Dimensional analysis and its applications.

XI- SCIENCE

SUBJECT : PHYSICS
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Quantities having the same dimensional formulae

1. Impulse and momentum
2. Work, energy, torque, moment of force
3. Angular momentum, Planck's constant, rotational impulse
4. Stress, pressure, modulus of elasticity, energy density
5. Force constant, surface tension, surface energy
6. Angular velocity, frequency, velocity gradient
7. Gravitational potential, latent heat
8. Thermal capacity, entropy, universal gas constant and Boltzmann's const.
9. Force, thrust
10. Power, luminous flux

Dimensional formulae for physical quantities often used in Physics are given at the end.

Dimensional analysis and its applications.

Principle of homogeneity : If an equation truly expresses a proper relationship between variables in a physical process, it will be dimensionally homogeneous; i.e., each of its additive terms will have the same dimensions.

- Two quantities can be added or subtracted if they are dimensionally same .
- In a correct equation all terms are dimensionally same .
- Trigonometric functions , logarithmic functions , exponential functions are dimensionless . Also the arguments or power of these functions are dimensionless .
- E.g. : In $\sin(\omega t + \theta_0)$, the whole function is dimensionless . Also ωt and (θ_0) are dimensionless .

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Dimensional analysis and its applications.

Question : In an equation ; $v = \frac{F}{a + bt}$

v = speed , F = force and t = time . Find dimensional formula of a and b .

Solution :

$$v = \frac{F}{a + bt}$$

$$\Rightarrow av + bvt = F$$

$$\Rightarrow [av] = [bvt] = [F]$$

$$\Rightarrow [a] = \left[\frac{F}{v}\right] = \left[\frac{MLT^{-2}}{LT^{-1}}\right] = ML^0T^{-1}$$

$$\Rightarrow [b] = \left[\frac{F}{vt}\right] = \left[\frac{MLT^{-2}}{LT^{-1}T}\right] = ML^0T^{-2}$$

Dimensional analysis and its applications.

Question : In an equation ; $\left(P+\frac{a}{V^2}\right)(V-b)=RT$

P = Pressure , V = volume and T =temperature . Find dimensional formula of a , b and R .

Solution :

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

$$\Rightarrow PV-Pb+\frac{a}{V}-\frac{ab}{V^2}=RT$$

$$\Rightarrow [PV]=[Pb]=\left[\frac{a}{V}\right]=\left[\frac{ab}{V^2}\right]=[RT]$$

$$\Rightarrow [b]=[V]=L^3$$

$$\Rightarrow [a]=[PV^2]=(ML^{-1}T^{-2})(L^3)^2=ML^5T^{-2}$$

$$\Rightarrow [R]=\left[\frac{PV}{T}\right]=\left[\frac{ML^{-1}T^{-2}.L^3}{K}\right]=ML^2T^{-2}K^{-1}$$

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Dimensional analysis and its applications.

Question : In an equation ; $y = \alpha \exp\left(-\frac{\alpha t}{\beta}\right)$

P = Pressure , V = volume and T =temperature . Find dimensional formula of a , b and R .

Solution : By principle of homogeneity ;

$\exp\left(-\frac{\alpha t}{\beta}\right)$ is dimension less .

So;

$$[y] = [\alpha] \Rightarrow [\alpha] = L^1$$

Again ; $\left(\frac{\alpha t}{\beta}\right)$ is dimension less .

$$\Rightarrow [\beta] = [\alpha t] = L^1 T^1$$

Dimensional analysis and its applications.

- **Applications of dimensional analysis :**

  Check the correctness of equation

  Derivation of expression for some quantities dimensionally

  Role of dimension in unit conversion

Dimensional analysis and its applications.

PROBLEM Let us consider an equation $\frac{1}{2}mv^2 = mgh$, where m is mass, v is velocity of the body, g is acceleration due to gravity and h is the height. Check whether this equation is dimensionally correct.

Solution

$$\left[\frac{1}{2}mv^2\right] = ML^2T^{-2}$$

$$[mgh] = MLT^{-2}L = ML^2T^{-2}$$

So the equation is dimensionally correct.

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Dimensional analysis and its applications.

Question : A book with many printing errors contain four different formulas for the displacement y of a particle undergoing a certain periodic motion :

(a) $y = a \sin(2\pi t / T)$

(b) $y = a \sin(vt)$

(c) $y = (a/T) \sin(t / a)$

(d) $y = a\sqrt{2}\{\sin(2\pi t / T) + \cos(2\pi t / T)\}$

a = max. displacement of the particle ,

v = speed of the particle

T = time period of motion

Rule out the wrong formula on dimensional grounds .

Solution : Left for students

Dimensional analysis and its applications.

Question : A dimensionally correct formula may o may not be actually correct , but an actually correct formula must be dimensionally correct .

Justify the statement by giving proper example

Solution : Let's take a formula ; $E_K = mv^2$

$$[E_K] = ML^2T^{-2}$$

$$[mv^2] = M(LT^{-1})^2 = ML^2T^{-2}$$

So the given eqn. is dimensionally correct , but we know this is actually incorrect .

Let's take a correct equation ; v = u + at

$$[v] = LT^{-1}, [u] = LT^{-1}, and [at] = LT^{-2}.T = LT^{-1}$$

So the equation is also dimensionally correct . Hence the statement is justified .

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Dimensional analysis and its applications.

Unit of a quantity in a system of unit from its dimensional formula :

E.g. :

Find the unit of K.E. in a system where unit of mass is 100 g , unit of length is 10 m and unit of time is 10 s .

Solution :

$$[K.E.] = [ML^2T^{-2}]$$

$$\Rightarrow 1u = (100g)(10m)^2(10s)^{-2} = 100gm^2s^{-2}$$

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Dimensional analysis and its applications.

Homework :

1. **Check correctness of the following equations :**

   (a) $mv^2 = mgh$

   (b) $y = A\exp(\omega t / x)$

   (c) $y = A\ln(at / x)$

2. **Find unit of pressure in a system of units where unit of mass is α kg , unit of length is βm , and unit of time is γ s .**
3. **NCERT exercise 2.1**
4. **NCERT exercise 2.2**
5. **NCERT exercise 2.3**

THANKING YOU

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