Chapter- 10

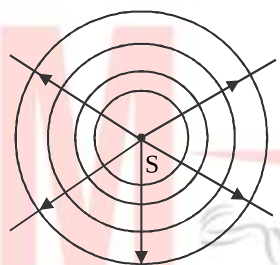
Wave Optics

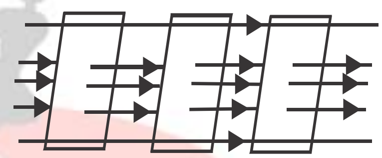
- A wavefront is defined as the continuous locus of all such particles which start vibrating from one instant and which are vibrating in the same phase at any instant. All particles on a wavefront have the same phase at every instant and hence phase difference is zero. Directions of propagation of wavefronts represent rays. Rays are parallel to the wavefront. Time taken by light to travel from one wavefront to another wavefront along any ray is the same.
- Different wavefronts are.

  (a) Spherical wavefront for point source

  (b) Cylindrical wavefront for elongated linear source.

  (c) Plane wavefront for sources at a large distance or parallel rays.



Huygen's Principle of secondary wavelets:-

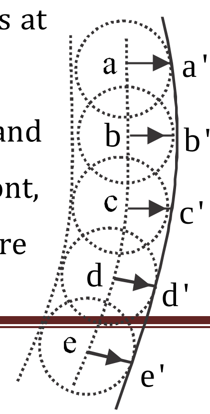
- This principle helps in constructing a secondary wavefront of a given wavefront after a certain time.
- Assumptions of the principle are

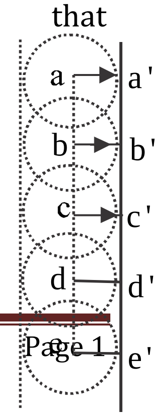
  (a) Each point on a wavefront act as a fresh source of the disturbance. Wavefronts produced by a particle of a wavefront are called as wavelets.

  (b) The wavelets spread out in all directions with speed of light in the given medium.

  (c) The new wavefront at any later time is given by the forward envelop (i.e tangential surface in the forward direction) of the secondary wavelets at that time.

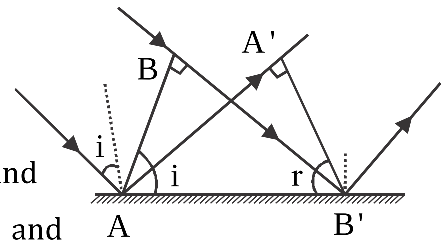
- Construction of secondary wavefront of a spherical wavefront and plane wavefront. If a, b, c, d, e are source points on a wavefront, then spheres drawn by taking these points as centre are





secondary wavelets. After a time t radii of these wavelets are cl each. Now forward envelop to these spheres is a'b'c'd'e' which represent the secondary wavefront after time t.

- **Verification of laws of reflection:-**



AB = Incident wavefront on a plane reflecting face

$A'B' =$ its reflecting wavefront after a time t.

$i =$ The angle of incidence = Angle between an incident ray and corresponding normal = angle between incident wavefront and surface = $\angle BAB'$.

$r =$ The angle of reflection = Angle between a reflecting ray and corresponding normal = Angle between reflected wavefront with surface $= \angle A'B'A$

Here, incident wavefront and reflected wavefront lie on one plane. This is the first law of reflection.

As $A'B'$ is the secondary wavefront of AB after time t

Hence $AA' = BB' = ct$. C = speed of light in vacuum

Now between triangles ABB' and $AA'B'$ we have

$AA' = BB' = ct$

$AB' =$ Common side

$\angle B = \angle A' = 90^o$

$\therefore \Delta AA'B' \cong \Delta B'BA$

$\Rightarrow \angle AB'A' = \angle B'AB$

$\Rightarrow i = r$

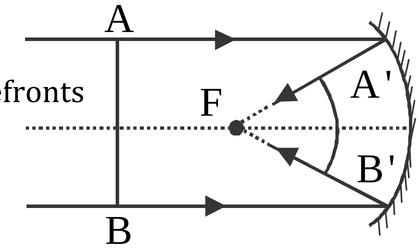
$\Rightarrow$ The angle of incidence = Angle of reflection

Laws of reflection are verified

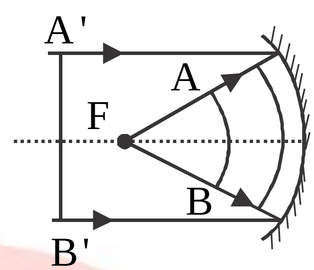
Examples of some reflected wavefronts:-

(i) From the concave mirror:-

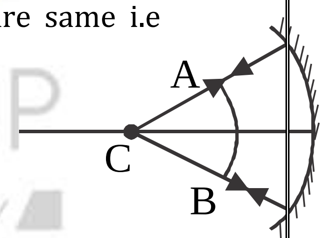
(a) For source at infinity or plane, wavefront reflected wavefronts are spherical with centre at the principal focus



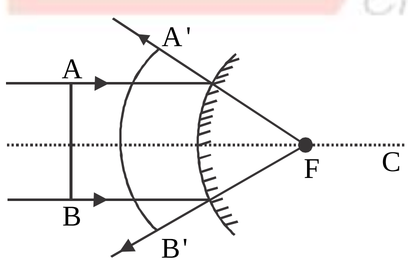
(b) For point object at F, reflected wavefronts are plane wavefronts.



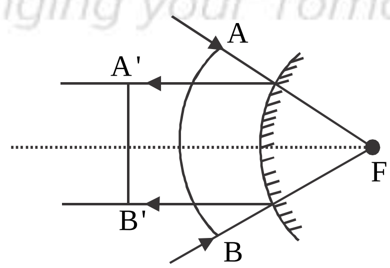
(c) For point object at c, incident wavefronts and reflected wavefronts are same i.e spherical with centre at C



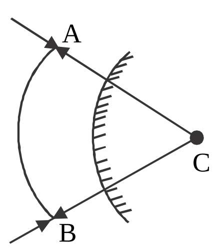
(ii) From convex mirror.



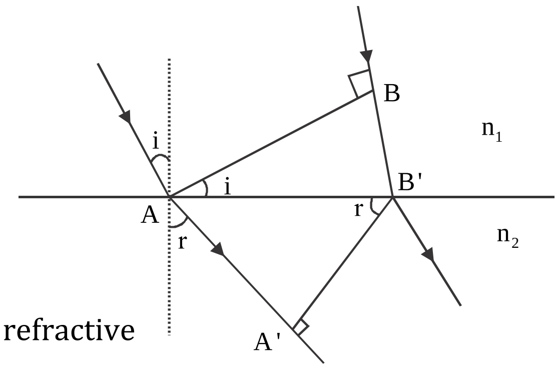
(a)



(b)



Verification of Snell's law:-



(a) For light travelling from denser to rare medium

AB = Incident wavefront in the medium of refractive index n_2

$A'B'$ = the corresponding refracted wavefront of refractive index n_1 after a time t.

Here incident wavefront and its refracted wavefront lie on one plane

$i = \angle BAB' =$ Angle of incidence

$r = \angle AB'A' =$ Angle of refraction

$\therefore BB' = V_2 t$ $\quad$ V_2 = speed of light in medium n_2

$AA' = V_1 t$ $\quad$ V_1 = speed of light in medium n_1

Now in $\Delta ABB', \sin i = \frac{BB'}{AB'}$

In $\Delta AA'B', \sin r = \frac{AA'}{AB'}$

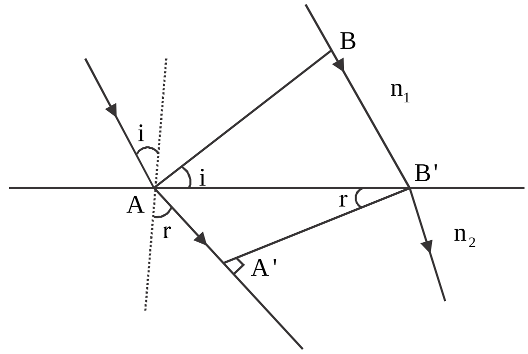
$$\therefore \frac{\sin i}{\text{sir } r} = \frac{BB'/AB'}{AA'/AB'} = \frac{BB'}{AA'} = \frac{V_2 t}{V_1 t}$$

$$\Rightarrow \frac{\sin i}{\sin r} = \frac{V_2}{V_1} = \frac{CV_2}{CV_1}$$ C = speed of light in vacuum

$$\Rightarrow \frac{\sin i}{\sin r} = \frac{C/V_1}{C/V_2} = \frac{n_1}{n_2} = n_{12}$$

$\Rightarrow$ Snell's law is verified

(b) For light travelling from rarer medium to denser need:-



AB = Incident wavefront

$A'B' =$ Corresponding refracted wavefront

$i =$ Angle incidence $= \angle BAB'$

$r =$ The angle of refraction $\angle AB'A'$ light, travelling from rarer medium of refractive index n_1 to denser medium of refractive index n_2.

$\therefore BB' = V_1 t$

$AA' = V_2 t$

In $\Delta ABB', \sin i = \frac{BB'}{AB'}$

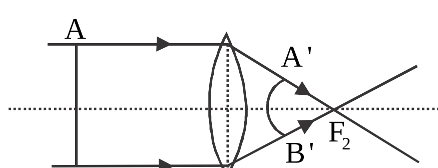
In $\Delta AA'B', \sin r = \frac{AA'}{AB'}$

$$\therefore \frac{\sin i}{\sin r} = \frac{BB'/AB'}{AA'/AB'} = \frac{BB'}{AA'} = \frac{V_1 t}{V_2 t} = \frac{V_1}{V_2} = \frac{CV_1}{CV_2} = \frac{C/V_2}{C/V_1} = \frac{n_2}{n_1} = n_{21}$$

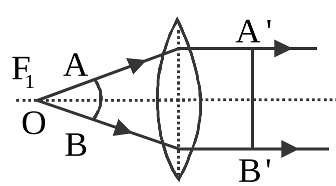
$\therefore$ Snell's law is verified

Examples of some refracted wavefronts:-

(a) Through convex lens

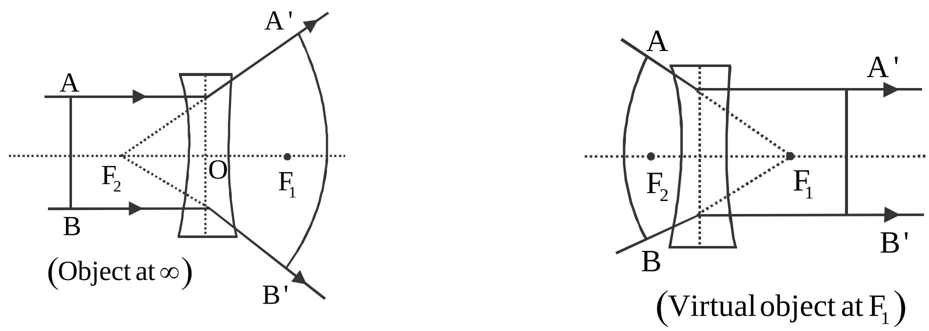


(Object at ∞)

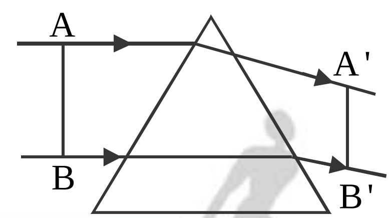


(Object at F_1)

(b) Through concave lens



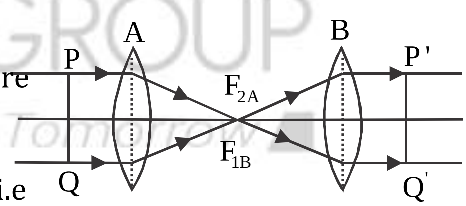
(c) Through a prism



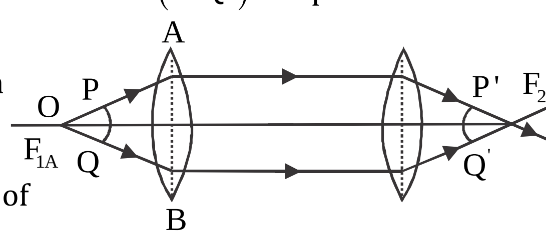
Question No. – 1:- Two convex lenses are held co-axial with the second focus of first lens coinciding with the first focus of the second lens. Draw the refracted wavefronts if. (a) The object is at infinity (b) Point object at the 1st principal focus of 1st lens.

Solution:-

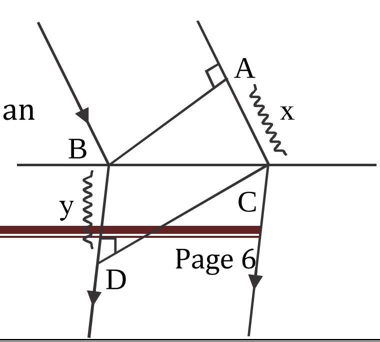
(a) As the object is at infinity incident wavefronts (PQ) are plane wavefronts. Image of A is a point at F_{2A} which is F_{1B} so the image of the second lens will be at ∞ i.e refracted rays from B are parallel rays. So refracted wavefronts $(P'Q')$ are plane.



(b) Here incident wavefronts are spherical with centre at F_{1A}. Refracted rays of A or incident rays of B are parallel to the principal axis. So refracted rays of lens B are meeting at the second principal focus of B. So refracted wavefronts are spherical with centre at F_{2B}



Question No. – 2:- A plane wavefront AB is incident from the air on an interface separating air from a medium. The refracted wavefront in the



medium is CD. If AC = x and BD = y, then represents the refractive index of the medium in term of x and y.

Solution:-

Let t = time in which wavelets from A reaches at C and wavelets from B reaches D

$\therefore$ x = ct, c = speed of light in vacuum or air

Y = vt v = speed light in the medium

$$\therefore \frac{x}{y} = \frac{ct}{vt} = \frac{c}{v} = \mu$$

$\Rightarrow \mu = \frac{x}{y} =$ the refractive index of the medium

Question No. – 3:-

(a) When monochromatic light is incident on a surface separating two media, then the reflected and refracted light both have the same frequency as incident frequency. Explain why?

(b) When light travels from a denser to a rarer medium, the speed decreases. Does this imply a reduction in the energy carried by the light wave?

(c) In the wave picture of light, the intensity of light is determined by the square of the amplitude of the wave. What determines the intensity of light in a photon picture of light?

Solution:-

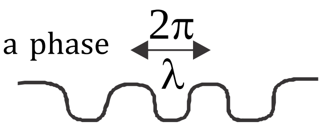
(a) When light propagates through a medium, the atoms of the medium may be viewed as oscillators, which take up the frequency from the external agency (light) causing forced oscillation. The frequency of light emitted by these oscillators equal to the frequency of to oscillation i.e frequency of lightwave incident so reflected and refracted waves have the same frequency as the incident wave.

(b) No energy does not depend upon velocity

(c) In the photon picture, intensity = number of photons per second.

Path difference and phase difference:-

In a sinusoidal wave, the same phrase is repeated after a path λ and a phase 2π



$\therefore$ path difference corresponding to phase difference $2\pi = \lambda$

$\Rightarrow$ Path difference (Δx) corresponding to a phase difference $'\phi' = \frac{\lambda}{2\pi} = \phi$

$\Rightarrow \Delta x = \frac{\lambda}{2\pi}\phi$ Or $\phi = \frac{2\pi}{\lambda}.\Delta x$

This is the relation between phase difference and path difference

Question No.-4:-

(a) What is the path differences corresponding to phase differences (i) $\frac{\pi}{2}$ (ii) π (iii) 2π

(b) What are the phase differences corresponding to path differences (i) $\frac{\pi}{4}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{7\pi}{2}$

Solution:-

(a) (i) $\Delta x = \frac{\pi}{2\pi}, \phi = \frac{\lambda}{2x}.\frac{\pi}{4} = \frac{\pi}{8}$ (ii) $\Delta x = \frac{\pi}{2\pi}, \phi = \frac{\lambda}{2\pi}.2\pi = \frac{\pi}{2}$

(iii) $\Delta x = \frac{\lambda}{2\pi}.\phi = \frac{\pi}{2\pi}.2\pi = \lambda$

(b) (i) $\phi = \frac{2\pi}{\lambda}.\Delta x = \frac{2\pi}{\lambda}.\frac{\lambda}{4} = \frac{\pi}{2}$ (ii) $\phi = \frac{2\pi}{\lambda}.\Delta x = \frac{2\pi}{\lambda}.\frac{\lambda}{3} = \frac{2\pi}{3}$

(iii) $\phi = \frac{2\pi}{\lambda}.\Delta x = \frac{2\pi}{\lambda}.\frac{7\lambda}{2} = 7\pi$

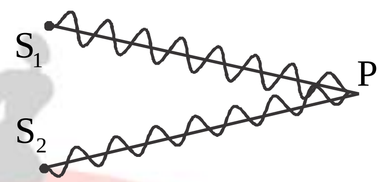
Principle of superposition of waves:-

At a particular point in the medium, the resultant displacement produced by many waves is the vector sum of the displacements produced by each of the waves. i.e $\vec{y} = \vec{y}_1 + \vec{y}_2 + ...$

Superposition of two waves of equal amplitudes coming from two very close point sources:-

Let displacement of P at any n stint due to waves from $s_1, y_1 = a\cos\omega t$ …………………(i)

From $s_2; y_2 = a\cos(\omega t + \phi)$ ……………………………… (ii)



Where ϕ = phase difference between the waves from S_1 and S_2

The resulting displacement of P by the principle of superposition is

$$y = y_1 + y_2$$

$$= a\cos\omega t + a\cos(\omega t + \phi)$$

$$= 2\cos\left(\frac{\omega t + \omega t + \phi}{2}\right)\cos\left(\frac{\omega t - \omega t - \phi}{2}\right)$$

$$= 2a\cos\left(\frac{2\omega t + \phi}{2}\right)\cos\left(\frac{-\phi}{2}\right)$$

$$= y = 2a\cos\frac{\phi}{2}\cos\left(\omega t + \frac{\phi}{2}\right) \text{ ………………………… (iii)} \qquad (\because \cos(-\theta) = \cos\theta)$$

Equation (iii) shows that resulting oscillation is sinusoidal with amplitude, $A = 2a\cos\frac{\phi}{2}$ ……… (iv)

And phase $= \omega t + \frac{\phi}{2}$ i.e $\frac{\phi}{2}$ differing from each wave

Intensity:- As intensity $\propto$ (amplitude)^2

$\therefore$ Individual sources must emit equal intensity say $I_0 \propto a^2$

$\therefore$ Resulting intensity $I \propto A^2$

$\Rightarrow I \propto 4a^2 \cos^2 \frac{\phi}{2}$ $\qquad \Rightarrow I = 4I_0 \cos^2 \frac{\phi}{2}$

This is the resultant intensity at the point where the phase difference is ϕ

Maxima:- Maxima are the points in the medium where intensity is maximum.

As intensity, $I = 4I_0 \cos^2 \phi/2$

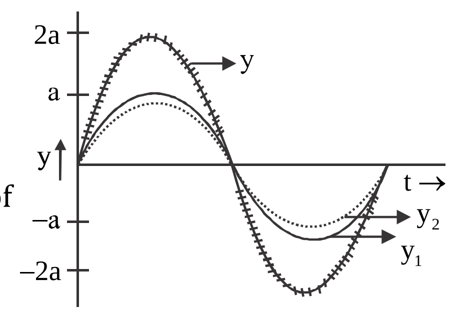
Maximum value of $\cos^2 \frac{\phi}{2} = 1$

$\therefore I_{max} = 4I_0$ $\qquad$ For maximum, $\cos^2 \frac{\phi}{2} = 1 \Rightarrow \cos \frac{\phi}{2} = \pm 1$

$\Rightarrow \frac{\phi}{2} = n\pi \quad n = 0, 1, 2, \ldots\ldots\ldots$

$\Rightarrow \phi = 2n\pi$ $\quad$ phase difference for maxima and $\Delta x = \frac{\lambda}{2\pi} . 2n\pi = n\lambda$

$\Rightarrow \Delta x = 2n . \frac{\lambda}{2}$ $\qquad$ path difference for maxima

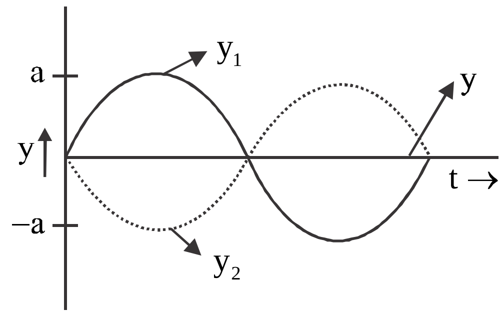


The amplitude at maxima, $\Delta_{max} = 2a \cos \pi = \pm 2a$

This happens when the crest of one wave coincides with crest of the other and trough of one wave coincides with trough of the other. This is called constructive interference.

Minima are the points where intensity is minimum.

As $I = 4I_0 \cos^2 \frac{\phi}{2}$ and the minimum value of $\cos^2 \frac{\phi}{2} = 0$



$\therefore I_{min} = 0$ $\therefore$ for minima, $\cos^2 \frac{\phi}{2} = 0 \Rightarrow \cos\frac{\phi}{2} = 0$

$\Rightarrow \frac{\phi}{2} = (2n-1)\frac{\pi}{2}$ n = 1, 2, 3, ……………..

$\Rightarrow \phi = (2n-1)\pi$ phase difference for minima

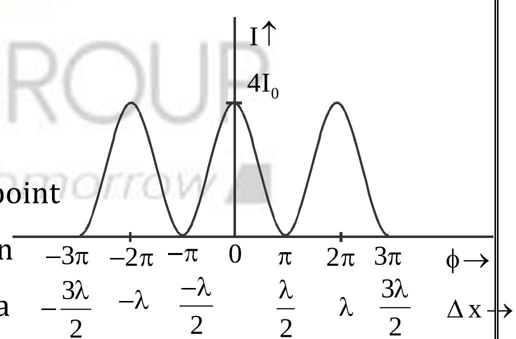
$$\Rightarrow \Delta x = \frac{\pi}{2\pi}.\phi = \frac{\lambda}{2\pi}.(2n-1)\pi$$

$\Rightarrow \Delta x = (2n-1)\frac{\lambda}{2}$ path difference for minima

The amplitude at minima, $\Delta_{min} = 2a\cos(2n-1)\frac{\pi}{2} = 0$

$= a - a$ i.e crest of one wave coincides with trough of the other. This is called destructive interference.

Intensity distribution curve:-



So it is concluded that due to superposition two waves every point of medium is not getting the equal intensity of light. But an alternate maxima and minima pattern is obtained. Such a phenomenon is called as interference and the pattern is an interference pattern.

Interference:- Interference is defined as the phenomenon due to which energy or intensity is redistributed among different points of the medium based on the principle of superposition.

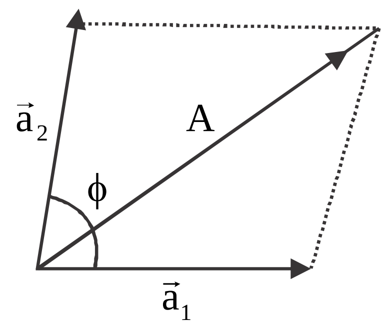
Notes:-

(a) When two close point sources emit monochromatic light of different intensities I_1 and I_2 then particles vibrate at different amplitudes a_1 and a_2 respectively, then resulting

Intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$ Amplitude, $A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\theta}$

This resultant amplitude can be obtained by vector addition method of amplitudes;

By Parallelogram Law:-



$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\theta}$

As $I \propto A^2$

$\Rightarrow I \propto \left(a_1^2 + a_2^2 + 2a_1 a_2 \cos\theta\right)$

Since $I_1 \propto a_1^2$ and $I_2 \propto a_2^2$

Combining, $I_1 I_2 \propto a_1^2 a_2^2$

$\Rightarrow \sqrt{I_1 I_2} \propto a_1 a_2$ $\therefore I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$

For constructive interference, $\Delta_{max} = a_1 + a_2$ $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

The destructive interference, $A_{min} = a_1 - a_2$ $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

The term $2\sqrt{I_1 I_2}\cos\theta$ is the interference term. If there were no interference, then $I = I_1 + I_2$

(b) For waves sending equal amplitudes or intensities, the resulting intensity is $I = 4I_0 \cos^2 \phi / 2$

Amplitude is $A = 2a\cos\phi/2$. If there were no interference, $I = 2I_0$

Question No. – 05:-

(a) The ratio of maximum and minimum intensities in an interference pattern is 225.1. Find the ratio between the intensities of interfering waves.

(b) The ratio between the intensities of interfering waves is 81:49. Find the ratio between the maximum and minimum intensities in the interference pattern.

Solution:-

(a) Given $\frac{I_{max}}{I_{min}} = \frac{225}{1}$

$$= \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \frac{225}{1} \qquad \Rightarrow \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = \frac{15}{1}$$

$$\Rightarrow \sqrt{I_1}+\sqrt{I_2} = 15\sqrt{I_1} - 15\sqrt{I_2} \qquad \Rightarrow -14\sqrt{I_1} = -16\sqrt{I_2}$$

$$\Rightarrow 196\,I_1 = 256\,I_2 \qquad \Rightarrow \frac{I_1}{I_2} = \frac{256}{196} = \frac{64}{49}$$

(b) Given $\frac{I_1}{I_2} = \frac{81}{49}$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \frac{\left(\sqrt{81}+\sqrt{49}\right)^2}{\left(\sqrt{81}-\sqrt{49}\right)^2} = \frac{(9+7)^2}{(9-7)^2} = \frac{256}{4} = 64:1$$

Question No. – 06:-

(a) Two sources emitting intensities I_0 and $4I_0$ produce an intensity $7I_0$ at a point. What is the phase difference between the waves at the point?

(b) Two sources produce equal amplitude 'a'. At a point in the medium waves suffer a path difference of $\frac{\lambda}{3}$. What are the resulting amplitude and intensity at the point?

Solution:-

(a) As $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$

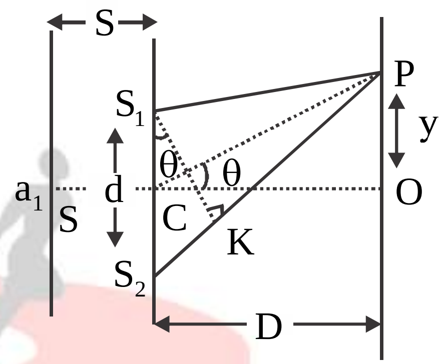
$$\Rightarrow 7I_0 = I_0 + 4I_0 + 2\sqrt{I_0.4I_0}\cos\theta$$

$$\Rightarrow 2I_0 = 4I_0\cos\phi \Rightarrow \cos\phi = \frac{2I_0}{4I_0} = \frac{1}{2} \qquad \Rightarrow \phi = \frac{\pi}{3}$$

(b) $A = 2a\cos\phi/2 = 2a\cos\left\{\frac{1}{2}.\frac{2\pi}{\lambda}\Delta x\right\} = 2a\cos\left\{\frac{1}{2}.\frac{2\pi}{\lambda}.\frac{\lambda}{3}\right\} = 2a\cos\frac{\pi}{3} = a$

$$\Rightarrow I = kA^2 = ka^2 = I_0$$

<u>Young's double-slit experiment:</u>



Experimental Arrangement:-

Monochromatic light passing through a narrow slit 'S' is allowed to fall on two slits S_1 and S_2 lying in a plane symmetrically to S.

S_1 and S_2 are very close to $S_1S_2 = d$

A screen is placed parallel to the plane of slits at a distance D. D >> d.

As S_1 and S_2 are at equal distances from S then they lie on same wavefront coming from S.

According to Huygen's principle, S_1 and S_2 send secondary wavelets at the same phase. For the region next to the plane of slits up to screen, the slits S_1 and S_2 are the sources. As they are vibrating in the same phase so they behave as coherent sources.

Now wavefronts from S_1 and S_2 superimpose with each other and interference pattern is formed on the screen.

Central Fringe:-

O is the point on the screen symmetrical w.r.t the slits

$\therefore$ Path difference between waves from S_1 and S_2 at O = 0

This satisfies the condition for maxima

So we get a maximum bright fringe at O. This is called a central bright fringe.

O is chosen as a reference point on the screen.

i.e all the positions on the screen are represented w.r.t O.

All angular positions of points on the screen are measured w.r.t to the line co.

Where C = midpoint of S_1S_2.

e.g. A point on the screen is P

Its position = y = OP

Its angular position = $\theta = \angle OCP$

The expression for path difference:-

Path difference between waves from S_1 and S_2 at any point P on the screen is.

$$\Delta x = S_2P - S_1P = S_2K = (S_1S_2)\sin\theta = d\sin\theta$$

$$\Rightarrow \frac{\Delta x}{d} = \sin\theta \text{ .... (i)}$$

Again in $\Delta COP, \tan\theta = \frac{OP}{OC} = \frac{y}{D}$ .... (ii)

$$\therefore d << D \Rightarrow \theta \to 0$$

$$\Rightarrow \sin\theta \to \theta \text{ and } \tan\theta \to \theta$$

$$\Rightarrow \frac{\Delta x}{d} = \frac{y}{D}$$

$$\Rightarrow \Delta x = \frac{yd}{D} \text{ .... (iii)}$$

The expression for intensity at any point:-

(a) if each slit is sending equal intensity I_1 9say), then at any point P on-screen intensity is

$$I = 4I_1 \cos^2 \phi/2$$

$$\Rightarrow I = 4I_1 \cos^2\left(\frac{2\pi}{\lambda}.\frac{\Delta x}{2}\right) = 4I_1 \cos^2 \frac{\pi \Delta x}{\lambda} = 4I_1 \cos^2\left(\frac{\pi y d}{\lambda D}\right)$$

At point O i.e central maxima intensity must be $I_0 = 4I_1$ at minima, $I = 0$

$\therefore$ At any point, $I = I_0 \cos^2 \phi/2$

(b) If each slit is not sending same intensity i.e $I_1 \neq I_2$ then intensity at any point is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

At maxima, $I_0 = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

At minima, $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

Positions for bright fringes and being fringe width:-

Let nth bright fringe be at P with position y_n w.r.t 0 and angular position θ_n w.r.t line CO

Now path difference, $S_2K = d(\sin\theta_n)$ or Δx

As $\theta_n \to 0 \Rightarrow \sin\theta_n \to \theta_n \to \tan\theta_n = \frac{y_n}{D}$

$$\therefore \Delta x = d.\theta_n = \frac{dy_n}{D}$$

As for nth maxima $\Delta x = n\lambda$

$$\Rightarrow \frac{d.y_n}{D} = n\lambda$$

$\Rightarrow y_n = \frac{n\lambda D}{d}$ n = 1, 2, 3, ………………..

$\Rightarrow \theta_n = \frac{y_n}{D} = \frac{n\lambda}{d}$ ……………………… (iv)

Bright fringe width (β) is the separation between two consecutive bright fringes

$\beta = y_{n+1} - y_n = (n+1)\frac{\lambda D}{d} - \frac{n\lambda D}{d}$

$\Rightarrow \beta = \frac{\lambda D}{d}$ ……………………… (v) Expression for bright fringe width

$\therefore$ Angular fringe width, $\Delta\theta = \frac{\beta}{D} = \frac{\lambda}{d}$ …………………………… (vi)

Positions for dark fringes and dark fringe width:-

Let nth dark fringe be at P with position $y_n^{'}$ and angular position $E_n^{'}$

$\therefore$ Path difference, $\Delta x = S_2k = d\left(\sin\theta_n^{'}\right)$

As $d << D \Rightarrow \theta_n^{'} \rightarrow \text{and} \sin\theta_n^{'} \rightarrow \theta_n^{'} \rightarrow \tan\theta_n^{'} = \frac{y_n^{'}}{D}$

$\therefore \Delta x = d.\frac{y_n^{'}}{D}$

For nth dark fringe, $\Delta x = (2n-1)\frac{\lambda}{2}$ n = 1, 2,3, ……………

$\Rightarrow \frac{d.y_n^{'}}{D} = (2n-1)\frac{\pi}{2}$

$\Rightarrow y_n^{'} = (2n-1)\frac{\lambda D}{2d}$

$$\Rightarrow \theta_n' = \frac{y_n'}{D} = (2n-1)\frac{\lambda}{2d} \text{ .... (vii)}$$

Dark fringe width (β') is the separation between two consecutive dark fringes.

i.e $\beta' = y_{n+1}' - y_n'$

$$= \{2(n+1)-1\}\frac{\lambda D}{2d} - (2n-1)\frac{\lambda D}{2d}$$

$$= \frac{\lambda D}{2d}[2n+2-1 \quad -2n+1]$$

$$= \frac{\lambda D}{2d}.2 = \frac{\lambda D}{d}$$

$$\Rightarrow \beta' = \frac{\lambda D}{d} \text{ .... (viii)}$$

Angular dark fringe width, $\Delta\theta' = \frac{\beta'}{D} = \frac{\lambda}{d}$ .... (ix)

Now from equations (v) and (viii) we have,

Fringe width; $\beta = \frac{\lambda D}{d}$

$$\Rightarrow \beta \propto \lambda, \beta \propto D \text{ and } \beta \propto \frac{1}{d}$$

Now from equations (vi) and (ix) we have angular fringe width $\Delta\theta = \frac{\lambda}{d}$

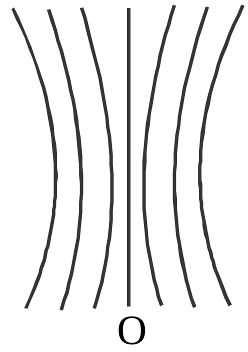
$\Rightarrow \Delta\theta \propto \lambda, \Delta\theta \propto \frac{1}{d}$ and $\Delta\theta$ is independent of D.

The shape of the fringes:-

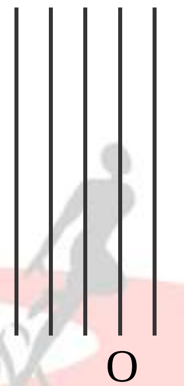
(a) Fringe shapes are hyperbola

(b) If D is very large in comparison to β the fringes appear like straight lines

Fringe shape for a set up with d = 0.005mm, D = 5cm and $\lambda = 5\times10^{-5}\text{cm} \Rightarrow \beta = 5\text{mm}$



Fringe shape for a set up with d = 0.025mm, D = 5cm and $\lambda = 5\times10^{-5}\text{cm} \Rightarrow \beta = 1\text{mm}$



Question No. – 7:- Two slits are made 1mm apart and the screen is placed 1m away in YDSE. When blue-green light of wavelength 500 nm is used, find

(a) Fringe separation

(b) Angular fringe width

(c) Position of 5th bright fringe

(d) Angular position of 9th dark fringe

Solution:-

(a) $\beta = \frac{\lambda D}{d} = \frac{500\text{nm}\times 1\text{m}}{1\text{mm}} = 5\times10^{-4}\text{m} = 0.5\text{mm}$

(b) $\Delta\theta = \frac{\lambda}{d} = \frac{500\text{nm}}{1\text{mm}} = 5\times10^{-4}\text{rad}$

(c) $y_5 = \frac{5\lambda D}{d} = 5\beta = 5\times 0.5\text{mm} = 2.5\text{mm}$

(d) $\Delta\theta_0' = (2\times9-1)\frac{\pi}{2d} = \frac{17}{2}.\Delta\theta = \frac{85}{2}\times10^{-4}\text{rad}$

Factors affecting the fringes in YDSE:-

(a) Slit separation (d):-

As $\beta \propto \frac{1}{d}$

$\Rightarrow$ Fringe width decreases with increase in slit separation

(b) The distance of screen (d):-

As $\beta \propto D$

$\Rightarrow$ Fringe width increases with an increase in distance of the screen

(c) The wavelength of light (λ) :-

As $\beta \propto \lambda$

$\Rightarrow$ Fringe width increases with an increase in wavelength

(d) The medium between slit and screen:-

As a medium between the slits and screen changes then $\lambda \to \frac{\lambda}{\mu}$

Where $\mu =$ the refractive index of the medium $\therefore \beta' = \frac{\beta}{\mu}$

(e) Width of source slit (a) and distance of source slit from slit plane (s):-

Fringe width is not affected by the width of source slit (a) and distance of source slit from slit plane (s). But the condition for fringes to be produced is $\frac{a}{s} \le \frac{\lambda}{d}$

If the source slit width is reduced or distance (s) of source slit from the slit plane is increased, then a/s gets reduced

$\Rightarrow$ Fringes are formed

But the intensity of fringes get reduced

If the source slit width (a) is increased or distance (s) of source slit is reduced then the intensity of fringes increase. Along with a/s also increases

$\therefore$ The condition $\frac{a}{s} \le \frac{\lambda}{d}$ may be violated

After $\frac{a}{s} > \frac{\lambda}{d}$ no fringes are formed. Till $\frac{a}{s} \le \frac{\lambda}{d}$ there are fringes

(f) By changing the monochromatic source by white light:-

At control fringe, all the colours have phase difference equal to 0. So all colours have maxima there. Hence central fringe is a bright white spot.

As minima position is $(2n-1)\frac{\lambda D}{2d}$, so different colours have different minima positions.

A point where a particular colour has minima, then that colour is absent there and the point appears coloured.

$\therefore$ coloured brings are obtained around the central white spot.

Question No. – 9:- In the double-slit experiment the angular width of fringe is found to be 0.2° on screen at 1m away. The wavelength of light is 600 nm. What will be the angular width of fringe if

(a) The whole setup is dipped in water (of refractive index 4/3)

(b) The source is replaced by another of wavelength 400 nm?

(c) The screen is brought to 2m distance? (NCERT)

Solution:- $\Delta\theta = \frac{\lambda}{d}$

(a) As dipped in water $\lambda \to \frac{\lambda}{\mu}$

$$\therefore \Delta\theta' = \frac{\lambda/\mu}{d} = \frac{\Delta\theta}{\mu} = \frac{0.2^o}{4/3} = 0.15^o$$

(b) $\therefore \Delta\theta' = \frac{\lambda'}{d} = \frac{\lambda'}{\lambda}.\frac{\lambda}{d} = \frac{\lambda'}{\lambda}.\Delta\theta$

$$= \frac{400nm}{600nm} \times 0.2^o = 0.13^o$$

(c) $\Delta\theta$ is not dependent on D

$$\therefore \Delta\theta' = \Delta\theta = 0.2^o$$

Question No – 10:- In YDSE set up with monochromatic light source if the screen is moved by 5cm towards the slit, the change in fringe width is $3\times10^{-5}m$. If slit separation is $3\times10^{-5}m$, calculate the length used.

Solution:- As $\beta = \frac{\lambda D}{d}$

$$\Rightarrow \Delta\beta = \frac{\lambda \Delta D}{d} \Rightarrow 3\times10^{-5}m = \frac{\lambda \times 5\times10^{-2}m}{10^{-3}m}$$

$$\Rightarrow \lambda = 6\times10^{-7}m = 600nm$$

Question No. – 11:- In YDSE set up maxima is obtained exact opposite to a slit. Obtain the order of slit in term of λ, D, d where symbols have their usual meaning

Solution:- Let nth order maxima are obtained exact opposite to a slit.

$$\therefore y_n = \frac{d}{2} \Rightarrow \frac{n\lambda D}{d} = \frac{d}{2}$$

$$\Rightarrow n = \frac{d^2}{2\lambda D}$$

Question No. – 12:- In YDSE, the intensity at a point is K unit, where path difference between waves is λ. What is the intensity at a point where (a) Path difference is $\frac{\lambda}{3}$ (b) Phase difference is $\frac{\pi}{3}$

Solution:- As $\Delta x = \lambda \Rightarrow \phi = \frac{2\pi}{\lambda} \times \lambda = 2\pi$

$$\therefore I = 4I_1 \cos^2 \frac{2\pi}{2} \Rightarrow K = 4I_1 \cos^2 \pi = 4I_1$$

$$\Rightarrow I_1 = \frac{k}{4}$$

(a) $\Delta x = \frac{\lambda}{3} \Rightarrow \phi = \frac{2\pi}{\lambda} . \frac{\lambda}{3} = \frac{2\pi}{3}$

$$\therefore I = 4I_1 \cos^2 \frac{2\pi/3}{2} = 4.\frac{K}{4}.\cos^2 \frac{\pi}{3} = K.\frac{1}{4} = \frac{K}{4}$$

(b) $\phi = \frac{\pi}{3} \Rightarrow 4I_1 \cos^2 \frac{\pi}{6} = 4 \times \frac{K}{4} . \frac{3}{4} = \frac{3k}{4}$

Question No. -13:- In YDSE set up, a beam of light consisting of two wavelengths 650 nm and 520 nm is used with D = 1m, d = 1mm. Find the least distance from the central maximum when bright fringe due to both wavelengths coincide? (NCERT)

Solution:- Let nth bright fringe of 650 nm wave coincides with the nth bright fringe of 520 nm wave

$$\Rightarrow \frac{n \times 650\text{nm} \times D}{d} = \frac{m \times 520\text{nm} \times D}{d}$$

$\Rightarrow \frac{n}{m} = \frac{520}{650} = \frac{A}{5}$

$\Rightarrow n = 4x \text{ and } m = 5x$

For least distance n = 4, m = 5

$\therefore$ The position = $\frac{4 \times 650\text{nm} \times D}{d} = \frac{4 \times 650 \times 10^{-9} \times 1}{10^{-3}} m = 2.6 \times 10^{-3} m$

Question No. – 14:- In YDSE set up, a beam of white light is used. Which wavelengths are found missing at a point just opposite to a slit? Obtain the expression in term of slit separation 'b' and screen distance'.

Solution:- A wavelength is missing mean, it has a minima

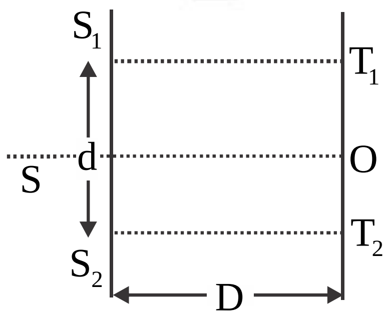
$\therefore y_n^1 = \frac{b}{2}$

$\Rightarrow (2n-1)\frac{\lambda d}{2b} = \frac{b}{2}$

$\Rightarrow \lambda = \frac{b^2}{(2n-1)d}$ n = 1, 2, 3, ...

$\therefore$ Wavelengths $\frac{b^2}{d}, \frac{b^2}{3d}, \frac{b^2}{5d}$ are missing

Question No. – 15:- In the given YDSE set up $D = \frac{d}{2}$. Obtain the expression for D in term of λ, such that 1st minima on the screen falls at distance D from centre O. (NCERT example)



Solution:- As $D = \frac{d}{2}$

$\Rightarrow$ At a distance D from O means point opposite to a slit.

As for 1st minima, $\Delta x = \frac{\lambda}{2}$

$$\Rightarrow S_2T_1 - S_1T_1 = \frac{\lambda}{2}$$

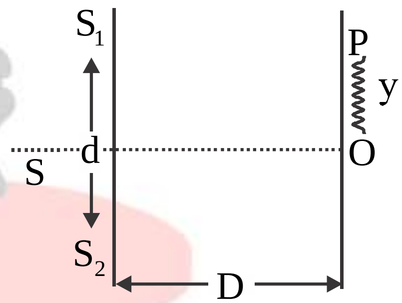
$$\Rightarrow \sqrt{S_2T_2^2 + T_2T_1^2} - S_1T_1 = \frac{\pi}{2}$$

$$\Rightarrow \sqrt{D^2 + (2D)^2} - D = \frac{\lambda}{2}$$

$$\Rightarrow \left(\sqrt{5} - 1\right)D = \frac{\lambda}{2}$$

$$\Rightarrow D = \frac{\lambda}{2\left(\sqrt{5} - 1\right)}$$

Question No. – 16:- In YDSE set up shown, $SS_2 - SS_1 = \frac{\lambda}{4}$ and D >> d



(a) State the condition for constructive and destructive interference

(b) Obtain an expression for fringe width

(c) Local the position of central fringe.

solution:-

(a) $\Delta x = (SS_2 + S_2P) - (SS_1 + S_1P) = (SS_2 - SS_1) + (S_2P - S_1P) = \frac{\lambda}{4} + \frac{yd}{D}$

For constructive interference, $\Delta x = n\lambda$

$$\Rightarrow \frac{\lambda}{4} + \frac{y_n d}{D} = n\lambda$$

$$\Rightarrow y_n = \left(n\lambda - \frac{\lambda}{4}\right)\frac{D}{d} = (4n - 1)\frac{\lambda D}{4d}$$

For destructive interference, $\Delta x = (2n - 1)\frac{\pi}{2}$

$$\Rightarrow \frac{\lambda}{4} + \frac{y_n' d}{D} = (2n - 1)\frac{\lambda}{2}$$

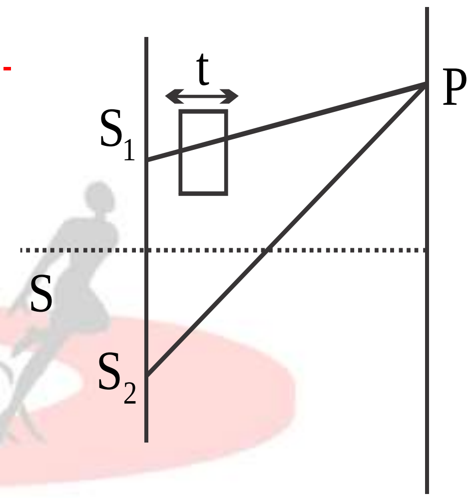
$$\Rightarrow y_n^1 = (4n-2-1)\frac{\lambda D}{4d} = (4n-3)\frac{\lambda D}{4d}$$

(b) $\beta = y_{n+1} - y_n = \left[\{4(n+1)-1\} - (4n-1)\right]\frac{\lambda D}{4d}$

$$= (4n+3-4n+1)\frac{\lambda D}{4d} \qquad = 4\frac{\lambda D}{4d} = \frac{\lambda D}{d}$$

(c) Position for central maxima, $y_0 = (4\times 0-1)\frac{\lambda D}{4d} = \frac{-\lambda D}{4d}$ i.e $\frac{\lambda D}{4d}$ below O

Fringe shift by keeping a transparent slab in front of a slit:-



Now $\Delta x' = S_2P - \left[S_1P + (\mu-1)t\right]$

$= (S_2P - S_1P) - (\mu-1)t$

$= \Delta x - (\mu-1)t$

(As t width through the slabs is equivalent to μt

distance in a vacuum, which is called an optical path)

For path different Δx, a particular fringe is at 'y'

As $\Delta x = \frac{yd}{D}$

$\Rightarrow y = \frac{D}{d}(\Delta x)$

Now the new position of the fringe is

$y^1 = \frac{D}{d}\Delta x' = \frac{D}{d}\Delta x - (\mu-1)t.\frac{D}{d}$

$\therefore$ Fringe shift, $\Delta y = y - y' = (\mu - 1) t . \frac{D}{d}$

$$\beta = \frac{\lambda D}{d}$$

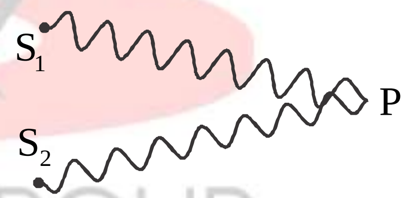
$$\Rightarrow \frac{\beta}{\lambda} = \frac{D}{d}$$

$$\Rightarrow y = (\mu - 1) \frac{t.\beta}{\lambda}$$

Coherent Sources:-

Two sources emitting continuous light waves of the same frequency and wavelength are said to be coherent if the sources are at the same phase or have a constant phase difference.

Suppose two sources S_1 and S_2 are emitting monochromatic light waves.



Let $\phi_1 =$ Phase difference between the vibration of S_1 and S_2

$\phi_2 =$ The phase difference between waves from S_1 and S_2 at P i.e phase difference due to path difference $S_2P - S_1P$ or Δx

$$\Rightarrow \phi_2 = \frac{2\pi}{\lambda} . \Delta x$$

$\therefore$ Now the total phase difference at P is $\phi = \phi_1 + \phi_2$

If ϕ_1 = constant or zero then sources are coherent.

Coherent addition of two waves:-

(a) if two sources send the light of same intensity I_0 then resulting intensity is

$$I = 4I_0 \cos^2 \frac{\phi}{2}$$

Where $\phi = \phi_1 + \phi_2 =$ time-independent or constant for a particular point in a medium i.e for each point there exist one ϕ and hence one I.

(b) If two sources send the light of different intensities I_1 and I_2, then $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Incoherent addition:-

Sources are incoherent t they vibrate at different phases and phase different varies from time to time.

i.e ϕ (in the above case) changes with time

$\Rightarrow \phi = \phi_1 + \phi_2$ is time-dependent

Resulting intensity

(a) if both sources send same intensity I_0

Then $I = 4I_0\langle\cos^2\frac{\phi}{2}\rangle \quad = 4I_0 . \frac{1}{2} = 2I_0$

i.e there is no interference and intensities are added algebraically

As ϕ is changing with time

$\langle\cos^2\phi/2\rangle$ = average of $\cos^2\phi/2$ overtime = $\frac{1}{2}$

(b) If both sources send unequal intensity I_1

Then $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle\cos\phi\rangle$

$\Rightarrow I = I_1 + I_2 \qquad$ As $\langle\cos\phi\rangle = 0$

$\Rightarrow I = I_1 + I_2$

i.e intensities are added algebraically and hence no interference.

Two independent sources of light can't be coherent:-

Light waves are emitted from every atom of a source when the atoms come back to its ground state from an excited state. As even a small source contains billions of atom, so they never can emit waves at the same phase or constant phase difference. So two independent waves can't be coherent.

Coherent sources are obtained from a single source of light:-

Generally by two methods

(a) Division of wavefronts:- For example in Young's double-slit experiment, Lloyd's mirror experiment, Fresnel's biprism experiment

(b) Division of amplitude:- e.g in thin films like soap film, in Newton's ring and Michelson's interferometer.

Conditions for obtaining two coherent sources of light:-

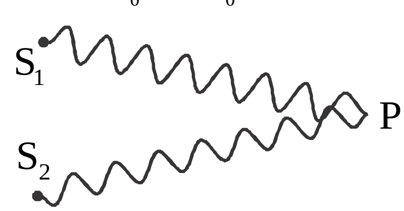
(a) Two sources must be obtained from a single source in such a way that any phase change of one source must be accompanied by a phase change of the other source. For this, we can take either.

(i) Source and its virtual image or (ii) two virtual images of the same source (iii) Two real images of the same source for two coherent sources

(b) Two sources should give monochromatic light

(c) Path difference between light waves from two sources should be small

Question No. – 17:- Two sources emit monochromatic lights of intensities I_0 and $4I_0$. What is the resulting intensity at point P if $S_2P - S_1P = \frac{\lambda}{3}$.



(a) Two sources are coherent with the same phase.

(b) Two sources are coherent with a constant phase difference of $\frac{\pi}{2}$

(c) Tow sources are incoherent

Solution:-

(a) As $S_2P - S_1P = \frac{\lambda}{3} \Rightarrow \phi_2 = \frac{2\pi}{\lambda}.\frac{\lambda}{3} = \frac{2\pi}{3}$

As S_1 and S_2 are in the same phase, $\phi_1 = 0$

$$\therefore \phi = \phi_1 + \phi_2 = \frac{2\pi}{3}$$

$$\therefore I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi = I_0 + 4I_0 + 2\sqrt{I_0.4I_0}\cos\frac{2\pi}{3}$$

$$= 5I_0 + 2 \times 2I_0 \times \left(-\frac{1}{2}\right) = 3I_0$$

(b) $\phi_1 = \frac{\pi}{2}$ and $\phi_2 = \frac{2\pi}{3}$

$$\therefore \phi = \phi_1 + \phi_2 = \frac{\pi}{2} + \frac{2\pi}{3} = \frac{3\pi + 4\pi}{6} = \frac{7\pi}{6}$$

$$\therefore I = I_0 + 4I_0 + 2\sqrt{I_0.4I_0}\cos\frac{7\pi}{6}$$

$$= 5I_0 + 2 \times 2I_0.\left(-\frac{\sqrt{3}}{3}\right) = \left(5 - 2\sqrt{3}\right)I_0$$

(c) For incoherent sources $I = I_1 + I_2 = I_0 + 4I_0 = 5I_0$

Question No. – 18:- n identical sources emitting intensity I_0 each is used simultaneously.

(a) What is the maximum intensity if all the sources are coherent?

(b) What is the resulting intensity if all the sources are incoherent?

Solution:-

(a) At maximum $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2} + ...\sqrt{I_n}\right)^2$

$$= \left(\sqrt{I_0} + \sqrt{I_0} + ...n\text{ times}\right)^2 = \left(n\sqrt{I_0}\right)^2 = n^2 I_0$$

(b) If sources are incoherent $I = I_1 + I_2 + ... + I_n = nI_0$

Conditions of r sustained interference:-

(a) Sources must be coherent

(b) Sources must be monochromatic

(c) Sources must be very close to each other

(d) Sources should emit light of same intensity

(e) Sources should be narrow or point sources

Energy conservation in interference:-

Let tow coherent sources emit intensities I_0 each. If there were no interference, at any point, $I = 2I_0$

If there is interference, at any point $I = 4I_0 \cos^2 \phi / 2$

As a point to point ϕ is changing so average intensity at any point is

$$I_{av} = 4I_0 \langle \cos^2 \theta / 2 \rangle = 4I_0 \times \frac{1}{2} = 2I_0 \left(\therefore \langle \cos^2 \frac{\theta}{2} \rangle = \frac{1}{2} \right)$$

So interference energy is conserved but is redistributed among points of the medium

Diffraction:-

Diffraction of light is the phenomenon of deviation of light from its rectilinear propagation or bending of light rays from sharp edges of an opaque obstacle/ aperture and spreading into geometrical shadow region.

Diffraction depends on two factors

(a) Size of obstacle or aperture (a)

(b) Wavelength of light (λ)

Condition for diffraction; $\frac{a}{\lambda} \approx 1$

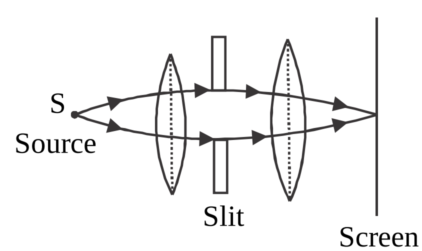
Practically diffraction will not occur if $a > 50\lambda$.

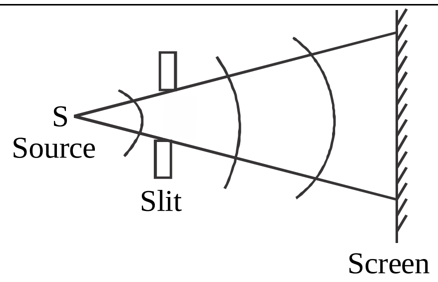
Sound wave shows more diffraction in comparison to light waves.

As for light waves λ is of the order of 10^{-7} m. Obstacles or apertures comparable to 10^{-7} m are very rarely present. So diffraction of the light wave is not observed in our daily life.

But for sound waves λ is of order 16mm to 16m. Obstacles or apertures of such size are practically occurring. So sound waves show more diffraction in our daily life.

Types of diffraction:-

Fresnel's diffraction	Fraunhofer diffraction
In this source and screen, both are at a finite distance from the diffracting device.	In this source and screen are effectively at ∞ distance from the diffracting device. 



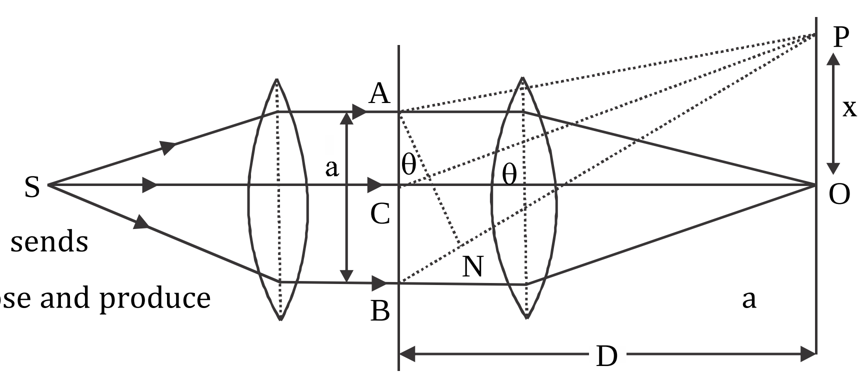
Comparison between Frsnel's and Fraunhofer diffraction:-

Fresnel's diffraction	Fraunhofer diffraction
(a) Source and screen are at a finite distance from the diffracting device	(a) Source and screen are at infinite distance from the diffraction device.
(b) Incident and diffracted wavefronts are spherical	(b) Incident and diffracted wavefronts are planes.
(c) Mirror or lenses are not used	(c) Lenses are used
(d) Centre of the diffraction pattern is sometimes bight and some limes dark	(d) Centre of diffraction is always bright
(e) The intensity of waves from different zones of the slit are unequal	(e) The intensity of waves from different zones is the same.

Diffraction due to single slit:-

AB = a single slit of width 'a'

Plane wavefronts strike the slit.



According to Huygen's principle, every point on the wavefronts AB sends secondary wavelets. The superimpose and produce diffraction pattern.

Path difference between waves from the edges A and B of the slit in a direction θ is

$\Delta x = BN = a\sin\theta$

Central Maximum point O:- O is a point on the screen at which waves coming from each half of wavefront AB suffer equal path. So path difference is zero and hence a maximum is produced at O called as central maxima. Intensity at central maxima is maximum say I_0.

Position of minima:- When path difference between the waves from the edges A and B is even multiple of $\frac{\lambda}{2}$, we get minima

$\therefore$ The angular position of nth minima = θ_n

$\Rightarrow a\sin\theta_n = 2n.\frac{\lambda}{2}$ where n = 1, 2, 3, ...

If its position on screen w.r.t O is x_n then

$\tan\theta_n = \frac{x_n}{D}$

If $\theta_n \to 0 \Rightarrow \sin\theta_n \to \theta_n$ and $\tan\theta_n \to \theta_n$

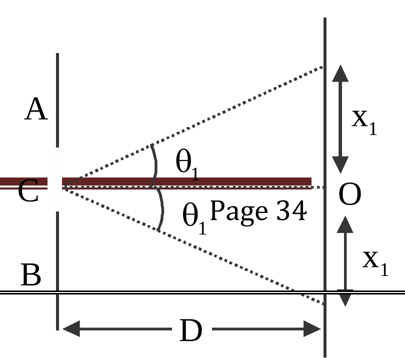
$\therefore a\theta_n = n\lambda \Rightarrow \theta_n = \frac{n\lambda}{a} = \frac{x_n}{D}$

$\Rightarrow x_n = \frac{n\lambda D}{a}$

For 1st minima, $a\sin\theta_1 = \lambda$ and $\tan\theta_1 = \frac{x_1}{D}$

If $\theta_1 \to 0 \Rightarrow \theta_1 = \frac{\lambda}{a} = \frac{x_1}{D}$

Width of Central maxima:-

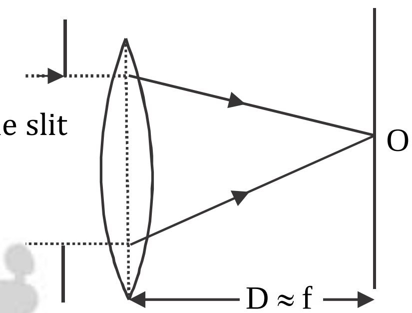


The separation between tow first minima in two opposite sides of central maximum point 0 is called the width of central maxima.

If θ_1 = Angular position of 1^{st} minima

$\Rightarrow$ The angular width of central maxima is $\omega_0 = 2\theta_1 = \frac{2\pi}{a}$ for $\theta_1 \to 0$

If x_1 = Position of 1^{st} minima w.r.t, 0 is , $\omega_x = 2x_1 = \frac{2\lambda D}{a}$



If the lens used to focus light on the screen is placed very close to the slit then D = f i.e focal length of the lens.

$\Rightarrow \omega_x = \frac{2\lambda f}{a}$ and $\omega_0 = \frac{2\lambda}{a}$

Position of secondary maxima:-

If secondary maxima are obtained in the direction θ_n^1 then $a\sin\theta_n^1 = (2n+1)\frac{\lambda}{2}$ n = 1, 2, 3.....

For $\theta_n^1 \to 0 \Rightarrow \theta_n^1 = (2n+1)\frac{\lambda}{2a}$

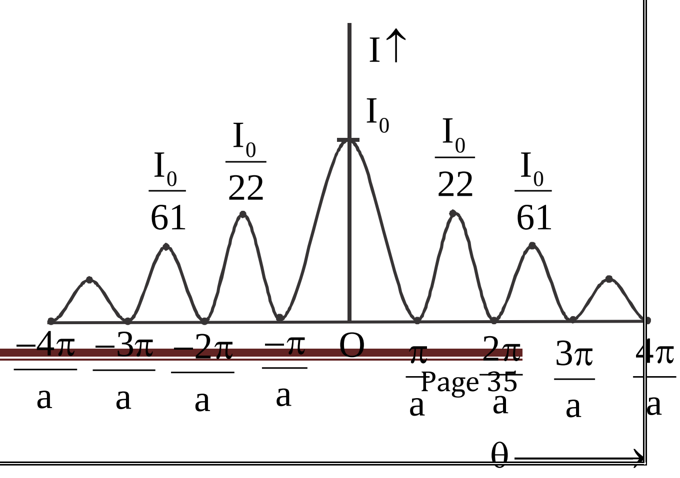
$\Rightarrow x_n^1 = D.\theta_n^1 = (2n+1)\frac{\lambda D}{2a}$

Fringe width is the separation between two consecutive secondary maxima or two consecutive minima's in either side of central maxima

$\therefore \beta = x_{n+1} - x_n = (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a} = \frac{1}{2}$ of ω_x

$= \frac{1}{2}$ of the width of central maxima

Intensity distribution curve:-



The intensity of the maxima point is given by the relation.

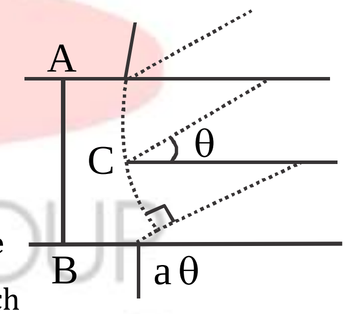
$$I = \left[\frac{2}{(2n+1)\pi}\right]^2 I_0$$

I_0 = Intensity at central maxima

$\therefore$ The intensity of secondary maxima are $\frac{I_0}{22}, \frac{I_0}{61}, \ldots\ldots\ldots\ldots$

Question No – 19:- For single slit of width 'a' the 1st minimum is obtained at an angle of $\frac{\lambda}{a}$, where at the same angle $\frac{\lambda}{a}$ we get a maximum for two narrow slits separated by a distance 'a'. Explain.

Solution:-



(a) For a single slit, at an angle of $\frac{\lambda}{a}$, path difference between waves from two edges = $a\theta = \lambda$

Here intensity at any point is due to the contribution of all points of the wavefront AB. If we imagine two parts of the wavefront then for each point of upper half i.e AC there exist a point on lower half BC at a distance a/2.

$\Rightarrow$ Waves from these two points have path difference = $\frac{a\theta}{2} = \frac{\lambda}{2}$

$\Rightarrow$ They destructively interfere

$\Rightarrow$ Effect of every point of the upper half is cancelled by corresponding points (at distance a/2) of the lower half

$\Rightarrow$ We get minima at an angle $\frac{\lambda}{a}$

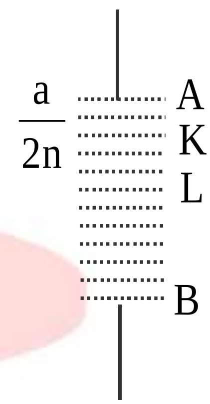
(b) For double slit, tow slits are treated as two-point sources

$\Rightarrow$ Angle $\frac{\lambda}{a}$ means path difference between waves from these two sources = $a\theta = a.\frac{\lambda}{a} = \lambda$

$\Rightarrow$ They constructively interfere and we have a maxima

Question No – 20:- Explain how, in the single-slit diffraction pattern, we have minima in the angle $\theta_n = \frac{n\lambda}{a}$

Solution:- If $\theta_n = \frac{n\lambda}{a}$



$\Rightarrow a\theta_n = n\lambda =$ path difference between waves from the two edges A and B.

Now if we imagine slit to be divided into 2n equal parts.

$\Rightarrow$ Width of each part = $\frac{a}{2n}$

Now for two consecutive parts AK and KL

Path different between waves from tops (i.e A and K) = $\frac{a}{2n}.\theta_n = \frac{a}{2n}.\frac{n\lambda}{a} = \frac{\lambda}{2}$

$\Rightarrow$ They destructively interfere

Similarly, the path difference between waves from the bottom points K and L = $= \frac{a}{2n}.\theta_n = \frac{a}{2n}.\frac{n\lambda}{a} = \frac{\lambda}{2}$

$\Rightarrow$ They also destructively interfere.

$\Rightarrow$ Two consecutive parts produce minima

As n such consecutive pairs are present on the wavefront, so we get minima in the direction $\theta_n = \frac{2\lambda}{a}$

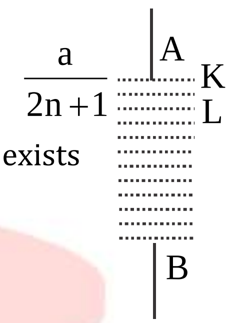
Question No. – 21:- Explain, why there are maxima at $\theta_n = \left(n+\frac{1}{2}\right)\frac{\lambda}{a}$ and why they go on weaker and weaker with an increase in 'n'?

Solution:- For $\theta_n = \left(n+\frac{1}{2}\right)\frac{\lambda}{a}$

$\Rightarrow a\theta_n = \frac{(2n+1)\lambda}{2}$ = Path difference between waves from the edges A and B of the slit.

Now imagine the wavefront AB to be divided into $(2n+1)$ parts.

$\Rightarrow$ Width of each part = $\frac{a}{2n+1}$



In two consecutive parts (like AK and KL), for every point of one part, there exists one point in the other part at a distance $\frac{a}{2n+1}$.

$\Rightarrow$ Waves from these two points have path difference = $\frac{a}{2n+1}.\theta_n = \frac{\lambda}{2}$

$\Rightarrow$ Destructive interference occurs between tow consecutive pairs.

As the whole wavefront is divided into $(2n+1)$ parts, for 1st '2n' part there exist 'n' such consecutive parts/ pairs those cause destructive interference.

$\Rightarrow$ Only $\frac{1}{2n+1}$ part of the wavefront send light and hence we get maxima.

As n increases then the width of the portion of wavefront contributing for secondary maximum gradual decreases. Hence the intensity of secondary maxima becomes gradually weaker and weaker.

Question No. – 22:- Find angular width of central maxima of a single slit diffraction pattern when the light incident has wavelength 600 nm and slit width is (a) 22×10^{-5}cm (b) 2 mm

Solution:-

(a) For 1st minimum, $\sin\theta_1 = \frac{\lambda}{a} = \frac{600\text{nm}}{120\times10^{-5}\text{cm}}$

$= \frac{6\times10^{-7}\text{m}}{12\times10^{-7}\text{m}} = \frac{1}{2}$

$\Rightarrow \theta_1 = 30^{o} = \frac{\pi}{6}$

$\therefore$ The angular width of central maxima = $2\theta_1 = 60^{o}$

(b) For 1st minimum, $\sin\theta_1 = \frac{\lambda}{a} = \frac{600\text{nm}}{2\text{mm}} = \frac{6\times10^{-7}\text{m}}{12\times10^{-7}\text{m}} = 3\times10^{-4}$

As $\frac{\lambda}{a}$ is very small

$\Rightarrow \sin\theta_1 \approx \theta_1 = \frac{\lambda}{a} = 3\times10^{-4}\text{rad}$

$\therefore$ The angular width of central maxima

$= 2\theta_1 = 6\times10^{-4}\text{rad}$

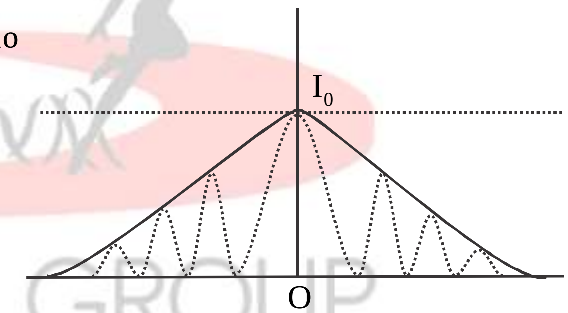
Difference between diffraction (single slit) pattern and interference (double slit) pattern:-

Double Slit Pattern	Single Slit Pattern
(a) Arises due to superposition of waves from two point sources (narrow slits)	(a) Arises due to superposition of waves from each point on the single slit (b) Central bright maximum has a width

(b) Bright and dark bands are equispaced	equal to twice of other fringe widths
(c) The intensity of all maxima are equal	(c) The intensity of maxima gradually fall on increasing the order
(d) At an angle, $\frac{\lambda}{a}$ we get a maximum, where a = separation between slits	(d) At an angle, $\frac{\lambda}{a}$ we get 1stl minimum. Where a = width of the slit.

Question No. – 22:- In the double-slit experiment the pattern on the screen is due to the superposition of single-slit diffraction from each slit. Justify this statement.

Solution:- The figure shows a broader diffraction peak in which there appear several fringes of smaller width due to double-slit interference. The number of interference fringes in the broad diffraction peak is dependent on the ratio $\left(\frac{d}{a}\right)$.



Where d = separation between the slits of the double slit

A = width of each slit

As $\frac{d}{a} \to \infty$ the broad diffraction peak become plane at I_0 i.e, all the interference fringe have the same intensity I_0. i.e we get the interference pattern.

If $\frac{d}{a}$ is small we will not have a clear interference pattern.

Question No. – 23:- In a double-slit experiment d = 1mm, D = 1m and $\lambda = 500\text{nm}$. What should be the width of each slit to obtain 10 maxima of double-slit pattern within the central maximum of single-slit pattern? **(NCERT)**

Solution:- Angular width of central maximum of the single-slit pattern is $\omega_\theta = \frac{2\pi}{a}$

Angular fringe width in double-slit pattern = $\frac{\lambda}{a}$

From question, $10\frac{\lambda}{d} = 2\frac{\lambda}{a}$

$\Rightarrow a = \frac{d}{5} = \frac{1mm}{5} = 0.2mm$

The validity of ray optics or Fresnel's distance:-

An aperture of width 'a' illuminated by a parallel beam sends diffracted light into an angle $\frac{\lambda}{a}$

After covering a distance, Z, width acquired by diffracted beam = $\frac{z\lambda}{a}$

When $Z = Z_F$ i.e Fresnel's distance then width acquired by diffracted beam = a i.e width of the slit

$\Rightarrow a = \frac{Z_F\lambda}{a}$

$\Rightarrow Z_F = \frac{a^2}{\lambda}$ Fresnel's distance

For distances more than Z_F, spreading due to diffraction dominates over that due to ray optics.

Hence for an aperture of width, "a" ray optics hold good up to a distance Z_F

Question No – 24:- For what distance is ray optics a good approximation when the aperture is 3mm wide and wavelength is 500nm?

Solution:- $Z_F = \frac{a^2}{\lambda} = \frac{(3mm)^2}{500nm} = \frac{9\times10^{-6}}{5\times10^{-7}} m = 18m$

Question No. – 25:- Two towers on top of two hills are 40km apart. The line joining them passes 50m above a hill halfway between the towers. What is the longest wavelength of radio waves, which can be sent between the towers without appreciable diffraction effect? (NCERT)

Solution:- Size of aperture = a = 50m

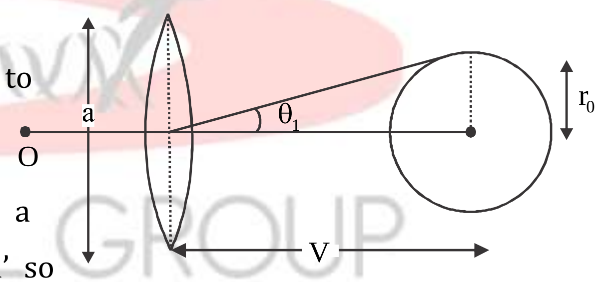
The distance of aperture from the tower is the fresnel's distance Z_F as the diffraction effect is to be neglected

$$\Rightarrow Z_F = \frac{40\text{km}}{2} = 20\text{km}$$

As $Z_F = \frac{a^2}{\lambda}$

$$\Rightarrow \lambda = \frac{a^2}{Z_F} = \frac{(50\text{m})^2}{(20\text{km})} = \frac{25}{2} \times 10^{-2}\text{m} = 12.5\text{cm}$$

Resolving power of optical instruments:-



Resolving power of an optical instrument is the ability to view distinctly to two closely placed objects

Generally, in optical instruments light enters into a convex lens with a circular aperture of diameter 'n' so when light enters into it diffraction occurs.

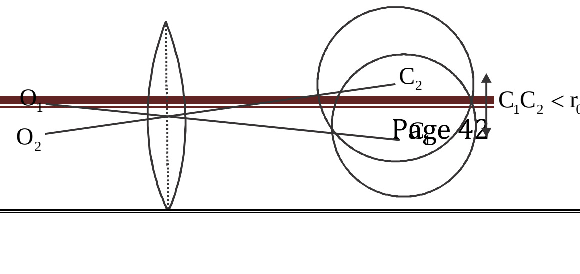
Due to diffraction effect image of a point object becomes a circular spot which is the region bounded by minima's.

It is obtained that the angular size of the radius of the image spot is $\theta_1 = \frac{122\lambda}{a}$

$\Rightarrow r_c = v\theta_1 = \frac{1.22\lambda v}{a}$, where V = image distance

If the object is at $\infty, v = f \Rightarrow r_0 = \frac{1.22\lambda f}{a}$

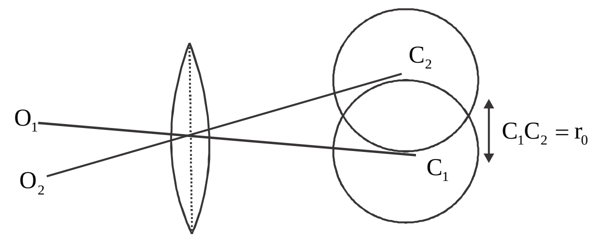
For two closely spaced objects:-



(a) if image spots are such that $C_1C_2 < r_0$ then, central maxima's overlap and image sports can't be identified as images of two objects

$\Rightarrow$ Images are not resolved

(b) If $C_1C_2 = r_0$

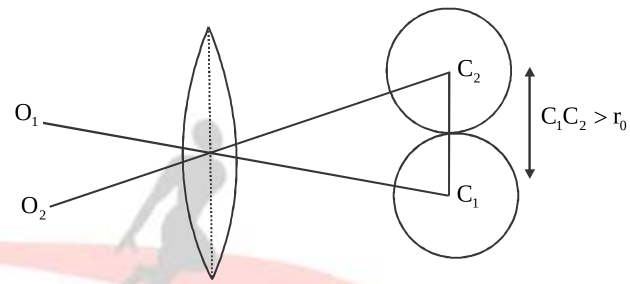


$\Rightarrow$ Central maximum of one is coinciding with 1st minimum of the 2nd

$\Rightarrow$ The intensity of one is not affected by that of the other

$\Rightarrow$ The images are said to just resolved

(c) If $C_1C_2 > r_0$



$\Rightarrow$ Central maximum are well apart

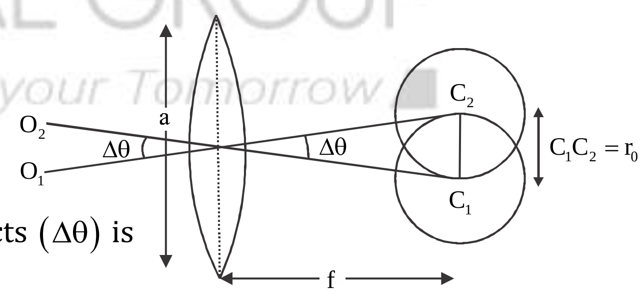
$\Rightarrow$ The images are said to be well resolved

Resolving power of a telescope (or human eye):-

For telescope objects are at infinite, so images are formed at the focus

$\Rightarrow r_0 = \frac{1.22\lambda f}{a}$ = the radius of an image spot

For images just to be resolved, $C_1C_2 = r_0$



At this stage, the angular separation between objects $(\Delta\theta)$ is called as resolving limit (R.L) of the telescope.

$$R.L = \Delta\theta = \frac{r_0}{f} = \frac{\frac{1.22\lambda f}{a}}{f}$$

$$\Rightarrow R.L = \frac{1.22\lambda}{a}$$

Resolving power of a telescope is the reciprocal of resolving limit

$\therefore R.P = \frac{a}{1.22\lambda}$ $\Rightarrow R.P \propto a$

$\Rightarrow R.P \propto \frac{1}{\lambda}$

Question No. – 26:- Assume that light of wavelength $6000\overset{o}{A}$ is coming from a star. What are the limit of resolution and resolving power of a telescope whose objective has a diameter of 100 inches? (NCERT)

Solution:- $\Delta\theta = \frac{1.22\lambda}{a} = \frac{1.22\times 6\times 10^{-7} m}{100\times 2.54\times 10^{-2} m} = 2.9\times 10^{-7} rad$

$R.P = \frac{1}{\Delta\theta} = \frac{1}{2.9}\times 10^{7} = 3.45\times 10^{6}$

Resolving power of a compound microscope:-

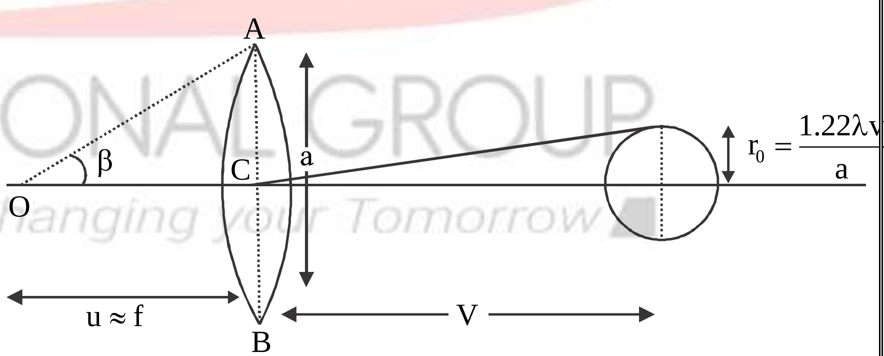
For compound microscope object is placed very close to focus

Magnification of objective, $m \approx \frac{v}{f}$

V = Image distance

If objects are separated by distance'

then image separation on image plane = $md = \frac{vd}{f}$



For image just to be resolved, $d = d_{min}$ and $\frac{vd_{min}}{f} = r_0$

$$\Rightarrow d_{min} = \frac{r_0 f}{v} = \frac{\left(\frac{1.22\lambda v}{a}\right)f}{v}$$

$$\Rightarrow d_{min} = \frac{1.22\lambda f}{a} \quad \text{.............................. (i)}$$

Again in $\Delta OCA, \tan\beta = \frac{a/2}{F} = \frac{a}{2f}$

If $\mu \to 0 \Rightarrow \tan\beta \approx \sin\beta$

$\therefore \sin\beta = \frac{a}{2f}$

$\Rightarrow F = \frac{a}{2\sin\beta}$ .... (ii)

Using equation (ii) in (i)

$d_{min} = \frac{1.22\lambda.\left(\frac{a}{2\sin\beta}\right)}{a}$ $\quad\Rightarrow d_{min} = \frac{1.22\lambda}{2\sin\beta}$

For any other medium $\lambda = \frac{\lambda}{\mu}$, $\mu =$ the refractive index of the medium

$\therefore d_{min} = \frac{1.22\lambda}{2\mu\sin\mu}$ R.L of microscope

$\therefore$ R.P of microscope $= \frac{1}{R.L} = \frac{2\mu\sin\beta}{1.22\lambda}$

$\therefore R.P \propto \mu$ i.e refractive index of the medium

$R.P \propto \sin\beta$ $2\beta =$ Angle made by the aperture of the objective at the focus

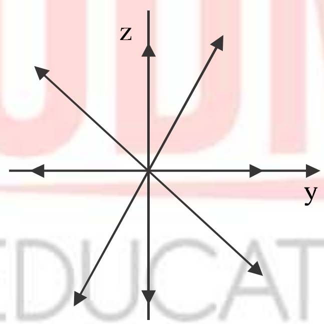
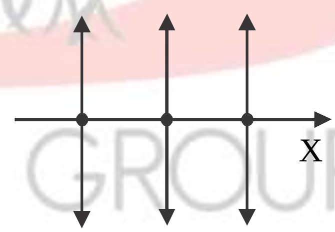
$R.P \propto \frac{1}{\lambda}$ $\quad\lambda =$ the wavelength of light entering into objective

R.P is independent of the focal length of the objective.

Polarisation:-

Upolarised light:- An ordinary light is unpolarized light

In this electric field, the vector $(\vec{E})$ can vibrate along with all possible directions in a plane perpendicular to the direction of propagation. Elongated/ ordinary light sources always produce unpolarised light because ordinary source emits light due to the vibration of its atoms. Each atom produces a wave of its orientation of $(\vec{E})$. So all possible directions $(\vec{E})$ are equally probable.

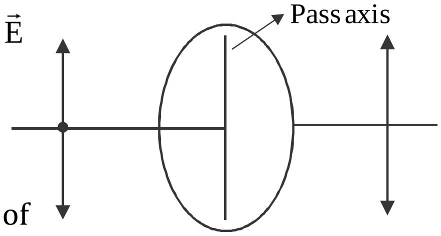
Unpoloarised light propagating along the x-axis  	Unpolarized light Component of $\vec{E}$ in the place of paper Component of $\vec{E}$ in the plane of the paper 

Polarisation:-

- Polarisation is the phenomenon of restricting electric vector of the light wave in a particular direction perpendicular to the direction of propagation
- If the wave is propagating along the x-axis and $\vec{E}$ is restricted along y-axis only then the wave is said as linearly polarized along y-axis or plane-polarized
- The devices used to produced polarized light are called as polarisers e.g tourmaline crystal, Nicol prism, Polaroid

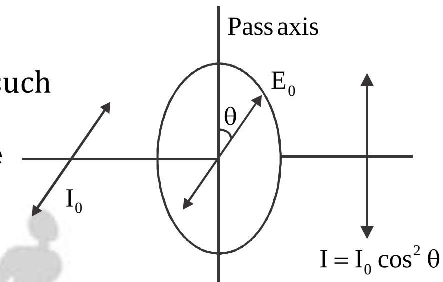
Polaroid:-

Polaroid is a plain sheet of a crystal having a long chain of the hydrocarbon chain in a particular direction. When light is incident the component of $\vec{E}$ along the hydrocarbon chain is used in producing a current in the Polaroid. So the component of $\vec{E}$ perpendicular to hydrocarbon chain only gets transmitted from Polaroid. This line to which $\vec{E}$ of transmitted light remains always parallel is called the pass axis.



Malus Law:-

When a polarized light of intensity I_0 is incident on a polaroid such that its $\vec{E}$ makes angle θ with the pass axis of Polaroid then the intensity of the transmitted light is given by $I = I_0 \cos^2\theta$



If the incident light is unpolarised, then the intensity of transmitted light is $I = I_0 \langle \cos^2\theta \rangle$

($\because \theta$ is changing time to time, an average of $\cos^2\theta$ is taken)

$$\Rightarrow I = I_0 \cdot \frac{1}{2}$$

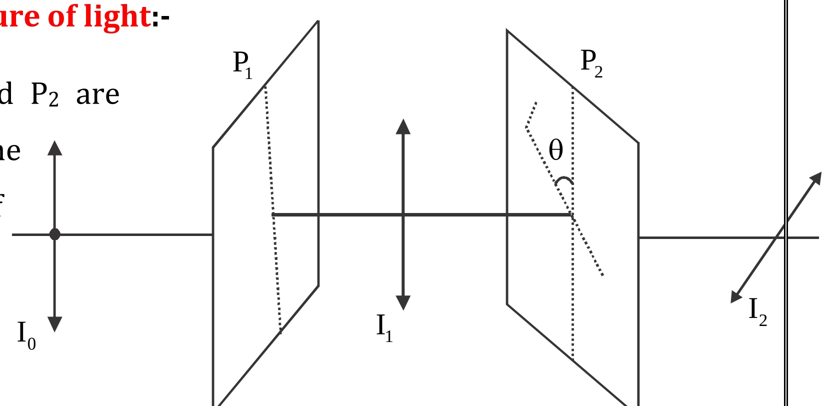
$$\Rightarrow I = \frac{I_0}{2}$$

Percentage of light transmitted by a Polaroid

$$= \frac{I}{I_0} \times 100\%$$

Experiment to verify the transverse nature of light:-

In this experiment two polaroids, P_1 and P_2 are placed parallel to each other in a plane perpendicular to the direction of propagation of light.



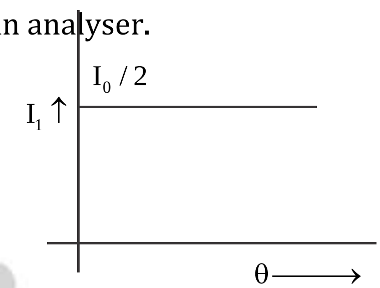
Unpolarised light of intensity I_0 is incident of P_1 called as the polarizer

Intensity coming out of P_1 is $I_1 = \frac{I_0}{2}$

The output of the polarizer (P_1) is independent of the rotation of polarizer about the axis along the direction of propagation of light.

Now polarized light of intensity I_1 from P_1 is incident on P_2 called as an analyser.

If analyser or polarizer is rotated then at any instant angle between their pass axes = θ

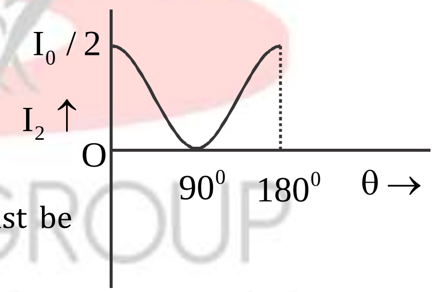


$\Rightarrow$ Intensity coming out of analyses is, $I_2 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2\theta$

This shows that when $\theta = 90^o \Rightarrow I_2 = 0$

i.e At a particular orientation of analyser

(i.e $\theta = 90^o$) $\vec{E}$ of the light, the wave is not passing



$\Rightarrow \vec{E}$ is not along the direction of propagation of light it must be perpendicular to the direction of propagation.

$\Rightarrow$ Light waves are transverse.

Question No. – 27:- Two Polaroid's are crossed to each other. When one of them is rotated through 60^o, then what percentage of incident unpolarized light will be transmitted by the polaroids?

Solution:- Let incident unpolarized light is of intensity I_0.

The light coming out of $P_1, I_1 = \frac{I_0}{2}$

As initially P_1 and P_2 are crossed, the angle between there pass axes was 90^o

Now one Polaroid is rotated through 60^0

$\Rightarrow \theta = 90^0 - 60^0 = 30^0$ = Angle between pass axes of P_1 and P_2

$$\therefore I_2 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2 30^0 = \frac{I_c}{2}.\frac{3}{4} = \frac{3I_c}{8}$$

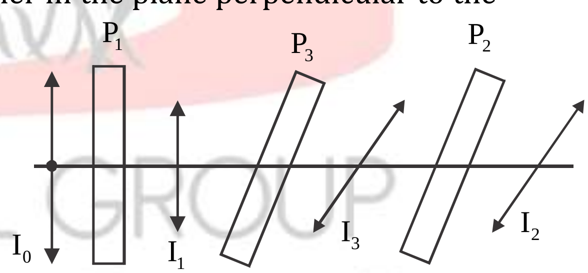
% transmitted = $\frac{I_L}{I_0} \times 100\% = \frac{3}{8} \times 100\% = 37.5\%$

Uses of Polarisation (Polaroids):-

- Refractive index of a medium can be obtained by measuring Brewster's angle.
- In CD player polarized layer bean acts as a needle for producing sound from a compact disc
- Polaroids can be used to control the intensity in sunglasses window panes etc
- Polaroids are also used in photographic cameras and 3D movie cameras.

Effect of rotation of a Polaroid in between two crossed polaroids:-

Two polaroids P_1 and P_2 are kept crossed with each other in the plane perpendicular to the direction of propagation of light.



Let unpolarised light of intensity I_0 is incident on I_0

$$\therefore I_1 = \frac{I_0}{2} \text{ .... (i)}$$

Let θ = Angle between pass axes of P_1 and P_3 at an instant

$$\therefore I_3 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2\theta \text{ .... (ii)}$$

Now angle between pass axes of P_3 and P_2 = $90^o - \theta$

$$\therefore I_2 = I_3 \cos^2\left(90^o - \theta\right) = I_3 \sin^2\theta$$

$$\Rightarrow I_2 = \left(\frac{I_0}{2}\cos^2\theta\right)\sin^2\theta \qquad \text{Using equation (ii)}$$

$\Rightarrow I_2 = \frac{I_0}{2}\cos^2\theta\sin^2\theta = \frac{I_0}{2}\left(\frac{\sin 2\theta}{2}\right)^2$ $\left(\because \cos\theta\sin\theta = \frac{\sin 2\theta}{2}\right)$

$\Rightarrow I_2 = \frac{I_0}{r}\sin^2 2\theta$ or $I_2 = \frac{I_1}{4}\sin^2 2\theta$

For maximum intensity to be transmitted from P_2

$\sin^2 2\theta = 1 \Rightarrow \sin 2\theta = 1 \Rightarrow 2\theta = 90^o$

$\Rightarrow \theta = 45^0$

$\therefore (I_2)_{max} = \frac{I_0}{8}$

Question No. – 28:- Two polaroids P_1 and P_2 are kept crossed in a plane perpendicular to the direction of propagation of light. A third Polaroid P_3 is kept in between P_1 and P_2 with its pass axis at an angle 30^0 with that of P_1. If unpolarized light is an incident on P_1 and intensity $20\omega / m^{2s}$ is emitted out of P_2, then find the intensity of incident light of P_1.

Solution;- Let unpolarised light incident on $P_1 = I_0$

$\Rightarrow I_1 = \frac{I_0}{2}$ = The intensity of light transmitted from P_1

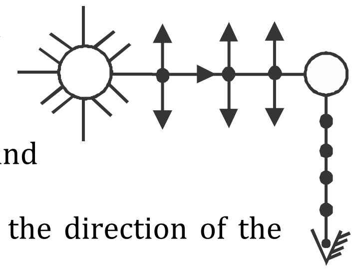
$\therefore I_3 = I_1\cos^2 30^o = I_1\frac{3}{4} = \frac{3I_0}{8}$ = Intensity transmitted from P_3

$\therefore I_2 = I_3\cos^2\left(90^o - 30^0\right) = \frac{3I_0}{8}\cos^2 60^o = \frac{3I_0}{8}\times\frac{1}{4} = \frac{3I_0}{32}$

Given that, $\frac{3I_0}{32} = 20 w/m^2 s$ $\Rightarrow I_0 = \frac{32\times 20}{3} w/m^2 s = \frac{640}{3} w/m^2 s$

Polarisation by Scattering:-

Unpolarised light from the sun has $\vec{E}$ with components both lying in the plane $(\updownarrow)$ of paper and perpendicular to the plane $(\bullet)$ of the paper. When unpolarized light from the sun strikes the atmospheric particles, then the electrons in the molecules acquire components of motion in both $(\updownarrow)$ and $(\bullet)$ direction due to $\vec{E}$ of light waves. When it is observed at 90^{o} to the direction of the sun, then charges accelerating parallel to $(\updownarrow)$ the component of $(\vec{E})$ don't radial energy towards this observer since their acceleration has no transverse component.

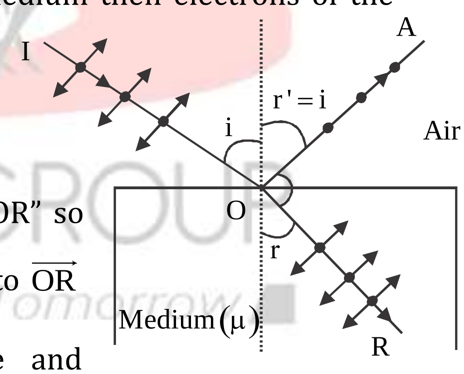


$\therefore$ Radiation scattered in this direction has $(\vec{E})$ along $(\bullet)$ only

$\Rightarrow$ The scattered light is polarized perpendicular to the plane of the figure

Polarisation by reflection:-

When light (unpolarised) from air strikes the surface of a medium then electrons of the medium vibrate under the effect of electric field vectors in the medium $(\vec{E}_m)$



In side medium direction of propagation changes i.e along 'OR" so components of $(\vec{E}_m)$ inside the medium must be transverse to $\overrightarrow{OR}$ component $(\updownarrow)$ transverse to $\overrightarrow{OR}$ is lying in the plane and component $(\bullet)$ is perpendicular to the plane.

Now both the reflected wave and refracted wave are emitted due to vibration of electrons of molecules of the emitted due to vibration of electrons of molecules of the medium under the influence of $\vec{E}_m$.

If the reflected wave is observed along the perpendicular to refracted wave i.e 11 to $(\updownarrow)$ a component of $\vec{E}_m$ then the electrons accelerating parallel to this component can not radiate energy along reflected wave direction 9since acceleration has no transverse component along this direction). So the electric field of the reflected wave has only one component $(\bullet)$ i.e perpendicular to the plane.

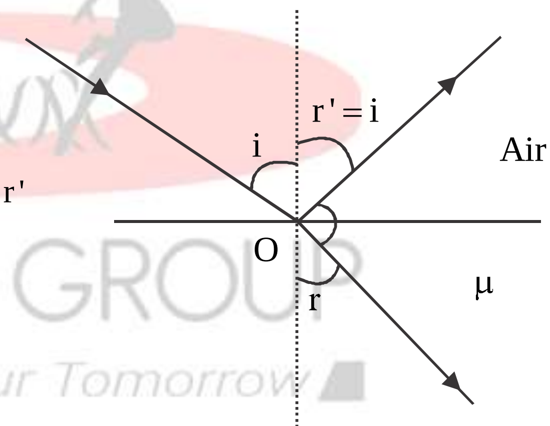
The reflected wave is polarized perpendicular to the plane if the reflected wave is perpendicular to the refracted wave.

Brewster's Law

Statement:- When light (unpolarised) from the air is incident on a medium of refractive index (μ) and reflected wave is polarized, the angle of incidence is called as polarizing angle (i_P) or Brewster's angle (i_B) obeying the relation

$$\tan i_p = \mu$$

Proof:- As the reflected wave is polarized reflected wave and reflected waves are perpendicular to each other



$\Rightarrow r' + r = 90^o$

$\Rightarrow i + r = 90^o$ $\quad \because$ By the law of reflection $i = r'$

$\Rightarrow r = 90^o - i$

$\Rightarrow r = 90^o - i_p$ $\quad \because i = i_p$

By Snell's law, $\dfrac{\sin i}{\sin r} = \mu \Rightarrow \dfrac{\sin i_p}{\sin(90^o - i_p)} = \mu$

$$\Rightarrow \frac{\sin i_p}{\cos i_p} = \mu \Rightarrow \tan i_p = \mu \text{ (Proved)}$$

The relation between polarization angle and the critical angle:-

As $\tan i_p = \mu$ and $\dfrac{1}{\sin i_C} = \mu$

$$\Rightarrow \tan i_p = \frac{1}{\sin i_c} = \operatorname{cosec} i_c$$

Question No. – 29:- What is Brewster's angle for the transition of light from medium 1 (with $\mu = 1.33$) to medium 2 (with = 1.5)?

Solution:- When light travels from rarer (μ_1) to denser (μ_2)

$$\tan i_p = \mu_{21} = \frac{\mu_2}{\mu_1} = \frac{1.5}{1.33} = 1.125$$

$$\Rightarrow i_p = \tan^{-1}(1.125)$$

Question No. – 30:- When light from airstrikes a medium at an angle i_p obeying $\tan i_p = \mu$ then show that reflected wave is polarized?

Solution:- As $\tan i_p = \mu \Rightarrow \sin i_p = \mu \cos i_p$

But by Snell's law, $\sin i_p = \mu \sin r$

$$\Rightarrow \sin r = \cos i_p \Rightarrow r + i_p = 90^o$$

$\Rightarrow$ Reflected wave and refracted waves are 1

$\Rightarrow$ The reflected wave is plane-polarized.