Chapter- 7

Alternating Current

Alternating Current

→ Current is said to be alternating current if its magnitude changes at every instant and direction changes at every instant and direction changes at every half cycle.

→ The equation representing AC :

The simple equation for current representing AC is

$$I = I_o \sin(\omega t + \varphi_o)$$

Where I_o = amplitude of current or peak value of current

ω = Angular frequency of AC

If ν the frequency of ac

$$\Rightarrow \boxed{\nu = \frac{\omega}{2\pi}}$$

If T = Time period of ac

$$\Rightarrow \boxed{T = \frac{1}{\nu} = \frac{2\pi}{\omega}}$$

$\omega t + \varphi_o$ = phase angle of ac at any instant t.

φ_o = phase angle of ac at t = 0

i.e. initial phase angle.

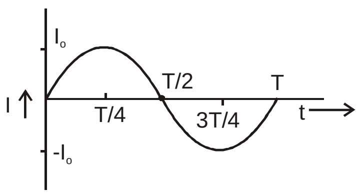
Similarly, an alternating voltage is represented by equation

$$V = V_o \sin(\omega t + \varphi_o)$$

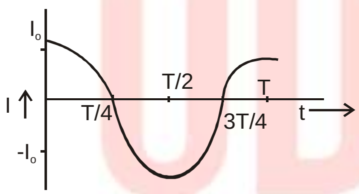
Where V_o = Peak value of voltage.

→ **Graphical representation of ac:** As current and voltage representing ac are sinusoidal functions of phase or time hence sine or cosine curves representing AC.

If current starts from zero $\Rightarrow \varphi_o = 0 \Rightarrow \boxed{I = I_o \sin \omega t}$



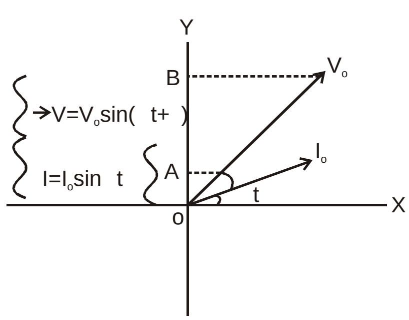
If current starts from its peak value $\Rightarrow \varphi_o = \frac{\pi}{2} \Rightarrow \boxed{I = I_o \cos \omega t}$



Phasor: The quantity which varies sinusoidally with time and represented as the projections of the rotating vectors is called a phasor.

- Alternating current and voltage are phasors.

Phasor diagram for alternating current, I = $I_o \sin \omega t$ and alternating voltage, $V = V_o \sin(\omega t + \varphi)$



Average current: for an interval t_1 to t_2 is defined by

$$I_{av} = \frac{\text{total charge } (Q) \text{ flowing in the internal}}{t_2 - t_1}$$

$$\Rightarrow I_{av} = \frac{\int_{t_1}^{t_2} I\, dt}{t_2 - t_1}$$

The average current is the reading shown by DC ammeter.

Average current for ac in full-cycle i.e. 0 to T.

$$I_{av} = \frac{\int_0^T I_o \sin(\omega t + \varphi_o)\, dt}{T - 0}$$

$$= \frac{I_o}{T}\left[\frac{-\cos(\omega t + \varphi_o)}{\omega}\right]_0^T$$

$$= \frac{-I_o}{\omega T}\left[\cos(\omega T + \varphi_o) - \cos(0 + \varphi_o)\right]$$

$$= \frac{-I_o}{2\pi}\left[\cos(2\pi + \varphi_o) - \cos\varphi_o\right] \qquad (\because \omega T = 2\pi)$$

$$= \frac{-I_o}{2\pi}\left[\cos\varphi_o - \cos\varphi_o\right] \qquad (\because \cos(2\pi + \varphi_o) = \cos\varphi_o)$$

= 0

$\therefore$ The average current in the full cycle of ac = 0

$\therefore$ DC ammeter gives reading 0 when connected to an ac source.

$\Rightarrow$ No charge is flowing in a particular direction through a cross-section in ac.

In ac, only energy blows from one point to another by setting charge into vibration about their mean position.

* Average current in +ve half cycle of ac $= \frac{2I_o}{\pi}$

* Average current in the –ve half cycle of ac $= -\dfrac{2I_o}{\pi}$

Virtual current (I_V) or RMS current (I_{rms}) :

* Virtual current is the equivalent steady current that produces the same heating effect as is produced by ac in a full cycle through same resistance R.

i.e. $I_v^2 RT = \int_0^T I^2 R dt$

$$\Rightarrow I_v^2 = \frac{\int_0^T I^2 dt}{T}$$

$$\Rightarrow \boxed{I_v = \sqrt{\frac{\int_0^T I^2 dt}{T}} = I_{rms}}$$

$\therefore$ 1 virtual ampere is defined as the virtual current of ac which produces 1J of heat through a resistor of resistance 1Ω in 1s.

* Relation between rms current and peak current of ac :

For ac, $I = I_o \sin(\omega t + \varphi_o)$

$$\therefore I_{rms}^2 = \frac{1}{T}\int_0^T \left[I_o \sin(\omega t + \varphi_o)\right]^2 dt$$

$$= \frac{I_o^2}{T}\int_0^T \sin^2(\omega t + \varphi_o) dt = \frac{I_o^2}{2T}\int_0^T \left(1 - \cos 2(\omega t + \varphi_o)\right) dt$$

$$= \frac{I_o^2}{2T}\left[t - \frac{\sin 2(\omega t + \varphi_o)}{2\omega}\right]_0^T$$

$$= \frac{I_o^2}{2T}\left[T - \frac{\sin(2\omega t + 2\varphi_o)}{2\omega} - 0 + \frac{\sin(0 + 2\varphi_o)}{2\omega}\right]$$

$$= \frac{I_o^2}{2T}\left[T - \frac{\sin(4x + 2\varphi_o)}{2\omega} + \frac{\sin(2\varphi_o)}{2\omega}\right]$$

$$= \frac{I_o^2}{2} \qquad \because \sin(4\pi + 2\varphi_o) = \sin 2\varphi_o$$

$$\Rightarrow \boxed{I_{rms} = \frac{I_o}{\sqrt{2}}}$$

* Similarly rms voltage, $V_{rms} = \frac{V_o}{\sqrt{2}}$

* AC ammeter gives rms current reading. So ac ammeter can measure both ac and dc as dc has also rms value.

Question-1: The peak value of an alternating current is 5A and its frequency is 50 Hz. Find its rms value. How long will the current, starting from zero, to reach (i) the peak value, (ii) the rms value for the first time.

Solution : $I_o = 5A$

$$\Rightarrow I_{rms} = \frac{I_o}{\sqrt{2}} = \frac{5}{\sqrt{2}} A$$

As current starts from 0 $\Rightarrow I = I_o \sin \omega t$

(i) For $I = I_o \Rightarrow I_o \sin \omega t = I_o$

$$\Rightarrow \sin \omega t = 1$$

$$\Rightarrow \omega t = \frac{\pi}{2}$$

$$\Rightarrow t = \frac{\pi}{2\omega} = \frac{\pi}{2 \times 2\pi\nu} = \frac{1}{4\nu} = \frac{1}{4 \times 50} s = \frac{1}{200} s$$

(ii) For $I = I_{rms} = \frac{I_o}{\sqrt{2}} \Rightarrow I_o \sin \omega t = \frac{I_o}{\sqrt{2}}$

$$\Rightarrow \sin\omega t = \frac{1}{\sqrt{2}} \Rightarrow \omega t = \frac{\pi}{4}$$

$$\Rightarrow t = \frac{\pi}{4\nu} = \frac{\pi}{8\pi\nu} = \frac{1}{8\nu} = \frac{1}{8\times 50}s = \frac{1}{400}s$$

Question-2: Current through a circuit is, $I = 3\sin 314t + 4\cos 314$. Find peak and rms current. Also, find its initial phase. After what instant current is at its peak value?

Solution : As $I = 3\sin 314t + 4\cos 314t$

Let's substitute, $3 = A\cos\theta$ ....(a)

$4 = A\sin\theta$ ....(b)

then $I = A\cos\theta\sin 314t + A\sin\theta\cos 314t$

$\Rightarrow I = A\sin(314t+\theta)$....... (i)

$(a^2)+(b^2) \Rightarrow 9+16 = A^2\cos^2\theta + A^2\sin^2\theta = A^2$

$\Rightarrow A = \sqrt{25} = 5$ = Peak value of current.

i.e. $I_o = 5A$

$\Rightarrow I_{rms} = \frac{I_o}{\sqrt{2}} = \frac{5}{\sqrt{2}}A$

New equation (b)/(a) $\Rightarrow \frac{4}{5} = \tan\theta \Rightarrow \theta = \tan^{-1}\frac{4}{3}$ = Initial phase angle

For $I = I_o \Rightarrow I_o\sin(314t+\theta) = I_o$

$$\Rightarrow 314t + \theta = \frac{\pi}{2} \Rightarrow t = \frac{\left(\frac{\pi}{2}-\theta\right)}{314} = \left(\frac{\frac{\pi}{2}-\tan^{-1}\frac{4}{3}}{314}\right)s$$

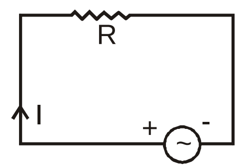
Question -3 : (a) Peak voltage of an ac supply is 300V. What is the rms voltage?

(b) RMS current in an ac circuit is 10A. What is the peak current? (NCERT)

Solution : (a) $V_o = 300V \Rightarrow V_{rms} = \frac{V_o}{\sqrt{2}} = \frac{300}{\sqrt{2}}V$

(b) $I_{rms} = 10A \Rightarrow I_o = I_{rms}\sqrt{2} = 10\sqrt{2}A$

A.C. voltage applied to a resistor :



Let voltage across resistor 'R' be, $V = V_o \sin\omega t$ ....(i)

As for the resistor, V = IR

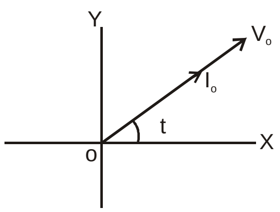
$\Rightarrow I = \frac{V}{R} = \frac{V_o}{R}\sin\omega t$

$\Rightarrow I = I_o \sin\omega t$ ....(ii)

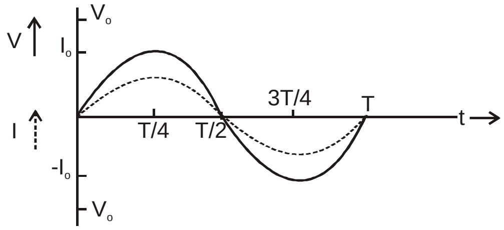
Where $\boxed{I_o = \frac{V_o}{R}}$ = peak current. So rms current, $\boxed{I_{rms} = \frac{V_{rms}}{R}}$

From equations (i) and (ii) it is evident that current and voltage are in the same phase for resistance.

$\Rightarrow$ Phasor representation is ;



$\Rightarrow$ The graphical representation is ;



Question-4: A light bulb is rated at 100W for a 220 V supply. Find (a) Resistance of the bulb, (b) the peak voltage of the source (NCERT), (c) the rms current through the bulb.

Solution : a) $R = \frac{V^2}{P} = \frac{(220V)^2}{100W} = 484\,\Omega$

b) $V_{rms} = 220V \Rightarrow V_o = V_{rms}\sqrt{2} = 220\sqrt{2}V$

c) $I_{rms} = \frac{V_{rms}}{R} = \frac{220V}{484\,\Omega} = \frac{5}{11}A$

Question-5 : A $100\,\Omega$ resistor is connected to a 220V – 50 Hz ac supply.

a) What are the rms and peak current in the circuit?

b) What is the net power consumed over a full cycle? (NCERT)

Solution : $V_{rms} = 220V$

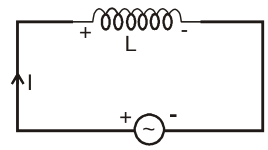
a) $I_{rms} = \frac{V_{rms}}{R} = \frac{220V}{100\,\Omega} = 2.2A$

$I_o = I_{rms}\sqrt{2} = 2.2\sqrt{2}A$

b) $P_{av} = I_{rms}^2 R = (2.2A)^2 \times 100\Omega = 484\,W$.

AC voltage applied to Inductor :

Let voltage drop across an inductor be $V = V_o \sin \omega t$ ....(i)



As for inductor, $V = L\frac{dI}{dt} \Rightarrow dI = \frac{V}{L}dt = \frac{V_o}{L}\sin \omega t dt$

Integrating both sides, $\int dI = \frac{V_o}{L}\int \sin \omega t\ dt$

$$\Rightarrow I = \frac{-V_o}{\omega L}\cos \omega t + C$$

Where C = constant of integration i.e. time-independent term.

As there is no time independent term in the source voltage.

So there must not be any time independent term in I.

$\therefore$ C = 0

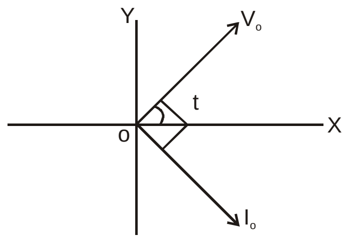
$$\Rightarrow I = -\frac{V_o}{\omega L}\cos \omega t$$

$$\Rightarrow \boxed{I = -I_o \cos \omega t = I_o \sin\left(\omega t - \frac{\pi}{2}\right)}$$ .....(ii)

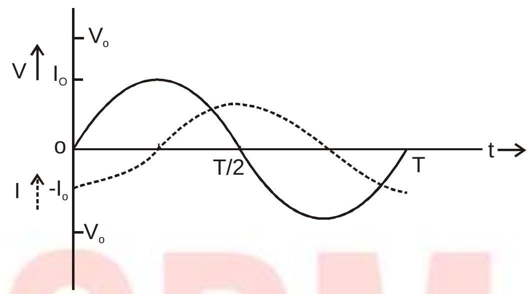
Where $I_o = \frac{V_o}{\omega L}$ = Peak current.

Equation (i) and (ii) show that for inductor voltage leads current by a phase $\frac{\pi}{2}$.

* Phasor diagram is :



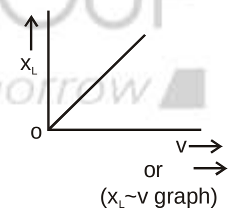
* Graphical representation :



* Inductive reactance : (X_L) i.e. opposition offered by inductor to AC.

$$\therefore \boxed{X_L = \frac{V_o}{I_o} = \frac{V_{rms}}{I_{rms}}}$$

As $I_o = \frac{V_o}{\omega L} \Rightarrow \boxed{X_L = \omega L = 2\pi\nu L} \Rightarrow X_L \; \alpha \; \nu$



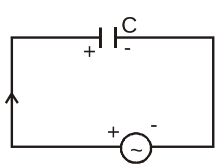
For DC; $\nu = 0 \Rightarrow X_L = 0$

$\Rightarrow$ Inductor shorts dc.

AC voltage applied to the capacitor :

Let voltage across the capacitor at any instant be, $V = V_o \sin\omega t$ ....(i)

As for the capacitor, $V = \frac{q}{C}$



$\Rightarrow q = CV = CV_o \sin\omega t$

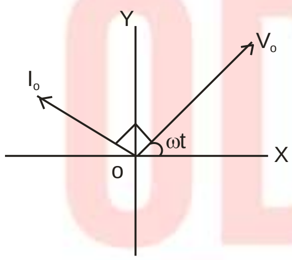
$$\Rightarrow I = \frac{dq}{dt} = \frac{d}{dt}(CV_o \sin\omega t)$$

$$\Rightarrow \boxed{I = \omega CV_o \cos\omega t = I_o \cos\omega t = I_o \sin\left(\omega t + \frac{\pi}{2}\right)} \quad ......(ii)$$

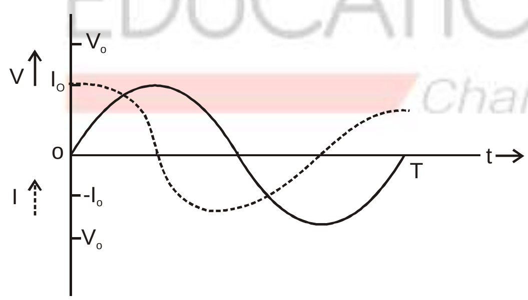
Where $\boxed{I_o = \text{Peak current} = \omega CV_o}$

From equations (i) and (ii) it is evident that current leads voltage or voltage lags behind current by a phase $\frac{\pi}{2}$ in a capacitor.

* Phasor diagram ;



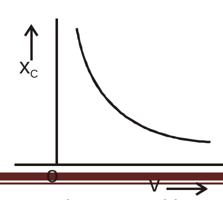
* Graphical representation ;



* Capacitive reactance : (X_C) is the opposition offered by the capacitor to ac.

$$\therefore \boxed{X_C = \frac{V_o}{I_o} = \frac{V_{rms}}{I_{rms}}}$$

As $I_o = \omega CV_o \Rightarrow X_C = \frac{V_o}{\omega CV_o}$



(X_C~v graph)

$$\Rightarrow \boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}}$$

$$\Rightarrow X_C \ \alpha \ \frac{1}{\nu}$$

For DC; $\nu = 0$

$$\Rightarrow X_C = \infty$$

i.e. capacitor offers infinite opposition to DC i.e. capacitor blocks dc.

Question-6: A pure inductor of 25.0 mH is connected to a source of 220V-50Hz. Find (i) Inductive reactance, (ii) rms and peak current through it.

What happens to the above results if the frequency is doubled?

Solution : (i) $X_L = 2\pi\nu L = 2\times3.14\times50Hz\times25.0\times10^{-3}H = 7.85\Omega$

(ii) $I_{rms} = \frac{V_{rms}}{X_L} = \frac{220V}{7.85\Omega}28A$

$$I_o = I_{rms}\sqrt{2} = 28\sqrt{2}A$$

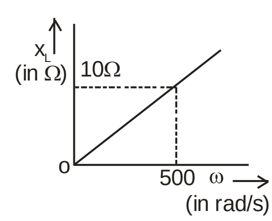
As $X_L \ \alpha \ \nu$. So if ν is doubled.

$$\Rightarrow X_L' = 2X_L = 2\times7.85\Omega = 15.7\Omega$$

and $I_{rms}' = \frac{I_{rms}}{2} = 14A$

$$I_o' = 14\sqrt{2}A$$

Question-7: The figure shows $X_L \sim \omega$ a graph for an inductor in the ac circuit. (i) Find self-inductance. (ii) If the inductor is used in another circuit so that current through it is given by $I = (5\sin1000t)A$ then write for voltage equation.



Solution : (i) From graph, L= slope

$$= \frac{10}{500} H = \frac{1}{50} H = 0.02H$$

(ii) For the other circuit, $\omega = 1000 \text{ rad/s}$.

$$\therefore X_L = \omega L = 1000 \times 0.02 = 20\Omega$$

$$\therefore V_o = I_o X_L = (5 \times 20) V = 100V$$

As for inductor voltage leads current by $\pi/2$, so

$$v = v_o \sin\left(1000t + \frac{\pi}{2}\right) = v_o \cos 1000t$$

$$\Rightarrow v = (100 \cos 1000t) \text{ volt.}$$

Question-8: A $15.0\mu F$ capacitor is connected to a 220V-50Hz source. Find the capacitive reactance and current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and current?

Solution : $X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 15.0 \times 10^{-6}} \Omega = 212\Omega$

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{220V}{212\Omega} = 1.04 A$$

$$I_o = I_{rms}\sqrt{2} = 1.04A \times \sqrt{2} = 1.47A$$

$\therefore$ Current oscillates between -1.47A to 1.47A with a phase leading voltage by $\pi/2$.

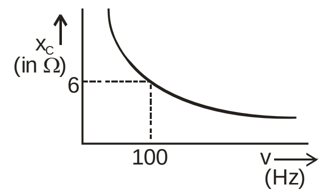
If ν is doubled as $X_C \ \alpha \ \frac{1}{\nu}$

$\Rightarrow X_C$ is reduced to $\frac{1}{2}$ i.e. becomes $\frac{212}{2}\Omega = 106A$

As $I_{rms} = \dfrac{V_{rms}}{x_C}$

$\Rightarrow I_{rms}$ is doubled i.e. becomes $2 \times 1.04A = 2.08A$.

Question-9: The graph shows $x_C \sim \nu$ a graph for a capacitor.



(a)Find capacitance, (b) An ideal inductor has the same reactance at 100Hz as of capacitor. Find its inductance, (c) Draw the graph between reactance of inductor with frequency.

Solution : (a) From the graph, At $\nu = 100Hz, x_C = 6\Omega$

As $x_C = \dfrac{1}{2\pi\nu C}$

$\Rightarrow C = \dfrac{1}{2\pi\nu x_C} = \dfrac{1}{2 \times 3.14 \times 100 \times 6}F$

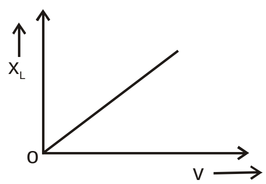
$\Rightarrow C = 2.65 \times 10^{-4}F$

(b) At $\nu = 100Hz$, $x_L = 6\Omega$

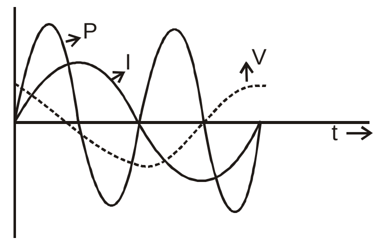
$x_L = 2\pi\nu L$

$\Rightarrow L = \dfrac{x_L}{2\pi\nu} = \dfrac{6}{2 \times 3.14 \times 100}H = \dfrac{3}{314}H$

(c)



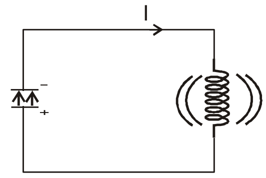
Question-10: Graphs represent the variations of instantaneous current I, voltage V and power P with time for an element in ac circuit.



(a) Identify the element.

(b) Find net opposition given by it to ac. Show its graphical variation with frequency.

Solution : (a) From the graphs we have,



$I = I_o \sin\omega t$

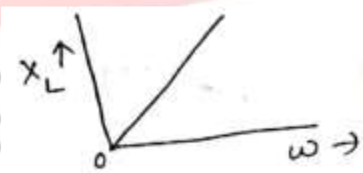
$V = V_o \cos\omega t = V_o \sin(\omega t + \pi/2)$

$P = P_o \sin 2\omega t$

i.e. voltage leads current by phase $\pi/2$.

$\Rightarrow$ Element is the inductor.

(b) As for inductor, $V = L\frac{dI}{dt} = L\frac{d}{dt}(I_o \sin\omega t)$



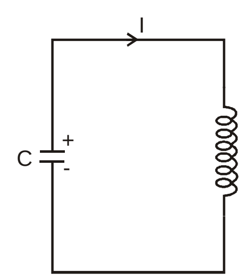
$\Rightarrow V = \omega L I_o \cos\omega t = V_o \cos\omega t$

$\therefore V_o = \omega L I_o$

The opposition offered to ac i.e. inductive reactance is

$x_L = \frac{V_o}{I_o} = \omega L$

LC Oscillation :



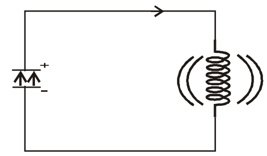
* When a capacitor initially charged to q_o is connected to an inductor then capacitor discharges through inductor and current is produced. The charge and current in the circuit vary sinusoidally with time and exhibit electrical oscillation as a mechanical oscillation of a spring-

block system.

* **Qualitative explanation :**

(i) At t = 0, charge on capacitor is $q = q_o$ and I = 0

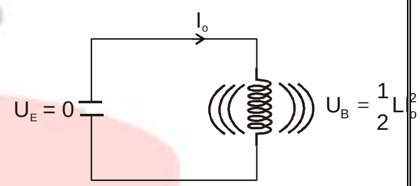
So total energy is electrostatic i.e. $\boxed{U_\varepsilon = U = \frac{q_o^2}{2C}}$



(ii) After this instant capacitor discharges through the inductor.

$\Rightarrow$ charge q decreases but polarity remains same and current I increases

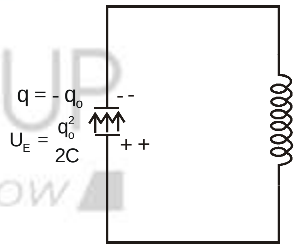
$\Rightarrow U_\varepsilon$ i.e. $\frac{q^2}{2C}$ decreases and U_B i.e. $\frac{1}{2}LI^2$ increases.



(iii) At $t = \frac{T}{4}$; q = 0 $I = I_o$

$\therefore U_E = 0$ and $U_B = \frac{1}{2}LI_0^2$

So total energy is magnetic.



(iv) After this instant; I $\rightarrow$ decrease but in the same direction

$\Rightarrow$ capacitor gets charged and q increases but with opposite polarity.

(v) At $t = \frac{T}{2}$, the capacitor is fully charged but with opposite polarity to that at t = 0

$\therefore q = -q_o$ and $I = 0$ Again total energy becomes electrostatic.

(vi) In the interval T/2 to T, the whole process is repeated in the opposite direction and the system regains the original state as at t = 0.

Hence we get electrical oscillation of (a) charge, (b) current

* Why LC oscillation is not realistic?

(i) Every inductor has resistance. So some energy is dissipated as heat in every cycle. So SHM is not achieving.

(ii) Even if R = 0, as we get oscillation of charge, so some energy must be radiated as an electromagnetic wave.

* Comparison between electrical oscillation and mechanical oscillation of a spring :

Quantities or terms	A mechanical system (spring-block)	Electrical oscillation (LC oscillation)
Inertia	Mass (m)	Inductance L
Force constant	Spring constant (k)	Elastance $\left(\frac{1}{C}\right)$
Oscillating quantities	Displacement (x)	Charge (q)
	Velocity $v = \frac{dx}{dt}$	Current $\left(I = \frac{dq}{dt}\right)$
Conserved quantity	Mechanical energy $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ $\frac{1}{2}kx^2 = P.E.$ $\frac{1}{2}mv^2 = K.E.$	Electromagnetic energy $U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$ $\frac{1}{2}\frac{q^2}{C} = \text{Electrostatic energy}$ $\frac{1}{2}LI^2 = \text{Magnetic energy}$
Frequency	$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	$v = \frac{1}{2\pi\sqrt{LC}}$
Instantaneous equations	$x = A\cos\omega t$ $v = -v_o \sin\omega t$ $v_o = \omega A$ $P.E. = E\cos^2 \omega t$ $K.E. = E\sin^2 \omega t$ $E = \frac{1}{2}kA^2$	$q = q_o \cos\omega t$ $I = -I_o \sin\omega t$ $I_o = \omega q_o$ $U_E = U\cos^2 \omega t$ $U_B = U\sin^2 \omega t$ $U = \frac{1}{2}\frac{q_o^2}{C}$

Question-11: Show that in free oscillations of an LC circuit, the sum of energies stored in the capacitor and inductor is constant in time.

Solution: Let initially charge on capacitor = q_o.

At any instant, $q = q_o \cos \omega t$ ……(i)

$I = \frac{dq}{dt} = -\omega q_o \sin \omega t = -I_o \sin \omega t$ …….(ii)

$\therefore I_o = \omega q_o$ …..(iii)

$\therefore$ Electrostatic energy, $U_E = \frac{q^2}{2C} = \frac{q_o^2}{2C}\cos^2 \omega t$ ….(iv)

Magnetic energy, $U_B = \frac{1}{2}LI^2 = \frac{1}{2}LI_o^2 \sin^2 \omega t = \frac{1}{2}L\omega^2 q_o^2 \sin^2 \omega t$ (using eqn(iv))

$= \frac{1}{2}Lq_o^2 \cdot \frac{1}{LC}\sin^2 \omega t \quad \left(\because \omega = 2\pi\nu = \frac{1}{\sqrt{LC}}\right) \quad = \frac{q_o^2}{2C}\sin^2 \omega t$

$\therefore$ Electromagnetic energy; $U = U_E + U_B = \frac{q_o^2}{2C}\left(\cos^2 \omega t + \sin^2 \omega t\right) = \frac{q_o^2}{2C}$ i.e. constant in time.

Question-12: An LC circuit contains a 20 mH inductor and $50\mu F$ capacitor with initial charge 10mC and negligible resistance. Let the instant circuit closed be t = 0.

a) What is the total energy stored initially? Is it conserved in LC oscillation?

b) What is the natural frequency of the circuit?

c) At what time is the energy stored (i) completely electrical (ii) completely magnetic

d) At what times is the total energy shared equally between the inductor and capacitor?

e) If a resistor is inserted, how much energy is eventually dissipated as heat?

Solution :

a) $U = \frac{q_o^2}{2C} = \frac{\left(10 \times 10^{-3} C\right)^2}{2 \times 50 \times 10^{-6} F} = 1J$

This energy is conserved during oscillation.

b) $\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{20\times10^{-3}H\times50\times10^{-6}F}} = \frac{500}{\pi}Hz$

$\omega = 2\pi\nu = 1000\,rad/s$

c) As $U_E = U\cos^2\omega t \Rightarrow U_E = U$ if $\cos^2\omega t = 1$

$\Rightarrow \cos\omega t = \pm1$

$\Rightarrow \omega t = n\pi \Rightarrow t = \frac{n\pi}{\omega} = \frac{n\pi}{1000}s$

As $U_B = U\sin^2\omega t \Rightarrow$ For $U_B = U;\ \sin^2\omega t = 1$

$\Rightarrow \sin\omega t = \pm1$

$\Rightarrow \omega t = (2n+1)\frac{\pi}{2}$

$\Rightarrow t = (2n+1)\frac{\pi}{2\omega} = \frac{(2n+1)\pi}{2000}s$

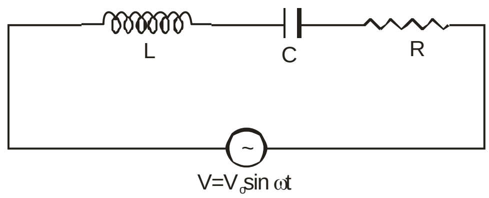
d) For $U_E = E_B \Rightarrow U\cos^2\omega t = U\sin^2\omega t$

$\Rightarrow \cos\omega t = \pm\sin\omega t \qquad \Rightarrow \omega t = (2n+1)\frac{\pi}{4} \qquad (n = 0, 1, 2\ldots\ldots\ldots)$

$\Rightarrow t = (2n+1)\frac{\pi}{4\omega} = \frac{(2n+1)\pi}{4000}s = \frac{\pi}{4000}s, \frac{3\pi}{4000}s, \ldots..$

e) $H = I^2R$

Series LCR circuit :



Using Kirchhoff's voltage law,

$$L\frac{dI}{dt}+\frac{q}{C}+RI=V$$

$$\Rightarrow \boxed{L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=V_o\sin\omega t}$$

This is the voltage equation of the LCR circuit.

(i) **Solution by Phasor diagram method :**

In series combination current through each element is constant.

Let $I=I_o\sin(\omega t+\varphi)$ …….. (i) be the current through each element.

$V=V_o\sin\omega t$ …..(ii) voltage across the combination.

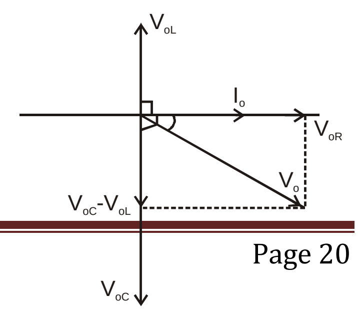
Now across L; $V_L=V_{oL}\sin\left(\omega t+\phi+\frac{\pi}{2}\right)$ where $V_{oL}=I_oX_L$ ……(iii)

Across C, $V_C=V_{OC}\sin\left(\omega t+\phi-\frac{\pi}{2}\right)$ where $V_{oc}=I_oX_C$ …..(iv)

Across R, $V_R=V_{oR}\sin(\omega t+\varphi)$ where $V_{oR}=I_oR$ …..(v)

Now phasor diagram;

From the phasor diagram;



$$V_o = \sqrt{(V_{oC} - V_{oL})^2 + V_{oR}^2}$$

$$= \sqrt{(I_o x_C - I_o x_L)^2 + (I_o R)^2}$$

$$= I_o \sqrt{(x_C - x_L)^2 + R^2}$$

Impedance of the circuit;

$$z = \frac{V_o}{I_o} = \sqrt{(x_C - x_L)^2 + R^2} = \sqrt{\left(\frac{1}{\omega C} - \omega L\right)^2 + R^2} \quad \text{....(vi)}$$

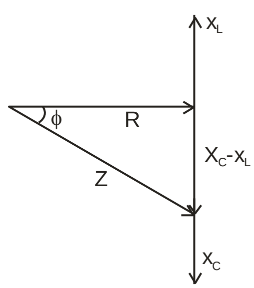
$$\tan\varphi = \frac{V_{oC} - V_{oL}}{V_{oR}} = \frac{I_o x_C - I_o x_L}{I_o R} = \frac{x_C - x_L}{R}$$

$$\Rightarrow \varphi = \tan^{-1}\left(\frac{x_C - x_L}{R}\right) \quad \text{...(vii)}$$

This is the phase difference between current and voltage.

* Impedance triangle: This is the triangle in which reactances and resistance are taken as vectors and represent the sides and impedance represents the resultant.

Using equations (vi) and (vii) impedance triangle is drawn as in the figure.



* **<u>Drawbacks of phasor diagram method</u> :**

(i) This gives the steady-state solution only and can't give the initial condition.

(ii) This can't give the transient growth or decay of current during switching on or off. The circuit.

(ii) **Analytical method :**

The voltage equation for series LCR circuit is

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=V_o\sin\omega t \quad \text{.......... (i)}$$

Let's assume the solution as $q=q_o\sin(\omega t+\theta)$ ....(ii)

$\therefore$ Current is, $I=\frac{dq}{dt}=\omega q_o\cos(\omega t+\theta)=\omega q_o\sin(\omega t+\varphi)$

$=I_o\sin(\omega t+\varphi)$ ....(iii)

Where $I_o=\omega q_o$ and $\varphi=\theta+\frac{\pi}{2}$

Now $\frac{d^2q}{dt^2}=\frac{d}{dt}\left(\frac{dq}{dt}\right)=\frac{d}{dt}\left(\omega q_o\cos(\omega t+\theta)\right)=-\omega^2 q_o\sin(\omega t+\theta)$ ...(iv)

Using equations (ii), (iii) and (iv) in equation (i), we get,

$$L\left[-\omega^2 q_o\sin(\omega t+\theta)\right]+R\omega q_o\cos(\omega t+\theta)+\frac{q_o}{C}\sin(\omega t+\theta)=V_o\sin\omega t$$

$$\Rightarrow \omega q_o\left\{\frac{1}{\omega C}-\omega L\right\}\sin(\omega t+\theta)+\omega q_o R\cos(\omega t+\theta)=V_o\sin\{(\omega t+\theta)-\theta\}$$

$\Rightarrow I_o\left\{\frac{1}{\omega C}-\omega L\right\}\sin(\omega t+\theta)+I_oR\cos(\omega t+\theta)=V_o\cos\theta\sin(\omega t+\theta)-V_o\sin\theta\cos(\omega t+\theta)$ (using $\omega q_o=I_o$)

Comparing the coefficients of $\cos(\omega t+\theta)$ and $\sin(\omega t+\theta)$ in both sides we have,

$$V_o \cos\theta = I_o\left\{\frac{1}{\omega C} - \omega L\right\} \qquad ....(v)$$

$$V_o \sin\theta = -I_o R \text{ ....}(vi)$$

Squaring adding (v) and (vi) we have

$$V_o^2 = I_o^2\left\{\left(\frac{1}{\omega C} - \omega L\right)^2 + R^2\right\}$$

$$\Rightarrow V_o = I_o\sqrt{\left(\frac{1}{\omega C} - \omega L\right)^2 + R^2}$$

$\therefore$ Impedance $Z = \dfrac{V_o}{I_o} = \sqrt{\left(\dfrac{1}{\omega C} - \omega L\right)^2 + R^2}$

Now equation (vi), (v) $\Rightarrow \tan\theta = \dfrac{-R}{\dfrac{1}{\omega C} - \omega L}$

$$\Rightarrow \tan\left(\phi + \frac{\pi}{2}\right) = \frac{-R}{\frac{1}{\omega C} - \omega L}$$

$$\Rightarrow -\cot\varphi = \frac{-R}{\frac{1}{\omega C} - \omega L}$$

$$\Rightarrow \cot\varphi = \frac{R}{\frac{1}{\omega C} - \omega L}$$

$$\Rightarrow \tan\varphi = \frac{\frac{1}{\omega C} - \omega L}{R}$$

$$\Rightarrow \boxed{\varphi = \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)}$$

This is the phase difference between current and voltage.

Note :

(i) For a series LCR circuit if $V = V_o \sin\omega t$

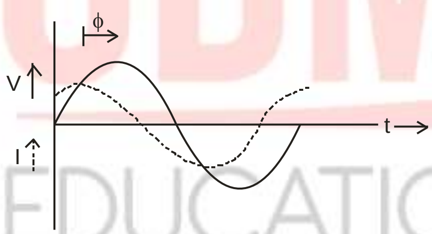
$\Rightarrow I = I_o \sin(\omega t + \varphi)$

Where $I_o = \frac{V_o}{Z} \Rightarrow I_{rms} = \frac{V_{rms}}{Z}$

Z = Impedance $= \sqrt{(x_C - x_L)^2 + R^2}$

And $\varphi = \tan^{-1}\left(\frac{x_C - x_L}{R}\right)$

(ii) Graphically;



(iii) If $x_C > x_L$ circuit is capacitive $\Rightarrow$ current leads voltage.

(iv) If $x_C < x_L$ circuit is inductive

$\Rightarrow$ voltage leads current

(v) If $x_C = x_L$ circuit is resistive

$\Rightarrow \varphi = 0$ i.e. current and voltage are in same phase.

(vi) For RL circuit; $x_C = 0$

$\Rightarrow z = \sqrt{x_L^2 + R^2}$

$\Rightarrow \varphi = \tan^{-1}\left(\frac{X_L}{R}\right)$ current lags behind the voltage

$\therefore$ If $V = V_o \sin\omega t \Rightarrow I = I_o \sin(\omega t - \varphi)$

(vii) For the RC circuit; $X_L = 0$

$\Rightarrow z = \sqrt{X_C^2 - R^2}$

$\Rightarrow \varphi = \tan^{-1}\frac{X_C}{R}$ current leads voltage

$\therefore$ If $V = V_o \sin\omega t \Rightarrow I = I_o \sin(\omega t + \varphi)$

(viii) For LC circuit; R = 0

$\Rightarrow z = X_C - X_L \quad \Rightarrow \varphi = \tan^{-1}\left(\frac{X_C - X_L}{0}\right) = \tan^{-1}\infty = 90^o \text{ or } \frac{\pi}{2}$

If $V = V_o \sin\omega t \Rightarrow I = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$ or $I_o \sin\left(\omega t - \frac{\pi}{2}\right)$

Question-13: A resistor of 200Ω and a capacitor of $15.0\mu F$ are connected in series to a 220V-50Hz source.

(a) Calculate the current in the circuit.

(b) Calculate the voltage (rms) across R and C. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox. (NCERT)

Solution: a) $X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\times3.14\times50\times15.0\times10^{-6}}\Omega = 212.3\Omega$

$\therefore z = \sqrt{R^2 + X_C^2} = \sqrt{(200\Omega)^2 + (212.3\Omega)^2} = 291.7\Omega$

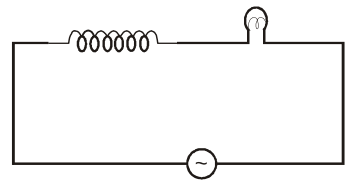
$\therefore I_{rms} = \frac{V_{rms}}{z} = \frac{220V}{291.7\Omega} = 0.755\,A$

b) $V_R = I_{rms}R = 0.755A \times 200\Omega = 151V$

$V_C = I_{rms}X_C = (0.755A) \times (212.3\Omega) = 160.3V \quad V_R + V_C = (151 + 160.3)V = 311.3V > 220V$

This is because V_R and V_C are not in phase. They are at a phase difference $\frac{\pi}{2}$.

So resulting voltage $= \sqrt{V_R^2 + V_C^2} = \sqrt{151^2 + (160.3)^2} = 220V$.



Question-14: The figure shows a bulb and an inductor will are in series across an ac source. With reason explain how glowing of bulb be affected by inserting (a) an iron rod (b) a bismuth rod into the interior of the inductor.

Solution: (a) As iron is a ferromagnetic material, hence by inserting it into the interior of inductor self-inductance 'L' increases

$\Rightarrow$ X_L i.e. ωL increases $\Rightarrow$ z increases as $z = \sqrt{X_L^2 + R^2}$

$\Rightarrow I = \frac{V}{z}$ decreases $\Rightarrow P = I^2R$ $\Rightarrow$ Glowing of bulb decreases.

(b) As bismuth is diamagnetic, hence by inserting it into the interior of inductor self-inductance L decreases

$\Rightarrow$ X_L decreases as $X_L = \omega L$ $\Rightarrow$ z decrease as $z = \sqrt{X_L^2 + R^2}$

$\Rightarrow$ I i.e. $\frac{V}{z}$ increases

$\Rightarrow P = I^2R$ increases $\Rightarrow$ Glowing of bulb increases.

Question-15: A lamp is connected in series with a capacitor. Predict your observations for dc and ac connections. What happens in each case if the capacitance of the capacitor is reduced?

Solution: For DC; there is no current through the lamp

$\Rightarrow$ The lamp doesn't glow

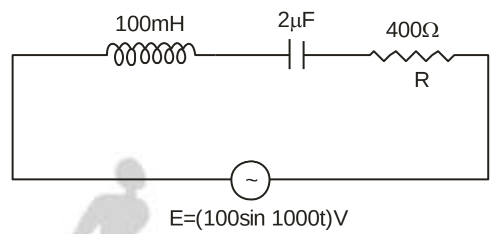
The reduction of capacitance does not affect. For AC; $z = \sqrt{X_C^2 + R^2}$ and $I = \dfrac{V}{z}$

$\Rightarrow$ Bulb glows. If C decreases $\Rightarrow X_C$ i.e. $\dfrac{1}{\omega C}$ increase

$\Rightarrow$ z increases $\Rightarrow$ I decreases $\Rightarrow$ P i.e. I^2R decreases

$\Rightarrow$ The glowing of bulb decreases.

Question-16: In the given circuit calculate the rms current phase difference between current and voltage. Hence write down the current equation.



Solution : $X_L = \omega L = \left(1000 \times 100 \times 10^{-3}\right)\Omega = 100\Omega$

$$X_L = \frac{1}{\omega C} = \frac{1}{1000 \times 2 \times 10^{-6}}\Omega = 500\Omega$$

$$\therefore\ z = \sqrt{\left(X_C - X_L\right)^2 + R^2} = \sqrt{\left(500-100\right)^2 + 400^2} = 400\sqrt{2}\,\Omega$$

$$\therefore\ I_{rms} = \frac{V_{rms}}{z} = \frac{\left(100/\sqrt{2}\right)}{400\sqrt{2}\Omega} = \frac{1}{8}A \qquad \Rightarrow I_o = I_{rms}\sqrt{2} = \frac{\sqrt{2}}{8}A$$

$$\varphi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right) = \tan^{-1}\left(\frac{400}{400}\right) = \frac{\pi}{4}$$

Since $X_C > X_L \Rightarrow$ I leads V $\quad \Rightarrow I = I_o \sin\left(\omega t + \varphi\right) = \dfrac{\sqrt{2}}{8}\sin\left\{1000t + \dfrac{\pi}{4}\right\}A$

Resonance :

- Resonance is defined as the phenomenon in which an oscillating source is driven by an energy source having a frequency equal to the natural frequency of the system. At this stage, the system oscillates with maximum amplitude.
- In series LCR circuit at resonance rms or peak current reaches its maximum value and hence impedance of the circuit becomes minimum.

Since $z = \sqrt{(x_C - x_L)^2 + R^2}$ z will be minimum if

$\boxed{x_C = x_L}$ → This is the condition for resonance for the series LCR circuit.

$$\Rightarrow \frac{1}{\omega_r C} = \omega_r L$$

$$\Rightarrow \omega_r^2 = \frac{1}{LC}$$

$\Rightarrow \boxed{\omega_r = \frac{1}{\sqrt{LC}}}$ → The expression for resonant angular frequency.

$\Rightarrow \boxed{\nu_r = \frac{\omega_r}{2\pi} = \frac{1}{2\pi\sqrt{LC}}}$ → The expression for resonant frequency.

- At resonant frequency ; (i) $z = z_{min} = \sqrt{o^2 + R^2}$

  $\Rightarrow \boxed{Z_{min} = R}$ → The circuit becomes resistive.

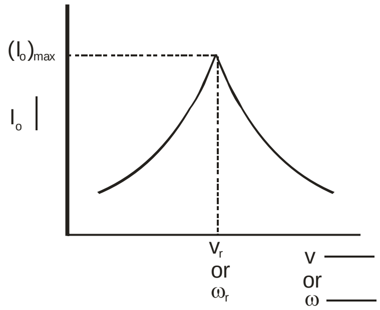
  (ii) $\boxed{I_o = (I_o)_{max} = \frac{V_o}{R}}$

  or $\boxed{I_{rms} = \frac{V_{rms}}{R}}$

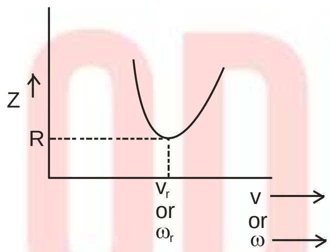
  (iii) Since $x_L = x_C \Rightarrow V_{OL} = V_{OC}$

  ∴ The voltage across the combination of LC = 0

- Graphical variation of peak current or rms current of LCR circuit with the frequency of applied source i.e. resonance curve.



The graphical variation between z and v or ω :



- **Application of series resonant circuit in tuning circuit of radio :**

  The tuning circuit of radio or TV set is a series LCR circuit. In tuning, we vary the capacitance of a capacitor in the tuning circuit such that the resonant frequency of the circuit becomes nearly equal to the frequency of a particular radio signal out of many signals reaching the antenna. When this happens then the amplitude of current with the frequency of the signal of the particular radio station is maximum in the circuit.

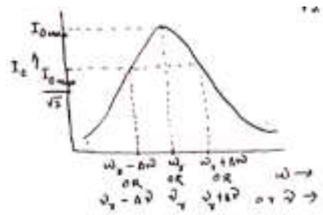
  Bandwidth $[(2\Delta\omega) \text{ or } (2\Delta\nu)]$:

- Bandwidth is defined as the range of frequency of source for which amplitude of the current in the resonant circuit is more than or equal to $\frac{1}{\sqrt{2}}$ times the peak current at resonance.

  If ν_r = resonant frequency then in the frequency range $\nu_r - \Delta\nu$ to $\nu_r + \Delta\nu$ peak current $(I_o) \geq \frac{(I_o)_{max}}{\sqrt{2}}$

At $\nu = \nu_r - \Delta\nu$ or $\nu = \nu_r + \Delta\nu$; $I_o = \frac{(I_o)_{max}}{\sqrt{2}} = \frac{V_o}{\sqrt{2}R}$ ....(i)

$\therefore$ Bandwidth = $2\Delta\nu$



- The expression for bandwidth :

  As we know, at $\nu = \nu_r - \Delta\nu$

  Or $\omega = \omega_r - \Delta\omega$, $I_o = \frac{I_{o\,max}}{\sqrt{2}}$ or $\frac{v_o}{R\sqrt{2}}$

$$\Rightarrow \frac{v_o}{z} = \frac{v_o}{R\sqrt{2}}$$

$$\Rightarrow z^2 = 2R^2$$

$$\Rightarrow (x_C - x_L)^2 + R^2 = 2R^2$$

$$\Rightarrow x_C - x_L = R$$

$$\Rightarrow \frac{1}{(\omega_r - \Delta\omega)C} - (\omega_r - \Delta\omega)L = R$$

$$\Rightarrow \frac{1}{\omega_r C\left(1 - \frac{\Delta\omega}{\omega_r}\right)} - \omega_r L\left(1 - \frac{\Delta\omega}{\omega_r}\right) = R$$

$$\Rightarrow \omega_r L\left[\left(1 - \frac{\Delta\omega}{\omega_r}\right)^{-1} - \left(1 - \frac{\Delta\omega}{\omega_r}\right)\right] = R \quad \left\{\because \omega_r L = \frac{1}{\omega_r C}\right.$$

$$\Rightarrow \omega_r L\left[1 + \frac{\Delta\omega}{\omega_r} - 1 + \frac{\Delta\omega}{\omega_r}\right] = R$$ (Using binomial expression, $(1+x)^n = 1 + nx$ for $x << 1$).

$$\Rightarrow \omega_r L \cdot \frac{2\Delta\omega}{\omega_r} = R$$

$$\Rightarrow \boxed{2\Delta\omega = \frac{R}{L}} \Rightarrow \boxed{2\Delta\nu = \frac{2\Delta\omega}{2\pi} = \frac{R}{2\pi L}}$$

This is the expression for bandwidth.

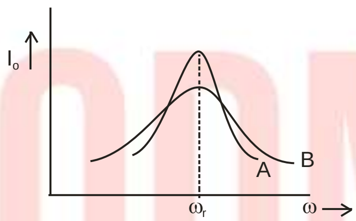
The sharpness of Resonance or Q-factor :

- Q-factor of a series resonant circuit is defined as the ratio between resonant frequency and bandwidth.

  i.e. $\boxed{Q = \frac{\omega_r}{2\Delta\omega} = \frac{\nu_r}{2\Delta\nu}}$

  More the Q-factor, sharper is the resonance curve and more is the selectivity of the circuit.

  e.g.



  If the figure shows resonance curves for two circuits A and B then $Q_A > Q_B$

  (As resonance curve of A is sharper)

- Expressions for Q-factor :

  As $2\Delta\omega = \frac{R}{L}$

  $\therefore\ Q = \frac{\omega_r}{R/L} = \frac{\omega_r L}{R} \quad \Rightarrow \boxed{Q = \frac{\omega_r L}{R}}$

  Again $\omega_r L = \frac{1}{\omega_r C}$ (as resonance condition)

  $\therefore \boxed{Q = \frac{1}{\omega_r CR}}$

As $\omega_r = \frac{1}{\sqrt{LC}}$

$\therefore Q = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$

Again $\omega_r L = (x_L)_{\text{at resonance}}$ and $\frac{1}{\omega_r C} = (x_C)_{\text{at resonance}}$

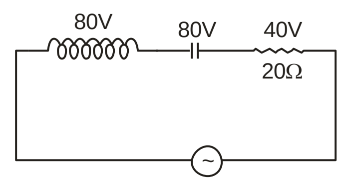
$$\therefore \boxed{Q = \frac{(x_L)_{\text{at resonance}}}{R} = \frac{(x_C)_{\text{at resonance}}}{R}}$$

Multiplying $(I_o)_{\text{at resonance}}$ with both numerator and denominator we have,

$$\boxed{Q = \frac{(V_{oL})_{\text{at resonance}}}{V_o} = \frac{(V_{oC})_{\text{at resonance}}}{V_o}}$$

Since V_o i.e. peak voltage of applied $ac = (V_{oR})_{\text{at resonance}}$.

Question-17: In the given figure the rms voltage drops across the elements shown are at source angular frequency 1000 rad/s.



Find (i) rms current and rms voltage of the applied source.

(ii) capacitance

(iii) the impedance of the circuit

(iv) Q-factor, bandwidth,

(v) Power and power factor

Solution: As the given circuit shows $V_L = V_C \Rightarrow$ Circuit is at resonance.

(i) As $V_R = I_{rms} \cdot R \Rightarrow I_{rms} = \frac{V_R}{R} = \frac{40V}{20\Omega} = 2A$

At this stage; V_{rms} of source = $V_R = 40V (\because V_L - V_C = 0)$

(ii) Now $x_L = \frac{V_L}{I_{rms}} = \frac{80V}{2A} = 40\Omega$

$$\Rightarrow \omega_r L = 40\Omega \Rightarrow L = \frac{40\Omega}{\omega_r} = \frac{40\Omega}{1000\,\text{rad/s}} = 40\,\text{mH}$$

As resonance, so $x_C = 40\Omega$

$$\Rightarrow \frac{1}{\omega_r C} = 40\Omega \Rightarrow C = \frac{1}{\omega_r \times 40\Omega} = \frac{1}{1000 \times 40} F = 25\mu F$$

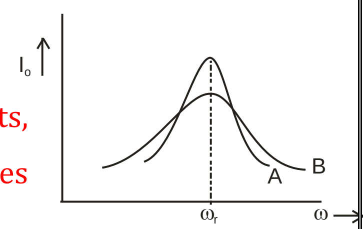
(iii) $Z = R = 20\Omega$

(iv) $Q = \dfrac{(V_L)_{\text{at resonance}}}{(V_{rms})\text{ of the source}} = \dfrac{80V}{40V} = 2$

Bandwidth $(2\Delta\omega) = \dfrac{\omega_r}{Q} = \dfrac{1000}{2}\text{ rad/s} = 500\text{ rad/s}$

(v) As resonance, power factor = 1

Power consumed $= V_{rms}I_{rms} = 40V \times 2A = 80W$



Question-18: The figure shows the resonance curves of two LCR circuits, with the same inductance. Compare their resistances and capacitances whose Q-value is more?

Solution: Given that $L_A = L_B$ From the figure, $(\omega_r)_A = (\omega_r)_B$

$\Rightarrow \sqrt{L_A C_A} = \sqrt{L_B C_B} \Rightarrow C_A = C_B \qquad \because L_A = L_B$

As the resonance curve of A is sharper than that of B.

$\Rightarrow Q_A > Q_B$

$\Rightarrow \dfrac{1}{R_A}\sqrt{\dfrac{L_A}{C_A}} > \dfrac{1}{R_B}\sqrt{\dfrac{L_B}{C_B}}$

$\Rightarrow \dfrac{1}{R_A} > \dfrac{1}{R_B} \Rightarrow R_A < R_B$

Question-19: At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If he/she is carrying anything made of metal, the metal detector emits a sound on what principle does this detector work?

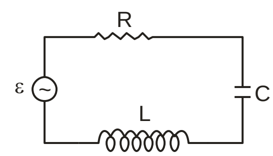
Solution: This works in the principle of resonance of the LCR circuit. A doorway of a metal detector is, in fact, a coil of many turns connected to a capacitor at resonance condition.

When someone walks through it with the metal piece then inductance of the circuit changes.

$\Rightarrow x_L$ changes $\Rightarrow z$ changes

$\Rightarrow$ significance change occurs in the current.

This change in current is detected and the electronic circulatory causes a sound to be emitted as an alarm.



Question-20: In the given circuit L= 5.0 H, $C = 80\mu F$ and $R = 40\mu F$. The ac source of 230 V has a variable frequency.

(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at resonating frequency.

(d) Obtain the bandwidth and Q-factor of the circuit.

Solution : (a) $\omega_r = \frac{1}{\sqrt{LC}} \Rightarrow \nu_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{80\times10^{-6}\times5.0}}\text{Hz} = \frac{25}{\pi}\text{Hz}$

(b) At resonance, $Z = R = 40\Omega$

$$I_o = \frac{V_o}{R} = \frac{230\sqrt{2}V}{40\Omega} = \frac{23\sqrt{2}}{4}A \Rightarrow I_{rms} = \frac{I_o}{\sqrt{2}} = \frac{23}{4}A$$

(c) At resonance, $(V_{rms})_L = I_{rms}x_L = \frac{23}{4}\times2\pi\times\frac{25}{\pi}\times5V = 1437.5V$

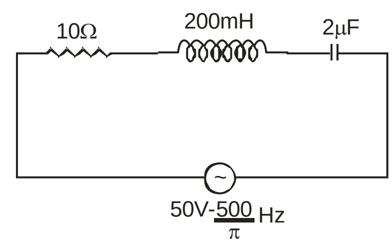
$$(V_{rms})_C = I_{rms}x_C = \frac{23}{4}\times\frac{1}{2\pi\times\frac{25}{\pi}\times80\times10^{-6}}V = 1437.5V$$

$$\left(V_{rms}\right)_R = I_{rms} \cdot R = \frac{23}{4} \times 40V = 230V$$

As $\left(V_{rms}\right)_L = \left(V_{rms}\right)_C$ and both are of opposite phase $\Rightarrow V_{LC} = V_C - V_L = 0$

(d) Bandwidth $\left(2\Delta\omega\right) = \frac{R}{L} = \frac{40}{5} \text{rad/s} = 8\text{rad/s}$ and $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{40}\sqrt{\frac{5}{80 \times 10^{-6}}} = \frac{25}{4}$

Question-21 :



In the given circuit, find the extra capacitance needed to make the power factor to be unity.

Solution: To make power factor to be unity means the circuit is to be at resonance.

$$\Rightarrow \omega_r L = \frac{1}{\omega_r C}$$

$$\Rightarrow C = \frac{1}{\omega_r^2 L} = \frac{1}{\left(2\pi \times \frac{500}{\pi}\right)^2 \times 200 \times 10^{-3}} F = \frac{10^{-6}}{0.2} F = 5\mu F$$

Capacitance present $= 2\mu F$

$\therefore$ Extra capacitance is needed = $5\mu F - 2\mu F = 3\mu F$ in parallel with the given capacitor.

Question-22: When an inductor of inductance L is connected in series with a resistor of resistance 50Ω across a source of frequency $\frac{500}{\pi}$Hz then the voltage of the combination leads the current by a phase of $\frac{\pi}{4}$. If a capacitor of capacitance C is connected in series to R and L in the given circuit the current and voltage are found to be in phase. Find L and

Solution: In the given LR circuit ; $\phi = \frac{\pi}{4}$

As $\tan\varphi = \frac{X_L}{R} \Rightarrow \tan\frac{\pi}{4} = \frac{X_L}{R} \Rightarrow 1 = \frac{X_L}{R} \Rightarrow X_L = R$

$\therefore\ \omega L = R$

$$\Rightarrow L = \frac{R}{\omega} = \frac{R}{2\pi\nu} = \frac{50\Omega}{2\pi \times \frac{500}{\pi} Hz} = 50\,mH$$

Now when C is connected in series, the circuit is an LCR circuit. Now as current and voltage are in phase, so the circuit is at resonance.

$$\therefore C = \frac{1}{\omega_r^2 L} = \frac{1}{\left(2\pi \times \frac{500}{\pi}\right)^2} \times 50 \times 10^{-3} F = 20 \times 10^{-6} F = 20\mu F$$

Power in ac circuit :

- Power at any instant; P = VI

  Let voltage at any instant, $V = V_o \sin\omega t$.

  Let current leads voltage by ϕ, given by $I = I_o \sin(\omega t + \phi)$

  $\therefore\ P = (V_o \sin\omega t)\{I_o \sin(\omega t + \phi)\}$

  $= V_o I_o \sin\omega t(\sin\omega t \cos\theta + \cos\omega t \sin\theta)$

  $= V_o I_o (\cos\theta \sin^2\omega t + \sin\theta \sin\omega t \cos\omega t)$

  $= \frac{V_o I_o}{2}[\cos\theta(1 - \cos 2\omega t) + \sin\theta \cdot \sin 2\omega t]$

- Average power in a full cycle of ac is

  $$P_{av} = \frac{\int_0^T P\,dt}{T} = \frac{V_o I_o}{2T}\int_0^T [\cos\theta(1 - \cos 2\omega t) + \sin\theta \cdot \sin 2\omega t]dt$$

  $$= \frac{V_o I_o}{2T}\left[\cos\theta\left\{\int_0^T 1 dt - \int_0^T \cos 2\omega t\,dt\right\} + \sin\theta \int_0^T \sin 2\omega t\,dt\right]$$

  $$= \frac{V_o I_o}{2T}[\cos\theta\{T - 0\} + \sin\theta \cdot 0] \qquad \left\{\because \int_0^T \cos 2\omega t\,dt = 0 \text{ and } \int_0^T \sin 2\omega t\,dt = 0\right.$$

$$= \frac{V_o I_o}{2} \cos\theta$$

$$= \frac{V_o}{\sqrt{2}} \cdot \frac{I_o}{\sqrt{2}} \cos\theta$$

$$= V_{rms} I_{rms} \cos\theta$$

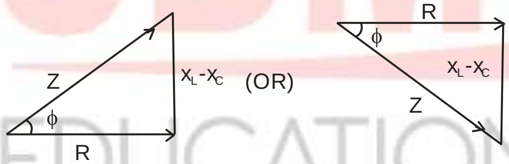
$$\therefore \boxed{P_{av} = \frac{V_o I_o}{2} \cos\theta = V_{rms} I_{rms} \cos\theta}$$

- So average power in an ac circuit depends upon the rms voltage, rms current, and cosine of the phase difference between current and voltage.
- **Power factor :** $\text{Power factor} = \frac{\text{Actual power}}{\text{Apparent power}} = \frac{V_{rms} I_{rms} \cos\theta}{V_{rms} I_{rms}} = \cos\theta$

  $\therefore$ The power factor is defined as the cosine of the phase angle difference between the current and voltage of the ac circuit.

  Again from impedance triangle of an ac circuit;

  $\boxed{\cos\theta}$



Some special cases of ac circuits :

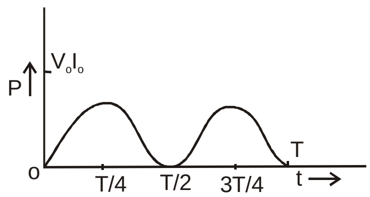
(i) For pure resistive circuit : $\theta = 0$

So if $V = V_o \sin\omega t$, then $I = I_o \sin\omega t$

- Power at any instant is, $P = VI = V_o I_o \sin^2 \omega t = \frac{V_o I_o}{2}(1 - \cos 2\omega t)$

$\Rightarrow$ Power is periodic but not sinusoidal.

Graphically;



- Average power in the full cycle of ac ;

$P_{av} = V_{rms}I_{rms}\cos\theta = V_{rms}I_{rms}\cos\theta$

$\Rightarrow \boxed{P_{av} = V_{rms}I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R}$

(ii) For pure inductive circuit: voltage leads current by phase $\frac{\pi}{2}$.

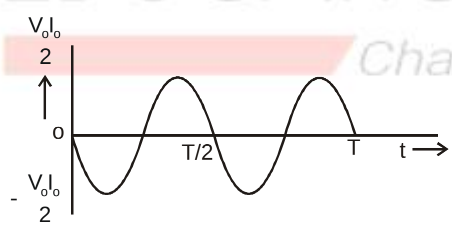
$\therefore$ If $V = V_o \sin\omega t \Rightarrow I = I_o \sin(\omega t - \pi/2) = -I_o \cos\omega t$

- Power at any instant ; $P = VI = -V_oI_o \sin\omega t \cos\omega t$

$= \frac{-V_oI_o}{2}\sin 2\omega t$

i.e. Power varies sinusoidally with time with angular frequency 2ω i.e. twice the frequency of ac.

Graphically;



- Average power in the full cycle of ac; $P_{av} = V_{rms}I_{rms}\cos\theta = V_{rms}I_{rms}\cos\frac{\pi}{2} = 0$

So current through inductor doesn't consume power. So it is wattles current.

(iii) For pure capacitive circuit: Currently leads voltage by phase $\frac{\pi}{2}$.

So if the voltage across the capacitor is $V = V_o \sin\omega t$ then $I = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$

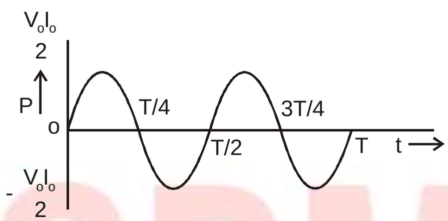
$\Rightarrow I = I_o \cos\omega t$

- Power at any instant; $P = VI = V_o I_o \sin\omega t \cos\omega t$

$\Rightarrow P = \frac{V_o I_o}{2}\sin 2\omega t$

i.e. power varies sinusoidally with time with an angular frequency 2ω equal to twice the frequency of ac.

Graphically;



- Average power in the full cycle of ac; $P_{av} = V_{rms} I_{rms} \cos\theta = V_{rms} I_{rms} \cos\frac{\pi}{2} = 0$

$\therefore$ Current through the capacitor doesn't consume power and hence is wattles.

(iv) For LR circuit ; $z = \sqrt{x_L^2 + R^2}$

$\therefore \cos\theta = \frac{R}{\sqrt{x_L^2 + R^2}} = \text{Power factor}$

(v) For the RC circuit : $z = \sqrt{x_C^2 + R^2}$

$\therefore \cos\theta = \frac{R}{\sqrt{x_C^2 + R^2}} = \text{Power factor}$

(vi) For LC circuit : R = 0

$\therefore \cos\theta = 0 \Rightarrow$ No power consumed

(viii) For LRC circuit at resonance; $x_L = x_C$

$\Rightarrow Z = R$

$$\therefore \cos\theta = \frac{R}{Z} = 1$$

$$\therefore P_{av} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

For general case (no resonance); $\cos\theta = \frac{R}{Z}$

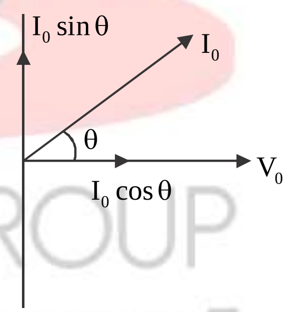
$$P_{av} = V_{rms} I_{rms} \cos\theta = V_{rms} \frac{V_{rms}}{z} \cdot \frac{R}{z} = \frac{V_{rms}^2 R}{z^2}$$

$$\text{Or } P_{av} = V_{rms} I_{rms} \cos\theta = \left(I_{rms} . z\right) \cdot I_{rms} \cdot \frac{R}{z} = I_{rms}^2 R$$

$\therefore$ Power is only consumed by the register in the LCR circuit.

Watt full and wattles components of current in ac circuit.

In an ac circuit if $V = V_o \sin\omega t$ then $I = I_o \sin\left(\omega t + \theta\right)$



In the phasor diagram, the current has two components,

(i) $I_o \cos\theta$ along with voltage

$$\therefore P_{av} = \frac{V_o I_o}{2} \cos\theta$$

$\Rightarrow I_o \cos\theta$ the component is called a wattful component.

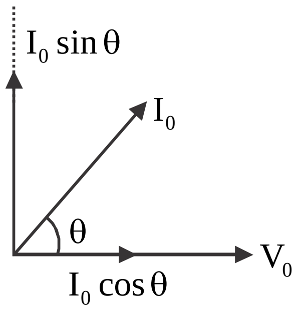
(ii) $I_o \sin\phi$ perpendicular to voltage. As this component doesn't consume power this is called a wattless component of current.

Question – 23:- (a) For circuits used for transporting electric power a low power factor implies large power loss in transmission. Explain. (b) The power factor can often be the capacitance in the circuit. Explain.

Solution:- (a) We know that, $P = VI\cos\phi$ where $\cos\theta$ = power factor

Now to supply a given power at a given voltage, if $\cos\theta$ is small then I will be large.

Since power loss in transmission = I^2R



$\Rightarrow$ Large power loss arises

(b) As current has two components (from phasor diagram)

(i) $I_0 \cos\theta = \frac{I_0 R}{z} =$ watt full component

(ii) $I_0 \sin\theta = \frac{I_0\left(x_C - x_L\right)}{z} =$ watt less component

$\therefore$ To improve power factor or to improve wall full component

We have to reduce watt less component

So we have to reduce X_C i.e $\frac{1}{\omega_C}$

$\Rightarrow$ we have to increase C

This can be done by connecting a capacitor of appropriate C in parallel.

Question No. – 24:- A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3\Omega$, L = 25.48 mH and $C = 796\mu F$. Find

(a) The impedance of the circuit

(b) The phase difference between voltage and current

(c) The power dissipated in the circuit

(d) Power factor

(e) Frequency of the source at which resonance occurs

(f) Impedance, current, and power dissipated at the resonance condition.

Solution:-

(a) $X_L = 2\pi\upsilon L = 2\times3.14\times50\times25.48\times10^{-3}\Omega = 8\Omega$

$$X_C = \frac{1}{2\pi\upsilon c} = \frac{1}{2\times3.14\times50\times796\times10^{-6}}\Omega = 4\Omega$$

$$\therefore Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{(8\Omega - 4\Omega)^2 + (3\Omega)^2}$$

$$= \sqrt{(4\Omega)^2 + (3\Omega)^2} = 5\Omega$$

(b) As $X_L > X_C \Rightarrow$ voltage leads current by phase θ And $\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

(c) $I_{rms} = \frac{V_{rms}}{Z} = \frac{283V}{\sqrt{2}\times5\Omega} = 40A$

$$\therefore P_{av} = I_{rms}^2 R \text{ or } V_{rms}I_{rms}\cos\theta = (40A)^2\times3\Omega \text{ or } \frac{283}{\sqrt{2}}\times40A\times\frac{3}{5} = 4800W$$

(d) Power factor = $\cos\theta = \frac{R}{Z} = \frac{3}{5}$

(e) $\upsilon_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times3.14\times\sqrt{25.48\times10^{-3}\times796\times10^{-6}}}Hz = 35.4Hz$

(f) At resonance, $Z = R = 3\Omega$ $\quad I_{rms} = \frac{V_{rms}}{R} = \frac{283/\sqrt{2}}{3}A = 66.7A$

$$P = V_{rms}I_{rms} = \frac{283}{\sqrt{8}}\times66.7\,W = 13.35kW$$

Question No.-25:- In ac ac circuit voltage and current are given by equations

(a) $V = (200\sin314t)$ volt $\quad I = 5\sin\left(315t - \frac{\pi}{3}\right)$ ampere

(b) $V = (200\sin314t)$ volt $\quad I = (5\cos314t)A$

(c) $V = (200\sin314t)$ volt $\quad I = 5\cos\left(314t = \pi/3\right)A$

Calculate the average power consumed in each case.

Solution:-

(a) $\theta = \frac{\pi}{3}$, $\quad \therefore P = V_{rms} I_{rms} \cos\theta = \left(\frac{200}{\sqrt{2}} V\right)\left(\frac{5}{\sqrt{2}} A\right).\cos\frac{\pi}{3}$

$$= \frac{200 \times 5}{2} \times \frac{1}{2} w = 250w$$

(b) $I = 5\cos 314t = 5\sin\left(314t + \pi/2\right)A \quad \therefore \theta = \pi/2 \quad \therefore P = V_{rms} I_{rms} \cos \pi/2 = 0$

(c) $I = 5\cos\left(314t - \pi/3\right)A = 5\sin\left(314t - \pi/3 + \pi/2\right)A$

$$= 5\sin\left(314t + \frac{\pi}{6}\right)A \quad \Rightarrow \theta = \pi/6 \qquad \therefore P = V_{rms} I_{rms} \cos\frac{\pi}{6}$$

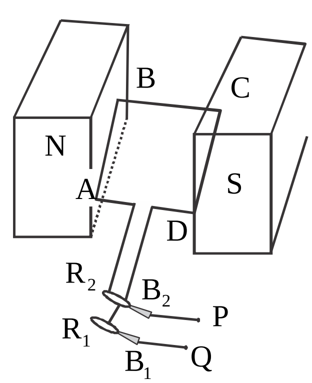
$$= \left(\frac{200}{\sqrt{2}} V\right).\left(\frac{5}{\sqrt{2}} A\right).\left(\frac{\sqrt{3}}{2}\right) \quad = 250\sqrt{3}\, w$$

A.C. Generator:-

A.C generator is a device that produces alternating current from mechanical energy

Principle:- It is based upon the principle of electromagnetic induction i.e when a coil is rotated in a magnetic field then emf is generated in it.

Construction:- An ac generator consists of following



(a) Armature:- It is a rectangular coil ABCD (as in the figure) having a large number of turns of insulated copper wire wound over a soft iron core. This soft iron core is used to increase the magnetic flux linked to the armature coil.

(b) Field magnet:- It is a strong permanent magnet with concave pole pieces. Armature coil is made to rotate about an axis perpendicular to the field lines between the pole pieces.

(c) Slip rings (R_1 and R_2):- Two slip rings are joined to the two ends of the armature coil and placed co-axial to the armature coil. This slip rings rotate along with the coil.

(d) Carbon Brushes (B_1 and B_2):- Two carbon brushes B_1 and B_2 are joined to slip rings R_1 and R_2 respectively. These are kept stationary and provide the terminals for the output of the ac generator.

Theory and Working:-

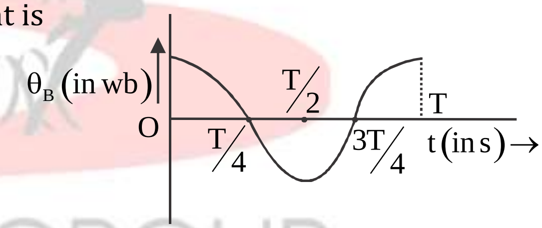
- As the coil ABCD rotates about an axis i.e perpendicular to the magnetic field $\vec{B}$, hence angle between normal $\hat{n}$ to the plane of coil and $\vec{B}$ changes. So magnetic flux changes with time and emf is induced in the coil.
- Let at t = 0, $\theta = 0$ = angle between $\hat{n}$ and $\vec{B}$, at any instant 't', $\theta = wt$

  Where w = angular speed of rotation of coil considered to be uniform

  Magnetic flux linked to each turn of the coil at any instant is

  $\theta_B = BA\cos\theta = BA\cos wt$ ....(i)

  So $\theta_B \sim t$ the graph is



- Emf induced in the coil at any instant t is

  $\varepsilon = -N\frac{d\theta_D}{dt}$ (N = number of turns of the coil)

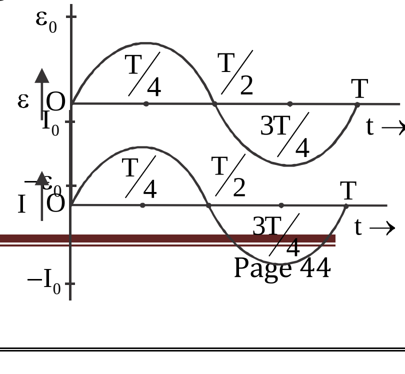
  $\Rightarrow \varepsilon = -N\frac{d}{dt}(BA\cos\omega t)$

  $\Rightarrow \varepsilon - NBA\omega\sin\omega t = \varepsilon_0 \sin\omega t$ .... (ii)

  Where $\varepsilon_0 = NBA\omega$ = the peak value of induced emf

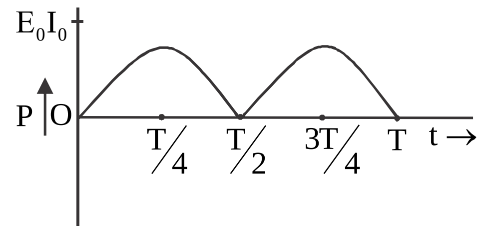
  Equation (ii) shows that induced emf is sinusoidal or alternating.

- $\varepsilon \sim t$ graph is
- If R = total resistance of the circuit, then-current at any instant



is, $I = \frac{\varepsilon}{R} = \frac{NBA\omega}{R}\sin\omega t = I_0 \sin\omega t$, where $I_0 = \frac{NBA\omega}{R}$ = plank constant

Graphically $I \sim t$



- Electrical power generated at any instant

  $P = \varepsilon I = \varepsilon_0 I_0 \sin^2\omega t = \frac{N^2B^2A^2\omega^2}{R}\sin^2\omega t$

  $P \sim t$ graph is

- The average power generated in one cycle of ac.

  $P_{av} = \langle \varepsilon_0 I_0 \sin^2\omega t\rangle = \varepsilon_0 I_0 \langle \sin^2\omega t\rangle$

  $P_{av} = \frac{\varepsilon_0 I_0}{2} = \frac{N^2B^2A^2\omega^2}{2R}$ since $\langle \sin^2\omega t\rangle = \frac{1}{2}$ = an average of $\sin^2\theta$ over a full cycle.

Different values at different position of coil in one complete rotation:-

Time (t)	Angle (θ)	Flux of each turn (ϕ_B)	Induced emf (ε)	Induced current (I)	Power (P)
0	0	BA	0	0	0
$T/4$	$\pi/2$	0	$NBA\omega$	$\frac{NBA\omega}{R}$	$\frac{N^2B^2A^2\omega^2}{R}$
$T/2$	π	$-BA$	0	0	0
$3T/4$	$3\pi/2$	0	$-NBA\omega$	$\frac{-NBA\omega}{R}$	$\frac{N^2B^2A^2\omega^2}{R}$
T	2π	BA	0	0	0

Where T = time period of rotation of coil = $\frac{2\pi}{\omega}$

Ac generator can be converted to de generator by using two splittings intend for slip rings. So in each half rotations, the split rings interchange their positions giving carbon brushes constant polarities.

Sources of energy for ac generator:-

(a) The potential energy of water in hydropower plants

(b) Nuclear energy in nuclear power plants

(c) 1 wind energy in windmills, etc

Advantages of AC over DC:-

(a) Ac transmission is more economical

(b) Ac can be easily converted to DC by using a rectifier

(c) AC voltage can easily be stepped up or down

(d) The magnitude of AC can be reduced by using a choke coil without much less energy

Question No.- 26:- An armature coil consists of 20 turns each of area 0.09 m^2 and total resistance 15.0Ω. It rotates in a magnetic field of 0.5 T at a constant frequency of $\frac{150}{\pi}$ Hz about an axis perpendicular to the magnetic field.

(a) Calculate the maximum values of induced emf, current, and power.

(b) Calculate the average values of induced emf, current, and power

(c) What will be the induced emf and current if the coil rotates about an axis parallel to the magnetic field?

Solution:-

(a) Maximum emf, $\varepsilon_0 = NBA\omega = 20\times0.5\times0.09\times2\pi\times\frac{150}{\pi}V = 270V$

Maximum current, $I_0 = \frac{\varepsilon_0}{R} = \frac{270}{15}A = 18A$

Maximum power, $P_0 = \varepsilon_0 I_0 = 270\times18\omega = 4860w$

(b) Average emf and current over full cycle = 0, as both are sinusoidal in nature.

Average power, $P_{av} = \frac{\varepsilon_0 I_0}{2} = \frac{4860}{2}w = 2430w$

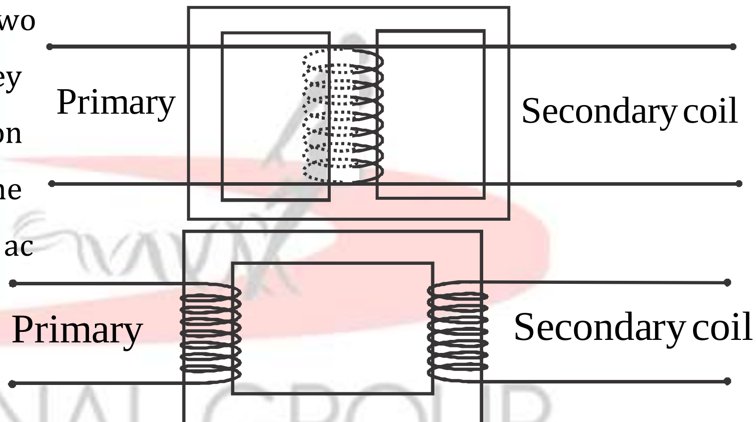
(c) If coil rotates about an axis parallel to the magnetic field then the angle between normal to the plane of coil and $\vec{B}$ remains constant i.e 0^0. So magnetic flux remains constant and hence no emf and currents are induced.

Transformer:-

The transformer is an electrical device that transfers power from one circuit to another by increasing or decreasing the voltage.

Principle:- it is based upon the principle of self and mutual induction.

Construction:- A transformer consists of two sets of coils insulated from each other. They are wound on a soft iron core, either one on the top of the other or separate limbs of the core. One of the coils is connected to the ac source.



This coil is called a primary coil with several turns N_P. The other coil is connected to the load. This coil is called a secondary coil with many turns N_S.

The primary coil is also called as input coil and the secondary coil is also called as an output coil.

Working and Theory:-

As alternating current passes in the primary coil then magnetic flux is linked to primary as well as secondary coil changes. If the primary coil has negligible resistance and there is no flux large, then the same magnetic flux ϕ_B links to each turn of the primary and secondary coil.

Due to self inductor, emf induced in the primary coil is $\varepsilon_P = -N_P \frac{d\theta_B}{dt}$ ……………………….. (i)

Due to mutual induction emf induced in the secondary coil is $\varepsilon_S = -N_S \frac{d\theta_B}{dt}$ .... (ii)

As the primary coil has negligible resistance then $\varepsilon_P = V_P$ i.e input voltage

If the secondary coil is open or has negligible resistance then output voltage or voltage across load resistance is $V_S = \varepsilon_S$

$$\therefore \frac{V_S}{V_P} = \frac{\varepsilon_S}{\varepsilon_P} = \frac{-N_S \frac{d\theta_B}{dt}}{-N_P \frac{d\theta_B}{dt}}$$

$$\Rightarrow \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

The efficiency of the transformer:-

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_S I_S}{V_P I_P}$$

Where I_S = Current in the output circuit

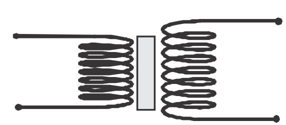
I_P = Current in the input circuit

For ideal transformer, $\eta = 100\%$

$$\Rightarrow V_S I_S = V_P I_P \quad \Rightarrow \frac{V_S}{V_P} = \frac{I_P}{I_S} \quad \therefore \frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P}$$

Step-up transformer:- Power is transferred by increasing the voltage i.e $V_S > V_P$

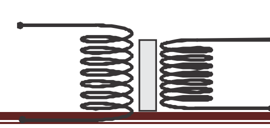
$\Rightarrow N_S > N_P$ and $I_P > I_S$



Circuit symbol is

Step down transformer:- Power is transferred by decreasing the voltage i.e $V_S < V_P$

$\Rightarrow N_S < N_P$ and $I_P < I_S$



Its circuit symbol is

Energy losses in Transformer:-

(a) Flux leakage:- The coupling of primary and secondary coils is not perfect so 100% flux linked to the primary coil will not pass to secondary. This causes some energy loss. This can be minimized by winding the primary and secondary coils over each other.

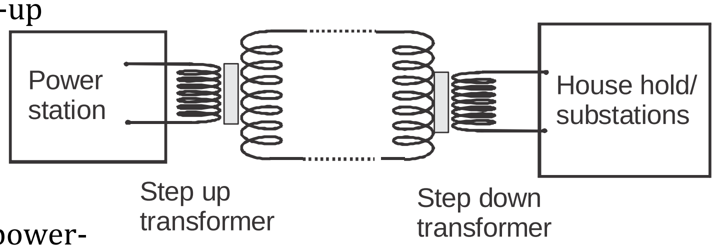
(b) Copper loss:- Windings of coils are made up of coppers which have some resistance. So some energy is dissipated in the form of it (equal to I^2Rt). This loss can be minimized by using thick copper wires for windings. As $R \propto \frac{1}{A}$

$\Rightarrow$ More thick means less R $\Rightarrow$ Less energy loss $\left(I^2Rt\right)$

(c) Eddy current loss (or Ironless):- As changing magnetic flux links to iron core, eddy current is produced. This causes the production of heat. This can be minimized by using a laminated iron core.

(d) Hysteresis loss:- Due to the flow of alternating current in coils, the iron core undergoes continuous magnetization and demagnetization i.e hysteresis cycle. So some energy is lost which is equal to the area of the hysteresis loop. This loss can be minimized by using a material with less retentively and less coercivity

Use of transformer for long-distance transmission of power:-



At power generating stations, step-up transformers are used to convert power into high voltage and low current form. So there will be less power loss along the transmission line (as power loss = I^2R). At power-consuming stations, again power is converted to low voltage and high current form for household use.

Question No.-27:- How much current is drawn by the primary coil of a transformer which steps down 220V to 22V to operate a device with an impedance of 220Ω?

Solution:- From the given information, $I_S = \frac{V_S}{R_L} = \frac{22V}{220\Omega} = \frac{1}{10}A$

As no loss of energy or ideal transformer $\frac{V_S}{V_P} = \frac{I_P}{I_S} \Rightarrow I_P = \frac{V_S}{V_P} \times I_S = \frac{22V}{220V} \times \frac{1}{10}A = \frac{1}{100}A$

Question No.-28:- A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary to get the power output at 220 V

Solution:- As $\frac{V_S}{V_P} = \frac{N_S}{N_P} \Rightarrow N_S = \frac{V_S}{V_P} \times N_P \quad \Rightarrow \frac{220V}{2200V} \times 3000 = 300$

Question No.-29:- Calculate the current drawn by the primary coil of a transformer which steps down 200V to 20V to operate a device of 20Ω resistance. Assume the efficiency of the transformer to be 80%.

Solution:- $I_S = \frac{E_S}{R_L} = \frac{20V}{20\Omega} = 1A$ As $\eta = 80\% \Rightarrow \frac{E_S I_S}{E_P I_P} = \frac{80}{100} \Rightarrow E_S I_S = 0.8 \times E_P I_P$

$\Rightarrow I_P = \frac{E_S I_S}{0.8 E_P} = \frac{20V \times 1A}{0.8 \times 200V} \quad \Rightarrow I_P = \frac{1}{8}A$

Question No.-30:- A small town with a demand of 800 kW of electric power at 220 V is situated 15km away from an electric plant generating power at 440 V. The resistance of two-line wires carrying power is $0.5\Omega/\text{km}$. The town gets power from the lines through a $4000V - 230V$ step-down transformer at a substation in the town.

(a) Estimate the line power loss in the form of heat

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(c) Characterize the step-up transformer at the plant. (NCERT)

Solution:-

(a) The total resistance of the transmission line is $R = (2 \times 15\text{km}) \times (0.5\Omega/\text{km}) = 15\Omega$

Current through the transmission line

$= \left(\frac{P}{E_P}\right)$ at the substance $\qquad = \frac{800\text{kw}}{4000\text{w}} = 200\text{A}$

$\therefore$ Power dissipation along the transmission line $P_{loss} = I^2R = (200\text{A})^2 \times 15\Omega = 600\text{kw}$

(b) Power supply by the plant = power supply by the substation + power loss in the transmission line

$= 800\text{kw} + 600\text{kw} = 1400\text{kw}$

(c) For the transformer at the plant, $E_P = 440\text{V}$

Current through its secondary coil = current in the transmission line = 200A

$\Rightarrow \frac{\text{Power at the plant}}{E_S} = 200\text{A}$

$\Rightarrow \frac{1400\text{kw}}{E_S} = 200\text{A} \Rightarrow E_S = \frac{1400\text{kw}}{200\text{A}} = 7000\text{V}$

$\therefore$ Transformer at the plant is a step-up transformer of $440 - 7000\text{V}$