Chapter- 6

Electromagnetic Induction

Electromagnetic Induction:- An Introduction

- In 1831, Michael Faraday suggested that if electricity moving in a wire produces magnetism, then the opposite might be true.
- He moved a magnet in and out of a coil of wire, and electricity flow was observed in the coil. This is called electromagnetic induction.
- E.M. I . is used in hundreds of machines and devices, like an electric motor, generator, etc.

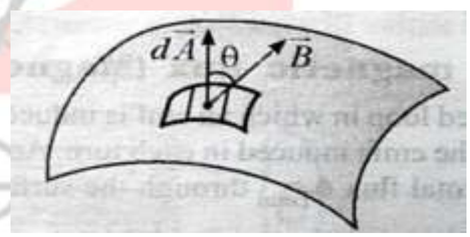
Magnetic Flux: -

The magnetic flux through a small element of the surface $d\vec{A}$ is defined as.

$$d\phi_B = B_\perp dA$$

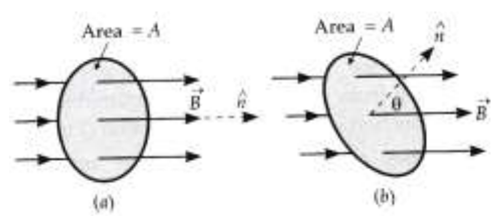
$$= B\cos\theta\, dA$$

$$= \vec{B}.d\vec{A}$$



- Total magnetic flux through the surface $\phi_B = \int \vec{B}.d\vec{A}$
- When $\vec{B}$ is uniform over a plane surface with total area A

  $\phi_B = B_\perp A = BA\cos\theta$



Note:-

- Physically it represents total lines of induction passing through a given area
- Through lines of force are imaginary, flux is a real scalar physical quantity with dimensions.

$$[\phi_B]=[B][S]=\left[\frac{F}{IL}\right][S]=\frac{\left[MLT^{-2}\right]}{[AL]}\left[L^2\right]$$

$$\Rightarrow[\phi_B]=\left[ML^2T^{-2}A^{-1}\right]$$

- S.I unit of magnetic flux

  As $\left[ML^2T^{-2}\right]$ corresponds to energy.

  Its S.I unit will be $\frac{\text{joule}}{\text{Ampere}}=\frac{\text{joule}\times\text{sec}}{\text{coulomb}}=\text{volt}\times\text{sec}$

and is called **weber** (wb) or $\frac{\text{Tesla}}{\text{meter}^2}$

  C.G.S unit $\rightarrow$ **Maxwell** (Mx)

  $1\text{wb}=10^8\text{Mx}$

- When the uniform field is along the surface (i.e field touches the surface tangentially)

$\theta=90^\circ$

$\phi_B=BA\cos 90^\circ=0$ (minimum)

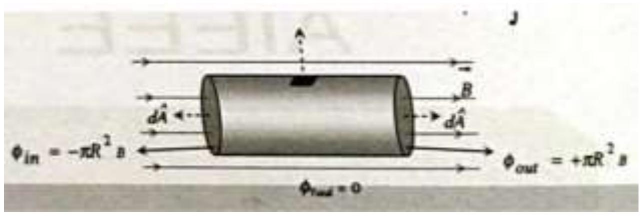
- When the uniform field is normal to the surface

$\theta=0^\circ$

$\phi=BA\cos 0^\circ$

$=BA(\text{maximum})$

- Positive and negative flux. In the case of a body present in a field either a uniform or non-uniform, outward flux is taken to be positive while inward negative.

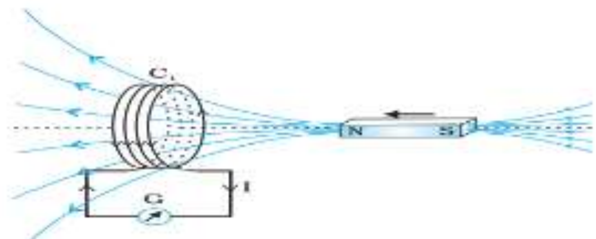


[Uniform field, Total flux = 0]

The Experiments of Faraday and Henry: **-**The understanding of electromagnetic induction is based on a long series of experiments carried out by Faraday and Henry.

Experiment – 1 (Current induced by a magnet)

(Bar magnet is pushed towards the coil which is connected to a sensitive galvanometer)



Observations:-

(a) Bar magnet of rest - No deflection in galvanometer

(b) The magnet moves toward coil - Deflection in one direction

(c) The magnet moves away from the coil - Deflection in the opposite direction

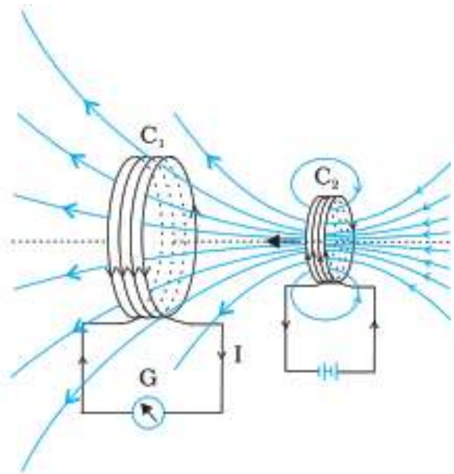
(d) When the south pole of the magnet is moved towards or away from the coil, the deflection in the galvanometer is opposite with the North Pole for similar movement.

(e) When the magnet is pushed towards or pulled away from the coil faster, the deflection is found to be larger.

Conclusion:-

It is the relative motion between the magnet and the coil that is responsible for the generation (induction) of electric current in the coil.

Experiment – 2 (Current induced by current)

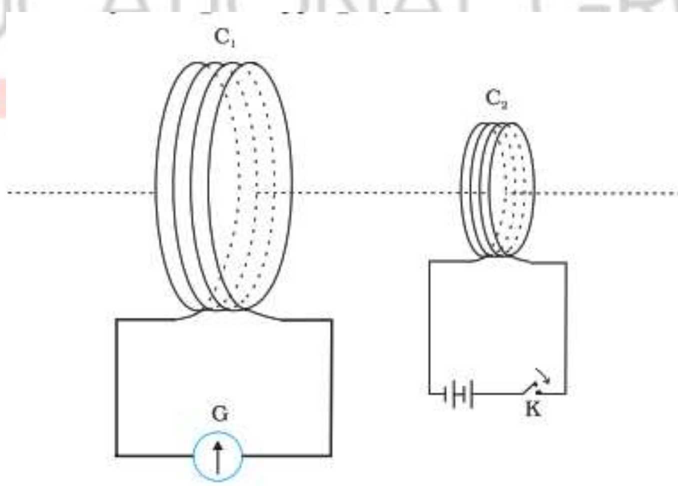


Here the bar magnet is replaced by a second coil C_2

Observations:-When C_2 is moved away or towards the coil C_1 the galvanometer shows the deflection like the previous expt.

Conclusion:-It is the relative motion between the coils that induce the electric current.

Experiment – 3, [Current induced by changing current]



Here relative motion is not an absolute requirement coils C_1 and C_2 held stationary

Observations:-

When tapping key is pressed galvanometer shows a momentary deflection

If the key is held pressed continuously, there is no deflection

When the key is released, a momentary deflection is observed again (but in opposite direction)

When an iron rod is inserted into the coil along their axis, the deflection increases drastically

Faraday's Laws of Electromagnetic Induction:-

Based on experiments (described above) Faraday concluded that.

(a) Whenever there is a change in magnetic flux linked with the circuit, an emf is induced in it. The phenomenon is called electromagnetic induction

(b) The induced emf last as long as the change in flux continues.

(c) The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.

Mathematically $|\varepsilon| = \frac{d\phi}{dt}$

(d) The direction of induced emf is such as to oppose the change or cause which creates it. This law is called **Lenz's Law.**

All the above statements taken together are known as Faraday's laws of electromagnetic induction and are expressed analytically as. $\varepsilon = -\frac{Nd\phi}{dt}$

-ve sign shows that if flux increases, ε is –ve and vice versa.

Note:-

- In the case of E.M.I, an emf $|\varepsilon| = \frac{d\phi}{dt}$ always existed, either the circuit is closed or open but the current will exist only if the circuit is closed.
- If the circuit is closed, induced current

$I = \frac{\varepsilon}{R} = -\frac{Nd\phi}{Rdt}$ (Where R is the total resistance of the circuit)

- Induced charge $dq = Idt = -\frac{Nd\phi}{R}$ (Independent of time)
- $\text{inducedpower}(P) = \varepsilon I = \frac{N^2}{R}\left(\frac{d\phi}{dt}\right)^2$
- Induced field: - Time-varying magnetic field induces electric field which is related to induced emf as $\varepsilon = \int E_{in}.d\ell$

Since $\int E.dl \neq 0$ this indicate induced electric field is a **non-conservative** field.

Question: - In Faraday's experiment

(a) What would you do to obtain a large deflection of the galvanometer?

(b) How would you demonstrate the presence of induced current in the absence of a galvanometer **[NCERT]**

Solution:-

(a) To obtain a large deflection, the steps that can be taken are

(i) Use of a soft iron rod inside the coil C_2

(ii) Use of powerful battery with coil

(iii) The motion of the arrangement rapidly towards the coil

(b) Using a small bulb. The relative motion between the two coils will cause the bulb to glow and thus demonstrate the presence of induced current.

Question:-

A square loop of side 10cm and resistance 0.5Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.1 T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.7 s at a steady rate.

(a) Determine the magnitudes of induced emf(b) Determine the induced current during this internal **[NCERT]**

Solution:-

$\phi = BA\cos\theta$

Initial flux = $BA\cos 45^o$ [$\therefore$ Area vector makes 45^o with field]

$= \left(0.1\times10^{-2}\right)\frac{1}{\sqrt{2}}$

Final flux = 0

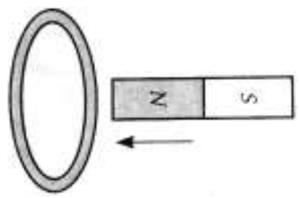
(a) $\varepsilon = \frac{\phi_{initial} - \phi_{final}}{\Delta t} = 1\,mv$

(b) $I = \frac{\varepsilon}{R} = \frac{10^{-3}\,v}{0.5\Omega} = 2\,mA$

Lenz's Law:-

Statement:-The direction of induced emf is such as to oppose the cause that creates it.

Example – 1, (Attraction and repulsion concept)



Suppose an N-pole of a bar magnet is being pushed towards the close coil.

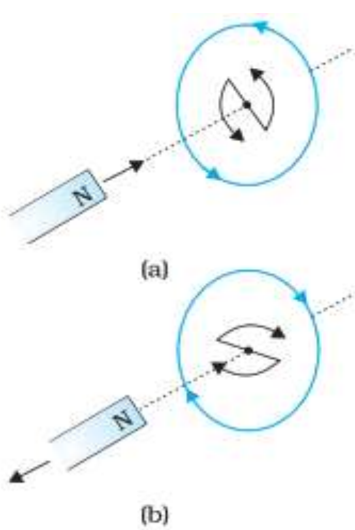
As it moves, the magnetic flux through the coil increases

Current is induced in the coil

The direction of induced current in the coil is in such a direction that it opposes the increase in flux

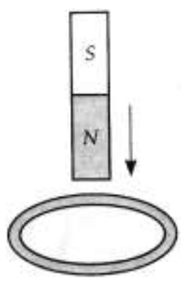
This is possible only if the current in the coil is in a counter-clockwise direction w.r.t an observer situated on the side of the magnet.

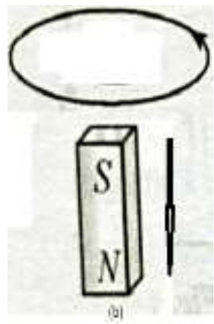
Similarly

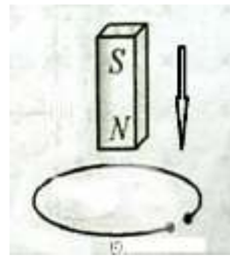


Note:-The direction shown by N and S indicate the directions of induced current.

Question: -A copper ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring as shown in the following diagrams. State whether its acceleration a is equal to greater than or less than the acceleration due to gravity g.







Solution:-

(a) $a = \frac{mg - F}{m} = g - \frac{F}{m}$

Whereof the bar magnet

$F \rightarrow$ Force of repulsion on approaching magnet

Hence a < g

(b) $a = \frac{mg - F}{m} = g - \frac{F}{m}$

$\Rightarrow a < g$

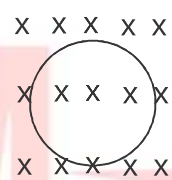
Here, F is the force of attraction on receding magnet

(c) Here an emf will be induced in the ring but no current will flow. So the coil can no more oppose the approach of the magnet. Hence a = g

Example – 2, (Cross or dot magnetic field increasing or decreasing concept) If cross magnetic field passing through a loop increases then the induced current will produce a dot magnetic field. Similarly, if the dot magnetic field passing through a loop decreases then the induced current will produce a dot magnetic field.

Question:-

A circular loop is placed in magnetic field B = 2t. Find the direction of the induced current produced in the loop.



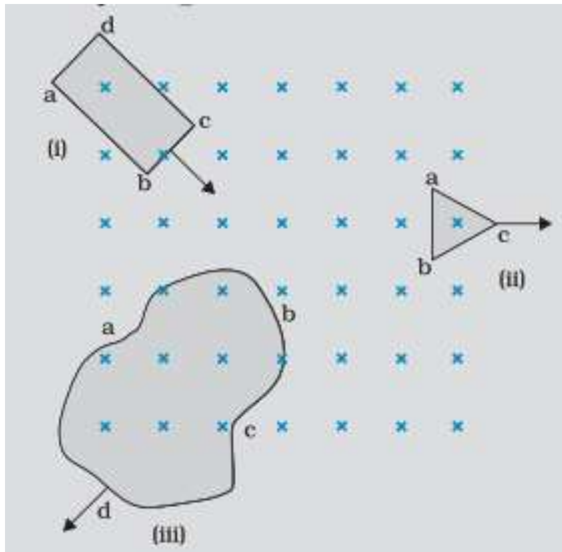
Solution:-B = 2l means, ⊗ a magnetic field passing through the loop is increasing

So induced current will produce dot magnetic field.

To produce a magnetic field, the induced current should be anti-clockwise.

Question:-

The figure shows planar loops of different shapes moving out of or into a region of the magnetic field which is directed normal to the plane of the loop away from the reader. Determine the direction of induced current in each loop using Lenz's law.



Solution:-

(a) Due to the motion of loop abcd into the region of the magnetic field. The cross magnetic field ⊗ through loop increases. Then induced current will produce dot magnetic field to produce a magnetic field, the induced current should be anti-clockwise (follow along the path bcdab)

(b) Due to the outward motion of the triangular loop (abc), the cross magnetic field ⊗ through abc decreases. Then induced current will produce a cross magnetic field to produce ⊗ magnetic field, the induced current should be clockwise (follow along the path bacb)

(c) Clockwise (along path cdabc)

Question:-

A close-loop is held stationary in the magnetic field between the north and south poles of two permanent magnets held fixed. Can we hope to generate current in the loop by using very strong magnets? (NCERT)

Solution:-

No, however strong the magnet may be, current can be induced by changing the magnetic flux through the loop.

Question:-

A closed-loop move normal to the constant electric field between the plates of a charged capacitor. Is a current induced in the loop

(a) When it is wholly inside the region between the capacitor plates?

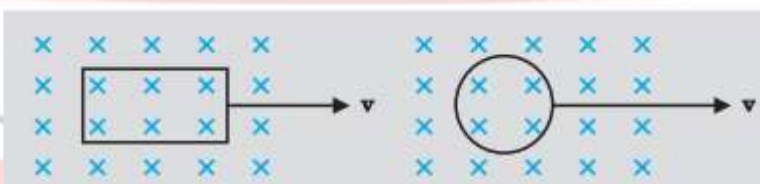
(b) When it is partially outside the plates of the capacitor?

Solution:-

No current is induced in either case. Because current cannot be induced by changing the electric flux

Question:-

A rectangular loop and a circular loop are moving out of a uniform magnetic field region to a field-free region with a constant velocity v. In which loop do you expect the induced emf to be constant during the passage out of the field region? The field is normal to the loop.

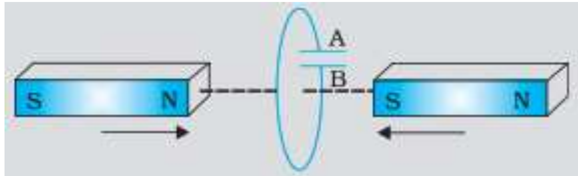


Solution:-

The induced emf is expected to be constant only in the case of a rectangular loop. In the case of the circular loop, the rate of change of the area of the loop during its passage out of the field region is not constant, hence induced emf will vary accordingly.

Question:-

Predict the polarity of the capacitor in the situation described by the following figure.

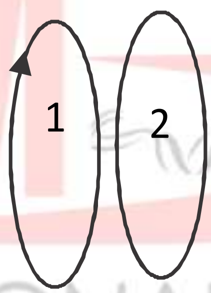


Solution:-The polarity of plate A will be +ve w.r.t plate B in the capacitor.

Example:-Attraction and repulsion between two loops facing each other of current in one loop is changed.

Question:-

Two loops are facing each other as shown in the figure state whether the loops will attract each other or repel each other if current I_1 is increased.



Solution:-

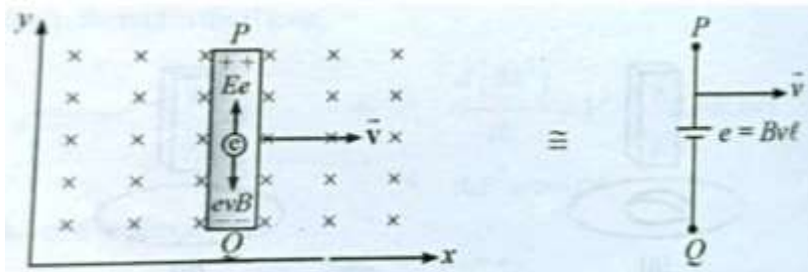
If current I_1 is increased then the induced current in loop 2 (say I_2) will be in the opposite direction. Now two loops having current in opposite directions repel each other.

Motional Electromotive Force:-

It is the emf induced in a conductor moving through a constant magnetic field.

Cause of motional emf:-

The following figure shows a conducting rod of length L moving with a constant velocity $\vec{v}$ which is $\perp r$ to the uniform magnetic field $\vec{B}$ directed into the paper.



From the diagram, the magnetic force on the free electrons $\vec{F}_m = -2\left(\vec{v}\times\vec{B}\right)$ move towards end Q within the rod, hence end Q becomes –ve w.r.t P.

Now an electric field $\vec{E}$ is set up within the rod which exerts a force on free electrons in opposite to $\vec{F}_m$.

At equilibrium $\vec{F}_e + \vec{F}_m = 0$

$$\Rightarrow -e\vec{E} + (-e)\left(\vec{v}\times\vec{B}\right) = 0$$

$$\Rightarrow \vec{E} = -\vec{v}\times\vec{B}$$

$\therefore$ The induced emf across the rod $\varepsilon = \int \vec{E}.d\vec{\ell}$

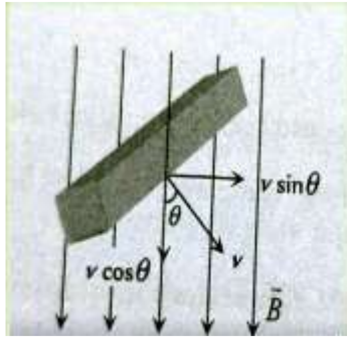
$$\Rightarrow \varepsilon = \int\left(\vec{v}\times\vec{B}\right).d\vec{\ell}$$

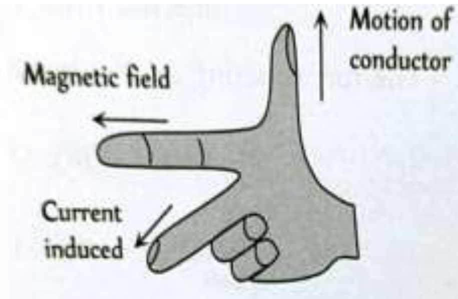
$$\Rightarrow \varepsilon = \int\left[v\hat{i}\times B\left(-\hat{k}\right)\right].d\ell\left(\hat{j}\right)$$

$$\Rightarrow \varepsilon = B\ell v$$

Fleming's right-hand rule:-

Though in general the direction of emf or current is determined by Lenz's law, in the case of motion of a straight conductor in a magnetic field it can also be determined by the so-called Fleming's right-hand rule.



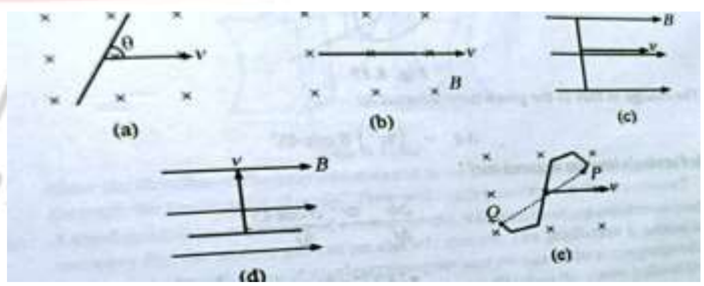


Statement: -

The thumb, forefinger, and the middle figure of the right hand are stretched mutually perpendicular to one another. If the thumb represents the direction of the conductor, the forefinger in the direction of the magnetic field, then the middle figure points in the direction of induced emf or current in the conductor.

Question:-

Find the induced emf across the ends of the conducting rod of length ℓ in the following situations.



Solution:-

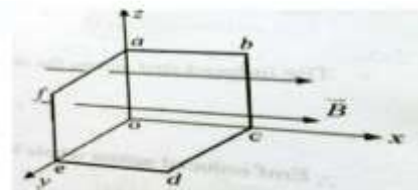
We know $\varepsilon = \int \left(\vec{v} \times \vec{B}\right).d\vec{\ell}$

(a) $Bv\ell \sin\theta$ (b) 0 (c) 0 (d) 0

(e) If PQ line makes angle θ with velocity vector then $\varepsilon = B\left(v\sin\theta\right)PQ$

Question:-

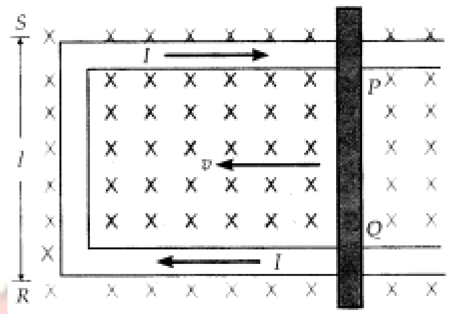
12 wires of equal length are connected in the form of a skeleton of the cube having velocity v in the direction of the magnetic field.



(a) Find the emf in each arm

(b) If the cube moves perpendicular to the field, what will be the induced emf in each arm?

Motional emf in a moving rod slides along a stationary U-shaped conductor.



The figure shows a rectangular conductor PQRS in which the arm PQ is free to move (to left) with constant velocity v

Method- I, the magnetic flux enclosed by the loop PQRS

$$\phi_B = B\ell x$$

Since x is changing with time

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d(Blx)}{dt} = -B\ell\frac{dx}{dt} = B\ell v$$

Where we have used $\frac{dx}{dt} = -v$, speed of conductor PQ

Method – II, when the conductor moves $\perp r \text{ to } \vec{B}$, all the charges experience the same force $(= qvB)$ in rod PQ. The work done in moving the charge from P to Q through a distance ℓ, is

$$w = F\ell = qvB\ell$$

Since emf is the work done per unit charge

$$\varepsilon = \frac{w}{q}$$

$$= \frac{qvB\ell}{q} = B\ell v$$

Note:-

(a) The induced current in the loop, $I_{in} = \frac{\varepsilon}{R} = \frac{B\ell v}{R}$

(Where r resistance of arm PQ. The total resistance of remaining arm u negligible compared to R)

(b) Magnetic force on the conductor opposing the motion of the rod.

$$F_m = BI_{in}\ell = B\left(\frac{B\ell v}{R}\right)\ell = \frac{B^2 v \ell^2}{R}$$

(c) Rate of doing work (power dissipated) in maintaining the motion of rod by pulling force F.

$$P = \frac{dw}{dt} = F.v = \frac{B^2\ell^2 v^2}{R}$$

(d) Electrical power dissipated through the resistor (Joule loss)

$$P_{thermal} = I_{in}^2 R = \left[\frac{B\ell v}{R}\right]^2 R = \frac{B^2 v^2 \ell^2}{R}$$

The phenomenon of electromagnetic induction in accordance with Lenz's law represents the conservation of energy.

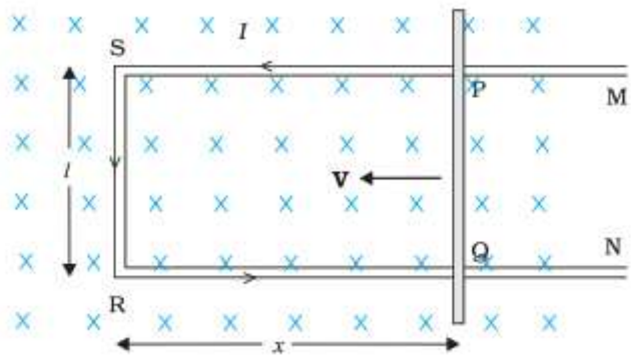
Qualitative treatment:-

Suppose the N-pole of the magnet is moved towards the coil, its upper face acquires north polarity. Therefore, work has to be done against the force of repulsion in bringing the magnet closer to the coil.

This mechanical work done in moving the magnet w.r.t the coil that changes into electric energy producing induced current.

Hence Lenz's law obeys the principle of conservation of energy.

Quantitative treatment: -If a conducting rod of length ℓ is moved in a magnetic field B and the current induced is I, then the magnetic force opposing the motion of the rod.



$$F_m = I\ell B = \left(\frac{B\ell v}{R}\right)\ell B = \frac{B^2\ell^2 v}{R}$$

Rate of doing work in maintaining the motion of the rod by pulling force F.

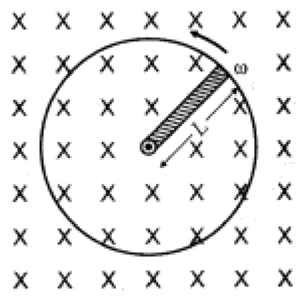
$$\frac{dw}{dt} = P_{much} = Fv = \frac{B^2\ell^2 v^2}{R} \qquad = \left[\frac{B\ell v}{R}\right]^2 R = I^2R = P_{thermal}$$

Hence mechanical energy applied to move the conductor is changed into electrical energy which dissipates into thermal energy. This is consistent with the law of conservation of energy.

Motional emf induced in a rotating bar: -

A conducting rod of length ℓ rotates with a constant angular speed ω about a point at one end.

A uniform magnetic field B is directed perpendicular to the plane of rotation



Let dr be a segment of the rod at a distance r from o

The induced emf in this segment $d\varepsilon = Bdrv = B(r\omega)dr$

Total emf across the rod

$$\varepsilon = \int d\varepsilon = \int_0^1 Br\omega.dr = \frac{B\omega\ell^2}{2}$$

$$\varepsilon = \frac{B\omega\ell^2}{2}$$

From the right-hand rule, we can see o is at a higher potential than P.

Thus $v_0 - v_p = \frac{B\omega\ell^2}{2}; \omega = 2\pi\upsilon$

Note:-

If the above metallic rod rotated about its axis of rotation, then the induced potential difference between any pair of identical located points of the rod is always 0.

Cycle wheel:-

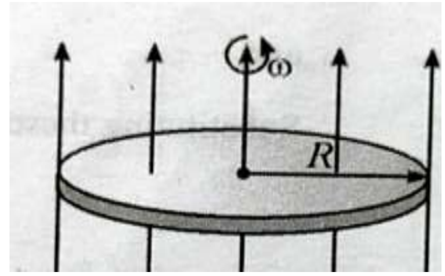
Flux cutting by each metal spoke is

same each spoke becomes cell of emf = $\frac{Bw\ell^2}{2}$

All such cells are parallel.

Therefore $\varepsilon_{net} = \frac{B\omega\ell^2}{\omega}; \omega = 2\pi\upsilon$

Faraday disc dynamo:-

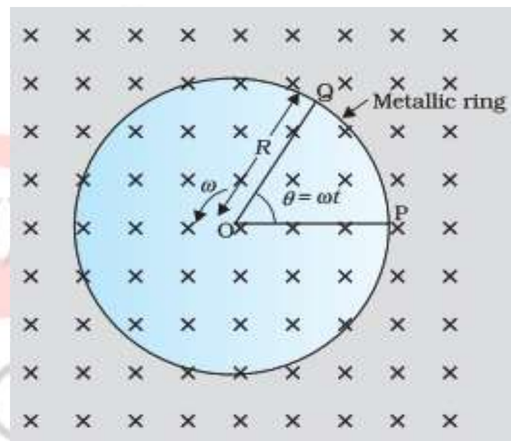


A metal disc can be assumed to be made up of several radial conductors. All such conductors behave like some cells in parallel.

Therefore $\varepsilon_{net} = \varepsilon = \frac{B\omega R^2}{2} = B(\pi R^2)\upsilon$

Question:-

A metallic rod of 1m is rotated with a frequency of 50 rev/s with one end hinged at the center and other end at the circumference of the circular metallic ring of radius 1m, about an axis passing through the center and perpendicular to the plane of the ring as shown in the figure, a constant and uniform magnetic field of 1T parallel to the axis is present everywhere. What is the emf between the center and the metallic ring?



Solution:-

$$\varepsilon = \int_0^R Bvdr = \int_0^r B(\omega r)dr = \frac{B\omega R^2}{2}$$

$$\Rightarrow \varepsilon = \frac{1}{2} \times 1.0 \times 2\pi \times 50 \times [1]^2 = 157v$$

Question:-

A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the earth's magnetic field at a place. If $H_E = 0.4$ G at the place, what is the induced emf between the axle and the rim of the wheel? Note that 1G = 10^4T

Solution:-

Induced emf = $\frac{1}{2}B\omega R^2$

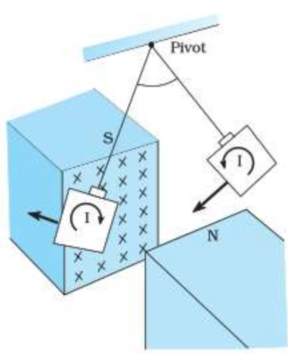
$$=\frac{1}{2}\times 4\pi\times 0.4\times 10^{-4}\times(0.5)^2$$

$$=6.28\times 10^{-5}\text{v}$$

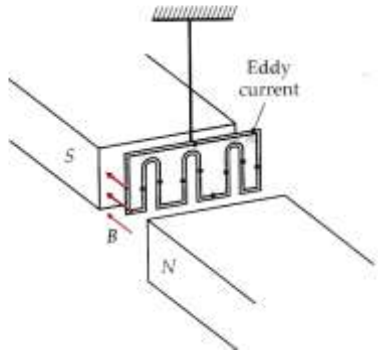
Since all the spokes are in parallel $\varepsilon_{total}=\varepsilon=6.28\times 10^{-15}\text{v}$

Eddy Current:-

It is a type of circulating current formed in a bulk piece of conducting material When it is subjected to a changing magnetic flux.



These are circulating currents like eddies in the water



Its magnitude is given by $I = \frac{\varepsilon}{R} = \frac{d\phi}{Rdt}$

Its direction is given by Lenz's law

The experimental concept was given by Foucault, hence also normed as Foucault current

Disadvantages:-

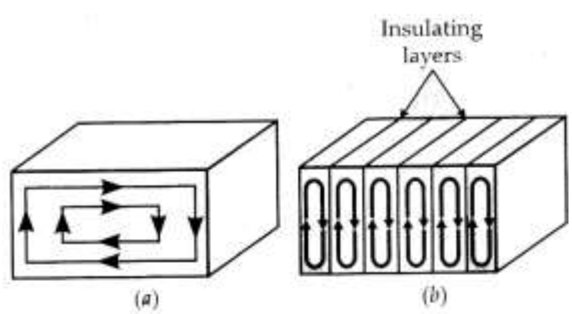
(a) The production of eddy currents in a metallic block leads to loss of electric energy in the form of heat

(b) The heat produced due to eddy currents breaks the insulation in electrical appliances.

(c) it may cause unwanted damping effect

Minimization of eddy current: -

By lamination By slotting process



Application of eddy current: -

(a) Magnetic breaking in trains

(b) Electromagnetic damping

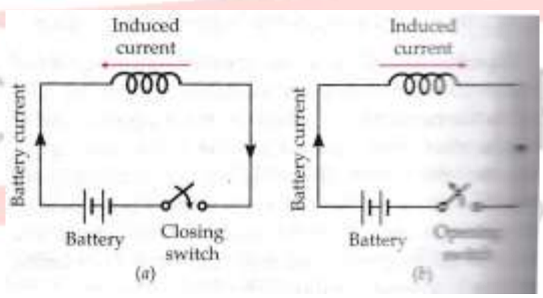
(c) Induction furnace

(d) Electric power meters

Self Induction:-

The phenomenon due to which, an opposing induced emf (back emf) is produced in a coil, as a result of a change in current in the coil itself is called self-induction.

Example:- (when the key is pressed)



The current starts growing the magnetic field (flux) also starts growing

A coil is connected to the battery with a tapping key. An induced emf is generated. The direction of induced emf is such that it opposed the growth of current.

When the key is released: -

The current, hence the field (flux) starts decaying which in turn produces induced emf whose direction is such that it opposes the decaying of current.

Both during growth and decay of current in a coil an opposing emf induced in the coil itself, which demonstrates the self-induction phenomenon.

Physical Significance: -

As the mass plays a role in mechanics, self-induction plays the same role in the electric circuit.

Self-induction is the inertia of electricity

Self-inductance/ co-efficient of self-induction: -

Flux linkage through a coil of N-turns is proportional to the current through it.

$N\phi \propto I$

$\Rightarrow N\phi = LI$ .... (1)

Where the proportionality constant L is called the coefficient of self-induction of the coil

Differentiating equation (1) w.r.t L we get $N\frac{d\phi}{dt} = L\frac{dI}{dt}$ ....(2)

From Faraday's law of e.m.i

$N\frac{d\phi}{dt} = -\varepsilon$ .... (3)

Comparing (2) and (3)

$\varepsilon = -L\frac{dI}{dt} \qquad \Rightarrow L = \dfrac{-\varepsilon}{\frac{dI}{dt}}$

Definition of self-inductance:-

If $\frac{dI}{dt} = 1$, then L = E (numerically). Thus self-inductance of a coil is numerically equal to back emf in the coil when the rate of change of current is unity.

Units of self-inductance: -S. I unit is $\frac{wb}{A} = henry(H)$

$\phi = LI \Rightarrow L = \frac{\phi}{I}$

Smaller units are

mili henry $(mH = 10^{-3} H)$

microhenry $(\mu H = 10^{-6} H)$

Definition of 1 henry of self-inductance:-

If $\frac{dI}{dt} = 1 A/s, E = 1 volt$, then L = 1 henry.

Self-inductance of a coil is said to be 1 henry

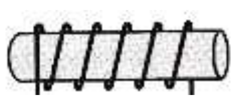
When a current changing at a rate of 1 A/s

causes a back emf of 1 volt in the coil itself

Dimensional formula (self-inductance):-

$$L = \varepsilon \frac{dt}{dI} \qquad = \frac{w}{q}\frac{dt}{dI} = \frac{[ML^2T^{-2}]}{[AT]}\frac{[T]}{[A]} = [ML^2T^{-2}A^{-2}]$$

Self-inductance of long solenoid: -



A = Cross-sectional area

n– number of turns per unit length

l= Length near the center of the solenoid

Magnetic field due to the current in coil $B = \mu_0 nI$ .... (1)

The flux linkage with the solenoid

$N\phi = (n\ell)(\mu_0 nI)A$ .... (2)

Comparing the above equation with $N\phi = LI$

$LI = (n\ell)(\mu_0 nI)A$ $\Rightarrow L = \mu_0 n^2 A\ell$

Note – 1 $\left[L_{end} = \frac{1}{2}L_{centre}\right]$

Note – 2 Step for finding L

(a) Assume current in the coil

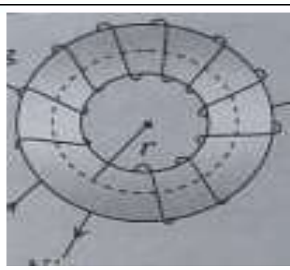
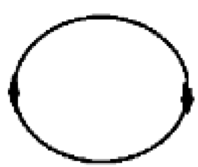
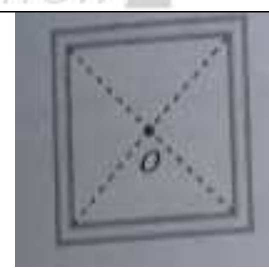
(b) Determine the magnetic field due to this current in the coil

(c) Obtain flux linkage $N\phi$

(d) Compare with $N\phi = LI$ to get L

Question:-Find the self-inductance of the toroid, circular coil, and a square coil

Solution

 $B = \frac{\mu_0 NI}{2\pi r}$ $N\phi = N\left(\frac{\mu_0 NI}{2\pi r}\right)A$ $L = \frac{\mu_0 N^2 A}{2\pi r}$	 $B = \frac{\mu_0 NI}{2r}$ $N\phi = N\left(\frac{\mu_0 NI}{2r}\right)\pi r^2$ $L = \frac{\mu_0 \pi N^2 r}{2}$	 $B = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}I}{a}N$ $N\phi = NBa^2$ $\Rightarrow L = \frac{2\sqrt{2}\mu_0 N^2 a}{\pi}$

The magnetic potential energy of inductor: -

The self-inductance plays the role of inertia. So, work needs to be done against the back emf in establishing the current.

This work done is stared as magnetic potential energy

For the current I at an instant in a circuit, the rate of work done

$$P = \frac{dw}{dt} = |\varepsilon| I$$

The energy dv supplied in time dt $\quad dw = pdt = |\varepsilon| I dt = LIdI$

The total energy supplied $\quad U = L\int_0^I I\,dI = \frac{1}{2}LI^2$

Question: -

(a) Obtain an expression for the magnetic energy stored in a solenoid in terms of magnetic field B, area A and length l of the solenoid.

(b) How does the magnetic energy compare with the electrostatic energy stored in a capacitor?

Solution: -(a) $U_B = \frac{1}{2}LI^2 = \frac{1}{2}L\left[\frac{B}{\mu_0 n}\right]^2$ $\quad \left[\begin{array}{l}\therefore B = \mu_0 nI \\ \Rightarrow I = \dfrac{B}{\mu_o n}\end{array}\right]$

$$\Rightarrow U_B = \frac{1}{2}\left(\mu_0 n^2 A\ell\right)\left[\frac{B}{\mu_0 n}\right]^2 \qquad \left[\therefore L = \mu_0 n^2 A\ell\right]$$

$$\Rightarrow U_B = \frac{1}{2\mu_0}B^2 A\ell$$

(b) Magnetic energy per unit volume (i. e magnetic energy density)

$$U_B = \frac{U_B}{v} = \frac{1}{2}\frac{\mu_0 B^2 A\ell}{A\ell} = \frac{B^2}{2\mu_0}$$

Electrostatic energy stored per unit volume in parallel plate capacitor is given by $U_E = \frac{1}{2}\varepsilon_0 E^2$

In both cases, the energy is proportional to the square of the field strength

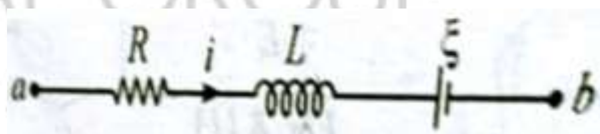
Question: -A 12H inductor carries a steady current of 2A. How can a 60v self-induced emf be made to appear in the inductor?

Solution:- $|\varepsilon| = L\frac{dI}{dt} \Rightarrow \frac{dI}{dt} = \frac{|\varepsilon|}{L} = \frac{60}{12} = 5\,A/s$

The current in the inductor should change at the rate of 5 A/s

Question:-

In the circuit as given, R = 10 Ω, L = 5 H, ξ = 20 V, *I* = 2A this current is decreasing at a rate of 1 A/s. Find v_{ab} at this instant



Solution:-Applying Kirchhoff's law

$$V_a - 10\times 2 + V_L - 20 = V_b$$

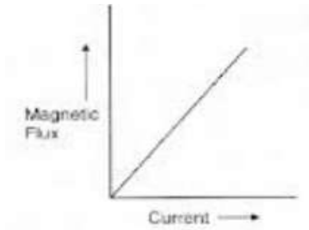
But $V_L = -L\frac{dI}{dt} = -5(-1) = +5V$

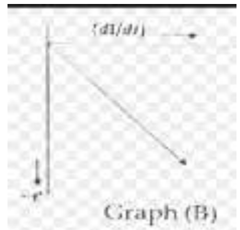
Thus $V_a - 20 + 5 - 20 = V_b \quad \Rightarrow V_a - V_b = 35\,\text{volts}$

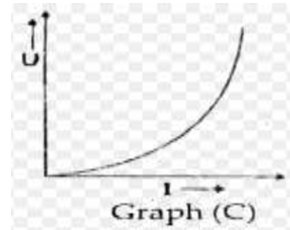
Question:-The current flowing through an inductor of self-inductance L is continuously increasing. Plot a graph showing the variation of

(a) Magnetic flux ~ current (b) Induced emf ~ dI/dt (c) Magnetic P.E ~ current

Solution:-



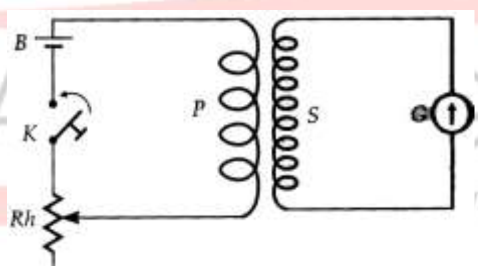




Mutual Induction:-

It is the phenomenon due to which an opposing emf (back emf) is produced in a coil, as a result, if a change in current (or change in magnetic flux) in the neighboring coil.

Example:-



When the key is pressed:-

Current in P begins to increase

Magnetic flux in P also increases

Magnetic flux throughs (which is coupled to P) also increase

This setup an induced emf in S, whose direction is such it opposes the growth of current in P

When the key is released:-In the similar method induced emf in S try to oppose the decay of current in P

Coefficient of Mutual Induction: -

Let ϕ be the magnetic flux linked with secondary due to current I in primary

Thus $\phi \propto I$

$\Rightarrow \phi \propto MI$ ……………………… (1)

Proportionality constant M is called mutual inductance

From (1) $\frac{d\phi}{dt} = M\frac{dI}{dt}$

$E = -\frac{d\phi}{dt} = -M\frac{dI}{dt}$ ………………………… (2)

$E = -M\frac{dI}{dt}$

From (2) if $\frac{dI}{dt} = 1\,{}^{Amp}\!/_{s}$, then E = -M

Hence, the mutual inductance of a coil is numerically equal to induced emf produced in one coil when the rate of change of current is unity in another coil.

Unit of Mutual Inductance:-

Henery – (SI unit)

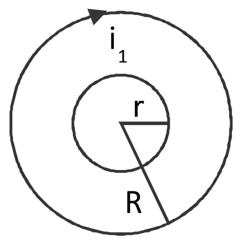
Ab – Henry (C.G.S unit)

mill. Henry microhenry

Mutual inductance for:-

(1) Two concentric co-planar circular coils

The magnetic field at the center due to current in the outer coil



$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi Ni}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

$N_2\phi_2 \rightarrow$ Total flux linkage with the secondary coil

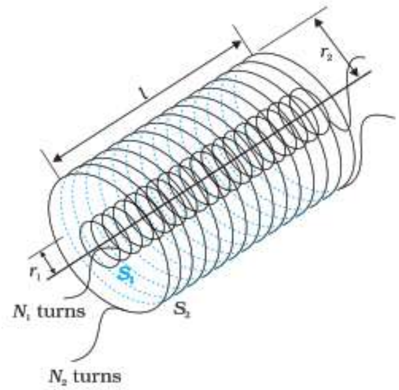
So $N_2\phi_2 = Mi_1$

$$\Rightarrow M = \frac{N_2(\phi_2)}{i_1} = \frac{N_2(B_1A_2)}{i_1} = N_2\frac{\frac{\mu_0}{4\pi}\frac{2\pi N_1 i_1}{r}(\pi r^2)}{i_1}$$

$$\Rightarrow M = \frac{\pi\mu_0 N_1 N_2 r^2}{2R}$$

$$\Rightarrow M \propto \frac{r^2}{R}$$

(ii) Tow solenoids



The magnetic field inside the primary solenoid

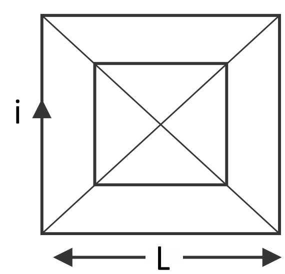
$B_1 = \mu_0 n_1 i_1$ .... (1)

$N_2 \phi_2 = M i_1$

$$\Rightarrow M = \frac{N_2 \phi_2}{i_1} = \frac{N_2 (B, A)}{i}$$

$M = \frac{\mu_0 N_1 N_2 A}{\ell}$ (A = Area of each solenoid)

Question:-Calculate the mutual inductance of two concentric co-planer square coils if sides L and ℓ respectively as shown in the figure.



Solution:- $M = \frac{\mu_0 2\sqrt{2} N_1 N_2 \ell^2}{\pi L}$

Dependence of mutual inductance:-

(a) Number of turns (N_1 N_2) of both coils

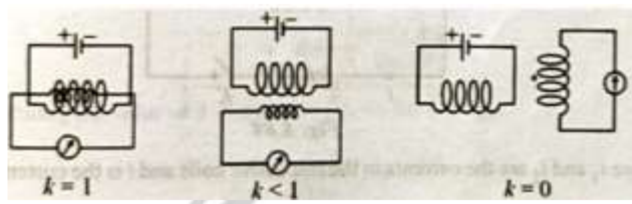
(b) coefficient of self-inductance (L_1 L_2) of both coils

(c) The common area of cross-section of coils

(d) Distance between two coils $\left(\text{As d}\uparrow = \text{M}\downarrow\right)$

(e) The orientation between primary and secondary coils

(f) Coupling factor (k) = $\dfrac{\text{flux in secondary}}{\text{flux in primary}}$



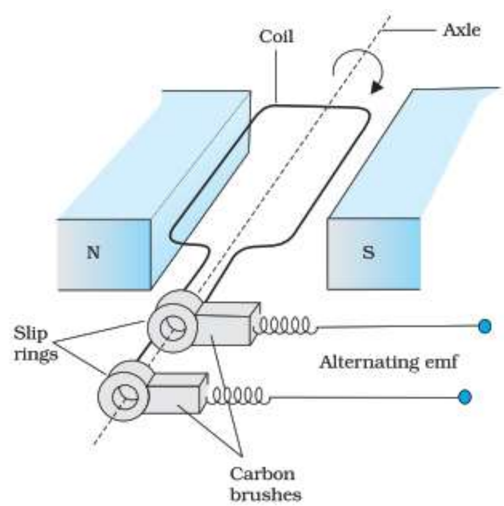
The relation between M, L_1, and L_2:-

For two magnetically coupled coil $M = K\sqrt{L_1L_2}$

AC Generator/ Dynamo: -It is an electric device which converts mechanical to electrical energy (A.C)

Principle:-

It is based on the principle of electromagnetic induction. (i.e when coil rotates in a uniform magnetic field, an emf is induced in it)



Construction:-The essential parts are

(a) Armature - Insulated copper wire, wound on a laminated cove of soft iron

(b) Field magnet – Horseshoe or strong magnet that provides a uniform magnetic field

(c) Slip ring - Metal shoe or strong magnet that provides a uniform magnetic field

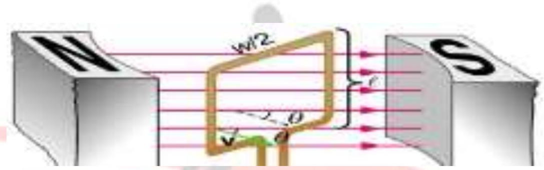
(d) Brushes – Two flexible metal rods which kept pressing against the slip ring. They are used to pass the current from armature to external resistance R.

Theory and working:-As the armature coils rotated, the flux linked with it changes

An emf induced which is given by $\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\theta)$

$= -\frac{d}{dt}(NBA\cos\omega t)$

$\Rightarrow E = \omega NBA\sin\omega t$ .... (1)



Where $\omega = \frac{\theta}{t} \rightarrow$ Angular velocity of the coil

From equation (1)

When $\sin\omega t = 1$

$E = NBA\omega(\max) \quad \Rightarrow E_0 = NBA\omega(\max)$

Thus from equation (1)

$E = E_0 \sin\omega t$

$\Rightarrow E = E_0 \sin 2\pi\upsilon t$ .... (2)

Where $\omega = 2\pi\upsilon t$ υ= frequency of radius of coil Equation (2) indicates the emf is alternating in nature.

Position-1	Position-2	Position-3	Position-4	Position-5
Coil $\perp$ r to $\vec{B}$ $\theta = \omega t = 0^{o}$ $\varepsilon = 0$	Coil $\parallel$ to $\vec{B}$ $\theta = \pi/2$ $\varepsilon = \varepsilon_{max}$	Again $\perp_r$ to B $\theta = \pi$ $\varepsilon = 0$	Again $\parallel$ $\vec{B}$ $\theta = 3\pi/2$ $\varepsilon = -\varepsilon_0$	$\perp$ r to B $\theta = 2\pi$ $\varepsilon = 0$