# ERRORS IN MEASUREMENT

XI- SCIENCE

**SUBJECT : PHYSICS**
**CHAPTER NUMBER: 2**
**CHAPTER NAME : UNITS AND MEASUREMENT**

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# COMBINATION OF ERRORS

In an experiment involving several measurements, the errors in all the measurements get combined.

**Example**:

- Density is the ratio of the mass to the volume of the substance.
- If there are errors in the measurement of mass and of the sizes or dimensions, then there will be error in the density of the substance.

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# COMBINATION OF ERRORS

**(a) Error of a Sum:**

Suppose two physical quantities A and B have measured values

$A \pm \Delta A$, $B \pm \Delta B$ respectively, where $\Delta A$ and $\Delta B$ are their absolute errors.

Let $Z = A + B$

$$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$

$$= (A + B) \pm (\Delta A + \Delta B)$$

$$= Z \pm (\Delta A + \Delta B)$$

$$\pm \Delta Z = \pm (\Delta A + \Delta B)$$

$$\boldsymbol{\Delta Z = (\Delta A + \Delta B)}$$

**Note**: When two quantities are added, the absolute error in the final result is the sum of the individual errors.

# COMBINATION OF ERRORS

## Numerical

The masses of two bodies measured are $m_1 \pm \Delta m_1 = (20 \pm 0.5)gm$ and $m_2 \pm \Delta m_2 = (50 \pm 0.5)gm$ Calculate the sum of the masses and the error therein.

**Solution**

Sum with error is;

$$M \pm \Delta M = \left(\mathrm{m}_2 + m_1\right) \pm \left(\Delta m_1 + \Delta m_2\right)$$

$$\Rightarrow M \pm \Delta M = 70g \pm 1g$$

# COMBINATION OF ERRORS

**(b) Error of a Difference**:

Suppose two physical quantities A and B have measured values

$A \pm \Delta A$, $B \pm \Delta B$ respectively, where $\Delta A$ and $\Delta B$ are their absolute errors.

Let, $Z = A - B$

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$

$$= (A - B) \pm \Delta A \mp \Delta B$$

$$= Z \pm (\Delta A + \Delta B) \quad \text{(since } \pm \text{ and } \mp \text{ are the same)}$$

$$\pm \Delta Z = \pm (\Delta A + \Delta B)$$

$$\mathbf{\Delta Z = (\Delta A + \Delta B)}$$

When two quantities are subtracted, the absolute error in the final result is the **sum** of the individual errors.

**Rule**:

When two quantities are added or subtracted, the absolute error in the final result is the **sum** of the absolute errors in the individual quantities.

## Numerical

**Question**: The temperature of two bodies measured by a thermometer are

$(20 \pm 0.5)^0 C = t_1 \pm \Delta t_1$

$(50 \pm 0.5)^0 C = t_2 \pm \Delta t_2$

Calculate the difference in temperature and the error there in:

**Solution**:

$T = t_2 - t_1 = 50 - 20 = 30^0 C$

$\pm \Delta T = \Delta t_2 + \Delta t_1 = \pm(0.5 + 0.5) = \pm 1^0 C$

$T \pm \Delta T = (30 \pm 1)^0 C$

# COMBINATION OF ERRORS

**(c) Error of a Product:**

Suppose two physical quantities A and B have measured values

$A \pm \Delta A$, $B \pm \Delta B$ respectively, where $\Delta A$ and $\Delta B$ are their absolute errors.

Let $Z = A \times B$

$$Z \pm \Delta Z = (A \pm \Delta A) \times (B \pm \Delta B)$$

$$Z \pm \Delta Z = AB \pm A\Delta B \pm B\Delta A \pm \Delta A\,\Delta B$$

Dividing LHS by Z and RHS by AB we have,

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A \Delta B}{AB}$$

$$\pm \frac{\Delta Z}{Z} = \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \ldots\ldots\ldots\ldots\ldots.. (\because \frac{\Delta A \Delta B}{AB} \text{ is very small and hence negligible})$$

$$\mathbf{\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}}$$

**Note**:

When two quantities are multiplied, the relative error in the final result is the **sum** of the relative errors of the individual quantities.

# COMBINATION OF ERRORS

**(d) Error of a Quotient:**

Suppose two physical quantities A and B have measured values

A ± ΔA, B ± ΔB respectively, where ΔA and ΔB are their absolute errors.

Let, $Z = \frac{A}{B}$

$$Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B}$$

$$Z\left(1 \pm \frac{\Delta Z}{Z}\right) = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$$

$$1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

$$1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$$

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \mp \left(\frac{\Delta A}{A} x \frac{\Delta B}{B}\right)$$

$$\pm \frac{\mathbf{\Delta Z}}{\mathbf{Z}} = \frac{\mathbf{\Delta A}}{\mathbf{A}} + \frac{\mathbf{\Delta B}}{\mathbf{B}}$$

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## Numerical

**Question**: The resistance $R = \frac{V}{I}$, $V = (100 \pm 5)V$, $I = (10 \pm 0.2)A$

Find the % error in R.

**Solution**:

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{5}{100} + \frac{0.2}{10} = \frac{7}{100}$$

$$\%\ \text{error} = \frac{\Delta R}{R} \text{x}100 = \frac{7}{100} \text{x}100 = 7\%$$

# COMBINATION OF ERRORS

**(f) Error of an Exponent (Power):**

Suppose a physical quantity A has measured values $A \pm \Delta A$ where $\Delta A$ is its absolute error.

Let $Z = A^p$ where p is a constant.

Applying log on both the sides, we have

$\log Z = |p| \log A$

Differentiating, we have

$$\frac{\Delta Z}{Z} = |p| \left( \frac{\Delta A}{A} \right)$$

(Whether p is positive or negative errors due to multiple quantities get added up only)

In general, if $Z = \dfrac{A^p \times B^q}{C^r}$, then

$$\frac{\Delta Z}{Z} = p \left( \frac{\Delta A}{A} \right) + q \left( \frac{\Delta B}{B} \right) + r \left( \frac{\Delta C}{C} \right)$$

**Note**:
$C^r$ is in Denominator, but the relative error is added up.

# Numerical

**Question**: Find the relative error in Z if $Z = \frac{A^4 B^{1/3}}{C D^{3/2}}$

**Solution**:

$$4\frac{\Delta A}{A} + \frac{1}{3}\frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{3}{2}\frac{\Delta D}{D}$$

# Numerical

**Question**: Time period of a simple pendulum is given by the formula, $T = 2\pi\sqrt{\frac{L}{g}}$. The length of the pendulum is measured to be 20.0 cm known to 1mm accuracy. The time for 100 oscillations is measured to be 90 s by using a watch of 1s resolution. Calculate the uncertainty in the determination of acceleration due to gravity.

**Solution**:

$$T = \frac{t}{100} \Rightarrow \frac{\Delta T}{T} = \frac{\Delta t}{t}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\Rightarrow g = 4\pi^2 \frac{L}{T^2}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T} = \frac{\Delta L}{L} + 2\frac{\Delta t}{t} = \frac{0.1\ \text{cm}}{20.0\ \text{cm}} + 2\frac{1\ \text{s}}{90\ \text{s}} = \frac{1}{200} + \frac{1}{45} = 0.0272 = 2.7\%$$

# Numerical

**Question**: $R_1 = (100 \pm 3)\Omega, R_2 = (200 \pm 4)\Omega$

Resistance are connected in (i) series (ii) parallel. Find the equivalent resistance in series and parallel connection.

**Solution**:

i. Series, $R = R_1 + R_2$

$R = (100 + 200) = 300\Omega$

$\Delta R = \Delta R_1 + \Delta R_2 = 3 + 4 = 7\Omega$

$R \pm \Delta R = (300 \pm 7)\Omega$

# Numerical

**Solution**:

ii. Parallel, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{3}$ =66.7ohm

$$\frac{\Delta R}{R^2} = \frac{\Delta R_1}{{R_1}^2} + \frac{\Delta R_2}{{R_2}^2}$$

$$\Delta R = R^2 \left(\frac{\Delta R_1}{{R_1}^2}\right) + R^2 \left(\frac{\Delta R_2}{{R_2}^2}\right)$$

$$\Delta R = \left(\frac{R^2}{R_1}\right)^2 \Delta R_1 + \left(\frac{R^2}{R_2}\right)^2 \Delta R_2$$

$$\Delta R = \left(\frac{66.7}{100}\right)^2 x3 + \left(\frac{66.7}{200}\right)^2 x4 = 1.8$$

$$R \pm \Delta R = (66.7 \pm 1.8)\Omega$$

# Home Assignment

1. The temperatures of two bodies measured by a thermometer are $t_1 = 20^oC \pm 0.5^oC$ and $t_2 = 30^oC \pm 0.5^oC$. Calculate the temperature difference and the error therein.

2. Density of a sphere is given by the formula $\rho = \frac{6m}{\pi D^3}$. Mass (m) of the sphere is measured to be 50g known to 1g accuracy. Diameter of the sphere is measured to be 2.50 cm by a slide caliper with 0.01 cm least count. What is the accuracy I the determination of density $\rho$.

3. A quantity is defined by the relation, $Q = \frac{AB^2}{\sqrt{C}D^3}$. The percentage error in A , B , C and D are measured to be 1% , 2% , 4% and 1% respectively . Calculate the percentage error in calculation of Q.

4. If error in measuring diameter of a sphere is 2%, then what is the error in calculating its volume?

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**THANKING YOU**

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