Chapter- 2

Electric Potential and Capacitance

ELECTROSTATIC POTENTIAL –

Electric field can also be represented in terms of a scalar quantity called Electrostatic Potential.

IMPORTANCE OF ELECTROSTATIC POTENTIAL –

Electric Potential represents :-

(i) The idea of potential energy possessed by a unit charge at that point.

(ii) The degree of Electrification of a body.

(iii) The direction of flow of charge between two bodies in contact.

Note :

- The actual value of potential energy is not physicallysignificant, it is only the difference of potential i.e significant.

ELECTROSTATIC POTENTIAL DIFFERENCE

Definition

Potential difference between two points in an electric field is defined as :

The amount of work done by an external force in carrying a unit +ve charge (test charge) from one point to other along any path (without acceleration)

MATHEMATICALLY; $V_P - V_R = \dfrac{W_{RP}}{q_0}$

Note – 1:

(i) $\left(W_{P\to R}\right)$ electric force = $q_0\left(V_p - V_R\right)$

(ii) $\left(W_{P\to R}\right)_{elec} + \left(W_{P\to R}\right)_{external} = (K)_R - (K)_P$

(iii) $\left(W_{P\to R}\right)_{ext} = (K_R - K_p) + q_0(V_R - V_P)$

(1) The work done by an electrostatic field in moving a charge from one point to another depends only on the positions of initial and final points. It does not depend on the path chosen in going from one point to other.

Thus, the electric field is usually conservative.

ELECTROSTATIC POTENTIAL AT A POINT:

- The infinity is taken as zero potential

  Thus at infinity $V_R = 0$

  Therefore

$$\Rightarrow V_P - 0 = \frac{W_{\infty P}}{q_o}$$

$$\Rightarrow \boxed{V_P = \frac{W_{\infty P}}{q_o}}$$

Thus, Electrostatic potential at any point in the electric field is defined as the work done in carrying a unit +ve charge from infinity to that point along any path without acceleration against the field.

SI unit $\rightarrow$ Volt = Joule/Columb

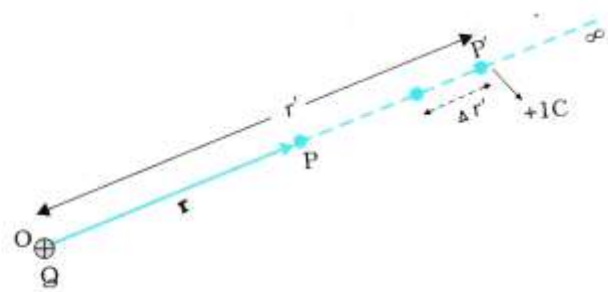
Define 1 Volt ?

Ans.: Electrostatic Potential at a point is said to be one, volt, when one joule of work is done in moving one Coloumb of positive charge from infinity to that point against the electrostatic force of the field without acceleration.

3. Dimensional formula: $V_P = \frac{W}{q_0} = \frac{[ML^2T^{-2}]}{AT} = [A^{-1}ML^2T^{-3}]$

POTENTIAL DUE TO A POINT CHARGE:

Let ‘P’ be any point in the field of a single point charge at ‘O’



The electrostatic force on unit positive charge at a distance ‘x’ at some intermediate point ‘A’ on this path:

$$F = \frac{1}{4\pi \in_O}\frac{Q}{x^2}\times q_0 \text{ .... (Along OA)--------------------(1)}$$

$\therefore$ Small amount of work done in moving aunity+vecharge from A to B through distance ‘dx’ is given by :

$$dW = \vec{F}\cdot\vec{dx}$$
$$= F(-dx)\cos 0^0 \text{ .... (2) } \textit{(as x is decreasing –dx is taken)}$$
$$= -Fdx$$

Total work done in moving unit +ve charge from ∞ to the point P is :

$$W = \int_{\infty}^{r} -Fdx$$
$$= \frac{-Qq_0}{4\pi \in_0}\int_{\infty}^{r} x^{-2}dx$$
$$= \frac{Qq_0}{4\pi \in_0 r}$$

By Definition ; $V = \frac{W}{q_0} = \frac{KQ}{r}$

$$V = \frac{Q}{4\pi \in_0 r}$$

GRAPH

For variation of V and E with r due to a point charge

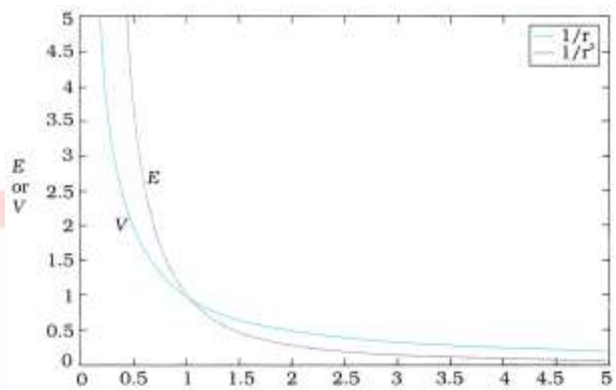


Fig shows the variation of electrostatic potential with distance i.e $V \propto \frac{1}{r}$: and also the variation of electrostatic field with distance i.e $E \propto \frac{1}{r^2}$

NUMERICAL

1. Can a metal sphere of radius 1centimeter hold a charge of 1C? Given that ionising potential of air is 3×10^4 volts :

Ans – We know.

$$V = \frac{Q}{4\pi \in_0 r} \; or \; \frac{KQ}{r}$$

now, $V = \frac{KQ}{r}$

$\Rightarrow 9\times10^{9}\times\frac{1}{1\times10^{-2}}$

$=\frac{9\times10^{9}}{1\times10^{-2}}$

$=9\times10^{11}V$

(The Potential is very much greater than the required to ionise the air $(3\times10^{4}V)$

N.C.E.R.T Example – 2.1

Q 2. (a) Calculate the potential at a point 'P' due to a charge of $4\times10^{-7}C$ located 9cm away

(b) Hence obtain the work done in bringing a charge of $2\times10^{-9}C$ from infinity to the point 'P'. Does the answer depend on the path along which the charge is brought.

Ans(a) $V=\frac{KQ}{r}=9\times10^{9}\times\frac{4\times10^{-7}}{9\times10^{-2}}$

$=\frac{36\times10^{2}}{9\times10^{-2}}$

$=4\times10^{4}V$

(b) $W=qV=2\times10^{-9}\times4\times10^{4}V=8\times10^{-5}J$

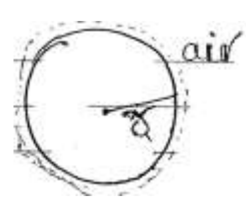
No, work done is path independent. Since the electric field is conservative.

Q 3. And isolated small spherical body is given a charge 'q' in air. What will be it's potential.

(i) in air?

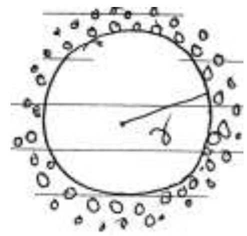
(ii) in a medium of a di-electric constant $(\in_r)$?

Ans – (i) Potential in air



$$V=\frac{KQ}{r}=\frac{Q}{4\pi\in_0 r} \quad -(1)$$

(ii) In a medium of di-electric : $V_m = \frac{KQ}{r}$



$$= \frac{1}{4\pi \in_r} \frac{Q}{} \left[\because \in_r = \frac{\in}{\in_0} \Rightarrow \in = \in_r \cdot \in \right]$$

$$\Rightarrow V_m = \frac{Q}{4\pi(\in_r \in_0) r} \quad - (2)$$

Comparing equations (1) and (2)

$$\Rightarrow \boxed{V_m = \frac{1}{\in_r}(V)}$$

Q. 3. A charge of 1mC is displaced from point A of potential 25V to another point B of potential 5V.

(i) Find the work done by electrostatic force on the charge for displacement $A \rightarrow B$.

(ii) If K.E. of the particle increases by 2mJ during displacement from $A \rightarrow B$, then calculate the work done by external force on the charge.

(iii) What would be the work done by the external force on the charge during the motion, if K.E of the charged particle remains constant?

Ans: (i) $(W_{el})_{A\rightarrow B} = q_0(V_A - V_B)$

$= 1(25-5)V$ = 20 m J

(ii) $(W_{ext})_{A\rightarrow B} = \{(K.E.)_B - (K.E.)_A\} - (W_{el})_{A\rightarrow B}$

$= 2mJ - 20mJ$ = -18 m J

(iii) As $(K.E.)_A = (K.E.)_B$

$\therefore (W_{ext})_{A\rightarrow B} = 0 - (W_{el})_{A\rightarrow B}$ = -20 mJ.

Q. 4. Electric field intensity and electric potential at a point due to a point charge are 10 N/C aid 100 V respectively.

(a) What is the magnitude of the charge?

(b) What is the distance of the point form the charge?

4 Ans.: (a) $E = \frac{kq}{r^2} = 10\,N/C$

$$V = \frac{kq}{r} = 100V$$

$$\therefore \frac{kq/r}{kq/r^2} = \frac{100}{10}$$

$$\Rightarrow r = 10m$$

(b) As $\frac{kq}{r} = 100$

$$\Rightarrow \frac{9\times10^9 \times q}{10} = 100$$

$$\Rightarrow q = \frac{1000}{9\times10^9}C$$

$$\Rightarrow q = \frac{1}{9}\times10^{-6}C$$

Note : Let charge q_0 moves from $A \to B$

$$W_{el} = q_0(V_A - V_B)$$

$$W_{ext} = ?$$

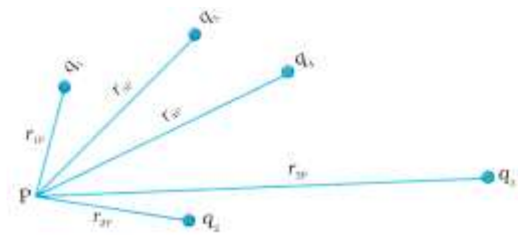
$$W_{el} + W_{ext} = k_B - k_A.$$

$$W_{ext} = (k_B - k_A) + q_0(V_B + V_A)$$

If charge is not accelerated $k_B - k_A = 0$

$\therefore W_{ext} = q_0(V_B - V_A).$

POTENTIAL DUE TO SYSTEM OF CHARGES :



Potential at P due to charge 'q_1' :-

$$V_1 = \frac{1}{4\pi \in_0} \frac{q_1}{r_{1P}}$$

Similarly values of potential due to other charges

$$V_2 = \frac{1}{4\pi \in_0} \frac{q_2}{r_{2P}}$$

$$V_3 = \frac{1}{4\pi \in_0} \frac{q_2}{r_{3P}} \quad ....$$

Similarly , $V_n = \frac{1}{4\pi \in_0} \frac{q_n}{r_{nP}}$

Using Super –position Principle:

$V = V_1 + V_2 + V_3 + ... + V_n$

$$= \frac{1}{4\pi \in_0} \left[\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \frac{q_3}{r_{3P}} .... + \frac{q_n}{r_{nP}} \right]$$

Therefore,

$$\boxed{V = \frac{1}{4\pi \in_0} \sum_{i=1}^{n} \frac{q_i}{r_{iP}}}$$

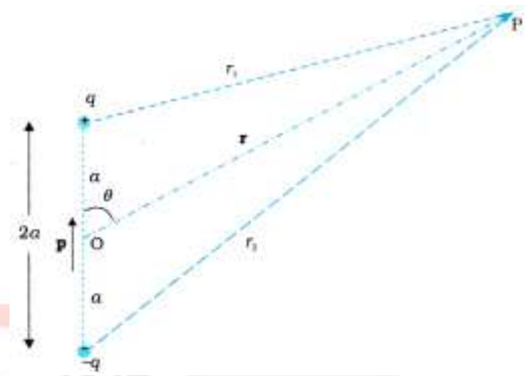
Note :-

- If we have to calculate electric potential due to a continuous charge distribution, characterised by volume charge density $\rho(\mathrm{T})$, we divide the entire volume into a large, number of small volume elements, each of size ΔV. Charge on each element = $\rho \Delta V = dq$

Find potential at the point due to the element, i.e $dV = \frac{kdq}{r}$

$\therefore$ Total potential due to the body, $V = \int \frac{kdq}{r}$

ELECTROSTATIC POTENTIAL AT A POINT DUE TO AN ELECTRIC DIPOLE



Potential At ‘P’ due to Q charge :

$$V_1 = \frac{1}{4\pi \in_0} \frac{(-q)}{r_1} = \frac{-1}{4\pi \in_0} \frac{q}{r_1}$$

Potential at ‘P’ due to +Q Charge

$$V_2 = \frac{1}{4\pi \in_0} \frac{q}{r_2}$$

Potential At ‘P’ due to dipole is given by

$$V = V_1 + V_2$$

$$= \frac{-1}{4\pi \in_0} \frac{q}{r_1} + \frac{1}{4\pi \in_0} \frac{q}{r^2}$$

$$= \frac{q}{4\pi \in_0} \left[\frac{1}{r_2} - \frac{1}{r_1} \right] \qquad .....(1)$$

Now by geometry:

$$r_1 = AP \approx CP = OP + OC = r + a\cos\theta$$

$$r_2 = PB \approx DP = OP - OD = r - a\cos\theta$$

Thus,

$$V = \frac{q}{4\pi \epsilon_0}\left[\frac{1}{r-a\cos\theta} - \frac{1}{r+a\cos\theta}\right]$$

$$= \frac{q}{4\pi \epsilon_0}\left[\frac{r+a\cos\theta - r - a\cos\theta}{(r^2 - a^2\cos^2\theta)}\right]$$

$$= \frac{q}{4\pi \epsilon_0}\frac{2a\cos\theta}{(r^2 - a^2\cos^2\theta)}$$

$$= \frac{P\cos\theta}{4\pi \epsilon_0 (r^2 - a^2\cos^2\theta)} \quad (\because p = q\times 2a)$$

If $r \gg a$, $a^2\cos^2 Q$ will be neglected in comparison to r^2

Hence, $\boxed{V = \frac{P\cos\theta}{4\pi \epsilon_0 r^2}}$

In vector notation $V = \frac{\vec{P}\cdot\hat{r}}{4\pi \epsilon_0 r^2}\left\{P\cos\theta = \vec{P}\cdot\hat{r}\right\}$

Special Cases :-

1. If the point 'P' lies on the axial line of the dipole. i.e $\theta = 0^0$ *or* 180^0

   Then, $V = \pm\frac{P}{4\pi \epsilon_0 r^2}$

2. If the point 'P' lies on the equatorial line of the dipole i.e $\theta = 90^0$

   Then, $V = 0$

Note :

- $V \propto \frac{1}{r}$ due to point charge
- $V \propto \frac{1}{r^2}$ due to an electric dipole
- Potential due to dipole depends upon :

  (i) Displacement

(ii) Angle between position vector and displacement vector

- The potential due to a dipole is axially symmetric about $\vec{P}$. i.e if we rotate the position vector $\vec{r}$ and $\vec{P}$ keeping θ fixed, the points corresponding to P on the cone so generated will have Same potential.

Numerical :

N.C.E.R.T Ex-2.2

Two charge $3\times10^{-8}C$ and $2\times10^{-8}C$ are located 15 cm apart. At what point on the line joining the two charges is the electric potential 0 ? Take the potential at infinity to be zero.

Potential at 'P' is on the axis at which V = 0i.e $V_A + V_B = 0$

Now,

$$V_A = \frac{K(3\times10^{-8})}{x} \text{ AND } V_B = \frac{-K(2\times10^{-8})}{(15-x)} = 0$$

$$\Rightarrow \frac{K\left(3\times10^{-8}\right)}{x} - \frac{K\left(2\times10^{-8}\right)}{15-x} = 0$$

$$\Rightarrow \frac{K(3\times10^{-8})}{x} = \frac{K(2\times10^{-8})}{15-x}$$

$$\Rightarrow \frac{3}{x} = \frac{2}{15-x}$$

$$\Rightarrow 45 - 3x = 2x$$

$$\Rightarrow 5x = 45$$

$$\Rightarrow x = 9cm \text{ (from charge A)}$$

Now if x lies in the extended line OA the required condition is :

Potential at 'P' on the extended line 'BP' where V = 0 i.e $V_A + V_B = 0$

$V_A = \frac{K\left(3\times10^{-8}\right)}{x}$ AND $V_B = \frac{K\left(2\times10^{-8}\right)}{x-15} = 0$

$\Rightarrow \frac{K\left(3\times10^{-8}\right)}{x} - \frac{K\left(2\times10^{-8}\right)}{15-x} = 0$

$\Rightarrow K\frac{3\times10^{-8}}{x} = K\frac{3\times10^{-8}}{x-15}$

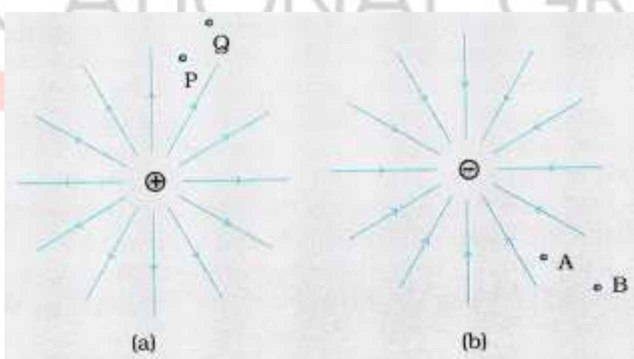
$\Rightarrow \frac{3}{x} = \frac{2}{x-15}$

$\Rightarrow 3x - 45 = 2x$

$\Rightarrow \boxed{x = 45cm}$

NCERT Ex – 2.3

Figure 2.8 (a) and (b) show the field lines of a positive and negative point charge respectively



(a) Give the signs of the potential difference $V_P - V_Q$; $V_B - V_A$

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

(d) Give the sign of the work done by the external agency in moving a small negative charge from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

Solution :

(a) As $V \propto \frac{1}{r}, V_P > V_Q$. Thus $(V_P - V_Q)$ is positive. Also V_B is less negative than V_A. Thus $V_B > V_A$ or $(V_B - V_A)$ is positive.

(b) A small negative charge will be attracted towards positive charge. The negative charge moves from higher potential energy to lower potential energy. Therefore the sign of potential energy difference of small negative charge between Q and P as positive.
Similarly $(\text{P.E})_A > (\text{P.E})_B$ and hence sign of potential energy difference is positive

(c) In moving a small positive charge from Q to P work has to be done by external agency against the electric field. Therefore, work done by the field is negative.

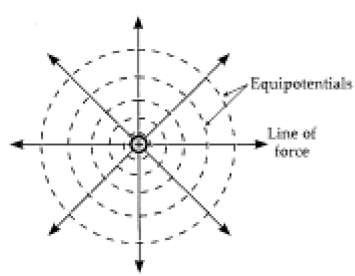
(d) In moving a small negative charge from B to A work has to be done by the external agency. It is positive.

(e) Due to force of repulsion on the negative charge velocity decreases and hence kinetic energy decreases in going from B to A.

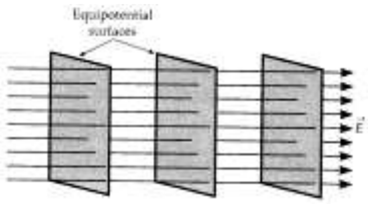
EQUIPOTENTIAL SURFACES

An equipotential is that surface at every point of which electric potential is the same.

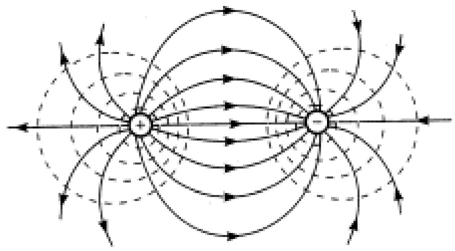
(i) For a single charge 'q':



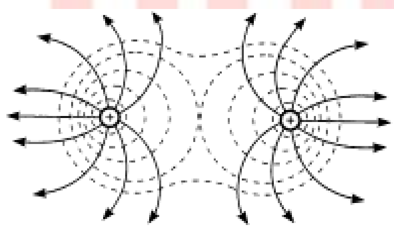
(ii) For uniform Electric Field :-



(iii) For Dipole:-



(iv) For Two Identical Positive Charges:-



PROPERTIES OF EQUIPOTENTIAL SURFACES:-

(1) NO WORK IS DONE IN MOVING THE TEST CHARGE OVER AN EQUI-POTENTIAL SURFACE:-

By definition, potential difference between two points B and A = Work done in carrying unit positive, test charge from A to B.

i.e $V_B - V_A = W_{AB}$

For Equipotential Surface:-

$V_B = V_A$ Therefore, $\boxed{W_{AB} = V_B - V_A = 0}$

(2) FOR ANY CHARGE CONFIGURATION, EQUIPOTENTIAL SURFACE THROUGH A POINT IS NORMAL TO THE ELECTRIC FIELD AT THAT POINT:-

If $d\ell$ is the small distance over the equipotential surface through which unit positive charge is carried:-

Then $dW = \vec{E}.d\vec{\ell}$

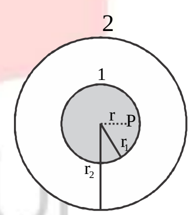
$= Ed\ell\cos\theta$

= 0

Therefore, $\cos\theta = 0 \text{ or } \theta = 90^0$

i.e $\vec{E} \perp d\vec{\ell}$

(3) EQUI-POTENTIAL SURFACE HELPS TO DISTINGUISH REGION OF STRONG FIELD FROM THOSE OF WEAK FIELD:- (i.e equipotential surface are close together in the region of stronger field)



$V_1 > V_2$

We know. $V_1 = \frac{kq}{r_1} \text{ and } V_2 = \frac{kq}{r_2}$

$$V_1 - V_2 = kq\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = kq\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

$$\Rightarrow r_2 - r_1 = \frac{v_1 - v_2}{kq}(r_1 r_2)$$

For constant $PD = (V_1 - V_2)$

$r_2 - r_1 \propto r_1 r_2$

At larger distance (where E decreases) $r_1 r_2$ is more. Hence $r_2 - r_1$ is more i.e, the spacing between two equipotential surface decreases as we move away from the charge.

(4) NO TWO EQUIPOTENTIAL SURFACES CAN INTEREST EACH OTHER:-

Incase, if they intersect there, will be two values of potential at a single point in field which is impossible.

(5) EQUI-POTENTIAL SURFACES OFFER AN ALTERNATIVE, VISUAL PICTURE IN ADDITION OF FIELD LINES AROUND A CHARGE FIELD.

Conceptual Questions:-

Question - 1:-What is the work done in moving a test charge 'q' through a distance of 1cm along the equatorial axis of an electric dipole?

Solution:-

On equatorial line of dipole,

V = 0

Then, $W = Q \times V = 0$

Question - 2:-What would be the work done if a point charge +Q is taken from a point A to B

(a) On the circumference of circle with another point charge $\pm q$ at the centre.

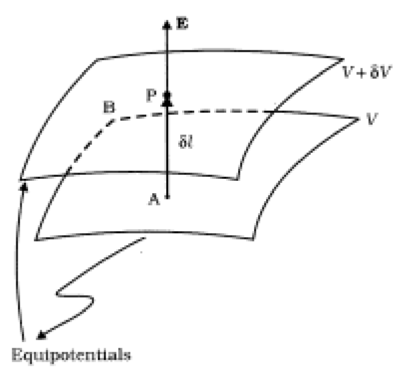
(b) Via C.

Solution:-

(a) As $V_A = V_B \Rightarrow W = Q(V_B - V_A) = 0$

(b) As $V_A = V_B \Rightarrow W = Q(V_B - V_A) = 0$ (As electrostatic force is conservative in nature, its work done is path independent.)

RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL:-



Let us consider two equipotential surfaces A and B spaced closely as shown. Let the potential of A be 'V' and B be V – dV.

$dV \rightarrow$ Decrease in potential in the direction of electric intensity $\vec{E}$

normal to A and B.

And, $d\vec{r}$ be the perpendicular distance between two equipotential surfaces. When a unit potitive charge is taken along this perpendicular distance from the surface B to the surface A against the electric field.

Work done, $W_{BA} = -\vec{E}.\vec{dr}$

By Definition,

$$W_{BA} = V_A - V_B$$

$$= V - (V - dV) = V - (V - dV)$$

$$= V - V + dV$$

$$\Rightarrow W_{BA} = dV$$

$$-\vec{E}.\vec{dr} = dV$$

$\Rightarrow \vec{E} = \Delta\vec{V}$ (= -ve gradient of potential)

In 1D polar $\vec{E} = -\frac{dV}{dr}$.

In 1D Cartesian co-ordinate, $\vec{E} = \frac{-dV}{dx}$

Note:-

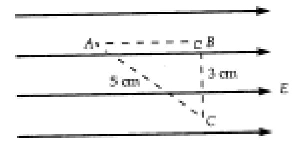
- (-ve) sign indicates the direction of electric field is in the direction of decreasing potential
- The electric potential is a scalar where as potential gradient is a vector quantity
- For uniform field we can write

$$E = -\frac{\Delta V}{\Delta r} = \frac{-(V_2 - v_1)}{d}$$

$$\boxed{\Rightarrow V_1 - V_2 = Ed}$$

Numerical :-

Question – 1:- Three points A, B and C lies in a uniform electric field $E = 5\times10^{-3} N/c$ **as shown in figure. Find out the potential difference between A and C.**



Solution:-

Electric field in A region is given by

$$E = \frac{-dV}{dr} \Rightarrow dV = -Edr$$

ATQ,

$$V_A - V_C = V_A - V_B$$

$$= +E(AB)$$

$$= +5\times10^3 \times 4\times10^{-2}$$

$$= +20\times10^1$$

$$= +200V$$

Question – 2 :-A test charge ' q_0 **' is moved from A to C along the path ABC as shown in figure. Find the P.D between points D and A.**

Solution:-

We know, $V_A = V_B$

And $V_D - V_A = V_D - V_B$

$$= -E(BD)$$

$$= -E(b\cos\theta)$$

$$= E\,b\cos\theta$$

Question – 3 :- Find the electric field between two metal plates 3mm apart connected to 12V battery

Solution:-

$$E = \frac{V}{d}$$

$$= \frac{12}{3\times10^{-3}} = 4\times10^{+3}\, N/C \,or\, volt/m$$

Question -4 :- Given $V = x^2y + yz$ **, calculate the magnitude of** $\vec{E}$ **at (1, 3, 1)**

Solution:-

$$E_x = -\frac{dV}{dx} = -2xy$$

$$= 2(1).(3) = -6\,unit$$

$$E_y = -\frac{dV}{dy} = -\left(x^2 + Z\right)$$

$$= \left((1)^2 + (1)\right) = -2\,unit$$

$$E_z = -\frac{dV}{dz} = -y = -3\,unit$$

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$= \sqrt{(-6)^2 + (-2)^2 + \left(-3^2\right)}$$

$$= \sqrt{36 + 4 + 9}$$

$$= \sqrt{49} = 7\,unit$$

Question - 5:- Equipotential, surface, with potential 2V, 4V, 6V and 8V parallel to y-axis as shown. Calculate the electric field intensity

Solution:-

We know, $|\vec{E}| = \frac{-dV}{dx}$

$$|\vec{E}| = \frac{dV}{dx}$$

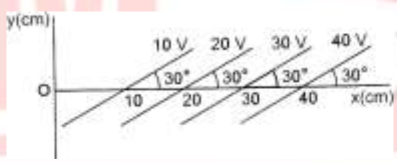
Now, $dV = 4 - 2 = 2$

$dx = 10cm = 0.1m$

$|\vec{E}| = \frac{2}{0.1} = 20\,V/m$

We know electric field is along the direction of decreasing potential.

Question - 6:-



In the above equi-potential surface. What can you say about magnitude and direction of E?

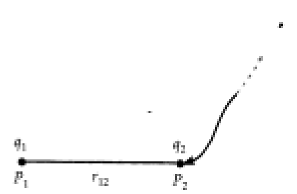
Electrostatic Potential Energy for a System of Charges:

(Definition)

Electrostatic Potential energy of a system of point charges is defined as the total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separations.

ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF TWO POINT CHARGES

Suppose a point charge q_1 is held at point with position vector $\vec{r}_1$ in space. Another point charge q_2 is at infinite distance from q_1. This is to be brought to the position $P_2(\vec{r}_2)$.



Where $P_1P_2 = (\vec{r}_{12})$

Now electrostatic potential at P_2 due to charge q_1 at P_1 is

V= $\frac{k}{r_{12}}q_1$

By definition work done in carrying charge q_2 from infinity to P_2

W = (Potential due to q_1) × charge (q_2)

W =$\frac{kq_1q_2}{r_{12}}$

This is stored in the system of two point charges q1 and q2 in the form of electrostatic potential energy U. Thus

$$U=W=\frac{kq_1q_2}{r_{12}}$$

FOR A THREE POINT CHARGE SYSTEM

Suppose a point charge $+q_1$ is at a point P in space.

NO WORK IS DONE, since other charge is at ∞

the charge $+q_2$ is brought from ∞ to P_2 at a distance r_{12}.

W_2= (potential due to q_1) × q_2

$=\frac{kq_1}{r_{12}}(q_2) =\frac{kq_1q_2}{r_{12}}$

When a charge $+q_3$ is brought from infinity to P_3at a distance of $\vec{r}_{13}$

Work has to be done against $q_1$and q_2.

W_3= (potential due to q_1 and q_2) × (charge (q_3))

$=k\left(\frac{q_1}{r_{13}}+\frac{q_2}{r_{23}}\right)q_3$

$=k\left(\frac{q_1q_3}{r_{13}}+\frac{q_2q_3}{r_{23}}\right)$

POTENTIAL ENERGY FOR THE SYSTEM

U= W= W_1 + W_2 + W_3

$= k\left(\frac{q_1q_3}{r_{13}}+\frac{q_2q_3}{r_{23}}+\frac{q_1q_2}{r_{12}}\right)$

NOTE :

For n charges $$U=\frac{1}{2}k\sum_{\substack{i=1\\k=1}}^{n}\frac{q_iq_k}{r_{ik}}$$

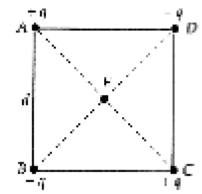
NUMERICAL

NCERT Example 2.4

Question – 1:- Four charges are arranged at the corners of a square ABCD of side "d"

(a) Find the work required to put together this arrangement.

(b) A charge q_0 is brought to the centre "E" of the square, the four charges being held fixed at its corners. How much extra work is needed to do this?



Solution:-

(a) (i) work needed to bring charge +q to A when no charge is present elsewhere = 0

(ii) work needed to bring charge –q to B when +q is at A

$$W = -q \times \left(\frac{q}{4\pi\varepsilon_0 d}\right) = -\frac{q^2}{4\pi\varepsilon_0}$$

(iii) work needed to bring charge "+q" to C when +q is at A and "–q "is at B

$$W = +q\left(\frac{+q}{4\pi\varepsilon_0 d\sqrt{2}} + \frac{-q}{4\pi\varepsilon_0 d}\right)$$

$$= -\frac{q^2}{4\pi\varepsilon_0 d}\left(1 - \frac{1}{\sqrt{2}}\right)$$

(iv) work needed to bring –q to D when +q is at A, -q is at B , and +q is at C

$$W = -q\left(\frac{q}{4\pi\varepsilon_0 d} + \frac{-q}{4\pi\varepsilon_0 d\sqrt{2}} + \frac{q}{4\pi\varepsilon_0 d}\right)$$

$$= -\frac{q^2}{4\pi\varepsilon_0 d}\left(2 - \frac{1}{\sqrt{2}}\right)$$

NET WORK DONE :

$$W = \frac{-q^2}{4\pi\varepsilon_0 d}\left(0 + 1 + \left(1 - \frac{1}{\sqrt{2}}\right) + \left(2 - \frac{1}{\sqrt{2}}\right)\right)$$

$$= \frac{-q^2}{4\pi\varepsilon_0 d}\left(4 - \sqrt{2}\right)$$

(b) The electrostatic potential at centre "E" is clearly 0 since potential due to A and C is cancelled by that due to B and D. Hence no work is required to bring any charge to point E.

Because $W = V(q_0)$

$= 0(q_0) = 0$

Potential Energy in an External Field:-

1. **For Single charge:-**

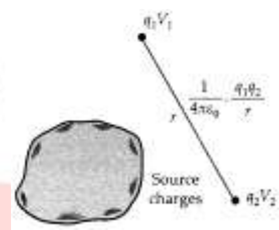
   Electric potential is different at different point of an external field. Let 'V' be the potential at any point 'P'.

   $\therefore$ potential energy of the charge 'q' = work done in bringing the charge from ∞ to that point.

   i.e $U = q(V)$

Potential Energy in terms of position vector:-

Therefore $u = q \times V(\vec{r})$



2.For Two Charges:-

Suppose q_1 and q_2 are two point charges at position vector r_1 and r_2 respectively in a uniform field E.

Work done in bringing charge q_1 from ∞ to position $\vec{r}_1$

$$W_1 = q_1 . V(\vec{r}_1)$$

Again work done in charge q_2 from ∞ to the position $\vec{r}_2$ against the external field.

$$W_2 = q_2 . V(\vec{r}_2)$$

Which q_2 is brought from ∞ to position $\vec{r}_2$. Work has also been done against the field due q_1

Thus, $W_3 = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

By the super position principle:-

P.E of the system = Total work done in assembling the charge configuration

Thus, $U = W_1 + W_2 + W_3$

$$= q_1.V(\vec{r}_1) + q_2.V(\vec{r}_2) + \frac{q_1.q_2}{4\pi\varepsilon_0 r_{12}}$$

The above equation represents Potential energy in terms of a position vector .

Numerical NCERT Example 2.5:-

Question -2:-

(a) Determine the electrostatic potential energy of a system consisting of two charges $7\mu C$ and $-2\mu C$ (and with no external field) placed at($-9cm, 0, 0$) and (9cm, 0, 0) respectively.

(b) How much work is required to separate the two charges infinitely away from each other?

(c) Suppose that the same system of charges is now placed in an external field $E = (1/r^2)$, $A = 9\times10^5 cm^2$. What would be the electrostatic energy of the configuration be?

Solution:-

(a) We know for a two point charge system

The potential energy is

$$U = K\frac{q_1 q_2}{r}$$

$$= \frac{9\times10^9 \times 7\times10^{-6} \times (-2)\times10^{-6}}{0.18}$$

$$= 0.7J$$

(b) $W = U_2 - U_1 = 0 - U = 0 - (-0,7) = 0.7J$

(c) The mutual interaction energy of the two charges remains unchanged. In addition, there, is the energy of interaction of the two charges with the external electric field. We find.

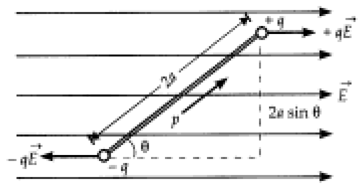
$$q_1V(r_1)+q_2V(r_2)=A\frac{7\mu C}{0.09m}+A\frac{-2\mu C}{0.09m}$$

And the net electrostatic energy is:-

$$=q_1V(r_1)+q_2V(r_2)+\frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$$

$$=A\frac{7\mu C}{0.09m}+A\frac{-2\mu C}{0.09m}-0.7J$$

WORK DONE IN ROTATING A DIPOLE IN AN EXTERNAL FIELD:-



Let a dipole of dipole moment $\vec{P}$ be placed in an electric field making an angle θ with the direction of $\vec{E}$.

Magnitude of Torque acting on dipole:-

$$\tau = PE\sin\theta$$

Work done in rotating the dipole in a field through $d\theta$ is given by.

$$dW=\tau d\theta$$

$$=PE\sin\theta\, d\theta$$

Work done in rotating the dipole from $\theta_1 \text{to} \theta_2$

$$W=\int dW=\int_{\theta_1}^{\theta_2} PE\sin\theta\, d\theta$$

$$=PE[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$=-PE(\cos\theta_2-\cos\theta_1)$$

Therefore $\boxed{W=PE(\cos\theta_1-\cos\theta_2)}$

Question – 3:- If a dipole is rotated from field direction to any position θ. (i.e
$\theta_1=0, \text{then}\ \theta_2=\theta$

Show that $W = PE(1-\cos\theta)$

Solution :- Left to the students

CONCEPTUAL QUESTION:-

Question – 1:-Calculate the amount of work done in rotating the dipole from direction to position normal to field.

Solution:-

$$\cos\theta = 90^0$$

Now, $W = PE[1-\cos\theta]$

$$= PE\left[1-\cos 90^0\right]$$

$$= PE[1-0]$$

= PE

Question – 2:-Calculate the amount of work done in deflections the dipole through an angle of 180^0**, if it was placed normal to field.**

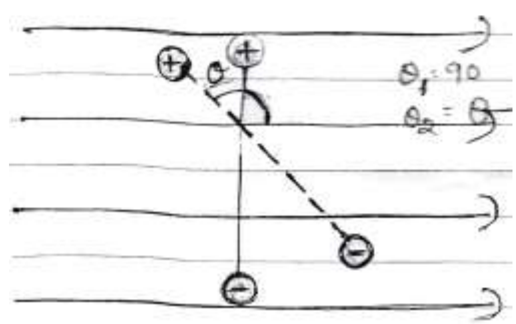
Solution:-

$$W = PE\left[\cos\theta_1 - \cos\theta_2\right]$$

$$= PE\left[\cos 90^0 - \cos(90+180)\right]$$

$$= PE[0-0] = 0$$

Potential energy of a dipole in an electric field :-



It is defined as the work done in rotating a dipole from direction perpendicular to the field to a given direction

Expression for potential energy of a dipole:-

Mathematically

$U = W_0 - W_{90^0}$ (by definition)

$= PE(1-\cos\theta) - PE$

$= PE - PE\cos\theta - PE$

$= -PE\cos\theta$

Therefore, $U = -PE\cos\theta$

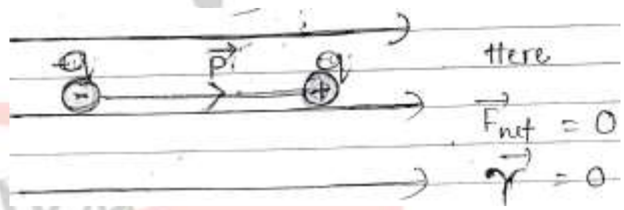
In vector form:- $\boxed{U = -\vec{P}.\vec{E}}$

CONCEPTUAL QUESTION:-

Question – 1:-

(a) When the dipole is said to be in stable equilibrium in an electric field?

Solution:-



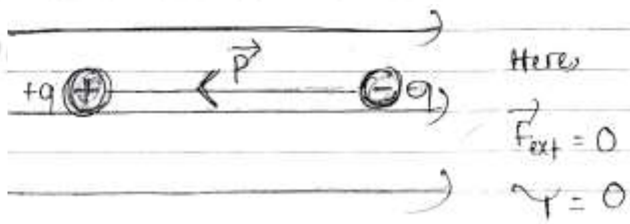
When, $\theta = 0^0$

$U = -\vec{P}.\vec{E} \; = -PE\cos\theta \; = -PE$ (minimum)

Here, $\vec{F}_{net} = 0, \vec{\tau} = 0$

(b) When the dipole is said to be in unstable equilibrium in an electric field?

Solution:-

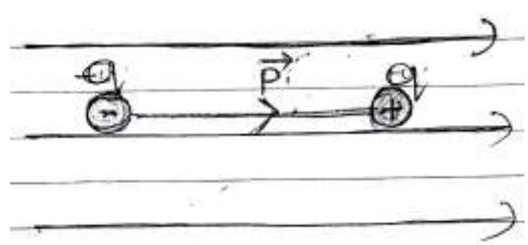


When, $\theta = 180^0$

$U = \vec{P}.\vec{E}$

$= -PE\cos\theta \; = -PE(-1) = PE$ (maximum)

Here $\vec{F}_{ext} = 0, \tau = 0$



Numerical:-Question – 2:-An electric dipole of dipole moment $(\vec{P})$ is placed in an uniform electric field $(\vec{E})$ in

stable equilibrium position. Its moment of inertia about the central axis is I. It is displaced slightly from its mean position. Find the period of small oscillation.

Solution:-

When displaced at an angle θ from its mean position. The magnitude of restoring force.

We know, $\tau = -PE\sin\theta$

For slight displacement:-

$$\sin\theta \approx \theta$$

Therefore, $\tau = -PE\theta$

We know, $\tau = I\propto$

$$\therefore \propto = \frac{\tau}{I} = \frac{-PE\theta}{I}$$

Now, $\alpha = -\omega^2\theta\left[\text{for SHM} \rightarrow \alpha = -\omega^2\theta\right]$

Substituting the values:-

$$\alpha = \frac{-PE\theta}{I}$$

$$\Rightarrow -\omega^2\theta = \frac{-PE\theta}{I}$$

$$\Rightarrow \omega^2 = \frac{PE}{I} \Rightarrow \omega = \sqrt{\frac{PE}{I}}$$

Also $\omega = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{PE}{I}}$

$$\Rightarrow T = 2\pi\sqrt{\frac{I}{PE}}$$

Question – 3 :- Draw the variation of potential energy of an electric dipole in electric field with θ

Unit of Electrostatic Potential Energy:-

SI unit – Joule

Define 1J potential Energy?

Solution:-

The energy which is required to move 1c of charge through a p.d of 1V is called 1J

Other common units:-

1eV – It is the energy gained by electron by moving through a field of p.d of V

Conversion:-

1eV = charge of 1e x 1 volt

$$=\left(1.6\times10^{-19}\text{C}\right)\times1\text{volt}$$

$$=1.6\times10^{-19}\text{C.V}$$

Therefore $1\text{eV}=1.6\times10^{-19}\text{J}$

Now, $1\text{meV}=10^{-3}\text{eV}$

$$=1.6\times10^{-19}\times10^{-3}$$

$$=1.6\times10^{-22}\text{J}$$

Kilo electron volt : $1\text{keV}=10^{3}\text{eV}=1.6\times10^{-19}\times10^{3}\ \ =1.6\times10^{-16}\text{J}$

Mega electron volt:- $1\text{MeV}=10^{6}\text{eV}\ =1.6\times10^{-19}\times10^{6}\ \ =1.6\times10^{-13}\text{J}$

Giga electron volt:- $1\text{GeV}=10^{9}\text{eV}\ \ =1.6\times10^{-10}\text{J}$

Tera electron volt:- $1\text{TeV}=10^{12}\text{eV}\ =1.6\times10^{-7}\text{J}$

Numerical (NCERT Book 2.6)

Question – 4:- A molecule of a substance has a permanent electric dipole moment of magnitude 10^{-29}cm. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude 10^{6}Vm^{-1}. The direction of the field is suddenly changed by an angle of 60^{0}. Estimate the heat released by the substance, in aligning its dipoles along the new direction of the field. For simplicity assume 100% polarization of the sample.

Solution:-

Here, dipole moment of each molecules = 10^{-29}Cm.

As 1 mole of the substance contains $6\times10^{23}\text{mol}$

Total dipole moment of all the molecules, $P = 6\times10^{23}\times10^{-29}\text{Cm}$

$$= 6\times10^{-6}\text{Cm}$$

Initial potential energy $U_t = -PE\cos\theta$

$$= -6\times10^{-6}\times10^{6}\cos\theta = -6\text{J}$$

Final potential energy (when $\theta = 60^{0}$)

$$U_f = -6\times10^{-6}\times10^{6}\cos60^{o} = -6\times\frac{1}{2} = -3\text{J}$$

Change in Potential Energy = $-3\text{J} - (-6\text{J}) = 3\text{J}$

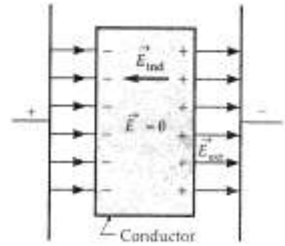
So, there is a loss in potential energy. This must be the energy released by the substance in the form of heat in aligning its dipoles.

ELECTROSTATICS OF A CONDUCTOR

Behaviour of metal conductor in electrostatic field.

Some of the important results regarding electrostatics of conductors are discussed below:-

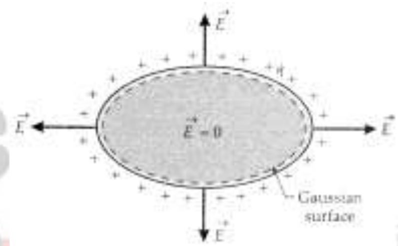
(1) Inside a conductor, electric field is zero



Suppose a conductor ABCD is held in an external electric field of intensity $\vec{E}_0$. Free electrons in the conductor move from AB to CD.

As a result, some net negative charge appears on CD and an equal positive charge appear on AB. These are called induced charges. They produce an induced electric field of intensity $\vec{E}_P$ opposite to the external field.

(2) The interior of a conductor can have no excess charge in static situation:-



Let us consider any arbitrary volume element 'V'

A Gaussian surface is imagined just inside the element

Then according to Gauss Law:-

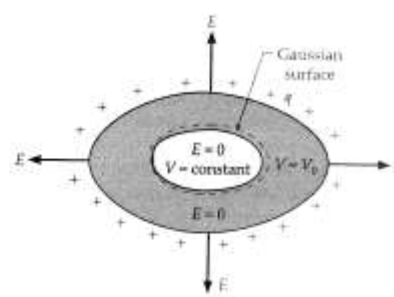
$$\oiint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0}$$

Inside the conductor E = 0

Therefore $q_{in} = 0$

Thus if an excess charge is placed on an isolated conductor the charge move quickly spreads over the surface due to the fact that like charges repel each other.

(3)Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point:-



Under electrostatic conditions, once the charges on a conductor are re-arranged, the flow of charges stops. Therefore component of electric field along the tangent to the surface of the conductor must be zero.

i.e $E\cos\theta = 0$, where θ is the angle which electric field intensity makes tangent to the surface.

As $E \neq 0 \therefore, \cos\theta = 0 \text{ or } \theta = 90^0$

Hence, the field is perpendicular to the surface of the conductor at every point. If a uncharged conductor is placed in external field inside the conductor reduced to zero making the net field at the surface is to the surface.

(4) Electrostatic Potential is constant through the volume of the conductor and has the same value as on its surface:-

As electric field $\vec{E} = 0$, inside the conductor, no work is done in moving a small test charge within the conductor. Therefore, there is no potential difference between any two points inside the conductor; i.e electrostatic potential is constant throughout the volume of the conductor.

Mathematically $\vec{E} = -\frac{dV}{dr}$

Inside the conductor $E = 0$

Therefore $\frac{dV}{dr} = 0 \Rightarrow V = \text{constant}$

Thus the interior of a charged conductor is an equipotential region

Note:-

<u>Surface of the conductor is a equipotential surface.</u>

This is because, there is no electric field along the surface (i.e $E\cos\theta = \theta$). This shows that there is no potential gradient along the surface.

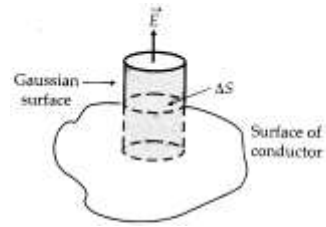
$\frac{dv}{dr} = 0 \Rightarrow \boxed{V = \text{Constant}},$

V is same everywhere on the surface

(5) Electric field at any point close to the charged conductor is σ / ε_0

Let us consider a short cylinder of small area of cross section ds and negligible height partly inside and partly outside the surface of a conductor of surface charge density 'σ'.

Just inside the surface, E = 0



Just outside, the field $\vec{E}$ is normal to the surface. The contribution to the total flux through the cylinder comes only from the outside circular cross section of the cylinder over the small area 'ds' taking E to be constant electric flux $= \pm E(ds)$. Positive for $\sigma > 0$ and negative for $\sigma < 0$,.

As charge enclosed by the element $\sigma.ds$

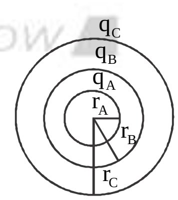
By Gauss theorem, $E(ds) = \frac{\sigma ds}{\varepsilon_0}$

or $E = \frac{\sigma}{\varepsilon_0}$

As electric field is normal to the surface we can write $\boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}}$

Question -1:-In the figure shown, find out the electric potential at A, B and C

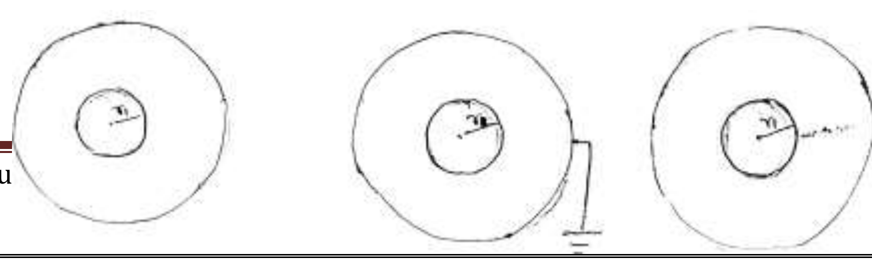
Solution:-



Potential at A $V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_A} + \frac{q_B}{r_B} + \frac{q_C}{r_c}\right]$

Similarly $V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_B} + \frac{q_B}{r_B} + \frac{q_C}{r_C}\right]$ and $V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_C} + \frac{q_B}{r_C} + \frac{q_C}{r_C}\right]$

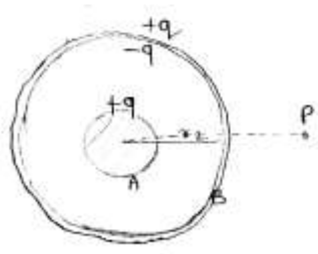
Question - 2:-A thin spherical conducting shell of radius 'r' carries a charge q, concentric with another thin metallic spherical shell of radius $r_2 (r_2 > r_1)$. Calculate the electric potential at point 'p' at a distance r in following cases for $r > r_2, r_1 > r > r_2$ and $r < r_1$.



Solution:

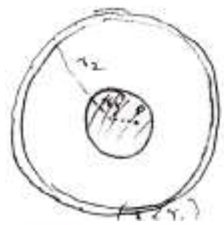
Case – I

$r > r_2$



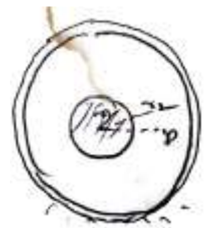
$$V_p = V_A + V_B$$
$$= \frac{kq}{r} + \frac{k(-q+q)}{r}$$
$$= \frac{kq}{r}$$

$(r_1 > r > r_2)$



$$V_p = V_A + V_B$$
$$= \frac{kq}{r} + \frac{k(-q+q)}{r_2}$$
$$= \frac{kq}{r}$$

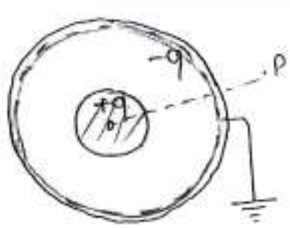
$(r < r_1)$



$$V_p = V_A + V_B$$
$$= \frac{kq}{r_1} + \frac{k(q-q)}{r_2}$$
$$= \frac{kq}{r_1}$$

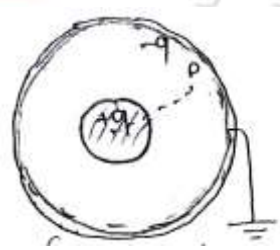
Case – II

$(r > r_2)$



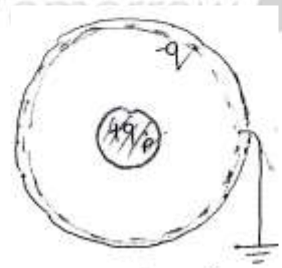
$$V_p = V_A + V_B$$
$$= \frac{kq}{r} - \frac{kq}{r} = 0$$

$(r_1 > r > r_2)$



$$V_P = \frac{kq}{r} + \frac{k(-q)}{r_2}$$

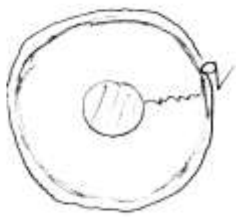
$(r < r_1)$

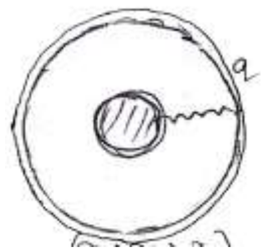


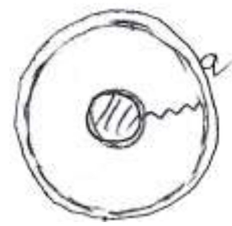
$$V_p = \frac{kq}{r_1} + \frac{k(-q)}{r_2}$$

Case – III

$(r_1 > r > r_2)$ $(r < r_1)$







$$V_p = V_A + V_B$$
$$= 0 + \frac{kq}{r}$$

$$V_p = V_A + V_B$$
$$= \frac{kq}{r_2}$$

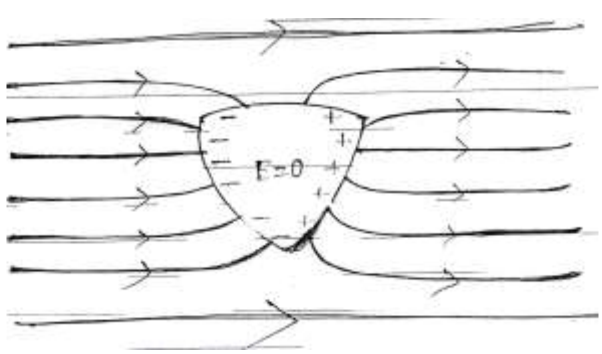
$$V = \frac{kq}{r_2}$$

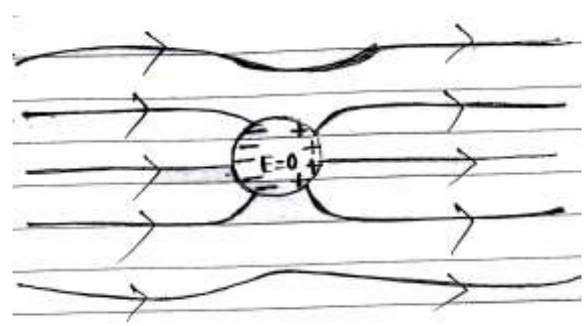
Note:-

In fact field gradually decreases from $\frac{\sigma}{\varepsilon_0}$ to zero in a small thickness of about 4 to 5 atmic layer at the surface.

Conceptual:-

A metallic solid sphere is placed in uniform electric field. Draw the electric field lines inside and outside of sphere.



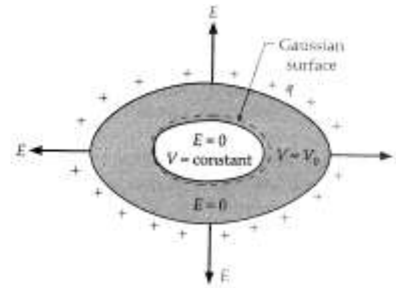


Electrostatic Shielding:-

Definition:- The phenomenon of making a region free from any electric field is called electrostatic shielding. it is based on the fact that electric field varnishes inside the charity of a hollow conductor.

Proof:-

For the Gaussian Surface inside the conductor



$$\oint \vec{E}.\overrightarrow{ds} = \frac{q_{in}}{\varepsilon_0}$$

We know, E = 0 (inside the conductor)

Therefore $q_{in} = 0$

Furthermore if we consider the surface of cavity as Gaussian surface

By Gaussis Theorem,

$$\oint \vec{E}.\overrightarrow{ds} = \frac{q_{in}}{\varepsilon_0} = \frac{0}{\varepsilon_0}$$

E = 0 (inside the cavity)

Applications of Electrostatic Shielding:-

- In a thunderstorm accompanied by lightning, it is safest to sit inside a car, rather than near a tree or on the open ground. The metallic body of the car becomes an electrostatic shielding from lightning.
- Sensitive components of electronic devices are protected or shielded from external electric disturbances by placing metal shields around them.
- In a coaxial cable, the outer conductor connected to ground provides an electrical shield to the signals carried by central conductor.

Note:-

Suppose that there is a conductor inside the cavity which has charge q but insulated from cavity.

Since $\vec{E}=0$ inside the conductor, which gives $q_{in}=0$. As the charge inside the cavity is q. So there must be a charge on cavity wall equal and opposite sign to q. If outer phase is initially unchanged there must be change of +q.

DIELECTRICS AND THEIR POLARIZATION:

Dielectric:- Dielectrics are insulating materials which transmits electric effect without actually conducting electricity.

Classification:- (1) Polar

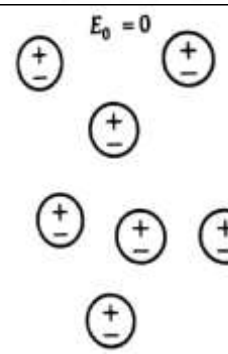
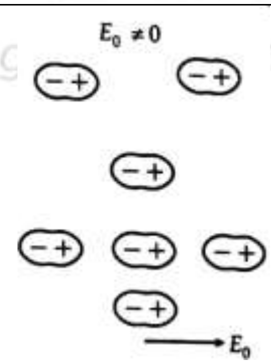
(2) Non-Polar

Non-Polar Dielectric

In such dielectrics the centre of mass of +ve charge coincides with C.M of –ve charge in the molecule.

Example:- Hydrogen nitrogen, oxygen, CO_2, Benzene, methane

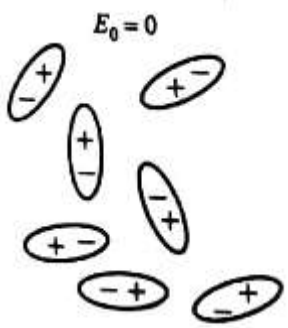
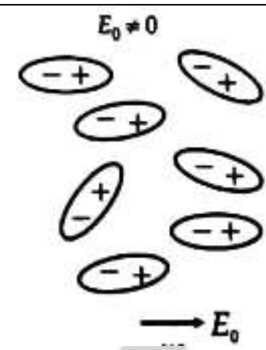
(Note:- Such molecules are symmetric)

 Each molecule has zero dipole moment in its normal state	 In presence of E the C.M of the charges are displaced in the direction of external field while C.M of –ve charges are displaced in the opposite direction. These induced dipole moments are add up to give net dipole moment.

Polar Dielectric:-

Such dielectric are made up of polar molecules

Example:- Water HCl NH_3, Alcohol, etc

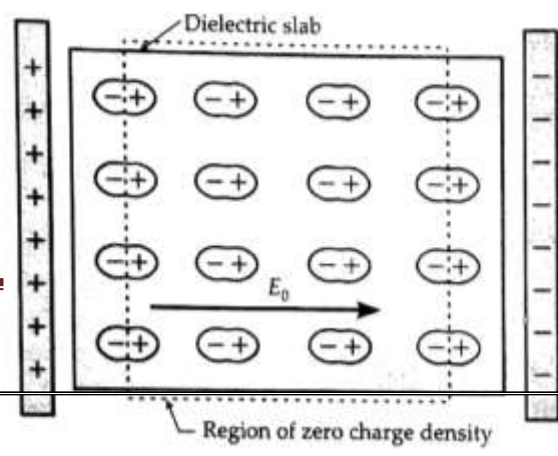
 	
➢ Under normal condition the cm of +ve and C.M of –ve charges do not coincide because of the asymmetric shape of the molecules ➢ Each molecule has a spontaneous or permanent electric dipole moment	➢ In the presence of an external electric field the cm of +ve charges and cm of –ve charges are displaced. ➢ The induced dipole moments of different molecules are added up giving a net dipole-moment to the dielectric in the direction of $\vec{E}$

Note:-

- ➢ Its magnitude refer to polarization density.
- ➢ Direction of 'P' is same as that of external field

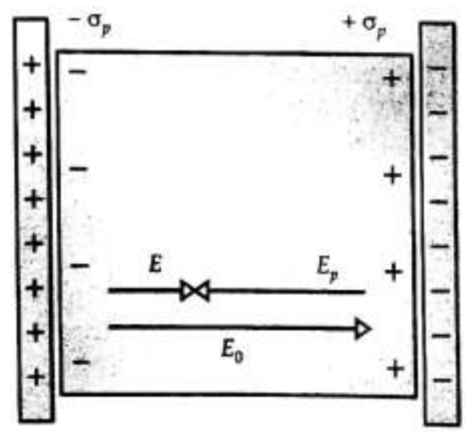
Conceptual Question:-

Question – 1:- Explain why the polarization of dielectric reduces the electric field inside the dielectric. Hence define dielectric constant.



Solution:- Consider a rectangular dielectric slab placed in a uniform electric field $\vec{E}_0$ acting parallel to two of its faces.

- Its molecular dipoles align themselves in the direction of $\vec{E}_0$. This results in uniform polarization of the dielectric.
- The positive charges of the dipoles of first vertical column cancel the negative charges of the dipoles of the second column and so on.
- Thus the volume charge density in the interior of the slab is zero. However there is a net uncancelled charge densities in the two both.
- The uncancelled charges are the induced surface charges due to the external field $\vec{E}_0$. Since the slab as a whole remaining electrically neutral, the magnitude of the induced positive charge is equal to that of negative induced charge.



Reduced field in a dielectric

$$E = E_0 - E_P$$

The field produced by the induced charge is opposite to external field. Therefore the total field in a dielectric is reduced from the case, when no dielectric is present.

Induced dipole moment P acquired by the single polar molecule may be written as

$$P = \alpha E_0 \varepsilon_0$$

where α is called atomic/ molecular polarizability.

$$\boxed{\alpha = \frac{P}{\varepsilon_0 E_0}}$$

unit of $\alpha = \dfrac{Cm}{(C^2N^{-1}m^{-2})(NC^{-1})} = m^3$

Dielectric constant: -

The ratio of the original field $\vec{E}_0$ and the reduced field $\vec{E}_0 - \vec{E}_P$ in the dielectric is called dielectric constant (χ)

Thus $$\chi = \frac{|\vec{E}_0|}{|\vec{E}_{net}|} = \frac{|\vec{E}_0|}{|\vec{E}_0 - \vec{E}_P|}$$

Electric Susceptibility:-

Thus the ratio of the polarization to ε_0 times the electric field is called the electric susceptibility of the dielectric.

Physical Significance:-

Like 'P' it also describes the electrical behavior of dielectric. The dielectrics with constant x are called linear dielectric.

Expression for Electric Susceptibility:-

If the field $\vec{E}$ is not large. Then the electric polarization $\vec{P}$ is proportional to the resultant field $\vec{E}$) existing in the dielectric i.e $\vec{P} \propto \vec{E}$

or $\vec{P} = \varepsilon_0 \chi_E \vec{E}$

Where χ_e (chi) is a proportionality constant called electric susceptibility. The multiplication factor ε_0 is used to keep x dimension less. Clearly,

$$\chi_e = \frac{|\vec{P}|}{|\varepsilon_0 \vec{E}|}$$

Note:-

Polarisation of vacuum = 0

Thus for vacuum χ = 0

Relation between polarization density and induced surface charge density:-

Suppose a dielectric slab of surface area 'A' and thickness 'd' acquires a surface charge density $\pm\sigma_P$ due to polarization in the electric field and its two faces acquire charges $\pm Q_P$. Then

$$\sigma_P = \frac{Q_P}{A}$$

We can consider the whole dielectric slab as a large dipole having dipole moment to $Q_P d$. The dipole moment per unit volume or the polarization density will be:-

$$P = \frac{\text{dipole moment of dielectric}}{\text{volume of dielectric}}$$

$$= \frac{Q_P d}{Ad} = \frac{Q_P}{A} = \sigma_P$$

<u>Dielectric Strength:-</u>

The maximum electric field that can exists in a dielectric without causing the breakdown of its insulating property is called dielectric strength of the material.

Unit:- Vm^{-1}. But the more common practical unit is $(kV)(mm^{-1})$

Note:- For air it is about $3\times10^6 Vm^{-1}$

Relation between dielectric constant and electric susceptibility of the material:-

$E = E_0 - E_P$ ....(1)

But $E_P = \frac{\sigma_P}{\varepsilon_0} = \frac{P}{\varepsilon_0}$

Thus from (1)

$$E = E_0 - \frac{P}{\varepsilon_0} = E_0 - \frac{\varepsilon_0 \chi_e E}{\varepsilon_0} = E_0 - \chi_e E$$

$\Rightarrow E_0 = E + \chi_e E$

$\Rightarrow E_0 = E(1+\chi_e)$

$\Rightarrow \frac{E_0}{E} = 1+\chi_e$

$\boxed{\chi = 1+\chi_e}$

CAPACITOR AND CAPACITANCE:

It is an arrangement, which can store more electric charge or potential energy in a small space compared to an isolated conductor.

Capacitance of an isolated conductor: -

When a conductor is given some q charge, it spreads over its outer surface. Hence its potential increases

Thus $V \propto q$

$\Rightarrow V = \frac{1}{C}q$ (where $\frac{1}{C}$ is called proportionality constant)

$\Rightarrow C = \frac{q}{V}$ ....(1)

Here constant C is called capacitance of conductor

Definition of capacitance:-

From equation (1)   when V = 1, then C = q

Thus capacitance of a conductor is the charge required to increase its potential by unity.

Units of capacitance:-

S.I Unit is Farad (F)

If 1C of charge is required to increase the potential by 1 volt the capacitance of conductor is said to be 1 Farad.

Thus,

1 Farad = $\frac{1\text{coulomb}}{1\text{volt}}$

Note: - 1 F is very large quantity. Generally smaller units like $\mu F, nF, PF$ are used.

Micro Farad $1\mu F = 10^{-6} F$

Nano Farad $1nF = 10^{-9} F$

Pico Farad $1PF = 10^{-12} F$

Dimensional formula

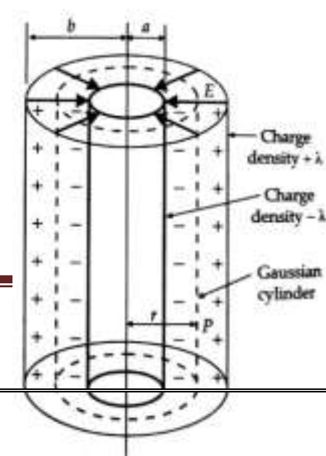
$$C = \frac{Q}{V} = \frac{Q}{W/Q} = \frac{Q^2}{W} = \frac{[AT]^2}{[ML^2T^{-2}]} = [M^{-1}L^{-2}T^4A^2]$$

Capacitance of spherical capacitor: -

Isolated sphere	Isolated earthed sphere
 $V = \frac{kq}{R}$, $C = \frac{q}{V} = 4\pi\varepsilon_0 R$	 $V = 0$, $C = \frac{q}{0} = \infty$

Exercise (NCERT – 2.32):-

Question-1: A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the



inner cylinder is given a charge of $3.5\mu C$. Determine the capacitance of the system.

Solution:-

Applying Gauss's law. Electric field in between plates

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ .... (1)}$$

$\therefore$ P.D between plates are

$$V = -\int_b^a E.dr$$

$$= -\int_b^a \frac{\lambda}{2\pi\varepsilon_0 r}.dr$$

$$= -\int_b^a \frac{Q}{2\pi\varepsilon_0 \ell}\frac{dr}{r}$$

$$= -\frac{Q}{2\pi\varepsilon_0 \ell}[\log r]_b^a$$

$$= -\frac{Q}{2\pi\varepsilon_0 \ell}(\log a - \log b)$$

$$= \frac{Q}{2\pi\varepsilon_0 \ell}(\log b - \log a)$$

$$\boxed{V = \frac{Q}{2\pi\varepsilon_0 \ell}\ln\left(\frac{b}{a}\right)} \text{ ....(2)}$$

Thus, $C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi\varepsilon_0 \ell}\ln\left(\frac{b}{a}\right)}$

$$\boxed{C = \frac{2\pi\varepsilon_0 \ell}{\ln\left(\frac{b}{a}\right)}} \text{ ....(3)}$$

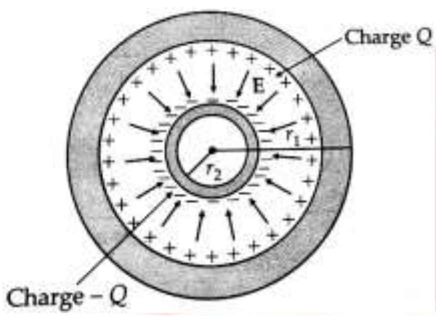
Question-2: Find out the capacitance of spherical concentric capacitor

(a) With earthed outer sphere

(b) With earthed inner sphere

Solution: -

(a) Here $q' = 0$



Because $V_B = 0$

$$\Rightarrow \frac{kQ}{b} + \frac{k(-Q+q')}{b} = 0$$

$$\Rightarrow q' = 0$$

∴ P.D between plates

$$V = V_A - V_B$$

$$V = \left[kQ\left[\frac{1}{a} - \frac{1}{b}\right] - 0\right]$$

$$C = \frac{Q}{V}$$

$$C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

(b) Left to the students

Question-3: Find the radius of an isolated spherical capacitor to achieve the capacitance of 1 micro farad.

Solution:-

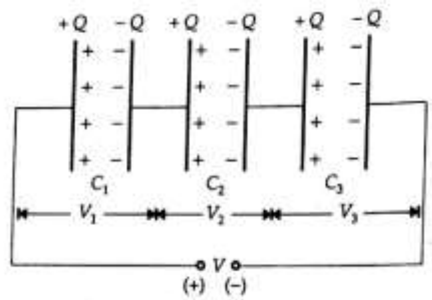
$$C = 4\pi\varepsilon_0 r$$

$$\Rightarrow r = \frac{C}{4\pi\varepsilon_0} = 1\times10^{-6}F\times9\times10^{9}$$

$$= 9\times10^{3}m = 9km$$

COMBINATIONS OF CAPACITORS

In series:



Here the magnitude of charge on all the plates same but the potential is distributed in the inverse ratio of the capacity.

$$\therefore V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$$

$$V = V_1 + V_2 + V_3$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \text{ .... (1)}$$

If we regard the combination as an effective capacitor with charge A and P.D 'V' then the effective capacitance of the combination.

$$C_s = \frac{Q}{V} \Rightarrow V = \frac{Q}{C_S} \text{ ....(2)}$$

Comparing equations (1) and (2)

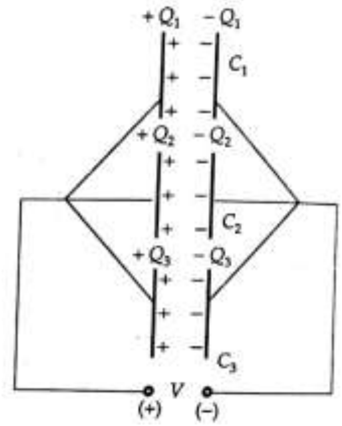
$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\Rightarrow \boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

For n no of capacitors $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$

For n identical $C_S = \frac{C}{n}$

Capacitors in Parallel:-



Here the P.D for all individual capacitors is same but the total charge 'Q' is distributed in the ratio of their capacitance.

$\therefore Q_1 = C_1V, Q_2 = C_2V, Q_3 = C_3V$

$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$ ....(1)

If C_P is the equivalent capacitance of the parallel combination

then $Q = C_PV$ ....(2)

Comparing equation (1) and (2)

$$C_PV = C_1V + C_2V + C_3V$$

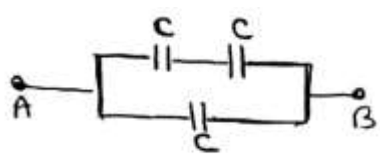
$$\Rightarrow \boxed{C_P = C_1 + C_2 + C_3}$$

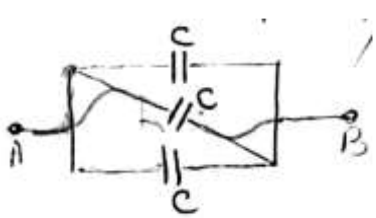
For n no of capacitors

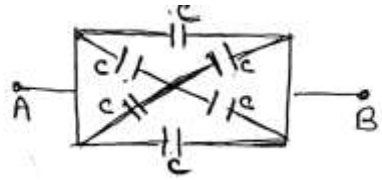
$$\boxed{C_P = C_1 + C_2 + C_3 + ... + C_n}$$

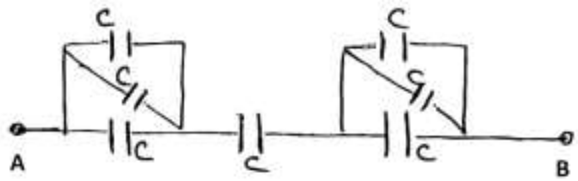
For n identical capacitors $\boxed{C_P = nC}$

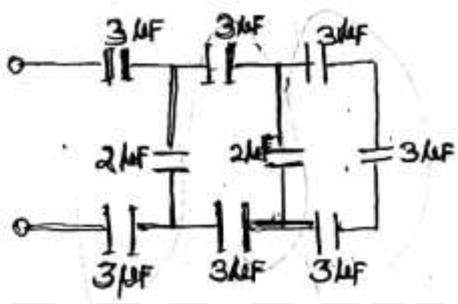
Question-1: Find the equivalent capacitance between points A and B

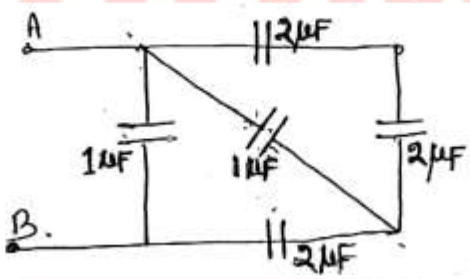
a) 

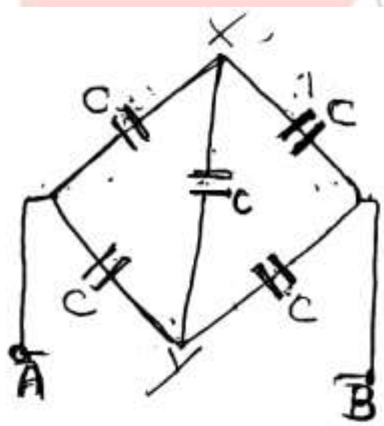
b) 

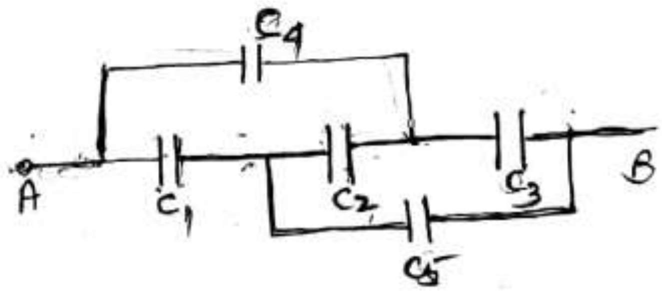
c) 
C
C
C
C
C
C
A
B

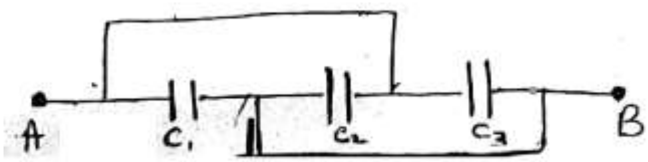
d) 
C
C
C
C
C
C
C
A
B

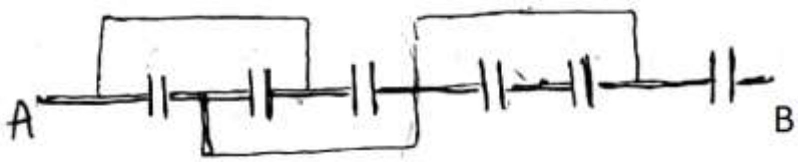
e) 
3μF
3μF
3μF
2μF
2μF
3μF
3μF
3μF
3μF

f) 
A
2μF
1μF
1μF
2μF
B
2μF

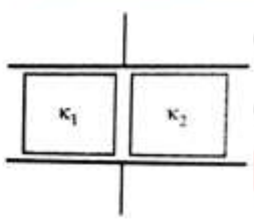
g) 
X
C
C
C
C
C
Y
A
B

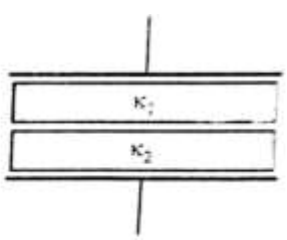
h) 

i) 

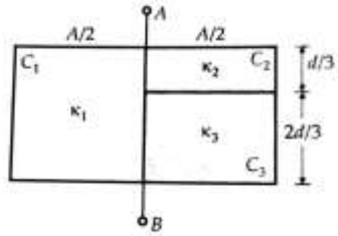
j) 

Question-2: Find the equivalent capacitance between points A and V

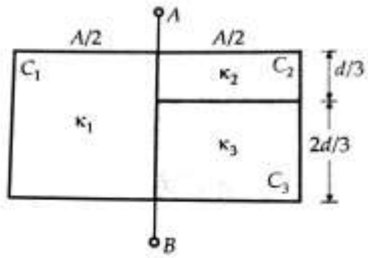
a) 

b) 

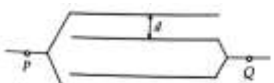
c)



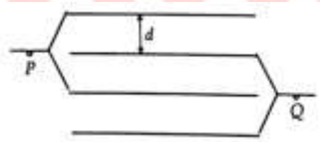
d)



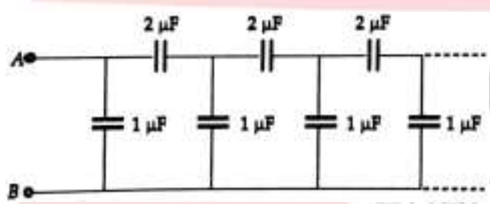
e)



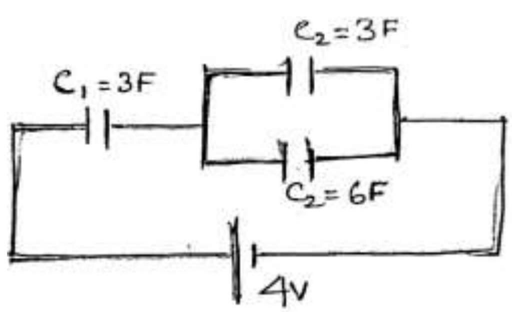
f)



g)



Question-3: In the circuit diagram find out the charge a capacitor C_1, C_2 and C_3.



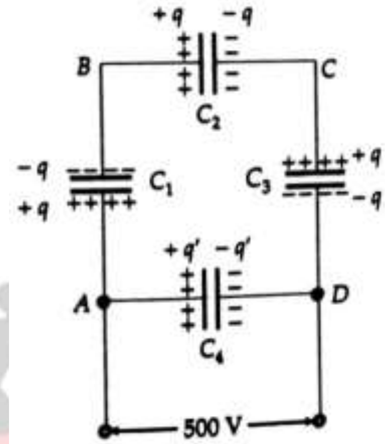
Solution:-

$$C_{eq} = \frac{q}{4} F$$

$$Q = V \times C_{eq} = qc$$

Thus $C_1 = 9C, C_2 = 3C, C_3 = 6C$

Question-4: Determine (a) the equivalent capacitance of the network
(b) Charge on each capacitor of the net work



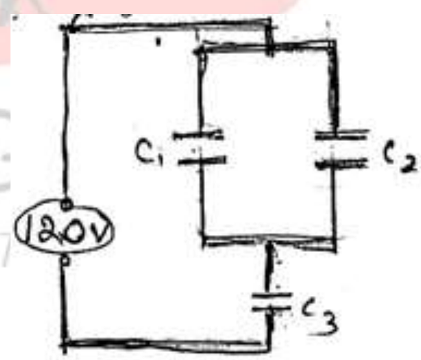
Solution: -

(a) $C_{eq} = 13.3\mu F$

(b) $Q = V \times C_{eq} = 500 \times 13.3\mu F$

Question-5: In the figure $C_1 = 10\mu F, C_2 = 20\mu F, C_3 = 15\mu F$.Find out the P.D across the capacitor

Solution: -



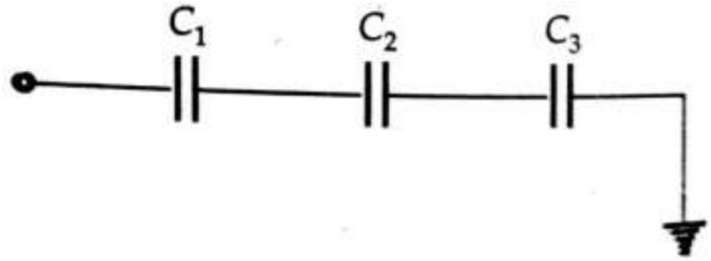
$$V_1 + V_2 = 120$$

$$2x + x = 120$$

$$x = 40$$

P.D across $C_3 = 40 \times 2 = 80$ volt

Questions-6: In the diagram find P.D between the plate C_2.



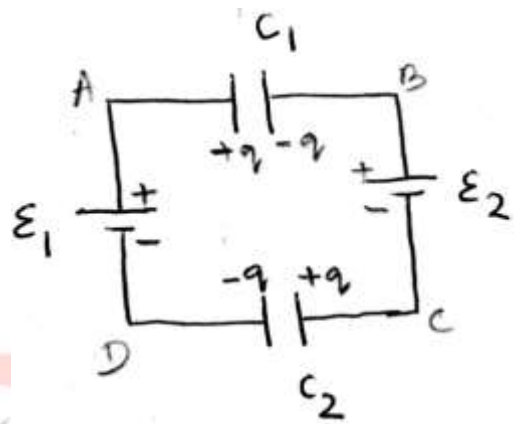
Solution: -

$$Q = C_{eq} \times V_{diff}$$

$$= \frac{20\mu F}{3} \times (90-0) = \frac{20}{3} \times 10^{-6} \times 90 = 600 \times 10^{-6} C$$

P.D across $C_2 = \frac{600 \times 10^{-6}}{30 \times 10^{-6}} = 20V$

Questions-7: In the given figure if $\varepsilon_1 > \varepsilon_2$ find potential difference across $C_1 \text{and} C_2$.



Solution:-

In DABCD Loop

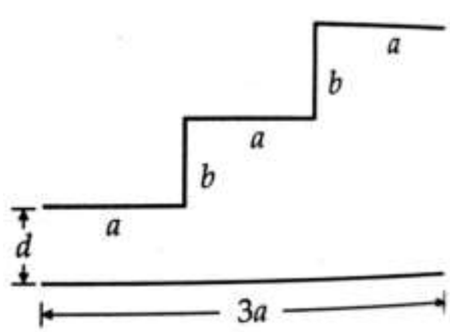
$$-\varepsilon_1 + \frac{q}{C_1} + \varepsilon_2 + \frac{q}{C_2} \Rightarrow q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \varepsilon_1 - \varepsilon_2$$

$$\Rightarrow q = \left(\frac{C_1 C_2}{C_1 + C_2}\right)(\varepsilon_1 - \varepsilon_2)$$

$$\therefore V_1 = \frac{q}{C_1} = \frac{C_2(\varepsilon_2 - \varepsilon_1)}{C_1 + C_2}$$

$$\therefore V_2 = \frac{q}{C_2} = \frac{C_1(\varepsilon_2 - \varepsilon_1)}{C_1 + C_2}$$

Questions-8: Find the capacitance of the capacitor shown in the figure?



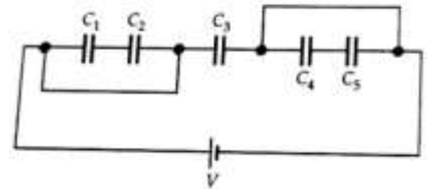
Solution:-

$$C = \frac{\varepsilon_0 A/3}{d} + \frac{\varepsilon_0 A/3}{2d} + \frac{\varepsilon_0 A/3}{3d}$$

$$= \frac{\varepsilon_0 A}{d}\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{9}\right)$$

$$= \frac{\varepsilon_0 A}{d}\left(\frac{18+9+6}{54}\right) = \frac{11\varepsilon_0 A}{18d}$$

Questions-9: In the given figure. Find C_{eq} and q and energy stored.



Solution:-

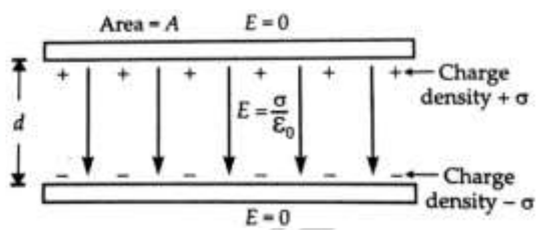
$$C_{eq} = C_3 = 2\mu F$$

As others are shorted

$$q = 5 \times 2 = 10\mu C$$

$$U = \frac{1}{2} \times 2\mu F \times (5V)^2 = 25\mu J$$

PARALLEL PLATE CAPACITOR:



A – Plate area

d – distance between plates

Outer face of 2nd plate is earthed because if charged conductor is placed near a earthed conductor its capacity increases.

Case – I : When only air is between Electric field in inner region between plates :

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

But the $V = Ed = \frac{Q}{A\varepsilon_0} d$

$$\therefore C = \frac{Q}{V} = \frac{Q}{Qd / A\varepsilon_0} \Rightarrow C_{air} = \frac{A\varepsilon_0}{d}$$

Case – II : When there is Di-electric in between the plates

$$C_{med} = \frac{A\varepsilon}{d} = \frac{A\varepsilon_0\varepsilon_r}{d} = \frac{A\varepsilon_0 k}{d}$$

$$\boxed{\frac{C_{med}}{C_{air}} = k}$$

Question-1: Justify that 1F too big unit in practice

OR

Calculate the area of the plates needed to have capacitance of 1F for separation of 1cm.

Solution: Given that

C=1F

d = 1cm=10^2m

C = $\frac{A\varepsilon_0}{d}$

A= $\frac{Cd}{\varepsilon_0}$ = $\frac{1\times10^{-2}}{8.8\times10^{-12}}$

A = $\frac{1}{8.85\times10^{10}} \simeq 10^9 m^2$

Which is the plate about 30km in length and breadth.

Capacitance of a parallel plate capacitor with a dielectric slab:

Let us consider a parallel plate capacitor of plates A and B, each of area A and distance of separation d between the plates. Let charge on plate A be +q and that of plate B be -q Let a dielectric slab of thickness t and dielectric constant K be introduced in the space between the plates (Let t < d). So the region between the plates with vacuum has width (d - t).

Now the electric field at any point in the vacuum region between the plates has magnitude

$E_0 = \frac{\sigma}{\varepsilon_0}$ .... (i) ($\sigma = \frac{q}{A}$ = surface charge density)

Now the dielectric slab will be polarized. So, a bound charge q_P is gathered at the surfaces of the slab.

The bound charge density has magnitude $\sigma_P = \frac{q_P}{A}$

So an electric field is induced within the dielectric slab opposite to the field E_0.

This is given by, Ep = $\frac{\sigma_P}{\varepsilon_0}$ ....(ii)

So net field inside the dielectric slab is $E = \frac{E_0}{K} = E_0 - E_P$...........(iii) (By definition $K = \frac{E_0}{E}$)

Potential difference between the plates i.e. potential of the capacitor is

$$V = Et + E_0(d - t) = \frac{E_0}{K}t + E_0(d - t) = E_0\left(d - t + \frac{t}{K}\right) = \left(d - t + \frac{t}{K}\right) = \frac{q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$$

Capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{q}{\frac{q}{A\varepsilon_0}(d - t + \frac{t}{K})} = \frac{A\varepsilon_0}{d - t + \frac{t}{K}}$$ ....(iv) (this is the expression for capacitance)

$$\Rightarrow C = \frac{\varepsilon_0 A/d}{1 - t/d + t/Kd} = \frac{C_0}{1 - t/d + t/Kd}$$

Where $C_0 = \frac{\varepsilon_0 A}{d}$ = capacitance of the capacitor when space between the plates is vacuum

If space between the plates is filled with dielectrics i.e. t = d, then eq.(iv)

gives, $C_m = \frac{KA\varepsilon_0}{d}$

$$\frac{C_m}{C_0} = \frac{\frac{KA\varepsilon_0}{d}}{\frac{A\varepsilon_0}{d}}$$

$$\frac{C_m}{C_0} = K$$

Since $K\varepsilon_r \geq 1$

$$\Rightarrow \frac{C_m}{C_0} \geq 1$$

Thus, capacitance of parallel plate capacitor increases due to introduction of dielectric slab between its plates (keeping the charge to be constant).

Note-1: Here electric field hence P.D decreases by a factor k (di electric constant)

$\therefore E = \frac{E_0}{K}$ and $V = \frac{V_0}{K}$ [Here E = Reduced Field = E_0 - E_P]

Note-2: Induced charge in di electric is given by

$q_i = q[1 - \frac{1}{K}]$ [for metallic K = infinity, $q_i = q$]

NUMERICALS

Question-1: A dielectric slab (di electric constant =k) is introduced between the plates of a charged air capacitor when battery remain connected what happens to

i. P.D between plates
ii. Electric field
iii. Capacitance
iv. Charge
v. Electrostatic potential energy

Solution

i. V becomes constant
ii. $E = \frac{V}{d} \Rightarrow E \propto \frac{1}{d}$, since d=constant so E=constant
iii. $\frac{C_m}{C_0} = K \Rightarrow C_m = KC_0$, capacitance becomes K times
iv. $q = CV \Rightarrow q \propto C$, thus charges will increase to K times.
v. $U = \frac{1}{2}CV^2 \Rightarrow U \propto C$, thusU will increase to K times.

Question-2: A di electric slab of di electric constant k is introduced between the plates of a charged air capacitor when the battery is disconnected , what happens to its

i. Electric charge
ii. P.D
iii. Capacitance
iv. Field
v. Electrostatic potential energy

Solution:

i. q=constant

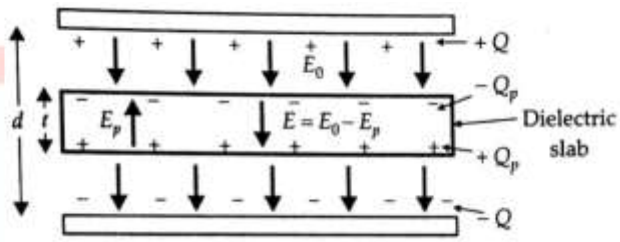
ii. $V = \frac{q}{C} \Rightarrow V \propto \frac{1}{C}$ therefore, V becomes 1/K times of its initial value

iii. $C = \frac{KA\varepsilon_0}{d} \Rightarrow C \propto \frac{K}{d}$

iv. If q=constant, $E = \frac{E_0}{K} \Rightarrow E \propto \frac{1}{K}$

v. If q=constant, $U = \frac{1}{2}\frac{q^2}{C} \Rightarrow U \propto \frac{1}{C}$ i.e. decreases by K times

CAPACITANCE OF A CAPACITOR PARTIALLY FILLED WITH DIELECTRIC:



Let E_0 is the electric field in the region where dielectric is absent. Therefore electric field inside the dielectric will be $E = \frac{E_0}{K}$

Potential difference between the plates i.e. potential of the capacitor is

$$V = Et + E_0(d - t) = \frac{E_0}{K}t + E_0(d - t) = E_0\left(d - t + \frac{t}{K}\right) = \left(d - t + \frac{t}{K}\right) = \frac{q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$$

Capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{q}{\frac{q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)} = \frac{A\varepsilon_0}{d - t + \frac{t}{K}}$$ ....(iv) (this is the expression for capacitance)

$$\Rightarrow \boxed{C = \frac{\varepsilon_0 A/d}{1 - t/d + t/Kd} = \frac{C_0}{1 - t/d + t/Kd}}$$

Note-1: If more than one di electric slabs are placed between the plates of capacitor.

$$C = \frac{\varepsilon_0 A}{(d - t_1 - t_2 ... t_n) + (\frac{t_1}{K_1} + \frac{t_2}{K_2} ... \frac{t_n}{K_n})}$$

Note-2: Ifspace between the plates is filled with dielectrics i.e. t = d , then

$$C = \frac{KA\varepsilon_0}{d}$$

Note-3: If a conducting slab ($K = \infty$) partially fills between plates, then

$$C = \frac{A\varepsilon_0}{d - t}$$

Note-4: If the metal slab fills the space between the plates i.e. t = d then

$$C = \frac{A\varepsilon_0}{0} = \infty$$

Question-1: A slab of material of dielectric constant 'K' has the same area as the plates of parallel plate capacitor, but has thickness 3d/4, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates.

Solution: When there is no dielectric

$$V_0 = E_0 d$$

But in presence of dielectric

$$V = Ed = E_0(d - t) + Et$$

$$\Rightarrow V = E_0(\frac{d}{4}) + E(\frac{3d}{4}) = E_0(\frac{d}{4}) + \frac{E_0}{K}(\frac{3d}{4})$$

$$\Rightarrow V = V_0(\frac{K+3}{4K})$$

$$\therefore C = \frac{Q_0}{V} = \frac{4K}{K+3}(\frac{Q_0}{V_0}) = \frac{4K}{K+3}C_0$$

ENERGY STORED IN A CAPACITOR:

Work has to be done in charging conductor against the force of repulsion by the already existing charge on it.

This work is stored as potential energy in the electric field of the conductor.

Suppose a conductor of capacity C is charged to a potential V and the charge at that instant be q..Therefore potential of the conductor V= $\frac{q}{C}$

Now the work done in bringing small charge dq at this potential is

dW=V dq=($\frac{q}{C}$) dq

therefore total work done in charging it from 0 to q is given by

$$W= \int_0^W dW = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$$

This work is stored as potential energy.

Thus $U = \frac{1}{2}\frac{Q^2}{C}$ ....(1)

Further by using Q = CV in eqn (1) we have

$$U = \frac{1}{2}\frac{(CV)^2}{C}$$

$$\boxed{U=\frac{1}{2} CV^2} \quad ....(2)$$

Again $C = \frac{Q}{V}$

From eqn (2) $U = \frac{1}{2} \left(\frac{Q}{V}\right)V^2$

$$\boxed{U = \frac{1}{2} QV} \quad ....(3)$$

Again, for parallel plate capacitor

$E = \frac{\sigma}{\varepsilon_0}$

$\sigma = \varepsilon_0 E$

and $Q = \sigma A = \varepsilon_0 E A$

Capacitance $C = \frac{A\varepsilon_0}{d}$

Putting the volume of Q and C in equation (1)

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(\varepsilon_0 EA)^2}{\frac{A\varepsilon_0}{d}} = \frac{1}{2}\varepsilon_0 E^2 Ad$$

Thus, $\boxed{U = \frac{1}{2}\varepsilon_0 E^2 Ad}$ ……………….(4)

In general if a conductor of capacity C is charged to a potential V by giving it a charge Q then.

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\varepsilon_0 E^2 Ad}$$

Conceptual Question:-

Question-1: In which form energy is stored inside a capacitor?

Answer: In form of electric field

Question -2: Where is the energy stored in a parallel plate capacitor?

Answer: In between the plates (space)

Question-3: A dielectric is introduced between plates of parallel plate capacitor. Has it any effect on the force?

Answer: No. Because, induced charges on opposite faces of dielectric are equal and opposite and the electric field in the vicinity of the plates depends only on net charge.

Note

Total energy stored in series combination or parallel combination of capacitors is equal to sum of the energies stored in individual capacitor.

i.e. $U = U_1 + U_2 + U_3 + \ldots$

Question-1: An unknown capacitor is connected to battery. Show that half of its energy supplied by the battery is lost as heat while charging the capacitor.

Solution: The work done by the battery in charging a capacitor

$$W = QV$$

But energy stored in the capacitor

$$U = \frac{1}{2}QV$$

Remaining energy = $QV - \frac{1}{2}QV = \frac{1}{2}QV$ is lost as heat radiation.

Thus, $\boxed{W_{ExternalSource} = 2U}$

Question-2: Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $\frac{1}{2}QE$ .where Q is the charge in the capacitor and E is the magnitude of the electric field between the plates.

Solution: Force between the plates of the capacitor

$$F = -\frac{du}{dx} = -\frac{d}{dx}\left(\frac{\varepsilon_0 E^2 Ad}{2}\right) = -\frac{1}{2}\varepsilon_0 E^2 A = \frac{1}{2}(\varepsilon_0 EA)E$$

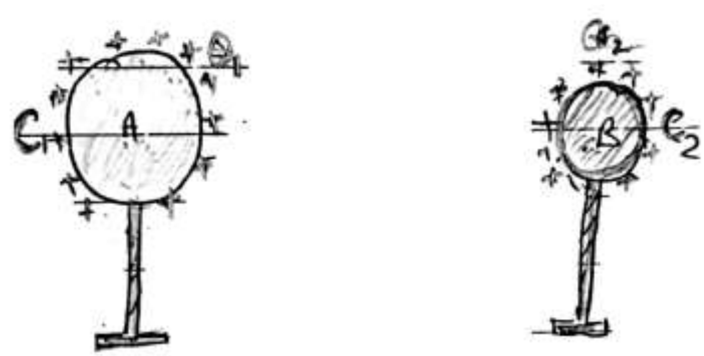
$\Rightarrow F = \frac{1}{2}QE$ (as $Q = \varepsilon_0 EA$)

Thus, $\boxed{F = \frac{1}{2}QE = \frac{1}{2}\varepsilon_0 E^2 A = \frac{Q^2}{2\varepsilon_0 EA}}$

Question-3: When two charged conductors having different capacity and different potentials are joint together. Show that there is always a loss of energy.

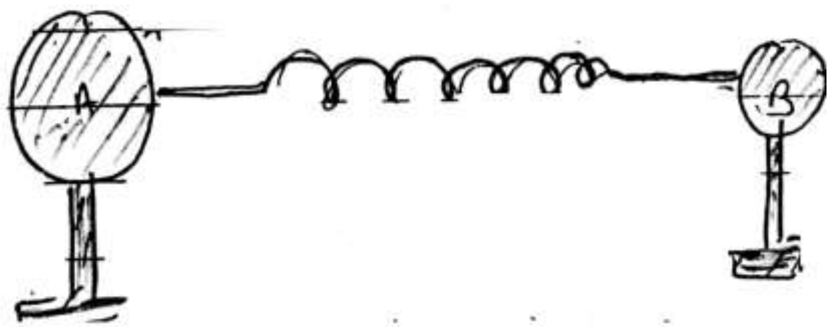
Solution: Sharing of charges

If two capacitors C1 and C2 at potential differences V1 and V2 respectively are connected in parallel then they share charge till both attain equal potential V .



Charges on capacitors before sharing are
$q_1 = C_1 V_1$ and $q_2 = C_2 V_2$

Charges on capacitors after sharing are $q'_1 = C_1 V$ and $q'_2 = C_2 V$



By law of conservation of charge $q_1 + q_2 = q'_1 + q'_2$

$$\Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$$

$$\Rightarrow V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$ (This is the expression for common potential .)

Loss of energy during sharing of charge

$\Delta U = U_i - U_f =$ total potential energy before sharing - total potential energy after sharing

$$= \frac{1}{2}\left[\left(C_1V_1^2 + C_2V_2^2\right) - \left(C_1 + C_2\right)V^2\right]$$

$$= \frac{1}{2}\left[\left(C_1V_1^2 + C_2V_2^2\right) - \left(C_1 + C_2\right)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2\right]$$

$$= \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_1C_2V_2^2 + C_2C_1V_1^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right]$$

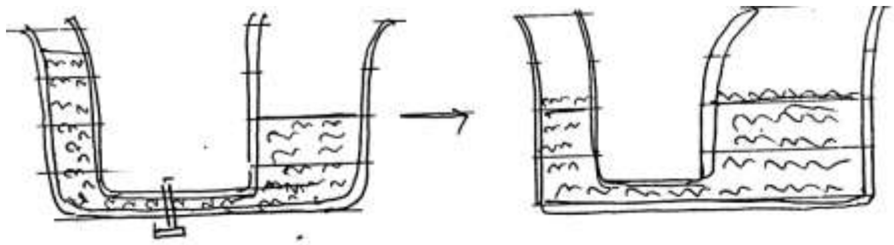
$$= \frac{1}{2}\left[\frac{C_1C_2V_2^2 + C_2C_1V_1^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right]$$

$$= \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)\left(V_2^2 + V_1^2 - 2V_1V_2\right)$$

$$= \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)\left(V_1 - V_2\right)^2$$

Now as C_1C_2 and $(V_1 - V_2)^2$ are always positive, $U_i > U_f$ i.e. there is a decrease in energy. Hence, energy is always lost in redistribution of charge.

Note: redistribution of charge is analogous to the following example.



When the value is open, the level in both the vessels become equal but the volume of liquid in the right vessel is more than the left vessel.

Question-1:

a) A 900 PF capacitor is charged by a 100 V battery. How much electrostatic energy is stored by the capacitor?

b) The capacitor is disconnected from the battery and connected to another 900 PF capacitor. What is the electro static energy stored.

c) Where has the remainder of the energy gone?

Solution:

a) $U_i=(1/2)CV^2$

$$=\frac{1}{2}\left(900x10^{-12}\right)(100)^2 = 4.5x10^{-6}\,J$$

b) After connection the common potential will be

$$V_{common} = \frac{Q_1+Q_2}{C_1+C_2} = \frac{CV+0}{900+900} = \frac{1}{2}x10^2\,volts$$

Now the final energy stored $U_f = \frac{1}{2}(C_1V_1^2 + C_2V_2^2) = 2.25x10^{-6}\,J$

c) Loss of energy = $\Delta U = U_i - U_f = (2.25x10^{-6} - 0.5x10^{-6}) = 1.75x10^{-6}\,J$

Question-2: A 600 PF capacitor is charged by a 200 V supply. It is then disconnected from the supplier and is connected to another 600 PF capacitor. How much electro static energy is lost in the process.

Solution: -

$C_1 = C_2 = 600PF$

$V_1 = 200V, V_2 = 0$

Putting the formula for energy loss $= \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2 = 6x10^{-6}J$

Question-3: A $4\mu F$ capacitor is charged by 200 V supply. It is then disconnected from the supply and is connected to another uncharged $2\mu F$ capacitor. How much electro static energy of the first capacitor is dissipated in the form of heat and electromagnetic radiation?

Solution: -

$U_i=(1/2)CV^2$

$= \frac{1}{2}\left(4x10^{-6}\right)(200)^2 = 8x10^{-2}J$

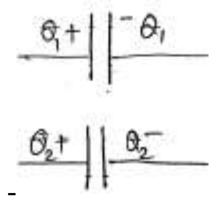
After connection $V_{common} = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+0}{C_1+C_2} = \frac{800}{6} volts$

Final electro static energy $U_f = \frac{1}{2}(C_1V_1^2 + C_2V_2^2) = 5.33x10^{-2}J$

Energy dissipated = $\Delta U = U_i - U_f = 2.67x10^{-2}J$

Question-4: Two parallel plate condenser A and B having capacitances of $1\mu F$ and $5\mu F$ are charged separately to the same potential of 100 V. Now the positive plate of A is connected to negative plate of B. And the negative plate of A is connected to positive plate of B. Find the final charge on each condenser and total loss of electric energy in the condenser?

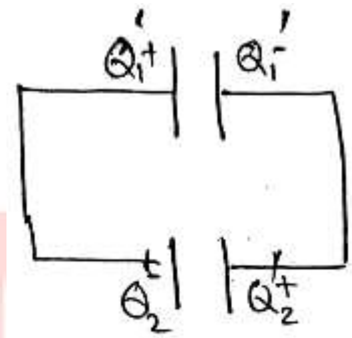
Solution:



$$Q_1 = C_1V = 100x10^{-6}C$$
$$Q_2 = C_2V = 500x10^{-6}C$$

Initial energy of the capacitor = $U_i = \frac{1}{2}\left(C_1V_1^2 + C_2V_2^2\right) = 0.03J$

After connection



$$V_{common} = \frac{Q_1+Q_2}{C_1+C_2} = \frac{200}{3}volts$$

Therefore, final energy of capacitance = $\frac{1}{2}(C_1+C_2)V_{common}^2 = \frac{0.04}{3}J$

Therefore, loss in energy = $0.03 - \frac{0.04}{3} = \frac{0.05}{3}J$

Notes:

For parallel plate capacitor

- When like plates are connected
  $$V_{common} = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$
- When unlike plates are connected
  $$V_{common} = \frac{C_1V_1 - C_2V_2}{C_1 + C_2} = \frac{Q_1-Q_2}{C_1-C_2}$$