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Dimensional analysis and its applications.

XI- SCIENCE

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Dimensional analysis and its applications.

Conversion of unit of a quantity from one system of unit to another :

Any quantity in any system is given by , $Q = nu$

In different systems , if same quantity is measured then its value remains unchanged although its numeric value (n) and unit (u) change . So ;

$$n_1 u_1 = n_2 u_2$$

$$\Rightarrow n_2 = \frac{n_1 u_1}{u_2}$$

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Conversion of unit of a quantity from one system of unit to another :

E.g. :

A calorie is a unit of heat energy and it equals about 4.2 J where $1J = 1kgm^2s^{-2}$ **. Suppose we employ a system of units in which the unit of mass equals α kg , the unit of length equals βm and the unit of time is γs . Show that a calorie has a magnitude** $4.2\alpha^{-1}\beta^{-2}\gamma^2$ **in terms of the new units .**

Solution : $[E] = ML^2T^{-2}$

In S.I. system ; $1J = 1kgm^2s^{-2}$

In new system ; $1u_2 = (\alpha kg)(\beta m)^2(\gamma s)^{-2} = \alpha\beta^2\gamma^{-2}kgm^2s^{-2}$

Let ; $1cal = 4.2J = xu_2$

$$\Rightarrow x = \frac{4.2J}{u_2} = \frac{4.2kgm^2s^{-2}}{\alpha\beta^2\gamma^{-2}kgm^2s^{-2}} = 4.2\alpha^{-1}\beta^{-2}\gamma^2$$

$$\Rightarrow 1cal = 4.2\alpha^{-1}\beta^{-2}\gamma^2u_2$$

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PROBLEM Consider a simple pendulum having a bob attached to a string that oscillates under the action of force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length l, mass of the bob m, and acceleration due to gravity g. Derive the expression for its time period using method of dimensions.

Solution

Let time period t.

Let

$t \propto l^x m^y g^z$ ---eq.(1)

$\Rightarrow t = k l^x m^y g^z$, where k has no dimension

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Writing eq.(1) in dimension form, we get

$$[t]=[k][l]^x[m]^y[g]^z \Rightarrow T = L^x M^y L^z T^{-2z} = M^y L^{x+z} T^{-2z}$$

$$\Rightarrow y=0, x+z=0, -2z=1$$

$$\Rightarrow y=0, x=\frac{1}{2}, z=-\frac{1}{2}$$

So

$$t = k\sqrt{\frac{l}{g}}$$

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Establishment of relation among physical quantities :

E.g. :

A planet moves around sun in circular orbit. Its time period of revolution only depends upon

(i) the radius (r) of the orbit .

(ii) mass of the sun (M)

(iii) the gravitational constant (G) .

Taking proportionality constant as 2π , find the expression for T , using dimensional analysis .

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Establishment of relation among physical quantities :

Solution : Let ; $T \alpha\, r^x M^y G^z$

$\Rightarrow T = 2\pi\, r^x M^y G^z$ ......(i) ($\because$ Proportionality constant $= 2\pi$)

Putting dimensional formula of both sides ,

$$T = L^x M^y \left(M^{-1} L^3 T^{-2}\right)^z = M^{y-z} L^{x+3z} T^{-2z}$$

$$\Rightarrow y - z = 0,\ x + 3z = 0,\ -2z = 1 \Rightarrow z = -\frac{1}{2}$$

$$\Rightarrow y = z = -\frac{1}{2}$$

$$\Rightarrow x = -3z = \frac{3}{2}$$

Using in equation (i) we get ;

$$T = 2\pi\, r^{3/2} M^{-1/2} G^{-1/2} = 2\pi \sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow T^2 \alpha r^3$$

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Establishment of relation among physical quantities :

E.g. :

In a new system of units universal gravitational constant (G) , planck's constant (h) and speed of light (c) are taken as the fundamental quantities . Obtain the dimensional formula for mass in the new system .

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Solution : Let ; $[\text{mass}] = [G^x h^y c^z]$ ......(i)

Putting dimensional formula of both sides ,

$$M = (M^{-1}L^3T^{-2})^x (M^1L^2T^{-1})^y (LT^{-1})^z = M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}$$

$$\Rightarrow -x + y = 1.....(a),\ 3x + 2y + z = 0...(b),\ -2x - y - z = 0.....(c)$$

Adding equations (b) and (c) we have ;

$$\Rightarrow x + y = 0.....(d)$$

Solving (a) and (d) we have ; $x = -\frac{1}{2}$ and $y = \frac{1}{2}$

Putting the values of x and y in the equation (c) we have ;

$$z = -2x - y = 1 - \frac{1}{2} = \frac{1}{2}$$

Now equation (i) becomes ;

$$[\text{mass}] = [G^{-1/2}h^{1/2}c^{1/2}]$$

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Limitations of dimensional analysis:

(i) It can’t give any information about the proportionality constant (k) .

(ii) It fails when a quantity depends upon more than 3 quantities .

(iii) It fails when a quantity is the sum or difference of two or more quantities .

(iv) It fails to establish relationship which involves trigonometric , logarithmic and exponential functions .

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Homework:

1. **1atm = 101300 Pa and** $1Pa = 1kgm^{-1}s^{-2}$

   Now let's imagine a system in which unit of mass is α kg , unit of length is β m and unit of time is γ s . What is the magnitude of 1 atm in the new system of units ?

2. **Assuming that mass (M) of the largest stone that can be moved by a flowing river depends upon the velocity (v) , density (ρ) of the water and acceleration due to gravity (g) . Show by dimensional analysis method that , M varies with the sixth power of v .**

3. **If force (F) , energy (E) and time (T) are taken as fundamental quantities in a system , then find the dimensional formula of mass in the new system .**

4. **Find the dimensions of ab in the relation;**

$$P = \frac{b - x^2}{at}$$

   where ; P is power, x is distance and t is the time .

THANKING YOU

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