosx, \forall\ x \in R$. Show that f is neither one-one nor onto.

20. Let $f(x) = \frac{x-1}{x+1}$ then $f\left(f(x)\right)$ is

a) $\frac{1}{x}$　　b) $-\frac{1}{x}$　　c) $\frac{1}{x+1}$　　d) $\frac{1}{x-1}$

21. If $f(x) = \frac{3x+2}{5x-3}$, then $(fof)(x)$ is

a) x　　b) $-x$　　c) $f(x)$　　d) $-f(x)$

22. Fill in the blank: Let $f = \{(1,2), (3, 5), (4,1)\}$ and $g = \{(2,3), (5,1), (1,3)\}$. Then $gof =$ __________ .

23. If $f(x) = x + 7$ and $g(x) = x - 7, x \in R$ then find $fog(7)$.

24. Let $f: R \rightarrow R$ be defined by $f(x) = x^2 - 3x + 2$, find $fof(x)$.

25. If functions $f: A \rightarrow B$ and $g: B \rightarrow A$ satisfy $gof = I_A$, then show that f is one-one and g is onto.

26. If $f(x) = \left(\frac{x-2}{x-1}\right), x \neq 1$ and $g(x) = \left(\frac{3x+1}{x+1}\right), x \neq -1$, find $gof(x)$.

27. If the function $f : R \to R$ is given by $f(x) = x^2 + 2$ and $g: R \to R$ is given by $g(x) = \frac{x}{x-1}$, $x \neq 1$, then find fog and gof and hence find $fog(2)$ and $gof(-3)$.
28. If $f : R \to R$ is defined as $f(x) = 10x + 7$. Find the function $g: R \to R$ such that $gof = fog = I_R$
29. If $f, g: R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x, \forall x \in R$. Then find fog and gof. Hence $fog(-3), fog(5)$ and $gof(-2)$.
30. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2+1}$ $\forall x \in R$ is neither one-one nor onto. Also, if $g: R \to R$ is defined as $g(x) = 2x - 1$, find $fog(x)$.
31. Let $f: R \to R$ be the functions defined by $f(x) = x^3 + 5$. Then $f^{-1}(x)$ is
    a) $(x+5)^{\frac{1}{3}}$ b) $(x-5)^{\frac{1}{3}}$ c) $(5-x)^{\frac{1}{3}}$ d) $5 - x$
32. If $f: [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals
    a) $\frac{x+\sqrt{x^2-4}}{2}$ b) $\frac{x}{1+x^2}$ c) $\frac{x-\sqrt{x^2-4}}{2}$ d) $1 + \sqrt{x^2 - 4}$
33. Let $f: A \to B$ and $g: B \to C$ be the bijective functions. Then $(gof)^{-1}$ is
    a) $f^{-1}og^{-1}$ b) fog c) $g^{-1}of^{-1}$ d) gof
34. Consider a bijective function $f: R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R_+ is the set of all positive real numbers. Find the inverse function of f.
35. If $f: W \to W$ is defined as $f(x) = x - 1$, if x is odd and $f(x) = x + 1$, if x is even. Show that f is invertible. Find the inverse of f, where W is the set of all whole numbers.
36. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}.
37. Consider $f: R^+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R^+ is the set of all non-negative real numbers.
38. Let $f: N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to S$, where S is the range of f, is invertible. Also, find the inverse of f.
39. A function $f: A \to B$ is defined as $f(x) = \frac{x-5}{x-4}$ and $g(x) = \frac{4x-5}{x-1}$, where $A = R - \{4\}$ and $B = R - \{1\}$. Show that (i) $gof(x) = x$ (ii) $fog(x) = x$ and hence find $(iii) f^{-1}(x)$ and $g^{-1}(x)$.