

ERRORS IN MEASUREMENT XI- SCIENCE

SUBJECT : PHYSICS CHAPTER NUMBER: 2 CHAPTER NAME : UNITS AND MEASUREMENT

CHANGING YOUR TOMORROW

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In an experiment involving several measurements, the errors in all the measurements get combined.

Example:

- Density is the ratio of the mass to the volume of the substance.
- If there are errors in the measurement of mass and of the sizes or dimensions, then there will be error in the density of the substance.



(a) Error of a Sum:

Suppose two physical quantities A and B have measured values

A $\pm \Delta A$, B $\pm \Delta B$ respectively, where ΔA and ΔB are their absolute errors.

Let Z = A + B $Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$ $= (A + B) \pm (\Delta A + \Delta B)$ $= Z \pm (\Delta A + \Delta B)$ $\pm \Delta Z = \pm (\Delta A + \Delta B)$ $\Delta Z = (\Delta A + \Delta B)$

Note: When two quantities are added, the absolute error in the final result is the sum of the individual errors.



Numerical

The masses of two bodies measured are $m_1 \pm \Delta m_1 = (20 \pm 0.5)gm$ and $m_2 \pm \Delta m_2 = (50 \pm 0.5)gm$ Calculate the sum of the masses and the error therein.

Solution

Sum with error is;

 $M \pm \Delta M = (\mathbf{m}_2 + m_1) \pm (\Delta m_1 + \Delta m_2)$ $\Rightarrow M \pm \Delta M = 70g \pm 1g$



(b) Error of a Difference:

Suppose two physical quantities A and B have measured values

A $\pm \Delta A$, B $\pm \Delta B$ respectively, where ΔA and ΔB are their absolute errors.

Let, Z = A - B $Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$ $= (A - B) \pm \Delta A \mp \Delta B$ $= Z \pm (\Delta A + \Delta B)$ (since \pm and \mp are the same) $\pm \Delta Z = \pm (\Delta A + \Delta B)$ $\Delta Z = (\Delta A + \Delta B)$

When two quantities are subtracted, the absolute error in the final result is the **<u>sum</u>** of the individual errors.

Rule:

When two quantities are added or subtracted, the absolute error in the final result is the **<u>sum</u>** of the absolute errors in the individual quantities.



Question: The temperature of two bodies measured by a thermometer are $(20 \pm 0.5)^0 C = t_1 \pm \Delta t_1$ $(50 \pm 0.5)^0 C = t_2 \pm \Delta t_2$

Calculate the difference in temperature and the error there in:

Solution:

 $T = t_2 - t_1 = 50 - 20 = 30^{\circ}C$ $\pm \Delta T = \Delta t_2 + \Delta t_1 = \pm (0.5 + 0.5) = \pm 1^{\circ}C$ $T \pm \Delta T = (30 \pm 1)^{\circ}C$



(c) Error of a Product:

Suppose two physical quantities A and B have measured values

A $\pm \Delta A$, B $\pm \Delta B$ respectively, where ΔA and ΔB are their absolute errors.

 $7 = A \times B$ let $Z \pm \Delta Z = (A \pm \Delta A) \times (B \pm \Delta B)$ $Z \pm \Delta Z = AB \pm A \Delta B \pm B \Delta A \pm \Delta A \Delta B$ Dividing LHS by Z and RHS by AB we have, $1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A \Delta B}{AB}$ $\pm \frac{\Delta Z}{Z} = \pm \frac{\Delta B}{R} \pm \frac{\Delta A}{A} \dots \dots \dots \dots (:: \frac{\Delta A \Delta B}{\Delta R}$ is very small and hence negligible) $\frac{\Delta \mathbf{Z}}{\mathbf{Z}} = \frac{\Delta \mathbf{A}}{\mathbf{A}} + \frac{\Delta \mathbf{B}}{\mathbf{B}}$ Note:

When two quantities are multiplied, the relative error in the final result is the **<u>sum</u>** of the relative errors of the individual quantities.



(d) Error of a Quotient:

Suppose two physical quantities A and B have measured values A $\pm \Delta A$, B $\pm \Delta B$ respectively, where ΔA and ΔB are their absolute errors. Let, $Z = \frac{A}{B}$ $Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B}$ $Z\left(1\pm\frac{\Delta Z}{Z}\right) = \frac{A\left(1\pm\frac{\Delta A}{A}\right)}{B\left(1\pm\frac{\Delta B}{R}\right)}$ $1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right) \left(1 \pm \frac{\Delta B}{B}\right)^{-1}$ $1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right) \left(1 \mp \frac{\Delta B}{B}\right)$ $1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \mp \left(\frac{\Delta A}{A} \times \frac{\Delta B}{B}\right)$ $\pm \frac{\Delta \mathbf{Z}}{\mathbf{Z}} = \frac{\Delta \mathbf{A}}{\mathbf{A}} + \frac{\Delta \mathbf{B}}{\mathbf{B}}$



Question: The resistance $R = \frac{V}{I}$, $V = (100 \pm 5)V$, $I = (10 \pm 0.2)A$ Find the % error in R.

Solution:

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{5}{100} + \frac{0.2}{10} = \frac{7}{100}$$

% error = $\frac{\Delta R}{R} \times 100 = \frac{7}{100} \times 100 = 7\%$



(f) Error of an Exponent (Power):

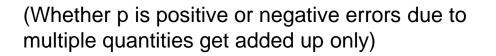
Suppose a physical quantity A has measured values A $\pm \Delta A$ where ΔA is its absolute error.

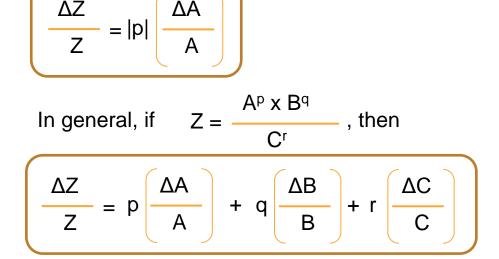
Let $Z = A^p$ where p is a constant.

Applying log on both the sides, we have

 $\log Z = |p| \log A$

Differentiating, we have





Note:

C^r is in Denominator, but the relative error is added up.



Question: Find the relative error in Z if $Z = \frac{A^4B^{1/3}}{CD^{3/2}}$

Solution:

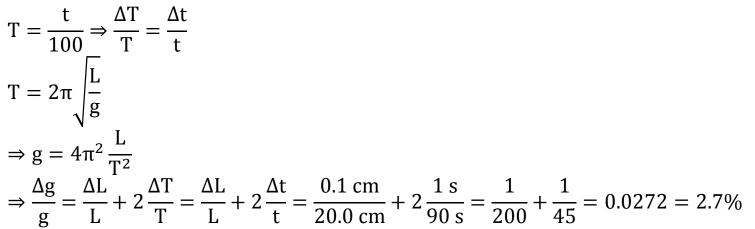
$$4\frac{\Delta A}{A} + \frac{1}{3}\frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{3}{2}\frac{\Delta D}{D}$$



Question: Time period of a simple pendulum is given by the formula, $T = 2\pi \sqrt{\frac{L}{g}}$. The length of

the pendulum is measured to be 20.0 cm known to 1mm accuracy. The time for 100 oscillations is measured to be 90 s by using a watch of 1s resolution. Calculate the uncertainty in the determination of acceleration due to gravity.

Solution:





Question: $R_1 = (100 \pm 3)\Omega$, $R_2 = (200 \pm 4)\Omega$

Resistance are connected in (i) series (ii) parallel. Find the equivalent resistance in series and parallel connection.

Solution:

i. Series, $R = R_1 + R_2$ $R = (100 + 200) = 300\Omega$ $\Delta R = \Delta R_1 + \Delta R_2 = 3 + 4 = 7\Omega$ $R \pm \Delta R = (300 \pm 7)\Omega$



Solution:

ii. Parallel,
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

 $R = \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{3} = 66.7$ ohm
 $\frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$
 $\Delta R = R^2 \left(\frac{\Delta R_1}{R_1^2}\right) + R^2 \left(\frac{\Delta R_2}{R_2^2}\right)$
 $\Delta R = \left(\frac{R^2}{R_1}\right)^2 \Delta R_1 + \left(\frac{R^2}{R_2}\right)^2 \Delta R_2$
 $\Delta R = \left(\frac{66.7}{100}\right)^2 x_3 + \left(\frac{66.7}{200}\right)^2 x_4 = 1.8$

 $\mathbf{R} \pm \Delta \mathbf{R} = (66.7 \pm 1.8)\Omega$



Home Assignment

- 1. The temperatures of two bodies measured by a thermometer are $t_1 = 20^{\circ}C \pm 0.5^{\circ}C$ and $t_2 = 30^{\circ}C \pm 0.5^{\circ}C$. Calculate the temperature difference and the error therein.
- 2. Density of a sphere is given by the formula $\rho = \frac{6m}{\pi D^3}$. Mass (m) of the sphere is measured to be 50g known to 1g accuracy. Diameter of the sphere is measured to be 2.50 cm by a slide caliper with 0.01 cm least count. What is the accuracy I the determination of density ρ .
- 3. A quantity is defined by the relation, $Q = \frac{AB^2}{\sqrt{c}D^3}$. The percentage error in A , B , C and D are measured to be 1% , 2% , 4% and 1% respectively . Calculate the percentage error in calculation of Q.
- 4. If error in measuring diameter of a sphere is 2%, then what is the error in calculating its volume?



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