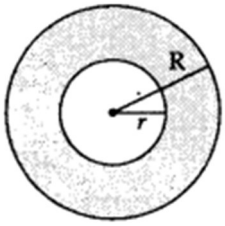
Chapter- 12

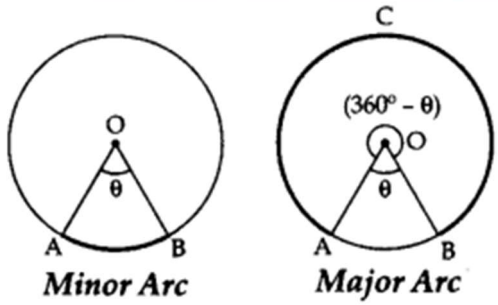
AREAS RELATED TO CIRCLES

STUDY NOTES

Circumference of a circle = $2\pi r$
Area of a circle = πr^2 ...[where r is the radius of a circle]
Area of a semi-circle = $\pi r^2 2$
Area of a circular path or ring:

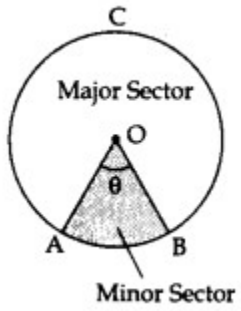


Let 'R' and 'r' he radii of two circles
Then area of shaded part = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r)$
Minor arc and Major Arc: An arc length is called a major arc if the arc length enclosed by the two radii is greater than a semi-circle.



If the arc subtends angle 'θ' at the centre, then the
Length of minor arc = $\theta/360 \times 2\pi r = \theta/180 \times \pi r$
Length of major arc = $(360-\theta)/360 \times 2\pi r$

Sector of a Circle and its Area
A region of a circle is enclosed by any two radii and the arc intercepted between two radii is called the sector of a circle.
(i) A sector is called a minor sector if the minor arc of the circle is part of its boundary.
$OAB\hat{}$ is minor sector.



Area of minor sector = $\theta/360(\pi r^2)$

Perimeter of minor sector = $2r+\theta/360(2\pi r)$

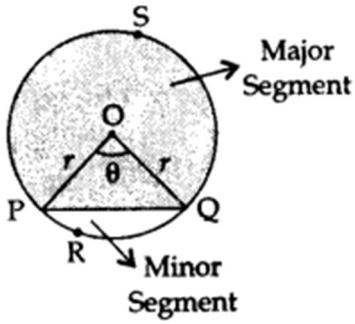
ii) A sector is called a major sector if the major arc of the circle is part of its boundary.

$OACB\hat{}$ is major sector

Area of major sector = $(360-\theta)/360)(\pi r^2)$

Perimeter of major sector = $2r+(360-\theta)/360)(2\pi r)$

Minor Segment: The region enclosed by an arc and a chord is called a segment of the circle. The region enclosed by the chord PQ & minor arc PRQ is called the minor segment.



Area of Minor segment = Area of the corresponding sector – Area of the corresponding triangle

$$= \left[\frac{\theta}{360}\pi r^2 - \frac{1}{2}r^2\sin\theta\right]$$

$$= \frac{1}{2}r^2\left[\frac{\theta}{180}\pi - \sin\theta\right] \quad \text{or} \quad \frac{1}{2}r^2\left[\frac{\theta}{180}\pi - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

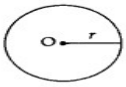
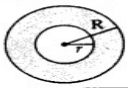
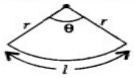
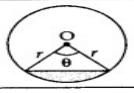
Major Segment: The region enclosed by the chord PQ & major arc PSQ is called the major segment.

Area of major segment = Area of a circle – Area of the minor segment

Area of major sector + Area of triangle

$$= \pi r^2 - \frac{\theta}{360}\pi r^2 + \frac{1}{2}r^2 \sin\theta = r^2\left[\pi - \frac{\theta}{360}\pi + \frac{\sin\theta}{2}\right]$$

TABLE FOR AREA AND PERIMETER

Figures	Area	Perimeter	
Circle 	πr^2 or $\frac{\pi d^2}{4}$	$2\pi r$ or πd	r : radius d : diameter $\pi = \frac{22}{7}$ or 3.14
Semicircle 	$\frac{\pi r^2}{2}$	$\pi r + 2r$	
Quadrant 	$\frac{\pi r^2}{4}$	$\frac{\pi r}{2} + 2r$	
Ring 	$\pi(R + r)(R - r)$	$2\pi R$ (Outer circumference) $2\pi r$ (Inner circumference)	R : Radius of bigger circle r : Radius of smaller circle
Sector 	(i) $\frac{\theta}{360} \times \pi r^2$ (ii) $\frac{1}{2}lr$	$\frac{\theta}{360} \times 2\pi r + 2r$	r : Radius of circle l : length of arc
Segment 	$\frac{\theta}{360}\pi r^2 - \frac{1}{2}r^2 \sin\theta$	$\frac{\pi r\theta}{180} + 2r \sin\frac{\theta}{2}$	θ : angle subtended by arc at centre