Chapter- 1
Electric Charge and Field

Electricity

It is originated from the Greek word “Elektron” which means Amber.

It is a form of energy associated with static or dynamic charges

Electricity

Static Electricity

Dynamic Electricity

Electrostatics: The branch of physics that deals with the study of force, field, and potential arising from static charges is called electrostatic

The points to be remembered:-

Q at rest	Q moving with constant $\vec{v}$	Accelerated charge
Produces Electric field	Produces Electric Field and magnetic field (B)	Produces E, B, and Radiant energy

Electric Charge:-

It is the basic property of charged particles like protons or electrons which give rise to the electric force between them.

- ✓ Represented by letter Q or q
- ✓ It is a scalar quantity
- ✓ Dimensional formula:- $[Q]=[It]=[AT]$
- ✓ S.I unit of charge is coulomb (C)
- ✓ 1C of charge: It is the charge carried by 6.25×10^{18} nos. of electrons

Points to ponder:

(a) Charge of 6.25×10^{18} nos. of electrons = 1C

Charge of 1 electrons = $\frac{1}{6.25\times10^{18}} = 1.6\times10^{-19}\,\text{C}$

(b) IC is practically a bigger unit. Therefore smaller units of charge are

Mili coulomb (mC) = 10^{-3} C

Micro coulomb (μC) = 10^{-6} C

Nano coulomb (nC) = 10^{-9} C

Question - 1

If 10^9 electrons move out of the body to another body every second, how much time is required to get a total charge of 1C on the other body? (NCERT)

Solution:-

10^9 electrons to be transferred, time taken = 1 s

1 electron to be transferred, time taken = $\frac{1}{10^9}\,\text{s}$

6.25×10^{18} electrons to be transferred, time taken = $\frac{1}{10^9}\times6.25\times10^{18}\,\text{s} = 6.25\times10^{8}$

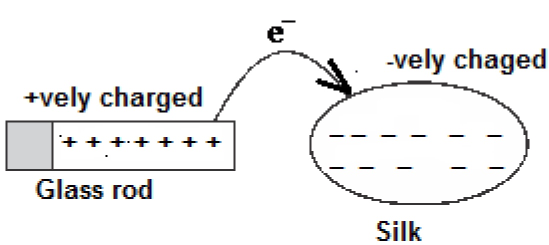
$$= \frac{6.25\times10^{8}}{365\times24\times3600} = 200 \text{ years.}$$

Thus one coulomb is a very large unit for many practical purposes.

Kinds of charges:-

(a) +ve charge (b) –ve charge

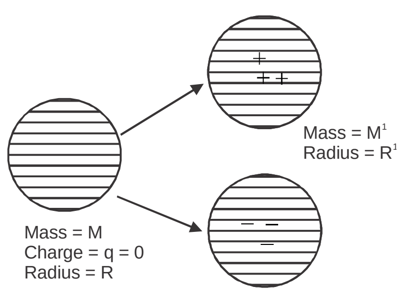
- An object can attain +ve charge by losing electrons while another can attain –ve charge by gaining electrons.



The body acquires +ve charge on rubbing	The body acquires –ve charge on rubbing
Glass rod	Silk
Cat fur	Ebonite
Woolen cloth	Amber
Woolen cloth	Rubber
Woolen cloth	Plastic bodies

- Electrons are transferred from the body of less work function to the body of more work function.
- We know that electron has mass = $9.1\times10^{-31}\text{kg}$. In this process mass of the positively charged by is slightly decreased.

  The mass lost by +vely charged body = mass gained by the negatively charged body.



Conductor and Insulators:-

- Conductors are those substances which can be used to conduct electric charge from one point to other.

  Examples:- Silver, metals (Copper, iron, Al), Human body

- Insulator are those substances which cannot conduct electric charge

  Examples:- Glass, Plastic, Rubber, Wood, etc.

Question – 2: How much +ve and –ve charge is there in a cup of water (250g)? (NCERT)

Solution:- 18g of H_2O contains 6.022×10^{23} molecules

1g of H_2O contains $\left[\frac{6.022\times10^{23}}{18}\right]$ molecules

$\therefore$ 250g of H_2O contains $\left[\frac{6.022\times10^{23}}{18}\times250\right]$ molecules

Each molecule of H_2O contains = 10 e^-

Thus $\left[\frac{6.022\times10^{23}}{18}\times250\right]$ molecules of H_2O contain = $\frac{6.022\times10^{23}\times250\times10\times1.6\times10^{-19}}{18}$

= $1.34\times10^{7}\,C$

Question – 3: A polythene piece rubbed with wool is found in have -ve charge of $3\times10^{-7}\,C$

(a) Estimate the no. of electrons. Transferred (from which to which)

(b) Is there a transfer of mass from wool to polythene? (NCERT)

Solution:-

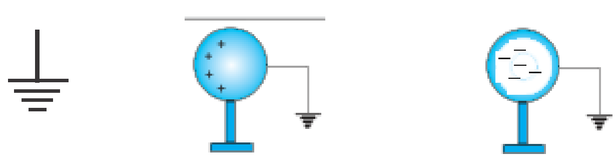
(a) To acquire 1C of charge, no. of electrons to be transferred = 6.25×10^{18}

To acquire $3\times10^{-7}\,C$, no of electrons to be transferred = $\left[6.25\times10^{18}\times3\times10^{-7}\right]$ = 18.75×10^{11}

(b) Yes, mass is transferred from wool to polythene

Mass to be transferred = $m_e\times\left(18.75\times10^{11}\right)$ = $9.1\times10^{31}\left(18.75\times10^{11}\right)kg$ = $1.7\times10^{-18}kg$

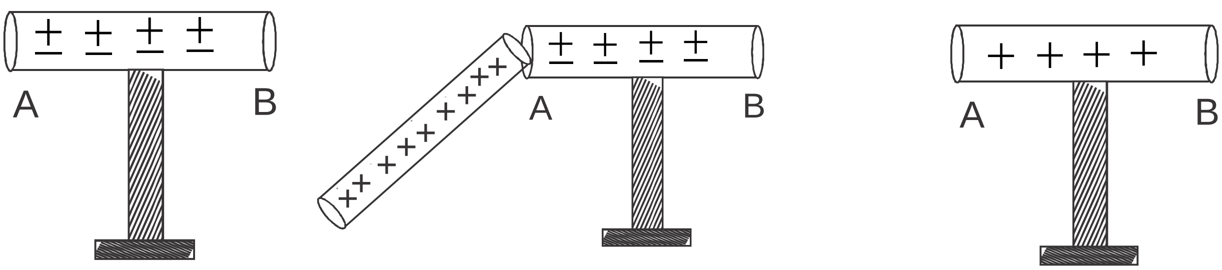
Earthing or Grounding:- The method of sharing of charge between the charged body and earth is called earthing or grounding.



Methods of charging:-

(a) Friction (b) Conduction (c) Induction

Conduction (To charge a body +vely) (Charging on touching)



+vely charged glass rod is kept in contact with AB. The electrons transferred to +vely charged rod from AB

Body AB, which is to be charged is placed on an insulating stand. It has equal +ve and –ve charges

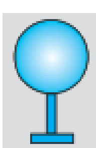
On removing the glass rod the +ve charges only remain in the body AB

Induction:-

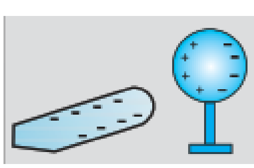
The process of charging of neutral body, by bringing an oppositely charged body nearer to it without contact is called Induction.

Question – 4: **How can you charge a metal sphere +vely without touching it?**

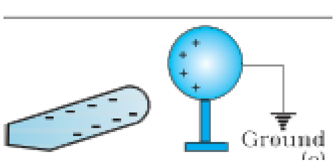
Solution:-



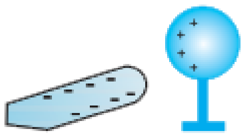
The spherical metal sphere is kept on an insulating stand.



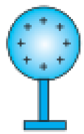
When ebonite rod (-ve) is brought nearer, +ve charges came to end A called as bound charges. Due to repulsion –ve charge comes to an end B called as free charges.



End B is now earthed. All the free charges are earthed away.



Keeping the ebonite rod at the same place, earthing is removed. (only +vely charges remain bound on the metal sphere)

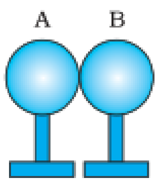


After removal of – ve ly charged rod and earthing all +ve charge is now spread over the metal sphere.

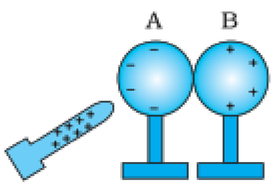
Question –5: You are given two spherical conductors and a +vely charged rod.

(a) How will you charge these spheres with equal and opposite charges.

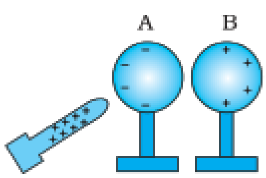
(b) What will happen if the glass rod is subsequently removed and finally charged spheres A and B are separated apart?



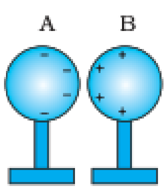
Two spheres to be charged are kept on insulating stand by touching each other.



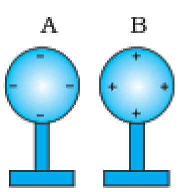
A +vely charged glass rod is brought nearer to A. Sphere A will – vely charged and B will be +vely charged.



The spheres are slightly separately keeping the charged rod undisplaced



After removal of the charged rod, the charges on the sphere will rearrange.

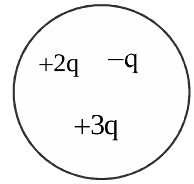


After separating far apart, the charges on each sphere get uniformly distributed.

Basic Properties of Electric charges:-

(a) Additive nature of Charge:-

The total electric charge of a body is equal to the algebraic sum of all the electric charges located anywhere on the body.



In the given figure total charge = +2q-q+3q=+4q

(b) Conservation of electric charge:-

- The total charge of an isolated system remains conserved
- Example – 1: Glass rod rubbed with silk.

  Before rubbing, total charge = 0

  Let due to rubbing two electrons are transferred from the glass rod to silk. So the charge on the glass rod is +2e and the charge on the silk is -2e.

  So after rubbing total charge = +2e – 2e = 0
- Example – 2: Radioactive decay of uranium ${}_{0}n^{1} + {}_{92}U^{235} \rightarrow {}_{56}B^{141} + {}_{36}Kr^{92} + 3\left({}_{0}n^{1}\right) + e$

  Proton no. before fission = 92. Proton no. after fission = 56 + 36 + 3(0) = 92

(c) Invariance of charge:- $q_{rest} = q_{motion}$

(d) Quantization of electric charge:-

Charge on any object is always an integral multiple of basic elementary charge, i.e electronic charge $\left(= \pm 1.6 \times 10^{-19} C\right)$.

$\therefore q = \pm ne$, where n = 1, 2, 3 ……

Points to ponder:-

- ✓ **Cause of quantization:** when one body is rubbed with another only integral no. of electrons is transferred.
- ✓ Protons and neutrons are made up of Up quark $\rightarrow +\frac{2}{3}e$ Down quark $\rightarrow -\frac{e}{3}$.
- ✓ So proton = ***uud*** and neutron = ***udd***

Question - 6: At the macroscopic level the quantization of charge can be ignored. Why?

Solution: At the microscopic level, where the charges involved are of the order of a few tens or hundreds of the number of electrons, they can be counted, they appear in discrete lumps. But at the macroscopic level, one deals with enormous charges. Hence the quantization of charge is ignored.

Points to ponder:

- ✓ When two identical spheres with charges q_1 and q_2 are touched together and separated, the charge on each sphere will be. $\frac{q_1 + q_2}{2}$
- ✓ If one is charged (=q) other has no charge then each sphere will have q/2
- ✓ The charge on a body is directly proportional to its size.

Question- 7: A sphere '1' with charge q is touched to another sphere 2 of half the radius of 1. Find out the charge on each sphere.

Solution: Let the charge on sphere 1 be q_1 and charge on 2 be q_2

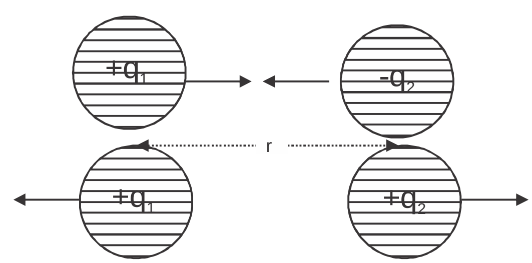
$q_1 + q_2 = q$ .... (1)

Again $q_2 = 2q_1$ .... (2) (Charge is proportional to size)

Solving (1) and (2) we get, $Q_2 = (2/3)\, q$ and $q_1 = (1/3)\, q$

Coulomb's Law of Electrostatics:-

- **Statement:-**The magnitude of the force of interaction between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance of separation between them



- Mathematically:- $F \alpha |q_1 q_2|$

$$F \alpha \frac{1}{r^2}$$

Combining; $$F = \frac{k|q_1 q_2|}{r^2}$$

k is called Coulomb's constant/ electrostatic constant.

Value of k:-

- In S.I system (medium air/ vacuum) $k = \frac{1}{4\pi \in_0} = \frac{1}{4\pi\left(8.85\times10^{-12}\right)} = 9\times10^{9}\,Nm^2/C^2$

  Here, $\in_0$ is called the permittivity of free space. (It represents the response of free space for interaction of charges)

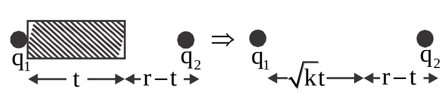
- Relative Permittivity/ Dielectric Constant:- $\varepsilon_r \text{ or } K = \frac{\varepsilon\left(\text{permitivity of medium}\right)}{\varepsilon_0\left(\text{permitivity of free space}\right)}$

Points to ponder:

- ✓ Force between two charged body when placed in the vacuum $F_0 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{r^2}$
- ✓ Force between two charged body when placed in a medium $F_m = \frac{1}{4\pi\varepsilon}\frac{|q_1q_2|}{r^2}$
- ✓ So, $\frac{F_0}{F_m} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$ or K
- ✓ If a dielectric is inserted in the space between the charges, the force between them,

  $F = \frac{F_0}{\varepsilon_r}$

  Since, $\varepsilon_r > 1(\text{except vacuum}), \text{so } F < F_0$ (i.e force between them decrease)
- ✓ If the dielectric is inserted in some part of the space:



$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{\left[\sqrt{kt}+(r-t)\right]^2}$$

Note:- The relative permittivity or dielectric constant is unitless and dimensional less quantity.

Medium	Dielectric Constant
Free space	1
Air	1.00546
Water	8.0
Metal	∞

Question:- 8: Define 1 coulomb of charge from coulomb's law

Answer:-From Coulomb's law, $F = k\frac{q_1 q_2}{r^2}$ ……………………………………. (1)

Let $q_1 = q_2 = q$ (say). Let them be separated by 1 meter in vacuum or air $\left(\text{i.e}\ k = 9\times10^9\right)$

So repulsive force between them = $9\times10^9\,\text{N}$

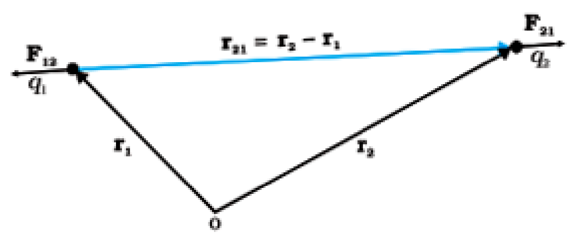
Substituting all the above in equation (1) we get

$$9\times10^9 = \frac{9\times10^9\left(q\times q\right)}{1^2} \Rightarrow q^2 = 1 \Rightarrow q \pm 1\text{C}$$

Thus, 1C is the charge that when placed at a distance of 1m from another charge of the same magnitude in vacuum experience an electric force of repulsion of magnitudes $9\times10^9\,\text{N}$

Question - 9: Prove that Coulomb's law agrees with Newton's third law of motion.

Answer: Let two point charges q_1 and q_2 be located in free space



$\vec{F}_{12} \rightarrow$ Force exerted on q_1 by q_2

$\vec{F}_{21} \rightarrow$ the force exerted on q_2 by q_1

In diagram using triangle law of vector addition, $\vec{r}_1 + \vec{r}_{21} = \vec{r}_2 \Rightarrow \vec{r}_{21} = \vec{r}_2 - \vec{r}_1$

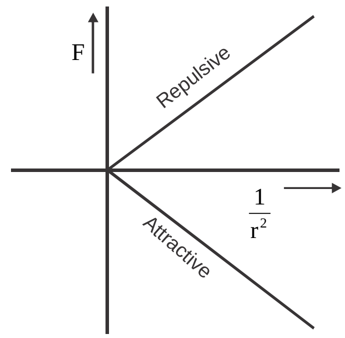
Similarly $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$, Thus $\vec{F}_{21} = k\dfrac{q_1 q_2}{r^2}\hat{r}_{21}$ .... (1)

Applying coulomb's law, $\vec{F}_{12} = k\dfrac{q_1 q_2}{r^2}\hat{r}_{12}$, $\vec{F}_{21} = k\dfrac{q_1 q_2}{r^2}\hat{r}_{21}$

Since $\hat{r}_{12} = -\hat{r}_{21} \Rightarrow \vec{F}_{21} = -\vec{F}_{12}$

Thus, they are equal in magnitude and opposite in direction, which is Newton's 3rd law of motion.

Note:- The graph between Coulomb's force and $1/r^2$ is :



Force due to Multiple Charge(Principle of superposition of force) :

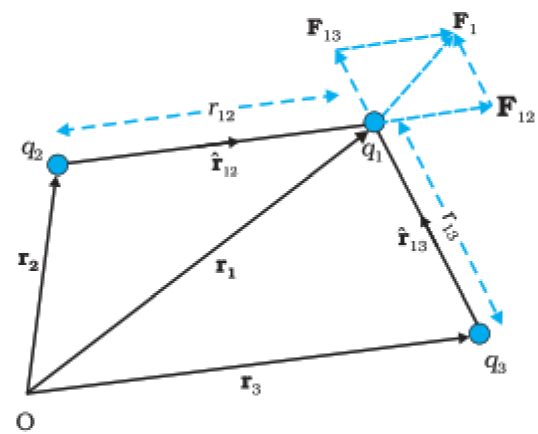
This principle enables us to find the force on a point charge due to a graph of charges.

Statement:-

When a no of charges is interacting the total force on a given charge is the vector sum of the forces exerted on it due to all other charges.

From the figure, the force on charge q_1

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} \Rightarrow \vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13}\right]$$

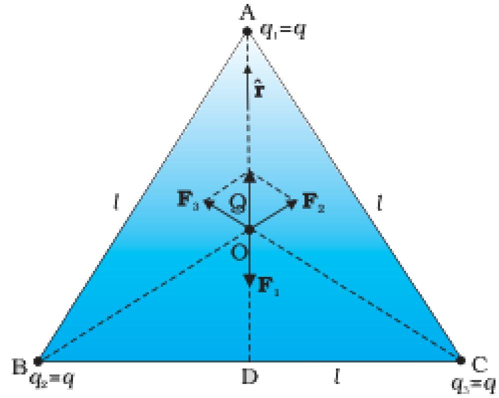


(a)

For a system consisting of no charge

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + ... + \vec{F}_{1n} = \frac{1}{4\pi\varepsilon}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + ... + \frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}\right] = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{qi}{r_{1i}}$$

Question – 10: Consider three charges q_1, q_2, and q_3 each equal to q at the vertices of an equilateral triangle of side '*l*'. What is the force on a charge 'Q' (with the same sign of q) placed at the centroid of the triangle? (NCERT)



Solution:-

$$|\vec{F}_1| = |\vec{F}_2| = |\vec{F}_3| = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{\left(\ell/\sqrt{3}\right)^2} = \frac{3}{4\pi\varepsilon_0} = \frac{Qq}{\ell^2}$$

As three forces act at angles 120^0 with each other, so

$$\left[\frac{F_1}{\sin 120^0} = \frac{F_2}{\sin 120^0} = \frac{F_3}{\sin 120^0}\right].$$

Hence these three forces acting on +Q satisfy Lami's theorem. Hence net force on +Q is zero.

Equilibrium of a system of charges:-

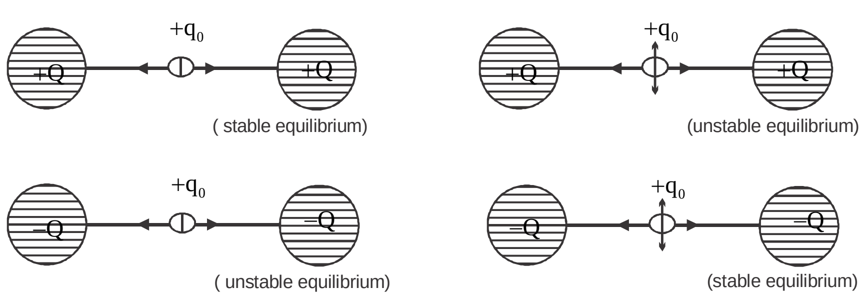
(a) Stable equilibrium

(b) Unstable equilibrium

Stable Equilibrium:-

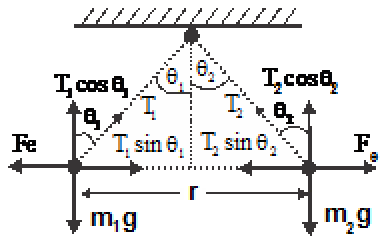
- $\vec{F}_{net}$ or $\vec{\tau}_{net}$ on charged particle must be zero
- After displacing the charged particle from its initial position it returns.
- Potential energy must be minimum.

Case – 1, For two charges of the same sign.



Case – 2, Equilibrium of two suspended charged balls (both at the same horizontal level).

Considering the equilibrium of 1st ball



$T_1\sin\theta_1 = F_e$ and $T_1\cos\theta_1 = m_1 g$

$\tan\theta_1 = \dfrac{F_e}{m_1 g}$ ....(1)

Similarly for the 2nd ball, $\tan\theta_2 = \dfrac{Fe}{m_2 g}$ ....(2).

From (1) and (2) $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{m_2}{m_1}$

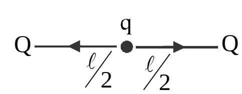
Thus, If $m_1 = m_2$ then $\theta_1 = \theta_2$

If $m_1 < m_2$ then $\theta_1 > \theta_2$

If $m_1 > m_2$ then $\theta_1 < \theta_2$

Question – 11: A charge q is placed at the center of the line joining two charges Q. The system of three charges to be in equilibrium, what should be the value of q?

Solution: The charge q is in equilibrium because it is at the center.

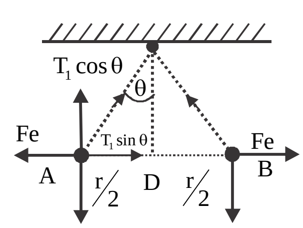


For equilibrium of charge Q, $\vec{F}_{net} = 0$

$$\Rightarrow \frac{Qq}{(\ell/2)^2} + \frac{QQ}{\ell^2} = 0 \Rightarrow q = -Q/4$$

Question – 12: Two small spheres each having mass *m* and charge *q* are suspended from a point by insulating threads each of length *l* but negligible mass. If θ is the angle made by each string with the vertical at equilibrium then show that $q^2 = \left(4mg\ell^2\sin^2\theta\tan\theta\right)4\pi\varepsilon_0$.

Solution: We have $T_1\sin\theta = F_e$, $T_1\cos\theta = mg$



$$\Rightarrow \tan\theta = \frac{Fe}{mg} \Rightarrow \tan\theta = \frac{kq^2}{r^2 mg} = \frac{q^2}{4\pi\varepsilon_0 r^2 mg}$$

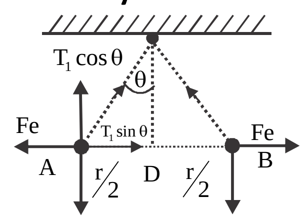
$$\Rightarrow q^2 = mg\tan\theta r^2 (4\pi\varepsilon_0)$$

In $\Delta ODA, \sin\theta = \frac{r/2}{\ell} \Rightarrow 2\ell\sin\theta = r$,

hence $q^2 = 4mg\ell^2\sin^2\theta\tan\theta\,4\pi\varepsilon_0$ Proved.

Question – 13

Two identical charged spheres are suspended by a string of equal length. The strings make an angle 30^0 with each other. When suspended in a liquid of density of 800 kg/m^3, the angle remains the same. What is the dielectric constant of the liquid? (Given – Density of material of sphere is 1600 kg/m^3)



Solution:- $\tan\theta = \frac{Fe}{mg}$ .... (1)

When the system is placed in liquid

$$\tan\theta = \frac{Fe/k}{mg - U} = \frac{Fe/k}{mg - V\rho_\ell g}$$

$$\Rightarrow \tan\theta = \frac{Fe/k}{\left[mg - \frac{mg}{1600}\times 800\right]} = \frac{Fe/k}{mg/2} \text{ .... (2)}$$

From equation (1) and (2), $\frac{Fe}{mg} = \frac{Fe/k}{mg/2}, \Rightarrow k = 2$

Continuous Charge Distribution:-

(a) **Linear charge distribution:** When the charge is distributed along a line e.g a straight line or circumference of a circle or a curved line, then it is linear charge distribution.

(b) **Surface charge distribution:** When the charge is distributed along a surface e.g. a sheet of charge or a charged conducting surface, then it is surface charge distribution.

(c) **Volume charge distribution:** When the charge is distributed throughout the volume of a charged body, then it has a volume charge.

Points to ponder:

- ✓ Linear charge density; $\lambda = \frac{dq}{dl}$
- ✓ Surface charge density; $\sigma = \frac{dq}{ds}$
- ✓ Volume charge density; $\rho = \frac{dq}{dV}$
- ✓ S.I. unit of linear charge density is C/m, that of surface charge density is C/m^2, and that of volume charge density is C/m^3.
- ✓ As per the nature of the distribution of charge the total charge on the body can be calculated as;

  For linear charge distribution, $q = \int_0^l \lambda dl$

  For surface charge distribution, $q = \int_0^s \sigma ds$

  For volume charge distribution, $q = \int_0^V \rho dV$
- ✓ For uniform charge distribution, λ, σ, and ρ are constants.

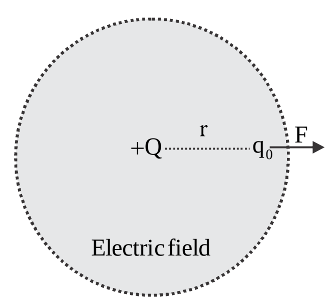
  In this case; $q = \lambda l$ (for linear charge distribution)

  $q = \sigma s$ (for surface charge distribution)

  $q = \rho V$ (for volume charge distribution)

Electric Field:- Something (invisible) existing in the space around every charge in which its effect is realized is called an electric field.

Electric Field Intensity:- Quantitatively it is defined as the force, a unit +vely charge would experience if it is placed at that point without disturbing the position of source charge.



Mathematically, $\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$

Points to remember:

- ✓ Here ‘Q’ is called source charge (it produces the field). q_0 is called test charge (it tests the effect of source charge)
- ✓ The presence of q_0 may disturb the charge distribution of the source. Hence q_0 must be small enough.

$$\therefore \vec{E} = \lim_{q_0 \to 0} = \frac{\vec{F}}{q_0}$$

- ✓ S.I. Unit – N/C or V/m
- ✓ $[E] = M^1L^1T^{-3}A^{-1}$

Physical Significance of Electric Field:-

It plays an intermediate role in the force between two charges.

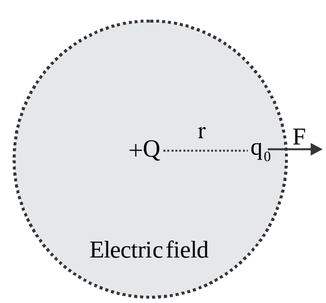
The charge q_1 produces an electric field which then propagates with speed c, reaches q_2, and causes a force on q_2.

$$\boxed{\text{Charge } q_1} \rightleftharpoons \boxed{\text{Electric field } (E)} \rightleftharpoons \boxed{\text{Charge } q_2}$$

$$\vec{F}_{21} = q_2\vec{E}_1$$

$$\vec{F}_{12} = q_1\vec{E}_2$$

Electric field due to an isolated charge:-



Let ‘P’ be a point at a distance r from the source charge ‘Q’.

Now, if a test charge q_0 is kept at the point then electrostatic force on it is; $F = \frac{1}{4\pi\varepsilon_0}\frac{|Qq_0|}{r^2}$

$$\Rightarrow E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2}$$ (Since test charge is +ve)

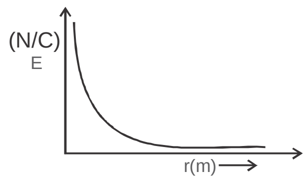
In vector form, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

We know $\vec{r} = r\hat{r} \Rightarrow \hat{r} = \frac{\vec{r}}{r}$.

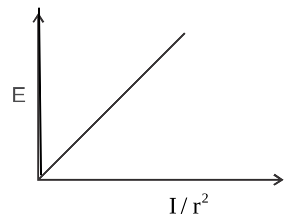
Thus $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q\vec{r}}{r^3}$

Graphical representation of the variation of the electric field with distance from the source charge:

Variation of E with r;



Variation of E with $\frac{1}{r^2}$;

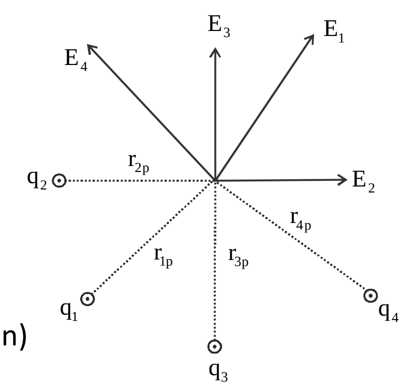


Electric field due to multiple charges:

$\vec{E}_1 = K\frac{q_1}{r_{1p}^2}\hat{r}_{1p}$

$\vec{E}_2 = K\frac{q_2}{r_{2p}^2}\hat{r}_{2p}$.

$\vec{E}_n = K\frac{q_n}{r_{np}^2}\hat{r}_{np}$

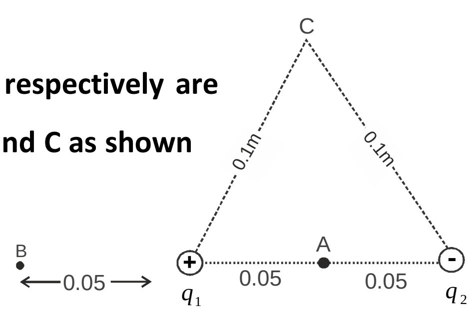


Hence net field at point 'P' (According to the principle of superposition)

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n \Rightarrow \vec{E}_{net} = K\left[\frac{q_1}{r_{1p}^2}\hat{r}_{1p} + \frac{q_2}{r_{2p}^2}\hat{r}_{2p} + \ldots\ldots + \frac{q_n}{r_{np}^2}\hat{r}_{np}\right]$$

Question - 14

Two-point charges q_1 and q_2 of magnitude $+10^{-8}$C and -10^{8}C respectively are placed 0.1 m apart. Calculate the electric field at points A, B, and C as shown in the figure.



Solution:-

Hints;

Field at A	Field at B	Field at C
$(E_{net})_A = E_{+q} + E_{-q}$	$(E_{out})_B = E_{+q} - E_{-q}$	$E_C = \sqrt{E_{+q}^2 + E_{-q}^2 + 2E_{+q}E_{-q}\cos\theta}$

Question - 15

If a charge is displaced from its position, then after what time, the field at a distance r will change?

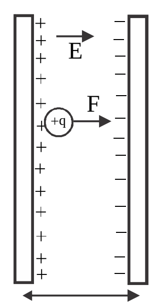
Solution:- t = r/c, where c is the speed of light

The motion of a Charged Particle in an electric field:-

Acceleration of charge in the uniform field is given by $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} =$ constant.

So the equation of motion is valid. Now there are two possibilities.

(a) If the particle is initially at rest.



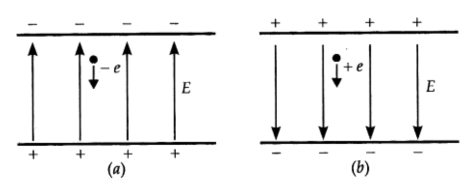
As u = 0,

v = at = $\frac{qE}{m}t$

$$S = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2$$

This motion is accelerated where $v \propto t$ and $s \propto t^2$.

Question –16:- An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude $2.0\times10^4\,NC^{-1}$. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance. Computer the time of fall in each case. Contrast the situation (a) with that of free fall under gravity. (NCERT)



Solution:-(a) The upward field exerts a downward force eE on the electron.

$\therefore$ Acceleration of the electron, $a_e = \dfrac{eE}{m_e}$

As u = 0, $s = ut + \dfrac{1}{2}at^2 = \dfrac{1}{2}at^2$

$\therefore$ The time of fall of the electron is

$$t_e = \sqrt{\frac{2s}{a_e}} = \sqrt{\frac{2sm_e}{eE}} = \sqrt{\frac{2\times1.5\times10^{-2}\times9.1\times10^{-31}}{1.6\times10^{-19}\times2.0\times10^{4}}} = 2.9\times10^{-9}s$$

(b) The downward field exerts a downward force eE on the proton

$\therefore a_p = \dfrac{eE}{m_p}$

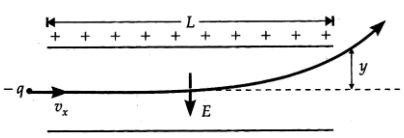
The time of fall of the proton is

$$t_p = \sqrt{\frac{es}{a_p}} = \sqrt{\frac{2sm_p}{eE}} = \sqrt{\frac{2\times1.5\times10^{-2}\times1.67\times10^{-27}}{1.6\times10^{-19}\times2.0\times10^{4}}} = 1.25\times10^{-7}s$$

Thus the heavier particle takes a greater time to fall through the same distance. This is in contrast to the situation of free fall under gravity, where the time of fall is independent of the mass of the body. Here the acceleration due to gravity g being negligibly small has been ignored.

(b) If the particle is projected perpendicular to the field with an initial velocity V_0.

The motion of charge in an electric field is analogous to the motion of a projectile in the gravitational field.



For motion along the x-axis, $u_x = u_0, a = 0$, so V_x = constant.

If X is the horizontal distance then $X = V_x t \Rightarrow t = \dfrac{x}{V_x}$

For motion along Y-axis

$$u_y = 0, a_y = \frac{F_y}{m} = \frac{qE}{m},$$

Thus $Y = \frac{1}{2} a_y t^2$

$$\Rightarrow Y = \frac{1}{2}\frac{qE}{m}\left(\frac{x}{V_x}\right)^2$$

$$\Rightarrow Y = Kx^2$$

Thus within the electric field, the charged particle follows a parabolic path, when enters perpendicularly.

Vertical deflection suffered by charge within the electric field.

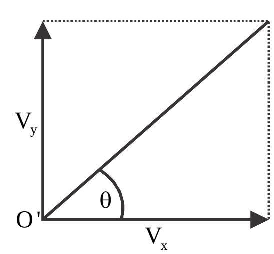
When x = L, Y = y_1

Thus $Y_1 = \frac{1}{2}\frac{qE}{m}\left(\frac{L}{V_x}\right)^2$ $\boxed{\Rightarrow Y_1 = \frac{qE}{2mV_x^2} L^2}$

Vertical deflection by charge outside the field.

As soon as the charge leaves the electric field at 'O' it follows a straight path and hits the screen at B.

Let V_y be the vertical velocity of charge of '0'



Then $V_y = a_y t = \frac{qE}{m}\left(\frac{L}{V_x}\right)$,

Clearly $\tan\theta = \frac{V_y}{V_x} = \frac{qE}{m}\frac{L}{V_x} / V_x$ .... (1)

From $\Delta O'AB$ $\tan\theta = \frac{y_2}{D}$ .... (2)

From (1) and (2) $\dfrac{V_2}{D} = \dfrac{qEL}{mV_x^2} \quad \Rightarrow Y_2 = \dfrac{qED}{mV_x^2}L$

Total deflection on screen

$$Y = Y_1 + Y_2$$

$$Y = \frac{qE}{2mV_x^2}L^2 + \frac{qED}{mV_x^2}L$$

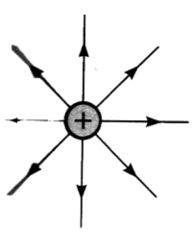
$$\boxed{\Rightarrow Y = \frac{qEL}{Mv_x^2}\left[\frac{L}{2} + D\right]}$$

Electric Field Lines:-

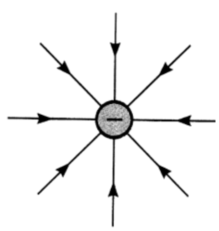
An electric field can be visualized graphically in terms of electric field lines.

Definition:- Electric field lines are defined as the paths along with a unit +ve charge moves if it free to do so.

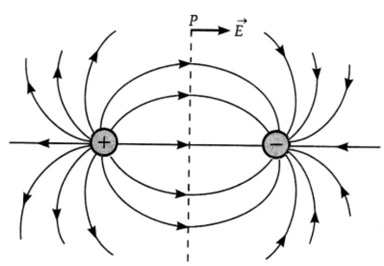
Electric Field lines due to some Charge Configuration:-



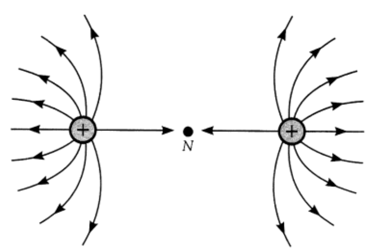
Field lines of a positive point charge.



Field lines of a negative point charge.



Field lines of an electric dipole.



Field lines of two equal positive charges.

Properties of Electric Field Lines:-

(a) Field lines originate from +ve charge and terminate at –ve charge. But for the isolated charge they many start or end at infinity.

(b) In a charge-free region, field lines can be taken to be continuous curve without any break

(c) Field lines never cross each other. (Reason - At the point of intersection, two tangents can be drawn which shows two directions at a single point which is impossible)

(d) Field lines do not form any close loops. i.e field lines never pass through the conductor that justify the absence of electric field within the conductor.

(e) Field lines are always normal to the surface of the conductor

(f) Relative closeness of field lines gives a measure of the strength of the field.

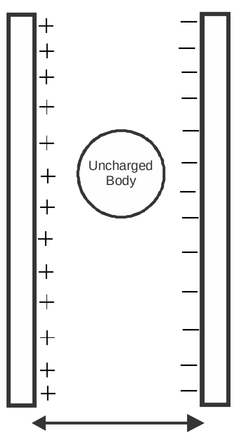
Questions:- 17

Draw the electric field lines for the following charge configuration

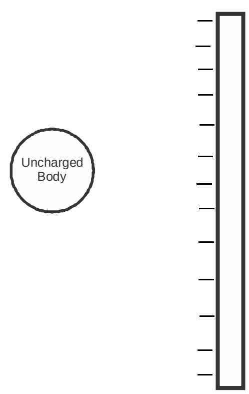
(a) Between $+q_1$ and $-q_2$ such that $|q_1|=2q_2$

(b) Between two parallel equal and oppositely charged plates.

(c)



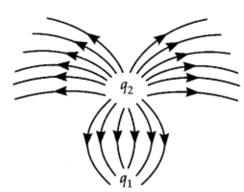
(d)



(d) For some non-uniform electric fields.

Solution: Left for discussion with the students.

Question:- 18



In the given figure what are the signs of q_1 and q_2?

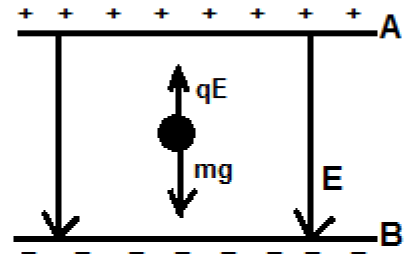
Also, find the ratio of q_1 / q_2

Solution:- q_2 is +ve and q_1 is –ve

$$\frac{q_1}{q_2} = 1:3$$

Millikan's Oil drop Expt. (Quantitative approach)

Determination of charge:- The electric field between the plates is adjusted in such a way that the drop remains at rest.



qE = mg,

$$\Rightarrow q = \frac{mg}{E} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)g}{E}$$

$$\boxed{\Rightarrow q = \frac{4}{3}\frac{\pi r^3 \rho g d}{V_{AB}}}$$

The radius of the drop:- $r^3 = \frac{qE}{\frac{4}{3}\pi\rho g} \Rightarrow r = \left\{\frac{3neE}{4\pi\rho g}\right\}^{1/3}$

Question – 19:- An oil drop of 12 excess electrons is held stationary under a constant electric field $2.55\times10^4\,\text{Vm}^{-1}$ in Millikan's oil drop experiment. The density of the oil is $1.26\,\text{g cm}^{-3}$. Estimate the radius of the drop. (g = 9.81 ms^{-2}). (NCERT)

Solution:-

Given n = 12e = $12\times1.6\times10^{-19}\,c$

$$E = 2.55\times10^4\,N/C$$

$$\rho = 1.26\times10^3\,kg/m^3$$

$$r = \left\{\frac{3neE}{4\pi\rho g}\right\}^{1/3}$$

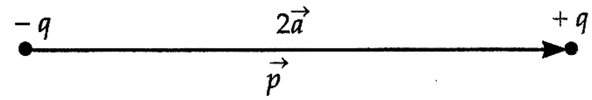
$$= \frac{3\times12\times\left(1.6\times10^{-19}\right)\times\left(2.55\times10^4\right)}{4(3.14)\left(1.26\times10^3\right)\times9.8}\,m$$

$$= 9.8\times10^{-4}\,mm$$

Electric Dipole:-

When two equal and opposite charges are separated by a certain distance they constitute an electric dipole.

2a is the length of the dipole.



Examples:-

Water (H_2O), ammonia (NH_3), Chloroform ($CHCl_3$) are examples of a permanent electric dipole.

In these molecules center of +ve and the center of – ve, charges do not coincide. So they have a permanent dipole moment.

Question No. 21:- What is an ideal or point dipole?

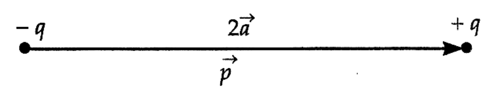
Solution:- The dipole for which $q \to \infty$ $2a \to 0$. Such that $q \times 2a$ has a finite value, is known as point diple.

Electric dipole moment:-

The strength of the electric dipole moment is measured by a vector quantity known as the electric dipole moment.

Definition:- The product of the magnitude of either charge and distance between two charges.

$\vec{P} = q \times \overrightarrow{2a}$



Points to remember:-

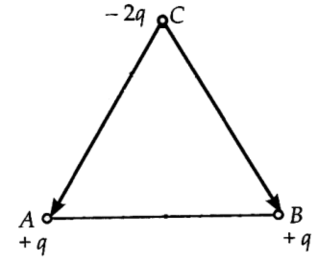
(a) The direction of dipole moment is from –ve to +ve charge.

(b) S.I unit C m, dimension = $\left[L^1 T^1 A^1\right]$

(c) Net charge in dipole is zero but the net field at any pt is non-zero. A complicated array of charges can be disrobed and analyzed in terms of the dipole.

Question No 22:-

Find out the net dispole moment of the system as given below.



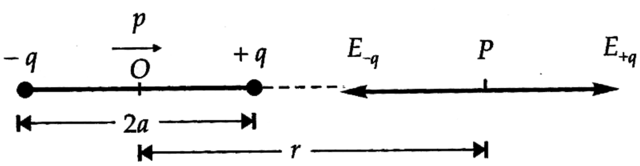
Solution:-

Net dipole $=\sqrt{P_1^2+P_2^2+2P_1P_2\cos 60^0}$

$=\sqrt{3P^2}=\sqrt{3}p=\sqrt{3}\left(Q\ell\right)$

Electric Field due to electric dipole:-

(1) For the point on axis i.e, at the point on axial end on position



Electric field at an axial point of dipole.

Find at P due to charge –q

$\vec{E}_{-q}=\dfrac{Kq}{\left(r+a\right)^2}$ (opposite to $\vec{P}$)

Field of P due to +q

$\vec{E}_{+q}=\dfrac{Kq}{\left(r-a\right)^2}$ (along $\vec{P}$)

The net field at P is given by

$$E_{net}=E_{+q}-E_{-q}=kq\left[\frac{1}{\left(r-a\right)^2}-\frac{1}{\left(r+a\right)^2}\right]=kq\left[\frac{r^2+a^2+2ra-r^2-a^2+2ra}{\left(r^2-a^2\right)^2}\right]=k\left[\frac{2r\left(q2a\right)}{\left(r^2-a^2\right)^2}\right]$$

$\Rightarrow E_{net}=\dfrac{k2rp}{\left(r^2-a^2\right)^2}$ .... (1)

If a <<r, a^2 is negligibly small in comparison to r^2. Hence neglected

Thus from equation (1)

$E=\dfrac{k2rp}{\left(r^2\right)^2}=\dfrac{k2p}{r^3}$,

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}}$$

In vector form $\vec{E}_{axial} = +\frac{1}{4\pi\varepsilon_0}\frac{2\vec{P}}{r^3}$.

Points to Ponder:

- ✓ +ve sign indicates $\vec{E}_{axial}$ and $\vec{P}$ is in the same direction. So the angle between $\vec{E}_{axial}$ and $\vec{P}$ is 0^0.
- ✓ $E_{axial} \alpha \frac{1}{r^3}$

(2) Electric field at any point on equatorial position (or Broad side on position):-

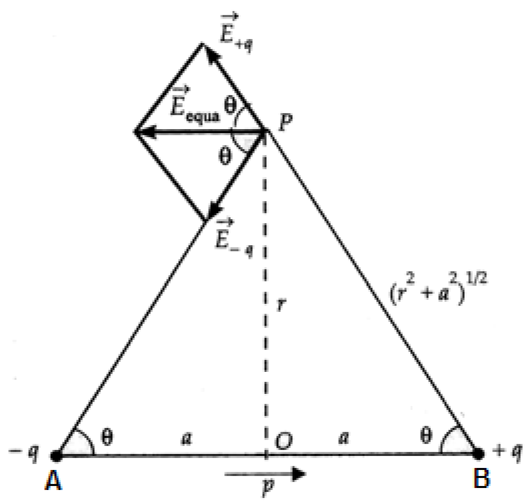
The electric field at P due to +q

$\vec{E}_{+q} = \frac{Kq}{\left(r^2+a^2\right)^2}$ (In the direction of BP).

The electric field at P due to –q

$\vec{E}_{-q} = \frac{Kq}{\left(r^2+a^2\right)^2}$ (In the direction of PA).

Here $|\vec{E}_{-q}| = |\vec{E}_{-q}| = E(say) = \frac{kq}{(r^2+a^2)^2}$



Electric field at an equatorial point of a dipole.

∴ The net electric field at P

$$E_{net} = \sqrt{E_{+q}^2 + E_{-q}^2 + 2E_{+q}E_{-q}\cos 2\theta} = \sqrt{E^2 + E^2 + 2EE\cos 2\theta}$$

$$= \sqrt{2E^2(1+\cos 2\theta)} = \sqrt{2E^2(1+2\cos^2\theta - 1)}$$

$= \sqrt{4E^2\cos^2\theta} = 2E\cos\theta$.................................. (1)

In $\Delta AOP, \cos\theta = \frac{a}{\sqrt{r^2+a^2}}$.

Substituting for E and $\cos\theta$ in equation (1) we get

$$E_{net} = 2\frac{kq}{(r^2+a^2)^2}\frac{a}{(r^2+a^2)^{1/2}} = \frac{kq2a}{(r^2+a^2)^{3/2}} = \frac{kp}{(r^2+a^2)^{3/2}}$$

For short dipole a << r. Neglecting a^2 in comparison to r^2 we have

$$E_{net} = \frac{kp}{(r^2)^{3/2}} \Rightarrow \boxed{E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}}$$

In vector form $\vec{E}_{net} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{P}}{r^3}$

Points to ponder:

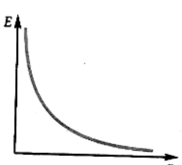
- ✓ For small dipole, the ratio between electric field intensities at the axial and equatorial points at the same distances from the center is 2: 1.
- ✓ The electric field intensity at the center is; $\vec{E} = -\frac{k\vec{p}}{a^3}$. (Since at the center, r = 0)
- ✓ The angle between the electric field intensity and the dipole moment at the equatorial point is 180^0.

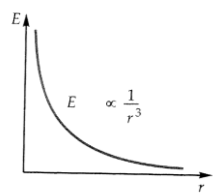
Question No 23:-

Graphically represent the variation of the electric field with distance for

(a) Isolated charge (b) Electric dipole

Solution:-

(a) 

(b) 

Question No. 24:-

What is the angle between electric field intensities at

(a) Axial end on position and electric dipole moment

(b) Equatorial bisector position and electric dipole moment

(c) Axial end on position and equatorial bisector position

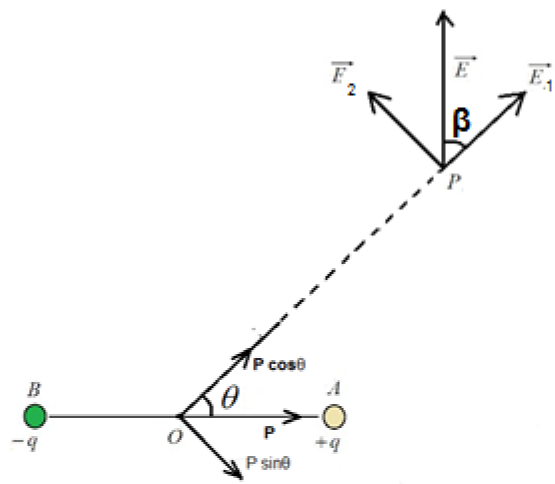
Solution:-

(a) 0^0 (b) 180^0 (c) 180^0

Question No. 25:-

Find out the electric field at a general point due to a short electric dipole

Solution:-



$$E_1 = k\frac{2p\cos\theta}{r^3}$$

$$E_2 = k\frac{p\sin\theta}{r^3}$$

$$E = \sqrt{E_1^2 + E_2^2} = K\frac{p}{r^3}\sqrt{[3\cos^2\theta + 1]}$$

If β is the angle which $\vec{E}$ makes with $\vec{E}_1$ then

$$\tan\beta = \frac{E_2}{E_1} = \frac{\tan\theta}{2} \Rightarrow \beta = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$$

Electric Multipole:-

According to the number of charges in the system, it is called

Monopole (one pole)

Dipole (Two poles),

Quadrupole (Four poles)

Octo pole (eight-poles)

Dipole in a uniform external field:-

Force on charge –q = -qE (opposite to $\vec{E}$)

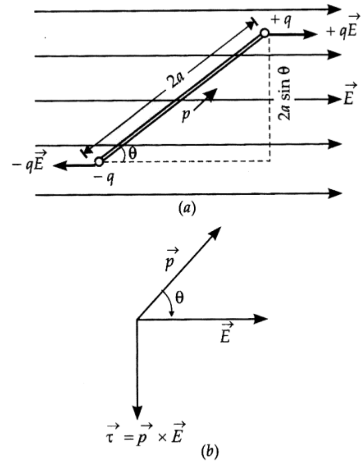
Force on charge +q = +qE (along $\vec{E}$)

These two forces are equal and opposite and act at different points of the dipole.

Thus, they form a couple that exerts a torque.

The magnitude of torque = (Magnitude of the force of either charge) X ($\perp$ r distance).

$$\Rightarrow \tau = (qE)\,AN \text{ .... (1)}$$



In triangle ANB,

$$\sin\theta = \frac{AN}{2a} \Rightarrow AN = 2a\sin\theta$$

From equation (1)

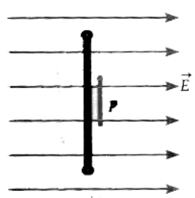
$$\tau = qE(2a\sin\theta)$$

$$\Rightarrow \tau = pE\sin\theta$$

Hence net force on the dipole is zero but net torque is non-zero.

Special Cases: -

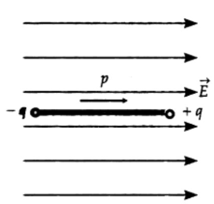
Maximum Torque: -



When $\theta = 90^o$

$$\Rightarrow \sin\theta = 1 \text{and}\, \tau = PE$$

Minimum Torque: -



When $\theta = 0^o$ or 180^0

$$\Rightarrow \sin\theta = 0 \,\text{and}\, \tau = PE\sin 0 = 0$$

In vector form $\vec{\tau} = \vec{P} \times \vec{E}$

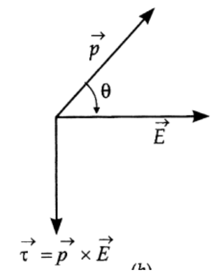
Vectorial proof for $\vec{\tau} = \vec{P} \times \vec{E}$

We know that $\vec{\tau} = \vec{r} \times \vec{F}$

$$= a\sin\theta\hat{j}\times qE\left(\hat{i}\right)+a\sin\theta\left(\hat{j}\right)\times qE\left(-\hat{i}\right)$$

$$= a\sin\theta qE\left(\hat{j}\times\hat{i}\right)+a\sin\theta qE\left(\hat{j}\times\hat{i}\right)$$

$$= q2aE\sin\theta\left(-\hat{k}\right),\ = PE\sin\theta\left(-\hat{k}\right)$$

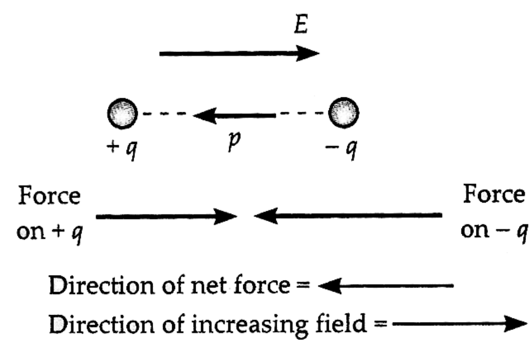


$$\Rightarrow \vec{\tau} = \vec{P}\times\vec{E}$$

Dipole in a non-uniform electric field: -

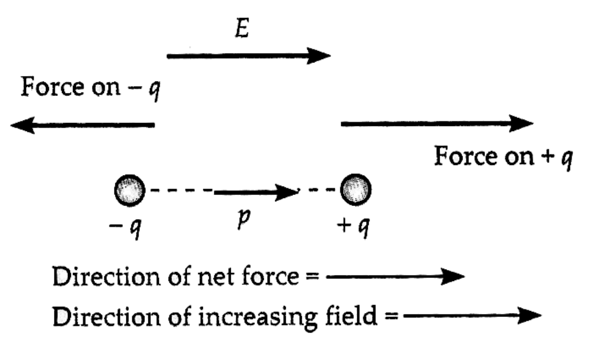
Case – I, when the dipole is placed in a decreasing field. Hence the net force is along the opposite direction of the electric field.

Here $\tau = 0$ But $F_{net} \neq 0$



Case – II, when the dipole is placed in an increasing field. Hence the net force is along the direction of the electric field.

Here $\tau = 0$ But $F_{net} \neq 0$



Case – III, If the dipole is placed making a certain angle with the field it experience force as well as torque.

Points to ponder: -

- ✓ If the electric dipole is placed in a field of changing magnitude but the same direction along the dipole axis then force on the dipole is,

$$\vec{F} = p\frac{d\vec{E}}{dx}$$

Where p = dipole moment

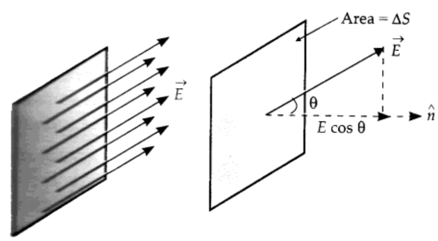
$\frac{d\vec{E}}{dx}$ = Rate of change of E with the axis of dipole.

- ✓ Except being parallel to the direction of the field, for any other configuration of the dipole, it experiences both force and torque.

Electric Flux:-

It is the measure of the numbers of electric field lines through a given surface.

Definition:- It is defined as the dot product of electric field and area vector.



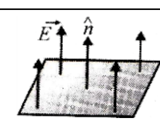
Flux through an inclined area.

Mathematically $d\phi = \vec{E}.d\vec{s} = Eds\cos\theta$

Points to remember:-

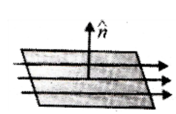
- ✓ It is a scalar quantity
- ✓ S.I unit $\rightarrow \frac{N}{c}m^2$ or $\left[\frac{V}{m}\times m^2 = V\times m\right]$
- ✓ Dimensional formula $\frac{[MLT^{-2}][L^2]}{[AT]} = [ML^3T^{-3}A^{-1}]$

Case – I: When $\theta = 0^0$ (i.e surface is normal to the field)



$d\phi = Eds\cos 0 = Eds$ (maximum). It is called emerging flux

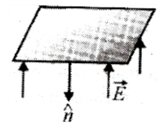
Case – II



when $\theta = 90^0$ (i.e surface is kept parallel to the field)

$d\phi = Eds\cos 90^o \Rightarrow d\phi = 0$

Case – III

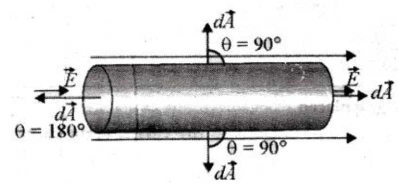


When $\theta = 180^o$, $d\phi = Eds\cos 180^0 = -Eds$

This –ve flux is called entering flux.

Question - 27:-

A uniform electric field exists in space. Find the electric flux through a cylindrical surface with the axis parallel to the field.



Solution:-

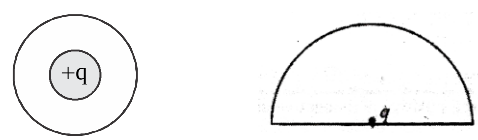
Net flux $\phi = \int \vec{E}.d\vec{s}_1 + \int \vec{E}.d\vec{s}_2 + \int \vec{E}.d\vec{s}_3$

$= \int Eds\cos 180^0 + \int Eds\cos 0^0 + \int Eds\cos 90^0$

$= \int Eds - \int Eds + 0$ = 0 (i.e total flux through a close surface is zero if no charge is enclosed by the surface)

Question - 28:-

Find the electric flux due to the electric field through the surface as given in the following figure.



Solution:-

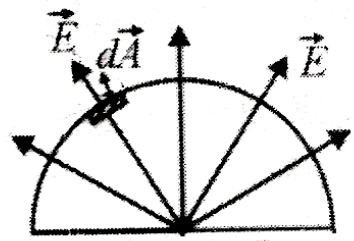
(a)Let ds be a small element of the spherical surface

Flux through the small element $d\phi = \vec{E}.d\vec{s} = Eds\cos 0^o, = Eds$ ………………….. (1)

Flux through the entire close surface

$\varphi = \oint d\varphi = \oint_s Eds = E\oint_s ds = E4\pi r^2 = \frac{q}{4\pi\varepsilon_0 r^2}(4\pi r^2)$

$\Rightarrow \phi = \frac{q}{\varepsilon_0}$



(b) $\phi = \phi_1 + \phi_2$

$= \int \vec{E}.d\vec{s}_1 + \int \vec{E}.d\vec{s}_2 = \int Eds_1 \cos 0^o + \int Eds_2 \cos 90^0$

$= \int Eds_1 + 0 = E\int ds_1 = E\left(2\pi r^2\right) = \frac{kq}{r^2} \times 2\pi r^2$

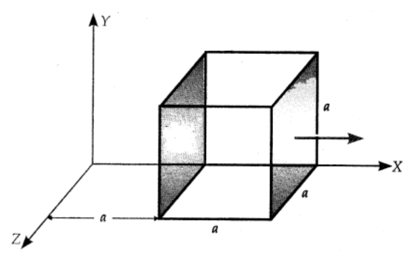
$\Rightarrow \phi = \frac{q}{2\varepsilon_0}$

Conclusion:-

Total flux through a close surface in free space is always $\frac{q}{\varepsilon_0}$ regardless of the shape and size of the surface.

Question - 29

The electric field components in the figure are $E_x = \alpha x^{1/2}, E_y = E_z = 0$ **in which** $\alpha = 800N/cm^{1/2}$



Calculate:-(a) Flux through the cube of side 0.1 m

(b) Also find the charge within the cube (NCERT)

Solution:-

(a) Net flux through the cube = $\phi_{leftface} + \phi_{rightface}$.................................(1)

$$\phi_L = \int \vec{E}_L.d\vec{s} = \alpha x^{1/2} a^2 \cos 1\,80^o = -[\alpha a^{1/2}]a^2 = -\alpha a^{5/2}$$

$$\phi_R = \vec{E}_R.d\vec{s}_R = [\alpha(2a)^{1/2}]a^2 \cos 0^0 = (\sqrt{2}\alpha a^{1/2})a^2 = \sqrt{2}\alpha a^{5/2}$$

From equation (1)

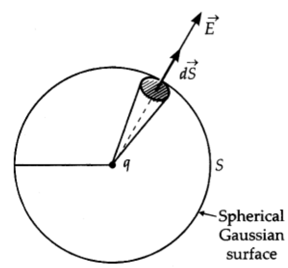
$\phi_{net} = -\alpha a^{5/2} + \sqrt{2}\alpha a^{5/2} = \alpha a^{5/2}(\sqrt{2} - 1) = (0.1)^{5/2}.800(1.414 - 1)$=$1.05Nm^2/c$

Gauss's Law:-

The net electric flux through any hypothetical close surface in free space is equal to $\frac{1}{\varepsilon_0}$ times of the net charge enclosed within the surface.

Mathematically, $\boxed{\phi = \frac{\sum q_{in}}{\varepsilon_0}}$

Proof of Gauss's law (for a spherical symmetric surface)



Flux through a sphere enclosing a point charge.

To find the field at distance r from charge 'q' let us assume a Gaussian surface (spherical). The electric flux through the small area element ds

$$d\varphi = \vec{E}.d\vec{s}$$

$$= (E\hat{r}).(ds\hat{n}) = Eds(\hat{r}.\hat{n}) = Eds = \frac{q}{4\pi\varepsilon_0 r^2}ds \text{ .... (1)}$$

The electric flux through the entire surface,

$$\varphi = \oint_S d\varphi = \oint_S \frac{q}{4\pi\varepsilon_0 r^2}ds = \frac{q}{4\pi\varepsilon_0 r^2}\oint_S ds = \frac{q}{4\pi\varepsilon_0 r^2}.4\pi r^2$$

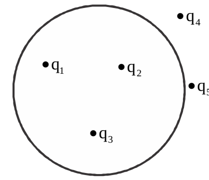
$$\Rightarrow \varphi = \frac{q}{\varepsilon_0}$$

Notes:

- It is regarded as the fundamental law in electrostatics
- It gives the relation between electric field at a point on the close surface and net charge enclosed
- The hypothetical surface on which Gauss law is obeyed is called Gaussian surfaces.
- Gauss law can be used to find the electric field intensities due to some symmetrical field arrangements like infinitely long uniformly charged wire, infinite charged sheet, etc.

Points to ponder:-

- ✓ If some charges are placed in a medium other than vacuum then the law takes the form as



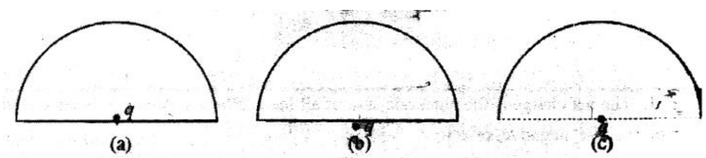
$$\phi = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

All the charges shown in the figure contribute to the electric field on the surface but only the charges within the surface contribute to the flux.

- ✓ Importance of Gauss law over Coulomb's law
  - Applicable for both static and dynamic charges.
  - Useful for calculation of field due to symmetric charge distribution whereas coulomb's law only for point charges.

Question 30:-

Find the electric flux due to charge q placed as shown in the following given figures.



Solution:-

(a) $\phi = \frac{q}{\varepsilon_0}$ (b) $\phi = \frac{q}{2\varepsilon_0}$ (c) $\phi = 0$

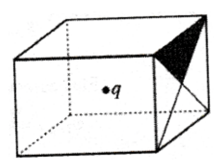
Question 31:-

A point charge q is placed at the center of a cubical box. Find

(a) Total flux associated with the box

(b) Flux emerging through each face of the box

(c) Flux through a shaded area of the surface



Solution:-

(a) Total flux $\phi_{net} = \frac{q}{\varepsilon_0}$

(b) Total flux is emerging equally from each face of the box (6 faces).

$\therefore$ Flux through each face = $\frac{q}{6\varepsilon_0}$

(c) The flux through the shaded portion $\phi' = \frac{\phi}{4} = \frac{q}{24\varepsilon_0}$

Application of Gauss's law:-

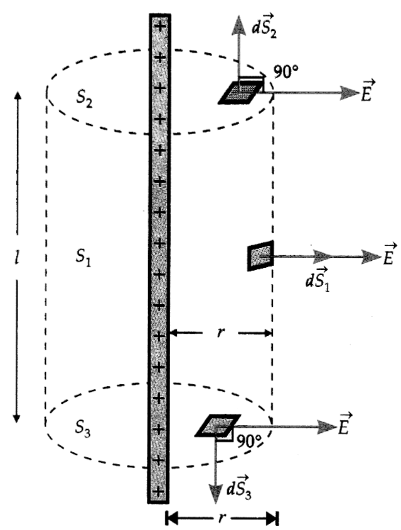
Electric field due to an infinitely long straight uniformly charged wire.

The figure shows a small path of infinitely long cylindrical non-conducting charged wire of linear charge density λ.

Thus $\lambda = \frac{q}{\ell} \Rightarrow q = \lambda\ell$. Here the direction of the field is radially outward.

Let P be the point of r distance from the axis of the wire.

Applying Gauss's theorem



$$\oint_s \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \int \vec{E}.d\vec{s}_1 + \int \vec{E}.d\vec{s}_2 + \int \vec{E}.d\vec{s}_3 = \frac{\lambda\ell}{\varepsilon_0}$$

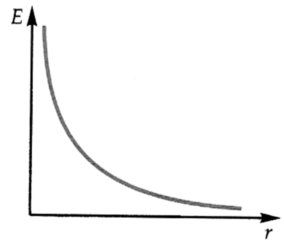
$$\Rightarrow 0 + 0 + \int \vec{E}.d\vec{s}_3 \cos 0^0 = \frac{\lambda\ell}{\varepsilon_0}.$$

$$\Rightarrow E\int ds_3 = \frac{\lambda\ell}{\varepsilon_0}$$

$$\Rightarrow E(2\pi r\ell) = \frac{\lambda\ell}{\varepsilon_0}$$

$$\boxed{\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

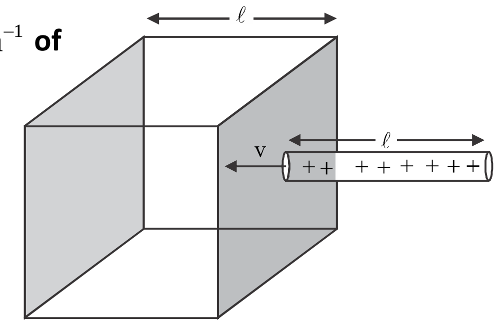
In vector form $\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}.\hat{r}}$



Variation of E with r :

Question 31:-

A uniformly charged rod with a linear charge density λCm^{-1} of length ℓ is inserted in the cube with constant velocity v and moves out of the opposite face. Draw the graph showing the variation of electric flux with time.

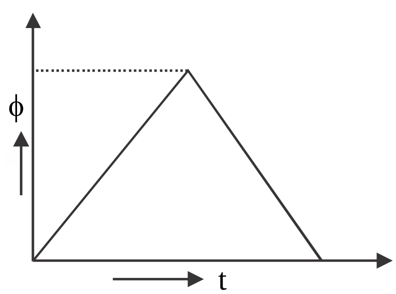


Solution:-

By Gauss's law

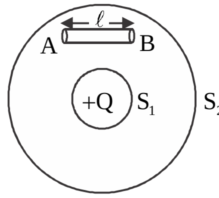
$\phi = \frac{q}{\varepsilon_0} = \frac{\lambda\ell}{\varepsilon_0} = \frac{\lambda(vt)}{\varepsilon_0} \therefore$ (after t sec length of the charged rod in cube = vt)

$\Rightarrow \phi \propto t$ till the entire rod enters the cube and thereafter ϕ decreases



Question32:-

In the figure shown, calculate the total flux of the electrostatic field through the surface S_1 and S_2. The wire AB shown has a linear charge density λ given $\lambda = kx$ where x is the distance measured along the wire from end A.



Solution:-

The total charge on wire AB

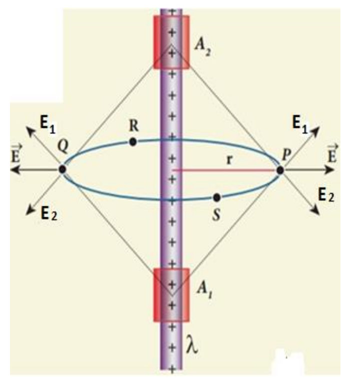
$$Q = \int_0^\ell \lambda dx = \int_0^\ell kxdx = \frac{1}{2}k\ell^2$$

By Gauss's theorem.

Total flux through $S_1 = \frac{Q}{\varepsilon_0}$ and total flux through $S_2 = \frac{Q+\frac{1}{2}k\ell^2}{\varepsilon_0}$

Question 33:-

Prove that the electric field due to an infinitely charged rod is radial with a plane normal to the length of the conductor.

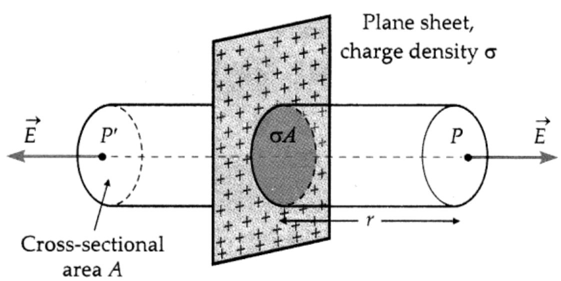


Solution:-

Two normal components of E_1 and E_2 canceled out only resultant vector acts radially outwards.

Electric field due to a uniformly charged infinite, non conducting, plane sheet:-

The figure shows a portion of a thin, infinite, non-conducting sheet with a uniform surface charge density σ. For non-conducting sheet charges are residing on only one of its two plane surfaces.



To find the field intensity at a distance *r* in front of the sheet we choose a Gaussian surface (cylindrical) with end caps of area A.

The charge enclosed in the Gaussian cylinder is; $q = \sigma A$

Applying Gauss's theorem

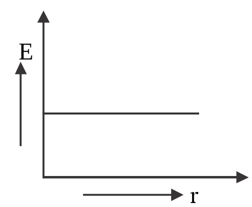
$\oint_s \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$

$$\Rightarrow \int \vec{E}.d\vec{s}_1 + \int \vec{E}.d\vec{s}_2 + \int \vec{E}.d\vec{s}_3 = \frac{\sigma A}{\varepsilon_0}$$

$$\Rightarrow \int Eds_1.\cos 0 + \int Eds_2 \cos 0^0 + \int Eds_3 \cos 9\,0^0 = \frac{\sigma A}{\varepsilon_0} \Rightarrow \int 2Eds = \frac{\sigma A}{\varepsilon_0}$$

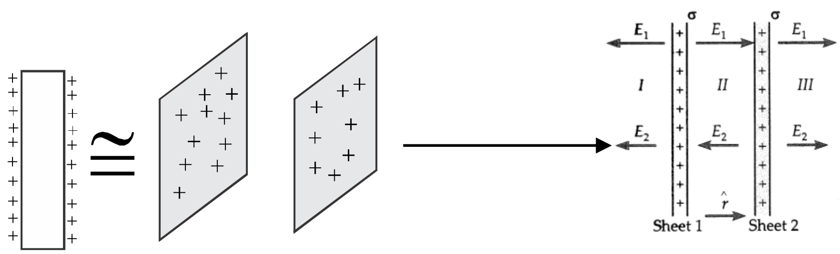
$$\Rightarrow 2E\int ds = \frac{\sigma A}{\varepsilon_0}$$

$\boxed{\Rightarrow 2EA = \frac{\sigma A}{\varepsilon_0}} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$



In vector form $\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}}$

For conducting sheet:-



In conducting sheet, charges reside on the entire outside surface. So a charged conducting sheet is equivalent to two sheets with equal charges.

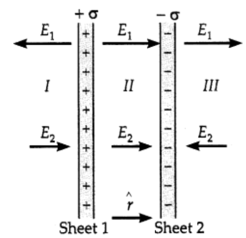
Field on the right of plates	Find in between the plates	Field on the left of plates
$E_{net} = E_1 + E_2$ $= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$	$E_{net} = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$	$E_{net} = E_1 + E_2$ $= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

Points to ponder:-

- ✓ When the equal and same sign of charges are given to parallel plates, the charges will appear on the far faces of the plates.
- ✓ When two parallel plates are given equal and opposite charges, the charge will appear on near faces of plates.

Question 34:-

Find the electric field in between and outside of two infinite parallel plates of surface charge densities $+\sigma \text{ and} -\sigma$**.**

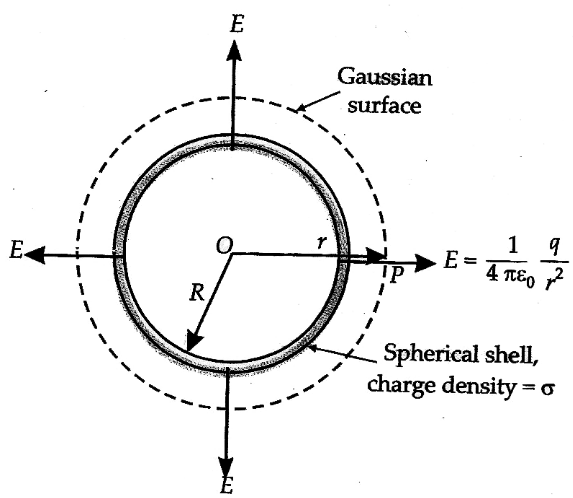


Solution:-

Find in between plates at P_1 $E_{net} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$.

The field outside the plates at P_2 $E_{net} = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

Electric field due to a uniformly charged thin spherical shell or solid conducting sphere.:-



The figure shows a thin spherical shell of radius R with surface charge density σ.

The field at any point P must be radial.

The field at a point outside the shell:-

To find the field at P we take the Gaussian surface to be a sphere of radius r.

According to Gauss's theorem

$$\oint_s \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \oint_s Eds\cos 0 = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E\oint_s ds = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E\left(4\pi r^2\right) = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q_{in}}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow E = \frac{\sigma s}{4\pi\varepsilon_0 r^2}.$$

$$\Rightarrow E = \frac{\sigma 4\pi R^2}{4\pi\varepsilon_0 r^2} = \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2}$$

Hence field outside the shell.

$$\boxed{\Rightarrow E = \frac{\sigma 4\pi R^2}{4\pi\varepsilon_0 r^2} = \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2}}$$

Field on the surface of a shell:-

On the surface; r = R.

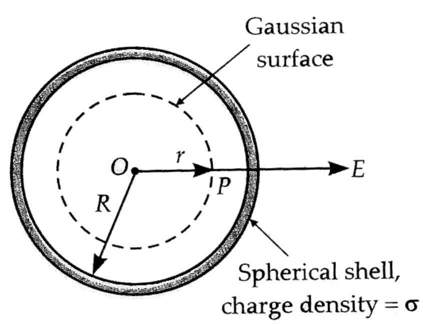
Substituting r = R in the above equation

$$\boxed{E_{on\,surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} = \frac{\sigma}{\varepsilon_0}}$$

The field at a point inside the shell:-

Here the Gaussian surface encloses no charge.

Applying Gauss's law.



$$\oint_s \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0} = \frac{0}{\varepsilon_0}$$

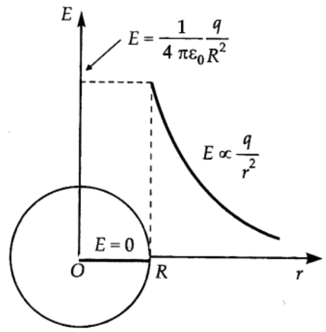
$$\Rightarrow \int Eds = 0$$

$$\boxed{\Rightarrow E = 0}$$

Question 35:-

Draw the graph showing the variation of the electric field with distance r from the center of a uniformly +vely charged thin spherical shell.

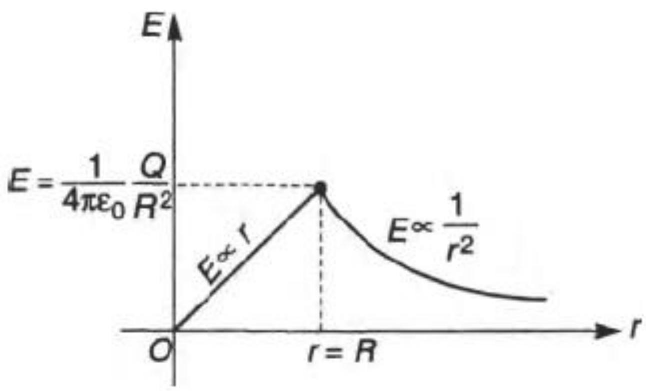
Solution:-



Question 36:-

Draw the graph showing the variation of the electric field with distance r from the center of uniformly charged non-conducting solid spheres.

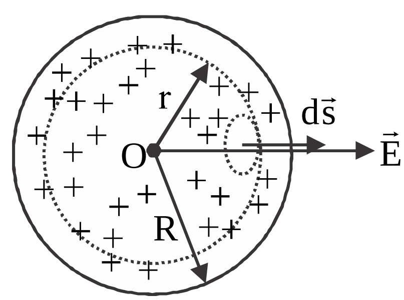
Solution:-



Question 37:-

Inside the surface of a charged non conducting solid sphere prove that electric field is directly proportional to the distance (r)

Solution:-



According to Gauss's law

$$\oint_s \vec{E}.d\vec{s} = \frac{q'}{\varepsilon_0}$$

$$\Rightarrow \oint_s E ds \cos 0 = \frac{q'}{\varepsilon_0}$$

Volume	Charge
$\frac{4}{3}\pi R^3$	q
1	$\frac{q}{\frac{4}{3}\pi R^3}$
$\frac{4}{3}\pi r^3$	$\left(\frac{q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right)$

$$\Rightarrow E \oint_s ds = \frac{q'}{\varepsilon_0}$$

$$\Rightarrow E(4\pi r^2) = q\left(\frac{r^3}{R^3}\right)/\varepsilon_0$$

$$\Rightarrow E = \frac{qr^3}{4\pi\varepsilon_0 r^2 R^3}$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 R^3} r$$

As $\frac{q}{4\pi\varepsilon_0 R^3}$ =constant

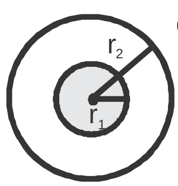
$\Rightarrow E \propto r$

Question 38:-

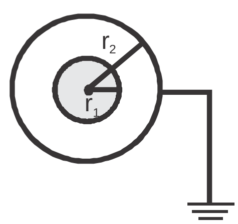
A metal sphere of radius r_1 with charge q is placed at the center of an uncharged thin conducting shell of radius r_2. Find the electric field in the following cases at the position

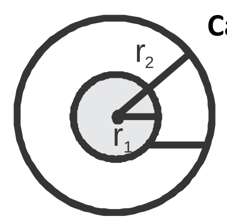
(a) $r > r_2$ **(b)** $r_2 > r > r_1$ **(c)** $r < r_1$

Case – I



Case– II

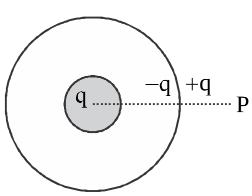




Case – III

Solution:-

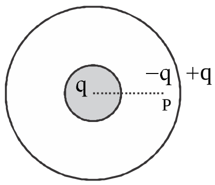
(a) At point r > r_2



$$E_p = E_1 + E_2$$

$$= \frac{kq}{r^2} + \frac{k(-q+q)}{r^2} = \frac{kq}{r^2}$$

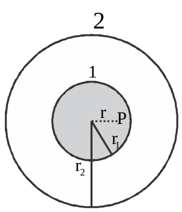
(b) At the point $r_1 < r < r_2$,



$$E_p = E_1 + E_2$$

$$= \frac{kq}{r^2} + 0 = \frac{kq}{r^2}$$

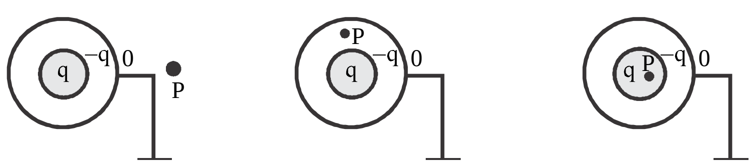
(c) $r < r_1$



$$E_p = E_1 + E_2 = 0 + 0 = 0$$

Case – II

$r > r_2$ $\quad$ $r_1 < r < r_2$ $\quad$ $r < r_1$



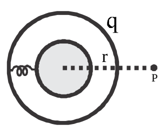
$$E_p = E_1 + E_2 = \frac{kq}{r^2} - \frac{kq}{r^2} = 0$$

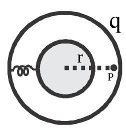
$$E_p = E_1 + E_2 = \frac{kq}{r^2} + 0 = \frac{kq}{r^2}$$

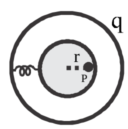
$$E_p = E_1 + E_2 = 0 + 0 = 0$$

Case – III

$r > r_2$







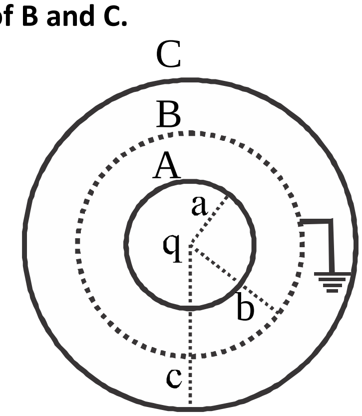
$$E_p = E_1 + E_2 = 0 + \frac{kq}{r^2} = \frac{kq}{r^2}$$

$$E_p = 0 + 0 = E_1 + E_2 = 0$$

$$E_p = E_1 + E_2 = 0 + 0 = 0$$

Question 39:-

The figure shows three concentric thin spherical shells A, B, and C of radii a, b, and c respectively. The shell A and C are given charges q and –q respectively and the shell B is earthed. Find the charge appearing on the surface of B and C.



Solution:-Left for the students.

FORMULA CORNER:

1. The total charge on a body; $Q = \pm ne$

   Where e = charge on electron = 1.6 x 10^{-19} C

2. The magnitude of the force between two point charges; $F = \frac{k|q_1q_2|}{r^2} = \frac{1}{4\pi\varepsilon}\frac{|q_1q_2|}{r^2}$

3. $\frac{F_{\text{vacuum}}}{F_{\text{medium}}} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K$

4. In vector form; $\vec{F} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^3}\vec{r}$ [Where $\vec{r}$ position of the charge experiencing force w.r.t. the charge exerting force]

5. The net electrostatic force on a point charge due to a large number of point charges is; $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots.$

6. The electric field intensity, at a position $\vec{r}$ due to a point charge, kept at origin is; $\vec{E} = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon}\frac{q}{r^3}\vec{r}$

   If the source charge is kept at a position $\vec{r}$, then $\vec{E} = \frac{1}{4\pi\varepsilon}\frac{q}{|\vec{r}-\vec{r}_0|^3}(\vec{r} - \vec{r}_0)$

7. The electric field intensity due to multiple point charges; $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$

8. The electric field due to a small dipole; at an end-on-position; $\vec{E} = \frac{1}{4\pi\varepsilon}\frac{2\vec{p}}{r^3}$

   At a broad-side-on position; $\vec{E} = -\frac{1}{4\pi\varepsilon}\frac{\vec{p}}{r^3}$

   At any point; $\vec{E} = \frac{1}{4\pi\varepsilon}\frac{2p\cos\theta}{r^3}\hat{r} - \frac{1}{4\pi\varepsilon}\frac{p\sin\theta}{r^3}\hat{\theta}$

9. When a point charge is kept in an electric field; the force on it is; $\vec{F} = q\vec{E}$

   Acceleration of the charged particle due to the electric field is; $\vec{a} = \frac{q\vec{E}}{m}$

10. When an electric dipole is placed in uniform electric field, then $\vec{F}_{net} = 0$; $\vec{\tau} = \vec{p} \times \vec{E} \Rightarrow \tau = pE\sin\theta$

11. When an electric dipole is placed parallel to an electric field whose magnitude is varying; then $\tau = 0$, and $\vec{\mathrm{F}} = p\frac{\mathrm{d}\vec{\mathrm{E}}}{\mathrm{dx}}$

12. The electric flux through a plane surface; $\varphi_E = \vec{E}.\vec{A} = EA\cos\theta$

13. The electric flux through any surface; $\varphi_E = \oint_S \vec{E}.d\vec{A} = \oint_S EdA\cos\theta$

14. The electric flux through a closed surface; $\varphi_E = \frac{q_{enclosed}}{\varepsilon_0}$

15. The electric field intensity due to an infinitely long uniformly charged wire is; $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$

16. The electric field intensity due to an infinitely large uniformly charged non-conducting sheet is; $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$

17. The electric field intensity due to an infinitely large uniformly charged conducting sheet is; $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$

18. The electric field due to a uniformly charged spherical shell or a conducting sphere of radius R ; $E_{in} = 0,\ E_{ext} = \frac{Q}{4\pi\varepsilon_0 r^2},\ E_{surface} = \frac{Q}{4\pi\varepsilon_0 R^2}$

19. The electric field due to a solid sphere; $E_{in} = \frac{Qr}{4\pi\varepsilon_0 R^3},\ E_{ext} = \frac{Q}{4\pi\varepsilon_0 r^2},\ E_{surface} = \frac{Q}{4\pi\varepsilon_0 R^2}$